Tutorial-1

Q.1 In each case, find an open interval about x_0 on which the inequality $|f(x)-L|<\epsilon$ holds. Then give a value for $\delta>0$ such that for all x satisfying $0<|x-x_0|<\delta$ the inequality $|f(x)-L|<\epsilon$ holds.

(a) $f(x)=\sqrt{x+1}$, $L=1$, $x_0=0$, $\epsilon=0.1$

(b) $f(x)=x^2-5$, $L=11$, $x_0=4$, $\epsilon=1$

(c) $f(x)=\frac{1}{x}$, $L=-1$, $x_0=-1$, $\epsilon=0.1$

Q.2 Show that (Use (ϵ,δ) definition)

(i) $\lim_{x\to 4} 9-x=5$

(ii) $\lim_{x\to -3} \frac{x^2-9}{x+3}=-6$

(iii) $\lim_{x\to 0} x\sin\frac{1}{x}=0$

Q.3 $f(x)=\begin{cases}\sin\frac{1}{x}, & x>0\\ 0, & x\le 0\end{cases}$

(a) Does $\lim_{x\to 0^+} f(x)$ exist? Why or why not?

(b) Does $\lim_{x\to 0^-} f(x)$ exist? Why or why not?

Q.4 Use (ϵ,δ) approach to prove that

(i) $\lim_{x\to 0^-} \frac{x}{|x|}=-1$

(ii) $\lim_{x\to 2^+} \frac{x-2}{|x-2|}=1$

Q-5 Using $\lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1$, find the following limits

~~Prove that~~

(i) $\lim_{x \to 0} \frac{x^2 - x + \sin x}{2x}$

(ii) $\lim_{t \to 0} \frac{\sin(1-\cos t)}{1-\cos t}$

Q-6 For what values of a and b, the function

$$g(x) = \begin{cases} -2, & x \le -1 \\ ax+b, & -1 < x < 1 \\ 3, & x \ge 1 \end{cases}$$

is continuous at every x?

Q-7 Show that the function

$$f(x) = \begin{cases} 1, & \text{if } x \text{ is rational} \\ 0, & \text{if } x \text{ is irrational} \end{cases}$$

is discontinuous at every point.

Is $f(x)$ is right continuous at any point?

Is $f(x)$ is left continuous at any point?

Q-8 If functions $f(x)$ and $g(x)$ are continuous for $0 \le x \le 1$, could $\frac{f(x)}{g(x)}$ possibly be discontinuous at a point of $[0,1]$? Give reason for your answer.

$x-1$ $x-2$

Solution

Tut-1

Q.1 (a)

Part-a $|f(x)-L| < \epsilon$

$|\sqrt{x+1}-1| < 0.1$

$\Rightarrow -0.1 < \sqrt{x+1}-1 < 0.1$

$\Rightarrow 0.9 < \sqrt{x+1} < 1.1$

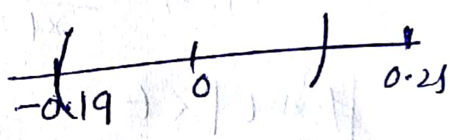
$\Rightarrow 0.81 < x+1 < 1.21 \Rightarrow -0.19 < x < 0.21$

$(-0.19, 0.21)$ interval

Part b

$|x-0| < \delta$

$\Rightarrow -\delta < x < \delta$



We take the ~~interval~~ $\delta = 0.19$

So that for all $x \in (a-\delta, a+\delta)$,

$|f(x)-L| < \epsilon$.

