CS201

* **Physic concepts:-**

m: 10^{-30} kg 10^{-3} kg - 10^{2} kg 10^{53} kg

L: 10^{-35} m 10^{-4} m - 10^{4} m 10^{25} m

T: 10^{-43} s 10^{-1} s - 10^{3} s 10^{17} s

The scale in which we leave

→ $u = \dfrac{v+u'}{1+(u'\cdot v)/c^2}$, v = speed of reference frame
u' = velocity of obj. w.r.t moving frame
u = velocity of obj. w.r.t. fixed frame.

→ $m = \dfrac{m_0}{\sqrt{1-(v/c)^2}}$, m_0 = rest mass

→ $l = l_0 * \sqrt{1-(v/c)^2}$, l_0 = proper length.

→ $t = \dfrac{t_0}{\sqrt{1-(v/c)^2}}$, time dilation to proper time.

→ $l_p = \sqrt{\dfrac{Gh}{c^3}}$, l_p = plank's length

→ $t_p = \dfrac{l_p}{c}$, t_p = plank's time

→ $\lambda = \frac{h}{p} = \frac{h}{mv}$, λ = de-Broglie wavelength.

→ $m_{eff} = \frac{E}{c^2} = \frac{h\nu}{c^2} = \frac{h}{\lambda c}$

→ $V_{esc} = \sqrt{\frac{2GM}{r_0}}$

→ $R_s = \frac{2GM}{c^2}$, R_s = schwarzchild radius / radius of event horizon

→ $\frac{PV}{N} = \frac{R}{N_A} \times T = K_B T$, $K_B = \frac{R}{N_A}$ = boltzman constant.

→ Entropy $S = K_B \ln \Omega$, Ω = multiplicity
= measure of disorder.

Page No.:
Date:

* Mathematical Theory :-

⇒ First order automomous systems :-

→ $\frac{dx}{dt} = f(x)$, $x \equiv x(t) \Rightarrow x(t_0) = x_0$

→ $x = x_0 + \left.\frac{dx}{dt}\right|_{t_0} (t-t_0) + \frac{1}{2!} \left.\frac{d^2x}{dt^2}\right|_{t_0} (t-t_0)^2 + \ldots$

(Taylor expansion)

→ Taylor expansion in terms of $f(x)$:-

$$x = x_0 + f(x_0)(t-t_0) + \frac{1}{2}\left.\left(f \cdot \frac{df}{dx}\right)\right|_{x_0} (t-t_0)^2 + \frac{1}{6}\left.\left[f\left(\frac{df}{dx}\right)^2 + f^2 \cdot \frac{d^2f}{dx^2}\right]\right|_{x_0} (t-t_0)^3 + \ldots$$

→ Euler Method :-

$x \simeq x_0 + f(x_0) \cdot (t-t_0)$, $x_{n+1} \simeq x_n + f(x_n) \cdot \Delta t$

→ Example :-

1. $\frac{dx}{dt} = \pm ax \Rightarrow x = x_0 \cdot e^{\pm at}$

2. $\frac{dx}{dt} = a - bx \Rightarrow x = x_0(1 - e^{-bt}) = \frac{a}{b}(1 - e^{-bt})$

Terminal value $= x_0 = a/b$

at $t \simeq 1/b$, $x \simeq 0.63 x_0$

Page No:
Date:

2. $\frac{dx}{dt} = a - bx \Rightarrow x = x_0(1-e^{-bt}) \Rightarrow x = \frac{a}{b}(1-e^{-bt})$

Terminal value $= x_0 = a/b$

at $t \approx 1/b$, $x \approx 0.63x_0$

3. $\frac{dx}{dt} = a + bx \Rightarrow x = \frac{a}{b}(e^{bt} - 1)$

$\Rightarrow$ First-order non-linear autonomous systems :-

1. $\frac{dx}{dt} = ax - bx^2 \Rightarrow x = \frac{K}{1 + c^{-1}.e^{-at}}$, $K = a/b$ = saturation, $C = \frac{x_0}{K - x_0}$

$$\Rightarrow x = \frac{x_0 e^{at}}{1 + \frac{x_0}{K}(e^{at} - 1)}$$

2. $\frac{dx}{dt} = a - bx^2 \Rightarrow x = \sqrt{\frac{a}{b}} \tanh(\sqrt{ab}\, t)$

