Superposition

- Equivalent resistance
- Voltage / current dividers
- Source transformations
- Node voltages
- Mesh currents
- Superposition

In a circuit having more than one independent source, we can consider the effects of the sources one at a time.

A math problem:

$$8v_a - 2v_b = 36 \text{ V}$$
$$-2v_a + 6v_b = 2 \text{ V}$$

Solving gives: $v_a = 5$ V, $v_b = 2$ V

$$8v'_a - 2v'_b = 0$$
$$-2v'_a + 6v'_b = 2 \text{ V}$$

Solving gives: $v'_a = 0.09091$ V, $v_b = 0.363637$ V

$$8v''_a - 2v''_b = 36 \text{ V}$$
$$-2v''_a + 6v''_b = 0$$

Solving gives: $v''_a = 4.909091 \text{ V}, v''_b = 1.636364 \text{ V}$

$$v'_a + v''_a = 5.0 \text{ V} = v_a$$
$$v'_b + v''_b = 2.0 \text{ V} = v_b$$

Mathematically, we can solve the simultaneous equations a bit at a time.

This means that we can solve a circuit a bit at a time.

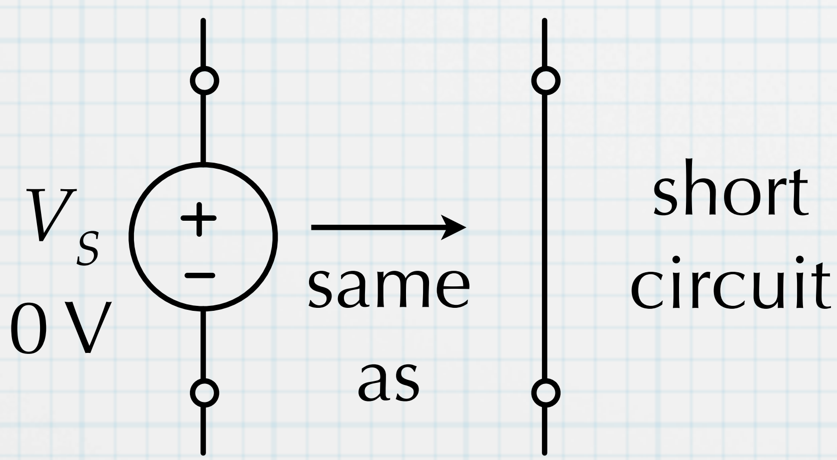
The superposition method

In a circuit having more than one independent source, we can consider the effects of the sources one at a time.

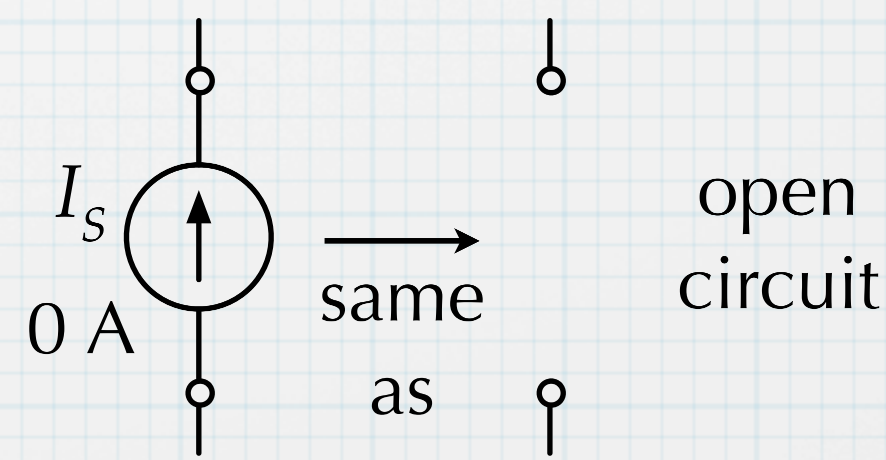
If a circuit has *n* independent sources, then we will have to solve the *n* circuits. It this easier? Perhaps. The resulting “partial” circuits will have one source and some resistors. We might be able to solve the partial circuits using the short-cut methods we saw earlier – each partial circuit may go very fast.

As we consider the effect of each source by itself, we must “turn off” (de-activate) all of the other sources – set their values to zero.

Replace voltage sources with shorts.

