Lecture - 16

insem 1

1. r = no. of red socks
b = no. of blue socks. = even

$\{0, 2, 4, 6, 8, \ldots\}$

Choosing 2 socks at random.

$P(\text{both are red}) = \frac{1}{2}$.

$$\frac{\binom{r}{2}}{\binom{r+b}{2}} = \frac{1}{2}$$

$$\frac{r(r-1)}{(r+b)(r+b-1)} = \frac{1}{2}$$

$2r^2 - 2r = r^2 + rb - r + rb + b^2 - b$ (2)

$r^2 - r(1+2b) + (b - b^2) = 0$

$b = 0$,

$b = 2$,

$b = 4$,

$b = 6$, $r = 15$

21

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2. X = no. of times you throw a dice to get a 6

$X = \{1, 2, 3, 4, \ldots\}$

$P \quad \frac{1}{6}, \frac{5}{6}\frac{1}{6}, \left(\frac{5}{6}\right)^2\frac{1}{6}, \ldots$

$E(X) = 1\left(\frac{1}{6}\right) + 2\left(\frac{5}{6}\right)\left(\frac{1}{6}\right) + 3\left(\frac{5}{6}\right)^2\left(\frac{1}{6}\right) + 4\left(\frac{5}{6}\right)^3\left(\frac{1}{6}\right) + \ldots$

$\left(\frac{5}{6}\right)^0 = 6$

3. D_1: 1 dice shows 1
D_2: " 2
⋮
D_6: " 6

} 6 mutually exclusive & exhaustive. (3)

(C): coin shows a head each time.

$$P(D_5|C) = \frac{P(C|D_5)\,P(D_5)}{P(C|D_1)\,P(D_1) + \cdots + P(C|D_6)\,P(D_6)}$$

$$= \frac{(1/2)^5}{\frac{1}{2} + \frac{1}{2}\cdot\frac{1}{2} + \frac{1}{2^3} + \cdots + \frac{1}{2^6}}$$

$$= \frac{2}{6.3}$$

(9). (4)

largest value of s satisfying

$$\sum p_i < \frac{s}{s+1} = \frac{100}{103}$$

Call this s^*, $s^* = 3$

$\Rightarrow \underline{s^*+1}$ is the no. of items that you stock

$\frac{2}{3}$, $\frac{2}{3} + \frac{2}{9}$, $\frac{2}{3} + \frac{2}{9} + \frac{2}{27}$ less

$\frac{2}{3} + \frac{2}{9} + \frac{2}{27} + \frac{2}{81}$

max

$s = 3$, 4 marks

$s = 4$, 5 marks.

(5)

F A F

f > a

f a f + 2f a (1-f)

a f a + 2f a (1-a) more

(5)

A F A

X = no. of games you win

→ $X \geq 2$ wrong

→ $E(X) \geq 2$ wrong

Uniform random variable.

$$f(x) = \begin{cases} 1 & 0 \le x \le 1 \\ 0 & \text{otherwise.} \end{cases}$$

$$f(x) = \begin{cases} \dfrac{1}{b-a} & a \le x \le b \\ 0 & \text{otherwise.} \end{cases}$$

2 6

$$P\left(X = \frac{a+b}{2}\right) = P(X=4) = 0$$

Cumulative distribution function

$$F_X(m) = P(X \le m) =$$

$$F_X(m) = P(X \le m) = \begin{cases} 0 & m \le a \\ \dfrac{m-a}{b-a} & a \le m \le b \\ 1 & m \ge b \end{cases}$$

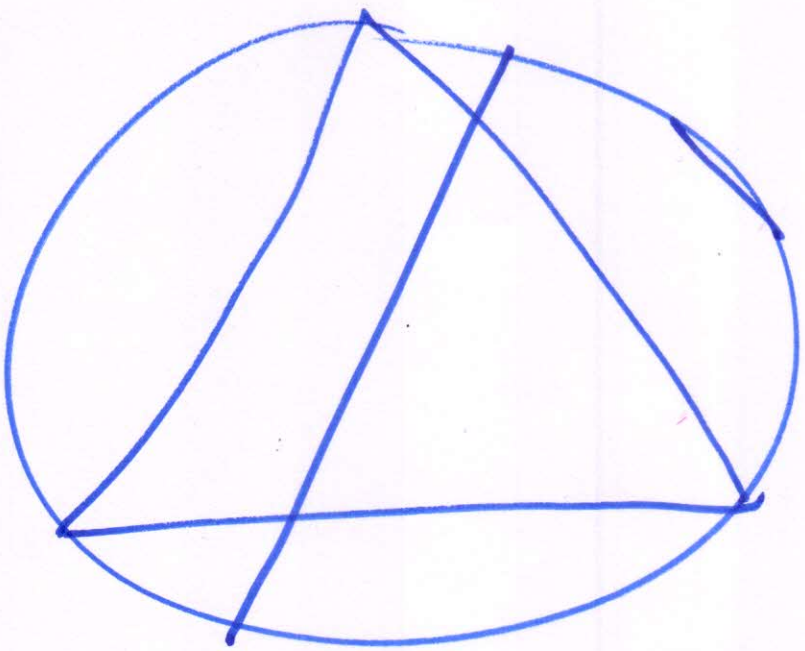
$$F_X(m) = \int_{-\infty}^{m} f(x)\,dx =$$

