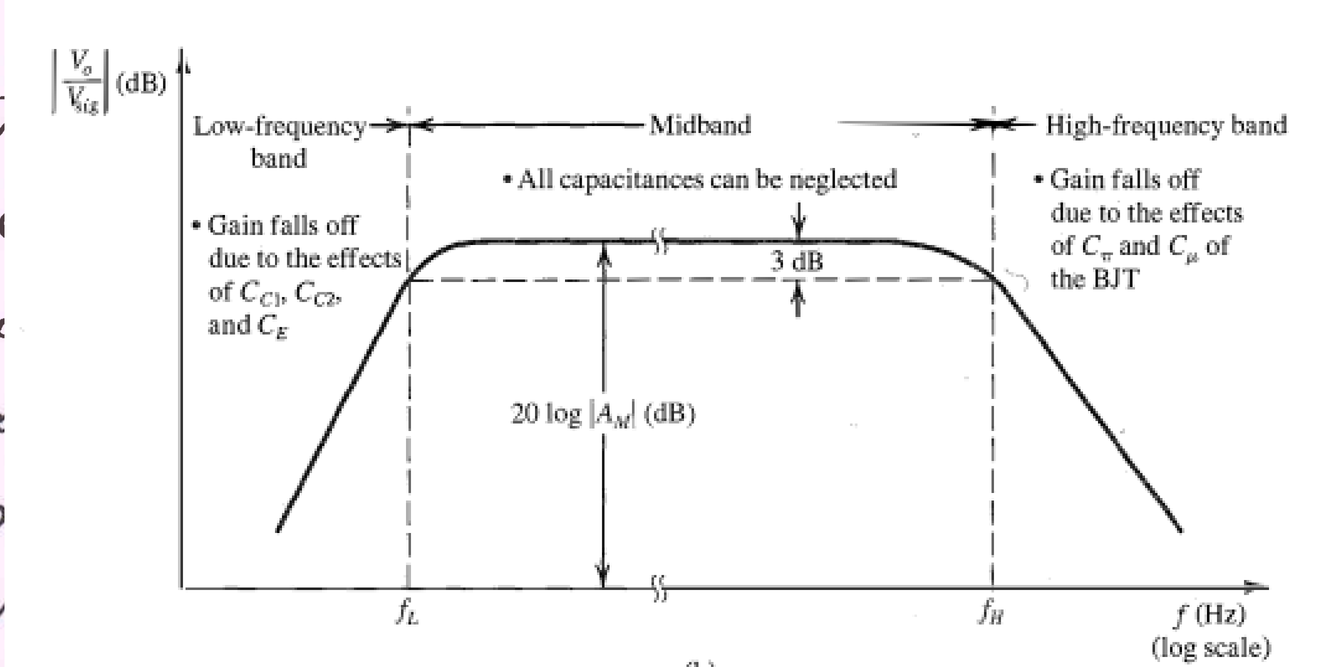
BJT Internal Capacitances & HIGH FREQUENCY MODEL

(a)



(b)

Th... se
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behaviour of

BASE CHARGING or Diffusion Capacitance C_{de}

When BJT is in active or saturation region, minority carrier charges are stored in base region

The charge Q_n in terms of collector current i_C

$$Q_n = \frac{W^2 i_C}{2 D_n} = \tau_F \cdot i_C$$

where τ_F = device constant = $W^2 / 2 D_n$

W = effective width of Base

D_n = electron diffusivity in Base

τ_F = FORWARD BASE TRANSIT TIME of the order of 10 pico s to 100 pico seconds.

Since i_C is related to v_{BE} exponentially Q_n also depends upon v_{BE} nonlinearly.
This mechanism can be represented by a SMALL SIGNAL DIFFUSION CAPACITANCE C_{de}

$$C_{de} = \frac{dQ_n}{dv_{BE}} = \tau_F \frac{di_C}{dv_{BE}} = \tau_F \cdot g_m$$

$$= \tau_F \cdot \frac{I_C}{V_T}$$

Thus C_{de} increases with I_C or Bias current.

—— x ——

BASE EMITTER JN CAPACITANCE

B-E jn. depletion layer capacitance C_{je} is given by

$$C_{je} = \frac{C_{jeo}}{\left(1 - \frac{V_{BE}}{V_{oe}}\right)^m}$$

C_{jeo} ← value of C_{je} at 0 V

V_{oe} = EBJ built-in Voltage ≈ 0.9 V

m = grading coeff. ≈ 0.5

V_{BE} = Forward junction DC voltage or bias voltage.

