10 Angle Modulation

10.1 Introduction

In angle modulation, the information signal may be used to vary the carrier frequency, giving rise to *frequency modulation,* or it may be used to vary the angle of phase lead or lag, giving rise to *phase modulation.* Since both frequency and phase are parameters of the carrier angle, which is a function of time, the general term *angle modulation* covers both. Frequency and phase modulation have some very similar properties, but also some marked differences. The relationship between the two will be described in detail in this chapter.

Compared to amplitude modulation, frequency modulation has certain advantages. Mainly, the signal-to-noise ratio can be increased without increasing transmitted power (but at the expense of an increase in frequency bandwidth required); certain forms of interference at the receiver are more easily suppressed; and the modulation process can take place at a low-level power stage in the transmitter, thus avoiding the need for large amounts of modulating power.

10.2 Frequency Modulation

The modulating signal $e_m(t)$ is used to vary the carrier frequency. For example, $e_m(t)$ may be applied as a voltage to a voltage-dependent capacitor, which in turn controls the frequency of an oscillator. (Some modulating circuits are described in Section 10.12). In a well-designed modulator the *change* in carrier frequency will be proportional to the modulating voltage and thus can be represented as $ke_m(t)$, where k is a constant known as the *frequency deviation constant.* The units for k are clearly *hertz / volt* or Hz/V. The instantaneous carrier frequency is therefore equal to

$$f_i(t) = f_c + ke_m(t) \tag{10.2.1}$$

where f_c is the unmodulated carrier frequency.

EXAMPLE 10.2.1

Sketch the instantaneous frequency–time curve for a 100-MHz carrier wave frequency modulated by a 1-kHz square wave that has zero dc component and peak-to-peak voltage of 20 V. The frequency deviation constant is 9 kHz/V.

SOLUTION The peak-to-peak frequency deviation is $20 \times 9 = 180$ kHz, and this is spaced symmetrically about the unmodulated carrier of 100 MHz. The resulting frequency–time curve is shown in Fig. 10.2.1.

As noted previously, the instantaneous frequency may be expressed as $f_i(t) = f_c + ke_m(t)$, and the corresponding instantaneous angular velocity is $\omega_i(t) = 2\pi f_i(t)$. The generation of the modulated carrier can be represented graphically by means of a rotating phasor as shown in Fig. 10.2.2 (a).

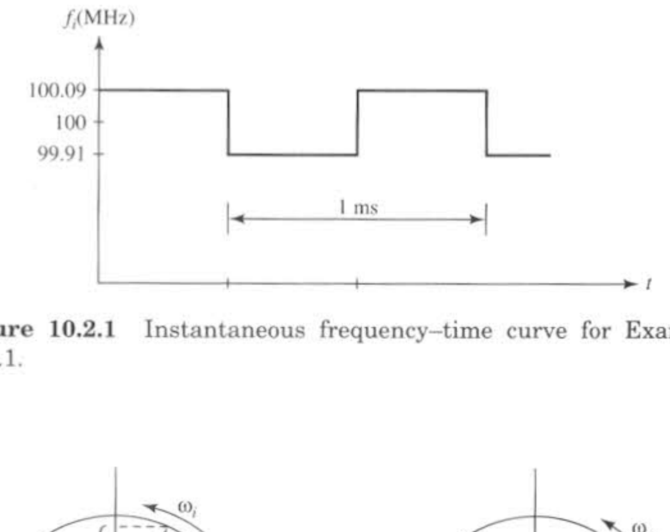

Figure 10.2.1 Instantaneous frequency–time curve for Example 10.2.1.

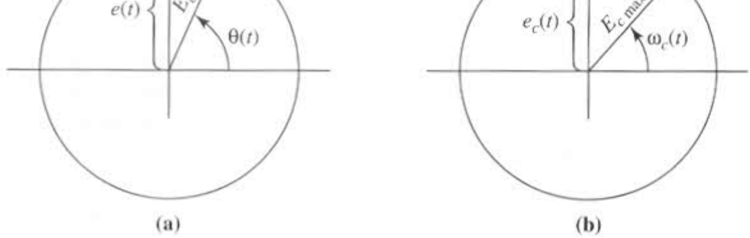

Figure 10.2.2 Rotating phasor representation of a carrier of amplitude $E_{c\ max}$ rotating (a) at instantaneous angluar velocity $\omega_i(t)$ and (b) at constant angular velocity ω_c.

The phasor, of constant length $E_{c\,\max}$, rotates in a counterclockwise direction at an angular velocity $\omega_i(t) = 2\pi f_i(t)$. The angle turned through in time t is shown as $\theta(t)$, where for convenience the positive x-axis is used as the reference axis. The angle $\theta(t)$ is found by noting that the angular velocity is the time rate of change of angle, or

$$\frac{d\theta(t)}{dt} = \omega_i(t) \tag{10.2.2}$$

and hence

$$\begin{aligned}\theta(t) &= \int_0^t \omega_i(t)\,dt \\ &= \int_0^t 2\pi(f_c + ke_m(t))\,dt \\ &= 2\pi f_c t + 2\pi k\int_0^t e_m(t)\,dt\end{aligned} \tag{10.2.3}$$

Thus the modulating signal is contained in the angle, in this rather indirect way. Note that the expression for the modulated angle could not have been obtained by simply substituting f_i for f_c in the sine-wave function $E_{c\,\max}\sin(2\pi f_c t)$, the reason being that this is derived on the basis of a constant frequency, which is not valid for frequency modulation. By setting $e_m(t) = 0$, the unmodulated angle is seen in Fig 10.2.2 (b) to be simply $\theta(t) = 2\pi f_c t$.

