Assignment 3
Design of Fixed Wing UAV
EC4.402 - Introduction to UAV Design

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1 FW-UAV Design for 3D Mapping

The following are the given requirement

$$\text{Endurance} = 60\,\text{min} \quad \text{Area of operation} = 10\,\text{Km rad.} \quad \text{Cruise speed} = 20\,\text{m/s}$$
$$\text{Flying altitude} = 100\,\text{m (from ground)} \quad \text{Climb rate} = 2\,\text{m/s} \quad \text{Descent rate} = 2\,\text{m/s}$$

It is assumed that these are the minimum values and with a greater budget these can be extended. It is assumed that some compromise on the area of operation (in terms of coverage area and not range) is acceptable.

1.1 CONOPS

CONOPS of *Concept of Operations* is the overview of the operations involved in the application of the Fixed Wing UAV (FW-UAV). The stages of operation are defined as

1. **Take-off** from ground. It is preferred to have a hand launch mechanism for take-off (bungee launch at best). Runway takeoff UAVs are least preferred (too much arrangement required for them).

2. **Climb** to the height of $100\,m$ from ground. The climb rate is $2\,m/s$.

3. **Cruise** in the area for performing mapping. It is assumed that the UAV travels in straight lines (back and forth) from one end to another end (confined in a circle - of $10\,Km$ radius).
   - It is possible to take multiple trials for covering the entire area.
   - It is preferred to have an autonomous solution, where the path is programmed and the UAV follows it.
   - A manual solution is only required as a backup.

4. **Descent** from $100\,m$ to the ground. The descent rate is $2\,m/s$.

5. **Landing** on the ground. The UAV will be at very low speeds when landing and can land on any grassland. It need not have a runway for landing (it doesn't even need wheels).

The *loitering* phase was not chosen because we decided on traveling (cruising) in straight lines, while taking turns. This is better if multiple views are required. Loitering can be chosen if the desired path is in form of expanding circles.

However, there will be no *aggressive flight maneuvers*, as the application does not desire it.

1.2 Requirement Specifications

The following can be considered the *requirement specifications* for the starting of the design phase

- **Operating velocity** can be the cruise speed of $20\,m/s$.
- **Range** can be assumed to be around $30\,Km$ (for traversals). This will depend on the sensor. Some sensors can work from far, while some will require close proximity (increasing traversal range).
- **Endurance** can be $60\,min$.
- **Payload** will be the cameras and/or LiDAR sensors (which will be used for 3D mapping). Usually, these will be well under $500\,gm$.
- **Wind** conditions will be around $5\,km/hr$ to $20\,km/hr$ [1]. They will hardly be expected to go beyond $25\,km/hr$.
- **Altitude** will be around $100\,m$ from ground, and $1000\,m$ to $3000\,m$ from sea level (mostly will be under $1500\,m$ for most commercial applications).

[1]Weather data from `https://www.meteoblue.com/en/weather/historyclimate/`

- **Safety**: The UAV must be certified for surveillance and mapping standards, as well as human operation standards. It also must have regulatory compliance with GOI (Government of India) guidelines. The following can be noted [2]
  - The drone falls in the *micro* category (weighing total of $250\,g$ to $2\,kg$).
  - Drone must have GPS, flight data logging, RTH (Return-to-home), anti-collision light, RFID and SIM with NPNT (No Permission No Takeoff) compliant software, and ID plate.
- **Maneuverability**: No agile maneuvers are needed. The drone will be oriented horizontally virtually always. Only turning maneuvers are needed.

1.3 Market Survey

senseFly eBee X

The senseFly eBee X fixed wing UAV is a hand-launched UAV suitable for mapping and surveillance applications. It is very mobile and convenient to use.

The product can be found at `https://www.sensefly.com/drone/ebee-x-fixed-wing-drone/`. Its specifications were obtained from the comparison table and datasheet at the same website.

1. *Operating velocity*: It can cruise from $11\,m/s$ to $30\,m/s$. Comfortably within our $20\,m/s$ bounds.
2. *Range*: It has a standard flight range of $37\,km$ (maximum is $55\,km$). This is well over our $30\,km$ requirement.
3. *Endurance*: It can fly for up to $90\,min$. Our requirement is only for $60\,min$.
4. *Payload*: It will carry a camera as payload. The total weight will be around $1.3\,kg$ to $1.6\,kg$. It is very easy to carry around in a bagpack.
5. *Wind*: It can tolerate wind up to $46\,km/hr$ (well beyond our maximum of $25\,km/hr$).
6. *Altitude*: Its optimal altitude is $120\,m$, we only require $100\,m$.
7. *Safety*: The UAV will require additional safety certification for India. It is certified for Canada, EU and the USA.

With the above specifications, the UAV almost exceeds our requirements. Choosing it should give very comfortable bounds, thereby allowing for greater needs in the future. Price would be around ₹ 10,00,000 (including all accessories and carrying case, with extra batteries and ground stations). This UAV is shown in figure 1a.

It has a $116\,cm$ wingspan.

Delair UX11

The Delair UX 11 fixed wing UAV is a compact hand-launched UAV suitable for mapping applications. The UAV has 3G and 4G capabilities giving it extended communication range and reducing the number of ground stations to one (receiver).

The product can be found at https://delair.aero/. Its specifications were obtained from the datasheet at the same website. Some information is also taken from its Amazon product page.