Approx. $C_{je} \approx 2\, C_{jeo}$

Collector Base Jn. Capacitance C_μ

∵ CB is reversed biased, its junction has a capacitance like effect modelled as

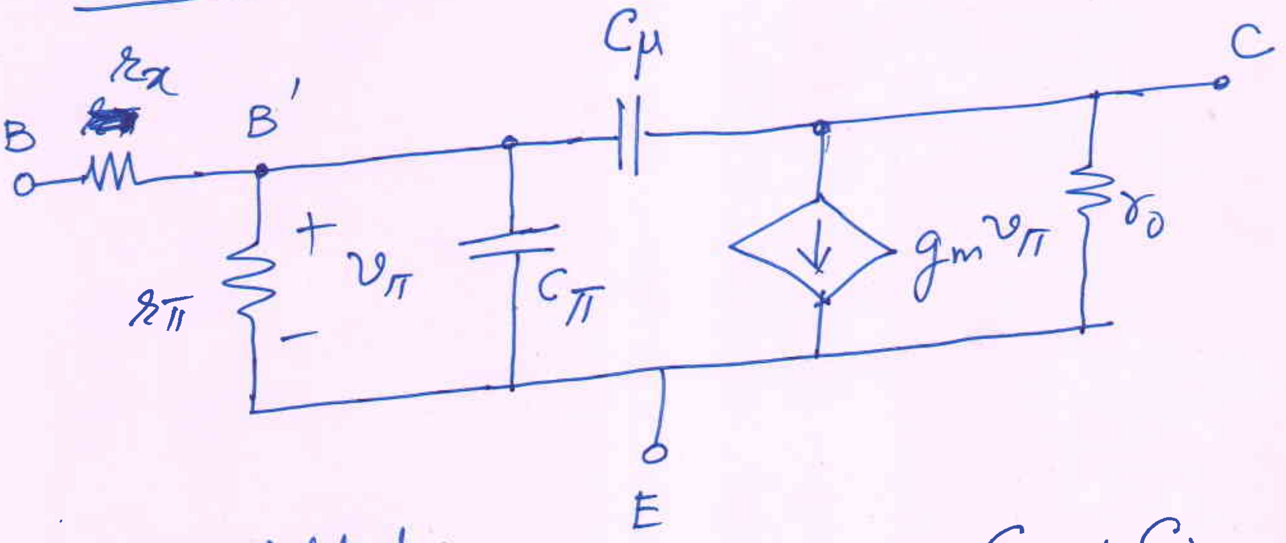
$$C_\mu = \frac{C_{\mu o}}{\left(1 + \frac{V_{CB}}{V_{oc}}\right)^m}$$

$C_{\mu o} \rightarrow C_\mu$ at 0V

$V_{oc} \rightarrow$ CBJ built-in Voltage $\simeq 0.75\,V$

$m \approx 0.2$ to $0.3V$.

HIGH-FREQUENCY HYBRID-π MODEL.



Newly Added:

C_π = emitter base capacitance = $C_{de} + C_{je}$

C_μ = Collector-base capacitance

r_x = small value to model resistance of silicon material $\approx$ few tens of ohms.

At low frequencies $r_x + r_\pi \approx r_\pi$ therefore it was never considered or brought in discussion.

At high freq. C_π will bypass r_π & r_x will matter.