(b) $f(x) = x^2-5$, $L=11$, $x_0=4$, $\epsilon=1$

Ans (i) $\sqrt{15} < x < \sqrt{17}$

(ii) $|x-4| < \delta \Rightarrow -\delta < x-4 < \delta \Rightarrow -\delta+4 < x < \delta+4$

$-\delta+4 = \sqrt{15} \Rightarrow \delta = 4-\sqrt{15} \approx 0.1270$.

or $\delta+4 = \sqrt{17} \Rightarrow \delta = \sqrt{17}-4 \approx 0.1231$

Take the smaller $\delta \approx 0.12$

(c) $f(x) = \frac{1}{x}$, $L=-1$, $x_0=-1$, $\epsilon=0.1$

Ans (i) $-\frac{10}{9} < x < -\frac{10}{11}$

(ii) $\delta = \frac{1}{11}$

Prob-2

Show that

(i) $\lim_{x\to 4} 9-x = 5$

(ii) $\lim_{x\to -3} \frac{x^2-9}{x+3} = -6$

(iii) $\lim_{x\to 0} x \sin 1/x = 0$

Sol^n

(ii) $\lim_{x\to -3} \frac{x^2-9}{x+3} = -6$

$f(x) = \frac{x^2-9}{x+3}$ $L = -6$, $x_0 = -3$

$|f(x)-L| < \epsilon$

$\left|\frac{x^2-9}{x+3} - (-6)\right| < \epsilon \Rightarrow |(x-3)+6| < \epsilon$

$\Rightarrow -\epsilon < x+3 < \epsilon \Rightarrow -\epsilon-3 < x < \epsilon-3$

$|x-(-3)| < \delta \Rightarrow -\delta < x+3 < \delta$

$\Rightarrow -\delta-3 < x < \delta-3$

$-\delta-3 = -\epsilon-3 \Rightarrow \delta = \epsilon$

or $\delta-3 = \epsilon-3 \Rightarrow \delta = \epsilon$ $\Bigg\} \Rightarrow \delta = \epsilon$

So For each $\epsilon > 0$, $\exists\, \delta > 0$ such that

for all $0 < |x-(-3)| < (\delta = \epsilon)$, $|f(x)-L| < \epsilon$.

(i) of (iii) Do it similarly.

Q-1

(i) Ans = 0

(ii) Ans 1

Q-6

$$\lim_{x \to -1^-} f(x) = -2$$

$$\lim_{x \to -1^+} f(x) = a(-1)+b = -a+b$$

$$\lim_{x \to 1^-} f(x) = a \cdot 1 + b = a+b$$

$$\lim_{x \to 1^+} f(x) = 3$$

For $f(x)$ to be continuous we must have

$-2 = -a+b$ and $a+b = 3$

$\Rightarrow b-a = -2$ & $a+b = 3$

$\Rightarrow 2b = 1 \Rightarrow b = \frac{1}{2}$

$a = 3 - \frac{1}{2} = \frac{5}{2}$

Q-7

Suppose we want to ~~check~~ the continuity at x_0

Let x_0 be rational $\Rightarrow f(x_0) = 1$.

Choose $\epsilon = \frac{1}{2}$.

For any $\delta > 0$, there is an irrational number x in the interval $(x_0 - \delta, x_0 + \delta) \Rightarrow f(x) = 0$

Then $0 < |x - x_0| < \delta$ but $|f(x) - f(x_0)| = |0-1| = 1 > \frac{1}{2} = \epsilon$.

So $\lim_{x \to x_0} f(x)$ does not exist.

$\Rightarrow f$ is discontinuous at rationals.

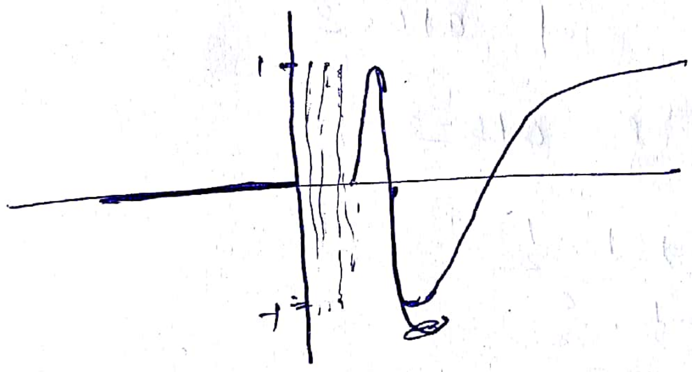
Q.3 $f(x) = \begin{cases} \sin \frac{1}{x}, & x > 0 \\ 0, & x \le 0 \end{cases}$

(a) Does $\lim_{x \to 0^+} f(x)$ exist? Why or why not?