Saturation $= x = \sqrt{a/b}$

$\Rightarrow$ Numerical error in integration:-

$\rightarrow$ $\frac{dx}{dt} = \lambda x \Rightarrow x = e^{\lambda x}$, $\lambda < 0$

We must take Δt such that

$$\boxed{0 < \Delta t < \frac{-2}{\lambda}}$$

⇒ Numerical integration of non-autonomous equation :-

→ $\frac{dx}{dt} = f(x, t)$, $x(t_0) = x_0$

→ Taylor expansion :-

$$x = x_0 + f(t_0, x_0) \cdot (t - t_0) + \frac{1}{2!} \left[\frac{\partial f}{\partial t} + f \frac{\partial f}{\partial x} \right]_{(t_0, x_0)} (t - t_0)^2$$

$+ \ldots$

→ Euler method :-

$$x_{n+1} = x_n + f(t_n, x_n) \cdot \Delta t$$

Page No.:
Date:

* **Physical system :-**

⇒ Radio activity :-

→ $\frac{dx}{dt} = -\lambda x$, $\lambda > 0$, λ = decay constant

⇒ $x = x_0 e^{-\lambda t}$ ⇒ $t = \frac{1}{\lambda} \ln\left[\frac{x_0}{x}\right]$

→ $T_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda}$

i) α- decay :-

$${}^{A}_{Z}X \longrightarrow {}^{A-4}_{Z-2}Y + {}^{4}_{2}He$$

ii) β- decay :-

$${}^{A}_{Z}X \longrightarrow {}^{A}_{Z+1}Y + {}^{0}_{-1}e$$ OR

$${}^{1}_{0}n \longrightarrow {}^{1}_{1}P + {}^{0}_{-1}e + \bar{\nu}$$, $\bar{\nu}$ = anti-neutrino

iii) Positron emission :-

$${}^{1}_{1}P \longrightarrow {}^{1}_{0}n + {}^{0}_{1}e + \nu$$, ν = neutrino

⇒ Nuclear reaction :-

i) Fission :-

$${}^{235}_{92}U + {}^{1}_{0}n \longrightarrow {}^{144}_{56}Ba + {}^{89}_{36}Kr + 3\,{}^{1}_{0}n$$

→ ${}^{14}_{6}C$ → radio carbon

$${}^{14}_{7}N + {}^{1}_{0}n \longrightarrow {}^{14}_{6}C + {}^{1}_{1}P$$

$$ {}^{14}_{6}C \longrightarrow {}^{14}_{7}N + {}^{0}_{-1}e \quad , \quad T_{1/2} = 5600 \text{ yr.} $$

$\rightarrow$ $t - t_0 = \dfrac{T_{1/2}}{\ln 2} \ln\left(\dfrac{x_0}{x}\right) = \dfrac{T_{1/2}}{\ln 2} \ln\left[\dfrac{\dot{x}(t_0)}{\dot{x}(t)}\right]$

$\Rightarrow$ Q-R-C circuit :-

$\rightarrow$ $\dfrac{dQ}{dt} = \dfrac{V_0}{R} - \dfrac{Q}{RC}$ $\Rightarrow$ $Q = Q_0(1 - e^{-t/RC})$, $\underbrace{Q_0 = V_0 \cdot C}_{\text{limiting value}}$

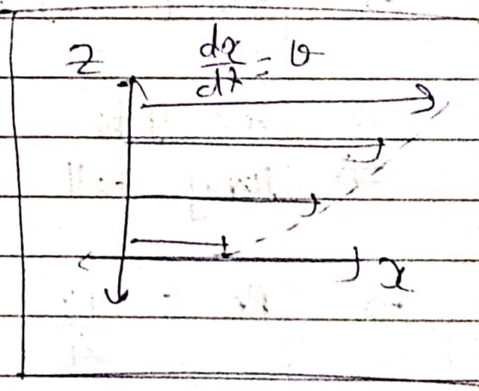
$\Rightarrow$ Viscoelasticity :-

i) Elastic property :-

$\rightarrow$ σ (stress) $= \dfrac{F}{A} \propto \epsilon$, $\epsilon = \dfrac{\Delta l}{l_0}$

