an increase in distance for clockwise movement and the other an increase for counterclockwise travel. These two scales are shown in Figure 11.17. Note that the one marked "wavelengths toward generator" (wtg) shows increasing values of l/λ for clockwise travel, as described previously. The zero point of the wtg scale is rather arbitrarily located to the left. This corresponds to input impedances having phase angles of 0° and $R_L < Z_0$. We have also seen that voltage minima are always located here.

EXAMPLE 11.14

The use of the transmission line chart is best shown by example. Let us again consider a load impedance, $Z_L = 25 + j50\ \Omega$, terminating a 50 Ω line. The line length is 60 cm and the operating frequency is such that the wavelength on the line is 2 m. We desire the input impedance.

Solution. We have $z_L = 0.5 + j1$, which is marked as A on Figure 11.18, and we read $\Gamma = 0.62\angle 82°$. By drawing a straight line from the origin through A to the circumference, we note a reading of 0.135 on the wtg scale. We have $l/\lambda = 0.6/2 = 0.3$, and it is therefore 0.3λ from the load to the input. We therefore find z_{in} on the $|\Gamma| = 0.62$ circle opposite a wtg reading of $0.135 + 0.300 = 0.435$. This construction is shown in Figure 11.14, and the point locating the input impedance is marked B. The normalized input impedance is read as $0.28 - j0.40$, and thus $Z_{\text{in}} = 14 - j20$. A more accurate analytical calculation gives $Z_{\text{in}} = 13.7 - j20.2$.

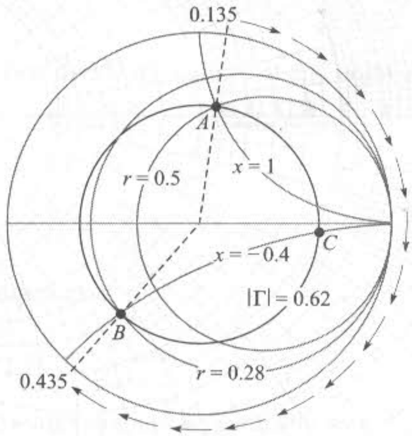

Fig. 11.18 The normalized input impedance produced by a normalized load impedance $z_L = 0.5 + j1$ on a line 0.3λ long is $z_{\text{in}} = 0.28 - j0.40$.

Information concerning the location of the voltage maxima and minima is also readily obtained on the Smith chart. We already know that a maximum or minimum must occur at the load when Z_L is a pure resistance; if $R_L > Z_0$ there is a maximum at the load, and if $R_L < Z_0$ there is a minimum. We may extend this result now by noting that we could cut off the load end of a transmission line at a point where the input impedance is a pure resistance and replace that section with a resistance R_{in}; there would be no changes on the generator portion of the line. It follows, then, that the location of voltage maxima and minima must be at those points where Z_{in} is a pure resistance. Purely resistive input impedances must occur on the $x = 0$ line (the Γ_r axis) of the Smith chart. Voltage maxima or current minima occur when $r > 1$, or at wtg $= 0.25$, and voltage minima or current maxima occur when $r < 1$, or at wtg $= 0$. In Example 11.14, then, the maximum at wtg $= 0.250$ must occur $0.250 - 0.135 = 0.115$ wavelengths toward the generator from the load. This is a distance of 0.115×200, or 23 cm from the load.

We should also note that since the standing wave ratio produced by a resistive load R_L is either R_L/R_0 or R_0/R_L, whichever is greater than unity, the value of s may be read directly as the value of r at the intersection of the $|\Gamma|$ circle and the r axis, $r > 1$. In our example this intersection is marked point C, and $r = 4.2$; thus, $s = 4.2$.

