2. Full Adder

Inputs			Output	
X	Y	C_{in}	C_{out}	S
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

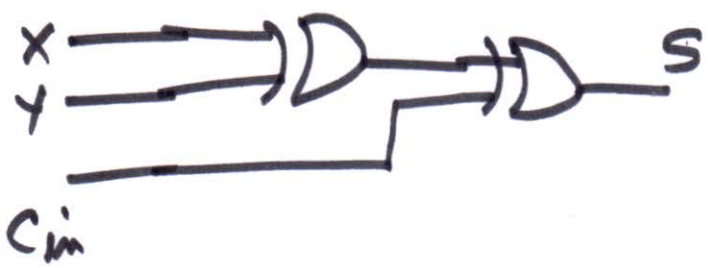
S =

$$S = X\bar{Y}\bar{C}_{in} + \bar{X}\bar{Y}C_{in} + XYC_{in} + \bar{X}Y\bar{C}_{in}$$

$$= \bar{Y}(X\bar{C}_{in} + \bar{X}C_{in}) + Y(XC_{in} + \bar{X}\bar{C}_{in})$$

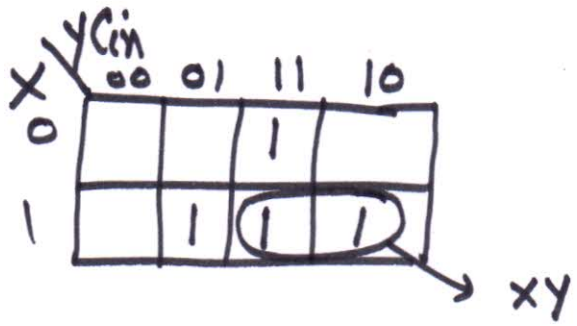
$$= \bar{Y}(X \oplus C_{in}) + Y\overline{(X \oplus C_{in})}$$

$$= Y \oplus X \oplus C_{in}$$



Full Adder (Cont.)

