2

Elementary Fields and Waves

At low frequencies, electrical phenomena are usually described and measured in terms of charge, voltage, and current; whereas at microwave frequencies, fields and waves are often used. Therefore, it is important for the microwave engineer to appreciate and understand these concepts. This chapter reviews some of the basic principles of electromagnetic fields and waves.

2–1 ELECTRIC AND MAGNETIC FIELDS

The concept of electric and magnetic fields is very useful in analyzing the forces associated with electric charges and their motion.

2–1a The Electric Field

The force between electric charges is governed by Coulomb's law. If the charges are of opposite sign, the force is one of attraction, while for those with like sign, the force is a repulsive one. It is convenient to express this force in terms of an electric field intensity $\vec{E}$ defined as the force per unit positive charge. That is,

$$\vec{E} \equiv \frac{\vec{F}}{q} \tag{2–1}$$

where the SI unit for $\vec{E}$ is V/m.[1] The electric field due to a single positive charge q_1 is shown in Fig. 2–1. The field lines at any point in the surrounding region indicate

[1] A list of symbols and their SI units is given in Appendix A.

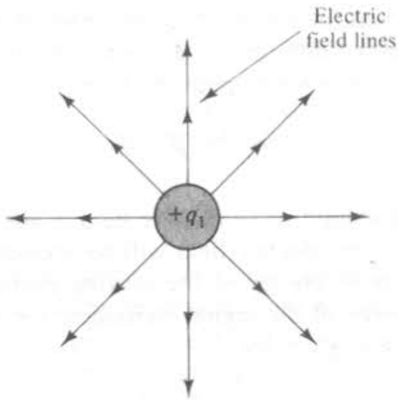

Figure 2-1 The electric field due to a positive point charge.

the direction of the force that would exist on a unit positive charge placed at that point. The magnitude of the field at a distance r from q_1 is given by

$$E = \frac{q_1}{4\pi\epsilon_0\epsilon_R r^2} \tag{2-2}$$

where ϵ_R is the relative permittivity or dielectric constant of the surrounding region. For free space, $\epsilon_R = 1.00$.

Since the density of E lines is the same in all directions, its value is inversely proportional to r^2, as is the magnitude of $\vec{E}$. Therefore, one can say that the density of lines is a relative indication of the electric field strength. This statement is true for any combination of charges, since with E linearly proportional to charge, superposition applies.

The concept of a flux density $\vec{D}$ that is proportional to $\vec{E}$ is useful when dealing with electric fields in a dielectric. It is defined as

$$\vec{D} \equiv \epsilon_R\epsilon_0 \vec{E} \tag{2-3}$$

where $\vec{D}$ is the electric flux density (C/m^2). With $\vec{E}$, and hence $\vec{D}$, inversely proportional to r^2, the relationship between flux density and charge can be expressed in terms of Gauss' law. Namely, the net outflow of electric flux through a closed surface S is equal to the net electric charge enclosed. Stated mathematically,

$$\oint \vec{D}\cdot d\vec{S} = \sum_1^m q_n \quad \text{or} \quad \int_v \rho_v dv \tag{2-4}$$

where ρ_v is the volume charge density within the volume v. The integral form on the right side of the equation is used when the charge distribution is continuous rather than an array of point charges.

Another useful concept in the study of electric fields is that of potential or potential difference. The potential V at a point P is defined as the work required to bring a unit positive test charge from infinity to the point P. This work is necessary to overcome the electric field created by one or more fixed charges. That is,

$$V \equiv -\int_\infty^P \vec{E}\cdot d\vec{l} \tag{2-5}$$

the vector $d\vec{l}$ being an element of length along the integration path. In most cases, one is interested in the potential difference V_{AB}, namely the work required to move a unit positive test charge from point A to point B. Thus,

$$V_{AB} \equiv -\int_A^B \vec{E} \cdot d\vec{l} \tag{2–6}$$

The concept of stored energy in an electric field is also useful. A free charge placed in a region containing an electric field will be accelerated or decelerated by the field. The change in kinetic energy of the moving charge can be said to have come from the potential energy of the region containing the field. This stored electric energy U_E in a volume v is given by

$$U_E = \frac{1}{2}\int_v \vec{D} \cdot \vec{E}\, dv \tag{2–7}$$

Proof of this relationship is given in most texts on electromagnetism (see, for example, Refs. 2–1 and 2–6).

2–1b The Magnetic Field

A method analogous to that described for electric fields may be used to define the magnetic field intensity $\vec{H}$. Namely, $\vec{H}$ represents the force exerted on a unit north pole. Given an isolated magnetic pole m_1, the force exerted by it on a unit north pole is inversely proportional to the square of the distance between them.[2] The magnitude is given by

$$H = \frac{m_1}{4\pi \mu_R \mu_0 r^2} \tag{2–8}$$

where μ_R is the relative permeability of the surrounding region. For free space and nonmagnetic materials, $\mu_R = 1.00$. In the SI system, m_1 is in webers and H in A/m.

Again, because of the inverse square relation with distance r, the density of lines is proportional to the magnitude of $\vec{H}$. The concept of a magnetic flux density $\vec{B}$ is useful when describing magnetic forces within a magnetic material. It is defined as

$$\vec{B} \equiv \mu_R \mu_0 \vec{H} \tag{2–9}$$

where its SI unit is the tesla, which is equivalent to a weber per square meter. One can also write Gauss' law for magnetic fields. Namely

$$\oint \vec{B} \cdot d\vec{S} = 0 \tag{2–10}$$

Note that the right-hand side is zero, since the existence of isolated magnetic poles has never been shown. As a result, magnetic flux lines form complete loops. On the

[2] Although no such physical entity is known to exist, the concept of an isolated magnetic pole is sometimes useful!

other hand, electric flux lines emerge from positive charge and terminate on negative charge, except for time-varying fields, where they may form complete loops.

It is an experimental fact that a magnetic field is associated with the movement of electric charge. As an example, consider the long wire shown in Fig. 2–2. For a current I, the magnetic field in the region surrounding the wire is given by

$$H = \frac{I}{2\pi r} \tag{2–11}$$

where r is the radial distance from the center of the wire to the point in question. This phenomenon can be generalized into Ampere's law, namely, the line integral of $\vec{H}$ around a closed path is equal to the net current passing through a surface $\vec{S}$ defined by the closed path. Stated mathematically,

$$\oint \vec{H} \cdot d\vec{l} = \sum_{1}^{m} I_n \qquad \text{or} \qquad \int \vec{J} \cdot d\vec{S} \tag{2–12}$$

The integral form on the right side of the equation is useful when the current passing through $\vec{S}$ has a nonuniform distribution. $\vec{J}$ is the current density (A/m^2) at any point on the surface $\vec{S}$. Later, this equation will be modified to include the effect of time-varying electric fields.

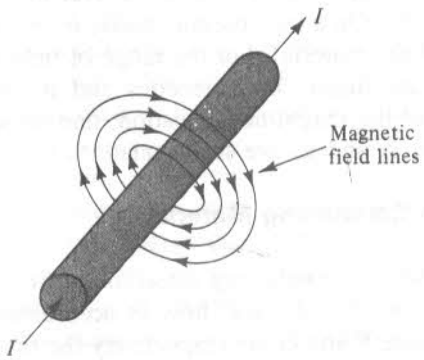

Figure 2–2 The magnetic field due to an infinitely long, current-carrying conductor.

Experimentally, a magnetic field is known to exert a force on moving charge and therefore one can construct an alternate definition of $\vec{B}$ (and hence $\vec{H}$). Referring to Fig. 2–3, the force on a charge q, moving with velocity $\vec{v}$ in a magnetic field of flux density $\vec{B}$ is given by

$$\vec{F} = q(\vec{v} \times \vec{B}) \tag{2–13}$$

The cross-product reflects the fact that the force is perpendicular to both $\vec{v}$ and $\vec{B}$. The magnitude of the force is $qvB \sin \theta$. Since force, charge, and velocity are measurable fundamental quantities, Eq. (2–13) may be considered a definition of $\vec{B}$. Most texts, in fact, present this as the primary definition, since it is useful in studying the motion of charged particles. The earlier-stated definition, referring to isolated magnetic poles, is useful in the analysis of ferromagnetic circuits.

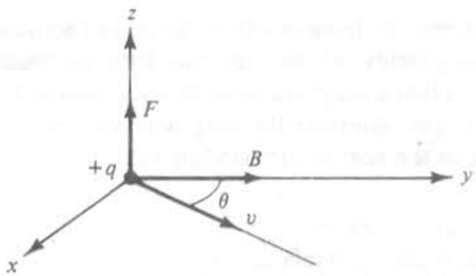

Figure 2–3 The force on a charge moving in a magnetic field.

Since the magnetic field can exert a force on either a magnetic pole or moving electric charge, the concept of stored energy associated with a field can again be utilized. The stored magnetic energy U_M in a volume v is given by

$$U_M = \frac{1}{2}\int_v \vec{B} \cdot \vec{H}\, dv \tag{2–14}$$

Note the similarity to Eq. (2–7), the expression for stored electric energy.

2–2 FIELDS IN CONDUCTORS AND INSULATORS

The flux density concept is useful in describing the interaction between matter and the electric and magnetic fields. On a macroscopic basis, σ, ϵ_R, and μ_R may be used to describe the properties of the material. For the range of field values normally encountered many materials are linear, homogeneous and isotropic. That is, their properties are independent of the magnitude, location, and direction of the applied field. For these materials, σ, ϵ_R and μ_R are scalar quantities.

2–2a The Electric Field in Conducting Materials

Consider the elemental cylinder of conducting material shown in Fig. 2–4. With a voltage V applied as shown, a current I will flow in accordance with Ohm's law. That is, $I = V/R = GV$, where R and G are respectively the resistance and conductance of the sample. If the current is uniformly distributed over the cross-sectional area A,

$$R = \rho\frac{l}{A} = \frac{l}{\sigma A} \tag{2–15}$$

where ρ is the resistivity of the material (ohm-m) and $\sigma \equiv 1/\rho$ is its conductivity (mho/m). Since it may be assumed that the electric field and the current distribution are uniform over the elemental volume, $\vec{E} = V/l$, $\vec{J} = I/A$ and hence

$$\vec{J} = \sigma\vec{E} = \vec{E}/\rho \tag{2–16}$$

This form of Ohm's law is particularly useful when analyzing nonuniform fields and currents. Values of conductivity for some commonly used metals are given in Appendix B, Table B–1.

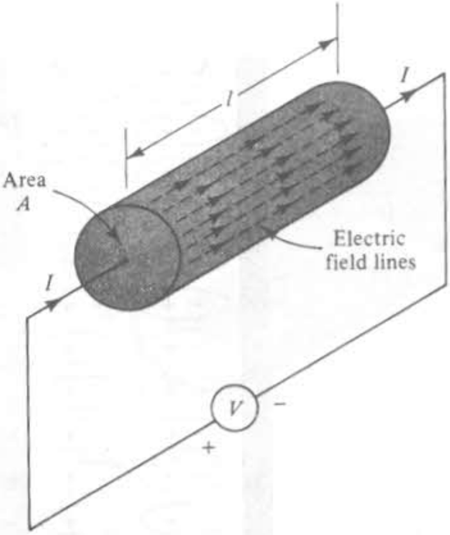

Figure 2–4 An elemental volume of conductive material.

2–2b The Electric Field in Dielectrics

An electric field in a dielectric causes its atoms to become polarized. On a macroscopic basis, this effect can be accounted for by the material's dielectric constant ϵ_R. To illustrate, consider the parallel-plate capacitor shown in Fig. 2–5a. If one plate is charged $+q_f$ and the other $-q_f$, an electric field is created. In the absence of dielectric material between the plates, $E = q_f/A\epsilon_0$. This result is obtained by applying Gauss' law to a closed surface surrounding one of the plates. If the capacitor contains a dielectric, the expression for electric field must be modified to account for the atomic polarization. Referring to part *b* of the figure, $E = (q_f/A\epsilon_0) - (q_p/A\epsilon_0)$, where q_f is the charge resulting from an external source (free charge) and q_p is the induced polarization charge due to the dielectric material. In most cases, the polarization effect is linearly proportional to the electric field. That is, $q_p/A\epsilon_0 = \chi_e E$, where the constant of proportionality χ_e is known as the *electric susceptibility* of the material. Substitution into the above expression for E yields

$$E = \frac{q_f}{A(1 + \chi_e)\epsilon_0} = \frac{q_f}{A\epsilon_R\epsilon_0} \tag{2–17}$$

where $1 + \chi_e \equiv \epsilon_R$ is the relative permittivity or dielectric constant of the material. Note that for a fixed value of free charge, the effect of dielectric polarization is to reduce the electric field by ϵ_R.[3] Conversely, for a fixed voltage between the plates, the electric field is unchanged, and hence the presence of the dielectric increases q_f.

Room temperature values of ϵ_R at 3.0 GHz for various insulators are given in Appendix B, Table B–2. For most dielectrics, ϵ_R is only a slight function of frequency and temperature.

[3] With the effect of dielectric polarization accounted for by ϵ_R, the right-hand side of Gauss' law, Eq. (2–4), is interpreted as the net *free* charge enclosed. A detailed discussion of dielectric polarization may be found in Chapter 5 of Ref. 2–1.

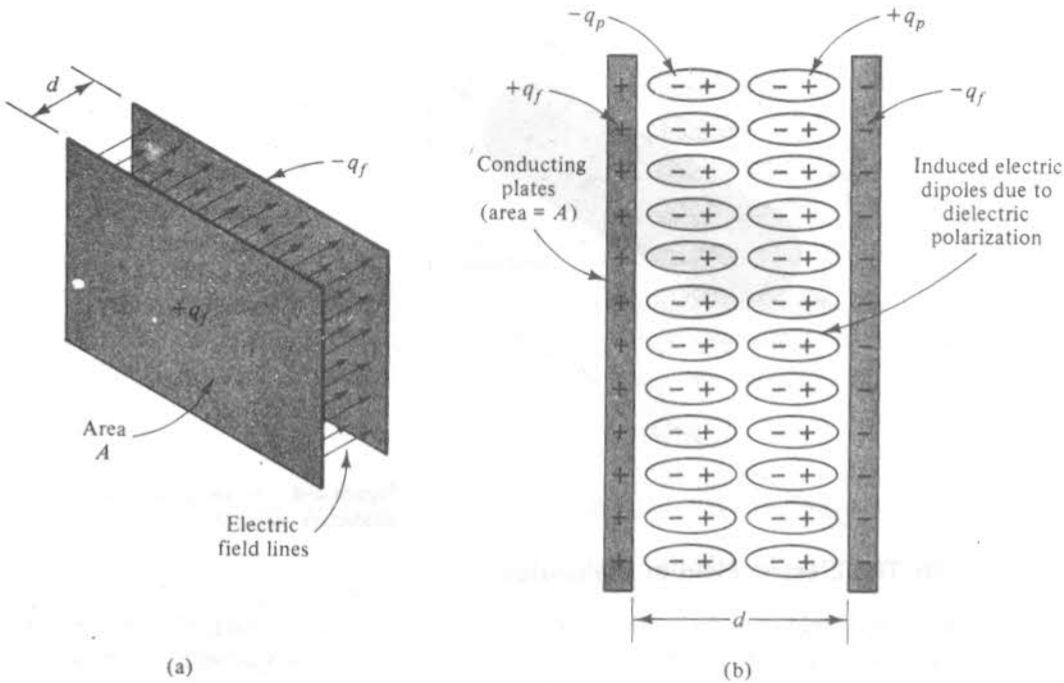

Figure 2–5 Dielectric polarization in a charged capacitor.

Another consequence of dielectric polarization is an increase in the value of capacitance. Capacitance is defined as

$$C \equiv \frac{q_f}{V} \tag{2–18}$$

where V is the voltage between the plates. For the parallel-plate capacitor, the electric field is uniform and hence $E = V/d$. Substitution into Eq. (2–17) yields the following expression for capacitance of a parallel-plate capacitor.

$$C = \frac{A\epsilon_R \epsilon_0}{d} \tag{2–19}$$

where A and d are defined in Fig. 2–5. Thus the capacitance value is directly proportional to the dielectric constant of the material between the capacitor plates. This is true for any configuration of capacitor plates.

The following example illustrates the use of Gauss' law and the capacitance definition.

Example 2–1:

Derive an expression for the capacitance of the coaxial configuration shown in Fig. 2–6. Ignore fringing at the ends by assuming $l \gg b$.

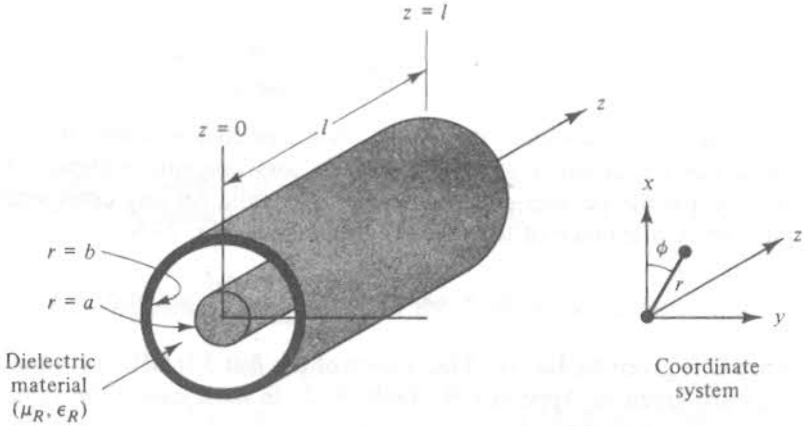

Figure 2–6 A coaxial transmission line. (See Exs. 2–1 and 2–2.)

