forecast

May 12, 2025

1 HMM Forecasting Walkthrough

1.0.1 This notebook shows the step by step process of generating hmm forecasts along with forecast validation using simulated data.

1.1 Setup

Load hmmTMB and other packages

[1]:
```
library(devtools)
library(ggridges)
set.seed(1)
```

```
Loading required package: usethis
```

[2]:
```
load_all("../../../hmmTMB")
```

```
 Loading hmmTMB
Loading required package: R6

Loading required package: mgcv

Loading required package: nlme

This is mgcv 1.9-3. For overview type 'help("mgcv-package")'.

Loading required package: TMB

Loading required package: ggplot2
```

1.2 Generate Data

Use the true model defined in true_mod.hmm to set the model functions. Set arbitrary fixed effect coefficients.

[3]:
```
# Simulate data ------------------------------------------------------------

# number of time steps
```

```r
n <- 1000

# Generate training data with random covariates
empty <- data.frame(
  ID = 1,
  count = rep(NA, n),
  size = rep(NA, n),
  covariate_1 = rnorm(n, mean = 0, sd = 1),
  covariate_2 = runif(n, min = -1, max = 1),
  covariate_3 = sample(c(-1, 0, 1), n, replace = TRUE)
)

# create true model
true_mod <- HMM$new(file = "forecast_true_mod.hmm")

# Create a list of random integers for the hidden Markov model coefficients
random_coeff_fe_obs <- replicate(
  length(true_mod$coeff_list()$coeff_fe_obs),
  sample(c(0.9, 1, 1.1, 1.2), 1, replace = TRUE)
)
random_coeff_fe_hid <- replicate(
  length(true_mod$coeff_list()$coeff_fe_hid),
  sample(c(0.9, 1, 1.1, 1.2), 1, replace = TRUE)
)

# Update the coefficients of the hidden Markov model with the random values
true_mod$obs()$update_coeff_fe(coeff_fe = random_coeff_fe_obs)
true_mod$hid()$update_coeff_fe(coeff_fe = random_coeff_fe_hid)

# View the model parameters
true_mod$coeff_list()
```

$coeff_fe_obs A matrix: 10 × 1 of type dbl

count.rate.state1.(Intercept)	0.9
count.rate.state1.covariate_1	1.2
count.rate.state2.(Intercept)	1.0
count.rate.state2.covariate_1	1.0
size.mean.state1.(Intercept)	0.9
size.mean.state1.covariate_2	1.1
size.mean.state2.(Intercept)	1.1
size.mean.state2.covariate_2	0.9
size.sd.state1.(Intercept)	1.1
size.sd.state2.(Intercept)	0.9

$log_lambda_obs

$coeff_fe_hid A matrix: 4 × 1 of type dbl

S1>S2.(Intercept)	1.2
S1>S2.covariate_3	1.2
S2>S1.(Intercept)	1.2
S2>S1.covariate_3	1.2

$log_lambda_hid

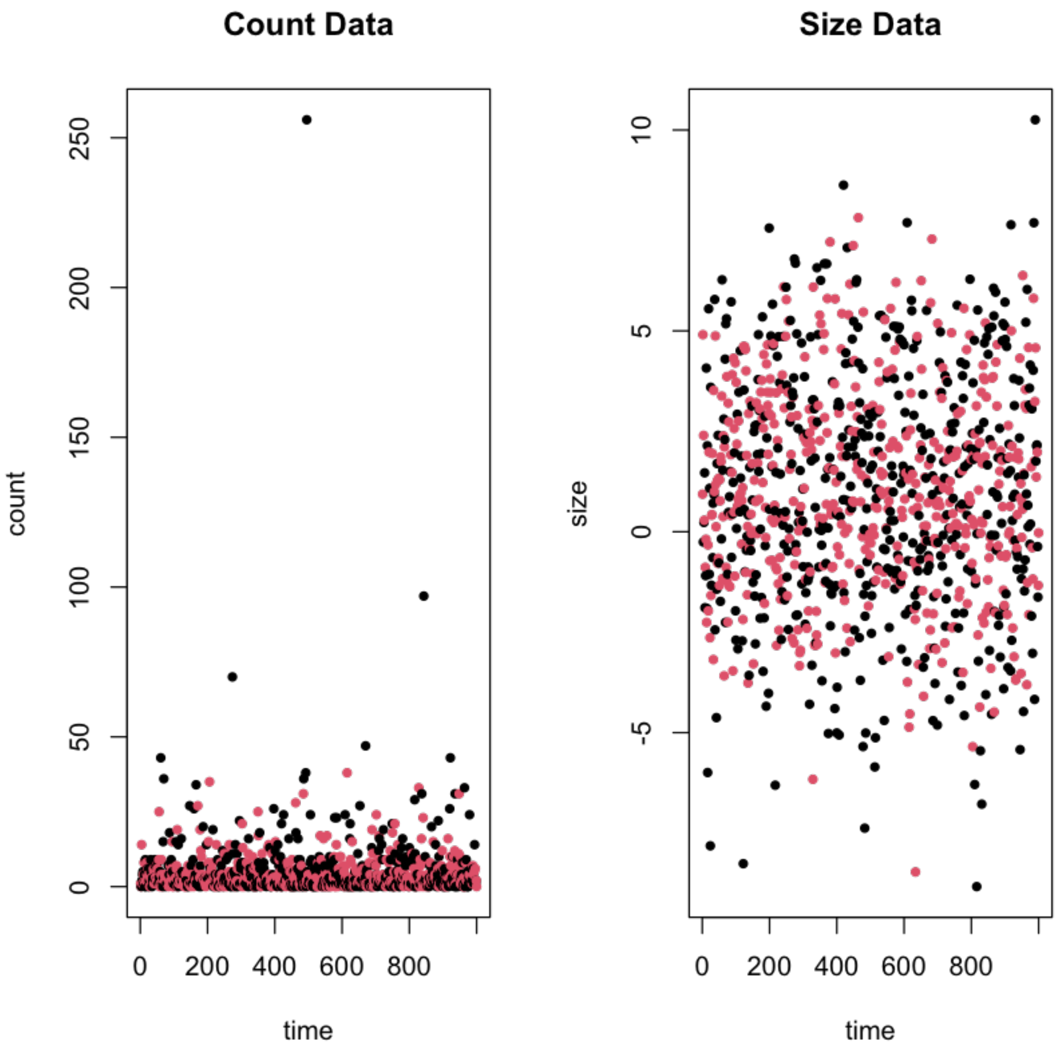
$log_delta0 A matrix: 1 × 1 of type dbl ID:1.state1 | 0

$coeff_re_obs

$coeff_re_hid

[4]:
```
# simulate from true model
training <- true_mod$simulate(n, data = empty)

# update data in true_mod with dat
true_mod$obs()$update_data(training)

# plot data
par(mfrow = c(1, 2)) # set outer margins to 0
plot(training$count, pch = 20, xlab = "time", ylab = "count", col =␣
↪attr(training, "state"), main = "Count Data")
plot(training$size, pch = 20, xlab = "time", ylab = "size", col =␣
↪attr(training, "state"), main = "Size Data")
par(mfrow = c(1, 1))
```

```
Simulating states… 100%
Simulating count… 100%
Simulating size… 100%
```



