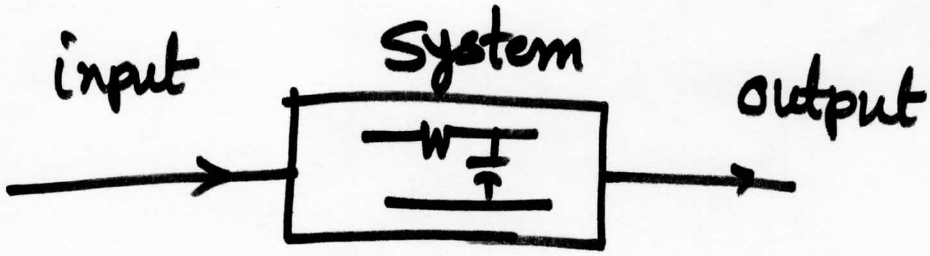


$$e^{st} \longrightarrow \frac{1}{1+sRC}\, e^{st}$$

$\underline{\underline{s = j\omega}}$

$$e^{j\omega t} \longrightarrow \frac{1}{1+j\omega RC}\, e^{j\omega t}$$

$$= \cos\omega t + j\sin\omega t$$

$$= \frac{1}{\sqrt{1+(\omega RC)^2}}\, e^{j(\omega t - \underbrace{\tan^{-1}\omega RC}_{=\theta})}$$

$$\cos\omega t = \mathrm{Re}\left[e^{j\omega t}\right] \longrightarrow \frac{1}{\sqrt{1+(\omega RC)^2}} \cos(\omega t - \theta)$$

$$\sin\omega t \longrightarrow \frac{1}{\sqrt{1+(\omega RC)^2}} \sin(\omega t - \theta)$$

Filters

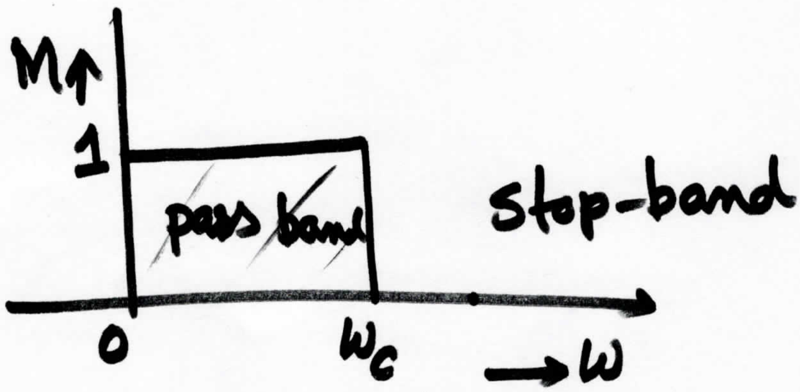
1) Low Pass Filter (LPF)

$$\frac{V_o}{V_i} = H(s)$$

$$H(s)\Big|_{s=j\omega} = H(j\omega) = M e^{j\theta}$$

where $M = |H(j\omega)|$ &

$\theta = \angle H(j\omega)$



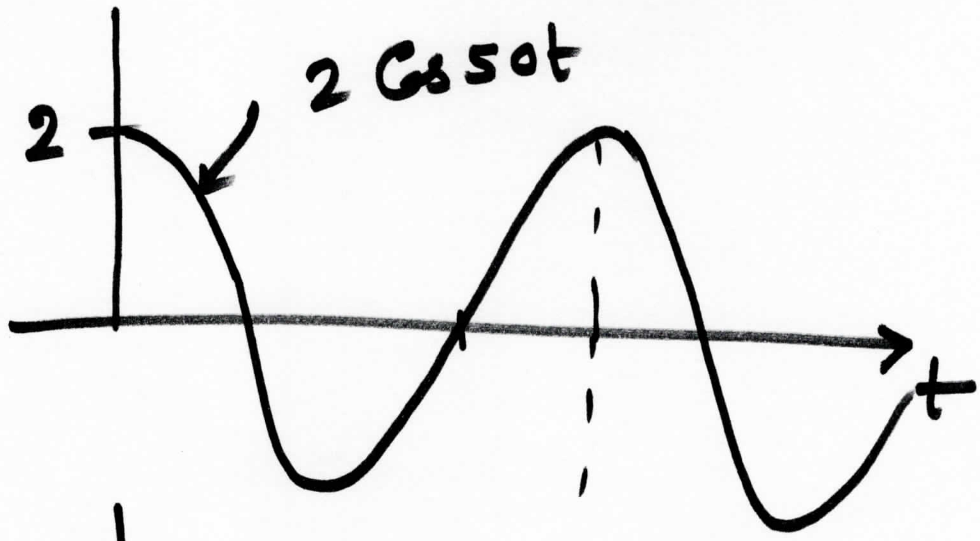
Cut-off frequency

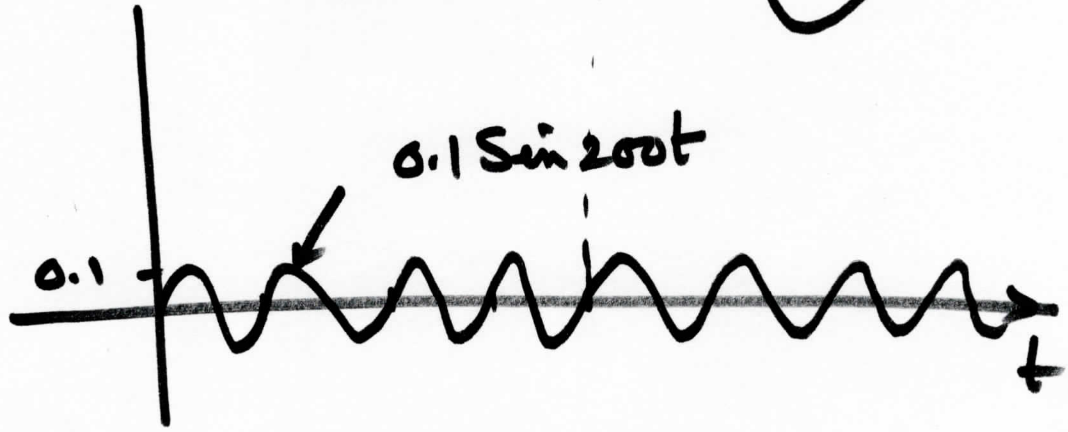
ideal LPF

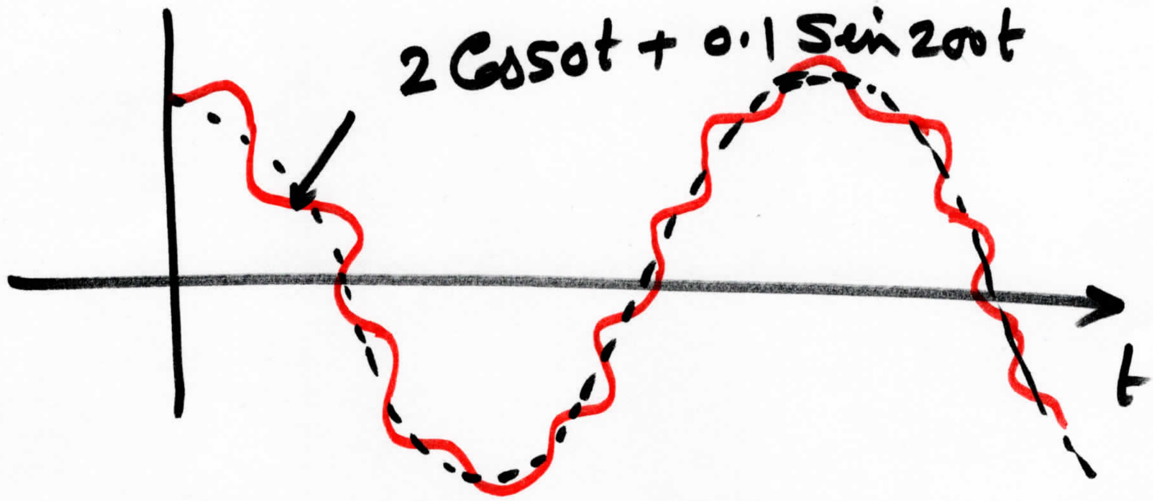
$\omega_c = 100$ rad/sec.

