Lecture - 10 P 1

Recap:

Variance

$Var(X) := E(X - E(X))^2$

Proof $= E(X^2) - (E(X))^2$

<u>e.g.</u> tossing a dice.

$X \in \{\underset{1/6}{1}, \underset{1/6}{2}, \ldots, \underset{1/6}{6}\}$

$Var(X) = E(X^2) - 49/4$

$E(X) = \frac{1}{6}(1+2+\ldots+6)$

$= \frac{1}{\cancel{6}} \cdot \frac{\cancel{6} \cdot 7}{2} = \frac{7}{2}$

$E(X^2) = \frac{1}{6}\begin{pmatrix}1+4+9+16+25 \\ +36\end{pmatrix}$

$= 91/6$

$Var(X) =$

$E(X^2) - (E(X))^2$

$= \frac{91}{6} - \frac{49}{4} = \frac{182 - 147}{12}$

$Var(X) = \frac{35}{12}$

$Var(aX+b) \; / \; Y = aX+b$

$= Var(Y) = E(Y - E(Y))^2$

$= E(aX+b - E(aX+b))^2$

$= E(aX+\cancel{b} - a \cdot E(X) - \cancel{b})^2$

$= E(aX - aE(X))^2$

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$Var(aX+b) =$

$E(aX - aE(X))^2$

$E(a(X - E(X)))^2$

$a^2 \boxed{E(X - E(X))^2}$

$\underline{a^2 Var(X)}$

In particular, if $a=1$,

then $Var(X+b) =$

$Var(X)$

This completes your syllabus for 1st insem.

Bernoulli random Variable. 4

Doing a experiment that has 2 outcomes

$P(X=1) = p \to$ success $\to H$

$P(X=0) = 1-p \to$ failure $\to T$

Coin toss, wherein the coin may be biased.

Binomial random variable.

$\to$ Repeat this experiment n times.

X = no. of times you succeed.

<u>e.g.</u>

toss a coin n times.

probability $(H) = p$

$X =$ no. of Heads.

$X \in \{0, 1, \ldots, n\}$.

$$P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}$$

$$\underline{T}\quad \underline{T}\quad \underset{\uparrow}{\underline{H}}\quad \underline{T}\qquad \underset{\uparrow}{\underline{H}}\qquad \underset{\uparrow}{\underline{H}}$$

$$\xleftrightarrow{\qquad\qquad n \qquad\qquad}$$

$$\sum_{k=0}^{n} \binom{n}{k} p^k (1-p)^{n-k}$$

$$= (p + (1-p))^n = 1^n$$

$$= 1$$

e.g.:

Salman Khan is on jury trial.

12 people in the jury.

Atleast 8 people need to say that he is guilty for him to be sentenced.

$P\left(\begin{array}{l}\text{juror will make}\\ \text{correct decision}\end{array}\right) = 0.9$

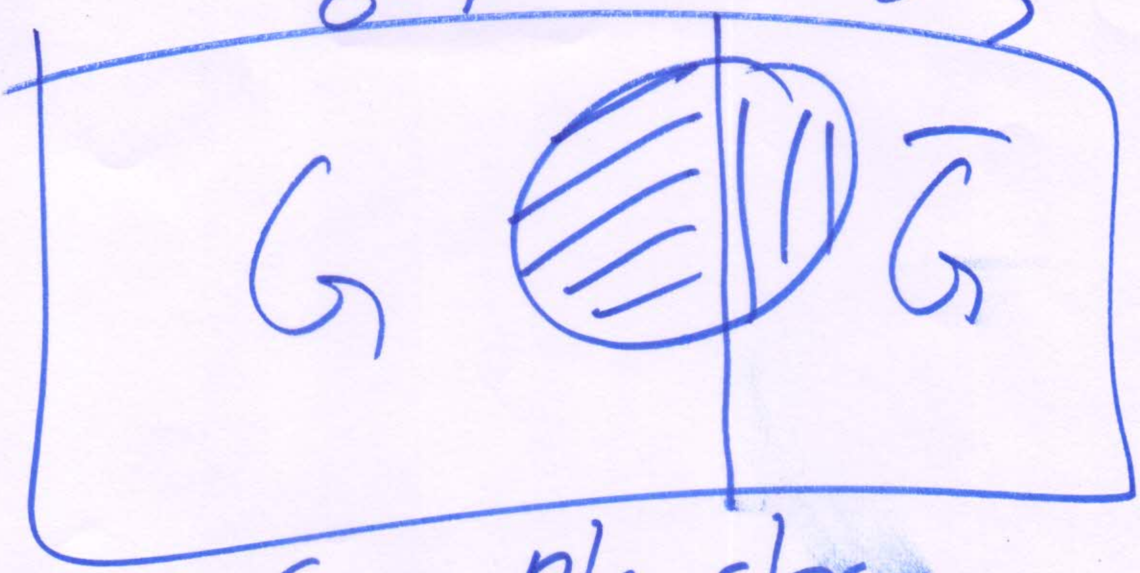
$P(\text{SK is guilty}) = 0.7$

What is the probability that the jury makes the right decision?

J: jury gives correct decision

G: he is guilty $= 0.7$

$$P(J) = P(J|G)\,P(G) + P(J|\bar{G})\,P(\bar{G})$$



sample space

$$\equiv J \cap G \cup \equiv J$$
$$\equiv J \cap \bar{G}$$

$$P(J \cap G) = P(J|G)\,P(G)$$
$$P(J \cap \bar{G}) = P(J|\bar{G})\,P(\bar{G})$$

$P(J|G) =$

what is the probability that the jury makes the correct decision given that he is guilty?

$n = 12$, $p = 0.9$

$k = 8, \cdots, 12$

$$P(J|G) = \sum_{k=8}^{12} \binom{n}{k} p^k (1-p)^{n-k}$$

$P(J|\bar{G}) =$ what is the probability that the jury makes the right decision given that he is innocent?

he is innocent,
correct decision
↓ set free

no. of
jurors giving
correct decision

1	α
2	α
3	α
4	α
5	

5 : innocent | 6 7 . . 12
7 : guilty

Binomial random variable.

Proof in the book

$E(X) = \cancel{np}\ np$

$Var(X) = np(1-p)$

$$P(X=k) = \binom{n}{k} p^k (1-p)^{n-k}$$

when is this probability highest? What value of k maximizes $P(X=k)$?

$\lceil \frac{n}{2} \rceil, \lfloor \frac{n}{2} \rfloor,$

$$\frac{P(X=k)}{P(X=k-1)} \geq 1$$

$$\frac{P(X=1)}{P(X=0)}, \frac{P(X=2)}{P(X=1)}, \dots, \frac{P(X=n)}{P(X=n-1)}$$

$$\frac{\binom{n}{k} p^{k} (1-p)^{n-k}}{\binom{n}{k-1} p^{k-1} (1-p)^{n-k+1}}$$

$$= \frac{\cancel{n!}\, p\, (k-1)!\, (n-k+1)!}{k!\, (n-k)!\, (1-p)\, \cancel{n!}}$$

$$= \frac{p\,(n-k+1)}{(1-p)\,k} \geq 1$$

$$p(n-k+1) \geq (1-p)k$$

$$np - \underset{\alpha}{kp} + p \geq k - \underset{\alpha}{kp}$$

$$\boxed{k \leq p(n+1)}$$