1.3 Fit the model

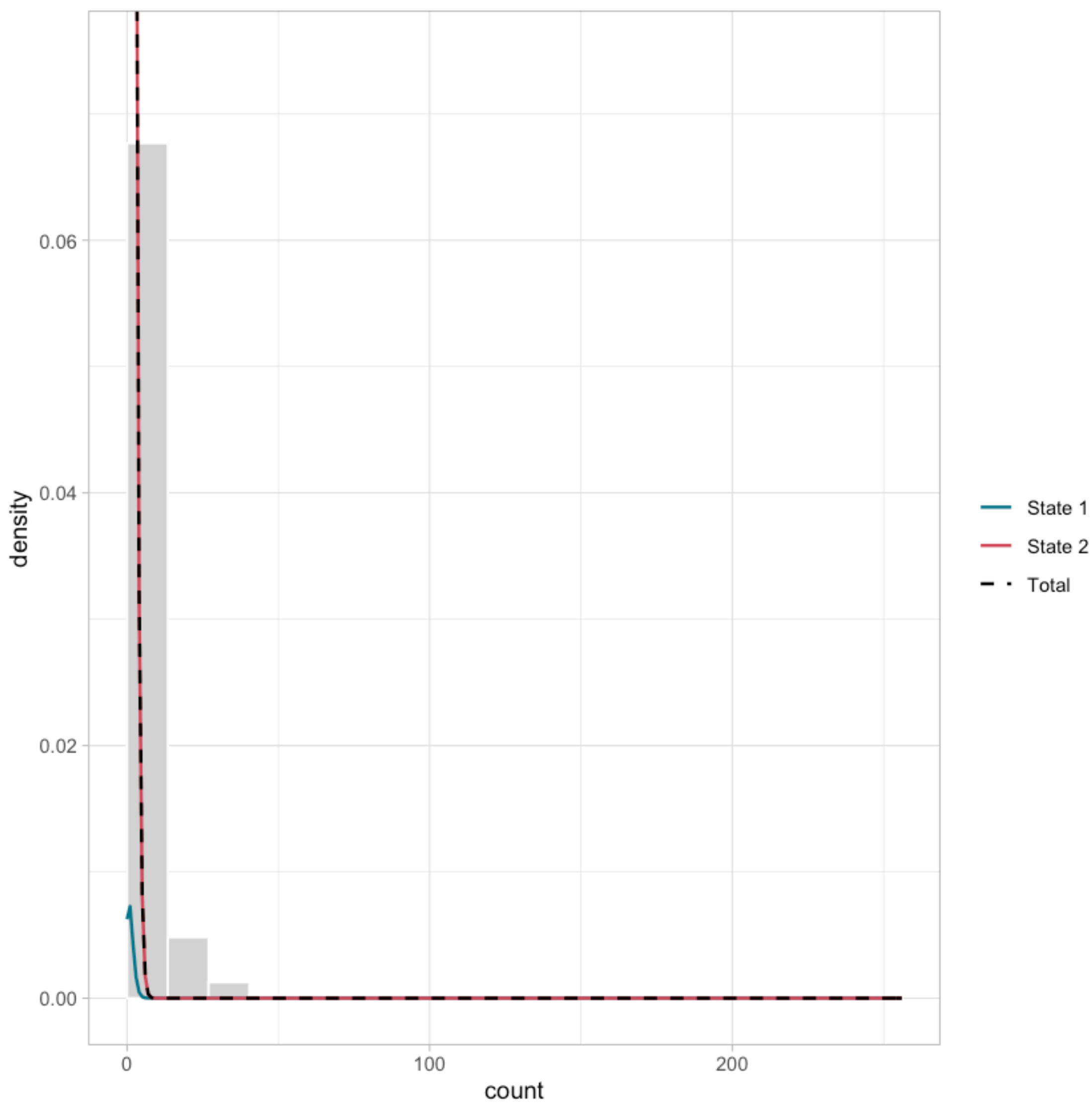
Fit a model using the model definition and starting parameters defined by mod.hmm

[5]:
```
# Fit model -----------------------------------------------------------

# create model to fit
mod <- HMM$new(file = "forecast_mod.hmm")

# suggest better starting parameters
ini <- mod$suggest_initial()

# set to new starting parameters (or you could edit the specification file)
```

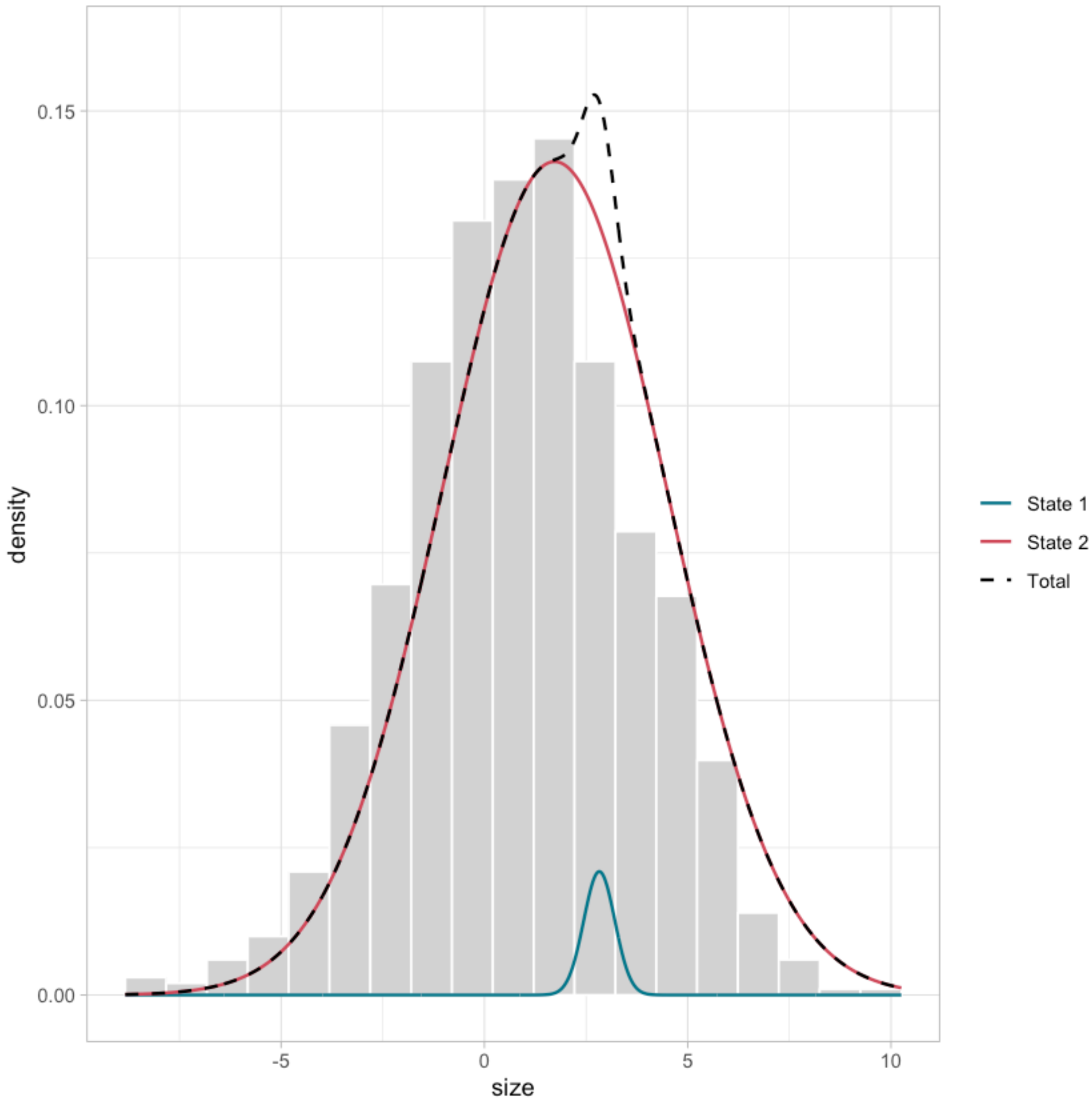
```
mod$obs()$update_par(ini)

# fit model
mod$fit(silent = TRUE)
```

