

1. Calculate the laplacian of the following:

(i) $F = x^2 + 2xy + 3z + 4$ (ii) $F = \sin(\hat{\mathbf{k}} \cdot \vec{\mathbf{r}})$ (iii) $F = \frac{1}{r}$

2. Evaluate $(\hat{\mathbf{r}} \cdot \vec{\nabla})r$ and $(\hat{\mathbf{r}} \cdot \vec{\nabla})\hat{\mathbf{r}}$

3. Find the volume of an ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ using the tripple integral $\int \int \int dx dy dz$ with appropriate limits.

4. Consider $\vec{\mathbf{A}} = x^2\hat{\mathbf{i}} + y^2\hat{\mathbf{j}} + z^2\hat{\mathbf{k}}$

(a) Evaluate $\oint_S \vec{\mathbf{A}} \cdot d\vec{\mathbf{a}}$ where S is a cubical surface given by the planes $x = a \pm l$; $y = b \pm l$; $z = c \pm l$.

(b) Verify that at the point (a, b, c) ,

$$\vec{\nabla} \cdot \vec{\mathbf{A}} = \lim_{l \rightarrow 0} \frac{1}{8l^3} \oint_S \vec{\mathbf{A}} \cdot d\vec{\mathbf{a}}$$

5. Evaluate $\int_P^Q \vec{\mathbf{A}} \cdot d\vec{\mathbf{l}}$ for $\vec{\mathbf{A}} = y\hat{\mathbf{i}} - x\hat{\mathbf{j}}$ along the following paths :
 $P \equiv (-a, 0)$; $Q \equiv (a, 0)$.

(a) $(-a, 0) \rightarrow (0, a) \rightarrow (a, 0)$

(b) $(-a, 0) \rightarrow (0, -a) \rightarrow (a, 0)$

(c) a loop, forward along (a) and backward along (b)

(d) Let I be the value of the loop integral evaluated in (c). Let S be the flat area enclosed by the loop. Verify that at the origin

$$(\vec{\nabla} \times \vec{\mathbf{A}}) = \left[\lim_{a \rightarrow 0} \frac{I}{S} \right] (-\hat{\mathbf{k}})$$

(e) Can we find a scalar function F such that $\vec{\nabla}F = y\hat{\mathbf{i}} - x\hat{\mathbf{j}}$?