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A fast Gaussian beam tracing method for reflection and refraction of general vectorial astigmatic Gaussian beams from general curved surfaces

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Abstract

A fast Gaussian beam tracing method for general vectorial astigmatic Gaussian beams based on phase matching has been formulated. Given the parameters of a vectorial Gaussian beam in its principal coordinate system the parameters of the reflected and refracted beams from a general curved surface (with general constitutive parameters) are found. The reflection and transmission of such beams from and through passive photonic structures such as lenses, mirrors and prisms can then be found by considering multiple reflections and transmissions.

Keywords: Gaussian beams; Beam tracing; General astigmatism

1. Introduction

Paraxial Gaussian beams (GB) are very good approximations for output of laser sources. Other sources of electromagnetic fields can be expanded as sum of GBs. Design optimization of photonic structures and photonic integrated devices requires fast methods for propagating or tracking such beams in structures such as lenses, prisms and mirrors of arbitrary shape. GB tracing and tracking has been investigated and reported in the literature. Felsen et al. [1,2] used the complex source point method to find the reflection and refraction of 2D GB from flat and cylindrical surfaces. Using ray tracing methods, they traced the reflected and refracted rays from a surface, originating from a 2D lines source. These were then converted to GB by transforming the source to the complex plane and applying the paraxial approximation. Relatively lengthy search procedures are needed to find the point of incidence and the results are more

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appropriate for far field calculation. Pathak et al. [3] approximated the physical optic approximation and came up with a closed form formula for the reflected GB from a reflector antenna. They also used a Gabor like expansion in order to expand the feed profile into a set of GBs making their method applicable to many real life problems to the author's knowledge, this is the only work so far which can treat General Astigmatic GB (GAGB). Seung and Lee [4] used the complex source point (CSP) method together with the rigorous solution to Maxwell's equations to find the scattered field from a sphere illuminated by a GB. This method which give a rigorous solution to the problem is only applicable for a sphere, the generalization to arbitrary surface is not possible. Siegman [5] used direct phase matching to find the reflected and refracted scalar GBs from a ellipsoid. The axis of the GB in their work was assumed to be aligned with one of the axes of symmetry of the ellipsoid and also General Astigmatism and the vectorial nature of the GB were not considered. The GB tracing method that is presented here, uses an extension of phase matching [6] as applied to GBs. It is an extremely fast, fully vectorial 3D method. Given an arbitrary Vectorial 3D General Astigmatic incident GB and the analytic equation of a surface (dielectric or conductor) we find the parameters (waist and center and direction of propagation and the complex angle of rotation), of the reflected and transmitted GBs using phase matching by approximating the surface with a quadratic function. Note that whenever, a GB is incident on a surface if the two radii of curvature of the surface at the point of incidence are not equal a GAGB is produced, so this kind of astigmatism is a very important and essential part of our GB tracing method. We then use Fresnel reflection and transmission coefficients to account for the vectorial nature of the beams considered and the method therefore, takes polarization of the beams into account. Lenses and other similar structures can be analyzed by considering multiple reflections and refraction from the different surfaces that make up such structures. We have also verified our method by comparing the results with Physical Optics method and applied the method to several practical problems.

2. General astigmatic GBs

As described in [7], a *simple* astigmatic vectorial GB (as opposed to *General* astigmatic) can be described in its principal coordinate system using the following

$$\vec{E}(x,y,z) = \vec{E}_0 \frac{\sqrt{w_{0x}w_{0y}}}{\sqrt{w_x(z)w_y(z)}} \exp\left\{ -\mathrm{j}k\left[z + \frac{1}{2}\left(\frac{x^2}{q_x(z)} + \frac{y^2}{q_y(z)}\right)\right] + \mathrm{j}\eta(z)\right\}, \tag{1}$$

where

$$\begin{aligned}
\frac{1}{q_i(z)} &= \frac{1}{R_i(z)} - \mathrm{j}\frac{\lambda}{\pi n w_i^2(z)},\\
w_i(z) &= w_{0i}\sqrt{1+\left(\frac{z-z_{0i}}{z_{ri}}\right)^2},\\
R_i(z) &= (z-z_{0i})\left[1+\left(\frac{z_{ri}}{z-z_{0i}}\right)^2\right],\\
z_{ri} &= \frac{w_{0i}^2 n\pi}{\lambda} \quad \text{for } i=x,y,\\
\eta(z) &= \frac{1}{2}\tan^{-1}\left(\frac{z-z_{0x}}{z_{rx}}\right) + \frac{1}{2}\tan^{-1}\left(\frac{z-z_{0y}}{z_{ry}}\right),\\
\vec{E}_0 &= E_{0x}\hat{x} + E_{0y}\hat{y},
\end{aligned} \tag{2}$$