Transmission line charts may also be used for normalized admittances, although there are several slight differences in such use. We let $y_L = Y_L/Y_0 = g + jb$ and use the r circles as g circles and the x circles as b circles. The two differences are: first, the line segment where $g > 1$ and $b = 0$ corresponds to a voltage minimum; and second, 180° must be added to the angle of Γ as read from the perimeter of the chart. We shall use the Smith chart in this way in Section 11.14.

Special charts are also available for non-normalized lines, particularly 50 Ω charts and 20 mS charts.

EXAMPLE 11.15

A lossless transmission line having characteristic impedance Z_0 is terminated into a load $Z_L = R_L + jX_L$. Find out with the help of Smith chart theory, the points on the line where the impedance becomes purely real, and determine the value of the real impedance at these points. Show how the standing wave ratio along the line can be obtained by plotting the constant $|\Gamma|$ circle on the Smith chart.

Solution. The conventional Smith chart with a typical load $Z_L = R_L + jX_L$ represented by the point L is shown in Figure 11.19. As you know, you can draw a circle with the origin as O and the radius as 'OL' on the chart, which would cut the real axis at two points denoted by the point M on the right-hand side, and the point m on the left-hand side. This circle is usually called the constant $|\Gamma|$ circle because of the fact that if you move along the perimeter of this circle then the magnitude of the reflection coefficient remains constant. Now, since the voltage standing-wave ratio s depends exclusively on the magnitude of the reflection coefficient, hence by this virtue, the parameter s also remains constant if you move on this circle. The points M and m represent the points where the impedance becomes purely real. These points would always be located on the real axis on the right-hand side and the left-hand side of the origin O, respectively, and the exact location of these two points would depend upon the value of the load impedance Z_L.

Now, we have to find out the actual value of normalized impedance at two points denoted by M and m. Let us first consider the point M on the right-hand side of this Smith chart, and let us assume that the point M intersects the constant r (normalized resistance) circle. It can easily be seen from the Smith chart that at any point on the right-hand side of the origin O, the value of the normalized resistance would be greater than unity, i.e., $r > 1$. With this background, let us try to determine the value of the standing wave ratio s at the point M which is given by the following expression