(b) Does $\lim_{x \to 0^-} f(x)$ exist? Why or why not?

Sol^n (b) $\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} 0 = 0$ ~~it exist~~ Yes.

(a) $\lim_{x \to 0^+} \sin \frac{1}{x}$ does not exist because $\sin \frac{1}{x}$ does not approach any single value as x approaches 0.



Q.4 (i) $\lim_{x \to 0^-} \frac{x}{|x|} = -1$

$\left| \frac{x}{|x|} - (-1) \right| < \epsilon \Rightarrow \left| \frac{x}{-x} + 1 \right| < \epsilon$

$\Rightarrow 0 < \epsilon$ which is always true independent of the value of x.

Hence we can choose any $\delta > 0$ with $-\delta < x < 0$

$\Rightarrow \lim_{x \to 0^-} \frac{x}{|x|} = -1$.

(ii) $\lim_{x \to 2^+} \frac{x-2}{|x-2|} = 1$

As $x \to 2^+$, $|x-2| = x-2$

$\left| \frac{x-2}{|x-2|} - 1 \right| \Leftrightarrow \left| \frac{x-2}{x-2} - 1 \right| < \epsilon \Rightarrow 0 < \epsilon$ which is always true

Hence choose any $\delta > 0$, such that $2 < x < 2 + \delta$

$\Rightarrow \left| \frac{x-2}{|x-2|} - 1 \right| < \epsilon$.

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Q.1

If x_0 is irrational $\Rightarrow f(x_0)=0$ & there is a rational number x in $(x_0-\delta, x_0+\delta)$

$\Rightarrow f(x)=1$

Again $|f(x)-f(x_0)| = 1 \nless \frac{1}{2}$

So $\lim_{x \to x_0} f(x)$ does not exist.

$\Rightarrow f$ is discontinuous at irrationals

$\Rightarrow f$ is discontinuous at every real number

(b)

f is neither right continuous nor left continuous at any point x_0 because in every interval $(x_0-\delta, x_0)$ or $(x_0, x_0+\delta)$ there exist both rationals and irrationals.

Thus $\lim_{x \to x_0^-} f(x)$ & $\lim_{x \to x_0^+} f(x)$ both does not exist.

Q.2

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Tutorial - 2

Q.1 Find the limits

(i) $\lim_{x\to\infty} \left(\sqrt{x^2+3x} - \sqrt{x^2-2x}\right)$

(ii) $\lim_{x\to\infty} \left(\sqrt{x^2+x} - \sqrt{x^2-x}\right)$

Q.2 Use formal definitions to prove that

(i) $\lim_{x\to 0} \frac{1}{|x|} = \infty$

(ii) $\lim_{x\to 1^-} \frac{1}{1-x^2} = \infty$

Q.3 Find the oblique asymptotes of

(i) $f(x) = \frac{x^2+1}{x-1}$

(ii) $f(x) = \frac{x^3+1}{x^2}$

Q.4

$$f(x) = \begin{cases} x, & \text{if } x \text{ is rational} \\ 0, & \text{if } x \text{ is irrational} \end{cases}$$

(a) Show that f is continuous at $x=0$.

(b) Also show that f is not continuous at any other real number.

Q.5 Which of the following statements are true, and which are false? If true, say why? If false, give counter example.

(a) If $\lim_{x\to a} f(x)$ exists but $\lim_{x\to a} g(x)$ does not exist, then $\lim_{x\to a} (f(x)+g(x))$ does not exist.

(b) If neither $\lim_{x \to a} f(x)$ nor $\lim_{x \to a} g(x)$ exists, then
$\lim_{x \to a} (f(x) + g(x))$ does not exist.

