

# Monad Transformers for the Easily Confused

...

Github Repo:

<https://github.com/TheWizardTower/monadTransformers>

**My friends have horrible  
senses of humor.**



**Ken Burnside** Transformers! Monads in Disguise!

(You may not be old enough for that to be a horrifying earworm.)



**TheWizardTower** @TheWizardTower · Mar 7

More good news:

I'm speaking at LambdaConf 2018!! I'll be giving a talk on Monad Transformers.  
:-D

cc @snoyberg @LambdaMom @lambda\_conf @mattoflambda



4



3



26



**Miriam Snoyman**

@LambdaMom

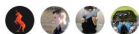
Following

Replying to @TheWizardTower @snoyberg and 2 others

## TRANSFORMERS: MONADS IN DISGUISE

9:23 PM - 7 Mar 2018

4 Likes



1



4



Tweet your reply



**TheWizardTower** @TheWizardTower · Mar 7

Replying to @LambdaMom @snoyberg and 2 others

You are literally the second person to make this joke to me.

Today.



2



2



**Miriam Snoyman** @LambdaMom · Mar 7

So it's finally catching on? 7 years after I made the Haskellers shop?

[shop.spreadshirt.com/haskellers/tra...](http://shop.spreadshirt.com/haskellers/tra...)



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# A word to begin

Don't fret if this talk doesn't make the lightbulb go off.

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Don't fret if this talk doesn't make the lightbulb go off.

A lot of this is sitting with the concepts for a while and letting them stew.

It's a lot like learning Algebra or Calculus for the first time. You have to give yourself time to make the cognitive leaps.

# A word to begin

Be patient! Keep trying!



# Talk Summary

Monad Transformers Transform Monads

...but how?

# Talk Summary

Monad Transformers Transform Monads

We're going to review Monoid, Functor, Applicative, and Monad laws, then see if we can figure out a way to get to what MTs do for us.

# Monoids

"associative, binary operations with an identity."

Source: <http://hackage.haskell.org/package/base-4.11.1.0/docs/Data-Monoid.html>

# Monoids -- Associative

Associative operations mean that:

$$(a \text{ <op> } (b \text{ <op> } c)) == ((a \text{ <op> } b) \text{ <op> } c)$$

# Monoids -- Binary

Binary operations are operations that take two values, and produce a result.

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To illustrate, exponentiation and absolute value are examples of **unary** operations.

# Monoids -- Binary

Binary operations are operations that take two values, and produce a result.

To illustrate, exponentiation and absolute value are examples of **unary** operations.

Binary operations include addition, subtraction, multiplication, division, list concatenation, and more.

# Monoid -- With Identity

Monoidal operations must have an identity value, such that

$$a \text{ <op> id } == a == \text{id <op> } a$$



# Monoid -- With Identity

Monoidal operations must have an identity value, such that

$$a \text{ <op> id } == a == \text{id <op> } a$$

Examples include 0 for addition, or the empty list for list concatenation.

# Functors

Functors obey two laws

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Functors obey two laws

```
fmap id = id
```

# Functors

Functors obey two laws

```
fmap id = id
```

```
fmap g . fmap h == fmap (g . h)
```

# Applicative

# Applicative

```
pure id <*> v = v                                -- Identity

pure f <*> pure x = pure (f x)                    -- Homomorphism

u <*> pure y = pure ($ y) <*> u                    -- Interchange

pure (.) <*> u <*> v <*> w = u <*> (v <*> w) -- Composition
```

(source: [https://en.wikibooks.org/wiki/Haskell/Applicative\\_functors](https://en.wikibooks.org/wiki/Haskell/Applicative_functors) )

# Applicative

Obviously, there's a lot going on, so let's unpack these one by one.

# Applicative -- Identity

```
pure id <*> v = v                                -- Identity
```

Much like the first functor law, this asserts that `pure id` does not distort the function in any way.

“Is your applicative instance doing sneaky things, Y/N?”



# Applicative -- Homomorphism

```
pure f <*> pure x = pure (f x)           -- Homomorphism
```

# Applicative -- Homomorphism

```
pure f <*> pure x = pure (f x)           -- Homomorphism
```

Remember the second functor law?

# Applicative -- Homomorphism

```
pure f <*> pure x = pure (f x) -- Homomorphism
```

Remember the second functor law?

```
fmap g . fmap h = fmap (g . h)
```

# Applicative -- Interchange

```
u <*> pure y = pure ($ y) <*> u           -- Interchange
```

# Applicative -- Interchange

```
u <*> pure y = pure ($ y) <*> u           -- Interchange
```

This becomes clearer with a concrete example.