1. *Operating velocity*: The UAV can cruise at speed upto $15\,m/s$. This is a little less than our $20\,m/s$ requirement.
2. *Range*: It has a range of $53\,km$. This is well over our $30\,km$ requirement.
3. *Endurance*: It can fly for up to $60\,min$, which our exact requirement.
4. *Payload*: It will not require any external payload as the cameras are built into the UAV body. The entire UAV weighs less than $1.5\,kg$.
5. *Wind*: It can tolerate wind up to $45\,km/hr$ (well beyond our maximum of $25\,km/hr$). It can even fly in light rain.

[2] Main reference from `https://uavcoach.com/drone-laws-in-india/` and `https://digitalsky.dgca.gov.in/home`

6. *Altitude*: Its optimal altitude is $120\,m$, we only require $100\,m$.
7. *Safety*: The UAV will require additional safety certification for India. It is certified for the USA.

With the above specifications, the UAV almost meets our requirements. Choosing it for the immediate needs is ideal. Price would be around ₹ 7,00,000 (including all accessories and carrying case). This UAV is shown in figure 1b. It has a $110\,cm$ wingspan.

(a) senseFly eBee X

(b) Delair UX11

(c) WingtraOne Gen II

(d) Aeromapper Talon LITE

Figure 1: FW-UAV Market Survey

The VTOL capable WingtraOne is also a good choice. Another FW-UAV is Aeromapper Talon LITE

1.4 Airfoil Selection and sizing

The design happens in the following stages

Specifications

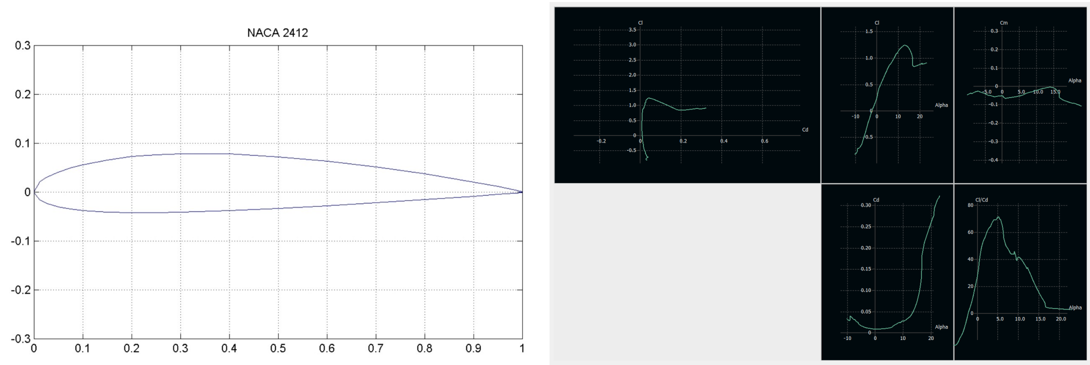
We assume a takeoff weight of $2\,kg$. The environment temperature is assumed to be $25°C$, where density of air is $\rho = 1.184\,kg/m^3$ an the dynamic viscosity is $\mu = 1.849 \times 10^{-5} kg/(ms)$ [3]. For now, the cruise speed can be $20\,m/s$.

For the starting, we will assume the wing to be rectangular and with chord length $c = \bar{c} = 0.2\,m$. Let us also assume the wingspan be to $b = 1.1\,m$. These are values taken from the UAVs in the market survey.

[3] Values obtained from `https://lynniezulu.com/what-is-the-dynamic-viscosity-of-air/`

Wing Design - Phase 1: Airfoil analysis

We will choose `NACA 2412` airfoil for starting the design phase (simply because it is common for remote controlled UAVs). It is shown in figure 2a.



(a) NACA-2412 Profile

(b) Airfoil analysis - 1

Figure 2: NACA-2412 Airfoil

We first analyze the airfoil in XFLR5, using `XFoil Direct Analysis`. The Reynold's number is calculated using

$$R_e = \frac{\bar{c}V_a\rho}{\mu} = \frac{0.2 \times 20 \times 1.184}{1.849 \times 10^{-5}} = 256138.45 \approx 256140 \tag{1.1}$$

The first analysis is shown in figure 2b.

We notice that highest ${}^{C_l}/_{C_d}$ occurs at $\alpha = 5°$ (best lift and least drag). The lift peaks at $\alpha = 13°$, with the drag elbowing and then increasing rapidly afterwards. This means that for this airfoil, the preferred angle of attack in the 3D plane design should be from $5°$ to $13°$.

Noting these observations, we proceed with selecting the `NACA 2412` airfoil.

Wing Design - Phase 2: Wing from airfoil

Running an `LTT` analysis on a *wing only* model of a plane (using the wingspan $b = 1.1$ and chord $\bar{c} = c = 0.2$) yields figure 3.