Solution: Assume that the center conductor has been charged to $+q_f$ and the outer conductor to $-q_f$. Ignoring end effects, the E lines will be directed radially outward. Applying Gauss' law to a cylindrical surface of length l and radius r, where $a < r < b$, yields

$$\int_0^l \int_0^{2\pi} D\, r\, d\phi\, dz = q_f$$

where standard cylindrical coordinates have been used. Because of symmetry, D is independent of ϕ and z. Hence

$$D = \frac{q_f}{2\pi r l} \qquad \text{and} \qquad E = \frac{q_f}{2\pi \epsilon_0 \epsilon_R r l} \tag{2–20}$$

From Eq. (2–6), the voltage between the plates is

$$V = -\int_a^b \vec{E} \cdot d\vec{r} = \frac{q_f}{2\pi \epsilon_0 \epsilon_R l} \ln \frac{b}{a}$$

and therefore

$$C \equiv \frac{q_f}{V} = \frac{2\pi \epsilon_0 \epsilon_R l}{\ln (b/a)} \tag{2–21}$$

Lossy dielectrics. All dielectrics have a finite amount of conductivity (σ). At microwave frequencies, this dissipative property is expressed in terms of the material's dielectric loss tangent ($\tan \delta$). To illustrate its meaning, consider the parallel-plate capacitor in Fig. 2–5. Its capacitance is given by Eq. (2–19). If the dielectric material is lossy, there will be some conductance G in parallel with C. Therefore, the ac admittance of the lossy capacitor becomes $Y = G + j\omega C$, where $G = \sigma A/d$. Utilizing Eq. (2–19),

$$Y = \omega C \tan \delta + j\omega C \tag{2–22}$$

where

$$\tan \delta \equiv \frac{\sigma}{\omega \epsilon_R \epsilon_0}$$

Note that the conductance due to the lossy dielectric is simply the capacitive susceptance (ωC) multiplied by the material's loss tangent. Although Eq. (2–22) was derived for the parallel-plate capacitor, it is valid for any configuration. For example, the conductance of the coaxial capacitor in Fig. 2–6 is

$$G = \omega C \tan \delta = \frac{2\pi\epsilon_0\epsilon_R l}{\ln (b/a)} \omega \tan \delta \qquad (2\text{–}23)$$

since C is given by Eq. (2–21). Values of $\tan \delta$ at 3.0 GHz for some dielectric materials are given in Appendix B, Table B–2. In most cases, the value is only slightly affected by frequency and temperature variations.[4]

2–2c The Magnetic Field in Magnetic Materials

Most materials exhibit some magnetic properties. The effect is quite pronounced in materials such as iron, nickel, cobalt, and ferrite. In others, such as the dielectrics listed in Table B–2, it is practically negligible. On a macroscopic basis, the magnetic properties of a material may be accounted for by its relative permeability (μ_R). To illustrate, consider the long solenoid in Fig. 2–7. For $l \gg r$, the magnetic field within the solenoid is essentially uniform and is given by

$$H = \frac{NI}{l} \qquad (2\text{–}24)$$

where I is the current in the coil. When the material within the solenoid is nonmagnetic, the flux density is given by $B = \mu_0 H$. However, when the material is paramagnetic or ferromagnetic, the flux density will be greater than $\mu_0 H$. This increase

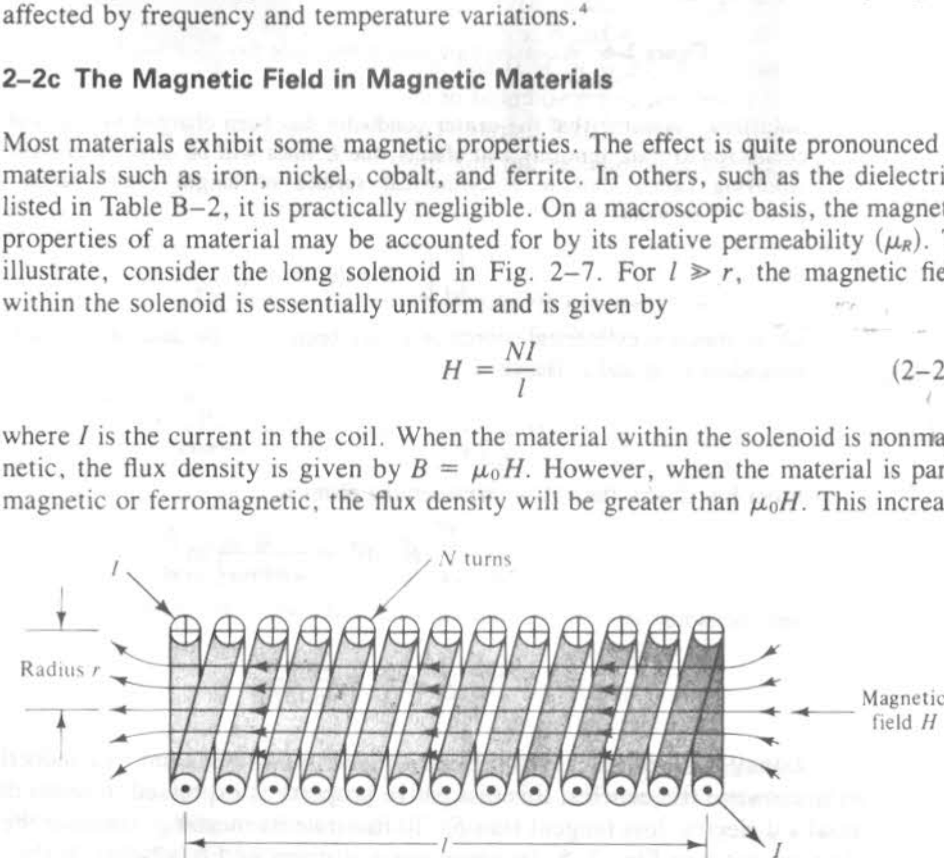

Figure 2–7 The magnetic field in a long, current-carrying solenoid.

[4] Values of ϵ_R and $\tan \delta$ for many materials may be found in Refs. 2–2 and 2–3. In these references, the term *dissipation factor* is used rather than *loss tangent*.

in B is associated with the spin motion of the electrons in the material. Since an electron possesses charge, the spin creates magnetic flux in a manner similar to a tiny current-carrying coil.[5] With current flowing in the coil, the total flux density is

$$B = \mu_0(H + M) \tag{2-25}$$

where the magnetization M is the net magnetic field contributed by the material. In certain cases, M is linearly proportional to the magnetic field H. That is,

$$M = \chi_m H \tag{2-26}$$

where χ_m is the *magnetic susceptibility* of the material. Substitution into Eq. (2–25) yields

$$B = \mu_0(1 + \chi_m)H = \mu_0 \mu_R H \tag{2-27}$$

where $1 + \chi_m \equiv \mu_R$, the relative permeability of the material. Thus the effect of the material is to increase the flux density by the factor μ_R. Note that for free space and nonmagnetic materials, $\chi_m = 0$ and hence $\mu_R = 1.00$.

The inductance of a solenoid or any other inductor is affected by the presence of magnetic material. To illustrate, consider again the long solenoid in Fig. 2–7. Inductance (L) is defined as the flux linkages per unit of current. That is,

$$L \equiv \frac{N\Phi}{I} \tag{2-28}$$

where N is the number of turns and Φ is the total magnetic flux. For a given flux density, Φ may be determined from

$$\Phi = \int_S \vec{B} \cdot d\vec{S} \tag{2-29}$$

where $\vec{S}$ is the surface through which the flux passes. Since in this case the field is uniform, $\Phi = BA$, where $A = \pi r^2$. Making use of Eqs. (2–24) and (2–27) results in the following expression for the inductance of a long solenoid ($l \gg r$).

$$L = \mu_0 \mu_R \frac{N^2 A}{l} \tag{2-30}$$

This equation is accurate to within ten percent when $l > 3r$. Thus, assuming a linear relation between M and H, the inductance is directly proportional to the relative permeability of the material.

The following example illustrates the use of Ampere's law and the inductance definition in Eq. (2–28).

Example 2–2:

Derive an expression for the inductance of the coaxial configuration in Fig. 2–6. Assume that the current flows along the surface of the inner conductor and returns via the inner surface of the outer conductor, which is the case in microwave applications.

[5] A detailed discussion of the magnetic properties of matter may be found in Refs. 2–1, 2–4, and 2–6.

Solution: A current flow I down the center conductor produces a magnetic field around it. Since the conductor is round, the application of Ampere's law to a circle of radius r, where $a \le r \le b$, yields

$$\int_0^{2\pi} H\, r\, d\phi = 2\pi r H = I$$

and hence $H = I/2\pi r$ for $a \le r \le b$. For $r < a$, $H = 0$ since the current enclosed is zero. For $r > b$, the net current enclosed is also zero, since the return current I flows along the $r = b$ surface. Thus $H = 0$ for $r > b$. From Eq. (2–29), the total magnetic flux passing between the conductors is

$$\Phi = \int_0^l \int_a^b B\, dr\, dz = \mu_R \mu_0 l \int_a^b \frac{I}{2\pi r}\, dr = \frac{\mu_0 \mu_R I l}{2\pi} \ln \frac{b}{a}$$

Substituting into Eq. (2–28) and noting that $N = 1$,

$$L = \frac{\mu_0 \mu_R l}{2\pi} \ln \frac{b}{a} \tag{2–31}$$

which is the *high-frequency* inductance of a coaxial line.

2–3 MAXWELL'S EQUATIONS AND BOUNDARY CONDITIONS

The contributions of James Clerk Maxwell to the understanding of electricity and magnetism were discussed in Sec. 1–2. His theoretical studies resulted in the formulation of a general theory of electromagnetism. The four relationships derived by him bear his name. This section reviews the ideas leading to these equations.

2–3a Maxwell's Equations

Maxwell's four equations are mathematical expressions of Gauss' laws for electric and magnetic fields, (Eqs. 2–4 and 2–10), Faraday's law, and Ampere's law. The mathematical form of these laws are developed here.

Faraday's law. Faraday's law of induction states that time-varying magnetic flux passing through a surface produces an emf around the perimeter of the surface. Since emf is the electric force exerted over a path and magnetic flux is the surface integral of $\vec{\mathscr{B}}$, Faraday's law may be written as

$$\oint \vec{\mathscr{E}} \cdot d\vec{l} = -\int_S \frac{\partial \vec{\mathscr{B}}}{\partial t} \cdot d\vec{S} \tag{2–32}$$

The minus sign is a result of Lenz's principle which states that changing magnetic flux produces an emf which tends to oppose the flux change that created the emf.

Ampere's law and displacement current. In developing his theory of electromagnetism, Maxwell suggested that a time-varying electric field would produce a magnetic field. In effect, he was saying that Ampere's law, Eq. (2–12), had to be modified to account for time-varying fields. The complete representation of Ampere's law is

$$\oint \vec{\mathcal{H}} \cdot d\vec{l} = \int_S \vec{\mathcal{J}} \cdot d\vec{S} + \int_S \frac{\partial \vec{\mathcal{D}}}{\partial t} \cdot d\vec{S} \qquad (2\text{–}33)$$

The first term on the right side of the equation is the total conduction current through a surface enclosed by the path, while the second term is the total displacement current enclosed. The quantity $\partial \vec{\mathcal{D}} / \partial t$ is the displacement current density (A/m^2). For good conductors, the first term is dominant, while the second term dominates for time-varying fields in an insulator.

To understand the displacement current concept, consider the capacitor in Fig. 2–8. With a dc voltage source V connected to the capacitor, the plates become charged. When the value of q_f reaches CV, the capacitor is fully charged and current flow ceases. The electric field (E) and flux density (D) between the plates are related to the charge q_f by Gauss' law. For a parallel-plate capacitor, $D = q_f/A$, where A is the area of one plate.

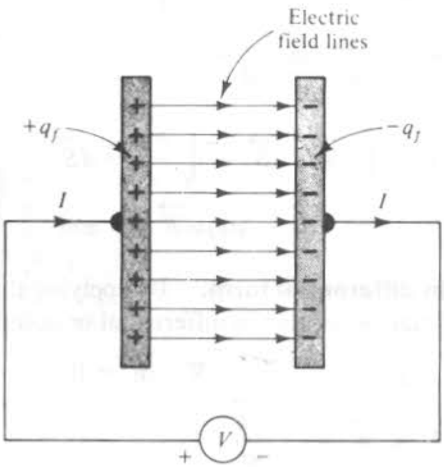

Figure 2–8 The displacement current concept in a capacitor.

Suppose the source voltage V is not constant but increasing with time. This requires that $q_f = CV$ also increases with time. The increase in charge is supplied by the current $I = dq_f/dt$. In this situation, it would appear that Kirchhoff's current law is violated. That is, current entering the plate is I, but current leaving via the insulating space is zero since a perfect insulator cannot support a conduction current. This dilemma is resolved by noting that the current flow that produces a change in q_f also causes a change in flux density, and that the unit for $d\mathcal{D}/dt$ is A/m^2, a current density. By considering this quantity as a current density, Kirchhoff's current law may be applied to time-varying fields. The quantity $\partial \vec{\mathcal{D}} / \partial t$ is known as the *displacement current density* and hence the displacement current through a surface S is

$$I_d = \int_S \frac{\partial \vec{\mathcal{D}}}{\partial t} \cdot d\vec{S} \qquad (2\text{–}34)$$

To verify that the current law is applicable, consider again the parallel-plate capacitor. In this case, $\mathscr{D} = q_f/A$ and hence the displacement current $A(d\mathscr{D}/dt)$ equals dq_f/dt. Thus the displacement current leaving via the insulating region equals the conduction current entering the capacitor plate.

Displacement current only exists when the electric field is time-varying. Since it is a current, one would expect it to create a magnetic field. Displacement current does indeed produce a magnetic field and hence Ampere's law must be restated to reflect this experimental fact. This has been done in Eq. (2–33). As explained in Sec. 2–4, the most important consequence of displacement current is its key role in electromagnetic wave propagation.

With Ampere's law modified to include the effect of displacement currents, all four of Maxwell's equations may now be stated. They are summarized here in both integral and differential forms.

Maxwell's equations in integral form.

$$\oint \vec{\mathscr{D}} \cdot d\vec{S} = \int_v \rho_v \, dv \quad , \quad \oint \vec{\mathscr{B}} \cdot d\vec{S} = 0 \tag{2–35}$$

$$\oint \vec{\mathscr{E}} \cdot d\vec{l} = -\int_S \frac{\partial \vec{\mathscr{B}}}{\partial t} \cdot d\vec{S} \tag{2–36}$$

$$\oint \vec{\mathscr{H}} \cdot d\vec{l} = \int_S \vec{\mathscr{J}} \cdot d\vec{S} + \int_S \frac{\partial \vec{\mathscr{D}}}{\partial t} \cdot d\vec{S} \tag{2–37}$$

where $\vec{\mathscr{D}} = \epsilon_R \epsilon_0 \vec{\mathscr{E}}$, $\vec{\mathscr{B}} = \mu_R \mu_0 \vec{\mathscr{H}}$ and $\vec{\mathscr{J}} = \sigma \vec{\mathscr{E}}$

Maxwell's equations in differential form. By applying the rules of vector calculus, Maxwell's equations may be written in differential or point form.[6] Namely,

$$\nabla \cdot \vec{\mathscr{D}} = \rho_v \quad , \quad \nabla \cdot \vec{\mathscr{B}} = 0 \tag{2–38}$$

$$\nabla \times \vec{\mathscr{E}} = -\frac{\partial \vec{\mathscr{B}}}{\partial t} \tag{2–39}$$

$$\nabla \times \vec{\mathscr{H}} = \vec{\mathscr{J}} + \frac{\partial \vec{\mathscr{D}}}{\partial t} \tag{2–40}$$

These relationships represent the fundamental equations of electromagnetism.

2–3b Boundary Conditions for Electric and Magnetic Fields

Most electromagnetic problems involve boundaries between regions having different electric and magnetic properties. The conditions that must exist at these boundaries may be deduced from Maxwell's equations. Table 2–1 lists the conditions for tangential (subscript T) and normal (subscript N) components of the fields.

[6] As explained in the preface, the use of the differential form of Maxwell's equations is limited to a few sections in the text. These sections are indicated in the table of contents by a star (✱).

TABLE 2-1 Boundary Conditions for Electromagnetic Fields at the Interface Between Two Materials

TANGENTIAL COMPONENTS	NORMAL COMPONENTS
$\mathscr{E}_{T_1} = \mathscr{E}_{T_2}$	$\mathscr{D}_{N_1} = \mathscr{D}_{N_2} - \rho_s$
$\mathscr{H}_{T_1} = \mathscr{H}_{T_2} + \mathscr{K}$	$\mathscr{B}_{N_1} = \mathscr{B}_{N_2}$
where $\mathscr{K}$ ≡ Surface current density (A/m)	where ρ_s ≡ Surface charge density (C/m^2)

NOTE: 1. If the boundary is charge-free, $\rho_s = 0$.
2. In the absence of conduction currents, $\mathscr{K} = 0$.

