Problem Set 1

1. Find the maxima and minima, if any, of the function
   $f(x) = 4x^3 - 18x^2 + 27x - 7$. x=3/2, point of inflection

2. Verify whether the following matrices is positive definite, negative definite, or indefinite.

   (a) $\begin{pmatrix} 3 & 1 & -1 \\ 1 & 3 & -1 \\ -1 & -1 & 5 \end{pmatrix}$ +ve

   (b) $\begin{pmatrix} 4 & 2 & -4 \\ 2 & 4 & -2 \\ -4 & -2 & 4 \end{pmatrix}$ +ve semidefinite

   (c) $\begin{pmatrix} -1 & -1 & -1 \\ -1 & -2 & -2 \\ -1 & -2 & -3 \end{pmatrix}$ -ve

3. Determine whether each of the following quadratic form is positive definite, negative definite, or neither

   (a) $f = x^2 - y^2$ indefinite

   (b) $f = 4xy$ indefinite

   (c) $f = x^2 + 2y^2$ +ve

   (d) $f = -x^2 + 4xy + 4y^2$ indefinite

   (e) $f = -x^2 + 4xy - 9y^2 + 2xz + 8yz - 4z^2$ indefinite

4. Match the following equations and their characteristics.

(a) $f = 4x - 3y + 2$	Relative maximum at $(1, 2)$ c
(b) $f = (2x - 2)^2 + (x - 2)^2$	Saddle point at origin d
(c) $f = -(x - 1)^2 - (y - 2)^2$	No minimum a
(d) $f = xy$	Inflection point at origin e
(e) $f = x^3$	Relative minimum at $(1, 2)$ b

5. State whether each of the following functions is convex, concave, or neither.

   (a) $f = -2x^2 + 8x + 4$ concave

(b) $f = x^2 + 10x + 1$ convex

(c) $f = x^2 - y^2$ neither

(d) $f = -x^2 + 4xy$ neither

(e) $f = e^{-x},\ x > 0$ convex

(f) $f = \sqrt{x},\ x > 0$ concave

(g) $f = xy$ neither

(h) $f = (x-1)^2 + 10(y-2)^2$ convex

6. Find the third-order Taylor's series approximation of the function $f(x, y, z) = y^2 z + xe^z$ at the point $(1, 0, -2)$.

7. Find the dimensions of a closed cylindrical soft drink can, that can hold soft drink of volume V for which the surface area (including top and bottom) is minimum.

8. An open rectangular box is to be manufactured from a given amount of sheet metal (area S). Find the dimensions of the box to maximize the volume.

9. Find the dimensions of a straight beam of circular cross section that can be cut from a conical log of height h and base radius r to maximize the volume of the beam.

10. Find the value of x, y and z that maximize the function $f(x, y, z) = \frac{6xyz}{x+2y+2z}$, where x, y and z are restricted by the relation $xyz = 16$.

11. Minimize

$$f(X) = \frac{1}{2}(x_1^2 + x_2^2 + x_3^2)$$

subject to the constraints

$$g_1(X) = x_1 - x_2 = 0$$
$$g_2(X) = x_1 + x_2 + x_3 = 1.$$