Typically C_π = few tens of Picofarads

C_μ = fraction to few Picofarads

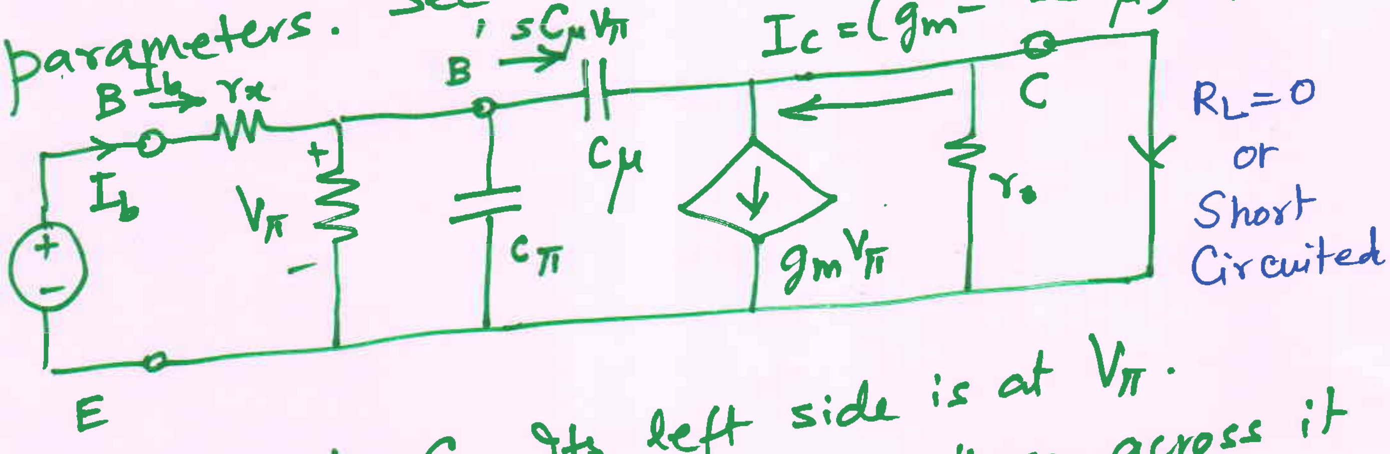
r_x = tens of ohms.

Voltages and currents described now onwards will be function of frequency hence we will use uppercase letters with lowercase subscripts like V_π or I_c.

Expression of Upper Cutoff Frequency

Derive an expression for short circuit current gain β (also called as h_{fe}) or CE S.C.C. Gain as a function of frequency, in terms of model parameters. See model below with $R_L = 0$. S.C.

$I_c = (g_m - sC_\mu) \cdot V_\pi$



See capacitor C_μ. Its left side is at V_π. Its right side is at GND ∴ voltage across it is V_π. ∴ Current through $C_\mu = \frac{V_\pi}{X_{C\mu}} = \frac{V_\pi}{1/C_\mu s}$

$= C_\mu s \cdot V_\pi$

∵ r_o is bypassed by a short circuit, it does not matter at all in calculations

Now look at currents in Node C :

coll. current $I_c = g_m V_\pi - sC_\mu V_\pi$.

$= (g_m - sC_\mu) V_\pi$

V_π is voltage across r_π due to I_b.

$$V_\pi = I_b \left(r_\pi \,//\, C_\pi \,//\, C_\mu \right)$$

$\because$ voltage across C_μ is also V_π.

$$V_\pi = \frac{I_b}{\frac{1}{r_\pi} + sC_\pi + sC_\mu}$$

Calculate h_{fe}

$$h_{fe} = \frac{I_c}{I_b} = \frac{(g_m - sC_\mu) \cdot V_\pi}{\left(\frac{1}{r_\pi} + sC_\pi + sC_\mu\right) \cdot V_\pi}$$

Assuming that at high frequencies $g_m >> \omega C_\mu$ & we neglect sC_μ in numerator.

