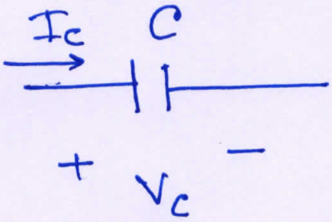
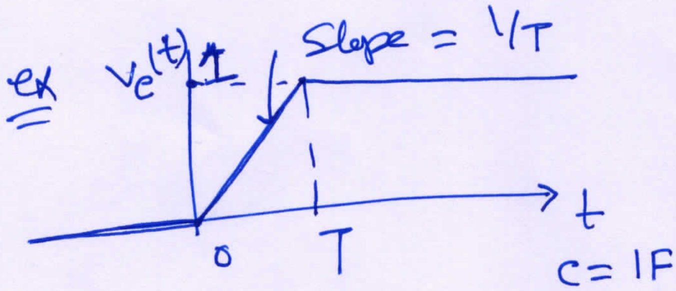
RC & RL Circuits

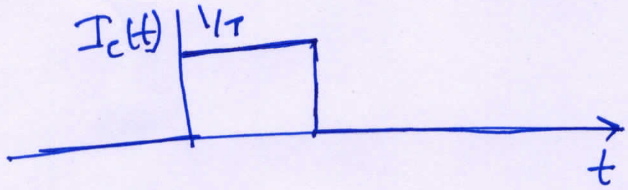
Capacitor



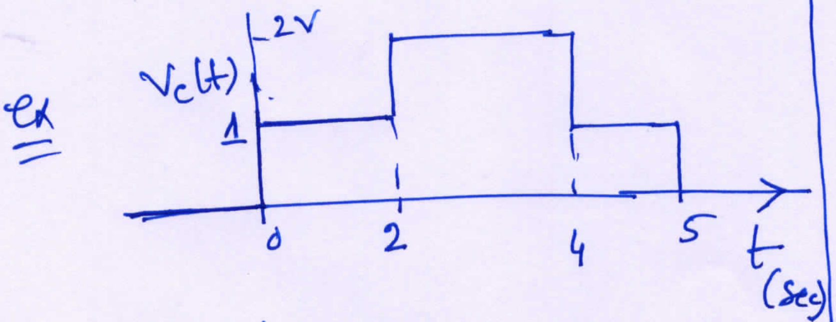
$$I_C = C\frac{dV_C}{dt}$$

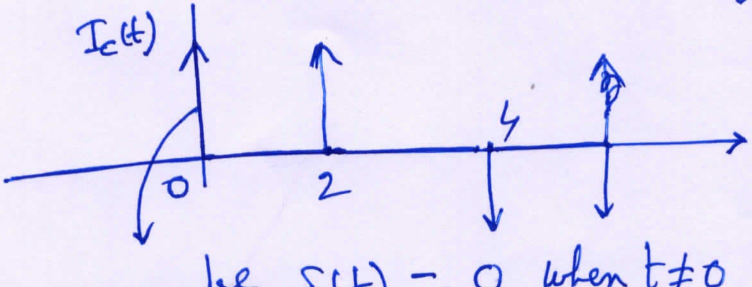
ex





ex

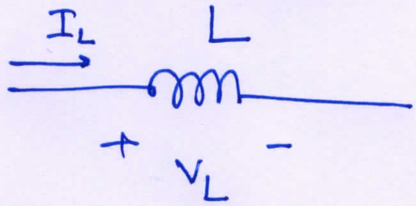




Impulse $\delta(t) = 0$ when $t \neq 0$

$$\int_{-\infty}^{\infty} \delta(t)\,dt = 1$$

Inductor ~~Inductance~~



$$V_L = L\frac{dI_L}{dt}$$

For finite $I_c(t)$,

$$V_c(t) \longrightarrow \text{no sudden change}$$

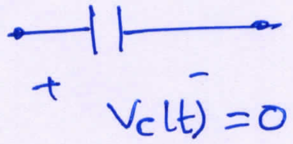
$$\Rightarrow V_c(t = T_-) = V_c(t = T_+)$$

Similarly,

$$I_L(t = T_-) = I_L(t = T_+)$$

Initial condition:

