<u>Assignment-4 (19/02/2019)</u>   deadline: 26^{th} February, 2019

Assignments must be submitted in the form of a report along with MATLAB codes (through moodle). For all the problems below, **provide graphs (from the simulation) in your report**. **Plots/graphs must be properly labeled with proper units and right choice of axis.**

For each problem: describe the model (governing mathematical equations), approximations made and conclusions based on the simulation (final data/plot).

You can do more investigations with your program/code in addition to what have been asked in the question for a particular problem (put these investigations under "<u>additional investigation</u>" section in your report below each problem.)

<u>Oscillatory motion</u>

<u>Recommended initial conditions are given for the following assignments, however you are free to choose your own initial conditions for investigating the system.</u>

System: Loaded horizontal spring-mass system within elastic limits, follows Hooke's law and has force constant k. Stable position x=0.

1. **Goal: investigate the motion of a body attached to a spring.**

Case 1 (SI units for all):

m=1; k=1; x0=0, v0=1, time-step=.01, time of simulation=25 s.

- Plot to summarize important observations.
- Increase m and see what happens to the solution, increase k and see again.
- To which type of functions do x(t), v(t) and a(t) belong? Compare it with built in trigonometric functions for comparison with exact ones. Note the inaccuracies in the numerically obtained solution and how does it depend on time-step.
- Try x0=1 and v0=1, do the position and velocity show any variation. Vary the initial conditions and see if the time dependence of physical quantities change with them.

- Calculate the amplitude and time period of the oscillation. Does it match with the analytical solution (accurately)?
- Phase-space trajectory: do you always get similar curves with different initial conditions. Do these always start from the same point? Does the trajectory trace itself in the same direction for different initial conditions? Comment on the observations.
- Compute the time dependence of the percentage of error (Ei-E0/E0) in the energy. Comment on error over one cycle and how does it depend on time-step. Does the numerical solution maintain the energy conservation? How does the choice of time step influence the accuracy and stability of the solution? What is the effect of the size of time step on the computation time (compute/observe for a long simulation time for different time-steps for a conclusion)?

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2. Goal: To study the effects of damping in the case of a pendulum by starting with some initial angular displacement and study how the motion decays with time. Repeat the simulations for different values of damping constant and investigate its effect on the oscillations. What is the effect of different initial conditions on freq. of damped oscillations?

Investigate all the cases - underdamped, overdamped and crticially damped oscillations as discussed in the class. Report about the choice of initial conditions which lead to the above mentioned cases. Plot your result in a single graph for 3 cases. In which of the above cases the pendulum comes to the equilibrium position - the fastest i.e. minimum time to reach at rest.

Plot Phase space plot for all the three cases and explain the nature of the curve.

For analytical solution, you can refer to (Marion and Thornton).

Damped Oscillator: ($ma = -kx - cv$; where c denotes the damping coefficient)

Start with the same initial conditions as the previous case but with c=.5

- Run the program for different values of k and m of the system, and find out the damping coefficient for which the equilibrium is reached most quickly. Towards which point the phase space curve reach finally. Comment on the dissipation of energy.
- Start with c=0, increase c in small steps and plot the x(t) vs t for different values of the damping coefficient c. How do the oscillations go from undamped to critically damped to over-damped. At critical damping, how does the amplitude vary with time?
- For different values of c (start with c=0 (no damping) to c=5-6 (highly damped)), do you find the amplitude decays with time? Define a parameter as maximum amplitude in one cycle; plot a graph of amplitude vs. number of cycles, what is the nature of the decay? Find out the time constant over which amplitude decays to 1/e of its maximum value?
- In the above problem, study the fall of amplitude for critical damping. Does it still obey the exponential behavior?
- Introduce friction between the mass and the horizontal surface on which it is vibrating. Let us assume that it is independent of the magnitude of the velocity (however remember the force acts in a direction opposite to that of velocity).

The force law is:

$a=-kx/m - c$ (from left to right)

$a=-kx/m + c$ (from right to left)

So you have to deal with the discontinuous nature of the force near the extremities, modify your program to solve this problem numerically.

- Study the motion of a loaded spring with sliding friction. Plot x vs t, compare the nature of damping with that of the previous case of damped oscillator. What is the influence of sliding friction on the period of oscillations, does the mass come to rest at the same spot where it started.

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3. Goal: Investigate the above problem (particularly the under damped case) with an external force with a given freq as discussed in the class. To compare the initial/transient behavior with the steady state behavior.

Forced/driven Oscillations

New force law : ma= -kx –cv + F0 cos(omega t)

- Two frequencies: natural frequency of the oscillator and the driving frequency.
- Two kind of responses: transient response at the beginning and steady state response which takes over after the transient effects fade away.
- You can start with similar initial conditions as previous problems (SI units), with c=2 , omega=1.01, F0=1.5, time-step=.01 and time of simulation =30 s.
- Run the program for more than 2 cycles, and check if the angular frequency of oscillations has changed after several cycles.
- Study time variations of potential and kinetic energies of the system. Draw phase space trajectory of the driven system.
- Study the behavior when "natural freq> omega" and when "natural freq< omega". How is the amplitude affected by the frequency? What happens at "natural freq= omega", does the system show resonance. What are the conclusions regarding the phase-space diagram for this case.
- Modify the program (requires if then conditions) for driven oscillator to plot a graph between maximum amplitude and omega. Does the graph have a peak, if yes at which omega does it occur.
- Also investigate the phase angle between the driving force and the resultant motion computationally, supported by figure.
- Summarize important observations for the above case from your computational results.