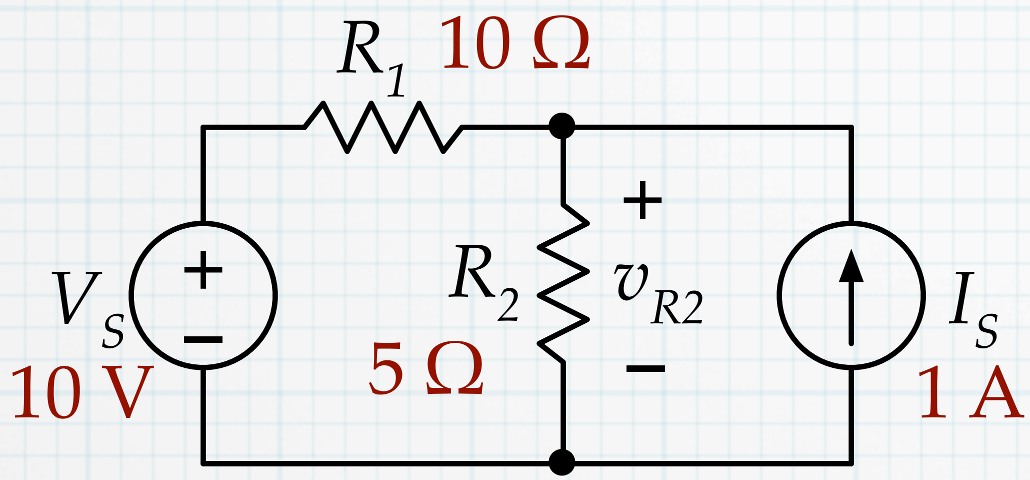


Replace current sources with opens.

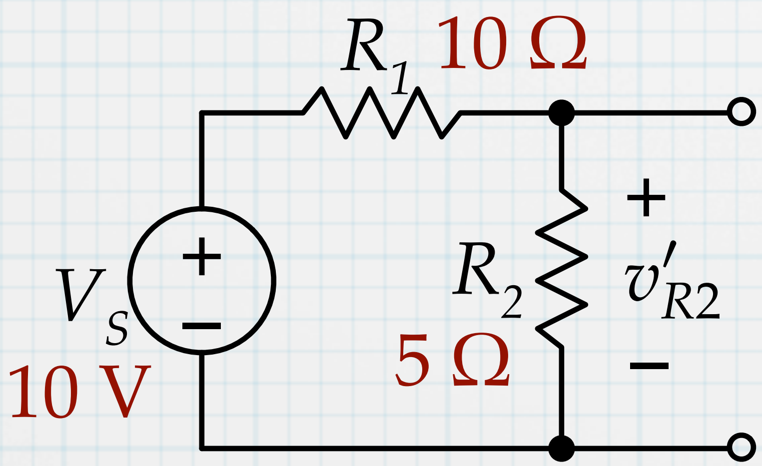


Example of the superposition method

Consider the 2-source, 2-resistor circuit one more time. Let's us use superposition to find the voltage v_{R2}.



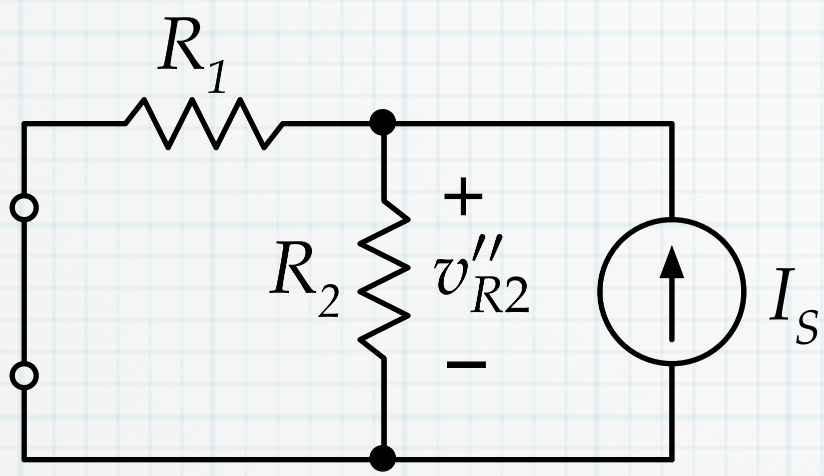
1. Turn off one of the sources – the order doesn't matter. So start by de-activating the current source. Set the value of I_S to zero, which has the same effect as replacing I_S with an open circuit.



$$v'_{R2} = \frac{R_2}{R_1 + R_2} V_S$$

$$= \frac{5\Omega}{10\Omega + 5\Omega} (10\text{V}) = 3.33\text{V}$$

2. Go back to the original circuit and turn off the other source – set V_S to zero, which is the same as replacing it with a short circuit.



Note that shorting V_S causes R_1 to be in parallel with R_2.