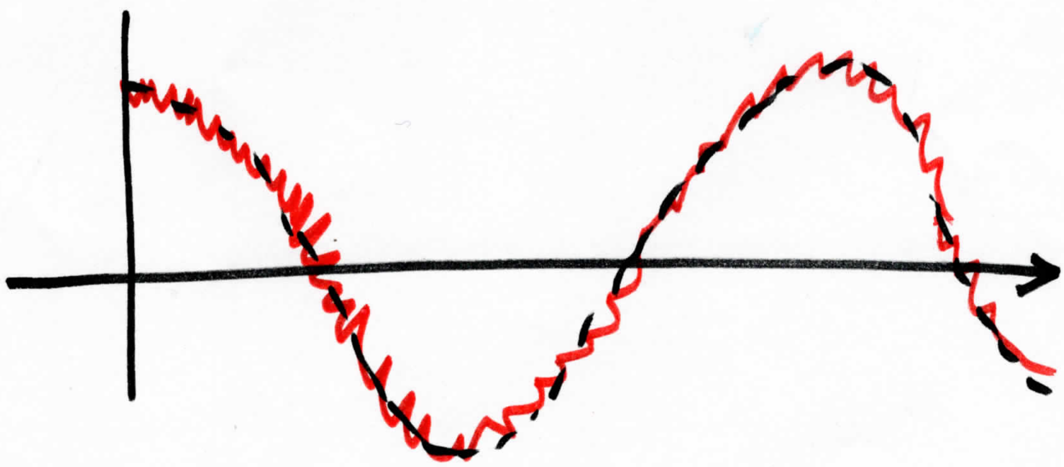
$v_i(t) = 2\cos 50t + 0.10 \sin 200t$

$v_o(t) = 2\cos 50t$

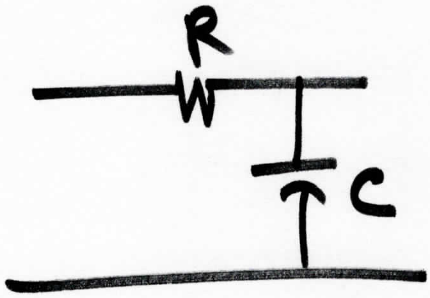

2 Cos 50t
2
t


0.1 Sin 200t
0.1
t


2 Cos 50t + 0.1 Sin 200t
t



Practical LPF :

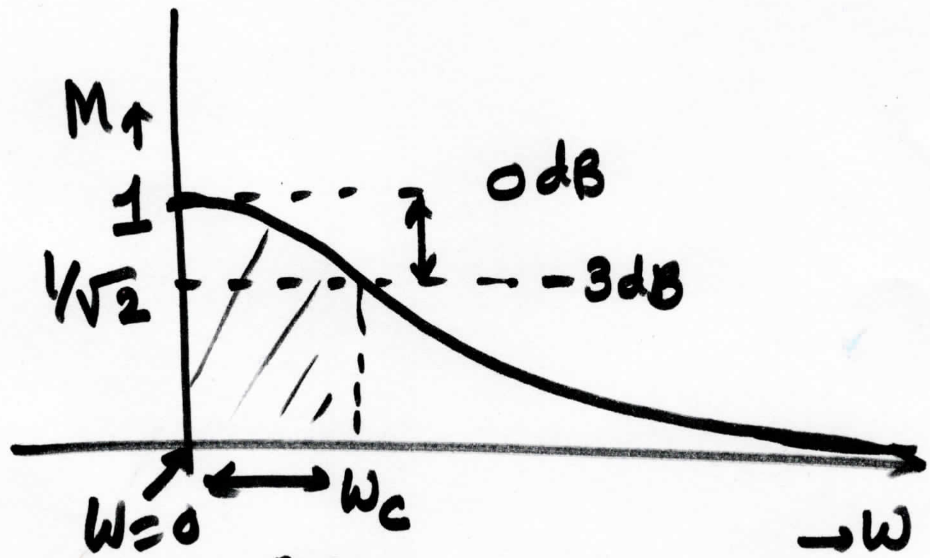


$$H(s) = \frac{1}{1+sRC}$$

$$H(j\omega) = \frac{1}{1+j\omega RC}$$

$$\Rightarrow M(\omega) = \frac{1}{\sqrt{1+\omega^2R^2C^2}} = \frac{1}{\sqrt{1+\left(\frac{\omega}{\omega_c}\right)^2}}$$

where $\omega_c = \frac{1}{RC}$



3dB BW

$20\log_{10} M \rightarrow$ in dB

$M \rightarrow$ voltage gain

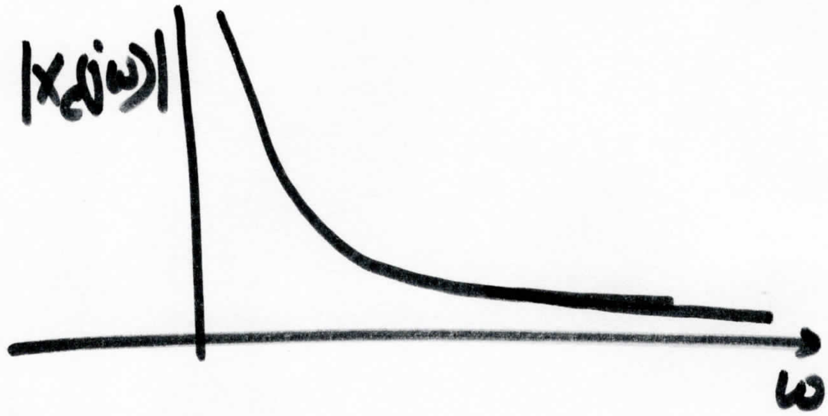
$20\log\frac{1}{\sqrt{2}}$

$= -10\log 2$

$= -3$dB

$$X_c(s) = \frac{1}{Cs}$$

$$|X_c(j\omega)| = \left|\frac{1}{j\omega C}\right| = \frac{1}{\omega C}$$

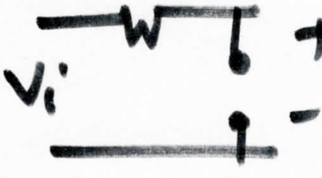


For DC : $|X_c| \rightarrow \infty$ (open ckt)

For high freq. AC: $|X_c| \rightarrow 0$ (Short ckt)

ω	M
0	1
∞	0

$\underline{\omega = 0}$



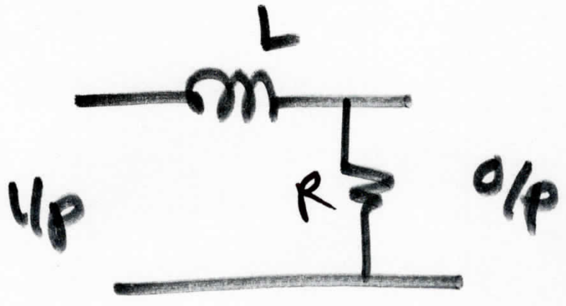
$V_o = V_i$

$\Rightarrow M = 1$

$\underline{\omega = \infty}$

$V_o = 0$

$\Rightarrow M = 0$

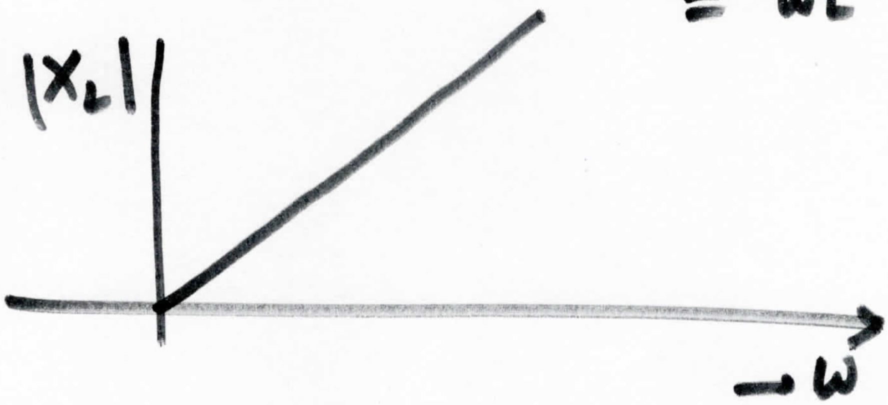


$$X_L(s) = sL$$

$$|X_L(j\omega)| = |j\omega L|$$

$$= \omega L$$

$$\frac{V_o}{V_i} = \frac{R}{R + Ls}$$



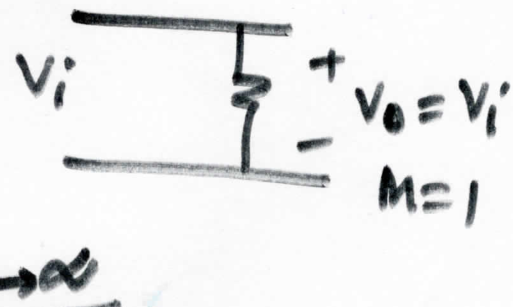
For DC : $|X_L| = 0$ (Short ckt)

For high freq AC : $|X_L| \rightarrow \infty$ (open ckt)