These conditions may be verified with the aid of Fig. 2–9. In part *a* of the figure, the application of Faraday's law to the closed path *a-b-c-d* results in $\mathscr{E}_{T_1}l - \mathscr{E}_{T_2}l \approx -(\partial\mathscr{B}/\partial t)(l\Delta n)$, where it is assumed $\Delta n \ll l$ and both approach zero. As $l\Delta n \rightarrow 0$, $\mathscr{E}_{T_1} = \mathscr{E}_{T_2}$, which proves the first tangential condition in Table 2–1. The second condition may be verified by applying Ampere's law to the same closed path *a-b-c-d*, except that $\mathscr{E}_{T_1}$ and $\mathscr{E}_{T_2}$ are replaced by $\mathscr{H}_{T_1}$ and $\mathscr{H}_{T_2}$, respectively. The result is that $\mathscr{H}_{T_1}l - \mathscr{H}_{T_2}l = \mathscr{K}l$, where $\mathscr{K}$ is the conduction current density (A/m) at the boundary surface. A positive value of $\mathscr{K}$ denotes a surface current directed into the page.

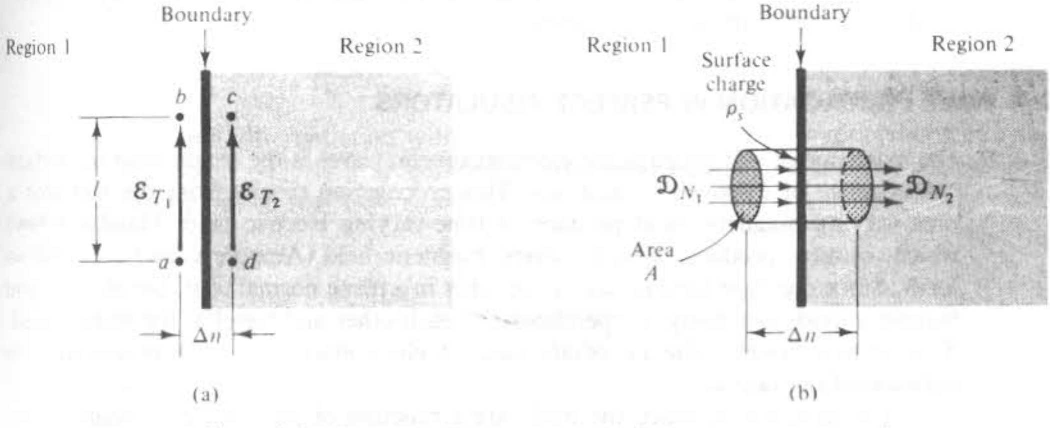

Figure 2–9 Tangential (T) and normal (N) field components at a boundary.

The conditions for the normal components may be verified with the aid of Fig. 2–9b. If both A and Δn are sufficiently small, the application of Gauss' law to the closed cylinder yields $\mathscr{D}_{N_2}A - \mathscr{D}_{N_1}A \approx \rho_s A$, where ρ_s is the surface charge density (C/m^2). As A and Δn approach zero, $\mathscr{D}_{N_1} = \mathscr{D}_{N_2} - \rho_s$. Replacing $\mathscr{D}_{N_1}$ and $\mathscr{D}_{N_2}$ with $\mathscr{B}_{N_1}$ and $\mathscr{B}_{N_2}$ and using Gauss' law for magnetic fields results in $\mathscr{B}_{N_1} = \mathscr{B}_{N_2}$, which means that magnetic flux lines are continuous. Electric flux lines are only continuous when the boundary is charge-free ($\rho_s = 0$).

Special conditions exist when one of the two regions is a perfect conductor (that is, $\sigma \rightarrow \infty$). This requires that $\mathscr{D}$ and $\mathscr{E}$ in that region be identically zero. If, for example, region 2 has infinite conductivity,

$$\mathscr{E}_{T_1} = 0 \qquad \text{and} \qquad \mathscr{D}_{N_1} = -\rho_s \tag{2–41}$$

Thus the direction of $\mathscr{E}$ lines terminating on the surface of a perfect conductor must be perpendicular to that surface.

When the fields are time-varying, $\mathscr{H}$ and $\mathscr{B}$ must also be zero within a perfect conductor. This is a direct consequence of Faraday's law, Eq. (2–36), and the fact that $\mathscr{E} \equiv 0$ in a perfect conductor. Thus for time-varying fields at the boundary of a perfect conductor, the magnetic field conditions become

$$\mathscr{H}_{T_1} = \mathscr{K} \qquad \text{and} \qquad \mathscr{B}_{N_1} = 0 \tag{2–42}$$

where again it is assumed that region 2 is the perfect conductor. If μ_{R_1} is a scalar, $\mathscr{H}_{N_1} = 0$, which means that the magnetic field must be tangential to the surface of a perfect conductor. Furthermore, if region 1 is a perfect insulator ($\tan \delta = 0$), the following boundary condition also holds for time-varying fields.

$$\frac{\partial \mathscr{H}_{T_1}}{\partial n} = 0 \tag{2–43}$$

where n represents a direction perpendicular to the boundary surface.

The analysis of microwave transmission lines and components make extensive use of the boundary conditions derived here. An appreciation of their meaning is also useful in estimating the electromagnetic field pattern in structures constrained by dielectric and metallic boundaries.

2–4 WAVE PROPAGATION IN PERFECT INSULATORS

The existence of self-propagating electromagnetic waves is the single most important consequence of Maxwell's equations. This propagation results from the fact that a time-varying magnetic field produces a time-varying electric field (Faraday's law) which, in turn, produces a time-varying magnetic field (Ampere's law) . . . and so forth. Since one type field produces the other in a plane normal to it, the electric and magnetic fields are always perpendicular to each other and travel at the same speed. This section reviews the important case of electromagnetic wave propagation in unbounded insulators.

The situation in which the fields are a function of only one coordinate represents the simplest application of Maxwell's equations. The resultant analysis describes the phenomena of uniform plane waves, which is of considerable engineering importance.

Consider the coordinate system shown in Fig. 2–10 and assume a time-varying electric field in the x direction that is independent of x and y. That is, $\mathscr{E}_x = f(z, t)$.

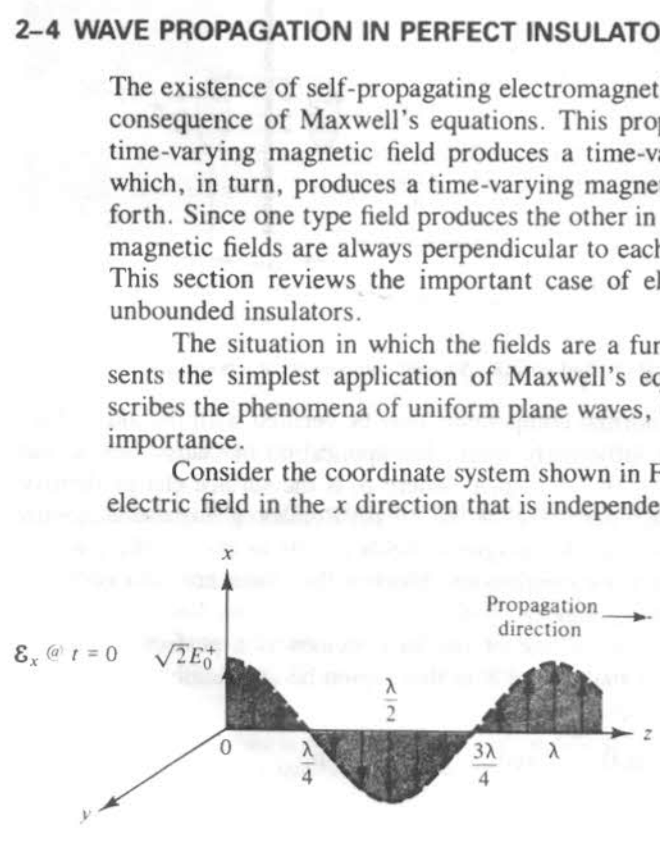

Figure 2–10 $\mathscr{E}_x$ at $t = 0$ as a function of position along the propagation axis.

Also assume that all space consists of a perfect insulator devoid of free charges, which means that both ρ_v and J are zero in Eqs. (2–35) to (2–40). Since $\overrightarrow{\mathscr{E}}$ is not a function of x and y and only has a component in the x direction, Maxwell's equations reduce to[7]

$$\frac{\partial \mathscr{E}_x}{\partial z} = -\mu_R \mu_0 \frac{\partial \mathscr{H}_y}{\partial t} \qquad \text{and} \qquad -\frac{\partial \mathscr{H}_y}{\partial z} = \epsilon_R \epsilon_0 \frac{\partial \mathscr{E}_x}{\partial t} \tag{2–44}$$

By differentiating the first equation with respect to z and the second one with respect to t, $\mathscr{H}_y$ can be eliminated, resulting in

$$\frac{\partial^2 \mathscr{E}_x}{\partial z^2} = \mu_R \mu_0 \epsilon_R \epsilon_0 \frac{\partial^2 \mathscr{E}_x}{\partial t^2} \tag{2–45}$$

This is the well-known wave equation of mathematical physics. Let us now solve this equation for steady-state sinusoidal excitation using the rms-phasor method.[8]

Rewriting Eqs. (2–44) and (2–45) in phasor form yields

$$\frac{d\boldsymbol{E}_x}{dz} = -j\omega\mu_R \mu_0 \boldsymbol{H}_y \quad , \quad -\frac{d\boldsymbol{H}_y}{dz} = j\omega\epsilon_R \epsilon_0 \boldsymbol{E}_x \tag{2–46}$$

and

$$\frac{d^2 \boldsymbol{E}_x}{dz^2} = -\omega^2 \mu_R \mu_0 \epsilon_R \epsilon_0 \boldsymbol{E}_x \tag{2–47}$$

where each differentiation with respect to time introduced a multiplying factor $j\omega$. Since Eq. (2–47) is a second-order differential equation, it has two independent solutions, which may be written as

$$\boldsymbol{E}_x = E_0^+ e^{-j\beta z} + E_0^- e^{+j\beta z} \tag{2–48}$$

where $\quad \beta \equiv \dfrac{\omega}{v} \quad , \quad v = \dfrac{1}{\sqrt{\mu_R \mu_0 \epsilon_R \epsilon_0}} \quad$ and $\quad \omega = 2\pi f = 2\pi/T.$

Substitution into the first of Eqs. (2–46) yields the two associated solutions for $\boldsymbol{H}_y$.

$$\boldsymbol{H}_y = H_0^+ e^{-j\beta z} - H_0^- e^{+j\beta z} \tag{2–49}$$

where $\quad \dfrac{E_0^+}{H_0^+} = \dfrac{E_0^-}{H_0^-} \equiv \eta = \sqrt{\dfrac{\mu_R \mu_0}{\epsilon_R \epsilon_0}}.$

To reconstruct the instantaneous form of the solutions, multiply the rms-phasor solutions by $\sqrt{2}\, e^{j\omega t}$ and take the real parts thereof. Thus,

$$\mathscr{E}_x = \sqrt{2}\, E_0^+ \cos(\omega t - \beta z) + \sqrt{2}\, E_0^- \cos(\omega t + \beta z) \tag{2–50}$$

and

$$\mathscr{H}_y = \sqrt{2}\, H_0^+ \cos(\omega t - \beta z) - \sqrt{2}\, H_0^- \cos(\omega t + \beta z) \tag{2–51}$$

[7] The reduction of the differential form of Maxwell's equations is found in most books on electromagnetic theory (for example, Chapter 11 in Ref. 2–4). An excellent treatment of the integral form and its simplification is given in Chapter 12 of Ref. 2–1.

[8] The rms-phasor is described in Sec. 1–4.

To understand the nature of the above equations, consider the first term in Eq. (2–50). Since $\beta = \omega/v$, it can be rewritten as $\sqrt{2}\, E_0^+ \cos[\omega(t - z/v)]$. At $z = 0$, the electric field is given by $\sqrt{2}\, E_0^+ \cos \omega t$ and is plotted in Fig. 2–11. This curve does *not* represent a wave, but merely the time variation of $\mathscr{E}_x$ at one place, the $z = 0$ plane. A wave implies the movement of a time function from one place to another. The term $\sqrt{2}\, E_0^+ \cos[\omega(t - z/v)]$ does represent a wave since its cosinusoidal variation at $z = 0$ is repeated at the plane $z = l$ with a time delay l/v. Thus it appears that the $\mathscr{E}_x$ time function has traveled a distance l with a velocity v. The conclusion is that the first term in Eq. (2–50) represents an electric wave traveling in the positive z direction with a velocity v. By a similar argument, the second term describes a wave traveling in the negative z direction. From ac theory, phase delay is merely normalized time delay multiplied by 2π rad (that is, $2\pi t_d/T$). Since $\beta z = (2\pi/T)(z/v)$, it represents a phase delay for the forward traveling wave ($+z$ direction). The quantity β is the phase shift per unit length and is known as the *phase constant* of the wave.

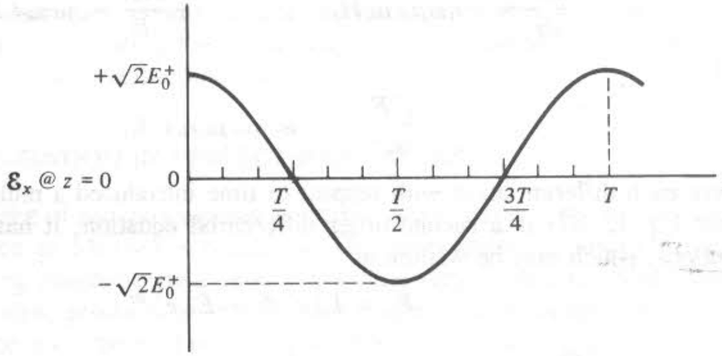

Figure 2–11 $\mathscr{E}_x$ at $z = 0$ as a function of time.

The wavelength λ and period T are related by the wave velocity. Wavelength is defined as the distance one must traverse for a function, periodic in space but fixed in time, to repeat itself. This quantity is described in Fig. 2–10 for the first term in Eq. (2–50) at $t = 0$. Period, on the other hand, is the time required for a function, periodic in time but at a fixed point in space, to repeat itself. It is described in Fig. 2–11. If the field pattern shown in Fig. 2–10 moves in the $+z$ direction with a velocity v, we have a wave. An observer situated at, say, $z = \lambda$ will see a cosinusoidal variation of $\mathscr{E}_x$ with time. When $t = 0$, its value will be a positive maximum. The time required for the next positive maximum, presently located at $z = 0$, to arrive at $z = \lambda$ represents one period. Thus, $T = \lambda/v$. Since $T = 1/f$, this leads to the familiar relationship for all wave propagation

$$f\lambda = v \tag{2–52}$$

For the electromagnetic wave described here, v is given by the expression below Eq. (2–48). With

$$\mu_0 = 4\pi \times 10^{-7} \text{ H/m} \qquad \text{and} \qquad \epsilon_0 \approx \frac{1}{36\pi} \times 10^{-9} \text{ F/m},$$

$$v = \frac{c}{\sqrt{\mu_R \epsilon_R}} \tag{2–53}$$

where $c \approx 3 \times 10^8$ m/s is the velocity of light in free space. If the wavelength in free space is denoted by λ_0,

$$\lambda_0 = \frac{c}{f} \tag{2–54}$$

and hence for any other insulator,

$$\lambda = \frac{\lambda_0}{\sqrt{\mu_R \epsilon_R}} \tag{2–55}$$

Keep in mind that for nonmagnetic insulators $\mu_R = 1.00$. Equations (2–53) and (2–55) show that both the velocity and wavelength of an electric wave in a dielectric are smaller than their values in free space. This effect is useful in reducing the size of microwave components. Also, since the phase constant $\beta = \omega/v$,

$$\beta = \frac{2\pi}{\lambda} = \frac{2\pi}{\lambda_0}\sqrt{\mu_R \epsilon_R} \qquad \text{rad/length} \tag{2–56}$$

Equation (2–51) reveals that the traveling waves also contain magnetic field components having the same velocity as the electric field. This is understandable because the two fields generate each other. Also, the ratio of their magnitudes is a constant. The relationship is given below Eq. (2–49). The ratio η is called the *intrinsic impedance* of the medium. Note that it is a real number since a lossless insulator has been assumed. This means that the traveling electric and magnetic waves are in phase. Substituting in the MKS values of μ_0 and ϵ_0 yields the equation

$$\eta = 120\pi\sqrt{\frac{\mu_R}{\epsilon_R}} = 377\sqrt{\frac{\mu_R}{\epsilon_R}} \qquad \text{ohms} \tag{2–57}$$

Note that for free space, the intrinsic impedance is 377 ohms.

Figure 2–12 shows a sketch of the forward traveling electromagnetic wave described by the first terms in Eqs. (2–50) and (2–51). The peak values are $\sqrt{2}\,E_0^+$ and $\sqrt{2}\,H_0^+$ and therefore E_0^+ and H_0^+ are the rms values. Keep in mind that since $\mathscr{E}_x$ and $\mathscr{H}_y$ are independent of x and y, the $\mathscr{E}$-$\mathscr{H}$ pattern shown in the figure is the same along any other line parallel to the z axis. For this reason, the wave is called a *uniform plane wave*.