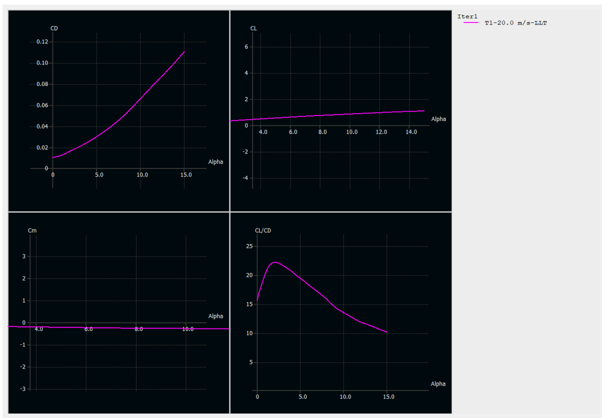


Figure 3: Wing analysis

Fitting the line and quadratic equations through graphs in figure 3, we get

$$C_{L_w} = 3.2944\alpha + 0.2951 \quad C_{M_w} = -0.5991\alpha - 0.1522 \quad C_{D_w} = 0.7666\alpha^2 + 0.2022\alpha + 0.0065 \tag{1.2}$$

We need to counter the weight for lift (when cruising at steady state). This gives us

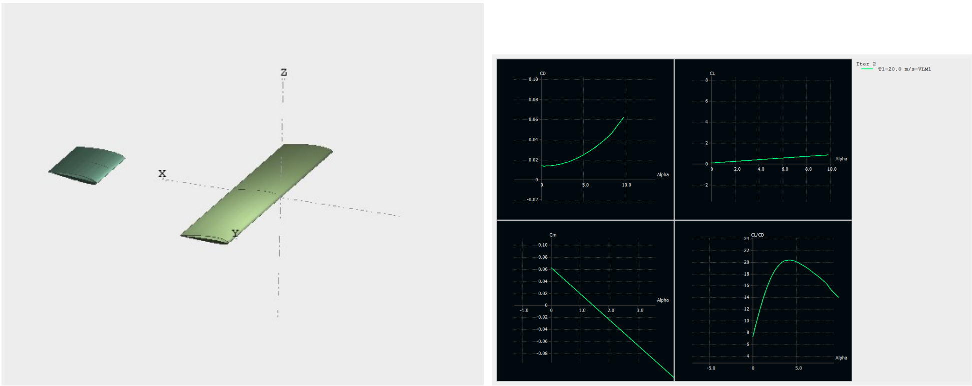
$$F_{L_w} = C_{L_w}\,\frac{1}{2}\rho V_a^2 S_w \Rightarrow C_{L_w} = \frac{mg}{\frac{1}{2}\rho V_a^2 S_w} \tag{1.3}$$

We get $C_{L_w} = 0.37622$ for $V_a = 20$, giving us $\alpha = 1.4°$. We can probably fly the plane with this α as well, as seen in figure 2b.

If we want the *most* optimal performance for the airfoil (highest $^{C_l}/_{C_d}$), we should choose to fly at around $16\,m/s$ (cruise). However, highest $^{C_L}/_{C_D}$ is found at around $\alpha = 2.1°$. Our $V_a = 20\,m/s$ should be manageable.

Tail Design - Phase 3: Horizontal Tail We place the tail such that at cruise speed, the pitching moment (along Y axis) is zero. The moment should be negative when α of wing increases, and should be positive when α of wing decreases from the cruise value. This will make the plane self stabilizing.

Through XFLR experimentation, we find that keeping $c = 0.2$, $b = 0.15$ for the horizontal tail is ideal (shape-wise). We place the tail $0.7\,m$ behind the wing, at a tilt of $-5°$. This is shown in figure 4a.



(a) Horizontal tail (b) XFLR Analysis

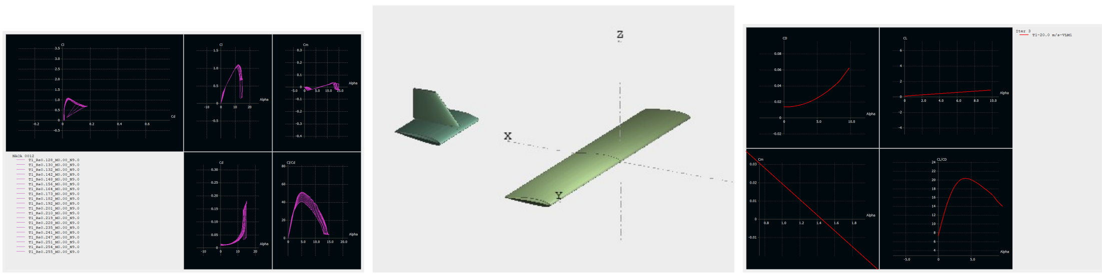
Figure 4: Wing with Horizontal tail

As is visible in the bottom left graph of figure b, the moment counters change in α and will make the plane passively stable. Minor adjustments can be done through control surface designing (elevator).

Tail Design - Phase 4: Vertical Tail

This needn't be too complicated, we don't need to make agile maneuvers. We can choose an airfoil with zero camber so that it passively doesn't add yaw, something like `NACA 0012` airfoil should work.

We run a batch analysis with a range of Reynolds number for `NACA 0012` and end up with the results in figure 5a.



(a) NACA 0012 (b) Final plane (c) XFLR Analysis

Figure 5: Final plane design
NACA 0012 airfoil for the vertical tail. The plane analysis is presented here.

We then attach the vertical tail at $0.7\,m$ X and $0.025\,m$ Z, with `NACA 0012` airfoil and taking chord 0.20 at base and 0.10 at the top (0.120 height). The final plane is visible in figure 5b.

After running analysis on the plane (we do not expect much change from previous phase), we get figure 5c.

Control Surface

The **elevator** and **rudder** are chosen at 35% and 25% of the chord length of horizontal and vertical tail respectively.

Based on historical data, we choose the **aileron** to span 37.5% of the wing, while spanning 25% of the chord.