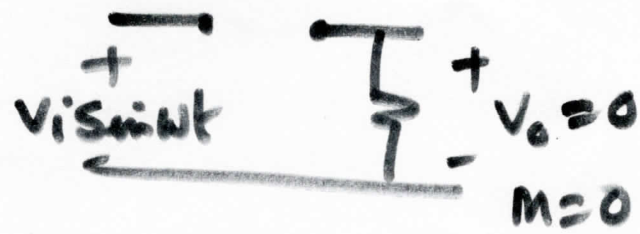
ω	M
0	1
∞	0

(LPF)

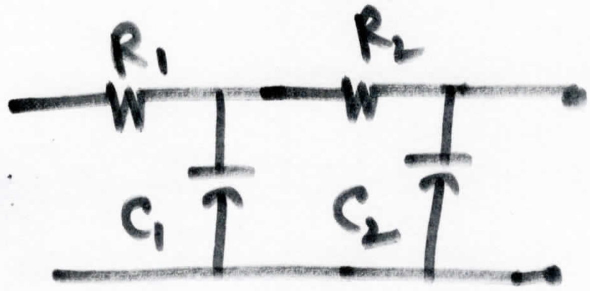
$\underline{\omega = 0}$

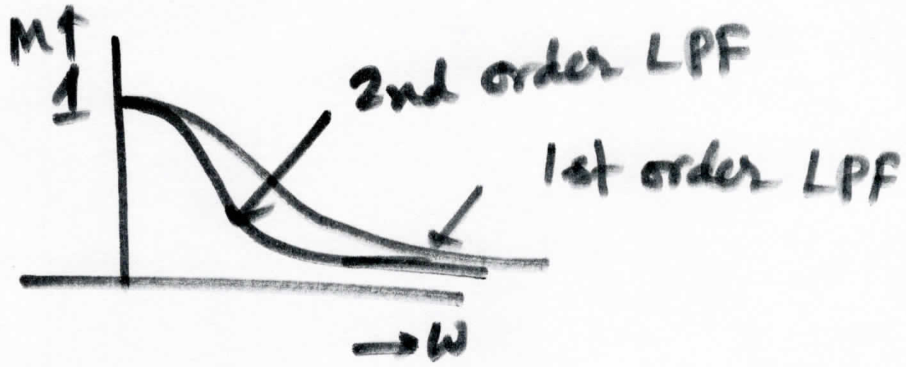


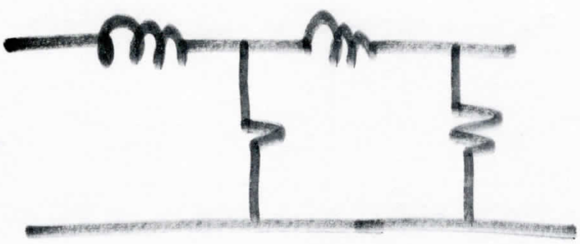
$\underline{\omega \rightarrow \infty}$

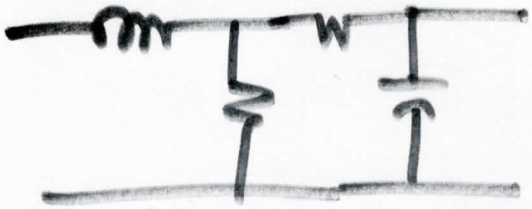


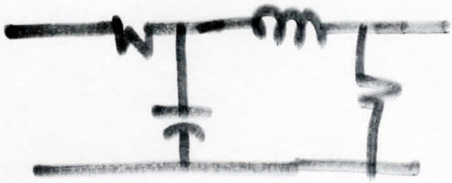
Higher order LPF

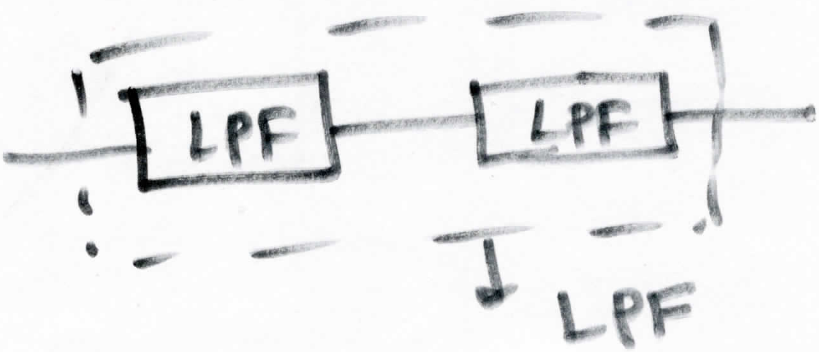




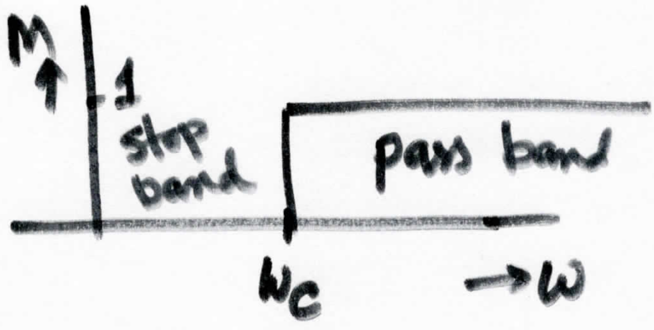






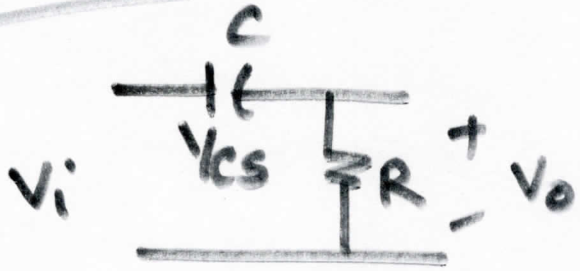


High Pass Filter (HPF)



ideal

Practical



$$\frac{V_o}{V_i} = H(s) = \frac{R}{R + 1/Cs}$$

$$= \frac{RCs}{1 + RCs}$$

$$H(s) = \frac{RCs}{1 + RCs}$$

$$= \frac{1}{1 + \frac{1}{RCs}}$$

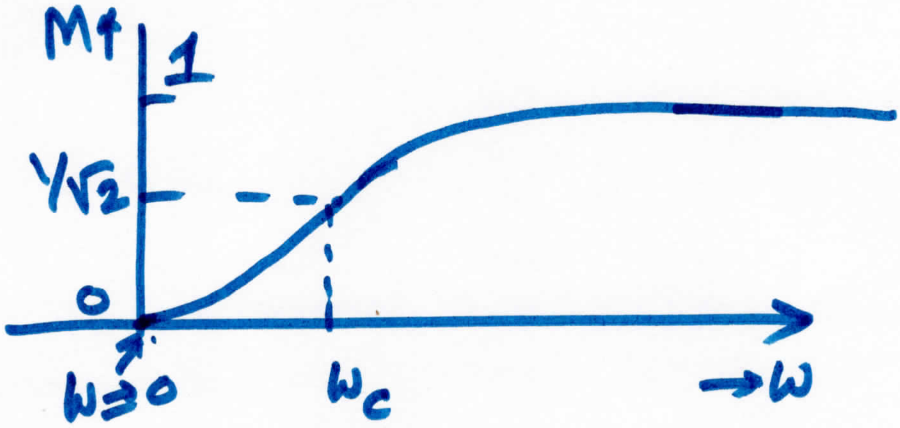
$$\Rightarrow H(j\omega) = \frac{1}{1 + \frac{1}{j\omega RC}} = \frac{1}{1 + \frac{1}{j}\left(\frac{\omega_c}{\omega}\right)} = \frac{1}{1 - j\left(\frac{\omega_c}{\omega}\right)}$$

