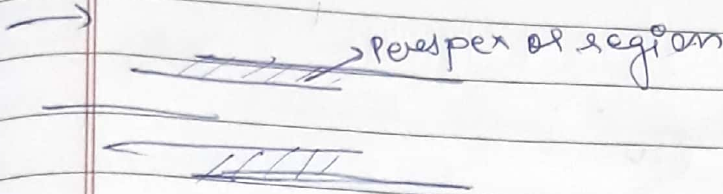
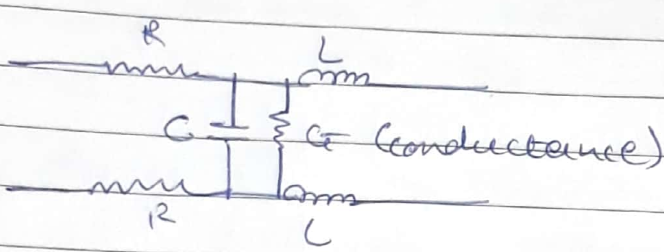
16/9/19



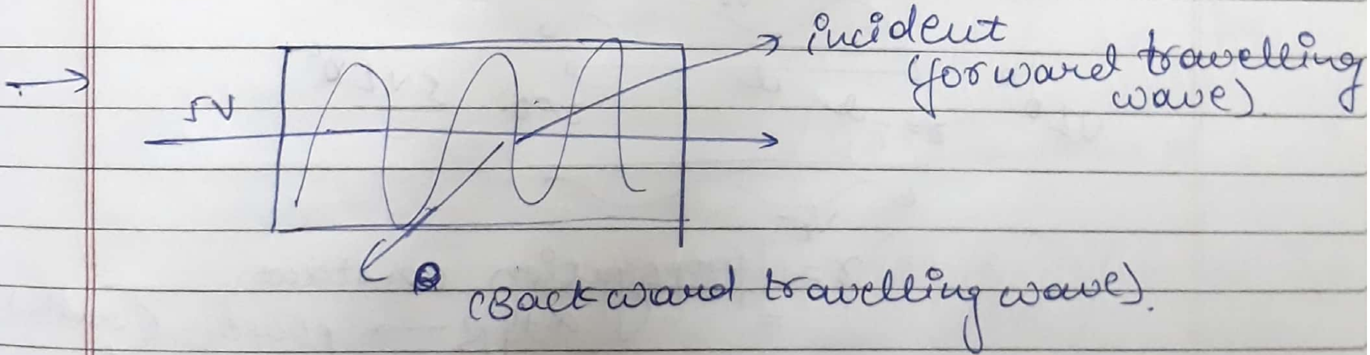


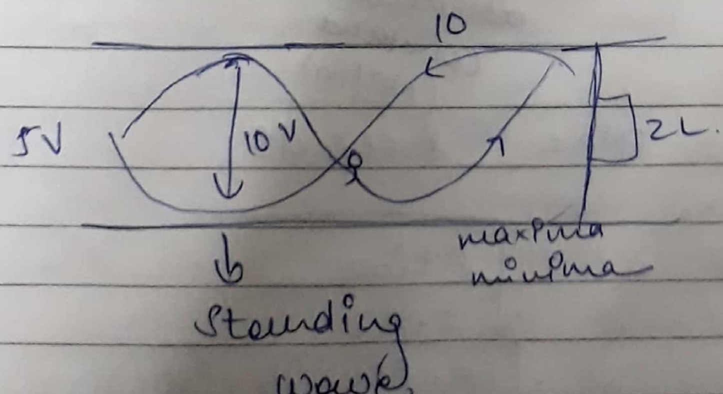
R → due to material by conductance.

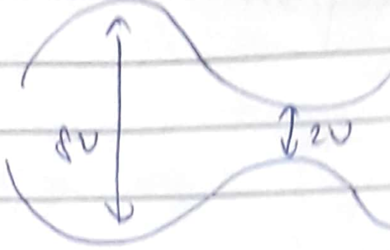
L → required magnetic flux generated by current on Tx line.

C → Capacitance (due to 2 conductors seperated by dielectric)

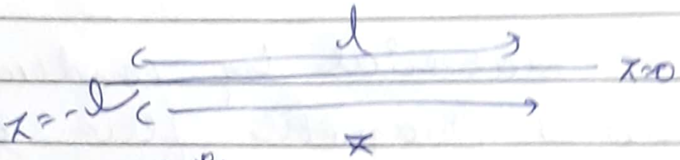
G → Conductance because of coupling ($\neq 1/R$) b/w 2 conductors.







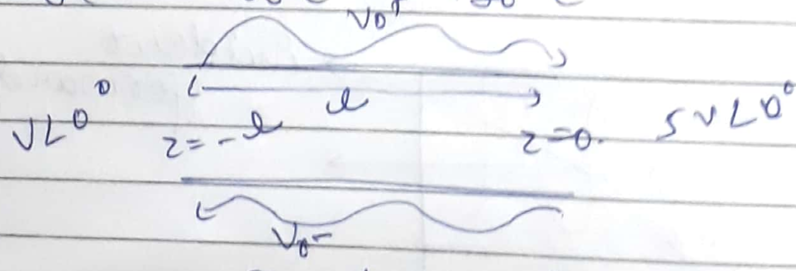
V(z)



(characteristic impedance) $Z_0 = 50\Omega$

$$V(z) = \underbrace{V_0^+ e^{-\gamma z}}_{\text{forward}} + \underbrace{V_0^- e^{\gamma z}}_{\text{Reverse}}$$

$$I(z) = I_0^+ e^{-\gamma z} + I_0^- e^{\gamma z}$$



γ = propogation constant
$= \alpha + j\beta$

α → attenuation constant (Np/m).

β → phase (constant)

1Np = 8.686 dB

→ $Z_0 \, (\Omega) = R_0 + jX_0 = \sqrt{\dfrac{R + j\omega L}{G + j\omega C}}$

$\gamma \, (m^{-1}) = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)}$

$$v_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}}$$

→ Case-1

Lossless line (ideal) $- R=0, G=0$

$$\gamma = j\omega\sqrt{LC} = \alpha + j\beta$$

$\therefore \alpha = 0, \beta = \omega\sqrt{LC}$

$$Z_0 = \sqrt{\frac{L}{C}} = R_0 + jX_0$$

$\therefore R_0 = \sqrt{\frac{L}{C}}, \quad X_0 = 0$

→ Case-2

Low loss line. $- R << \omega L, \; G << \omega C$

$$\gamma = \alpha + j\beta = \sqrt{(R+j\omega L)(G+j\omega C)}$$

$$= j\omega\sqrt{LC}\left(1 + \frac{R}{j\omega L}\right)^{1/2}\left(1 + \frac{G}{j\omega C}\right)^{1/2}$$

$$\left\{(1+x)^n = 1 + nx + \dots\right\}$$

$$= j\omega\sqrt{LC}\left(1 + \frac{R}{2j\omega L}\right)\left(1 + \frac{G}{2j\omega C}\right) + \dots$$

$$= j\omega\sqrt{LC}\left[1 + \frac{1}{2}\left(\frac{R}{L} + \frac{G}{C}\right)\right] + \dots$$

$$= \underbrace{j\omega\sqrt{LC}}_{\beta} + \underbrace{\frac{1}{2}R\sqrt{\frac{C}{L}} + \frac{1}{2}G\sqrt{\frac{L}{C}}}_{\alpha}$$

$$Z_0 = R_0 + jX_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

Distortionless

Losses of electric field = losses of magnetic field

$\boxed{RC = GL}$ $\boxed{\frac{R}{L} = \frac{G}{C}}$

$$= \sqrt{\frac{\frac{R}{j\omega L}+1}{\frac{G}{j\omega C}+1} \cdot \frac{j\omega L}{j\omega C}}$$

$$= \sqrt{\frac{L}{C}}\left(1+\frac{R}{j\omega L}\right)^{1/2}\left(1+\frac{G}{j\omega C}\right)^{-1/2}$$

$$\approx \sqrt{\frac{L}{C}}\left[1+\frac{1}{2j\omega}\left(\frac{R}{L}-\frac{G}{C}\right)\right]$$

$$\therefore R_0 = \sqrt{\frac{L}{C}}, \quad X_0 = \sqrt{\frac{L}{C}} \cdot \frac{1}{2j\omega}\underbrace{\left(\frac{R}{L}-\frac{G}{C}\right)}_{\approx 0}$$

$$v_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}}.$$

$$\gamma = \alpha + j\beta = \sqrt{(R+j\omega L)(G+j\omega C)}$$

$$= \sqrt{(R+j\omega L)\left(\frac{RC}{L}+j\omega C\right)}$$

$$= \sqrt{\frac{R^2C}{L} - \omega^2 LC + j\omega RC + j\omega CR}$$

$$= \sqrt{\frac{R^2C}{L} - \omega^2 LC + 2j\omega CR}$$

$$= \sqrt{\frac{C}{L}(R^2 - \omega^2L^2 + 2j\omega RL)}$$

$$= \sqrt{\frac{C}{L}(R+j\omega L)^2}$$

$$= \sqrt{\frac{C}{L}}(R+j\omega L) = \gamma = \alpha + j\beta$$

$$\therefore \alpha = R\sqrt{\frac{C}{L}} = \sqrt{RG} \quad \left(\because G = \dots\right.$$

$$\beta = \omega\sqrt{LC}$$

$$v_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}}$$

$$Z_0 = R_0 + jX_0 = \sqrt{\frac{(R+j\omega L)}{(G+j\omega C)}}$$

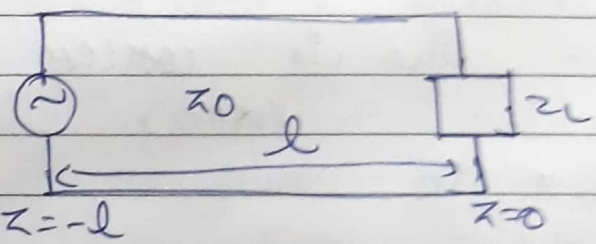
$$= \sqrt{\frac{R+j\omega L}{\left(\frac{RC}{L}\right)+j\omega C}}$$

$$R_0 = \sqrt{\frac{L}{C}} = \sqrt{\frac{R}{G}}$$

$$X_0 \approx 0$$

19/9/19

→



$$\gamma = \alpha + j\beta = \sqrt{(R+j\omega L)(G+j\omega C)}$$

γ : propagation constant (m^{-1})

α : attenuation constant (Np/m)

β : phase constant (rad/m)

$$\underset{(\Omega)}{Z_0} = \sqrt{\frac{R+j\omega L}{G+j\omega C}} \text{ (complex)}$$

characteristic impedance

- Lossless line , ($R=0$, $G=0$)
- Lowloss line ($R<<\omega L$, $G<<\omega C$)
- Distortionless line ($RL = GL$)

$$v_p = \frac{\omega}{\beta} \quad , \beta = \frac{2\pi}{\lambda}$$

$$V(z) = \underset{\substack{\downarrow \\ \text{forward} \\ \text{(incident)}}}{V_0^+ e^{-\gamma z}} + \underset{\substack{\downarrow \\ \text{backward} \\ \text{(reflected)}}}{V_0^- e^{+\gamma z}}$$

$$I(z) = I_0^+ e^{-\gamma z} + I_0^- e^{+\gamma z}$$
$$= \frac{V_0^+}{Z_0} e^{-\gamma z} + I_0^- e^{+\gamma z}$$

$$\left[\because Z_0 = \frac{V_0^+}{I_0^+} = \frac{-V_0^-}{I_0^-} \right]$$

→ Assumption:- line is lossless ($\alpha = 0$)

$$\gamma = \alpha + j\beta = j\beta$$

$$\left[\begin{aligned} V(z) &= V_0^+ e^{-j\beta z} + V_0^- e^{+j\beta z} \\ I(z) &= \frac{V_0^+}{Z_0} e^{-j\beta z} - \frac{V_0^-}{Z_0} e^{+j\beta z} \end{aligned}\right] \begin{aligned} &— \text{(A)} \\ &— \text{(B)} \end{aligned}$$

→ Voltage reflection co-efficient (Γ or Γ)
load is at $z=0$.