Conclusion Notes

- We could have chosen a *sweepback* wing planform instead of a *rectangular* one. This will require a detailed batch analysis for the `NACA 2412` airfoil, as well as a more detailed review of the pitch and yaw control (as there will be no need of a tail). This will save a lot of space, but will make the design process more complicated (hence not attempted here).
- The horizontal tail could also be a zero camber one, but with a different tile angle. We went with choosing the same airfoil as the wing.
- We can add 10% wing tips at both sides of the wing to prevent vortex drag. But this will require some advanced analysis (for new drag and lift calculations).
- Sometimes during analysis, we could gett a `Point out of flight envelop` error (this is because of interpolation issues). Seems like, the best fix is running a detailed batch analysis beforehand [4].

1.5 Component Identification

Identifying all parts of the FW-UAV. The total weight will come around $2\,kg$.

Common parts

The plane **airfoils** are decided in the previous subsection. We will also need an *aerodynamic body* to join all parts, as well as host all other components.

We will need a **GPS** module (for location information). We could use the `NEO-6M GPS Module` (more info here).

We will also need two airspeed sensors (mounted in a perpendicular fashion). We could use the `MATEKSYS Digital Airspeed Sensor (ASPD-4525)` (more info here), which will give us the differential pressure (and the airspeed)

We use the `Ardupilot firmware on BeagleBone Blue` (reference here). We can also have a telemetry sensor (or store odometry data on the beagle bone).

We can use the `EMAX ES08MA II` metal servo (more info here) for the control surfaces.

Propeller, ESC and Battery

We could use the `F3P T90-4D` propeller, and the `AM20 F3P-A` motor [5]. The ESC can be `AM66A + AM Link 3D` [6].

A battery with high enough C rating is needed [7]. We can choose the `Turnigy 5000mAh 4S, 20C` battery.

Sensor payload

We can use multiple `OV2640 Binocular Camera` sensors along with `Walkera G-2D` gimbal (will require some modifications). This will give high field of view in 3D and require fewer traversals [8].

[4] From xflr5.tech/docs/
[5] More about motor here, and about propeller here
[6] More about the ESC here
[7] The C Rating gives an estimate of charging and discharging current of a battery $I = C_r \times E_{Ah}$. More here
[8] More about the camera here and the gimbal here

1.6 Performance Analysis

We got the following when we were creating the wing

$$C_{L_w} = 3.2944\alpha + 0.2951 \quad C_{M_w} = -0.5991\alpha - 0.1522 \quad C_{D_w} = 0.7666\alpha^2 + 0.2022\alpha + 0.0065 \tag{1.4}$$

Assuming that the drag is only due to the wing and the fuselage.

$$F_{D_w} = C_{D_w}\frac{1}{2}\rho V_a^2 S_w \qquad D_f = C_{Df}0.5\rho V_a^2 S_w \qquad C_{Df} = C_{fe}\frac{S_{wa}}{S_w} = 2.535 \times 10^{-3}$$

We substitute the following

$$S_{wa} = \pi d_f l_f \left(1 - \frac{2}{\lambda_f}\right)^{2/3}\left(1 + \frac{1}{\lambda_f^2}\right) = 0.18594\, m^2 \qquad S_w = 0.22\, m^2 \qquad C_{fe} = 0.003$$

Where $\lambda_f = {l_f}/{d_f} = 8$, where $l_f = 0.8\, m$ is the length of the fuselage and $d_f = 0.1\, m$. The total drag will be given by $D = F_{D_w} + D_f$.

For this analysis, we assume that $V_a = 20$ (airspeed). As observed in the design phase (of the wing), this yields $\alpha = 1.4°$.

This gives us the total drag as

$$\begin{aligned} D &= F_{D_w} + D_f = C_{D_w}\frac{1}{2}\rho V_a^2 S_w + C_{D_f}\frac{1}{2}\rho V_a^2 S_w = \left(C_{D_w} + C_{D_f}\right)\frac{1}{2}\rho V_a^2 S_w \\ &= \left(0.7666\alpha^2 + 0.2022\alpha + 0.0065 + 2.535 \times 10^{-3}\right)\frac{1}{2}\rho V_a^2 S_w = 0.014433 \times \frac{1}{2}\rho V_a^2 S_w \\ &= 0.7519\, N \end{aligned}$$

The diameter of our propeller is $13\, in$ and assuming the pitch to be $10\, in$. This puts it equal to the `Aeronaut 13 x 10` [9], which will be used for analysis hereon.

Using an RPM of 4000, we get the following values for propeller thrust

$$J = \frac{V_a}{nD} = \frac{20}{({4000}/{60})\,(13 \times {2.54}/{100})} = 0.908 \qquad C_T = 0.013 \qquad T = C_T \rho n^2 D^4 = 0.815\, N$$

Keeping some additional drag into account, this thrust seems fine for the steady state flight.

Power The power consumed when cruising is $P = {T_a V_a}/{\eta} = 29.26\, W$. The battery can give $4 \times 4.2 \times 5 \times 3600 = 302.4\, kJ$.

Even if the combined efficiency of the power system is 60%, this means a total cruising time of around $1.5\, hr$.

Climb We need to climb at $2\, m/s$, we know that $mg\dot{h} = (T_a - D)V_a$. We will need $T_a = 2.72\, N$ for climbing. This is achieved at approximately 4500 RPM.