$$s \equiv \frac{1 + |\Gamma|}{1 - |\Gamma|} \tag{i}$$

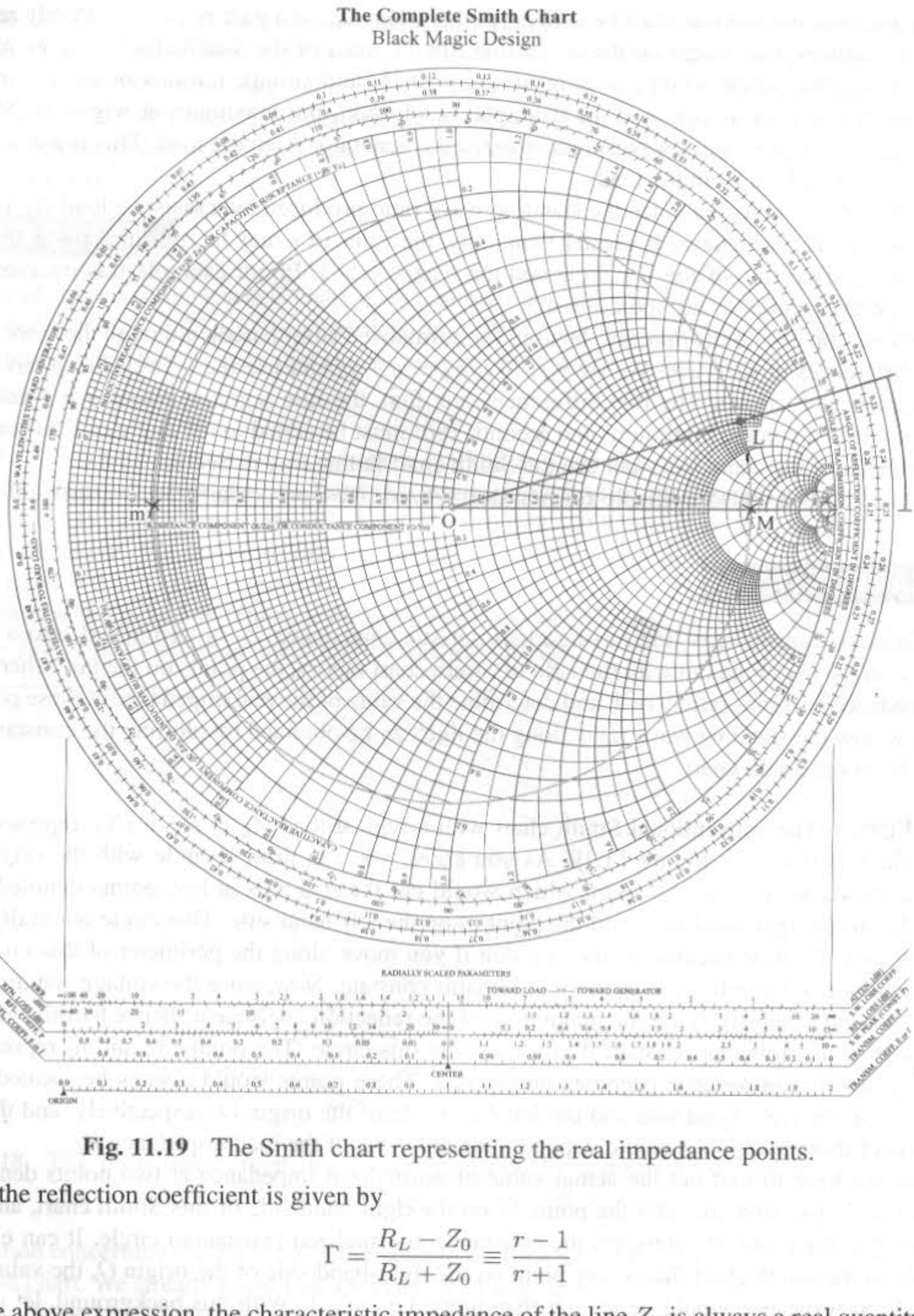

Fig. 11.19 The Smith chart representing the real impedance points.

where, the reflection coefficient is given by

$$\Gamma = \frac{R_L - Z_0}{R_L + Z_0} \equiv \frac{r-1}{r+1} \tag{ii}$$

In the above expression, the characteristic impedance of the line Z_0 is always a real quantity, and the load impedance R_L is considered purely real or resistive in the present situation because of the fact that at the point M, the imaginary part of the impedance is zero. It means that the normalized impedance $r \equiv R_L/Z_0$ would also be purely real in Eq. (ii). Now, since r is a purely real quantity in (ii), and its value is greater than unity, (ii) can also be written as

$$|\Gamma| \equiv \frac{r-1}{r+1} \tag{iii}$$

Finally, the value of $|\Gamma|$ can be substituted from (iii) into (i) to obtain the following expression for the voltage standing wave ratio s:

$$s = \frac{1+|\Gamma|}{1-|\Gamma|} \equiv \frac{1+\frac{r-1}{r+1}}{1-\frac{r-1}{r+1}} \equiv r \tag{iv}$$

From the above equation, we can say that the value of the normalized impedance at the point M, where the constant $|\Gamma|$ circle cuts the real axis, is purely real, and is equal to the standing-wave ratio.

