Lecture-20

Recap:

Std. Normal Distribution

Exponential distribution

Normal approximation to Binomial.

n is large
σ is high } DeMoivre- Laplace ($p = 1/2$)

A coin is tossed 40 times.

X = no. of heads., $p = 1/2$

$$P(X = 20) = \binom{40}{20}\left(\frac{1}{2}\right)^{20}\left(\frac{1}{2}\right)^{20}$$

$$= \frac{40!}{20!\,20!\,2^{40}} = 0.1254$$

Consider this a Normal distribution.

$$\mu = np = 40 \cdot \frac{1}{2} = 20$$

$$\sigma = \sqrt{np(1-p)}$$

$$= \sqrt{40 \cdot \frac{1}{2} \cdot \frac{1}{2}} = \sqrt{10}$$

$P(X = 20) = 0$

continuity correction

$$P(19.5 \leq x \leq 20.5) \quad \begin{array}{l} \mu = 20 \\ \sigma = \sqrt{10} \end{array}$$

$$= P\left(\frac{19.5 - 20}{\sqrt{10}} \leq Y \leq \frac{20.5 - 20}{\sqrt{10}}\right)$$

$$= P(-0.158 \leq Y \leq 0.158)$$

3

$= \phi(0.158) - \phi(-0.158)$

$= \phi(0.158) - [1 - \phi(0.158)]$

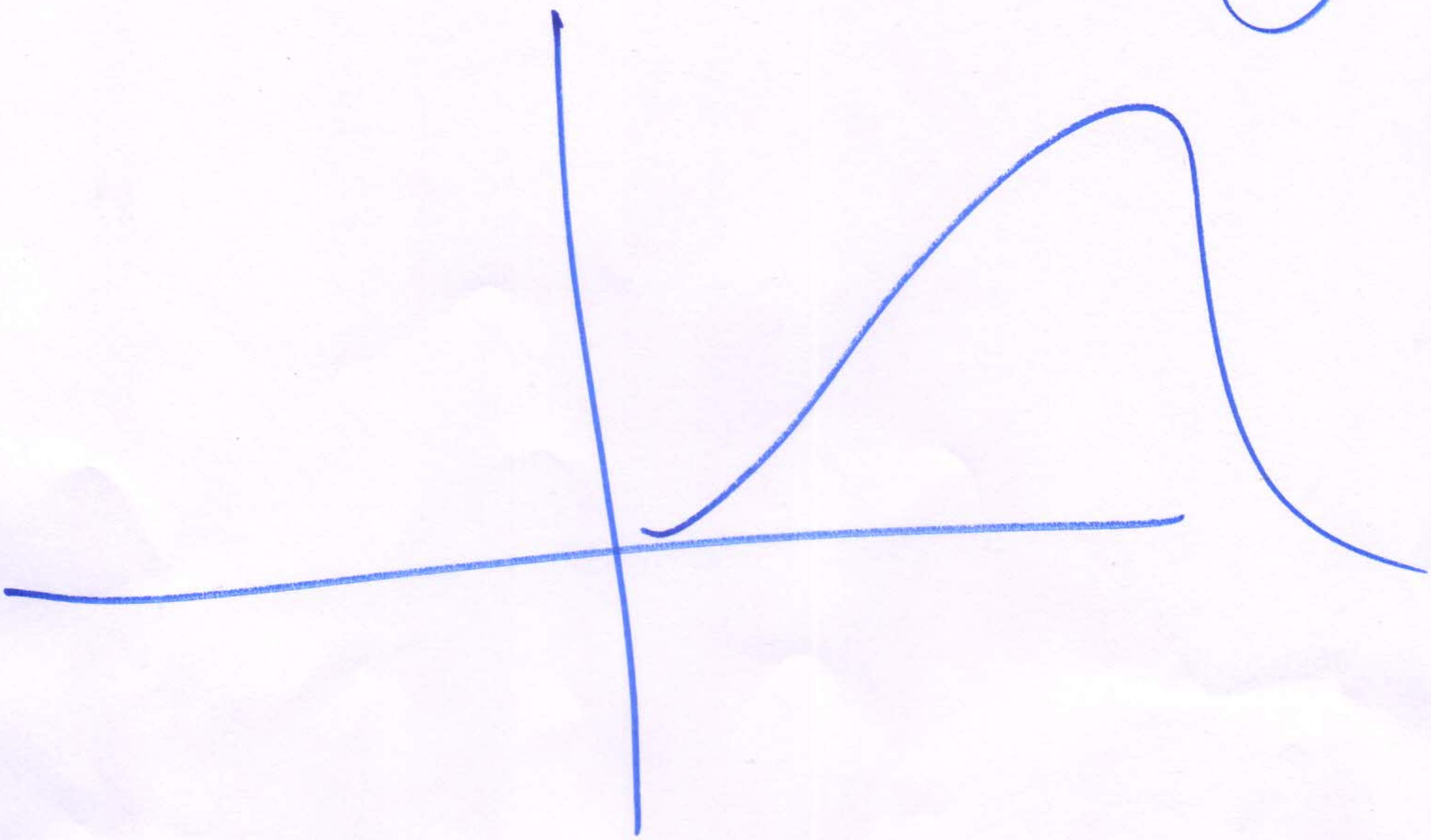
$= 2\phi(0.158) - 1$

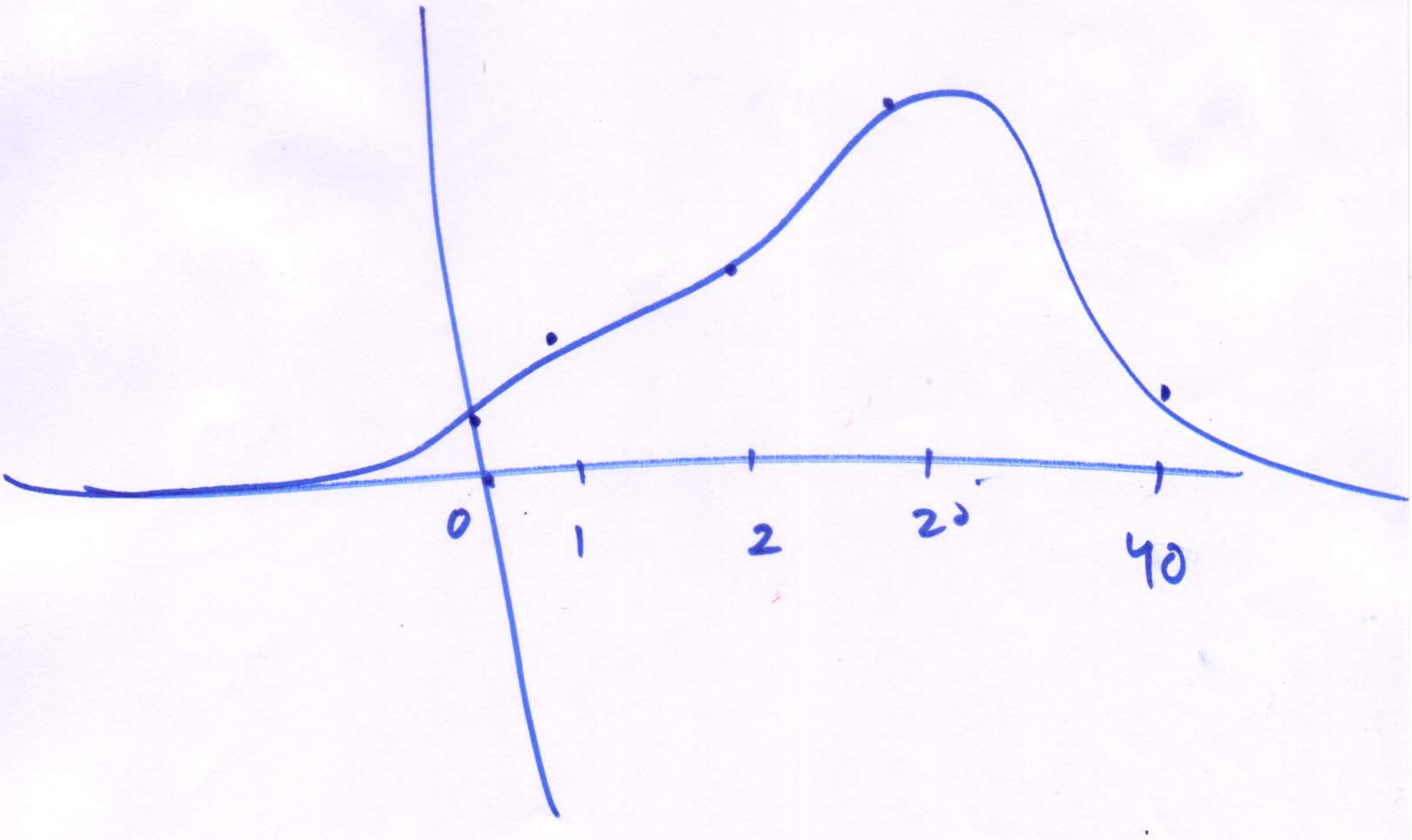
$= 2 * 0.5636 - 1$

$= 1.1272 - 1$

$= 0.1272$ (0.1254)