# Applicative -- Interchange

```
*Main> pure ($ 5) <*> Just (+10)
```

```
Just 15
```

```
*Main> Just (+10) <*> pure 5
```

```
Just 15
```

```
*Main> (pure ($ 5) <*> Just (+10)) == (Just (+10) <*> pure  
5)
```

```
True
```

# Applicative -- Interchange

(source:

<https://stackoverflow.com/questions/27285918/applicatives-in-interchange-law> )

# Applicative -- Interchange

Referencing the typeclassopedia, we find:

"Intuitively, this says that when evaluating the application of an effectful function to a pure argument, the order in which we evaluate the function and its argument doesn't matter."

(source: [https://wiki.haskell.org/Typeclassopedia#Laws\\_2](https://wiki.haskell.org/Typeclassopedia#Laws_2) )



# Bonus Round!

`u <*> pure y = pure ($ y) <*> u` `-- Interchange`

Why `($ y)` ?

# Bonus Round!

```
u <*> pure y = pure ($ y) <*> u           -- Interchange
```

Why (\$ 5) ?

Let's ask GHCi

# Bonus Round!

```
*Main> :t ($ 5)
```

```
($ 5) :: Num a => (a -> b) -> b
```

```
*Main>
```

# Bonus Round!

We can rewrite

`($ 5)`

Into:

`(\f -> f 5)`

Even GHC agrees!

# Bonus Round!

```
*Main> :t (\f -> f 5)
```

```
(\f -> f 5) :: Num t1 => (t1 -> t2) -> t2
```

```
*Main>
```

# Bonus Round!

```
*Main> :t (\f -> f 5)
```

```
(\f -> f 5) :: Num t1 =>  
(t1 -> t2) -> t2
```

```
*Main>
```

```
*Main> :t ($ 5)
```

```
($ 5) :: Num a => (a -> b)  
-> b
```

```
*Main>
```

# Bonus Round!

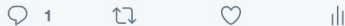


**TheWizardTower** @TheWizardTower · May 28  
Haskell Peeps!

Applicative laws. Specifically the interchange law.

$u \leq^* \text{pure } y = \text{pure } (\$ y) \leq^* u$

Can someone explain why the \$ is necessary in  $(\$ y)$  ?



**Harry Pan** @xingzhipan · May 28  
So that  $y$  is the second argument to  $\$$ .



**TheWizardTower** @TheWizardTower · May 29  
So,

Just  $(\$ 5) \leq^* \text{Just } (+10)$

would get rewritten as

Just  $\$ +10\ 5$

Right?



**Michael Snoyman**  
@snoyberg

Following

Replying to @TheWizardTower @xingzhipan

Not quite, you need some parens. But  $(\$ x)$  is equivalent to  $(\lambda f \rightarrow f \$ x)$ , if that helps. It's called an operator section. And  $(f \$ x)$  is the same as  $(f x)$

12:04 AM - 29 May 2018

# Applicative -- Composition

```
pure (.) <*> u <*> v <*> w = u <*> (v <*> w) -- Composition
```



# Applicative -- Composition

```
pure (.) <*> u <*> v <*> w = u <*> (v <*> w) -- Composition
```

Again, concrete examples will make this clearer

# Applicative -- Composition

```
*Main> pure (.) <*> Just (+10) <*> Just (+20) <*> Just 30
```

```
Just 60
```

```
*Main> Just (+10) <*> (Just (+20) <*> Just 30)
```

```
Just 60
```

```
*Main> (pure (.) <*> Just (+10) <*> Just (+20) <*> Just 30)  
== (Just (+10) <*> (Just (+20) <*> Just 30))
```

```
True
```

# Applicative -- Composition

Let's walk through that again.