Assuming that we climb at $2\, m/s$, it'll take $50\, s$ to attain this. Considering that the thrust is $T_a = 2.72\, N$, and $D = 0.7519\, N$ (drag), the energy requirement for the climb phase is $(2.72 - 0.7519) \times 20 \times 50 = 1968.1\, J$.

Cruise Assuming a cruise time of $60\, min = 3600\, sec$, the power required during cruise phase is $P = {T_a V_a}/{\eta} = 29.26\, W$ (actual thrust comes at around $T_a \approx 0.878\, N$, $\eta = 0.6$).

The total energy consumption is $29.26 \times 3600 + 1968.1 = 107304.1\, J$ (per flight). The battery stores $302.4\, kJ$, it can easily power **two** complete flights.

The total flight time possible here is ${(302.4 \times 10^3 - 1968.1)}/{29.26} = 10267.66\, sec = 2.8\, hr$ (in the air). If one is ready to move communication stations, or attaches 4G to the drone, it can theoretically fly for about $10200 \times 20 = 204\, km$.

[9] Find it here

1.7 Design Optimization

In order to maximize the range, we assume that the drone is flying at the best C_L/C_D value. This was already found to be $\alpha = 2.1°$, $V_a = 16\,m/s$. Using the same calculations as before, we have the new $C_L = 0.4158$ and $C_{D_w} = 0.01494$. Note that $C_{D_f} = 2.535 \times 10^{-3}$ remains unchanged.

The lift force is $F_{L_w} = C_L \times 0.5 \times \rho V_a^2 S_w = 13.9\,N$. This means that our new mass is around $1.5\,kg$. We can shed some weight from the battery and make the components out of a lighter material. Some UAVs in our market survey have even lesser weight.

The new drag is $D = (C_{D_w} + C_{D_f}) \times 0.5 \times \rho V_a^2 S_w = 0.584\,N$. We can now use the `Aeronaut 12 x 13` propeller and keep the flight cruising at just 3000 RPM.

The power consumed when cruising is $P = {}^{T_a V_a}/_{\eta} = {}^{(0.584 \times 16)}/_{0.6} = 15.5733\,W$.

During the climb phase, the thrust required is given by $T_a = {}^{mgh}/_{V_a} + D \Rightarrow T_a = 2.4241$. At 4000 RPM, we get a thrust of $T_a = 3.050\,N$ (more than enough for takeoff and climb of $2\,m/s$).

The new energy requirement for climb is $(3.050 - 0.584) \times 16 \times 50 = 1972.8\,J$. The power requirement for cruising is $15.5733\,W$. If we assume an endurance of $60\,min$, we will consume $1972.8 + 15.5733 \times 3600 = 58036.68\,J$. With a $302.4\,kJ$ battery, we can now power **five** flights!

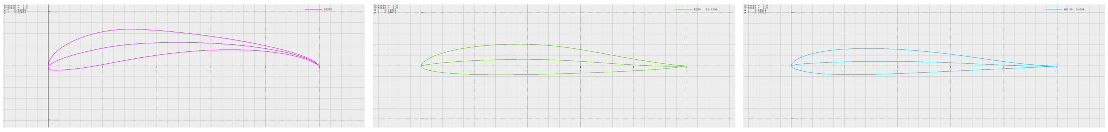
The total flight time possible here is ${}^{(302.4 \times 10^3 - 1972.8)}/_{15.5733} = 19291.17\,sec = 5.35\,hr$ (in the air). If one is ready to move communication stations, or attaches 4G to the drone, it can theoretically fly about $19200 \times 16 = 307.2\,km$.

2 Comparing airfoils

We compare the following airfoils (as asked in the question)

- Selig **S1223** high lift low Reynolds number airfoil
- Eppler **E220** low Reynolds number airfoil
- Martin Hepperle **MH45** for flying wings

These data files (coordinates) are available on the UIUC airfoil database. The profiles are shown in figure 6.



(a) S1223 (b) E220 (c) MH45

Figure 6: Different airfoils
All profiles as visible in `Direct Foil Design` of XFLR.

2.1 S1223

The Selig **S1223** has a high positive camber in the middle. This will give it superior lift (compared to other two). The results can be seen in figure 7.

