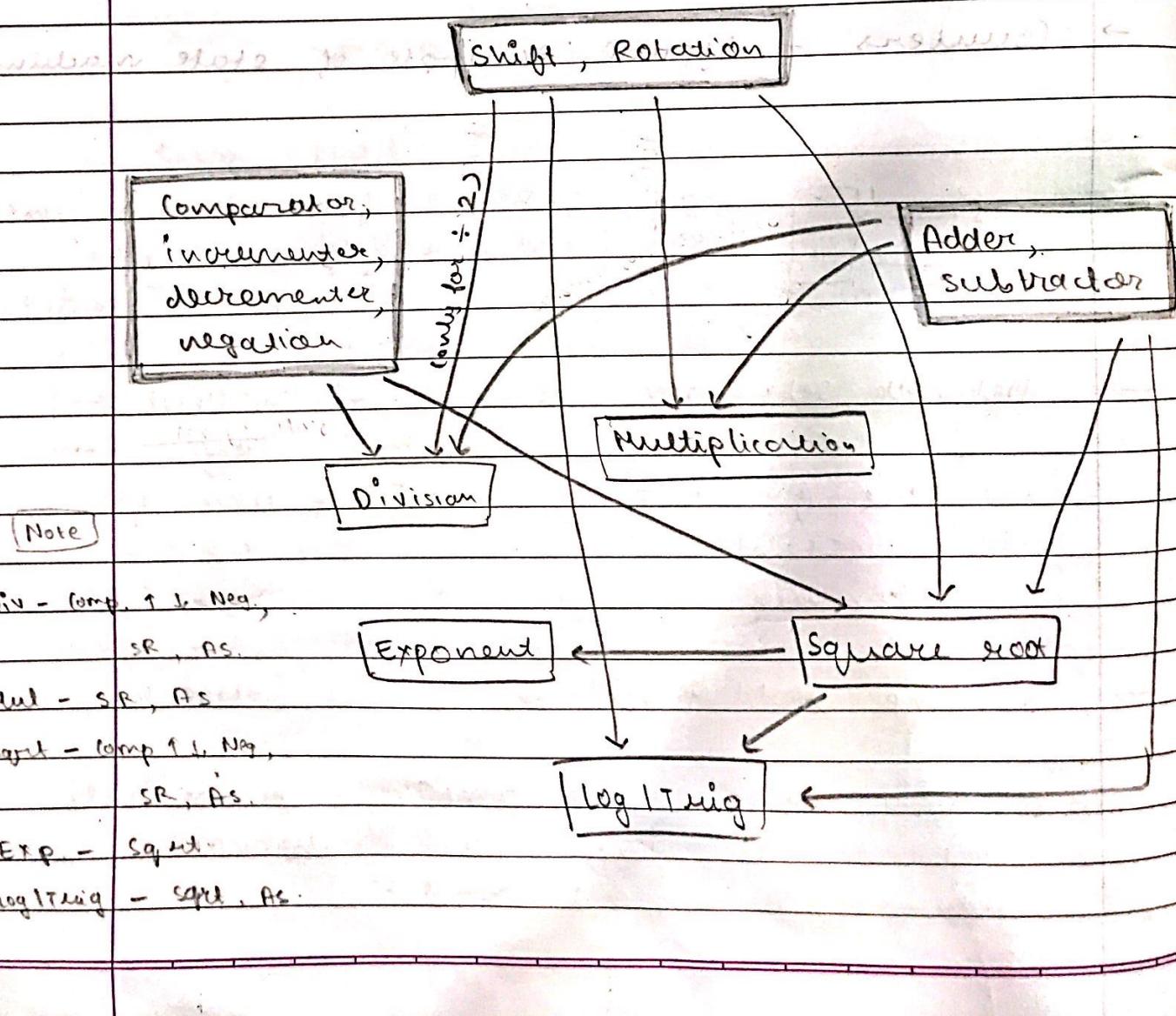


Arithmetic Circuits

- Does any kind of arithmetic operations.
- They are heart of every digital circuit.
- Used in optimisers to improve performance.
- Most microcontrollers have ALU - multiplication, addition, ALU, shifter, incrementer, decrementer etc.

Types of arithmetic functions - shift, rotation, increment, decrement, compare, negation, add, subtract,  $*$ ,  $\div$ ,  $\sqrt$ , exp, log, tiny max complexity.



→ Most operators are using the basic adder circuit, then optimising them is a great idea.

11 0 111 0 11 → 7 ones.

→ We found this 7 by adding, which is essentially counting.

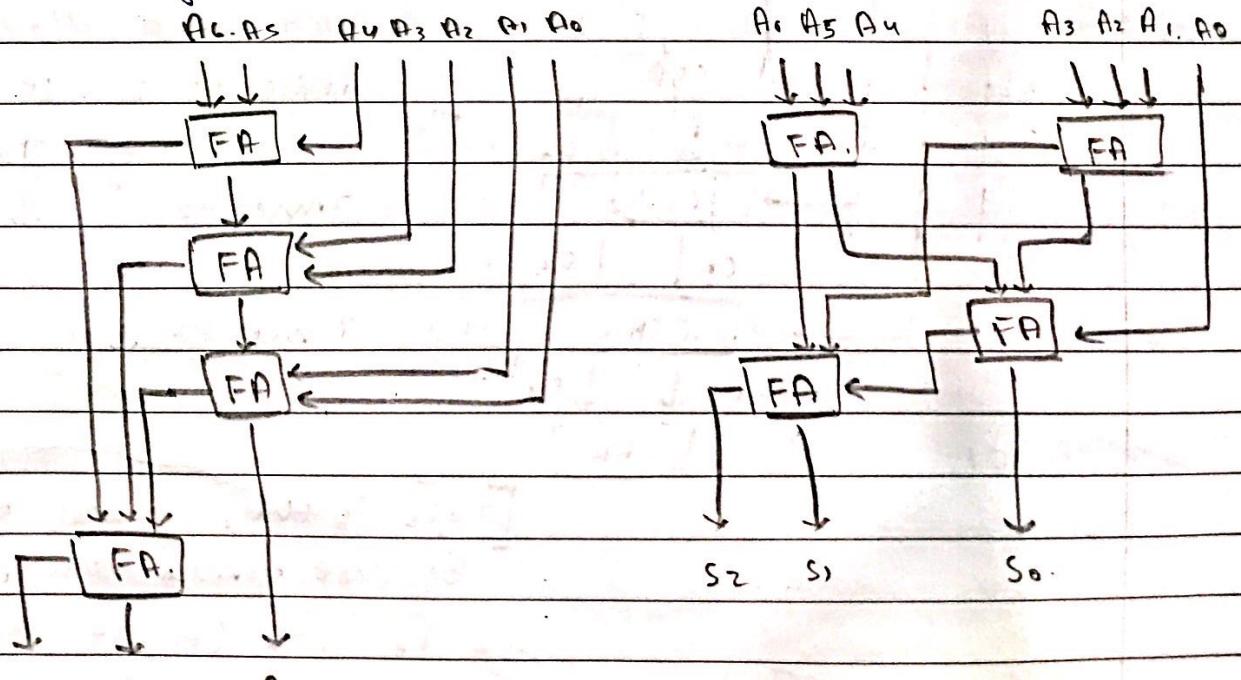
→ Therefore, adders are counters.

→ Half adder is a  $(2, \underline{2})$  counter.

It can add  $\underline{2}$  one bit numbers and result in a  $\underline{2}$  bit number.

→ Full adder is a  $(3, 2)$  counter. It is also called compressor.

→ Adding multiple one bit numbers.

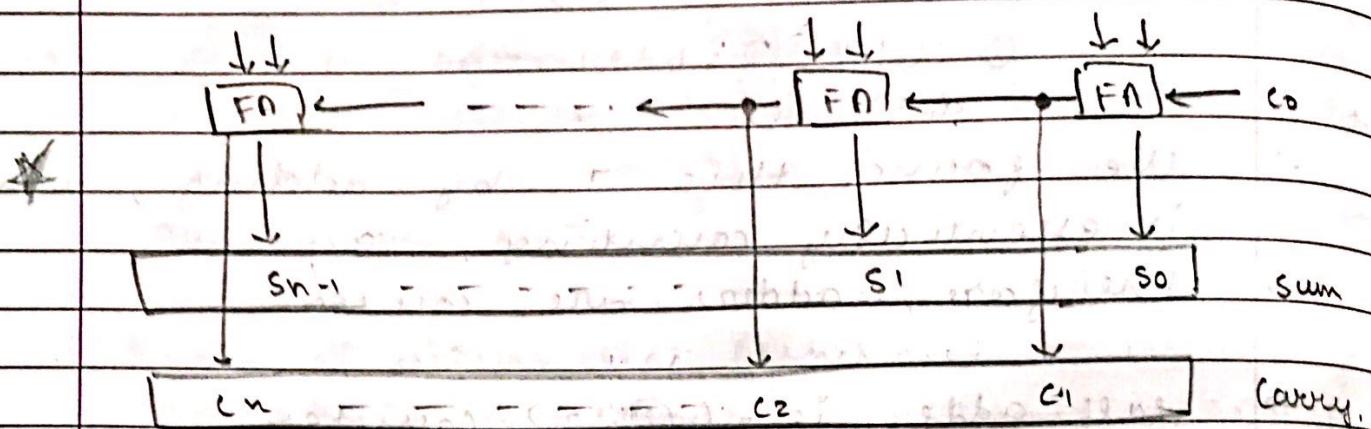


Symmetric.

