CT111 Introduction to Communication Systems
Lecture 9: Digital Communications

Yash M. Vasavada

Associate Professor, DA-IICT, Gandhinagar

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Overview of Today's Talk

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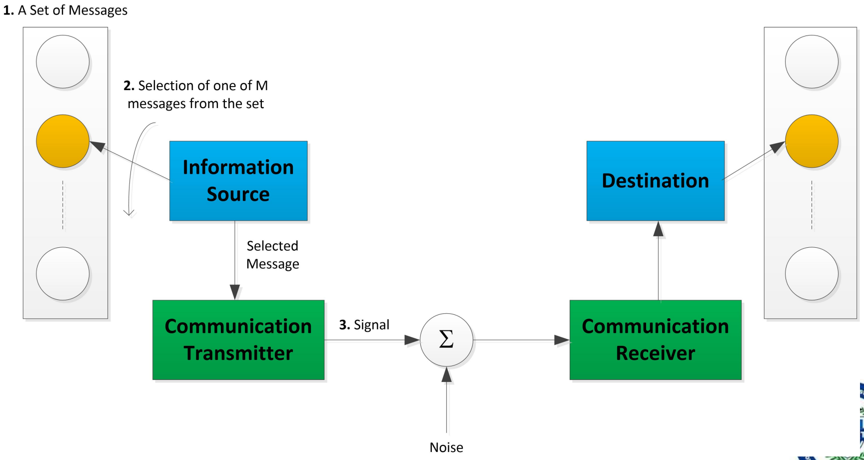
A Model of Digital Communication Systems
A Simple Block Diagram



A Model of Digital Communication Systems
A Simple Block Diagram

Key design parameters:

1. Size M of the message set: determines number of bits N required to convey the message
   - $\rightarrow$ $N = \log_2 M$
2. How fast the messages are selected: determines the number of messages per second
   - $\rightarrow$ the larger the message set size and/or the greater the speed of the message transfer, the bit rate R = the number of bits per second, increases
   - $\rightarrow$ Greater the bit rate R, the greater the information that gets conveyed. However, greater also is the work that the communication system has to do.
3. The power P_s that the communication receiver gets (determined by the power that the transmitter can put in the transmitted signal), the spectral bandwidth W that it has and the power P_n of the noise that the communication channel introduces

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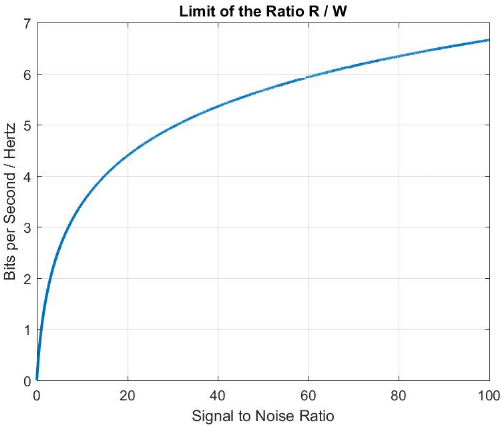
A Fundamental Limit on Communications

Shannon Information Theory

→ The celebrated relationship:

$$R \leq W \log_2 \left(1 + \frac{P_s}{P_n}\right)$$



Bandwidth Efficiency η_B

- As data rate R increases, the pulse width of transmitted signal reduces and therefore the bandwidth B, which is inversely proportional to the transmitted pulse width, increases.
- This cannot be avoided; however some schemes use the available bandwidth more efficiently than the others
- We will denote the ratio R/W as the bandwidth efficiency η_B.
- It is obviously better to have η_B as large as possible. However, there is a cost associated to making η_B large.

Energy Efficiency η_E

- Communication systems are characterized by the signal to noise ratio (SNR) P_s/P_n required to attain a certain performance
- Typically improving η_B (making it large) requires SNR P_s/P_n to be increased
- We will define energy efficiency η_E as $\left(\frac{P_s}{P_n}\right)^{-1}$ required to attain some excellent communication performance (e.g., only one bit out of 10^5 bits is in error on average).
- Greater the required $\frac{P_s}{P_n}$, the smaller the energy efficiency.

