1. **Let** $f(x,y,z) = \sqrt{x^2+y^2+z^2}$. **Find** $\vec{\nabla} f$. **Find the rate of change of** f **at the point** $(1,1,0)$ **along a direction specified by the unit vector** $\frac{1}{\sqrt{2}}(\hat{\mathbf{i}} - \hat{\mathbf{j}})$.

2. **Let** $\vec{\mathbf{r}}$ **be the separation vector from a fixed point** (x', y', z') **to the point** (x,y,z). **Show that**
   (a) $\vec{\nabla}(1/r) = -\hat{\mathbf{r}}/r^2$
   (b) Evaluate $\vec{\nabla}(r^n)$

3. **Find the gradient of the function** $f(\vec{\mathbf{r}}) = \sin(\vec{\mathbf{k}} \cdot \vec{\mathbf{r}})$ **where** $\vec{\mathbf{k}}$ **is a fixed vector. Why do you think is the direction of gradient vector fixed in space?**

4. **A real square matrix** M **is orthogonal if** $M^{-1} = M^T$. **Using the fact that the magnitude of a vector doesn't change under rotation prove that a rotation matrix is orthogonal.**

5. *This question tries to give an idea of what a scalar quantity is.*
   The electric potential at a point on a horizontal plate with respect to a given coordinate system is given as $V(x,y) = xy$. **If someone work with a coordinate system that is rotated by** 45°, **the new coordinates** (x', y') **are given in terms of the old ones as** $x' = \frac{x+y}{\sqrt{2}}$ **and** $y' = \frac{y-x}{\sqrt{2}}$. **Let's write this as** $\vec{r'} = R\vec{r}$. **Potential is a scalar quantity. If** $V'(x', y')$ **is the functional form of the potential function in the new coordinate system then** $V'(x', y') = V(x,y)$.

   (a) Find the form of the function $V'(x', y')$.
   (b) Verify that $\vec{\nabla}' V' = R\vec{\nabla} V$, **i.e., components of a gradient transform as a vector quantity.**

6. **Let**
$$D = \begin{pmatrix} \frac{\partial A_x}{\partial x} & \frac{\partial A_y}{\partial x} \\ \frac{\partial A_x}{\partial y} & \frac{\partial A_y}{\partial y} \end{pmatrix}$$
   Under a rotation of the coordinate system
$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = R \begin{pmatrix} x \\ y \end{pmatrix}$$

show that

$$D' = \begin{pmatrix} \frac{\partial A'_x}{\partial x'} & \frac{\partial A'_y}{\partial x'} \\ \frac{\partial A'_x}{\partial y'} & \frac{\partial A'_y}{\partial y'} \end{pmatrix} = RDR^T$$