$\therefore \sigma \propto \epsilon \Rightarrow \boxed{\sigma = Y \cdot \epsilon}$

ii) Viscocity property :-



$\rightarrow$ drag force $F \propto A \dfrac{dv}{dz}$

$\Rightarrow$ σ (viscus stress) $= \dfrac{F}{A} \propto \dfrac{dv}{dz}$

$$ \therefore \boxed{\sigma = \eta \frac{dv}{dz}} $$

$$ \therefore \sigma = \eta \frac{d}{dz}\left(\frac{dx}{dt}\right) $$

$\therefore \boxed{\sigma = \eta \dfrac{d\epsilon}{dt}}$ where $\epsilon = \dfrac{dx}{dz}$

Page No.:
Date:

→ Viscoelastic system;

$$\boxed{\sigma = Y\epsilon + \eta\frac{d\epsilon}{dt}}$$

$$\Rightarrow \frac{d\epsilon}{dt} = \frac{\sigma}{\eta} - \frac{Y\epsilon}{\eta} \Rightarrow \epsilon = \frac{\sigma}{Y}\left(1 - e^{-\frac{Y t}{\eta}}\right)$$

⇒ Object falling through a liquid coloum:-

→ $$m\frac{dv}{dt} = mg - \rho_l V g - Kv$$

$$\Rightarrow v = v_T\left(1 - e^{-Kt/m}\right), \quad v_T = \frac{m\bar{g}}{K}, \quad \bar{g} = g\left(1 - \frac{\rho_l}{\rho}\right)$$

$$, \quad K = 6\pi\eta r$$

→ $$z = v_T \cdot t + v_T t_0\left(e^{-t/t_0} - 1\right)$$

→ $$\zeta = (\tau - 1) + e^{-\tau}, \quad \zeta = \frac{z}{v_T t_0}, \quad \tau = \frac{t}{t_0}$$

→ $v_T = \bar{g} \cdot t_0$

⇒ Long fall through air:-

→ $$Re = \frac{lv}{\mu} = \frac{\rho l v}{\eta}$$

→ $D \propto v^r$,
$Re \sim 10 \Rightarrow r = 1$
$Re \sim 10^3 \Rightarrow r = 2$
$10 < Re < 10^3 \Rightarrow r$ is uncertain

→ $$m\frac{dv}{dt} = mg - Kv^2$$

$$\Rightarrow v = \underbrace{\sqrt{\frac{mg}{K}}}_{\text{terminal value}} \tanh\left(\sqrt{\frac{Kg}{m}}\, t\right)$$

⇒ Bicycle motion :-

i) Without air resistance :-

→ $m\frac{dv}{dt} = \frac{1}{v}\frac{dE}{dt}$

⇒ $v^2 = v_0^2 + \frac{2Pt}{m}$ (no limit of v)

ii) With air resistance

→ Drag force $D \approx -\frac{1}{2}\rho A v^2$ OR

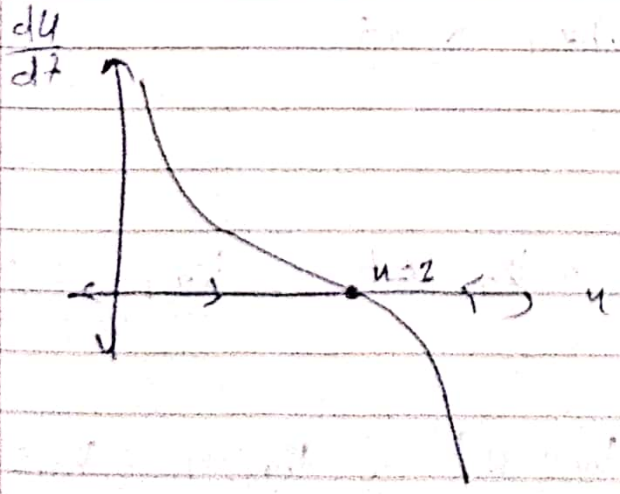
$D = -b\rho A v^2$

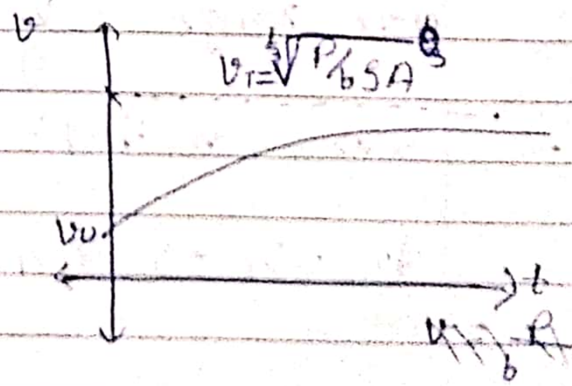
→ $m\frac{dv}{dt} = \cancel{\frac{P}{v}}\ \frac{P}{v} - b\rho A v^2$

