Lecture 8

Projectile Motion : Computational Approach

Canon shell:

- ODE
- Initial condition given
- Euler’s method; Finite Difference

Canon shell (goal: to understand motion in two spatial dimensions)

Case 1: simple case without air resistance

Write down the equation of motion.

Newtons second Law of motion

d2x/dt2=0

d2y/dt2=-g

x,y→ horizontal and vertical components.

Second order differential eqns. How to solve?

Alternate approach?

Two first order differential eqns.

$$\frac{dx}{dt} = v_x$$

$$\frac{dv_x}{dt} = 0$$

$$\frac{dy}{dt} = v_y$$

$$\frac{dv_y}{dt} = -g\,,$$

$$x_{i+1} = x_i + v_{x,i}\,\Delta t$$

$$v_{x,i+1} = v_{x,i}$$

$$y_{i+1} = y_i + v_{y,i}\,\Delta t$$

$$v_{y,i+1} = v_{y,i} - g\,\Delta t\,.$$

What about drag

Including drag

$$F_{\text{drag}} = -B_2 v^2$$

$$v = \sqrt{v_x^2 + v_y^2}$$

Opposite to the velocity, so we need vector components.

Write down the vector components.

Components

$$F_{\mathrm{drag},x} = F_{\mathrm{drag}} \cos\theta = F_{\mathrm{drag}}\, v_x / v$$

$$F_{\mathrm{drag},x} = -B_2\, v\, v_x$$
$$F_{\mathrm{drag},y} = -B_2\, v\, v_y$$

$$x_{i+1} = x_i + v_{x,i}\,\Delta t$$
$$v_{x,i+1} = v_{x,i} - \frac{B_2\, v\, v_{x,i}}{m}\,\Delta t$$
$$y_{i+1} = y_i + v_{y,i}\,\Delta t$$
$$v_{y,i+1} = v_{y,i} - g\,\Delta t - \frac{B_2\, v\, v_{y,i}}{m}\,\Delta t$$

MATLAB Code

Modify the code to understand the following

- role of air resistance.
- Role of firing angle.
- Density of atmosphere.