$$h_{fe} \simeq \frac{g_m \cdot r_\pi}{1 + s(C_\pi + C_\mu) r_\pi}$$

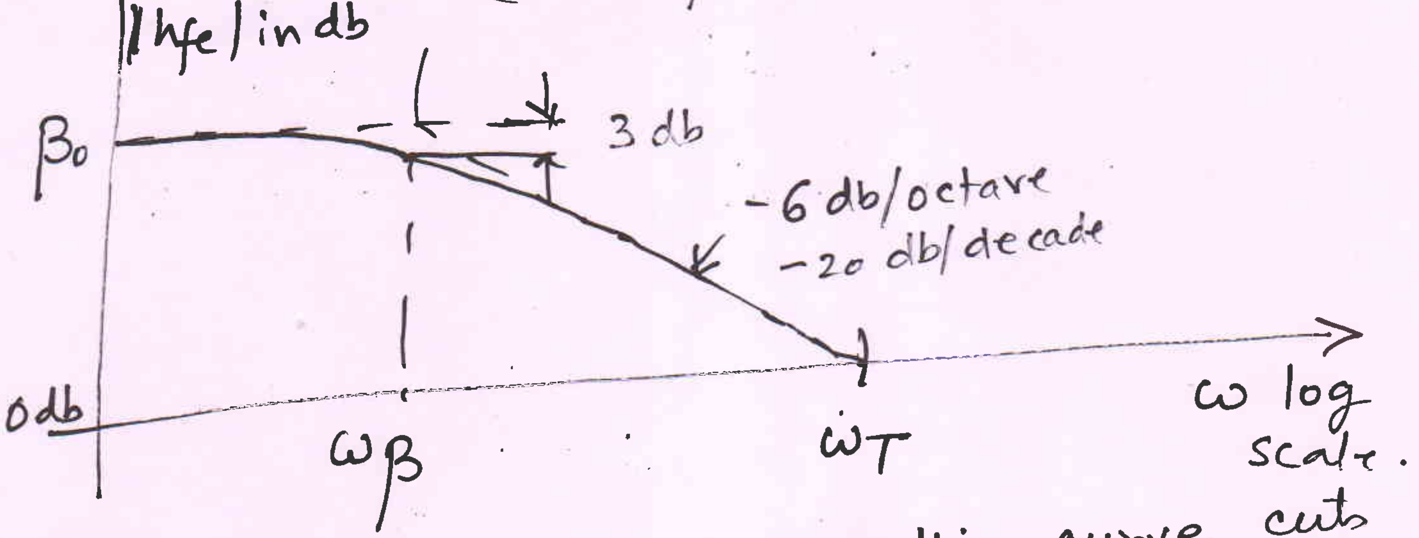
$$= \frac{\beta_0}{1 + s(C_\pi + C_\mu) r_\pi}$$

where β_0 = low frequency β.
$= g_m \cdot r_\pi$

Now, we can see this expression as an STC function at a 3-db or corner freq. $\omega = \omega_\beta$ where

$$\omega_\beta = \frac{1}{(C_\pi + C_\mu)\, r_\pi}$$

See Bode Plot



The frequency at which this curve cuts 0-db line is called frequency where $|h_{fe}| = 1$ or UNITY GAIN BANDWIDTH ω_T such that $\omega_T = (\beta_0 \cdot \omega_\beta)$.

Thus $\omega_T = \dfrac{\beta_0}{(C_\pi + C_\mu)\, r_\pi} = \dfrac{g_m}{(C_\pi + C_\mu)}$

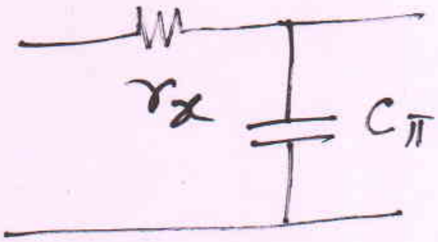
$$\therefore \boxed{f_T = \frac{\omega_T}{2\pi} = \frac{g_m}{2\pi(C_\pi + C_\mu)}}$$

f_T is usually specified in BJT datasheets.
It is in MHz to GHz range.

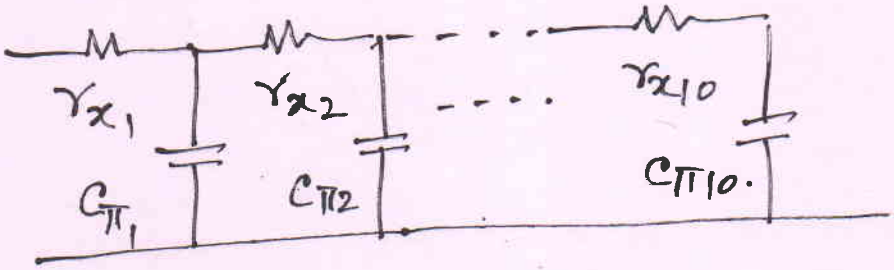
From f_T, we can get C_π & C_μ.
We can measure practically C_μ between B & C and get a measure of C_π.

This STC model we discussed, works well upto $0.2 f_T$. At higher frequencies, we must use distributed R-C model for better estimates:

lumped model.



distributed model. for say 10 taps...



Numerical on C_μ C_π and f_T.