Fight between η_E and η_B

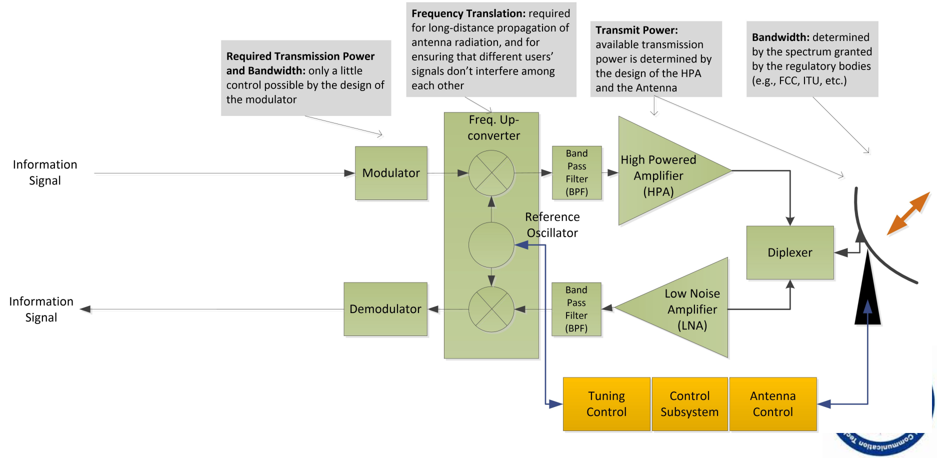
- As it often is the case in the life, it is hard to get best of both the worlds.
  - An increase in η_B translates to a decrease in η_E and vice versa.
- Trade-off between bandwidth and energy efficiencies can be viewed as the *equivalence* between the power and the bandwidth
  - If the system designer has a fixed transmit power (i.e., the design is limited or handicapped by the transmit power), this limit can be overcome to some extent by increasing the bandwidth
  - Vice versa, if the bandwidth is limited, the power can be increased to obtain the desired data rate

Comparison

of a Digital with Analog Method of Communication

- In both the digital and the analog communications. . .
  - the information (or the message) signal that is getting transferred over the communication link is often analog
  - Actual (physical) signal that is transmitted over the communication channel is also almost always *analog*, i.e., continuous in time and often continuous in the amplitudes (voltage)
- It is the modulation scheme that determines whether the communication system is called analog or digital.
  - In digital communications, the analog information signal is digitized (discrete-time and discrete-levels of amplitude), and it is this digitized message that modulates the transmitted analog signal
  - In analog communication, the analog information directly modulates the transmitted analog signal

Block Diagram
of an *Analog* Communication Transceiver



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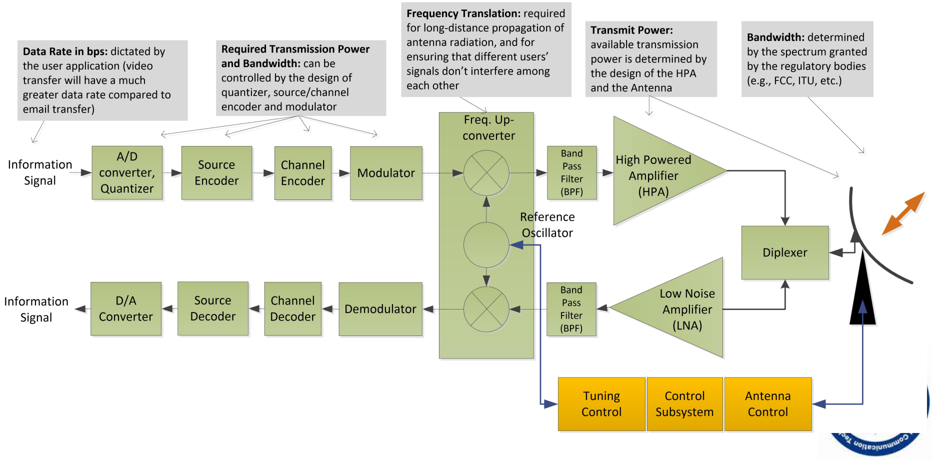

Why Digital Communications
Instead of Analog

Compared to analog communication systems, the digital communications. . .

1. Allows approaching the Shannon Capacity bound more closely (results that are only 0.04 dB away from Shannon bound have been obtained)
2. Provides a better tradeoff of bandwidth efficiency against energy efficiency (i.e., exchange the power with the bandwidth).
3. Has a better ability to compensate for the effect of noise (any noise introduces irrecoverable distortion in the analog signal. In comparison, the a digital receiver needs to distinguish only a finite number of transmitted data. Thus, it is possible to completely remove the effect of noise)
4. Allows use of many performance enhancing signal processing techniques, such as source coding, channel coding, encryption, etc.