EXAMPLE 10.2.2

Sketch $\theta(t)$ as a function of time for a 100-MHz carrier wave frequency modulated by a 1-kHz square wave that has zero dc component and peak-to-peak voltage of 20 V. The frequency deviation constant is 9 kHz/V.

SOLUTION The peak-to-peak frequency deviation is $20 \times 9 = 180$ kHz, and this is symmetrical about the unmodulated carrier of 100 MHz. Thus $\Delta f = \pm 90$kHz about the carrier, where the plus sign is used for the positive half-cycles and the negative sign for the negative half-cycles of the modulating waveform. Over the positive half-cycles the integral term gives $+\Delta f \cdot t$, and over the negative half-cycles $-\Delta f \cdot t$. Thus the angle is given by $\theta(t) = 2\pi(f_c \pm \Delta f)t$, where the plus sign applies to positive half-cycles and the negative sign to negative half-cycles. The waveforms are sketched in Fig. 10.2.3.

The cosine function representing the carrier wave is given by the projection of the phasor on the x-axis and is seen to be

$$e_c = E_{c\,\max}\cos\theta(t) \tag{10.2.4}$$

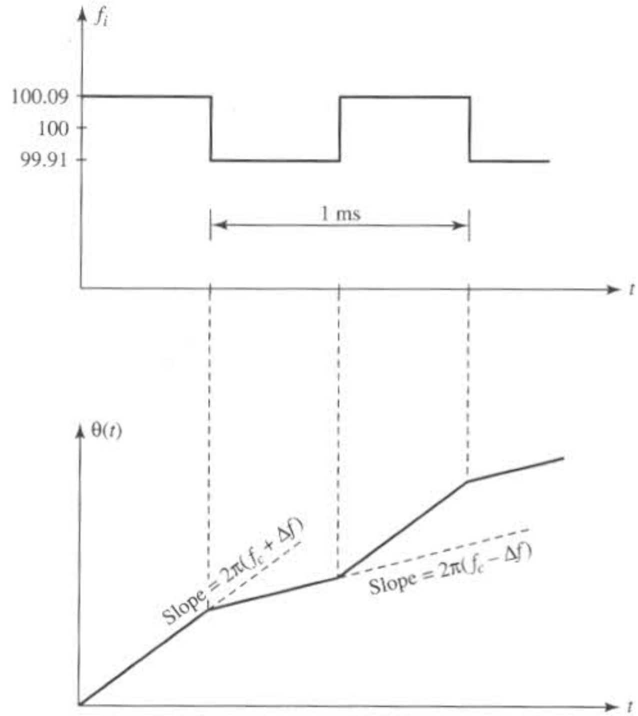

Figure 10.2.3 Solution to Example 10.2.2

Thus, in the unmodulated case, this reduces to the sinewave $E_{c\ max} \cos 2\pi f_c t$, while for the modulated case, the full expression for $\theta(t)$, including the integral term, must be used. For the square-wave modulation in the previous example, the modulated carrier would appear as sketched in Fig. 10.2.4.

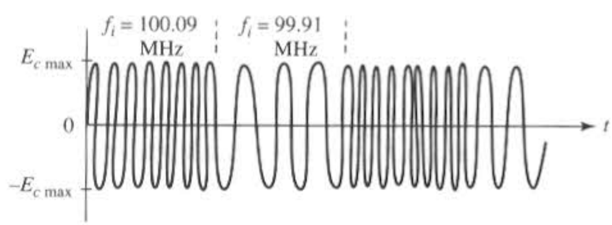

Figure 10.2.4 Square-wave FM.

10.3 Sinusoidal FM

Many important characteristics of FM can be found from an analysis of sinusoidal modulation. For sinusoidal modulation, $e_m(t) = E_{m\ max} \cos 2\pi f_m t$ and hence

$$\begin{aligned} f_i(t) &= f_c + ke_m(t) \\ &= f_c + kE_{m\,\max} \cos 2\pi f_m t \\ &= f_c + \Delta f \cos 2\pi f_m t \end{aligned} \tag{10.3.1}$$

where the *peak frequency deviation* Δf is proportional to the peak modulating signal and is

$$\Delta f = kE_{m\,\max} \tag{10.3.2}$$

The instantaneous frequency as a function of time is sketched in Fig. 10.3.1.