$$V_c(t = 0_-) = 0$$



Short ckt

$$I_L(t = 0_-) = 0\ A$$

$$\Rightarrow I_L(t = 0_+) = 0\ A$$

open ckt

Final condition for DC input

$I_c(t), V_c(t) \longrightarrow$ Constant at steady-state

$$I_c = \frac{c\,dv_c(t)}{dt} = 0$$

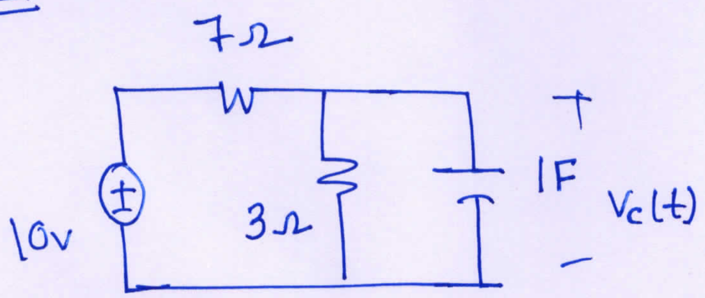
$\overrightarrow{I_c = 0}$ open ckt

$I_L(t), V_c(t) \longrightarrow$ Constant

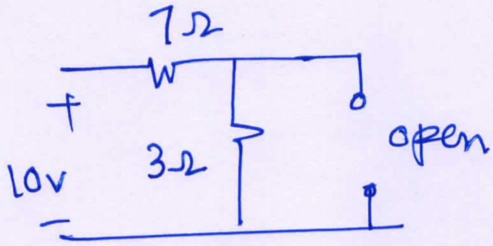
$$V_L(t) = L\frac{dI_L}{dt} = 0$$

Short ckt

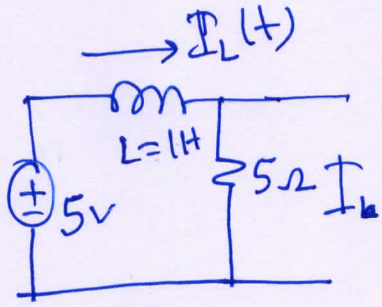
ex



$V_c(\infty) = ?$

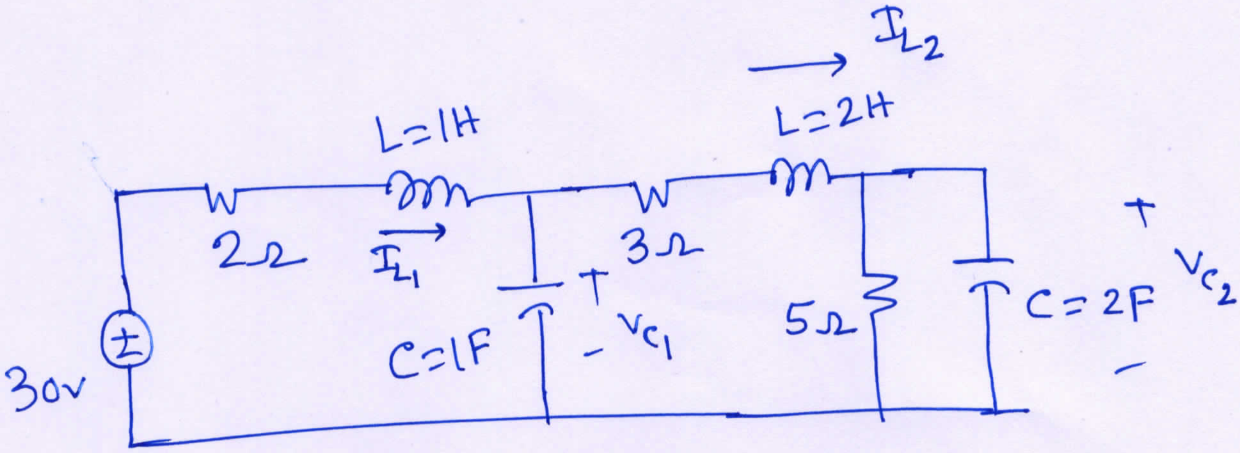


$$V_c(\infty) = \frac{3}{3+7} \times 10 = 3V$$



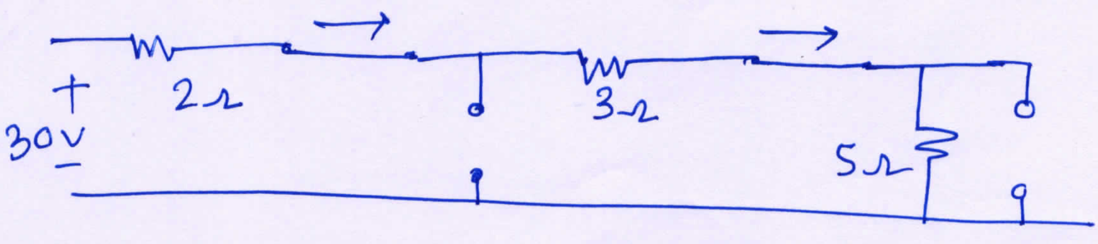
$$I_L(\infty) = \frac{5}{5} = 1A$$

ex

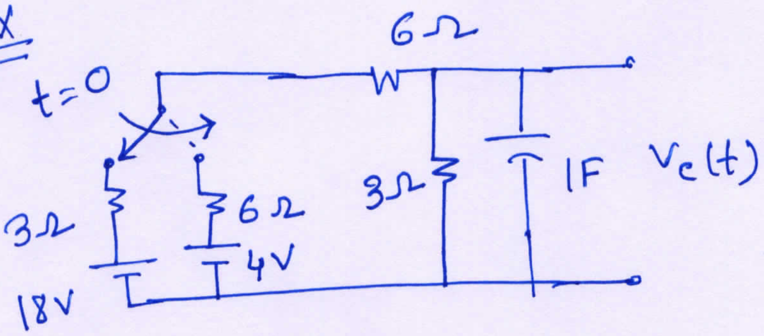


$V_{c_1}(\infty) = 24V$ $\quad$ $I_{L_1}(\infty) = 3A$

$V_{c_2}(\infty) = 15V$ $\quad$ $I_{L_2}(\infty) = 3A$



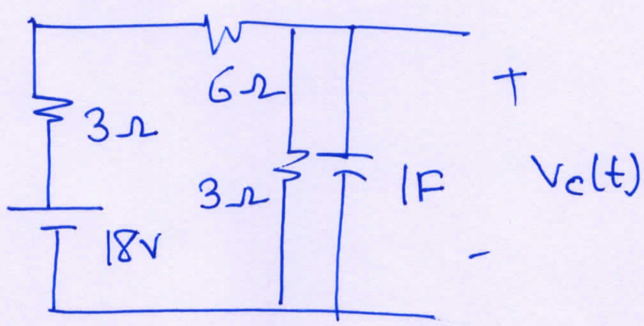
ex



$V_c(0+) = ?$

$V_c(\infty) = ?$

$t < 0$

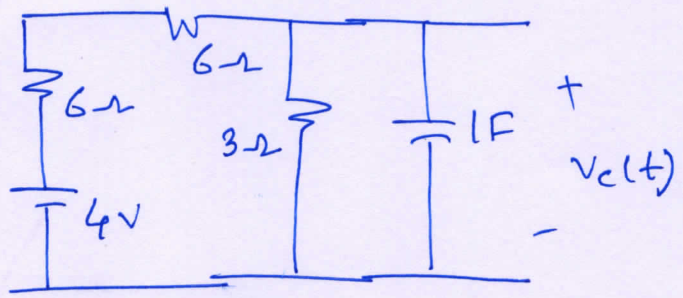


