AC analysis - many examples

The basic method for AC analysis:

1. Represent the AC sources as complex numbers:

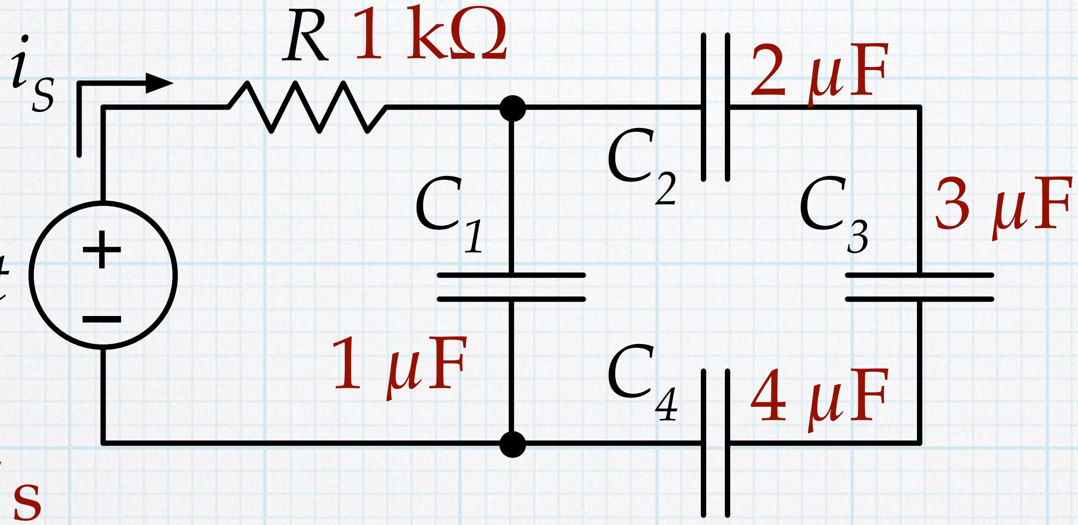
$$V_s(t) \rightarrow \tilde{V}_S = V_m e^{j0^\circ} = V_m$$

2. Convert resistors, capacitors, and inductors into their respective impedances: resistor → $Z_R = R$; capacitor → $Z_C = 1/(j\omega C)$; inductor → $Z_L = j\omega L$.
3. Re-draw the circuit using the complex sources and impedances.
4. Use your favorite method to find expressions for the complex currents and voltages (phasors) in terms of the sources and impedances
5. Do whatever complex math is needed. (This is the longest part!)
6. Express the answer in magnitude/phase form (usually).
7. If needed, re-express the voltage and currents as sinusoids. (Often un-necessary.)

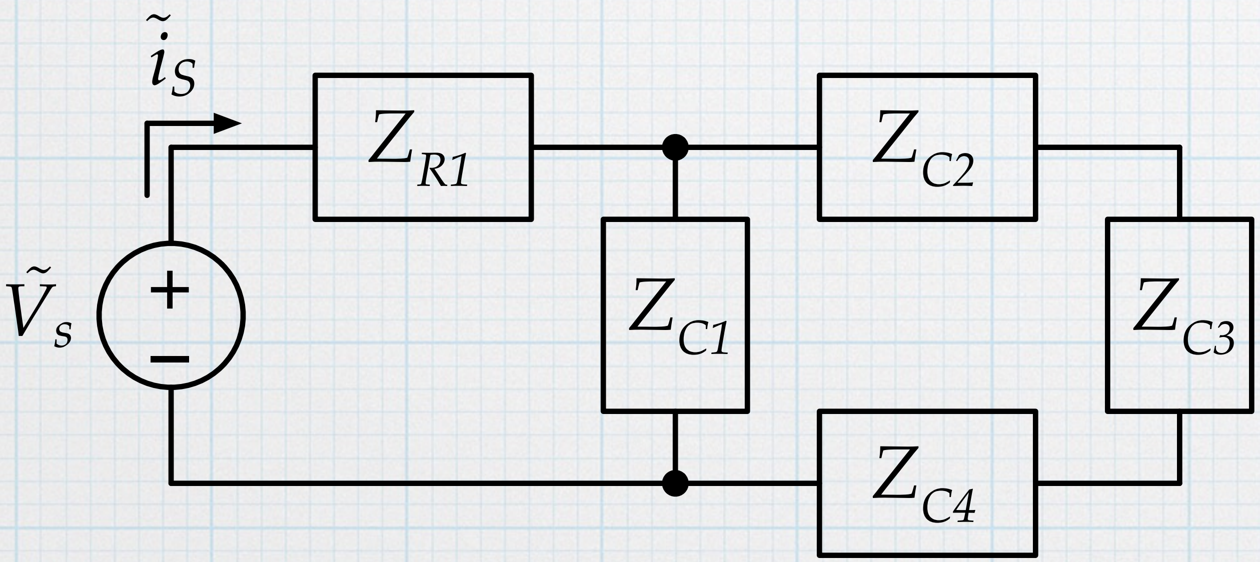
Example - equivalent impedances.

