

①

We had

$$N(E)dE = \frac{\sqrt{2}}{\pi^2} \left(\frac{m^*}{\hbar^2} \right)^{3/2} \sqrt{E} dE$$

$$= \frac{m^* \sqrt{2m^* E}}{\pi^2 \hbar^3} dE$$

We may replace this E by $(E - E_c)$ as we are focusing on occupied energy states in CB only.

$$\Rightarrow N(E)dE = \frac{m^* \sqrt{2m^* (E - E_c)}}{\pi^2 \hbar^3} dE \quad ; \quad E \geq E_c$$

$$f(E) = \frac{1}{1 + e^{(E - E_F)/kT}}$$

Substituting the two in

$$n_0 = \int_{E_c}^{E_{top}} N(E) f(E) dE$$

$$= \frac{m^* \sqrt{2m^*}}{\pi^2 \hbar^3} \int_{E_c}^{E_{top}} \frac{\sqrt{E-E_c}}{1 + e^{(E-E_F)/kT}} dE$$

$$= \frac{m_e^* \sqrt{2m_e^*}}{\pi^2 \hbar^3} e^{(E_F-E_c)/kT} \int_{E_c}^{\infty} \frac{\sqrt{E-E_c}}{1 + e^{(E-E_c)/kT}} dE$$

let's assume

$$\eta = \frac{E-E_c}{kT} \Rightarrow E-E_c = \eta kT \Rightarrow \sqrt{E-E_c} = (kT)^{1/2} \sqrt{\eta}$$

$$\eta_c = \frac{E_F-E_c}{kT}$$

and $E_{top} \rightarrow \infty$

$$\text{and } d\eta = \frac{dE}{kT} \Rightarrow dE = kT d\eta$$

and similarly limits

$$\text{when } E \rightarrow E_c \quad \eta \rightarrow 0$$

$$E \rightarrow \infty \quad \eta \rightarrow \infty$$

The denominator of the integral is $\frac{1}{1 + e^{(E-E_F)/kT}} \rightarrow$

$\frac{1}{e^{(E-E_F)/kT}}$ for $E \gg 3kT$

$$= e^{-(E-E_F)/kT} = e^{(E_F-E)/kT}$$

$$= e^{(E_F-E_c + E_c-E)/kT} = e^{(E_F-E_c)/kT} \times e^{-(E-E_c)/kT}$$

$$n_0 = \frac{m_e^* \sqrt{2m_e^*} (kT)^{3/2}}{\pi^2 \hbar^3} \int_0^\infty \frac{\eta^{1/2} d\eta}{1 + e^{\eta - \eta_c}} \quad \xrightarrow{\text{change of variable}} \quad \int_0^\infty \eta^{1/2} e^{-\eta} d\eta \quad \xrightarrow{\text{result}} \quad \sqrt{\pi}/2$$

New put

$$N_c = 2 \left(\frac{2\pi m_e^* kT}{h^2} \right)^{3/2} ; \text{ effective density of CB states}$$

one obtains

$$n_0 = N_c e^{-(E_c - E_f)/kT}$$

$$\begin{aligned} \hbar^3 &= \frac{h^3}{8\pi^3} = \frac{(h^2)^{3/2}}{8(\pi^2)^{3/2}} \\ \Rightarrow \frac{\sqrt{2}}{\pi^2 \hbar^3} \cdot \frac{\sqrt{\pi}}{2} &= \frac{\sqrt{2}}{\pi^2} \frac{4\pi^3}{(h^2)^{3/2}} \cdot \frac{\sqrt{\pi}}{2} \\ &= 2 \cdot \left(\frac{2\pi}{h^2} \right)^{3/2} \end{aligned}$$