$$V_c(0-) = \frac{3}{3+6+3} \times 18$$

$$= \frac{\cancel{3}}{\cancel{12}_4} \times 18 = 4.5V$$

So, $V_c(0+) = V_c(0-) = 4.5V$

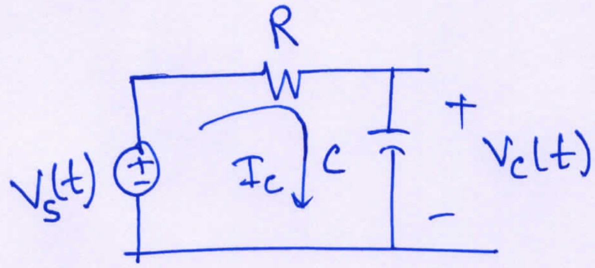
$t > 0$



$$V_c(\infty) = \frac{3}{6+6+3} \times 4$$

$$= \frac{3}{15} \times 4 = 0.8V$$

RC circuit



$V_c(t=0_-) = V_i$
(initial voltage across capacitor c)

$$I_c(t) = C\frac{dV_c}{dt}$$

KVL

$$V_s(t) = I_c(t)R + V_c(t)$$

$$= RC\frac{dV_c(t)}{dt} + V_c(t)$$

$$\boxed{V_c(t) + RC\frac{dV_c(t)}{dt} = V_s(t)}$$

$$V_c(t) = C.F + P.I$$

C.F (Natural response)

$$V_c(t) + RC\frac{dV_c}{dt} = 0$$

$$\Rightarrow \frac{dV_c}{dt} = -\frac{1}{RC}V_c$$

$$\Rightarrow \int\frac{dV_c}{V_c} = -\int\frac{1}{RC}dt$$

$$\Rightarrow \ln(V_c(t)) = -t/RC + K_1$$

put $K_1 = \ln(K)$

$$\Rightarrow \ln(V_c(t)) = -\frac{t}{RC} + \ln(K)$$

$$\Rightarrow \ln_e\left(\frac{V_c(t)}{K}\right) = -\frac{t}{RC}$$

$$\Rightarrow \frac{V_c(t)}{K} = e^{-\frac{t}{RC}}$$

$$\Rightarrow \boxed{V_c(t) = K e^{-t/RC}}$$

P.I (Forced response)