Figure 2–12 shows the electromagnetic wave at two instances in time, namely, $t = 0$ and $t = T/4$. Since it is traveling in the $+z$ direction, the second wave is merely the wave at $t = 0$ displaced one quarter of a wavelength in the positive z direction. Note that the electric and magnetic fields are perpendicular to each other and both lie in a plane transverse to the direction of propagation. For this reason the uniform plane wave is called a *transverse electromagnetic* (TEM) wave. The fact that

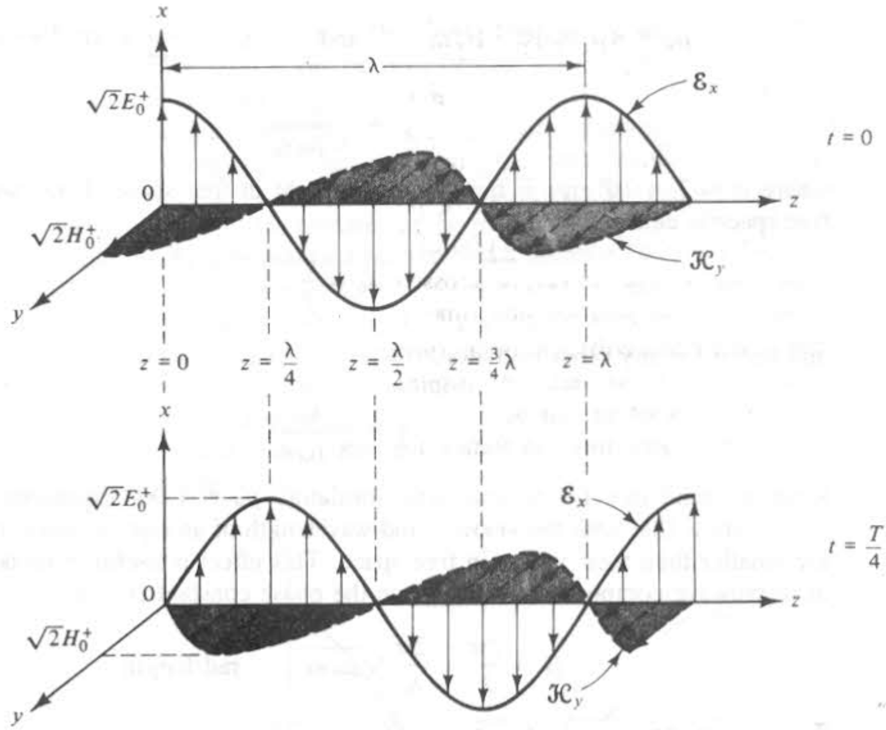

Figure 2–12 Description of an electromagnetic wave traveling in the positive z direction (shown at $t = 0$ and $t = T/4$).

the electric and magnetic fields are perpendicular to each other is a direct consequence of Faraday's and Ampere's laws. To understand this, consider the electric field at $z = \lambda/4$ in Fig. 2–12. Note that when $t = 0$, $\mathcal{E}_x = 0$ at that point. However, as the wave travels to the right, $\mathcal{E}_x$ immediately starts to increase to some finite positive value. This means that its time rate of change is positive and hence the displacement current is in the positive x direction when $t = 0$. In other words, although $\mathcal{E}_x = 0$ at $t = 0$, its time derivative is at a positive maximum. Ampere's law states that a line integral of $\mathcal{H}$ enclosing this displacement current must also be positive. By applying the right-hand rule, the magnetic field when $t = 0$ must be in the positive y direction for z slightly less than $\lambda/4$ and in the negative y direction for z slightly greater than $\lambda/4$. This is indeed the case as shown. If the wave were traveling in the negative z direction, the displacement current would be reversed and hence the direction of the magnetic field would be reversed. A similar argument, utilizing Faraday's law, shows that the movement of the magnetic wave leads to a finite line integral of $\mathcal{E}$ in a plane perpendicular to $\mathcal{H}$. In developing this argument, remember that Faraday's law has a minus sign, while Ampere's law does not.

Let us now consider the power flow associated with the electromagnetic wave propagation we have been describing. Since $\mathcal{E}$ and $\mathcal{H}$ are force fields and contain stored energy, it makes sense that electromagnetic waves involve the propagation of energy. A formal derivation of the energy in an electromagnetic field results in the following vector relationship for power flow.[9]

$$\vec{\mathscr{p}} = \vec{\mathscr{E}} \times \vec{\mathscr{H}} \tag{2-58}$$

$\vec{\mathscr{p}}$ is the instantaneous vector power density (W/m^2) and is known as the *Poynting vector*. Its magnitude represents the value of instantaneous power density, while its direction indicates the direction of power flow. Because Eq. (2–58) involves the vector cross product, the direction of $\vec{\mathscr{p}}$ is always perpendicular to both $\vec{\mathscr{E}}$ and $\vec{\mathscr{H}}$. In our example (Fig. 2–12), $\vec{\mathscr{E}}$ crossed into $\vec{\mathscr{H}}$ is in the positive z direction for any and all values of position and time. To reverse the direction of propagation and hence power flow, either $\vec{\mathscr{E}}$ or $\vec{\mathscr{H}}$ (not both) must be reversed. This agrees with the conclusion arrived at using the displacement current concept in combination with Faraday's and Ampere's laws.

The average power densities for the forward and reverse traveling waves described by Eqs. (2–50) and (2–51) are

$$p_z^+ = E_0^+ H_0^+ \qquad \text{and} \qquad p_z^- = E_0^- H_0^- \tag{2-59}$$

where E_0 and H_0 represent rms values. The average power flow through a surface S perpendicular to the direction of propagation is given by

$$P = \int_S p_z \, dS \qquad \text{where} \qquad p_z = p_z^+ - p_z^- \tag{2-60}$$

For uniform plane waves, p_z is independent of position and hence $P = p_z S$.

The following illustrative example is intended to reinforce the various ideas discussed in this section.

Example 2–3:

A 500 MHz electromagnetic wave is propagating through a perfect nonmagnetic dielectric having $\epsilon_R = 6$.

(a) Calculate the wavelength and the phase constant.

(b) With the wave traveling in the $+z$ direction, the sinusoidal electric field at $z = 80$ cm is delayed relative to the field at $z = 65$ cm. Calculate the time delay (in nanoseconds) and the phase delay (in degrees).

(c) Calculate the average power density in the wave if the peak value of magnetic field is 0.5 A/m.

Solution:

(a) $\lambda_0 = c/f = (3 \times 10^8)/(500 \times 10^6) = 0.6$ m.
Since $\mu_R = 1$, $\lambda = 0.6/\sqrt{6} = 0.245$ m
and $\beta = 2\pi/0.245 = 25.65$ rad/m or 1470°/m.

(b) $t_d = \Delta z/v$, where $v = 3 \times 10^8/\sqrt{6} = 1.22 \times 10^8$ m/s.
Thus, $t_d = (0.80 - 0.65)/v = 1.23 \times 10^{-9}$ s or 1.23 ns.
Phase Delay $= \beta\Delta z = 1470(0.15) = 220°$.

(c) With $\mu_R = 1$, $\eta = 377/\sqrt{6} = 154$ ohms.
$p_z^+ = E_0^+ H_0^+ = \eta(H_0^+)^2$, where $H_0^+ = 0.5/\sqrt{2} = 0.354$ A/m.
Therefore, $p_z^+ = 154(0.354)^2 = 19.3$ W/m^2.

[9] See, for example, Chapter 12 in Ref. 2–1 or Chapter 4 in Ref. 2–6.

2-5 WAVE POLARIZATION

There are four types of wave polarization: linear, circular, elliptic, and random. The wave described in Fig. 2–12 is an example of a linear or plane polarized wave. By convention, the direction of electric field is used to denote the polarization. Therefore the wave in Fig. 2–12 is *vertically polarized* because the electric field is always and everywhere in the vertical direction. A horizontally polarized wave would, of course, be one in which the E lines are horizontal. Linear polarized waves are thus characterized by the fact that the orientation of the field is the same everywhere in space and is independent of time.

It is very useful to describe an electromagnetic wave in terms of phasor quantities. Figure 2–13 shows the phasor representation at five points along the propagation axis for the forward traveling wave depicted in Fig. 2–12. The rms-phasor representation of $\mathscr{E}_x$ at $z = 0$ is given by $E_0^+\underline{/0}$. With the wave traveling in the $+z$ direction, $\mathbf{E}$ at $z = l$ will be phase delayed βl. For instance at $z = \lambda/4$, the phase delay is $(2\pi/\lambda)(\lambda/4) = \pi/2$ rad and hence $\mathbf{E} = E_0^+\underline{/-\pi/2}$. For a given propagation direction, knowledge of $\mathbf{E}$ at one point defines its value at all other points along the propagation axis. The magnetic field phasors are also shown in the figure. As explained previously, the electric and magnetic fields are perpendicular to each other, have the same velocity and are related by the intrinsic impedance η. Note that at any point, $\mathbf{E}$ and $\mathbf{H}$ are in phase, which is the case for η real.

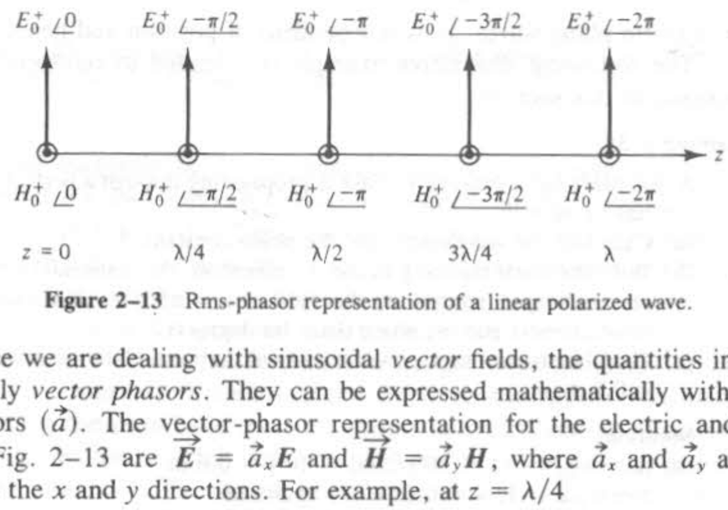

Figure 2–13 Rms-phasor representation of a linear polarized wave.

Since we are dealing with sinusoidal *vector* fields, the quantities in the figure are actually *vector phasors*. They can be expressed mathematically with the aid of unit vectors ($\vec{a}$). The vector-phasor representation for the electric and magnetic fields in Fig. 2–13 are $\overrightarrow{E} = \vec{a}_x E$ and $\overrightarrow{H} = \vec{a}_y H$, where $\vec{a}_x$ and $\vec{a}_y$ are the unit vectors in the x and y directions. For example, at $z = \lambda/4$

$$\overrightarrow{E} = \vec{a}_x E_0^+\underline{/-\pi/2} = \vec{a}_x E_0^+ e^{-j\pi/2} \quad \text{and} \quad \overrightarrow{H} = \vec{a}_y H_0^+\underline{/-\pi/2} = \vec{a}_y H_0^+ e^{-j\pi/2}$$

This method of describing an electromagnetic wave is a very useful analytical tool. In most cases, it is sufficient to specify the electric field and η, since $\boldsymbol{H} = \boldsymbol{E}/\eta$ and, for a given propagation direction, the magnetic field direction can be deduced from Eq. (2–58).

The rules of vector decomposition may be applied to vector phasors as illustrated by the following example.

Example 2–4:

The metal grating in Fig. 2–14 has the property that a wave with E lines perpendicular to the metal strips propagates through it, while one with E lines parallel to the strips is completely reflected by it. This assumes that the spacing between the strips is much less than the wavelength.

A linearly polarized wave propagating in free space and oriented 30 degrees from vertical ($\theta = 30°$) impinges upon the grating. Determine the rms value and the direction of the electric field at the output side of the grating. The incoming wave has an average power density of 50 W/m^2.

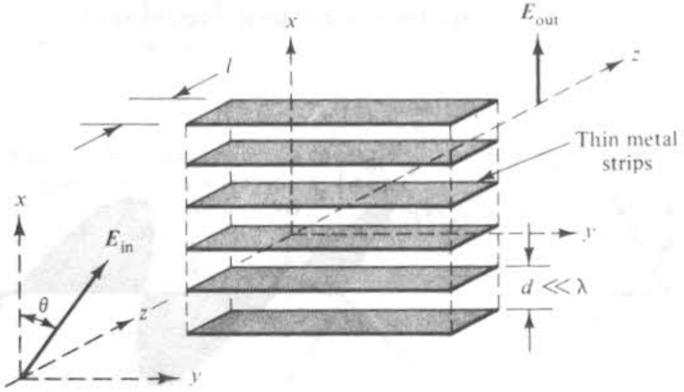

Figure 2–14 Propagation of a linear polarized wave through a metal grating. (See Ex. 2–4.)

Solution: For free space, $\eta = 377$ ohms and hence $p_{in} = (E_{in}^+)^2/377 = 50$ W/m^2. Therefore, $E_{in}^+ = 137.3$ V/m and its phasor value may be written as $\boldsymbol{E}_{in}^+ = 137.3\underline{/0}$. The incoming vector wave may be decomposed into two components, one parallel and one perpendicular to the metal strips. That is,

$$\vec{\boldsymbol{E}}_{in} = \vec{a}_x \boldsymbol{E}_{in}^+ \cos 30° + \vec{a}_y \boldsymbol{E}_{in}^+ \sin 30° = \vec{a}_x(119\,\underline{/0}) + \vec{a}_y(68.7\,\underline{/0})$$

Since only the x component passes through the grating, $\vec{\boldsymbol{E}}_{out} = \vec{a}_x(119\underline{/-\beta l})$, where l is the length of the grating and βl is the phase delay through it. The rms value of the output electric field is 119 V/m and it is oriented in the x direction as shown. Thus the output wave is vertically polarized.

Vector addition and subtraction can also be applied to the vector-phasor representation of linearly polarized waves as long as the waves all have the same frequency and propagation direction. If the individual waves are all in phase (cophasal), a vector diagram may be used to determine the resultant wave. Figure 2–15 describes the addition of two such waves. The individual linear waves and their vector phasors are shown in part a. By utilizing the properties of right triangles, their vector addition yields the resultant phasor $\mathbf{E}_r = 2E_0\underline{/0}$ which is oriented 30° from the vertical. The wave associated with $\mathbf{E}_r$ is shown in part b. It has a peak value of $2\sqrt{2}\,E_0$ and its plane of polarization is 30° from the vertical (the x'-z plane). Although the magnetic field is not shown, it is perpendicular to the electric field and its direction may be deduced from the Poynting vector.

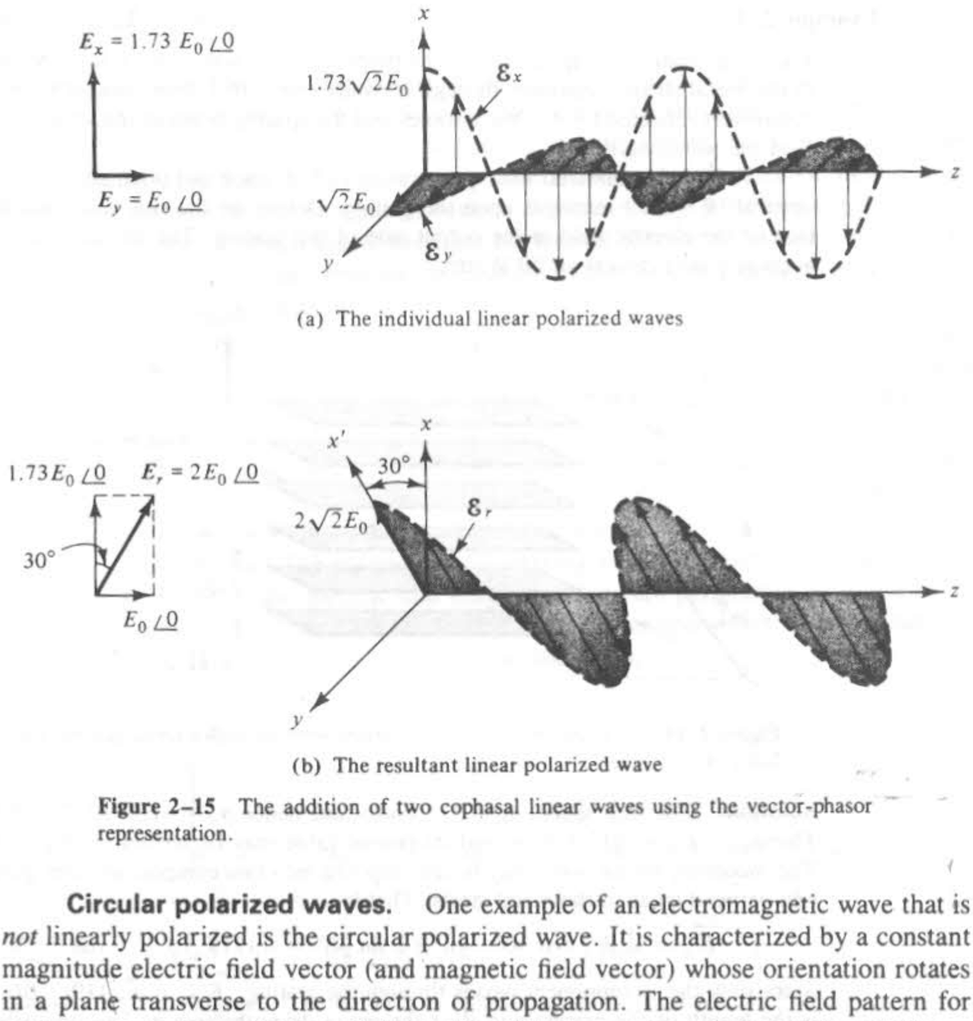

Figure 2–15 The addition of two cophasal linear waves using the vector-phasor representation.

