Lec - 19

P ①

Show that Normal distribution is a probability density fn?

→ non-negative

→ $\int_{-\infty}^{\infty} f(x)\,dx = 1$

$$f(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$\int_{-\infty}^{\infty} f(x)\,dx = 1 \rightarrow$ H.W.

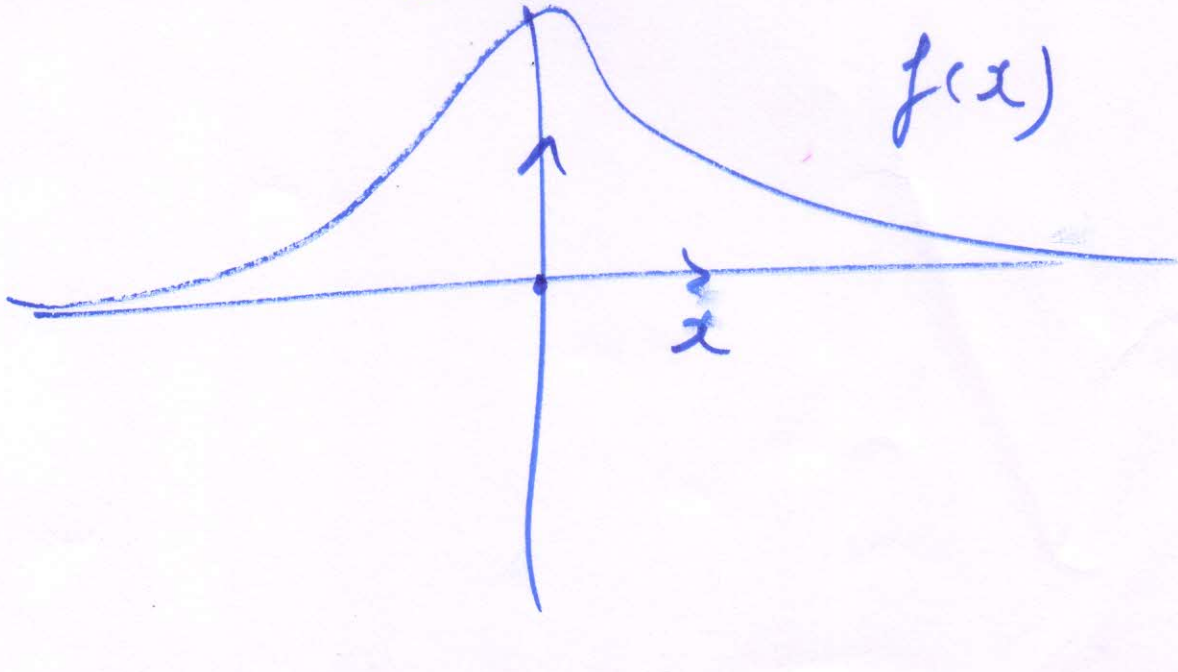
$$\left.\begin{aligned} E[X] &= \mu \\ Var[X] &= \sigma^2 \end{aligned}\right] \text{H.W.}$$

$aX + b$

$$\left.\begin{aligned} E[aX+b] &= a\mu + b \\ Var[aX+b] &= a^2\sigma^2 \end{aligned}\right] \text{H.W.}$$

