

Arithmetic Circuits :

Circuits which perform arithmetic function.
(adding 2 binary nos: standard half adder).

What and why do we use arithmetic circuits?

- They are the heart of every digital circuit.
- Optimize to improve performance.
- Most microcontrollers have ALU.

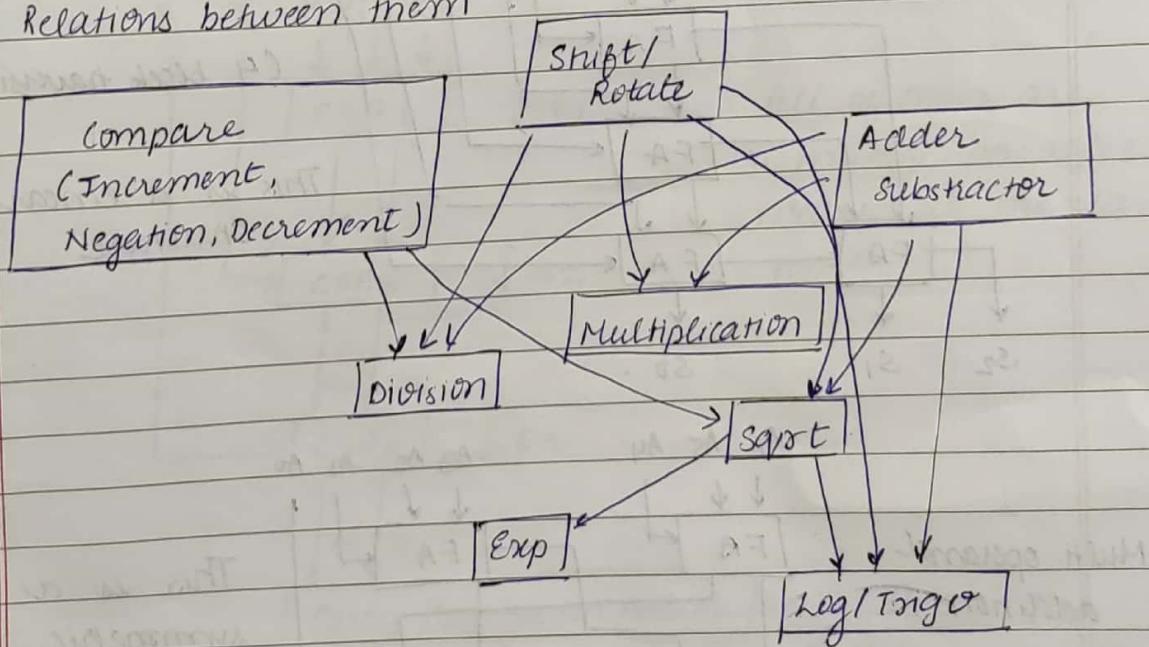
Multiplication, Addition ALU, shifter (incrementor, decrementor, etc).

Types of Arithmetic functions:

min complexity: shift, rotation, incrementor, decrementor, compare, negation, add, sub, -, *, ÷, sqrt, exp, log, trig

max complexity .

Relations between them :



Most important block : adder subtractor

1 1 0 1 1 1 0 0 1 1

no. of ones = 7

∴ I can infer that my add operation is nothing but a counter.

Half adder is a (2, 2) counter.

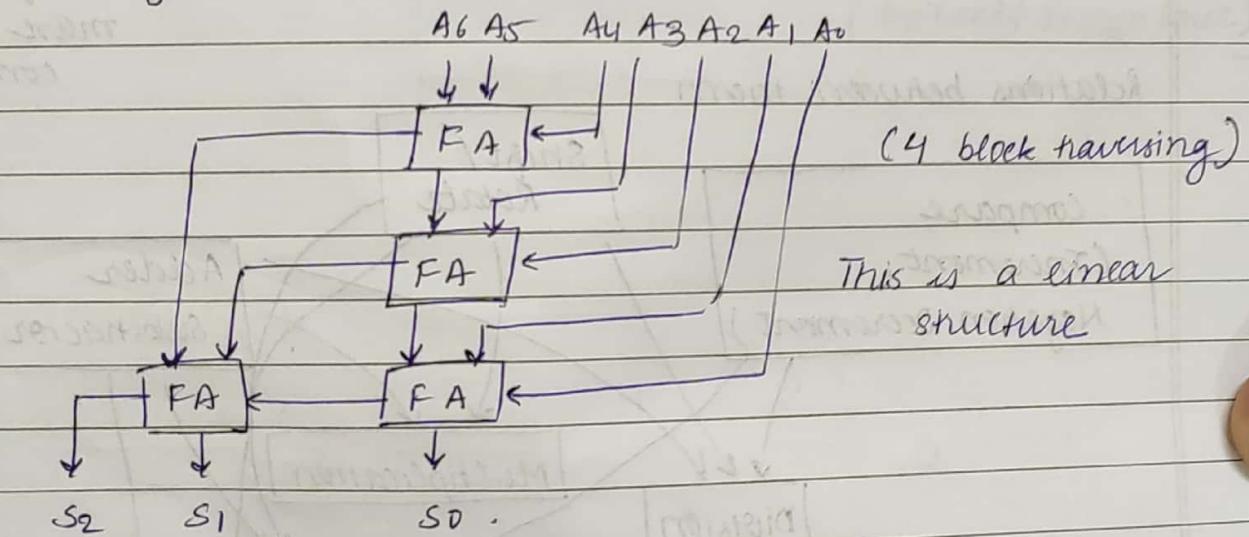
It can add two 1 bit numbers and the result is a 2 bit number.

What is a (3, 2) counter?

It can add 3 1 bit numbers and present it in a 2 bit number (full adder).

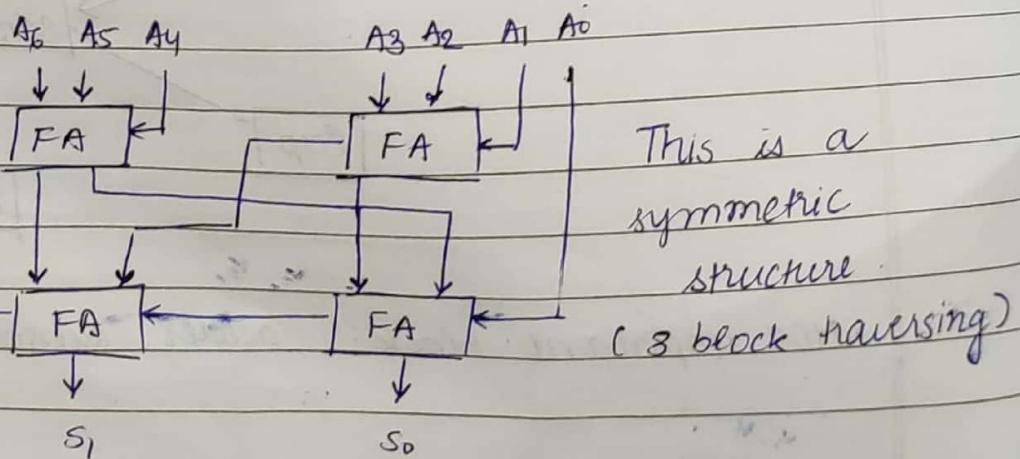
They are compressor type counters (3, 2).