where apart from the term $\eta(z)$ the phase is

$z + \frac{1}{2}\mathbf{x}\bar{\mathbf{Q}}\mathbf{x}$,

with

$$\bar{\mathbf{Q}} = \begin{bmatrix} \frac{1}{q_x} & 0 \\ 0 & \frac{1}{q_y} \end{bmatrix} \quad \text{and} \quad \mathbf{x} = [x\ y],$$

and $\bar{\mathbf{Q}}$ being the complex curvature matrix of the beam. When such a beam passes through a non-orthogonal system it turns into a *General* Astigmatic beam. Arnaud and Kogelnik [8] showed that, a GAGB can be formally obtained "by attaching a complex value to the angular orientation" of coordinate system in which the GB is expressed in, and a typical ray fixed coordinate system. Under such complex rotation angle ϕ of the coordinate systems the GB curvature matrix becomes:

$$\bar{\mathbf{Q}}_\phi = \bar{\mathbf{J}}_\phi \bar{\mathbf{Q}} \bar{\mathbf{J}}_{-\phi}, \tag{3}$$

$$\bar{\mathbf{J}}_\phi = \begin{bmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{bmatrix}, \tag{4}$$

where $\phi = a + \mathrm{j}b$ is complex. For such a beam we cannot define a principal coordinate system as we can not eliminate the cross term in both phase and amplitude functions in any coordinate system. It has been shown that, for such a GAGB the ellipses of irradiance and also the ellipse of phase rotate as the beam propagates in free space, while they maintain a fixed angle relative to one another and that, the axes of these quadratic forms can never be aligned. Although a principal coordinate system is not defined for such a beam it is completely specified by its curvature matrix $\bar{\mathbf{Q}}$ and the complex angle ϕ in a specified ray fixed coordinate system.

3. Reflection and transmission from a general curved surface

Consider a Vectorial GAGB that is incident upon a general curved surface, Fig. 1. We assume that, at the point of incidence the waist of the incident beam is smaller than the radii of curvature of the surface. It can be shown that, in this case, the reflected and transmitted beams are very close to GBs. We have verified that, if the radii of curvature of the surface is twice the beam waist at point of incidence our method yields very good results. Our goal is to find the reflected and transmitted beams from the interface once the incident beam is known. For this problem we consider two different coordinate systems:

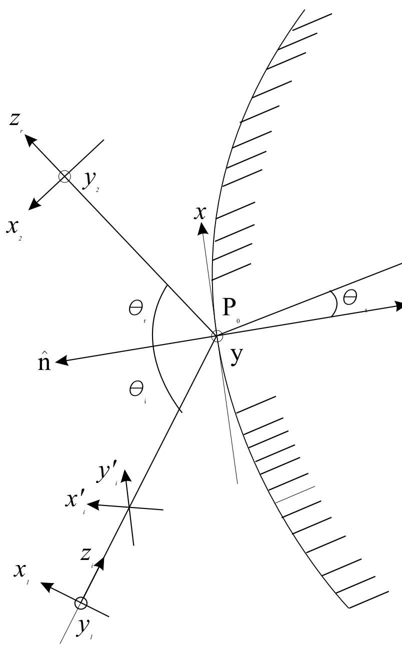

Fig. 1. The geometry of GB reflection and transmission.

1. main coordinate system (x, y, z) is a fixed coordinate system in which the equation of the interface is given,
2. (x_l, y_l, z_l) refers to the ray fixed coordinate system [9] for $l = 1, 2, 3$ for incident, reflected and transmitted, respectively. We denote $[x_1, y_1]$ by $\mathbf{x}_1$. The point of incidence with the surface $z = f(x, y)$ is P_0. The normal vector to the surface at the point of incidence is given by

$$\hat{n} = \vec{\nabla}(f(x,y) - z).$$

The reflection and transmission directions are given using Snell's laws:

$$\widehat{s_i} \cdot \hat{n} = \widehat{s_r} \cdot \hat{n} \quad \text{or} \quad \theta_i = \theta_r$$

$$\text{(Law of reflection)}, \tag{5}$$

$$n_1(\widehat{s_i} \times \hat{n}) = n_2(\widehat{s_t} \times \hat{n}) \quad \text{or} \quad n_1 \sin\theta_i = n_2 \sin\theta_t$$
$$\text{(Law of refraction)}, \tag{6}$$

where n_1 and n_2 are the refractive indices of the two medium shown in Fig. 1. The case of reflection from a perfect conductor can also be easily handled. The angle of incidence, reflection and transmission are denoted by θ_i, θ_r and θ_t. Once these directions are known the ray fixed coordinate system of the incident, reflected and transmitted beams can be found from (see for example [7]):

$$\begin{aligned}
\widehat{x_1} &= \frac{\widehat{z_i} \times (\hat{n} \times \widehat{z_i})}{|\widehat{z_i} \times (\hat{n} \times \widehat{z_i})|}, \\
\widehat{x_2} &= -\frac{\widehat{z_r} \times (\hat{n} \times \widehat{z_r})}{|\widehat{z_r} \times (\hat{n} \times \widehat{z_r})|}, \\
\widehat{x_3} &= \frac{\widehat{z_t} \times (\hat{n} \times \widehat{z_t})}{|\widehat{z_t} \times (\hat{n} \times \widehat{z_t})|}, \\
\widehat{y_l} &= \widehat{z_l} \times \widehat{x_l} \quad \text{for } l = i, r, t.
\end{aligned} \tag{7}$$