standard Normal distribution

$\mu = 0, \quad \sigma = 1$

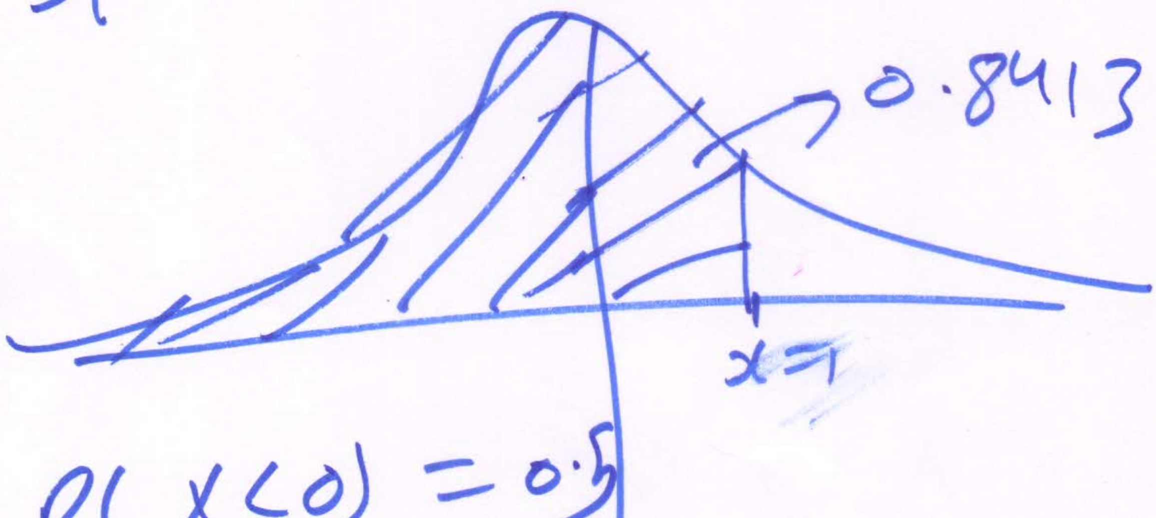


$$P(a < X < b) = \cancel{\phi(b) - \phi(a)}$$

$$\int_a^b \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx$$

$$P(a < x < b) = F(b) - F(a)$$

St. Normal $N(0,1)$



$$P(X < 0) = 0.5$$

e.g. X is n.d.

$\underset{\substack{\shortparallel \\ 0}}{\mu} = 3 \quad \underset{\substack{\shortparallel \\ 1}}{\sigma} = 3$

$P(2 < X < 5)$

$\rightarrow E[aX + b] = 0 = a\mu + b$

$Var(aX + b) = 1$

$a^2\sigma^2 = 1 \quad a^2\sigma^2 = 9$

$a = +\frac{1}{3}$ or $-\frac{1}{3}$

$a\mu + b = 0$

$\frac{1}{3} \cdot 3 + b = 0 \Rightarrow \underline{b = -1}$

$X \rightarrow \frac{X}{3} - 1 = \frac{X-3}{3}$

$$a^2 \sigma^2 = 1$$

$$a = \frac{1}{\sigma}$$

$$\frac{1}{\sigma} \cdot \mu + b = 0$$

$$b = \frac{-\mu}{\sigma}$$

$$X \Rightarrow \frac{1}{\sigma} \cdot X - \frac{\mu}{\sigma}$$

$$\boxed{X = \frac{X - \mu}{\sigma}}$$

$$P(2 < X < 5)$$

$$P\left(\frac{2-3}{3} < \underbrace{\frac{X-3}{3}}_{Y} < \frac{5-3}{3} \right)$$

6

$$P\left(-\frac{1}{3} < Y < \frac{2}{3}\right)$$

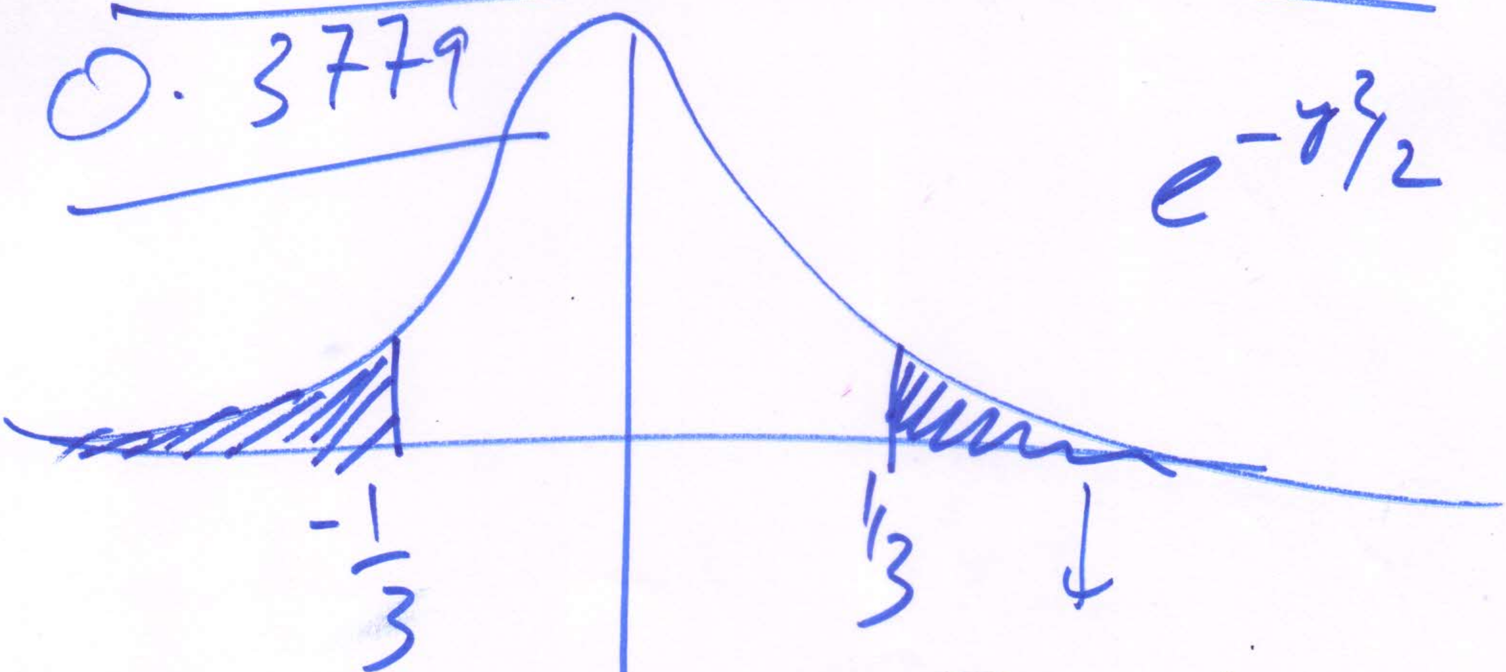
$$= \Phi\left(\frac{2}{3}\right) - \Phi\left(-\frac{1}{3}\right)$$

