Lecture - 13 P (1)

Recap:

Poisson random variable

Negative binomial

↳ Banach Match Problem

Hypergeometric random variable

Bag with N balls.

m white, $N-m$ black

Draw n balls (without replacement)

X = no. of white balls.

$$P(X=i) = \frac{\binom{m}{i}\binom{N-m}{n-i}}{\binom{N}{n}}$$

e.g. Estimate the number of ~~tigers~~ lions in Gir forest.

Let N is total no. of ~~tigers~~. lions. You randomly catch m lions, mark them and then release them.

Randomly catch n lions. You count how many are marked, its X

$$P(X=i) = \frac{\binom{m}{i}\binom{N-m}{n-i}}{\binom{N}{n}}$$

$i=0,\ldots,n$

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Estimate N.

MLE: maximum likelihood estimate

3

$$f(N) = \frac{\binom{m}{i}\binom{N-m}{n-i}}{\binom{N}{n}}$$

$$\frac{f(N)}{f(N-1)} \geq 1$$

$$\shortparallel$$

$$\frac{\binom{N-m}{n-i}\binom{N-1}{n}}{\binom{N}{n}\binom{N-1-m}{n-i}} =$$

$$\frac{\cancel{(N-m)!}\,\cancel{(N-1)!}\,\cancel{n!}\,\cancel{(N-n)!}\,\cancel{(n-i)!}\,\cancel{(N-1-m-n+i)!}}{\cancel{(n-i)!}\,\cancel{(N-m-n+i)!}\,\cancel{(N-1-n)!}\,\cancel{n!}\,\cancel{N!}\,\cancel{(N-1-m)!}}$$

$$= \frac{(N-m)(N-n)}{N(N-m-n+i)} \geq 1$$

$$\cancel{N^2} - \cancel{Nn} - \cancel{Nm} + mn \geq \cancel{N^2} - \cancel{Nm} - \cancel{Nn} + iN$$

$$N \leq \frac{mn}{i}$$

$N = \frac{mn}{i}$

assumption that the lions are equally probable to be anywhere in the forest.

$m = 10$ lions.

$n = 50$ lions.
$i = 5$ are marked.

$\frac{m}{N} = \frac{i}{n} \Rightarrow$

$N = \frac{mn}{i}$

$= 100$

Sums of random variables.

Expected value of sum of random variables = Sum of expected values.

$X_1, X_2, \dots, X_n$

$E[X_1 + X_2 + \cdots + X_n] =$

$E[X_1] + E[X_2] + \cdots + E[X_n]$

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toss 3 dice. X = total. $E[X]$.

$\frac{7}{2} * 3 = \frac{21}{2}$

X	
3	1/216
4	3/216
5	6/216 1,1,3 * 3 1,2,1 * 3
18	1/216

Cumulative distribution function

$$F(a) = \sum_{x \leq a} p(x)$$

= Probability that X is $\leq a$.

1. Its a nondecreasing function.
   if $a < b$, then $F(a) \leq F(b)$

2. $\lim_{b \to \infty} F(b) = 1$

3. $\lim_{b \to -\infty} F(b) = 0$

Continuous random variable.

Discrete was <u>countable</u>.

$\mathbb{Q}$ is countable.

A set is countable if $\exists$ a bijection with $\mathbb{N}$.

You can list down (enumerate) all the elements of the set.

$\mathbb{R}$ is not countable.

continuous random variable

Defn: X is a continuous random variable if $\exists$ a nonnegative function f, defined over R, s.t.

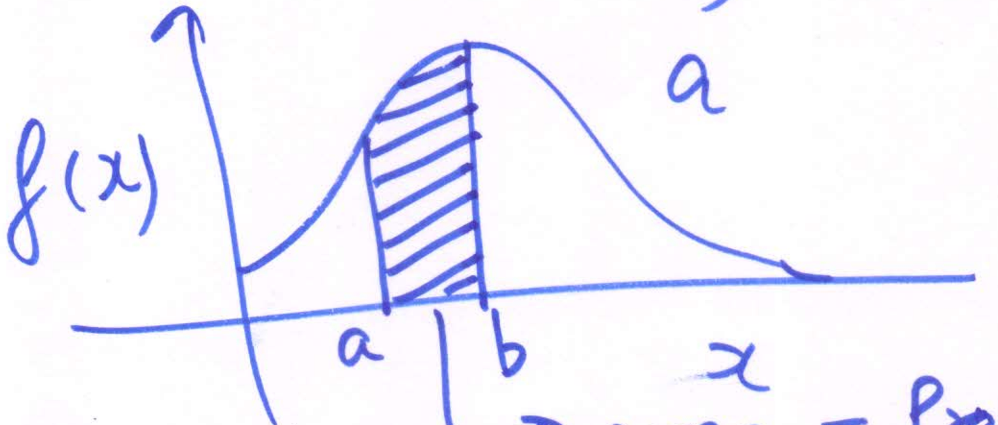
$$P(X \in B) = \int_B f(x)\,dx,$$

$B \subseteq R$, any subset of R.

f: probability density function.

