Tut 6 Solution

Solution 1)

$h(n) = h_1(n) * h_2(n)$

$= \sum_{k=-\infty}^{\infty} h_1(n) h_2(n-k)$

$= \sum_{k=-\infty}^{\infty} \left(\frac{1}{3}\right)^k u(k) \left(\frac{1}{6}\right)^{n-k} u(n-k)$

$= \sum_{k=0}^{n} \left(\frac{1}{3}\right)^k \cdot \left(\frac{1}{6}\right)^n \cdot \left(\frac{1}{6}\right)^{-k}$

$= \left(\frac{1}{6}\right)^n \sum_{k=0}^{n} \left(\frac{1}{3}\right)^k \cdot \left(\frac{1}{6}\right)^{-k}$

$$= \left(\frac{1}{6}\right)^n \sum_{k=0}^{n} (2)^k$$

$$= \frac{\left(\frac{1}{6}\right)^n (2^n - 1)}{2 - 1}$$

$$= \left(\frac{1}{6}\right)^n (2^n - 1)$$

Solution 2)

(a)

$h(n) = \delta(n - n_d)$

$\sum_{n=-\infty}^{\infty} h(n) = 1$ ∴ System is stable.

$h(n) = 0 \quad for \ n < 0$ ∴ System is causal.

(b)

$h(n) = \sum_{k=-\infty}^{n} \delta(k)$

$h(n) = 0 \quad for \ n < 0$ ∴ System is causal.

$\sum_{n=-\infty}^{\infty} h(n) = \infty$ ∴ System is Unstable.

(C) $h(n) = \delta(n+1) - \delta(n)$

$\sum_{n=-\infty}^{\infty} h(n) = 2 < \infty$ $\therefore$ System is Stable.

$h(n) = 1 \quad for\ n = -1$ $\therefore$ System is not Causal.

(d)

$h(n) = \delta(n) - \delta(n-1)$

$\sum_{n=-\infty}^{\infty} h(n) = 2 < \infty$ $\therefore$ System is Stable.

$h(n) = 0\ for\ n < 0$ $\therefore$ System is Causal.

Solution 3)

$$y(t) = x(t) * \delta(t)$$

$$= \int_{-\infty}^{\infty} x(\tau)\delta(t-\tau)d\tau$$

$= x(t)$ (sifting property)

$\therefore x(t) = x(t) * \delta(t)$ (1)

Let $x(t) = \delta(t)$ then

$\delta(t) = \delta(t)* \delta(t)$ (2)

Using equation (2) in equation (1) we get,

$\delta(t) = \delta(t)* \delta(t) * \delta(t)$

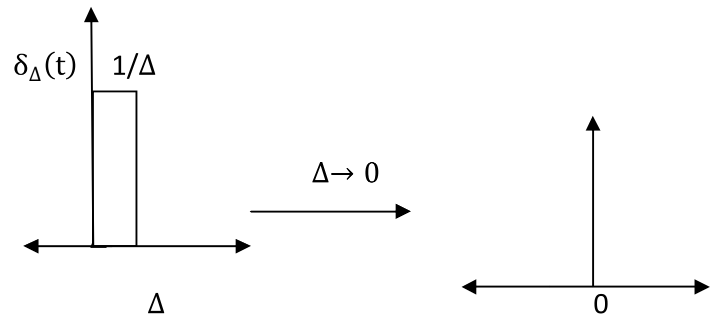
$\vdots$

$\vdots$

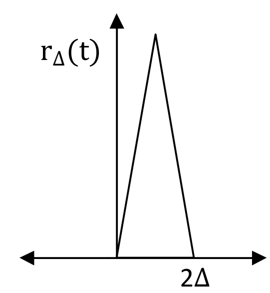
$\delta(t) = \delta(t)* \delta(t) * \delta(t) \cdots\cdots \ \delta(t)* \delta(t) * \delta(t)$

(b) We have
1

$\lim_{\Delta\to 0} \delta_\Delta(t) = \delta(t)$



$\delta_\Delta(t) * \delta_\Delta(t) = r_\Delta(t)$



$r_\Delta(t)$ can be considered as impulse if the pulse width Δ is sufffuciently small for the system. This means that convolution of impulse with itself is considered as impulse if the pulse width of impulse is sufficiently small for the system.

Solution 4)

Autocorrelation of periodic signal is periodic with same period. Here we will consider one period for calculating autocorrelation for periodic signal.

$$r_{xx}(l) = \sum_{<n>} x(n) \cdot x(n-l)$$

Here period =4. So we will calculate autocorrelation for $l = 0,1,2,3$, which will be periodically repeated.

$$r_{xx}(0) = 1 \cdot 1 + 2 \cdot 2 + 3 \cdot 3 + 4 \cdot 4 = 30$$

$$r_{xx}(1) = 1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + 4 \cdot 1 = 24$$

$$r_{xx}(2) = 1 \cdot 3 + 2 \cdot 4 + 3 \cdot 1 + 4 \cdot 2 = 22$$

$$r_{xx}(3) = 1 \cdot 4 + 2 \cdot 1 + 3 \cdot 2 + 4 \cdot 3 = 24$$

This will repeat with period 4. Also look at the reference material for cross-correlation kept in lecture folder.

Solution 5)

$$r_{xy}(l) = \sum_{<n>} x(n) \cdot y(n-l)$$

As per above example,

$r_{xy}(0) = 7$

$r_{xy}(1) = 13$

$r_{xy}(2) = -17$

$r_{xy}(3) = 16$

$r_{xy}(4) = -8$

$r_{xy}(5) = 5$

$r_{xy}(6) = -3$

Solution 6)

$$r_{xy} = \sum_{n=-\infty}^{\infty} x(n)y(n-l) \qquad (1)$$

$$r_{yx}(l) = \sum_{n=-\infty}^{\infty} y(n)x(n-l) \qquad (2)$$

Let $n - l = m$ in equation (1)

$$r_{xy}(l) = \sum_{m=-\infty}^{\infty} x(m+l)y(m)$$

$$r_{xy}(l) = \sum_{m=-\infty}^{\infty} y(m)\, x(m+l) \qquad (3)$$

Here m is a dummy variable which can be replaced as n

$$r_{xy}(l) = \sum_{m=-\infty}^{\infty} y(n)\, x(n+l)$$

Replace $l = -l$ in equation (2) we will get

$$r_{yx}(-l) = \sum_{n=-\infty}^{\infty} y(n)x(n+l) \quad = r_{xy}(l)$$

$$\therefore r_{xy}(l) = r_{yx}(-l)$$