

## **Lecture 7**

# **Investigating Motion : Computational Approach**

# Realistic motion :

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- ODE
- Initial condition given
- Euler's method; Finite Difference

## Bicycle Racing (goal: to understand what determines speed)

Case 1: simple case without friction

Write down the equation of motion. (Law?)

# Newton's second Law of motion

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$$dv/dt = F/m$$

$F \rightarrow$  force that comes from the effort of the rider

Accurate expression for force??

Alternate approach?

# Alternate approach

In terms of power.

Power output over a period of time !!

Write down the previous eqn. in terms of power.

$P \rightarrow$  power output of the rider. ( $\sim 400$  watts over  $\sim 1$  hour)

# Eqns.

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- $dE/dt=P$  ;  $E \rightarrow$  total energy of the combination
- $E=1/2 (mv^2)$
- $dE/dt=mv(dv/dt)$
- $dv/dt=P/mv$
- **If  $P$  is constant, write down the Finite Difference form**

# Analytical soln.

- If  $P$  is constant, what is the solution

$$\int_{v_0}^v v' dv' = \int_0^t \frac{P}{m} dt'$$

$$v = \sqrt{v_0^2 + 2 P t / m}$$

FD form  $\rightarrow$

$$v_{i+1} = v_i + \frac{P}{m v_i} \Delta t$$

# Write MATLAB program for the bicycle problem

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Recall the algorithm for growth or decay problem.

initialize *simulationLength*

initialize *number\_atoms*

initialize *decay-Rate*

initialize length of time step  $\Delta t$

$Num\_of\_Iterations \leftarrow simulationLength / \Delta t$

**for  $i$  going from 1 through *num\_of\_iterations***

do the following:

$decay \leftarrow decay-Rate * number\_atoms$

$number\_atoms \leftarrow number\_atoms (+/-) decay * \Delta t$

$t \leftarrow i * \Delta t$

display  $t$ ,  $decay$ , and *number\_atoms*

# Result of computation

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Take reasonable initial conditions and run the program for different values of time-step.

- Velocity grows indefinitely !!
- Some mechanism of energy loss needs to be included.

Main loss mechanism → atmospheric drag !! How to include.

Physics of air resistance – complex !!

Modify the code and include the drag term !!



# Realistic model.

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In general; **Drag force**  $\sim -C_1v - C_2v^2$

Second term dominates at reasonable velocities; how to approximate  $C_2$ .

As objects moves, it push the air in front of it.

The mass of air moved in time  $dt$  is  $\sim ??$

This air is given a velocity  $v$ , therefore its kinetic energy is  $(1/2)m_{\text{air}}v^2$

This is the work done by the drag force in time  $dt$ .

# Realistic model

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Drag-force  $\times v \times dt = \text{KE air}$

$m_{\text{air}} \sim \text{density of air} \times [\text{frontal area of object} \times (v \times dt)]$

$$F_{\text{drag}} \approx -C\rho A v^2$$

C- drag coefficient; reasonable estimate in this case .5

$A \sim .3 \text{ m}^2$

New FD eqn.  $\rightarrow$

$$v_{i+1} = v_i + \frac{P}{m v_i} \Delta t - \frac{C\rho A v_i^2}{m} \Delta t$$

# Question

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**Why bicycle racers move in a group??**

Can you explain your answer using the code with more **investigations (numerical experiments) !!**



# Use of aerobars !! How it helps

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Speed with and without aerobars (approximate calculations)