$$V_c(t) + RC \frac{dv_c(t)}{dt} = V_s(t)$$

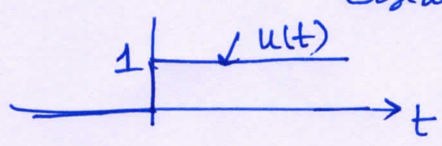
For DC input

$$V_s(t) = V\,u(t)$$

$u(t) \rightarrow$ step input

$u(t) = 1$ for $t \geq 0$

$= 0$ elsewhere



$\Rightarrow V_c(t) \rightarrow$ Constant

$$\Rightarrow \frac{dv_c}{dt} = 0$$

$$\Rightarrow V_c(t) = V$$

So, voltage across the capacitor

$$V_c(t) = C.F + P.I$$

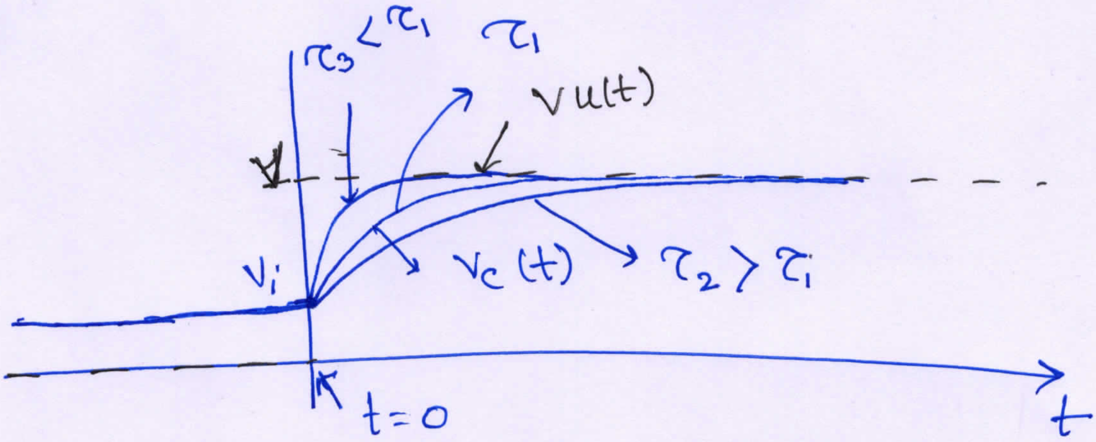
$$= K e^{-\frac{t}{RC}} + V$$

at $t=0$, $V_c(t) = V_i = V_c(0-) = V_c(0+)$

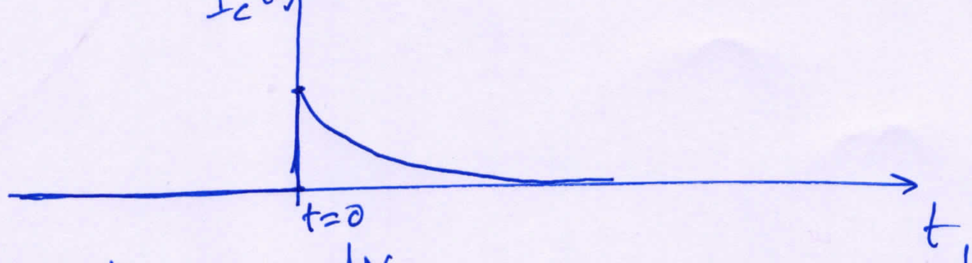
$$\Rightarrow V_i = K . e^{-0} + V$$

$$\Rightarrow K = (V_i - V)$$

So, $\boxed{V_c(t) = V + (V_i - V)\, e^{-t/RC}}$ for $t \geqslant 0$



$RC = \tau =$ time constant



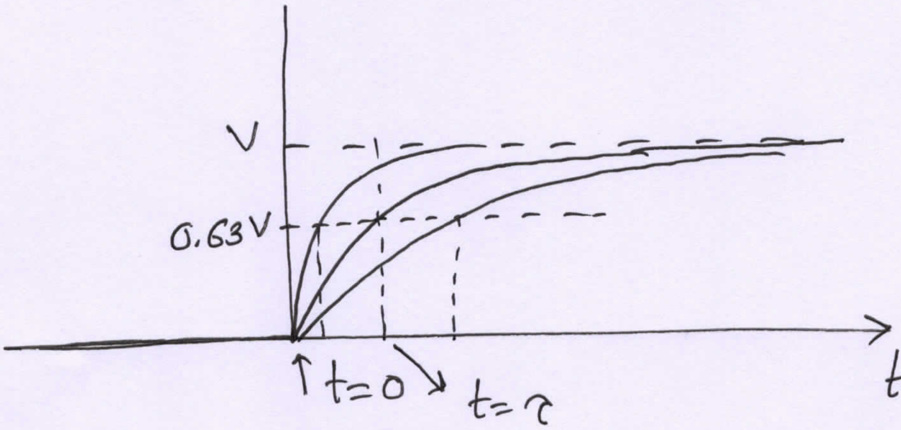
$$I_c(t) = C\frac{dV_c}{dt} = \cancel{C}(V_i - V)\left(-\frac{1}{R\cancel{C}}\right) e^{-t/RC}$$

For $V_i = 0$,

$$V_c(t) = V + (0 - V)\, e^{-t/RC}$$

$$= V\left(1 - e^{-t/RC}\right) = V\left(1 - e^{-t/\tau}\right)$$

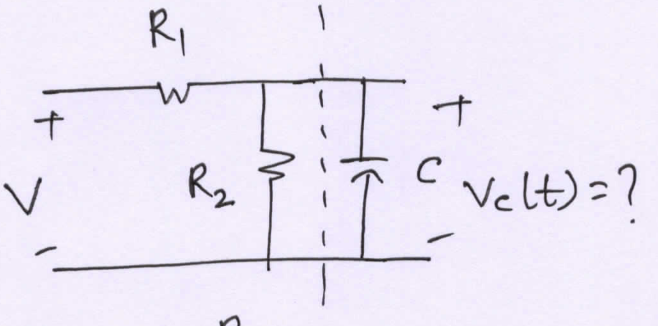
where $\tau = RC$



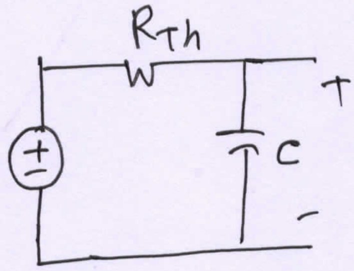
at $t = \tau$,

$$V_c(t) = V\left(1 - e^{-1}\right) = 0.63\,V$$

<u>ex</u>



V_i = intial potential across C.



$$V_c(t) = V_{Th} + (V_i - V_{Th})\, e^{-t/(R_{Th} C)}$$

$V_{Th} = \dfrac{R_2}{R_1 + R_2} V$

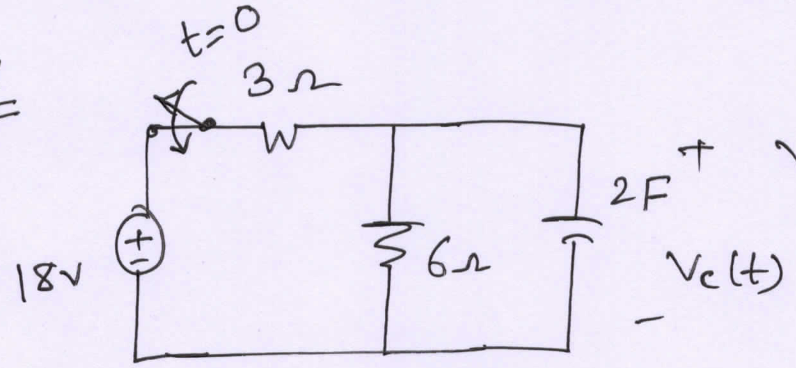
$R_{Th} = R_1 \parallel R_2 = \dfrac{R_1 R_2}{R_1 + R_2}$