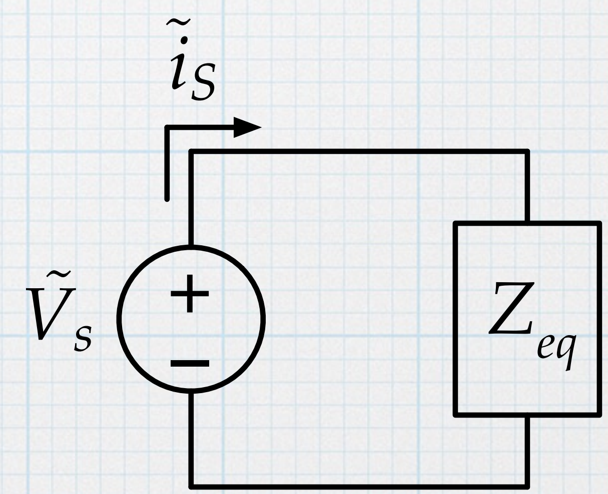
Impedances can be combined and reduced just like resistors from our earlier work.

$$V_S(t) = V_m \cos \omega t$$



$$V_m = 10 \text{ V}$$
$$\omega = 1000 \text{ rad/s}$$





$$Z_{eq} = Z_{R1} + Z_{C1} \| (Z_{C1} + Z_{C2} + Z_{C3})$$

$$\tilde{i}_S = \frac{\tilde{V}_S}{Z_{eq}}$$

Oftentimes, it is useful to carry through with symbols for as long as possible, but in this case we may as well go to numbers now.

$$Z_R = R = 1000\ \Omega$$

$$Z_{C1} = \frac{1}{j\omega C_1} = \frac{-j}{\omega C_1} = \frac{-j}{(1000\ \mathrm{rad/s})\,(10^{-6}\ \mathrm{F})} = -j1000\ \Omega$$

$$Z_{C2} = \frac{1}{j\omega C_2} = -j500\ \Omega \qquad Z_{C3} = \frac{1}{j\omega C_3} = -j333\ \Omega \qquad Z_{C4} = \frac{1}{j\omega C_4} = -j250\ \Omega$$

$$Z_{234} = Z_{C2} + Z_{C3} + Z_{C4} = (-j250\ \Omega) + (-j250\ \Omega) + (-j250\ \Omega) = -j1083\ \Omega$$