(A) V_L (Voltage at load) $= V(z=0)$
$= V_0^+ + V_0^-$

(B) I_L (Current at load) $= I(z=0)$
$= \frac{1}{Z_0} [V_0^+ - V_0^-]$

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(Anechoic chamber)

$$Z_L = \frac{V_L}{I_L}$$

$$Z_L = \frac{V_0^+ + V_0^-}{\frac{1}{Z_0}[V_0^+ - V_0^-]}$$

$$\Rightarrow V_0^- = \left(\frac{Z_L - Z_0}{Z_L + Z_0}\right) V_0^+$$

$$\Rightarrow \frac{V_0^-}{V_0^+} = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$\Rightarrow \boxed{\Gamma = \frac{V_0^-}{V_0^+} = \frac{Z_L - Z_0}{Z_L + Z_0}} \quad (Z_0 = 50 + j0)$$

complex quantity.

$$\boxed{\frac{I_0^-}{I_0^+} = \frac{-V_0^-}{V_0^+} = -\Gamma}$$

Γ = complex quantity. ($\because Z_L$ is complex).

$$= \underbrace{|\Gamma|}_{\text{Magnitude of reflection co-efficient}} e^{j\theta_r} = |\Gamma| \underbrace{\angle \theta_r}_{\text{angle of ref. co-efficient}}$$

→ Matched line – ($Z_L = Z_0$)

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = 0.$$

– Short-circuited line. ($Z_L = 0$)

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = -1 = 1 \angle 180^\circ$$

- open-circuited line : $Z_L = \infty$.

$\Gamma = 1 \quad = 1\angle 0^\circ$.

→ TDR (Time Domain Reflectometer).
OTDR (Optical ").

$-1 < \Gamma < 1$

$0 \le |\Gamma| \le 1$

- VSWR (Voltage Standing Wave Ratio)

$VSWR = \dfrac{1+|\Gamma|}{1-|\Gamma|}$.

(Matched load)

ML - 1
SC - ∞
OC - ∞

$1 < VSWR < \infty$

→ Standing wave (Lossless $\alpha = 0 \Rightarrow \gamma = j\beta$)

(A) : $V(z) = V_0^+ e^{-j\beta z} + V_0^- e^{+j\beta z}$

(B) : $I(z) = \dfrac{V_0^+}{Z_0} e^{-j\beta z} - \dfrac{V_0^-}{Z_0} e^{+j\beta z}$

$\Rightarrow \left[\begin{aligned} V(z) &= V_0^+ (e^{-j\beta z} + \Gamma e^{j\beta z}) \\ I(z) &= I_0^+ (e^{-j\beta z} - \Gamma e^{j\beta z}) \end{aligned} \right]$

$\left[\because \dfrac{V_0^-}{V_0^+} = \Gamma \Rightarrow V_0^- = \Gamma V_0^+ \right]$

$$|V(z)| = [V(z) \cdot \underbrace{V^*(z)}_{\text{complex conjugate}}]^{1/2}$$

$$= [\{(V_0^+)(e^{-j\beta z} + \Gamma e^{j\beta z})\}. \{(V_0^+)^* (e^{j\beta z} + \Gamma e^{-j\beta z})\}]^{1/2}$$

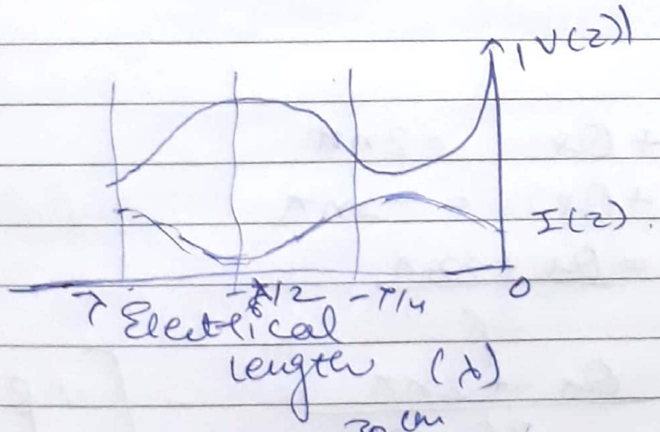
$$= [\{V_0^+ (e^{-j\beta z} + |\Gamma| e^{j\theta_\Gamma} e^{j\beta z})\}. \{(V_0^+)^* (e^{j\beta z} + |\Gamma| e^{-j\theta_\Gamma} e^{-j\beta z})\}]^{1/2}$$

$$[\because \Gamma = |\Gamma| e^{j\theta_\Gamma}]$$

$$= |V_0^+| [1 + |\Gamma|^2 + |\Gamma| e^{j(2\beta z + \theta_\Gamma)} + e^{-j(2\beta z + \theta_\Gamma)}]$$

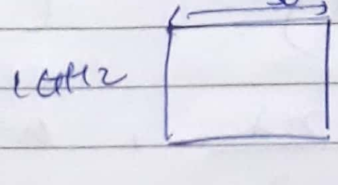
$$|V(z)| = |V_0^+| [1 + |\Gamma|^2 + 2|\Gamma| \cos(2\beta z + \theta_\Gamma)]^{1/2}$$

$$[\because e^{x} + e^{-jx} = 2\cos x].$$



$\beta = \frac{2\pi}{\lambda}$

$z \rightarrow (\lambda)$



$l = \lambda$ at 1GHz.

(7.5 cm) $l = \frac{\lambda}{4}$ at 1GHz.

(15 cm) $l = \frac{\lambda}{2}$ @ 1GHz.

20/9/19

* Standing Waves (condition):

→ $|V(z)| = |V_0^+| [1 + |\Gamma|^2 + 2|\Gamma| \cos(2\beta z + \theta_r)]^{1/2}$

$\beta = \frac{2\pi}{\lambda}$, θ_r = angle of reflection Co-efficient (Γ)

– Our objective:
1. To find positions of maxima & minima.
2. To find amplitudes of maxima & minima.

→ Maxima:- The max^m value of standing wave pattern of $|V(z)|$ corresponds to positions on line at which incident &

$\rightarrow \quad 2\beta P_{max} + \theta_r = 2n\pi$

$\rightarrow \quad -2\beta P_{max} + \theta_r = -2n\pi$

$\Rightarrow \quad P_{max} = \frac{\theta_r + 2n\pi}{2\beta}$

$= \frac{\theta_r + 2n\pi}{2\left(\frac{2\pi}{\lambda}\right)} \qquad \left[\because \beta = \frac{2\pi}{\lambda}\right]$

$P_{max} = \frac{\theta_r \lambda}{4\pi} + \frac{n\lambda}{2}, \quad n = 0, 1, 2, \ldots$

first max^m will occur at,

$\frac{\theta_r \lambda}{4\pi}$ (for $n=0$)

2nd ...... $\frac{\theta_r \lambda}{4\pi} + \frac{\lambda}{2}$ (for $n=1$)

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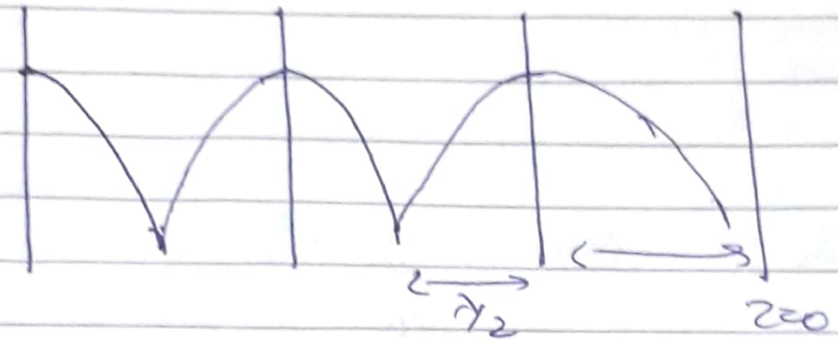
3rd ...... $\frac{\phi_s \lambda}{4\pi} + \lambda$ (for $n=2$)



Distance b/w 2 successive maxima = $\lambda/2$

→ Minima :-

$(2\beta z + \phi_s) = (2n+1)\pi$, $n =$

↓

P_{min}

$$l_{min} = \frac{\theta_r + (2n+1)\pi}{2\beta}$$

$=$

1^{st} min^m :

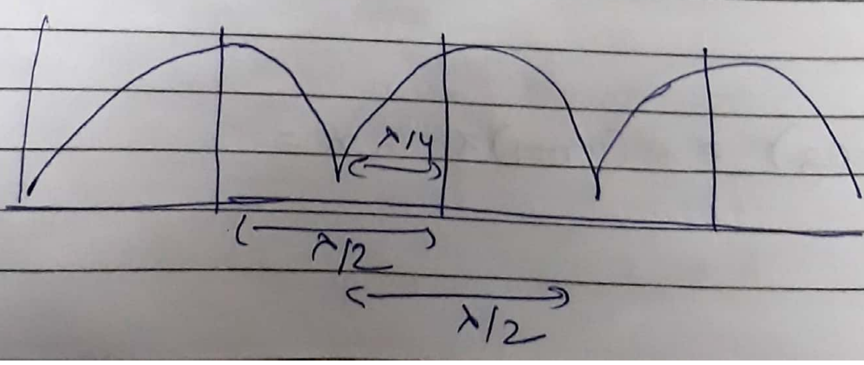
$$l_{min_1} = \frac{\theta_r \lambda}{4\pi} + \frac{\pi}{4\pi/\lambda} \quad (n=0)$$