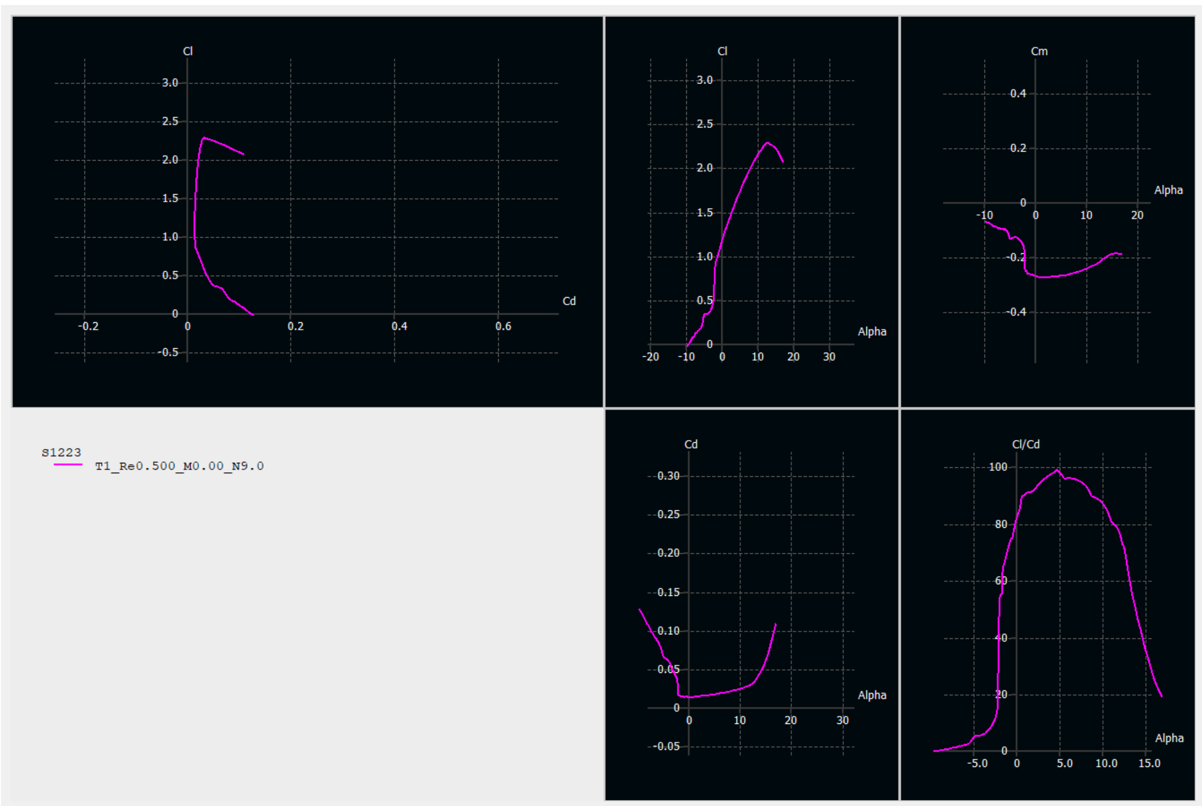


Figure 7: XFLR Analysis of S1223 Airfoil

For C_l The maximum C_l occurs at $\alpha = 12.6°$.

For C_l/C_d The maximum C_l/C_d occurs at $\alpha = 4.7°$.

For C_m The C_m graph follows a linear path for most of the positive α (small angles). It reaches the peak (most favorable) at $\alpha = 15.8°$.

2.2 E220

The Eppler **E220** has a lower camber than S1223. It has much lesser moment, but the lift is lower. The results can be seen in figure 8.

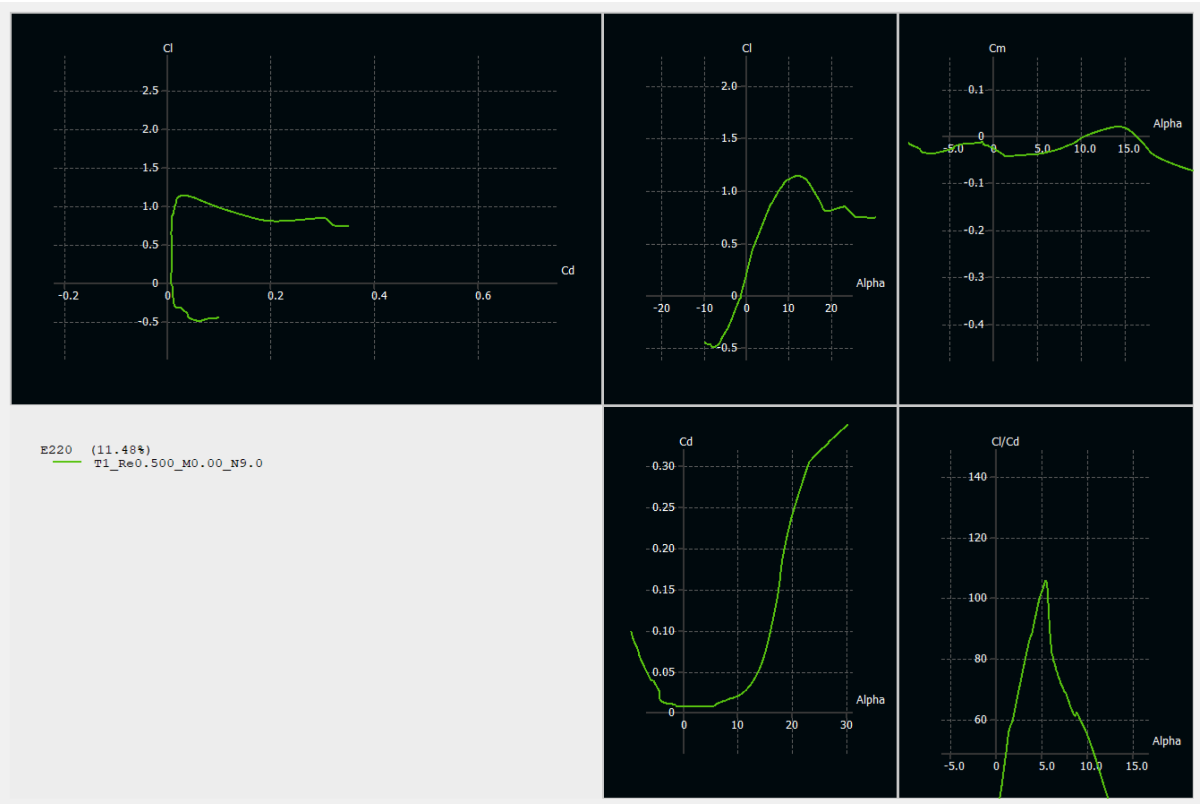


Figure 8: XFLR Analysis of E220 Airfoil

For C_l The maximum C_l occurs at $\alpha = 12.1°$.

