CT111 Introduction to Communication Systems
Lecture 6: Fourier Transform

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Overview of Today's Talk

1. Correlation between Complex Phasors
2. Fourier Transform
3. Examples
4. Effect of Sampling
5. Fourier Series
6. Filtering Effect
7. Properties
8. Mid-Term Exam Syllabus

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Time Average of Complex Phasors

- Time integral of a Complex Phasor $\exp\left(i\left(2\pi ft\right)\right)\, dt$ over a time interval T is zero, if $T = m \times \frac{1}{f} = m \times T_{cycle}$:

$$\frac{1}{T_{cycle}} \int_{t=-T_{cycle}/2}^{T_{cycle}/2} \exp\left(i\left(2\pi ft\right)\right)\, dt = 0$$

- Why?
  - → Complex phasor $\exp\left(i\left(2\pi ft\right)\right) = \cos\left(2\pi ft\right) + i\sin\left(2\pi ft\right)$ is comprised of two sinusoidal waveforms.
  - → When cycles are allowed to complete, the sinusoidal waveforms have equal positive and negative valued areas, which cancel out.

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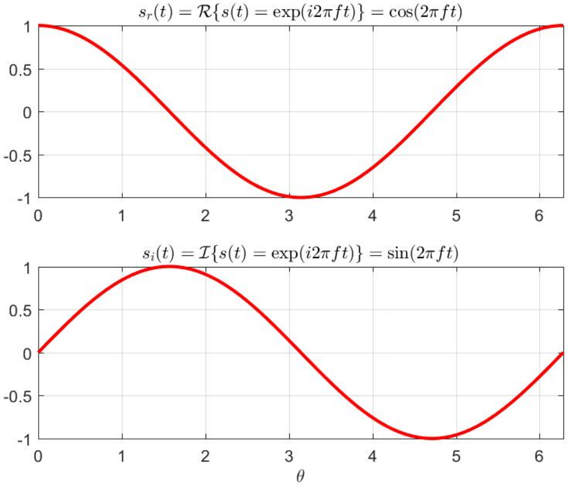

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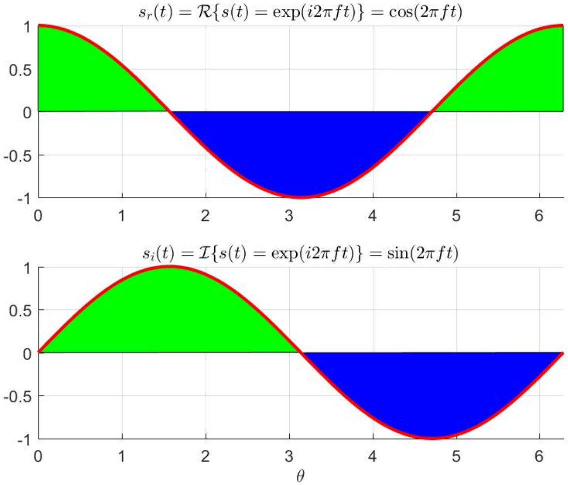
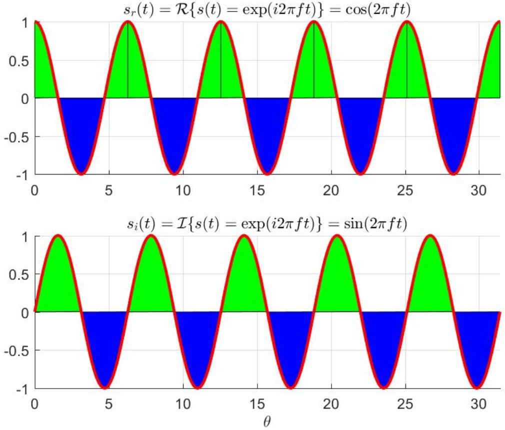


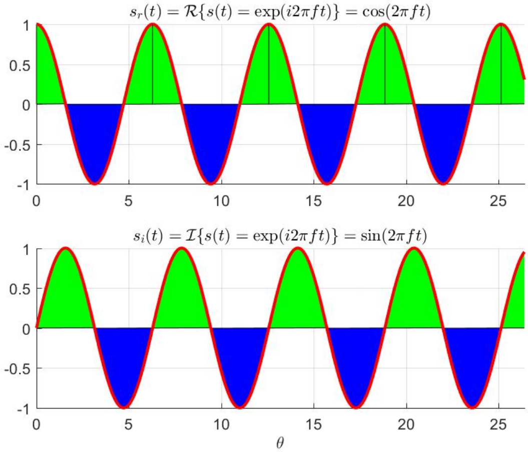
Time Average of Complex Phasors

- Time average remains zero with multiple cycles as well, as long as they're allowed to complete



Time Average of Complex Phasors

- Time average does become nonzero if the cycle is cut short



Time Average of Complex Phasors

- However, the time integral still approaches zero as $T \to \infty$

$$\frac{1}{T}\int_{t=-T/2}^{T/2} \exp\left(i\left(2\pi ft\right)\right)\, dt \to 0$$

- Why?
  - $\rightarrow$ Maximum value of the integral is limited by the area under *half* cycle.
  - $\rightarrow$ Let us call this constant q. Note that q does not increase with T.
  - $\rightarrow$ Therefore, as $T \to \infty$, the ratio $\frac{q}{T} \to 0$.

