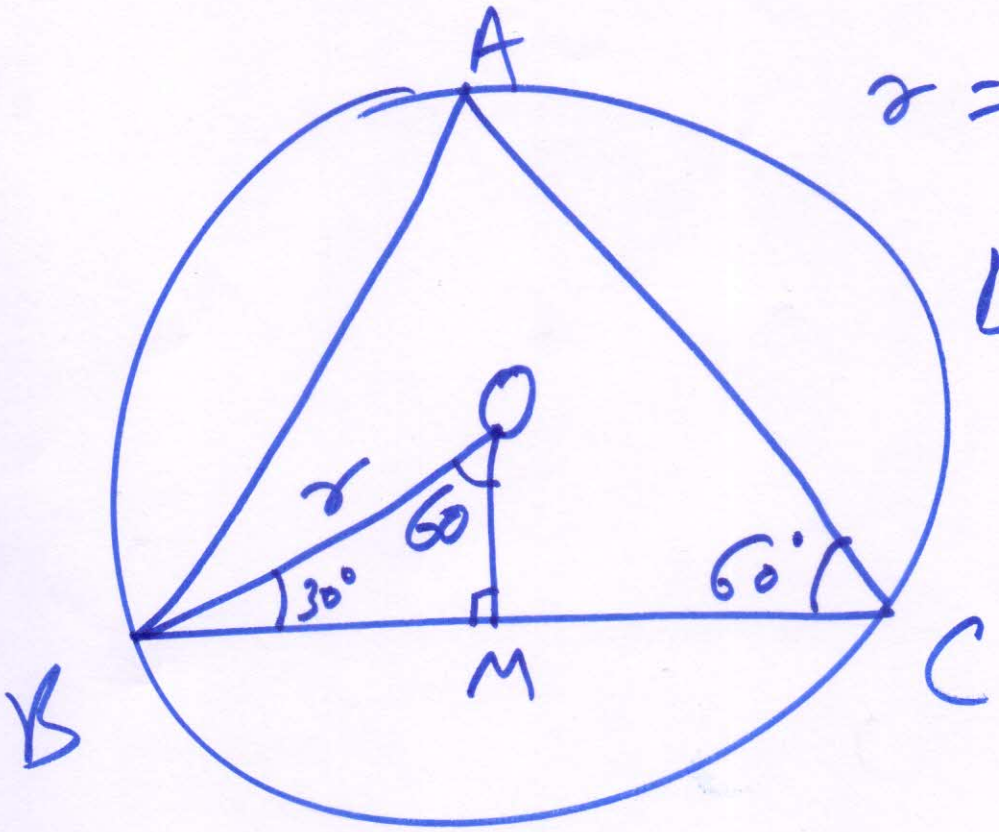
Lec-17

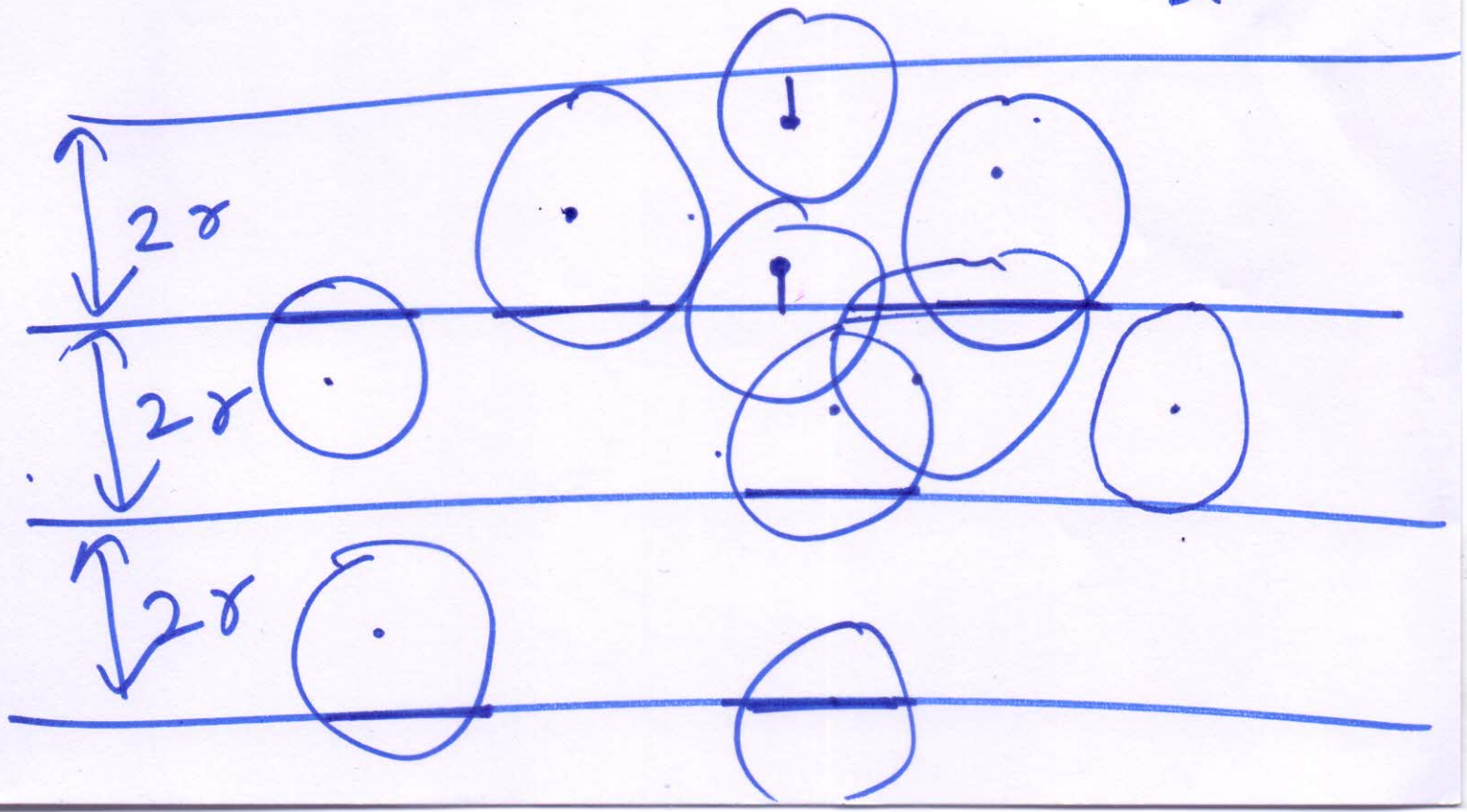


r = radius

$BC = \sqrt{3}\,r$

$$\sin 60 = \frac{BM}{r} \Rightarrow BM = r \sin 60$$