$\Rightarrow B = [a, b]$ ~~ff~~

$$P(a \le x \le b) = \int_a^b f(x)\,dx$$



$\hookrightarrow$ area = Probability

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

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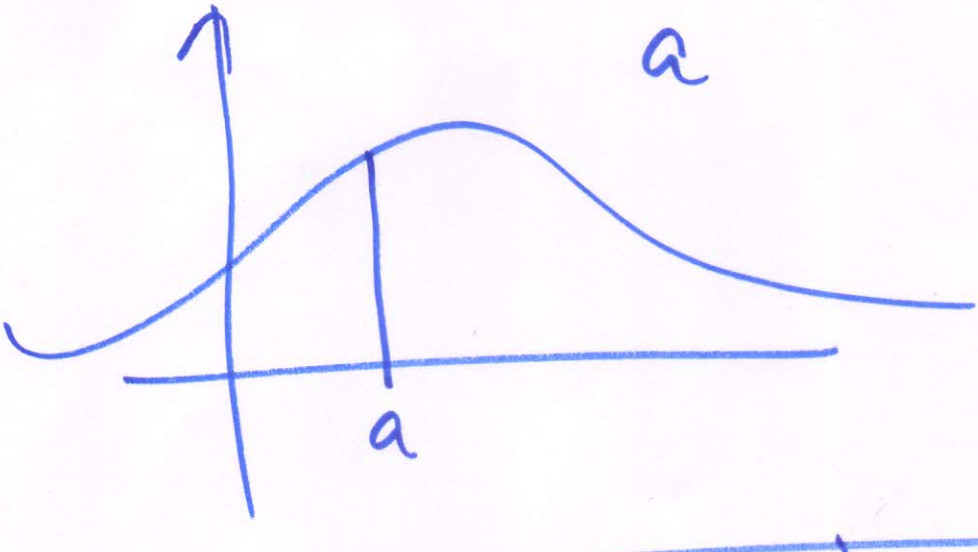
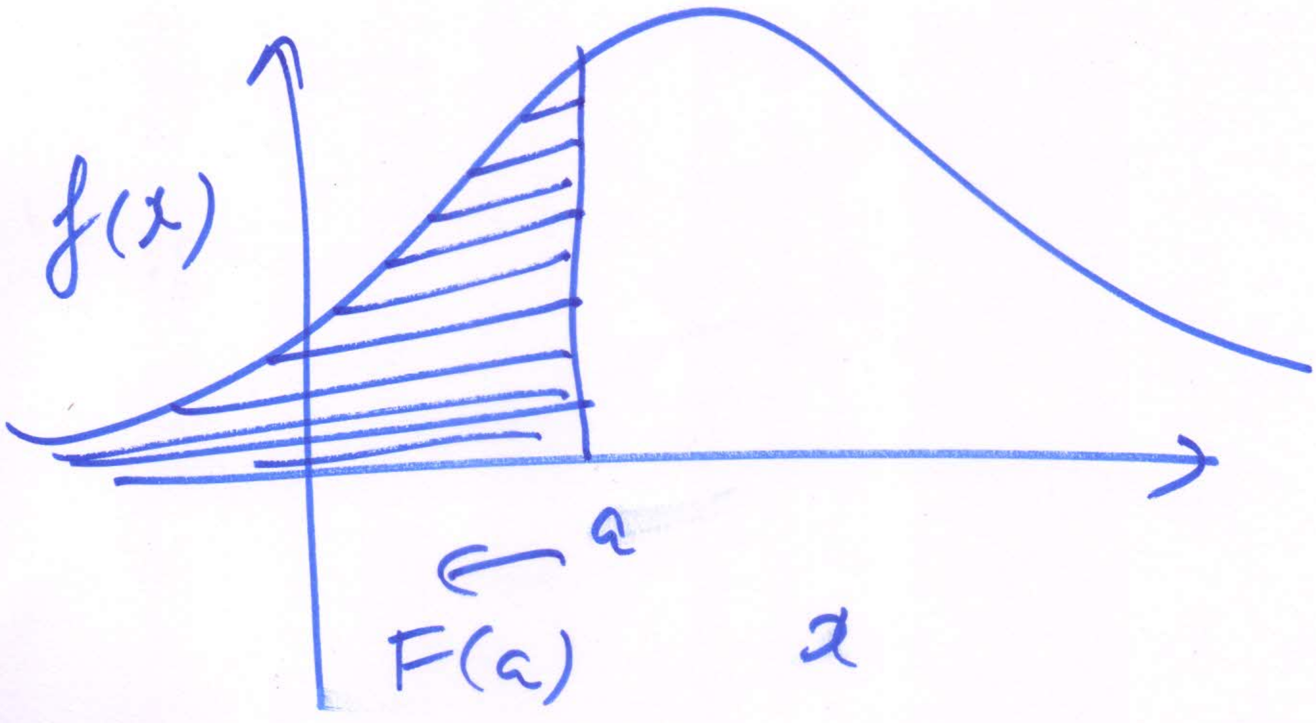
if $b = a \rightarrow \underset{0}{\underset{\shortparallel}{P(X=a)}}$

$$P(a \le X \le a) = \int_a^a f(x)\,dx = 0$$



$$\underset{\underset{F(a)}{\shortparallel}}{\boxed{P(X \le a) = P(X < a)}} + \underset{\underset{0}{\shortparallel}}{P(X=a)}$$

$F(a)$ probability distribution function $= \int_{-\infty}^{a} f(x)\,dx$



f(x)
a
F(a)
x