Circular polarized waves. One example of an electromagnetic wave that is *not* linearly polarized is the circular polarized wave. It is characterized by a constant magnitude electric field vector (and magnetic field vector) whose orientation rotates in a plane transverse to the direction of propagation. The electric field pattern for such a wave is shown in Figs. 2–16 and 2–17. The angular velocity of the rotation ω is equal to $2\pi f$, where f is the frequency of the wave. The rotation in a fixed transverse plane can be either clockwise or counterclockwise.

Let us assume that the pattern shown in Fig. 2–16 represents a wave traveling in the positive z direction at $t = 0$. This pattern with its fixed E vectors, all having the same magnitude, moves in the positive z direction with a velocity v. Therefore, an observer looking down the propagation axis sees the rotating field pattern described in Fig. 2–17. Note that the vectors rotate counterclockwise with time. This is usually referred to as *left-hand circular polarization,* while clockwise rotation is called *right-hand circular polarization*. The solid arrows in the figure represent the electric field vectors at $t = 0$ and the dashed arrows indicate their direction of rotation with time. As in the case of linear polarization, one symbol is sufficient to completely describe the circular polarized wave when the direction of propagation is known. If the first symbol in Fig. 2–17 represents the counterclockwise wave at $z = 0$ and $t = 0$, then the wave at any other time and position is defined. At a fixed

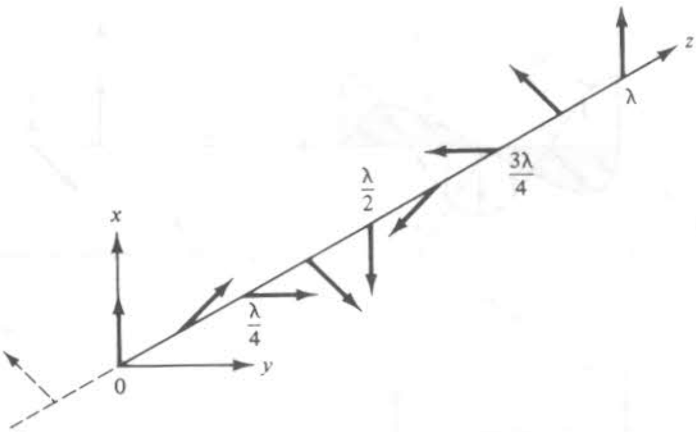

Figure 2–16 Description of a circular polarized wave at one instant in time. (Magnetic vectors not shown.)

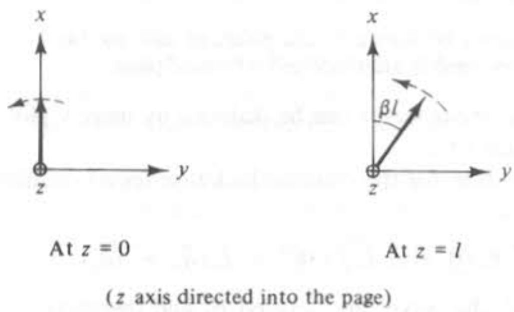

Figure 2–17 Rotating field vectors at $z = 0$ and $z = l$ for the circular polarized wave in Fig. 2–16.

time, the vector at $z = l$ is displaced *clockwise* by an angle βl relative to the vector at $z = 0$. Also, at any fixed point, say $z = l$, the vector is defined for any future time t, since it is rotated *counterclockwise* by an angle $\omega t = 2\pi t/T$ relative to its position at $t = 0$.

A circularly polarized wave may be represented by two orthogonal, linearly polarized waves equal in amplitude and 90° out-of-phase. The frequency of the waves must be identical and equal to $\omega/2\pi$, where ω is the angular velocity of the rotating field vectors. Figure 2–18 shows two such waves $\mathscr{E}_x$ and $\mathscr{E}_y$ at $t = 0$. Addition of their $\mathscr{E}$ vectors verifies that they are equivalent to the circular polarized wave shown at the right. Observe that the two linear waves are orthogonal in space and at any point are 90° out-of-phase in time (when one is maximum, the other is zero). If the peak value of each linear wave is $\sqrt{2}\,E_0$, then the amplitude of the circular polarized vector is $\sqrt{2}\,E_0$. The vector phasors for the two linear waves and the equivalent circular polarized representation are also shown, assuming propagation in the $+z$ direction.[10] In this example, $\mathbf{E}_y$ leads $\mathbf{E}_x$ by 90° which results in a counterclockwise circular polarized wave. For a clockwise wave, $\mathbf{E}_y$ would have to lag $\mathbf{E}_x$ by 90°.

[10] Caution must be exercised in adding or subtracting vector phasors by means of a vector diagram. This technique is valid *only* when the waves are cophasal. For example, one might conclude that the sum of the vector-phasors in Fig. 2–18 results in a linear wave polarized 45° from the vertical. This is incorrect! With the waves 90° out-of-phase, the resultant wave is *circularly polarized*.

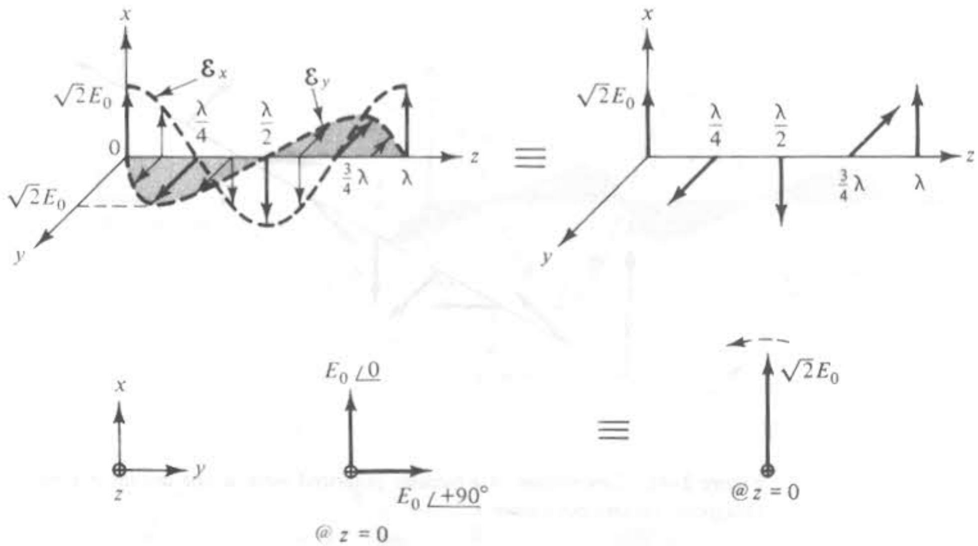

Figure 2–18 The equivalence between a circular polarized wave and two orthogonal, linear polarized waves equal in amplitude and 90° out-of-phase.

The relative time phase of the linear waves can be deduced by merely plotting $\mathcal{E}_x$ and $\mathcal{E}_y$ versus time for a fixed value of z.

The mathematical expression for the counterclockwise (ccw) circular polarized wave in Fig. 2–18 at $z = 0$ is

$$\vec{E}_{\text{ccw}} = \vec{a}_x E_0\underline{/0} + \vec{a}_y E_0\underline{/+90°} = E_0(\vec{a}_x + j\vec{a}_y)$$

At any other point, say $z = l$, the waves are delayed βl and therefore

$$\vec{E}_{\text{ccw}} = (E_0\underline{/-\beta l})(\vec{a}_x + j\vec{a}_y) \tag{2–61}$$

For a clockwise (cw) wave propagating in the $+z$ direction, the y component lags the x component. Therefore at $z = l$,

$$\vec{E}_{\text{cw}} = (E_0\underline{/-\beta l})(\vec{a}_x - j\vec{a}_y) \tag{2–62}$$

A circular polarized wave can be represented by any pair of orthogonal linear waves that are equal in amplitude and 90° out-of-phase. Figure 2–19 shows two equivalent representations of a clockwise wave. The following analysis verifies the equivalence. The pair on the left are given by $E_0(\vec{a}_x - j\vec{a}_y)$. The pair on the right are given by

$$\vec{E}_{\text{cw}} = \vec{a}_{x'}(E_0\underline{/-\theta}) + \vec{a}_{y'}(E_0\underline{/-90° - \theta}) = (E_0\underline{/-\theta})(\vec{a}_{x'} - j\vec{a}_{y'})$$

where $\vec{a}_{x'}$ and $\vec{a}_{y'}$ are the unit vectors in the x' and y' directions. These can be written in terms of $\vec{a}_x$ and $\vec{a}_y$ as follows.

$$\vec{a}_{x'} = \vec{a}_x \cos\theta + \vec{a}_y \sin\theta \qquad \text{and} \qquad \vec{a}_{y'} = -\vec{a}_x \sin\theta + \vec{a}_y \cos\theta$$

Therefore,

$$\begin{aligned}\vec{E}_{\text{cw}} &= E_0 e^{-j\theta}\,[\vec{a}_x \cos\theta + \vec{a}_y \sin\theta + j\vec{a}_x \sin\theta - j\vec{a}_y \cos\theta] \\ &= E_0 e^{-j\theta}\,[(\cos\theta + j\sin\theta)(\vec{a}_x - j\vec{a}_y)] = E_0(\vec{a}_x - j\vec{a}_y)\end{aligned}$$

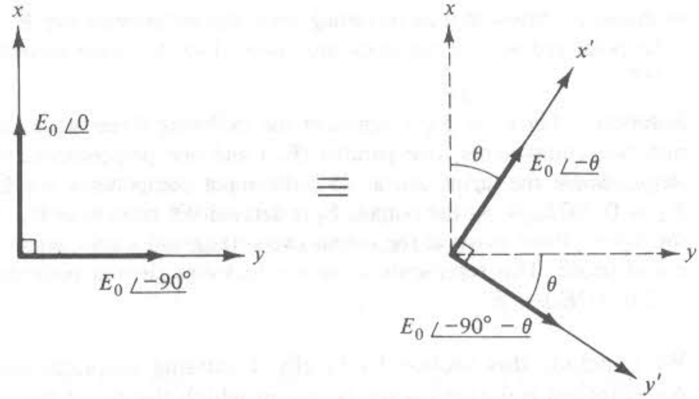

Figure 2–19 Two equivalent representations of a clockwise circular polarized wave.

which verifies that the two representations are equivalent. Thus, for a clockwise circular polarized wave, a *clockwise* rotation θ of the polarization of the orthogonal linear waves requires that phase *delays* equal to θ be added to their phasor values.

In practice, a circular polarized wave can be generated from a single linear wave. The following example illustrates the technique.

Example 2–5:

The array of dielectric strips described in Fig. 2–20 has the property that a wave with E lines parallel to the strips is phase delayed relative to one with E lines perpendicular

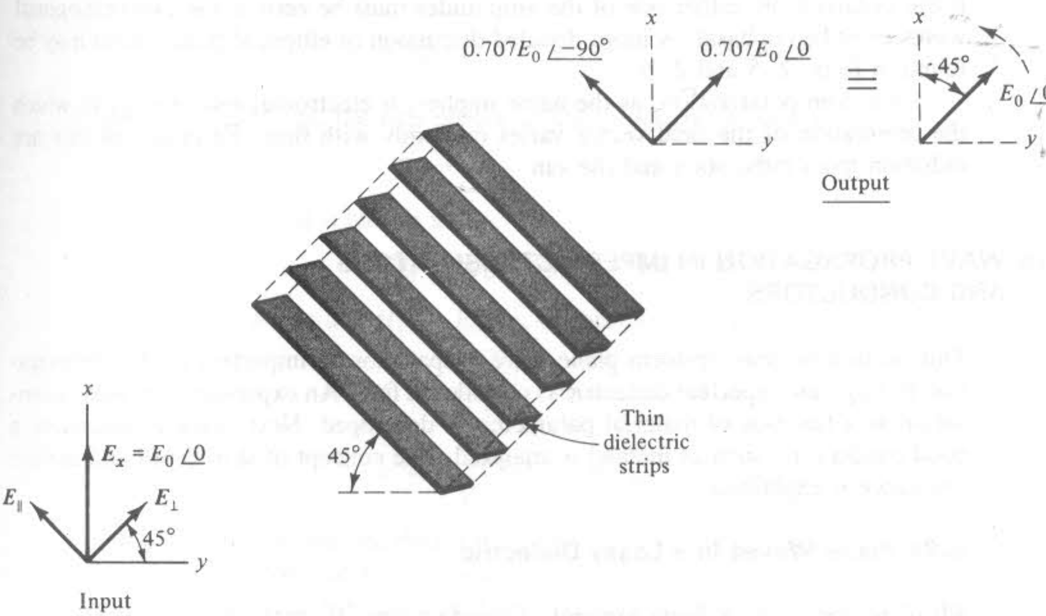

Figure 2–20 A dielectric grating for converting a linear polarized wave into a circular polarized wave. (See Ex. 2–5.)

to the strips. Show that an incoming vertically polarized wave $\vec{\mathbf{E}}_x$ is converted to a circular polarized wave if the strips are oriented 45° from the vertical and the phase delay is 90°.

Solution: Let $\mathbf{E}_x = E_0\underline{/0}$ represent the incoming linear wave. It can be decomposed into two components, one parallel ($\mathbf{E}_\parallel$) and one perpendicular ($\mathbf{E}_\perp$) to the dielectric strips. Since the strips are at 45°, the input components are $\mathbf{E}_\parallel = 0.707E_0\underline{/0}$ and $\mathbf{E}_\perp = 0.707E_0\underline{/0}$. At the output, $\mathbf{E}_\parallel$ is delayed 90° relative to $\mathbf{E}_\perp$. Thus, as indicated in the figure, there exists at the output two orthogonal waves, equal in amplitude and 90° out-of-phase. This represents a counterclockwise circular polarized wave of amplitude $\sqrt{2}(0.707E_0) = E_0$.

We conclude this section by briefly discussing elliptical and random polarization. An elliptical polarized wave is one in which the tip of the rotating field vector in a fixed transverse plane traces an elliptical path with time. The ellipticity (in dB) of the wave is defined as

$$\text{Ellipticity (dB)} \equiv 20 \log \frac{E_{\max}}{E_{\min}} \tag{2-63}$$

where $E_{\max}$ and $E_{\min}$ are, respectively, the electric field amplitudes along the major and minor axes of the ellipse. A circularly polarized wave has 0 dB ellipticity since $E_{\max} = E_{\min}$.

One method of generating an elliptical wave is to combine two orthogonal linearly polarized waves, *unequal* in amplitude and 90° out-of-phase. Actually, circular and linear polarization are special cases of elliptical polarization. In the case of circular polarization, the amplitudes of the two orthogonal waves must be equal. For linear polarization, either one of the amplitudes must be zero or the two orthogonal waves must be cophasal. A more detailed discussion of elliptical polarization may be found in Refs. 2–5 and 2–6.

Random polarization, as the name implies, is electromagnetic energy in which the orientation of the field vector varies randomly with time. Examples of this are radiation from radio stars and the sun.

2–6 WAVE PROPAGATION IN IMPERFECT INSULATORS AND CONDUCTORS

This section reviews uniform plane wave propagation in imperfect media. Propagation through an imperfect dielectric is considered first. An expression for wave attenuation as a function of material parameters is developed. Next, wave propagation in good conductors (such as metals) is analyzed. The concept of skin depth and surface resistance is explained.

2–6a Plane Waves in a Lossy Dielectric

All dielectrics have a finite amount of conductivity. If $\tan \delta < 0.10$, its effect on electromagnetic propagation is mainly some attenuation of the wave. This is verified by the following analysis of uniform plane waves in a lossy dielectric.

For sinusoidal time variations, Faraday and Ampere's laws may be written in phasor form. That is,

$$\frac{d\mathbf{E}_x}{dz} = -j\omega\mu_R\mu_0\mathbf{H}_y \qquad \text{and} \qquad -\frac{d\mathbf{H}_y}{dz} = (\sigma + j\omega\epsilon_R\epsilon_0)\mathbf{E}_x \tag{2-64}$$

Note that these expressions are similar to those in Eq. (2–46) except that a conduction current term $\mathbf{J} = \sigma\mathbf{E}_x$ has been added to Ampere's law. Eliminating $\mathbf{H}_y$ results in the following second order differential equation.

$$\frac{d^2\boldsymbol{E}_x}{dz^2} = -\omega^2\mu_R\mu_0\epsilon_R\epsilon_0(1 - j\tan\delta)\boldsymbol{E}_x \tag{2-65}$$

where, as explained in Sec. 2–2, $\tan\delta \equiv \sigma/\omega\epsilon_R\epsilon_0$ is known as the *loss tangent* of the material.