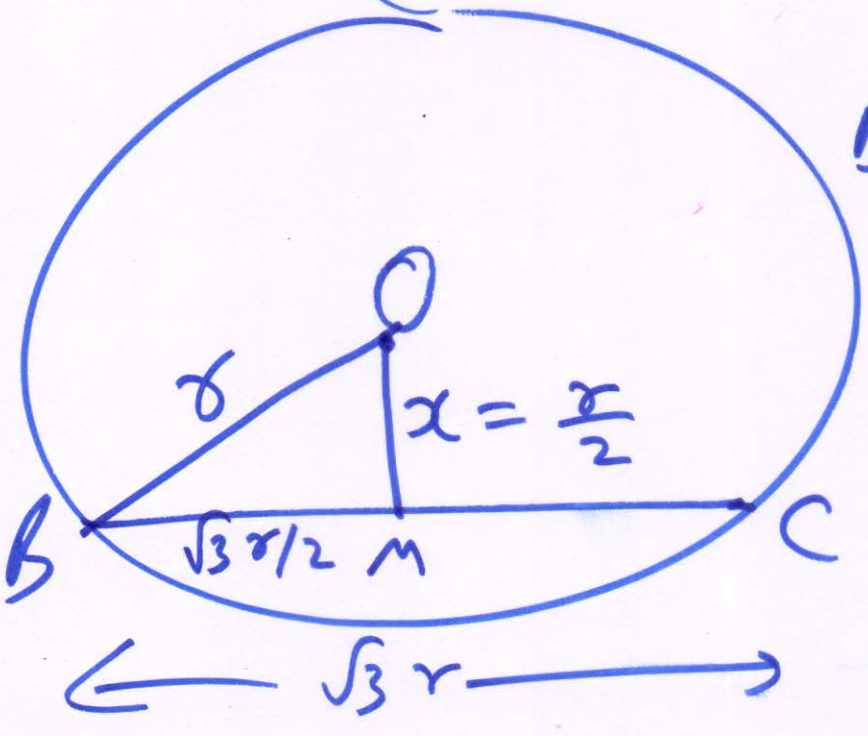
$$= \frac{r \cdot \sqrt{3}}{2}$$



X = distance of the nearer line from the center of the circle (where you drop the pen)

