

Tutorial 5

1. Find the limit of f as $(x, y) \rightarrow (0, 0)$ or show that the limit does not exist (By using polar coordinates).

(a) $f(x, y) = \frac{x^3 - xy^2}{x^2 + y^2}$

(b) $f(x, y) = \cos\left(\frac{x^3 - y^3}{x^2 + y^2}\right)$

(c) $f(x, y) = \frac{y^2}{x^2 + y^2}$

(d) $f(x, y) = \frac{2x}{x^2 + x + y^2}$

(e) $f(x, y) = \tan^{-1}\left(\frac{|x| + |y|}{x^2 + y^2}\right)$

(f) $f(x, y) = \frac{x^2 - y^2}{x^2 + y^2}$

2. Show that the following functions have no limit as $(x, y) \rightarrow (0, 0)$.

(a) $f(x, y) = -\frac{x}{\sqrt{x^2 + y^2}}$

(b) $f(x, y) = \frac{x+y}{x-y}$

(c) $f(x, y) = \frac{x^4}{x^4 + y^2}$

(d) $f(x, y) = \frac{x^2 + y}{y}$

(e) $f(x, y) = \frac{x^4 - y^2}{x^4 + y^2}$

(f) $f(x, y) = \frac{x^2}{x^2 - y}$

(g) $f(x, y) = \frac{xy}{|xy|}$

(h) $f(x, y) = \frac{x-y}{x+y}$

3. Show that the following limits do not exist.

(a) $\lim_{(x,y) \rightarrow (1,1)} \frac{xy^2 - 1}{y - 1}$

(b) $\lim_{(x,y) \rightarrow (1,-1)} \frac{xy+1}{x^2 - y^2}$

4. Use (ϵ, δ) definition for the following problems. Given $f(x, y)$ of and a positive ϵ . In each of the following problem show that there exist $\delta > 0$ such that for all (x, y) , $\sqrt{x^2 + y^2} < \delta \implies |f(x, y) - f(0, 0)| < \epsilon$.

(a) $f(x, y) = x^2 + y^2$, $\epsilon = 0.01$

(b) $f(x, y) = \frac{y}{x^2 + 1}$, $\epsilon = 0.05$

(c) $f(x, y) = \frac{x+y}{x^2 + 1}$, $\epsilon = 0.01$

(d) $f(x, y) = \frac{x+y}{2 + \cos x}$, $\epsilon = 0.02$