

BAIRSTOW'S METHOD (SIR LEONARD BAIRSTOW, 1880–1963)

How to find roots of a polynomial  $P(x)$  with real coefficients *without using complex arithmetic*.

**The idea:** search for a quadratic divisor of  $P$ . If the degree of the quotient  $Q$  is greater or equal 2, reiterate with this procedure with  $P = Q$ .

*How to find a quadratic divisor of  $p$ ?* Given real numbers  $B$  and  $C$ , we have:

$$P(x) = (x^2 + Bx + C) Q(x) + (Rx + S),$$

This defines a mapping:

$$\mathcal{F} : \begin{pmatrix} B \\ C \end{pmatrix} \mapsto \begin{pmatrix} R \\ S \end{pmatrix}.$$

Just cancel  $\mathcal{F}(B, C)$  using Newton's method.

**The beauty of this method:** how to compute the partial derivatives of  $\mathcal{F}$ ? The idea of Bairstow is nice and simple:

Once  $P$  has been divided by  $(x^2 + Bx + C)$ , that is:

$$P(x) = (x^2 + Bx + C) Q(x) + (Rx + S),$$

just cancel the first variations of  $P$ . This gives for  $C$ :

$$0 = \frac{\partial P}{\partial C} = (x^2 + Bx + C) \frac{\partial Q}{\partial C} + Q(x) + x \frac{\partial R}{\partial C} + \frac{\partial S}{\partial C}.$$

Then, we just need to divide  $-Q$  by the same quadratic  $(x^2 + Bx + C)$  to get the partial derivatives of  $R$  and  $S$ :

$$-Q(x) = (x^2 + Bx + C) \frac{\partial Q}{\partial C} + x \frac{\partial R}{\partial C} + \frac{\partial S}{\partial C}.$$

For  $B$ , we get:

$$-xQ(x) = (x^2 + Bx + C) \frac{\partial Q}{\partial B} + x \frac{\partial R}{\partial B} + \frac{\partial S}{\partial B}.$$

That's all.