Time Average of Complex Phasors

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Time Average of Complex Phasors

- A general conclusion: time integral of a Complex Phasor $\exp\left(i\left(2\pi ft\right)\right)\,dt$ over a time interval T approaches zero as $T \to \infty$

$$\frac{1}{T}\int_{t=-T/2}^{T/2} \exp\left(i\left(2\pi ft\right)\right)\,dt \to 0$$

- An exception to the above is when $f = 0$. In this case

$$\frac{1}{T}\int_{t=-T/2}^{T/2} \exp\left(i\left(2\pi 0t\right)\right)\,dt = \frac{1}{T}\int_{t=-T/2}^{T/2} 1\,dt = 1$$

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A Short Form Notation of Complex Phasors

Let us use the following short-form notation to denote a complex phasor:

$$
\begin{aligned}
s_{A,f} &\stackrel{\text{def}}{=} a \exp\left(i(2\pi f t + \theta)\right) \\
&= a \exp\left(i\theta\right) \exp\left(i(2\pi f t)\right) \\
&= A \exp\left(i(2\pi f t)\right)
\end{aligned}
$$

Note that a is the real-valued amplitude, whereas $A \stackrel{\text{def}}{=} a \times \exp\left(i\theta\right)$ is complex-valued amplitude

Two Complex Phasors

- Let us denote two complex phasors as follows:
  1. $w(t) = s_{A,f_1}(t) = A\exp\left(i(2\pi f_1 t)\right)$
     - $w(t)$, in general, will represent the signal that is given to us (whose frequency content we are interested in evaluating)
  2. $s_{1,f}(t) = \exp\left(i(2\pi f t)\right)$
     - $s_{1,f}(t)$ is the signal that is locally (on our computer or using our hardware) generated. It has unit amplitude and a frequency f that is swept over a range of interest

Dot Product between Two Complex Phasors

Also Called Correlation

- Let us define the dot product between these two complex phasors as follows:

$$W(f) = \frac{1}{T} \int_{t=-T/2}^{T/2} w(t)\, s_{1,f}^{*}(t)\, dt$$

- Here $s_{1,f}^{*}(t)$ denotes the *complex-conjugate* of signal $s_{1,f}(t)$

$\rightarrow$ Conjugate x^* of any complex number x is defined as follows:

$$|x^*| = |x|$$
$$\theta_{x^*} = -\theta_x$$

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Dot Product between Two Complex Phasors

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- The dot product can be calculated as follows:

$$\begin{aligned} W(f) &= \frac{1}{T}\int_{t=-T/2}^{T/2} \underbrace{A\exp\left(i\left(2\pi f_1 t\right)\right)}_{w(t)} \underbrace{\exp\left(-i\left(2\pi f t\right)\right)}_{s_{1,f}(t)} \, dt \\ &= \frac{1}{T}\int_{t=-T/2}^{T/2} A\exp\left(i\left(2\pi (f_1 - f)t\right)\right) \, dt \end{aligned}$$

- We notice the integrand itself is just a complex phasor at a frequency $f_1 - f$. Therefore, we can write

$$W(f) = \begin{cases} A, & \text{when } f_1 - f = 0, \text{ i.e., } f = f_1 \\ 0, & \text{otherwise} \end{cases}$$

Dot Product between Two Complex Phasors

Also Called Correlation

- When $w(t) = s_{A,f_1}(t) = A\exp\left(i(2\pi f_1 t)\right)$,

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- A short-hand notation for the above (involves a more deeper concept of Dirac Delta functions, which we will not study in this course):

$$W(f) = A\,\delta(f - f_1)$$

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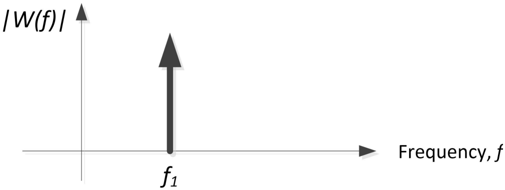
Fourier Transform

Analysis Equation

- Pictorial view of the Fourier Transform of $w(t) = s_{A,f_1}(t) = A\exp\left(i(2\pi f_1 t)\right)$:



- Note that the F.T. $W(f)$ is a *complex* number in general. It has both magnitude and phase (polar coordinates) or (in Cartesian coordinates) real and imaginary part

Fourier Transform

Analysis Equation

- Suppose a communication signal $w(t)$ is made up of two complex phasors, at different frequencies, and different complex-valued amplitudes:

$$w(t) = s_{A_1,f_1}(t) + s_{A_2,f_2}(t)$$

- The dot-product between $w(t)$ and $s_{1,f}(t)$ is given as follows:

$$W(f) = \frac{1}{T}\int_{t=-T/2}^{T/2} w(t)\, s_{1,f}^{*}(t)\, dt = \begin{cases} A_1, & f = f_1 \\ A_2, & f = f_2 \\ 0, & \text{otherwise} \end{cases}$$

$$= A_1\,\delta(f - f_1) + A_2\,\delta(f - f_2)$$

Fourier Transform

Analysis Equation

- Suppose any arbitrary communication signal $w(t)$ is made up of an infinite number of complex phasors, at different frequencies, and complex-valued amplitudes:

$$w(t) = \sum_{k=-\infty}^{\infty} s_{A_k, f_k}(t)$$

- The dot-product between $w(t)$ and $s_{1,f}(t)$ is given as follows:

$$W(f) = \frac{1}{T} \int_{t=-T/2}^{T/2} w(t)\, s_{1,f}^{*}(t)\, dt = \begin{cases} A_k, & f = f_k \\ 0, & \text{otherwise} \end{cases}$$

$$= \sum_{k=-\infty}^{\infty} A_k\, \delta(f - f_k)$$

Fourier Transform
Analysis Equation

- For technical reasons, when T is replaced by ∞, the Fourier Analysis equation becomes as follows:

$$W(f) = \int_{t=-\infty}^{\infty} w(t)\, s_{1,f}^{*}(t)\, dt$$
$$= \int_{t=-\infty}^{\infty} w(t)\, \exp\left(-i\,(2\pi f t)\right)\, dt$$