Adding multiple 1 bit numbers:

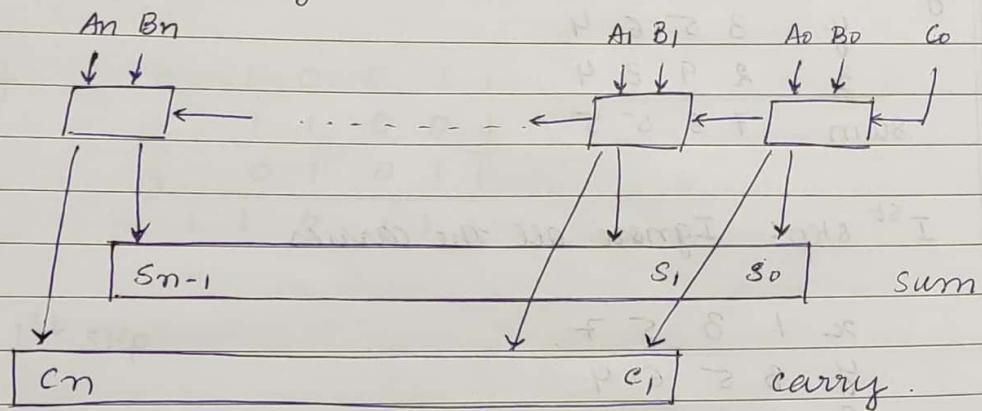


Multi operand addition.

They are popularly called as carry propagate adder.



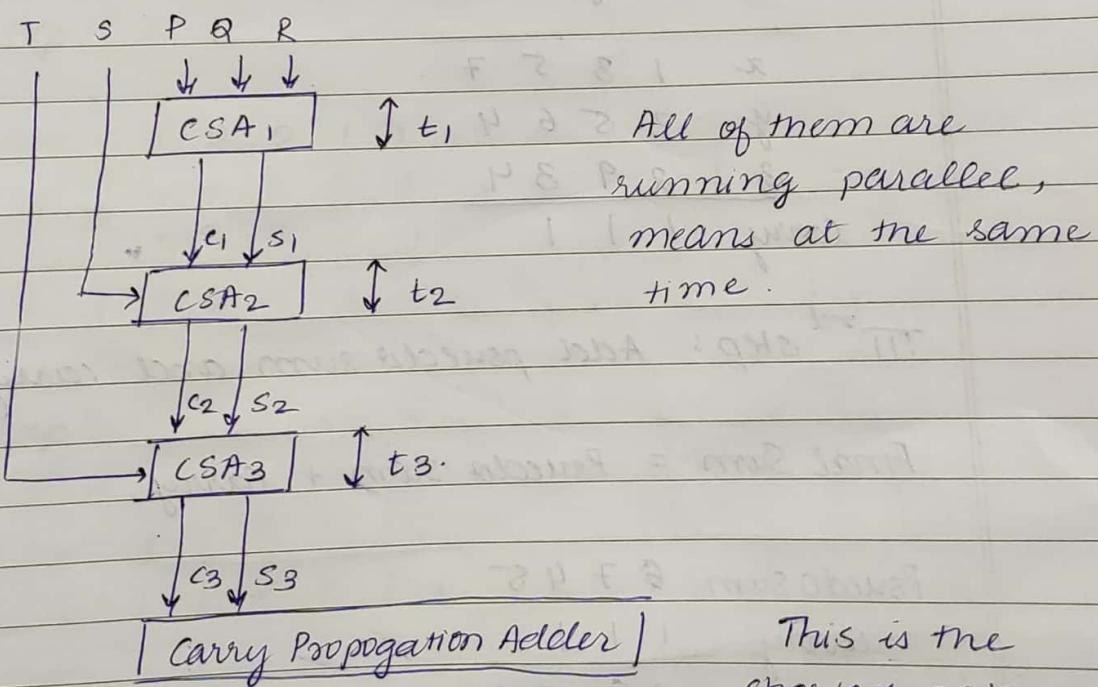
Instead of carry to be propagated we can use carry save adders to reduce multiple I/P to 2 and then single CSA is used.



$$\text{carry} + \text{sum} = A + B + C$$

$$c_1 + s_0 = A_0 + B_0 + C_0$$

$$C + S = P + Q + R + S + T$$



This is the shortest path.

$$\max(t_1, t_2, t_3)$$

They are inherently parallel to each other generated Asynchronously.

Carry Save Adder:

eg $\begin{array}{r} x \quad 1 \ 3 \ 5 \ 7 \\ y \quad 3 \ 5 \ 6 \ 4 \\ z \quad 2 \ 9 \ 3 \ 4 \\ \hline \text{sum} \quad 7 \ 8 \ 5 \ 5 \end{array}$

Ist step: Ignore all the carries

$$\begin{array}{r} x \quad 1 \ 3 \ 5 \ 7 \\ y \quad 3 \ 5 \ 6 \ 4 \\ z \quad 2 \ 9 \ 3 \ 4 \\ \hline \text{pseudo sum} \quad 6 \ 7 \ 4 \ 5 \\ \text{sum} \end{array}$$

IInd step: Only write the carries

$$\begin{array}{r} x \quad 1 \ 3 \ 5 \ 7 \\ y \quad 3 \ 5 \ 6 \ 4 \\ z \quad 2 \ 9 \ 3 \ 4 \\ \hline \text{carry} \quad 1 \ 1 \ 1 \end{array}$$

IIIrd step: Add pseudo sum and carry

$$\text{Final Sum} = \text{Pseudo sum} + \text{carry}$$

$$\text{Pseudo sum} \quad 6 \ 7 \ 4 \ 5$$

$$\text{carry} \quad 1 \ 1 \ 1$$

$$\text{Final sum} \quad 7 \ 8 \ 5 \ 5$$

Carry and Pseudo sum can be computed independently.