$$v''_{R2} = I_S\left(R_1 \| R_2\right)$$

$$= (1\text{A})\,(5\Omega \| 10\Omega) = 3.33\text{V}$$

3. The complete answer is the sum (superposition) of the two partial answers.

$$v_{R2} = v'_{R2} + v''_{R2} = 3.33\text{V} + 3.33\text{V} = 6.66\text{V}$$

Summary of the superposition method

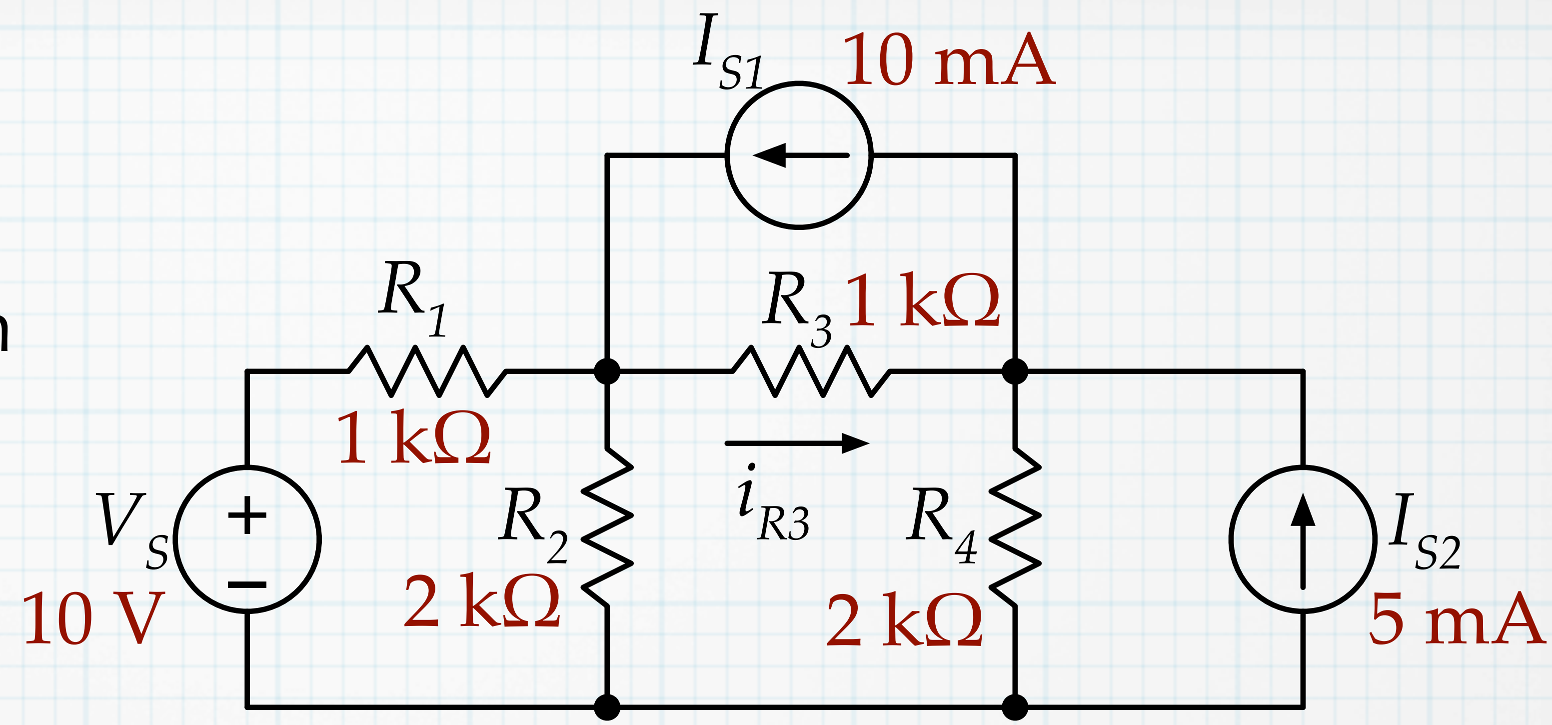
1. Identify all of the independent sources in the circuit.
2. Choose one source that will remain active. De-activate all of the others. (Remove current sources, leaving open circuits. Replace voltage sources with short circuits.)
3. Using whatever techniques are appropriate, solve for the desired quantity (current or voltage) in the circuit. This will be a "partial" result, due only to the one active source in the circuit.
4. Return to the original circuit. Choose a different source to remain active and de-activate all of the others.
5. Solve again for the desired quantity, which will be a second partial result.
6. Continue in this manner, working sequentially through each of the sources in the circuit, finding a partial result for each.
7. Add together all of the partial results to obtain the total result corresponding to all of the sources working simultaneously in the circuit.

Cautions in using superposition

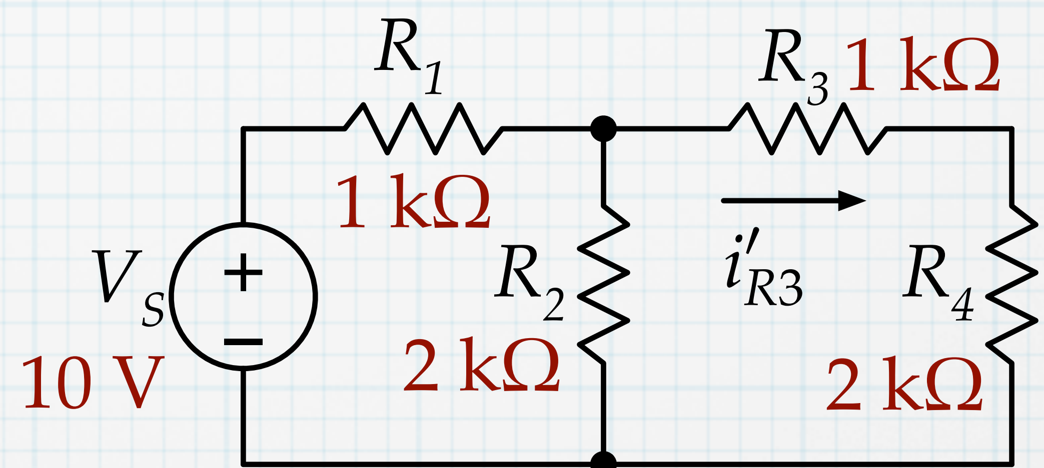
1. Superposition only works with *linear* circuits. (Linear circuits contain only sources, resistors, capacitors, inductors, linear amplifiers, etc.) Most electronic devices (diodes and transistors) are non-linear, so superposition will not be applicable.
2. Because the method relies on linearity, you cannot add powers directly using the superposition method. (Power goes as v^2 or i^2 – it is not linear.) Use superposition to find the total current or voltage and then calculate power from that result.
3. When finding the partial solutions, be sure to maintain the same voltage polarity and current direction in each case. For example, one source may induce the current in a particular resistor to flow in one direction while another source causes to a current flowing in the opposite direction. You must keep the proper signs when adding to the partial results to obtain the correct total result.

