CT111 Introduction to Communication Systems
Lecture 6: Channel Coding

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Overview of Today's Talk

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1. Block Diagrams
2. Practical Channel Coding
3. Hamming Distance

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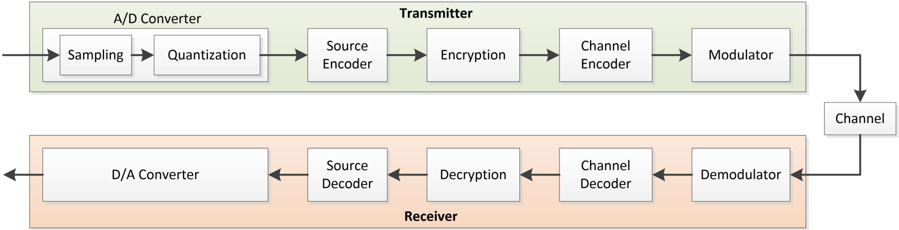
1. Block Diagrams
2. Practical Channel Coding
3. Hamming Distance

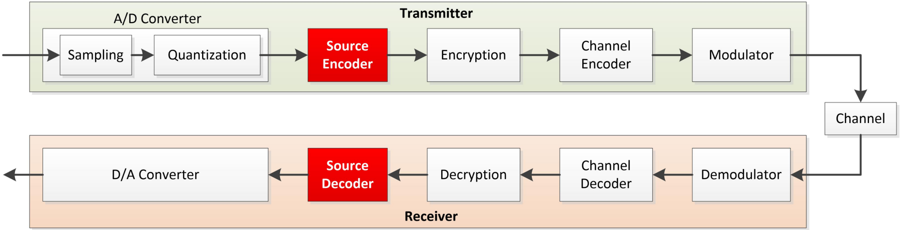
Digital Communication Transceiver
Block Diagram

- We have earlier seen this block diagram model of a digital communication transceiver



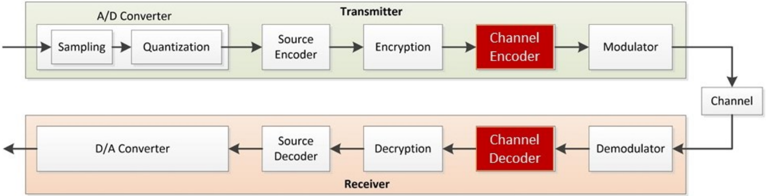
Digital Communication Transceiver
Block Diagram

- We have studied the mathematics and algorithms of source encoding



Digital Communication Transceiver
Channel Coding

- We now focus on channel encoding/decoding, also known as FEC (forward error correction) coding



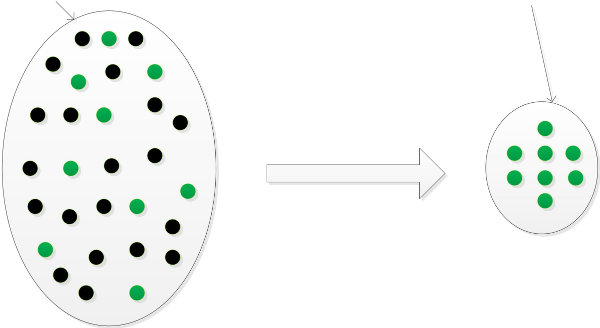
A Non-uniform PMF

Data Compression Possible

- A view of data compression

Before data compression:
Requires M bits
Total size of the set: 2^{M}

After data compression:
Requires M× H(X) bits
Total size of the set: $2^{M \times H(X)}$



● Members of the Typical Set

● Remaining, belong to subsets with vanishing probability as N becomes large

Data Compression and Error Correction

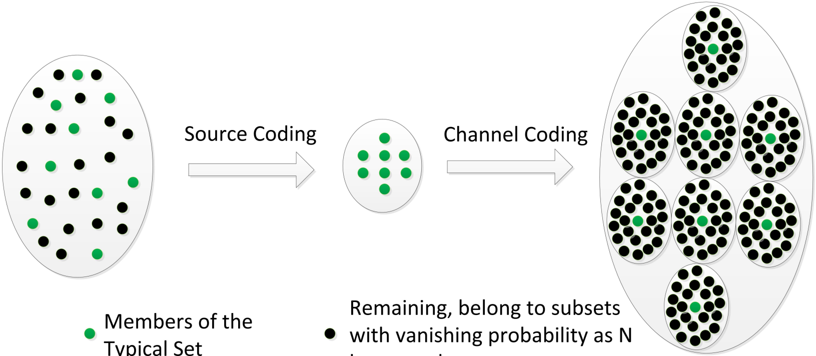
- Data compression and error correction are dual to each other
  - ▷ Uncompressed data allows for error correction
  - ▷ Compressed data leaves no room for correcting for the errors

