Tute 8 sol^n

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Sol^n - 1 Here your company needs to make a sealed bid for a construction project. And There are many more companies which are participating with us..

Now, the company with the lowest bid will win the ~~$~~ contract.

Minimum bid of another companies can be modeled as a random variable that is uniformly distributed on (70,140) (Thousand ~~of~~ dollars).

$$f(x) = \begin{cases} \frac{1}{70} & 70 < x < 140 \\ 0 & \text{else} \end{cases}$$

If you win the contract you will have to spend 100,000 \$ to do the work.

- Assume that you bid x thousand \$. $70 < x < 140$.
- If you win the contract you will make a profit of $(x-100)$ k\$.
- If you lose your profit will be 0 \$.
- Now, you will win iff your bid is lowest and the probability of that event is

$$P(X>x) = \int_x^{140} f(x)\,dx$$

$$= \int_x^{140} \frac{1}{70}\,dx$$

$$\therefore \boxed{P(\text{win}) = \frac{140-x}{70}}$$

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$$E[x-100] = \frac{(x-100)(140-x)}{70} + 0$$

↳ Lose.

$$\therefore \boxed{E[x-100] = \frac{1}{70}(240x - x^2 - 14000)}$$

To maximise the profit.

$$\frac{d}{dx} E[x-100] = 0 \qquad \left[\text{sinse } \frac{d^2 E[x-100]}{dx^2} = (-2)\right]$$

$$\therefore 240 - 2x = 0$$

$$\therefore \boxed{x = 120}$$

$$\text{Expected profit} = E[x-100]\Big|_{x=120} = \boxed{\frac{40}{7}}$$

∴ You should bid 120 thousand dollars and your expected profit will be $\left(\frac{40}{7}\right)$ thousand dollars.

Sol^{n}2 → X : Travel time from your home to your office. → in minutes.

↳ normally distributed with $\mu = 40$, $\sigma = 7$.

Suppose you ~~take~~ ~~x~~ leave home x minutes before 1 pm. You want to be sure–

$$P(X < x) = 0.95$$

$$\therefore P\left(\frac{X-40}{7} < \frac{x-40}{7}\right) = 0.95$$

$$\therefore P\left(Z < \frac{x-40}{7}\right) = 0.95$$

$$\therefore \Phi\left(\frac{x-40}{7}\right) = 0.95$$

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$$\therefore \frac{x-40}{7} = 1.645$$

$$\therefore \boxed{x = 51.515}$$

The latest time you should leave home is 51.515 minutes before 1 pm. i.e. 8.485 minutes after 12 p.m.

Sol^n 3 There are 38 slotes in a roulette wheel.

$\therefore$ winning probability $= \frac{1}{38}$ on a single bet. (equali likely cases).

if the number of bets $= n$ & if you win X bets

$\Rightarrow$ winning amount $= 35X - \underbrace{(n-X)}_{\text{lose } 1}$

$= (36X - n)$

You will be winning after n bets $\Rightarrow 36X - n > 0$

we need $P(36X - n > 0)$

$\boxed{= P(X > n/36)}$

here X is a binomial random variable with parameters $n = n$ & $p = \frac{1}{38}$.

Since n will be large & $\sqrt{np(1-p)}$ is large, we'll be using the normal approximation to the binomial distribution.

$\mu = np \quad \sigma = \sqrt{np(1-p)}$

(a) $n = 34$

$p = P(X \geq 1)$

$= P(X > 0.5)$ (Continuity correction)

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$$\therefore p = P\left\{ \frac{X - 34/38}{\sqrt{34(1/38)(37/38)}} > \frac{0.5 - 34/38}{\sqrt{34(1/38)(37/38)}} \right\}$$

$$= P\{Z > -0.4229\}$$

$$= 1 - \Phi(-0.4229)$$

$$= \Phi(0.4229)$$

$$\boxed{= 0.6638}$$

(b) $n = 1000.$

$$\therefore p = P(X \geqslant 30)$$

$$= P(X > 27.5) \quad \text{(continuity correction)}$$

$$= P\left(\frac{X-\mu}{\sigma} > \frac{27.5 - 1000/38}{\sqrt{1000(1/38)(37/38)}} \right)$$

$$= P(Z > 0.2339)$$

$$= 1 - \Phi(0.2339)$$

$$\boxed{= 0.4075}$$

(c) $n = 100{,}000.$

$$\therefore p = P(X \geqslant 2777.78)$$

$$= P(X > 2777.5) \quad \text{(continuity correction)}$$

$$= P\left(\frac{X-\mu}{\sigma} > \frac{2777.5 - 100{,}000/38}{\sqrt{100000(1/38)(37/38)}} \right)$$

$$= P(Z > 2.883)$$

$$= 1 - \Phi(2.883)$$

$$\boxed{= 0.0020}$$

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Sol^n-4] X = lifetime of a battery

$$P\{X > s+t \mid X > t\} = \frac{P\{X > s+t \wedge X > t\}}{P\{X > t\}}$$

$$= \frac{P\{X > s+t\}}{P\{X > t\}}$$

$$= \frac{P\{X > s+t \mid \text{battery is type 1}\} P_1 + P\{X > s+t \mid \text{battery is type 2}\} P_2}{P\{X > t\}}$$

for exponential random variable

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & \text{else} \end{cases}$$

$$\therefore F(a) = P(X \leq a) = \int_{-\infty}^{a} \lambda e^{-\lambda x}\, dx \qquad (a > 0)$$

$$= \int_0^a \lambda e^{-\lambda x}\, dx$$

$$= \left[e^{-\lambda x}\right]_a^0$$

$$\therefore P(X \leq a) = 1 - e^{-\lambda a}$$

$$\boxed{\therefore P(X > a) = e^{-\lambda a}}$$

$$\therefore \boxed{P\{X > s+t \mid X > t\} = \frac{e^{-\lambda_1(s+t)} P_1 + e^{-\lambda_2(s+t)} P_2}{e^{-\lambda_1 t} P_1 + e^{-\lambda_2 t} P_2}}$$