(c) If f is continuous at x, then so is $|f|$.

(d) If $|f|$ is continuous at a, then so is f.

Q6 ~~The function $f(x) = |x|$ has an~~ absolute ~~minimum at $x = 0$ even though f is not~~ differentiable ~~at $x = 0$~~.

Test the differentiability of the following function

(i) $x^2 \sin\frac{1}{x}$ at $x = 0$

(ii) $x|x|$ at $x = 0$

If Differentiable what will be the derivative for each function. Is the derivative again differentiable?

Tutorial-3

Q-1

(i) $\lim_{x\to\infty}\left(\sqrt{x^2+3x}-\sqrt{x^2-2x}\right)$

$= \lim_{x\to\infty}\left(\sqrt{x^2+3x}-\sqrt{x^2-2x}\right)\left(\frac{\sqrt{x^2+3x}+\sqrt{x^2-2x}}{\sqrt{x^2+3x}+\sqrt{x^2-2x}}\right)$

$= \lim_{x\to\infty}\frac{x^2+3x-(x^2-2x)}{\sqrt{x^2+3x}+\sqrt{x^2-2x}}$

$= \lim_{x\to\infty}\frac{5x}{\sqrt{x^2+3x}+\sqrt{x^2-2x}} = \lim_{x\to\infty}\frac{5\cancel{x}}{\cancel{x}\left(\sqrt{1+\frac{3}{x}}+\sqrt{1-\frac{2}{x}}\right)}$

$= \frac{5}{1+1} = \frac{5}{2}$

Similarly
(ii) Ans $\Rightarrow$ 1

Q.2

(i) $\lim_{x\to 0}\frac{1}{|x|}=\infty$

Defn: For every real number $B>0$, we must find a $\delta>0$ such that for all x, $0<|x-0|<\delta \Rightarrow \frac{1}{|x|}>B$.

Now $\frac{1}{|x|}>B>0 \Rightarrow |x|<\frac{1}{B}$

Choose $\delta=\frac{1}{B}$; Then $0<|x-0|<\delta$

$\Rightarrow |x|<\frac{1}{B} \Rightarrow \frac{1}{|x|}>B$.

$\Rightarrow \lim_{x\to 0}\frac{1}{|x|}=\infty$.

(ii) $\lim_{x\to 1^-}\frac{1}{1-x^2}=\infty$

For $B>0$ & $0<x<1$, $\frac{1}{1-x^2}>B \Leftrightarrow 1-x^2<\frac{1}{B}$

$\Leftrightarrow (1-x)(1+x)<\frac{1}{B}$

Now $\frac{1+x}{2}<1$ since $x<1$.

Choose $\delta=\frac{1}{2B}$. Then $1-\delta<x<1 \Rightarrow -\delta<x-1<0$

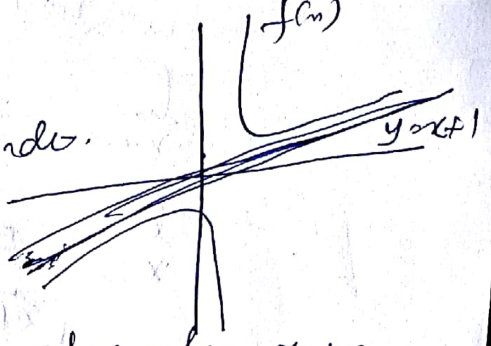
$\Rightarrow 1-x<\delta<\frac{1}{2B}$

$\Rightarrow (1-x)(1+x)<\frac{1}{B}\left(\frac{1+x}{2}\right)<\frac{1}{B}$.

$\Rightarrow \frac{1}{1-x^2} > B$ for $0<x<1$ of x near 1.

$\Rightarrow \lim_{x\to 1^-} \frac{1}{1-x^2} = \infty$.

Q.3 (1) $f(x) = y = \frac{x^2+1}{x-1} = \underset{g(x)}{(x+1)} + \frac{2}{x-1}$ remainder.