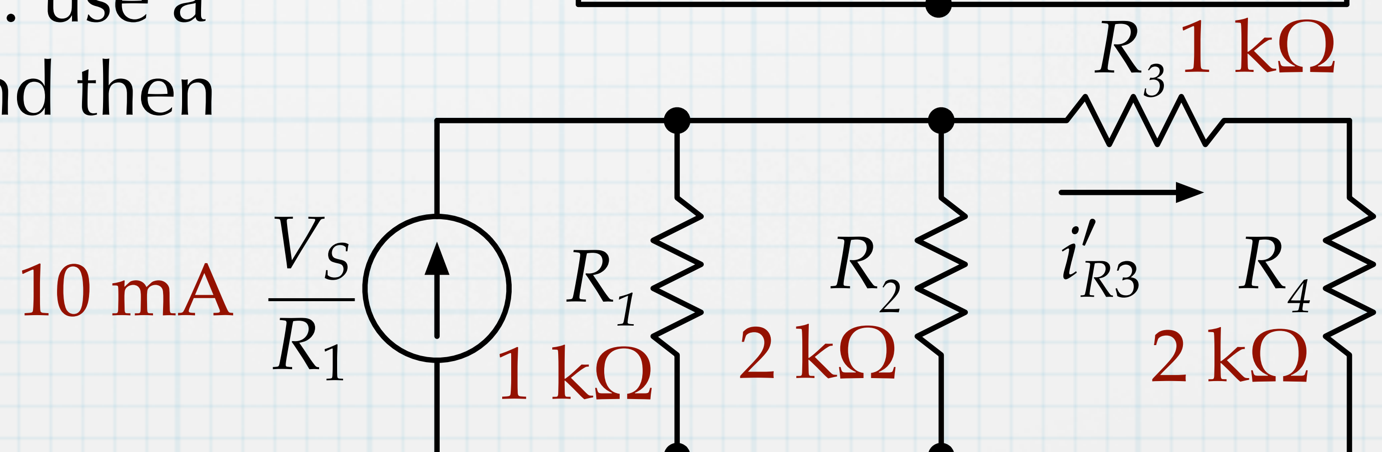
Example

In the circuit, find i_{R3}. With three sources, there will be three partial solutions.



1. Start with V_S. De-activate the two current sources. Find i'_{R3} in the partial circuit, using whatever short-cut methods you like. One approach: use a source transformation and then a current divider.

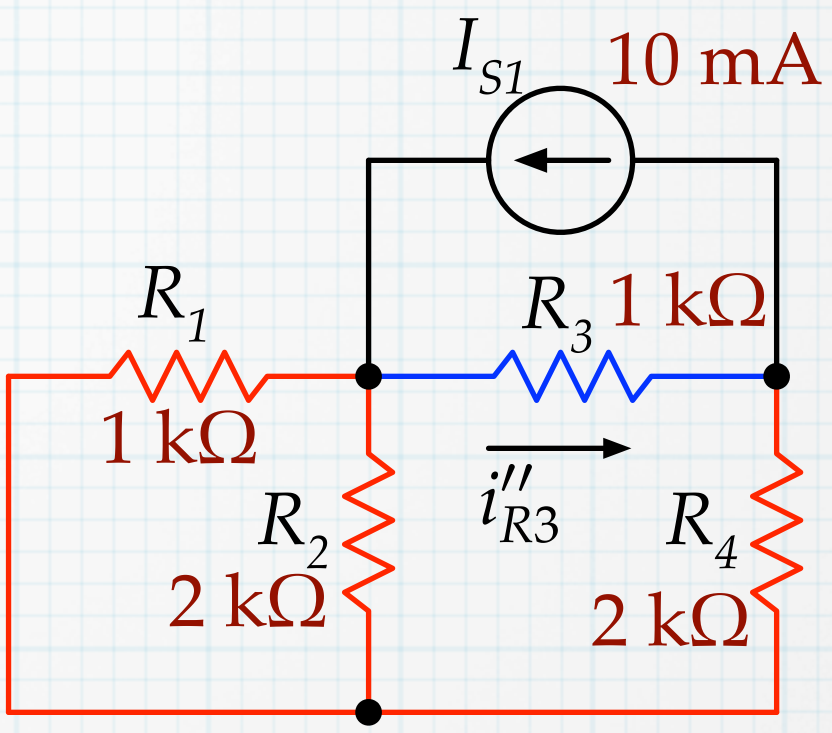




$$i'_{R3} = \frac{\frac{1}{R_3+R_4}}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3+R_4}}\left(\frac{V_S}{R_1}\right) = \frac{\frac{1}{3\text{k}\Omega}}{\frac{1}{1\text{k}\Omega}+\frac{1}{2\text{k}\Omega}+\frac{1}{3\text{k}\Omega}}(10\text{mA}) = 1.818\text{mA}$$