Show fig 3-16, page 85 of Streetman

Similarly,

concentration of holes in the VB is

$$p_0 = N_v [1 - f(E_v)]$$

$$\text{where } f(E_v) = \frac{1}{1 + e^{(E_v - E_F)/kT}}$$

$$\Rightarrow 1 - f(E_v) \approx e^{-(E_F - E_v)/kT}$$

$$\text{for } E_F > E_v + 3kT$$

$$\Rightarrow p_0 = N_v e^{-(E_F - E_v)/kT}$$

$$\text{with } N_v = 2 \left(\frac{2\pi m_p^* kT}{h^2} \right)^{3/2}$$

Maxwell-Boltzmann Stats

$$f(E) = \frac{1}{e^{(E-E_0)/kT}}$$

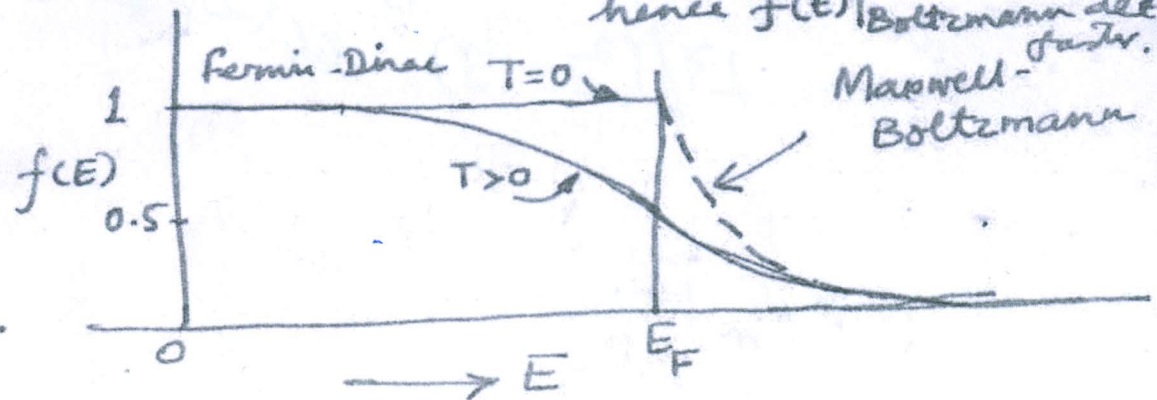
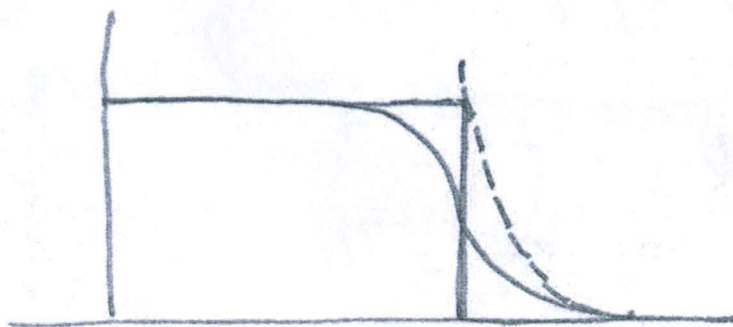
at $E=E_0$, $f(E)=1$

Fermi-Dirac Stats

$$f(E) = \frac{1}{e^{(E-E_F)/kT} + 1}$$

at $E=E_F$, $f(E)=\frac{1}{2}$
 here $f(E)$ is always
 smaller than that of
 Maxwell-Boltzmann.

Que: Which one decays faster?
 Meaning, for a given E ,
 $f(E)|_{\text{Fermi}} < f(E)|_{\text{Boltz}}$
 hence $f(E)|_{\text{Boltzmann}}$ decays
 faster.



for intrinsic material

6

$$n_i = p_i$$

We have just obtained

$$n_o = n_i = \frac{4\sqrt{2}}{h^3} (\pi m_e^* kT)^{3/2} e^{(E_F - E_c)/kT}$$

$$\text{and } p_o = p_i = \frac{4\sqrt{2}}{h^3} (\pi m_p^* kT)^{3/2} e^{(E_v - E_F)/kT}$$

if we divide the two and take $\frac{n_i}{p_i} = 1$

$$\Rightarrow \left(\frac{m_e^*}{m_p^*} \right)^{3/2} e^{(2E_F - E_c - E_v)/kT} = 1$$

$$\Rightarrow \frac{2E_F - E_c - E_v}{kT} = \ln \left(\frac{m_p^*}{m_e^*} \right)^{3/2} = \frac{3}{2} \ln \left(\frac{m_p^*}{m_e^*} \right)$$

⑦

This will define the position of

$$\Rightarrow 2E_F - (E_c + E_v) = \frac{3}{2} kT \ln \left(\frac{m_p^*}{m_e^*} \right)$$

$$\Rightarrow E_F = \frac{E_c + E_v}{2} + \frac{3}{4} kT \ln \left(\frac{m_p^*}{m_e^*} \right)$$

Therefore, for $m_p^* = m_e^*$

$$E_F = \frac{E_c + E_v}{2} \quad \text{which is in the centre of the bandgap.}$$

Even there is slight difference in the masses m_p^* & m_e^* since the ratio is on logarithmic scale, it does not change the situation much and we can conclude, in general, that

$$E_F = \frac{E_c + E_v}{2}$$

for intrinsic material.