Thus gives us a parallel implementation. This can give a better and faster implementation than carry ripple adder.

eg

x 1 0 0 1 0 0 1 1

y 1 1 0 0 1

z 0 1 0 1 1 0

1 1 0 1 1 1 1 1 1

1st step:

1 0 0 1 1 1 0 1 0 1

1 1 0 0 1

0 1 0 1 1

0 0 0 0 1

2nd step:

1 0 0 1 1

1 1 0 0 1

0 1 0 1 1

1 1 0 1 1

3rd step:

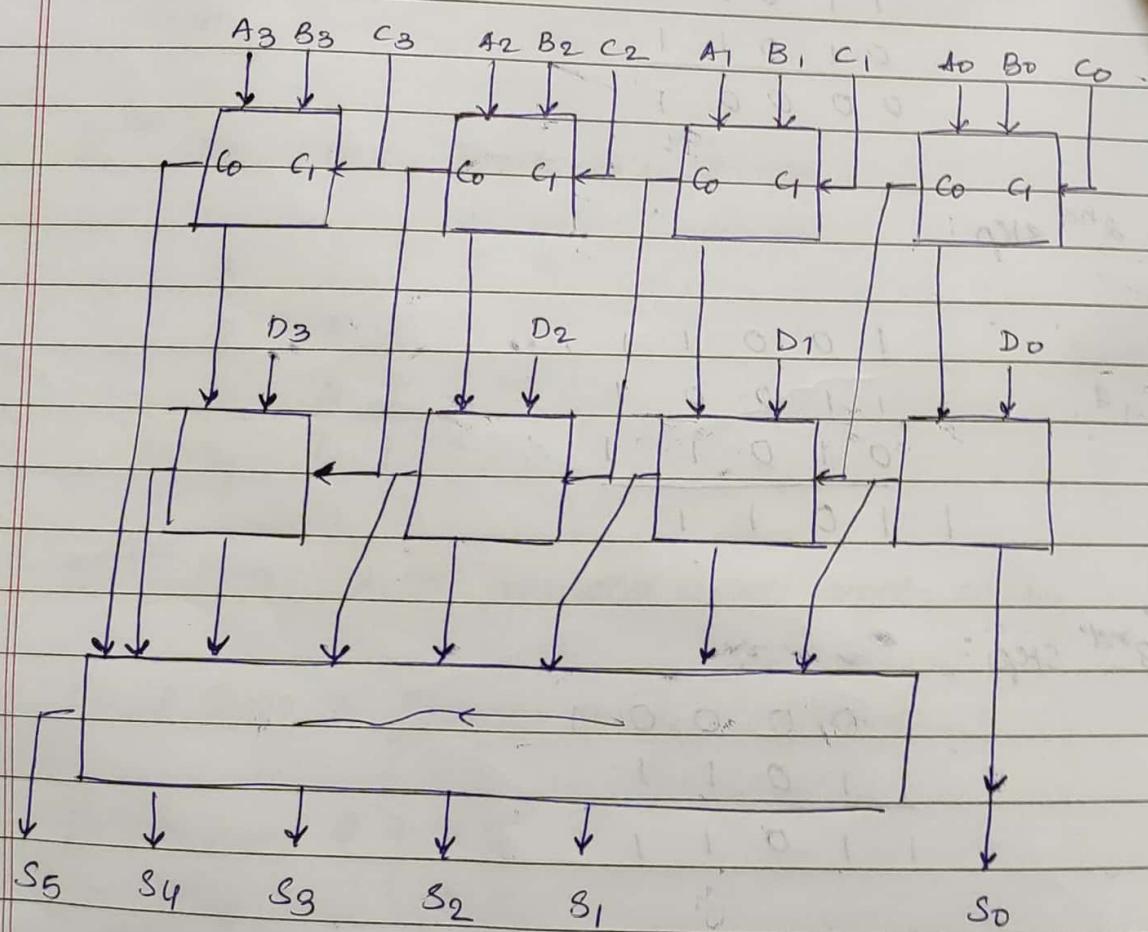
0 0 0 0 1

1 1 0 1 1

1 1 0 1 1 1

I take 3 operands (to build my hardware)

	A_3	A_2	A_1	A_0	
A	0	1	1	1	
B	B_3	B_2	B_1	B_0	
C	C_3	C_2	C_1	C_0	
	1	1	1	1	
	0	0	1	0	S_1^*
	1	1	1	1	C_0'
D	D_3	D_2	D_1	D_0	
	1	0	1	1	
	1	0	1	1	S_2
	1	0	1	0	C_0''
	1	0	1	0	
	1	0	1	1	
	1	0	1	1	$Final\ sum$

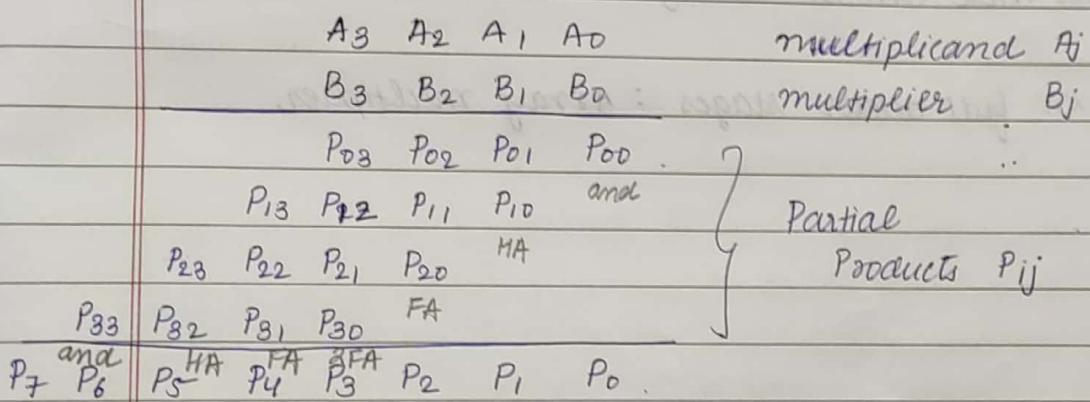


For m integer operands, involves $(m-2)$
CSA additions

in RCA,
delay } $(m-1)$ tRCA

in CSA,
 $(m-2)$ tCSA + tRCA $m \geq 3$

For multiplication :



$$P_0 = P_{00}$$

$$P_1 = P_{01} \oplus P_{10}$$

$$P_2 = P_{02} \oplus P_{11} \oplus P_{20} \oplus C_{12}$$

$$P_3 = P_{03} \oplus P_{12} \oplus P_{21} \oplus P_{30} \oplus [C_{23} \oplus C_{23}']$$

$$P_4 = P_{13} \oplus P_{22} \oplus P_{31} \oplus [C_{34} \oplus C_{34}' \oplus C_{34}'']$$