For C_l/C_d The maximum C_l/C_d occurs at $\alpha = 5.4°$.

For C_m The C_m graph follows a linear path for most of the positive α (small angles). It reaches approximately zero at $\alpha = 10.3°$.

2.3 MH45

Martin Hepperle **MH45** has a low camber (like E220). It has better lift and moment curves. The results can be seen in figure 9.

For C_l The maximum C_l occurs at $\alpha = 13.6°$.

For C_l/C_d The maximum C_l/C_d occurs at $\alpha = 5.8°$.

For C_m The C_m graph follows a linear path for most of the positive α (small angles). It is very close to zero. A plane designed using this airfoil might not even need a tail (only change center of gravity a little and have big ailerons). It is near zero at $\alpha = 10.7°$.

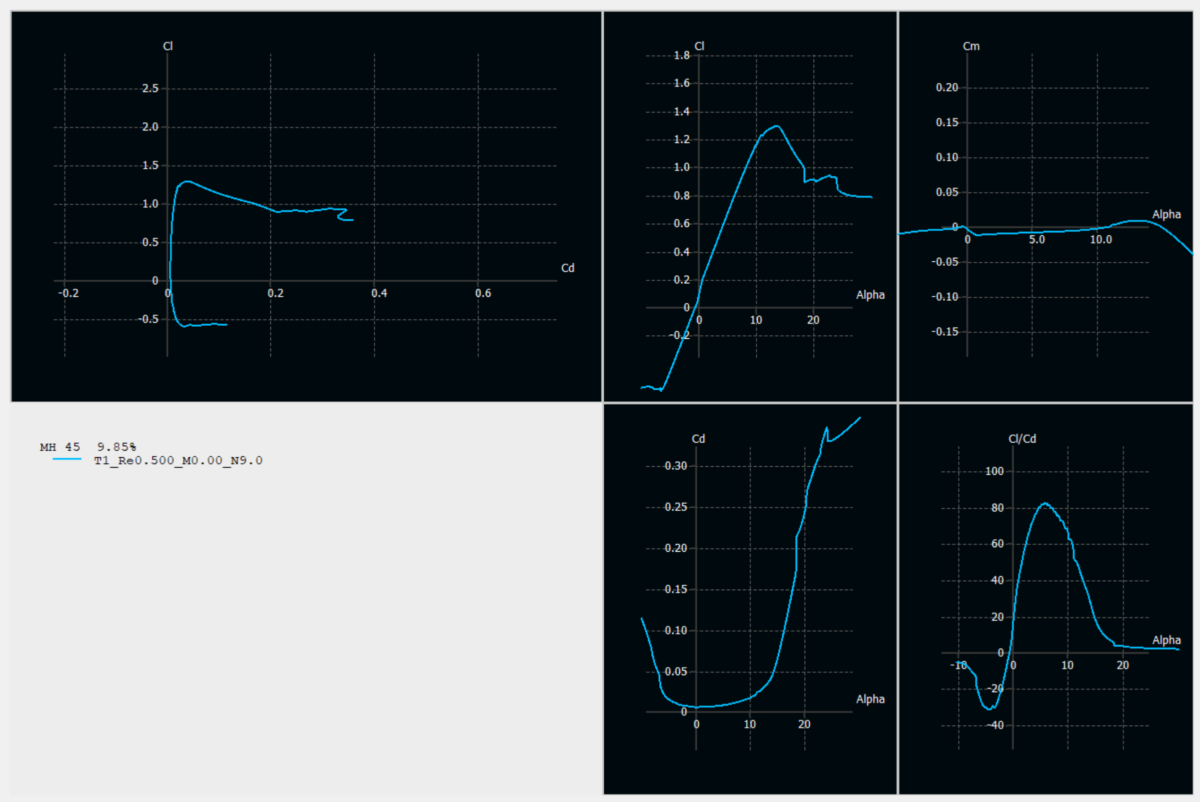


Figure 9: XFLR Analysis of MH45 Airfoil

Summary

A summary of the performance of all airfoils is shown in figure 10

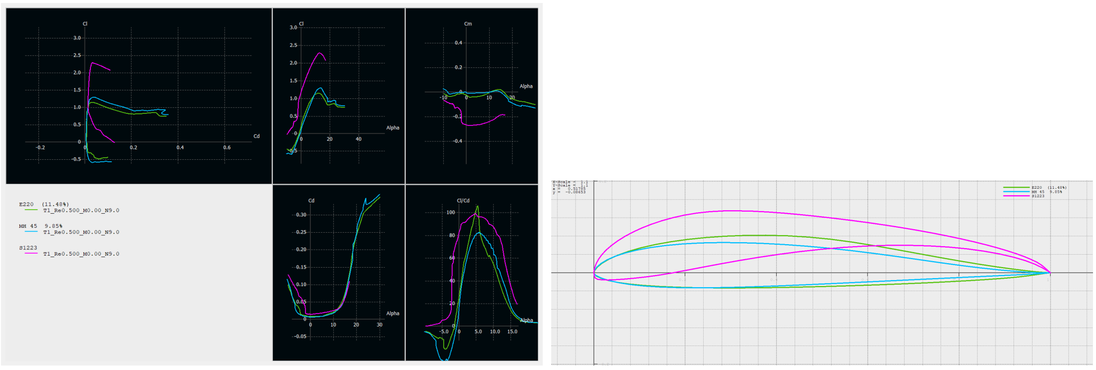


Figure 10: Comparing all airfoils

3 MCDM

This section covers **MCDM** - Multi-Criteria Decision Making - for the airfoils discussed in the previous section