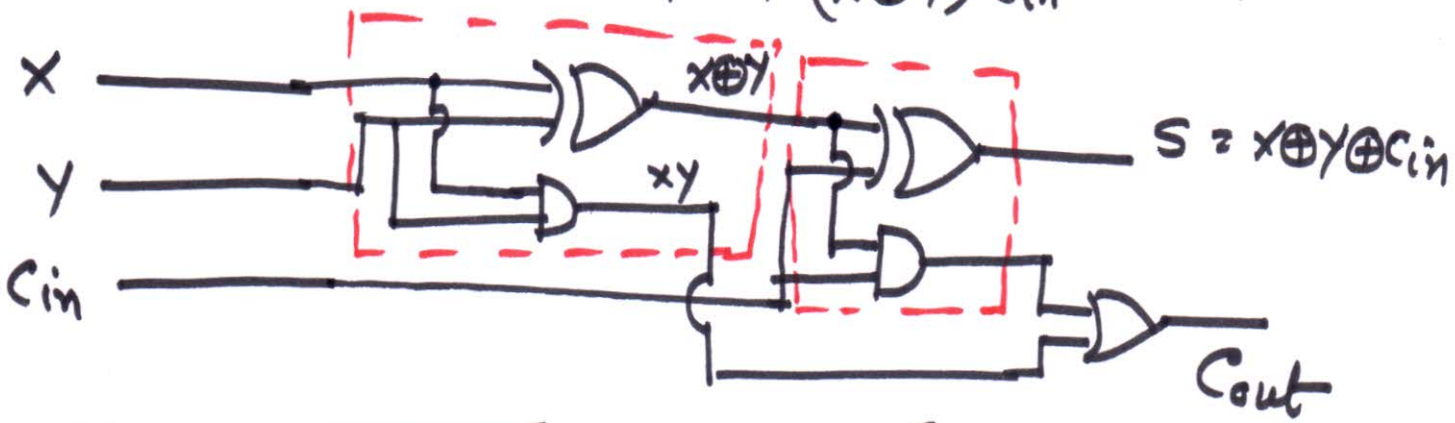
Cout

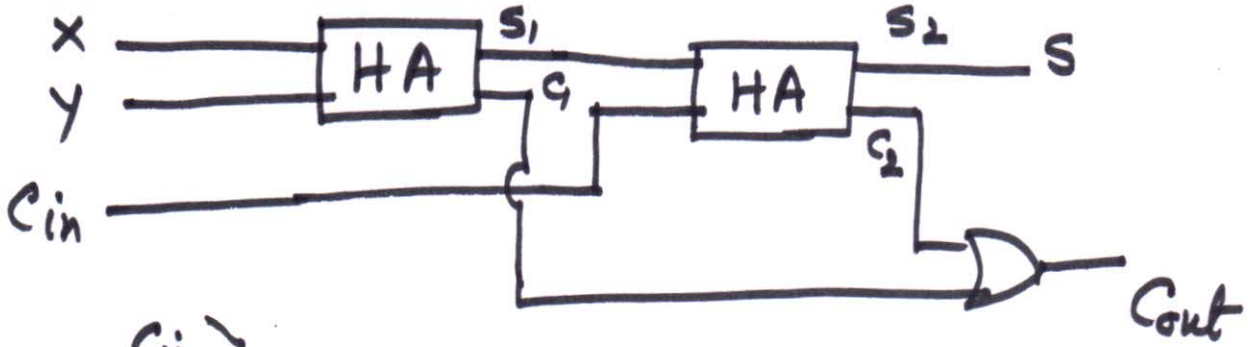


$$C_{out} = xy + \bar{x}yC_{in} + x\bar{y}C_{in}$$

$$= xy + (\bar{x}y + x\bar{y})C_{in}$$

$$= xy + (x \oplus y)C_{in}$$

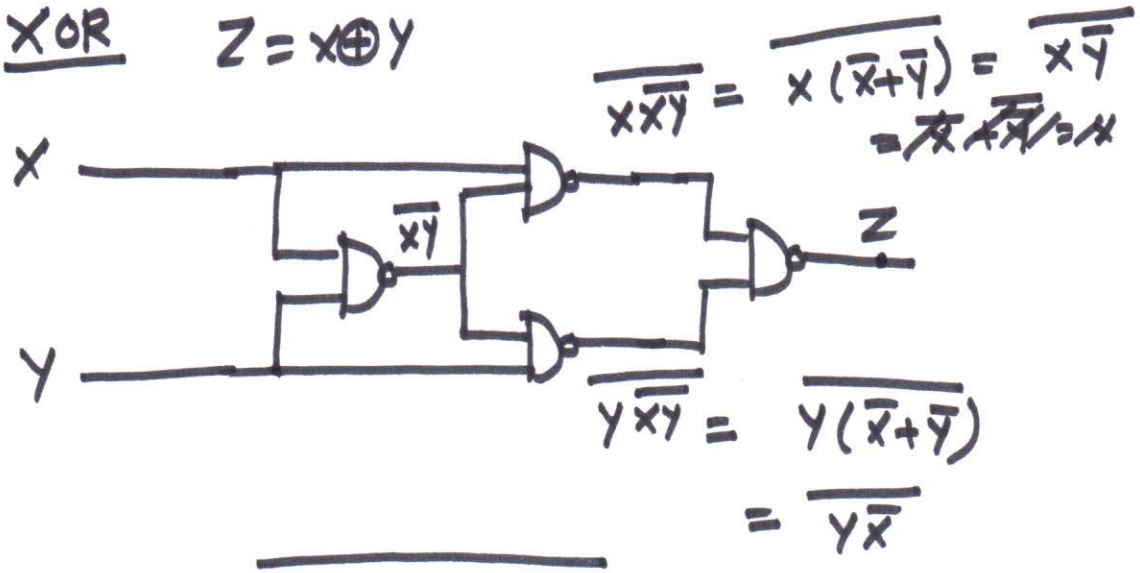




Cin
+ X } S
+ Y }

Full Adder with NAND Gates:

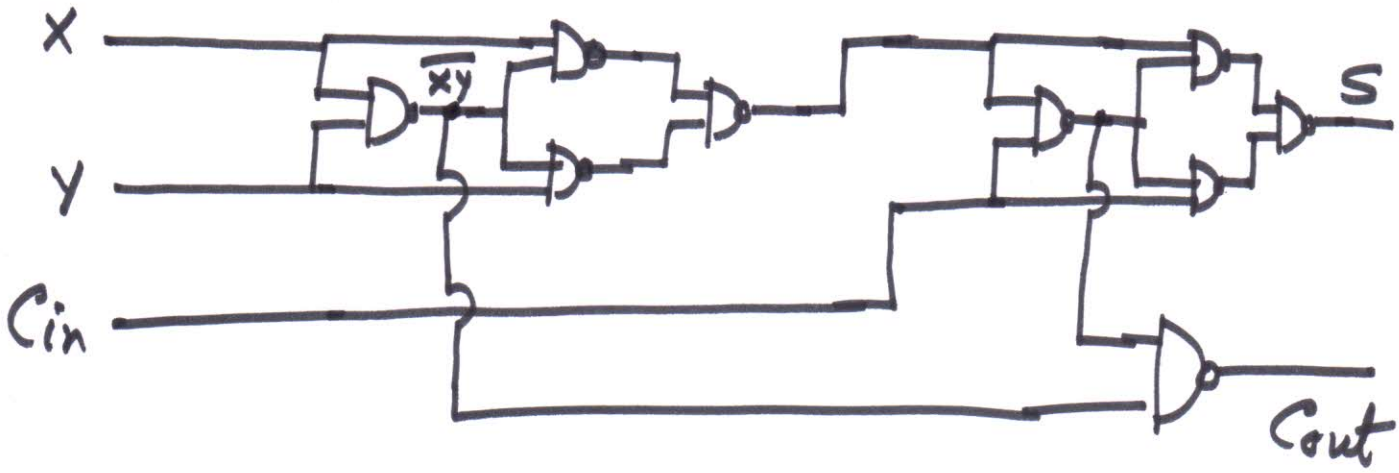
XOR $Z = x \oplus y$



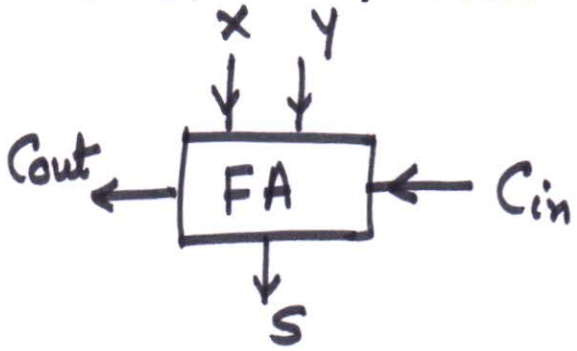
$$Z = \overline{\overline{x\overline{y}} \cdot \overline{y\overline{x}}}$$

$$= \overline{\overline{x\overline{y}}} + \overline{\overline{y\overline{x}}}$$

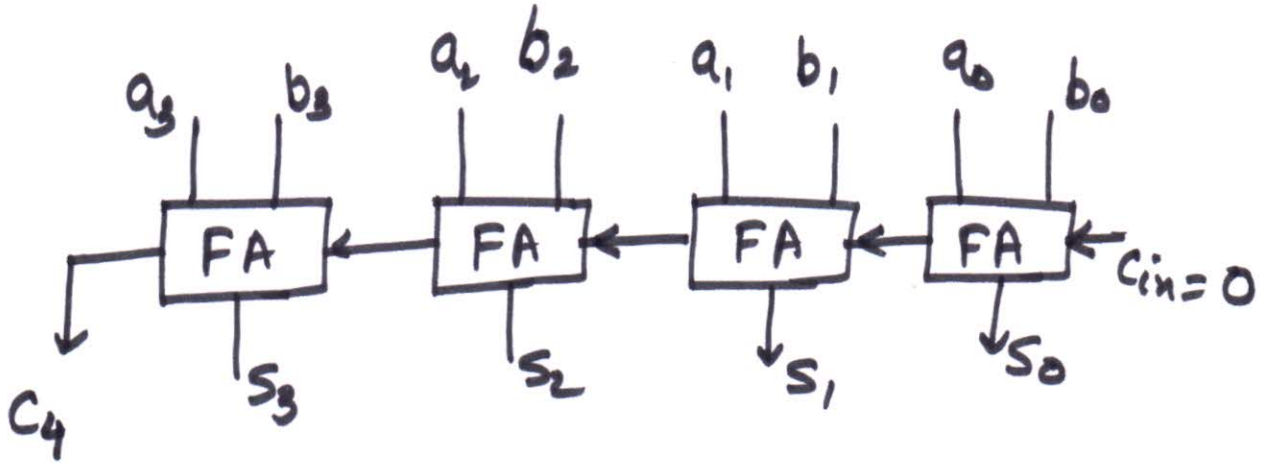
$$= x\overline{y} + y\overline{x} = x \oplus y$$

