Tutorial 2

1. Find the limit
   (a) $\lim_{x\to\infty}(\sqrt{x^2+3x}-\sqrt{x^2-2x})$
   (b) $\lim_{x\to\infty}(\sqrt{x^2+x}-\sqrt{x^2-x})$
2. Use formal definition, to prove that
   (a) $\lim_{x\to 0}\frac{1}{|x|}=\infty$
   (b) $\lim_{x\to 1^-}\frac{1}{1-x^2}=\infty$
3. Find the oblique asymptotes of
   (a) $f(x)=\frac{x^2+1}{x-1}$
   (b) $f(x)=\frac{x^3+1}{x^2}$
4. For the function
$$f(x)=\begin{cases} x, & x \text{ is rational} \\ 0, & x \text{ is irrational} \end{cases}$$
   (a) Show that f is continuous at $x=0$.
   (b) Also show that f is not continuous at any of the real number.
5. Which of the following statements are true, and which are false? If true, say why, if false, give counter example.
   (a) If $\lim_{x\to a} f(x)$ exists but $\lim_{x\to a} g(x)$ does not exist, then $\lim_{x\to a}(f(x)+g(x))$ does not exist.
   (b) If neither $\lim_{x\to a} f(x)$ nor $\lim_{x\to a} g(x)$ exist, then $\lim_{x\to a}(f(x)+g(x))$ does not exist.
   (c) If f is continuous at x, then so is $|f|$.
   (d) If $|f|$ is continuous at a, then so is f.
6. Test the differentiability of the following functions
   (a) $x^2\sin\frac{1}{x}$ at $x=0$
   (b) $x|x|$ at $x=0$

   If differentiable then what will be the derivative of each function. Is the derivative again differentiable?
7. From a paper with dimension $a\times b$, a cuboid is constructed with a window of dimension $a/2\times b/2$ on one surface. What is the dimension of the cuboid such that volume of the cuboid is maximum? Justify your answer.