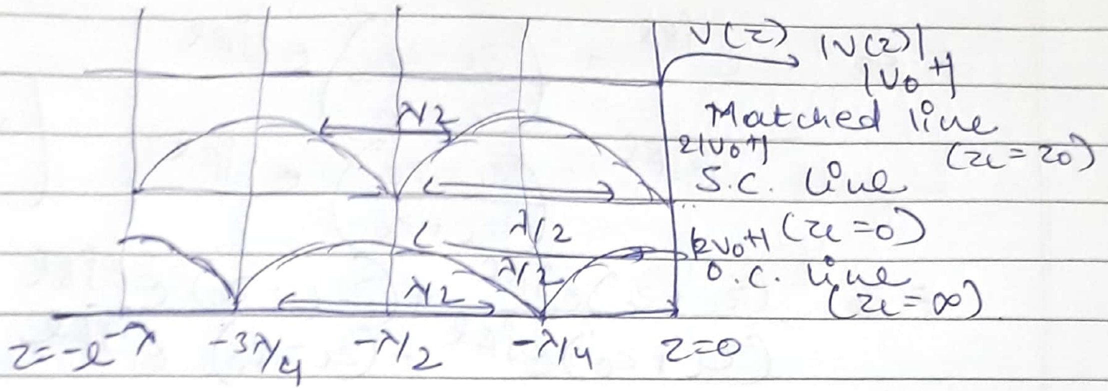
$$l_{min_1} = \frac{\theta_r \lambda}{4\pi} + \frac{\lambda}{4} \quad (n=0)$$

$$l_{min_2} = \frac{\theta_r \lambda}{4\pi} + \frac{3\lambda}{4} \quad (n=1)$$

Distance b/w 2 successive minima $= \lambda/2$

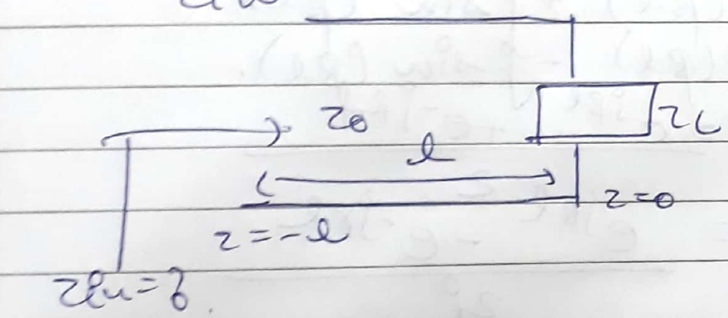
Distance b/w max^m & subsequent min^m $= \lambda/4$





$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$$

→ Z_{in}



(A) : $V(z) = V_0^+ e^{-j\beta z} + V_0^- e^{+j\beta z}$

(B) : $I(z) = \frac{V_0^+}{Z_0} e^{-j\beta z} - \frac{V_0^-}{Z_0} e^{+j\beta z}$

$Z_{in}\,(z=-l)$ from load $= \frac{V\,(z=-l)}{I\,(z=-l)}$

$$Z_{in} = Z_0 \, \frac{V_0^+ e^{j\beta l} + V_0^- e^{-j\beta l}}{V_0^+ e^{j\beta l} - V_0^- e^{-j\beta l}}$$

$$= Z_0 \, \frac{V_0^+ \left(e^{j\beta l} + V_0^-/V_0^+ \, e^{-j\beta l}\right)}{V_0^+ \left(e^{j\beta l} - \frac{V_0^-}{V_0^+} \, e^{-j\beta l}\right)}$$

$$= Z_0 \frac{e^{j\beta l} + \Gamma e^{-j\beta l}}{e^{j\beta l} - \Gamma e^{-j\beta l}} \quad \left[\frac{V_0^-}{V_0^+} = \Gamma\right]$$

$$= Z_0 \cdot \frac{e^{j\beta l} + \left(\frac{Z_L - Z_0}{Z_L + Z_0}\right) e^{-j\beta l}}{e^{j\beta l} - \left(\frac{Z_L - Z_0}{Z_L + Z_0}\right) e^{-j\beta l}} \quad \left[\because \Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}\right]$$

$$= Z_0 \cdot \frac{(Z_L + Z_0) e^{j\beta l} + (Z_L - Z_0) e^{-j\beta l}}{(Z_L + Z_0) e^{j\beta l} - (Z_L - Z_0) e^{-j\beta l}}$$

$$= Z_0 \cdot \frac{Z_L (e^{j\beta l} + e^{-j\beta l}) + Z_0 (e^{j\beta l} - e^{-j\beta l})}{Z_L (e^{j\beta l} - e^{-j\beta l}) + Z_0 (e^{j\beta l} + e^{-j\beta l})}.$$

$$e^{j\beta l} = \cos(\beta l) + j \sin(\beta l)$$

$$e^{-j\beta l} = \cos(\beta l) - j \sin(\beta l).$$

$$\rightarrow \cos(\beta l) = \frac{e^{j\beta l} + e^{-j\beta l}}{2}$$

$$\rightarrow \sin(\beta l) = \frac{e^{j\beta l} - e^{-j\beta l}}{2j}$$

$$Z_{in} = Z_0 \cdot \frac{Z_L \cos(\beta l) + j Z_0 \sin(\beta l)}{j Z_L \sin(\beta l) + Z_0 \cos(\beta l)}$$

$$Z_{in}(z = -l) = Z_0 \cdot \frac{Z_L + j Z_0 \tan(\beta l)}{Z_0 + j Z_L \tan(\beta l)}$$

$$\beta = \frac{2\pi}{\lambda}$$

$R = \Omega / m$

$L = mH/m.$

→ distributed elements.

lumped elements.

If line is lossy. ($\alpha \neq 0$, $\gamma = \alpha + j\beta$)

$$Z_{in}\ (\Omega) = Z_0 \cdot \frac{Z_L + Z_0 \tanh(\gamma l)}{Z_0 + Z_L \tanh(\gamma l)}$$

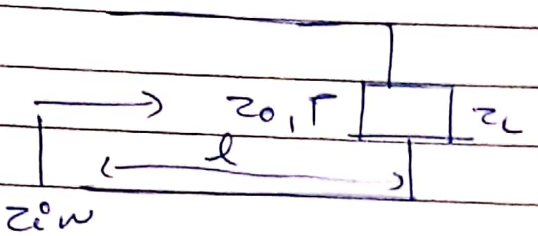
$\tanh(\gamma l) = \tanh(j\beta l) = j \tan\beta l$

↳ for lossless ($\gamma = j\beta$)

23/9/19

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$\boxed{Z_{in} = Z_0 \cdot \frac{Z_L + jZ_0 \tan(\beta l)}{Z_0 + jZ_L \tan(\beta l)}}$$ lossless



$, \beta = \frac{2\pi}{\lambda}$

Case-5 Lossless short-circuited line:-

$Z_L = 0$ (short-circuited).

(b) (SC).

$$Z_{in}^{SC} = Z_0 \cdot \frac{jZ_0 \tan\beta l}{Z_0}$$

$$\boxed{Z_{in}^{SC} = jZ_0 \tan\beta l}$$

purely reactive.
(Lossc).

→ At $l = 0$, $Z_{in}^{SC} = 0$ [$\because \tan 0 = 0$]

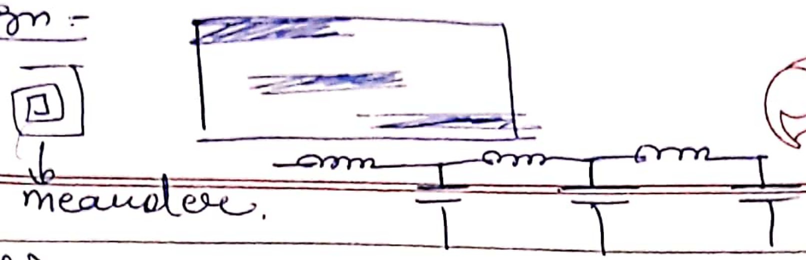
$l = \frac{\lambda}{4}$, $\beta l = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{4}$

$= \frac{\pi}{2}$

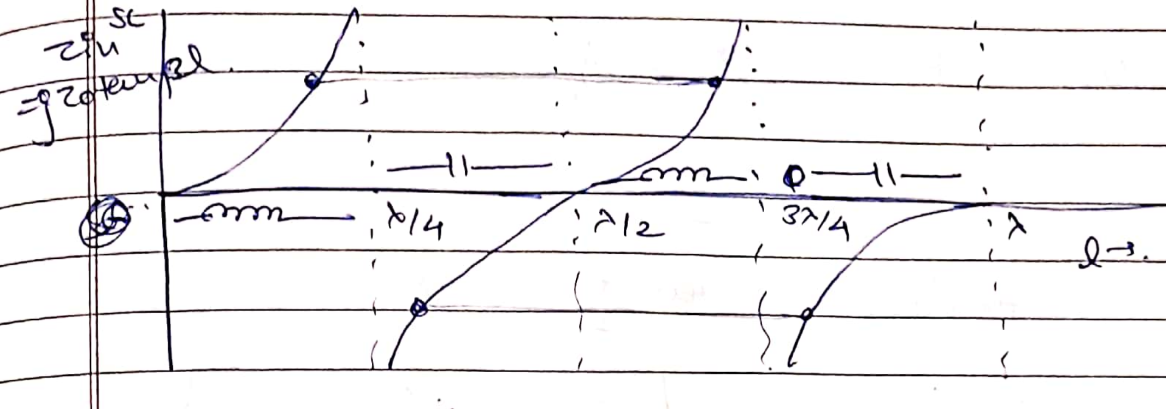
$\tan(\beta l) = \tan\left(\frac{\pi}{2}\right) = \infty$

$l = \frac{\lambda}{2}$, $\beta l = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{2} = \pi.$

