Lecture - 05

Recap:

what is the probability of you having an accident in the next one year?

E.g. A student taken a MCQ test. The question has 4 choices.

p(knows correct answer) = 0.8

p(don't know) = 0.2

↓ guess

You answered the question correctly. B

What is the probability that you knew the correct answer? A

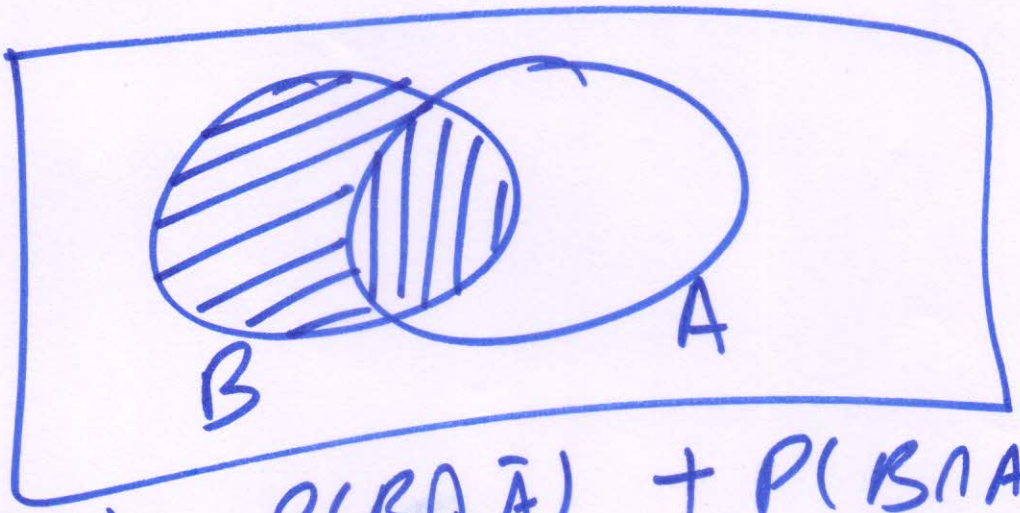
its not a guess?

A: you knew the correct answer

B: you answered correctly.

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(B|A)P(A)}{P(B)}$$

?



$$P(B) = P(B \cap \bar{A}) + P(B \cap A)$$

(by axiom 3)

$$= P(B|\bar{A})P(\bar{A}) + P(B|A)P(A)$$

$$= 0.25 * 0.2 + 1 * 0.8$$

A, B, C are mutually exclusive and exhaustive.

(3)

(A, $\bar{A}$)

$A \cap B = \phi$

$B \cap C = \phi$

$C \cap A = \phi$

$A \cup B \cup C = S$

A, B, C form a partition.

If this is True, then you can use Bayes theorem.

E

$$P(A|E) = \frac{P(A \cap E)}{P(E)}$$

$$= \frac{P(E|A)\,P(A)}{P(E|A)\,P(A) + P(E|B)\,P(B) + P(E|C)\,P(C)}$$

↓

Bayes theorem

e.g.

Three shops in your neighborhood.

Shop	P(good bulb)
A	0.8
B	0.4
C	0.1

You buy a bulb from one of these shops at random.

The bulb is defective.

P(it is from shop A) | P(A|E)

E: the bulb is defective.

A: bulb is from shop A

B:

C:

$$P(A|E) = \frac{P(A \cap E)}{P(E)}$$

(5)

$$\frac{P(E|A)\,P(A)}{P(E|A)\,P(A) + P(E|B)\,P(B) + P(E|C)\,P(C)}$$

$$= \frac{0.2 * 1/3}{0.2 * 1/3 + 0.6 * 1/3 + 0.9 * 1/3}$$

eg: A witness sees a green-color taxi run away from an accident. He correctly identifies colors 80% of the time.

100

90% - Blue
10% - Green

What is the probability that the taxi was indeed of Green color?

A: taxi is of Green color.
B: witness says it is a Green color taxi.

$P(A|B) =$

(7)

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{P(B|A)\,P(A)}{P(B|A)\,P(A) + P(B|\bar{A})\,P(\bar{A})}$$

$$= \frac{0.8 * 0.1}{0.8 * 0.1 + 0.2 * 0.9}$$

$$= \frac{8}{8 + 18} = \frac{8}{26}$$

$$= \frac{4}{13} \leq 0.31$$

e.g.

