

Large K-map

- To minimize the Boolean function with variables more than four.

Ex

$$F(A, B, C, D, E)$$

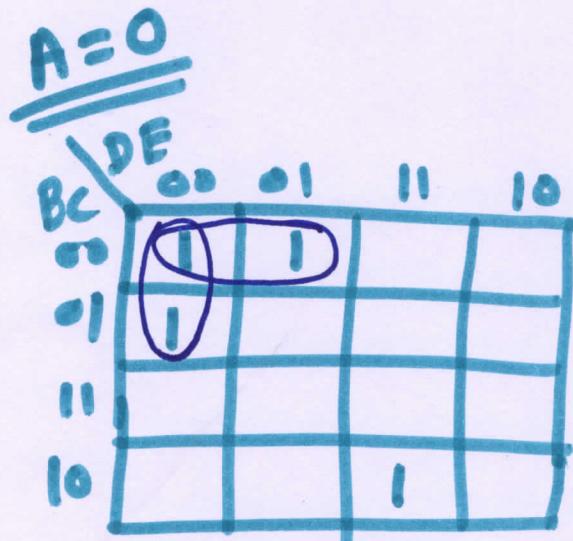
$$= \sum m(0, 1, 4, 11, 17, 21, 27)$$

Proc.

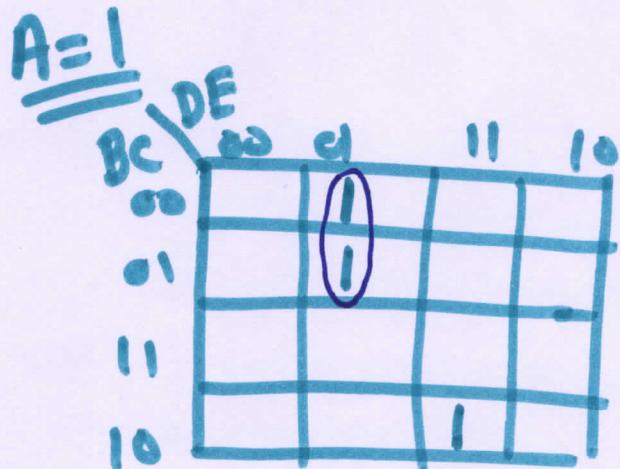
$$F(A, B, C, D, E)$$

$$= \overline{A} \cdot F(\overline{0}, B, \overline{C}, D, E) +$$

$$A \cdot F(1, B, \overline{C}, D, E)$$



$$\overline{B}\overline{D}\overline{E} + \overline{B}\overline{C}\overline{D} + B\overline{C}DE$$



$$\overline{B}\overline{D}E + B\overline{C}DE$$

So,

$$F(A, B, C, D, E)$$

$$= \bar{A} \cdot (\bar{B}\bar{D}\bar{E} + \bar{B}\bar{C}\bar{D} + B\bar{C}DE) +$$

$$A \cdot (\bar{B}\bar{D}E + B\bar{C}DE)$$

$$= \bar{A}\bar{B}\bar{D}\bar{E} + \bar{A}\bar{B}\bar{C}\bar{D} + \bar{A}\underline{\bar{B}\bar{C}DE}$$
$$+ A\bar{B}\bar{D}E + A\underline{\bar{B}\bar{C}DE}$$

$$= \bar{A}\bar{B}\bar{D}\bar{E} + \bar{A}\bar{B}\bar{C}\bar{D} + B\bar{C}\bar{D}E$$
$$+ A\bar{B}\bar{D}E$$

Quine - McCluskey Method

$F(A, B, C, D, \bar{E})$

$$= \sum m(2, 3, 7, 9, 11, 13) +$$

$$\sum d(1, 10, 15)$$

Step: 1

| min-terms | A B C D |
|-----------|-----------|
| 1 | 0 0 0 1 |
| 2 | 0 0 1 0 |
| 3 | 0 0 1 1 - |
| 7 | 0 1 1 1 * |
| 9 | 1 0 0 1 - |
| 10 | 1 0 1 0 - |
| 11 | 1 0 1 1 * |
| 13 | 1 1 0 1 * |
| 15 | 1 1 1 1 |

Step: 2

| Group | Minterms | A B C D |
|-----------------------|---------------|---------------------------------|
| A (no one) | | |
| B (one one (1)) | 1 2 | 0 0 0 1 0 0 1 0 - |
| C (two ones) | 3 9 10 | 0 0 1 1 - 1 0 0 1 1 0 1 0 |
| D (Three Ones) | 7 11 13 | 0 1 1 1 1 0 1 1 1 1 0 1 |
| E (Four Ones) | 15 | 1 1 1 1 |

Step : 3

| Group | Minterms | A B C D |
|-------|--------------------------------------|---|
| B-C | 1-3 1-9 2-3 2-10 | 0 0 -1- - 0 0 1 = 0 0 1 0 - - 0 1 0 |
| C-D | 3-7 3-11 9-11 9-13 10-11 | 0 - 1 1 - - 0 1 1 = 1 0 - 1 - 1 - 0 1 1 0 1 - - |
| D-E | 7-15 11-15 13-15 | - 1 1 1 1 - 1 1 ✓ 1 1 - 1 |