As before, the two solutions represent traveling waves in the + and − z directions. In rms-phasor form, they are

$$\boldsymbol{E}_x = E_0^+e^{-\gamma z} + E_0^-e^{+\gamma z}$$

where $\gamma = (j\omega\sqrt{\mu_R\mu_0\epsilon_R\epsilon_0})(\sqrt{1 - j\tan\delta})$ is known as the *propagation constant*. For dielectric materials with small conductivity ($\tan\delta < 0.10$), the approximation $\sqrt{1 - j\tan\delta} \approx 1 - j\frac{1}{2}\tan\delta$ may be used. Therefore,

$$\gamma \approx \tfrac{1}{2}\omega\sqrt{\mu_R\mu_0\epsilon_R\epsilon_0}\tan\delta + j\omega\sqrt{\mu_R\mu_0\epsilon_R\epsilon_0}$$

The real part of γ is called the *attenuation constant* (α), while the imaginary part is the previously defined phase constant β. Note that

$$\alpha = \tfrac{1}{2}\beta\tan\delta = \frac{\pi}{\lambda}\tan\delta \qquad \text{Np/length} \tag{2-66}$$

Since $\gamma = \alpha + j\beta$, the rms-phasor solutions may be written as

$$\mathbf{E}_x = E_0^+e^{-\alpha z}e^{-j\beta z} + E_0^-e^{+\alpha z}e^{+j\beta z} \tag{2-67}$$

The magnetic field solutions are

$$\mathbf{H}_y = H_0^+e^{-\alpha z}e^{-j\beta z} - H_0^-e^{+\alpha z}e^{+j\beta z} \tag{2-68}$$

where $H_0^+ = E_0^+/\eta$ and $H_0^- = E_0^-/\eta$. In this case, the intrinsic impedance is

$$\eta = 377\sqrt{\frac{\mu_R}{\epsilon_R}}\left(\frac{1}{\sqrt{1 - j\tan\delta}}\right)$$

$$\approx 377\sqrt{\frac{\mu_R}{\epsilon_R}}(1 + j\tfrac{1}{2}\tan\delta) \qquad \text{ohms} \tag{2-69}$$

What conclusions may be drawn from this analysis? First, since $\beta = \omega/v$, the wave velocity is approximately the same as in the lossless case, namely, $v = c/\sqrt{\mu_R\epsilon_R}$. Also, the fact that η is complex means that $\mathbf{H}_y$ is phase shifted relative to $\mathbf{E}_x$. For $\tan\delta < 0.10$, the phase shift is less than three degrees, a negligible value. Thus for good insulators v, β, λ, and η are unaffected by the material's finite conductivity and hence Eqs. (2–53), (2–55), (2–56), and (2–57) may be used with negligible error.

The only significant effect of finite (but small) conductivity is that the amplitude of the traveling waves are attenuated as they propagate along the z axis. For the forward traveling wave, the ratio of amplitudes at any two points ($z = l_1$ and $z = l_2$) is given by

$$\frac{E_0^+ e^{-\alpha l_1}}{E_0^+ e^{-\alpha l_2}} = \frac{H_0^+ e^{-\alpha l_1}}{H_0^+ e^{-\alpha l_2}} = e^{-\alpha(l_1 - l_2)}$$

Since the unit for α is Np/length, the exponent $\alpha(l_1 - l_2)$ is the total attenuation in nepers. Nepers and decibels are reviewed in Sec. 1–4. The following example gives some indication of the relation between attenuation and the properties of the dielectric.

Example 2–6:

A uniform plane wave at 1000 MHz is propagating through a non-magnetic dielectric whose properties are defined by $\epsilon_R = 4$ and $\tan\delta = 0.02$. The rms value of the forward traveling wave at $z = 10$ cm is 100 V/m. Calculate the loss in power density between $z = 10$ cm and $z = 70$ cm.

Solution: Since $\lambda_0 = 0.3$ m at 1000 MHz and $\mu_R = 1$, $\lambda = \lambda_0/\sqrt{4} = 0.15$ m. Therefore, $\beta = 2\pi/0.15$ rad/m and $\alpha \approx (\pi/0.15)\tan\delta = 0.42$ Np/m. The ratio of rms values is

$$\frac{E^+ \text{ at } z = 70 \text{ cm}}{E^+ \text{ at } z = 10 \text{ cm}} = e^{-0.42(0.70-0.10)} = e^{-0.252} = 0.78$$

and thus E^+ at $z = 70$ cm is 78 V/m.

With both $\boldsymbol{E}_x$ and $\boldsymbol{H}_y$ being attenuated at the same rate, the average power density p_z of the traveling wave is attenuated by

$$\frac{p_z \text{ at } z = l_1}{p_z \text{ at } z = l_2} = e^{-2\alpha(l_1 - l_2)}$$

For $l_1 = 0.70$ m and $l_2 = 0.10$ m, the power ratio is $(0.78)^2 \approx 0.60$, which represents an attenuation of 0.252 Np or 2.19 dB. Thus over a distance of 60 cm (four wavelengths), about 40% of the power in the uniform plane wave has been lost due to I^2R losses in the dielectric.

Microwave devices and lines often extend over many wavelengths, which means that the material chosen in this example would not be suitable for low-loss applications. For such applications, microwave engineers usually select materials with loss tangents less than 0.001. Most of the dielectrics listed in Table B–2 of Appendix B satisfy this condition. The conclusions described here apply equally well to a wave traveling in the negative z direction since it will be attenuated at the same rate as a forward traveling wave.

2–6b Plane Waves in a Good Conductor

Let us now consider the case of uniform plane waves in a good conductor (such as a metal). Since these materials are characterized by very high conductivity, the conduction current ($\sigma \mathbf{E}$) is much greater than the displacement current ($j\omega\epsilon_R \epsilon_0 \mathbf{E}$). Therefore Eqs. (2–64) reduce to

$$\frac{d\mathbf{E}_x}{dz} = -j\omega\mu_R\mu_0\mathbf{H}_y \qquad \text{and} \qquad -\frac{d\mathbf{H}_y}{dz} = \sigma\mathbf{E}_x \tag{2–70}$$

Eliminating $\mathbf{H}_y$ results in the following second order differential equation

$$\frac{d^2\mathbf{E}_x}{dz^2} = j\omega\mu_R\mu_0\sigma\mathbf{E}_x \tag{2–71}$$

Although this equation has two solutions, only the one associated with a forward traveling wave will be considered, namely,

$$\mathbf{E}_x = E_0^+ e^{-\gamma z} \qquad \text{where} \qquad \gamma = \sqrt{j\omega\mu_R\mu_0\sigma}$$

Since $\omega = 2\pi f$ and $\sqrt{j} = 1\underline{/45^\circ} = (1 + j1)/\sqrt{2}$,

$$\gamma = \alpha + j\beta = \sqrt{\pi f \mu_R\mu_0\sigma} + j\sqrt{\pi f\mu_R\mu_0\sigma}$$

Thus,

$$\mathbf{E}_x = E_0^+ e^{-z/\delta_s} e^{-jz/\delta_s} \tag{2–72}$$

where

$$\delta_s \equiv \frac{1}{\sqrt{\pi f \mu_R \mu_0 \sigma}} \tag{2–73}$$

is known as the *skin depth* of the material.

To understand the meaning of Eqs. (2–72) and (2–73), consider the sketch in Fig. 2–21. Assume that an ac source in region 1 is connected to the conductor at the $z = 0$ plane resulting in a rms electric field E_0^+ as shown. Recalling that tangential E must be continuous across a boundary, the electric field just inside the conductor also equals E_0^+. From Eq. (2–72), the rms value of $\mathbf{E}_x$ decreases as the wave attempts to penetrate the conductor. If δ_s is small, the decay of E_x with distance is very rapid. Part *b* of the figure shows a plot of the rms value of current density as a

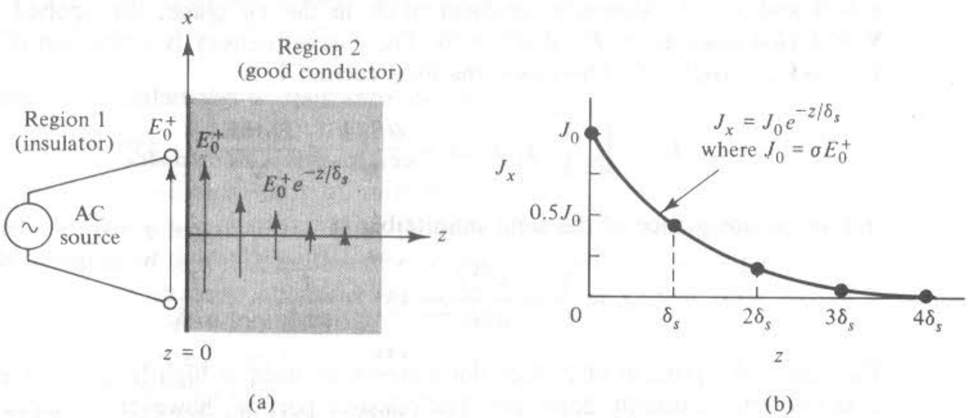

Figure 2–21 Description of skin effect in a good conductor.

function of penetration into the conductor. Its rate of decay with distance is the same as E_x since $J_x = \sigma E_x$.

The magnetic field also decreases at the same rate since $\mathbf{H}_y = \mathbf{E}_x/\eta_s$, where

$$\eta_s = \frac{1+j}{\sigma\delta_s} = (1+j)R_s = \sqrt{2}\, R_s\underline{/45^\circ} \quad \text{ohms} \tag{2–74}$$

and R_s is defined in Eq. (2–78). This result is obtained by substituting $\mathbf{E}_x$ into the first of Eqs. (2–70). Its significance is that in a good conductor, the magnetic field lags the electric field by 45°.

At one skin depth ($z = \delta_s$), E_x is e^{-1} or 37 percent of its surface value. This, in fact, may be considered a definition of skin depth. Three skin depths into the conductor, the electric field amplitude is down to e^{-3} or 5 percent of its surface value. These comments also apply to J_x and H_y. Since power density is proportional to E_x^2, its value at three skin depths is e^{-6} or 0.25 percent of the surface power density. Thus, practically speaking, all of the power is located within a few skin depths of the surface.

For copper, $\mu_R = 1$ and $\sigma = 5.8 \times 10^7$ mho/m. Therefore at a frequency f (Hz), the expression for skin depth reduces to

$$\delta_{cu} = \frac{0.066}{\sqrt{f}} \qquad \text{meter} \tag{2–75}$$

For example, at 10,000 MHz, the skin depth of copper is 6.6×10^{-7} m or 0.66 micron. Thus a few skin depths represents only 2 microns or *80 millionths of an inch*. In other words, at microwave frequencies, power and current flow in metals is essentially a surface phenomenon. Microwave engineers take advantage of this fact by plating a fairly good conductor (brass or aluminum) with several skin depths of an excellent conductor (silver or gold). In this manner, the electrical properties of the excellent conductor are obtained with minimum cost.

Skin-Effect resistance. Figure 2–22a shows an ac source connected to a semi-infinite bar of conducting material. The connection is via two strips located at $x = 0$ and $x = l$. Assuming uniform fields in the x-y plane, the applied voltage $\mathbf{V} = E_0^+ l\underline{/0}$ since $\mathbf{E}_x = E_0^+\underline{/0}$ at $z = 0$. The current density is a function of z since $\mathbf{J}_x = \sigma\mathbf{E}_x = \sigma E_0^+ e^{-\gamma z}$. Therefore, the total current $\mathbf{I}_x$ is

$$\mathbf{I}_x = \int_0^\infty \int_0^b \mathbf{J}_x\, dy\, dz = \frac{\sigma E_0^+ b}{\gamma} = \frac{\delta_s \sigma b E_0^+}{\sqrt{2}}\underline{/-45^\circ}$$

and the ac impedance of the semi-infinite bar is

$$Z = \frac{\mathbf{V}}{\mathbf{I}_x} = \frac{\sqrt{2}\, l}{\sigma b \delta_s}\underline{/+45^\circ} = \frac{l}{\sigma b \delta_s} + j\frac{l}{\sigma b \delta_s} \tag{2–76}$$

The imaginary portion of Z does not concern us since at high frequencies external inductive effects usually dominate. The resistive portion, however, is important. It

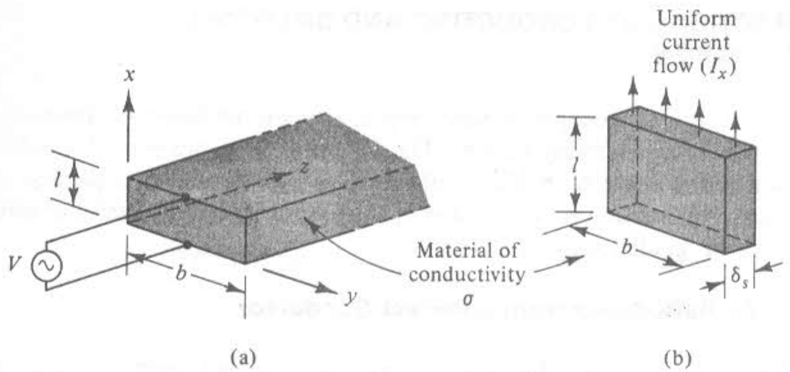

Figure 2–22 The influence of skin effect on the ac resistance of a good conductor.

is given by

$$R_{ac} = \frac{l}{\sigma b \delta_s} \quad \text{ohms} \tag{2–77}$$

Comparing this expression with Eq. (2–15) leads us to an interesting and useful conclusion. Namely, the ac resistance of the semi-infinite conductor is the same as the dc resistance of a rectangular bar of length l and transverse dimensions b and δ_s. In other words, the skin-effect resistance may be computed by assuming that the total current is uniformly distributed over a thickness of *one* skin depth. This concept is illustrated in part b of Fig. 2–22. The same conclusion may be obtained by analyzing the power dissipated in the conductor (Refs. 2–6 and 2–7).

The resistance of a square section ($l = b$), denoted by R_s, is given by

$$R_s = \frac{1}{\sigma \delta_s} = \sqrt{\frac{\pi f \mu_R \mu_0}{\sigma}} \quad \text{ohms/square} \tag{2–78}$$

When the skin depth is small compared to the conductor's thickness (that is, its z dimension), R_s is called the *surface resistivity*.

The phenomenon of skin effect can also be explained using circuit concepts. An excellent treatment is found in Chapter 7 of Ref. 2–7. Also included is a derivation of the skin-effect resistance of round wire. At high frequencies (that is, when $\delta_s << a$, the radius of the wire), the concept associated with Eq. (2–77) may be used to calculate the ac resistance. Namely,

$$R_{ac} \approx \frac{l}{\sigma (2\pi a) \delta_s} \quad \text{ohms} \tag{2–79}$$

where l is the length of the wire. In other words, at high frequencies, the wire may be considered as a thin hollow tube of length l, radius a and wall thickness δ_s.

As an example, consider a 100-m length of copper wire having a radius $a = 0.005$ m. At 1000 MHz, $\delta_s = 2.09 \times 10^{-6}$ m and therefore $R_{ac} = 26.3$ ohms. The dc resistance, from Eq. (2–15), is only 0.022 ohms, which means that the 1000 MHz resistance is more than a thousand times greater. Thus because of skin effect, I^2R losses are significantly greater at the higher frequencies.

2-7 REFLECTIONS AT CONDUCTING AND DIELECTRIC BOUNDARIES

When an electromagnetic wave impinges upon the boundary between two materials, a reflection invariably occurs. The magnitude and form of this reflection is a function of the angle of incidence of the wave as well as the properties of the two materials. This section reviews some of the cases that will be useful in subsequent discussions on guided waves.

2-7a Reflections from a Perfect Conductor

Let us first consider the case of a linearly polarized, uniform plane wave in a perfect insulator propagating toward a perfectly conducting surface (Fig. 2-23). The direction of propagation has been chosen perpendicular to the surface of the conductor (normal incidence). By virtue of Poynting's vector, both the electric and the magnetic vectors are parallel to the plane of the boundary. For the coordinate system shown, the electric and magnetic fields are given by Eqs. (2-50) and (2-51). Since a perfect insulating medium has been assumed, the attenuation in region 1 is zero. It is convenient to choose the $z = 0$ plane at the boundary between the insulating and conducting regions. This choice of coordinates is indicated in Fig. 2-23. Since a perfect conductor has been assumed for region 2, the tangential component of electric field must be zero at the boundary and thus $E_0^- = -E_0^+$. With $E_0^+/H_0^+ = E_0^-/H_0^-$, $H_0^- = -H_0^+$ and hence the magnetic field at $z = 0$ is $2\sqrt{2}H_0^+ \cos \omega t$. As a result, the electric field is zero at the conducting surface, while the magnetic field is double the value of $\mathcal{H}$ associated with the incident wave. For a perfect conductor, $\delta_s = 0$ and therefore ac current flow is restricted to the surface. By virtue of Eq. (2-42), the surface current density $\mathcal{K}$ is in the x direction and equal to $2\sqrt{2}\, H_0^+ \cos \omega t$.