Des Rgn :-



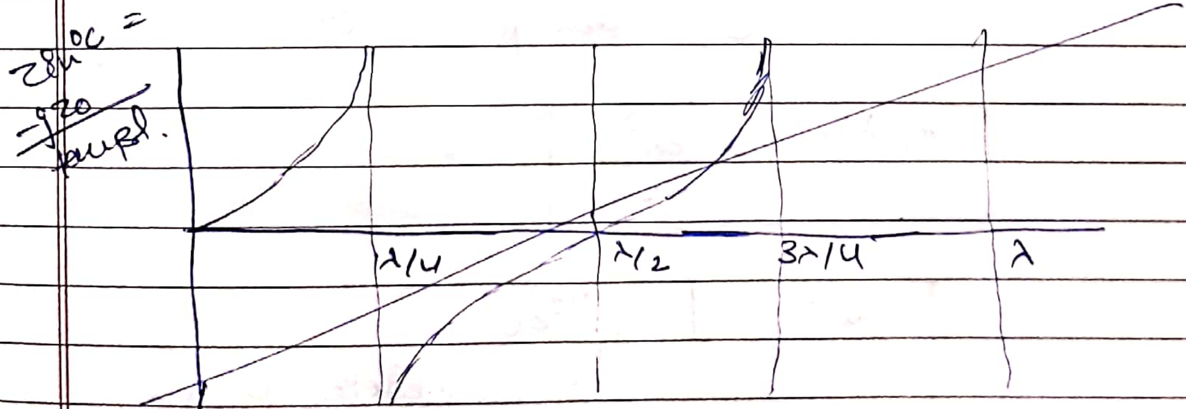
$\tan(\beta l) = \tan \pi = 0.$

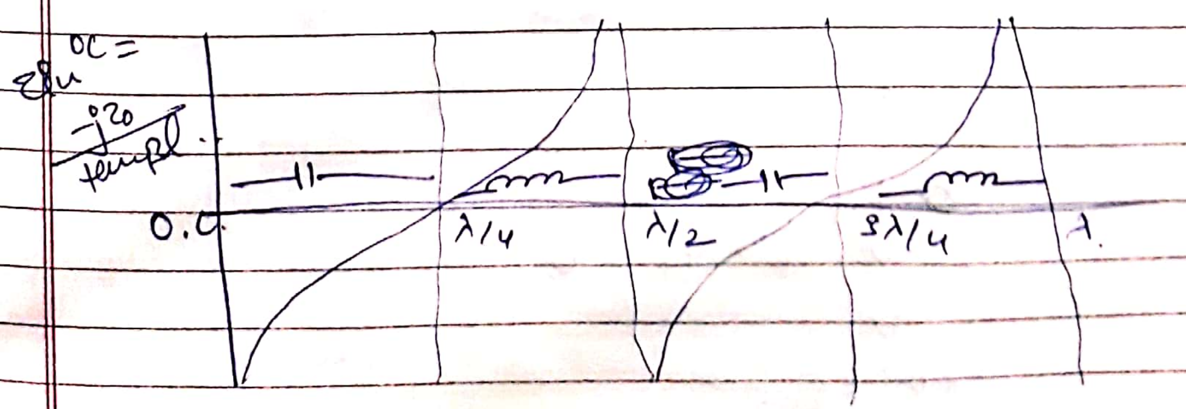


→ Case-2 - Lossless O.C. line.
$(Z_L = \infty)$

(b) : $Z_{in}^{OC} = \dfrac{-jZ_0}{\tan\beta l} = -jZ_0 \cot\beta l.$

~~Lossy line~~





→ Case-3 – $l = \frac{\lambda}{4}$

(i) : $\beta l = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{4}$

$= \frac{\pi}{2}$

$\tan(\beta l) = \tan\frac{\pi}{2} = \infty$

(ii) : $Z_{in}^{\lambda/4} = Z_0 \left[\dfrac{\dfrac{Z_L}{\tan\beta l} + jZ_0}{\dfrac{Z_0}{\tan\beta l} + jZ_L}\right]$

$Z_{in}^{\lambda/4} = Z_0 \left[\dfrac{jZ_0}{jZ_L}\right]$

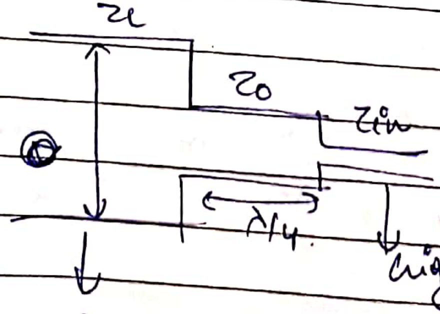
$\left[\because \dfrac{1}{\tan\beta} = \dfrac{1}{\infty} \to 0\right]$

$$\boxed{Z_{in}^{\lambda/4} = \frac{Z_0^2}{Z_L}}$$

$$\boxed{Z_0 = \sqrt{Z_{in}^{\lambda/4} Z_L}}$$

→ geometric mean.

→



low impedance

high impedance.

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$= \frac{75-50}{75+50}$$

$$= \frac{25}{125}$$

$$= 20\%$$

$$Z_0 = \sqrt{Z_{in} Z_L}$$

$$= \sqrt{50 \times 75}$$

$$= 65\Omega$$

— Impedance Transformer.
(Quarter Wave Transformer) ($\lambda/4$)
(Impedance inverter).

$OC \rightleftharpoons SC$

induction $\rightleftharpoons$ capacitive.

→ Case-4 — $l = \frac{\lambda}{2}$

$$\beta l = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{2}$$

$$\tan(\beta l) = \tan\pi = 0$$

(D): $Z_{in}^{\lambda/2} = Z_0 \cdot \frac{Z_L + 0}{Z_0 + 0}$

$$\boxed{Z_{in}^{\lambda/2} = Z_L}$$

↓

Half wave transformer.

30/9/19

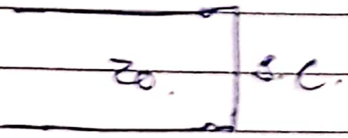
→ $Z_{in}^{sc} = jZ_0 \tan\beta l$

$Z_{in}^{oc} = \dfrac{-jZ_0}{\tan(\beta l)}$

$\dfrac{\lambda}{4} : Z_0 = \sqrt{Z_{in} Z_L}$

$\lambda/2 : Z_0 = Z_{in}$

* Equivalent Reactive Elements (L/C).

→ 

Equivalent L (or) $Z_{in} = jZ_0 \tan\beta l$

$\Rightarrow jX_{in} = jZ_0 \tan\beta l$

$\Rightarrow j\omega L_{eq} = jZ_0 \tan\beta l$

$\Rightarrow L_{eq} = \dfrac{Z_0 \tan\beta l}{\omega}$ (H).

$Z = R + jX,$
$X = j\omega L$

$\downarrow$

$0 \le l \le \lambda/4$

$\frac{\lambda}{2} \le l \le 3\lambda/4$

$\omega = 2\pi f, \ \beta = \dfrac{2\pi}{\lambda}.$

$l = \dfrac{1}{\beta} \tan^{-1}\left(\dfrac{\omega L_{eq}}{Z_0}\right)$ (m).

Equivalent C (or).

$Z_{in} = jZ_0 \tan\beta l$

$\Rightarrow jX_{in} = jZ_0 \tan\beta l$

$\Rightarrow \dfrac{-j}{\omega C_{eq}} = jZ_0 \tan\beta l = \dfrac{1}{j\omega C_{eq}}$

$\downarrow$

$\frac{\lambda}{4} \le l \le \lambda/2, \ \frac{3\lambda}{4} \le l \le \lambda, \ldots$

classmate

Date

Page

$l = \frac{-1}{\beta} \tan^{-1} \left(\quad \right)$

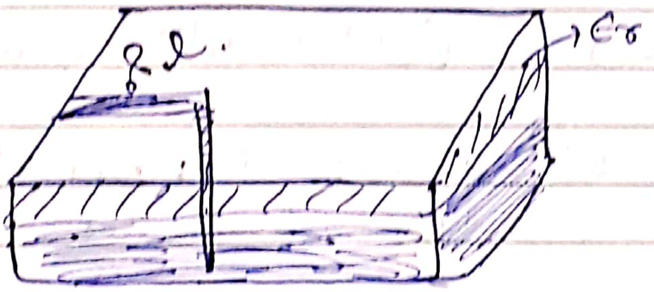
eg

$C_{eq} = 4pF$ @ 2.025GHz.

using a short-circuited Tx line

wave velocity $= \frac{c}{\sqrt{\epsilon_r}} = 0.75c$.

(or phase velocity)



$Z_{in}^{sc} = jZ_0 \tan\beta l = \frac{-j}{\omega C_{eq}} = jZ_0 \tan\beta l.$

$\Rightarrow \frac{1}{j\omega C_{eq}} = \frac{-j}{\omega C_{eq}}$

$\Rightarrow \tan\beta l = \frac{-1}{Z_0 \omega C_{eq}}$

$\omega = 2\pi f.$

$f = 2.25 \times 10^9$

$v_p = 0.75c = 0.75 \times 3 \times 10^8$

$= 2.25 \times 10^8 \text{ m/s}$

$\lambda = \frac{v_p}{f}$

$\beta = \frac{2\pi}{\lambda} = \frac{2\pi f}{v_p} = \frac{2\pi \times 2.25 \times 10^9}{2.25 \times 10^8}$

$= 20\pi$

$\beta = 62.8$ radian..

$$\tan\beta l = \frac{-1}{Z_0 \omega C_{eq}}$$

$$\tan\beta l = \frac{1}{\underbrace{50\times 2\pi\times 2.25\times 10^{9}}_{\omega = 2\pi f} \times \underbrace{4\times 10^{-12}}_{C=}}$$

$$\tan\beta l = -0.03054$$

tan(-ve) (2^{nd} & 4^{th} quadrant)

Sol^n in 2^{nd} quadrant : $\beta l_1 = 2.8$ rad.

or, $l_1 = \frac{2.8}{\beta} = \frac{2.8}{62.8}$

$= \underline{4.46 \text{ cm}}$

Sol^n in 4^{th} quadrant : $\beta l_2 = 5.94$ rad.

$\Rightarrow l_2 = \frac{5.94}{62.8} = \underline{9.46 \text{ cm}}$

$l_2 > l_1 = 5\text{cm}$ $(= \lambda/2)$

$\lambda = 10\text{cm}$..

$$\boxed{l = 4.46\text{cm} + \frac{n\lambda}{2}} \quad n = 0, 1, 2, \ldots$$

* Analysis of s.c. and o.c. mean :

→ $Z_{in}^{sc} = jZ_0 \tan\beta l$ ——— ①

$Z_{in}^{oc} = -jZ_0 \cot\beta l = \frac{-jZ_0}{\tan\beta l}$ ——— ②

Date
Page

① × ② $\rightarrow Z_0 = \sqrt{Z_{in}^{sc} Z_{in}^{oc}}$.

$\frac{①}{②} \Rightarrow \tan\beta l = \sqrt{\frac{-Z_{in}^{sc}}{Z_{in}^{oc}}}$.

$\beta = \frac{2\pi}{\lambda}$.

eg $Z_{in}^{sc} = j40.42\,\Omega$

$Z_{in}^{oc} = -j121.24\,\Omega$

$Z_0 = ?$

$\longrightarrow Z_0 = \sqrt{Z_{in}^{sc} Z_{in}^{oc}}$.

$= \sqrt{(j40.42)(-j121.24)}$

$= \underline{70\,\Omega}$

$\tan\beta l = \sqrt{\frac{-Z_{in}^{sc}}{Z_{in}^{oc}}} = \sqrt{\frac{1}{3}}$

$\beta = \frac{2\pi}{\lambda}$.

$\Rightarrow |\Gamma| = ?$ for a purely reactive load (inductive).

$Z_L = jX_L$

$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{jX_L - Z_0}{jX_L + Z_0}$.

$= \frac{-(Z_0 - jX_L)}{Z_0 + jX_L}$

$$\Gamma = \frac{-\sqrt{Z_0^2+X_L^2}\; e^{-j\theta}}{\sqrt{Z_0^2+X_L^2}\; e^{j\theta}}$$

$$= -e^{-j2\theta}.$$

$$\theta = \tan^{-1}\frac{X_L}{Z_0}.$$

$$|\Gamma| = |-e^{j2\theta}|$$

$$= \left[(e^{-j2\theta})(e^{-j2\theta})^*\right]^{1/2}.$$

$$\boxed{|\Gamma| = 1}$$

eg A $100\,\Omega$ Tx line is connected (terminated) to a load consisting of $50\,\Omega$ resistor in series with 10 pF Capacitor. Find Γ for 100 MHz sig.