(This has lesser delay than linear)

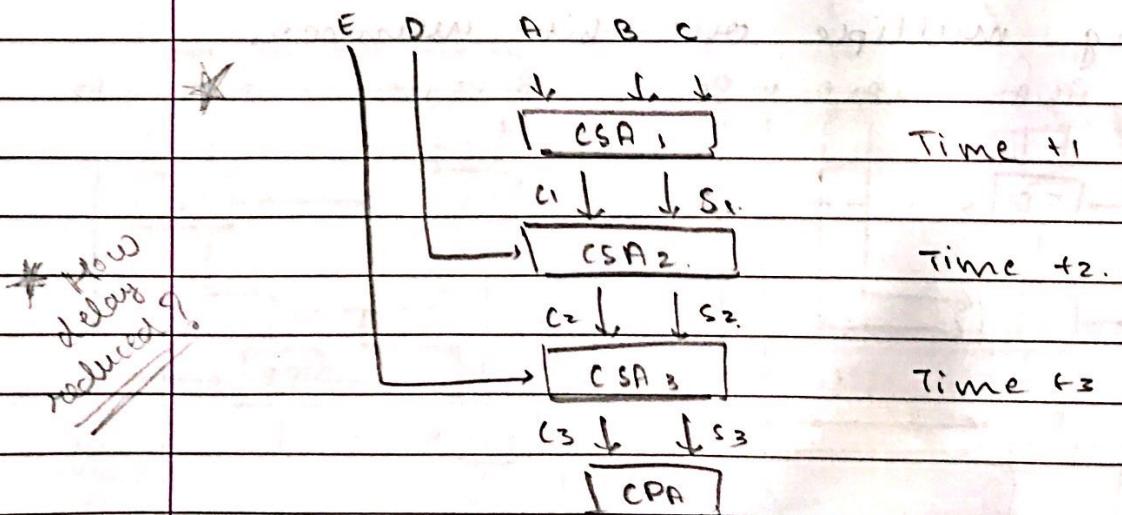
Linear

→ instead of carries to be propagated, we can use carry save adders to reduce multiple inputs to 2 and then single CPA is used.



Here we compressed sum and carry stage.

$$A + B + C = \text{sum} + (\text{carry})$$



[This is the slowest path; because others even parallelly in max (t1, t2, t3)]

Read! [Carry Save adder  
Carry skip. adder]

## Carry save Adder

Eg.  $x = 1357$

$y = 3564$

$z = 2934$

Ans = 7855

sum without :  $1357 + 3564 = 4921$  carry :  $1357$

carry ie pseudo sum :  $3564 + 0111 = 3564$

sum + (sum) :  $2934 + 0111 = 2934$

6745 + 01110

$$\text{Ans} = x + y + z = \text{sum} + \text{carry}$$

so now, here carry and sum can be computed independently and at last stage both are added using carry ripple adder.

Eg.  $x = 10011$

$y = 11001$

$z = 01011$

Ans = 110111

sum = 10011

carry = 10011

$11001 + 01011 = 11011$

01011

00001

110110

Ans = 00001

110110

110111

→ We need 3 operands for the hardware

$$\begin{bmatrix} A & 0111 \\ B & 1010 \\ C & 1111 \end{bmatrix}$$

Two operands for first carry save stage

$$\begin{bmatrix} \text{sum} & 0010 & s_0 \\ \text{carry} & 11110 & c_0 \\ \text{PDS} & 1011 & 0 \end{bmatrix}$$

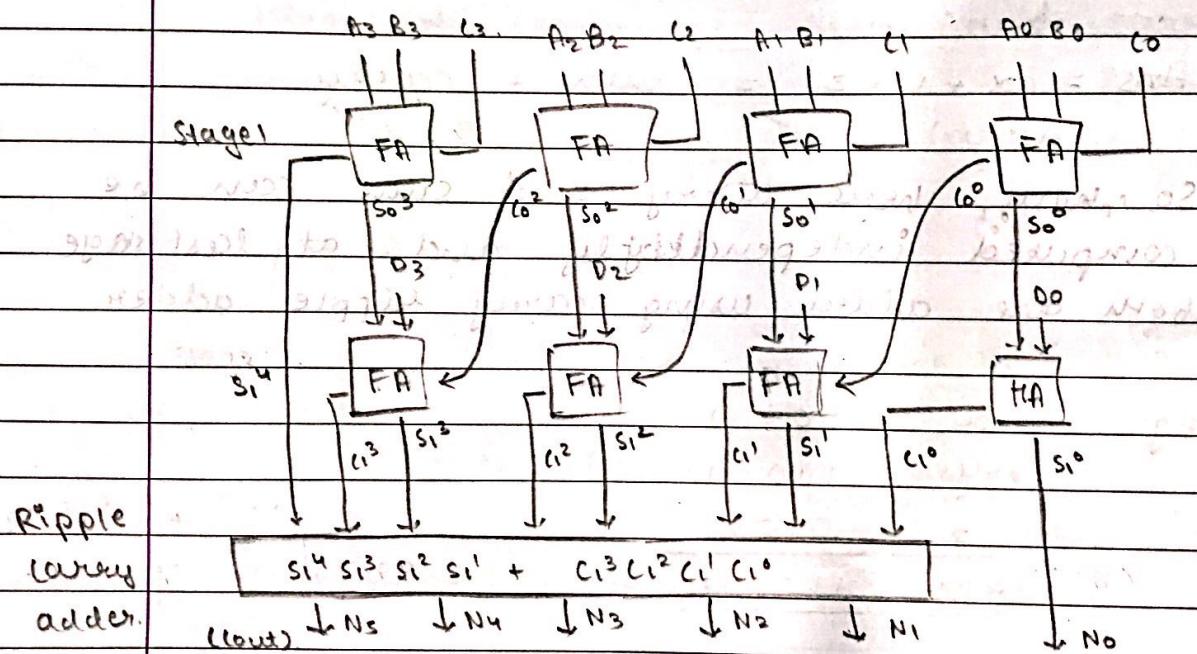
Three operands for second carry save stage

$$\begin{bmatrix} \text{sum} & 10111 \\ \text{carry} & 010100 \\ \text{Ans.} & 101011 \end{bmatrix}$$

Pseudo sum  $s_i$

Final carry out  $c_i$

N

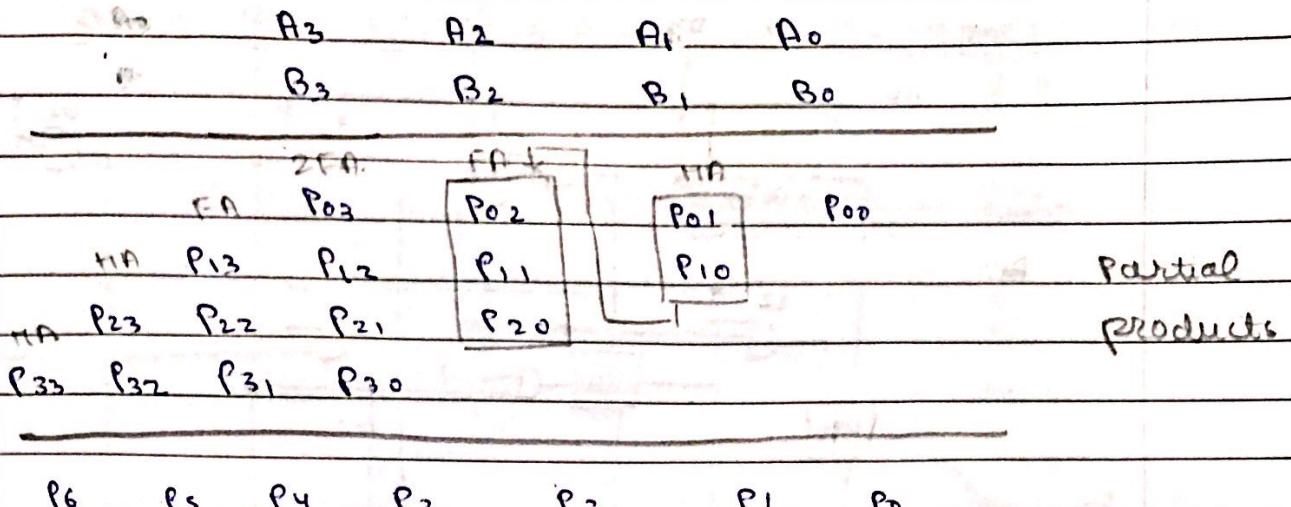


→ For m integer operands. - involves  $(m-2)$  CSA additions.

① In RCA:  $(m-1) + \text{RCA delay}$

② In CSA:  $(m-2) + \text{CSA} + \text{tRCA delay. } (m > 3)$

→ For multiplication of  $A_3 A_2 A_1 A_0$  and  $B_3 B_2 B_1 B_0$ , we addition of partial products.



