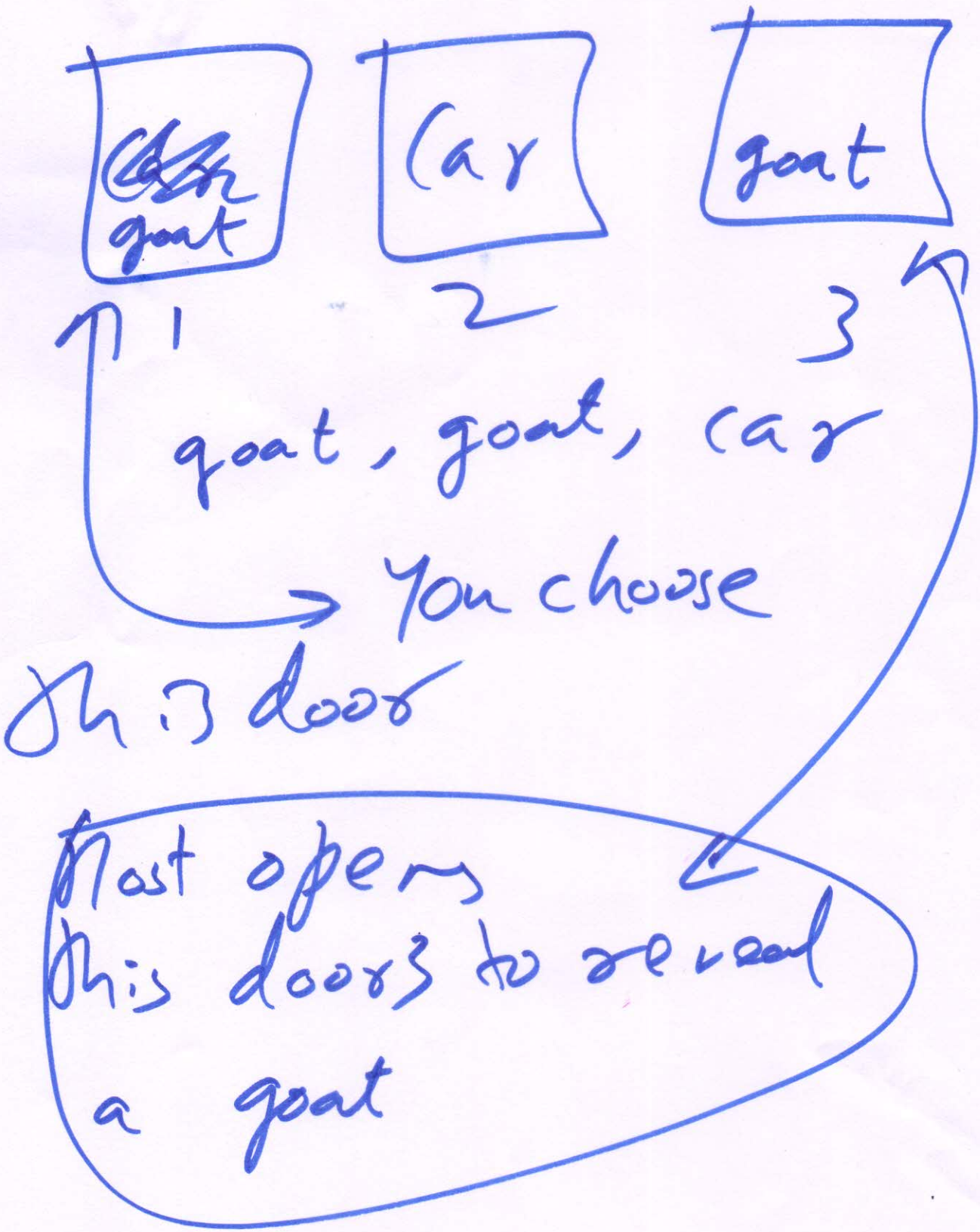
Lecture-6

Monty Hall Problem



A: car behind door 1 ②

B: 2

C: ③

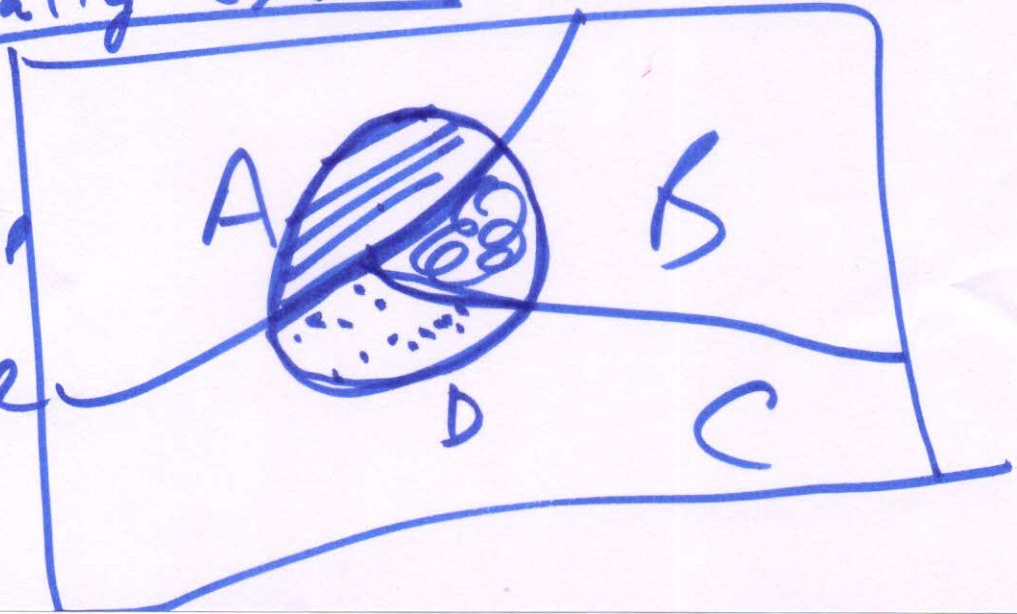
D: Host opens door 3 to reveal a goat.

$P(A|D) = \frac{1}{3}$ | $P(\text{Host opens 3 to reveal a goat} | \text{car is behind 1})$

$\rightarrow P(B|D) = \frac{2}{3}$ |

$$P(A|D) = \frac{P(A \cap D)}{P(D)} = \frac{P(D|A)\,P(A)}{P(D)}$$

mutually exclusive and exhaustive



$P(D) = P((D \cap A) \cup (D \cap B) \cup (D \cap C))$ (3)

mutually exclusive.

axiom (iii)

$P(D) = P(D \cap A) + P(D \cap B) + P(D \cap C)$

$P(D|A)\,P(A) + P(D|B)\,P(B) + P(D|C)\,P(C)$

$$P(A|D) = \frac{P(D|A)\,P(A)}{P(D|A)\,P(A) + P(D|B)\,P(B) + P(D|C)\,P(C)}$$

$P(A) = \frac{1}{3}$

$$= \frac{\frac{1}{2} * \frac{1}{3}}{\frac{1}{2} * \frac{1}{3} + 1 * \frac{1}{3} + 0 * \frac{1}{3}}$$

Q2. (4)

There has been a murder.
You are 60% sure
that the gardner committed
the crime.

A strand of brown hair
is found near the victim.
The gardner also has brown hair.
20% of the population has
brown hair.

g: gardner has brown hair. → taken place.

(C) gardner committed the murder.

→ interested in

$P(C) = 0.6$

$P(C|g) = \frac{P(C \cap g)}{P(g)}$

$$P(c|g) = \frac{P(c \cap g)}{P(g)}$$

conditional probability

$$= \frac{P(g|c)\,P(c)}{P(g)}$$

$$= \frac{P(g|c)\,P(c)}{P(g|}$$

C: gardener committed the murder.

g: Person with brown hair committed the murder.

Brown hair was found near the body ⟹ a person with brown hair did the murder.

C: gardener is the murderer

g: the particular strand of brown hair is the gardener's

$P(C) = 0.6$ (before new evidence)

S: strand of brown hair is found

$P(C \mid S)$ = What is the probability that the gardener is the murderer given that a strand of brown hair is found near the victim?