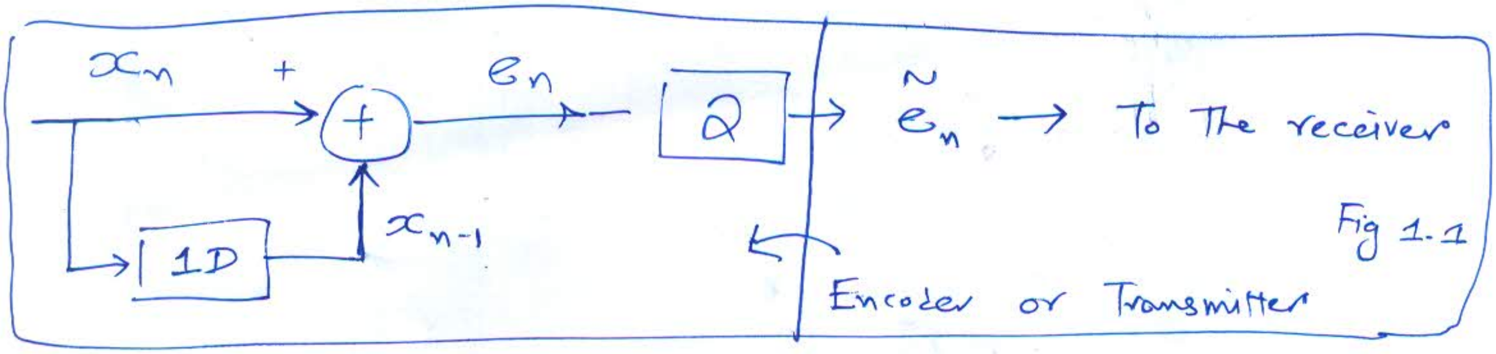
Differential PCM :

Main Idea: Quantize the difference between consecutive samples.

One Possible Design (not the best way) :



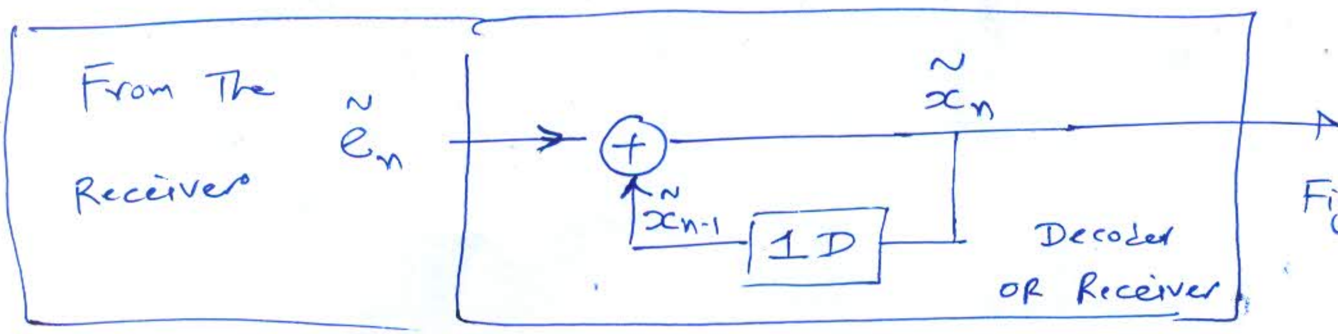


Fig 1.2

$\boxed{1D}$ one sample delay. $\boxed{Q}$ Quantization function.

In this scheme, decoded sample at the receiver can be represented as

$$\begin{aligned}\tilde{x}_n &= \tilde{x}_{n-1} + \tilde{e}_n \\ &= \tilde{x}_{n-2} + \tilde{e}_{n-1} + \tilde{e}_n \\ &= \tilde{x}_{n-3} + \tilde{e}_{n-2} + \tilde{e}_{n-1} + \tilde{e}_n \\ &\ \ \vdots \\ &= \tilde{x}_0 + \sum_{k=1}^{n} \tilde{e}_k\end{aligned}$$

Thus transmitter implements derivative function followed by quantization, and receiver is an accumulator (or integrator) function.