In the second step, let us consider the point m on the left-hand side of the Smith chart intersecting the constant r circle. It can be observed from the Smith chart that at any point on the left-hand side of the origin O, the value of the normalized resistance would be less than unity, i.e., $r < 1$. Hence, at the point m, the normalized impedance is purely resistive, and the value of the normalized resistance r is less than unity, which then modifies Eq. (iii) as follows:

$$|\Gamma| \equiv \frac{1-r}{1+r} \tag{v}$$

The expression given by Eq. (5) is based on the simple fact that the magnitude of the reflection coefficient, i.e., the value of $|\Gamma|$ should always be a positive number which is not possible using the form given by (iii) if the value of 'r' is assumed to be less than unity. Once the value of $|\Gamma|$ is defined using (v), the voltage standing-wave ratio 's' can be computed using (i), i.e.,

$$s = \frac{1+|\Gamma|}{1-|\Gamma|} \equiv \frac{1+\frac{1-r}{1+r}}{1-\frac{1-r}{1+r}} \equiv \frac{1}{r} \tag{vi}$$

Hence, the value of the normalized impedance at the point 'm', where the constant $|\Gamma|$ circle cuts the real axis on the left-hand side of the origin 'O', is purely real, and is equal to the inverse of the standing wave ratio.

From Eq. (iv), it can also be postulated that the standing-wave ratio along the line can be obtained by plotting the constant $|\Gamma|$ circle on the Smith chart, and noting the point of intersection of this circle with the positive real axis represented by the point M in Figure 11.19. The value of the normalized resistance r circle, which passes through the point M, then provides the value of the voltage standing wave ratio s on the line.

EXAMPLE 11.16

What are the limitations of a quarter-wave transformer? Explain whether it is possible to match a reactive load having $Z_L = (100 + j150)\Omega$ with a lossless transmission line having $Z_0 = 50\,\Omega$ using a quarter-wave transformer. If yes, find out the characteristic impedance of the transformer and other relevant parameters required to achieve this matching.

Solution. The quarter-wave transformer is a piece of transmission line, which is used either for matching two different impedances or for matching a load with a transmission line of different characteristic impedance. It has two major limitations when used for matching purposes. The first limitation is that it can only be used to match the *real* impedances. The second limitation is that the matching is achieved in the very narrow frequency band with the condition that the ideal matching is achieved only at those frequencies where the length of the line is an odd multiple of quarter wavelength, i.e., $l \equiv (2m+1)\lambda/4$.

Now, if one wants to match a reactive load with a transmission line having a *real* value of the characteristic impedance then it can only be achieved if the reactive load is first converted into a *resistive* load. In other words, a piece of transmission line of appropriate length should be connected at the load so that the input impedance becomes *resistive* at the new position. Hence, you have to locate the position on the line shown in Figure 11.20 such that if a transmission line of length l is connected at the load, then the impedance Z_B seen from the new position towards the load becomes purely resistive. Now, you know from the Smith chart theory that the impedance can be purely resistive either at the position of the voltage maximum or at the voltage minimum. In principle, the quarter-wave section can be inserted at any of these two points. However, from the practical point of view, one tries to find a position which is closer to the load in order to minimize the length l of the extra line. The whole procedure is explained below with the help of the Smith chart shown in Figure 11.21.

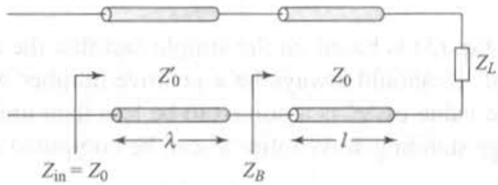

Fig. 11.20 Load line.

Major steps

1. First of all, connect a line of length l shown in Figure 11.20 so that the impedance seen from this point towards the load $Z_L = (100 + j150)\ \Omega$ becomes purely resistive. The characteristic impedance of this line is assumed to be same as that of the actual transmission line, i.e., $Z_0 = 50\ \Omega$.
2. Normalize the load impedance $Z_L = (100 + j150)\ \Omega$ with respect to the characteristic impedance of the line, i.e., $Z_L = \frac{(100+j150)}{50} = (2 + j3)$.
3. Plot $Z_L = (2 + j3)$ point on the Smith chart shown by L in Figure 11.21. Draw a constant $|\Gamma|$ circle on the Smith chart towards the generator (anti-clockwise direction) from the point L with the origin as O and the radius as OL as shown in this figure. This circle intersects the real axis on the right-hand side of the chart at the point M, where the voltage is maximum and the impedance is purely real.
4. The value of the normalized resistance at the point M may be read as $r = 7$. The distance between the load position L and the voltage maximum position M may be determined from the WTG scale provided on the Smith chart, which is given by $l = (0.25 - 0.214)\lambda \equiv 0.036\lambda$. It means that if you move a distance of $0.036\ \lambda$ from the load towards the generator, then you would arrive at a position where the impedance becomes purely resistive, and this impedance can now be matched with the quarter-wave transformer.