$$P_5 = P_{23} \oplus P_{32} \oplus [C_{45} \oplus C_{45}' \oplus C_{45}'']$$

$$P_6 = P_{33} \oplus [C_{56} \oplus C_{56}']$$

$$P_7 = C_{67}$$

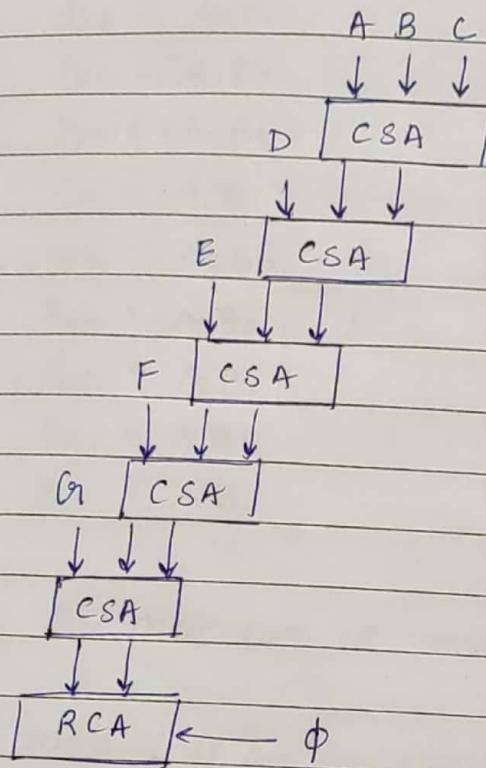
$$\begin{array}{ll} P_{00} = A_0 B_0 & P_{13} = A_3 B_1 \\ P_{10} = A_0 B_1 & P_{22} = A_2 B_2 \\ P_{01} = A_1 B_0 & P_{31} = A_1 B_3 \\ P_{20} = A_0 B_2 & P_{23} = A_3 B_2 \\ P_{11} = A_1 B_1 & P_{32} = A_2 B_3 \\ P_{02} = A_2 B_0 & P_{33} = A_3 B_2 \\ P_{03} = A_3 B_0 & \\ P_{12} = A_2 B_1 & \\ P_{21} = A_1 B_2 & \\ P_{30} = A_0 B_0 & \end{array}$$

All these can be realized with 'AND'.

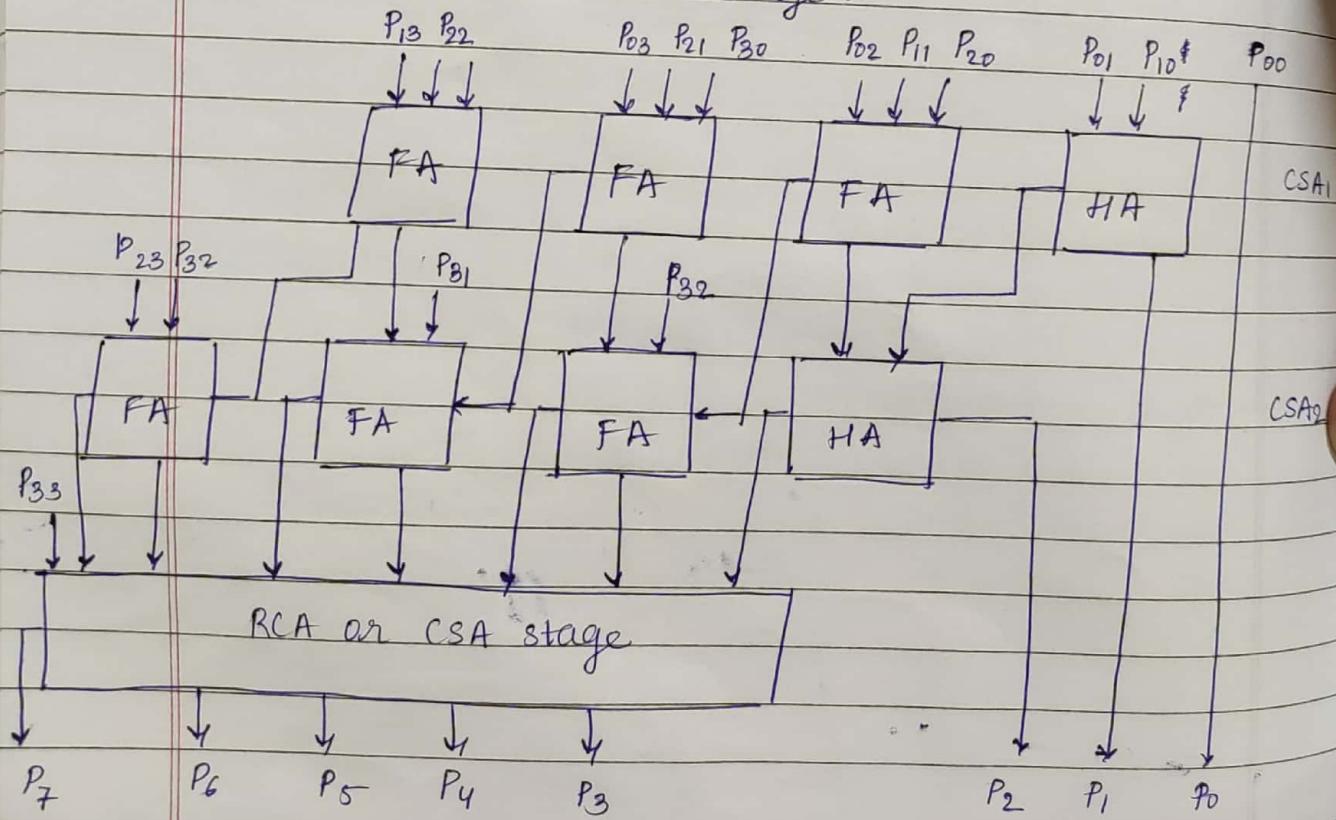
$m \times n$ full adder stages : Array multiplier.

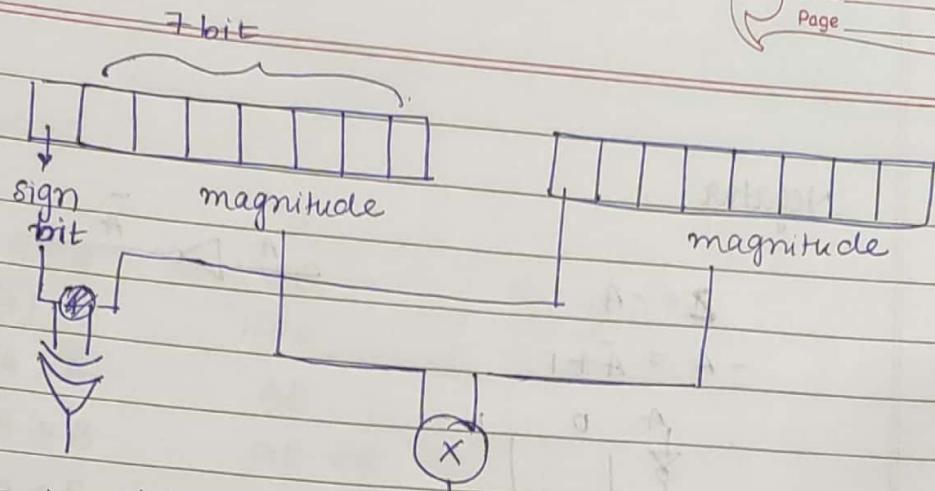


$$A + B + C + D + E + F + G$$



Sure, I can replace a FA or RCA stage to CSA stage. I can replace my parallel multiplier with CSA stage.





