Lecture 5

Investigating Motion : Computational Approach

Realistic motion :

- ODE
- Initial condition given
- Euler's method; Finite Difference

Bicycle Racing (goal: to understand what determines speed)

Case 1: simple case (flat surface without friction)

Write down the equation of motion. (Law?)

Newtons second Law of motion

dv/dt=F/m

F→ force that comes from the effort of the rider

Accurate expression for force??

Alternate approach?

Alternate approach

In terms of power.
Power output over a period of time !!

Write down the previous eqn. in terms of power.

P→ power output of the rider. (~400 watts over ~1hour)

Eqns.

- $dE/dt=P$; $E \rightarrow$ total energy of the combination
- $E=1/2\ (mv^2)$
- $dE/dt=mv(dv/dt)$

- **$dv/dt=P/mv$**

- **If P is constant, write down the Finite Difference form**

Analytical soln.

- **If P is constant, what is the solution**

$$\int_{v_0}^{v} v'\,dv' = \int_0^t \frac{P}{m}\,dt'$$

$$v = \sqrt{v_0^2 + 2\,P\,t/m}$$

FD form →

$$v_{i+1} = v_i + \frac{P}{m v_i}\Delta t$$

Write MATLAB program for the bicycle problem

Recall the algorithm for growth or decay problem.

initialize *simulationLength*
initialize *number_atoms*
initialize *decay-Rate*
initialize length of time step Δt
Num_of_Iterations ← *simulationLength* / Δt

for *i* going from 1 through *num_of_iterations*
do the following:
decay ← *decay-Rate* * *number_atoms*
number_atoms ← *number_atoms* (+/-) *decay* * Δt
$t \leftarrow i * \Delta t$

display *t*, *decay*, and *number_atoms*

Result of computation

Take reasonable initial conditions and run the program for different values of time-step.

- Velocity grows indefinitely !!
- Some mechanism of energy loss needs to be included.

Main loss mechanism → atmospheric drag !! How to include.

Physics of air resistance – complex !!

Modify the code and include the drag term !!

Realistic model.

In general; **Drag force ~ - $C_1 v$ – $C_2 v^2$**

Second term dominates at reasonable velocities; how to approximate C2.

As objects moves, it push the air in front of it.

The mass of air moved in time dt is ~ ??

This air is given a velocity v, therefore its kinetic energy is $(½)m_{air}v^2$

This is the work done by the drag force in time dt.

Realistic model

Drag-force x (v x dt) = KE air

m_{air}~ density of air x **[frontal area of object x (v x dt)]**

$$F_{\text{drag}} \approx -C\rho A v^2$$

C- drag coefficient; reasonable estimate in this case .5

A ~ .3 m2

New FD eqn. →

$$v_{i+1} = v_i + \frac{P}{m v_i}\Delta t - \frac{C\rho A v_i^2}{m}\Delta t$$

Question

Can you explain the following picture and explain your answer using the code with more **investigations (numerical experiments) !!**

Use of aerobars !! How it helps

Speed with and without aerobars (approximate calculations)