Thus multiplication of 2 4 bit numbers gives at max 8 bit number.

$$P_0 = P_{00}$$

$$P_1 = P_{01} \oplus P_{10}$$

$$P_2 = P_{02} \oplus P_{11} \oplus P_{20} \oplus C_{12}$$

$$P_3 = P_{03} \oplus P_{12} \oplus P_{21} \oplus P_{30} \oplus (C_{23} \oplus C_{23}')$$

$$P_4 = P_{13} \oplus P_{22} \oplus P_{31} \oplus (C_{34} \oplus C_{34}' \oplus C_{34}'')$$

$$P_5 = P_{23} \oplus P_{32} \oplus (C_{45} \oplus C_{45}' \oplus C_{45}'')$$

$$P_6 = P_{33} \oplus (C_{56} \oplus C_{56}')$$

$$P_7 = C_{67}$$

$$P_{00} = A_0 B_0$$

$$P_{10} = A_0 B_1$$

$$P_{01} = A_1 B_0$$

$$P_{20} = A_0 B_2$$

$$P_{11} = A_1 B_1$$

1

$$P_{33} = A_3 B_3$$

Initial stages are:

realised using AND gates.

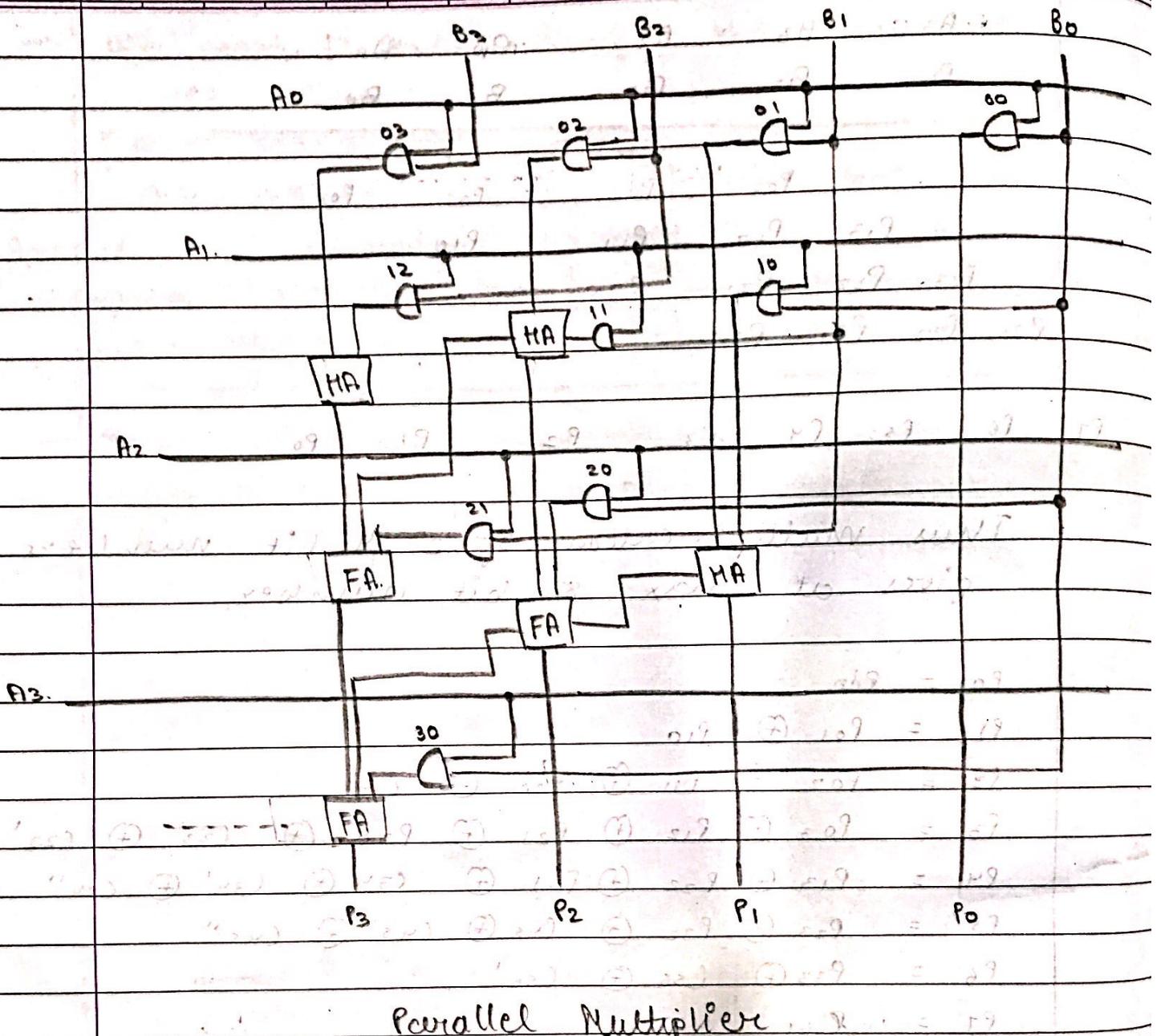
Then subsequently

adders are also

used.