V_{Th} = final potential across $C = V_f$

$$V_c(t) = V_f + (V_i - V_f)e^{-t/R_{eq}C}$$

$$\begin{cases} V_c(0+) = V_i \\ V_c(\infty) = V_f \end{cases}$$

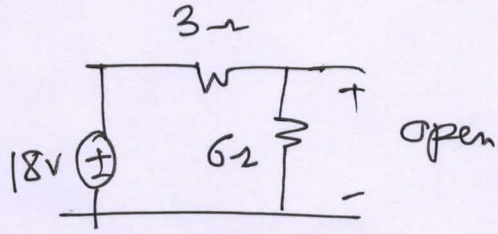
<u>ex</u>



$V_i = 2V$

$R_{eq} = 3 \| 6 = 2\Omega$

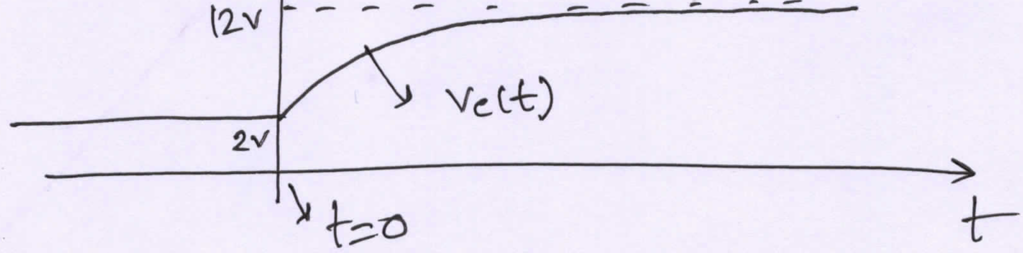
$V_f = \frac{6}{3+6} \times 18 = 12V$



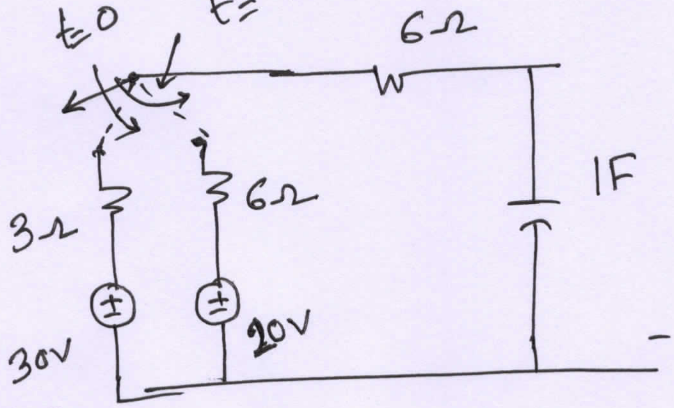
$$V_c(t) = V_f + (V_i - V_f)e^{-t/R_{eq}C}$$

$$= 12 + (2 - 12)\, e^{-t/4}$$

$$= 12 - 10e^{-t/4}$$

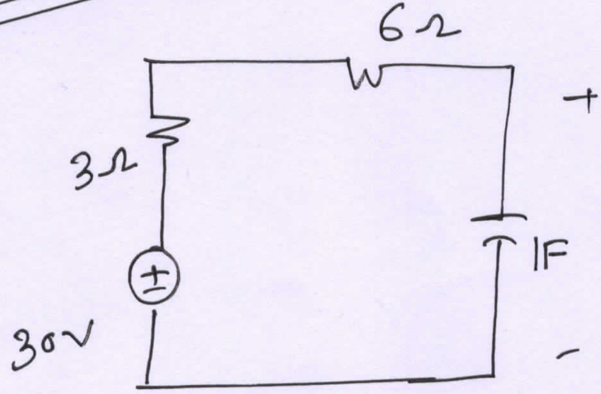


ex



$$V_i = V_c(0-)$$
$$= 5V$$

$0 \leq t \leq 2$

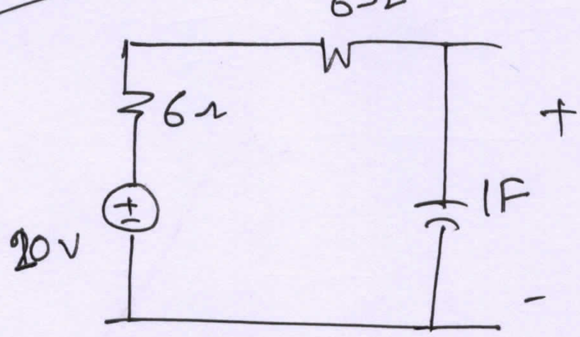


$V_f = 30V$

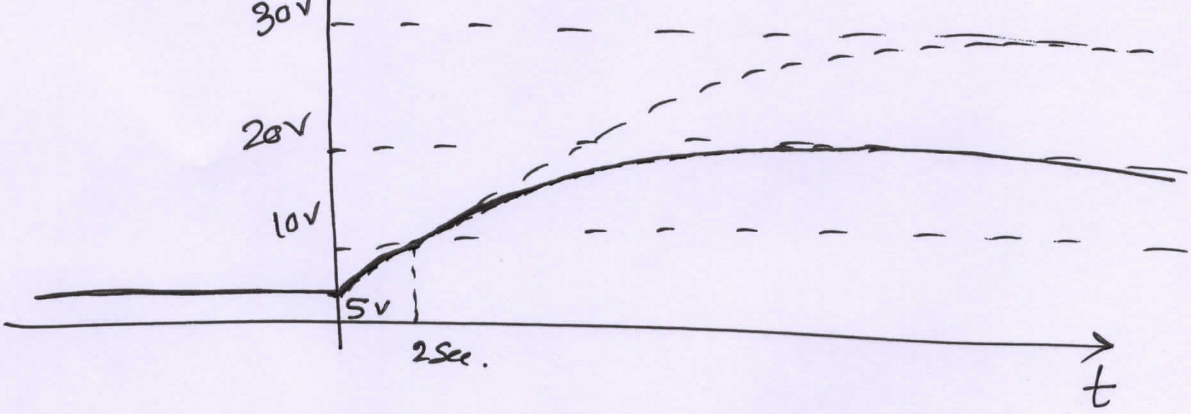
$$V_i = 5V$$

$$v_c(t) = V_f + (V_i - V_f)\, e^{-t/R_{eq}C}$$
$$= 30 + (5-30)\, e^{-t/9}$$
$$= 30 - 25 e^{-t/9}$$
$$v_c(2) = 30 - 25 e^{-2/9} = 9.98V$$

