Tutorial 1

1. In each case, find an open interval about x_0 on which the inequality $|f(x) - L| < \epsilon$ holds. Than give a value $\delta > 0$ such that for all x satisfying $0 < |x - x_0| < \delta$ the inequality $|f(x) - L| < \epsilon$ holds
   (a) $f(x) = \sqrt{x+1}, L = 1, x_0 = 0, \epsilon = 0.1$
   (b) $f(x) = x^2 - 5, L = 11, x_0 = 4, \epsilon = 1$
   (c) $f(x) = \frac{1}{x}, L = -1, x_0 = -1, \epsilon = 0.1$

2. Using (ϵ, δ) definition, show that
   (a) $\lim_{x\to 4} 9 - x = 5$
   (b) $\lim_{x\to -3} \frac{x^2-9}{x+3} = -6$
   (c) $\lim_{x\to 0} x \sin\frac{1}{x} = 0$

3. For

$$f(x) = \begin{cases} \sin\frac{1}{x}, & x > 0 \\ 0, & x \leq 0 \end{cases}$$

   (a) Do[illegible]$_{x\to 0^+}$ exist? Why or Why not?
   (b) D[illegible]im$_{x\to 0^-}$ exist? Why or Why not?

4. Use (ϵ, δ) approach to prove that
   (a) $\lim_{x\to 0^-} \frac{x}{|x|} = -1$
   (b) $\lim_{x\to 2^+} \frac{x-2}{|x-2|} = 1$

5. Using $\lim_{\theta\to 0} \frac{\sin\theta}{\theta} = 1$, find the following
   (a) $\lim_{x\to 0} \frac{x^2 - x + \sin x}{2x}$
   (b) $\lim_{t\to 0} \frac{\sin(1-\cos t)}{1-\cos t}$

6. For what value[illegible] and b, the function

$$g(x) = \begin{cases} -2, & x \leq -1 \\ ax + b, & -1 < x < 1 \\ 3, & x \geq 1 \end{cases}$$

   is continuous at every point?

7. Show that the function

$$f(x) = \begin{cases} 1, & x \text{ is rational} \\ 0, & x \text{ is irrational} \end{cases}$$

   is discontinuous at every point.
   (a) [illegible]) is right continuous at any point?
   (b) [illegible]) is left continuous at any point?

8. If functions $f(x)$ and $g(x)$ are continuous for $0 \leq x \leq 1$, could $\frac{f(x)}{g(x)}$ possibly be discontinues at a point of $[0,1]$? Give reason for your answer.