# Applicative -- Composition

```
(pure (.) <*> Just (+10) <*> Just (+20))
```

becomes

```
Just (+20 . +10)
```

# Applicative -- Composition

```
Just (+20 . +10)
```

Adding back the `<*>` `Just 30` :

```
Just (+20 . +10) <*> Just 30
```

# Applicative -- Composition

```
Just (+20) . +10) <*> Just 30
```

is simplified down to

```
Just 60
```

# Monad

# Monad

- Must be burritos
- Must be delicious
- Can swap out burrito type with monadic burrito actions



# Monad

(okay, okay)

# Monad

```
return a >>= k = k a -- left identity
```

```
m >>= return = m      -- right identity
```

```
m >>= (\x -> k x >>= h) = (m >>= k) >>= h -- associativity
```

# Monad -- Left Identity

```
return a >>= k = k a
```

```
*Main> return 5 >>= (\x ->  
Just (x + 5))
```

```
Just 10
```

```
*Main>
```

# Monad -- Right Identity

```
m >>= return = m
```

```
*Main> Just 5 >>= return
```

```
Just 5
```

```
*Main>
```

# Monad -- Associativity

$$m \gg= (\backslash x \rightarrow k \ x \gg= h) \quad = \quad (m \gg= k) \gg= h$$

# Monad -- Associativity

```
*Main> let f = (\x -> Just (5 - x))
```

```
*Main> return 5 >>= f
```

```
Just 0
```

```
*Main> return (-5) >>= f
```

```
Just 10
```

# Monad -- Associativity

```
*Main> let g = (\x -> Just (10 - x))
```

```
*Main> return 5 >>= f >>= g
```

```
Just 10
```

```
*Main> return 5 >>= g >>= f
```

```
Just 0
```

# Monad -- Associativity

```
*Main> Just 5 >>= (\x -> f x >>= g)
```

```
Just 10
```

```
*Main> (Just 5 >>= f) >>= g
```

```
Just 10
```

```
*Main>
```



# Monad -- Remix

```
return >=> m    = m                -- left identity

m >=> return    = m                -- right identity

(f >=> g) >=> h = f >=> (g >=> h)    -- associativity
```

# Monad -- Remix

```
return >=> m    = m           -- left identity

m >=> return    = m           -- right identity

(f >=> g) >=> h = f >=> (g >=> h) -- associativity
```

(source: [https://wiki.haskell.org/Monad\\_laws](https://wiki.haskell.org/Monad_laws) )

# A Closer Look at $>>=$

Courtesy of Category Theory, we know that  $>>=$  can be expressed as

```
m >>= k = join $ fmap k m
```

# A Closer Look at $\gg=$

```
*Main> :t (>>=)
```

```
(>>=) :: Monad m => m a ->  
(a -> m b) -> m b
```

# A Closer Look at `>>=`

`(>>=) :: Monad m =>`

`m a`

`-> (a -> m b)`

`-> m b`

# A Closer Look at >>=

```
(>>=) :: Monad m =>
```

```
    m a
```

```
  -> (a -> m b)
```

```
  -> m b
```

```
*Main> :t fmap
```

```
fmap :: Functor f =>
```

```
    (a -> b)
```

```
  -> f a
```

```
  -> f b
```

# A Closer Look at $>>=$

Remember that `m >>= k = join $ fmap k m`

```
*Main> fmap (\a -> Just (a + 10)) (Just (10))  
  
Just (Just 20)
```

# A Closer Look at >>=

```
*Main> join $ fmap (\a -> Just (a + 10)) (Just (10))
```

```
Just 20
```

```
*Main> (flip (>>=)) (\a -> Just (a + 10)) (Just (10))
```

```
Just 20
```



## A Closer Look at >>=

```
*Main> join $ fmap (\a -> Just (a + 10)) (Just (10))
```

```
Just 20
```

```
*Main> (= <<) (\a -> Just (a + 10)) (Just (10))
```

```
Just 20
```

# A Closer Look at $>>=$

```
*Main> :t join
```

```
join :: Monad m => m (m a) -> m a
```

# A Closer Look at $>>=$

```
*Main> :t join
```

```
join :: Monad m => m (m a) -> m a
```

There's an important note here: The monadic contexts you're joining have to match!

# A Closer Look at >>=

```
*Main> join $ Just $ Just 10
```

```
Just 10
```

```
*Main> join $ Just Nothing
```

```
Nothing
```

```
*Main> join $ Just $ Just Nothing
```

```
Just Nothing
```

# A Closer Look at >>=

```
*Main Control.Monad> join [[1],[2],[3]]
```

```
[1,2,3]
```

```
*Main Control.Monad> join [[[1,2,3],[4,5,6],[7,8,9]]]
```

```
[[1,2,3],[4,5,6],[7,8,9]]
```



Let's make  $>>=$  for IdentityT!

# Let's make $\gg=$ for IdentityT!

First step, define our arguments.