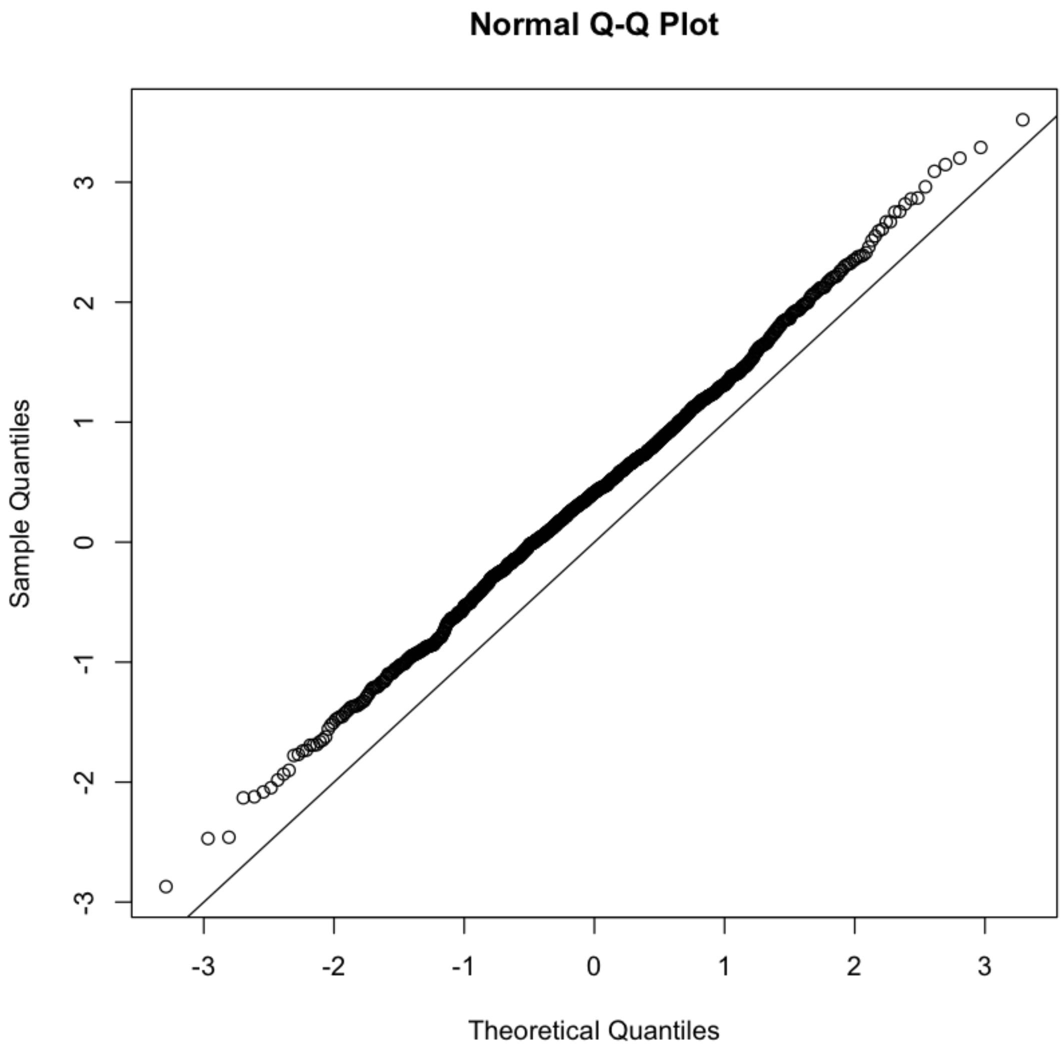
Check that the model has converged and fit properly

[6]:
```
par(mfrow = c(1, 2))
mod$plot_dist("count")
mod$plot_dist("size")
par(mfrow = c(1, 1))

pr <- mod$pseudores()

qqnorm(pr$count)
abline(0, 1)
```



```
Computing CDFs… done
Computing residuals for count … done
Computing residuals for size … done
```





1.4 Forecasting

Use the fitted model to generate a probability distribution for each future state.

Set the number of future time steps to predict. Calculate the parameter values at future time steps accounting for covariates using the predict method

[7]:
```
# Set number of observations to predict
n <- 10

# Create new data for prediction, optionally with covariates
forecasts <- data.frame(
  ID = 1,
```

```
  count = rep(NA, n),
  size = rep(NA, n),
  covariate_1 = rnorm(n, mean = 0, sd = 1),
  covariate_2 = runif(n, min = -1, max = 1),
  covariate_3 = sample(c(-1, 0, 1), n, replace = TRUE)
)

forecasts
```

A data.frame: 10 × 6

ID	count	size	covariate_1	covariate_2	covariate_3
<dbl>	<lgl>	<lgl>	<dbl>	<dbl>	<dbl>
1	NA	NA	-0.27225030	0.2506252	1
1	NA	NA	-0.71720824	0.8678033	0
1	NA	NA	-0.19532618	-0.2374188	1
1	NA	NA	1.31326420	0.5469202	0
1	NA	NA	0.01268243	0.3950817	-1
1	NA	NA	-1.12421159	0.8369966	-1
1	NA	NA	0.85662803	-0.7556111	1
1	NA	NA	0.31418212	0.7315667	-1
1	NA	NA	0.33667190	0.7975774	1
1	NA	NA	0.26202593	-0.1804867	0

[8]:
```
# Predict parameters for each future time step accounting for covariates
obs_par_forecast <- mod$predict("obspar", newdata = forecasts)
tpm_forecast <- mod$predict("tpm", newdata = forecasts)

# Set the forecasted parameters in the forecasts object
attr(forecasts, "obs_par") <- mod$predict("obspar", newdata = forecasts)
attr(forecasts, "tpm") <- mod$predict("tpm", newdata = forecasts)

# Print the first three time steps of the forecast
cat("Observation Parameter Forecast:\n")
print(obs_par_forecast[1, ,1:3])
cat("\nTransition Probability Matrix Forecast:\n")
print(tpm_forecast[, ,1:3])
```

```
Observation Parameter Forecast:
            [,1]     [,2]     [,3]
state 1 1.614333 1.070022 1.733282
state 2 1.938500 1.166783 2.116320

Transition Probability Matrix Forecast:
, , 1

           state 1   state 2
state 1 0.16877528 0.8312247
state 2 0.05528666 0.9447133
```

```
, , 2

            state 1   state 2
state 1 0.360825935 0.6391741
state 2 0.009623971 0.9903760

, , 3

           state 1   state 2
state 1 0.16877528 0.8312247
state 2 0.05528666 0.9447133
```

Hidden State Forecast Forecast the hidden states based on the TPM matrix. Use the last hidden state of the fitted model as the starting point.

TODO: The starting distribution should be a variable that can either be set as the last fitted state distribution, or the stationary distribution or a custom starting point.

