Answer 1(d)

$f_c = 10\,MHz$, $f_m = 5\,KHz$, $E_{c\,max} = 10V$, $E_{m\,max} = 5V$, $R = 20\Omega$

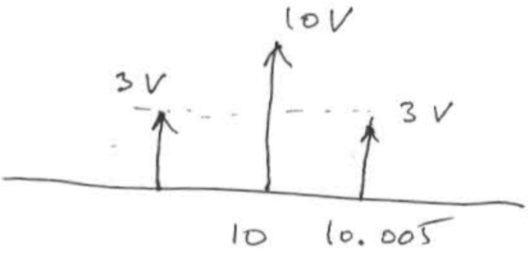
$m = \frac{6}{10} = 0.6$ — 1

$P_c = \frac{E_{c\,max}^2}{2R} = \frac{100}{2\times20} = 2.5\ W$ — 1

$P_{SF} = \frac{m^2}{4} P_c = \frac{0.36}{4} \times 2.5 = 0.225\ W$ — 1

Total AM Power

$P_T = P_c\left(1+\frac{m^2}{2}\right) = 2.5 \times (1.18) = 2.95\,W$ — 1




frequency MHz — 1

1(f)

Bandwidth $= 2\ f_{m\,max}$ — 2

2 (b)

$\delta = 20\ KHz$, $f_c = 100\ KHz$, $A = 10V$, $f_m = 5KHz$

$m_f = \frac{\delta}{f_m} = \frac{20}{5} = 4$ — 1

No. of sidebands from Bessel Function Table

$n = 7$

Bandwidth Bessel Function $= 2n\,f_m$

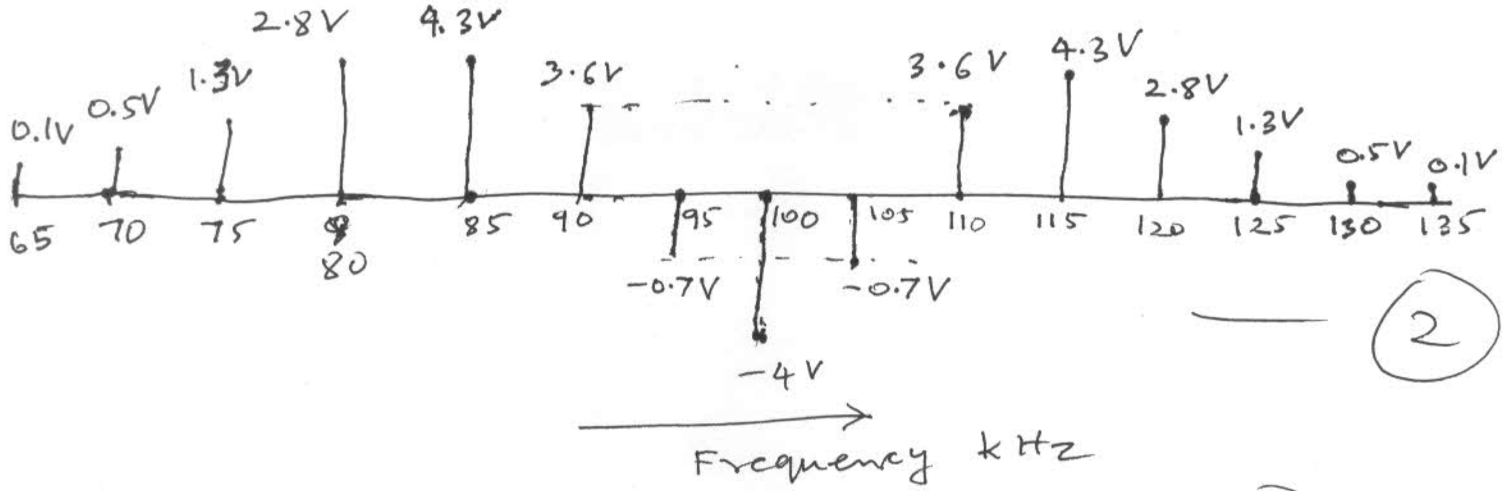
$= 2 \times 7 \times 5$

$= 70\ KHz$ — 1

Bandwidth Carson's Rule $= 2(\delta + f_m)$

$= 2 \times (20+5) = 50\ KHz$ — 1

Frequency Spectrum




3 (d)