# Let's make `>>=` for `IdentityT`!

```
(\a ->
```

```
  IdentityT (Just (a + 10))
```

# Let's make >>= for IdentityT!

```
(\a -> IdentityT (Just 10))  
  
IdentityT (Just (a + 10))
```

# Let's make $>>=$ for IdentityT!

```
*Main> fmap (\a -> IdentityT (Just (a + 10))) (IdentityT  
(Just 10))
```

```
IdentityT (Just (IdentityT (Just 20)))
```

# Let's make `>>=` for IdentityT!

```
IdentityT (Just (IdentityT (Just 20)))
```

`join` can't reduce any part of this -- none of the adjacent monads match.

# Let's make `>>=` for `IdentityT`!

```
IdentityT (Just (IdentityT (Just 20)))
```

`join` can't reduce any part of this -- none of the adjacent monads match.

There is a function that can help, though.

# Let's make $\gg=$ for IdentityT!

```
*Main> :t runIdentityT
```

```
runIdentityT :: IdentityT f a -> f a
```

# Let's make $>>=$ for IdentityT!

```
*Main> fmap (\a -> IdentityT (Just (a + 10))) (IdentityT  
  (Just 10))
```

```
IdentityT (Just (IdentityT (Just 20)))
```

# Let's make $\gg=$ for IdentityT!

```
*Main> fmap (runIdentityT . (\a -> IdentityT (Just (a +  
10)))) (IdentityT (Just 10))
```

```
IdentityT (Just (Just 20))
```



## Let's make $>>=$ for IdentityT!

```
*Main> fmap (runIdentityT . (\a -> IdentityT (Just (a +  
10)))) (IdentityT (Just 10))
```

```
IdentityT (Just (Just 20))
```

Progress! Still not correct, but progress all the same!

# Let's make $\gg=$ for IdentityT!

```
*Main> runIdentityT $ fmap (runIdentityT . (\a -> IdentityT  
(Just (a + 10)))) (IdentityT (Just 10))
```

```
(Just (Just 20))
```

# Let's make >>= for IdentityT!

```
*Main> join $ runIdentityT $ fmap (runIdentityT . (\a ->  
IdentityT (Just (a + 10)))) (IdentityT (Just 10))
```

```
Just 20
```

# Let's make >>= for IdentityT!

```
*Main> IdentityT $ join $ runIdentityT $ fmap (runIdentityT  
  . (\a -> IdentityT (Just (a + 10)))) (IdentityT (Just 10))
```

```
IdentityT Just 20
```

# Let's make >>= for IdentityT!

```
*Main> IdentityT $ join $ runIdentityT $ fmap (runIdentityT  
  . (\a -> IdentityT (Just (a + 10)))) (IdentityT (Just 10))
```

```
IdentityT Just 20
```

We did it!

# Let's make $>>=$ for IdentityT!

However, we did it the *really really ugly* way.

# Let's make $>>=$ for IdentityT!

```
iTBind :: Monad f =>
```

```
    IdentityT f a
```

```
  -> (a -> IdentityT f b)
```

```
  -> IdentityT f b
```

```
iTBind (IdentityT fa) ab = IdentityT $ join $ fmap  
  (runIdentityT . ab) fa
```

# Let's make `>>=` for IdentityT!

We can also look at the monad instance for IdentityT in the transformers library!





# Let's make $>>=$ for IdentityT!

```
m >>= k = IdentityT $ runIdentityT . k =<< runIdentityT m
```

# Let's make $\gg=$ for IdentityT!

Reminder:

```
*Main> :t (>>=)
```

```
(>>=) :: Monad m => m a -> (a -> m b) -> m b
```

# Let's make $>>=$ for IdentityT!

Reminder V2: We've seen  $=<<$  before!

```
*Main> :t (=<<)
```

```
(=<<) :: Monad m => (a -> m b) -> m a -> m b
```

```
*Main> :t (flip (>>=))
```

```
(flip (>>=)) :: Monad m => (a -> m b) -> m a -> m b
```

# Let's make $>>=$ for IdentityT!

```
(>>=)  :: Monad m =>                runIdentityT . k
        m a
-> (a -> m b)
-> m b
```

# Let's make $>>=$ for IdentityT!

```
(>>=) :: Monad m =>
```

```
    m a
```

```
  -> (a -> m b)
```

```
  -> m b
```

```
runIdentityT . k :: (a -> m b)
```

# Let's make $>>=$ for IdentityT!

```
(>>=) :: Monad m =>
```

```
    m a
```

```
  -> (a -> m b)
```

```
  -> m b
```

```
runIdentityT . k :: (a -> m b)
```

```
runIdentityT m
```

