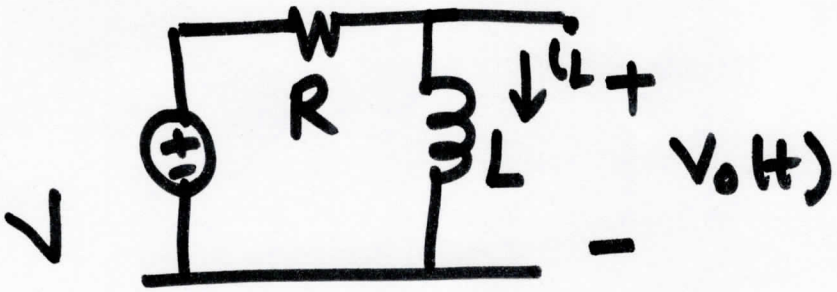
RL Circuit



$$i_L(0^-) = I_i$$

KVL

$$V = i_L R + L\frac{di_L}{dt}$$

$$\Rightarrow \quad i_L(t) = C.F + P.I$$

C.F

$$i_L R + L\frac{dI_L}{dt} = 0$$

$$\Rightarrow \int \frac{dI_L}{I_L} = \int -\frac{R}{L}dt$$

$$\Rightarrow \ln(I_L) = -\frac{R}{L}t + \ln K$$

$$\Rightarrow \ln\left(\frac{i_L}{K}\right) = -\frac{Rt}{L}$$

$$\Rightarrow i_L(t) = K e^{-\frac{Rt}{L}}$$

$$= K e^{-t/\tau}$$

where $\tau = \frac{L}{R}$

P.I

$$V = i_L R + L\frac{di_L}{dt}$$

$V \rightarrow$ Dc input

$i_L(t)$ = Constant at $t = \infty$

$$\frac{di_L}{dt} = 0$$

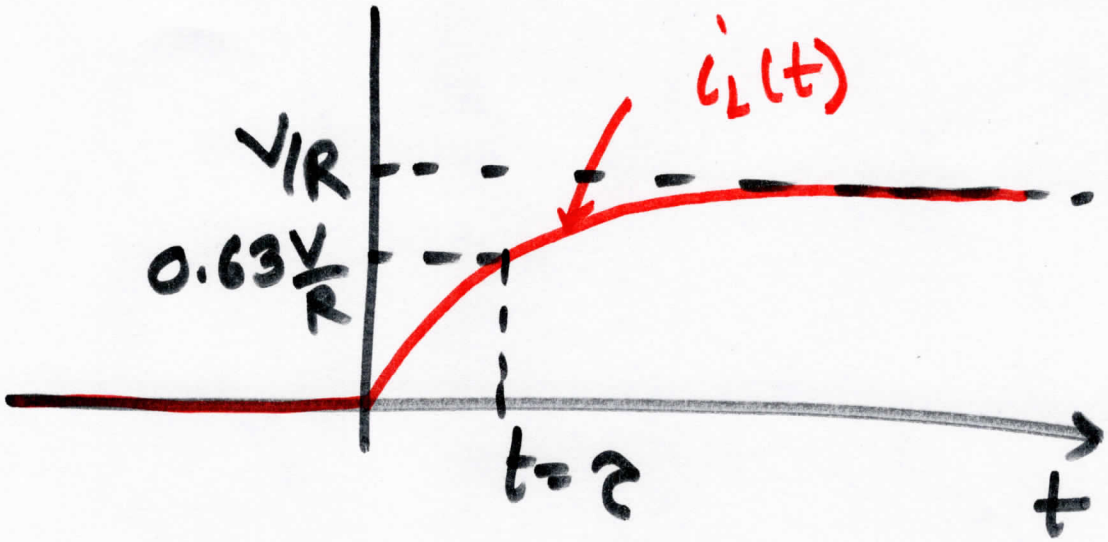
$$\Rightarrow i_L(t) = \frac{V}{R}$$

$$\Rightarrow i_L(t) = K e^{-t/\tau} + \frac{V}{R}$$

$$\boxed{i_L(t) = i_f + (i_i - i_f) e^{-t/\tau}}$$

$$\begin{cases} i_L(0^-) = 0A = i_i \\ i_f = \dfrac{V}{R} \\ \tau = L/R \end{cases}$$

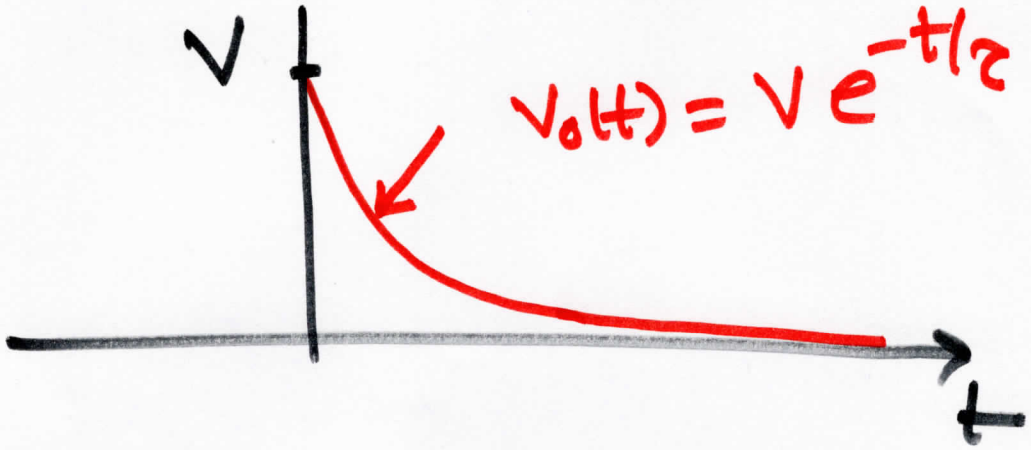
$$i_L(t) = \frac{V}{R}\left[1 - e^{-t/(L/R)}\right]$$