$$H = \sum_{i=1}^{5} p_i \log_2\left(\frac{1}{p_i}\right) \quad \text{— } 1$$

$$= \frac{1}{2}\log_2 2 + \frac{1}{4}\log_2 4 + \frac{1}{8}\log_2 8 + \frac{1}{16}\log_2 16 \quad \text{— } 1$$

$$+ \frac{1}{16}\log_2 16 \quad \text{— } 2$$

$$= \frac{1}{2} + \frac{1}{2} + \frac{3}{8} + \frac{4}{16} + \frac{4}{16} = 1.875 \text{ bits/message}$$

$R = 1500 \times 1.875 = 2812.5$ bits/sec. — 1

$C = 2500$ bits/sec.

So, $R > C$

Probability error = 1 from Shannon's theorem. — 1

3(e)

The capacity C is given by

$$C = B \log_2\left(1 + \frac{S}{N}\right)$$

$$= 3000 \cdot \log_2(1 + 10)$$

$$= 10378 \text{ bits/sec.} \quad \text{— } 1$$

Entropy $H = \log_2 128 = 7$ bits/character. — 1

$R = r_s H < C$ ($r_s$ = message rate)

$7 r_s < 10378$ → 1

$r_s < 1482$ characters/sec. — 2

Question 1 (b)



Fortunately, many of the characteristics of AM can be examined using sinusoidal modulation as described in the following sections.

## 8.4 Modulation Index for Sinusoidal AM

For sinusoidal AM, the modulating waveform is of the form

1/2

$$e_m(t) = E_{m\,max}\cos(2\pi f_m t + \phi_m) \tag{8.4.1}$$

In general the fixed phase angle $\phi_m$ is unrelated to the fixed phase angle $\phi_c$ for the carrier, showing that these two signals are independent of each other in time. However, the amplitude modulation results are independent of these phase angles, which may therefore be set equal to zero to simplify the algebra and trigonometry used in the analysis. The equation for the sinusoidally modulated wave is therefore

1/2

$$e(t) = (E_{c\,max} + E_{m\,max}\cos 2\pi f_m t)\cos 2\pi f_c t \tag{8.4.2}$$

Since in this particular case $E_{max} = E_{c\,max} + E_{m\,max}$ and $E_{min} = E_{c\,max} - E_{m\,max}$ the modulation index is given by

$$m = \frac{E_{max} - E_{min}}{E_{max} + E_{min}}$$

1/2

$$= \frac{E_{m\,max}}{E_{c\,max}} \tag{8.4.3}$$

The equation for the sinusoidally amplitude modulated wave may therefore be written as

1/2

$$e(t) = E_{c\,max}(1 + m\cos 2\pi f_m t)\cos 2\pi f_c t \tag{8.4.4}$$

Figure 8.4.1 shows the sinusoidally modulated waveforms for three different values of $m$.