Represent in 2's complement and then multiply.

eg

0 0 1 1 3

1 0 1 1 -5

Put n (no. of digits) of,
in front on 2's complement

0 0 0 0 0 0 1 1 3

1 1 1 1 1 0 1 1 -5

Operations : (Adder)

- Incrementor
- Negator
- Subtractor
- Adder Subtractor
- Comparator

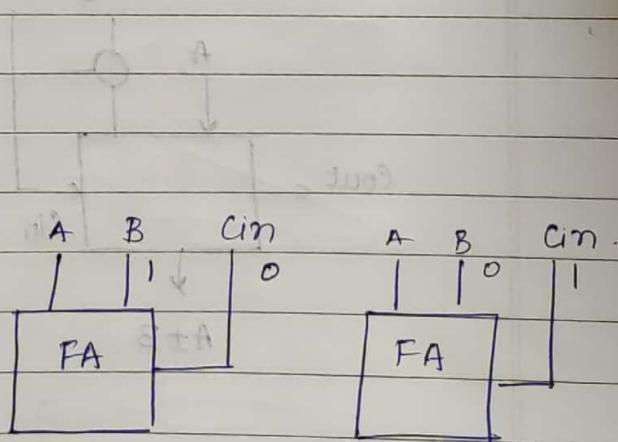
Incrementor:

$$Z = A + 1$$

$$Z = A + B + \text{Cin}$$

$$B=1 \quad \text{Cin}=0$$

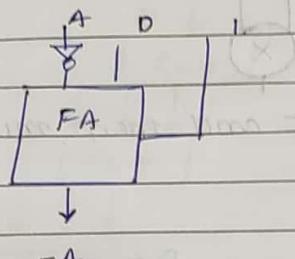
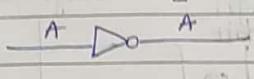
$$B=0 \quad \text{Cin}=1$$



Negator:

$$Z = -A$$

$$-A = \bar{A} + 1$$

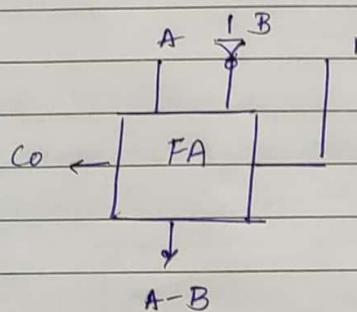


$$B = 1100$$

$$B = 1101$$

Subtractor:

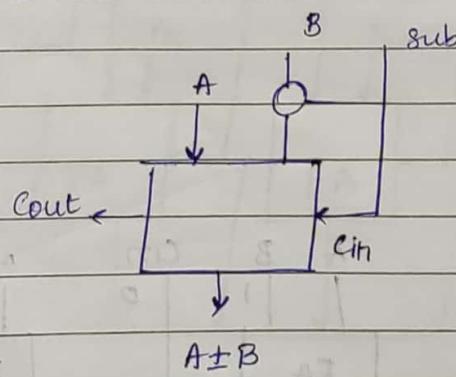
$$Z = A - B$$



$$B = 1100\ 0000$$

$$B = 1101\ 1111$$

Adder Subtractor:



Cin is used to complement B.

comparator:

$A = B$ 011 E6? ? Parity

$A \neq B$ $N \neq Q$

$$A > B \quad GE$$

$A \geq B$ GE. EQ

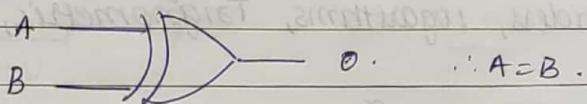
$A < B$ LT / GE

$$A \leq B \quad GE + EG$$

$$A > B : A - B \Rightarrow \text{cont} = 1.$$

$$\therefore A > B$$

$$C_{out} = 0$$



else.

$$A < B$$

Without adders:

- Shifters
 - Rotators

Logical shifter: (true) nicht genutzt in Funktion

$$\begin{array}{r} 11001 \\ \rightarrow \\ 00110 \end{array}$$

Arithmetic shifter

$$11001 \ggg 2 = \underline{\underline{11110}}$$

Same as LS but fills the empty space with the MSB.

Rotator:

$$11001 \text{ ROR } 2 = 01110$$

Rotates bits in circle. Bits shifted off one end are shifted to the other end.

We can use the shifter as multiplier or divider.

$$\begin{array}{r} <<_2 \\ >>_2 \end{array} \left| \begin{array}{l} \times 2^2 \\ \div 2^{-2} \end{array} \right.$$

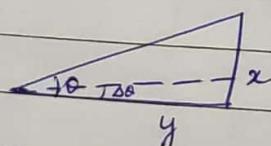
Adder, shifter,
comparator

Dividers, logarithms, Trigonometric, Exponential

$$\exp(x) \text{ or } e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$$

- x^2 Squaring fn $\rightarrow *$
- $x^{1/2}$ Dividing fn $\rightarrow +, -, \text{comp}, *$
- $x^{1/2} +$ Adding fn $\rightarrow +$

Division is expensive, thus we do multiplication instead of division by storing values in the Look Up Table (LUT) / ROM / Memory.



$$\tan \theta = \frac{x}{y}$$

$$\theta = \tan^{-1} \frac{x}{y}$$

$\boxed{\quad}$ \rightarrow Linear transformation
 (2^{-1})

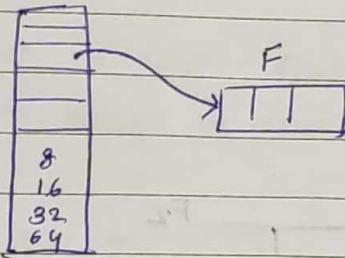
Fetch 1

8
16
32
64

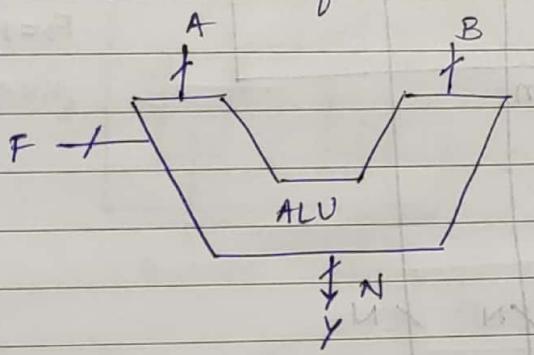
Fetch from
some
to know

F -

Fetch / Decode / Execute :



Fetch from an instruction to ALU. There would be some field F which would be decoded by ALU to know which function is to be performed.