# Let's make `>>=` for IdentityT!

```
(>>=) :: Monad m =>
```

```
    m a
```

```
  -> (a -> m b)
```

```
  -> m b
```

```
runIdentityT . k :: (a -> m b)
```

```
=<<
```

```
runIdentityT m :: m a
```



# Let's make $>>=$ for IdentityT!

```
(>>=) :: Monad m =>
```

```
    m a
```

```
  -> (a -> m b)
```

```
  -> m b
```

```
runIdentityT . k :: (a -> m b)
```

```
=<<
```

```
runIdentityT m :: m a
```

```
:: m b
```

# Let's make $>>=$ for IdentityT!

```
(>>=) :: Monad m =>
```

```
    m a
```

```
  -> (a -> m b)
```

```
  -> m b
```

```
IdentityT $
```

```
runIdentityT . k :: (a -> m b)
```

```
=<<
```

```
runIdentityT m :: m a
```

```
:: IdentityT m b
```

That's great, but how do we use it?

# That's great, but how do we use it?

Glad you asked. Let's build a basic  
MaybeT computation.

# That's great, but how do we use it?

```
myPrompt :: String  
         -> IO String
```

```
myPrompt prompt = do  
    putStr prompt  
    result <- getLine  
    return result
```

# That's great, but how do we use it?

```
getName :: MaybeT IO String
```

```
getName = do
```

```
    input <- myPrompt "Name? "
```

```
    if input == ""
```

```
        then MaybeT $ return Nothing
```

```
        else MaybeT $ return $ Just $ "Name: " ++ input ++ "\n"
```

# This Does Not Typecheck!

```
getName :: MaybeT IO String
```

```
getName = do
```

```
    input <- myPrompt "Name? "
```

```
    if input == ""
```

```
        then MaybeT $ return Nothing
```

```
        else MaybeT $ return $ Just $ "Name: " ++ input ++ "\n"
```

# Let's imagine the function we want.

We want a function that has this type signature:

```
IO String -> MaybeT IO String
```



# Let's imagine the function we want.

```
IO String -> MaybeT IO String
```

# Let's imagine the function we want.

`IO a -> MaybeT IO a`

# Let's imagine the function we want.

```
Monad m => m a -> MaybeT m a
```

# Let's imagine the function we want.

```
(Monad m, Control.Monad.Trans.Class.MonadTrans t) =>
```

```
    m a
```

```
-> t m a
```

# SURPRISE! This is called `lift`

```
(Monad m, Control.Monad.Trans.Class.MonadTrans t) =>
```

```
    m a
```

```
-> t m a
```

# SURPRISE! This is called `lift`

```
*Main> :t lift
```

```
lift
```

```
:: (Monad m, Control.Monad.Trans.Class.MonadTrans t) =>
```

```
  m a -> t m a
```

# This typechecks now!

```
getName :: MaybeT IO String
```

```
getName = do
```

```
    input <- lift $ myPrompt "Name? "
```

```
    if input == ""
```

```
        then MaybeT $ return Nothing
```

```
        else MaybeT $ return $ Just $ "Name: " ++ input ++ "\n"
```

# Let's make a few more.

```
getNumber :: String -> MaybeT IO String
```

```
getNumber str = do
```

```
    input <- lift $ myPrompt "Phone number? "
```

```
    if (length input) /= 7
```

```
        then MaybeT $ return Nothing
```

```
        else MaybeT $ return $ Just $ str ++ "Phone Number: " ++  
input ++ "\n"
```



# Let's make a few more.

```
getStreetName :: String -> MaybeT IO String
```

```
getStreetName str = do
```

```
    input <- lift $ myPrompt "Street Name? "
```

```
    if input == ""
```

```
        then MaybeT $ return Nothing
```

```
        else MaybeT $ return $ Just $ str ++ "Street Name: " ++  
input ++ "\n"
```

# Let's make a few more.

```
compositionMethod :: MaybeT IO (String)
```

```
compositionMethod = getName >>=
```

```
    getNumber >>=
```

```
    getStreetName
```

# (Just to prove you can still do this with do-notation)

```
compAlt :: MaybeT IO (String)
```

```
compAlt = do
```

```
  a <- getName
```

```
  b <- getNumber a
```

```
  c <- getStreetName b
```

```
  return c
```

That's great, but how do you run it? It's not an IO action, it's a MaybeT IO action.

You use `runMaybeT!`

