1. (a) Whether the quadratic form $f = -x_1^2 + 4x_1x_2 - 9x_2^2 + 2x_1x_3 + 8x_2x_3 - 4x_3^2$ is positive definite/negative definite/indefinite? Give reason. (2)

   (b) Find the stationary points of $f = x^3 - 3xy^2$ and identify their nature (local maximum/local minimum/saddle point). (2)

2. A window is being built, the bottom is a rectangle and the top is a semi circle. If there is 12 meters of framing materials available. What will be the dimensions of the window so that it can allow maximum light? (4)

3. Using KKT conditions find the value(s) of β for which the point $x_1^* = 1$, $x_2^* = 2$ will be an optimal solution to the problem

$$\text{Maximize } f(x_1, x_2) = 2x_1 + \beta x_2$$

   subject to

$$x_1^2 + x_2^2 \leq 5$$
$$x_1 - x_2 \leq 2.$$

   (4)

4. Solve the following problem by the method of Lagrange multipliers

$$\text{Minimize } f(X) = \frac{1}{2}(x_1^2 + x_2^2 + x_3^2)$$

   subject to

$$g_1(X) = x_1 - x_2 = 0$$
$$g_2(X) = x_1 + x_2 + x_3 - 1 = 0$$

   (4)

5. Consider the following problem:

$$\text{Minimize } \ f = (x_1 - 2)^2 + (x_2 - 1)^2$$

   subject to

$$2 \geq x_1 + x_2$$
$$x_2 \geq x_1^2$$

   Using Kuhn-Tucker conditions, find which of the following vectors is/are local minima:
$$X_1 = \begin{pmatrix} 1.5 \\ 0.5 \end{pmatrix}, \ X_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}.$$
   (5)

6. Find the admissible and constrained variations at the point $X = (0,4)^T$ for the following problem

$$\text{Minimize} \quad f = x_1^2 + (x_2 - 1)^2$$

subject to

$$-2x_1^2 + x_2 = 4.$$

(4)