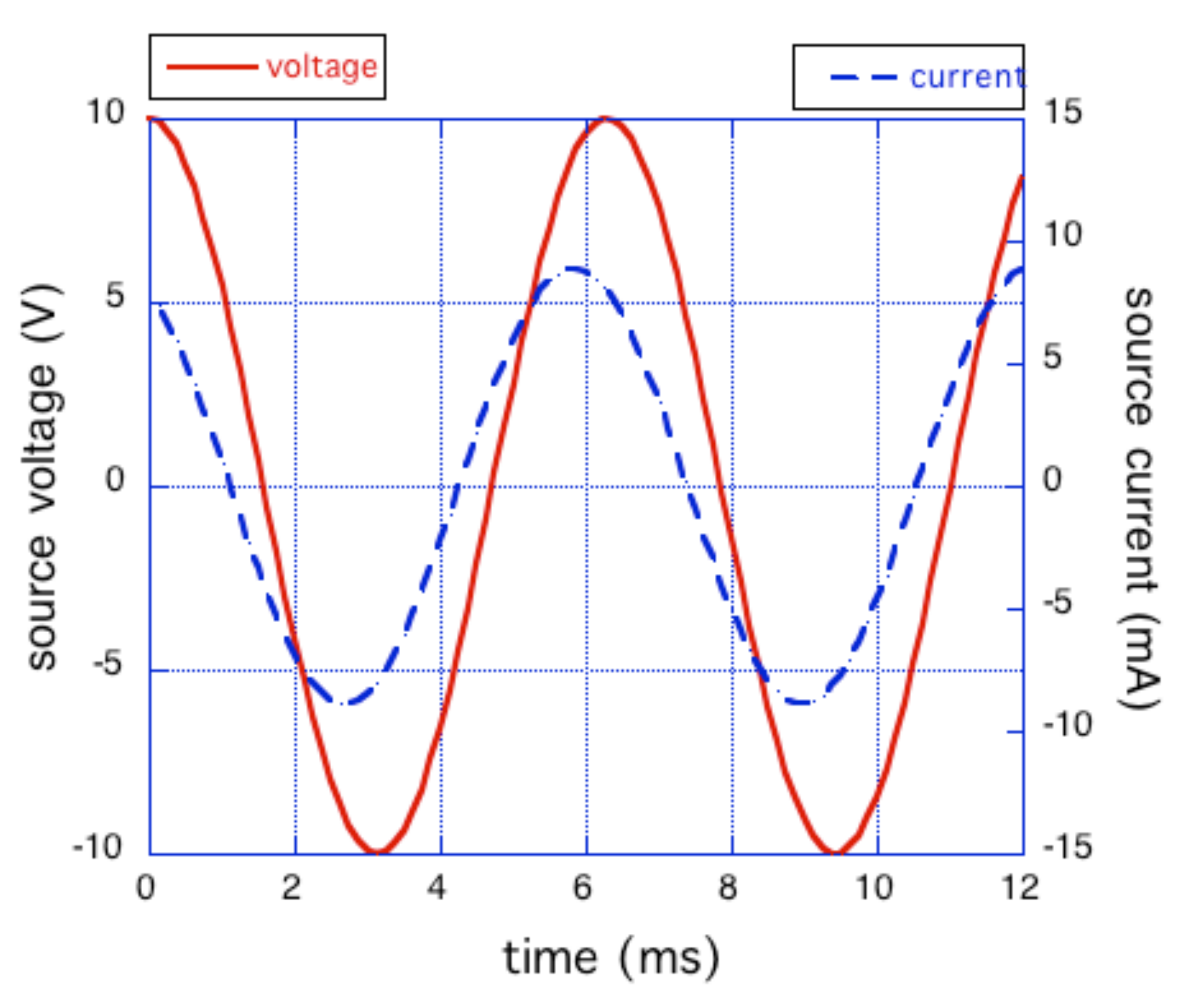
$$Z_{C1} \| Z_{234} = \left(\frac{1}{-j1000\ \Omega} + \frac{1}{-j1083\ \Omega} \right)^{-1} = -j520\ \Omega$$

$$Z_{eq} = Z_R + Z_{C1} \| Z_{234} = 1000\ \Omega - j520\ \Omega$$

$$Z_{eq} = (1127\ \Omega) \exp(-j27.5^\circ)$$

$$\tilde{i}_S = \frac{\tilde{V}_S}{Z_{eq}} = \frac{10\ \mathrm{V}}{(1127\ \Omega) \exp(-j27.5^\circ)} = (8.87\ \mathrm{mA}) \exp(+j27.5^\circ)$$