You have been identified as a terrorist by the image recognition s/w. 99% of the time their s/w correctly identifies

$P($ you are a terrorist $\mid$ s/w says that you are a terrorist $)$

(A) (B)

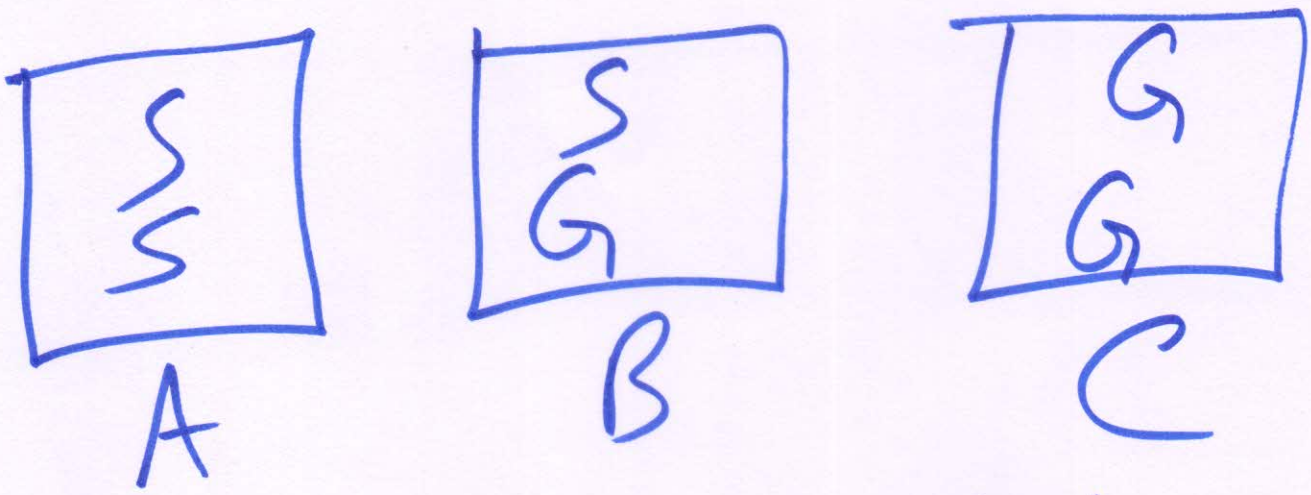
$P($ a random person is a terrorist $) = \frac{0.01\ (r}{700(r}$

$= \frac{1}{70000} = P(A)$

$$P(A|B) = \frac{P(B|A)\,P(A)}{P(B|A)\,P(A) + P(B|\bar{A})\,P(\bar{A})}$$

$P(B|A) = \frac{99}{100}$, $P(A) = \frac{1}{70k}$, $P(B|\bar{A}) = \frac{1}{100}$, $P(\bar{A}) = \frac{69999}{70000}$

e.g.



Bertrand's Box Problem

i) you randomly choose a box

ii) you take out a coin

iii) its a Gold coin.

iv) What is the probability that the other coin in the same box is also a Goldcoin?

E: its a Goldcoin	A: you chose box A B: " B C: " C P(C | E)

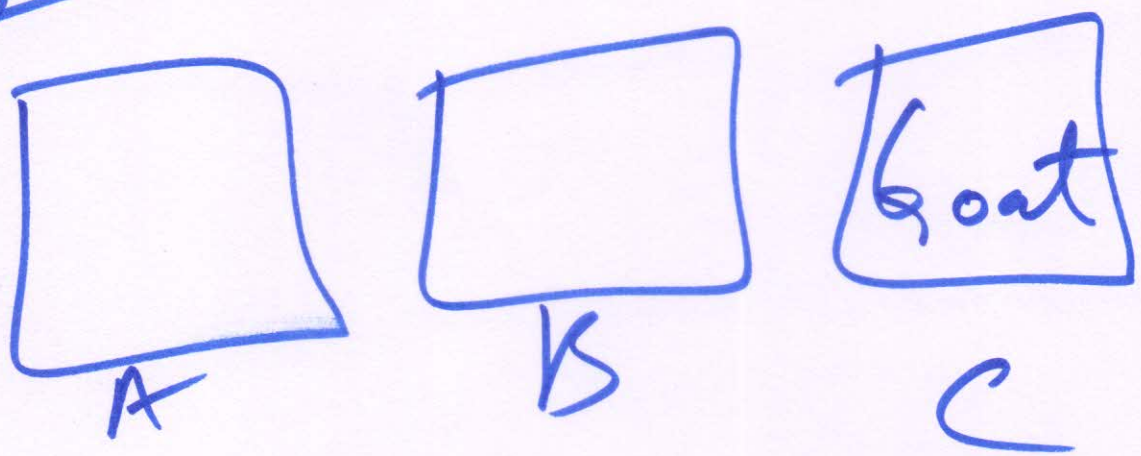
$$P(C|E) = \frac{P(C \cap E)}{P(E)}$$

$$= \frac{P(E|C)\,P(C)}{P(E|A)\,P(A) + P(E|B)\,P(B) + P(E|C)\,P(C)}$$

$$= \frac{1 * \frac{1}{3}}{0 * \frac{1}{3} + \frac{1}{2} * \frac{1}{3} + 1 * \frac{1}{3}}$$

$$= \frac{1}{\frac{1}{2} + 1} = \frac{2}{3}$$

e.g.



↑
(hose
this

↑
host opens
this

(A). car is behind door A

B': ↗ ↖ B

C': C

(E): host opens door C
to reveal a goat

P(A|E) (N.W.)