CT111: Introduction to Communication Systems

In-Class Quiz 1

Date: Friday 19^{th} January, 2018

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1. A complex exponential phasor is a mathematical function of four parameters: amplitude A, time t, frequency f, and initial phase θ.

   (a) Write the mathematical expression of this complex phasor, and

   → $w(t) = A \exp\left(j\left(2\pi f t + \theta\right)\right)$.

   (b) Show what this phasor looks like in the complex-number plane.

   → See Fig. 1.

   Take $A = 2$ Volts, $f = 2$ Hz, and $\theta = 45^0$.

   (a) Draw the time-domain diagram over a range of time t from 0 seconds to 1 second.

   → See Fig. 2.

   (b) Draw its frequency-domain diagram over a range of frequency f from -4 Hz to $+4$ Hz.

   → See Fig. 3.

2. A complex exponential phasor completes one cycle in 3 seconds. This phasor is sampled in time domain once every 0.5 seconds.

   (a) Draw in the complex-number plane the samples of this phasor collected over 3 seconds.

   → The phasor completes one cycle in $T_{cycle} = 3$ second. Therefore, when it is sampled every 0.5 second, there will be $3/0.5 = 6$ equispaced samples over unit circle, spaced $360/6 = 60^o$ apart. This is shown in Fig. 4.

   (b) *What* is the *polynomial* equation that these samples are the solutions of?

   → All of these 6 equispaced points along the unit circle are the solutions of the polynomial $p_6(x) : x^6 - 1 = 0$.

   (c) *Why* are they the solutions of the polynomial equation that you have provided as the answer to Question 2b?

   → Relationship between a complex number x and its sixth power $y = x^6$ is as follows. The magnitude of $y = x^6$ is given as $|y| = |x|^6$ and the phase angle of y is given as $\theta_y = 6\,\theta_x$.

   → Index the six points by m, where $m = 0, 1, \ldots, 5$. The m^{th} point has a magnitude of 1 and phase of $2\pi m/6 = \pi m/3$. Therefore, the magnitude of $y = x^6$ is given as $|y| = 1,\ \forall m$, and the phase angle of y is given as $\theta_y = 6\,\theta_x = 6\pi m/3 = 2\pi m,\ \forall m$. Thus, all these six points map to the same $y = x^6 = 1\, e^{j2\pi\, m} = 1$.

   → Therefore, these are the solutions of $p_6(x) : x^6 = 1$.

3. Explain the following:

   (a) What is the meaning, or definition, of frequency f?

   → Frequency is the number of cycles per second of a periodic signal or waveform in the time domain.

   (b) How can the frequency f be negative?