where $\omega_c = \frac{1}{RC}$

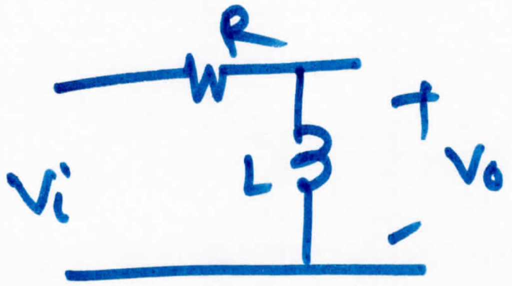
$$M = |H(j\omega)|$$

$$= \frac{1}{\sqrt{1 + \left(\frac{\omega_c}{\omega}\right)^2}}$$

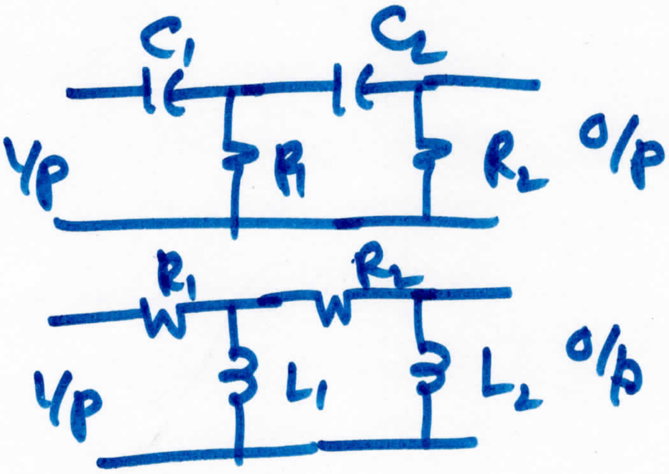
ω	M
0	0
∞	1

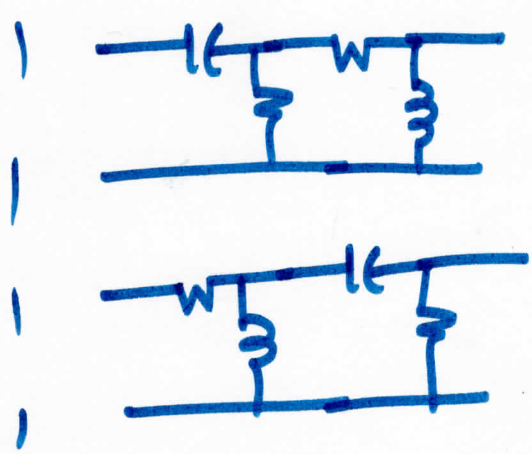


with L & R



ω	M
0	0
∞	1



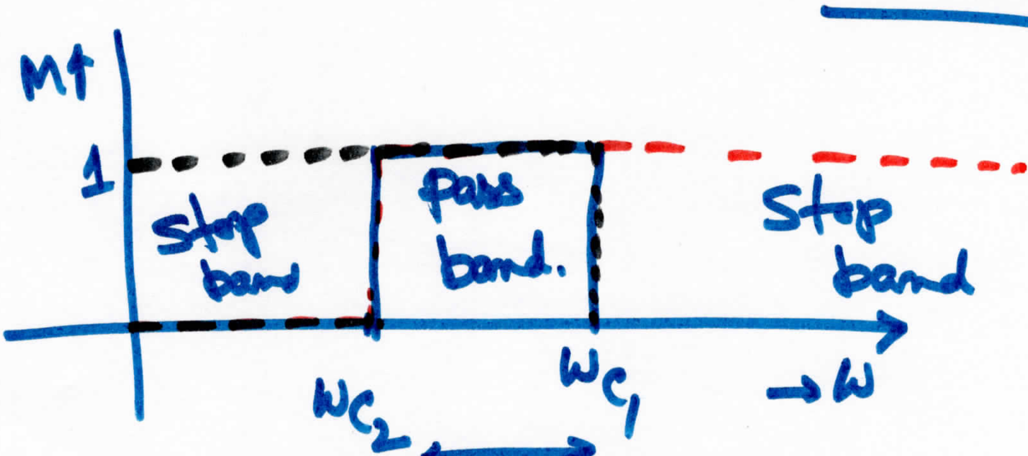


Band Pass Filter (BPF)

$$V_i \longrightarrow \boxed{\text{LPF}} \longrightarrow \boxed{\text{HPF}} \longrightarrow V_o$$

LPF: ω_{c_1} HPF: ω_{c_2}

$\omega_{c_2} < \omega_{c_1}$

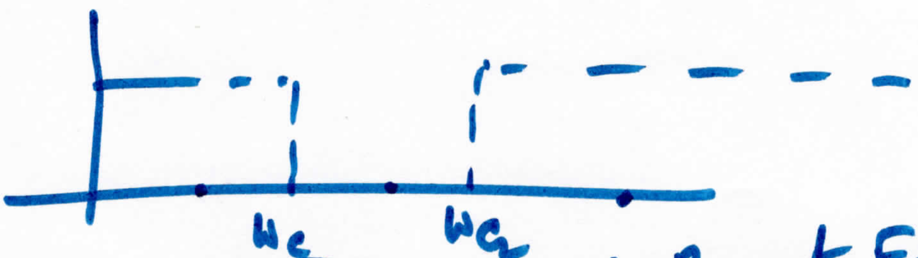


$BW = \omega_{c_1} - \omega_{c_2}$

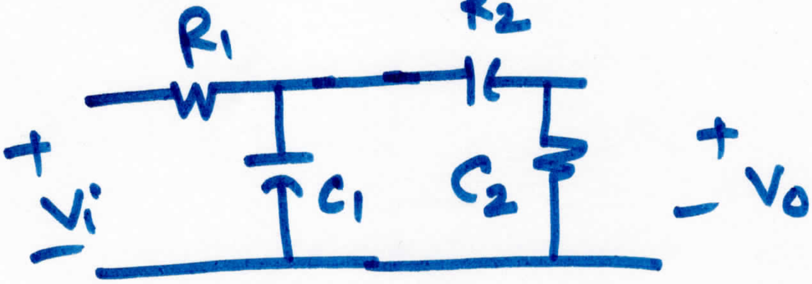
ideal

$$\longrightarrow \boxed{\text{HPF}} \longrightarrow \boxed{\text{LPF}} \longrightarrow$$

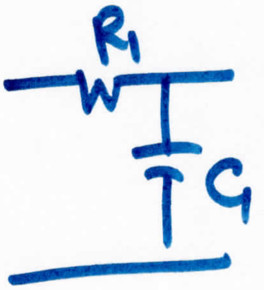
$\omega_{c_2} > \omega_{c_1} \longrightarrow V_o = 0$



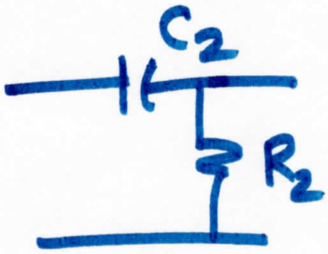
NOT a Band Reject Filter



$$H(s) \neq H_1(s)H_2(s)$$



$$\Rightarrow \quad H_1(s) = \frac{\frac{1}{C_1 s}}{R_1 + \frac{1}{C_1 s}} = \frac{1}{1 + R_1 C_1 s}$$



$$\Rightarrow \quad H_2(s) = \frac{R_2}{R_2 + \frac{1}{C_2 s}}$$

$$= \frac{R_2 C_2 s}{1 + R_2 C_2 s}$$

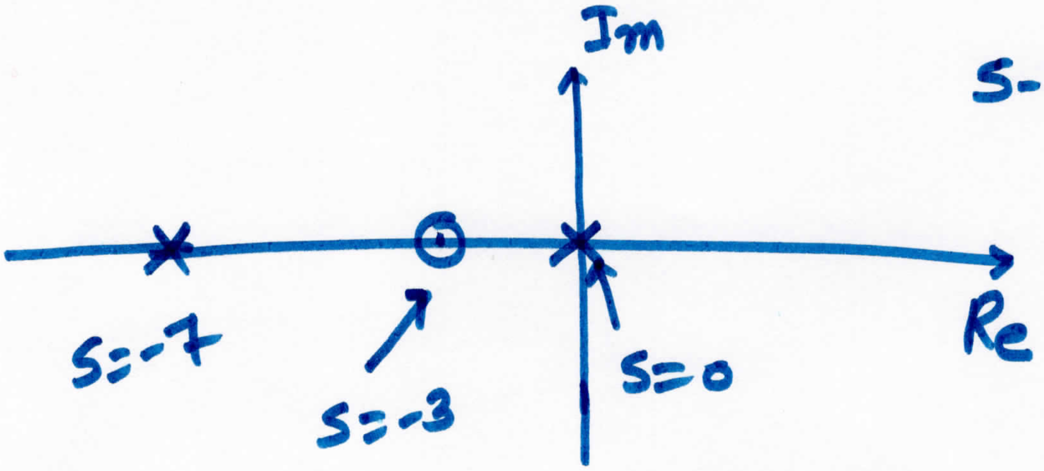
$$H(s) = \frac{P(s)}{Q(s)}$$

$P(s) = 0 \Rightarrow s = ?$ Zeros. ('o')