Step: 4

| Group | Minterms | A B C D |
|---------|----------------------|---------|
| BC - CD | 1-3-9-11 | - 0 - 1 |
| | 2-3-10-11 | - 0 1 - |
| CD - DE | 3-7-11-15 | |
| | 3-7-11-15 | - - 1 1 |
| | 9-11-13-15 | 1 - - 1 |

Step: 5

Prime Implicants

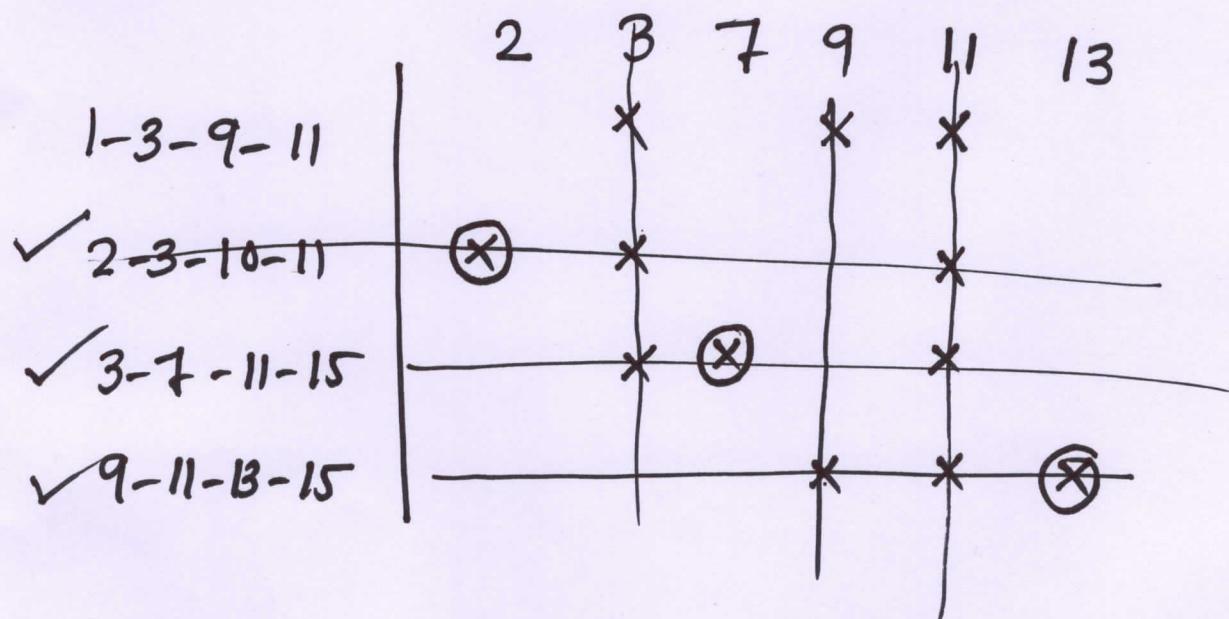
$$1-3-9-11 \Rightarrow \overline{B}D$$

$$2-3-10-11 \Rightarrow \overline{B}C$$

$$3-7-11-15 \Rightarrow CD$$

$$9-11-13-15 \Rightarrow AD$$

Essential Prime Implicants



$$F(A, B, C, D) = \overline{B}C + CD + AD$$

The Quine- McCluskey Algorithm

- The algorithm is used to minimize Boolean functions(especially, those are having many variables(≥ 4))
- We may write computer program for this algorithm.

Example:

$$F(A, B, C, D) = \sum m(0,1,2,8,10,11,14,15) + d(9,12,13)$$

Step 1: Write all the minterms and don't care terms in ascending order as follows:

| Minterms and don't care terms | A B C D |
|----------------------------------|---------|
| 0 | 0 0 0 0 |
| 1 | 0 0 0 1 |
| 2 | 0 0 1 0 |
| 8 | 1 0 0 0 |
| 9 | 1 0 0 1 |
| 10 | 1 0 1 0 |
| 11 | 1 0 1 1 |
| 12 | 1 1 0 0 |
| 13 | 1 1 0 1 |
| 14 | 1 1 1 0 |
| 15 | 1 1 1 1 |

Step 3: Combine two consecutive groups as following

| Group | Minterms | A B C D |
|-------|----------|---------|
| A-B | 0-1 ✓ | 0 0 0 - |
| | 0-2 | 0 0 - 0 |
| | 0-8 ✓ | - 0 0 0 |
| B-C | 1-9 ✓ | - 0 0 1 |
| | 2-10 | - 0 1 0 |
| | 8-9 ✓ | 1 0 0 - |
| | 8-10 | 1 0 - 0 |
| C-D | 8-12 | 1 - 0 0 |
| | 9-11 | 1 0 - 1 |
| | 9-13 | 1 - 0 1 |
| | 10-11 | 1 0 1 - |
| | 10-14 | 1 - 1 0 |
| | 12-13 | 1 1 0 - |
| D-E | 12-14 | 1 1 - 0 |
| | 11-15 | 1 - 1 1 |
| | 13-15 | 1 1 - 1 |
| | 14-15 | 1 1 1 - |