We drop the $\eta(z)$ which being a slowly varying function of z [10] does not contribute to the changes of phase on the surface that we are interested in, and has only a bulk effect (for example produces a phase shift of $\frac{\pi}{4}$ at the Raleigh range z_r). We take into account the effect of η at the last stage of the method Eq. (17). The phases of the incident, reflected and refracted GB's can then be expressed as:

$$\begin{aligned} &k_1(z_i + \tfrac{1}{2}\mathbf{x}_1\bar{\mathbf{Q}}_i\mathbf{x}_1^T), \\ &k_1(z_r + \tfrac{1}{2}\mathbf{x}_2\bar{\mathbf{Q}}_i\mathbf{x}_2^T), \\ &k_1(z_t + \tfrac{1}{2}\mathbf{x}_3\bar{\mathbf{Q}}_i\mathbf{x}_3^T), \end{aligned} \tag{8}$$

respectively. The phases of the incident, reflected and transmitted beams are matched at points near P_0 in a manner described in [6,9]. The only difference here is that the components of the curvature matrix are complex numbers, but this does not alter the results in any way. The curvature matrix of the reflected and transmitted beams are found from:

$$\bar{\mathbf{Q}}_r = (\bar{\mathbf{K}}_r)^{-1}[\bar{\mathbf{K}}_i^T\bar{\mathbf{Q}}_i\bar{\mathbf{K}}_i + \bar{\mathbf{C}}(\cos\theta_i + \cos\theta_r)](\bar{\mathbf{K}}_r)^{-1},$$

$$\bar{\mathbf{Q}}_t = \frac{n_1}{n_2}(\bar{\mathbf{K}}_t)^{-1}[\bar{\mathbf{K}}_i^T\bar{\mathbf{Q}}_i\bar{\mathbf{K}}_i + \bar{\mathbf{C}}(\cos\theta_i - \frac{n_2}{n_2}\cos\theta_t)](\bar{\mathbf{K}}_t)^{-1},$$

with

$$\bar{\mathbf{K}}_r = \begin{bmatrix} -\cos\theta_r & 0 \\ 0 & 1 \end{bmatrix},$$

$$\bar{\mathbf{K}}_t = \begin{bmatrix} \cos\theta_r & 0 \\ 0 & 1 \end{bmatrix}.$$

And $\bar{\mathbf{C}}$ is the curvature matrix of the surface defining the interface relative to the ray fixed coordinate system. The fact that the off diagonal element of $\bar{\mathbf{Q}}_r$ and $\bar{\mathbf{Q}}_t$ are given by

$$2C^{11}\cos\theta_i + Q_i^{12},$$

and

$$\frac{\sec\theta t}{k_2}(k_1\cos\theta_i Q_i^{12} - (k_1\cos\theta_i - k_2\cos\theta_t)C^{12}),$$

(Q^{ij} and C^{ij} represent the elements of $\bar{\mathbf{C}}$ and $\bar{\mathbf{Q}}$) shows that for any arbitrary choice of $\bar{\mathbf{Q}}_i$ and $\bar{\mathbf{C}}$ (real or imaginary) the curvature matrix of the reflected and transmitted beams are symmetric and can therefore, be diagonalized. Once these matrices are diagonalized the eigenvalues or the diagonal elements of the resultant matrix yield the reciprocal of q_x and q_y's of these beams. The real part is the reciprocal of the $R(z)$ and the imaginary part can be used to find the spot size $w(z)$ Eq. (2). The waist and the position of the waist of the beams can then be found from (we have dropped all the subscripts for convenience)

$$g = \frac{\lambda(z - z_0)}{\pi w_0^2 n},$$

then

$$g = -\frac{\mathrm{Re}[\frac{1}{q}]}{\mathrm{Im}[\frac{1}{q}]},$$

and we have

$$w_0 = \sqrt{\frac{w(z)}{(1+g^2)}}, \tag{9}$$

$$z_0 = -\frac{R(z)g^2}{(1+g^2)}. \tag{10}$$

Now assume that

$$\bar{\mathbf{V}} = \begin{bmatrix} v_{11} & v_{12} \\ v_{21} & v_{22} \end{bmatrix},$$

is a matrix whose columns are eigenvectors of $\bar{\mathbf{Q}}_r$ or $\bar{\mathbf{Q}}_t$. We normalize this matrix by dividing each element by $v_{11} * v_{11} + v_{21} * v_{21}$ (not by the norm of the eigenvector). The complex rotation matrix of each beam relative to its ray fixed coordinate system is obtained by finding the inverse cosine (on the appropriate branch) of the first element of the normalized matrix. Note that, the origin of the reflected and refracted beams are taken to be the point of incidence. The only remaining unknowns are the amplitudes of the reflected and refracted beams.