$Q(s) = 0 \Rightarrow s = ?$ poles ('x')

ex

$$H(s) = \frac{2(s+3)}{s(s+7)} \qquad s = \sigma + j\omega$$

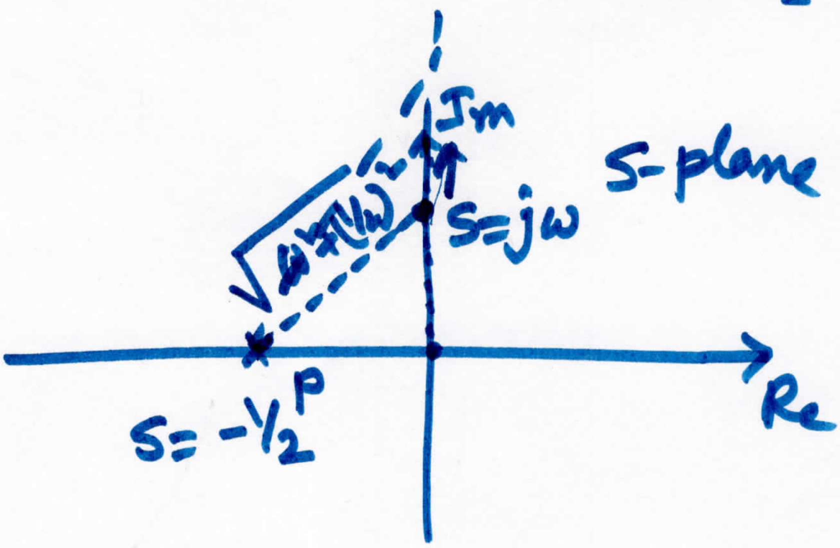


s-plane

ex **LPF**

$$H(s) = \frac{1}{1+2s} = \frac{1/2}{s+1/2}$$

$$M = |H(j\omega)| = \left| H(s)\big|_{s=j\omega} \right|$$

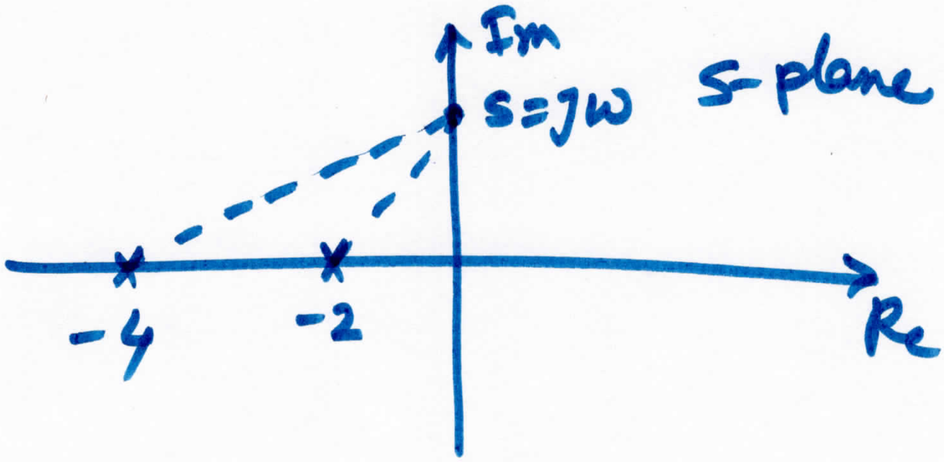


$$|H(j\omega)| = \frac{1/2}{\sqrt{\omega^2 + (1/2)^2}} = \frac{1/2}{SP}$$

$$H(s) = \frac{K(s+z_1)(s+z_2)\cdots}{(s+p_1)(s+p_2)\cdots}$$

ex

$$H(s) = \frac{1}{(s+2)(s+4)}$$



$$M = |H(j\omega)|$$

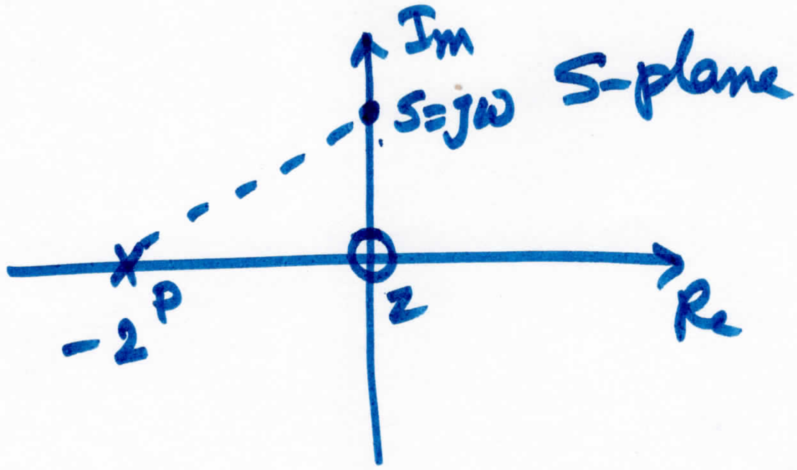
$$= \frac{1}{\sqrt{2^2+\omega^2}\cdot\sqrt{4^2+\omega^2}}$$

$\omega\uparrow \longrightarrow M\downarrow$ (LPF)

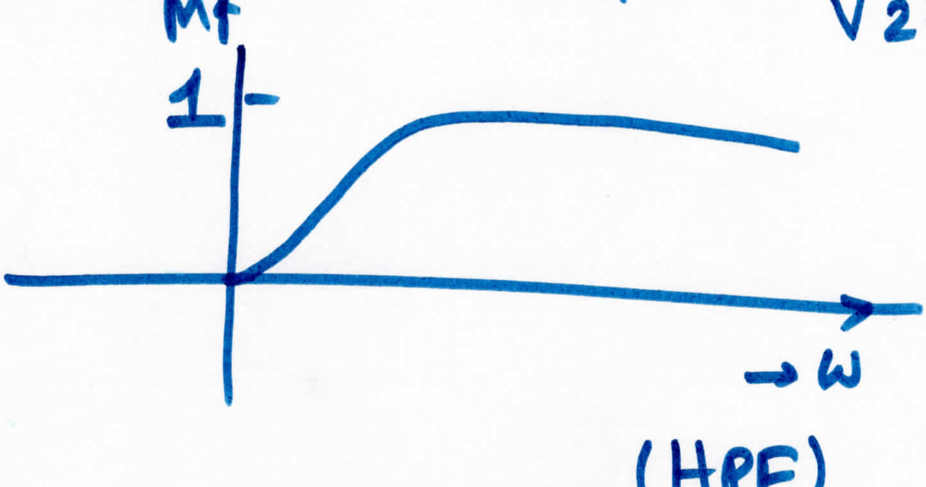
All poles system : $\longrightarrow$ LPF

Ex

$$H(s) = \frac{s}{s+2}$$



$$M = |H(j\omega)| = \frac{\overline{SZ}}{\overline{SP}} = \frac{\omega}{\sqrt{2^2+\omega^2}}$$



(HPF)

$$H(s) = \frac{s^2}{(s+2)(s+4)}$$

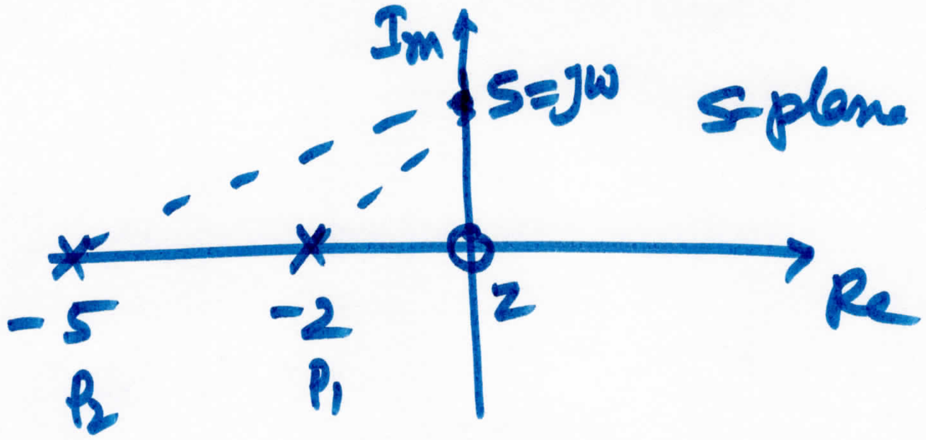
No. of zeros at origin = No. of poles on the LHS of s-plane

$\Downarrow$

HPF

ex

$$H(s) = \frac{s}{(s+2)(s+5)} = \left(\frac{s}{s+2}\right) \times \frac{1}{(s+5)}$$