Now, from Fig 1.1, $\tilde{e}_n = Q(e_n) = Q(x_n - x_{n-1})$

Therefore, at the receiver, in reconstruction process

$$\tilde{x}_n = \tilde{x}_{n-1} + \tilde{e}_n = \tilde{x}_0 + \sum_{K=1}^{n} \tilde{e}_K$$

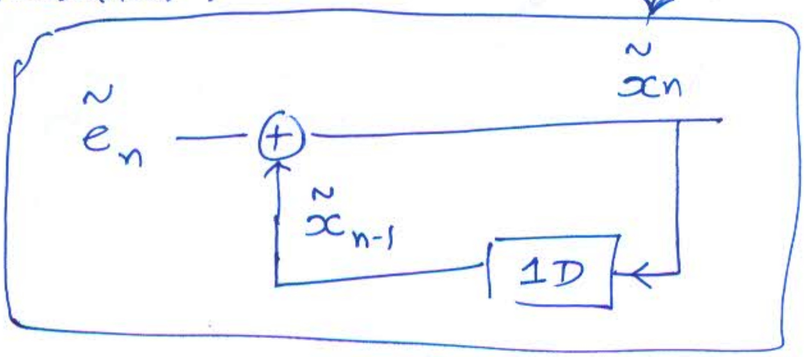
The quantization noise introduced by $Q(e_n)$ function accumulates. (due to accumulative summation $\sum_{K=1}^{n} \tilde{e}_K$).

This is undesirable, and it motivates an alternate design shown below.

Alternate Design (Better) : Main idea

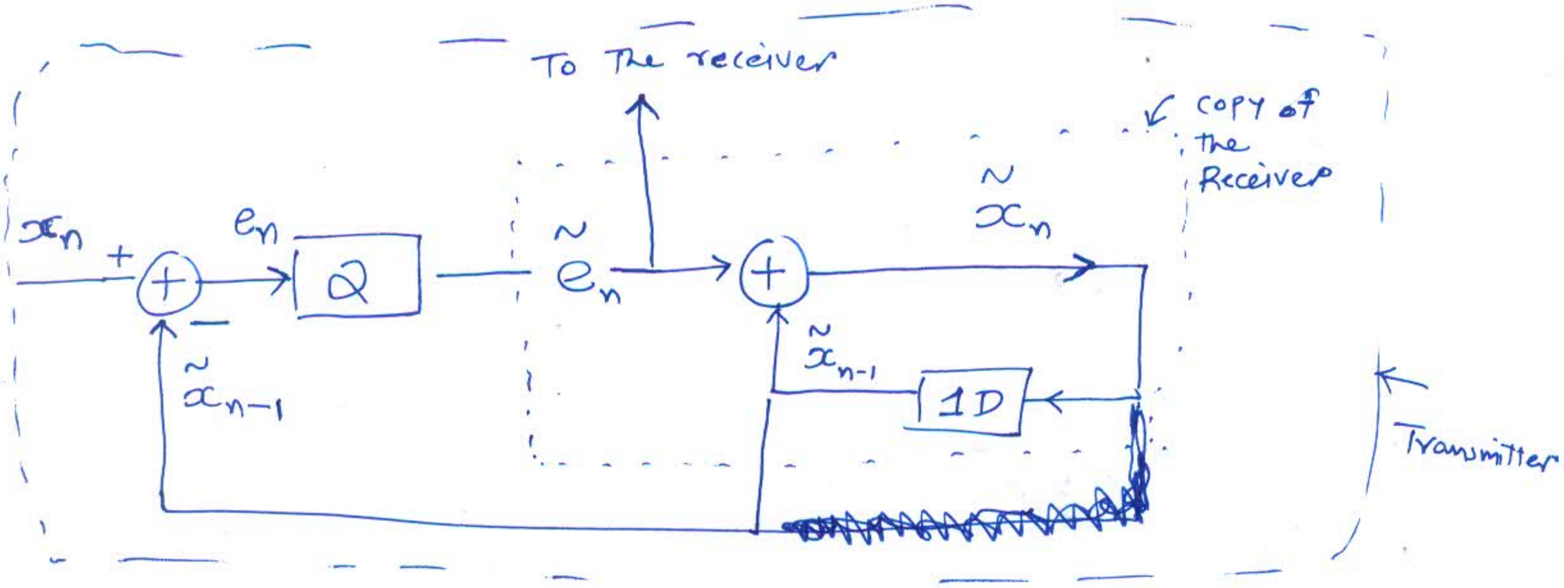
Implement a replica of the accumulative process (of the receiver) at the transmitter.