[9]:
```
# Hidden State Forecasting␣
↪------------------------------------------------------------
# Get starting distribution of hidden states
last_state_distribution <- tail(mod$state_probs(), 1)
last_training_tpm <- mod$hid()$tpm(nrow(mod$obs()$data()))[, , 1]

# Initialize the forecast matrix to store results
hidden_state_forecast <- array(NA, dim = c(mod$hid()$nstates(),␣
↪nrow(forecasts)))

# Set the initial distribution
hidden_state_forecast[, 1] <- last_state_distribution %*% last_training_tpm

# Loop through the remaining time steps
for (t in 2:nrow(forecasts)) {
  hidden_state_forecast[, t] <- hidden_state_forecast[, t - 1] %*%␣
↪tpm_forecast[, , t-1]
}

# Set hidden_state_forecast as an attribute of forecasts
attr(forecasts, "hidden_state_forecast") <- hidden_state_forecast

# Print the forecasted hidden state probabilities
dim(hidden_state_forecast)
print(hidden_state_forecast[ , 1:3])
```

1. 2 2. 10

```
            [,1]       [,2]       [,3]
[1,] 0.009624429 0.05637892 0.02942436
```

```
[2,] 0.990375571 0.94362108 0.97057564
```

Check Dimensions Get list of observation distributions in the case were the model is predicting multiple outputs.

Check that all the dimensions match.

[10]:
```r
obs_dists <- mod$obs()$dists()

length(obs_dists)
dim(obs_par_forecast)
dim(hidden_state_forecast)

# Check dimensions
if (dim(obs_par_forecast)[2] != dim(hidden_state_forecast)[1]) {
  stop("Error: Number of hidden states in obs_par_forecast does not match the␣
↪number of hidden states in hidden_state_forecast.")
}
if (dim(obs_par_forecast)[3] != dim(hidden_state_forecast)[2]) {
  stop("Error: Number of forecasts in obs_par_forecast does not match the␣
↪number of forecasts in hidden_state_forecast.")
}

cat("Dimensions checks passed.\n")
```

2

1. 3 2. 2 3. 10

1. 2 2. 10

```
Dimensions checks passed.
```

[11]:
```r
# Get hidden state covariate names
hid_covariates <- unique(unlist(lapply(true_mod$hid()$formulas(), all.vars)))

# Get emission variable names
emission_vars <- colnames(true_mod$obs()$obs_var())

obs_covariates <- c()
for (emission_var in emission_vars) {
  for (attribute in names(true_mod$obs()$formulas()[[emission_var]])) {
    obs_covariates <- c(
      obs_covariates,
      unlist(lapply(true_mod$obs()$formulas()[[emission_var]][[attribute]], all.
↪vars))
    )
  }
}
obs_covariates <- unique(obs_covariates)
```

```r
# Print results
cat("Observation Model Covariates:", obs_covariates, "\n")
cat("Hidden State Model Covariates:", hid_covariates, "\n")
cat("Emission Variables:", emission_vars, "\n")
```

```
Observation Model Covariates: covariate_1 covariate_2
Hidden State Model Covariates: covariate_3
Emission Variables: count size
```

1.4.1 Generate Forecasts

1. **Defining Evaluation Points:** Establishing a range of x values at which the forecast probability density will be evaluated.
2. **Iterating Through Dimensions and Time:** Looping through each output dimension and each time step in the forecast horizon.
3. **Calculating Weighted Probabilities:** For each time step, the forecast probability is computed as a weighted sum of the probability density of each hidden state multiplied by the probability of being in that hidden state.

[ ]:
```r
x_count <- 100
s <- 1
n <- 1
nstates <- mod$hid()$nstates()
# Requires mod object
# requires forecasts dataframe

preset_x_vals <- list(
  'count' = seq(0, 100, by = 1)
)

for (dimension in emission_vars) {

  # Step 1 - generate x values based on pm 10% of the training data range
  if (exists("preset_x_vals") && (dimension %in%
      names(preset_x_vals)) && !is.null(preset_x_vals[[dimension]])) {
    x_vals <- preset_x_vals[[dimension]]
    attr(forecasts, paste0(dimension, "_x_vals")) <- x_vals
  } else {
    max_range <- max(mod$obs()$data()[[dimension]], na.rm = TRUE)*1.1
    min_range <- min(mod$obs()$data()[[dimension]], na.rm = TRUE)*0.9
    x_vals <- seq(min_range, max_range, length.out = x_count)
    attr(forecasts, paste0(dimension, "_x_vals")) <- x_vals
  }


  # Step 2 - Get distribution function and parameters list
  obs_dists <- mod$obs()$dists()[[dimension]]
```

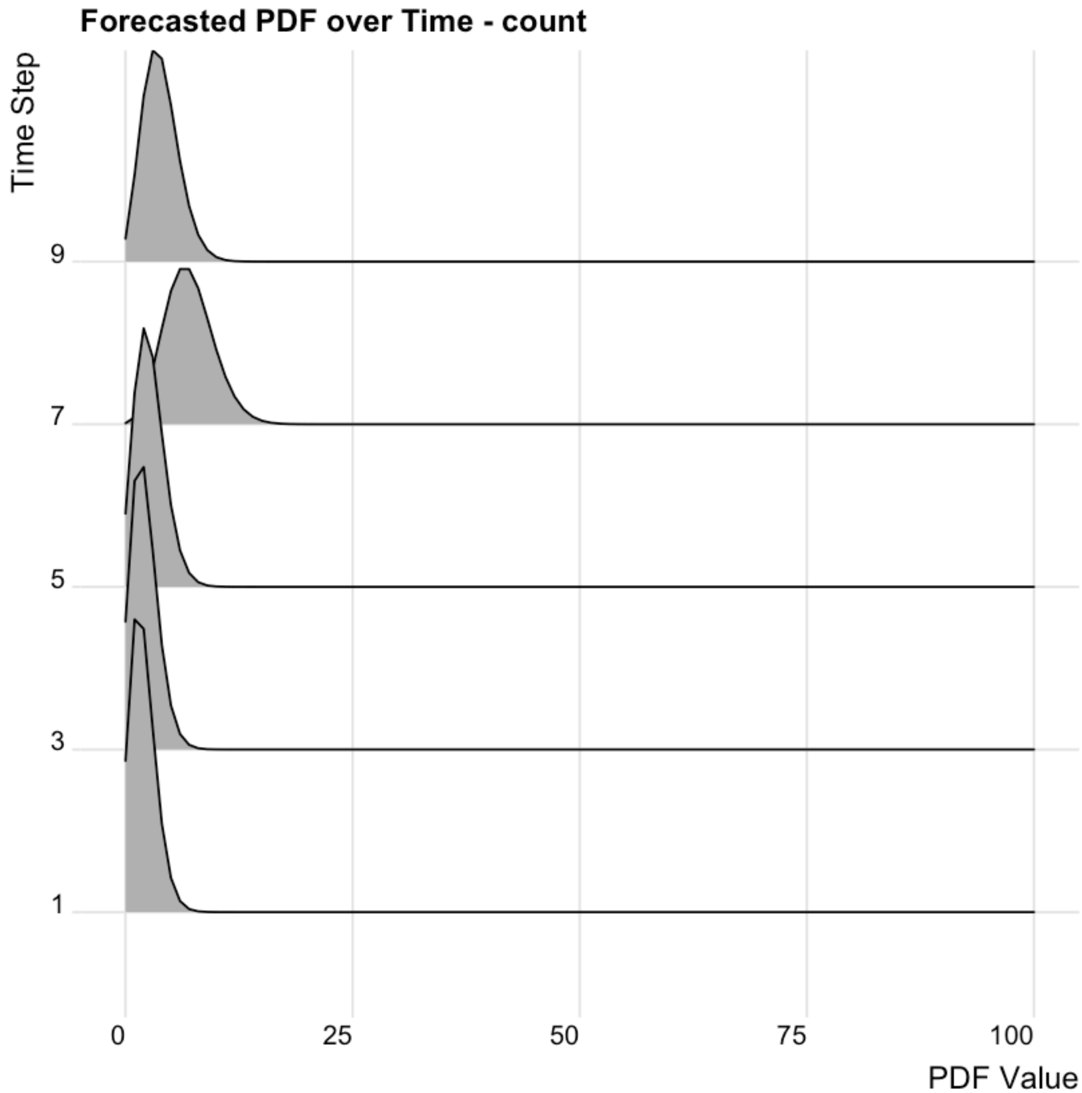
```
  pdf_params <- paste0(dimension, ".", names(formals(obs_dists$pdf())))
  model_params <- names(obs_par_forecast[ , 1, 1])
  current_params <- intersect(pdf_params, model_params)

  # Step 3 - Loop through the forecasted parameters and calculate the weighted
  # pdf
  forecast_pdfs <- array(NA, dim = c(length(x_vals), nrow(forecasts)))
  for (n in seq_len(nrow(forecasts))) {
    pdf_matrix <- sapply(seq_len(nstates), function(s) {
      obs_dists$pdf_apply(
        x = x_vals,
        par = setNames(
          obs_par_forecast[current_params, s, n],
          obs_dists$parnames()
        )
      )
    })
    # Normalize the probabilities
    forecast_pdfs[, n] <- as.vector(pdf_matrix %*% hidden_state_forecast[, n])
    forecast_pdfs[, n] <- forecast_pdfs[, n] / sum(forecast_pdfs[, n])
  }
  # Step 4 - Save the pdfs to the forecasts object
  attr(forecasts, paste0(dimension, "_pdfs")) <- forecast_pdfs
}
```