2. Go back to the original circuit and de-activate V_S and I_{S2}, keeping I_{S1}. Note that with V_S replaced by a short, R_1 is in parallel with R_2.

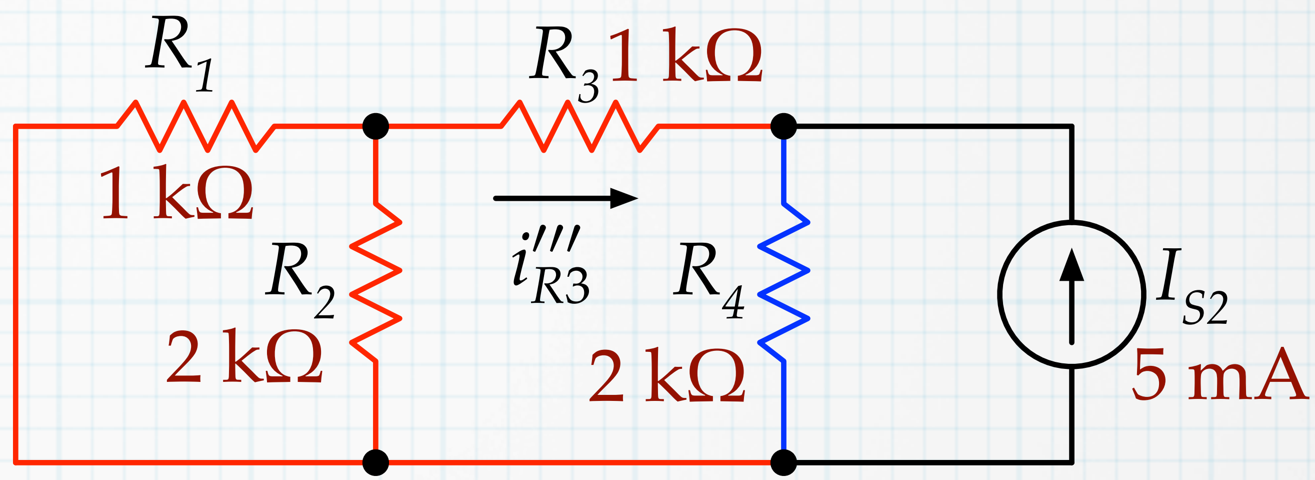
Examining the partial circuit carefully, we see that it is essentially a current divider. The source current splits between two paths – one with R_3 and another that is a combination of R_1, R_2, and R_4.



$$R_{14} = R_1 \| R_2 + R_4 \ = 1\text{k}\Omega \| 2\text{k}\Omega + 2\text{k}\Omega = 2.67\text{k}\Omega$$

$$i''_{R3} = \frac{\frac{1}{R_3}}{\frac{1}{R_3} + \frac{1}{R_{14}}} I_{S1} \quad = \frac{\frac{1}{3\text{k}\Omega}}{\frac{1}{3\text{k}\Omega} + \frac{1}{2.67\text{k}\Omega}} (10\text{mA}) = 4.71\text{mA}$$

3. Now we need the final partial circuit. Go back to the original circuit and de-activate V_S and I_{S1}, keeping I_{S2}. Again, R_1 is in parallel with R_2.



Again, we can handle the calculation with a current divider. The source current splits between two paths – one with R_4 and another that is a combination of R_1, R_2, and R_3. Also, note carefully the direction – i'''_{R3} will be negative!

$$R_{13} = R_1 \| R_2 + R_3 \quad = 1\text{k}\Omega \| 2\text{k}\Omega + 2\text{k}\Omega = 1.67\text{k}\Omega$$