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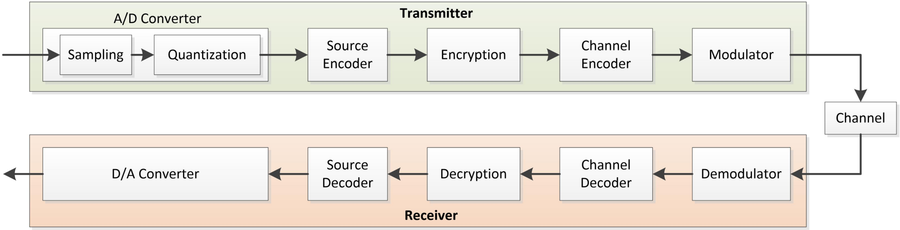
Why Digital Communications
Instead of Analog

In addition

- Digital technology, for which the primary currency is bits (Digital Integrated Circuits (ICs), and in general, the computers and the smartphones and their networks), has become very powerful and are inexpensive to manufacture.
- Digital Communications allows integration of voice, video and data on a single packet networking system.

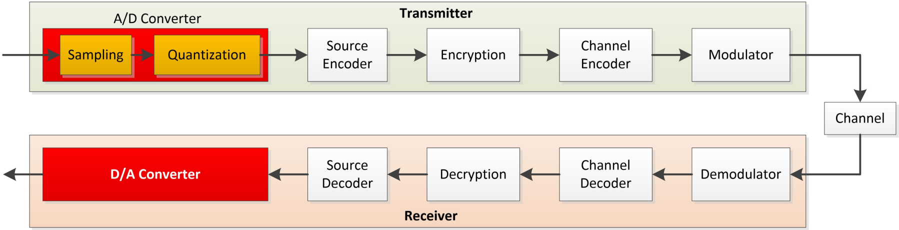
Digital Communication Transceiver
Block Diagram



Digital Communication Transceiver

Block Diagram

- We will now be looking at the process of quantization of an analog information source



Discrete-Time Representations of Continuous-Time Signals

- Examples of Continuous Time (C-T) signals: video, voice, image, etc.
- Sampling theorem says that if the C-Tsignal is band limited, it can be exactly recovered by its time domain samples taken sufficiently close together
  - → Sampling analog signals makes them discrete in time
  - → For band-limited signal, the ideal sampling scheme introduces no distortion

Sampling Theorem

- Let $x(t)$ be a band-limited signal with Fourier Transform $X(f)$ which is zero if $|f| > B$
- Time domain signal $x(t)$ can be perfectly reconstructed from its uniformly spaced samples provided these samples are taken at a rate $R > 2B$. Here, $2B$ is called the Nyquist Rate
- If time-domain samples are collected at a rate less than $2B$, aliasing occurs and it is not possible to perfectly reconstruct the analog signal from its samples
- We have studied the proof of sampling theorem in an earlier lecture.

Digital Representations of C-T Signals

- Quantization of sampled analog signals makes the samples discrete in amplitude
  - → Quantization introduces some distortion. There is a trade off between the bandwidth requirement and distortion
- In quantization, the amplitude of an analog signal within $x_{\max}$ volts and $x_{\min}$ volts is converted to one of L levels: $\{\tilde{x}_k, \ldots, \tilde{x}_k, \ldots, \tilde{x}_L\}$

An Example
of a Quantizer

An example quantizer with $L = 8$.

k	x_{k-1}	x_k	$\tilde{x}_k$	**Output Bits**
1	-4	-3	-3.5	000
2	-3	-2	-2.5	001
3	-2	-1	-1.5	010
4	-1	0	-0.5	011
5	0	1	0.5	100
6	1	2	1.5	101
7	2	3	2.5	110
8	3	4	3.5	111

Notations (Contd...)