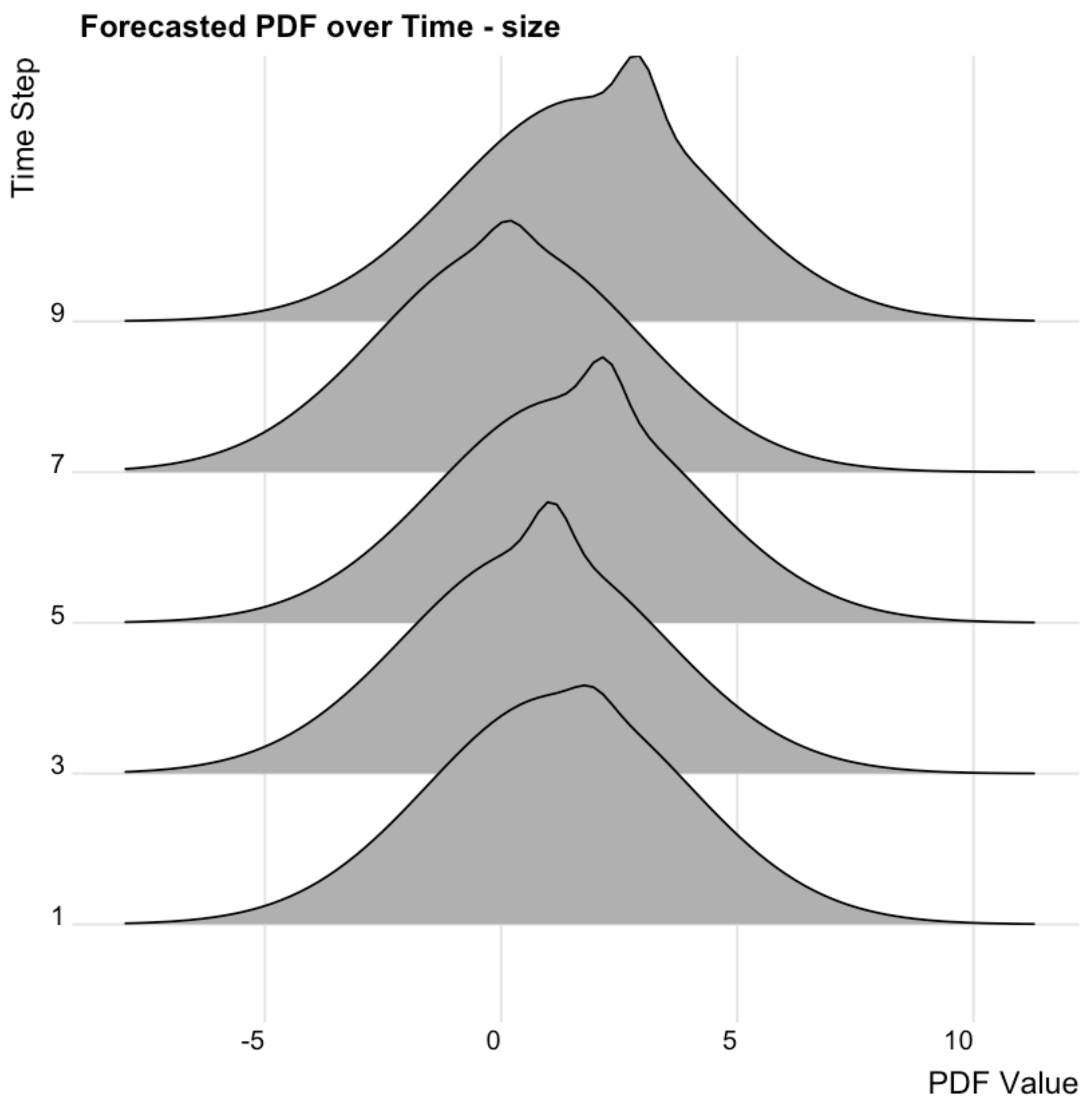
1.4.2 Plot the forecasted PDFs

Generate a ridge plot to show the probability distribution for each time step. The plot shows every second time step to help declutter the plot.

[13]:
```
n_steps <- nrow(forecasts)
plot_steps <- seq(1, n_steps, by = 2)

for (dimension in emission_vars) {
  # Prepare data for ridge plot
  ridge_data <- data.frame(
    x = rep(
      attr(forecasts, paste0(dimension, "_x_vals")),
      times = length(plot_steps)
    ),
    y = as.vector(attr(forecasts, paste0(dimension, "_pdfs"))[ , plot_steps]),
    time = factor(rep(plot_steps, each = length(attr(forecasts,␣
 ↪paste0(dimension, "_x_vals")))))
  )

  print(ggplot(ridge_data, aes(x = x, y = time, height = y, group = time)) +
```

```
    geom_density_ridges(stat = "identity", position = "identity") +
    labs(x = "PDF Value", y = "Time Step", title = paste("Forecasted PDF over␣
↪Time -", dimension)) +
    theme_ridges() +
    theme(axis.text.x = element_text(angle = 0, hjust = 1)))
}
```





1.5 Validate

Create 1000 simulations of the true model using the ending state and the same covariates used in forecasting.