Source Coding versus Channel Coding

Uncompressed Source Output

After Compression

After Channel Encoding: Transmitter selects one of green cicrles; Receiver receives any one of green and black circles



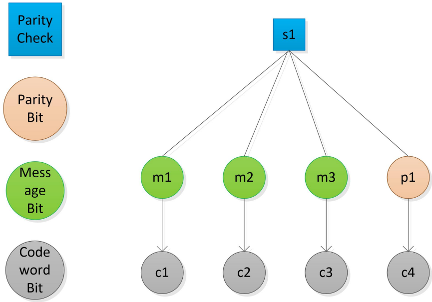
- Members of the Typical Set
- Remaining, belong to subsets with vanishing probability as N becomes large

Example Channel Coding Techniques
Single Parity Check (SPC) Code

- Encoding scheme:
  1. Take a block of $n-1 \geq 1$ bits as input.
  2. Add an extra n^{th} bit, called the parity check bit, which is 0 if the input block of n bits has even number of 1's, and it is 1 otherwise
  3. Transmit the resultant n bit long codeword
- Let $\{m_1, m_2, \ldots, m_{n-1}\}$ be the input block. The parity bit added is given as $p_n = \left(\sum_{k=1}^{n-1} m_k\right)_{mod-2}$
- Rate of this code is $r = \dfrac{n-1}{n}$.
  - → Rate r is the ratio of the number of information bits to total number of encoded bits

Example Channel Coding Techniques
Single Parity Check (SPC) Code

- Notations:
  - $\{m_k\}$: info bits,
  - $\{p_k\}$: parity bits,
  - $\{s_k\}$: check bits,
  - $\{c_k\}$: codeword bits
  - If $\mathbf{c} \stackrel{\text{def}}{=} [c_1, c_2, \ldots, c_n]^T$ is an SPC codeword, $s_1 = \sum_{i=1}^{n} c_i$ has to be zero, where this sum is modulo two.



Example Channel Coding Techniques

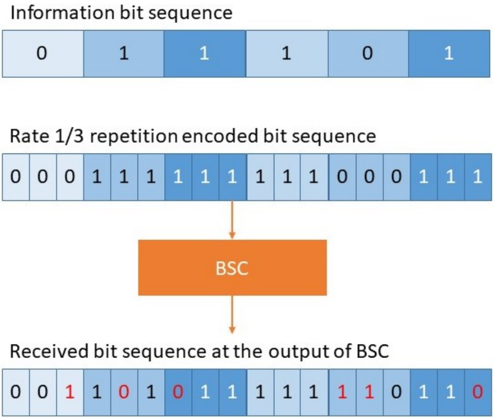
Single Parity Check (SPC) Code

- Decoding scheme:
  1. Take a block of n bits as input.
  2. Compute the modulot-two sum of these bits
  3. If this sum is zero, determine that zero or an even number of bit errors have occurred. If nonzero, determine that one or an odd number of bit errors have occurred
- Rate $(n-1)/n$ SPC code can detect one bit of error, but it cannot correct it

Example Channel Coding Techniques
Repetition Code

- Encoding scheme:
  1. Take one bit at a time as the input.
  2. Repeat this bit $n-1$ times
  3. Transmit the resultant n bit long codeword
- Rate of this code is $r = \dfrac{1}{n}$.
  - $\rightarrow$ Rate r is the ratio of the number of information bits to total number of encoded bits

Example Channel Coding Techniques
Repetition Code

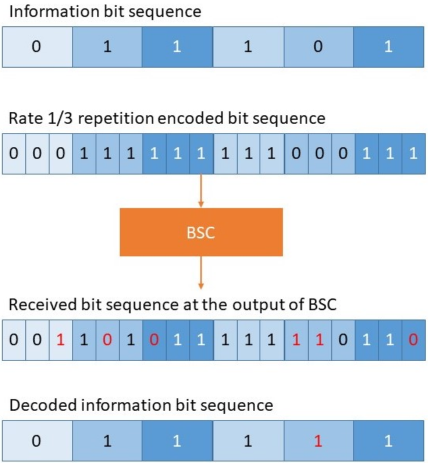


Example Channel Coding Techniques

Repetition Code

What would be a good decoding strategy for this code?

