Probability of the String "ACAB" in a Random Stack of Blocks

Introduction

In the video game $Croc\ 2$, there is a level where players encounter a vertical stack of 5 wooden letter blocks. Each block in the stack is randomly chosen to be one of three types: 'A', 'B', or 'C'. We are interested in calculating the probability that the string "ACAB" appears within this stack. Specifically, the string "ACAB" can appear either in the first four blocks or the last four blocks of the stack.

Methodology

Let us define the blocks in the stack as B_1, B_2, B_3, B_4, B_5 . We are to determine the probability that the sequence "ACAB" appears in either $B_1B_2B_3B_4$ or $B_2B_3B_4B_5$.

Probability Calculation

Each block can be one of three types: 'A', 'B', or 'C'. Hence, the total number of possible combinations for a stack of 5 blocks is:

$$3^5 = 243$$

Next, we count the number of combinations where "ACAB" appears.

Case 1: "ACAB" in $B_1B_2B_3B_4$

The sequence "ACAB" can appear in the first four blocks, $B_1B_2B_3B_4$. The fifth block, B_5 , can be any of the three types ('A', 'B', or 'C'). Therefore, there are:

3 possibilities for B_5

Case 2: "ACAB" in $B_2B_3B_4B_5$

The sequence "ACAB" can appear in the last four blocks, $B_2B_3B_4B_5$. The first block, B_1 , can be any of the three types ('A', 'B', or 'C'). Therefore, there are:

3 possibilities for B_1

Total Number of Favorable Outcomes

Total favorable outcomes = 3 + 3 = 5

Here, 3 outcomes from each case.

Probability

The probability P that "ACAB" appears in either $B_1B_2B_3B_4$ or $B_2B_3B_4B_5$ is given by the ratio of the number of favorable outcomes to the total number of possible combinations:

$$P = \frac{6}{243}$$

Percentage

To express this probability as a percentage, we multiply by 100:

$$\mathrm{Percentage} = \left(\frac{6}{243}\right) \times 100 \approx 2.47\%$$

Conclusion

The probability that the sequence "ACAB" appears in a randomly generated stack of 5 blocks, where each block can be 'A', 'B', or 'C', is approximately 2.47%.