Quantization

- Rate R of a quantizer is the number of *bits* required to represent a sample
- *Uniform quantizer* is the one in which $\tilde{x}_k - \tilde{x}_{k-1} = \Delta, \forall k \in 1, \ldots, L-1$
  - $\rightarrow$ $R = \log_2 L$ bits / sample for uniform quantizer
  - $\rightarrow$ In the previous example, $R = 3$ bits / sample
- In a *non-uniform quantizer*, the quantization regions are of unequal widths
  - $\rightarrow$ when would a non-uniform quantizer be preferred over a uniform quantizer?
- Uniform versus Non-uniform quantization is sometimes also called linear versus nonlinear quantization coding

Mean Squared Error or MSE

in the Quantization Process

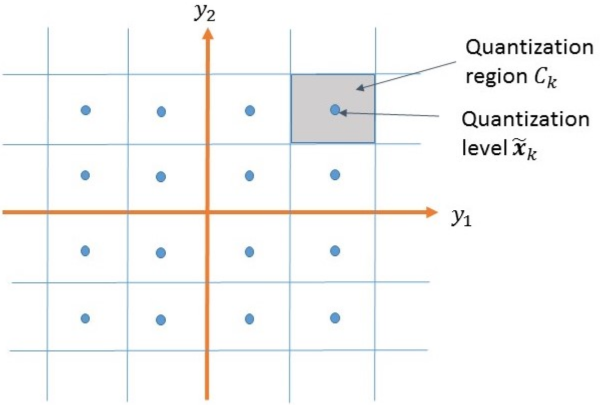
- A common choice for the distortion function: $d(x, \tilde{x}) = (x - \tilde{x})^2$
- $D = MSE = E\left[\left(X - \tilde{X}\right)^2\right] = \sum_{k=1}^{L} \int_{x_{k-1}}^{x_k} (x - \tilde{x})^2 \, p(x) \, dx$
- An interpretation of MSE: $\tilde{X} = f_Q(X) = x + \tilde{n}$, where $\tilde{n} = x - \tilde{x}$ is the noise introduced by the quantization operation. In this case, MSE $= E\left[\tilde{n}^2\right]$ may be interpreted as the power of the quantization noise
- $\left(\frac{P_S}{P_N}\right) = \frac{\text{Signal Power}}{\text{Noise Power}} = \frac{E\left[X^2\right]}{D}$
- When quantization noise $\tilde{n}$ is uniformly distributed, quantization-induced SNR is given approximately as $6 \times R$ dB.

Vector Quantization

- Quantize blocks of n samples at a time $\mathbf{Y} = \{y_1, y_2, \ldots, y_n\}$, where
  - → n is the *dimension* of the quantizer
  - → L quantization levels $\{\tilde{x}_k, k = 1, \ldots, L\}$ are replaced by L quantization vectors $\{\tilde{\mathbf{x}}_\mathbf{k}, k = 1, \ldots, L\}$
  - → Quantization intervals $((x_k, x_{k+1}), k = 1, \ldots, L)$ are replaced by quantization regions $(C_k, k = 1, \ldots, L), C_k \in R^n$
- Rate of VQ $R = \dfrac{\log_2 L}{n}$ bits per sample (fractional rates are now possible)

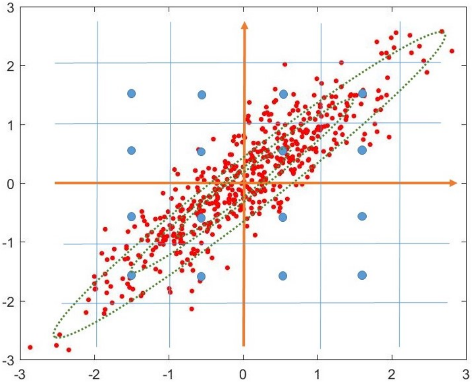
Example VQ

with $L = 16$ and $n = 2$



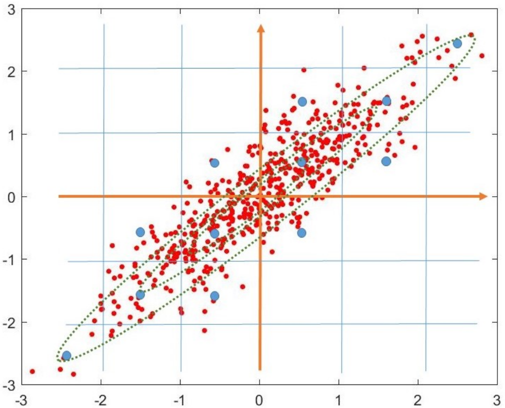
Why VQ Works?

