Lecture - 4

Birthday paradox:

n people in a room

365 days in a year.

A: atleast 2 people in the room share a birthday. →

$\bar{A}$: no two people share a birthday

n	$p(\bar{A})$
1	1
2	$\frac{365 \cdot 364}{365 \cdot 365}$
3	$\frac{365 \cdot 364 \cdot 363}{365^3}$

n people in the room

$$p(\bar{A}) = \frac{365 \cdot 364 \cdot \cdots (365-n+1)}{365^n}$$

$$p(A) = 1 - \frac{365 \cdot 364 \cdots (365-n+1)}{365^n}$$

$n = 70, \quad p = 0.999$

$n = 23, \quad p = 0.5$

$$1 - \frac{365 \cdot 364 \cdots (365-n+1)}{365^n} \geq 0.5$$

Conditional Probability. 3

<u>Definition:</u>

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

past

?

Probability of event A taking place given that event B has already taken place.

$$\boxed{\begin{aligned} P(A \cap B) &= P(A|B)\, P(B) \\ P(A \cap B) &= P(B|A)\, P(A) \end{aligned}}$$

$$P(B|A) = \frac{P(B \cap A)}{P(A)} = \frac{P(A \cap B)}{P(A)}.$$

e.g. Box: 25 bulbs.

5 are good: light for 1000 hours.

10 are partially defective : ,, 1 hour.

10 are totally defective : will not light up at all.

You randomly choose a bulb and it lights up. What is the probability that its a good bulb?

A : its a good bulb

B : it lights up

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{5/25}{15/25}$$

$$= \frac{5}{15} = \frac{1}{3}$$

Q: A student needs to choose between 2 electives

Machine Learning.

IOT

You choose one of the courses by tossing a coin.

if you choose ML, then $p(\text{getting AA}) = 1/2$.

if you choose IOT, $p(\text{getting AA})$ is $2/3$.

what is the probability that you get AA in ML?

A: get AA

B: you choose ML

$P(A|B)$ $\propto$ $P(B|A)$ $\propto$

$P(A\cap B) =$

$P(A|B) \cdot P(B) = \frac{1}{2}\cdot\frac{1}{2}$

$P(B|A)\, P(A)$ $\propto$ $\quad = \frac{1}{4}$

eg. Box has 12 balls

8 red

4 white.

2 are drawn (without replacement)

$$P(\underline{\text{both are red}}) = \frac{{}^{8}C_{2}}{{}^{12}C_{2}}$$

R_1: 1st ball drawn is Red

R_2: 2nd ” ”

$$P(R_1 \cap R_2) = P(R_2 | R_1)\, P(R_1)$$

$$\frac{7}{11} * \frac{8}{12}$$

$P(E_1 \cap E_2 \cap E_3 \ldots \cap E_n) =$

$P(E_1)\, P(E_2 \mid E_1)\, P(E_3 \mid E_1 \cap E_2)$

$P(E_4 \mid E_1 \cap E_2 \cap E_3) \ldots\ldots$

$P(E_n \mid E_1 \cap E_2 \cap E_3 \ldots \cap E_{n-1})$

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e.g.: Insurance

accident prone: P(having an accident in the next one year) $= 0.4$

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not accident prone: P(having an accident in the next one year) $= 0.2$

30% of the population is accident prone.

70% is not.

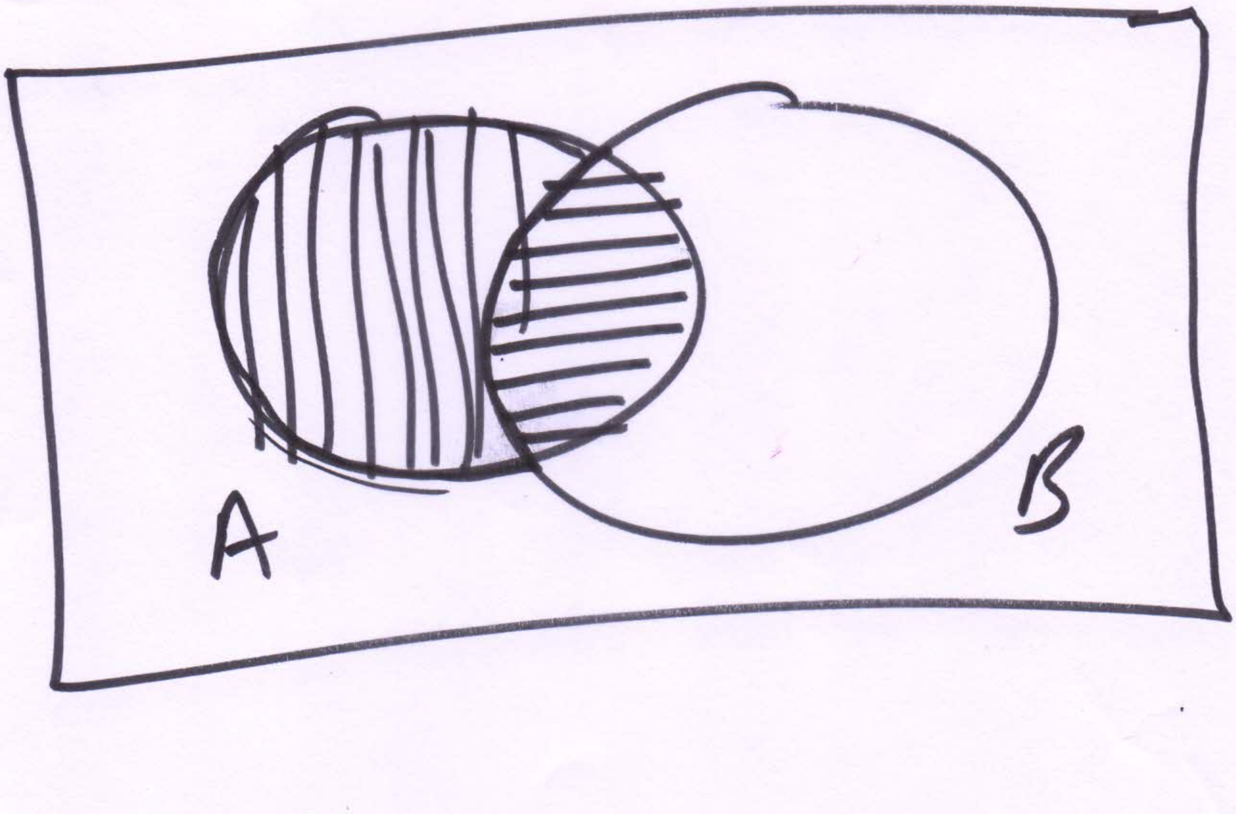
You buy a new policy. ①

What is the probability that you have an accident in the next one year?

A : you have an accident in the next one year.

B : You are accident prone.

$$P(A) = P(\underline{A \cap B}) + P(\underline{A \cap \bar{B}})$$



$$P(A) = \underline{P(A \cap B)} + P(A \cap \bar{B})$$

$$= P(A|B)\underline{\underline{P(B)}} + P(A|\bar{B})\,P(\bar{B})$$

$$= 0.4 * 0.3 + 0.2 * 0.7$$

$$= 0.12 + 0.14$$

$$= 0.26$$