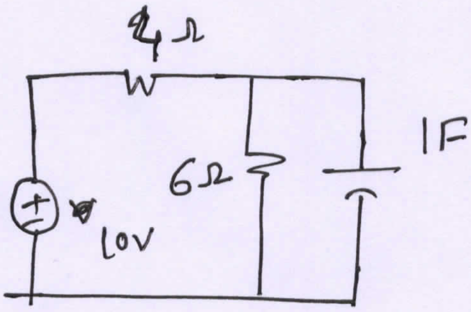
$2 \leq t < \infty$



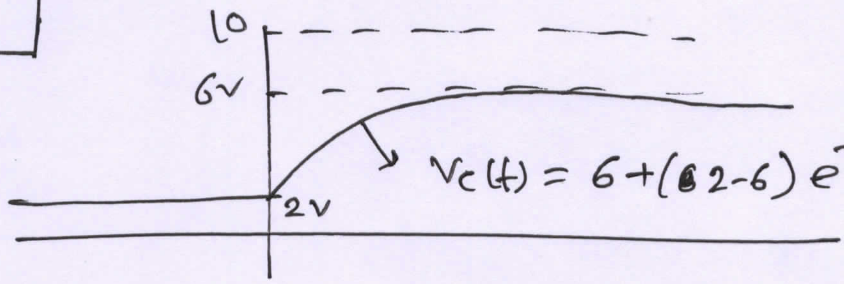
$$V_i = 9.98V$$
$$V_f = 20V$$
$$v_c(t) = V_f + (V_i - V_f)\, e^{-(t-2)/R_{eq}}$$
$$= 20 + (9.98 - 20)\, e^{-(t-2)/12}$$
$$= 20 - 10.02\, e^{-(t-2)/12}$$



ex



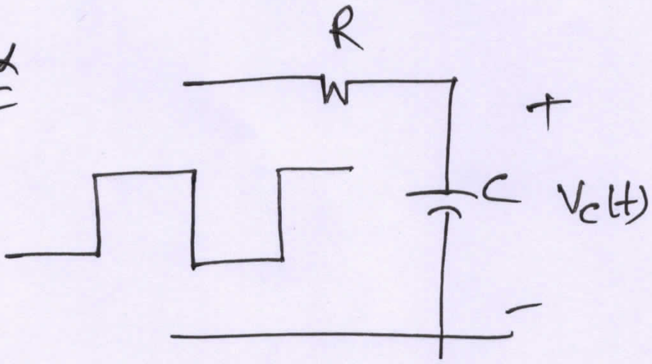
$V_i = 2V$

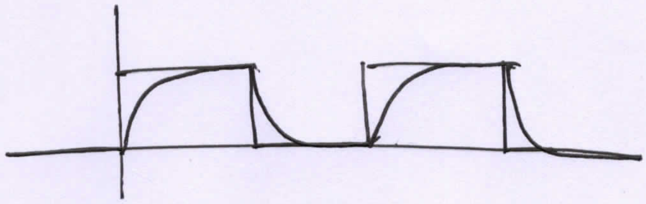


$$V_c(t) = 6 + (2-6)\, e^{-t/2.4}$$

$$R_{eq} = \frac{4 \times 6}{4+6} = 2.4$$

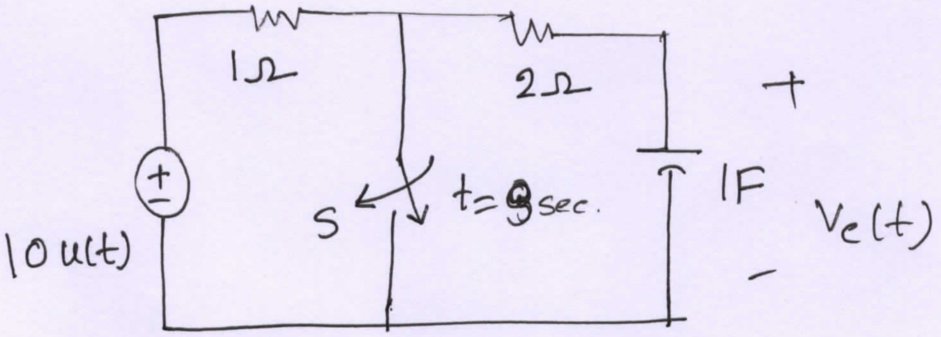
ex



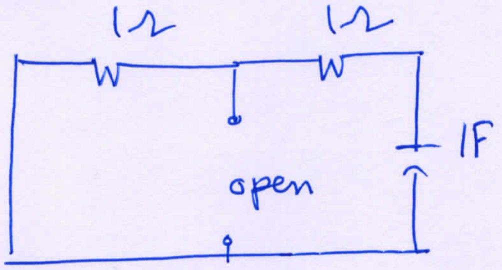


RC → small

ex

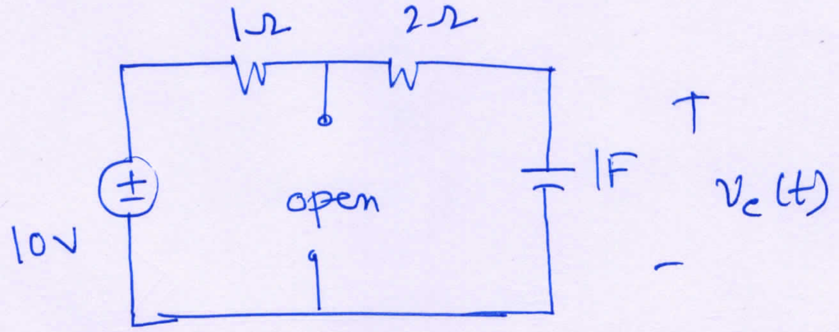


1) $t < 0$



$V_c(0_-) = 0V = V_i$

2) $0 \leq t \leq 3 \text{ sec.}$



$V_f = 10v$

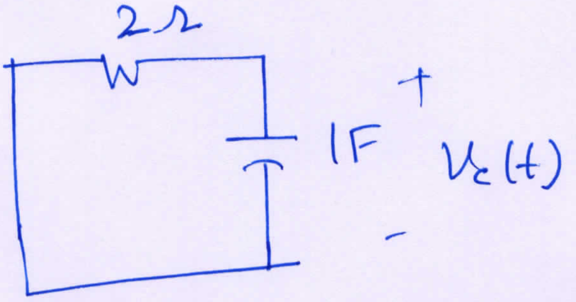
$$v_c(t) = V_f + (V_i - V_f)\, e^{-t/RC}$$