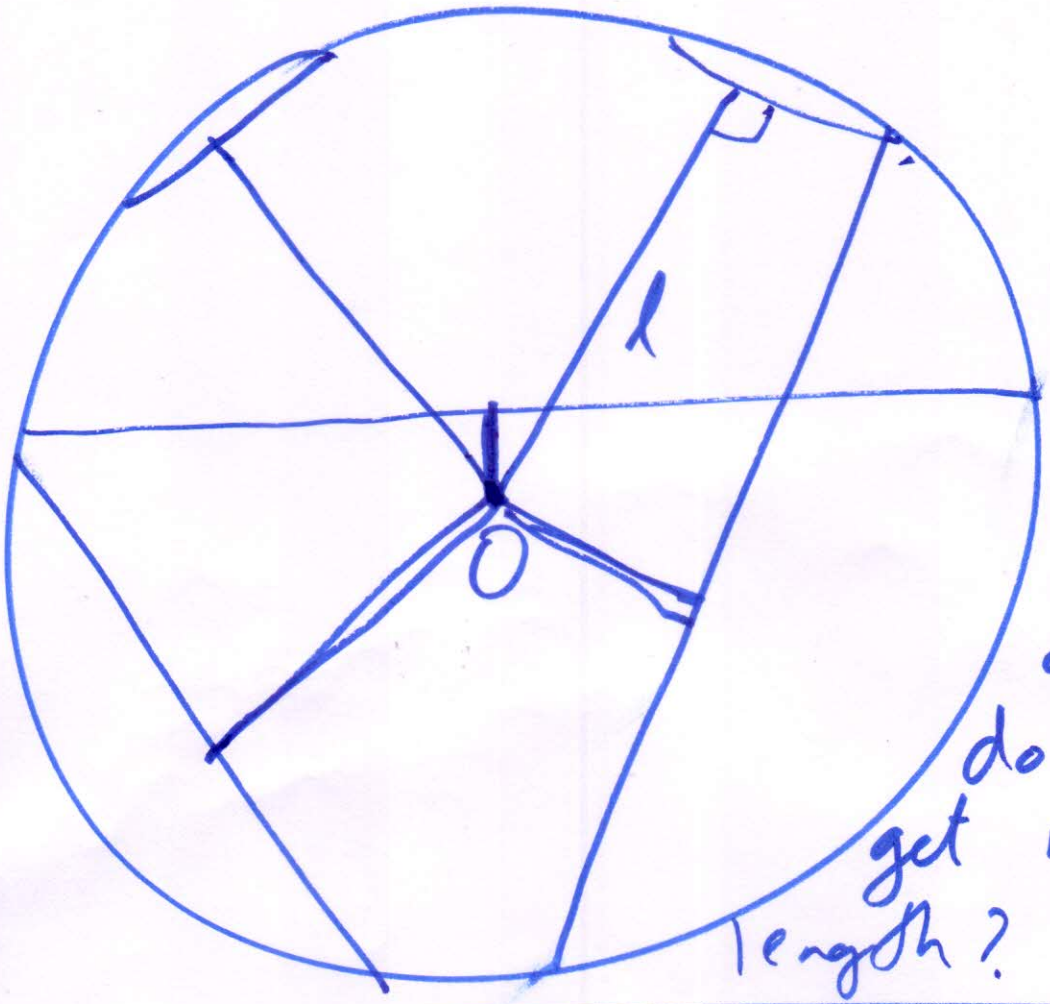
$$\int_{-\infty}^{a} f(x)\,dx + \int_{a}^{m} f(x)\,dx$$

$$\| \qquad \|$$

$$0 \qquad \frac{1}{b-a}(m-a)$$

Q: Consider a random chord of a circle. What is the probability that the length of the chord is greater than the side of the equilateral triangle inscribed in the same circle?

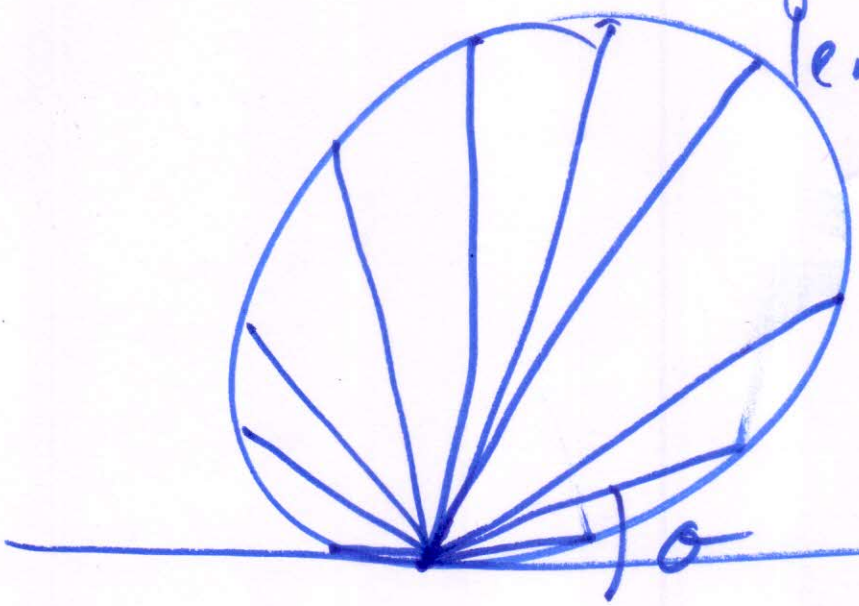




l
o to r

For what values of l do you get more length?

For which values of θ will the length



0 to 180