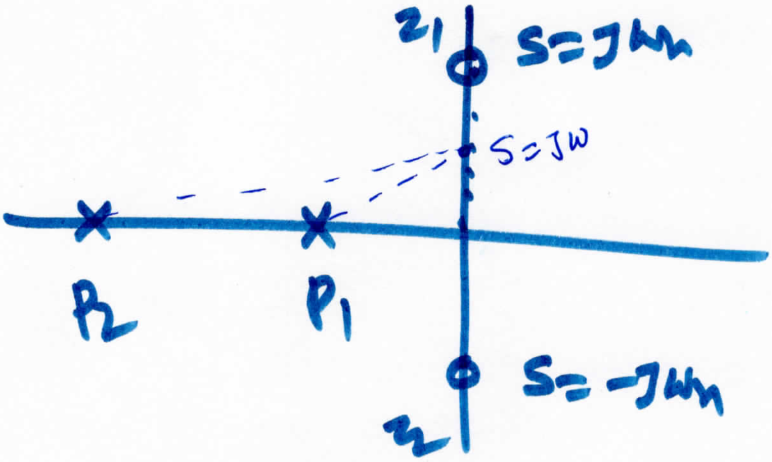
$$M = |H(j\omega)|$$

$$= \frac{\overline{sz}}{\overline{sp_1} \cdot \overline{sp_2}}$$

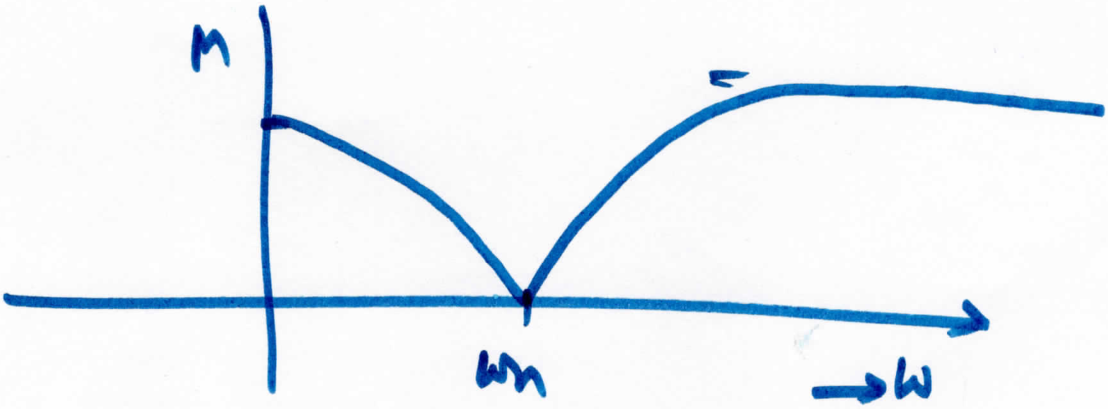
$$= \frac{\omega}{\sqrt{2^2+\omega^2}\,\sqrt{5^2+\omega^2}}$$

ex

$$H(s) = \frac{s^2 + \omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$



$$M = \frac{\overline{sz_1} \cdot \overline{sz_2}}{\overline{sp_1} \cdot \overline{sp_2}}$$



Notch Filter
(Notch)

$$H(s) = \frac{Q(s)}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

LPF: $Q(s) = \omega_n^2$

HPF: $Q(s) = s^2$

BPF: $Q(s) = s$

BRF: $Q(s) = s^2 + \omega_n^2$

APF: $Q(s) = s^2 - 2\zeta\omega_n s + \omega_n^2$

(All pass Filter)

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$$

$$s = \frac{-2\zeta\omega_n \pm \sqrt{4\zeta^2\omega_n^2 - 4\omega_n^2}}{2}$$

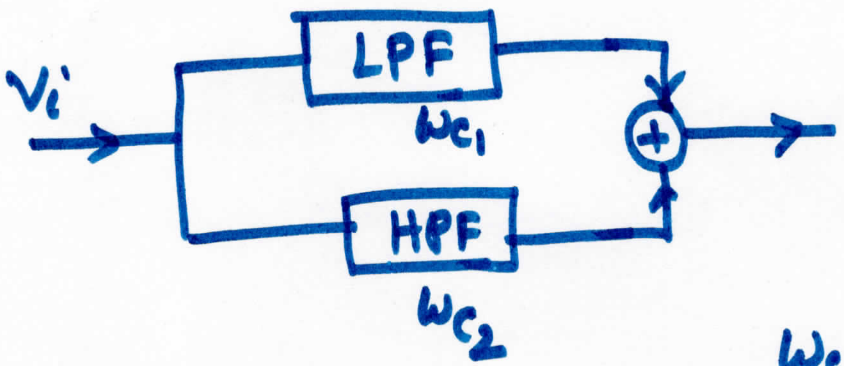
$$= -\zeta\omega_n \pm \omega_n\sqrt{\zeta^2 - 1}$$

$\zeta > 1 \rightarrow$ real & unequal — over-damped

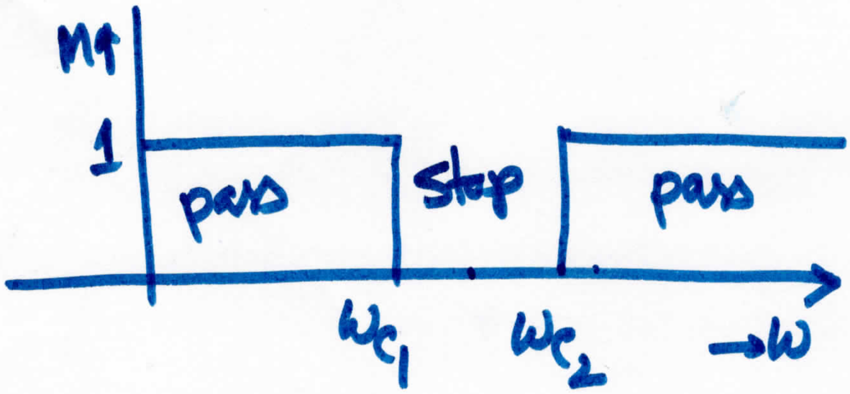
$\zeta = 1 \rightarrow$ real & equal — Critically "