X = uniformly distributed. $[0, r]$

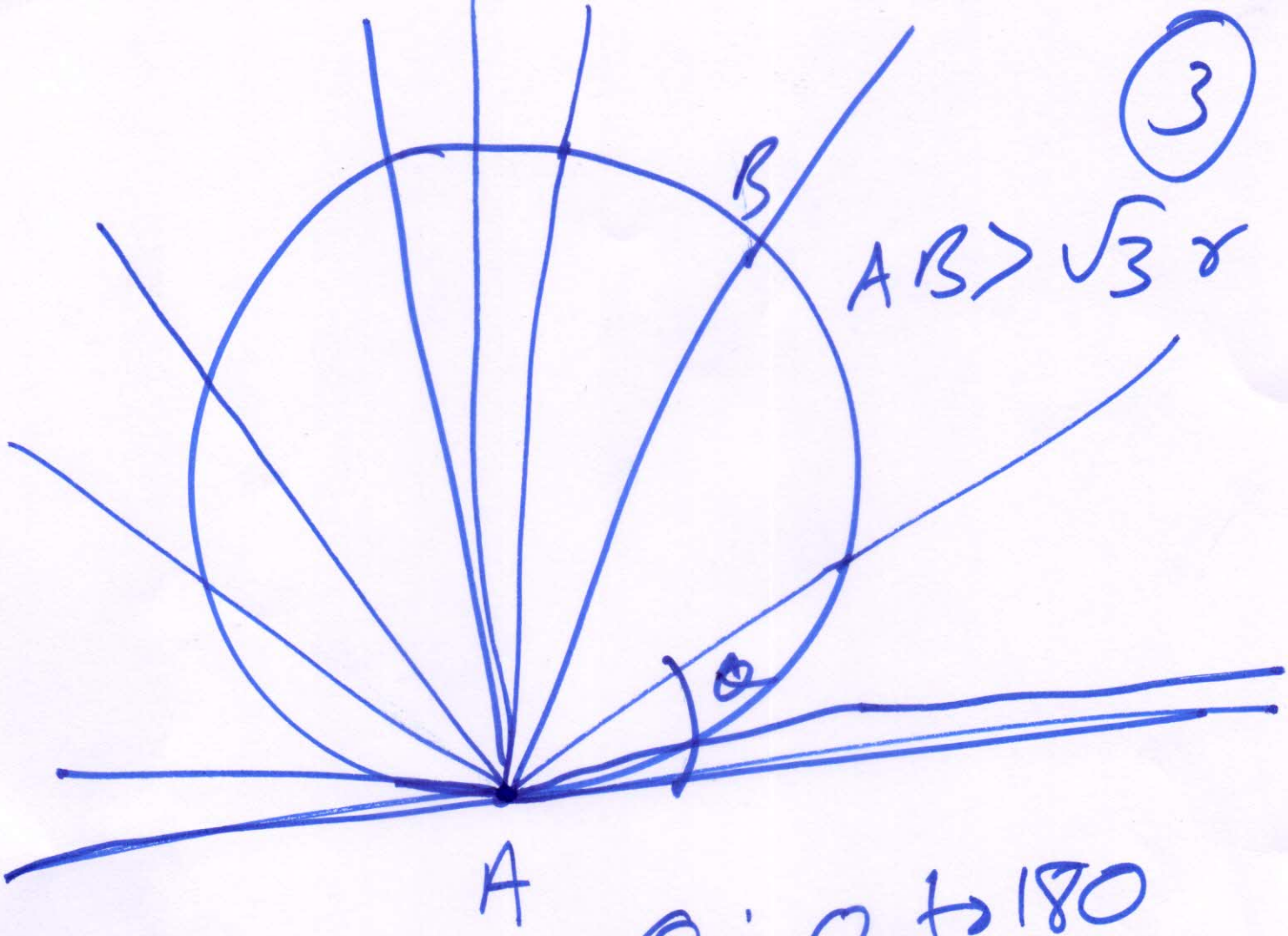
$$f(x) = \begin{cases} \frac{1}{r} & 0 \le x \le r \\ 0 & \text{otherwise} \end{cases}$$