4. Determination of the amplitude of the reflected and refracted beams

The amplitudes of the reflected and transmitted beams are found from Fresnel coefficients:

$$\begin{bmatrix} E_y^r(P_0) \\ E_x^r(P_0) \end{bmatrix} = \bar{\mathbf{R}}(P_0) \begin{bmatrix} E_y^i(P_0) \\ E_x^i(P_0) \end{bmatrix}, \tag{11}$$

$$\begin{bmatrix} E_y^t(P_0) \\ E_x^t(P_0) \end{bmatrix} = \bar{\mathbf{T}}(P_0) \begin{bmatrix} E_y^i(P_0) \\ E_x^i(P_0) \end{bmatrix}, \tag{12}$$

where the matrices $\bar{\mathbf{R}}(P_0)$ and $\bar{\mathbf{T}}(P_0)$ are defined as:

$$\bar{\mathbf{R}}(P_0) = \begin{bmatrix} R_{\mathrm{TE}} & 0 \\ 0 & R_{\mathrm{TM}} \end{bmatrix}, \tag{13}$$

$$\bar{\mathbf{T}}(P_0) = \begin{bmatrix} T_{\mathrm{TE}} & 0 \\ 0 & T_{\mathrm{TM}} \end{bmatrix}, \tag{14}$$

$$\begin{aligned} R_{\mathrm{TE}} &= \frac{\sin(\theta_t - \theta_i)}{\sin(\theta_t + \theta_i)}, \\ R_{\mathrm{TM}} &= \frac{\tan(\theta_t - \theta_i)}{\tan(\theta_t + \theta_i)}, \end{aligned} \tag{15}$$

$$\begin{aligned} T_{\mathrm{TE}} &= \frac{2\cos\theta_i \sin\theta_t}{\sin(\theta_t + \theta_i)}, \\ T_{\mathrm{TM}} &= \frac{2\cos\theta_i \sin\theta_t}{\sin(\theta_i + \theta_t)\cos(\theta_i - \theta_t)}. \end{aligned} \tag{16}$$

Assuming l is the distance between the origin of the incident beam and the electric vector of this beam is given in its ray fixed coordinate system then we define:

$$\begin{aligned} E_{0i}(P_0) &= \frac{\sqrt{w_{0ix}w_{0iy}}}{\sqrt{w_{ix}(l)w_{iy}(l)}} \exp(-\mathrm{j}k_1 l + \mathrm{j}\eta_i(l)), \\ A_r &= \frac{\sqrt{w_{0rx}w_{0ry}}}{\sqrt{w_{rx}(0)w_{ry}(0)}} \exp(\mathrm{j}\eta_r(0)), \\ A_t &= \frac{\sqrt{w_{0tx}w_{0ty}}}{\sqrt{w_{tx}(0)w_{ty}(0)}} \exp(\mathrm{j}\eta_t(0)). \end{aligned} \tag{17}$$

Taking the effect of the ignored $\eta(z)$ term into account we obtain:

$$\vec{E}_r = \vec{E}_r(0) \frac{\sqrt{w_{0rx}w_{0ry}}}{\sqrt{w_{rx}(z_r)w_{ry}(z_r)}} \times \exp\left\{-\mathrm{j}k_1\left[z_r + \frac{1}{2}\left(\frac{x_2^2}{q_{rx}(z_r)} + \frac{y_2^2}{q_{ry}(z_r)}\right)\right] + \mathrm{j}\eta_r(z_r)\right\},$$

$$\vec{E}_t = \vec{E}_t(0) \frac{\sqrt{w_{0tx}w_{0ty}}}{\sqrt{w_{tx}(z_t)w_{ty}(z_t)}} \times \exp\left\{-\mathrm{j}k_2\left[z_t + \frac{1}{2}\left(\frac{x_3^2}{q_{tx}(z_t)} + \frac{y_3^2}{q_{ty}(z_t)}\right)\right] + \mathrm{j}\eta_t(z_t)\right\},$$

where

$$\begin{aligned} \vec{E}_r(0) &= (E_x^i(0)R_{\mathrm{TM}}\hat{x}_2 + E_y^i(0)R_{\mathrm{TE}}\hat{y}_2)\frac{E_{0i}(P_0)}{A_r}, \\ \vec{E}_t(0) &= (E_x^i(0)T_{\mathrm{TM}}\hat{x}_3 + E_y^i(0)T_{\mathrm{TE}}\hat{y}_3)\frac{E_{0i}(P_0)}{A_t}, \\ \vec{E}_i(0) &= E_x^i(0)\hat{x}_1 + E_y^i(0)\hat{y}_1. \end{aligned}$$