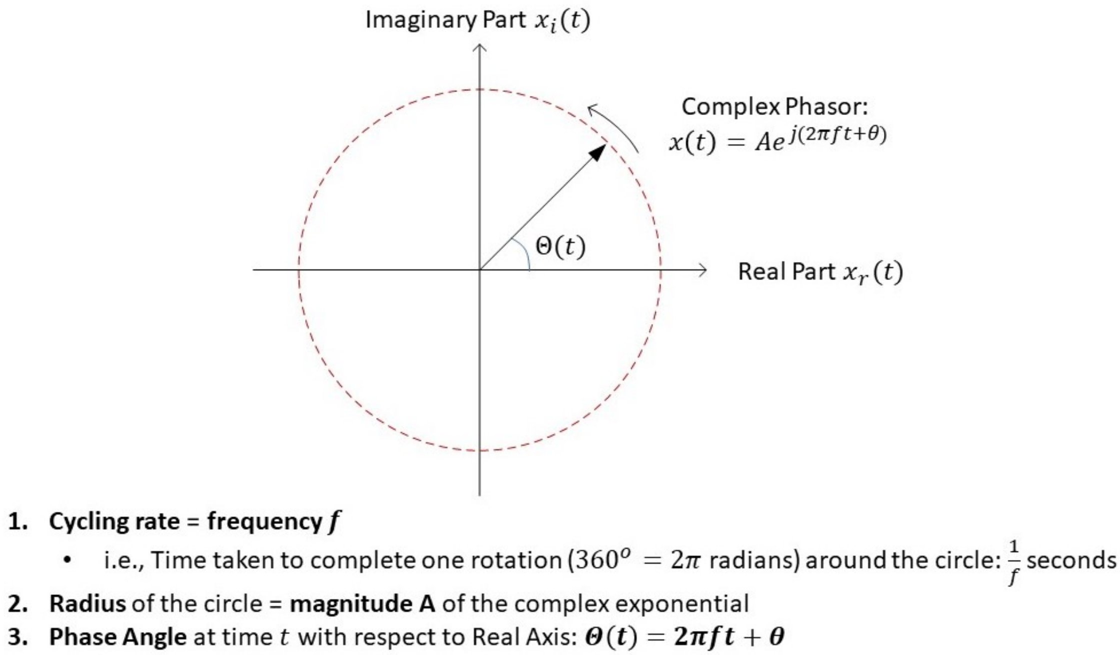

Figure 1: Question 1b: Diagrammatic representation of the complex phasor.

→ Frequency f can be 0 Hz ($f = 0$ Hz for a DC signal); it can be less than 1 Hz ($0 < f < 1$ Hz for a signal which takes more than 1 second to complete one cycle); it can be 1 Hz ($f = 1$ Hz if the cycle time for a signal is exactly 1 second); and it can be greater than 1 Hz ($f \geq 1$ Hz for signals whose cycle time is less than 1 second).

- ▷ For example, WiFi signal frequency is often at $f = 2.4$ GHz, i.e., $f = 2.4 \times 10^9$ Hz. This implies T_{cycle} for WiFi signal is 0.41 ns or 0.41×10^{-9} second. If you think that that is a complex-phasor rotating very fast, think of the frequency of color red, which is around 400 THz $= 400 \times 10^{12}$ Hz. The cycle duration for this electromagnetic (EM) radiation is only 2.2 femtoseconds $= 2.2 \times 10^{-15}$ second. The cycle duration for the color yellow is 2×10^{-15} second, not that different. Still somehow our eyes are able to make out such tiny changes in the duration of the EM wave that impinges upon the retina and clearly distinguish yellow from red!

→ Coming to the question asked, *can the frequency be negative?* For the real-world signal, it does not make sense to have negative frequency $f < 0$. However, complex phasors can have negative frequency f, which occurs when they rotate in the clockwise direction instead of anti-clockwise (since $f < 0$, the phase angle $\Theta(t) = 2\pi f t + \theta$ rotates in the opposite direction compared to when $f > 0$). In complex number plane, it is perfectly meaningful to talk about $f < 0$; for example, $\cos(2\pi f t)$ is a signal whose frequency f cannot be negative, but it can be thought of as comprised of two complex phasors, one at f and another at $-f$. Finally, complex phasor itself is to be thought of a pair of sinusoids (cosine and sine functions, which are) phase-shifted by 90^o with respect to each other. When $f < 0$, the sine function acquires negative sign compared to when $f > 0$ (the cosine function remains un-affected).

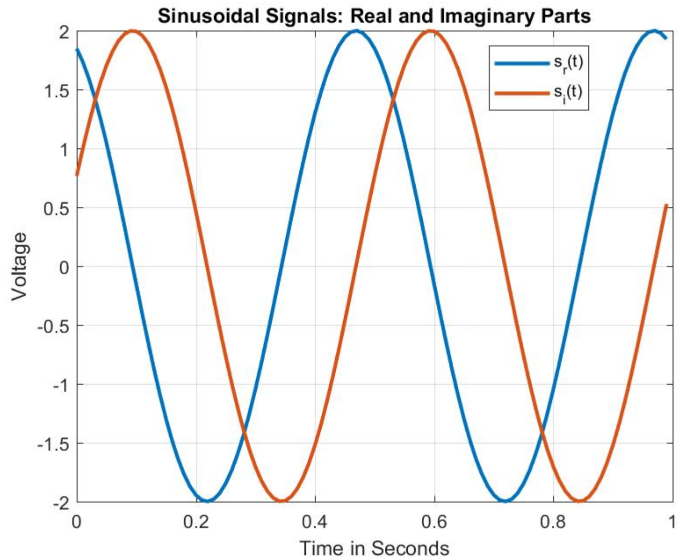

Figure 2: Question 1a: Time-domain diagram over a range of time t from 0 to +1 second.

