Lecture - 9

Recap:

random variable,
expected value.

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eg.

$$P(X=i) = \frac{c\,\lambda^i}{i!} \quad i=0,1,2,\ldots \rightarrow$$

$$P(X=0) = \underline{\underline{c}} = e^{-\lambda}$$

$$\boxed{\begin{array}{c} P(X>0) \\ \shortparallel \\ 1-P(X=0) \\ \shortparallel \\ 1-e^{-\lambda} \end{array}}$$

$$\sum_{i=0}^{\infty} P(X=i) = 1$$

$$c \cdot \sum_{i=0}^{\infty} \frac{\lambda^i}{i!} = 1$$

$$c \cdot e^{\lambda} = 1 \Rightarrow \underline{c = e^{-\lambda}}$$

Gamble A		Gamble B	2
89%	1 Crores	100%	1 Crores
4%	5 Crores		
7%	Nothing		
10		80	
1.09 Crores		1 Crore.	

<u>Note:</u> 2017
like to take risk, hypothetical

batch doesn't even in a thought experiment.

Gamble C		Gamble D	
4%	5 Crores.	11%	1 Crore.
96%	Nothing.	89%	Nothing.
80	20 lacs.	3	11 lacs.

Allais Paradox

Bag: 30 red balls. 3

60: black or yellow

b $\quad 60-b$

A	B
win if you get a red ball	win if you get a black ball
~~5~~ 30	~~3~~ 30
$30 > b$	$b > 30$?
love marriage	arranged marriage
C	**D**
red or yellow ball	black or yellow
25	50
$30 + (60-b) > (60-b) + b$	$(60-b) + b > 30 + (60-b)$

e.g. Quiz Show

q_1	0.6	200	p_1	r_1
q_2	0.8	100	p_2	r_2

X = amount of money that you win

Maximize $E(X)$

if $q_1 \to q_2$

X	
0	$1-p_1$
r_1	$p_1(1-p_2)$
r_1+r_2	$p_1 p_2$
	1

$$E_1(X) = r_1 p_1 (1 - p_2) + (r_1 + r_2) p_1 p_2$$

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if q_2, then q_1

X	p
0	$1 - p_2$
r_2	$p_2(1 - p_1)$
$r_1 + r_2$	$p_1 p_2$
	1

$$E_2(X) = r_2 p_2 (1 - p_1) + (r_1 + r_2) p_1 p_2$$

You answer q_1 first (6)

if $E_1(X) > E_2(X)$

$r_1 p_1 (1-p_2) + \cancel{(r_1 + r_2) p_1 p_2} >$

$r_2 p_2 (1-p_1) + \cancel{(r_1 + r_2) p_1 p_2}$

$r_1 p_1 (1-p_2) > r_2 p_2 (1-p_1)$

$$\frac{r_1 p_1}{1-p_1} > \frac{r_2 p_2}{1-p_2}$$

$$\frac{200 \cdot 0.6}{0.4} \qquad \frac{100 \; 0.8}{0.2}$$

300   400

Prefer to answer q_2 first.

functions of a r.v.

e.g.

X	p
-1	0.2
0	0.5
1	0.3

$E(X^2)$

$Y = X^2$

Y	p
0	0.5
1	0.5

$E(Y) = 0.5$

$E(X) = \sum x_i p_i$

$E(g(X)) = \sum g(x_i) p_i \rightarrow$

$= (-1)^2 \, 0.2 + (0)^2 (0.5) +$

$(1)^2 (0.3) = 0.5$

~~Thursday's~~ Tuesday's lecture is Tutorial 4

No tutorial during the afternoons.

e.g. You are wine dealer in Himachal. Your business is seasonal.

Each bottle you sell, you make a profit of b Rs.

Each bottle unsold, you lose l Rs.

X = no. of bottles you sell

↳ random varible. H.W.

X	p
0	p_0
1	p_1
2	p_2
⋮	
n	p_n

How many bottles will you stock? Tutorial 4

Property of $E(X)$

$$g(X) = ax + b$$

$$E(g(x)) = a\,E(x) + b$$

Variance:

$X = 0$ with $p = 1$

$Y = \begin{cases} -1 & p = 0.5 \\ +1 & p = 0.5 \end{cases}$

$Z = \begin{cases} -100 & p = 0.5 \\ +100 & p = 0.5 \end{cases}$

$$E(X) = E(Y) = E(Z) = 0$$

Kohli: 30, 40, 50, 50, 55,
45, 60, 70, 45, 55

Dhoni: 10, 10, 90, 80, 10, 90,
90, 20, 20, 80

$E(\text{kohli}) = E(\text{Dhoni})$
$= 50$

$Var(x) = E(X - E(X))^2$

DEFINITION

$\mu = E(X)$

$Var(x) = E(X - \mu)^2$

Thm:

$$E(X-\mu)^2 = E(X^2) - \mu^2$$

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Proof:

$$E(X-\mu)^2$$

$$= \sum (X-\mu)^2 \, p(x)$$

$$= \sum \left(\left(X^2 + \mu^2 - 2\mu X \right) \right) p(x)$$

$$= \boxed{\sum x^2 \, p(x)} + \mu^2 \boxed{\sum p(x)} - 2\mu \boxed{\sum x \, p(x)}$$

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$$= E(X^2) + \mu^2 - 2\mu^2$$

$$= E(X^2) - \mu^2$$

$$\text{''}$$

$$var(X)$$