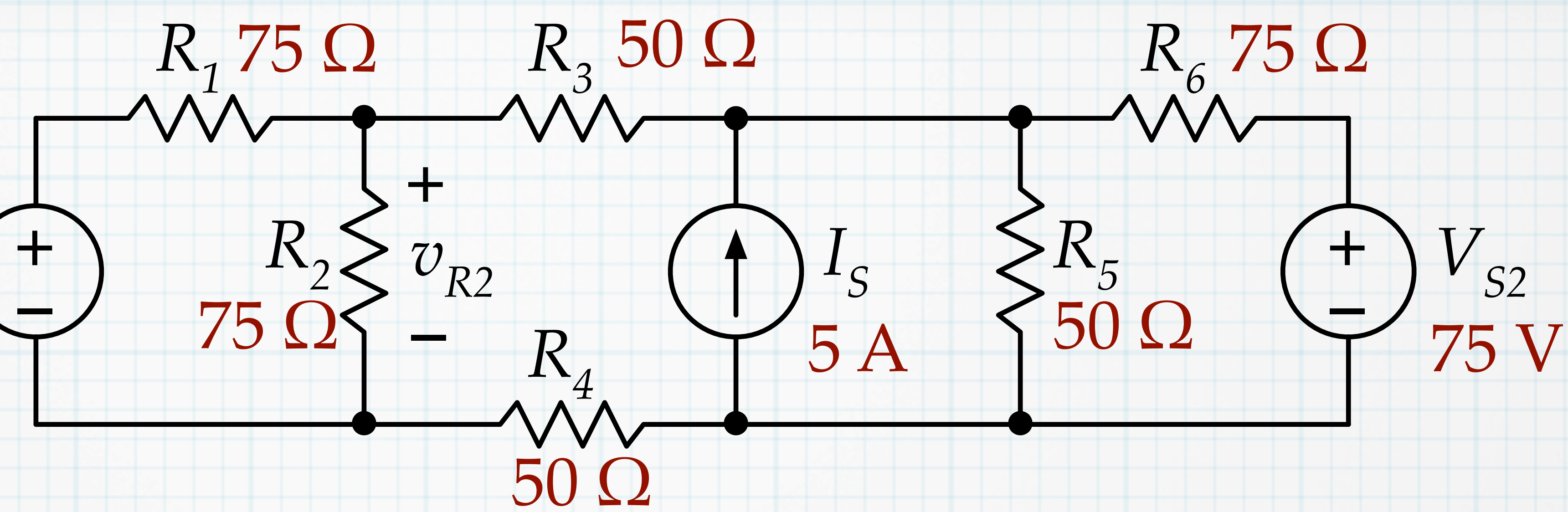
$$i'''_{R3} = -\frac{\frac{1}{R_{13}}}{\frac{1}{R_{13}} + \frac{1}{R_4}} I_{S2} \quad = -\frac{\frac{1}{1.67\text{k}\Omega}}{\frac{1}{1.67\text{k}\Omega} + \frac{1}{2\text{k}\Omega}} (5\text{mA}) = -2.72\text{mA}$$

4. To complete the calculation, add the three partial results to get the total.

$$i_{R3} = i'_{R3} + i''_{R3} + i'''_{R3} \quad = 1.818\text{mA} + 4.71\text{mA} - 2.72\text{mA} = 3.81\text{mA}$$

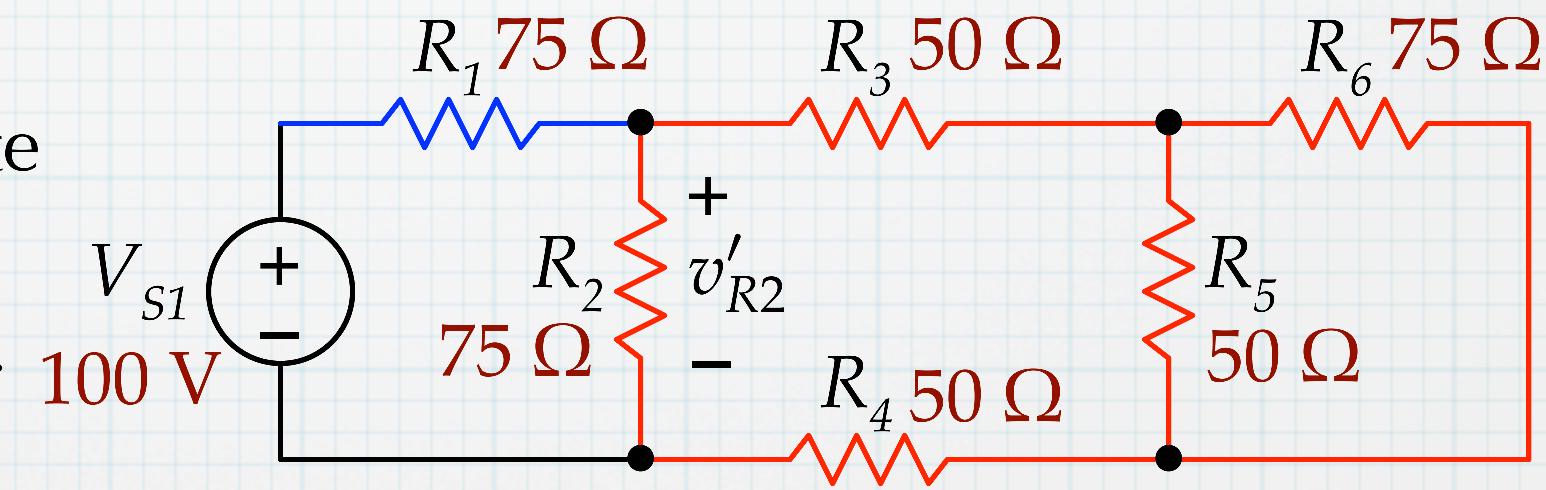
Example

Find v_{R2}.



Looks nasty – 6 nodes, 4 meshes. Either NV or MC will be tough. Might be able to do a bunch of source transformations. Since there are only three sources, let's try superposition.