⇒ ~~v u dv~~

⇒ $u\frac{du}{dT} = 1-u^3$, $T = \frac{ab\rho A}{m}t$

$a^3 = \frac{P}{b\rho A}$, $u = \frac{V}{a}$





Page No.:
Date:

⇒ Projectile motion :-

⇒ Euler method :-

→ $v_{i+1} = v_i + \frac{P}{m v_i} \Delta t - \frac{b \delta A}{m} \cdot v_i^2 \Delta t$

⇒ Projectile motion :-

i) Without drag :-

→ $\frac{d^2x}{dt^2} = 0$, $\frac{d^2y}{dt^2} = -g$

→ $\frac{dx}{dt} = v_x = v_0 \cos\alpha$ $\frac{dy}{dt} = v_y = v_0 \sin\alpha - gt$

→ $x = v_x t = v_0 \cos\alpha\, t$, $y = v_0 \sin\alpha t - \frac{gt^2}{2}$

→ $\boxed{y = (\tan\alpha) x - \frac{g x^2}{2 v_0^2 \cos^2\alpha}}$

→ Range $= x = \frac{2 v_0^2}{g} \cdot \tan\alpha \cos^2\alpha = \frac{v_0^2}{g} \cdot \sin(2\alpha)$

Max range $= \frac{v_0^2}{g}$ when $\alpha = 45^\circ$.

→ Euler's method :-

$\frac{dx}{dt} = v_x \Rightarrow x_{i+1} = x_i + v_{x,i} \Delta t$, $v_{x,i+1} = v_{x,i}$

$\frac{dy}{dt} = v_y \Rightarrow y_{i+1} = y_i + v_{y,i} \cdot \Delta t$, $v_{y,i+1} = v_{y,i} - g\Delta t$

ii) With drag :-

→ drag force $= D = -Bv^2$

$D_x = -B\sqrt{v_x^2+v_y^2} \cdot v_x$, $D_y = -B\sqrt{v_x^2+v_y^2} \cdot v_y$

→ $m\dfrac{dv_x}{dt} = -B\sqrt{v_x^2+v_y^2} \cdot v_x$, $\dfrac{dx}{dt} = v_x$

$m\dfrac{dv_y}{dt} = -mg - B\sqrt{v_x^2+v_y^2} \cdot v_y$, $\dfrac{dy}{dt} = v_y$

→ Euler method :-

- $v_{x,i+1} = v_{x,i} - \left(\dfrac{B}{m}\sqrt{v_{x,i}^2+v_{y,i}^2}\right) \cdot v_{x,i} \cdot \Delta t$

- $v_{y,i+1} = v_{y,i} - g\Delta t - \left(\dfrac{B}{m}\sqrt{v_{x,i}^2+v_{y,i}^2}\right) \cdot v_{y,i} \cdot \Delta t$

- $x_{i+1} = x_i + v_{x,i} \cdot \Delta t$

- $y_{i+1} = y_i + v_{y,i} \cdot \Delta t$

⇒ Vertical variation of air density:-

→ $\Delta P = -\rho g \cdot dz$ $\quad (P \downarrow \text{ as } z \uparrow)$

→ $P = \dfrac{\rho k_B \cdot T}{\bar{m}}$, $\rho = \dfrac{N\bar{m}}{V}$

$\Delta P = \Delta\rho \left(\dfrac{k_B T}{\bar{m}}\right) \Rightarrow \dfrac{\Delta \rho}{\rho} = \dfrac{-\bar{m}g}{k_B T} dz$

→ $\rho = \rho_0 \cdot \exp\left(\dfrac{-\bar{m}gz}{k_B T}\right) \simeq P_0 \exp\left(\dfrac{-mgz}{k_B T}\right)$

$\cancel{\simeq \rho_0\left(1 - \dfrac{\bar{m}gz}{k_B T}\right)} \simeq P_0\left[1 - \dfrac{\bar{m}gz}{k_B T}\right]$

⇒ Bernoulli eqn:-

$\dfrac{v^2}{2} + \dfrac{P}{\rho} + gz = \text{constant}$