5. Numerical examples

5.1. Example I

As a first example we have found the reflected and transmitted beams from a rotated cylindrical surface. The incident beam is an elliptical GB with waists of 5 and 20 μm. This beam hits a cylindrical lens, which is rotated about the z-axis with at an angle of 45°. Because, of this rotation the expression of the cylindrical surface would have a cross product term (xy) in the ray fixed coordinate system and thus, is an nonorthogonal system. The beam is therefore, transformed into a GAGB. The geometry of the problem is shown in Fig. 2. The parameters of the incident, transmitted and reflected beams are given below (see Figs. 3, 4 and Tables 1, 2).

5.2. Example II

As a second example we used our method to find the collimating effect of a hyperbolic surface

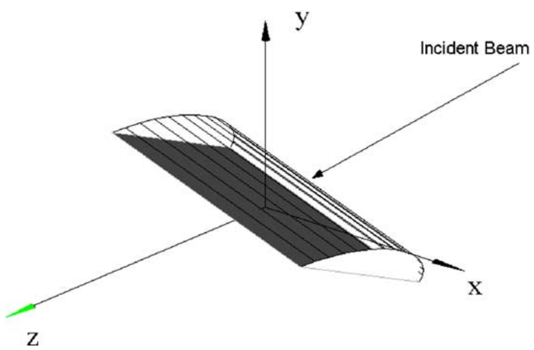

Fig. 2. The geometry of the problem.

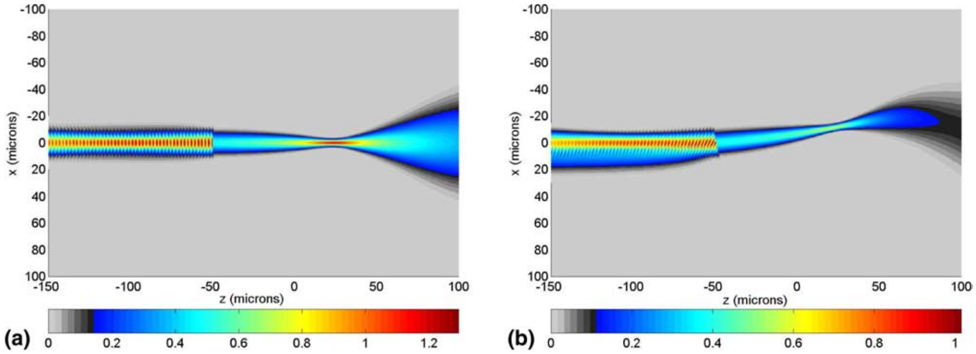

Fig. 3. Top XZ view of the field at $y = 0$ and 10 μm.

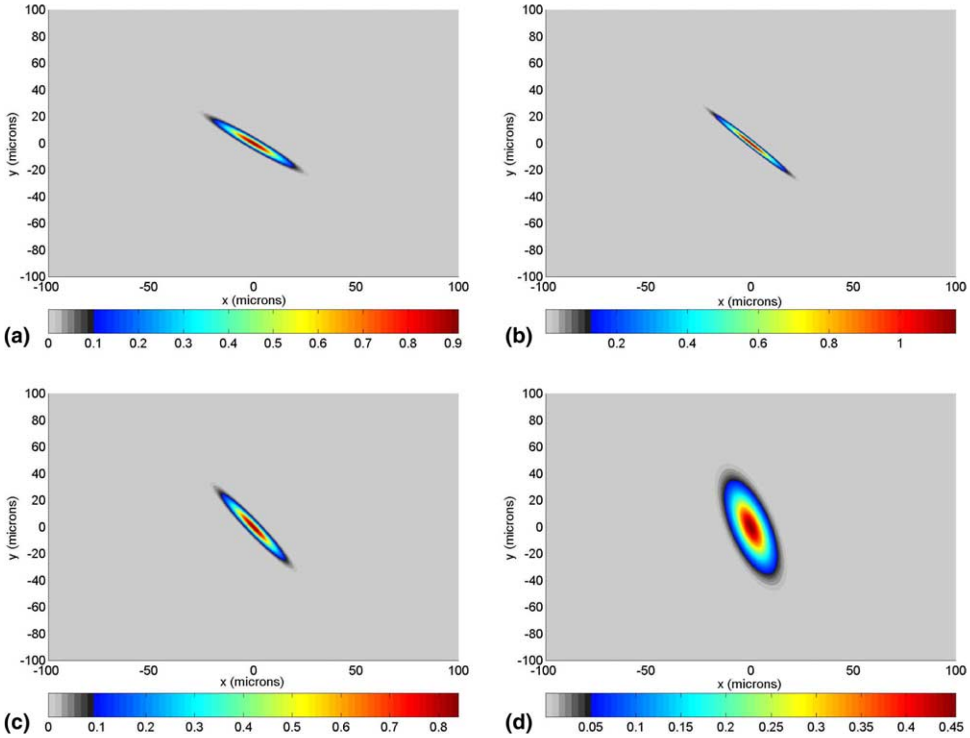

Fig. 4. Top XY view of the field at $z = 10$, 25, 40 and 80 μm.