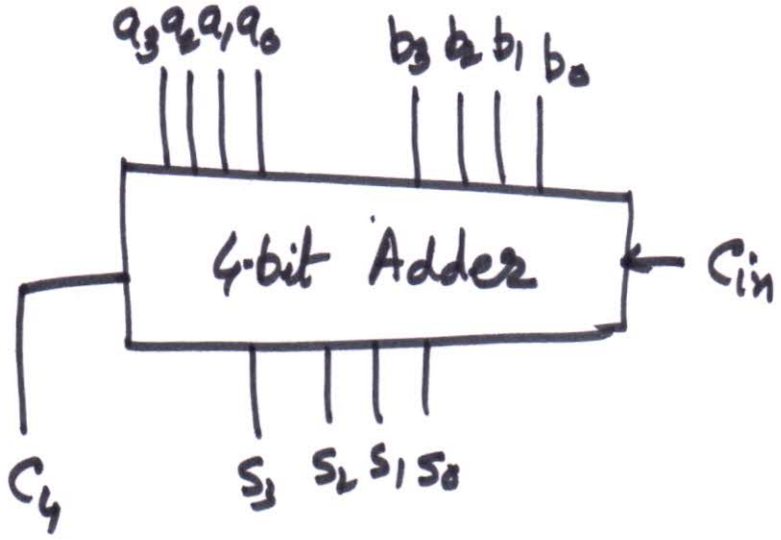


3. Binary Ripple Carry Adder:



$$\begin{array}{r} a_3\,a_2\,a_1\,a_0 \\ +\ b_3\,b_2\,b_1\,b_0 \\ \hline \end{array}$$





Binary Subtractor :

1. Half Subtractor

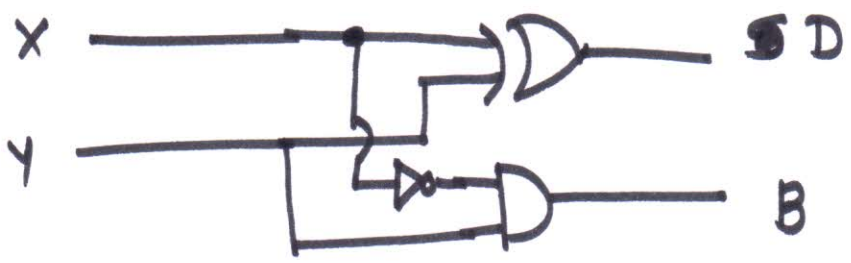
Inputs		Outputs	
X	Y	B	D
0	0	0	0
0	1	1	1
1	0	0	1
1	1	0	0

D: Difference (X-Y)

B: Borrow

$$D = X \oplus Y$$

$$B = \overline{X}Y$$



4. Full Subtractor

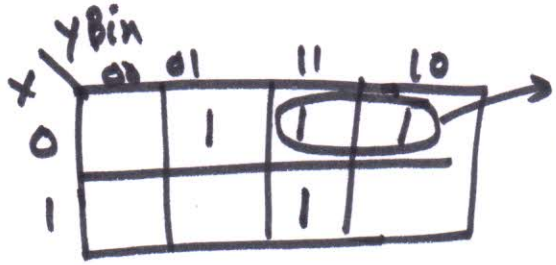
Inputs			Output	
X	Y	B_{in}	B_{out}	D
0	0	0	0	0
0	0	1	1	1
0	1	0	1	1
0	1	1	1	0
1	0	0	0	1
1	0	1	0	0
1	1	0	0	0
1	1	1	1	1

$X - Y - B_{in}$

D

$D = X \oplus Y \oplus B_{in}$

Bout



$\bar{x}y$

$B_{out} = \bar{x}y + \bar{x}\bar{y}B_{in} + xyB_{in}$

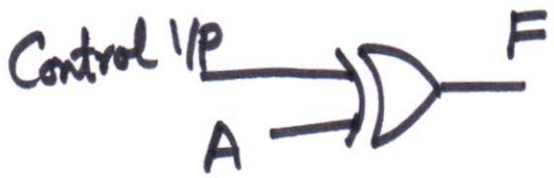
$= \bar{x}y + (\bar{x}\bar{y} + xy)B_{in}$

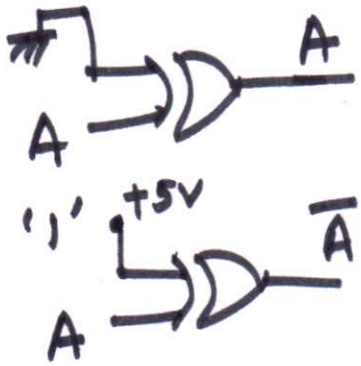
$= \bar{x}y + \overline{x \oplus y}\,B_{in}$

Adder-Subtractor:

Subtraction of two numbers can be accomplished by adding 2's complement of the subtrahend to the minuend and disregarding the final carry, if any.

Controlled inverter-





$a_3\ a_2\ a_1\ a_0$

$\pm\ b_3\ b_2\ b_1\ b_0$