$BC > \sqrt{3}\, r$

$0 \le x \le \frac{r}{2}$

$$\int_0^{r/2} f(x)\,dx = \frac{r/2}{r} = \frac{1}{2}$$



(3)

$AB > \sqrt{3}\, r$

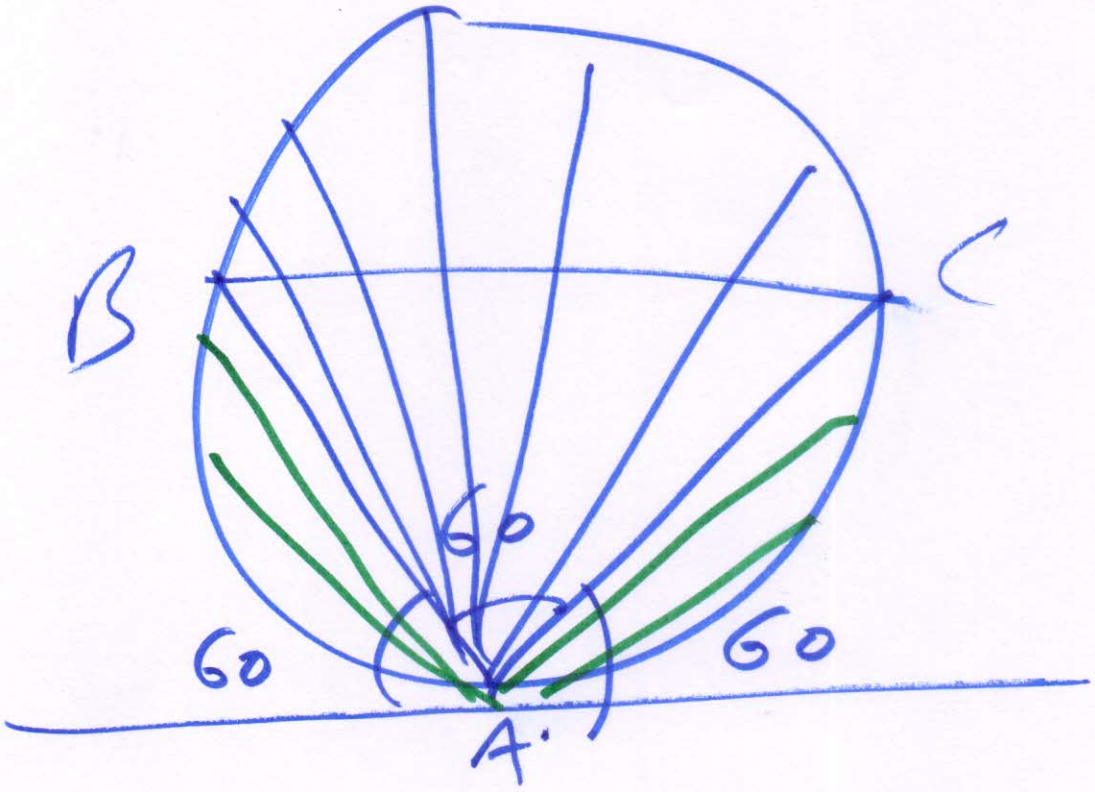
$\theta : 0$ to 180

uniform

long needle

$$f_\theta(x) = \begin{cases} \frac{1}{180} & 0 \le x \le 180 \\ 0 & \text{otherwise} \end{cases}$$