$0 < \zeta < 1 \rightarrow$ Complex Conjugate — under damped

BRF

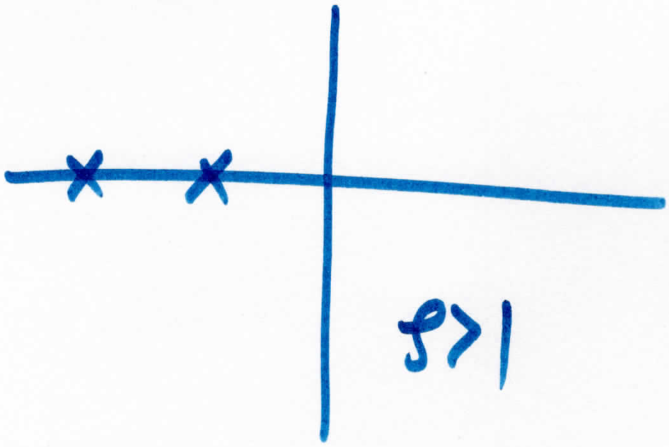


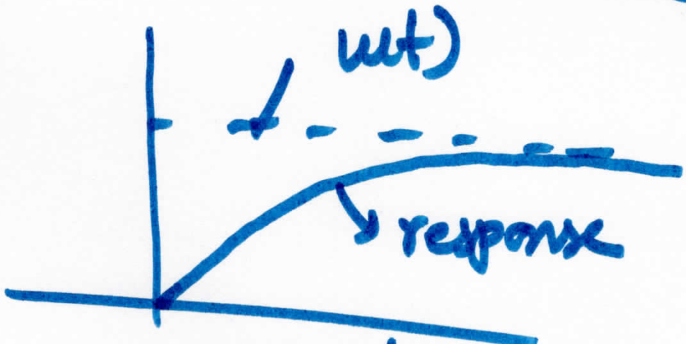
$\omega_{c_2} > \omega_{c_1}$



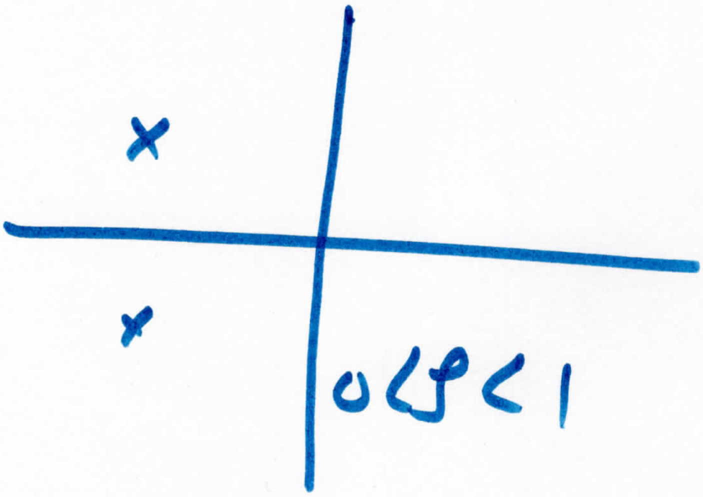
LPF

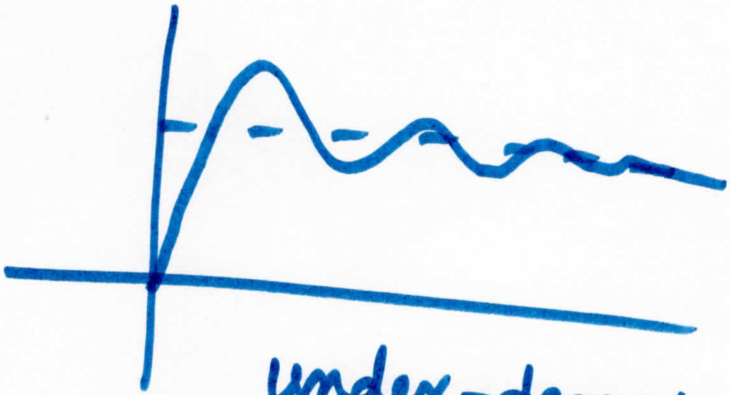
Step-response



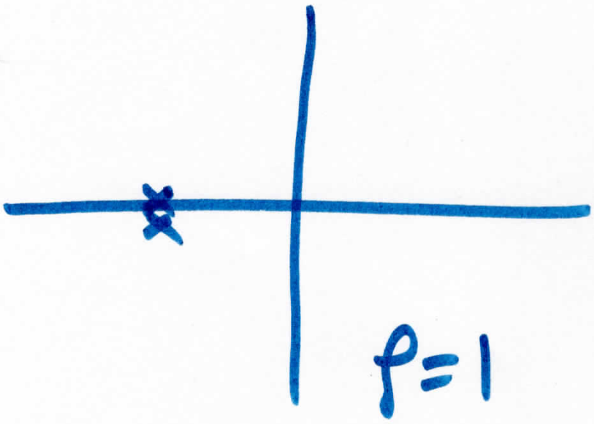


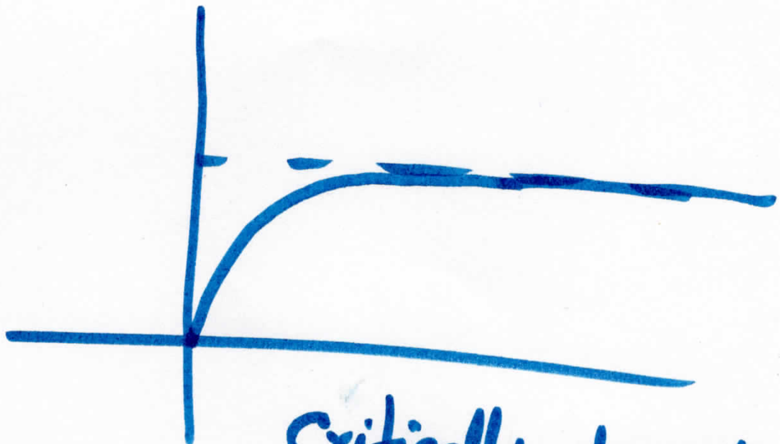
overdamped





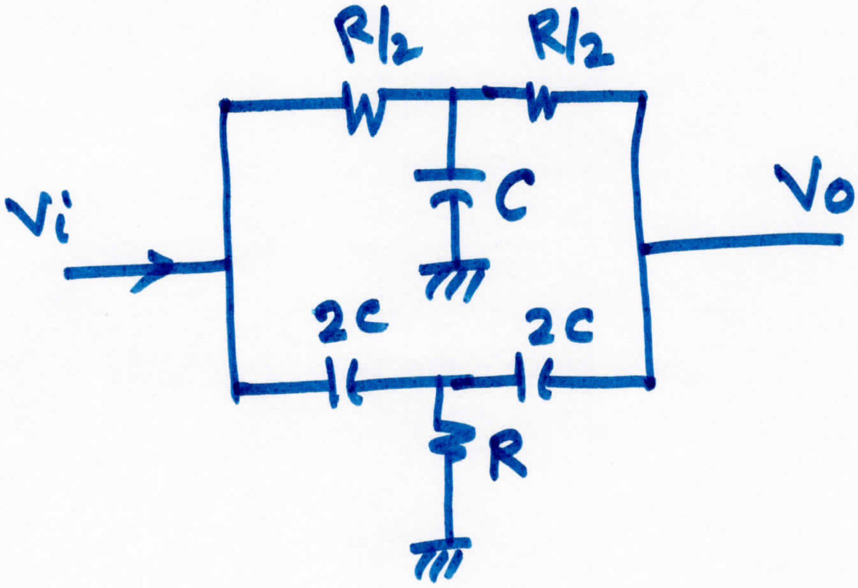
under-damped



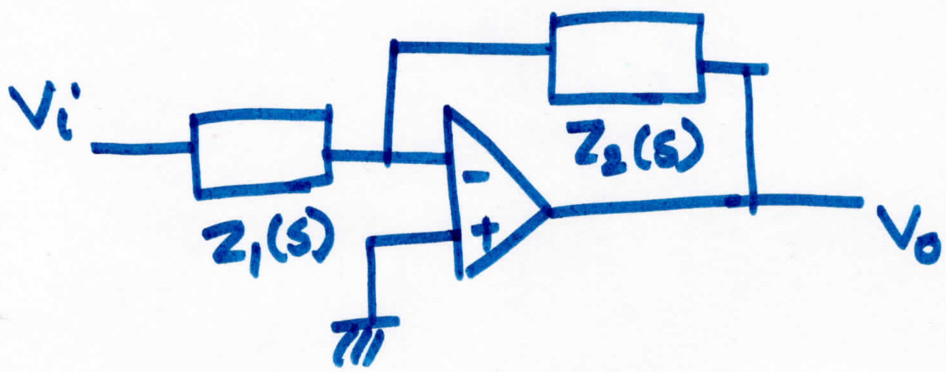


Critically-damped

Design of BRF



Active Filters



$$\frac{V_o(s)}{V_i(s)} = -\frac{Z_2(s)}{Z_1(s)}$$

For $Z_1(s)$ or $Z_2(s)$:

1) 

$\Rightarrow Z(s) = R$

2) 

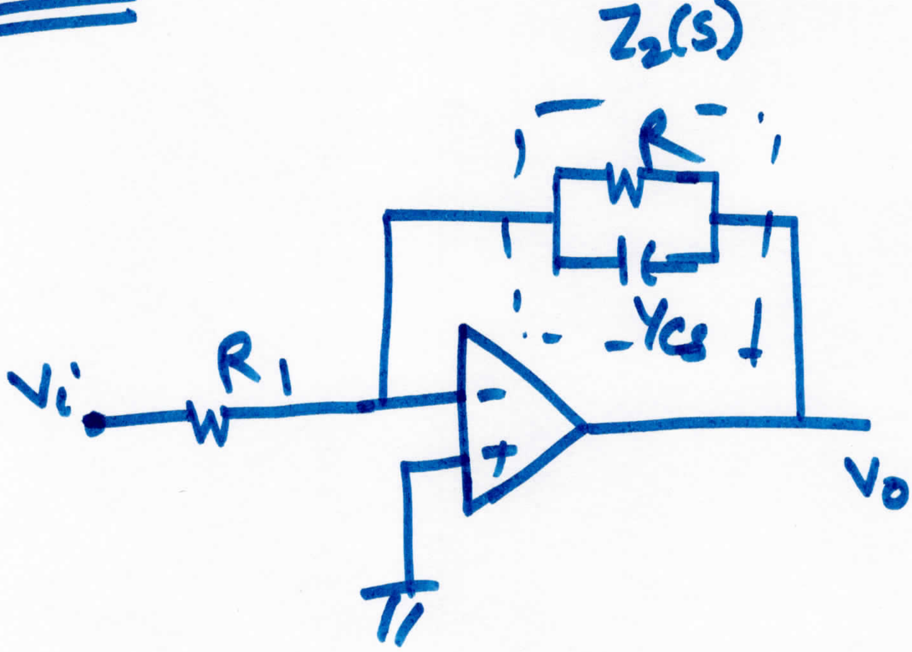
$\Rightarrow Z(s) = 1/Cs$

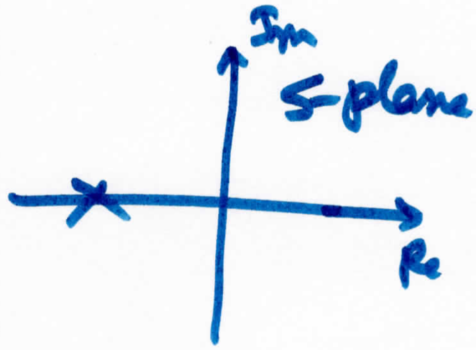
3) 