The complex current tells us that the current sinusoid has an amplitude of 8.87 mA with a phase of 26.5° relative to the source.



$$i_S(t) = I_m \cos(\omega t + \theta_i)$$

$$I_m = 8.87 \text{ mA}$$

$$\theta_i = 26.5°$$

With this circuit, we might find the equivalent capacitance of the small network capacitors *before* switching over to impedance. In finding equivalent capacitance, we must us the rules for combining capacitors. (As we recall, series capacitors add like resistors in parallel, and parallel capacitors add like resistors in series.)

$$C_{eq} = C_1 + \left(\frac{1}{C_2} + \frac{1}{C_3} + \frac{1}{C_4} \right)^{-1} = 1.92\ \mu\text{F}$$

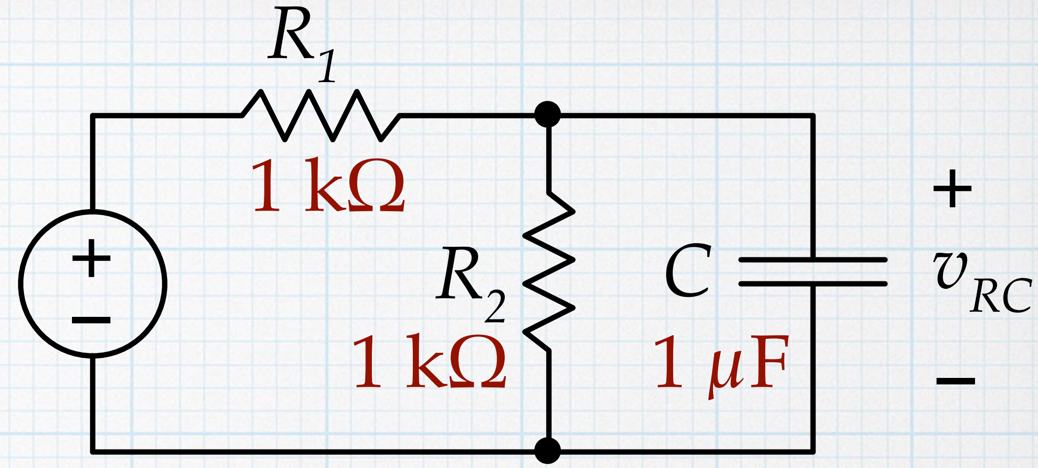
Then $Z_{eq} = R + \dfrac{1}{j\omega C_{eq}} = 1000\ \Omega - j520\ \Omega$ same as the other way

Example - voltage divider

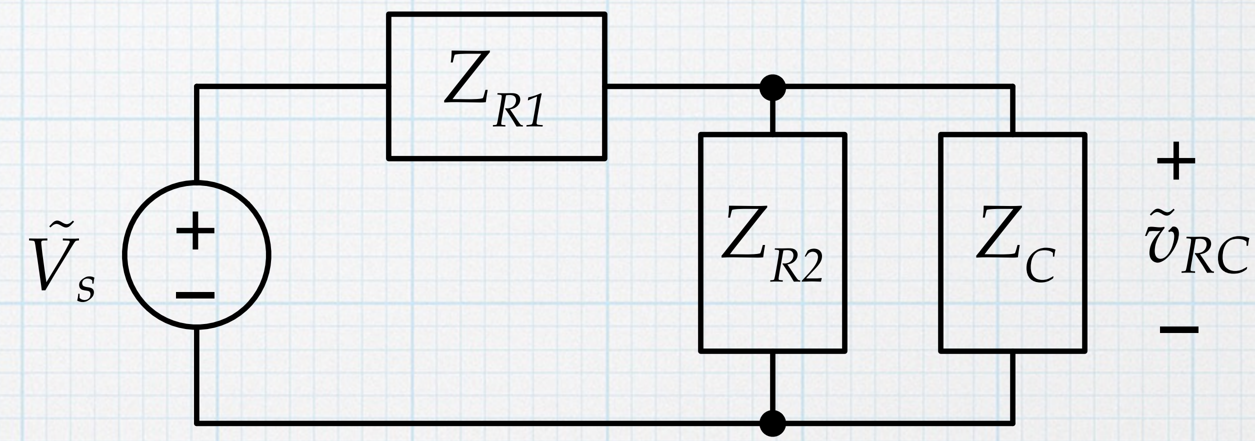
For the circuit at right, find v_{RC} for sinusoidal frequencies of 200, 2,000, and 20,000 rad/s.

