Signals and Systems (CT 203)

Tutorial Sheet-09 DA-IICT, Gandhinagar.

1. Find the Fourier transform of
   (a) $\delta(t+1)+\delta(t-1)$
   (b) $e^{-2t}u(t)$
   (c) $e^{-3(t-1)}u(t-1)$

2. Consider a causal LTI system with frequency response
$$H(j\omega)=\frac{1}{4+j\omega}.$$
   For a particular input $x(t)$ this system is observed to produce the output
$$y(t)=e^{-3t}u(t)-e^{-4t}u(t)$$

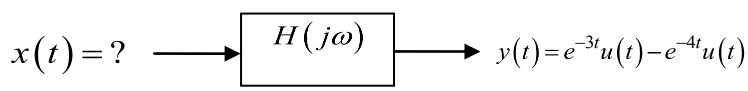

Fig.3. A causal LTI system

   Determine $x(t)$.

3. Consider a causal LTI system implemented as the RLC circuit shown in Fig. In this circuit $x(t)$ is the input voltage. The voltage $y(t)$ across the capacitor is considered the system output.
   a) Find the differential equation relating $x(t)$ and $y(t)$
   b) Determine the frequency response using Fourier transform
   c) Determine the impulse response of this electric circuit using inverse Fourier transform and convolution theorem

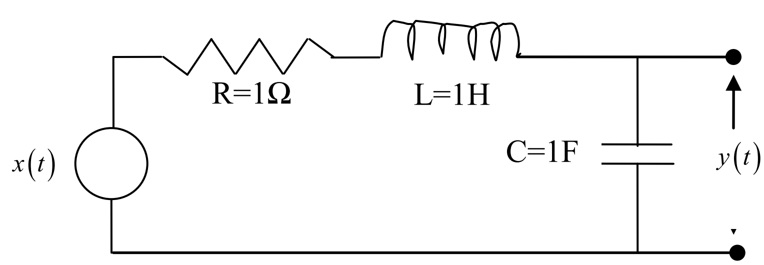

Fig.1. An RLC circuit

4. Let $x(t)$ and $y(t)$ be two real signals. Then the cross-correlation function of $x(t)$ and $y(t)$ is defined by
$$\phi_{xy}(t)=\int_{-\infty}^{+\infty}x(t+\tau)y(\tau)\,d\tau$$

and the autocorrelation of $x(t)$ is defined as

$$\phi_{xx}(t) = \int_{-\infty}^{+\infty} x(t+\tau)x(\tau)\,d\tau$$

(a) What is the relationship between $\Phi_{xy}(\omega)$ and $\Phi_{yx}(\omega)$?

(b) Find expression for $\Phi_{xy}(\omega)$ in terms of $X(\omega)$ and $Y(\omega)$.

(c) Show that $\Phi_{xx}(\omega)$ is real and nonnegative for every ω

(d) Suppose that $x(t)$ is the input to an LTI system with a real-valued impulse response and with frequency response $H(\omega)$ and that $y(t)$ is the output. Find expressions for $\Phi_{xy}(\omega)$ and $\Phi_{yy}(\omega)$ in terms of $\Phi_{xx}(\omega)$ and $H(\omega)$.