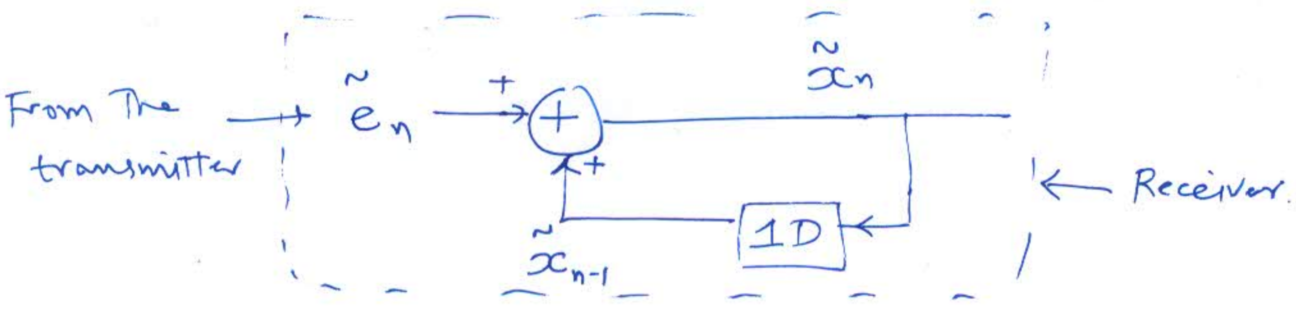
From Fig 1.2 :



Contd... on 3

In The alternate design, Fig 1.2 block is not only implemented at The ~~transmitter~~ receiver, but it is also "copied" at The transmitter.