The expression for the sinusoidally modulated carrier therefore becomes

$$\begin{aligned} e(t) &= E_{c\,\max} \cos \theta(t) \\ &= E_{c\,\max} \cos \left(2\pi f_c t + 2\pi \Delta f \int_0^t \cos 2\pi f_m t \, dt\right) \\ &= E_{c\,\max} \cos \left(2\pi f_c t + \frac{\Delta f}{f_m} \sin 2\pi f_m t\right) \end{aligned} \tag{10.3.3}$$

The modulation index for FM, usually denoted by β, is defined as

$$\beta = \frac{\Delta f}{f_m} \tag{10.3.4}$$

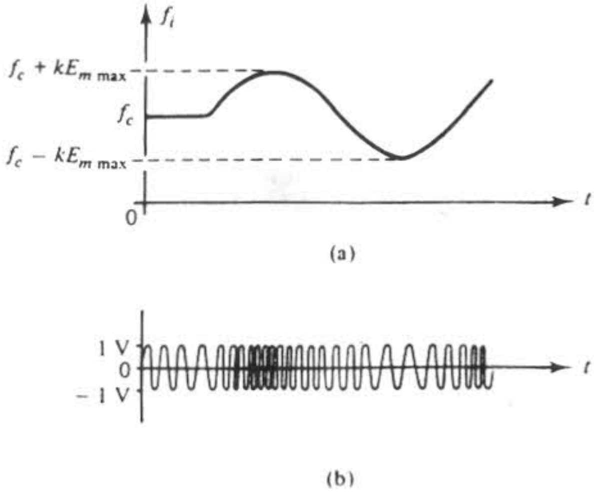

Figure 10.3.1 Instantaneous frequency–time curve for a sinusoidally frequency modulated wave.

and hence the equation for the sinusoidally modulated wave becomes

$$e(t) = E_{c\,\max} \cos (2\pi f_c t + \beta \sin 2\pi f_m t) \tag{10.3.5}$$

EXAMPLE 10.3.1

Determine the modulation index, and plot the sinusoidal FM wave for which $E_{c\,\max} = 10$ V, $E_{m\,\max} = 3$ V, $k = 2000$ Hz/V, $f_m = 1$ kHz, and $f_c = 20$ kHz. On the same set of axes, plot the modulating function. The plot should extend over two cycles of the modulating function.

SOLUTION The peak deviation is $\Delta f = 2000 \times 3 = 6000$ Hz. The modulation index is $\beta = 6$ kHz/1 kHz $= 6$. The functions to be plotted are $e_m(t) = 3 \cos 2\pi 10^3 t$ and $e(t) = 10 \cos (4\pi 10^4 t + 6 \sin 2\pi 10^3 t)$ over a range $0 \le t \le 2$ ms. The graphs, obtained using Mathcad, are shown in Fig. 10.3.2.

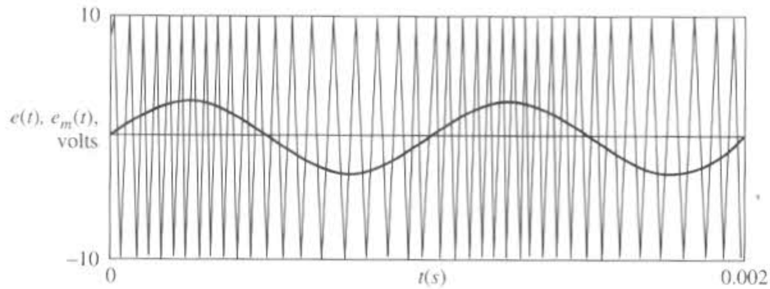

Figure 10.3.2 Solution to Example 10.3.1.

10.4 Frequency Spectrum for Sinusoidal FM

Equation (10.3.5) may be analyzed by Fourier methods in order to obtain the spectrum. The actual analysis is quite involved and only the results will be presented here. The trigonometric series contains a carrier term $J_0(\beta)E_{c\,\max} \cos \omega_c t$, a first pair of side frequencies $J_1(\beta)E_{c\,\max} \cos (\omega_c \pm \omega_m)t$, a second pair of side frequencies $J_2(\beta)E_{c\,\max} \cos (\omega_c \pm 2\omega_m)t$, a third pair of side frequencies $J_3(\beta)E_{c\,\max} \cos (\omega_c \pm 3\omega_m)t$, and so on. The amplitude coefficients $J_n(\beta)$ are known as *Bessel functions of the first kind of order n*. Values for these functions are available in both tabular and graphical form and are also available as built-in functions in programs for calculators and computers (such as Mathcad). From the point of view of applications here, the Bessel function gives the amplitude of the carrier ($n = 0$) and side frequencies ($n = 1, 2, 3$ ...). Some values are shown in Table 10.4.1, where for convenience $E_{c\,\max}$ is set equal to unity. The graphs of the carrier and the first three side frequencies are shown in Fig. 10.4.1 for values of β up to 10.

TABLE 10.4.1

Bessel Functions for a Sinusoidally Frequency-modulated Carrier of Unmodulated Amplitude, 1.0 V (Amplitude Moduli Less than |0.01| not shown.)

		Side Frequencies											
Modulation Index β	*Carrier* J_0	*1st* J_1	*2nd* J_2	*3rd* J_3	*4th* J_4	*5th* J_5	*6th* J_6	*7th* J_7	*8th* J_8	*9th* J_9	*10th* J_{10}	*11th* J_{11}	*12th* J_{12}
0.25	0.98	0.12	0.01										
0.5	0.94	0.24	0.03										
1.0	0.77	0.44	0.11	0.02									
1.5	0.51	0.56	0.23	0.06	0.01								
2.0	0.22	0.58	0.35	0.13	0.03	0.01							
2.4	0	0.52	0.43	0.20	0.06								
3.0	−0.26	0.34	0.49	0.31	0.13	0.04	0.01						
4.0	−0.40	−0.07	0.36	0.43	0.28	0.13	0.05	0.02					
5.0	−0.18	−0.33	0.05	0.36	0.39	0.26	0.13	0.05	0.02	0.01			
5.5	0	−0.34	−0.12	0.26	0.40	0.32	0.19	0.09	0.03	0.01			
6.0	0.15	−0.28	−0.24	0.11	0.36	0.36	0.25	0.13	0.06	0.02	0.01		
7.0	0.30	0	−0.30	−0.17	0.16	0.35	0.34	0.23	0.13	0.06	0.02	0.01	
8.0	0.17	0.23	−0.11	−0.29	−0.10	0.19	0.34	0.32	0.22	0.13	0.06	0.03	0.01
8.65	0	0.27	0.06	−0.24	−0.23	0.03	0.26	0.34	0.28	0.18	0.10	0.05	0.02