$$= 10 + (0 - 10)\, e^{-t/(3\times1)}$$

$$= 10(1 - e^{-t/3})$$

At $t = 3\text{sec.}$, $v_c(t) = 10(1 - e^{-1}) = 6.3\,v$

3) $t \geq 3$ Sec



$v_c(3-) = 6.3 = V_i$

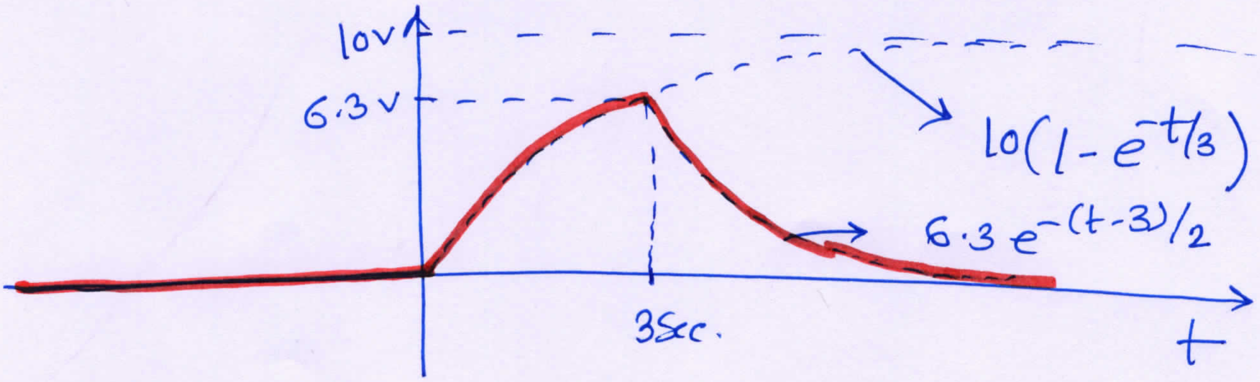
$V_f = 0$

$$v_c(t) = V_f + (V_i - V_f)e^{-(t-3)/RC}$$

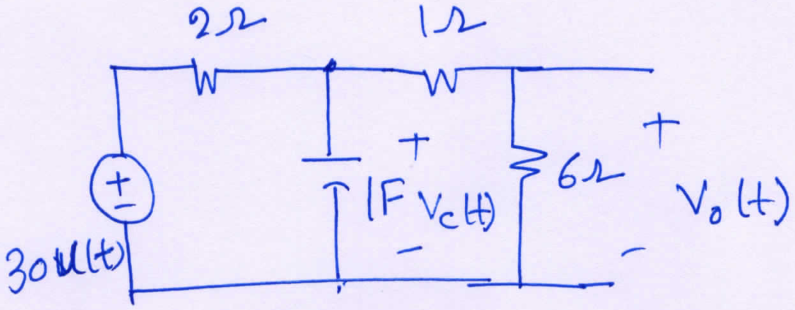
$$= 0 + (6.3 - 0)\, e^{-\frac{(t-3)}{2}}$$

$$= 6.3\, e^{-(t-3)/2}$$

$$v_c(t) = \left.\begin{array}{ll} 0 & \text{for } t \leq 0 \\ 10(1 - e^{-t/3}) & \text{for } 0 \leq t \leq 3 \\ 6.3\, e^{-(t-3)/2} & \text{for } t \geq 3 \end{array}\right\}$$



ex



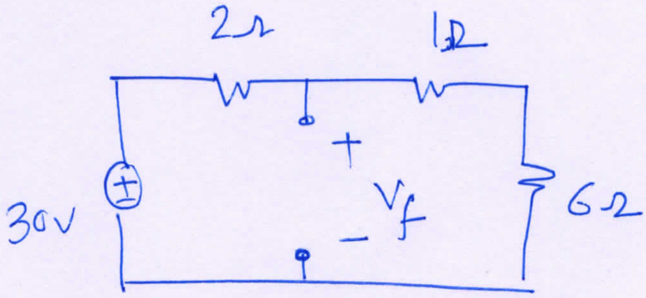
$$V_o(t) = \frac{6}{1+6} V_c(t)$$

$$= \frac{6}{7} V_c(t)$$

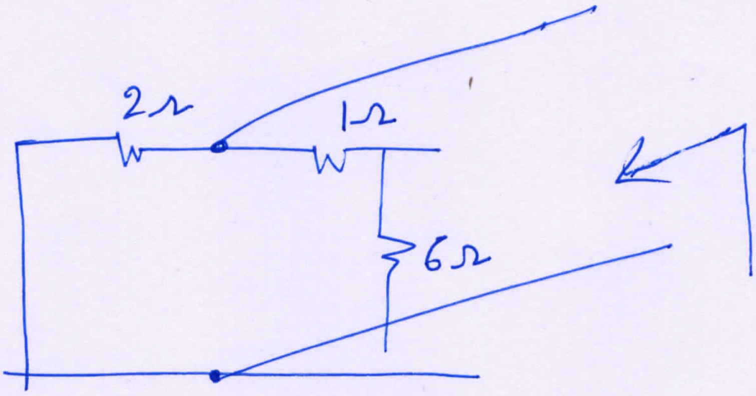
$$V_i = 0$$

$$V_f = \frac{7}{7+2} \times 30$$

$$= \frac{70}{3} \text{ V}$$



Req



Req

$$= 2 \parallel 7$$

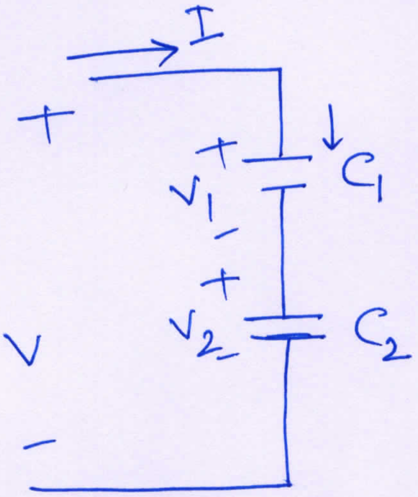
$$= \frac{2 \times 7}{2+7} = \frac{14}{9}\,\Omega$$

$$V_c(t) = \frac{70}{3} + \left(0 - \frac{70}{3}\right) e^{-t/(14/9 \times 1)}$$

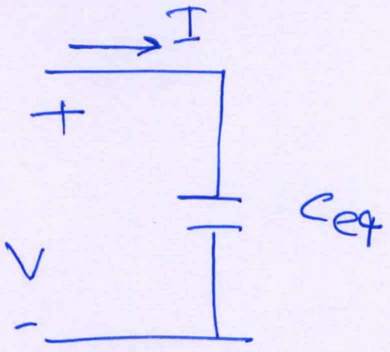
$$= \frac{70}{3}\left(1 - e^{-9t/14}\right)$$

$$\Rightarrow V_o(t) = 6/7\, V_c(t) = 20\left(1 - e^{-9t/14}\right) \; ; \; t \geq 0$$