3.149 Given npn BJT $I_c = 0.5\,mA$

$V_{CB} = 2V$; $\beta_o = 100$; $V_A = 50V$

$\tau_p = 30\,ps$; $C_{jeo} = 20$ femto farad $= 20\,fF = .02\,pF$

$C_{\mu o} = \cancel{20}\ 30$ femto F;

$V_{oc} = 0.75V$, $m_{CBJ} = 0.5$; $r_x = 100\,\Omega$

Draw hybrid-π model & find f_T.

— x —

$r_x = 100\,\Omega$; $g_m = I_c / V_T = 0.5\,mA / 25\,mv = 20\,m\mho$

$r_\pi = \beta_o / g_m = 100 / 20\,m\mho = 5\,K\Omega$

$r_o = V_A / I_c = 50V / 0.5\,mA = 100\,K\Omega$

$$C_\mu = C_{\mu o} / \left(1 + \frac{V_{CB}}{V_{oC}}\right)^m = \frac{\cancel{20}\ 30\,fF}{\left(1 + \frac{2V}{0.75V}\right)^{0.5}} = 15.66\,fF$$

$C_{je} \simeq 2\,C_{jeo} = 20 \times 20 = 40\,fF$

$C_{de} = \tau_F \cdot g_m = 30\,ps \cdot 20\,m\mho = 600$ femto F

$C_\pi = C_{je} + C_{de} = 40 + 600 = 640\,fF = 0.640\,pF$

$$f_T = \frac{g_m}{2\pi(C_\pi + C_\mu)} = \frac{20\ \cancel{m\mho} \times 10^{-3}\,\mho}{2 \times 3.14\,(640 + 15.66) \times 10^{-15}}$$

$$\boxed{= 4.857 \text{ Giga Hz}}$$

Q: 3.150 At 500 MHz signal, $|h_{fe}| = 2.5$ @ $I_C = 0.2$ mA and at same frequency $|h_{fe}| = 11.6$ @ $I_C = 1.0$ mA

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Given: $C_\mu = 0.05$ pF or 50 fF.

Find f_T at both Collector Currents.

What is τ_F and C_{je} value?

—— x ——

Note that

$$|h_{fe}| = \frac{f_T}{\text{freq of measurement}}$$

@ $I_C = 0.2$mA

$$\therefore f_T = |h_{fe}| \cdot f = 2.5 \times 500\text{ MHz} = 1.25\text{ GHz}.$$

at another I_C value @ $I_C = 1.0$ mA

$$f_T = 11.6 \times 500\text{ MHz} = 5.8\text{ GHz}$$

Note: Change in bias current from 0.2mA to 1.0 mA increases f_T from 1.25 to 5.8 GHz.

Thus if we want a higher frequency amplifier, we should keep I_C = higher value.

$$\because f_T = g_m / 2\pi(C_\pi + C_\mu)$$

$$\therefore C_\pi = \left(g_m / (2\pi f_T)\right) - C_\mu$$

at $I_C = 0.2$mA $\longrightarrow g_m = 0.2/25 = 8$ m℧

$$C_\pi = \left[8\text{ m℧} / (2 \times 3.14 \times 1.25\text{ GHz})\right] - 50\text{ fF} = 968\text{ fF}$$

at $I_C = 1.0\,mA$ $g_m = 40\,m\mho$

$$C_\pi = \left[40\,m\mho / (2 \times 3.14 \times 5.8\,GHz)\right] - 50\,fF$$

$$= 1047.6\,fF.$$

Note that C_π also increases with I_C.

— x —

Next, $C_\pi = C_{je} + \tau_F \cdot g_m$

@ $I_C = 0.2\,mA$

$$968\,fF = C_{je} + \tau_F \cdot 8\,m\mho \quad — (1)$$

@ $I_C = 1.0\,mA$

$$1047.6\,fF = C_{je} + \tau_F \cdot 40\,m\mho \quad — (2)$$

Solving (1) and (2) $C_{je} = 950\,fF$ and

$$\tau_F = 2.47\,ps$$

Q: 3.151 Given: $I_C = 2\,mA$; $C_\mu = 1\,pF$, $C_\pi = 10\,pF$, $\beta = 150$. Find f_T and f_β.

$$f_T = g_m / (2\pi(C_\mu + C_\pi)) = 80\,m\mho / (2 * 3.14 * (1+10)\,pF)$$

$$= \boxed{1.158\,GHz}$$

$$f_\beta = \frac{f_T}{\beta_0} = \frac{1.158\,GHz}{150} = \boxed{7.725\,MHz.}$$