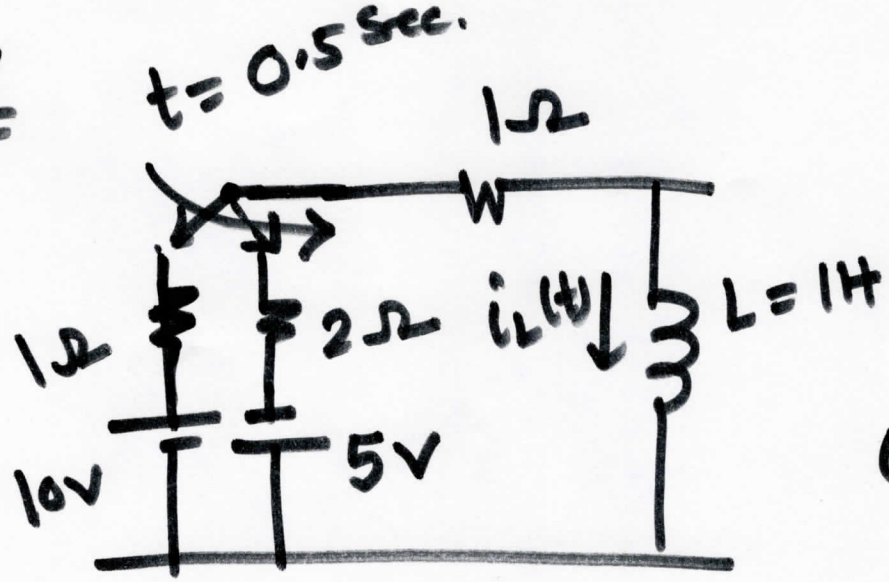
$$V_o(t) = L\frac{di_L}{dt}$$

$$= L \times \frac{V}{R} \times (-1)\left(-\frac{R}{L}\right) e^{-t/(L/R)}$$

$$= V e^{-t/\tau}$$

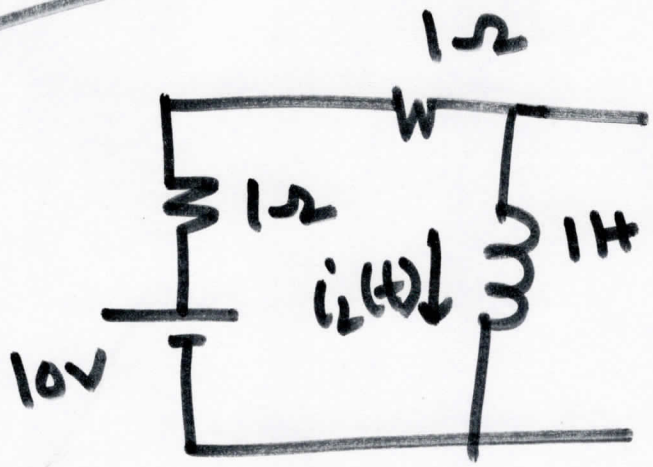


Ex



$i_L(0^-) = 0A$

$0 \le t \le 0.5$ sec

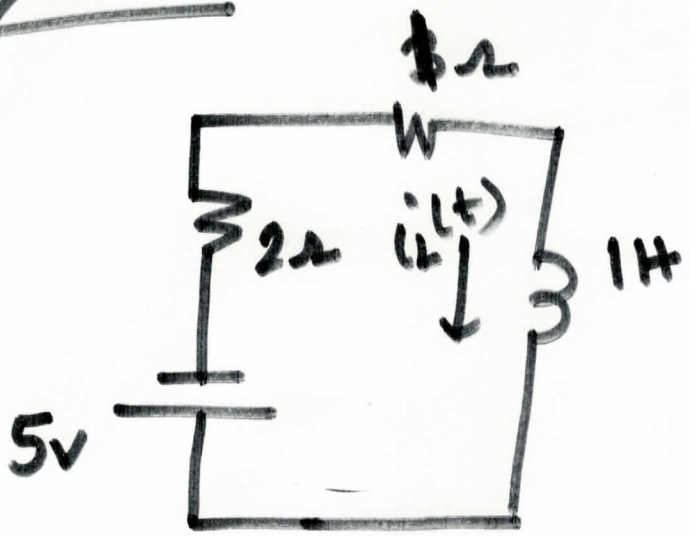


$I_f = \frac{10}{2} = 5A$

$\tau = \frac{L}{R} = \frac{1}{2} = 0.5$ sec.

$i_L(t) = 5(1 - e^{-2t})$

$t \geq 0.5$



$I_i = i_L(t=0.5-)$

$= 5 \times 0.63$ A

$I_f = -\frac{5}{3}$ A

$\tau = L/R = 1/3$ sec.

$i_L(t) = (-5/3) + (3.15 + 5/3)\, e^{-(t-0.5)/(1/3)}$

$= -5/3 + \frac{14.45}{3} e^{-3(t-0.5)}$