Lecture - 23

Recap: jointly distributed random variables

e.g.

$$f(x,y) = \begin{cases} e^{-x} e^{-y} & 0 < x < \infty \\ & 0 < y < \infty \\ 0 & \text{otherwise.} \end{cases}$$

Find the density function for $\frac{X}{Y}$.

1. Cumulative

$$P\left(\frac{X}{Y} \leq a\right) = P(X \leq aY)$$

$$= \int_0^\infty \left[\int_0^{aY} f(x,y)\,dx \right] dy = \frac{a}{a+1}$$

2. Computing the density fn. ②

$$\frac{d}{da}\left(\frac{a}{a+1}\right) = \frac{1}{(a+1)^2}$$

<u>Independence for random variables</u>

X & Y are independent r.v.

if for any two sets $A, B \subseteq \mathbb{R}$

$$P(X \in A, Y \in B) = P(X \in A)\, P(Y \in B)$$

This implies that

$$P(X \leq a, Y \leq b) = P(X \leq a) \cdot P(Y \leq b)$$

$$F_{X,Y}(a,b) = F_X(a)\, F_Y(b)$$

$p(x,y) = p(x)\, p(y)$ discrete

$f(x,y) = f_X(x)\, f_Y(y)$ continuous

e.g. discrete

$x \backslash y$	0	1	
1	0.1	0.2	0.3
2	0.2	0.2	0.4
3	0.2	0.1	0.3
	0.5	0.5	

$$P(X=1, Y=0) = P(X=1) \cdot P(Y=0)$$

$$0.1 \neq 0.3 * 0.5$$

$$P(X=2, Y=0) = P(X=2)\, P(Y=0)$$

$$0.2 = 0.4 * 0.5 \checkmark$$

e.g.

X \ Y	1	2	3
0	0.1	0.2	0.2
1	0.1	0.2	0.2

X & Y are independent

e.g. $f(x,y) = 4xy$ if

$0 < x < 1$

$0 < y < 1$

$$f_X(x) = \int_0^1 4xy \, dy = 2x$$

$$f_Y(y) = \int_0^1 4xy \, dx = 2y$$

$\therefore$ X & Y are independent

e.g.

$$f(x,y) = x + y \quad \alpha$$

$$0 < x < 1$$

$$0 < y < 1$$

$$f_X(x) = \int_0^1 (x+y)\,dy = x + \frac{1}{2}$$

$$f_Y(y) = y + \frac{1}{2}$$

not indePendent.

e.g.

$$f(x,y) = 2$$

$$0 < x < y < 1$$

$$f_X(x) = \int_{\text{over } Y} f(x,y)\,dy = \int_x^1 2\,dy = 2 - 2x$$

2

$$f_Y(y) = \int_{\text{over } X} f(x,y)\,dx$$

2

$$\int_0^1 (2-2x)\,dx = 1$$

$$f_Y(y) = \int_0^y 2\,dx = 2y$$

Not independent.

$$P(X|Y) \neq P\left(\frac{X}{Y}\right) \rightarrow \text{division}$$

given that

<u>e.g.</u>: 2 people A & B decide to meet at cafeteria at 1PM. <u>Each one</u> will come at a time uniformly distributed between 1-2PM. What is the probability that the 1st person to arrive has to wait more than 10 minutes?

X = no. of minutes after 1PM when A arrives. $[0, 60]$

Y = " B arrives $[0, 60]$

H.W.