$$= \Phi(0.67) - \Phi(-0.33)$$

$$= 0.7486 - (1 - \Phi(0.33))$$

$$= 0.7486 + 0.6293 - 1$$

$$0.3779$$



$$\Phi(-x) = 1 - \Phi(x)$$

$$1 - \Phi(1/3)$$

Grading (μ, σ)

Grade	Marks	No. of Students
AA	$> \mu + 3\sigma$	1
AB	$\mu + 2\sigma$ to $\mu + 3\sigma$	7
BB	$\mu + \sigma$ to $\mu + 2\sigma$	46
BC	μ to $\mu + \sigma$	116
CC	$\mu - \sigma$ to μ	116
CD	$\mu - \sigma$ to $\mu - 2\sigma$	46
DD	$\mu - 2\sigma$ to $\mu - 3\sigma$	7
DE	$< \mu - 3\sigma$	1
F		

8

$$P(\mu < X < \mu + \sigma)$$

$$P\left(\frac{\mu - \mu}{\sigma} < \frac{X - \mu}{\sigma} < \frac{\mu + \sigma - \mu}{\sigma}\right)$$

$$P(0 < Y < 1)$$

$$= \Phi(1) - \Phi(0)$$

$$= 0.8413 \quad 0.5$$

$$= 0.3413 \quad \times 340$$

$$= 116$$

$$\phi(2) - \phi(1) = 0.9772 \; 0.8413$$

$$= \underline{0.1359} \quad \times 340$$

$\phi(3) - \phi(2)$

$0.9987 - 0.9772$

$= 0.0215 \times 340$

$1 - \phi(3) = 1 - 0.9987$

$= 0.0017 \times 340$

e.g. Communication

Binary $\Rightarrow$ Volts

$$\left.\begin{array}{l} 0 \rightarrow -2V \\ 1 \Rightarrow +2V \end{array}\right\} X$$

You receive R

if $R < 0.5$, its 0

if $R \geq 0.5$, its 1

$$R = X + \underset{\substack{\downarrow \\ N(0,1)}}{N}$$

$P(X=0) = 1/3$

$P(X=1) = 2/3$

Probability of error

$\parallel$

P(error | 0 was sent)
P(0 was sent) +

P(error | 1 was sent) ~~+~~
P(1 was sent)

$P(\underline{e \mid X=0}) P(X=0) +$

$P(\underline{e \mid X=1}) P(X=1)$

$P(e \mid X = 0)$

$\parallel$

$R = X + N$

Sending -2

$R > 0.5$

$X + N > 0.5$

$N > 0.5 - X$

$N > 2.5$

$1 - \Phi(2.5) = 1 - 0.9938$

$= 0.0062$

$P(e \mid X=1)$

Sending 2

$R < 0.5$

$X + N < 0.5$

$N < 0.5 - X$

$N < 0.5 - 2$

$N < -1.5 = \phi(-1.5)$

$= 1 - \phi(1.5)$

$= 1 - 0.9332 = 0.0668$

Exponential random variable

$\lambda > 0$

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

It is a density function.

$$\int_0^\infty f(x)\,dx = 1$$

$$= \lambda \int_0^\infty e^{-\lambda x}\,dx = 1$$

Cumulative.

$$F(a) = P(X \leq a)$$