Lecture - 2

Definitions:

Sample Space: $\boxed{\text{Set}}$ of all possible outcomes of an experiment.

e.g. toss a coin

sample space = $\{H, T\}$

throw a dice.

$$S = \{1, 2, 3, 4, 5, 6\}$$

throw 2 dice at the same time.

$$\left| \left\{ \begin{matrix} 11, & \cdots, & 16, \\ 21, & \cdots & 26, \\ \cdots & \cdots & 66 \end{matrix} \right\} \right| = |S| = 36$$

Event: any subset of the sample space

$E \subseteq S$

→ Event is also a set.

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if $|S| = n$, then you may define 2^n different events on this sample space.

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10 horses are running in a race.

A: Horse SeaBiscuit comes first.

B: Alpha Boy is first and Sea Biscuit is second.

S = set of all possible permutations/orderings.

a, b, c, d, e, f, g, h, i, j

decbaghjif $\in S$

jfighabedc $\in S$

S = set of all possible permutations of these 10 horses.

A = Seabiscuit, $\underbrace{\cdot \quad \cdot \quad \cdot}_{\text{9 other horses.}}$

= set of all possible orderings where Seabiscuit is 1^{st}

B = Alpha Boy, Sea Biscuit, $\underbrace{- \quad \cdot \quad - \quad \cdot}_{\text{others 8}}$

Set operations
on Events.

$E \cup F$, $E \cap F$, E^c

Laws:

Commutative law

Associative laws

$$(A \cup B) \cup C = A \cup (B \cup C)$$

Distributive laws

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

De Morgan's laws

$$\overline{(A \cup B)} = \bar{A} \cap \bar{B}$$

3 axioms of probability 5

i) $0 \leq P(E) \leq 1$

ii) $P(\text{sample space}) = 1$

iii) if $E_1, \ldots, E_n$ are mutually exclusive events,

$$P\left(\bigcup_{i=1}^{\infty} E_i\right) = \sum_{i=1}^{\infty} P(E_i)$$

e.g. if $A \cap B = \phi$

$$P(A \cup B) = P(A) + P(B)$$

Probability of Union = Sum of probabilities.

Theorem:

$$P(\phi) = 0$$

$$\underbrace{S \ , \ \phi}$$

Are these mutually exclusive?

$$S \cap \phi = \phi \checkmark$$

$$P(A \cup B) = P(A) + P(B)$$

$$A = S, \quad B = \phi$$

$$P(\underbrace{S \cup \phi}) = P(S) + P(\phi)$$

$$\underline{P(S)} = \underline{P(S)} + P(\phi)$$

$$P(\phi) = 0$$

e.g.

$$P(\bar{E}) = 1 - P(E)$$

$$\underset{A}{E} \cap \underset{B}{\bar{E}} = \phi$$

$$P(E \cup \bar{E}) = P(E) + P(\bar{E})$$

$$P(S) = 1 =$$

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e.g. if $E \subseteq F$, then

$$P(E) \le P(F)$$

$A = E$

$B = \bar{E} \cap F$

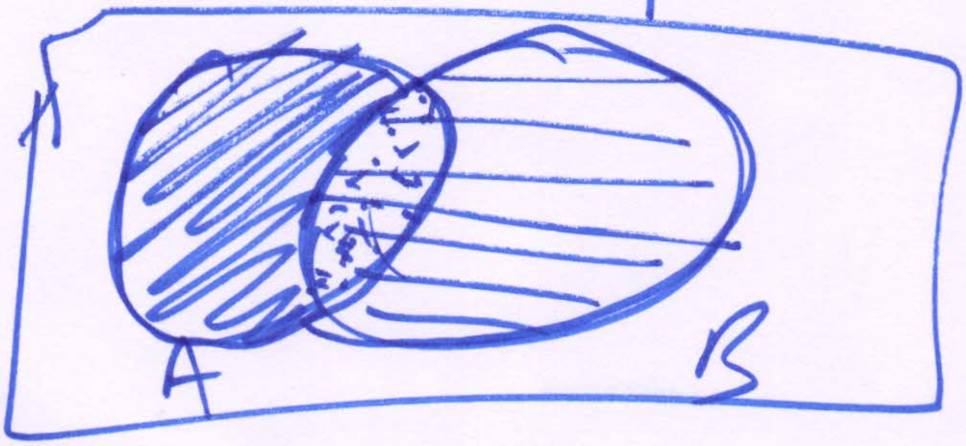
$P(A \cup B) = P(A) + P(B)$

Not a proof

$$P(F) = P(E) + \underbrace{P(\bar{E} \cap F)}_{\ge 0}$$

$$\boxed{P(F) \ge P(E)}$$

$$P(A\cup B) = P(A) + P(B) - P(A\cap B)$$



$$\left.\begin{array}{l} E_1 = A\cap \overline{B} \\ E_2 = B \end{array}\right\} E_1\cap E_2 = \phi$$

$$P(E_1 \cup E_2) = P(E_1) + P(E_2)$$

$$\cancel{P(A\cap \overline{B})}$$

$$P(A\cup B) = P(A\cap\overline{B}) + P(B)$$

$$B = (A\cap B)\cup(\overline{A}\cap B)$$

$$P(B) = P(A\cap B) + P(\overline{A}\cap B)$$

$$P(A\cup B) = \underbrace{\boxed{P(A\cap\overline{B}) + P(A\cap B)}}_{P(A)} + P(\overline{A}\cap B) + P(A\cap B) - P(A\cap B)$$