# Lecture-8

## Random Variables:

Its a real-valued function on the sample space.

e.g. toss a coin 3 times

no. of times you get a Head.

$S = \{HHH, \dots, TTT\}$

$X : S \to \mathbb{R}$ range $\subseteq$ codomain

range of $X$, i.e., what are the possible values that $X$ can take.

$X \in \{0, 1, 2, 3\}$

②

| $X=i$ | $P(X=i)$ |
|---|---|
| 0 | 1/8 |
| 1 | 3/8 |
| 2 | 3/8 |
| 3 | 1/8 |
| total | 1 → self-check |
| | |

e.g.
A box has 20 balls, numbered from 1 to 20. You randomly choose 3 balls. What is the probability ~~that~~ at least one of the balls $\geq 17$?

2, 7, 10 ✗
5, 10, 17 ✓
18, 19, 20 ✓

$P(E) = 1 - P(\bar{E})$

$\bar{E}$: all the balls are chosen from 1-16

$$P(E) = 1 - \frac{\binom{16}{3}}{\binom{20}{3}}$$

X: the highest number among the 3 chosen balls.

$X \in \{3, 4, \cdots, 20\}$

i = 3 to 20

$$P(X=i) = \frac{\binom{1}{1}\binom{i-1}{2}}{\binom{20}{3}}$$

$P(E) = P(X=17) + P(X=18) +$
$P(X=19) + P(\cancel{X} = 20)$

H.W. $\;\; 1 - {}^{16}C_3 / {}^{20}C_3$

e.g.: biased coin

$p(\text{Head}) = p$

$p(\text{Tail}) = 1-p$

Toss this coin again & again until

→ you get a Head OR

→ you have tossed $n$ times.

$X$ = no. of times you toss the coin.

$X \in \{1, 2, \cdots, n\}$

$p(X = i)\,, \quad i = 1, \cdots, n$

distribution of probability.

$P(\underset{H}{X} = 1) = p$

$P(\underset{TH}{X} = 2) = (1-p)\,p$

$P(\underset{TTH}{X} = 3) = (1-p)^2\,p$

$P(X = n-1) = (1-p)^{n-2} \cdot p$

$P(X = n) = \underline{(1-p)^{n-1}\,(p + (1-p))}$

total $\qquad$ 2 $\quad$ 1

$\overline{T \;\; T \;\; T \; \ldots \; T} \;\; H/T$

$\longleftarrow n-1 \longrightarrow$

e.g.: You throw 3 dice.
You bet on the no. of 6's.
if no 6, you lose 100 Rs.
if 1 6, you get 100 Rs.
if 2 6, you get 200
if 3 6, you get 300

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Let $X$ be the amount that you win.

6 · · · · ·
1 2 3
1 5 5

| $X=i$ | $P(X=i)$ | no. of 6's |
|---|---|---|
| $-100$ | $5^3/6^3$ | 0 |
| $100$ | $75/6^3$ | 1 |
| $200$ | $15/6^3$ | 2 |
| $300$ | $1/6^3$ | 3 |

$\underline{1}$

6 6 1..5

We are interested in Expected value of X.

$$E(\underline{X}) := \sum_{i=1}^{n} x_i \, p_i$$

DEFINITION

discrete random variable / countable

continuous " " / uncountable

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all finite sets,

all bijections of $N$.

$$E(x) = \sum_{i=1}^{4} x_i p_i$$

$$= (-100)\frac{5^3}{6^3} + (100)\left(\frac{75}{6^3}\right) +$$

$$(200)\left(\frac{15}{6^3}\right) + 300\left(\frac{1}{6^3}\right)$$

$$= \frac{-12500 + 7500 + 3000 + 300}{6^3}$$

$$= -\frac{1700}{6^3} \simeq -8 \text{ Rs}$$

if $E(X) = 0$, its a fair game.

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e.g: 3 wh