We use the following as the criteria of airfoil selection

1. C_l: The maximum C_l value (regardless of α) of the airfoil. The higher, the better.
2. ${}^{C_l}/_{C_d}$: The maximum ${}^{C_l}/_{C_d}$ value (regardless of α) of the airfoil. The higher, the better.
3. C_m: The C_m value at the α that has the highest ${}^{C_l}/_{C_d}$ ratio. This should be as close to zero as possible. All observations are negative, so higher the better.

Going by the above metrics, we estimate the following *decision matrix*

Airfoil	C_l	${}^{C_l}/_{C_d}$	C_m
Selig S1223	2.286	98.95	-0.266
Eppler E220	1.142	105.74	-0.037
Martin Hepperle MH45	1.295	82.5	-0.007

The weights given to the categories (second through fourth columns) is $W_1 = 0.3$, $W_2 = 0.35$, and $W_3 = 0.35$.

3.1 WSM

The *Weighted Sum Model* value is given by $A_i = \sum_{j=1}^{N} q_{ij} w_j$. We pick the one with highest A_i.

$$
\begin{aligned}
A_1 &= 2.286 * 0.30 + 98.95 * 0.35 + (-0.266) * 0.35 = 35.2252 \\
A_2 &= 1.142 * 0.30 + 105.74 * 0.35 + (-0.037) * 0.35 = 37.33865 \\
A_3 &= 1.295 * 0.30 + 82.5 * 0.35 + (-0.007) * 0.35 = 29.26105
\end{aligned}
$$

From the above, it is clear that **Eppler E220** should be chosen over others. The order is `Eppler E220` > `Selig S1223` > `Martin Hepperle MH45`.

3.2 WPM

The *Weighted Product Model* value is given by $R({}^{A_K}/_{A_L}) = \Pi_{j=1}^{N} ({}^{a_{Kj}}/_{a_{Lj}})^{w_j}$, if $R > 1$, we choose the numerator over denominator. We do ${}^{3}\mathrm{C}_2 = 3$ comparisons. We take $-$ for the last weight because the ratio of two negative numbers will be positive and we want the value to be *minimized* (high ratio means worse mark)

$$
\begin{aligned}
R({}^{A_1}/_{A_2}) &= ({}^{2.286}/_{1.142})^{0.30} \times ({}^{98.95}/_{105.74})^{0.35} \times ({}^{-0.266}/_{-0.037})^{-0.35} = 0.6032 < 1 \\
R({}^{A_1}/_{A_3}) &= ({}^{2.286}/_{1.295})^{0.30} \times ({}^{98.95}/_{82.5})^{0.35} \times ({}^{-0.266}/_{-0.007})^{-0.35} = 0.3537 < 1 \\
R({}^{A_2}/_{A_3}) &= ({}^{1.142}/_{1.295})^{0.30} \times ({}^{105.74}/_{82.5})^{0.35} \times ({}^{-0.037}/_{-0.007})^{-0.35} = 0.5864 < 1
\end{aligned}
$$

This yields us the comparisons $A_1 < A_2$, $A_1 < A_3$, and $A_2 < A_3$. Which means $A_3 > A_2 > A_1$.

This basically gives us `Martin Hepperle MH45` > `Eppler E220` > `Selig S1223`. We choose the **Martin Hepperle MH45**.

3.3 AHP

The *Analytic Hierarchy Process* is given by $A_i = \sum_{j=1}^{N} q_{ij} w_j$, with the particular note being that $\sum_{i=1}^{N} q_{ij} = 1 \forall j$. We use $-$ for the last feature as higher magnitude ratio is worse. The rest is same as weighted sum model (WSM).

$$
\begin{aligned}
A_1 &= \left(\frac{2.286}{2.286+1.142+1.295}\right)\times 0.30+\left(\frac{98.95}{98.95+105.74+82.5}\right)\times 0.35\\
&\quad -\left(\frac{-0.266}{-0.266-0.037-0.007}\right)\times 0.35=-0.0345\\
A_2 &= \left(\frac{1.142}{2.286+1.142+1.295}\right)\times 0.30+\left(\frac{105.74}{98.95+105.74+82.5}\right)\times 0.35\\
&\quad -\left(\frac{-0.037}{-0.266-0.037-0.007}\right)\times 0.35=0.1596\\
A_3 &= \left(\frac{1.295}{2.286+1.142+1.295}\right)\times 0.30+\left(\frac{82.5}{98.95+105.74+82.5}\right)\times 0.35\\
&\quad -\left(\frac{-0.007}{-0.266-0.037-0.007}\right)\times 0.35=0.1749
\end{aligned}
$$

This yields $A_3 > A_2 > A_1$ (same as in the case of WPM). This basically gives us `Martin Hepperle MH45` > `Eppler E220` > `Selig S1223`. We choose the **Martin Hepperle MH45**.