- While scalar quantization can be wasteful of bits with correlated samples,. . .



Why VQ Works?

- . . . VQ is better able to exploit correlation between samples



Speech Encoding

- All speech coding techniques require quantization.
- Many of them employ additional properties of speech

  [Temporal Waveform Coding:] attempts to represent time domain samples of speech

  [Spectral Waveform Coding:] attempts to represent spectral characteristics of speech

  [Model based Coding:] replicates a model of the process by which the Human auditory tract generates the speech

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Temporal Waveform Coding

1. *Pulse Code Modulation (PCM):* directly quantizes the samples of speech
   - *Companding:*
     - → In human speech, probability of low amplitudes is greater than that of the high amplitudes.
     - → Need to use non-uniform quantization.
     - → An alternate is nonlinear compander followed by the uniform quantizer
     - → Nonlinearity for μ-law companding: $y = \dfrac{\log(1+\mu|x|)}{\log(1+\mu)}$
   - Example: 64 kbps PCM for telephone lines:
     - → Speech signal (with a bandwidth of 4 kHz) is sampled at 8 ksps
     - → Encodes each sample with 8 bit uniform quantizer
     - → Uses companding with $\mu = 255$
2. *Differential PCM (DPCM):* speech samples are strongly correlated from one sampling instant to the next. DPCM quantizes the difference between the current sample and the predicted value of this sample using the past samples.

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Temporal Waveform Coding

3. Adaptive PCDM
   - Perceptual quality is determined by the relative accuracy of the quantization to the size of speech
   - Adaptive quantizers vary the step size between the quantization levels depending on whether the speech is loud or soft
   - Adaptive DPCM at 32 kbps is a common and easily implemented, and used in Digital European Cordless Telephone or DECT standard
4. Delta Modulation
   - An extreme case of PCM in which signal is oversampled and $R = 1$ bit/sample, i.e., needs a very simple one bit quantizer
   - Equivalent to just knowing the zero-crossing of the speech signal
   - Adaptive Delta Modulation can produce reasonable quality speech at 16 kbps

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Spectral Waveform Coding

1. Subband Coding
   - Human perception of speech quality depends on the frequency band. Higher MSE may be tolerated at very low and very high frequencies
   - Subband coders filter the speech signal into multiple bands using Quadrature Mirror Filters (QMFs)
   - Filtered signals are quantied using standard PCM, with a different value of R for different bands
2. Adaptive Transform Coding
   - Correlated time domain signals are transformed into frequency domain samples using FFT or Discrete Cosine Transform
   - These frequency domain signals are represented using their perceptual importance
   - Often combined with VQ and Linear Prediction

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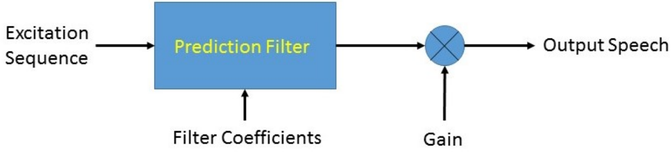
Linear Predictive Speech Coding

- Human speech is modeled as noise (air from the lungs) exciting a linear filter (throat, vocal cords and the mouth)
- Following are quantized and transmitted to the receiver: excitation sequence, filter coefficients, gain



- VSELP (Vector Sum Excited Linear Prediction)
  - → 20 ms frames with 159 bits per frame. Data rate of approx 8 kbps.
  - → Two stage VQ is used to quantize the excitation sequence
  - → Bits (such as those representing the filter gain) are more critical for perceptual quality. These are protected by error correction coding

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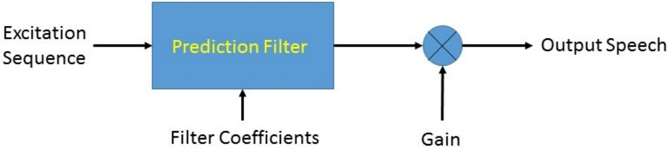

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Relative Performance

Table: Vocoder Comparison

Type	Rate (kbps)	Complexity (MIPS)	Delay (ms)	Quality
PCM	64	0.01	0	High
ADPCM	32	0.1	0	High
Subband	16	1	25	High
VSELP	8	$\sim$100	35	Fair

- How to measure the quality or the distortion of the speech encoder and quantizers?
  - MSE is a possible measure, however, it does not relate to how good the encoded and quantized speech sounds to the human ear
  - Alternative distortion measures: Perceptually Weighted MSE, Segmental SNR, Itakura-Saito, Log Spectral Distance, etc.
  - No single measure has been found to accurately quantify the perceptual quality
  - Subjective assessment of speech quality involves Mean Opinion Score (MOS) testing: subjects rate speech on a scale of 1 (unintelligible) to 5 (perfect). Toll quality telephone speech rates around 4.3.