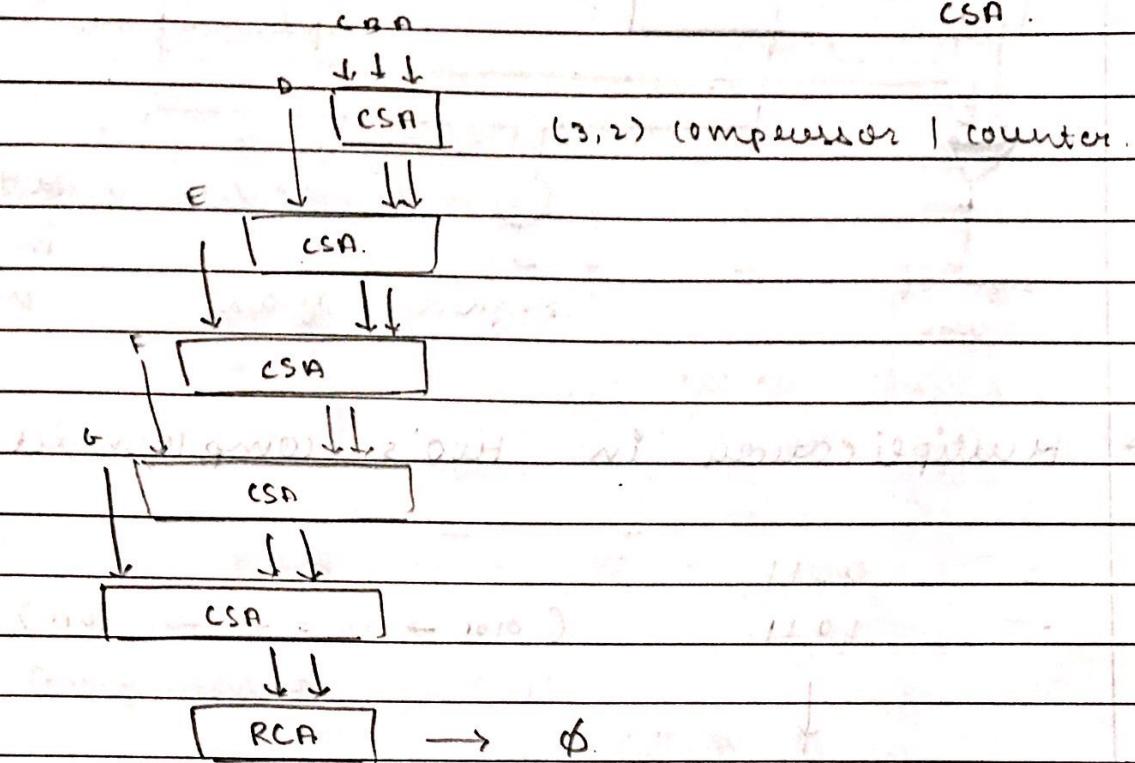
# Worst case delay - 7 structures

Page No.:  
Date:  
YOUVA

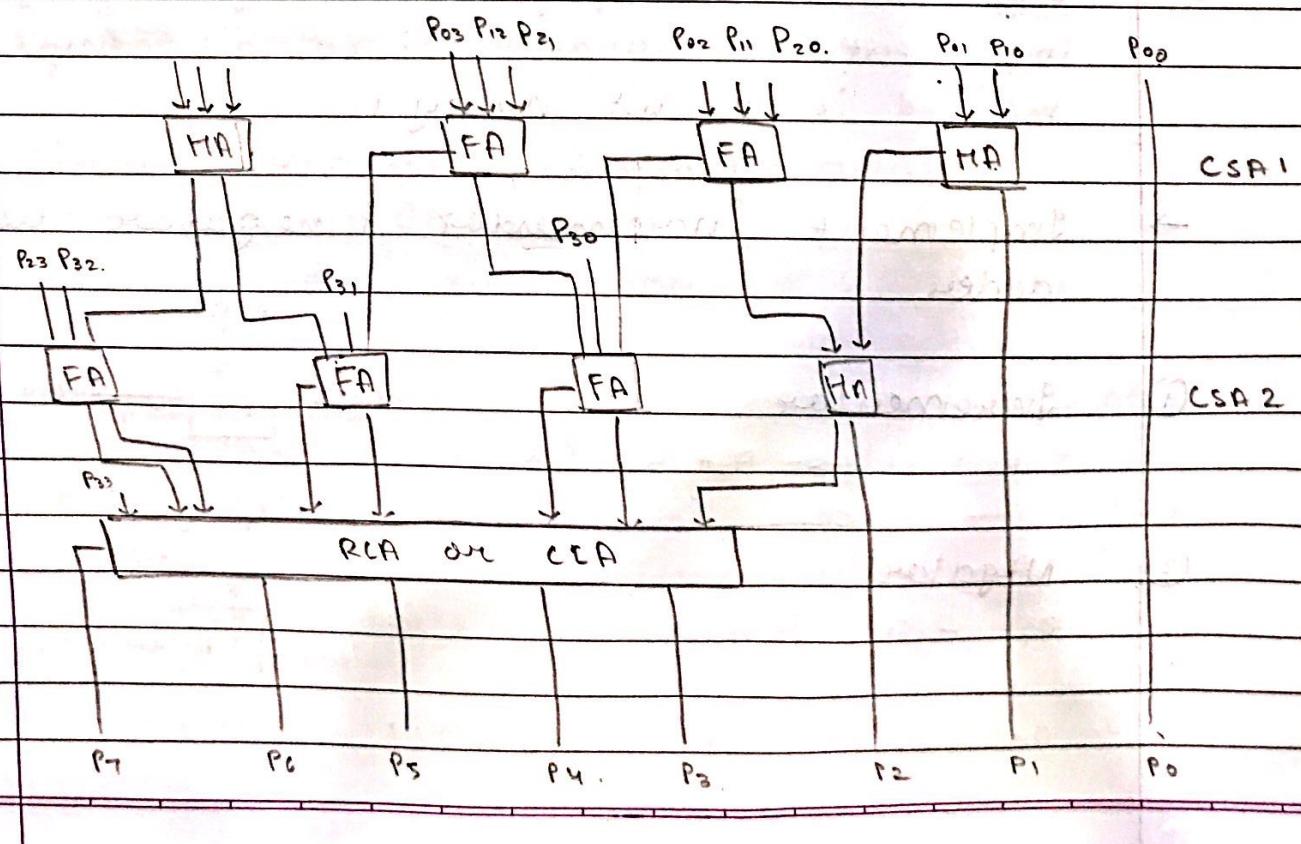


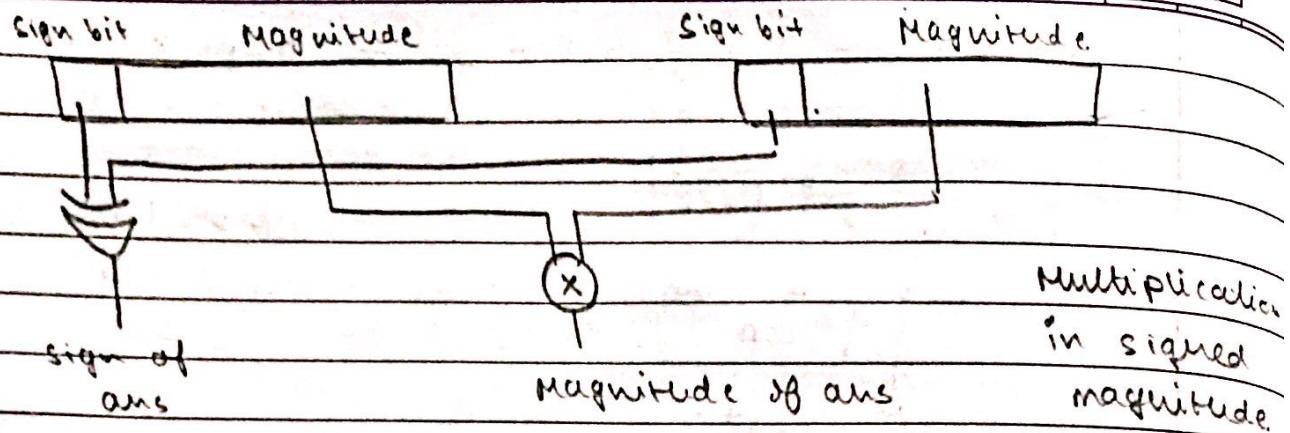
## Parallel Multiplier

$A + B + C + D + E + F + G \rightarrow$  Implement with CSA.



Since I can replace a FA or RCA stage to CSA stage, I can replace parallel multipliers with CSA stage.





→ Multiplication in Two's complement

$$\begin{array}{r}
 3 \quad 0011 \\
 -5 \quad 1011 \\
 \hline
 \end{array}
 \quad (0101 \rightarrow 1010 + 1 \rightarrow 1011).$$

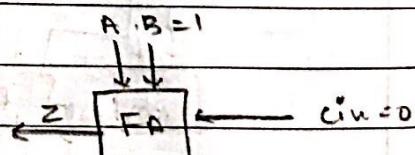
This is basically  $16 - 5 = 11$ . Had it been a 3 bit number, we can take  $8 - 5 = 3$  (011) — shortcut for 2's complement.

Difficult to multiply manually. Costly to implement in hardware. So 2's complement method is a bit messy.

→ Implement incrementor / negator using adder.

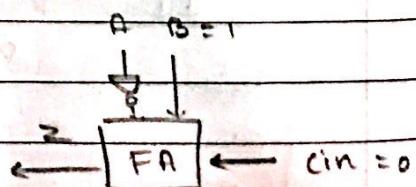
(1) Incrementor

$$Z = A + 1 = A + B + \text{cin}$$



(2) Negator

$$Z = -A = \bar{A} + 1.$$



## (3) Subtractor

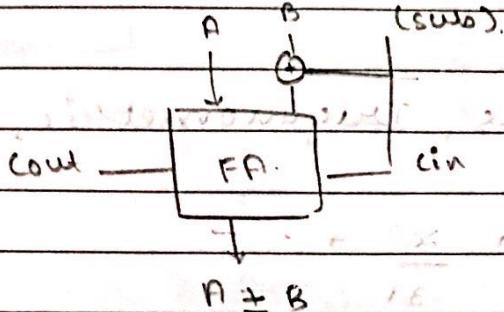
$$Z = A - B$$

$$= A + \bar{B} + 1$$



$$\text{cin} = 1$$

## (4) Adder-subtractor.



When  $\text{cin} = 1$ , it works as subtractor.

## (5) Comparator.

$$A = B \rightarrow \text{EQ.}$$

$$\text{DO } A - B : \quad A' = B \rightarrow \text{NEQ}$$

$$\textcircled{1} \quad \text{If } \text{cout} = 1 ; A > B \quad A > B \quad \text{GE}$$

$$\textcircled{2} \quad \text{If } \text{cout} = 0 ; A < B \quad A < B \quad \text{LT.}$$

$$A > B. \quad \text{GE. EQ}$$

We check  $A = B$  using  
XOR separately.

$$A \leq B \quad \overline{\text{GE}} + \text{EQ.}$$

→ Circuits realised without adders.

- shifters / Rotators.

- and, or, xor, nor, nand, not.

## (6) Logical shifter.

$$11001 \gg 2 \rightarrow 00110$$

## (7) Arithmetic shifter

$$11001 \gg \rightarrow 11110$$

same as logical, but fills  
front places with old MSB.

## (8) Rotator

$$11001 \text{ ROR } 2 \rightarrow 01110$$

(rotate right)

(bits shifted off one end are shifted into other end)

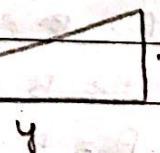
- shifters can be used for \* or  $\div$
- $$\ll 1 = x \cdot 2^1$$
- $$\gg 1 = \frac{x}{2^1}$$

⑨ Dividers, logarithms, Trigonometric, Exponential

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$$



use adder, divider, multiplier and look up table to implement this by storing some values.



$$\tan \theta = \frac{x}{y}$$

If  $\theta$  is very small, its almost like linear transformation / translation.