Example Channel Coding Techniques
Decoding of Repetition Code



Example Channel Coding Techniques
Decoding of Repetition Code

- The decoding algorithm is called the majority-vote decoding
  - ▷ Take $n = 3$ bit block at a time, and decode it as that bit (either 0 or 1) that occurs the majority of times
  - ▷ To avoid the confusion in decoding, it maybe preferred to make n an odd number

Example Channel Coding Techniques
Probability of Decoding Error

- Decoding error will occur in rate $r = 1/3$ repetition code if 2 or 3 bits are in error
- What is the probability of that occuring? The answer is given by the Binomial PMF

$$p_{error} = \binom{3}{2} p^2(1-p) + \binom{3}{1} p^3 = 3p^2 + p^3$$

$\rightarrow$ If $p = 0.1$, $p_{error} = 3 \times 0.1^2 \times 0.9 + 0.1^3 \approx 0.03$

- Generalization to rate $1/n$ repetition code:

$$p_{error} = \sum_{k=(n+1)/2}^{n} \binom{n}{k} p^k (1-p)^{n-k}$$

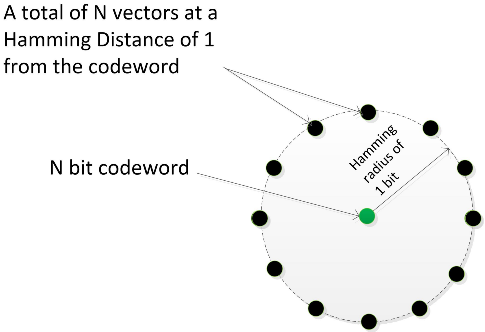
Hamming Weight and Hamming Distance
for Binary Sequences

- *Hamming Weight* is simply the number of ones in the sequence
- *Hamming Distance* d:
  - → Let $\mathbf{c}_m$ and $\mathbf{c}_n$ be two codewords, and $\mathbf{e}_{m,n} = \mathbf{c}_m \oplus \mathbf{c}_n$ be the difference vector (also binary) between these codewords
    - ▷ $\mathbf{e}_{m,n}$ has ones only in those places where the bits of $\mathbf{c}_m$ and $\mathbf{c}_n$ are differing; $\mathbf{e}_{m,n}$ is zero otherwise
  - → Hamming Distance d_H (or, more accurately $d_H^{m,n}$) between $\mathbf{c}_m$ and $\mathbf{c}_n$ is the Hamming Weight of $\mathbf{e}_{m,n}$
    - ▷ Hamming Distance is simply the number of places in which two binary sequences differ

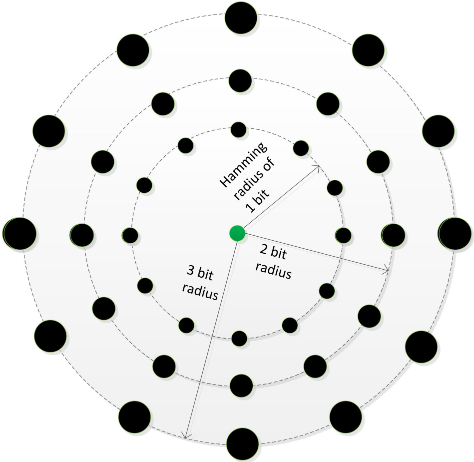
Minimum Hamming Distance d_{min}
for A Channel Code

- $d_H^{\min}$ is defined for a channel code with rate $r = \dfrac{K}{N}$
  - → This code takes K bit information sequence and generates N bit codeword
  - Thus, there are a total of 2^K codewords
- $d_H^{\min}$ for this channel code is the minimum Hamming distance between any two pairs of this channel code
- $d_H^{\min}$ relates to the error *detection* and error *correction* capabilities of the channel code. This can be visualized by drawing Hamming Circles as shown next

Hamming Circle of Radius 1 around a codeword



Hamming Circles around a codeword



Hamming Circles around several codewords