Lecture - 22 P 1

Recap:

Exponential random variables

functions of random variables.

Syllabus for insem 2

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Jointly Distributed random variables

X	Y		
height	weight	attendance	marks
height	marks		
rainfall	cropyield		
age	cholestrol level		

Joint distribution function

$$F(a,b) = P\left(\begin{matrix} X \le a \; , \\ Y \le b \end{matrix} \right)$$

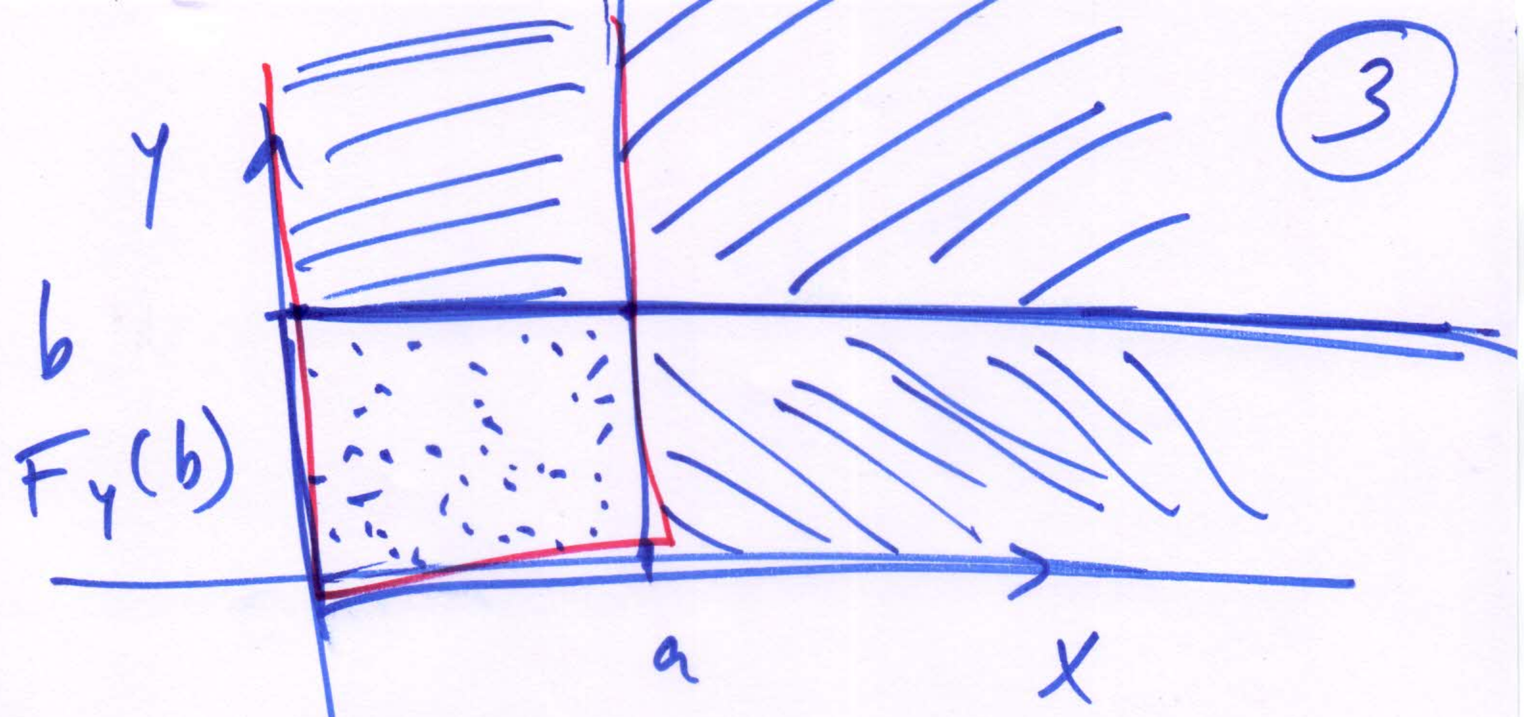
AND

$$F_X(a) = F(a, \infty) = \lim_{b \to \infty} F(a,b)$$

$$F_Y(b) = F(\infty, b) = \lim_{a \to \infty} F(a,b)$$

$$P(X > a, Y > b) =$$

$$1 - \left[\begin{matrix} F_X(a) - F(a,b) + \\ F_Y(b) - F(a,b) + \\ F(a,b) \end{matrix} \right]$$