$\rightarrow$ $\Gamma = \dfrac{Z_L - Z_0}{Z_L + Z_0}$

$Z_0 = 100\,\Omega$

$Z_L = 50\,\Omega$ in series with 10pF C $\rightarrow jX_C$.

$$= 50 - j\frac{1}{2\pi \times 10^{-8} \times 10^{-11}}$$

$$= 50 - j119\,\Omega$$

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = -0.76\, e^{j119.3^\circ}$$

($R+jX$
$R\angle\theta$
e^{θ})

$\Gamma = 0.76\, e^{j119.3}\, e^{-j180°}$

$= 0.76\, e^{-j60.7}$

$= 0.76 \angle -60.7°$

$|\Gamma| = 0.76$

* Powerflow in a Tx line :: (lossless)

→ (A) $V(z) = V_0^+ e^{-j\beta z} + V_0^- e^{+j\beta z}$

(B) $I(z) = \underbrace{\frac{V_0^+}{Z_0} e^{-j\beta z}}_{\text{forward wave}} - \underbrace{\frac{V_0^-}{Z_0} e^{-j\beta z}}_{\text{Backward wave}}$.

At load (z=0), incident and reflected voltages & currents.

$V^{in} = V_0^+$ $\qquad V^{ref} = V_0^-$

(1)

→ Defination of Time average power -

$$P_{av} = \frac{1}{2} \text{Re}[V \cdot I^*]$$ → complex conjugate

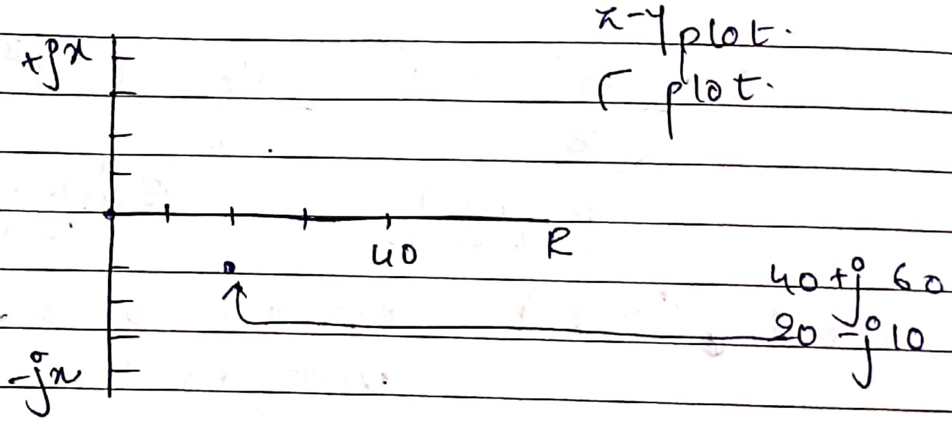
Incident

$$P_{av} = \frac{1}{2} \text{Re}\left[V_0^+ \cdot \frac{V_0^{+*}}{Z_0} \right]$$

7/10/19

x-y plot.
Γ plot.

→



$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$\Gamma_r + j\Gamma_i = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$= \frac{(Z_L - Z_0)/Z_0}{(Z_L + Z_0)Z_0}$$

$$\Gamma_r + j\Gamma_i = \frac{z - 1}{z + 1}$$

$$\Gamma_r + j\Gamma_i = \frac{r + jx - 1}{r + jx + 1}$$

($\because Z = R + jx$)

($\because z = r + jx$)

$$z = \frac{Z}{Z_0}$$

$$r = R/R_0$$

$$z_L = z_0\left(\frac{1+\Gamma}{1-\Gamma}\right)$$

$$z_L = \frac{1+\Gamma}{1-\Gamma}$$

$$r+jx = \frac{1+\Gamma}{1-\Gamma}$$

$$r+jx = \frac{(1+\Gamma_r)+j\Gamma_i}{(1-\Gamma_r)-j\Gamma_i}$$

$$r+jx = \frac{(1+\Gamma_r)+j\Gamma_i}{(1-\Gamma_r)-j\Gamma_i} \times \frac{(1-\Gamma_r)+j\Gamma_i}{(1-\Gamma_r)+j\Gamma_i}$$

$$= \frac{(1-\Gamma_r^2-\Gamma_i^2)+j(1+\Gamma_r)\Gamma_i+j(1-\Gamma_r)\Gamma_i}{1+\Gamma_r^2-2\Gamma_r+\Gamma_i^2}$$

$$\boxed{r+jx = \frac{(1-\Gamma_r^2-\Gamma_i^2)+2\Gamma_i}{1+\Gamma_r^2-2\Gamma_r+\Gamma_i^2}}$$

$\rightarrow$ r(real)

$$r = \frac{1-\Gamma_r^2-\Gamma_i^2}{1+\Gamma_r^2-2\Gamma_r+\Gamma_i^2}$$

$$\Rightarrow r + r\Gamma_r^2 - 2r\Gamma_r + r\Gamma_i^2 = 1-\Gamma_r^2-\Gamma_i^2$$

$$\Rightarrow \Gamma_r^2 + r\Gamma_r^2 - 2r\Gamma_r + r\Gamma_i^2 + \Gamma_i^2 = 1-r$$

$$\Rightarrow (1+r)\Gamma_r^2 - 2r\Gamma_r + (1+r)\Gamma_i^2 = 1-r$$

$$\Rightarrow \Gamma_r^2 - \frac{2r}{r+1}\Gamma_r + \Gamma_i^2 = \frac{1-r}{1+r}$$

Add

Subtract

$$\Rightarrow \Gamma_r^2 - \frac{2r}{r+1}\Gamma_r + \frac{r^2}{(r+1)^2} + \Gamma_i^2 - \frac{r^2}{(r+1)^2} = \frac{1-r}{1+r}$$

$$\Rightarrow \left(\Gamma_r - \frac{r}{r+1}\right)^2 + \Gamma_i^2 = \frac{1-r}{1+r} + \frac{r^2}{(1+r)^2}$$

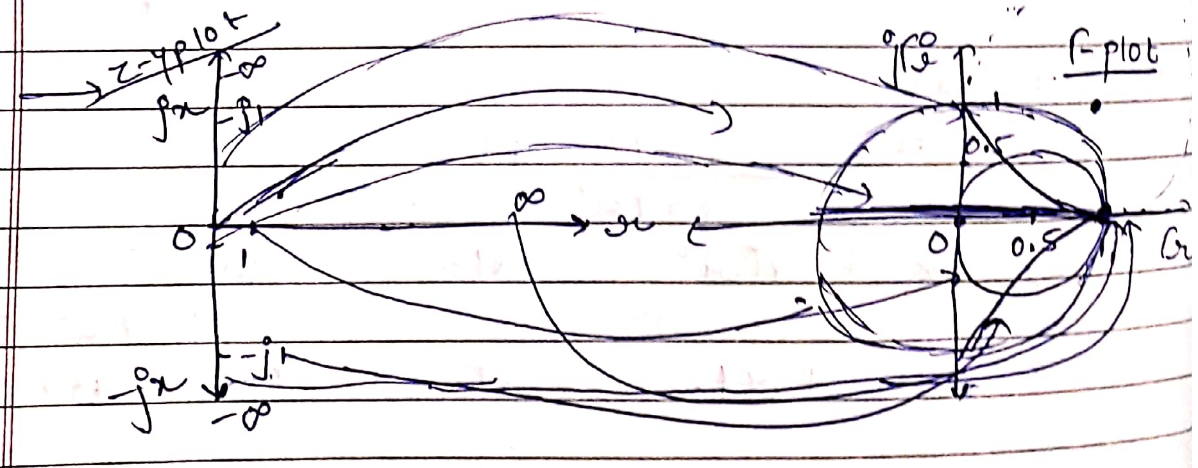
$$\Rightarrow \left(\Gamma_r - \frac{r}{r+1}\right)^2 + \Gamma_i^2 = \frac{1}{(1+r)^2} \quad -r^2 + r^2 \longrightarrow (A)$$

Eqn of a circle.

$(x-a)^2 + (y-b)^2 = r^2$
centre (a,b) & radius r

$$\therefore (a,b) = \left(\frac{r}{r+1}, 0\right)$$

$$r = \frac{1}{1+r}$$



for $r = 0$

(A) $(\Gamma_r - 0)^2 + \Gamma_i^2 = 1$

$\rightarrow$ center $(0,0)$, $r = 1$

for $r = 1$,

(A) $(\Gamma_r - 1/2)^2 + \Gamma_i^2 = (1/2)^2 \rightarrow$ center $(0.5, 0)$, $r = 0.5$

classmate

Date ____

Page ____

for $r = \infty$

(A) $\left(\frac{r}{r+1}, 0\right) \longrightarrow (1, 0)$

radius $\frac{1}{1+r} \longrightarrow 0$

$\Rightarrow$ x (imaginary)

$$x = \frac{2r_i}{1 + r_r^2 - 2r_r + r_i^2}$$

$$\Rightarrow x + x r_r^2 - 2x r_r + x r_i^2 = 2r_i$$

$$\Rightarrow 1 + r_r^2 - 2r_r - r_i^2 = 2r_i / x$$

$$\Rightarrow r_r^2 - 2r_r - 1 + r_i^2 - \frac{2r_i}{x} = 0.$$

$$\Rightarrow r_r^2 - 2r_r + 1 + r_i^2 - \frac{2r_i}{x} + \frac{1}{x^2} - \frac{1}{x^2} = 0$$

Add subtract.

$$\Rightarrow (r_r - 1)^2 + \left(r_i - \frac{1}{x}\right)^2 = \frac{1}{x^2} \longrightarrow \text{(B)}$$

Eqn of a circle.

$$(a, b) = \left(1, \frac{1}{x}\right)$$

$$r = \frac{1}{x^2}$$

for $x = \infty$

(B) center $(a, b) = \left(1, \frac{1}{x}\right)$, radius, $r = \frac{120}{x}$.

Scanned by CamScanner

for $x = 1$

(B) center $(a, b) = (1, 1)$, radius, $r = 1$

for $x = -1$

(B) $C = (1, -1)$, $r = 1$

for $x = 0$

$C = (1, \infty)$

$r = \infty$.