[ ]:
```
n_dimensions <- length(emission_vars)
n_simulations <- 1000
n_steps <- nrow(forecasts)

# Get last hidden state
true_last_state <- tail(attr(true_mod$obs()$data(), "state"), 1) # last state
# get TPM of last observation
last_tpm <- true_mod$hid()$tpm(nrow(mod$obs()$data()))[, , 1]
```

```r
# Update delta0 to use distribution of first forecasted state
# TODO: Allow for first distribution to be stationary or custom
true_mod$hid()$update_delta0(last_tpm[true_last_state, ])

# Initialize a named list of 2D arrays to store the simulation results
simulated_data <- list()
for (dimension in emission_vars) {
  simulated_data[[dimension]] <- array(NA, dim = c(n_simulations, n_steps))
}

# Simulate data and store it in the array
for (i in 1:n_simulations) {
  sim <- true_mod$simulate(n = n_steps, data = forecasts, silent = TRUE)

  for (dimension in emission_vars) {
    # Get the simulated data for the current dimension
    simulated_data[[dimension]][i, ] <- sim[[dimension]]
  }
}

# Optionally, print a subset of the simulated data
print(simulated_data[[1]][1:5, ]) # Print the first 5 simulations
```

```
     [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8] [,9] [,10]
[1,]    2    4    2   15    3    1    8    1    1     2
[2,]    2    1    0    9    3    1    4    4    4     5
[3,]    4    2    0   10    3    0    5    5    4     3
[4,]    1    1    2   11    2    1    6    2    4     7
[5,]    0    1    1    9    2    0    8    1    5     3
```

[ ]:
```r
# Convert simulated data to a histogram at each time step
n_dimensions <- length(emission_vars)
n_steps <- nrow(forecasts)

# Initialize a array to store histograms for each time step
simulated_pdfs <- list()

# Define bin width
bin_width <- x_vals[2] - x_vals[1]

# Define bin edges such that the x_vals are centered in the bins
bin_edges <- c(x_vals - bin_width / 2, max(x_vals) + bin_width / 2)

# loop through each dimension and time step
for (dimension in emission_vars) {
```

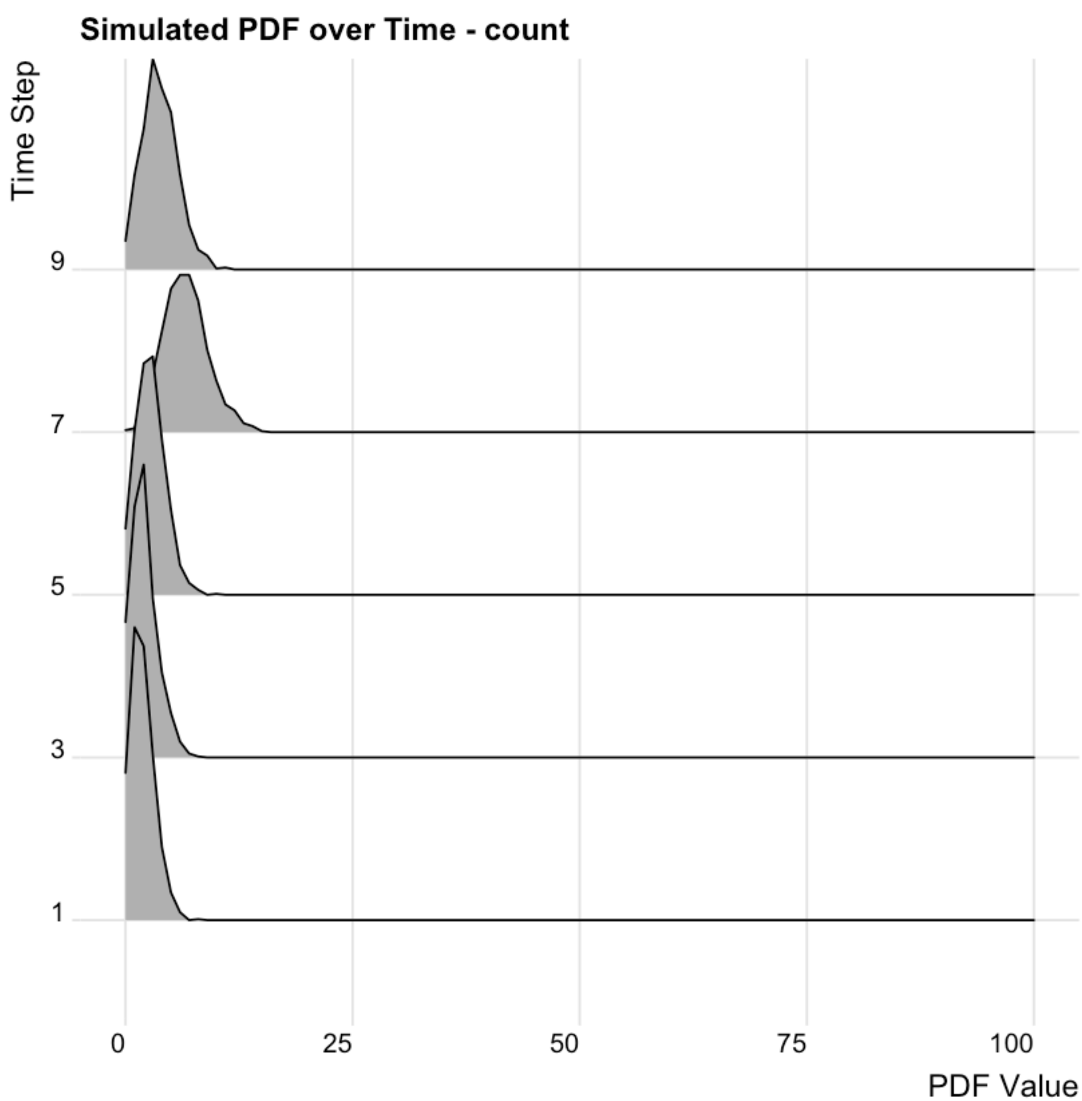
```r
  # Get locations to create histogram bins
  x_vals <- attr(forecasts, paste0(dimension, "_x_vals"))

  # Initialize a array to store histograms for each time step
  simulated_pdfs[[dimension]] <- array(NA, dim = c(length(x_vals), n_steps))

  # Define bin width
  bin_width <- x_vals[2] - x_vals[1]

  # Define bin edges such that the x_vals are centered in the bins
  bin_edges <- c(x_vals - bin_width / 2, max(x_vals) + bin_width / 2)

  for (t in 1:n_steps) {
    # Filter simulated data to be within the range of bin edges
    filtered_data <- simulated_data[[dimension]][, t]
    filtered_data <- filtered_data[
      filtered_data >= min(bin_edges) & filtered_data <= max(bin_edges)
    ]

    # Create histogram with specified bin edges
    hist_obj <- hist(filtered_data, plot = FALSE, breaks = bin_edges)

    hist_obj$density <- hist_obj$counts / sum(hist_obj$counts)

    simulated_pdfs[[dimension]][, t] <- hist_obj$density
  }

}

simulated_pdfs[[1]][1:5, 1:5]
```