- Points to note:
  - → Frequency f is a variable, it is moved from a low value to a high value, and for each value of f, $W(f)$ is calculated as above
  - → $W(f)$ calculated at a given value of f gives the complex-valued amplitude of the complex phasor at f in the signal $w(t)$

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Inverse Fourier Transform

Also known as Synthesis Equation

- Suppose you're given the complex-valued amplitudes A_k and the frequencies f_k of the corresponding complex phasors that make up a signal $w(t)$
- Using this information, the signal $w(t)$ can be generated, or *synthesized*

$$w(t) = \sum_{k=-\infty}^{\infty} s_{A_k,f_k}(t) = \sum_{k=-\infty}^{\infty} A_k \exp\left(i(2\pi f_k t)\right)$$

- In continuous-frequency domain, A_k become $W(f)$, f_k is simply the integral variable f and the summation is replaced by integration

$$w(t) = \int_{-\infty}^{\infty} W(f) \exp\left(i(2\pi f t)\right) df$$

- The above is called the Synthesis equation or Inverse Fourier Transform

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Fourier Transform

Summary

- Fourier Transform (Analysis Equation):
  - $\rightarrow$ $W(f) = \int_{t=-\infty}^{\infty} w(t) \exp\left(-i\left(2\pi f t\right)\right) dt$
  - $\rightarrow$ Moves the viewpoint of looking at the signal from time domain to frequency domain
- Inverse Fourier Transform (Synthesis Equation):
  - $\rightarrow$ $w(t) = \int_{f=-\infty}^{\infty} W(f) \exp\left(i(2\pi f t)\right) df$
  - $\rightarrow$ Moves the viewpoint back to time domain from the frequency domain

An Example: A Sinusoidal Signal

- Let $w(t) = a\sin(2\pi f_c t)$
- This is also written as

$$
\begin{aligned}
w(t) &= \frac{a}{2i}\left(\exp(i2\pi f_c t) - \exp(-i2\pi f_c t)\right) \\
&= \frac{a}{2}\exp(-i\pi/2)\exp(i2\pi f_c t) + \frac{a}{2}\exp(i\pi/2)\exp(-i2\pi f_c t)
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- Therefore, its Fourier Transform is given as

$$W(f) = A_1\,\delta(f - f_c) + A_2\,\delta(f + f_c)$$

where $A_1 = a\exp(-i\pi/2)/2$ and $A_2 = a\exp(i\pi/2)/2$

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Fourier Transform
for a Sinusoidal Signal

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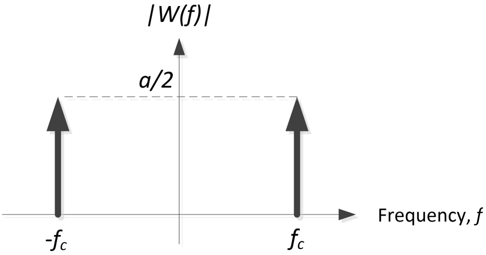
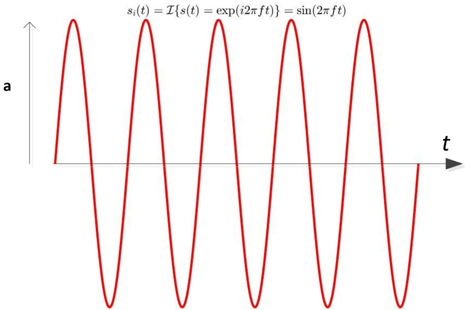


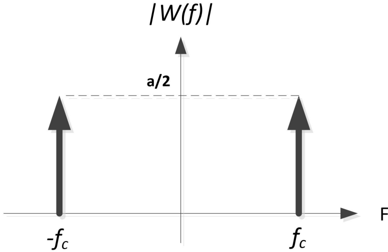
- Above is the magnitude of the F.T.
  $\rightarrow$ Need also to specify the phase of the F.T. to define it completely.

Sinusoidal Signals and Impulses Form F.T. Pair

Time Domain



Frequency Domain



A Property of the Fourier Transform

Duality between Time and Frequency Domains

- If $w(t) \Longleftrightarrow W(f)$, then $W(t) \Longleftrightarrow w(-f)$
- An intuitive explanation:
  - → In the mathematical expression for the complex phasor $\exp(i2\pi ft)$, f and t are interchangeable

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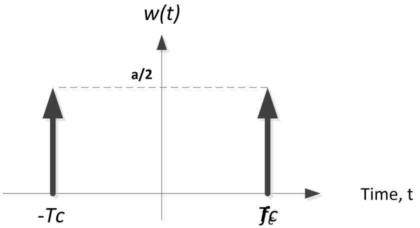
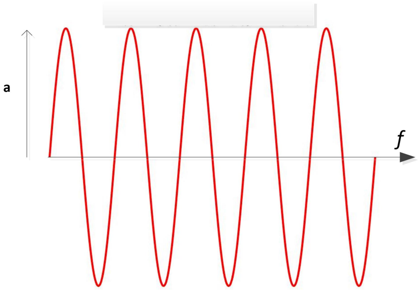


