Lecture-12

Recap:

Poisson Random Variable

Geometric random Variable

→ $E(X) = ?$

Repeat an experiment until 1st success

$p(\text{success}) = p,$

$p(\text{failure}) = 1-p$

$X =$ no. of trials required to get the 1st success.

$$p(X=k) = \underbrace{(1-p)^{k-1}}_{\longleftarrow\ k-1\ \longrightarrow\ \text{failure}} \underbrace{p}_{k\ \text{success}}$$

$\Sigma x_i p_i$

$$E(X) = \sum_{k=1}^{\infty} k\,(1-p)^{k-1} \cdot p \quad (A.G.P.)$$

$S = \sum_{n=1}^{\infty} n(1-p)^{n-1} \cdot p$ ②

$S = 1 \cdot (1-p)^0 \cdot p + 2 \cdot (1-p)^1 \cdot p + 3(1-p)^2 \cdot p + \dots$

$(1-p)S = \qquad 1 \cdot (1-p)^1 p + 2(1-p)^2 \cdot p + \dots$

$p \cdot S = 1(1-p)^0 \cdot p + 1(1-p)^1 \cdot p + 1 \cdot (1-p)^2 \cdot p + \dots$

$p \cdot S = p[1 + (1-p) + (1-p)^2 + \dots]$

$pS = \frac{p}{1-(1-p)} = 1$

$\Rightarrow \boxed{S = \frac{1}{p}} = E(X)$

$Var(X) = \underline{\underline{E(X^2)}} - [E(X)]^2$

$$\underbrace{E(x^2)} - \underbrace{(E(x))^2}_{\frac{1}{p^2}}$$

3

$$E(x^2) = \sum_{k=1}^{\infty} k^2 (1-p)^{k-1} \cdot p$$

$$k^2 = [(k-1) + 1]^2$$

$$= (k-1)^2 + 1^2 + 2(k-1)$$

1st term

$$\sum_{k=1}^{\infty} (k-1)^2 (1-p)^{k-1} \cdot p \qquad m = k-1$$

$$\sum_{m=0}^{\infty} m^2 (1-p)^m \cdot p = \sum_{m=1}^{\infty} m^2 (1-p)^m \cdot p$$

$$= (1-p) \sum_{m=1}^{\infty} m^2 (1-p)^{m-1} \cdot p$$

$$= (1-p) E(x^2)$$

2nd term

$$\sum_{k=1}^{\infty} 1 \cdot (1-p)^{k-1} \cdot p = 1$$ (4)

$$2 \cdot \sum_{k=1}^{\infty} (k-1)\,(1-p)^{k-1} \cdot p$$

$m = k-1$

$$2 \cdot \sum_{m=0}^{\infty} m \cdot (1-p)^{m} \cdot p$$

$$2 \cdot \sum_{m=1}^{\infty} m \cdot (1-p)^{m} \cdot p$$

$$2\,(1-p) \sum_{m=1}^{\infty} m\,(1-p)^{m-1} \cdot p$$

$E(x) = 1/p$

$$= \frac{2(1-p)}{p}$$

$$E(X^2) = (1-p)\,E(X^2) + 1 + \frac{2(1-p)}{p}$$

$$Var(X) = E(X^2) - (E(X))^2 = \frac{1-p}{p^2}$$

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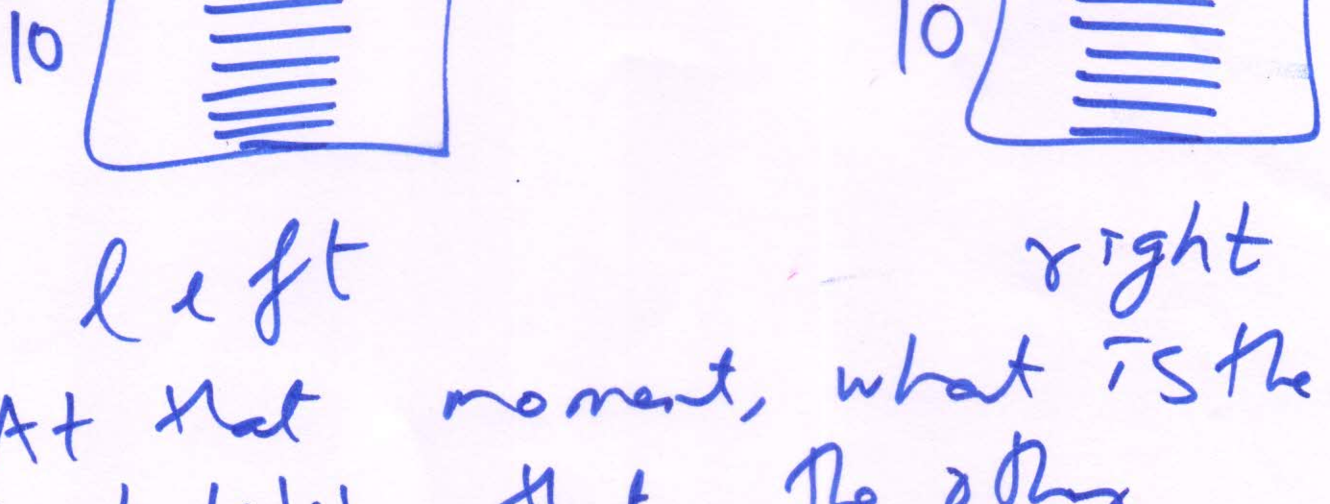
e.g. an absent-minded chain-smoking mathematician

Banach match Problem



left right

At that moment, what is the probability that the other matchbox has 3 matches?

Negative Binomial random variable.

Keep repeating an experiment until you have accumulated r successes.

X = no. of trials.

$P(\text{success}) = p$

$P(\text{failure}) = 1-p$

$$P(X = i) = \binom{i-1}{r-1} p^{r-1} (1-p)^{i-r} \cdot p$$

$\longleftarrow r-1 \longrightarrow$ Success

$\overline{1}$ $\overline{2}$ $\overline{3}$ $\overline{4}$ . . . $\overline{i-1}$ $\overline{i}$

$\longleftarrow r$ successes $\longrightarrow$

Banach Match Problem

what is success? putting hand in the left pocket.

$p(\text{success}) = 1/2$

$\rightarrow r = 11$

$\rightarrow i = 18$

$$\underbrace{11}_{\text{left pocket}} + \underbrace{(10-3)}_{\text{right pocket}}$$

$$\frac{\text{left pocket is empty.}}{P(\overline{\text{right pocket has 3 matches}})}$$