1. Keep V_{S1} and de-activate I_S and V_{S2}. Shorting V_{S2} puts R_6 in parallel with R_5.

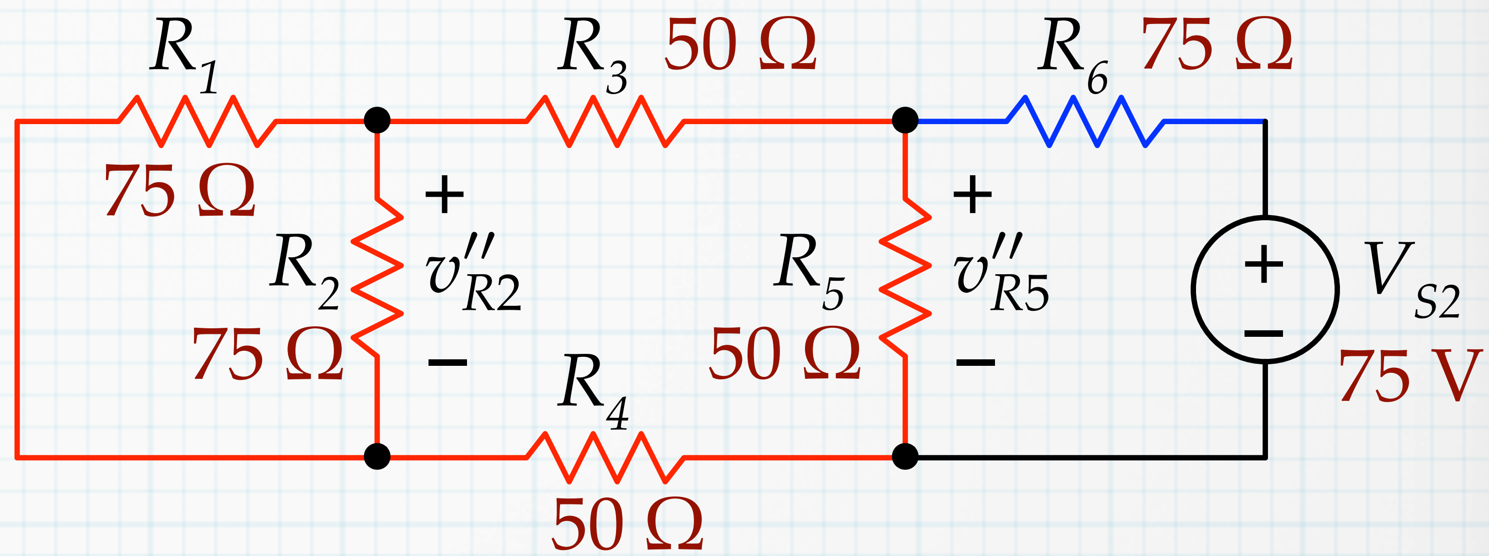


Use a voltage divider between R_1 and all the rest.

$$R_{26} = R_2 \| \left(R_3 + R_4 + R_5 \| R_6\right) = 47.6\Omega$$

$$v'_{R2} = \frac{R_{26}}{R_1 + R_{26}} V_{S1} = \frac{47.6\Omega}{75\Omega + 47.6\Omega}(100\text{V}) = 38.8\text{V}$$

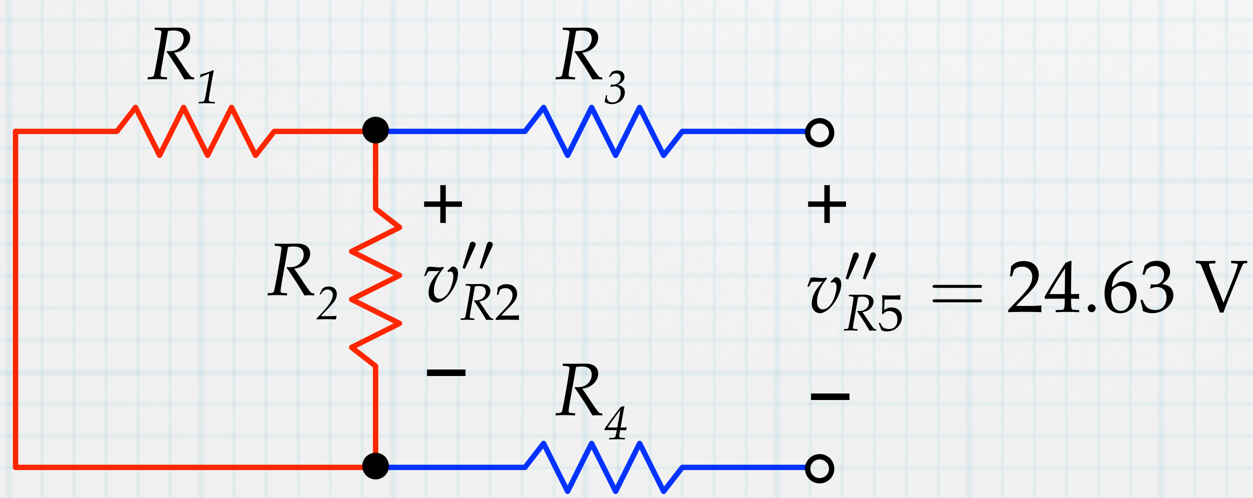
2. Go back to the original circuit. Keep V_{S2} and de-activate I_S and V_{S1}. Shorting V_{S1} puts R_1 in parallel with R_2.