of revolution. As it can be seen in the following figures the method correctly predicts a nearly constant phase which is an indication of flatness of the wavefront. The incident GB was placed at the focal plane of the hyperbola and because, the waist of it was chosen to be small its behavior is like a spherical wave (see Fig. 5). The equation of the hyperbola is given by

$$z = \frac{n_2 F}{n_2 + 1} + r_c \sqrt{\left(1 + \frac{n_2 + 1}{n_2 - 1} \frac{(x^2 + y^2)}{F^2}\right)},$$

where n_2 is the refractive index of the lens and was taken to be 1.435, F is the focal length of the lens and is equal 100 mm and $r_c = F/(n_2 + 1)$ the wavelength was 2.667 mm. The waist of the beam is 4.14 mm and is placed at 95.8 from the

Table 1
Parameters of the incident GB

Surface	$z = -\sqrt{(50\text{e}-6)^2 - \frac{x^2}{2} - \frac{y^2}{2} - \sqrt{(2)xy}}$
w_{0x}	5 μm
w_{0y}	20 μm
z_{0xi}	−100 μm
z_{0yi}	−100 μm
E_{0xi}	1
E_{0yi}	1
λ	1.31e−6
n_1	1
n_2	2.5
$\phi = \alpha + j\beta$	0+0j

Table 2
Reflected and transmitted beams' parameters

w_{0xt}	0.931 μm	w_{0xr}	4.04 μm
w_{0yt}	6.457 μm	w_{0yr}	1.31 μm
z_{0xt}	−18.41 μm	z_{0xr}	−171.9 μm
z_{0yt}	103.5 μm	z_{0yr}	74.3 μm
E_{0xt}	0.96−j0.93	E_{0xr}	1.84−j0.47
E_{0yt}	−0.96 + j0.93	E_{0yr}	0
$\phi = \alpha + j\beta$	0.71 + j0.074	$\phi = \alpha + j\beta$	0.69 + j0.097

surface to achieve the desired 100 mm curvature at incidence.

5.3. Example III

As a third example we found the effect of a ball lens placed in front of a GB. Our routine was used repeatedly to find the output field. Note the standing wave inside the ball lens due to multiple reflections and also the imperfect collimation of the ball lens. Our method is as mentioned before, extremely fast and it takes around 5 s to analyze this structure on a PIII 450 MHz. Therefore, it can be used to optimize for example the position of the ball lens for optimum collimation (see Fig. 6).