5. The actual value of the resistive impedance at a distance $l = .036\lambda$ from the load in Figure 11.20 is then given by $Z_B = 50 \times 7 = 350\ \Omega$.
6. This impedance can now be matched with a lossless transmission line having characteristic impedance of $Z_0 = 50\ \Omega$ with the help of a quarter-wave transformer. The characteristic impedance of the quarter-wave section in this case would be given by

$$Z_0' = \sqrt{Z_B \times Z_0} = \sqrt{350 \times 50} = 132.29\ \Omega.$$

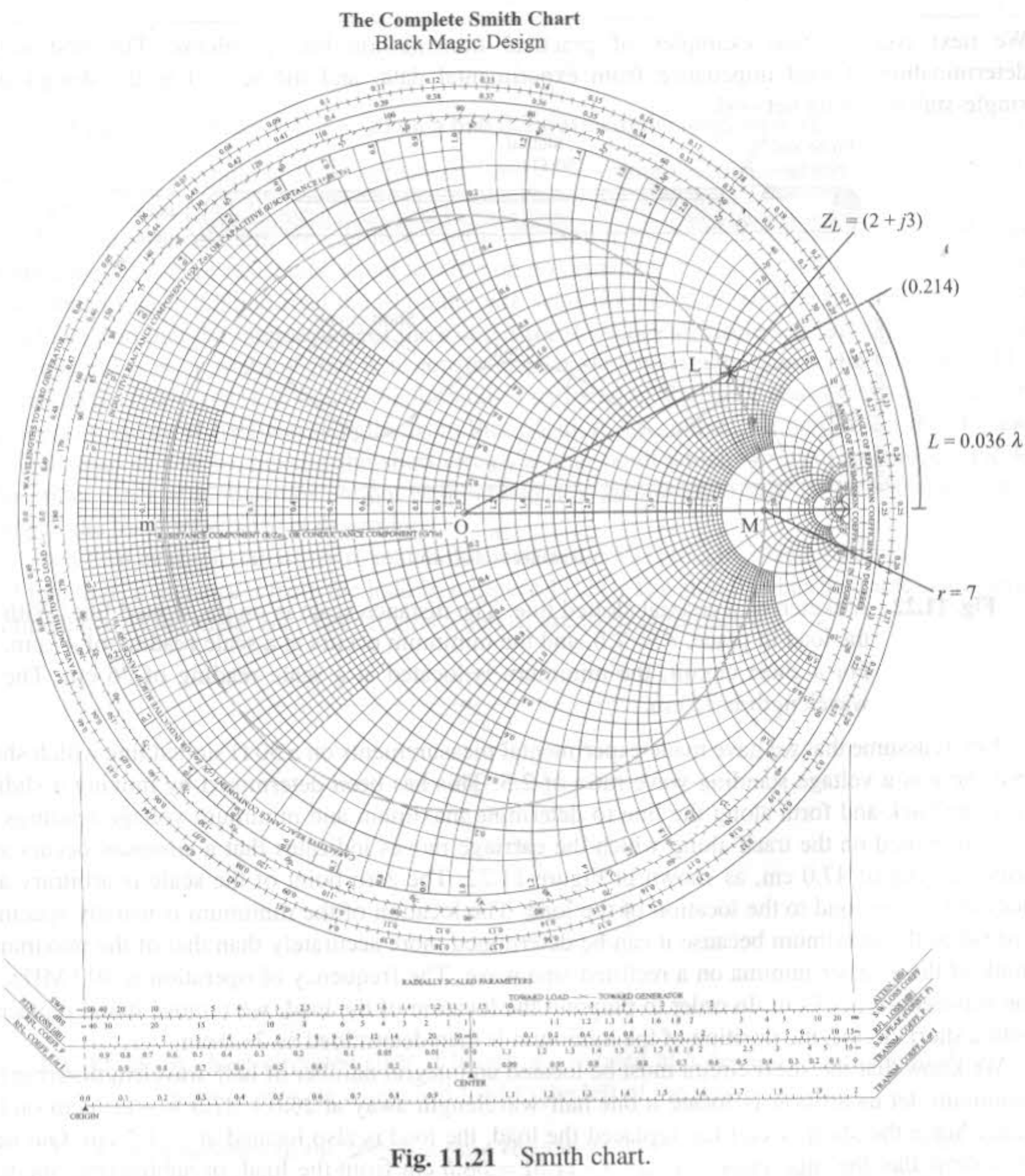

Fig. 11.21 Smith chart.

D11.6 A load $Z_L = 80 - j100\,\Omega$ is located at $z = 0$ on a lossless 50 Ω line. The operating frequency is 200 MHz and the wavelength on the line is 2 m. (*a*) If the line is 0.8 m in length, use the Smith chart to find the input impedance. (*b*) What is s? (*c*) What is the distance from the load to the nearest voltage maximum? (*d*) What is the distance from the input to the nearest point at which the remainder of the line could be replaced by a pure resistance?

Ans. $79 + j99\,\Omega$: 4.50; 0.0397 m; 0.760 m

We next consider two examples of practical transmission line problems. The first is the determination of load impedance from experimental data, and the second is the design of a single-stub matching network.