Use voltage dividers twice: first to find the voltage across R_5, then again to find voltage across R_2.

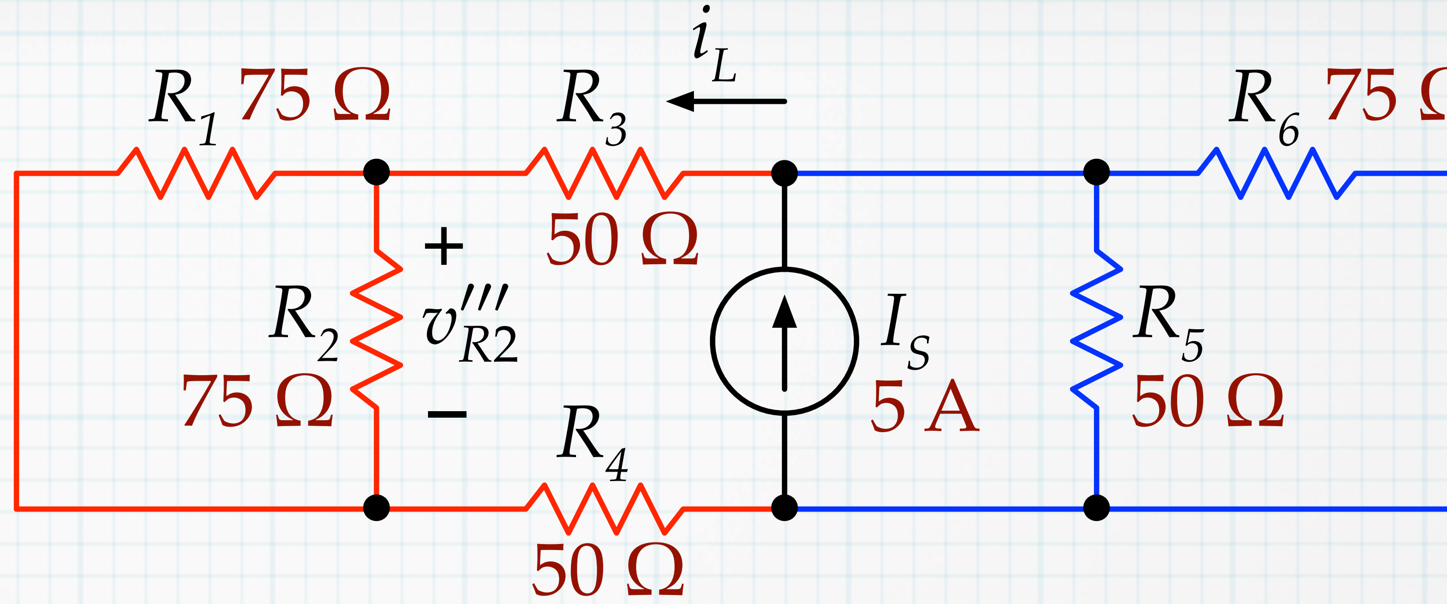
$$R_{51} = R_5 \| \left(R_3 + R_4 + R_1 \| R_2\right) = 36.67\ \Omega$$

$$v''_{R5} = \frac{R_{51}}{R_6 + R_{51}} V_{S2} = \frac{36.67\Omega}{75\Omega + 36.67\Omega}(75\text{V}) = 24.63\text{V}$$



$$v''_{R2} = \frac{R_1 \| R_2}{R_3 + R_4 + R_1 \| R_2} v''_{R5} = \frac{37.5\Omega}{50\Omega + 50\Omega + 37.5\Omega}(24.63\text{V}) = 6.72\text{V}$$

3. Finally, keep I_S and de-activate V_{S1} and V_{S2} in the original circuit.



$$R_{41} = R_4 + R_3 + R_1 \| R_2 = 137.5\ \Omega \qquad R_{56} = R_5 \| R_6 = 30\ \Omega$$

Source current divides to the left and right according to the equivalent resistances on either side. Use a current divider to find the current to the left.

$$i_L = \frac{\frac{1}{R_{41}}}{\frac{1}{R_{41}} + \frac{1}{R_{56}}} I_S = \frac{\frac{1}{137.5\Omega}}{\frac{1}{137.5\Omega} + \frac{1}{30\Omega}} (5\text{A}) = 0.90\text{A}$$

$$v'''_{R2} = i_L \left(R_1 \| R_2\right) = (0.90\ \text{A}) \left(75\ \Omega \| 75\ \Omega\right) = 33.6\ \text{V}$$

4. Add the three partial results to get the total.

$$v_{R2} = v'_{R2} + v''_{R2} + v'''_{R2} = 38.8\text{V} + 6.72\text{V} + 33.6\text{V} = 79.1\text{V}$$