# BAIRSTOW'S METHOD (SIR LEONARD BAIRSTOW, 1880–1963)

How to find roots of a polynomial $P(x)$ with real coefficients *without using complex arithmetic.*

**The idea:** search for a quadratic divisor of $P$. If the degree of the quotient $Q$ is greater or equal 2, reiterate with this procedure with $P = Q$.

*How to find a quadratic divisor of p?* Given real numbers $B$ and $C$, we have:

$$P(x) = (x^2 + Bx + C)\ Q(x) + (Rx + S),$$

This defines a mapping:

$$\mathcal{F} : \begin{pmatrix} B \\ C \end{pmatrix} \mapsto \begin{pmatrix} R \\ S \end{pmatrix}.$$

Just cancel $\mathcal{F}(B, C)$ using Newton's method.

**The beauty of this method.** How to compute the partial derivatives of $\mathcal{F}$? Bairstow's idea is nice and simple:

Once $P$ as been divided by $(x^2 + Bx + C)$, that is:

$$P(x) = (x^2 + Bx + C)\ Q(x) + (Rx + S),$$

just cancel the first variations of $P$. This gives for $C$:

$$0 = \frac{\partial P}{\partial C} = (x^2 + Bx + C)\ \frac{\partial Q}{\partial C} + Q(x) + x\frac{\partial R}{\partial C} + \frac{\partial S}{\partial C}.$$

Then, we just need to divide $-Q$ by the same quadratic $(x^2 + Bx + C)$ to get two of the partial derivatives of $R$ and $S$:

$$-Q(x) = (x^2 + Bx + C)\ \frac{\partial Q}{\partial C} + x\frac{\partial R}{\partial C} + \frac{\partial S}{\partial C}.$$

For $B$, we get the last two partial derivatives:

$$-xQ(x) = (x^2 + Bx + C)\ \frac{\partial Q}{\partial B} + x\frac{\partial R}{\partial B} + \frac{\partial S}{\partial B}.$$

That's all.