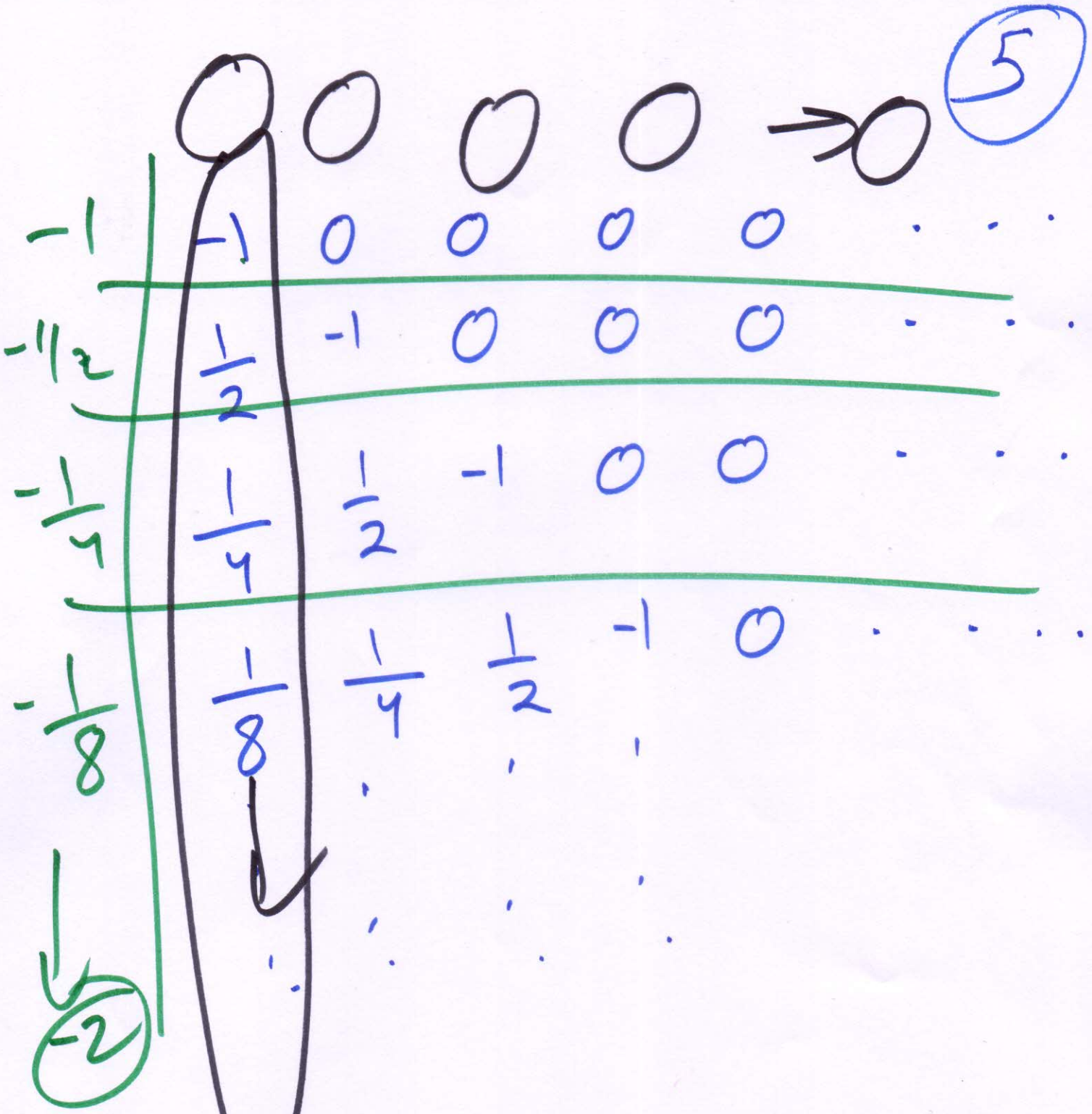
(4)



θ : 60 to 120

$$P(60 \leq \theta \leq 120) =$$

$$\int_{60}^{120} f(x)\,dx = \frac{120-60}{180} = \frac{1}{3}$$


5
-1
-1/2
-1/4
-1/8
-2

Cardinality	Set
aleph 0	$\mathbb{N}$
aleph 1	$\mathbb{R}$
aleph 2	$2^{\mathbb{R}}$
⋮	