## 8.5 Frequency Spectrum for Sinusoidal AM

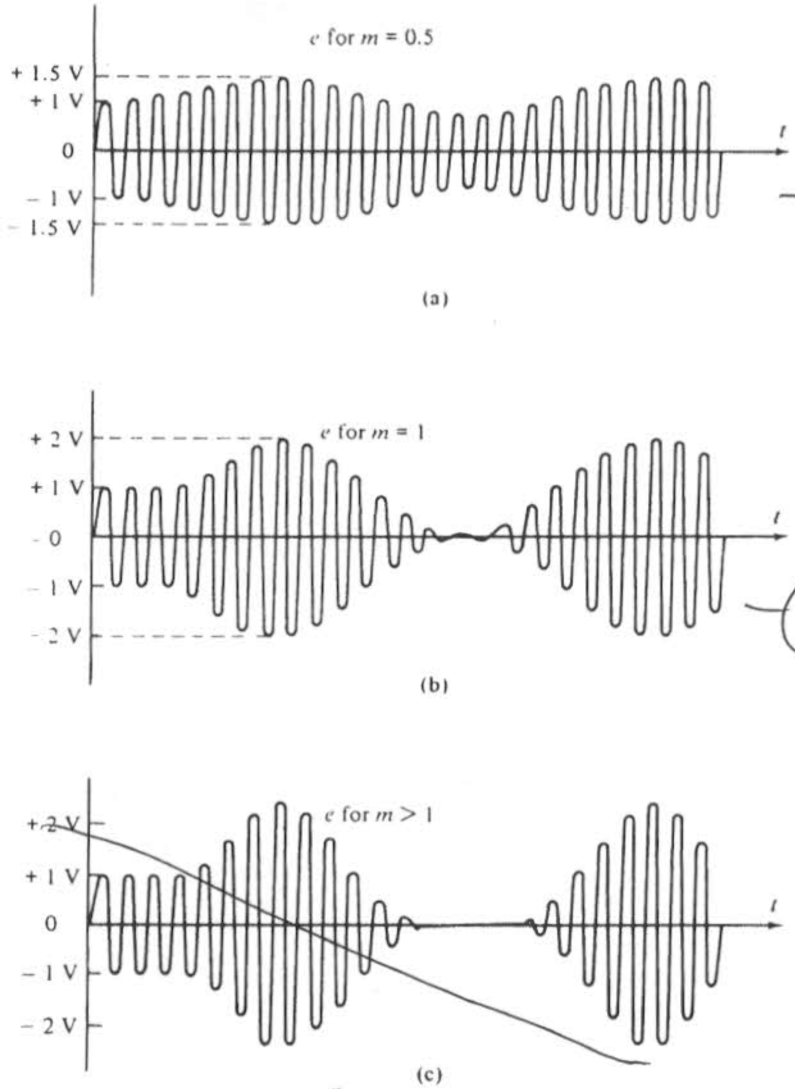
Although the modulated waveform contains two frequencies $f_c$ and $f_m$, the modulation process generates new frequencies that are the sum and difference of these. The spectrum is found by expanding the equation for the sinusoidally modulated AM as follows:

$$\begin{aligned} e(t) &= E_{c\,max}(1 + m\cos 2\pi f_m t)\cos 2\pi f_c t \\ &= E_{c\,max}\cos 2\pi f_c t + mE_{c\,max}\cos 2\pi f_m t \times \cos 2\pi f_c t \end{aligned}$$

4

Question 1(b)






1

1

**Figure 8.4.1** Sinusoidally amplitude modulated waveforms for (a) $m = 0.5$ (undermodulated), (b) $m = 1$ (fully modulated), and (c) $m > 1$ (overmodulated).

1

$$= E_{c\,\max} \cos 2\pi f_c t + \frac{m}{2} E_{c\,\max} \cos 2\pi (f_c - f_m)t + \frac{m}{2} E_{c\,\max} \cos 2\pi (f_c + f_m)t \tag{8.5.1}$$

It is left as an exercise for the student to derive this result making use of the trigonometric identity

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B \tag{8.5.2}$$

5

DSBSC - Spectrum Frequency:

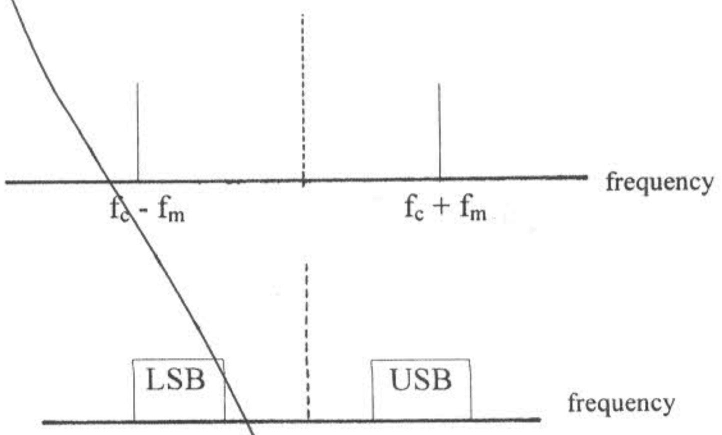



Figure 2.9 Frequency spectrum of a Double Sideband Suppressed Carrier System

i) Power content in DSB : ≤ Power as in standard AM.

ii) Bandwidth : same as in standard AM.

iii) Disadvantages of DSB : Receiver is complex and expensive. It requires the re-insertion of the carrier at the receiving end. The carrier must be of the correct frequency and phase as the original carrier.