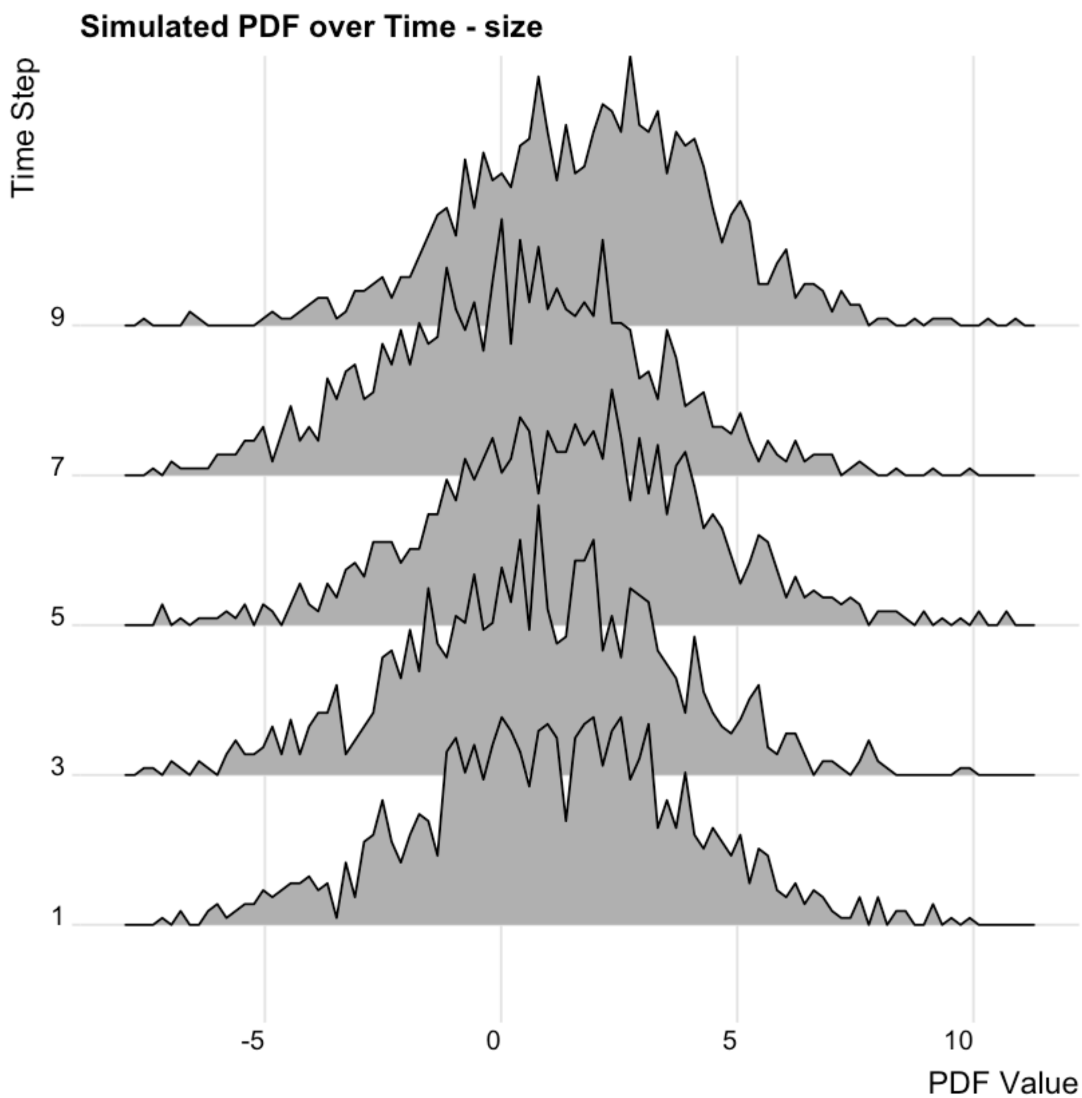
A matrix: 5 × 5 of type dbl

0.149	0.278	0.137	0.000	0.067
0.296	0.374	0.254	0.000	0.167
0.277	0.212	0.296	0.000	0.234
0.167	0.096	0.161	0.008	0.241
0.074	0.028	0.086	0.014	0.157

[16]:
```r
n_steps <- nrow(forecasts)
plot_steps <- seq(1, n_steps, by = 2)

for (dimension in emission_vars) {
  # Prepare data for ridge plot
  ridge_data <- data.frame(
    x = rep(
      attr(forecasts, paste0(dimension, "_x_vals")),
      times = length(plot_steps)
    ),
```

```
    y = as.vector(simulated_pdfs[[dimension]][, plot_steps]),
    time = factor(rep(plot_steps, each = length(attr(forecasts,␣
  ↪paste0(dimension, "_x_vals")))))
  )

  print(ggplot(ridge_data, aes(x = x, y = time, height = y, group = time)) +
    geom_density_ridges(stat = "identity", position = "identity") +
    labs(x = "PDF Value", y = "Time Step", title = paste("Simulated PDF over␣
  ↪Time -", dimension)) +
    theme_ridges() +
    theme(axis.text.x = element_text(angle = 0, hjust = 1)))
}
```