$F(a,b)$

$F_X(a)$, $F_Y(b)$

$F(a,b) \rightarrow$

$F_Y(b) - F(a,b) \rightarrow$

$F_X(a) - F(a,b) \rightarrow$

e.g.: Consider a circle of radius R. Choose a point at random within the circle uniformly. Center is origin, Point (X,Y)

$$f(x,y) = \begin{cases} c & \text{if } \underline{x^2+y^2 < R^2} \\ 0 & \text{otherwise} \end{cases}$$

i) determine c

$$c \iint_{x^2+y^2<R^2} dx\,dy = 1$$

$$c = \frac{1}{\pi R^2}$$

$$f(x,y) = \begin{cases} \dfrac{1}{\pi R^2} & \text{if } x^2+y^2<R^2 \\ 0 & \text{otherwise} \end{cases}$$

b) <u>Marginal density</u> for X & Y.

$$f_X(x) = \int_{-\infty}^{\infty} f(x,y)\, dy$$

$$f_Y(y) = \int_{-\infty}^{\infty} f(x,y)\, dx$$

$$\cancel{\int_{-\infty}^{\infty} \frac{1}{\pi R^2} \cdot dy}$$

$$\cancel{= \frac{1}{\pi R^2} \int_{-\infty}^{\infty} dy}$$

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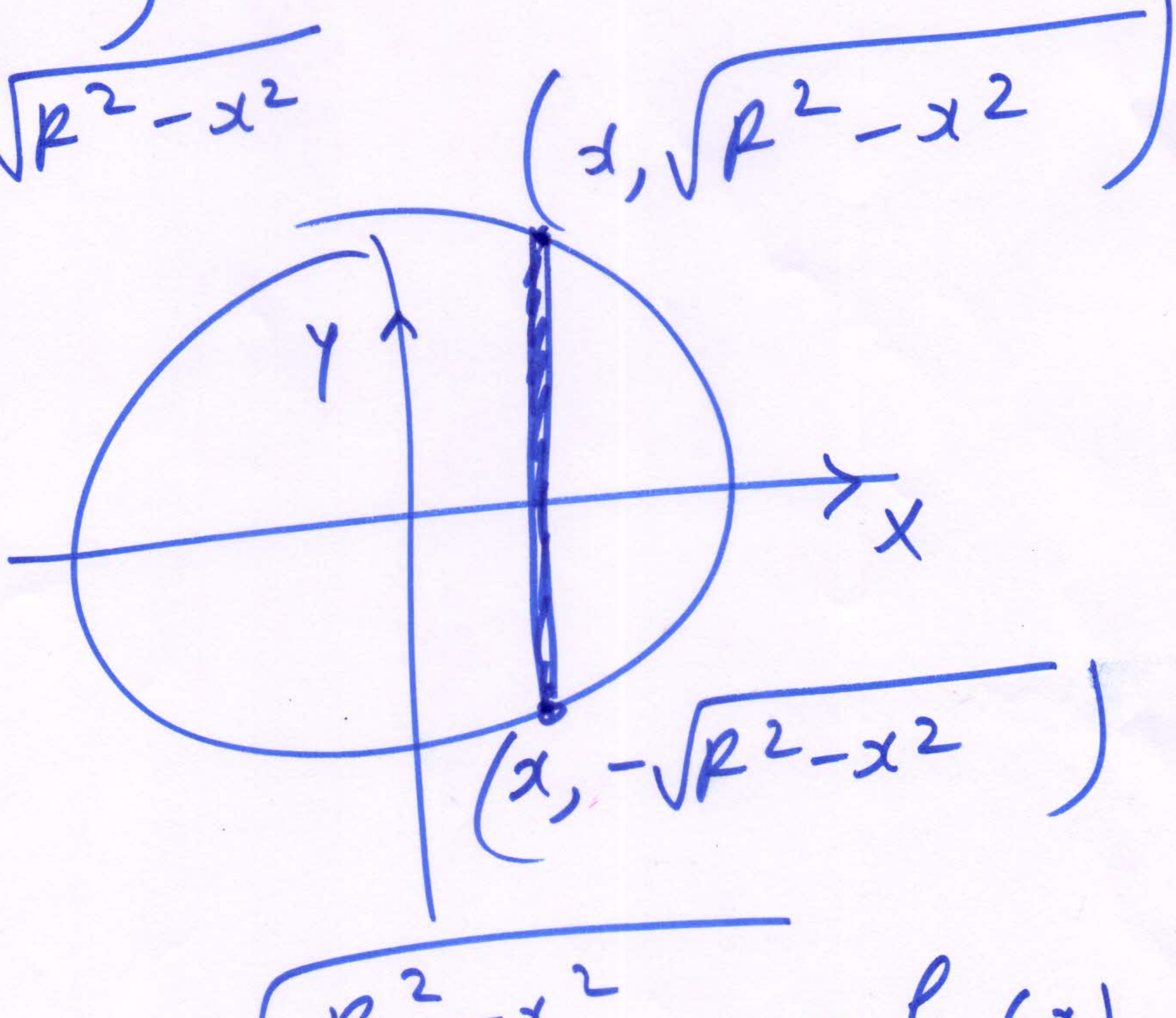
$$\int_{-\infty}^{\infty} f(x,y)\,dy$$