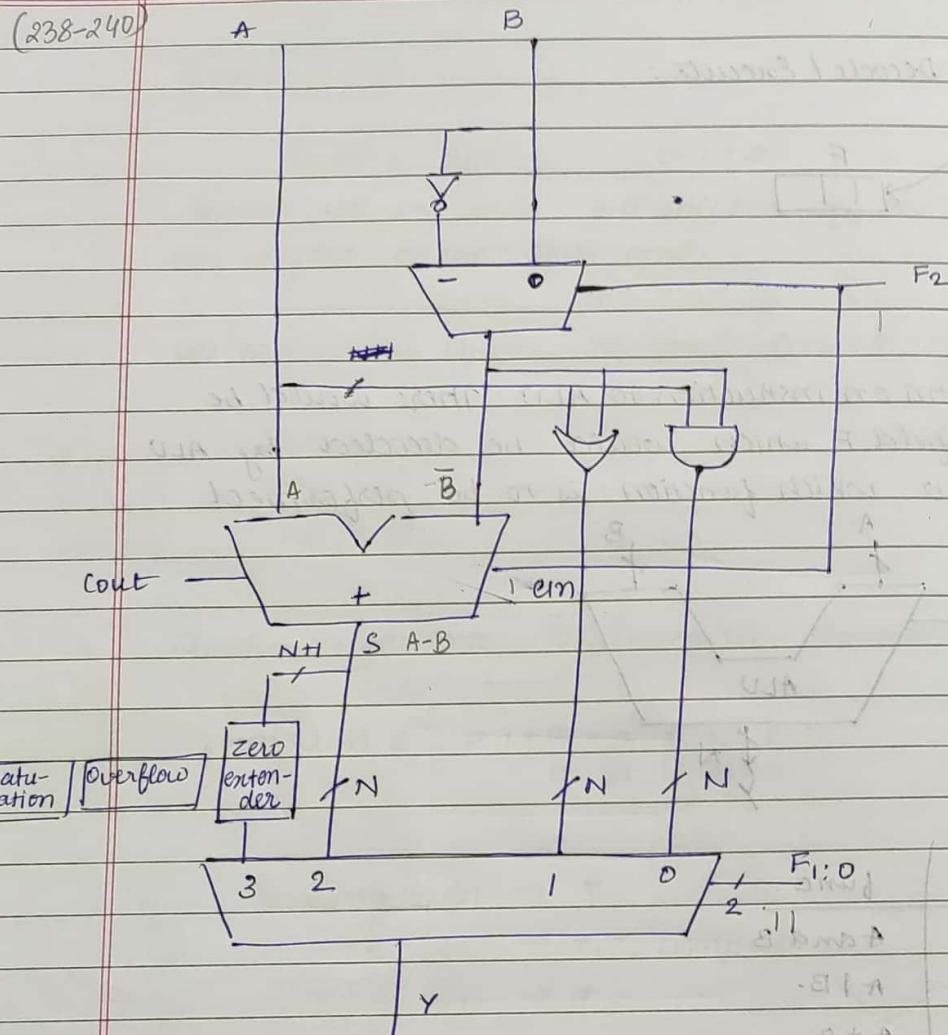


<u>f_{2:0}</u>	<u>func.</u>
000	A and B
001	A B
010	A + B
011	X
100	AB - B
101	A1 - B
110	A - B
111	SL

(238-240)

A

B



$$A = 25$$

$$B = 32$$

Compute this 32 bit ALU for shift left (SL) operation.