$V_S(t) = V_m \cos \omega t$

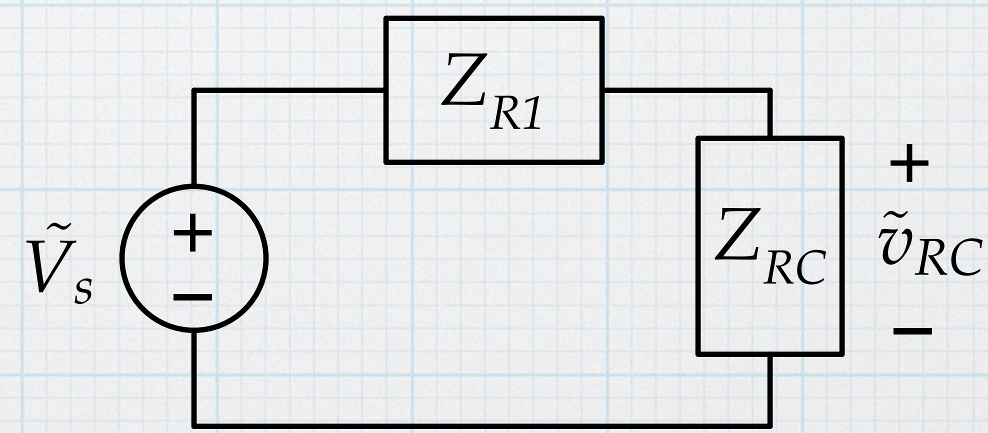
$V_m = 5$ V



Make the complex version of the circuit.

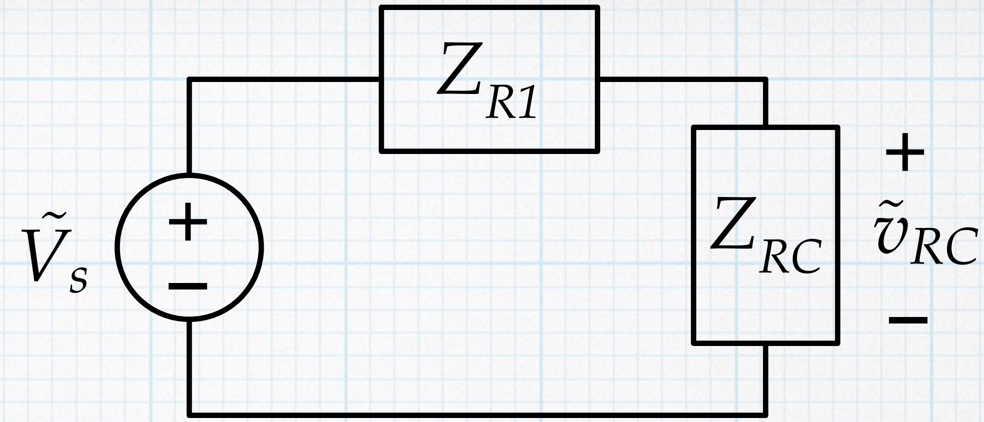


Combine the parallel combination into a single impedance.



$$Z_{RC} = Z_{R2} \| Z_C = \frac{(R_2)\left(\frac{1}{j\omega C}\right)}{R_2 + \frac{1}{j\omega C}} = \frac{R_2}{1 + j\omega R_2 C}$$