**(b) Single Sideband ( SSB )**

As both DSB and standard AM waste a lot of power and occupy large bandwidth, SSB is adopted. SSB is a process of transmitting one of the sidebands of the standard AM by suppressing the carrier and one of the sidebands.

<u>Advantages</u>

- Considerable saving of power . 1
- Increase signal to noise ratio, which increases the efficiency. 1/2
- bandwidth requirement is reduced by 50 %, which reduces frequency congestion 1

<u>Disadvantage</u>

- complex circuitry , as it requires frequency stability. 1/2

Question 2(a)



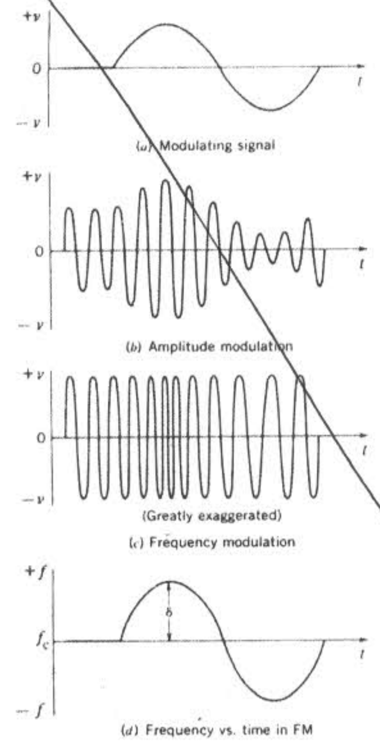



**FIGURE 5-1 Basic modulation waveforms.**

carrier at the same rate of 2000 times per second, no matter what their individual amplitudes. *The amplitude of the frequency-modulated wave remains constant at all times.* This is the greatest single advantage of FM.

## 5-1.2 Mathematical Representation of FM

From Figure 5-1*d*, it is seen that the instantaneous frequency $f$ of the frequency-modulated wave is given by

$$f = f_c\,(1 + kV_m \cos \omega_m t) \qquad \text{(5-2)}$$

(1)

where $f_c$ = unmodulated (or average) carrier frequency
$k$ = proportionality constant
$V_m \cos \omega_m t$ = instantaneous modulating voltage (cosine being preferred for simplicity in calculations)

The maximum deviation for this particular signal will occur when the cosine term has its maximum value, ±1. Under these conditions, the instantaneous frequency will be

$$f = f_c\,(1 \pm kV_m) \qquad \text{(5-3)}$$

1/2

7

Question 2 (a)



so that the maximum deviation $\delta$ will be given by

$$\delta = kV_m f_c \qquad \text{— } 1/2 \qquad (5\text{-}4)$$

The instantaneous amplitude of the FM signal will be given by a formula of the form

$$v = A \sin\,[F\,(\omega_c,\omega_m)] = A \sin\theta \qquad (5\text{-}5) \text{ — } 1$$

where $F\,(\omega_c,\omega_m)$ is some function of the carrier and modulating frequencies. This function represents an angle and will be called $\theta$ for convenience. The problem now is to determine the instantaneous value (i.e., formula) for this angle.