→ Z_L → complex

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$VSWR = \frac{1 + |\Gamma|}{1 - |\Gamma|} = (1, \infty)$$

10/10/19

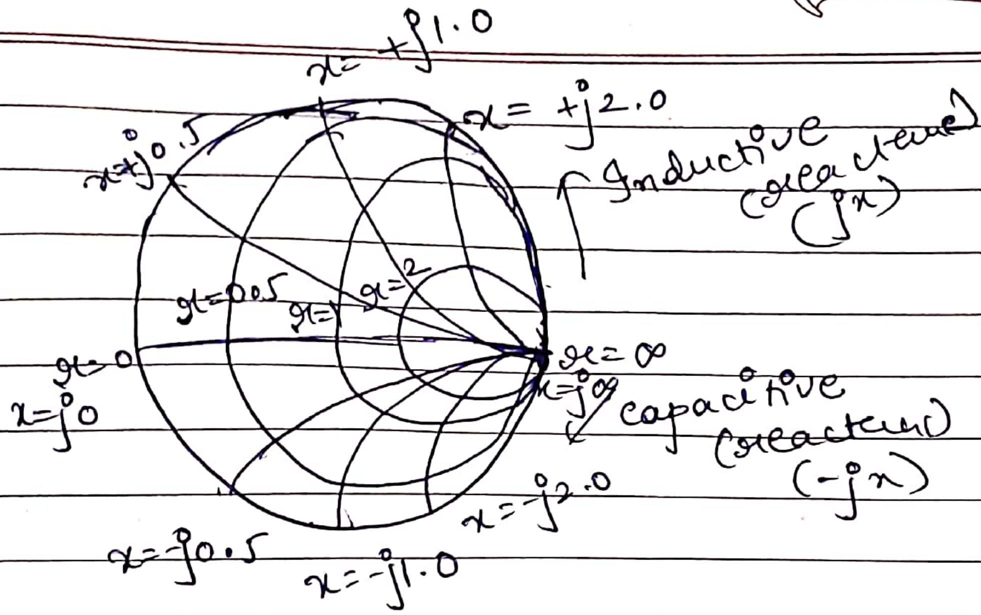
→ Rtn loss (dB) $= -20 \log (\Gamma)$

voltage ref. constant.

$$\frac{Z_L - Z_0}{Z_L + Z_0}.$$

→ Z - Y Smith Chart.

Γ plot



$Z = R + jx$

$\rightarrow \quad \Gamma = \dfrac{Z_L - Z_0}{Z_L + Z_0}$

$Z_L = Z_0 \left(\dfrac{1+\Gamma}{1-\Gamma}\right)$

$\dfrac{Z_L}{Z_0} = \dfrac{1+\Gamma}{1-\Gamma}, \quad z_L = \dfrac{1+\Gamma}{1-\Gamma}, \quad y_L = \dfrac{1}{z_L} = \dfrac{1-\Gamma}{1+\Gamma}$

$[-\Gamma = (\Gamma)\exp(j\pi)]$

Pr can now be

* Antenna :-

→ Radiation Eqn. :-

$\dot{I}L = Q\dot{v}$ → acceleration of charge (m/s^2) → (coulomb)

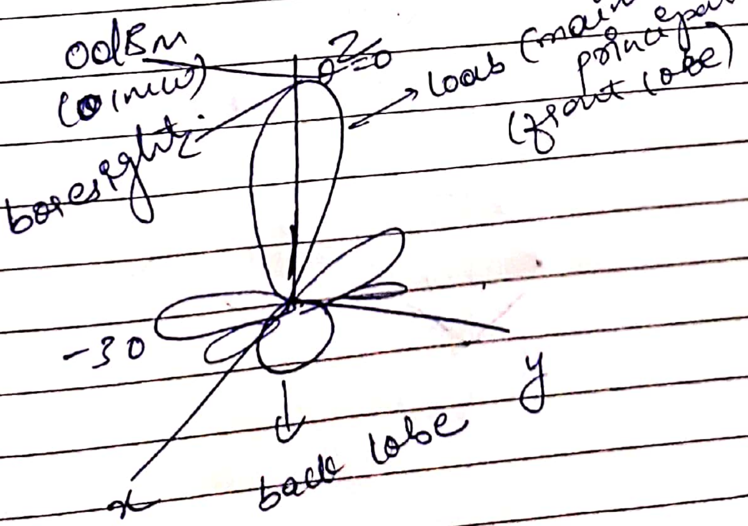
↓ Length of antenna (m)

↓ Time changing current (A/s)

- Time-changing current radiation
- accelerated charge radiation.

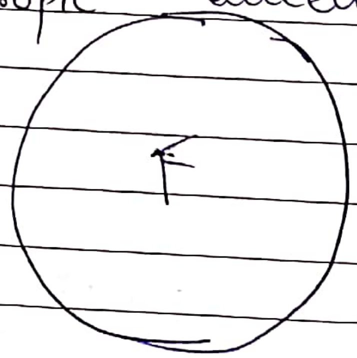
① Antenna should be able to search.
② detect
③ track

Radiation pattern



→ Omnidirection antenna -

↳ Isotropic antenna.



BW

⇓

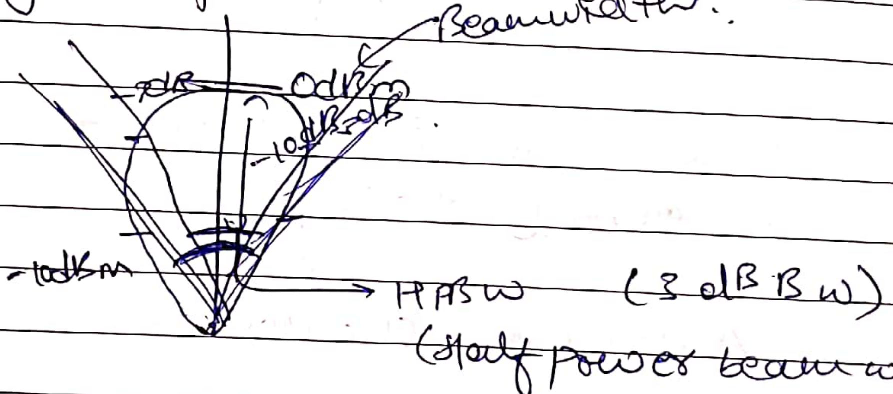
Δf Hz

$Q = \frac{f_0}{\Delta f}$

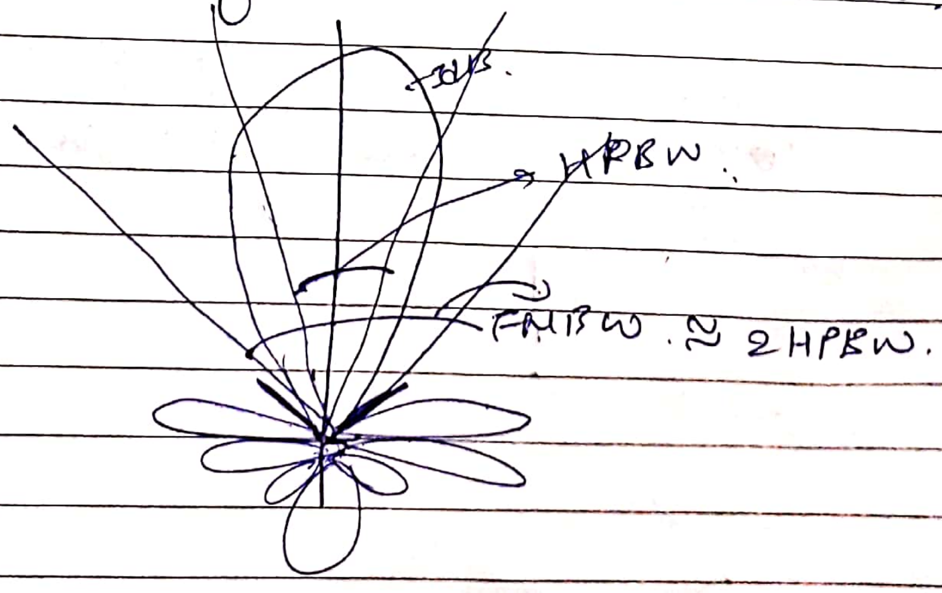
Beamwidth

↓

angle dependent.



FNBW (first null Beam width).

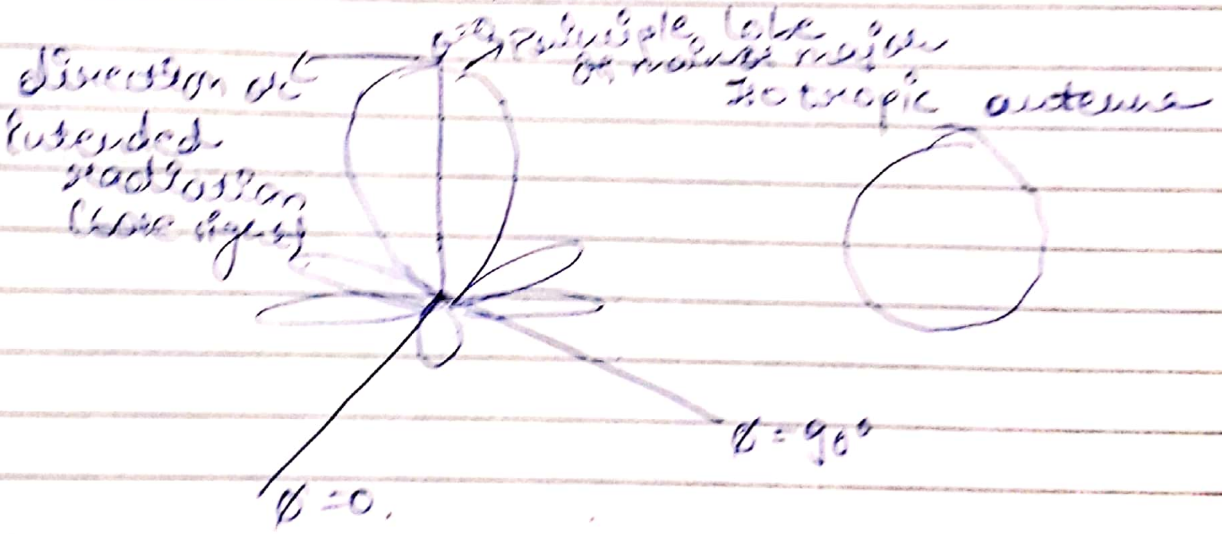


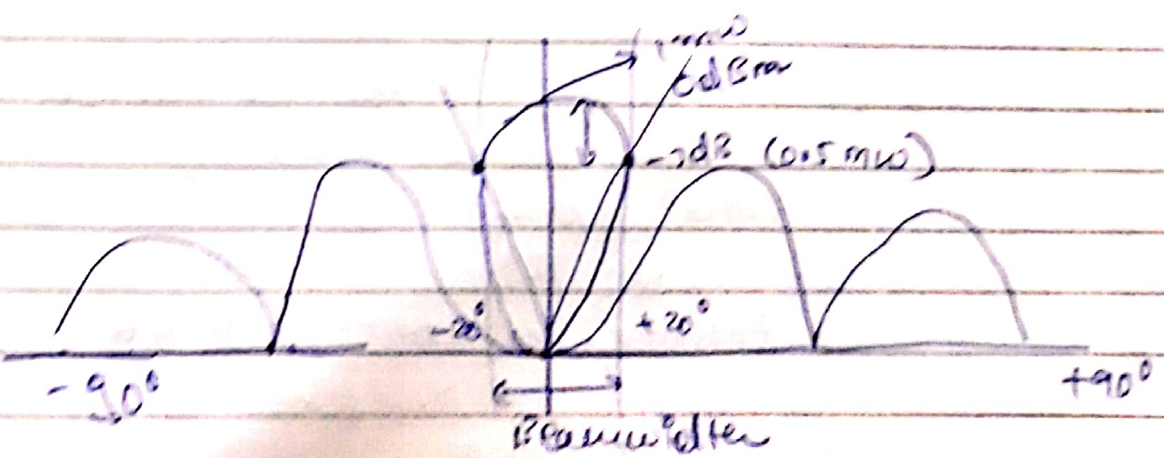
7/11/19

* Antennas.–
→ Radiators
→ Sensors (Temperature)
→ Transducers
→ Impedance matching device
(120π ≈ 377 Ω)