$$A - B = 25 - 32 = -7$$

\therefore The 2's complement represents -7, the MSB is 1

A < B

$$\therefore S_{SL} \text{ or } Y_3 = 1$$

zero

extender

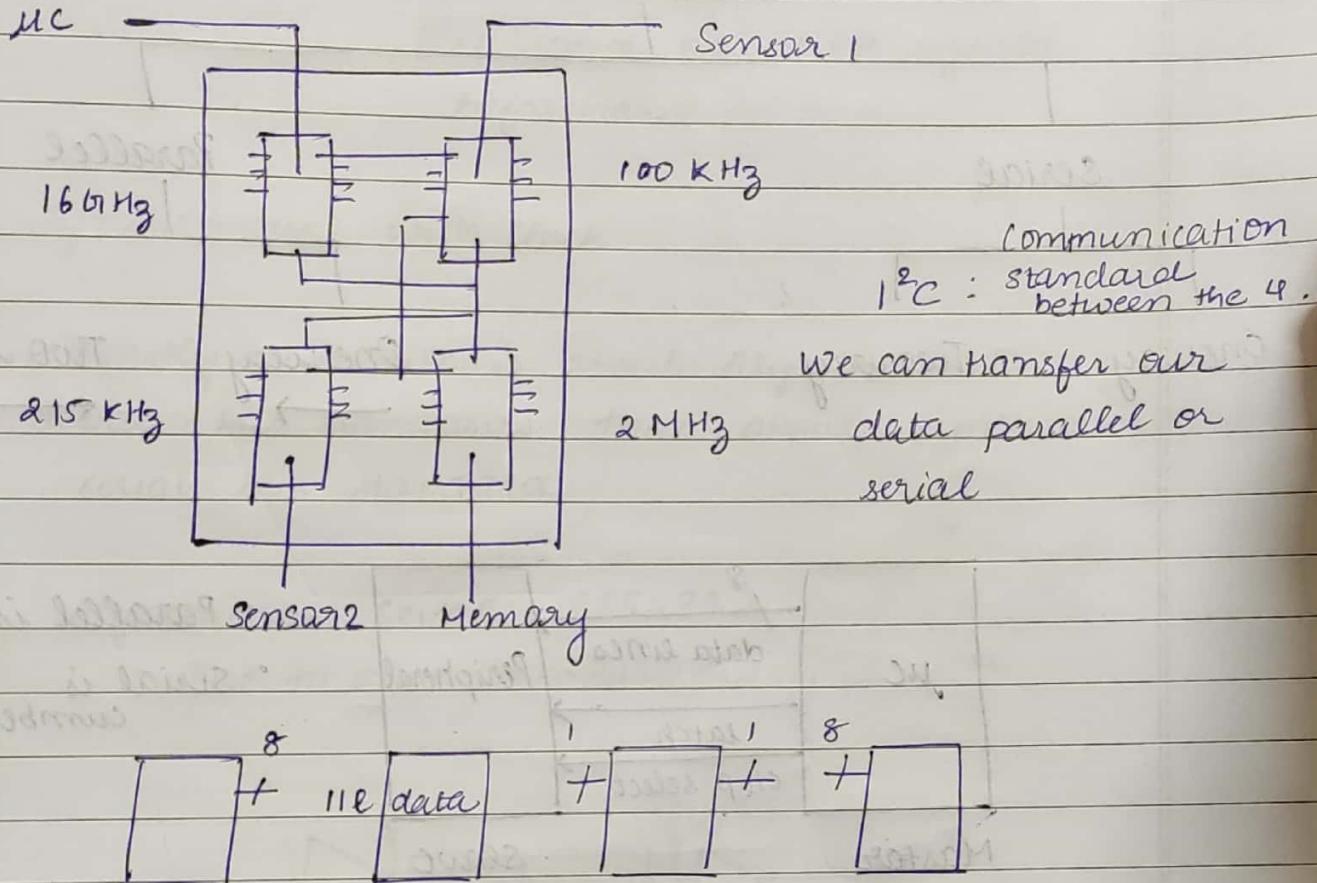
: 000.....1

32 bit

31 zeroes added
as $F_{1:0} = 11$, 3rd line
is selected.

Communication Protocol / Interface

- On chip - chip to chip
- Device to device



- serial - UART (Universal Asynchronous Receiver Transmitter)
- SPI (Serial Peripheral Interface)
 - I²C (Inter Integrated Circuit communication)
 - I2S (Inter Integrated sound Bus)

Device to Device:

- USB

- Ethernet

- VGA

- SCSI

- HDMI

- PS2

- S-Video

Two ways of communication:

- Serial
- Parallel

Communications

Serial

One way Two way

Parallel

One way Two way

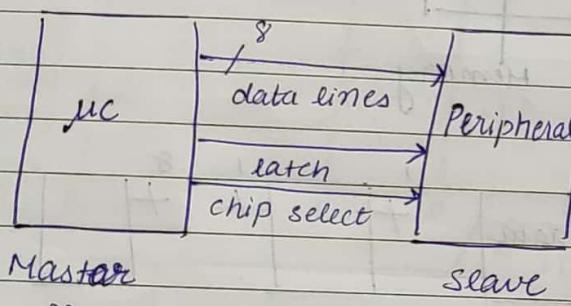
Serial

Async

Sync

FIFO

Series



- Parallel is fast.
- Serial is cumbersome.

Master
or

Controller

Parallel

Fast

Pin (Real estate) is
expensive

Serial

Slow

cheap

Serial is usually used as there are many pins on the chip.

Serial connection - Due to the lesser no. of pins
the complexity of communication
is tugh.

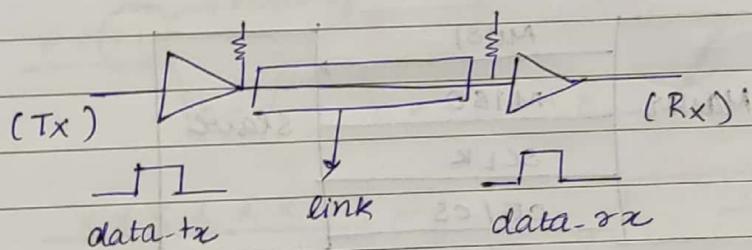
Asynchronous : without a clock.
But speed has to be agreed
beforehand

Synchronous : with clock

Firstly, we discuss about Asynchronous
Serial bus standards then about synchronous
serial bus standard.

Asynchronous serial (RS232)

- One to one communication.



- one Tx, one Rx

- Transmission process (9600 baud rate)
freq at which data
is transmitted

$$\frac{1}{9600} = 0.104 \text{ ms} = 104 \mu\text{s}$$

- Transmit idles high

- It goes for 1 bit.

- Sends LSB first MSB last

- Even or odd parity

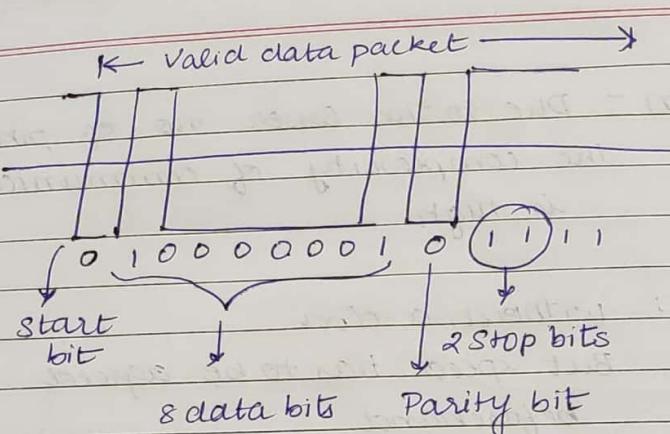
you get

2 data.

start and stop

bit for

communication



wires / lines
are unidirectional

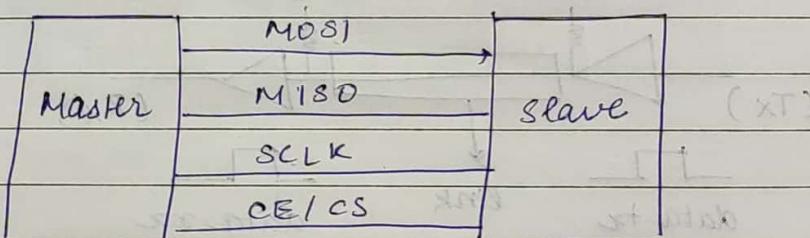
Key points of serial communications:

- 10-30 Gbps

Typically it started with < 115.2 kbps.

SPI (Serial - Peripheral Interface):

4 wires



High data rate (1 MHz to 70 MHz clk)

MOSI - Master Out slave in } can be shared
MISO - Master In slave Out } between any no. of peripherals