Normal Binomial




0
1
2
20
40

$\textcircled{5}$

$P(19.8 \leq x \leq 20.8)$

$P\left(\frac{19.8-20}{\sqrt{10}} \leq x \leq \frac{20.8-20}{\sqrt{10}}\right)$

$P\left(\frac{-0.2}{\sqrt{10}} \leq x \leq \frac{0.8}{\sqrt{10}}\right)$

$P(-0.063) \leq x \leq 0.253)$

$= \phi(0.253) - \phi(-0.063)$

$= \phi(0.253) - [1 - \phi(0.063)]$

$= \phi(0.253) + \phi(0.063) - 1$

$= 0.5987 + 0.5239 - 1$

$= 0.1226$

Exponential random variables.

$\lambda > 0$

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & , x \geq 0 \\ 0 & , x < 0 \end{cases}$$

$\int_0^\infty f(x)\,dx = 1$

$F(a) = P(X \leq a)$

$$= \int_0^a \lambda \cdot e^{-\lambda x} \cdot dx = 1 - e^{-\lambda a}$$

$$E[X] = \int_{-\infty}^{\infty} x \cdot f(x) \cdot dx = 1/\lambda$$

$$Var(X) = E[X^2] - (E[X])^2 = \frac{1}{\lambda^2}$$

H.W.

e.g. Suppose that the length of a phone call, in minutes, is an exponential r.v. with $\lambda = \frac{1}{10}$.

Someone arrives at a P.C.O just before you. What is the probability that you need to wait for more than 10 minutes?