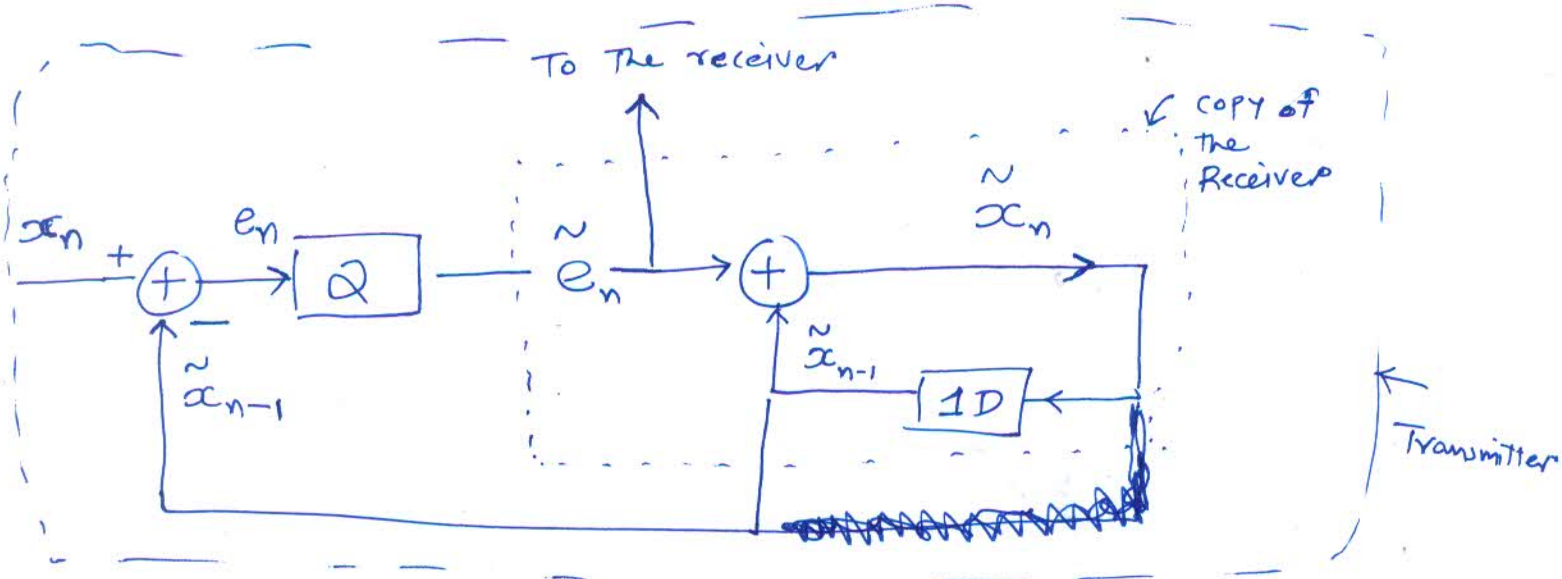


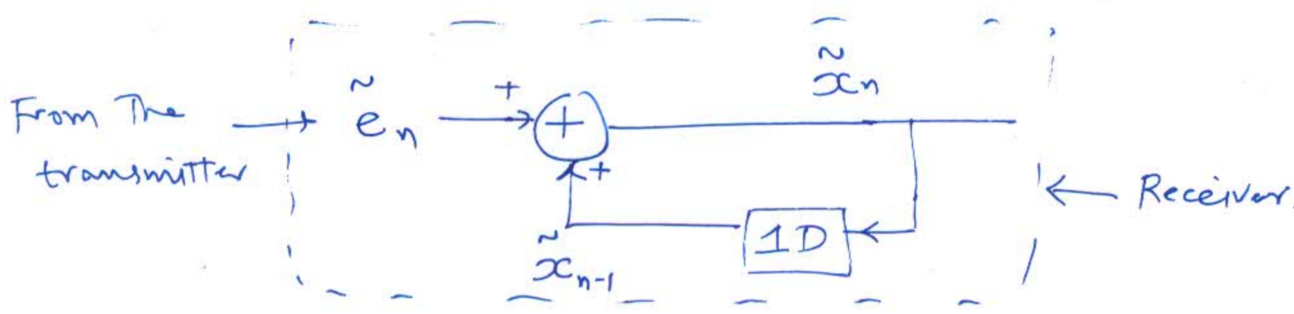


Thus, The receiver of This alternate design is The same as The receiver of The first design in Fig. 1.2.

However, although it implements The same cumulative process, The quantization noise does not grow.

In The alternate design, Fig 1.2 block is not only implemented at The receiver ~~transmitter~~, but it is also "copied" at The transmitter.





Thus, The receiver of This alternate design is The same as The receiver of The first design in Fig. 1.2.

However, although it implements The same cumulative process, The quantization noise does not grow.

This can be proven as follows:

$$\tilde{x}_n = \tilde{x}_{n-1} + \tilde{e}_n$$

$$\cancel{\tilde{x}_{n-1} + x_n - Q(x_n - \tilde{x}_{n-1})}$$

$$= x_n + \tilde{x}_{n-1} + \tilde{e}_n - x_n$$

$$= x_n + \tilde{e}_n - (x_n - \tilde{x}_{n-1})$$

$$= x_n + \tilde{e}_n - e_n$$

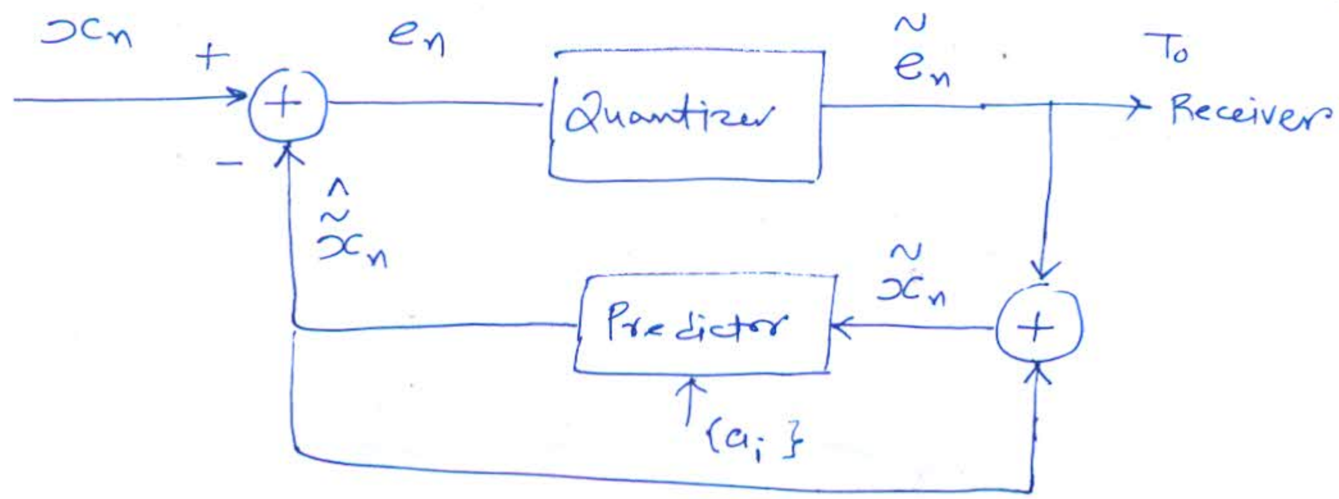
$$= x_n + q_n \quad , \quad \text{where } q_n = \tilde{e}_n - e_n$$

is The quantization error at n^{Th} sample.

This proves That The quantization error at n^{Th} sample does not grow with n.

So far, we have considered DPCM where x_n is predicted using The past sample. This is generalized next where a more general predictor is used.

DPCM with generalized prediction :

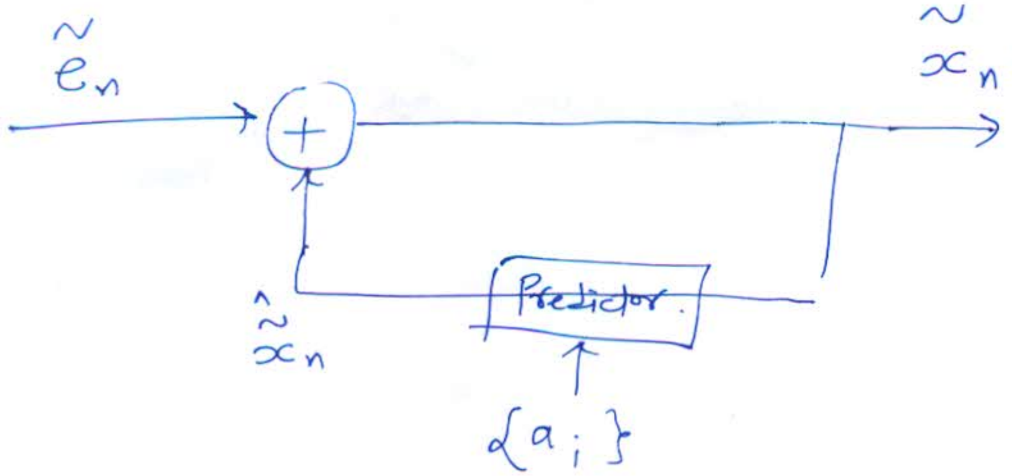


$$\tilde{e}_n - e_n = \tilde{e}_n - (x_n - \hat{\tilde{x}}_n)$$

TRANSMITTER

$$\Rightarrow \quad q_n = \tilde{e}_n + \hat{\tilde{x}}_n - x_n$$

$$\Rightarrow \quad q_n = \tilde{x}_n - x_n \Rightarrow \tilde{x}_n = x_n + q_n$$



$$\tilde{x}_n = \hat{\tilde{x}}_n + e_n$$

RECEIVER.