Series Connection of Capacitors



$\Rightarrow$



$$I = C_{eq} \frac{dv}{dt}$$

$$I = C_1 \frac{dv_1}{dt} \Rightarrow \frac{dv_1}{dt} = \frac{1}{C_1} I$$

$$I = C_2 \frac{dv_2}{dt} \Rightarrow \frac{dv_2}{dt} = \frac{1}{C_2} I$$

$$V = V_1 + V_2$$

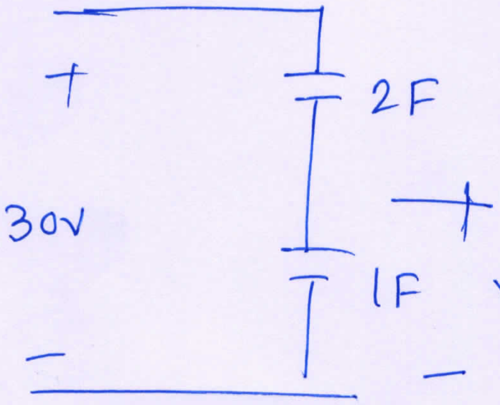
$$\Rightarrow \frac{dv}{dt} = \frac{dv_1}{dt} + \frac{dv_2}{dt} = \frac{I}{C_1} + \frac{I}{C_2}$$

$$= \left[\frac{1}{C_1} + \frac{1}{C_2}\right] I = \frac{1}{C_{eq}} I$$

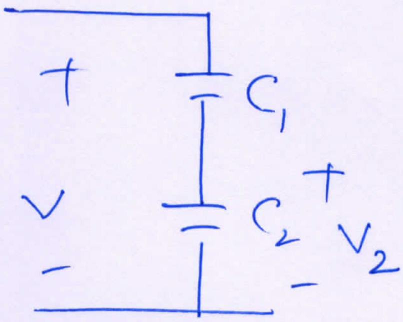
$$\Rightarrow \boxed{I = C_{eq} \frac{dv}{dt}}$$

where $C_{eq} = \dfrac{C_1 C_2}{C_1 + C_2}$

ex



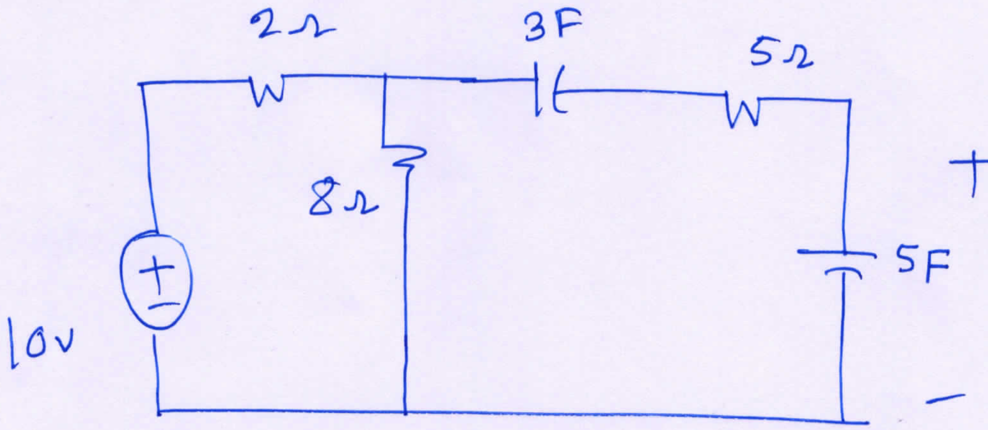
$V_o = ?$



$$V_o = \frac{2}{3} \times 30$$
$$= \underline{\underline{20V}}$$

$$V_2 = \frac{C_1}{C_1 + C_2} V$$

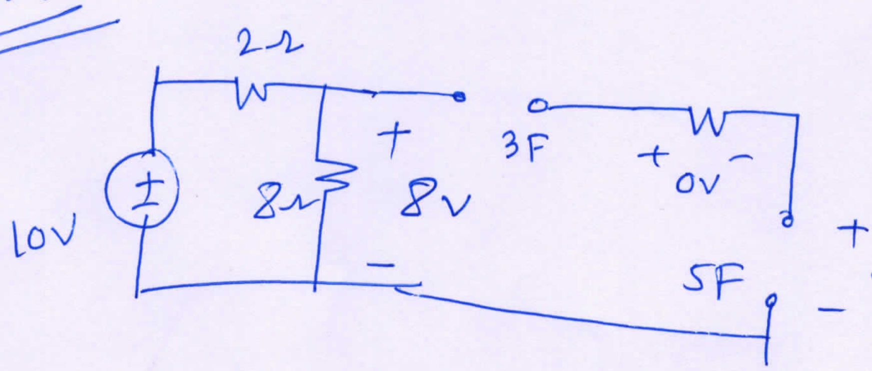
ex



at $t = \infty$

$V_o = ?$

At $t = \infty$



$$+ V_o - = \frac{3}{3+5} \times 8$$
$$= \underline{\underline{3V}}$$