Now use a voltage divider:



$$\tilde{v}_{RC} = \frac{Z_{RC}}{Z_{RC} + Z_{R1}} \tilde{V}_S \quad = \frac{\frac{R_2}{1+j\omega R_2 C}}{\frac{R_2}{1+j\omega R_2 C} + R_1} V_m \quad = \frac{R_2}{R_1 + R_2 + j\omega R_1 R_2 C} V_m$$

In magnitude and phase form:

$$\tilde{v}_{RC} = \left[\frac{R_2 V_m}{\sqrt{(R_1 + R_2)^2 + (\omega R_1 R_2 C)^2}} \right] \exp(j\theta)$$

$$\theta = -\arctan\left(\frac{\omega R_1 R_2 C}{R_1 + R_2} \right)$$

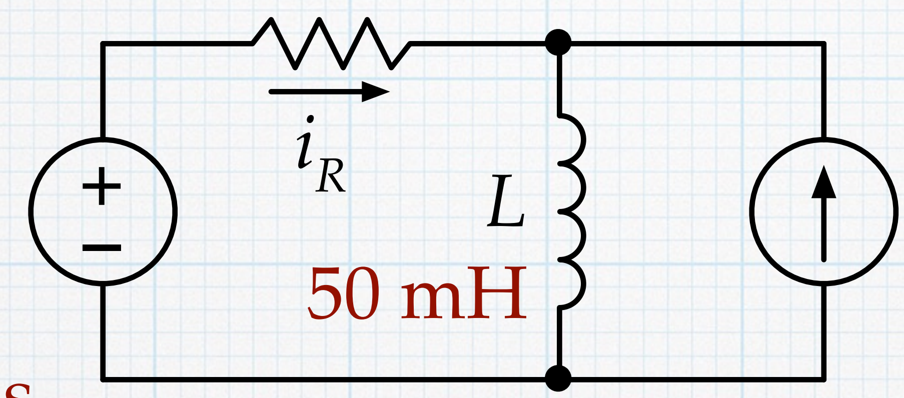
at ω = 200 rad/s: $\tilde{v}_{RC} = (2.49 \text{ V}) \exp(-j5.7^\circ)$ → $v_{RC}(t)$ = (2.49 V)cos(ωt – 5.7°)

ω = 2,000 rad/s: $\tilde{v}_{RC} = (1.77 \text{ V}) \exp(-j45.0^\circ)$ etc.

ω = 20,000 rad/s: $\tilde{v}_{RC} = (0.249 \text{ V}) \exp(-j84.3^\circ)$

Example - source transformation

For the circuit below, use a source transformation to find the resistor current.



$V_S(t) = V_m \cos \omega t$

$V_m = 25$ V

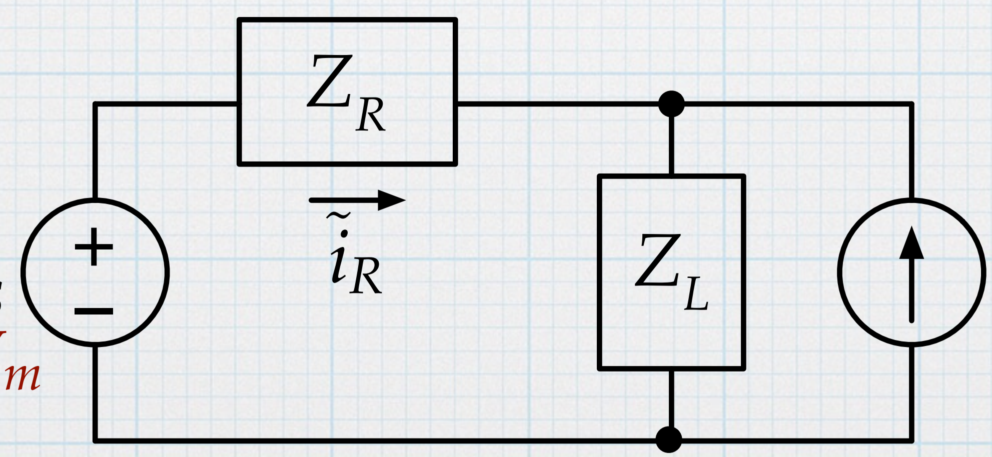
$\omega = 5000$ rad/s

$I_S(t) = I_m \cos(\omega t + \theta_i)$

$I_m = 50$ mA

$\theta_i = -45°$ ($\pi/4$ rad)