$d\theta$

→  $\tan \theta$  found by shifting.  
(equivalent to  $2^{-1}$ )

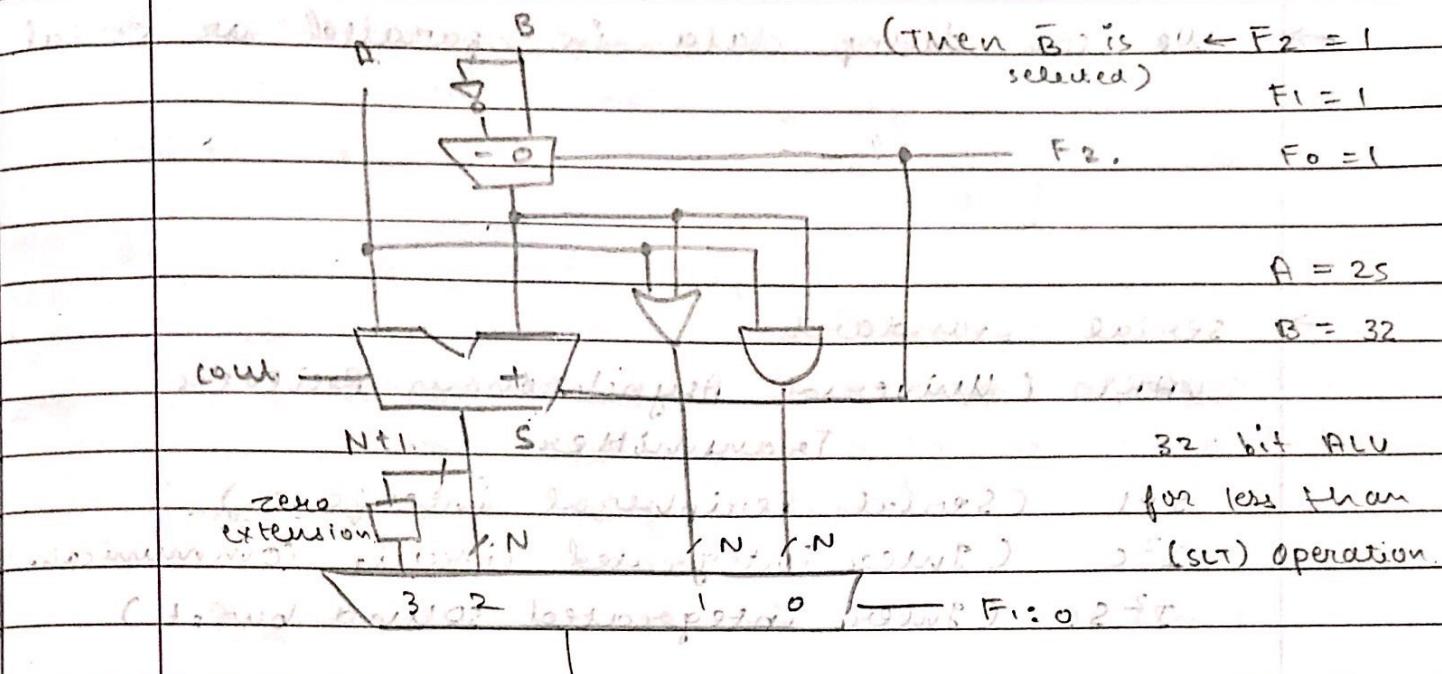
Different stages ...

Fetch — Decode — Execute.

Get the  
instruction  
from memory

Extract from the  
instruction what  
the ALU has  
to do

ALU actually  
works.



Actual implementation of ALU.

$F_{2:0}$  Function

000	$A \& B$	$A - B = -7$
001	$A \mid B$	2's complement
010	$A + B$	Representation
011	Not Used.	has MSB = 1
100	$A \& -B$	$\therefore S_3, \text{ or } Y_3 = 1$
101	$A \mid -B$	
110	$A - B$	$Y = \underbrace{000 \dots 001}_{\text{zero extension}}$
111	SLT (shift left).	to make 32 bits

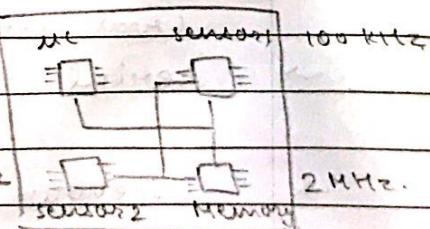
### Communication Protocols / Interfaces (few)

① → On chip : (chip to chip).

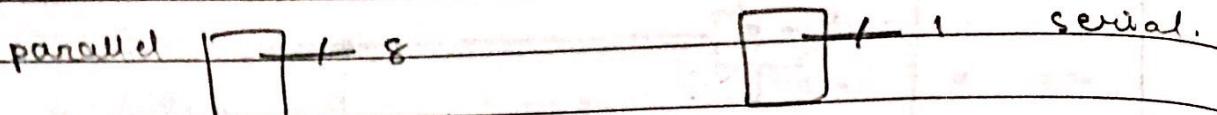
② → Device to device

③ Chip to chip : communicate ←

using I<sup>2</sup>C.



→ We can dump data in parallel or serial



→ serial standards

UART (Universal Asynchronous Receiver Transmitter)

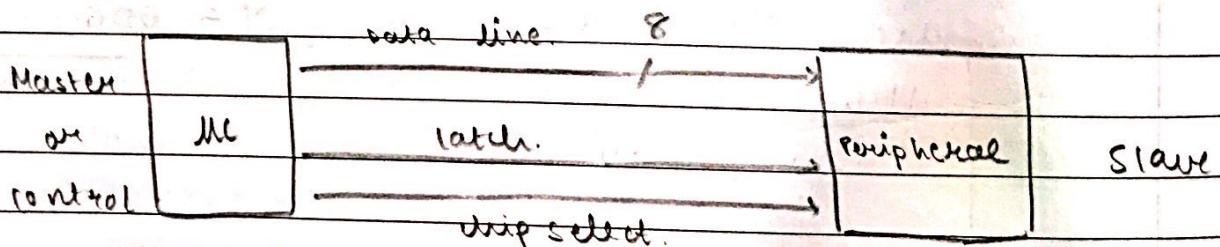
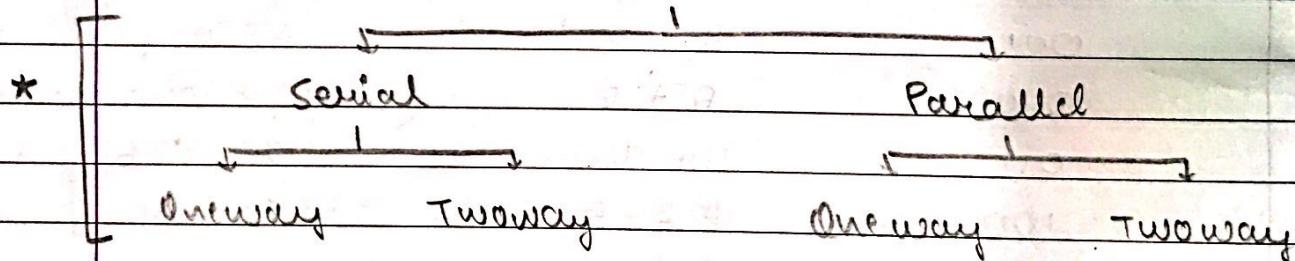
SPI (Serial Peripheral interfaces)

I<sup>2</sup>C (Inter integrated circuit communication)

I<sup>2</sup>S. (Inter integrated sound burst)

② Device to device - standards for communication : USB, Ethernet, VGA, SCSI, HDMI, PS2, sonet etc

### Communication



→ Parallel is fast, but expensive (pin real estate) is expensive.

→ serial is slow, but now expensive.

→ somehow, always parallel is not preferred

→ Serial connection - due to lesser no. of pins, the complexity of communication is high.

\* Can be both types:

(1) Synchronous (with clock)

(2) Asynchronous (without a clock -

but speed i.e. datarate has to be agreed beforehand).

### Asynchronous serial bus standards

(1) RS 232 : (No clock - synchronisation hardware).

→ used for one-one communication



data Tx

data Rx

→ One Tx, one Rx and data rate decided earlier, (using Hyperterminal).

→ Transmission process starts at 9600 baud rate.

$$\frac{1}{9600} \text{ s} = \frac{0.104 \text{ ms}}{\downarrow} \quad \begin{matrix} \text{speed of data} \\ \text{transmission} \end{matrix}$$

receive new info

every 0.104 ms

→ Transmitter idles high

→ It goes low for 1 bit.

→ send LSB first and MSB last.

→ Even parity / odd parity

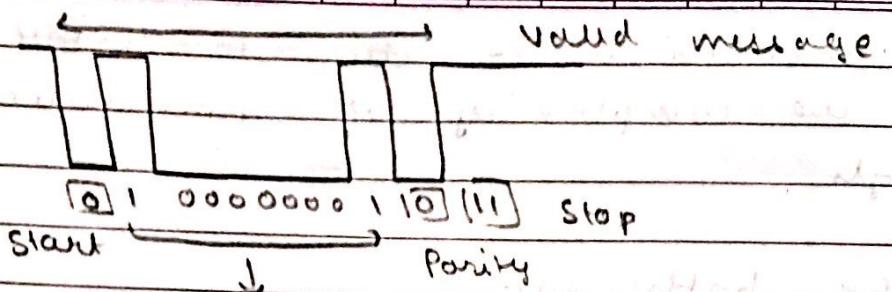
→ start and stop bits for communication

1 bit

2 bits

(0)

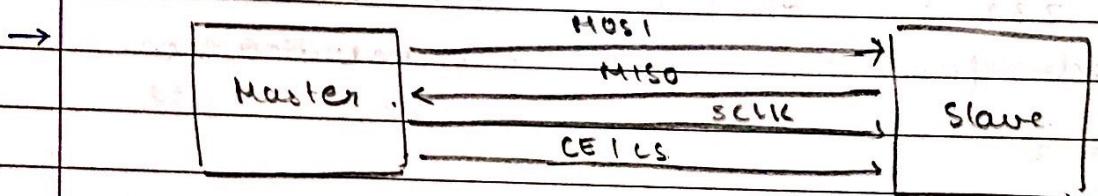
(1)



- Key points of serial communications:
  - Started with < 115.2 Kbps
  - Now typically 10 - 30 Gbps

## ② serial peripheral interface :

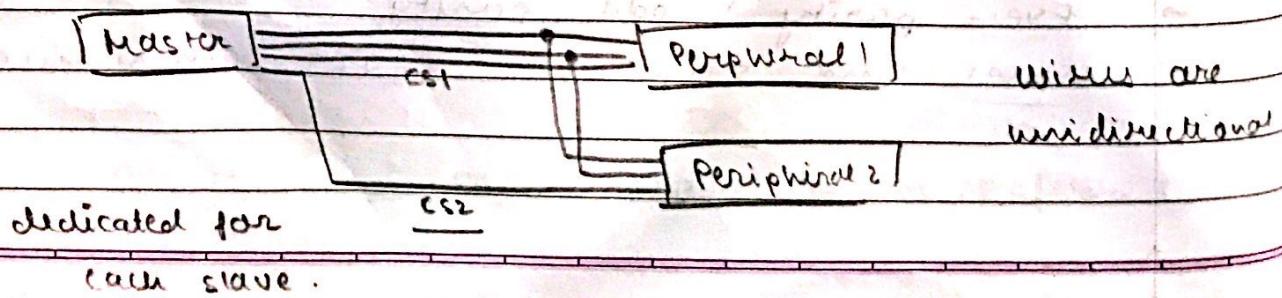
- 4 wires.