$\lim_{x\to\infty} \frac{2}{x-1} = 0$

So $f(x)$ & $g(x)$ become very close when $x\to\infty$.

So $g(x) = 1+x$ is the oblique asymptote.

(4) $f(x) = y = \frac{x^3+1}{x^2} = x + \frac{1}{x^2}$

As $x\to\infty$, $\frac{1}{x^2} \to 0$.

$\Rightarrow f(x)$ & x became very close as $x\to\infty$.

$\Rightarrow y = x$ is the oblique asymptote.

Q.4 (a) Let $\epsilon > 0$ be given.

If x is rational, then $f(x) = x$

$\Rightarrow |f(x)-0| = |x-0| < \epsilon$ x

Chose $\delta = \epsilon$. Then $|x-0| < \delta \Rightarrow |f(x)-0| < \epsilon$. for x rational.

If x is irrational, $f(x) = 0$

$\Rightarrow |f(x)-0| < \epsilon \Leftrightarrow 0 < \epsilon$. which is true no matter how close irrational x is to 0.

So again choosing δ any number, we have $|f(x)-0| < \epsilon$.

$\Rightarrow f$ is continuous at $x=0$.

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(b) Chose $x = c > 0$.

Then within any interval $(c-\delta, c+\delta)$ there are both rational & irrational numbers.

If c is rational, pick $\epsilon = \frac{c}{2}$. No matter how small we chose $\delta > 0$, there is an irrational number x in $(c-\delta, c+\delta) \Rightarrow |f(x) - f(c)| = |0 - c| = c > \frac{c}{2} = \epsilon$.

$\Rightarrow f$ is not continuous at any rational $c > 0$.

If c is irrational $\Rightarrow f(c) = 0$.

Again pick $\epsilon = \frac{c}{2}$.
No matter how small we chose $\delta > 0$, there is rational number x in $(c-\delta, c+\delta)$ with ~~$\frac{c}{2}$~~

~~$|f(x) - f(c)| < \epsilon \Rightarrow |x - 0| < \epsilon$~~

$|x - c| < \frac{c}{2} \Rightarrow \epsilon \iff \frac{c}{2} < x < \frac{3c}{2}$.

Then $|f(x) - f(c)| = |x - 0| = |x| > \frac{c}{2} = \epsilon$

$\Rightarrow f$ is not continuous at any irrational $c > 0$.

If $x = c < 0$, repeat the argument picking $\epsilon = \frac{|c|}{2} = \frac{-c}{2}$.

Therefore f fails to be continuous at any nonzero value $x = c$.

Q.5 (a) True, because if $\lim_{x \to a} (f(x) + g(x))$ exists, then

$$\underbrace{\lim_{x \to a} (f(x) + g(x))}_{\text{exists}} - \underbrace{\lim_{x \to a} f(x)}_{\text{exists}} = \lim_{x \to a} (f(x) + g(x) - f(x))$$

$= \lim_{x \to a} g(x)$ exists.

Q5 (b)

False $\quad f(x) = \frac{1}{x} \qquad g(x) = -\frac{1}{x}$

$\lim_{x\to 0} f(x)$ does not exist.

$\lim_{x\to 0} g(x)$ does not exist.

but $\lim_{x\to 0} (f(x)+g(x)) = \lim_{x\to 0}\left(\frac{1}{x}-\frac{1}{x}\right) = \lim_{x\to 0} 0 = 0$ exists.

(c) True.

$g(x) = |x|$ is continuous.

$\Rightarrow g(f(x)) = |f(x)|$ is continuous

(It is composition of two continuous functions.

(d) False

$$f(x) = \begin{cases} -1, & x \le 0 \\ 1, & x > 0 \end{cases}$$

$f(x)$ is discontinuous at $x=0$.

But $|f(x)| = 1$ which is continuous at $x=0$.

Q.6

Tutorial-3

① Find the volume of the largest right circular cylinder that can be inscribed in a sphere of radius 5.

