Generating Normal Variates In Python

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Currently, the report which investigates the generation of exponential and normal observations in the programming languages is missing the implementation to replicate the normal variates from NumPy's normal function. I currently have a function that uses the same methods (I believe) as NumPy's normal function to generate a normal variate from the outputs of the generator. However, upon running my implementation the observations I find are different despite the generators being set in the same way. The algorithm which I have written I believe to be correct as running it a large number of times I see that the output resembles that of a normal distribution, so the discrepancy I think lies in the random aspect of the algorithm. The particular algorithm we are implementing is the Ziggurat method, where the random element comprises of generating a 64-bit integer from our generator and uses this to generate our normal observation. Therefore, I think the issue lies in the 64-bit integer I am using in the algorithm rather than the algorithm itself and I haven't found a way to check that this is really the case. Despite this below, I will give the coded implementation that I have achieved so far, with the intention that the underlying algorithm can still be analyzed despite it not being able to replicate the observations we see in practice.

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def python_next_64(state):
    integer_one, state = mt19937_next(state)
    integer_two, state = mt19937_next(state)
    return (integer_one<<32 | integer_two), state

def python_random_normal(state, mu=0, sigma=1, size=1):
    output = []
    def python_random_normal_variate(state):
        while True:
            r, state=python_next_64(state)
            idx = r & 0xff
            r >>= 8
            sign = r & 0x1
            rabs = (r >> 1) & 0x000fffffffffffff
            x = rabs * wi_double[idx]
```

```
            if (sign & 0x1):
                x = -x
            if (rabs < ki_double[idx]):
                return x, state
            if (idx==0):
                while True:
                    u1, state = python_random_uniform(state)
                    uniform_1 = u1[0]
                    xx = -0.2736612373297582779*np.log(1-uniform_1)
                    u2, state = python_random_uniform(state)
                    uniform_2=u2[0]
                    yy = -np.log(1-uniform_2)
                    if (yy+yy>xx*xx):
                        if (rabs>>8) & 0x1:
                            return -(3.654152885361008796+xx), state
                        else:
                            return 3.654152885361008796+xx, state
            else:
                u3, state = python_random_uniform(state)
                uniform_3=u3[0]
                if  (((fi_double[idx - 1] - fi_double[idx]) * uniform_3 +␣
 ↪fi_double[idx]) < np.exp(-0.5 * x * x)):
                    return x, state
    for n in range(size):
        n1, state = python_random_normal_variate(state)
        output.append(n1)
    return output, state
```

```
rng.seed(1)
print(rng.normal(size=1))
u2, state = python_random_normal(python_state_from_seed(1), size=50000)
print(u2[0])

import scipy.stats as st

plt.hist(u2, bins=50, density=True)

x = np.linspace(-4,4,1000)
y = st.norm.pdf(x)
plt.plot(x,y)
plt.title('Histogram plot generated random normal variate function, with 50000␣
 ↪observations')
plt.show()
```

```
[1.62434536]
0.8705532693399859
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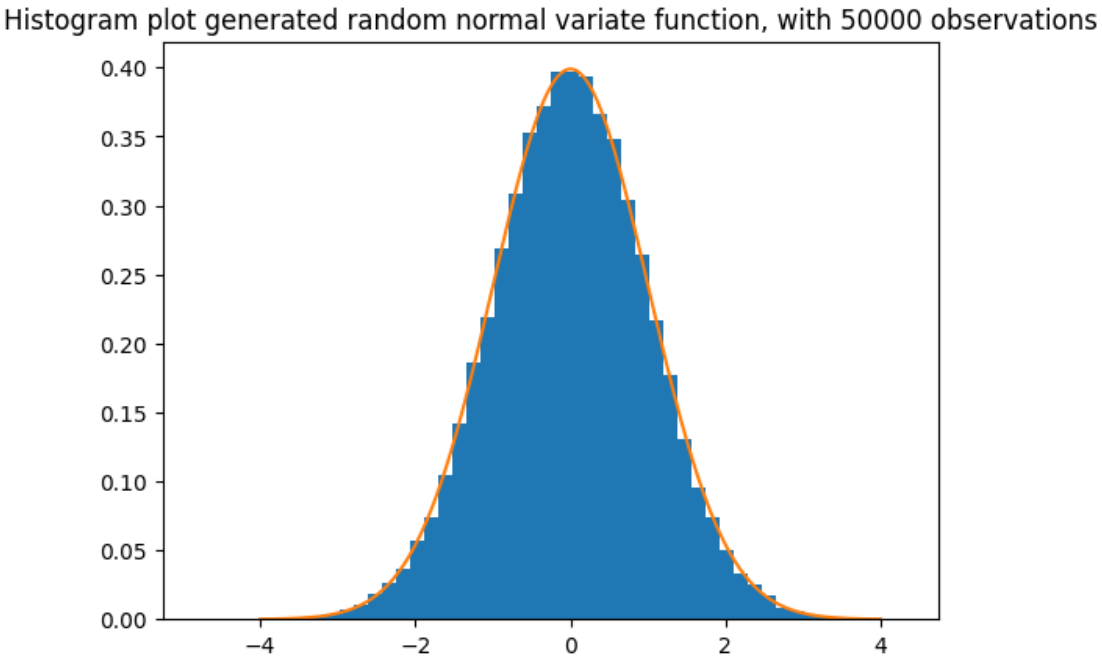
Histogram plot generated random normal variate function, with 50000 observations
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0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
−4
−2
0
2
4