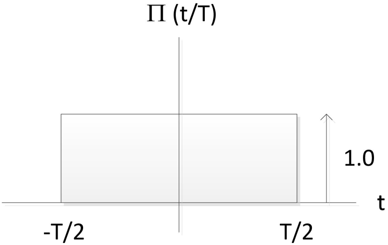
Frequency Domain



An Example: A Rectangular Signal

- A rectangular pulse is denoted by function $\Pi(\cdot)$:

$$\Pi\left(\frac{t}{T}\right) = \begin{cases} 1, & |t| \leq \dfrac{T}{2} \\ 0, & \text{otherwise} \end{cases}$$



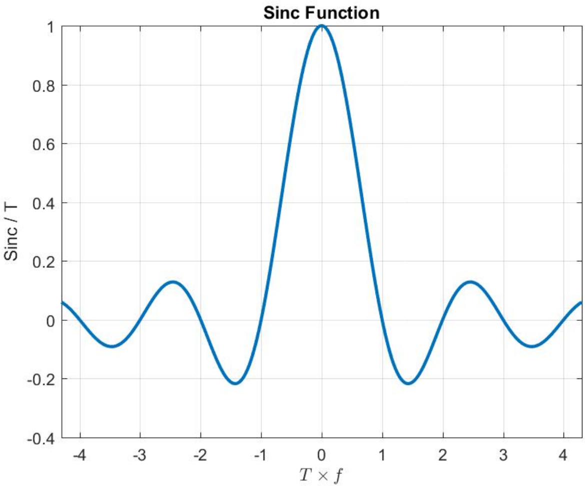
An Example: A Rectangular Signal
Fourier Transform

- Fourier Transform of the rectangular pulse is easy to calculate:

$$\begin{aligned} W(f) &= \int_{-T/2}^{T/2} 1 \exp\left(-i2\pi ft\right) \, dt \\ &= \frac{\exp\left(-i2\pi fT/2\right) - \exp\left(i2\pi fT/2\right)}{-j2\pi f} \\ &= T \operatorname{sinc}\left(Tf\right) \end{aligned}$$

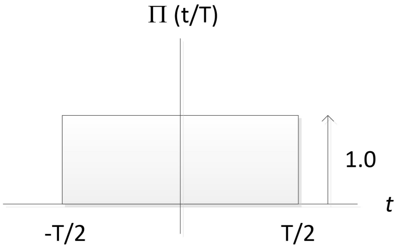
- Here $\operatorname{sinc}(x) \stackrel{\text{def}}{=} \frac{\sin(\pi x)}{\pi x}$
  $\rightarrow$ sinc is essentially a dampened sinusoid; the dampening is due to divide-by-x

An Example: A Rectangular Signal
Magnitude of F.T.



Rectangular and Sinc Signals Form F.T. Pair

Time Domain



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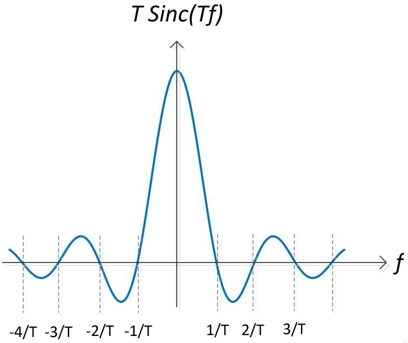
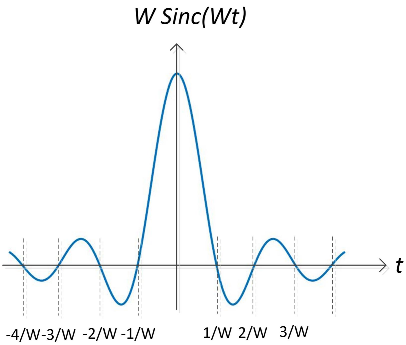


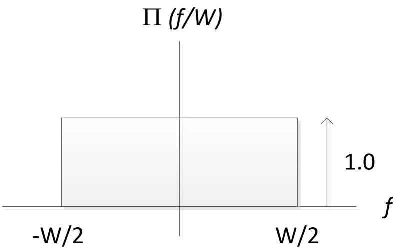
Rectangular and Sinc Signals Form F.T. Pair

Application of Duality Property of the F.T.

Time Domain



Frequency Domain

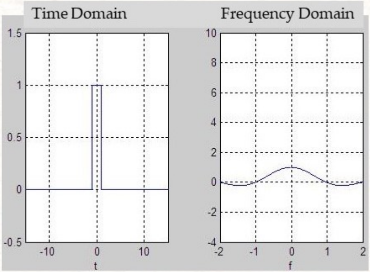


Rectangular and Sinc Signals Form F.T. Pair

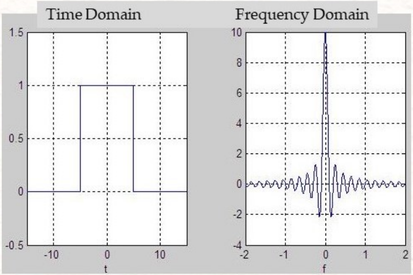
- Both the rectangular and Sinc signals are used in the communication system design and modeling
  - → These are ideal signals, hard to implement them in the real life
  - → In actual implementation, their practically feasible versions are instead used
- These functions allow us to appreciate the concepts of spectrum and bandwidth
  - → It is seen, for example, that enlarging the spectral bandwidth implies making the time domain pulse narrower, and vice versa

Rectangular and Sinc Signals Form F.T. Pair

Application of Duality Property of the F.T.