$$= \int_{-\sqrt{R^2-x^2}}^{\sqrt{R^2-x^2}} \frac{1}{\pi R^2} \cdot dy$$



$$= \frac{2\sqrt{R^2-x^2}}{\pi R^2} = f_X(x)$$

c) D is the distance of (X,Y) from the center. 7

$$P(D < a) =$$

$$P\left(\sqrt{x^2+y^2} < a\right)$$

$$\|$$

$$P(x^2+y^2 < a^2)$$

$$\|$$

$$\iint_{x^2+y^2<a^2} f(x,y)\,dx\,dy \overset{P(D<a)}{=}$$

$$\|$$

$$\frac{1}{\pi R^2}\cdot \pi a^2 = \frac{a^2}{R^2}$$

iv) $$E[D] = \int_{-\infty}^{\infty} p(D) \cdot D \cdot dD$$ ⑧

$$P(D < a) = \frac{a^2}{R^2}$$

$$f_D(a) = \frac{d}{da}\left(P(D<a)\right)$$

$$= \frac{d}{da}\left(\frac{a^2}{R^2}\right) = \frac{2a}{R^2}$$

$$E[D] = \int_0^R \frac{2a}{R^2} \cdot a \cdot da = \frac{2R}{3}$$

discrete, joint

e.g. A bag has
3 Red, 4 white, 5 Blue balls.
You randomly choose 3 balls.

X: no. of Red balls
Y: no. of White balls

Joint probability mass function

$P(X = x, Y = y)$

$$P(X = 1, Y = 1) = \frac{{}^{3}C_{1} \; {}^{4}C_{1} \; {}^{5}C_{1}}{{}^{12}C_{3}}$$