The following argument may help in understanding the above conclusions. As the E_0^+ wave impinges upon the conductor, a surface current flows in the same direction as the electric field. The resultant charge separation creates an E_0^- equal and opposite to E_0^+. By Ampere's law (right-hand rule), the surface current creates a magnetic field in the y direction that adds to that associated with the incident wave. Thus

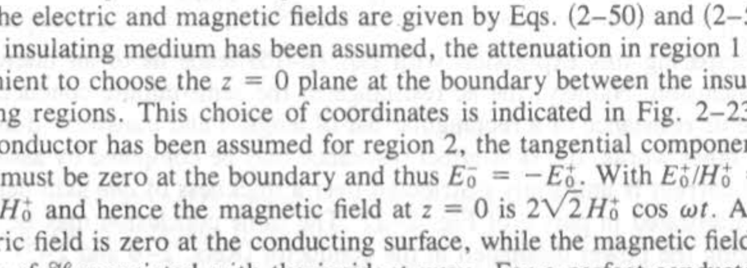

Figure 2-23 Wave propagation from a perfect insulator toward a perfect conductor (normal incidence).

the electric field experiences a reversal but the magnetic field does not, which means that the direction of propagation of E_0^- and H_0^- is opposite to that of the incident wave. Furthermore, since the magnitudes of E_0^- and H_0^- are the same as E_0^+ and H_0^+, the power density in the reflected wave is equal to that in the incident wave. This result is a direct consequence of the fact that a perfect conductor cannot absorb power.

Let us now consider the expressions for $\mathscr{E}_x$ and $\mathscr{H}_y$ at a distance d from the boundary plane. For $z = -d$ (region 1),

$$\begin{aligned} \mathscr{E}_x &= \sqrt{2}\,E_0^+ \cos(\omega t + \beta d) - \sqrt{2}\,E_0^+ \cos(\omega t - \beta d) \\ \mathscr{H}_y &= \sqrt{2}\,H_0^+ \cos(\omega t + \beta d) + \sqrt{2}\,H_0^+ \cos(\omega t - \beta d) \end{aligned} \tag{2–80}$$

Using the trigonometric identity $\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$, the above equations become

$$\begin{aligned} \mathscr{E}_x &= -\{2\sqrt{2}\,E_0^+ \sin \beta d\} \sin \omega t \\ \mathscr{H}_y &= \{2\sqrt{2}\,H_0^+ \cos \beta d\} \cos \omega t \end{aligned} \tag{2–81}$$

These expressions do not represent traveling waves since the arguments of the sin ωt and cos ωt terms do not contain βd terms. What they do represent are sinusoidal time functions whose amplitudes (the bracketed quantities) are a function of position along the z axis. They are called *standing waves*, the implication being that there is no net power flow in the z direction. This is understandable since it has already been shown that the power in the reflected wave is exactly equal to the power in the incident wave. Note that since Eqs. (2–80) and (2–81) are equivalent, two oppositely directed traveling waves and a standing wave are merely different ways of viewing the same phenomenon.

To better understand the nature of these standing waves, consider the expression for $\mathscr{E}_x$ in Eq. (2–81). Since $\beta = 2\pi/\lambda$, then $\mathscr{E}_x = 0$ at $d = 0, \lambda/2, \lambda$, etc. On the other hand, at $d = \lambda/4, 3\lambda/4, 5\lambda/4$, etc., $\mathscr{E}_x = \pm 2\sqrt{2}\,E_0^+ \sin \omega t$, which is an ac electric field having an rms value of $2E_0^+$. At other positions along the z axis, the rms value of $\mathscr{E}_x$ is less than $2E_0^+$. Thus, the largest possible rms value of $\mathscr{E}_x$ is *twice* the rms value of the incident wave. The expression for $\mathscr{H}_y$ in Eq. (2–81) also represents a standing wave. Note that for any fixed value d, $\mathscr{E}_x$ and $\mathscr{H}_y$ are 90° out-of-phase, which means that at any point in space, the average power in the electromagnetic field is zero. This is analogous to the 90° phase relation between voltage and current in a pure reactance wherein average power is also zero. Also note that nulls of $\mathscr{H}_y$ occur at $d = \lambda/4, 3\lambda/4, 5\lambda/4$, etc., while maximums occur at $d = 0, \lambda/2, \lambda$, etc. Thus the standing wave pattern for $\mathscr{H}_y$ is displaced one quarter wavelength relative to the $\mathscr{E}_x$ standing wave. A plot of E_x and H_y (rms values) as a function of distance d is shown in Fig. 2–24. Unlike the case of a pure traveling wave, the ratio of E_x to H_y for the standing wave is a function of position along the propagation axis and does *not* equal η, the intrinsic impedance of the medium. A plot of $\mathscr{E}_x$ and $\mathscr{H}_y$ versus position at several instants of time is shown in Fig. 2–25. The time intervals are one-eighth of a period apart and cover one period of the ac cycle. The pattern of the $\mathscr{E}_x$ curve as a function of time is similar to that of a vibrating string pinned at $d = 0$. The condition that the string's displacement is always zero at $d = 0$ is

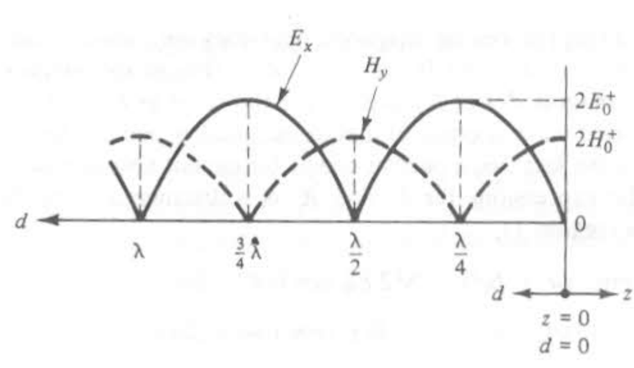

Figure 2–24 The rms values of $\mathscr{E}_x$ and $\mathscr{H}_y$ as a function of position for the case of a conducting surface located at $d = 0$.

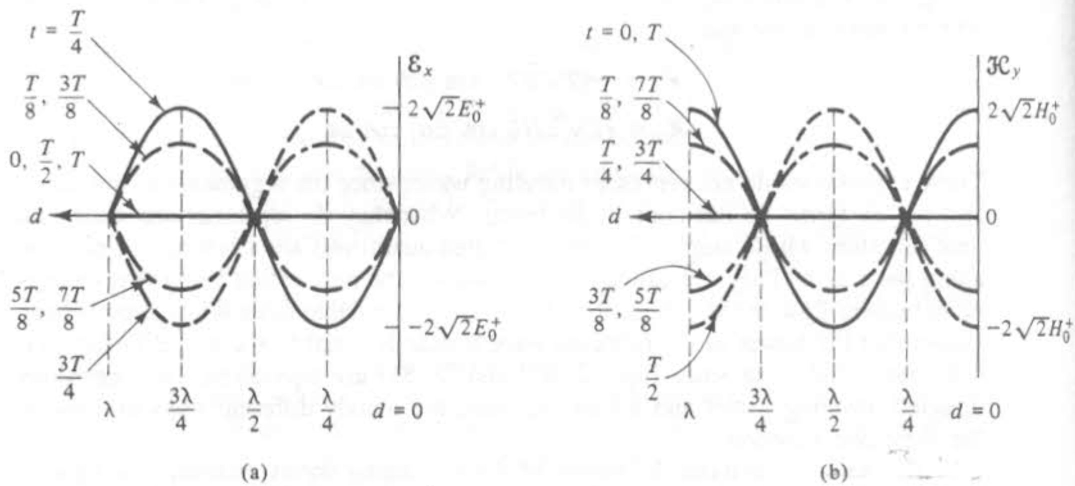

Figure 2–25 A plot of the instantaneous values of $\mathscr{E}_x$ and $\mathscr{H}_y$ as a function of position and time. The conducting surface is located at $d = 0$.

analogous to the boundary condition $\mathscr{E}_x = 0$ at the conducting surface. For any fixed value of d, both $\mathscr{E}_x$ and $\mathscr{H}_y$ are sinusoidal time functions having a period T. Observe that between a successive pair of nulls, all the sinusoidal time functions of the standing wave are in phase. That is, they all reach their maximum (and minimum) values at the same time. This contrasts with a pure traveling wave ($\alpha = 0$) wherein phase is a function of position but amplitude is not.

The analysis described here could have been carried out using phasor notation. The case of reflections at a dielectric boundary is analyzed using this approach.

Oblique incidence. Let us briefly consider the case in which the wave impinges upon the conducting surface at some angle θ_i. This angle, known as the *angle of incidence*, represents the angular distance between the propagation axis of the incident wave and a line normal to the reflecting surface. As in the case of optics, the angle of reflection θ_r equals the angle of incidence θ_i. This is true for any type polarization. The power density of the reflected wave equals that of the incident wave since a perfect conductor cannot absorb power. The proof of these results is given in Ref. 2–8.

2-7b Reflections and Refractions at a Dielectric Boundary

Consider now the case of a uniform plane wave propagating from one perfect insulator into another. The situation for the case of normal incidence is shown in Fig. 2-26. When the incident wave in region 1 arrives at the boundary between the two dielectrics (the $z = 0$ plane), some of it is reflected and some is transmitted into region 2. The expressions for $\boldsymbol{E}$ and $\boldsymbol{H}$ in region 1 are given by Eqs. (2-48) and (2-49). In region 2, only a forward traveling wave exists since the material extends to infinity in the positve z direction. The continuity condition for the tangential components of $\mathbf{E}$ and $\mathbf{H}$ at the boundary require that $\mathbf{E}_i + \mathbf{E}_r = \mathbf{E}_t$ and $\mathbf{H}_i - \mathbf{H}_r = \mathbf{H}_t$, where the incident and reflected waves in region 1 are indicated by the subscripts i and r, respectively. The transmitted wave in region 2 is indicated by the subscript t. With $\mathbf{H}_i = \mathbf{E}_i/\eta_1$, $\mathbf{H}_r = \mathbf{E}_r/\eta_1$, and $\mathbf{H}_t = \mathbf{E}_t/\eta_2$, solving for $\mathbf{E}_t$ and $\mathbf{H}_t$ results in

$$\frac{\mathbf{E}_t}{\mathbf{E}_i} = \frac{2\eta_2}{\eta_1 + \eta_2} \quad \text{and} \quad \frac{\mathbf{H}_t}{\mathbf{H}_i} = \frac{2\eta_1}{\eta_1 + \eta_2} \tag{2-82}$$

These ratios indicate the fraction of the incident fields that are transmitted into region 2. Since $\mathbf{E}_r = \mathbf{E}_t - \mathbf{E}_i$,

$$\Gamma \equiv \frac{\mathbf{E}_r}{\mathbf{E}_i} = \frac{\mathbf{H}_r}{\mathbf{H}_i} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} \tag{2-83}$$

The fraction of the incident wave that is reflected at the boundary is called the *reflection coefficient* (Γ). Because it is the ratio of phasor quantities, the reflection coefficient may be complex. The magnitude of the reflection coefficient $|\Gamma|$ is the amplitude ratio of reflected to incident wave, while the angle of the reflection coefficient indicates the phase shift of the reflected wave relative to the incident wave. When η_2 and η_1 are real, Γ is real and its value is restricted to $-1 \leq \Gamma \leq +1$.

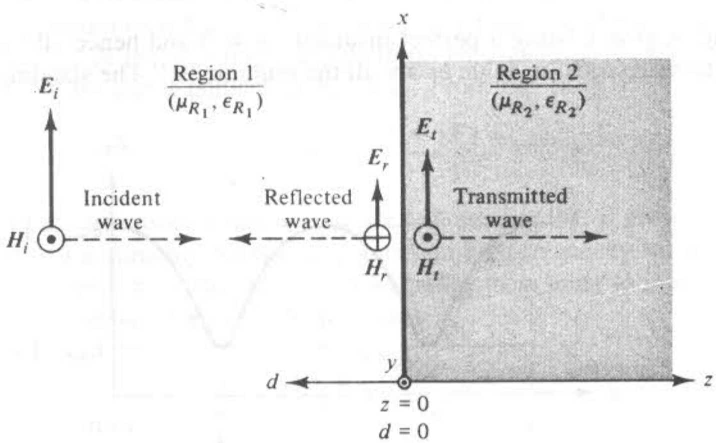

Figure 2-26 Transmission and reflection at the boundary between two ideal dielectrics (normal incidence).

At the boundary between the two dielectrics, part of the incident wave is reflected whenever $\eta_2 \neq \eta_1$. With power density proportional to the square of E, the reflected power density (p_r) is related to the incident power density (p_i) by

$$p_r = |\Gamma|^2 p_i \tag{2-84}$$

Utilizing Eqs. (2-82) and (2-83), the following relation is obtained for the transmitted power density.

$$p_t = \{1 - |\Gamma|^2\} p_i \tag{2-85}$$

This equation states that the power delivered to region 2 is equal to the incident power minus the reflected power. On the basis of a conservation of energy argument, this conclusion is plausible.

The reflected wave that is created when $\eta_2 \neq \eta_1$ produces a standing wave pattern in region 1. At a distance d to the left of the boundary plane (Fig. 2-26), the fields are given by Eqs. (2-48) and (2-49), where $z = -d$. That is,

$$\mathbf{E}_{x_1} = E^+_{0_1}\underline{/\beta d} + E^-_{0_1}\underline{/-\beta d} \quad \text{and} \quad \mathbf{H}_{y_1} = H^+_{0_1}\underline{/\beta d} - H^-_{0_1}\underline{/-\beta d}$$

where the subscript 1 denotes fields in region 1. Since the reflection coefficient at the boundary $\Gamma = E^-_{0_1}/E^+_{0_1} = H^-_{0_1}/H^+_{0_1}$, the above equations may be restated as

$$\mathbf{E}_{x_1} = E^+_{0_1}\underline{/\beta d} + \Gamma E^+_{0_1}\underline{/-\beta d} \quad \text{and} \quad \mathbf{H}_{y_1} = H^+_{0_1}\underline{/\beta d} - \Gamma H^+_{0_1}\underline{/-\beta d} \tag{2-86}$$

These phasor equations represent standing waves of electric and magnetic fields. The electric field pattern (rms values) for Γ positive is shown in Fig. 2-27. If Γ were negative, the pattern would be shifted one quarter wavelength. In either case, the maximum and minimum values of E_{x_1} are given by

$$E_{\max} = \{1 + |\Gamma|\} E^+_{0_1} \quad \text{and} \quad E_{\min} = \{1 - |\Gamma|\} E^+_{0_1} \tag{2-87}$$

With region 1 being a perfect insulator, $\alpha = 0$ and hence all the maximums in the pattern are equal in value as are all the minimums.[11] The standing wave patterns rep-

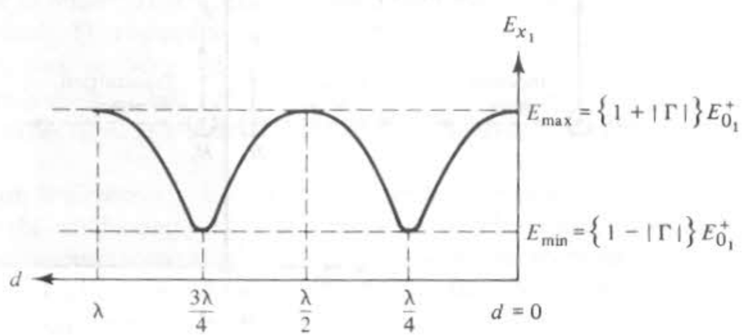

Figure 2-27 The standing wave pattern of E_{x_1} for Γ positive.

resent the amplitude variation resulting from the phasor addition of incident and reflected waves. The phasor additions for $\mathbf{E}_{x_1}$ at $d = 0$, $\lambda/8$ and $\lambda/4$ are shown in Fig. 2-28. Observe that as d increases, the incident phasor ($\mathbf{E}_i$) rotates counterclockwise while the reflected wave phasor ($\mathbf{E}_r$) rotates clockwise. The magnitude of the resultant phasor is the rms value of $\mathbf{E}_{x_1}$ at the particular position d. Note that unlike Fig. 2-24, the minimums are not zero and the maximums are not equal to $2E_0^+$. There are some similarities between the patterns in Figs. 2-24 and 2-27. Successive minimums are spaced $\lambda/2$ apart and adjacent maximums and minimums are separated by $\lambda/4$. Also, if the H_y pattern were plotted in Fig. 2-27, it would be shifted $\lambda/4$ relative to the E_x pattern.

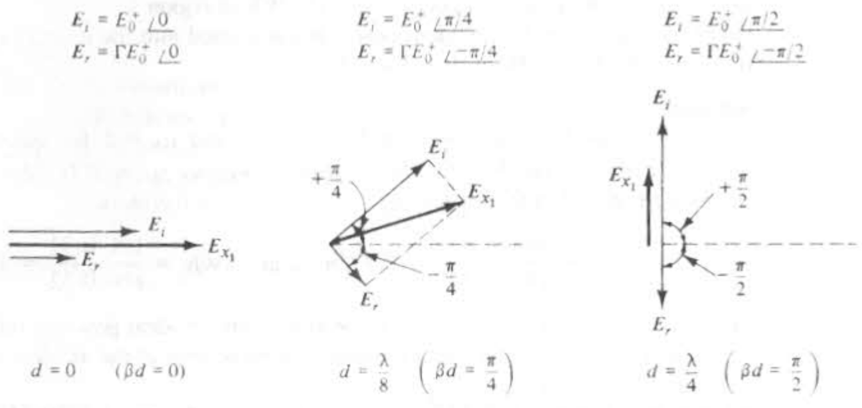

Figure 2-28 The phasor diagram of E_{x_1} at $d = 0$, $\lambda/8$, and $\lambda/4$ for the case where Γ is positive.