$T = 1$



$T = 10$

Sampling in Time Domain

- A Continuous-Time (C-T) domain signal $w(t) = A\exp\left(j(2\pi f_c t + \theta)\right)$ is *aperiodic* in the frequency domain
  - $\rightarrow$ Its spectrum is just a delta function centered at $f = f_c$, does not repeat with f
- If this signal is sampled at the rate of $T_s = 1/F_s$ seconds per sample (F_s is the sample rate in number of samples per second), the time t becomes discretized $t_n = n \times T_s$, for $n = 0, 1, 2, \ldots$
- The Discrete-Time (D-T) version of the original C-T signal is as follows:

$$
\begin{aligned}
w(t_n) &= A\exp\left(j(2\pi f_c n T_s + \theta)\right) \\
&= A\exp\left(j(2\pi \frac{f_c}{F_s} n + \theta)\right) \\
&= A\exp\left(j(2\pi \not{f}_c n + \theta)\right)
\end{aligned}
$$

Sampling in Time Domain

Periodicity in Frequency Domain

- Here $\ell_c = \dfrac{f_c}{F_s}$ is a ratio of frequencies
  - → Both f_c and F_s have units of Hertz, whereas ℓ_c does not have any unit
  - → Similarly, the time t has unit of seconds, but the discretized time index n does not have any unit (it's just an integer)
- The D-T signal $w(t_n)$ has now become periodic in the *frequency* domain
  - → Recall that the C-T signal $w(t)$ was not periodic in the frequency domain

Sampling in Time Domain

Periodicity in Frequency Domain

- Replace ℓ_c by $\ell_c + m$, where m is an arbitrary (positive or negative) integer:

$$
\begin{aligned}
A\exp\left(j(2\pi(\ell_c + m)n + \theta)\right) &= A\exp\left(j(2\pi\ell_c n + \theta)\right) \times \exp\left(j2\pi mn\right) \\
&= A\exp\left(j(2\pi\ell_c n + \theta)\right)
\end{aligned}
$$

  Thus, adding any integer m to ℓ_c gives the same signal. This implies $w(t_n)$ has a periodic spectrum with a period of 1.

- Since $\ell_c = \dfrac{f_c}{F_s}$, a period of 1 when the D-T signal is viewed as a function of ℓ_c is equivalent to a period of F_s when the D-T signal is viewed as a function of f_c in Hertz
  - $\rightarrow$ the original aperiodic spectrum as a function of f_c has become periodic with a period of F_s

Sampling in Time Domain
Periodicity in Frequency Domain

- Recall Fourier *Synthesis* operation (i.e., Inverse Fourier Transform):
  - → Any time domain signal $w(t)$ is viewed as a summation of multiple complex exponentials after they are weighted (i.e., multiplied) by different complex-valued scaling factors, as shown below:

$$w(t) = \sum_{k=-\infty}^{\infty} s_{A_k, f_k}(t) = \sum_{k=-\infty}^{\infty} A_k \exp\left(i(2\pi f_k t\right)$$

  - → In continuous-frequency domain, A_k become $W(f)$, f_k is simply the integral variable f and the summation is replaced by integration:

$$w(t) = \int_{-\infty}^{\infty} W(f) \exp\left(i(2\pi f t)\right) \, df$$

- In D-T domain, each of $s_{A_k, f_k}(t_n)$ becomes periodic with a period of F_s Hertz.
- Therefore, the summed signal $w(t)$ also exhibits a periodic spectrum with a period of F_s Hertz

Sampling in Time Domain

Periodicity in Frequency Domain

Summary:

- When a C-T signal $w(t)$ is sampled in time domain with a sampling duration of $1/F_s$ seconds...
- Its spectrum $W(f)$ becomes periodic with a period of F_s Hertz

Corollory: The Sampling Theorem

- A C-T signal $w(t)$ can be *exactly* recovered from its D-T samples $w(t_n)$ provided the sampling rate F_s is equal to or greater than the bandwidth B of $w(t)$, i.e., $F_s \geq B$

Sampling in Time Domain

Periodicity in Frequency Domain

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Sampling in Frequency Domains

- If an aperiodic C-T signal $w(t)$ is turned into a periodic signal with a period $T_0 = 1/f_0$ seconds, the effect in frequency domain is that the original spectrum $W(f)$ gets sampled
- These samples are multiples of the inverse of the period

$$f_n = n \times \frac{1}{T} = n \times f_0 \text{ Hz}$$

  - → The Fourier Transform $W(f)$ becomes zero at all frequencies f other than f_n, for $n \in \mathscr{Z}$
  - → $f_0 = 1/T_0$ is called the fundamental frequency and $n \times f_0$ are called the Harmonics

Sampling in Frequency Domains

- A periodic signal with a period of $T_0 = 1/f_0$ second *cannot* have any complex exponential whose period is not the same as T_0
  - Note that the fundamental frequency signal as well as the Harmonics all have a period of T_0
- Think of this as:
  - (synthesis or IFT) *by adding and combining items colored in yellow, you can make another item that is yellow-colored only, it cannot have any other color*, or vice versa
  - (analysis or FT) *if you dissect something that is yellow colored, you will find the constituent items that are also yellow colored only, there won't be any green or red items*

Here items of the same color are analogous to the complex phasor at the fundamental frequency and all of its Harmonics

Sampling in Frequency Domains
Fourier Series

- Fourier Transform, because it is now *sampled*, is given as follows:

$$W(f) = \sum_{n=-\infty}^{\infty} c_n \, \delta(f - nf_0)$$

→ Here, $c_n = W(f_n) = \dfrac{1}{T} \displaystyle\int_0^{T_0} w(t) \exp\left(i2\pi n f_0 t\right)$

▷ Note that the fraction $\dfrac{1}{T}$ appeared back now that the integration limits are *finite*