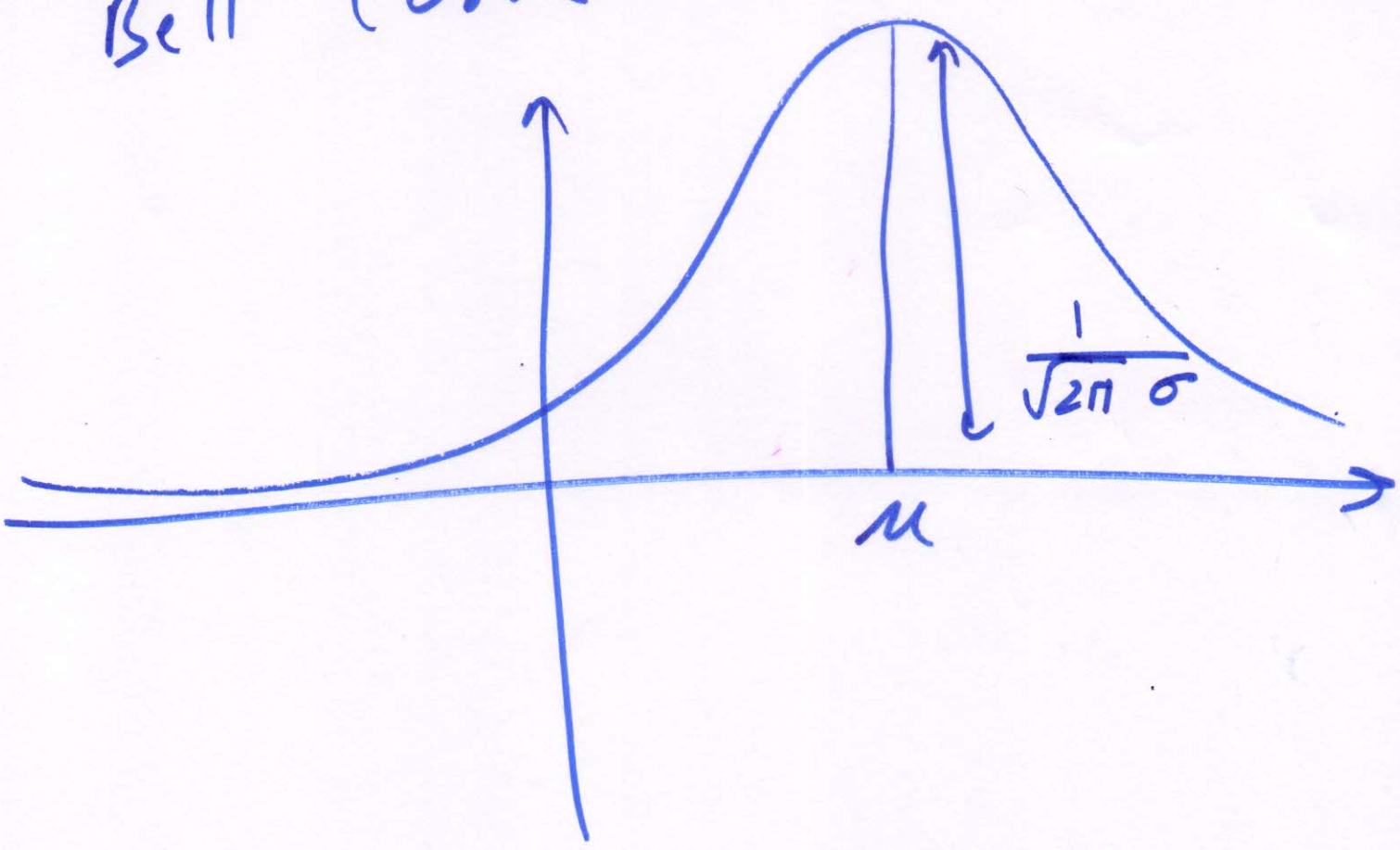
Normal Distribution

e.g. heights, weight, income, marks

X is normally distributed with parameters μ and σ^2 if density of X is

$$f(x) = \frac{e^{-\frac{(x-\mu)^2}{2\sigma^2}}}{\sqrt{2\pi}\,\sigma}$$

Bell Curve



When $\mu = 0$

$\sigma = 1$,

its called Standard Normal distribution

$N(0,1) \rightarrow \frac{e^{-x^2/2}}{\sqrt{2\pi}}$

$N(\mu, \sigma)$

N.W.

Show that it is a probability density function.