As Figure 5-2 shows, $\theta$ is the angle traced out by the vector $A$ in time $t$. If $A$ were rotating with a constant angular velocity, for example, $\rho$, this angle $\theta$ would be given by $\rho t$ (in radians). In this instance the angular velocity is anything but constant. It is governed by the formula for $\omega$ obtained from Equation (5-2), that is, $\omega = \omega_c\,(1 + kV_m \cos \omega_m t)$. In order to find $\theta$, $\omega$ must be integrated with respect to time. Thus

$$\begin{aligned}\theta &= \int \omega\, dt = \int \omega_c\,(1 + kV_m \cos \omega_m t)\, dt = \omega_c \int (1 + kV_m \cos \omega_m t)\, dt \\ &= \omega_c \left(t + \frac{kV_m \sin \omega_m t}{\omega_m}\right) = \omega_c t + \frac{kV_m \omega_c \sin \omega_m t}{\omega_m} \\ &= \omega_c t + \frac{kV_m f_c \sin \omega_m t}{f_m} \\ &= \omega_c t + \frac{\delta}{f_m} \sin \omega_m t \qquad \text{— } 1 \qquad (5\text{-}6)\end{aligned}$$

The derivation utilized, in turn, the fact that $\omega_c$ is constant, the formula $\int \cos nx\, dx = (\sin nx)/n$ and Equation (5-4), which had shown that $kV_m f_c = \delta$. Equation (5-6) may now be substituted into Equation (5-5) to give the instantaneous value of the FM voltage; therefore

$$v = A \sin\left(\omega_c t + \frac{\delta}{f_m} \sin \omega_m t\right) \qquad (5\text{-}7)$$

The modulation index for FM, $m_f$, is defined as

$$m_f = \frac{\text{(maximum) frequency deviation}}{\text{modulating frequency}} = \frac{\delta}{f_m} \qquad (5\text{-}8)$$

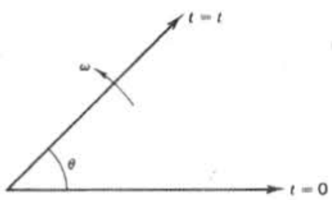



**FIGURE 5-2 Frequency-modulated vectors.**

$$v = A \sin\left(\omega_c t + m_f \sin(\omega_m t)\right) \quad \text{— } 1$$

8

Question 2(c)



(c) $v = 4 \sin (1.57 \times 10^8 t + 5 \sin 12{,}565t)$ (FM)
(d) $v = 4 \sin (1.57 \times 10^8 t + 25 \sin 12{,}565t)$ (PM)

Note that the difference between FM and PM is not apparent at a single modulating frequency. It reveals itself in the *differing behavior of the two systems when the modulating frequency is varied.*

The practical effect of all these considerations is that if an FM transmission were received on a PM receiver, the bass frequencies would have considerably more deviation (*of phase*) than a PM transmitter would have given them. Since the output of a PM receiver would be proportional to phase deviation (or modulation index), the signal would appear unduly bass-boosted. Phase modulation received by an FM system would appear to be *lacking in bass*. This deficiency could be *corrected by bass boosting the modulating signal prior to phase modulation*. This is the practical difference between phase and frequency modulation.

**Frequency and amplitude modulation** Frequency and amplitude modulation are compared on a different basis from that for FM and PM. These are both practical systems, quite different from each other, and so the performance and characteristics of the two systems will be compared. To begin with, frequency modulation has the following advantages:

1. The amplitude of the frequency-modulated wave is constant. It is thus independent of the modulation depth, whereas in AM modulation depth governs the transmitted power. This means that, in FM transmitters, low-level modulation may be used but all the subsequent amplifiers can be class C and therefore more efficient. Since all these amplifiers will handle constant power, they need not be capable of managing up to four times the average power, as they must in AM. Finally, *all* the transmitted power in FM is useful, whereas in AM most of it is in the transmitted carrier, which contains no useful information. 1
2. FM receivers can be fitted with amplitude limiters to remove the amplitude variations caused by noise, as shown in Section 5-2.2; this makes FM reception a good deal more immune to noise than AM reception. 1
3. It is possible to reduce noise still further by increasing the deviation (see Section 5-2.1). This is a feature which AM does not have, since it is not possible to exceed 100 percent modulation without causing severe distortion. 1
4. Commercial FM broadcasts began in 1940, decades after their AM counterparts. They have a number of advantages due to better planning and other considerations. The following are the most important ones:
   a. Standard frequency allocations (allocated worldwide by the International Radio Consultative Committee (CCIR) of the I.T.U.) provide a guard band between commercial FM stations, so that there is less adjacent-channel interference than in AM; 1/2
   b. FM broadcasts operate in the upper VHF and UHF frequency ranges, at which there happens to be less noise than in the MF and HF ranges occupied by AM broadcasts; 1/2