Intrinsic Concentration

Start with

$$n_i = \frac{4\sqrt{2}}{h^3} (\pi m_e^* kT)^{3/2} e^{(E_F - E_c)/kT}$$

Substitute $E_F = \frac{E_c + E_v}{2}$; $E_F - E_c = \frac{E_c + E_v}{2} - E_c = \frac{E_v - E_c}{2} = -\frac{E_g}{2}$

$$n_i = \frac{4\sqrt{2}}{h^3} (\pi m_e^* kT)^{3/2} e^{-E_g/2kT}$$

put std. values of $h = 6.62 \times 10^{-34} \text{ Js}$

$$kT = 0.0466 \times 10^{-19} \text{ J}$$

$$m_e^* = m_e = 9.1 \times 10^{-31} \text{ kg}$$

at room temp.

$$\frac{118.3 \times 10^{-2}}{\Rightarrow}$$

$$4\sqrt{2} = 5.657$$

$$\pi = 3.14$$

$$m_e = 9.1 \times 10^{-31} \text{ kg}$$

$$eT = 0.0416 \times 10^{-19} \text{ J}$$

$$h = 6.62 \times 10^{-34} \text{ Js}$$

$$\frac{(\text{kg} \cdot \text{J})^{3/2}}{(\text{Js})^3} = \frac{(\text{kg} \cdot \text{J})^{3/2} (\text{J} \cdot \text{s})^{-3}}{(\text{J} \cdot \text{s})^3}$$

$$= (\text{kg} \cdot \text{J} \cdot \text{J}^{-2} \cdot \text{s}^{-2})^{3/2}$$

$$= (\text{kg} \cdot \text{J}^{-1} \cdot \text{s}^{-2})^{3/2}$$

$$\begin{aligned} \text{Joule} &= \text{Force} \times \text{distance} \\ &= \text{mass} \times \text{distance} \\ &= \text{kg} \cdot \text{m} \cdot \text{s}^{-2} \cdot \text{m} \\ &= \text{kg} \cdot \text{m}^2 \cdot \text{s}^{-2} \end{aligned}$$

$$\begin{aligned} & \frac{5.657 \times (3.14 \times 9.1 \times 10^{-31} \times 0.0416 \times 10^{-19})^{3/2}}{(6.62 \times 10^{-34})^3} \\ &= \frac{5.657 \times (1.189 \times 10^{-50})^{3/2}}{290 \times 10^{-102}} \\ &= \frac{5.657 \times 1.3 \times 10^{-75}}{290 \times 10^{-102}} \\ &= 0.025 \times 10^{27} \\ &= 2.5 \times 10^{25} \\ &= 2.5 \times 10^{19} \text{ cm}^{-3} \end{aligned}$$

$$\begin{aligned} \frac{1}{\text{m}^3} &= \frac{1}{10^6 \text{ cm}^3} \\ &= 10^{-6} \text{ cm}^{-3} \end{aligned}$$

This is in the units of m^{-3} . We consider vol. in cm^{-3} . Hence unit $= 10^{-6} \text{ cm}^{-3}$

$$\begin{aligned} \text{J}^{-1} &= \text{kg}^{-1} \text{m}^{-2} \text{s}^2 \Rightarrow \text{kg} \text{J}^{-1} \text{s}^{-2} = \text{m}^{-2} \\ \Rightarrow (\text{kg} \cdot \text{J}^{-1} \text{s}^{-2})^{3/2} &= \text{m}^{-3} \end{aligned}$$

We get

$$n_i = 2.5 \times 10^{25} e^{-E_g/2kT} \text{ per m}^3 \text{ volume}$$

n_i is generally expressed in cm^{-3} volume and so

$$\text{m}^{-3} = 10^{-6} \text{ cm}^{-3}$$

$$\Rightarrow n_i = 2.5 \times 10^{19} e^{-E_g/2kT}$$

where both sides are in per cm^3 volume of the crystal

Again we know,

$$E_g = 1.1 \text{ eV for Si}$$

$$\text{and } kT = 0.026 \text{ eV at room temp.}$$

Substituting these two values yields

$$n_i \sim 1.6 \times 10^{10} \text{ cm}^{-3}$$

The actual value is found to be $1.4 \times 10^{10} \text{ cm}^{-3}$.

For Ge

$$E_g = 0.7 \text{ eV}$$

$$\Rightarrow n_i \sim 3.7 \times 10^{13} \text{ cm}^{-3}$$

and the measured value of n_i for Ge = $2.4 \times 10^{13} \text{ cm}^{-3}$.

What could be the reasons of these deviations?

- a) Error in E_g value, which may get amplified since concentrations depend exponentially on E_g .
- b) Our assumption that $m_e^* =$ mass of ^{a free} electron in vacuum

In fact electrons remain in a potential well even in CB. So 'free' is not the best term to describe their state. However, effective mass is almost close to the rest mass for Si and Ge.