X \ Y	0	1	2	3	
0	10	40	30	4	84
1	30	60	18	0	108
2	15	12	0	0	27
3	1	0	0	0	1
	56	112	48	4	↓

marginal for X

$P(Y=2) = \frac{48}{220}$

$^{12}C_3$ = denominator $= 220$

$= \frac{\cancel{12}^{2} \cdot 11 \cdot 10}{\cancel{6}}$

$^4C_1 \cdot {}^5C_2$

$= 4 \cdot 10$

$= 40$

$^5C_3 = \frac{5 \cdot \cancel{4}^{2}}{\cancel{2}} = 10$

$^4C_2 \cdot {}^5C_1 = \frac{\cancel{4}^{2} \cdot 3 \cdot 5}{\cancel{2}}$

e.g. Continuous, joint

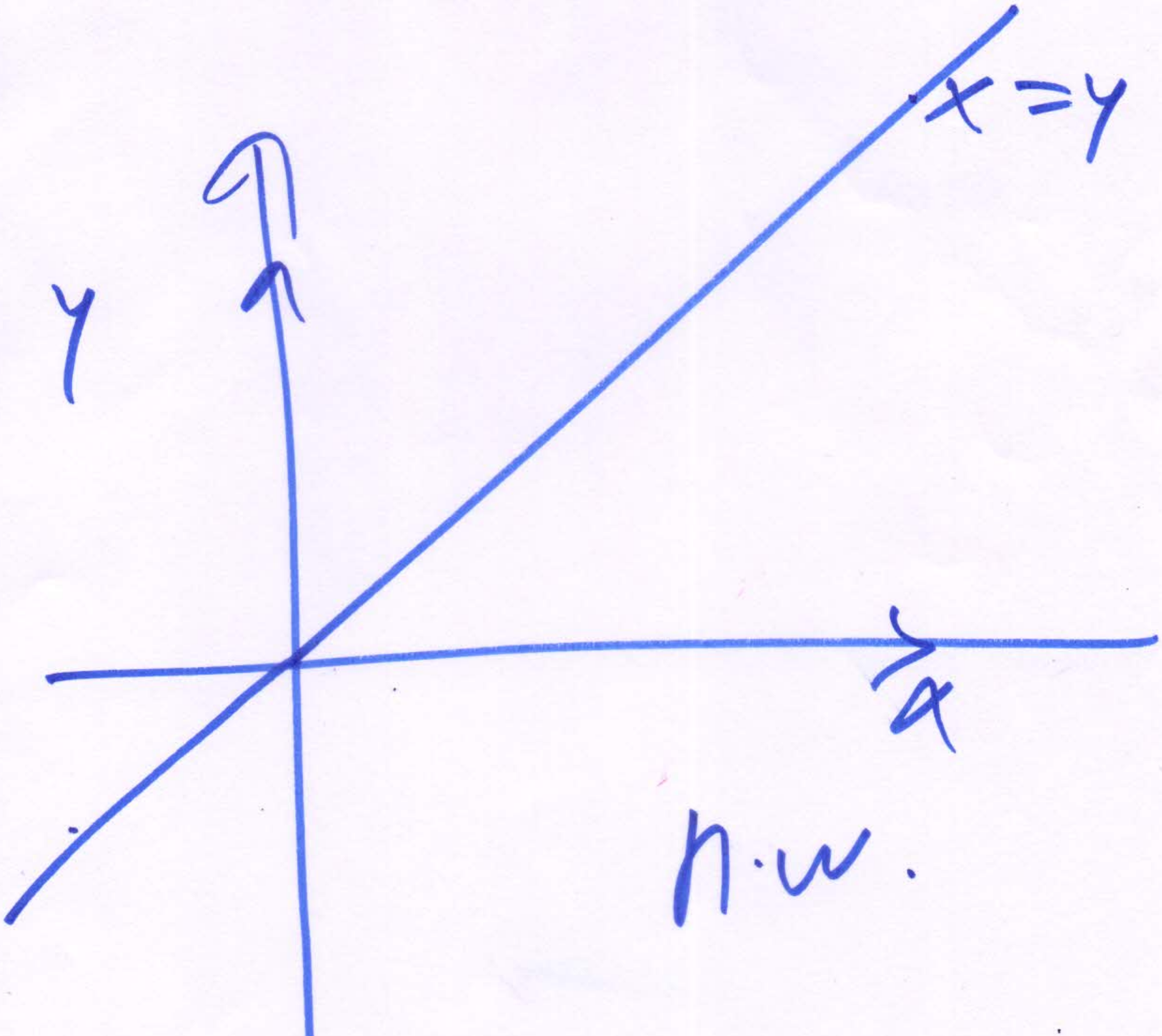
$$f(x,y) = \begin{cases} 2e^{-x}e^{-2y} \\ 0 < x < \infty \\ 0 < y < \infty \end{cases}$$

$P(X > 1 \text{ AND } Y < 1) =$

$$\int_{x=1}^{\infty}\int_{y=0}^{1} f(x,y)\,dx\,dy$$

$= e^{-1} - e^{-3}$

$$P(X < Y) = \iint f(x,y)\,dx\,dy$$



h.w.