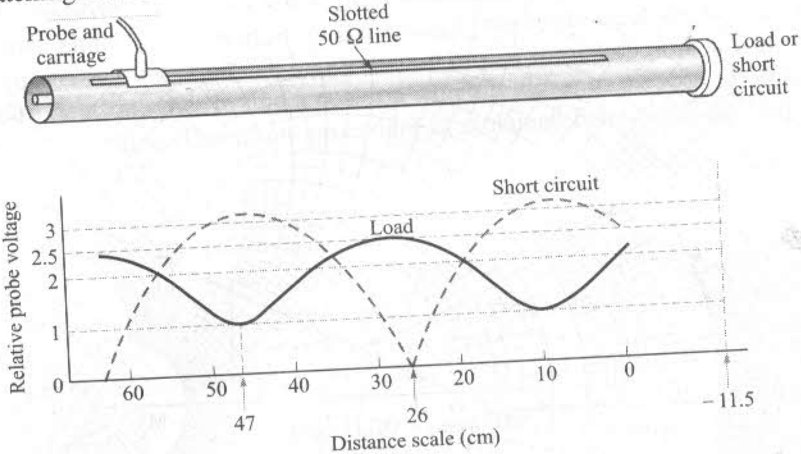

Fig. 11.22 A sketch of a coaxial slotted line. The distance scale is on the slotted line. With the load in place, $s = 2.5$, and the minimum occurs at a scale reading of 47 cm. For a short circuit, the minimum is located at a scale reading of 26 cm. The wavelength is 75 cm.

Let us assume that we have made experimental measurements on a 50 Ω slotted line which show that there is a voltage standing wave ratio of 2.5. This has been determined by moving a sliding carriage back and forth along the line to determine maximum and minimum voltage readings. A scale provided on the track along which the carriage moves indicates that a *minimum* occurs at a scale reading of 47.0 cm, as shown in Figure 11.22. The zero point of the scale is arbitrary and does not correspond to the location of the load. The location of the minimum is usually specified instead of the maximum because it can be determined more accurately than that of the maximum; think of the sharper minima on a rectified sine wave. The frequency of operation is 400 MHz, so the wavelength is 75 cm. In order to pinpoint the location of the load, we remove it and replace it with a short circuit; the position of the minimum is then determined as 26.0 cm.