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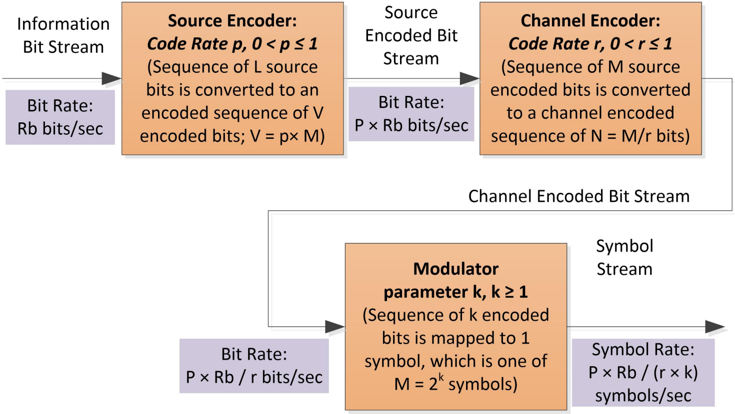
Digital Communication Transceiver

Encoding and Modulation



Digital Modulation
Elementary Methods

Parameters that can be modulated:

- Amplitude: this is called On Off Keying (OOK) or Amplitude Shift Keying (ASK).

$$1 \Rightarrow s(t) = A\cos(2\pi f_c t)$$
$$0 \Rightarrow s(t) = 0$$

- Frequency: this is called Frequency Shift Keying (FSK).

$$1 \Rightarrow s(t) = A\cos(2\pi f_{c,1} t)$$
$$0 \Rightarrow s(t) = A\cos(2\pi f_{c,2} t)$$

- Phase: this is called Phase Shift Keying (PSK).

$$1 \Rightarrow s(t) = A\cos(2\pi f_c t)$$
$$0 \Rightarrow s(t) = A\cos(2\pi f_c t + \pi) = -A\cos(2\pi f_c t)$$

Digital Modulation
Quadrature Method

There are three, completely equivalent, ways of representing the transmitted signal:

1. **Quadrature Notation:**

$$s(t) = x(t)\cos(2\pi f_c t) - y(t)\sin(2\pi f_c t)$$

Here $x(t)$ and $y(t)$ are real-valued baseband signals. These are called inphase and quadrature components.

2. **Complex Envelope Notation:**

$$s(t) = Re\left[(x(t) + jy(t))\exp\{-j2\pi f_c t\}\right] = Re\left[s_l(t)\exp\{-j2\pi f_c t\}\right]$$

Here $s_l(t) = x(t) + jy(t)$ is called the *complex envelope* of $s(t)$

3. **Magnitude and Phase Representation:**

$$s(t) = a(t)\cos(2\pi f_c t + \theta(t))$$

Here $a(t) = \sqrt{x^2(t) + y^2(t)}$ is the *magnitude* of $s(t)$, and $\theta(t) = \tan^{-1}\left(\frac{y(t)}{x(t)}\right)$ is the *phase* of $s(t)$.

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$$s(t) = a(t)\cos(2\pi f_c t + \theta(t))$$

Here $a(t) = \sqrt{x^2(t) + y^2(t)}$ is the *magnitude* of $s(t)$, and $\theta(t) = \tan^{-1}\left(\frac{y(t)}{x(t)}\right)$ is the *phase* of $s(t)$.

Digital Modulation
Quadrature Method

There are three, completely equivalent, ways of representing the transmitted signal:

1. **Quadrature Notation:**

$$s(t) = x(t)\cos(2\pi f_c t) - y(t)\sin(2\pi f_c t)$$

   Here $x(t)$ and $y(t)$ are real-valued baseband signals. These are called inphase and quadrature components.

