

1. Find the equation of the tangent plane to the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ at the point (x_0, y_0, z_0) on the ellipsoid.
2. Prove that for any vector field \vec{A} , $\vec{\nabla} \cdot \vec{A}$ is a scalar.
3. Let $\vec{A} = \vec{\omega} \times \vec{r}$ where $\vec{\omega}$ is a fixed vector in space. Find $\vec{\nabla} \times \vec{A}$.
4. Find the divergence of the following:
 - (a) $\vec{A} = \hat{r}$,
 - (b) $\vec{A} = \frac{\hat{r}}{r}$ in 2 dimension
 - (c) $\vec{A} = \frac{\hat{r}}{r}$ in 3 dimension
 - (d) $\vec{A} = \frac{\hat{r}}{r^2}$ in 3 dimension. Plot this field.
 - (e) $\vec{A} = \frac{\hat{r}}{r^3}$ in 3 dimension
5. Find the curl of the following:
 - (a) $\vec{A} = y\hat{i} - x\hat{j}$
 - (b) $\vec{A} = \frac{1}{\sqrt{x^2+y^2}}(y\hat{i} - x\hat{j})$
 - (c) $\vec{A} = \frac{1}{x^2+y^2}(y\hat{i} - x\hat{j})$
 - (d) $\vec{A} = (x^2 + y^2)\hat{k}$
6. For any vector field \vec{A} and any scalar field F show that
 - (i) $\vec{\nabla} \cdot (\vec{\nabla} \times \vec{A}) = 0$;
 - (ii) $\vec{\nabla} \times (\vec{\nabla} F) = 0$.
7. Can we find a scalar function F such that $\vec{\nabla} F = y\hat{i} - x\hat{j}$?
What about $\vec{\nabla} F = \frac{1}{x^2+y^2}(y\hat{i} - x\hat{j})$?