Tutorial 6

1. Find all the local maxima, local minima and saddle points of the functions.
   (a) $f(x,y) = e^{x^2+y^2-4x}$
   (b) $f(x,y) = ln(x+y) + x^2 - y$
   (c) $f(x,y) = 1 - (x^2+y^2)^{1/3}$

2. Find the absolute maxima and minima of the function $f(x,y) = 2x^2 - 4x + y^2 - 4y + 1$ on the closed triangular area bounded by the lines $x = 0$, $y = 2$, $y = 2x$ in the first quadrant.

3. Find two numbers a and b with $a \leq b$ such that $\int_a^b (6 - x - x^2)dx$ has its largest value.

4. Let $f(x,y) = x^2 - xy + y^2 - y$. Find the direction u and the value of $D_u f(1,-1)$ for which
   (a) $D_u f(1,-1)$ is largest
   (b) $D_u f(1,-1)$ is smallest
   (c) $D_u f(1,-1) = 0$
   (d) $D_u f(1,-1) = 4$
   (e) $D_u f(1,-1) = -3$

5. Is there a direction u in which the rate of change of $f(x,y) = x^2 - 3xy + 4y^2$ at the point P$(1,2)$ equals 14? Give reason for your answer.

6. A flat circular plate has the shape of the region $x^2 + y^2 \leq 1$. The plate including the boundary is heated so that the temperature at the point (x,y) is $T(x,y) = x^2 + 2y^2 - x$. Find the temperature at the hottest and coldest points on the plate.