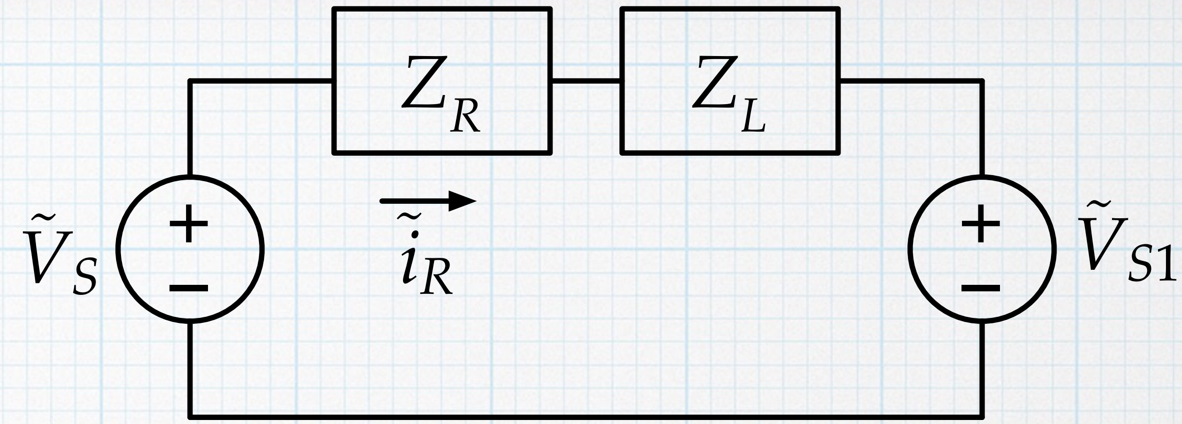
Form the complex version of the circuit. Note that there is a phase difference between the the two source and that difference must be show up in the complex numbers (phasors) describing the sources.



$\tilde{V}_S = V_m$

$\tilde{I}_S = I_m e^{j45°}$

Then do the source transformation. Since we are interested in the resistor, we cannot transform it and so we must transform the current-source / inductor pair.



$$\tilde{V}_{S1} = Z_L \tilde{I}_S = (j\omega L)\left(I_m e^{j\theta_i}\right) = \left(\omega L e^{j90^\circ}\right)\left(I_m e^{j\theta_i}\right) = (\omega L I_m)\, e^{j(\theta_i + 90^\circ)}$$

$$\tilde{V}_{S1} = (12.5\text{ V})\, e^{j45^\circ} = 8.84\text{ V} + j8.84\text{ V} \;\; \left(= V_{m1} e^{j\theta_v}\right)$$

The circuit analysis is trivial – use KVL around the loop.

$$\tilde{V}_S - \tilde{i}_R Z_R - \tilde{i}_R Z_L - \tilde{V}_{S1} = 0$$

$$\tilde{i}_R = \frac{\tilde{V}_S - \tilde{V}_{S1}}{Z_R + Z_L} = \frac{V_m - V_{m1} e^{j\theta_v}}{R + j\omega L}$$

$$\tilde{i}_R = \frac{25\text{ V} - (8.84\text{ V} + j8.84\text{ V})}{250\Omega + j250\Omega} = \frac{(18.4\text{ V})\, e^{-j28.7^\circ}}{(353.6\ \Omega)\, e^{j45^\circ}} = (52.1\text{ mA})\, e^{-j73.7^\circ}$$

$$i_R(t) = (52.1\text{ mA})\cos(\omega t - 73.7^\circ)$$