The magnitude of the reflection coefficient represents the fraction of the incident wave that is reflected at the boundary. Another quantity that is often used to indicate the amount of reflection is the ratio of the maximum field (rms) to the minimum field (rms) in the standing wave pattern. This quantity is called the *standing-wave ratio* (SWR) and is defined as E_{max}/E_{min}. From Eq. (2-87),

$$\text{SWR} = \frac{1 + |\Gamma|}{1 - |\Gamma|} \tag{2-88}$$

By virtue of its definition, SWR is always a real number that is greater than or equal to unity. It is a commonly measured quantity in high-frequency work. Since values of $|\Gamma|$ range from 0 to 1, the values of SWR range from unity to infinity. The above equation can be solved for $|\Gamma|$, which yields

$$|\Gamma| = \frac{\text{SWR} - 1}{\text{SWR} + 1} \tag{2-89}$$

[11] If $\alpha \neq 0$, the maximum and minimum values are functions of z. This case is discussed in Sec. 11.7 of Ref. 2-4.

An SWR of unity indicates no reflections and hence a pure traveling wave, while infinite SWR indicates full reflection and hence a pure standing wave. In general, the wave pattern consists of some combination of a traveling wave and a standing wave. For $|\Gamma| < 1$, there will be a net power flow in the propagation direction and therefore $\mathbf{E}_x$ and $\mathbf{H}_y$ cannot be 90° out-of-phase. The following example verifies this fact and illustrates the use of phasors in calculating the electric and magnetic standing wave patterns.

Example 2–7:

A 1250 MHz uniform plane wave propagating in free space (region 1) with an electric field equal to 10 V/m rms impinges on a semi-infinite volume of quartz (region 2) with normal incidence (Fig. 2–26).

(a) Calculate the reflection coefficient and SWR in region 1.
(b) What fraction of the incident power is transmitted into the quartz region?
(c) Calculate $\mathbf{E}_x$ and $\mathbf{H}_y$ at $d = 3$ and 6 cm.

Solution:

(a) From Table B–2 in Appendix B, $\epsilon_R = 3.8$ and $\mu_R = 1$ for quartz. Neglecting losses (that is, $\tan \delta \approx 0$), the intrinsic impedance $\eta_2 = 377/\sqrt{3.8} = 193$ ohms. Since region 1 is free space, $\eta_1 = 377$ ohms and therefore

$$\Gamma = \frac{193 - 377}{193 + 377} = 0.32\underline{/\pi} \quad \text{and} \quad \text{SWR} = \frac{1 + 0.32}{1 - 0.32} = 1.94.$$

(b) Since $|\Gamma|^2 = (0.32)^2 = 0.10$, 10 percent of the incident power is reflected. Therefore, $1 - |\Gamma|^2 = 0.90$, which means that 90 percent of the incident power is transmitted to region 2.

(c) For free space, $\eta_1 = 377$ ohms and $\lambda = \lambda_0 = 24$ cm at 1250 MHz. With $\beta = 2\pi/\lambda$, $E_{0_1}^+ = 10$ V/m, $H_{0_1}^+ = 10/377$ A/m and $\Gamma = -0.32$, Eq. (2–86) reduces to $\mathbf{E}_{x_1} = 10\underline{/\pi d/12} - 3.2\underline{/-\pi d/12}$ and $\mathbf{H}_{y_1} = 0.027\underline{/\pi d/12} + 0.009\underline{/-\pi d/12}$.
At $d = 3$ cm,
$\mathbf{E}_{x_1} = 10\underline{/\pi/4} - 3.2\underline{/-\pi/4} = 10.5\underline{/1.09 \text{ rad}}$ or $10.5\underline{/62.7°}$ V/m
$\mathbf{H}_{y_1} = 0.027\underline{/\pi/4} + 0.009\underline{/-\pi/4} = 0.028\underline{/27.5°}$ A/m
At $d = 6$ cm,
$\mathbf{E}_{x_1} = 10\underline{/\pi/2} - 3.2\underline{/-\pi/2} = 13.2\underline{/90°}$ V/m
$\mathbf{H}_{y_1} = 0.027\underline{/\pi/2} + 0.009\underline{/-\pi/2} = 0.018\underline{/90°}$ A/m
Note that at this position, $\mathbf{E}_{x_1}$ is a maximum and $\mathbf{H}_{y_1}$ is a minimum.

As discussed earlier, $\mathbf{E}_x$ and $\mathbf{H}_y$ are in phase for a pure traveling wave and 90° out-of-phase for a pure standing wave. In the more general situation, the phase difference is a function of position along the propagation axis. In the above example, $\mathbf{E}_x$ and $\mathbf{H}_y$ are in phase at $d = 6$ cm but 35.2° out-of-phase at $d = 3$ cm. However, the net power flow at both points must be the same since the region is lossless. In both cases, the net power density must equal $\{1 - |\Gamma|^2\}p_i$, where $p_i = (10)(0.027) = 0.27$ W/m^2 is the power density of the incident wave. Since $|\Gamma| = 0.32$, the net power density at all points must be 0.24 W/m^2. Let us verify this at $d = 3$ cm. The net power density is given by $E_x H_y \cos \theta_{pf}$, where θ_{pf} is the power factor angle between $\mathbf{E}_x$ and $\mathbf{H}_y$. Its value at $d = 3$ cm is $(10.5)(0.028) \cos 35.2° = 0.24$ W/m^2. A similar calculation at any other value of d yields the same result.

Oblique incidence. The case of oblique incidence at a dielectric interface is shown in Fig. 2–29. There are two possible cases, one with the E field perpendicular to the plane of incidence and the other with the E field parallel to it.[12] The first situation is described in the figure. As in the case for a conducting surface, the angle of incidence is equal to the angle of reflection. The relationship between the angle of incidence θ_i and the angle of refraction θ_t is given by Snell's law. Namely,

$$\frac{\sin\theta_i}{\sin\theta_t} = \frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2} = \sqrt{\frac{\mu_{R_2}\epsilon_{R_2}}{\mu_{R_1}\epsilon_{R_1}}} \tag{2-90}$$

If region 1 is free space, Snell's law reduces to

$$\frac{\sin\theta_i}{\sin\theta_t} = \frac{c}{v_2} = \frac{\lambda_0}{\lambda_2} = \sqrt{\mu_{R_2}\epsilon_{R_2}} = n_i \tag{2-91}$$

where n_i is known as the *index of refraction* of the material. Since $\mu_R \approx 1$ for practically all dielectrics, $n_i \approx \sqrt{\epsilon_R}$. This law indicates that a wave bends toward the normal to the boundary surface when propagating into a region with a higher index of refraction. For example, if region 1 is free space and region 2 has an index of refraction $n_2 = 3$, then for $\theta_i = 60°$, $\theta_t = \arcsin 0.289 = 16.8°$. On the other hand, if region 1 has the higher index of refraction, the transmitted wave bends away from the normal. In fact, under certain circumstances, it is possible that no real solution exists

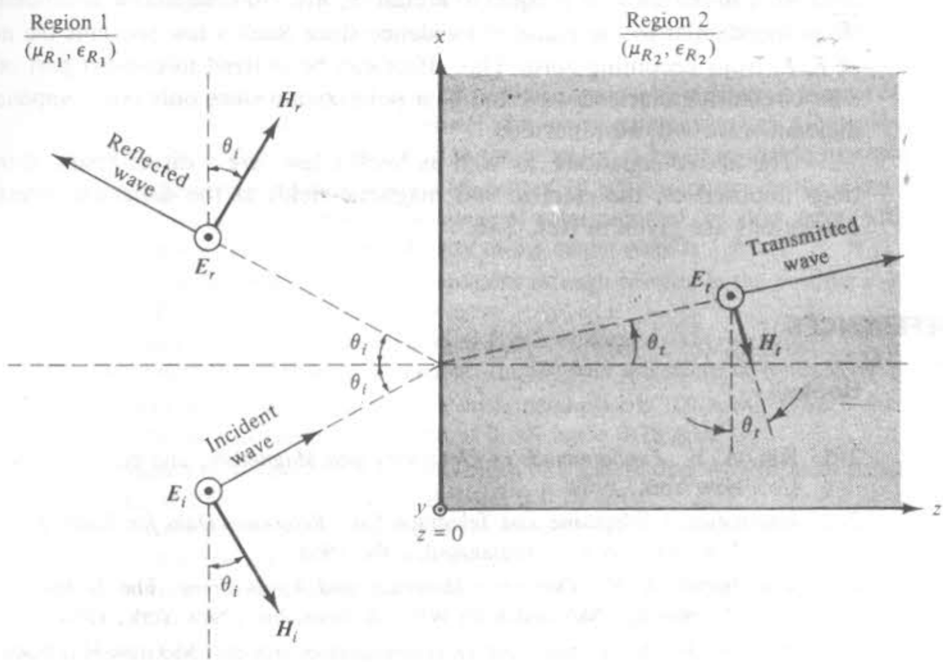

Figure 2–29 Transmission and reflection at the boundary between two ideal dielectrics (oblique incidence).

[12] Any other polarization may be considered as a linear combination of these two cases.

for θ_t. For instance, if $n_1 = 3$ and $n_2 = 1$, then for $\theta_i = 60°$, Eq. (2–90) yield θ_t = arcsin 2.6. Since this is not a real solution for θ_t, there can be no transmitte wave and hence the dielectric-air surface acts like a perfect reflector. This situatior wherein the wave does not escape the region of higher index of refraction, provides qualitative explanation for the dielectric waveguides and resonators discussed i Chapters 5 and 9.

In general, there are both transmitted and reflected waves at the dielectr boundary. The relationships governing their amplitudes are given by Fresnel's equa tions. Assuming lossless, nonmagnetic dielectrics in both regions, the equations a

For $\vec{E}_i$ perpendicular to the plane of incidence,

$$\frac{E_t}{E_i} = \frac{2\, n_1 \cos\theta_i}{n_1 \cos\theta_i + n_2 \cos\theta_t} \quad \text{and} \quad \frac{E_r}{E_i} = \frac{n_1 \cos\theta_i - n_2 \cos\theta_t}{n_1 \cos\theta_i + n_2 \cos\theta_t} \qquad \text{(2-9}$$

For $\vec{E}_i$ parallel to the plane of incidence,

$$\frac{E_t}{E_i} = \frac{2\, n_1 \cos\theta_i}{n_2 \cos\theta_i + n_1 \cos\theta_t} \quad \text{and} \quad \frac{E_r}{E_i} = \frac{n_2 \cos\theta_i - n_1 \cos\theta_t}{n_2 \cos\theta_i + n_1 \cos\theta_t} \qquad \text{(2-9}$$

A particular case of interest for $\vec{E}_i$ parallel to the plane of incidence is wh $n_2 \cos\theta_i = n_1 \cos\theta_t$. In this situation, $E_r = 0$ which means there is no reflect wave. The incident angle at which this occurs is called *Brewster's angle*. For an inc dent wave in region 1, it is equal to arctan (n_2/n_1). No comparable situation exists f $\vec{E}_i$ perpendicular to the plane of incidence since Snell's law prevents the numerat of E_r/E_i from becoming zero. This effect can be utilized to convert part of an inc dent circularly polarized wave to linear polarization since only one component of t incident wave will be reflected.

The above equations, as well as Snell's law, are a direct result of the cond tions imposed on the electric and magnetic fields at the dielectric interface. T derivations are given in Ref. 2–8.

REFERENCES

Books

2–1. Kip, A. F., *Fundamentals of Electricity and Magnetism*, 2nd ed., McGraw-Hill Bo Co., New York, 1969.

2–2. International Telephone and Telegraph Co., *Reference Data for Radio Engineers*, ed., H.W. Sams & Co., Indianapolis, IN, 1968.

2–3. von Hippel, A. R., *Dielectric Materials and Applications*, The Technology Press MIT, Cambridge, MA and John Wiley & Sons, Inc., New York, 1954.

2–4. Hayt, W. H., Jr., *Engineering Electromagnetics*, 4th ed., McGraw-Hill Book Co., N York, 1981.

2–5. Berkowitz, B., *Basic Microwaves*, Hayden Book Co., New York, 1966.

2–6. Rao, N. N., *Basic Electromagnetics with Applications*, Prentice-Hall, Inc., Englewo Cliffs, NJ, 1972.

2-7. Skilling, H. H., *Electric Transmission Lines*, McGraw-Hill Book Co., New York, 1951.

2-8. Zahn, M., *Electromagnetic Field Theory*, John Wiley & Sons, Inc., New York, 1979.

PROBLEMS

2-1. A teflon-filled, parallel-plate capacitor is fully charged by a 12 V battery. The plate separation is 0.10 cm and the dimensions of each plate is 0.8 cm by 1.2 cm. Neglecting fringe effects, calculate the energy stored in the electric field. Compare the result with that obtained from the formula for energy stored in a capacitor ($\frac{1}{2}CV^2$).

2-2. A quartz-filled, parallel-plate capacitor consists of two identical circular metal discs spaced 0.05 cm apart.

(a) Calculate the radius of the discs for $C = 6.0$ pF.

(b) What is the conductance of the structure at 3.0 GHz?

2-3. N turns of #24 copper wire are wound as a single layer on a 0.25 cm diameter glass rod. If the turns are uniformly distributed over a length of 2.0 cm, how many turns are needed for an inductance of 177 nH? Calculate the dc resistance of the coil.

2-4. The wavelength of a 600 MHz wave propagating through a nonmagnetic dielectric is 20 cm. What is the dielectric constant of the material?

2-5. An electromagnetic wave propagates through a lossless insulator with a velocity 1.8×10^{10} cm/s. Calculate the electric and magnetic properties of the insulator if its intrinsic impedance is 260 ohms.

2-6. An electromagnetic wave propagates through free space with a power density of 60 W/m^2. Another wave with the same power density propagates through a nonmagnetic dielectric. Show that the electric field in the dielectric is less than that in free space.

2-7. A linear polarized wave with a power density of 27 W/m^2 impinges on the metal grating shown in Fig. 2–14. What is the plane of polarization of the input wave (with respect to the x axis), if the power density of the output wave is 3 W/m^2?

2-8. A 3000 MHz uniform plane wave propagates through rexolite in the positive z direction. The **E** field at $z = 0$ is $100\underline{/0}$ V/m.

(a) Calculate the rms value and phase of **E** at $z = 4$ cm.

(b) Determine the total wave attenuation (in dB) over a distance of 6 wavelengths.

2-9. The current density at the surface of a thick metal plate is 100 A/m^2. What is the skin depth if the current density at a depth of 0.001 cm is 0.28 A/m^2?

2-10. It is proposed to silver-plate a 10 foot length of stainless steel wire so as to reduce its resistance at 1000 MHz. The wire diameter is 0.20 cm.

(a) Approximate the minimum plating thickness required to insure that the 1000 MHz current in the stainless steel is negligible.

(b) Assuming sufficient plating, calculate the 1000 MHz resistance of the wire. Compare the result to the resistance of the wire before plating.

2-11. An electromagnetic wave propagating in free space is reflected by a metal plate. The distance between successive nulls in the standing wave pattern is 12 cm. Calculate the frequency of the wave.

2-12. A 2000 MHz standing wave pattern exists in a nonmagnetic dielectric. What is ϵ_R if the distance between a maximum and an adjacent minimum is 1.5 cm?

2–13. Referring to Fig. 2–26, region 2 is free space and the properties of region 1 are $\mu_{R_1} = 1$ and $\epsilon_{R_1} = 9$. The net electric field at the boundary ($d = 0$) is $30\underline{/0}$ V/m. Use the phasor method to calculate $\mathbf{E}_{x_1}$ and $\mathbf{H}_{y_1}$ at $d = 1$ cm and 2 cm if the wave frequency is 1250 MHz. How far from the boundary is the first magnetic field maximum?

2–14. Regions 1 and 2 of Fig. 2–26 contain nonmagnetic dielectrics with $\epsilon_{R_1} = 6$ and $\epsilon_{R_2} = 3$. Calculate the reflection coefficient and SWR for a wave propagating from region 1 toward the dielectric interface.

2–15. A 500 MHz incident wave propagates as shown in Fig. 2–26. Region 1 contains a lossless insulator and region 2 is free space. Determine μ_{R_1} and ϵ_{R_1} if the SWR in region 1 is 1.60 and the wavelength is 35 cm. Assume $\epsilon_{R_1} > \mu_{R_1}$.

2–16. An electromagnetic wave with a power density of 5.0 W/m^2 impinges on a dielectric boundary causing a SWR of 1.90. Calculate the power density of the wave transmitted into the dielectric.

2–17. Calculate the SWR for the cases of 25 percent and 50 percent power reflected at a dielectric boundary.

2–18. Referring to Fig. 2–29, $\theta_i = 35°$ and region 2 is free space. What is the minimum value of index of refraction for region 1 that results in no transmission into the free space region?