2. **Complex Envelope Notation:**

$$s(t) = Re\left[(x(t) + jy(t))\exp\left\{-j2\pi f_c t\right\}\right] = Re\left[s_l(t)\exp\left\{-j2\pi f_c t\right\}\right]$$

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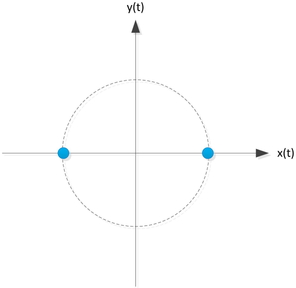
Digital Modulation
Quadrature Method

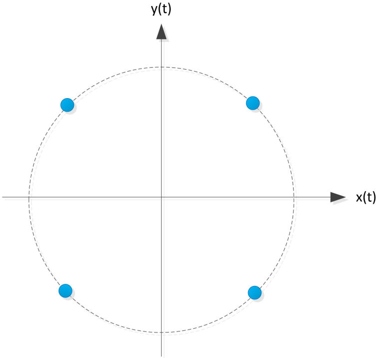
Quadrature method of signal representation allows us to:

- Look at the bandpass signal independent of the carrier frequency
- Set up coordinate system for looking at the common modulation types; this is called signal *constellation* diagram
- **Constellation diagram** for Binary Phase Shift Keying or BPSK: $x(t) \in \{\pm 1\}, y(t) = 0$



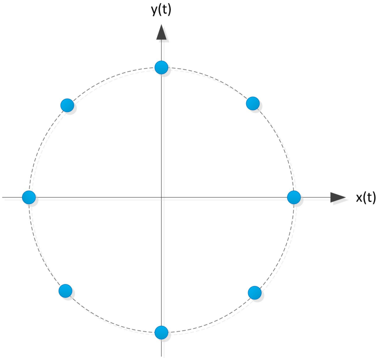
QPSK

→ Constellation diagram for Quadrature Phase Shift Keying or QPSK: $x(t) \in \{\pm 1\}, y(t) \in \{\pm 1\}$



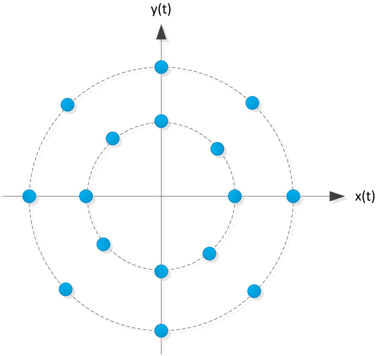
M-ary PSK

→ Constellation diagram for 8-ary Phase Shift Keying or 8-APSK:
$x(t) \in \left\{\pm 1, \pm\sqrt{2}\right\}, y(t) \in \left\{\pm 1, \pm\sqrt{2}\right\}$



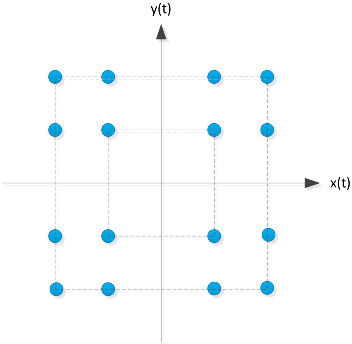
16-APSK

→ Constellation diagram for 16-ary Amplitude Phase Shift Keying or 16-APSK:
$x(t) \in \left\{\pm 1, \pm\sqrt{2}, \pm r, \pm r\sqrt{2}\right\}, y(t) \in \left\{\pm 1, \pm\sqrt{2}, \pm r, \pm r\sqrt{2}\right\}$



QAM

→ Constellation diagram for 16-ary Quadrature Amplitude Modulation or 16-QAM: $x(t) \in \{\pm 1, \pm 3\}, y(t) \in \{\pm 1, \pm 3\}$



Summary
of Constellation Diagrams

- Signals $x(t)$ and $y(t)$ make up the coordinate system
  - → Recall, $x(t)$ modulates the carrier $\cos(2\pi f_c t)$, and $y(t)$ modulates the quadrature carrier $\sin(2\pi f_c t)$
- Possible values of $[x(t), y(t)]$ are plotted on $2D$ plane with the above coordinate system as different points
- Probability of mistaking one transmitted symbol with another is dependent on the distance between the points
- At the receiver, the decision regarding which symbol was transmitted is made by choosing that point of the constellation diagram that is *closest* to the actual received signal location (recall the landmark point mentioned at the beginning of the lecture)

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