Tutorial 3

1. Find the volume of the largest right circular cylinder that can be inscribed in a sphere of radius 5.

2. You operate a tour service that offers the following rates.

   (a) 200 Rs per person if 50 people (the minimum number to book the tour) go on the four.

   (b) For each additional person, up to a maximum of 80 people total, the rate per person is reduced by 2 Rs.

   It costs 6000 Rs plus 32 Rs per person to conduct the tour.
   How many people does it take to maximize your profit?

3. Consider
$$f(x) = \begin{cases} x, & 0 \leq x < 1 \\ 0, & x = 1 \end{cases}$$
   $f(0) = 0 = f(1)$. According to Rolle's Theorem $\exists c \in (0,1)$ such that $f'(c) = 0$. But its derivative on $(0,1)$ is never zero. How can this be?

4. Show that if $f'' > 0$ throughout the interval $[a,b]$, then f' has at most one zero in $[a,b]$. What if $f'' < 0$ throughout $[a,b]$ instead?

5. A trucker handed in a ticket at a tool booth showing that in 2 hours he had covered 159 miles on a toll road with speed limit 65 mph. The trucker was cited for speeding. why?

6. A marathoner ran the 26.2 mile marathon in 2.2 hours. Show that at last twice the marathoner was running at exactly 11 mph, assuming the initial and final speeds are zero.

7. If $f'(x) = 0$ for all $x \in (a,b)$, then $f(x)$ is constant. Prove.

8. Show that $|\cos x - 1| \leq |x|$ for all x-values. (Hint: Consider $f(t) = \cos t$ on $[0,x]$ and apply MVT.)