Tutorial 7 Solution

Solution (1)

$$f(t)=\sum_{n=-\infty}^{\infty}F_n\cdot e^{jnw_0t}$$

$$=2\cdot e^{jw_0t}+2\cdot e^{-jw_0t}+4j\cdot e^{j3w_0t}+4j\cdot e^{-j3w_0t}$$

$$=4\cdot\cos(w_0t)+8j\cdot\cos(3w_0t)$$

$$=4\cdot\cos(w_0t)+8\cos\left(3w_0t+\frac{\pi}{2}\right)$$

$$now\qquad w_0=\frac{2\pi}{T}=\frac{2\pi}{8}=\frac{\pi}{4}$$

$$=4\cdot\cos\left(\frac{\pi}{4}t\right)+8\cos\left(3\frac{\pi}{4}t+\frac{\pi}{2}\right)$$

Solution (2)

$$f(t)=2+\frac{1}{2}e^{j\left(\frac{2\pi}{3}\right)}+\frac{1}{2}e^{-j\left(\frac{2\pi}{3}\right)}+\frac{1}{2}e^{j\left(\frac{5\pi}{3}\right)}-\frac{1}{2}e^{-j\left(\frac{5\pi}{3}\right)}$$

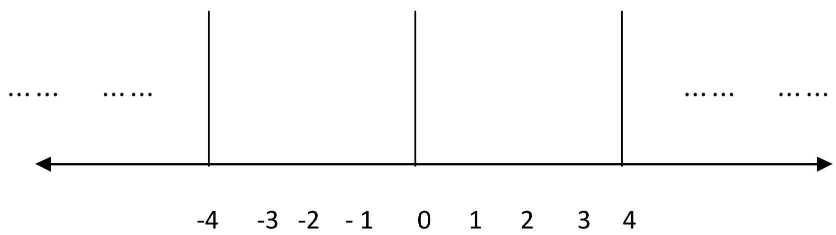
$$=2+\frac{1}{2}e^{j2\left(\frac{2\pi}{6}\right)}+\frac{1}{2}e^{-j\left(\frac{2\pi}{6}\right)}+\frac{1}{2}e^{j5\left(\frac{2\pi}{6}\right)}-\frac{1}{2}e^{-j5\left(\frac{2\pi}{6}\right)}$$

$$w_0=\frac{2\pi}{6}$$

$F_0=2$ $\qquad F_2=\frac{1}{2}=F_{-2}$ $\qquad F_5=\frac{1}{2j},\ F_{-5}=-\frac{1}{2j}$

Solution (3)

$$x(n) = \delta(n - 4k)$$



(periodic impulse sequence)

$x(n)$ can be written as Fourier series representation as follows

$$x(n) = \sum_{k} F_k e^{j\left(\frac{2\pi}{N}\right)kn}$$

Where $F_k = \frac{1}{4}\sum_{n=0}^{3} x(n) e^{-j\left(\frac{2\pi}{4}\right)kn}$

$$= \frac{1}{4}\sum_{n=0}^{3} \delta(n) e^{-j\left(\frac{2\pi}{4}\right)kn}$$

$$= \frac{1}{4} \qquad ; every\ k$$

$$\therefore x(n) = \frac{1}{4}\, e^{j\left(\frac{2\pi}{4}\right)k\,n}$$

By using the concept of Fourier series in LTI system framework , (i.e Eigen function theory)

$y(n) = \sum_{k=<N>} H(e^{j\left(\frac{2\pi}{4}\right)k})\, F_k e^{j\left(\frac{2\pi}{4}\right)k\,n}$ (over a period of N)

$$y(n) = \sum_{k=<N>} H(e^{j\left(\frac{2\pi}{4}\right)k})\, \frac{1}{4}\, e^{j\left(\frac{2\pi}{4}\right)k\,n}$$

$$y(n) = \cos\left(\frac{5\pi}{2}n + \frac{\pi}{4}\right)$$

$$\therefore \cos\left(\frac{5\pi}{2}n + \frac{\pi}{4}\right) = \sum_{k=0}^{3} H(\, e^{j\left(\frac{2\pi}{4}\right)k}\,)\, \frac{1}{4}\, e^{j\left(\frac{2\pi}{4}\right)k\,n}$$

$$\therefore \quad \frac{1}{2}e^{j\left(\frac{5\pi}{2}\right)n}e^{j\left(\frac{\pi}{4}\right)}+\frac{1}{2}e^{-j\left(\frac{5\pi}{2}\right)n}e^{-j\left(\frac{\pi}{4}\right)}$$

$$=\frac{1}{4}H\left(e^{j0}\right)e^{j0}+\frac{1}{4}H\left(e^{j\left(\frac{\pi}{2}\right)}\right)e^{j\left(\frac{\pi}{2}\right)}+\frac{1}{4}H\left(e^{j(\pi)}\right)e^{j(\pi)}++\frac{1}{4}H\left(e^{j\left(3\frac{\pi}{2}\right)}\right)e^{j\left(3\frac{\pi}{2}\right)}$$

Comparing on both sides

(note: $e^{\frac{j\pi}{2}}=e^{\frac{j5\pi}{2}}$; $e^{\frac{j3\pi}{2}}=e^{\frac{-j\pi}{2}}$)

$$H\left(e^{j0}\right)=H\left(e^{j\pi}\right)=0$$

$$H\left(e^{j\left(\frac{\pi}{2}\right)}\right)=2e^{j\left(\frac{\pi}{2}\right)}$$

$$H\left(e^{j\left(\frac{3\pi}{2}\right)}\right)=2e^{-j\left(\frac{\pi}{4}\right)}$$