Step 2: Make groups as follows:

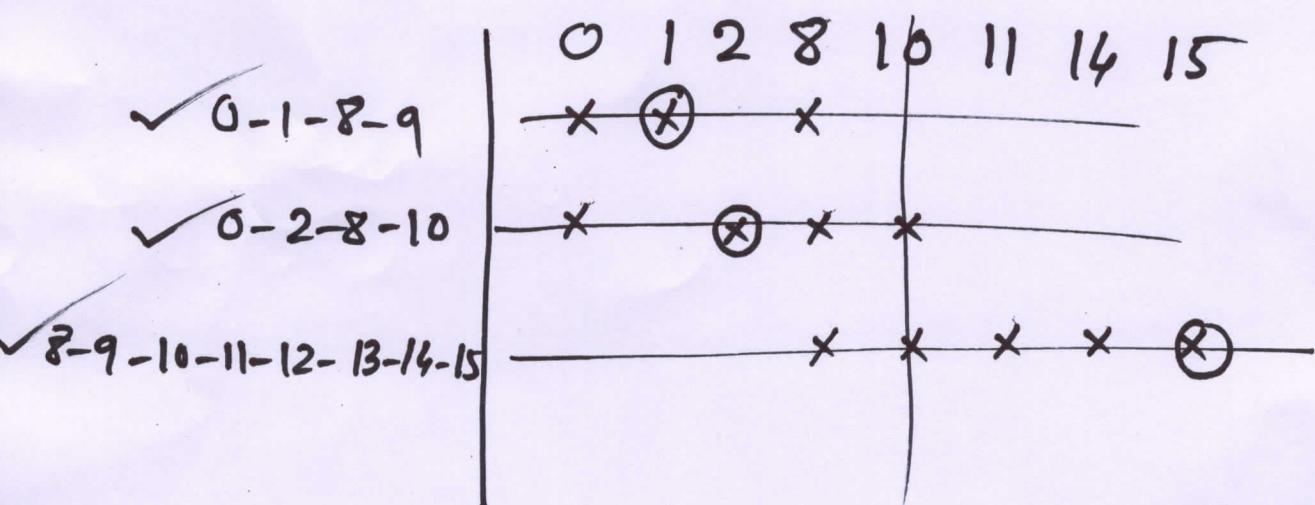
| Group | Minterms | A B C D |
|-----------------------|----------|---------|
| A (with no '1') | 0 | 0 0 0 0 |
| B (with one '1') | 1 | 0 0 0 1 |
| | 2 | 0 0 1 0 |
| | 8 | 1 0 0 0 |
| C (with two '1') | 9 | 1 0 0 1 |
| | 10 | 1 0 1 0 |
| | 12 | 1 1 0 0 |
| D (with three '1') | 11 | 1 0 1 1 |
| | 13 | 1 1 0 1 |
| | 14 | 1 1 1 0 |
| E (with four '1') | 15 | 1 1 1 1 |

Step 4: Continue the same procedure as you did in Step 3.

| Group | Minterms | A B C D |
|-----------------|------------------------|---------------------------------------|
| AB-BC (=ABC) | 0-1-8-9 | - 0 0 - → $\overline{B} \overline{C}$ |
| | 0-2-8-10 | - 0 - 0 → $\overline{B} \overline{D}$ |
| | 0 8 1 9 | - 0 0 |
| | 0 8 2 10 | 0 - 0 |
| BC-CD (=BCD) | 8-9-10-11 | 1 0 - - ✓ |
| | 8-9-12-13 | 1 - 0 - - ✓ |
| | 8 10 9 11 | 1 0 - - |
| | 8-10-12-14 | 1 - - 0 ✓ |
| | 8 12 9 13 | 1 - 0 |
| CD-DE (=CDE) | 9-11-13-15 | 1 - - 1 ✓ |
| | 9 13 11 15 | 1 - - 1 |
| | 10-11-14-15 | 1 - 1 - - ✓ |
| | 10 14 11 15 | 1 - 1 - |
| | 12-13-14-15 | 1 1 - - ✓ |
| | 12 14 13 15 | 1 1 - - |

Step 5: Continue the same procedure as you did in Step 4.

| Group | Minterms | A B C D |
|---------|---|-------------------------|
| ABC-BCD | | |
| BCD-CDE | 8-9-10-11-12-13-14-15 8-9-12-13-10-14-11-15 8-10-12-14-9-11-13-15 | 1 --- 1 --- 1 --- |
| | | $\} \rightarrow A$ |



$$F = \bar{B}\bar{C} + \bar{B}\bar{D} + A$$