- Inverse Fourier Transform (also known as *Fourier Series*) is given as follows:

$$w(t) = \sum_{n=-\infty}^{\infty} c_n \exp\left(i2\pi n f_0 t\right)$$

Fourier Transform
Summary

- Fourier Transform (Analysis Equation):
  - → Aperiodic Signals: $W(f) = \int_{t=-\infty}^{\infty} w(t) \exp\left(-i\left(2\pi f t\right)\right) dt$
  - → Periodic Signals with period T_0: $W(f_0) = c_n = \int_{t=-\infty}^{\infty} w(t) \exp\left(-i\left(2\pi n f_0 t\right)\right) dt$, where $f_0 = 1/T_0$ is the fundamental frequency, and nf_0 are the Harmonics
  - → Moves the viewpoint of looking at the signal from time domain to frequency domain
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Sampling in Frequency Domains
Fourier Series

- Alternative (*completely* equivalent) representations of Fourier Series:

$$\begin{aligned} w(t) &= a_0 + \sum_{n=1}^{\infty} \left(a_n \cos 2\pi n f_0 t + b_n \sin 2\pi n f_0 t \right) \\ &= D_0 + \sum_{n=1}^{\infty} D_n \cos\left(2\pi n f_0 t + \psi_n\right) \end{aligned}$$

→ First one uses the Cartesian (also known as Quadrature) Coordinates $((a_n, b_n))$, and the second one uses the Polar coordinates $((D_n, \psi_n))$ of the complex numbers c_n

Sampling in Frequency Domains

Fourier Series

- Relationships between Fourier Series Coefficients (assumes that the signal $w(t)$ is real-valued):
  - $\rightarrow$ $n = 0$:

$$a_0 = c_0 = D_0.$$

  - $\rightarrow$ $n \geq 1$:

$$\begin{aligned} a_n &= 2\Re\left(c_n\right) = D_n \cos\left(\psi_n\right) \\ b_n &= -2\Im\left(c_n\right) = -D_n \sin\left(\psi_n\right) \\ D_n &= \sqrt{a_n^2 + b_n^2} = 2|c_n| \\ \psi_n &= \theta_{c_n} = -\tan^{-1}\left(\frac{b_n}{a_n}\right) \end{aligned}$$

Sampling in Frequency Domains

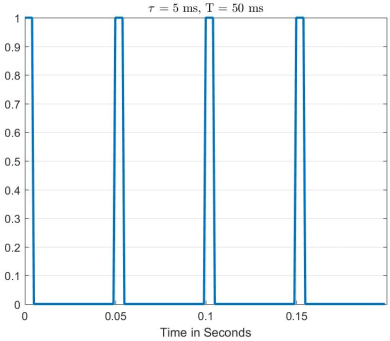
Relationships between Fourier Series Coefficients

- (Assumes that the signal $w(t)$ is real-valued)
- Complex numbers in terms of Cartesian Coordinates:

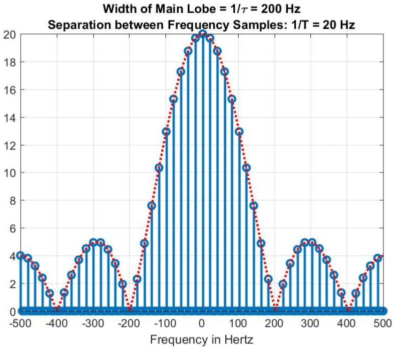
$$\rightarrow \; c_n = \begin{cases} 0.5\left(a_n - i\, b_n\right), & n > 0 \\ a_0, & n = 0 \\ 0.5\left(a_{-n} + i\, b_{-n}\right), & n < 0 \end{cases}$$

F.T. of Rectangular Periodic Waveform

- Time Domain



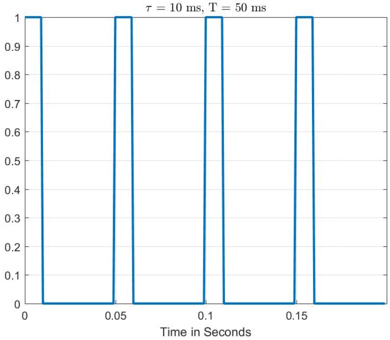
- Frequency Domain (magnitude of Fourier Series)

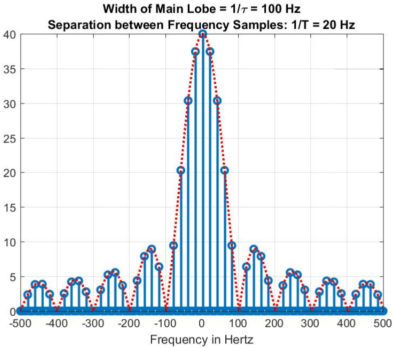


F.T. of Rectangular Periodic Waveform

Effect of Changing the Duty Cycle = ON duration / T_{cycle}

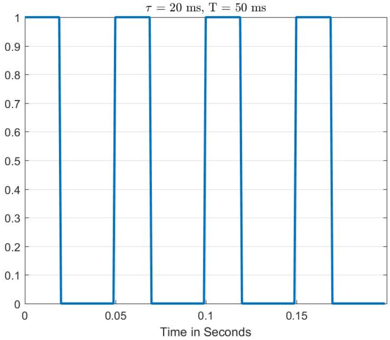
- Time Domain
- Frequency Domain



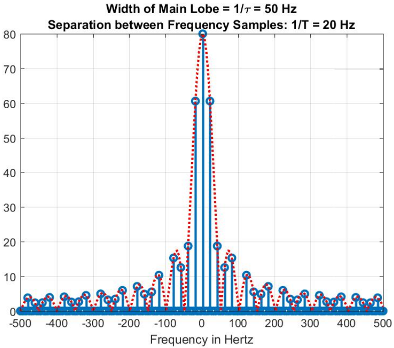


F.T. of Rectangular Periodic Waveform
Effect of Changing the Duty Cycle

- Time Domain



- Frequency Domain



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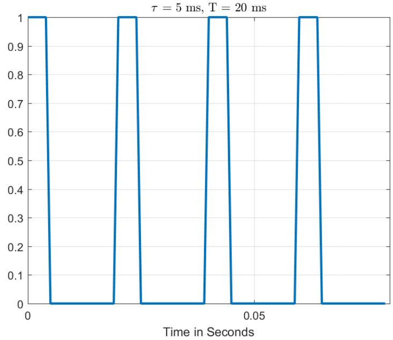