As an example of the use of Table 10.4.1, it can be seen that, for $\beta = 0.05$, the spectral components are

Carrier (f_c)	$J_0(0.5) = 0.94$
First-order side frequencies ($f_c \pm f_m$)	$J_1(0.5) = 0.24$
Second-order side frequencies ($f_c \pm 2f_m$)	$J_2(0.5) = 0.03$

The fact that the spectrum component at the carrier frequency decreases in amplitude does *not* mean that the carrier wave is amplitude modulated. The carrier wave is the sum of all the components in the spectrum, and these add up to give a constant amplitude carrier as shown in Fig. 10.3.2. The distinction is that the modulated carrier is not a sine wave, whereas the spectrum component at carrier frequency is. (All spectrum components are either sine or cosine waves.) It will be noted from Table 10.4.1 that amplitudes can be negative in some instances. It will also be seen that for certain values of β (2.4, 5.5, 8.65, and higher values not shown), the carrier amplitude goes to zero. This serves to emphasize the point that it is the sinusoidal component of the spectrum at carrier frequency, *not* the modulated carrier, that goes to zero and that varies from positive to negative peak (1 V in this case) as the frequency varies.

The spectra for various values of β are shown in Fig. 10.4.2(a), (b), and (c). In each case the spectral lines are spaced by f_m, and the bandwidth occupied by the spectrum is seen to be

$$B_{FM} = 2nf_m \tag{10.4.1}$$

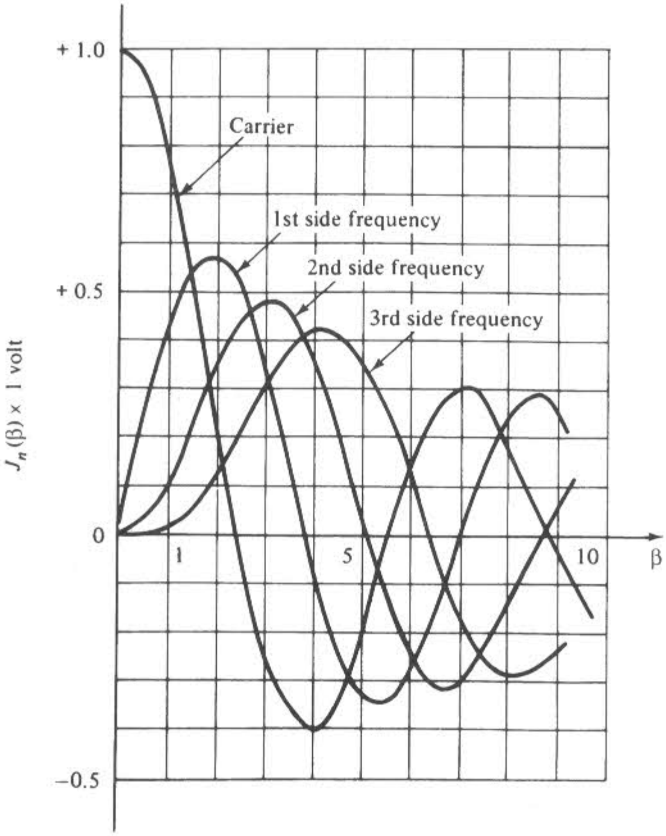

Figure 10.4.1 Graphs of the carrier amplitude and the first three side frequencies for a sinusoidally frequency modulated carrier ($E_{c\,max} = 1$ V).

where n is the highest order of side frequency for which the amplitude is significant. From Table 10.4.1 it can be seen that, where the order of side frequency is greater than $(\beta + 1)$, the amplitude is 5% or less of unmodulated carrier amplitude. Using this as a guide for bandwidth requirements, Eq. (10.4.1) can be written as

$$B_{FM} = 2(\beta + 1)f_m \tag{10.4.2}$$

or, substituting for β from Eq. (10.3.4)

$$B_{FM} = 2(\Delta f + f_m) \tag{10.4.3}$$

To illustrate the significance of this, three examples will be considered:

1. $\Delta f = 75$ kHz, $f_m = 0.1$ kHz

$$B_{FM} = 2(75 + 0.1)$$
$$= 150 \text{ kHz}$$

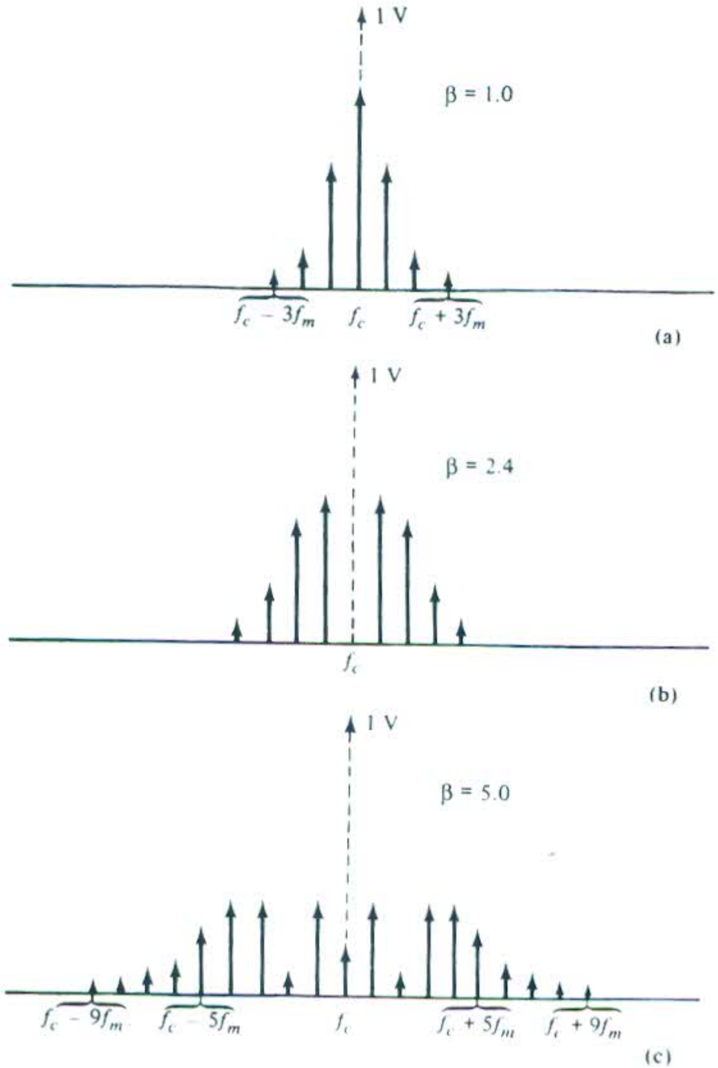

Figure 10.4.2 Spectra for sinusoidal FM with (a) $\beta = 1.0$, (b) $\beta = 2.4$ (note missing carrier), and (c) $\beta = 5.0$.

2. $\Delta f = 75$ kHz, $f_m = 1.0$ kHz

$$B_{FM} = 2(75 + 1) = 152 \text{ kHz}$$

3. $\Delta f = 75$ kHz, $f_m = 10$ kHz

$$B_{FM} = 2(75 + 10) = 170 \text{ kHz}$$

Thus, although the modulating frequency changes from 0.1 to 10 kHz, or by a factor of 100 : 1, the bandwidth occupied by the spectrum alters very little, from 150 to 170 kHz. These examples illustrate why frequency modulation is sometimes referred to as a constant-bandwidth system.

10.5 Average Power in Sinusoidal FM

The peak voltages of the spectrum components are given by $E_{n\,\max} = J_n(\beta)E_{c\,\max}$. Since the rms values denoted by E_n and E_c are proportional to the peak values, these are also related as

$$E_n = J_n(\beta)E_c \tag{10.5.1}$$

For a fixed load resistance R the average power of any one spectral component is $P_n = E_n^2/R$. The total average power is the sum of all such components. Noting that there is only one carrier component and a pair of components for each side frequency, the total average power is

$$P_T = P_o + 2(P_1 + P_2 + \cdots) \tag{10.5.2}$$

In terms of the rms voltages this becomes

$$P_T = \frac{E_o^2}{R} + \frac{2}{R}(E_1^2 + E_2^2 + \cdots) \tag{10.5.3}$$

In terms of the unmodulated carrier and the Bessel function coefficients, this is

$$\begin{aligned} P_T &= \frac{E_c^2 J_o^2(\beta)}{R} + \frac{2E_c^2}{R}(J_1^2(\beta) + J_2^2(\beta) + \cdots) \\ &= \frac{E_c^2}{R}[J_o^2(\beta) + 2(J_1^2(\beta) + J_2^2(\beta) + \cdots)] \\ &= P_c[J_o^2(\beta) + 2(J_1^2(\beta) + J_2^2(\beta) + \cdots)] \end{aligned} \tag{10.5.4}$$

Here, the unmodulated power is $P_c = E_c^2/R$. A property of the Bessel functions is that the sum $[J_o^2(\beta) + 2(J_1^2(\beta) + J_2^2(\beta) + \cdots)] = 1$, so the total average power is equal to the unmodulated carrier power. This result might have been expected because the amplitude of the wave remains constant whether or not it is modulated. In effect, when modulation is applied, the total power that was originally in the carrier is redistributed between all the components of the spectrum. As previously pointed out, at certain values of β the carrier component goes to zero, which means that in these instances the power is carried by the side frequencies only.

EXAMPLE 10.5.1

A 15-W unmodulated carrier is frequency modulated with a sinusoidal signal such that the peak frequency deviation is 6 kHz. The frequency of the modulating signal is 1 kHz. Calculate the average power output by summing the powers for all the side-frequency components.