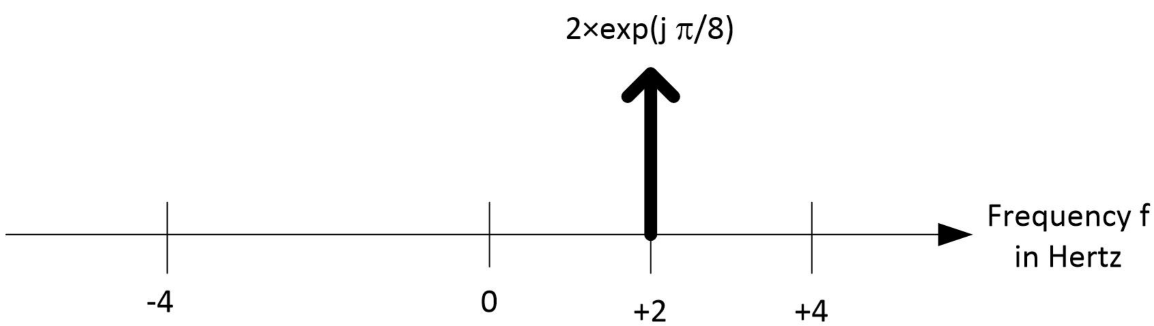

Figure 3: Question 1b: Frequency-domain diagram over a range of frequency f from -4 Hz to $+4$ Hz.

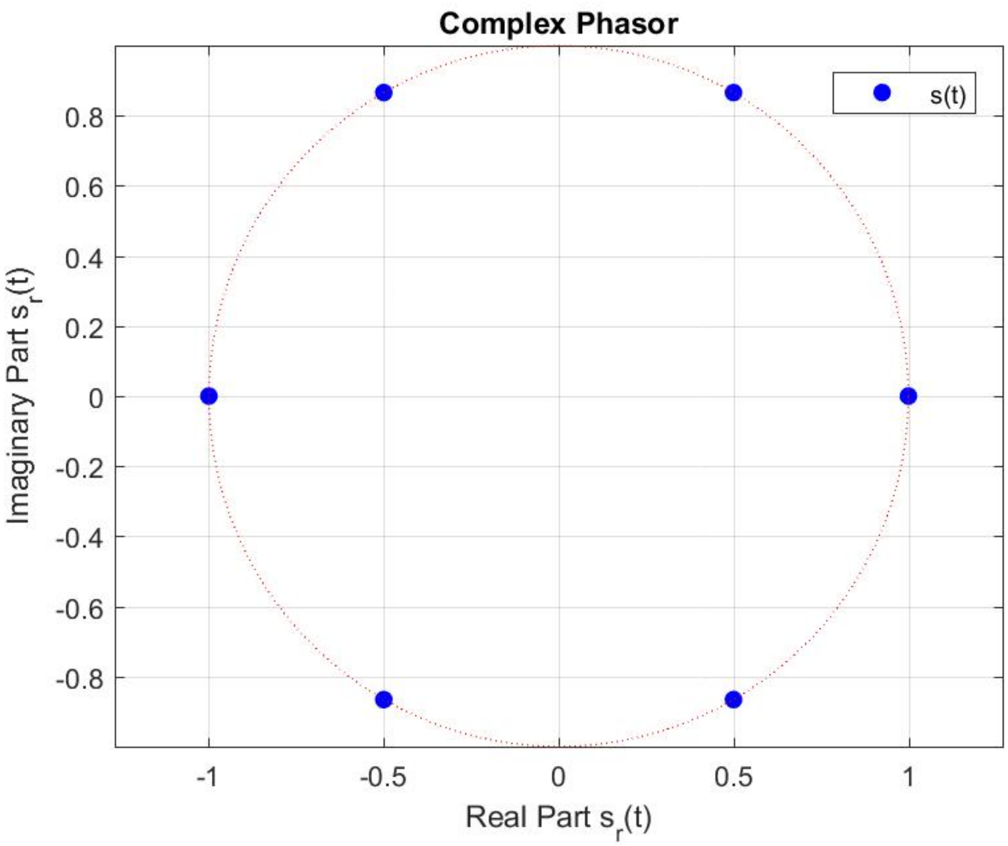

Figure 4: Question 2a: Samples of a complex phasor collected over 3 seconds.