② You operate a tour service that offers the following rates:

200 Rs per person if 50 people (the minimum number to book the tour) go on the tour.

For each additional person, up to a maximum of 80 people total, the rate per person is reduced by 2 Rs.

It costs 6000 Rs plus 32 Rs per person to conduct the tour. How many people does it take to maximize your profit?

③

$$f(x) = \begin{cases} x, & 0 \le x < 1 \\ 0, & x = 1 \end{cases}$$

$f(0) = 0 = f(1)$. According to Rolle's Th^m $\exists\, c \in (0,1)$ such that $f'(c) = 0$.
But its derivative on $(0,1)$ is never zero.
How can this be?

Q.4 ~~Suppose~~ Show that if $f'' > 0$ throughout interval $[a,b]$, then f' has at most one [illegible] in $[a,b]$. What if $f'' < 0$ throughout $[a,b]$ in[illegible]

Q.5 A trucker handed in a ticket at a toll booth showing that in 2 hours he had covered 159 mile on a toll road with speed limit 65 mph. The trucker was cited for speeding. why?

Q.6 A marathoner ran the 26.2 mile Marathon in 2.2 hours. Show that at least twice the marathoner was running at exactly 11 mph, assuming the initial and final speeds are zero.

Q.7 If $f'(x) = 0$ for all $x \in (a,b)$, then $f(x)$ is constant. Prove.

Q.8 Show that ~~Functions~~ which have same derivatives differ by a constant.

That is if $f'(x) = g'(x)$ $\forall x \in (a,b)$, then there exist c such that $f(x) = g(x) + c$ $\forall x \in (a,b)$.

That is $f-g$ is a constant on (a,b).

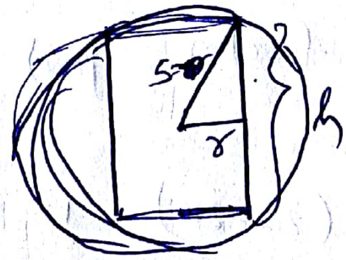
Q.9 Show that $|\cos x - 1| \le |x|$ for all x-values. (Hint: Consider ~~[illegible]~~ $f(t) = \cos t$ on $[0,x]$, apply MVT.

$|\cos x - 1| \le x = |x|$. ~~(proof)~~

Unit-3 Solutions

(1)

Let the radius of the cylinder be r cm.



Then the height is $2\sqrt{25-r^2}$

The volume of the cylinder

$$V(r) = 2\pi r^2\sqrt{25-r^2}$$

$$V'(r) = 2\pi r^2 \frac{1}{\cancel{2}\sqrt{25-r^2}}(-\cancel{2}r) + 2\pi\sqrt{25-r^2}\cdot 2r$$

$$= \frac{-2\pi r^3 + 4\pi r(25-r^2)}{\sqrt{25-r^2}}$$

$$= \frac{2\pi r(-r^2+50-2r^2)}{\sqrt{25-r^2}}$$

$$= \frac{2\pi r(50-3r^2)}{\sqrt{25-r^2}}$$

$V'(r)=0 \Rightarrow \cancel{r=0}$ or $3r^2=50$

$\Rightarrow r^2=\frac{50}{3}$

$\Rightarrow r=\sqrt{\frac{50}{3}} = 5\sqrt{\frac{2}{3}}$

$V'(r)>0$ for $0<r<5\sqrt{\frac{2}{3}}$

$V'(r)<0$ for $5\sqrt{\frac{2}{3}}<r<5$ (r can be max 5)

$\Rightarrow r=5\sqrt{\frac{2}{3}}$ corresponds to maximum volume (or check for $V''(r)$)

$r=5\sqrt{\frac{2}{3}}$, $h = 2\sqrt{25-r^2} = 2\sqrt{25-\frac{50}{3}}$

Max. volume $= V = \pi r^2 h$

Q2

Let x represents number of people over 50.