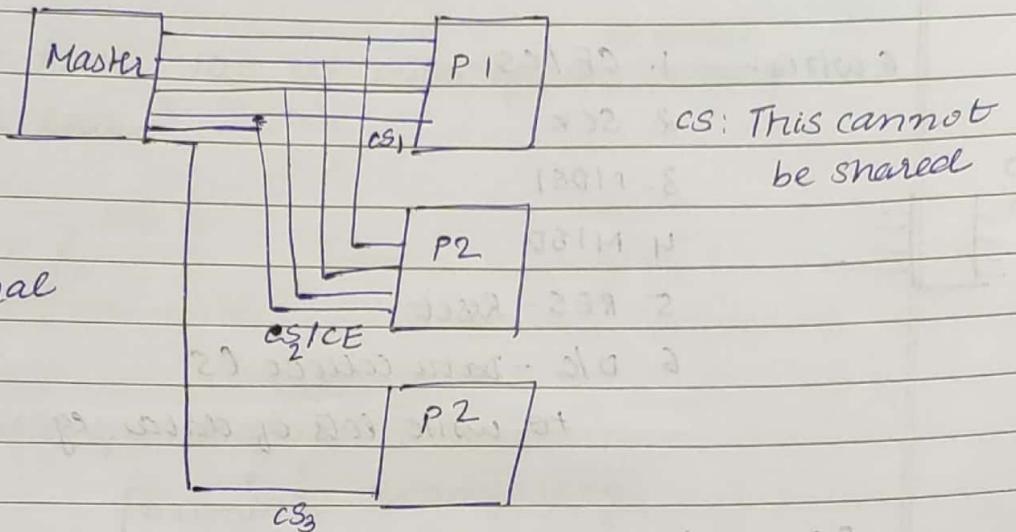
SCK/SCLK - sync. / serial clock

CE/CS - chip enable / chip select

MOSI:

difference VART - (SCK)

wires / Lines
are unidirectional



Q Why do the radio devices have SPI interface?

SPI:

State / Chip select

CS → / select a slave device

$$\bar{CS} = 0.$$

$$CS = 1$$

MOS

→ Reading

1 select a slave device

01 } write

11 → Reading

MISI

✓✓✓✓✓✓✓

~~X X X X X~~

→ data
bits
(1B)

8c

1 1 1 1

7

MOSI:

000

107

00 1

三

1

annals -

101

~~5-6 channels~~

6 wires: 1. CE/CS

2. SCK

3. MISO

4. MOSI

5. RES - Reset

6. I²C - Data collect CS

decreases

to write lots of data eg

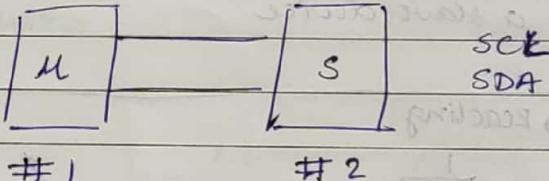
3 for 2 : drop MISO

wires
drop CE/CS (only one device)

I²C : Inter Integrated Circuit

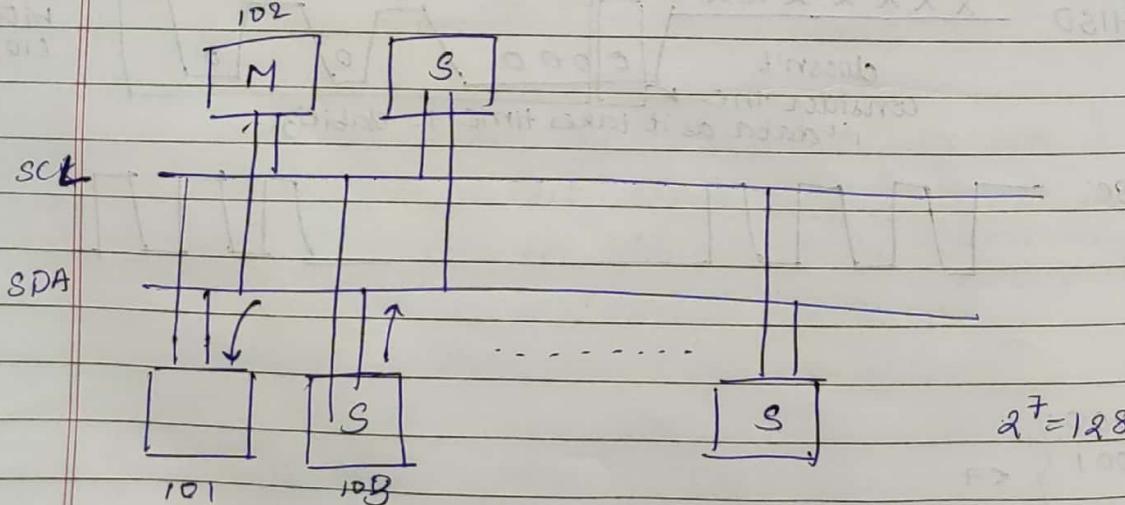
Communication Protocol developed in

1980 by Philips



100 kHz, 400 kHz, 3.4 MHz

Support up to 2⁷ devices.



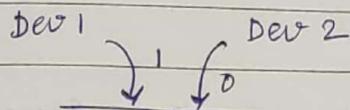
Video: Fun and Easy I²C

classmate

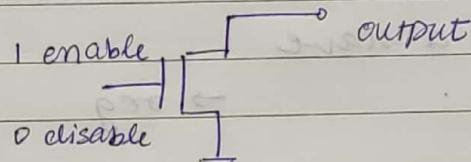
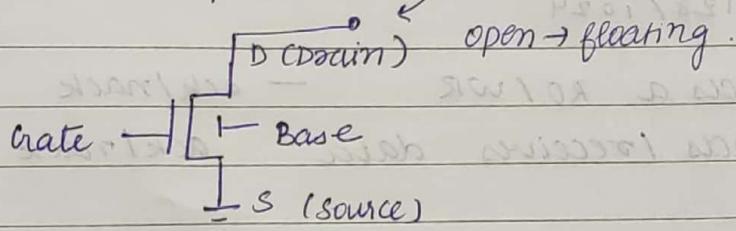
Date _____

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Select these 10's by pulling ^{external} extended pin high or low.

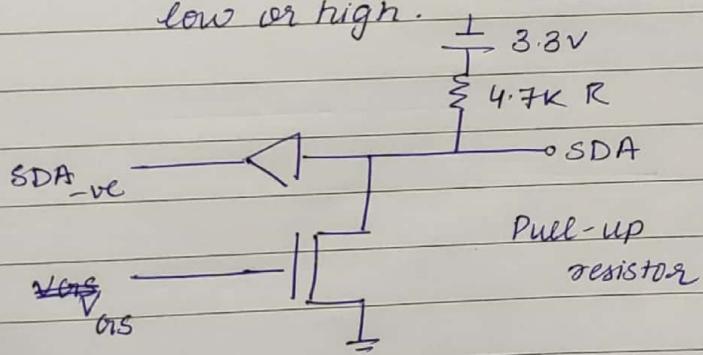


I²C uses open drain.



I²C uses open drain:

→ both master and slave are either low or high.



1^o:

1. Master is incharge of SCI frequency (MHz) $\hookrightarrow 3.4 \text{ MHz}$

100 KHz

400 KHz

2. Master generates the address of the slaves.

If _____ is ack/nack

128/1024

3. If sends a R/W/R — ack/nack

4. It sends /receives data — ack/nack

Sequence is as following :

Master is to write to a slave

a. Start → write → . $\rightarrow \text{reg}$

b. Master to read from slave

Start →

