Tutorial 10

1. Prove that $P\{a_1 < X \leq a_2, b_1 < Y \leq b_2\} = F(a_2, b_2) + F(a_1, b_1) - F(a_1, b_2) - F(a_2, b_1)$, whenever $a_1 < a_2,\ b_1 < b_2$.

2. The joint density function of X and Y is given by

$$f(x,y) = \begin{cases} 2e^{-x}e^{-2y} & ;\quad 0 < x < \infty\ ,\ 0 < y < \infty \\ 0 & ;\quad \text{otherwise} \end{cases}$$

   Compute $P\{X < Y\}$.

3. A man and a woman decide to meet at a certain location. If each of them independently arrives at a time uniformly distributed between 12 noon and 1 P.M., find the probability that the first to arrive has to wait longer than 10 minutes.

4. The continuous (discrete) random variables X and Y are independent if and only if their joint probability density (mass) function can be expressed as

$$f_{X,Y}(x,y) = h(x)g(y) \qquad -\infty < x < \infty\ ,\ -\infty < y < \infty$$

5. Suppose that X and Y are independent, continuous random variables having probability density functions f_X and f_Y. Prove that, $f_{X+Y}(a) = \int\limits_{-\infty}^{\infty} f_X(a-y)f_Y(y)dy$.

6. Show that, if $X_i, i = 1, \ldots, n$, are independent random variables that are normally distributed with respective parameters $\mu_i, \sigma_i^2,\ i = 1, \ldots, n$, then $\sum\limits_{i=1}^{n} X_i$ is normally distributed with parameters $\sum\limits_{i=1}^{n} \mu_i$ and $\sum\limits_{i=1}^{n} \sigma_i^2$.

7. A basketball team will play a 44-game season. Twenty-six of these games are against class A teams and 18 are against class B teams. Suppose that the team will win each game against a class A team with probability .4 and will win each game against a class B team with probability .7. Suppose also that the results of the different games are independent. Approximate the probability that
   (a) the team wins 25 games or more;
   (b) the team wins more games against class A teams than it does against class B teams.