SOLUTION The total average power output P is 15 W modulated. To check that this is also the value obtained from the sum of the squares of the Bessel functions, from Eq. (10.3.4) we have

$$\beta = \frac{\Delta f}{f_m} = \frac{6}{1} = 6$$

The Bessel function values for $\beta = 6$ are read from Table 10.4.1 and substituted in Eq. (10.5.4) to give

$$\begin{aligned} P_T &= 15[0.15^2 + 2(0.28^2 + 0.24^2 + 0.11^2 + 0.36^2 + 0.36^2 + 0.25^2 + 0.13^2 \\ &\qquad + 0.06^2 + 0.02^2 + 0.01^2)] \\ &= 15(1.00) \\ &= 15 \text{ W} \end{aligned}$$

It follows that, since the average power does not change with frequency modulation, the rms voltage and current will also remain constant, at their respective unmodulated values.

10.6 Non-sinusoidal Modulation: Deviation Ratio

In the frequency-modulation process, intermodulation products are formed; that is, beat frequencies occur between the various side frequencies when the modulation signal is other than sinusoidal or cosinusoidal. It is a matter of experience, however, that the bandwidth requirements are determined by the maximum frequency deviation and maximum modulation frequency present in the modulating wave. The ratio of maximum deviation to maximum frequency component is termed the *deviation ratio,* which is defined as

$$D = \frac{\Delta F}{F_m} \tag{10.6.1}$$

where ΔF is the maximum frequency deviation and F_m is the highest frequency component in the modulating signal. The bandwidth is then given by Eq. (10.4.2) on substituting D for β, with the same limitations on accuracy, as

$$\begin{aligned} B_{\max} &= 2(D + 1)F_m \\ &= 2(\Delta F + F_m) \end{aligned} \tag{10.6.2}$$

This is known as *Carson's rule.*

EXAMPLE 10.6.1

Canadian regulations state that for FM broadcast the maximum deviation allowed is 75 kHz and the maximum modulation frequency allowed is 15 kHz. Calculate the maximum bandwidth requirements.

SOLUTION Using Eq. (10.6.2),

$$\begin{aligned} B_{\max} &= 2(\Delta F + F_m) \\ &= 2(75 + 15) \\ &= 180 \text{ kHz} \end{aligned}$$

Examination of Table 10.4.1 shows that side frequencies of 1% amplitude extend up to the ninth side-frequency pair, so Carson's rule underestimates the bandwidth required. For D equal to 5 or greater, a better estimate is given by $B_{\max} = 2(D + 2)F_m$. In this example, this would result in a maximum bandwidth requirement of 210 kHz. The economic constraints on commercial equipment limit the bandwidth capabilities of receivers to about 200 kHz.

10.7 Measurement of Modulation Index for Sinusoidal FM

Commercially available frequency deviation meters are available that enable Δf to be measured for a known value of f_m. The frequency-modulation index is then simply the ratio of these two quantities as given by Eq. (10.3.4). As a further check, the spectrum can be examined with the aid of a spectrum analyzer and the conditions at which the carrier component of the spectrum goes to zero noted. These should occur at modulation indexes of 2.4, 5.5, and so on, as shown in Table 10.4.1.

10.8 Phase Modulation

Referring once again to the expression for an unmodulated carrier, this is

$$e_c(t) = E_{c\max} \cos(\omega_c t + \phi_c) \tag{10.8.1}$$

The phase angle ϕ_c is arbitrary and is included in the general case to show that the reference line for the rotating phasor of Fig. 10.2.2 is arbitrary. Figure 10.8.1(a) shows the situation for $\phi_c = 25°$.

When phase modulation is applied, it has the effect of moving the reference line (circuits for phase modulation are described in Section 10.12), as shown in Fig. 10.8.1(b). Mathematically, the phase modulation may be written as

$$\phi(t) = \phi_c + Ke_m(t) \tag{10.8.2}$$

where K is the *phase deviation constant,* analogous to the frequency deviation constant k introduced for frequency modulation. It will be seen that K must have units of radians per volt when ϕ_c is measured in radians. The constant phase angle ϕ_c has no effect on the modulation process, and this term can be dropped without loss of generality. Thus the equation for the phase modulated wave becomes

$$e(t) = E_{c\max} \cos(\omega_c t + Ke_m(t)) \tag{10.8.3}$$

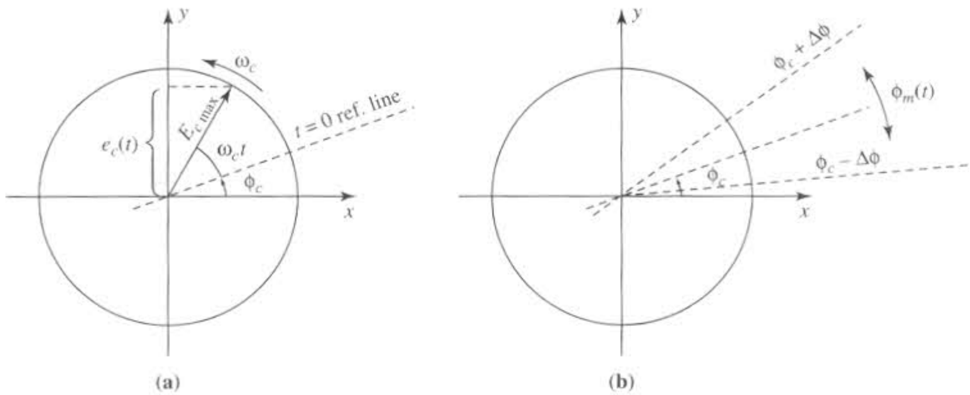

Figure 10.8.1 (a) Rotating phasor representation of a carrier of amplitude $E_{c\ \max}$ and phase lead $\phi_c = 25°$. (b) Effect of applying phase modulation.