We know that the short circuit must be located an integral number of half-wavelengths from the minimum; let us arbitrarily locate it one half-wavelength away at $26.0 - 37.5 = -11.5$ cm on the scale. Since the short circuit has replaced the load, the load is also located at -11.5 cm. Our data thus show that the minimum is $47.0 - (-11.5) = 58.5$ cm from the load, or subtracting one-half

Since it is much easier to combine admittances in parallel than impedances, let us rephrase our goal in admittance language: the input admittance of the length d containing the load must be $1 + jb_{\text{in}}$ for the addition of the input admittance of the stub jb_{stub} to produce a total admittance of $1 + j0$. Hence the stub admittance is $-jb_{\text{in}}$. We shall therefore use the Smith chart as an admittance chart instead of an impedance chart.

The impedance of the load is $2.1 + j0.8$, and its location is at -11.5 cm. The admittance of the load is therefore $1/(2.1 + j0.8)$, and this value may be determined by adding one-quarter wavelength on the Smith chart, since Z_{in} for a quarter-wavelength line is R_0^2/Z_L, or $z_{\text{in}} = 1/z_L$, or $y_{\text{in}} = z_L$. Entering the chart (Figure 11.25) at $z_L = 2.1 + j0.8$, we read 0.220 on the wtg scale; we add (or subtract) 0.250 and find the admittance $0.41 - j0.16$ corresponding to this impedance. This point is still located on the $s = 2.5$ circle. Now, at what point or points on this circle is the real part of the admittance equal to unity? There are two answers, $1 + j0.95$ at wtg $= 0.16$, and $1 - j0.95$ at wtg $= 0.34$, as shown in Figure 11.25. Let us select the former value since this leads to the shorter stub. Hence $y_{\text{stub}} = -j0.95$, and the stub location corresponds to wtg $= 0.16$. Since the load admittance was found at wtg $= 0.470$, then we must move $(0.5 - 0.47) + 0.16 = 0.19$ wavelength to get to the stub location.

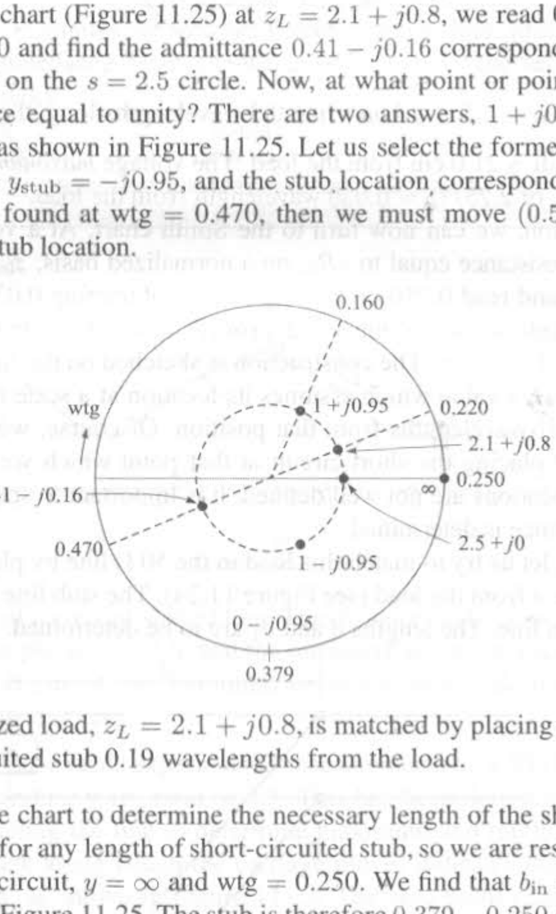

Fig. 11.25 A normalized load, $z_L = 2.1 + j0.8$, is matched by placing a 0.129-wavelength short-circuited stub 0.19 wavelengths from the load.