The profit function $P(x) = (50+x)(200-2x) - 32(50+x) - 6000$

$\begin{cases} (50+x)(200-2x) & \text{is total money you get.} \\ 32(50+x)+6000 & \text{is expenditure you have to make for the trip} \end{cases}$

$P(x) = -2x^2 + 68x + 2400$

$P'(x) = -4x+68 = 0 \Rightarrow 4x = 68 \Rightarrow x = 17$

$P''(x) = -4 < 0$

So at $x = 17$, maximum profit will be there

So it would take 67 people to maximise the profit.

Q-3

$$f(x) = \begin{cases} x, & 0 \le x < 1 \\ 0, & x = 1 \end{cases}$$

$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} x = 1$

$f(1) = 0$

So function is not continuous at 1.

So Rolle's theorem can't be applied here.

Since f'' exists throughout $[a,b]$, the derivative function f' is continuous there.

If f' has more than one zero in $[a,b]$, say $f'(r_1) = f'(r_2) = 0$ for $r_1 \neq r_2$, then by Rolle's theorem there is a $c \in (r_1, r_2) \subset (a,b)$ such that $f''(c) = 0$. Contrary to the assumption that $f'' > 0$ throughout $[a,b]$.

Therefore f' has at most one zero in (a,b).

The same argument valid if $f'' < 0$ throughout $[a,b]$.

Q-5 The trucker average speed is 79.8 mph. By MVT, the trucker must have been going that speed at least once during the trip.

(MVT says $\exists$ $c \in (a,b)$ at which the ~~average~~ ~~speed~~ instantaneous speed at c is equal the average speed)

So the trucker was cited for speeding as the speed limit is 65 mph.

Q-6 The runner's average speed for the marathon was approximately 11.909 mph. By the MVT, the runner must have been going that speed at least once during the marathon. Since the initial speed and final speed both are 0 mph of the runner's speed, by intermediate value theorem, the speed is continuous, the runner's speed must have been 11 mph at least twice.

Q.1 $f'(x) = 0$ for all $x \in (a,b)$.

Let x_1, and x_2 be two points in (a,b) such $x_1 < x_2$.

$\Rightarrow$ ~~ff~~ Consider $[x_1, x_2]$

f is differentiable on (x_1, x_2)

f is continuous on $[x_1, x_2]$.

$\Rightarrow \exists\, c \in (x_1, x_2)$ such that

$$f'(c) = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

But $f'(c) = 0$ [As $f'(x) = 0 \;\forall x \in (a,b)$].

$\cancel{\Rightarrow f(x_2) = f(x_1)}$

$\Rightarrow f(x_2) = f(x_1)$

Since x_1 and x_2 are arbitrary points in (a,b),

$f(x)$ is constant throughout.

Q.2 $f(t) = \cos t$ on $[0, x]$

$\cos t$ is continuous on $[0,x]$, differentiable on $(0,x)$.

$\Rightarrow \exists\, c \in (0,x)$ such that

$$f'(c) = \frac{f(x) - f(0)}{x - 0}$$

$$\Rightarrow -\sin c = \frac{\cos x - \cos 0}{x - 0} = \frac{\cos x - 1}{x}$$

$-1 \le \sin c \le 1$

$\Rightarrow -1 \le -\sin c \le 1 \quad \Rightarrow \quad -1 \le \dfrac{\cos x - 1}{x} \le 1$

If $x > 0$, $-1 \le \dfrac{\cos x - 1}{x} \le 1 \Rightarrow -x \le \cos x - 1 \le x$
$\Rightarrow |\cos x - 1| \le x = |x|$

If $x < 0$, $-1 \le \dfrac{\cos x - 1}{x} \le 1 \Rightarrow -x \ge \cos x - 1 \ge x$
$\Rightarrow x \le \cos x - 1 \le -x \Rightarrow -(-x) \le \cos x - 1 \le -x$
$\Rightarrow |\cos x - 1| \le -x = |x|$.