$\Rightarrow Z(s) = \dfrac{1+RCs}{Cs}$

4) 
$\Rightarrow Z(s) = \dfrac{R}{1+RCs}$

LPF

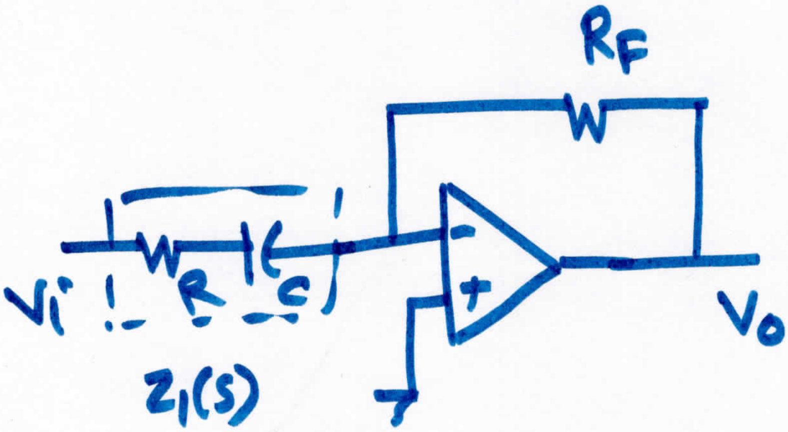


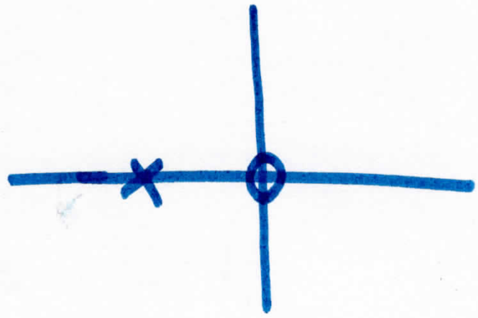


$$\frac{V_o}{V_i}(s) = -\frac{Z_2(s)}{Z_1(s)} = -\frac{R/(1+RCs)}{R_1}$$

$$= -\frac{(R/R_1)}{(1+RCs)}$$

HPF

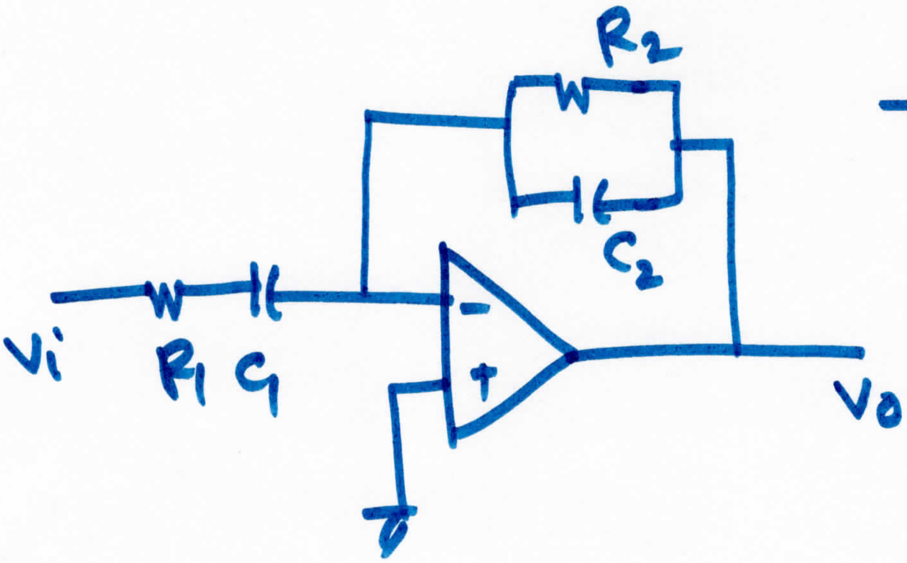


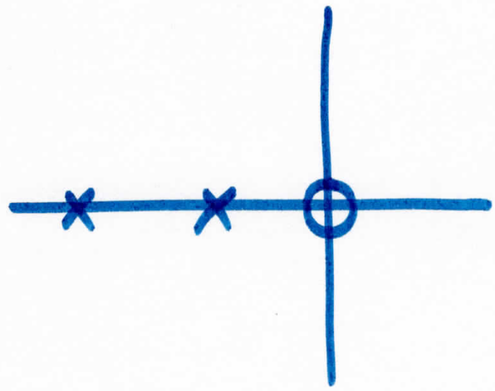


$Z_1(s) = \frac{1+RCs}{Cs}$

$$\frac{V_o}{V_i} = -\frac{R_F}{\frac{(1+RCs)}{Cs}} = \frac{-R_FCs}{1+RCs}$$

BPF



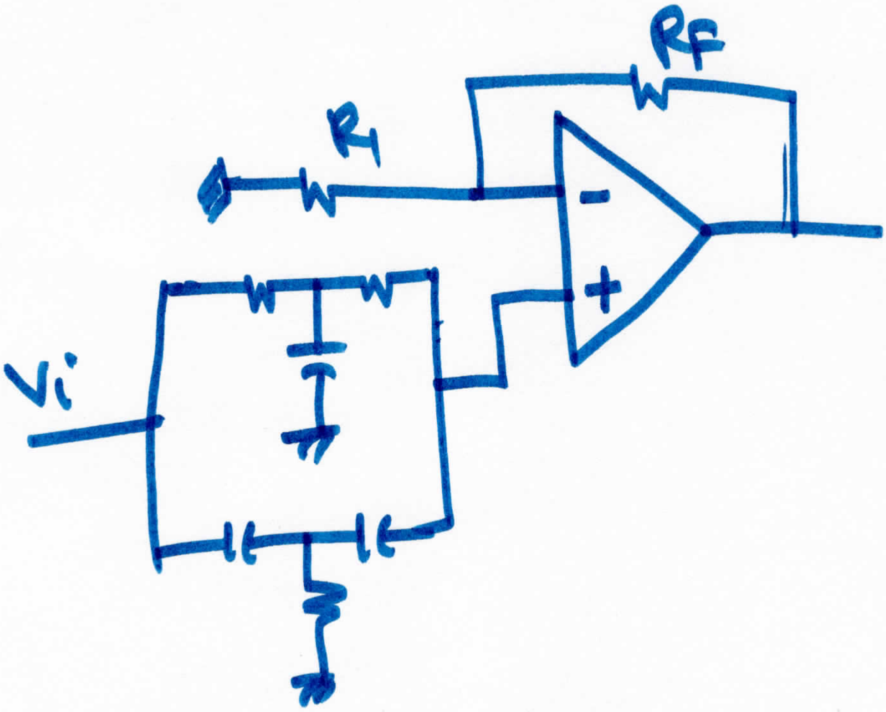


$$\frac{V_o}{V_i} = -\frac{Z_2(s)}{Z_1(s)}$$

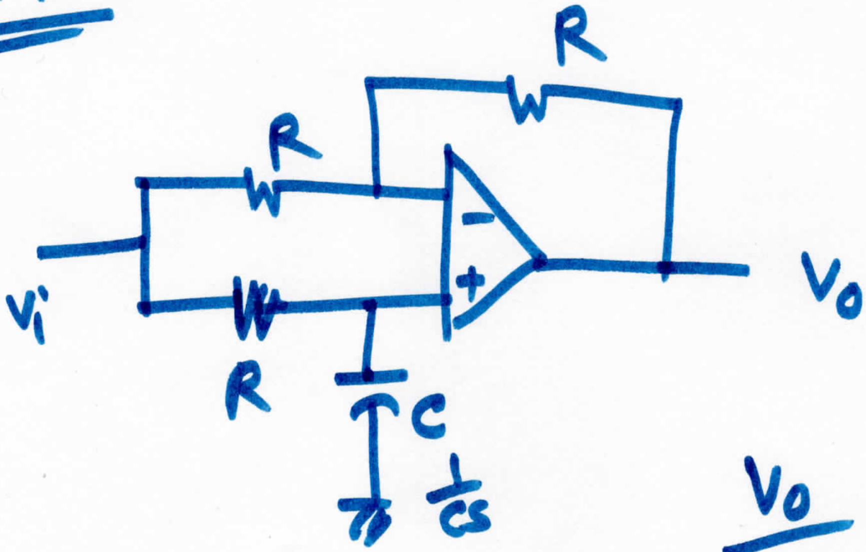
$$= -\frac{R_2/(1+R_2C_2s)}{\dfrac{(1+R_1C_1s)}{C_1s}}$$

$$= -\frac{R_2C_1s}{(1+R_1C_1s)(1+R_2C_2s)}$$

B NOTCH Filter



APF



$$\frac{V_o}{V_i} = ? = H(s)$$