9

Question 2(c)



c. At the FM broadcast frequencies, the *space wave* is used for propagation, so that the radius of operation is limited to slightly more than line of sight, as shown in Section 8-2.3. It is thus possible to operate several independent transmitters on the same frequency with considerably less interference than would be possible with AM.

The advantages are not all one-sided, or there would be no AM transmissions left. The following are some of the disadvantages of FM: 1

1. A much wider channel is required by FM, up to 10 times as large as that needed by AM. This is the most significant disadvantage of FM.
2. FM transmitting and receiving equipment tends to be more complex, particularly for modulation and demodulation. 1/2
3. Since reception is limited to line of sight, the area of reception for FM is much smaller than for AM. This may be an advantage for cochannel allocations, but it is a disadvantage for FM mobile communications over a wide area. Note that this is due not so much to the intrinsic properties of FM, but rather to the frequencies employed for its transmission. 1/2

## 5-2 NOISE AND FREQUENCY MODULATION

Frequency modulation is much more immune to noise than amplitude modulation and is significantly more immune than phase modulation. In order to establish the reason for this and to determine the extent of the improvement, it is necessary to examine the effect of noise on a carrier.

### 5-2.1 Effects of Noise on Carrier—Noise Triangle

A single-noise frequency will affect the output of a receiver only if it falls within its bandpass. The carrier and noise voltages will mix, and if the difference is audible, it will naturally interfere with the reception of wanted signals. If such a single-noise voltage is considered vectorially, it is seen that the noise vector is superimposed on the carrier, rotating about it with a relative angular velocity $\omega_n - \omega_c$. This is shown in Figure 5-5. The maximum deviation in amplitude from the average value will be $V_n$, whereas the maximum phase deviation will be $\phi = \sin^{-1}(V_n/V_c)$.

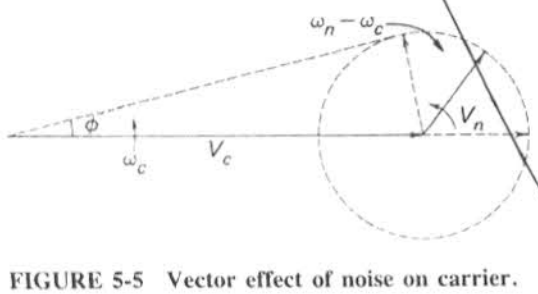



**FIGURE 5-5 Vector effect of noise on carrier.**

Q. 2(d)



Table 5.1–1 Comparison of PM and FM

| | Instantaneous phase $\phi(t)$ | Instantaneous frequency $f(t)$ |
|---|---|---|
| PM | $\phi_\Delta x(t)$ | $f_c + \frac{1}{2\pi}\phi_\Delta \dot{x}(t)$ |
| FM | $2\pi f_\Delta \int^t x(\lambda)\,d\lambda$ | $f_c + f_\Delta x(t)$ |

Therefore, regardless of the message $x(t)$, the average transmitted power is

$$S_T = \tfrac{1}{2}A_c^2 \tag{8}$$

For another, the zero crossings of an exponentially modulated wave are *not periodic*, whereas they are always periodic in linear modulation. Indeed, because of the constant-amplitude property of FM and PM, it can be said that

The message resides in the *zero crossings alone*, providing the carrier frequency is large.

Finally, since exponential modulation is a nonlinear process,

The modulated wave does not look at all like the message waveform.

Figure 5.1–2 illustrates some of these points by showing typical AM, FM, and PM waves. As a mental exercise you may wish to check these waveforms against the corresponding modulating signals. For FM and PM this is most easily done by considering the instantaneous frequency rather than by substituting $x(t)$ in Eqs. (3) and (7).