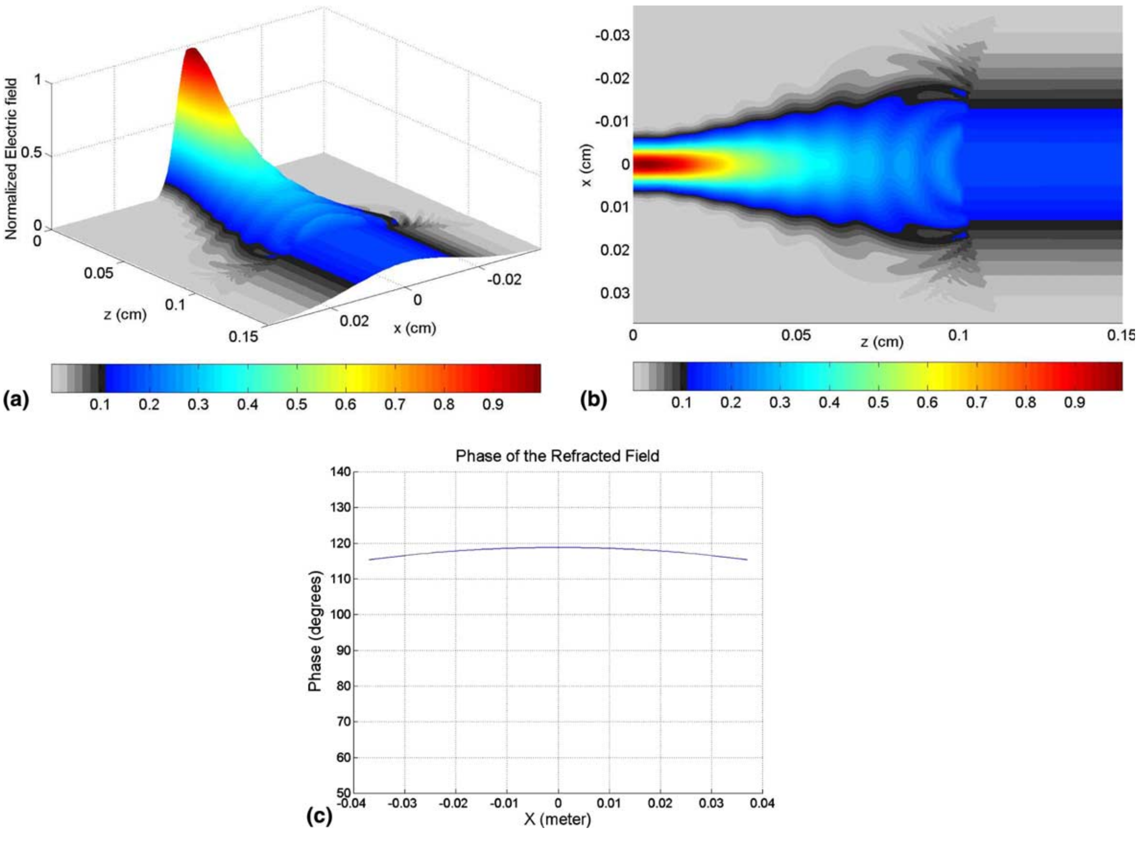

Fig. 5. (a) 3D View of the amplitude of electric field. (b) Top XZ view. (c) Phase of the electric field.

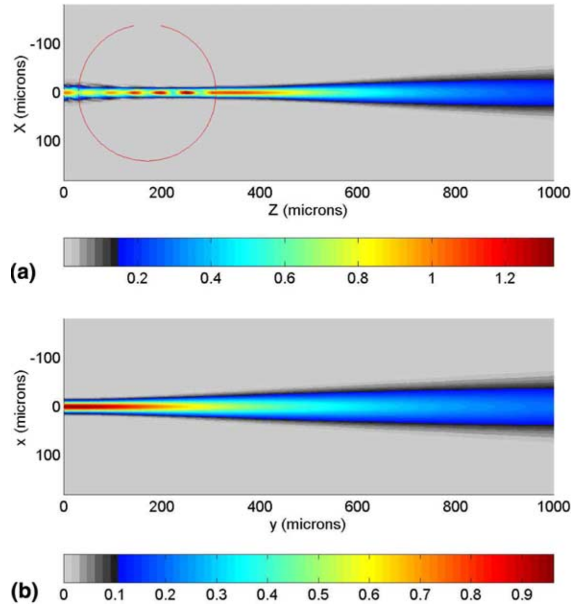

Fig. 6. (a) The imperfect collimation of a GB passing through a 280 μm ball lens. (b) The same beam without the ball lens.

6. Verification

We compared the results of our method with the physical optics (PO) method in a manner similar to [11]. To apply PO to this problem (reflection and refraction of a GB from a curved surface) we have to find the equivalent electric and magnetic sources on the surface;

$$\vec{J}_{\text{eq}} = \hat{n} \times \vec{H},$$

$$\vec{M}_{\text{eq}} = -\hat{n} \times \vec{E},$$

where $\vec{E}$ and $\vec{H}$ are the total fields on the surface and $\hat{n}$ is the outward normal (towards region including the incident beam). Determining these rigorously is not an easy task. Following the method of [11] we approximate these fields by the physical optics fields over the surface:

$$\vec{J}_{\text{eq}} = \vec{J}_{\text{PO}} = \hat{n} \times (\vec{H}_i + \vec{H}_r),$$

$$\vec{M}_{\text{eq}} = \vec{M}_{\text{PO}} = -\hat{n} \times (\vec{E_i} + \vec{E_r}),$$

and $\vec{E}_r$ and $\vec{H}_r$ are found under the assumption that the fields act locally as plane waves and Fresnel coefficients are applicable. The fields inside S is found from the equivalent sources:

$$\vec{J}_{\text{eq}} = -\vec{J}_{\text{PO}},$$

$$\vec{M}_{\text{eq}} = -\vec{M}_{\text{PO}}.$$

Radiation integrals are then used to find the fields inside and outside of S [12]. Two of these integrals are given below and the rest of the components can be found in [12]:

$$\begin{aligned} E_{Ax} = \frac{-\mathrm{j}\eta}{4\pi\beta} \int\int_s & G_1 J_x + (x - x')G_2 \\ & \times [(x - x')J_x + (y - y')Jy \\ & + (z - z')Jz]\mathrm{e}^{-\mathrm{j}\beta R}\,\mathrm{d}s', \end{aligned}$$

$$E_{Fx} = \frac{-1}{4\pi} \int\int_s [(z - z')M_y - (y - y')M_z] G_0 \mathrm{e}^{-\mathrm{j}\beta R}\,\mathrm{d}s',$$