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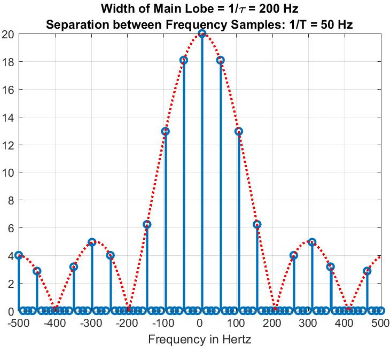


Bandwidth of a Communication Channel

- Suppose the transmitter sends a communication signal $s(t) \Longleftrightarrow S(f)$
- Receiver receives this signal after it passes through the communication channel
- Communication channel itself can be thought of as sort of like a signal $h(t) \Longleftrightarrow H(f)$
- Let us call the received signal $r(t) \Longleftrightarrow R(f)$
- Effect of many practical, real-world, communication channels (such as wireline as well as wireless channels) can be *modeled* as follows:

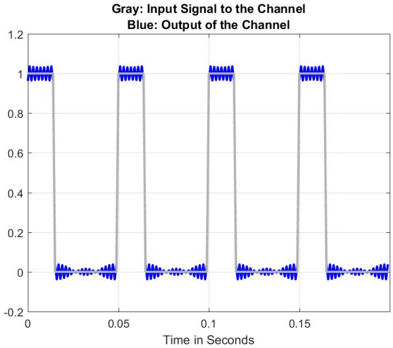
$$R(f) = H(f) \times S(f)$$

→ Above is multiplication in frequency domain

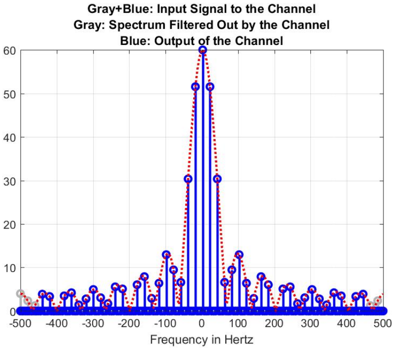
→ Channel acts like a gate; won't let any signal s(t) pass through accurately if $S(f)$ has a larger size (i.e., bandwidth) along frequency axis compared to $H(f)$

Effect of Signal Filtering
by Communication Channel

- Time Domain



- Frequency Domain



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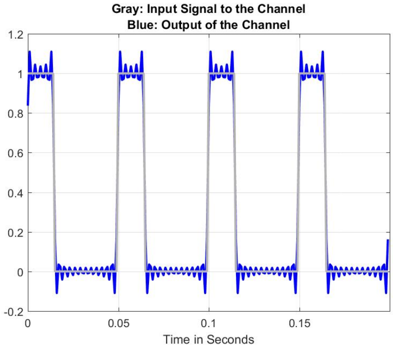

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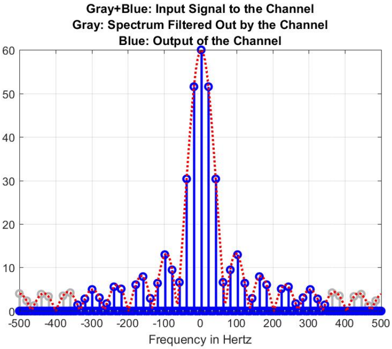

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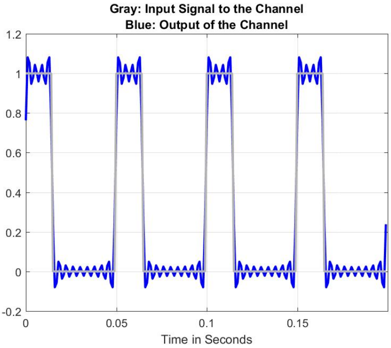

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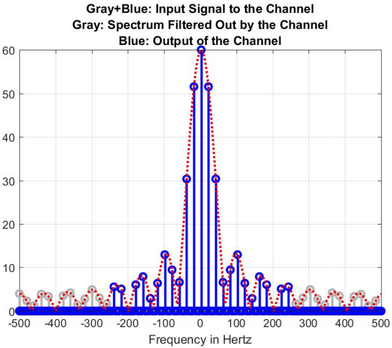

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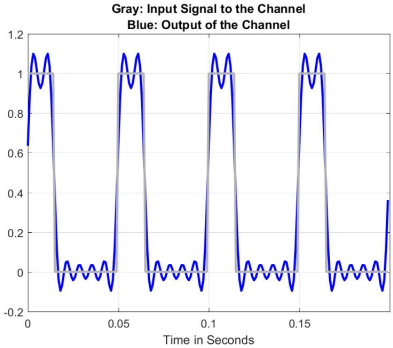