$f \geq 0$ (nonnegative)

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$\int_{-\infty}^{\infty} \frac{e^{-x^2/2}}{\sqrt{2\pi}}\,dx = 1$$

If X is normally distributed, then so is

$$aX + b$$

$E[X] = \mu$, $Var[X] = \sigma^2$ } Proof given in the book

$$E[aX+b] = a\mu + b$$

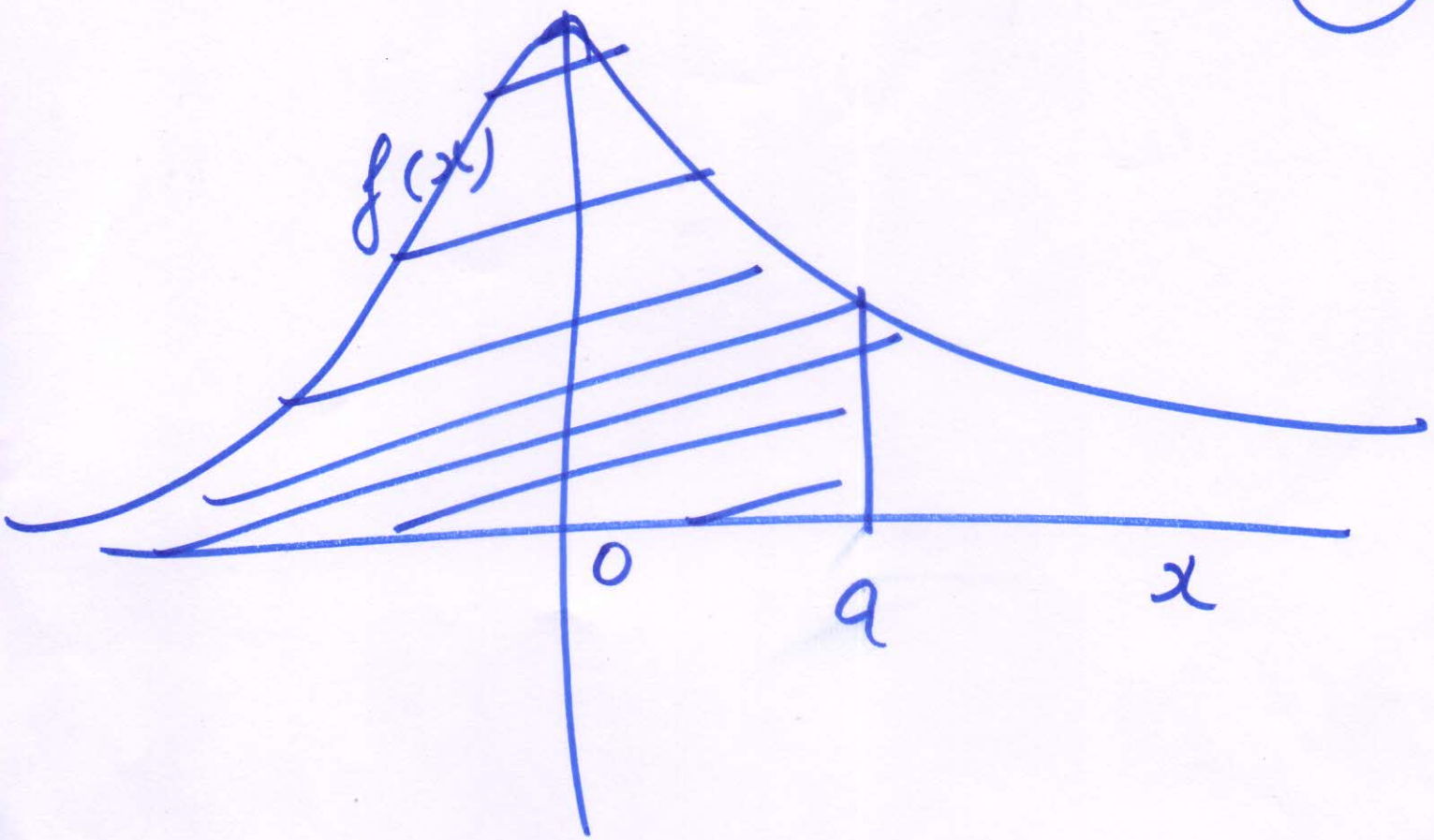
$$Var[aX+b] = a^2\sigma^2$$

Cumulative distribution fn:

$$\Phi(x) = \int_{-\infty}^{x} \frac{e^{-x^2/2}\,dx}{\sqrt{2\pi}}$$

$$N(0,1)$$

$N(0,1)$



$$\Phi(a) = P(x \le a)$$

$$\Phi(0) = \frac{1}{2} = 0.5$$