CT 203: Signals and Systems
Tutorial on Fourier Transform and Hilbert Transform
(Week of October 02, 2011)

1. Let $g(t) = Ae^{-bt}u(t)$. What are the range of frequencies that would contain $x\%$ of the total energy of $g(t)$?

2. Using the duality theorem compute the FT of $z(t) = A\text{Sinc}(2Wt)$.

3. Denote $x(t)$ and $\hat{x}(t)$ as the signal and its Hilbert transform (HT). With this notation prove the following properties of HT: (a) $x(t)$ and $\hat{x}(t)$ have the same amplitude spectrum, (b) $-x(t)$ is the Hilbert transform of $\hat{x}(t)$ and (c) $x(t)$ and $\hat{x}(t)$ are orthogonal to each other.

4. Compute the Hilbert transform of the causal rectangular pulse $x(t) = A\Pi\left(\frac{t-\frac{\tau}{2}}{\tau}\right)$ (where as before $\Pi\left(\frac{t}{\tau}\right)$ denotes the rectangular pulse of duration τ seconds centered around zero).