EXAMPLE 10.8.1

A modulating signal given by $e_m(t) = 3\cos(2\pi 10^3 t - 90°)$ volts is used to phase modulate a carrier for which $E_{c\ \max} = 10$ V and $f_c = 20$ kHz. The phase deviation constant is $K = 2$ rad/V. Plot the modulated waveform over two cycles of the modulating function.

SOLUTION The phase modulation function is

$$\begin{aligned}\phi_m(t) &= Ke_m(t)\\ &= 2 \times 3\cos(2\pi 10^3 t - 90°)\\ &= 6\sin 2\pi 10^3 t\end{aligned}$$

Hence the modulated wave function is

$$e(t) = 10\cos(4\pi 10^4 t + 6\sin 2\pi 10^3 t)$$

This is identical to the modulated wave in Example 10.3.1 and hence the graph of Fig. 10.3.2 applies.

10.9 Equivalence between PM and FM

It is seen that for phase modulation the angular term is given by

$$\theta(t) = \omega_c t + Ke_m(t) \tag{10.9.1}$$

Now, the corresponding instantaneous frequency in general is obtained from

$$f_i(t) = \frac{1}{2\pi}\frac{d\theta(t)}{dt} \tag{10.9.2}$$

Hence the equivalent instantaneous frequency for phase modulation is, on differentiating Eq. (10.9.1),

$$f_{iPM}(t) = f_c + \frac{K}{2\pi}\frac{de_m(t)}{dt} \tag{10.9.3}$$

The importance of this equation is that it shows how phase modulation may be used to produce frequency modulation. The differentiation is nullified by passing the modulating signal through an integrator before it is applied to the phase modulator, as shown in Fig. 10.9.1. The time constant of the modulator is shown as τ, so the actual voltage applied to the phase modulator is

$$v_m(t) = \frac{1}{\tau}\int_0^t e_m(t)\,dt \tag{10.9.4}$$

The equivalent frequency modulation is then

$$\begin{aligned} f_{i\mathrm{PM}}(t) &= f_c + \frac{K}{2\pi}\frac{dv_m(t)}{dt} \\ &= f_c + \frac{K}{2\pi\tau}e_m(t) \end{aligned} \tag{10.9.5}$$

This is identical to the instantaneous frequency expression for frequency modulation, Eq. (10.2.1), with a frequency deviation constant given by $k = K/(2\pi\tau)$. This equivalence between FM and PM has already been illustrated in Example 10.8.1 for sinusoidal modulation. Frequency modulators that utilize the equivalence between FM and PM are described in Section 10.12.

As with frequency modulation, many important characteristics can be found from an analysis of a sinusoidally phase-modulated carrier. However, at this point it is instructive to compare the three methods of modulation, amplitude, frequency, and phase, for a modulating signal that is a step function.

In the case of amplitude modulation (Fig. 10.9.2[a]), the amplitude follows the step change, while the frequency and phase remain constant with time. The amplitude change could be observed, for example on an oscilloscope. With frequency modulation, shown in Fig. 10.9.2(b), the amplitude and phase remain constant while the frequency follows the step change. Again, this change could be observed, for example on a frequency counter.

With phase modulation, the amplitude remains constant while the phase angle follows the step change with time, as shown in Fig. 10.9.2(c). The phase change is measured with reference to what the phase angle would have been with no modulation applied. After the step change in phase, the sinusoidal carrier appears as though it is a continuation of the dashed curve

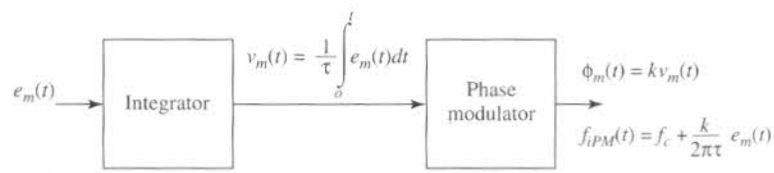

Figure 10.9.1 How FM may be obtained from PM.

