A quick review of Probability Theory:

Motivation: why are we studying Probability Theory in CTIII?

→ As both P_s and W are reduced, the effect of noise at power P_n becomes more noticeable.

→ However, noise is entirely random. Its effect can be evaluated using Probablistic framework.

→ The Bayesian framework (observed effects used to ascertain the possible cause) is entirely based on the Prob. Theory

A quick Review of Prob. Theory:

We will typically define a Random Variable (R.V.) with letter X, Y, n, e, etc.

- An R.V. is always a number.
- An R.V. can take one of either finite or infinite set of numbers value/member
- Associated with each such case is a probability.

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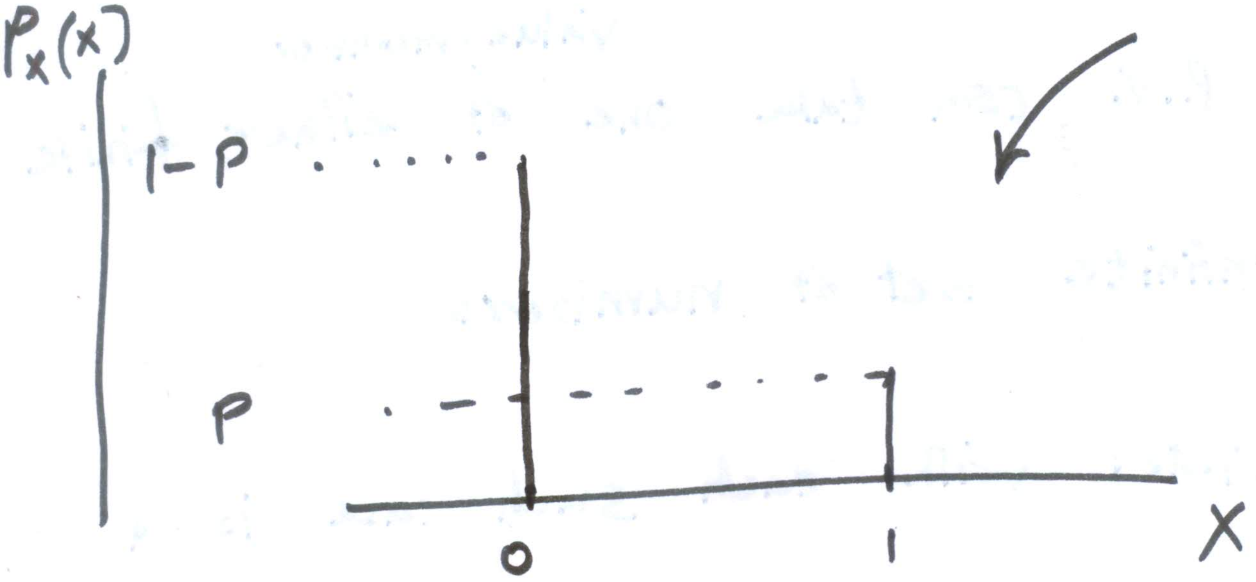
- An Example: X takes values from a finite binary-valued set $\{0, 1\}$. Such an X is called Bernoulli RV.

$$P(X=0) : 1-p$$

$$P(X=1) : p$$

This can be plotted in a function, Prob. Mass Function (PMF)



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Define $E[X] : \sum x P_X(x) \leftarrow \mu_X$

Therefore $E[X^2] : \sum x^2 P_X(x)$

$$VAR[X] : E[(X-\mu_X)^2]$$

Question: Evaluate for Bernoulli (P)

RV, $E(X)$, $Var(X)$.

$$E(X) = P$$

$$E(X^2) = P$$

$$Var(X) = P(1-P) = P - P^2 .$$ 4

Average of N different realizations

$\{x_1, x_2, \dots, x_N\}$ of RV X

$$\bar{X} = \mu_N = \frac{1}{N} \sum_{i=1}^{N} x_i$$

Let us denote the set of possible values that the RV X takes

as $\{x^{(1)}, x^{(2)}, \dots, x^{(M)}\}$.

For example, for Bernoulli RV,

$$m = 2 , \{ x^{(1)}, x^{(2)} \} = \{ 0, 1 \}$$

$$\bar{X} = \mu_N = \frac{1}{N} \sum_{i=1}^{N} x_i$$

$$= \frac{1}{N} \sum_{m=1}^{M} N_m x^{(m)} \quad (5)$$

N_m : denotes the number of times the RV X takes the value $x^{(m)}$ in N experiments.

Take an example:

$$N = 5 , \{ x_1, x_2, x_3, x_4, x_5 \}$$

$$= \{ 0, 0, 1, 1, 1 \}$$

$$\bar{X} = \mu_5 = \frac{1}{5}(0+0+1+1+1) = \frac{1}{5}(2 \times 0 + 3 \times 1)$$

$$\bar{X} = \mu_N = \sum_{m=1}^{M} \frac{N_m}{N} x^{(m)}$$

How did we define the ratio

$$\lim_{N \to \infty} \frac{N_m}{N} = P(X = x^{(m)})$$

Relative frequency definition of Prob.

$$\lim_{N \to \infty} \mu_N = \sum P_X(x) \cdot x$$

$$= E[X]$$

A key Point :

For finite values of N, μ_N is itself a RV.

How to characterize μ_N ?

Two important Theorems of Probability Theory :

If $X_1, X_2, \ldots, X_N$ are independent RVs : (7)

(i)

$$E[X_1 + X_2 + \ldots + X_N] = E[X_1] + E[X_2] + \ldots + E[X_N]$$

(ii)

$$Var[X_1 + X_2 + \ldots + X_N] = Var[X_1] + Var[X_2] + \ldots + Var[X_N]$$

$$E[\mu_N] = \underline{E[X]}$$

$$Var[\mu_N] = \underline{Var[X]/N}$$

A non-binary discrete-valued

RV :

Let us say That $X \sim$ Bernoulli(p)

RV.

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Let us define

$$\hat{X} = \{x_1, x_2, \ldots, x_N\}$$

$P(\hat{X}$ having K ~~zeros~~ ones and $N-K$ zeros$)$

$$= \underline{p^K (1-p)^{N-K}}$$

Assume That X_i are independent.

Let us define ($X_i \sim$ Bernouilli(p))

$$Y = \sum_{i=1}^{N} X_i$$: is called Binomial RV

— What are The Values of Y ?

$$K \in \{0, 1, \dots, N\}.$$

— What is $P(Y = K)$? 9

— Example: $N = 3$.

Y	x_1	x_2	x_3	$P(y)$
0	0	0	0	$(1-p)^N$
1	0	0	1	$\binom{N}{K}(1-p)^{N-K} p^K$
	0	1	0	
	1	0	0	

— What is $E[Y]$? What is $Var[Y]$?

$$r = s + n$$

zero mean & σ_n^2 variance

$$P\left(r/s\right) \Rightarrow$$ Gaussian PDF with mean of s and variance of σ_n^2

Cond. Prob. of r given s is transmitted

$\underbrace{\text{Cond. Prob. of } r \text{ given } s \text{ is transmitted}}$

The likelihood function

"Forward Prob."

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