Despite the many similarities of PM and FM, frequency modulation turns out to have superior noise-reduction properties and thus will receive most of our attention. To gain a qualitative appreciation of FM noise reduction, suppose a demodulator simply extracts the instantaneous frequency $f(t) = f_c + f_\Delta x(t)$ from $x_c(t)$. The demodulated output is then proportional to the frequency deviation $f_\Delta$, which can be increased without increasing the transmitted power $S_T$. If the noise level remains constant, increased signal output is equivalent to reduced noise. However, noise reduction does require increased transmission bandwidth to accommodate large frequency deviations.

Ironically, frequency modulation was first conceived as a means of **bandwidth reduction,** the argument going somewhat as follows: If, instead of modulating the

(11)

Question 3 (a)



Shannon's theorem says that it is possible, in principle, to devise a means whereby a communications system will transmit information with an arbitrarily small probability of error provided that the information rate $R$ is less than or equal to a rate $C$ called the *channel capacity*. The technique used to approach this end is called *coding* and is discussed beginning with Sec. 13.9. To put the matter more formally, we have the following:

**Theorem** Given a source of $M$ equally likely messages, with $M \gg 1$, which is generating information at a rate $R$. Given a channel with channel capacity $C$. Then, if

$$R \leq C$$

there exists a *coding* technique such that the output of the source may be transmitted over the channel with a probability of error in the received message which may be made arbitrarily small.

$1\frac{1}{2}$

The important feature of the theorem is that it indicates that for $R \leq C$ transmission may be accomplished without error in the presence of noise. This result is surprising. For in our consideration of noise, say, gaussian noise, we have seen that the probability density of the noise extends to *infinity*. We should then imagine that there will be some times, however infrequent, when the noise *must* override the signal thereby resulting in errors. However, Shannon's theorem says that this need not cause a message to be in error.

There is a negative statement associated with Shannon's theorem. It states the following:

**Theorem** Given a source of $M$ equally likely messages, with $M \gg 1$, which is generating information at a rate $R$; then if

$$R > C$$

the probability of error is close to unity for every possible set of $M$ transmitter signals.

$1\frac{1}{2}$

This *negative* theorem states that if the information rate $R$ exceeds a specified value $C$, the error probability will increase toward unity as $M$ increases, and that also, generally, in this case where $R > C$, increasing the complexity of the coding results in an increase in the probability of error.

## 13.7 CAPACITY OF A GAUSSIAN CHANNEL

A theorem which is complementary to Shannon's theorem and applies to a channel in which the noise is gaussian is known as the Shannon–Hartley theorem.

(12)

Question 3(b)



**Theorem** The channel capacity of a white, bandlimited gaussian channel is

$$C = B \log_2 \left(1 + \frac{S}{N}\right) \text{ bits/s} \qquad (13.7\text{-}1)$$

3

where $B$ is the channel bandwidth, $S$ the signal power, and $N$ is the total noise within the channel bandwidth, that is, $N = \eta B$, with $\eta/2$ the (two-sided) *power spectral density*.

This theorem, although restricted to the gaussian channel, is of fundamental importance. First, we find that channels encountered in physical systems generally are, at least approximately, gaussian. Second, it turns out that the results obtained for a gaussian channel often provide a *lower bound* on the performance of a system operating over a nongaussian channel. Thus, if a particular encoder-decoder is used with a gaussian channel and an error probability $P_e$ results, then with a nongaussian channel another encoder-decoder can be designed so that the $P_e$ will be smaller. We may note that channel capacity equations corresponding to Eq. (13.7-1) have been derived for a number of nongaussian channels.

The derivation of Eq. (13.7-1) for the capacity of a gaussian channel is rather formidable and will not be undertaken. However, the result may be made to appear reasonable by the following considerations. Suppose that, for the purpose of transmission over the channel, the messages are represented by fixed voltage levels. Then, as the source generates one message after another in sequence, the transmitted signal $s(t)$ takes on a waveform similar to that shown in Fig. 13.7-1.

