Tutorial-1

1.

a) $S = \{1, 2, \ldots N\}$

b) $S = \{(1, \text{black}), (2, \text{black}), (3, \text{white}), (4, \text{white})\}$

c) $S = \{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT\}$

d) $S = (0, 1)$

2. $P(\text{getting } 1) = 1/n$
$P(\text{getting } 2) = 1/n-1$
$\vdots$
$P(\text{getting } n) = 1/1$

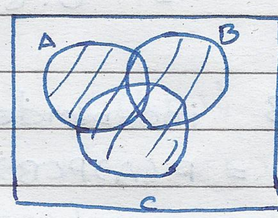
$P\{1, 2, \ldots n\} = \frac{1}{n} \cdot \frac{1}{n-1} \cdot \ldots \cdot \frac{1}{1}$

$= \frac{1}{n!}$

3. $P(\text{getting } 1) = 1/n$
$P(\text{getting } 2) = 1/n$
$\vdots$
$P(\text{getting } n) = 1/n$

$P\{1, 2, \ldots n\} = \frac{1}{n^n}$

4. $P(A) = 0.5$
$P(B) = 0.4$
$P(C) = 0.3$
$P(AB) = 0.35$
$P(BC) = 0.2$
$P(CA) = 0.25$
$P(ABC) = 0.15$



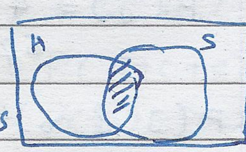
At least one cource

$P(A \cup B \cup C)$

$= P(A) + P(B) + P(C) - P(AB) - P(BC) - P(CA) + P(ABC)$

$= 0.55$

5. $S = \{DD, DND, DNND, DNNN,$
$NDD, NDND, NDNN$
$NNDD, NNDN, NNNN, NNND\}$



6. $P(H) = 0.8$, $P(H+S)' = 0.15$; $P(H+S) = 1 - 0.15$
$P(S) = 0.85$ $P(HS) = ?$ | $P(H+S) = P(H) + P(S) - P(HS)$

$\therefore P(HS) = \cancel{P(H)+P(S)}$
$P(H) + P(S) - P(H+S) = 0.8$

7. B_i : Selecting i box.

$P(B_i) = 1/4 \; ; i = 1,2,3,4$

D : Defective product choosen

$P(D/B_1) = 0.05$

$P(D/B_2) = 0.4$

$P(D/B_3) = 0.1$

$P(D/B_4) = 0.1$

(a) $P(D) = P(B_1).P(D/B_1)$
$\quad + P(B_2).P(D/B_2)$
$\quad + P(B_3).P(D/B_3)$
$\quad + P(B_4).P(D/B_4)$
$= 0.1625$

(b) $P(B_2/D) = \dfrac{P(D/B_2).P(B_2)}{P(D)}$
$= 0.6153$

8. A & B independent $\Rightarrow P(AB) = P(A)P(B)$

$A = B$
$\Rightarrow P(AB) = P(AA) = P(A)$
$\Rightarrow P(AB) = P(BB) = P(B)$
$\Rightarrow P(A) = P(B)$ — ①

$\Rightarrow P(AB) = P(A).P(A)$ (∵ 1)
$= (P(A))^2$

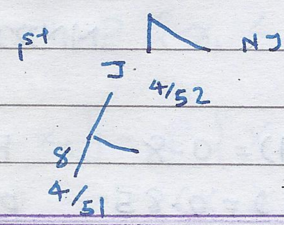
$\Rightarrow P(A) = [P(A)]^2$

$\Rightarrow P(A) = 1$ or $P(A) = 0$

$\Rightarrow A = S$ or $A = \emptyset$

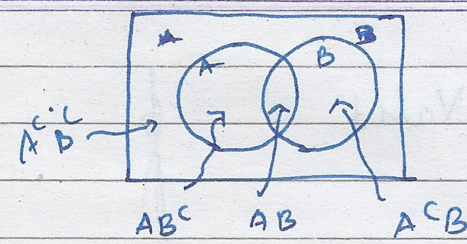
$\Rightarrow A = B = S$ or $A = B = \emptyset$

9. $P(J) = \dfrac{4}{52}$, $P(8/J) = \dfrac{4}{52}$



$P(J \,\&\, 8) = \dfrac{4}{52}.\dfrac{4}{52} = \dfrac{1}{169}$

10. $P(AB) = P(A).P(B)$ —①



a) $AB^c = A - AB$

$\therefore P(AB^c) = P(A) - P(AB)$
$= P(A) - P(A).P(B)$
$= P(A)[1 - P(B)]$
$= P(A).P(B^c)$

$\Rightarrow$ A and B^c are independent.

b) $A^cB = B - AB$

$\therefore P(A^cB) = P(B) - P(AB)$
$= P(B) - P(A).P(B)$
$= P(B)[1 - P(A)]$
$= P(B).P(A^c)$

$\Rightarrow$ A^c and B are independent.

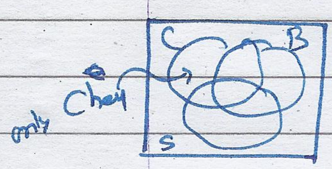
c) $A^cB^c = 1 - \{A + B - AB\}$

$P(A^cB^c) = 1 - \{P(A) + P(B) - P(AB)\}$
$= 1 - P(A) - P(B) + P(A)P(B)$ $(\because 1)$
$= (1 - P(A))(1 - P(B))$
$= P(A^c).P(B^c)$

$\Rightarrow$ A^c and B^c are independent.

11. C : like cherry
B : like banana
S : like strawberry

$P(B) = 22/35$, $P(BC) = 7/35$
$P(C) = 18/35$, $P(BS) = 8/35$
$P(S) = 13/35$, $P(CS) = 5/35$
$P(B+C+S) = 1$



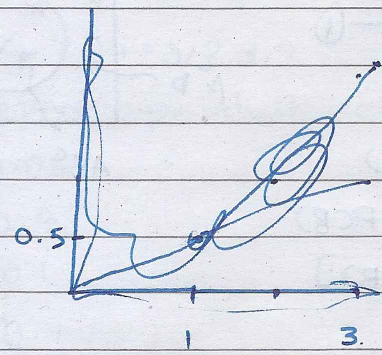
only C $= C - BC - SC + BSC$

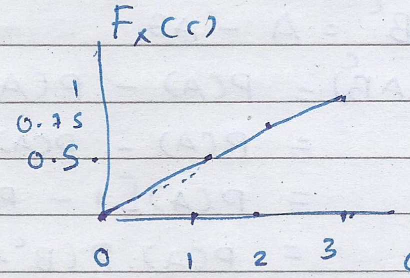
$P\{\text{only } C\} = P(C) - P(BC) - P(SC) + P(BSC)$
$= P(C) - P(BC) - P(SC) + P(B+C+S) - P(B) - P(C) - P(S) + P(BC) + P(CS) + P(BS)$

$= 8/35$

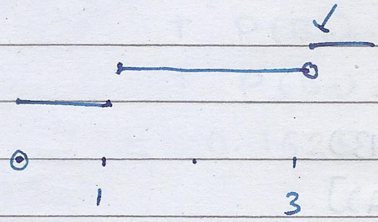
12.

a) Valid





b)



Not Valid

Not right continuous

c) $F_X(-\infty) \neq 0$, Not valid

d) Valid.

e) decreasing, Not Valid

f) $F_X(\infty) \neq 1$, Not valid