Finally, we may use the chart to determine the necessary length of the short-circuited stub. The input conductance is zero for any length of short-circuited stub, so we are restricted to the perimeter of the chart. At the short circuit, $y = \infty$ and wtg $= 0.250$. We find that $b_{\text{in}} = -0.95$ is achieved at wtg $= 0.379$, as shown in Figure 11.25. The stub is therefore $0.379 - 0.250 = 0.129$ wavelength, or 9.67 cm long.

EXAMPLE 11.17

It is desired to measure the impedance of an unknown load using the 50 Ω coaxial slotted line set-up shown in Figure 11.22. The experiment is performed by terminating the slotted line in the unknown impedance. The voltage standing-wave ratio measured using this set-up is found to be 3.0, and the voltage minimum is recorded at the scale reading of 50 cm.

Afterwards, the load is removed and a short is placed at the load position. The two consecutive voltage minima are now observed at scale readings of 20 cm and 45 cm, respectively. Using the Smith chart, determine (a) the wavelength, (b) the load impedance, and (c) the reflection coefficient of the load. (d) What would be the location of the first voltage minimum if the actual load is replaced by an open circuit?

Solution.

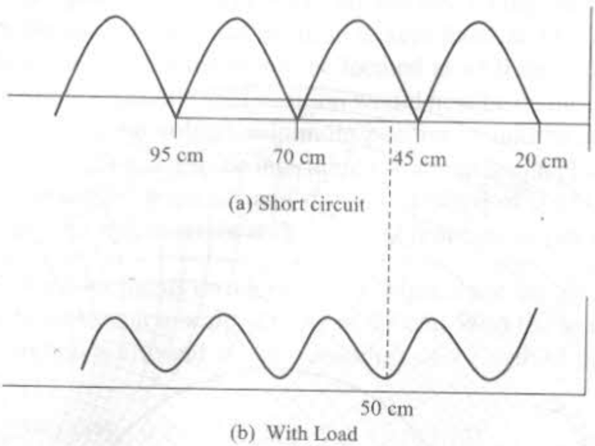

Fig. 11.26 Coaxial slotted line measurement.

The positions of the minimum under the load and the short conditions is shown in Figure 11.26. The detailed procedure is explained below.

(*a*) The consecutive voltage minima under the short conditions are at observed at 45 cm and 20 cm as seen in Figure 11.26(a). As these minima are spaced half wavelength apart, hence the wavelength may be calculated as

$$\lambda = (45 - 20) \times 2 = 50 \text{ cm}$$

(*b*) For finding the load impedance, first draw the constant VSWR circle with $s = 3$ as shown on the Smith chart. Mark the point A, where the $s = 3$ intersects the left-hand side of the real axis. This point A basically represents the voltage minimum point corresponding to the load. In other words, you can consider the point $z = 50$ cm in Figure 11.26 (b) mapped to the point A on the Smith chart.

Now, you can assume the load to be located at any of the minima point corresponding to the short, i.e., the load position can be either at $z = 20$ cm or 45 cm as shown in Figure 11.26 (a). It is convenient to consider the point which is nearest to the load minimum. Hence, the load can be assumed to be positioned at $z = 45$ cm.

It basically means that if you start from the load minima position 'A' on the Smith chart ($z = 50$ cm), and move a distance of $d = 5$ cm $= 0.1\ \lambda$ towards load then you would arrive at the load position ($z = 45$ cm).

On the Smith chart, move on the constant $s = 3$ circle from the point 'A' towards the load until you reach 0.1λ marked on the 'Wavelength towards Load' scale. Draw a line from the centre of