- Supports high data rate (1 MHz ~ 70 MHz etc.)
- MOSI - Master out slave in
- MISO - Master in slave out
- CE / CS - Chip enable / chip select
- SCLK - synchronous clock.

SPI does not have this. There speed decided apriori. Here clock decides data rate.

- MOSI, MISO, SCLK can be shared across peripherals but CE / CS is dedicated.



- Radio channels have SPI interfaces. - Need
- All sensors have SPI interfaces. order of MHz. (can't use USB)

$\overline{CS}$  = 0 enable (neg logic)

$CS$  = 1 enable

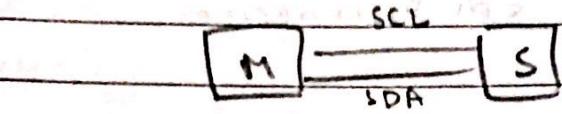
- CS used to make a dedicated connection between one slave and master (i.e. selects one particular slave)
- MISO - master accepts input from slave
- MOSI - master sends data to slave
- SPI Pins - There are typically 6 pins.
  - 1) CE (CS)
  - 2) SCK
  - 3) MOSI
  - 4) MISO
  - 5) RES (Reset).
  - 6) D/C - Data command (seen in devices to write lots of data e.g. display).

(2 optional pins - VDD and ground)

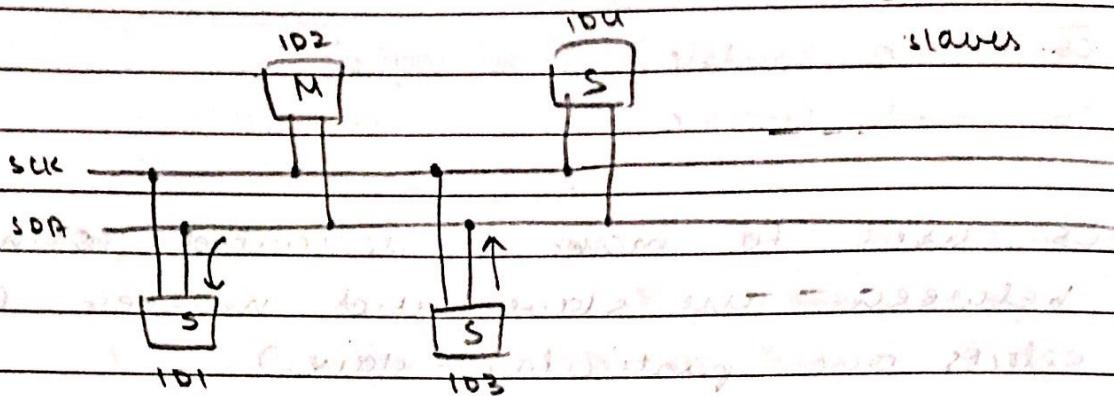
- For only one slave device, we can drop chip select pin.
- We can also drop RES pin and simply use soft reset.
- Drop D/C from simpler application.

Thus SCK, MOSI, MISO are only imp.

### ③ I<sup>2</sup>C - Inter Integrated Circuit.



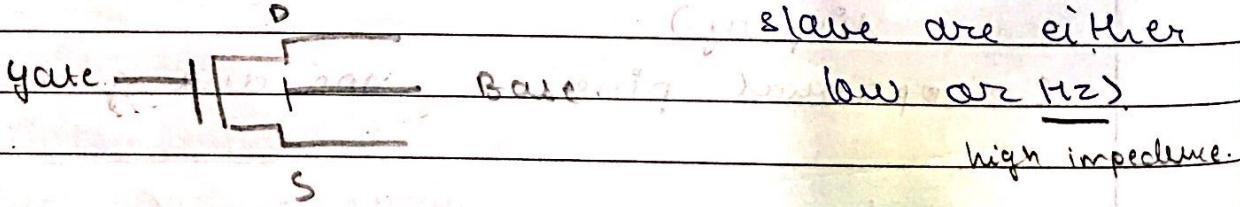
uses 7 bit addressing  
so can connect  $2^7$  slaves



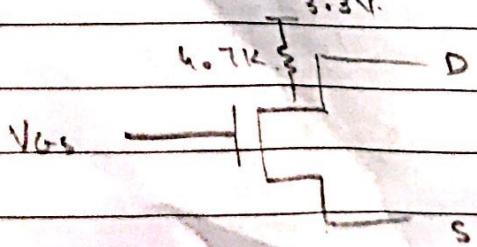
Two blocks cannot write on bus at same time. Suppose M want to write to S, it takes control of data bus. At that point, no other block can write.

After done, the bus again collapses in high impedance state and other block can now use it.

- Select  $IO_2$  by pulling external pins H<sub>2</sub> or low
- I<sup>2</sup>C uses open drain. (both master and



When no input at gate, D is floating. hence is H<sub>2</sub> state. so solve this add a pull up resistor.



- Master is in charge of SCK frequency (typically 100 kHz or 400 kHz)
- Master generates the address of the slave.
- If receives an ACK or NACK (if it gets a NACK it means the slave address is invalid, so regenerate).
- It sends a R0 | WR depending on whether it has to read or write.
- If will send | receive data.

① Sequence of events when master is to write to a slave.

Start → write → device address → register you want to write to → send data → stop

② Sequence is as following when master is to read from slave.

Start → <sup>write = 1</sup> read → send device address → send the message you want to read from → restart communication → read = 1 → send device address → read bits → after every byte ACK to the slave → continue till NACK

→ Difference in SPI and I<sup>2</sup>C: Clock speed - SPI has 100 MHz while I<sup>2</sup>C has 100 kHz. (But problem with SPI - it is dedicated. Once it is connected to sensor, it cannot be shared.)

→ Advantage of SPI - bidirectional. We can configure master into slave at even time.

\* SPI VS I<sup>2</sup>C

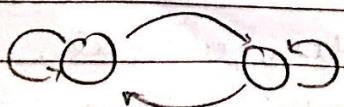
VIDEO!

- SPI - single source, single destination.
- I<sup>2</sup>C - single source, multiple destinations
- ↓
- fanout - 127 devices (limitation)  
(Now 1023 slaves are supported)
- Data Collision - when 2 devices try to use the same bus, data collides. Here, one device will have to back up.
- For repetitive transfer (same source and destination), we can remove source, destination, checksum bytes, but we always have to send preamble.
- Optical Tx Rx standards - beam angle = 30° (half angle = 15°)
- Isochronous data transfer - happening at same time.

### State Machines

Controls a digital system that carries a step by step procedure (algorithm).

state diagrams: Transitions are called arcs



(state graph)

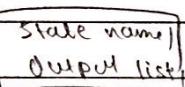
Typical flow chart used that are called Algorithmic State Machines (ASM)

- Mainly used to design control units.
- A given state machine chart can be converted to several forms. It gives rise to several different digital (network) implementations.
- SM (state machine) — certain rules are to be followed.
- When these rules are followed, SM chart is equal to a state graph. (leads directly to network implementation.)
- 3 parts (the 3 SM blocks) that make a SM chart:

(1) State box.

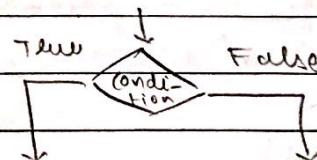


Assign optional state code



Optional output list

(2) Decision box



Boolean Exp.

(3) Conditional list

Cond. list



contains

conditional

output list.