Since $P\{X > t\} = P\{X > t \mid \text{type} = 1\} P_1 + P\{X > t \mid \text{type} = 2\} P_2$,

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Sol^{n}5 X: Random variable (exponential)

$$\mu = \frac{1}{\lambda} = \begin{cases} 1 & \text{, defendant is innocent} \\ 2 & \text{, defendant is guilty.} \end{cases}$$

(a) $P\{X < c \mid \text{defendant is innocent}\} = 0.95$

$\therefore 1 - e^{-c} = 0.95$ $\quad (\because \lambda = 1)$

$\therefore e^{-c} = 0.05$

$\therefore c = -\ln(0.05)$

$\therefore \boxed{c = 2.996}$

(b) $P\{X > c \mid \text{defendant is guilty}\} = e^{-\frac{1}{2}c}$ $\quad (\lambda = 1/2)$

$= e^{-\frac{1}{2} \times 2.996}$

$\boxed{= 0.2236}$

Sol^{n}6 A point is uniformly distributed on a circle.

Radious = R

Origin = center of a circle.

(X, Y) coordinates of the point choosen.

Density function of (X, Y):

$$f(x,y) = \begin{cases} c & , x^2+y^2 \le R^2 \\ 0 & , x^2+y^2 > R^2. \end{cases}$$

(a) c = ?

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\,dx\,dy = 1$$

$x^2+y^2 \le R^2$

$\therefore -\sqrt{R^2-y^2} \le x \le \sqrt{R^2+y^2}$

$-R \le y \le R$

$$\therefore c\int_{-R}^{R}\int_{-\sqrt{R^2-y^2}}^{\sqrt{R^2-y^2}} dx\,dy = 1$$

$$\therefore c\int_{-R}^{R} 2\sqrt{R^2-y^2}\,dy = 1$$

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$$\therefore c\left[x\left(\frac{y(\sqrt{R^2-y^2})}{x}+\frac{R^2}{x}\sin^{-1}\frac{y}{R}\right)\right]_{-R}^{R}=1$$

$$\therefore c\left[0+R^2\left[\frac{\pi}{2}-\left(-\frac{\pi}{2}\right)\right]\right]=1$$

$$\therefore c\pi R^2=1$$

$$\therefore \boxed{c=\frac{1}{\pi R^2}} \quad \text{—— (1)}$$

(b) Marginal density function of X & Y

$$f_X(x)=\int_{-\infty}^{\infty} f(x,y)\,dy$$

$$=\frac{1}{\pi R^2}\int_{-\sqrt{R^2-x^2}}^{\sqrt{R^2-x^2}} dy \qquad (x^2\le R^2)$$

$$\boxed{=\frac{2}{\pi R^2}\sqrt{R^2-x^2}}$$

$$\therefore f_X(x)=\begin{cases}2/\pi R^2\,(\sqrt{R^2-x^2}), & (x^2\le R^2)\\ 0, & (x^2>R^2)\end{cases}$$

Same way.

$$f_X(y)=\begin{cases}\frac{2}{\pi R^2}\sqrt{R^2-y^2} & y^2\le R^2\\ 0 & y^2>R^2.\end{cases}$$

(c) $D=\sqrt{x^2+y^2}$

$$F_D(a)=P(\sqrt{X^2+Y^2}\le a)$$

$$=P(X^2+Y^2\le a^2)$$

$$=\int_{-a}^{a}\int_{-\sqrt{a^2-y^2}}^{\sqrt{a^2-y^2}} f(x,y)\,dx\,dy$$

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$$\therefore F_D(a) = \frac{1}{\pi R^2} \int_{-a}^{a} \int_{-\sqrt{a^2+y^2}}^{\sqrt{a^2-y^2}} dx\, dy$$

$$= \frac{\pi a^2}{\pi R^2}$$ (using result I.)

$$\boxed{= \frac{a^2}{R^2}}$$

(d) $E[D] = ?$

now $f_D(a) = \dfrac{d[F_D(a)]}{da}$

$$\boxed{\therefore f_D(a) = \frac{2a}{R^2}}$$

$$\therefore E[D] = \int_{-\infty}^{\infty} x\, f_D(x)\, dx$$

$$= \frac{2}{R^2} \int_0^R x^2\, dx \qquad 0 \leq \sqrt{x^2+y^2} \leq R.$$

$$= \frac{2}{R^2} \left[\frac{x^3}{3}\right]_0^R$$

$$\boxed{\therefore E[D] = \frac{2R}{3}}$$

—X—.

All the best for the second insem!