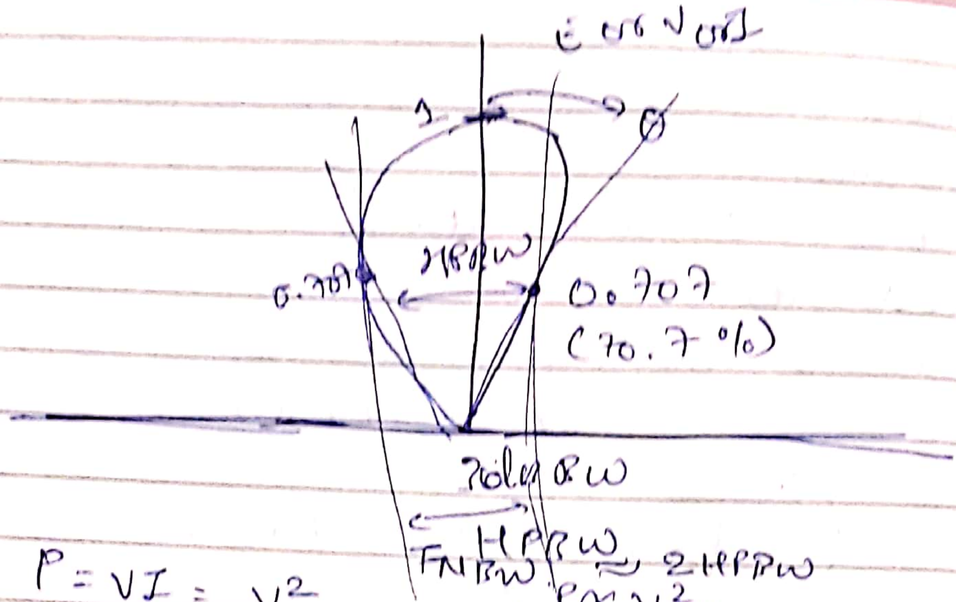
→ fictious resistance
→ radiation resistance

$\dot{I}L = 0\dot{i}$ (Am)





3dB BW (HPBW) = 40°



$\text{FNBW} \approx 2\,\text{HPBW}$

$P = VI = \frac{V^2}{R}$

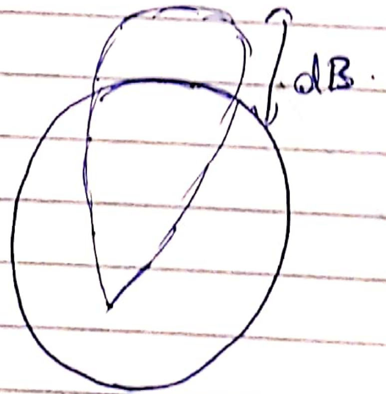
$P \propto V^2$

$(0.707)^2 = 0.5$

$P(\theta) = E^2(\theta)$

dBi

↳ dB w.r.t. isotropic antenna



$E(\theta) = \cos^2\theta$ for $0° \leq \theta < 90°$

HPBW ?

$\cos\theta = \sqrt{0.707}$

$\theta = 33°$

$\text{HPBW} = 2\theta = 2(33) = 66°$

$E(\theta) = \cos\theta \cos 2\theta$

for $0° \leq \theta \leq 90°$

HPBW = ?

$\cos\theta \cos 2\theta = 0.707 = \frac{1}{\sqrt{2}}$

$$\cos 2\theta = \frac{1}{\sqrt{2}\cos\theta}$$

$$2\theta = \cos^{-1}\left(\frac{1}{\sqrt{2}\cos\theta}\right)$$

$$\theta = \frac{1}{2}\cos^{-1}\left(\frac{1}{\sqrt{2}\cos\theta'}\right)$$

$\theta' = 0° \Rightarrow \theta = 22.5°$

$\theta' = 22.5° \rightarrow \theta = 30°$

$\theta' = 30.5° \rightarrow \theta = 30.5°$

⇒ <u>Beam Area</u> - (beam solid angle) (Ω_A)

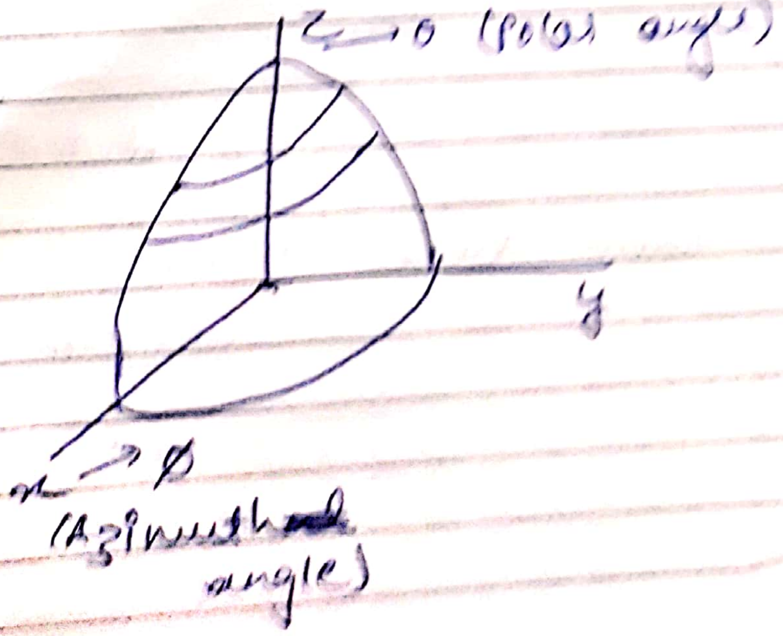
Steradian (sr) = $\frac{\text{solid angle subtended by sphere}}{4\pi}$

$= 1\,\text{rad}^2$

$= \left(\frac{180}{\pi}\right)^2 \text{deg}^2$

$= 3282.8064\ \text{deg}^2$ (square degrees)

$\approx 3283 □$



Area of sphere $= 2\pi r^2 \int_0^{\pi} \sin\theta \, d\theta$

$= 2\pi r^2 \left[-\cos\theta\right]_0^{\pi}$

$= 4\pi r^2$

↓

Solid angle subtended by sphere (Sr)

$1 \text{ rad}^2 = 1 \text{ Sr} = 3283^{\square}$

$4\pi \text{ Sr} = 3283^{\square} \times 4\pi$

$$\boxed{4\pi \text{ Sr} = 41,253^{\square} = \text{solid angle in sphere}}$$

Beam area (or solid angle) = integral of normalized power pattern over a sphere (4π Sr)

$$\boxed{\Omega_A = \int_0^{2\pi}\int_0^{\pi} P_n(\theta,\phi) \underbrace{\sin\theta \, d\theta \, d\phi}_{d\Omega}}$$

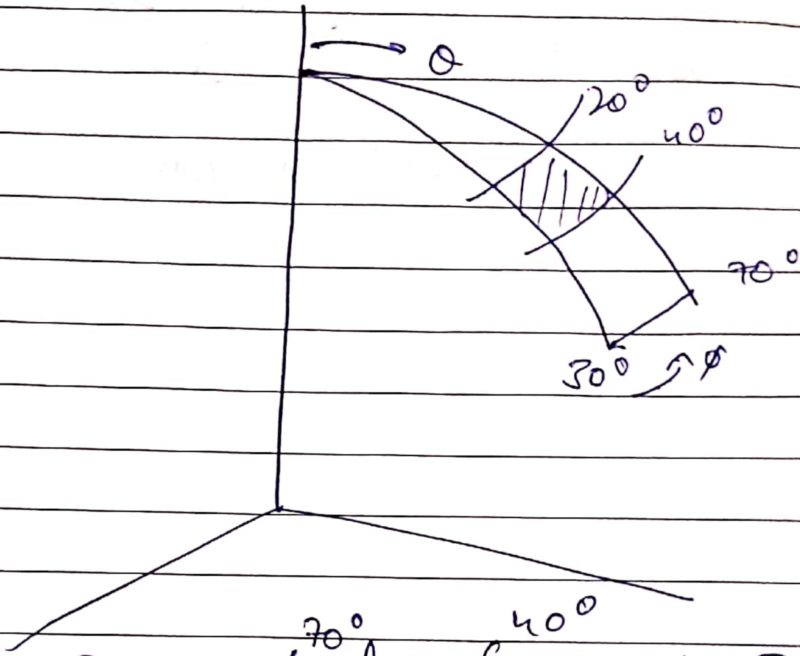
$$\boxed{\Omega_A = \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} P_n(\theta,\phi) \, d\Omega.}$$

$d\Omega = \sin\theta \, d\theta \, d\phi$

Beam Area $= \Omega_A$

$= \theta_{HP} \, \phi_{HP}$

→ solid angle Ω_A on a spherical surface between $\theta = 20°$ & $40°$ and $\phi = 30°$ & $70°$



$$\Omega_A = \int_{\phi=30°}^{70°} d\phi \int_{\theta=20°}^{40°} \sin\theta \, d\theta$$

$$\Omega = \iint_{\phi\ \theta} \sin\theta \, d\theta \, d\phi$$

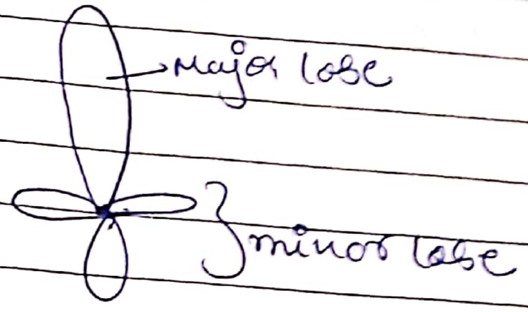
$$= \frac{40}{360} \cdot 2\pi \left[-\cos\theta\right]_{20}^{40}$$

$$= 0.222\pi \times 0.173 \text{ sr}$$

$$= 0.121 \text{ sr}$$

$$= 0.121 \times 3283 \text{ sq deg.} = 397$$

→ Beam efficiency –



Beam efficiency, $\eta = \frac{\Omega_M}{\Omega_A} \times 100\%$

→ Directivity – (dB) ratio of max^m power density $P_n(\theta,\phi)_{max}$ (W/m^2) to its average value over a sphere,

$$D = \frac{P(\theta,\phi)_{max^m}}{P(\theta,\phi)_{avg}} \geq 1$$

$D = 1$
(0dB)

$$P(\theta,\phi)_{av} = \frac{1}{4\pi}\int_{\phi=0}^{4\pi}\int_{\theta=0}^{\pi} P(\theta,\phi)\underbrace{\sin\theta\, d\theta\, d\phi}_{d\Omega}$$