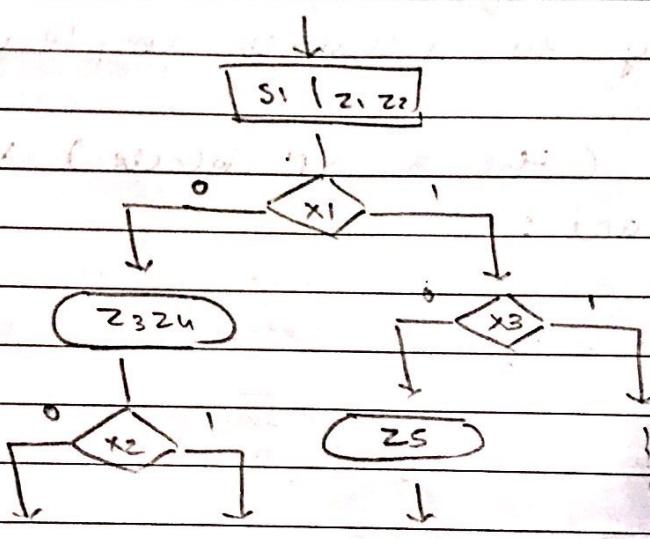
Conditional output depends both on state of system and inputs.

→ Thus, SM chart has 3 SM blocks (an input and an output — it describes that the machine is in one state)

- ① One state box
- ② Few decision blocks
- ③ Conditional output boxes (associated with state)

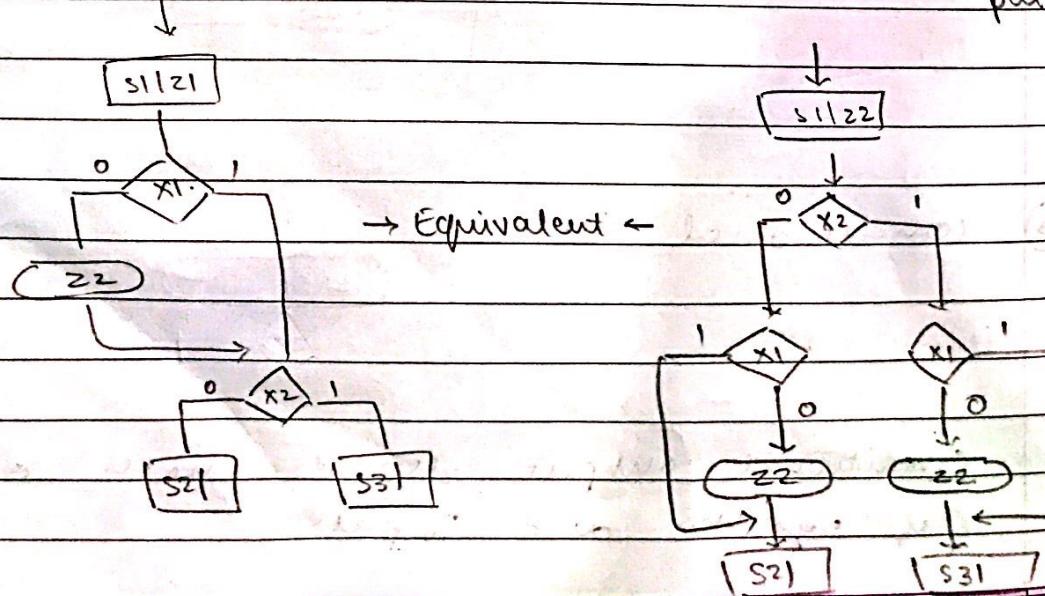
→ A path through an SM block from entrance to exit is known as finite path

Eg 1

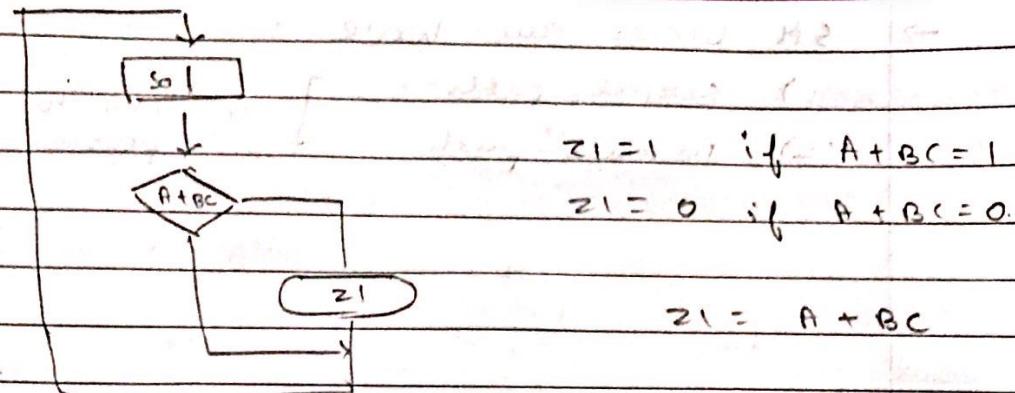


1      2      3      n      There can  
There can be n possible exit paths.

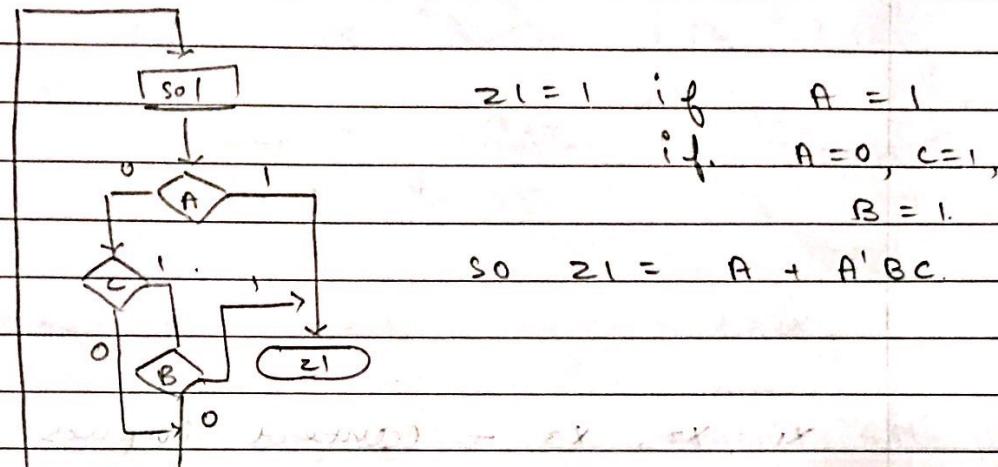
Eg 2



Eg 3

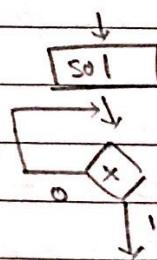


Eg 4

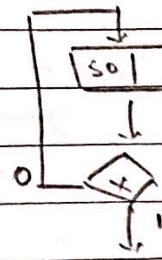


### Rules

- (1) For valid combinations of input variables, there must be exactly one path. This is because a single input can lead to a single next state.
- (2) Internal feedback is not allowed.