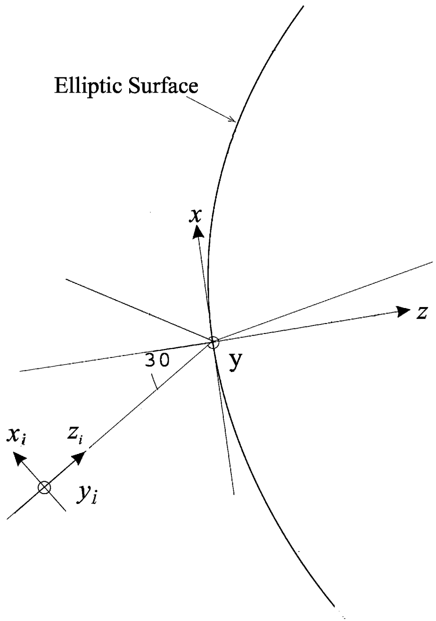

Fig. 7. Geometry of the problem used for verification.

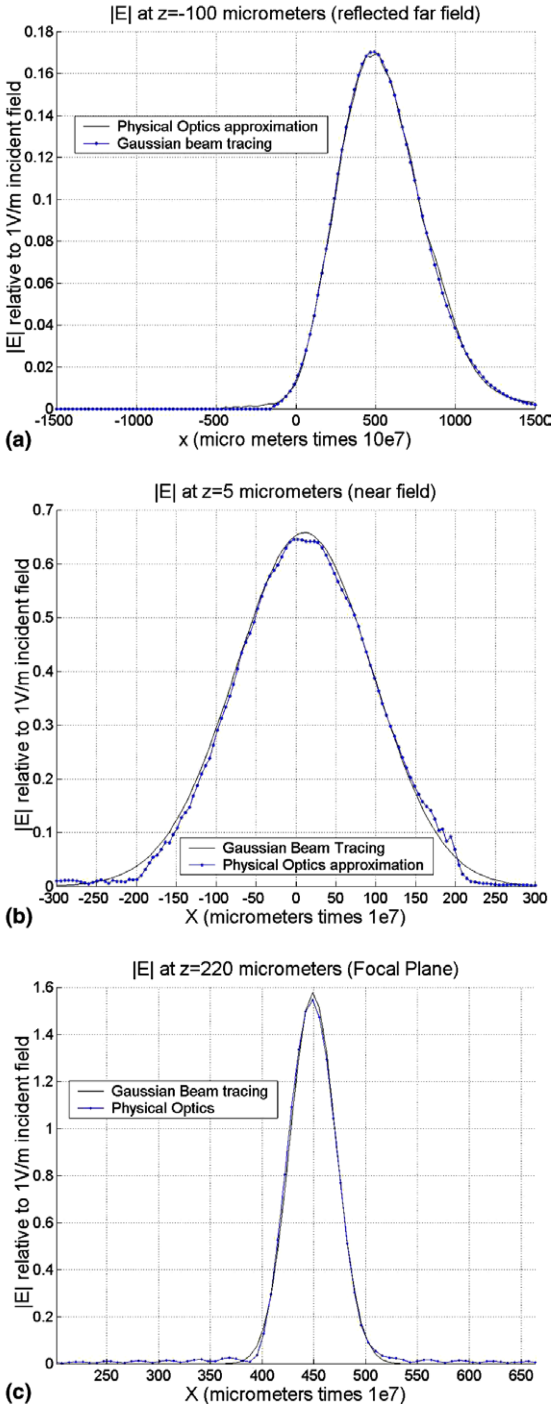

Fig. 8.

where

$$G_0 = \frac{1 + j\beta R}{R^3},$$

$$G_1 = \frac{-1 - j\beta R + \beta^2 R^2}{R^3},$$

$$G_2 = \frac{3 + j3\beta R - \beta^2 R^2}{R^5},$$

where ds' is the element of area of the surface over which this integrals must be evaluated. If this surface is given in terms of $(x', y', f(x', y')$ then this element of area is given by

$$ds' = \sqrt{1 + \left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2}\, dx'\, dy',$$

and therefore, the integrals become normal double integrals in terms of x' and y'. We must also note that z' in the above formulas has to be expressed in terms of x', y'. As a specific example we considered the oblique incidence at 30° of a GB with $w_{0x} = 10$ μm and $w_{0y} = 5$ μm on an elliptic surface. The distance of the waist to the surface is 100 μm and the equation of the surface is $z = \frac{1}{200\lambda}(\frac{x^2}{1} + \frac{y^2}{2})$. The geometry is shown in Fig. 7. Below are three plots showing the reflection and refraction of a GB from an elliptic surface together with the Physical Optics solution, as it can be seen from the two methods yield very close results (see Fig. 8).

7. Conclusion

A very fast method for tracing GAGBs based on phase matching has been developed. This method which is 3D and takes the vectorial nature of the GB into account can be used to find the transmission and reflection of GAGBs off general curved surfaces forming the boundaries of arbitrary smooth isotropic media. The method can be a good basis for hybrid methods. The hybridization with other methods is the subject of another paper.

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