$$= \frac{1}{4\pi}\iint_{4\pi} P(\theta,\phi)\, d\Omega \quad (W/sr)$$

$$\therefore D = \frac{P(\theta,\phi)_{max}}{\frac{1}{4\pi}\iint_{4\pi} P(\theta,\phi)\, d\Omega}$$

$$= \frac{1}{\frac{1}{4\pi}\iint_{4\pi}\left[\frac{P(\theta,\phi)}{P(\theta,\phi)_{max}}\right] d\Omega}$$

$$= \frac{4\pi}{\iint_{4\pi} P(\theta,\phi)\, d\Omega}$$

$$D = \frac{4\pi}{\Omega_A}$$

for an antenna radiation over any half a sphere,

$$\Omega_A = 2\pi \text{ Sr}$$

$$D = \frac{4\pi}{2\pi} = 2$$

$\downarrow$

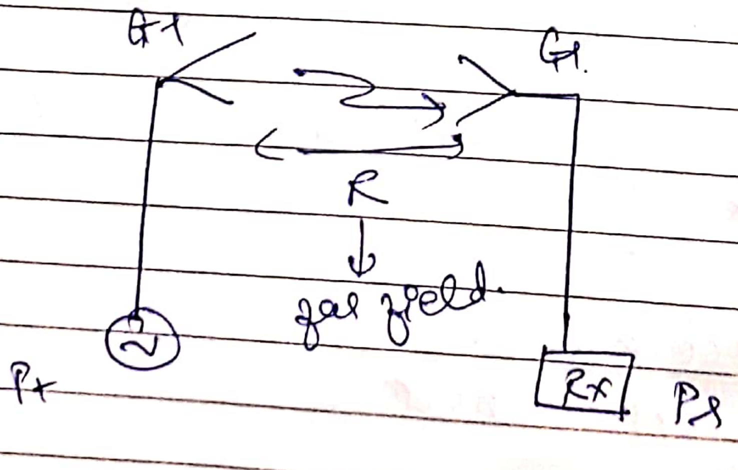
3dB

3dBi

→ gain.-

→ RF or radio link - (freespace)

NFC

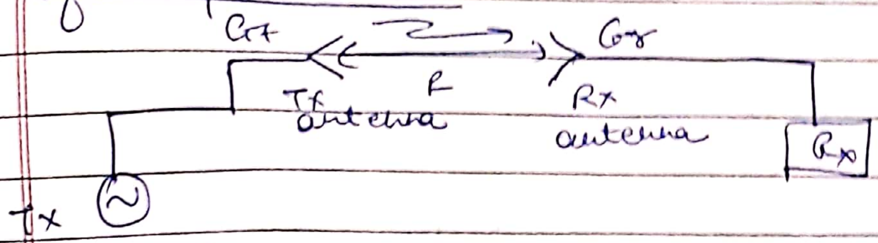


Anechoic chamber

11/1/19

* Gain : ?
RF link / commⁿ link / M/w link / satellite

Free-space link –



$P_r = ?$

Surface Area of sphere of radius R is
$A = 4\pi R^2$
But as sphere expands (i.e. R increases) power decreases, i.e.

$$S_{av} = \frac{P_t}{A} = \frac{P_t}{4\pi R^2} \quad W/m^2 \quad \text{—— (1)}$$

If Gt is gain of Tx antenna, power density radiated by Tx antenna is given by,

$$S_{av} = \frac{P_t G_t}{4\pi R^2} \quad W/m^2 \quad \text{—— (2)}$$

This power is incident on Rx antenna, we use concept of effective aperture area of antenna (capture area)
(Ae)

Ae is related to gain by
$4\pi A_e/\lambda^2$

$$\Rightarrow A_e = \frac{G_r \lambda^2}{4\pi} \qquad \text{——— (3)}$$

$P_r = ?$

$$P_r = A_e . S_{av} \qquad (2) \; \& \; (3)$$

$$P_r = \frac{P_t G_t}{4\pi R^2} \; \frac{G_r \lambda^2}{4\pi} \qquad (w)$$

$$\boxed{P_r = P_t G_t G_r \left(\frac{\lambda}{4\pi R}\right)^2}$$

Friis equation.

$$P_r \propto \frac{1}{R^2}$$

→ Geostationary orbit,

$R = 36,900$ km.

$$P_r = \frac{P_t G_t G_r}{(4\pi)^2 R^2} \; \frac{c^2}{f^2}$$

$$= \frac{P_t G_t G_r \; (3\times10^8)^2}{158 \times (10^3)^2 R^2 \times (10^6)^2 f^2}$$

$f \to$ MHz

$R \to$ km.

$$P_r = P_t G_t G_r \left[\frac{0.057 \times 10^{-2}}{\underset{\text{MHz}}{f^2} \underset{\text{km}}{R^2}}\right]$$

$$P_r|_{dBm} = P_t|_{dBm} + G_t|_{dB} + G_r|_{dB} + \left[10\log(0.057\times10^{-2})\right] - 10\log(f^2) \; 10\log(R^2)$$

$$P_r|_{dBm} = P_t|_{dBm} + G_t|_{dB} + G_r|_{dB} - [32.57 + 20\log f + 20\log R$$

$20\log R$ law or $20\log D$ law ← free space path loss (dB)

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Date
Page

f (in GHz) , R (km)

$$FSPL = 92.5 + 20\log f + 20\log R.$$

14/11/19

→ $P_r = \underbrace{P_t G_t}\, G_r \left(\frac{\lambda}{4\pi R}\right)^2$

↓ free space path loss

$EIRP = P_t G_t$

↓

Effective Isotropic Radiated power (W)

$$EIRP\Big|_{dBm} = P_t\Big|_{dBm} + G_t\Big|_{dB}$$

$$D = \frac{4\pi}{\Omega_A} = \frac{4\pi}{\int_0^{\pi}\int}$$

$$D = \frac{4\pi A_e}{\lambda^2}$$

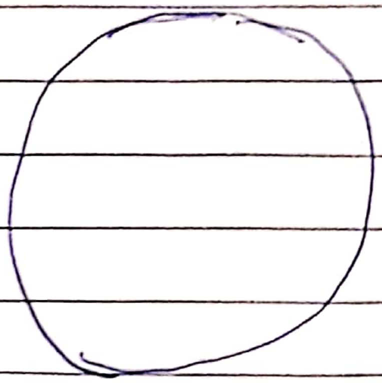
$$G = \frac{4\pi A_e}{\lambda^2}$$

$G = kD$

↓

Efficiency

→ A_e for an ideal isotropic antenna -



$D = 1$

$$A_e = \frac{D\lambda^2}{4\pi} = \frac{\lambda^2}{4\pi} = 0.0796\lambda^2$$

2 identical antennas

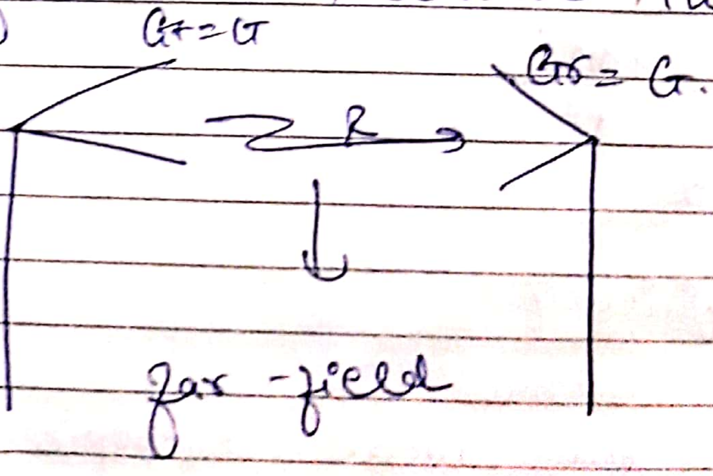
$$G_t = G_r = G$$

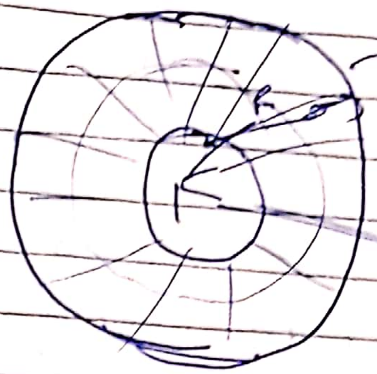
$$P_r = P_t G^2 \left(\frac{\lambda}{4\pi R}\right)^2$$

NFC	far field
fresnel	fraunhofer

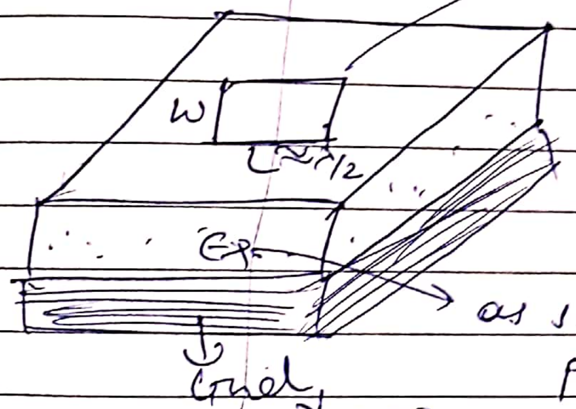
- Near field = Reactive in nature (non - radiative)

- far field = Radiative in nature.

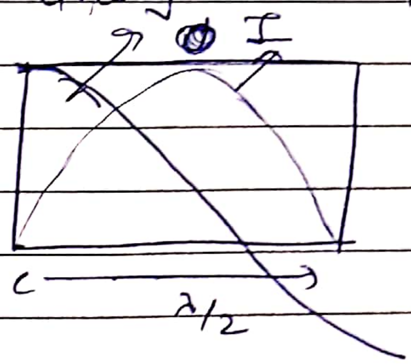




* Microstrip or patch Antenna:- (Radiator)
↓
planar
↓
printed



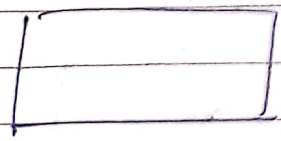
Aluminium
$\epsilon_r = 10$

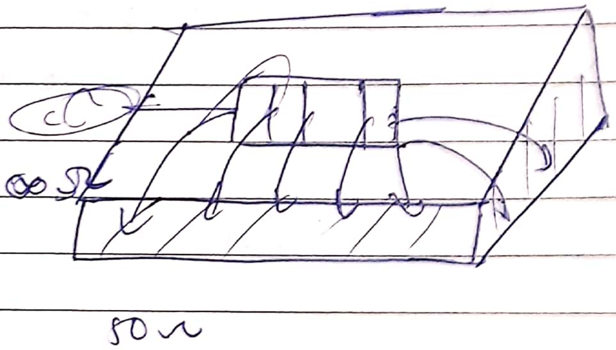


Ideal Resonant antenna

→ Disadvantage ① Limited Bandwidth. (1-3%)

② Low power handling
③ Low power gain. (Radiate in half plane)




50Ω