Not allowed



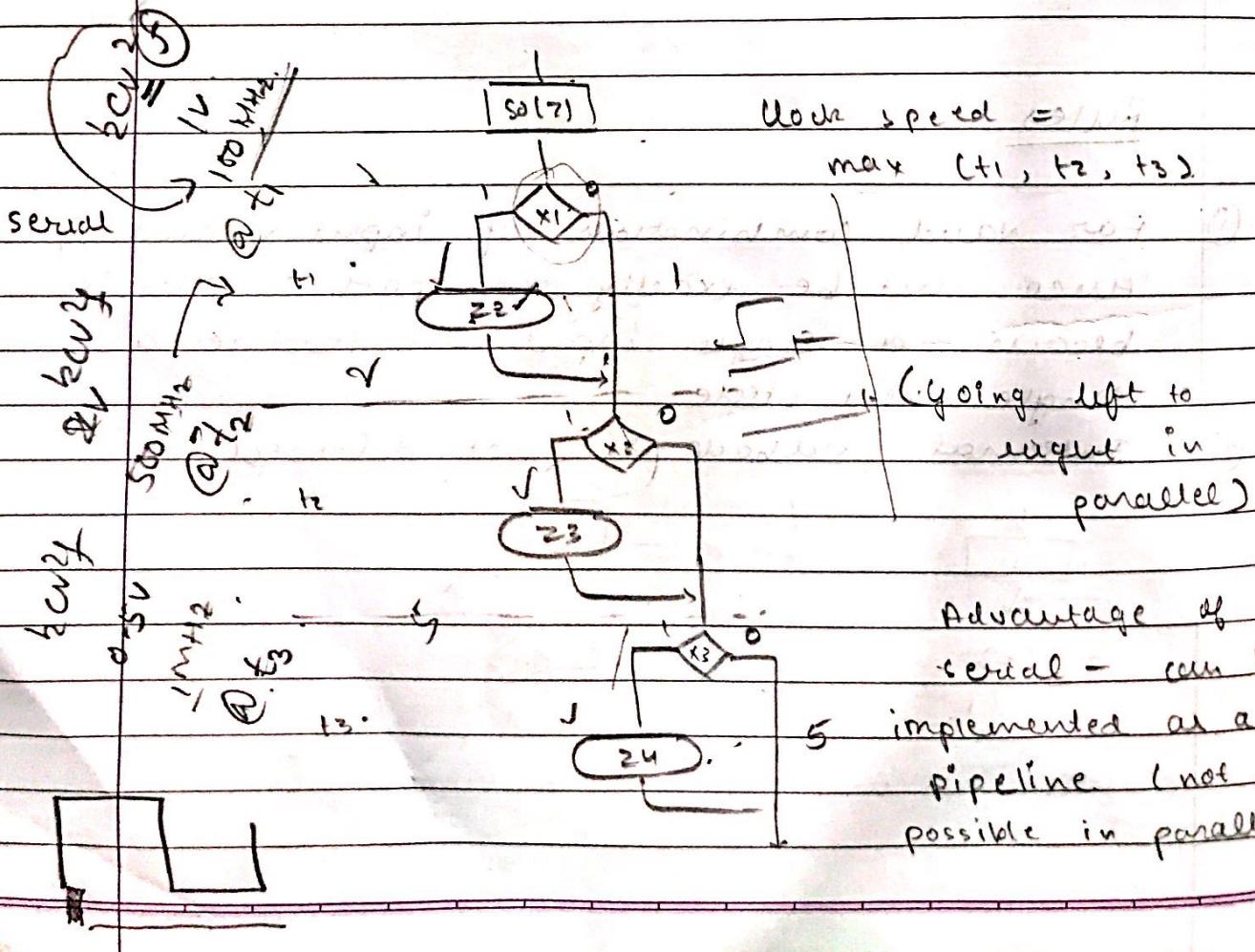
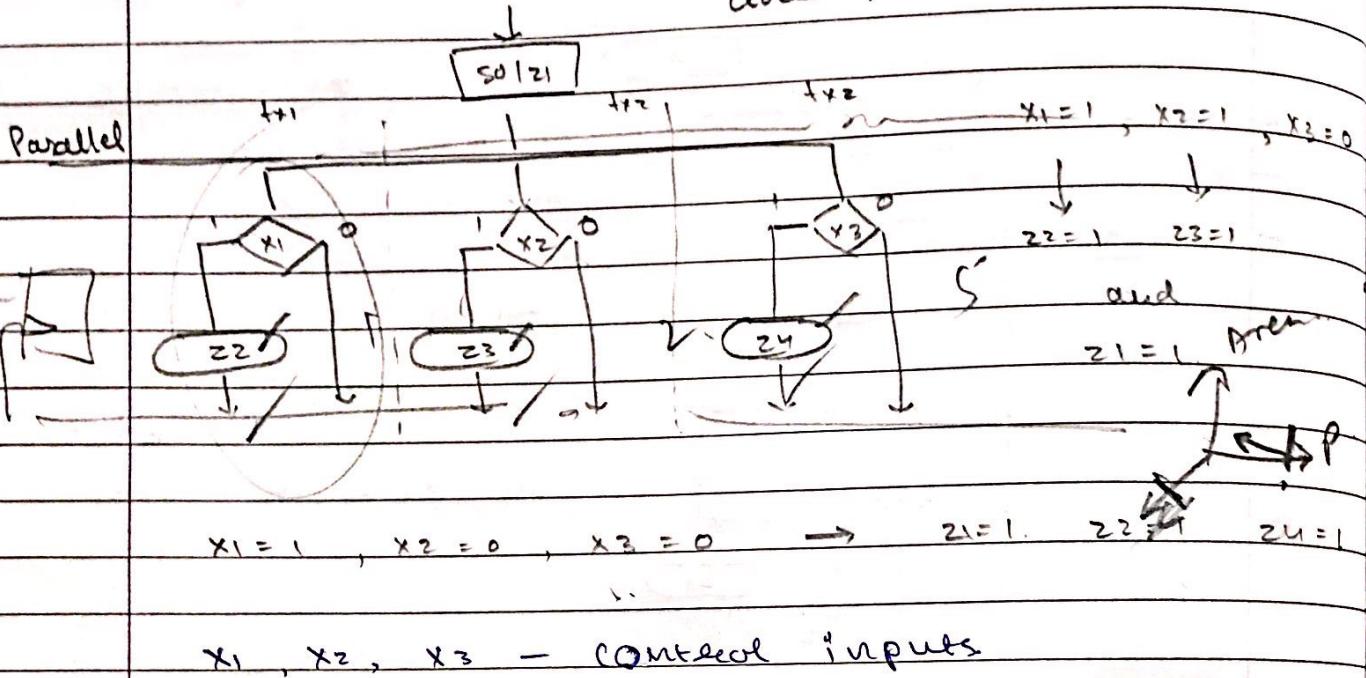
Allowed.

→ SM block can have :

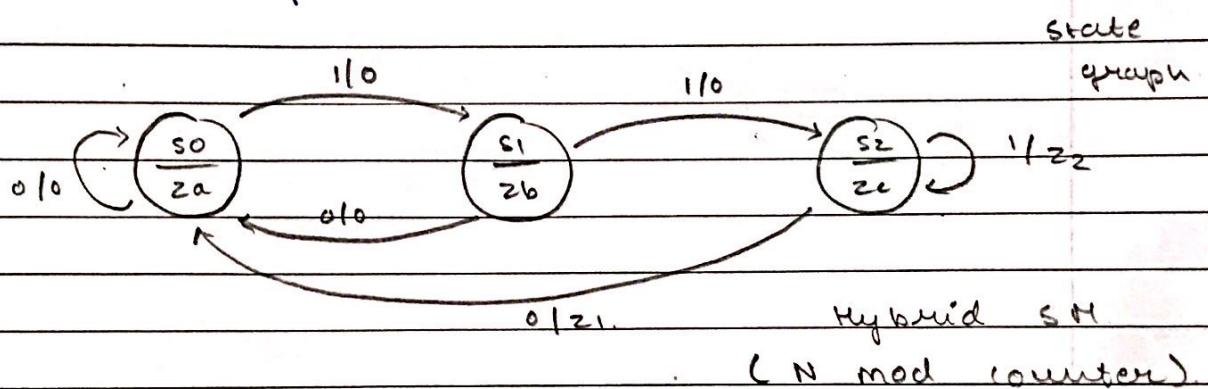
- 1) serial path
- 2) parallel path

reaching to same exit path

$$\text{Clock speed} = \max(t_{x1}, t_{x2}, t_{x3})$$

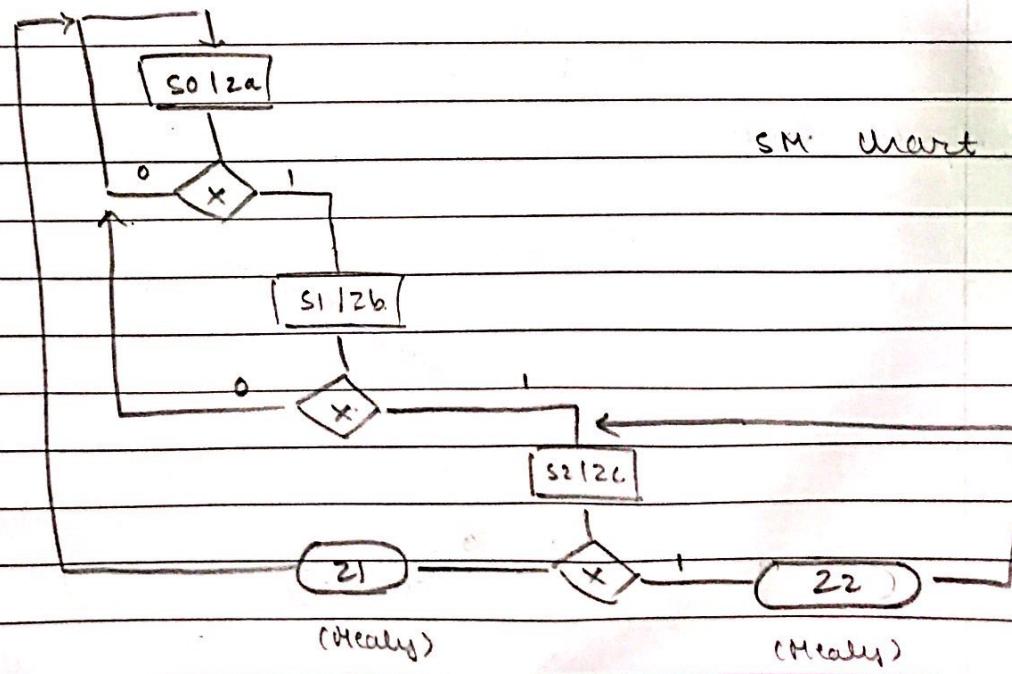


- state graph  $\rightleftharpoons$  SM chart.
  - state graph can have a Mealy machine / Moore machine
  - Moore machine : Output  $z_a, z_b, z_c$  placed inside state boxes. Independent of inputs. Depends only on current state
  - Mealy machine : Output  $z, z$ . outside state boxes. Depend on both input and current state.



$$S_2 : \quad z_c = 1 \quad (\text{Muore})$$

If  $x = 0$      $z_1 = 1$  ,     $NS = S_0$     ( Mealy )



# NOTES

you've

## Insem 1

- Number systems (binary | hex | octal ...)
- State machines
- Counters
- Difference between latch and flip flop
- sequential | combinational
- Set-up and hold time
- n-bit address
- Floating point | fixed point
- Hardware.

## Insem 2

- Adders , Adders to ALU , CLA , C save A  
→ Multiplier  
→ Protocols. (RS232, I<sup>2</sup>C, SPI - sure!) - standard
- Given ALU and an operation, find data path and no of clock cycles needed for it
- SM chart