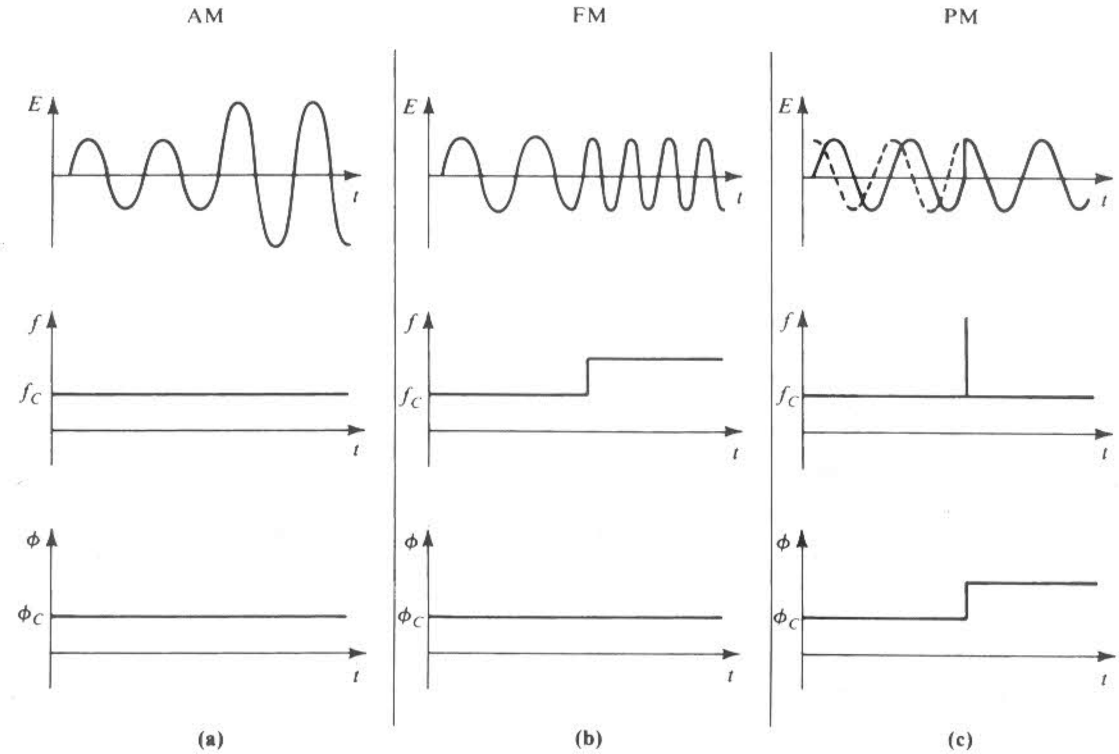

Figure 10.9.2 Modulating with a step waveform: (a) AM, (b) FM, (c) PM.

shown on the amplitude–time graph of Fig. 10.9.2(c). Also, from the amplitude–time graph it is seen that the frequency of the wave before the step change is the same as after the step change. However, at the step change in phase, the abrupt displacement of the waveform on the time axis makes it appear as though the frequency undergoes an abrupt change. This is shown by the spike in the frequency–time graph in Fig. 10.9.2(c). A phase meter could be used to measure the change in phase, but this requires the reference waveform and is not as direct as the measurement of amplitude or frequency. The spike change in frequency could be measured directly on a frequency counter. In principle, the apparent change of frequency with phase modulation will occur even where the source frequency of the carrier is held constant, for example by using a crystal oscillator. In practice, it proves to be more difficult to achieve large frequency swings using phase modulation.

10.10 Sinusoidal Phase Modulation

For sinusoidal modulation, $e_m(t) = E_{m\,\max} \sin 2\pi f_m t$, and hence

$$\begin{aligned} Ke_m(t) &= KE_{m\,\max} \sin 2\pi f_m t \\ &= \Delta\phi \sin 2\pi f_m t \end{aligned} \tag{10.10.1}$$

where the *peak phase deviation* $\Delta\phi$ is proportional to the peak modulating signal and is

$$\Delta\phi = KE_{m\,\max} \tag{10.10.2}$$

The sine expression is used for the modulation signal rather than the cosine expression, because this brings out more clearly the equivalence in the spectra for FM and PM, as will be shortly shown. The equation for sinusoidal PM is therefore

$$e(t) = E_{c\,\max} \cos(\omega_c t + \Delta\phi \sin \omega_m t) \tag{10.10.3}$$

This is identical to Eq. (10.3.5) with $\Delta\phi = \beta$, and therefore the trigonometric expansion will be similar to that for sinusoidal FM, containing a carrier term, and side frequencies at $f_c \pm nf_m$. The amplitudes are also given in terms of Bessel functions of the first kind, $J_n(\Delta\phi)$. In this case the argument is the peak phase deviation $\Delta\phi$, rather than the frequency modulation index β. It follows therefore that the magnitude and extent of the spectrum components for the PM wave will be the same as for the FM wave for which $\Delta\phi$ is numerically equal to β. It also follows that the power relationships developed in Section 10.5 for sinusoidal FM apply to the equivalent sinusoidal PM case.

For analog modulating signals, phase modulation is used chiefly as a stage in the generation of frequency modulation, as previously described. It should be noted that the demodulators in analog FM receivers (even the phase discriminators described in Section 10.14) interpret the received signal as frequency modulation, real or equivalent. The effect this has on the reception of a phase modulated carrier is illustrated in Problem 10.17.

With sinusoidal phase modulation, application of Eq. (10.9.3) gives the equivalent frequency modulation as

$$\begin{aligned} f_{i\,eq}(t) &= f_c + \Delta\phi f_m \cos \omega_m t \\ &= f_c + \Delta f_{eq} \cos \omega_m t \end{aligned} \tag{10.10.4}$$

The equivalent peak deviation is seen to be

$$\Delta f_{eq} = \Delta\phi \cdot f_m \tag{10.10.5}$$

The other major area of application for phase modulation lies in the digital modulation of carriers.

10.11 Digital Phase Modulation

Phase modulation is very widely used in digital systems. In the simplest system, the voltage levels $\pm V$ representing the binary digits 1 and 0 may be used to multiply the carrier. If the digital signal is represented by $p(t)$ and a balanced modulator is used (see Section 8.9), the modulated signal is essentially a DSBSC wave given by

$$e(t) = Ap(t)E_{c\,\max} \cos \omega_c t \tag{10.11.1}$$