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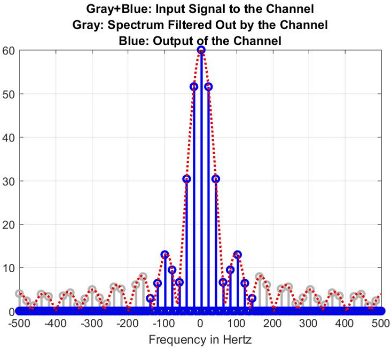

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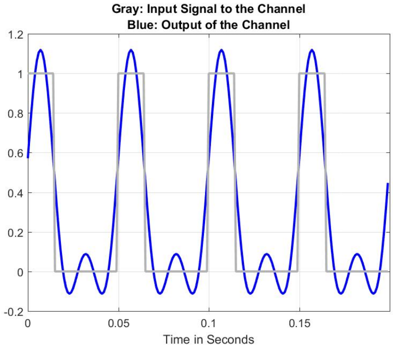

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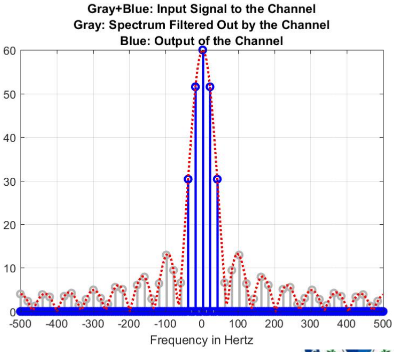

Several Properties
of Fourier Transform

- Duality:
  - → If $w(t) \Longleftrightarrow W(f)$, then $W(t) \Longleftrightarrow w(-f)$
- Time or Frequency Shifts:
  - → Time Shift or Delay:
    - ▷ $w(t - T_d) \Longleftrightarrow W(f)\exp(-i2\pi f T_d)$
  - → Frequency Shift or Translation (also known as Modulation Property):
    - ▷ (Complex): $w(t)\exp(i2\pi f_c t) \Longleftrightarrow W(f - f_c)$
    - ▷ (Real): $w(t)\cos(2\pi f_c t + \theta) \Longleftrightarrow \frac{1}{2}\left[e^{i\theta}W(f - f_c) + e^{-i\theta}W(f + f_c\right]$
- Spectral symmetry of real-valued signals:
  - → If $w(t)$ is real: $W(-f) = W^*(f)$
  - → If $w(t)$ is real and symmetric: $W(f)$ is real-valued and symmetric

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    - $\triangleright$ (Complex): $w(t) \exp(i2\pi f_c t) \Longleftrightarrow W(f - f_c)$
    - $\triangleright$ (Real): $w(t) \cos(2\pi f_c t + \theta) \Longleftrightarrow \frac{1}{2}\left[e^{i\theta} W(f - f_c) + e^{-i\theta} W(f + f_c\right]$
- Spectral symmetry of real-valued signals:
  - $\rightarrow$ If $w(t)$ is real: $W(-f) = W^*(f)$
  - $\rightarrow$ If $w(t)$ is real and symmetric: $W(f)$ is real-valued and symmetric

Several Properties
of Fourier Transform

- Duality:
  - $\rightarrow$ If $w(t) \Longleftrightarrow W(f)$, then $W(t) \Longleftrightarrow w(-f)$
- Time or Frequency Shifts:
  - $\rightarrow$ Time Shift or Delay:
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Several Properties
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- Duality:
  - → Applications are everywhere
- Time or Frequency Shifts:
  - → Time Shift or Delay:
    - ▷ Communication channels introduce a delay. This property tells us what to expect in frequency domain, i.e., a linearly changing phase as a function of frequency. Alternatively, if the communication channel introduces nonlinear phase shift, that tells us that it is introducing a time distortion in the signal instead of a simple time delay
  - → Frequency Shift or Translation (also known as Modulation Property):
    - ▷ (Complex): a message signal $w(t)$ is typically centered at 0 Hz. Its frequency is translated to Radio Frequency or RF in the manner shown by the property of complex frequency shift.
    - ▷ (Real): In real world systems, a sinusoidal signal is used for RF frequency conversion
- Spectral symmetry of real-valued signals:
  - → Effect of turning a complex time-domain signal into a real signal is that the negative (mirror-image) frequencies show up

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- Convolution and Multiplication:
  - $\rightarrow \; w_1(t) * w_2(t) = \int_{-\infty}^{\infty} w_1(\tau) w_2(t-\tau)\, d\tau \Longleftrightarrow W_1(f) W_2(f)$
  - $\rightarrow \; w_1(t) w_2(t) \Longleftrightarrow \int_{-\infty}^{\infty} W_1(\nu) W_2(f-\nu)\, d\nu$
- Scale Change:
  - $\rightarrow \; w(m\,t) \Longleftrightarrow \frac{1}{|m|} W\left(\frac{f}{m}\right)$
- Parseval's Theorem:
  - $\rightarrow \; \int_{-\infty}^{\infty} w_1(\tau) w_2^*(\tau)\, d\tau \Longleftrightarrow \int_{-\infty}^{\infty} W_1(f) W_2^*(f)\, df$
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- Convolution and Multiplication:
  - → Convolution in time domain is used to perform filtering in frequency domain
  - → Effect of time domain sampling can be thought of convolution in frequency domain, which makes the frequency spectra periodic
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  - → Doppler effect
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Syllabus
for the First Mid Term Exam

Tomasi Book: Electronic Comm Systems: Fundamentals Through Advanced (Fifth Edition)

1. Chapter 1: Sections 1.2 to 1.6
2. Chapter 2 (entire)

Couch Book: Digital and Analog Comm Systems (Sixth Ed)

1. Chapter 1: Sections 1.2, 1.6 to 1.10
2. Chapter 2: Sections 2.1, 2.2, 2.5 and 2.6