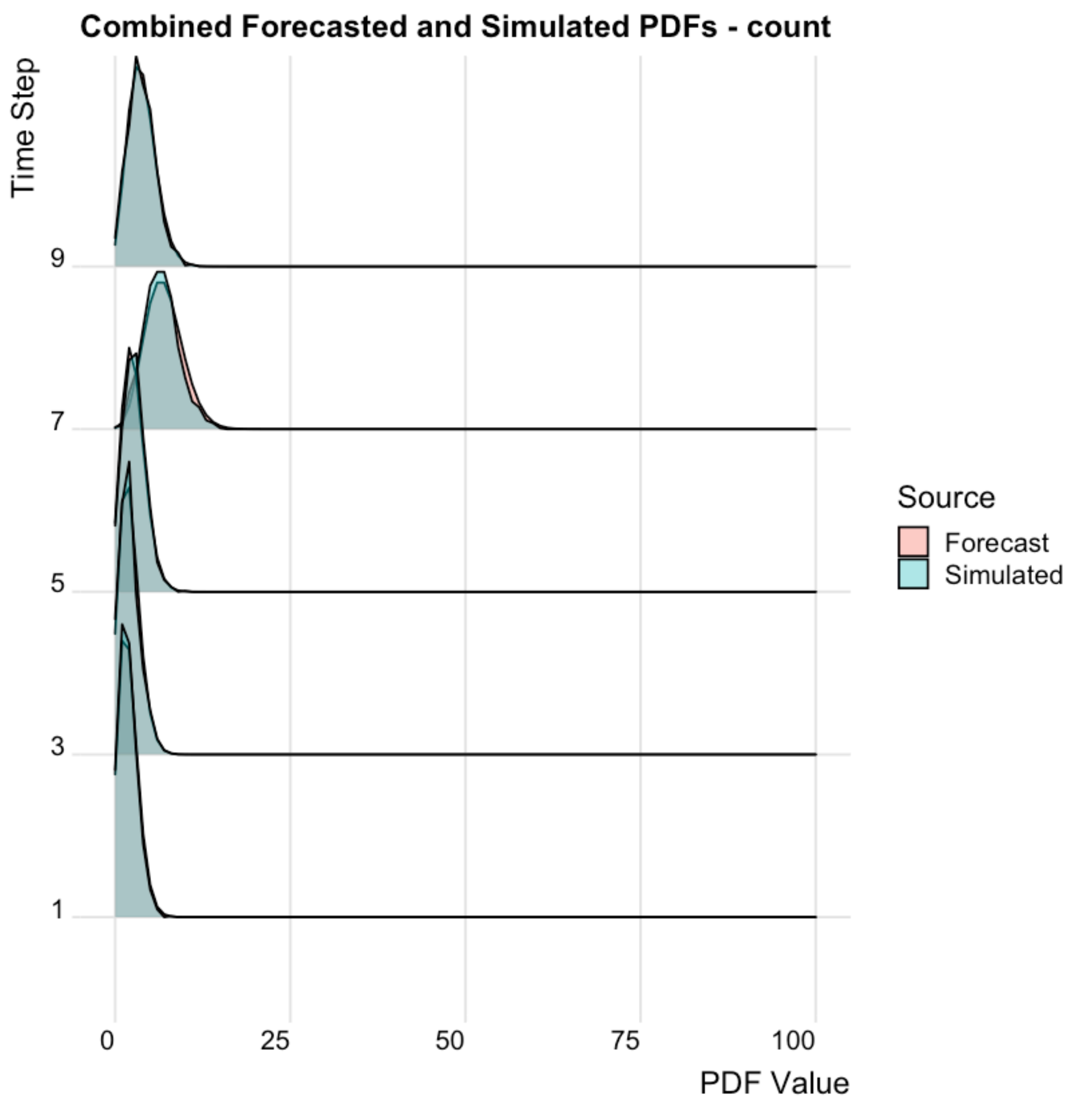




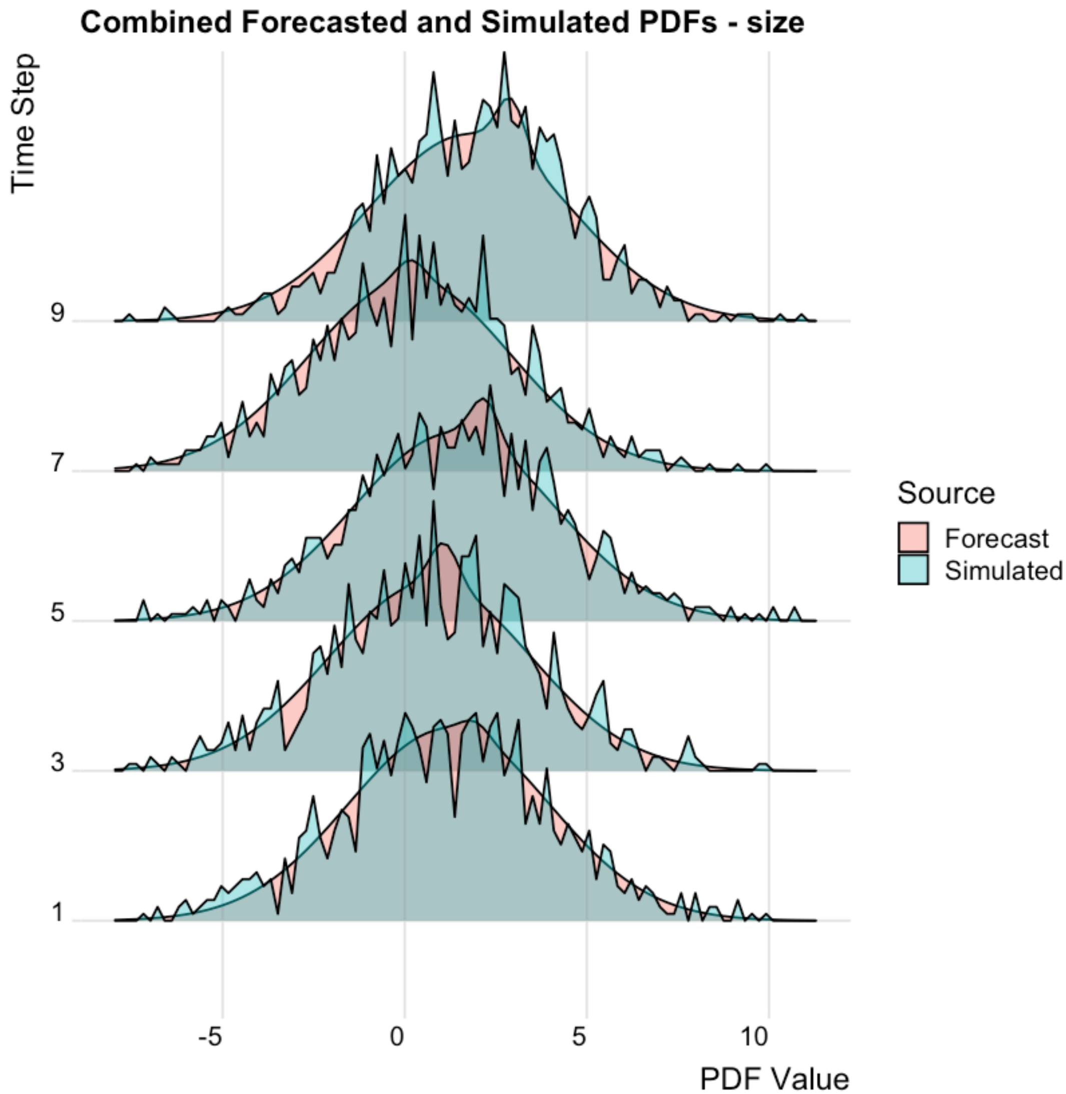
Compare forecasted PDFs with simulated PDFs

[17]:
```
# Define the number of steps and the dimension to plot
n_steps <- nrow(forecasts)
plot_steps <- seq(1, n_steps, by = 2)  # Plot every second step for clarity

for (dimension in emission_vars) {
  # Get forecasted PDFs and simulated PDFs
  x_vals <- attr(forecasts, paste0(dimension, "_x_vals"))
  forecast_pdfs <- attr(forecasts, paste0(dimension, "_pdfs"))
  simulated_pdfs_dim <- simulated_pdfs[[dimension]]

  # Prepare data for ridge plot
```

```r
  ridge_data <- data.frame(
    x = rep(x_vals, times = length(plot_steps) * 2),
    y = c(
      as.vector(forecast_pdfs[, plot_steps]),
      as.vector(simulated_pdfs_dim[, plot_steps])
    ),
    time = factor(rep(plot_steps, each = length(x_vals))),
    source = rep(c("Forecast", "Simulated"), each = length(x_vals) *␣
↪length(plot_steps))
  )

  # Create the ridge plot
  print(
    ggplot(ridge_data, aes(x = x, y = time, height = y, group =␣
↪interaction(time, source), fill = source)) +
      geom_density_ridges(stat = "identity", position = "identity", alpha = 0.
↪4) +
      labs(
        x = "PDF Value",
        y = "Time Step",
        title = paste("Combined Forecasted and Simulated PDFs -", dimension),
        fill = "Source"
      ) +
      theme_ridges() +
      theme(axis.text.x = element_text(angle = 0, hjust = 1))
  )
}
```



Combined Forecasted and Simulated PDFs - count
Time Step
9
7
5
3
1
0
25
50
75
100
PDF Value
Source
Forecast
Simulated



1.5.1 Appendix:

I originally had the idea of making the code more general, so instead of requiring the user to input the values of where evaluate the forecast PDF (i.e x_vals), it would instead return a PDF function for each forecasted time step, allowing the user to set arbitrary forecast precision. However I wasn't able to get the functions to work since the iterated varables (d, n) were always globally scoped so the PDFs never behaved properly. However I still think its a good method that I might try to revisit.

[18]:
```
create_pdf_forecasts <- function(obs_dists, obs_par_forecast,␣
↪hidden_state_forecast) {
  # Extract dimensions
```

```r
  dimensions <- length(obs_dists)              # Number of observation␣
↪dimensions
  states <- dim(obs_par_forecast)[2]       # Number of hidden states
  n_steps <- dim(obs_par_forecast)[3]       # Number of time steps

  # Extract PDF functions and parameter names from distribution objects
  pdfs <- lapply(obs_dists, function(dist) dist$pdf())

  # Initialize output: list of length dimensions, each containing a list of␣
↪length n_steps
  output <- vector("list", dimensions)

  # Loop over each observation dimension
  for (d in seq_len(dimensions)) {
    output[[d]] <- vector("list", n_steps)
    # Loop over each time step
    for (n in seq_len(n_steps)) {
      # Use local() to create a new environment for each iteration
      local({
        d_val <- d
        n_val <- n
        output[[d_val]][[n_val]] <- function(x) {
          pdf_matrix <- sapply(seq_len(states), function(s) {
            do.call(pdfs[[d_val]], c(list(x = x), as.
↪list(obs_par_forecast[d_val, s, n_val])))
          })
          as.vector(pdf_matrix %*% hidden_state_forecast[1, , n_val])
        }
      })
    }
  }

  return(output)
}

pdf_forecasts <- create_pdf_forecasts(obs_dists, obs_par_forecast,␣
↪hidden_state_forecast)
```

```
Error in dist$pdf: object of type 'closure' is not subsettable
Traceback:

1. lapply(obs_dists, function(dist) dist$pdf())
2. FUN(X[[i]], …)
```