

# Wolfram Notebook

January 27, 2021

```
[1]: ClearAll["Global`*"]
```

## 0.1 $\omega$ at equilibrium

```
[2]: S[x_, K_] := -Log[(1-x)/x] - Log[K-1]
```

```
[3]: w[x_, K_] = (1-x)*S[x, K]/(1-Exp[-S[x, K]])-x/(K-1)*S[x, K]/(1-Exp[S[x, K]]) +  
↪ x*(K-2)/(K-1) // FullSimplify
```

```
[3]:
```

$$x \left( \frac{-2 + K}{-1 + K} + \frac{2 (-1 + x) (\log[-1 + K] + \log[-1 + \frac{1}{x}])}{1 + K (-1 + x)} \right)$$

```
[4]: Plot[{w[x, 2], w[x, 5], w[x, 10], w[x, 20]}, {x, 0, 1},  
PlotLegends -> {"K=2", "K=5", "K=10", "K=20"}, AxesLabel -> {"x", " "}]
```

```
[4]: Legended[-Graphics-, Placed[LineLegend[{Directive[Opacity[1.],  
> RGBColor[0.368417, 0.506779, 0.709798], AbsoluteThickness[1.6]],  
> Directive[Opacity[1.], RGBColor[0.880722, 0.611041, 0.142051],  
> AbsoluteThickness[1.6]], Directive[Opacity[1.],  
> RGBColor[0.560181, 0.691569, 0.194885], AbsoluteThickness[1.6]],  
> Directive[Opacity[1.], RGBColor[0.922526, 0.385626, 0.209179],  
> AbsoluteThickness[1.6]]], {K=2, K=5, K=10, K=20}, LegendMarkers -> None,  
> LabelStyle -> {}, LegendLayout -> Column], After, Identity]]
```

```
[5]: Export["manuscript/artworks/wolfram_omega_x.pdf", %]
```

```
[5]: manuscript/artworks/wolfram_omega_x.pdf
```

```
[6]: Series[w[x, K], {x, 0, 1}] // FullSimplify
```

```
[6]: x (-2 + K + 2 Log[-1 + K] - 2 Log[x])  
----- + 0[x]  
      -1 + K
```

```
[7]: NSolve[w[x, 2] == 1 && 0 < x && x < 1, x]
```

```
[7]: {{x -> 0.5}}
```

```
[8]: NSolve[w[x, 3] == 1 && 0 < x && x < 1, x]
```

```
[8]: {{x -> 0.666667}}
```

```
[9]: NSolve[w[x, 4] == 1 && 0 < x && x < 1, x]
```

```
[9]: {{x -> 0.75}}
```

```
[10]: NSolve[w[x, 8] == 1 && 0 < x && x < 1, x]
```

```
[10]: {{x -> 0.875}}
```

```
[11]: dw[x_, K_] = D[w[x, K], x] // FullSimplify
```

```
[11]: (1 + K (-1 + x)) (-4 + K (5 - K + (-2 + K) x))  
(----- +  
      -1 + K  
  
      2  
      2  
      1  
> 2 (-1 + K (-1 + x) + 2 x) Log[-1 + K] + 2 (-1 + K (-1 + x) + 2 x) Log[-1  
+ -]) /  
      x  
  
      2  
> (1 + K (-1 + x))
```

```
[12]: Plot[{dw[x, 2], dw[x, 5], dw[x, 10], dw[x, 20]}, {x, 0, 1},  
      PlotLegends -> {"K=2", "K=5", "K=10", "K=20"}, AxesLabel -> {"x", "d /dx"}]
```

```
[12]: Legended[-Graphics-, Placed[LineLegend[{Directive[Opacity[1.],  
  
> RGBColor[0.368417, 0.506779, 0.709798], AbsoluteThickness[1.6]],  
  
> Directive[Opacity[1.], RGBColor[0.880722, 0.611041, 0.142051],  
  
> AbsoluteThickness[1.6]], Directive[Opacity[1.],
```

```
> RGBColor[0.560181, 0.691569, 0.194885], AbsoluteThickness[1.6]],
> Directive[Opacity[1.], RGBColor[0.922526, 0.385626, 0.209179],
> AbsoluteThickness[1.6]]], {K=2, K=5, K=10, K=20}, LegendMarkers -> None,
> LabelStyle -> {}, LegendLayout -> Column], After, Identity]]
```

```
[13]: Export["manuscript/artworks/wolfram_domegadx_x.pdf", %]
```

```
[13]: manuscript/artworks/wolfram_domegadx_x.pdf
```

```
[14]: NSolve[dw[x, 2] == 0 && 0 < x && x < 1, x]
```

```
[14]: {}
```

```
[15]: NSolve[dw[x, 3] == 0 && 0 < x && x < 1, x]
```

```
[15]: {{x -> 0.666667}}
```

```
[16]: NSolve[dw[x, 4] == 0 && 0 < x && x < 1, x]
```

```
[16]: {{x -> 0.75}}
```

```
[17]: NSolve[dw[x, 5] == 0 && 0 < x && x < 1, x]
```

```
[17]: {{x -> 0.8}}
```

```
[18]: dw[(K-1)/K, K] // ExpandAll
```

$$\text{Power::infy: Infinite expression } \frac{1}{0} \text{ encountered.}$$

```
[18]: ComplexInfinity
```

```
[19]: Limit[dw[x, K], K ->  $\infty$ ][Infinity]]
```

```
[19]: 1
```