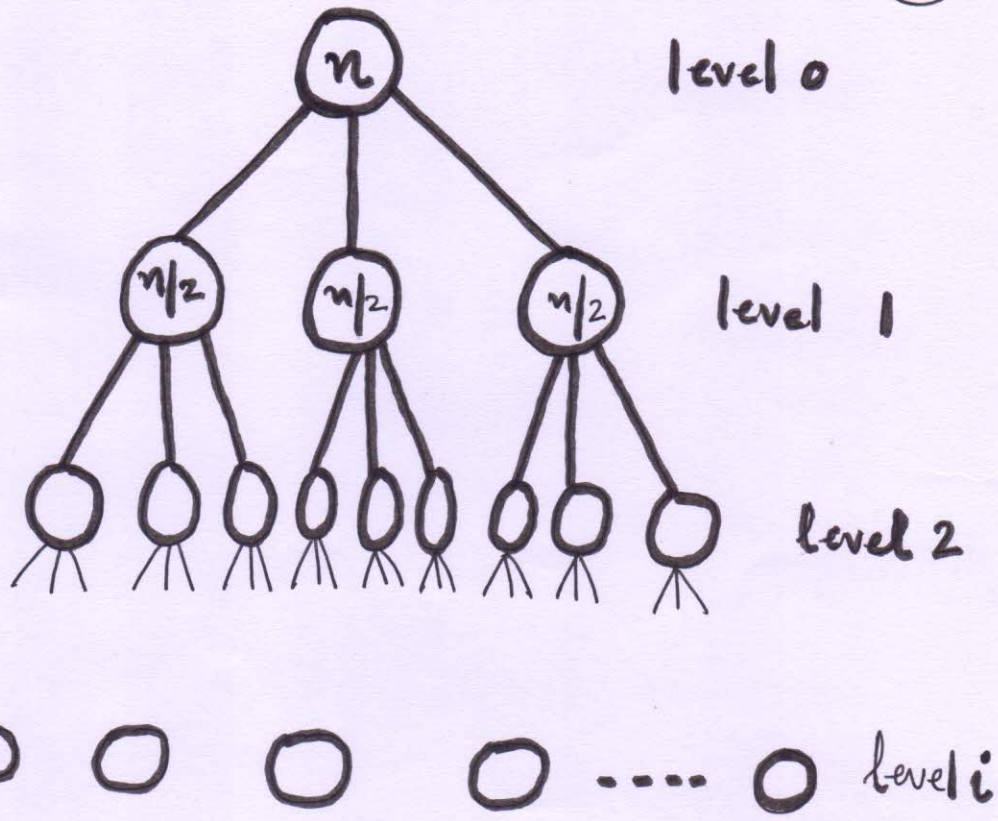
We just solved:

$$T(n) \leq 2 \cdot T\left(\frac{n}{2}\right) + cn$$
$$T(2) \leq c$$

Now solve:

$$T(n) \leq 3 \cdot T\left(\frac{n}{2}\right) + cn$$
$$T(2) \leq c$$



What is i ?

__Answer__ : $\frac{n}{2^i} = 2$

$i = \log_2 \frac{n}{2} = (\log_2 n - 1)$

$$T(n) \leq \sum_{j=0}^{\log_2 n - 1} (c \cdot n) \left(\frac{3}{2}\right)^j$$

$$\leq (c \cdot n) \left[\frac{\left(\frac{3}{2}\right)^{\log_2 n} - 1}{\left(\frac{3}{2} - 1\right)} \right]$$

$$\leq 2 \cdot c \cdot n \left[\left(\frac{3}{2}\right)^{\log_2 n} - 1 \right]$$

We know $a^{\log b} = b^{\log a}$, $a, b > 1$

$$\leq 2 \cdot c \cdot n^{\log_2 \left(\frac{3}{2}\right) + 1} - 1$$

$$\leq 2 \cdot c \cdot n^{\log_2 \left(\frac{3}{2}\right) + 1}$$

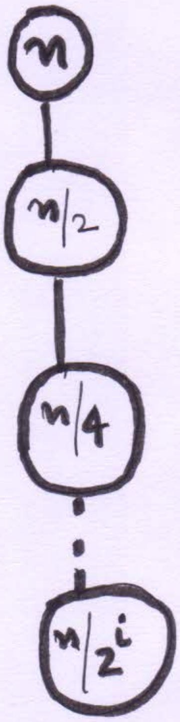
$$\leq 2 \cdot c \cdot n^{\log_2 3}$$

$$= O\left(n^{\log_2 3}\right) = O\left(n^{1.59}\right)$$

THE CASE OF ONE SUB PROBLEM

$$T(n) \leq T\left(\frac{n}{2}\right) + cn$$

$$T(2) \leq c$$



$$\frac{n}{2^i} = 2$$

$$i = \log_2 n - 1$$

$$T(n) \leq \sum_{j=0}^{\log_2 n - 1} \frac{c \cdot n}{2^j}$$

$$\leq c \cdot n \sum_{j=0}^{\log_2 n - 1} \frac{1}{2^j}$$

$$\leq c \cdot n \sum_{j=0}^{\infty} \frac{1}{2^j}$$

$$\leq 2 \cdot c \cdot n$$

$T(n)$ is $O(n)$.