The received signal is accompanied by noise whose root-mean-square voltage is $\sigma$. The levels have been separated by an interval $\lambda\sigma$, where $\lambda$ is a number presumed large enough to allow recognition of individual levels with an acceptable probability of error. Assuming an even number of levels, the levels are located at voltages $\pm \lambda\sigma/2$, $\pm 3\lambda\sigma/2$, etc. If there are to be $M$ possible messages, then there

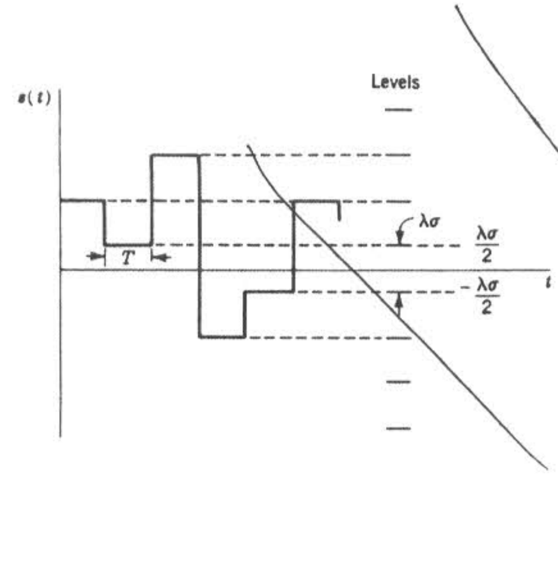



**Figure 13.7-1** A sequence of messages is represented by a waveform $s(t)$ which assumes voltage levels corresponding to the messages.

Question 3 (c)

13



Since the messages are independent, the probability of the composite message is $p_k p_l$ with corresponding information content of messages $m_k$ and $m_l$ is

$$I_{k,l} = \log_2 \frac{1}{p_k p_l} = \log_2 \frac{1}{p_k} + \log_2 \frac{1}{p_l} = I_k + I_l \tag{13.2-3}$$

It is of interest to note that the term information applied to the symbol $I_k$ in Eq. (13.2-1) is rather aptly chosen, since there is some correspondence between the properties of $I_k$ and the meaning of the word information as used in everyday speech. For example, suppose that an airplane dispatcher calls the weather bureau of a distant city during daylight hours to inquire about the present weather. If he receives, in response, the message, "There is daylight here," he will surely judge that he has received no information since he was certain beforehand that such was the case. On the other hand, if he hears "It is not raining here," he will consider that he has received information since he might anticipate such a situation with less than perfect certainty. Further, suppose the dispatcher receives some weather information on two different days. Then he might indeed consider that the total information received was the sum of the information received in the individual weather reports.

Furthermore, consider that the weather bureau of a city located in the desert, where it has not rained for 25 years, is called before each flight. The call is required although everyone "knows" that the weather will be clear. However, the day the call is made and the reply is that a very heavy rainstorm is in progress and that the flight must be canceled. The information received is then very great.

## 13.3 AVERAGE INFORMATION, ENTROPY

Suppose we have $M$ different and independent messages $m_1, m_2, \ldots,$ with probabilities of occurrence $p_1, p_2, \ldots$. Suppose further that during a long period of transmission a sequence of $L$ messages has been generated. Then, if $L$ is very large, we may expect that in the $L$ message sequence we transmitted $p_1 L$ messages of $m_1$, $p_2 L$ messages of $m_2$, etc. The total information in such a sequence will be

$1\frac{1}{2}$

$$I_{\text{total}} = p_1 L \log_2 \frac{1}{p_1} + p_2 L \log_2 \frac{1}{p_2} + \cdots \tag{13.3-1}$$

The average information per message interval, represented by the symbol $H$, will then be

$$H \equiv \frac{I_{\text{total}}}{L} = p_1 \log_2 \frac{1}{p_1} + p_2 \log_2 \frac{1}{p_2} + \cdots = \sum_{k=1}^{M} p_k \log_2 \frac{1}{p_k} \tag{13.3-2}$$

$1\frac{1}{2}$

This average information is also referred to by the term *entropy*.