# Lecture-3



## Recap:

3 axioms of probability

$$P(\phi) = 0$$

$$P(\bar{E}) = 1 - P(E)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

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$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$$

H.W.

# Sample Spaces with equally likely outcomes

e.g. Box has

6 white balls

5 black balls

draw 3 balls at random.

1 white

2 black

$w_1 w_2 w_3 \rightarrow$ outcome 1

$w_1 B_1 B_2 \rightarrow$ outcome 2

$B_1 B_2 w_6$

. . . . . outcome no. $^{11}C_3$

$$p = \frac{|E|}{|S|} = \frac{{}^6C_1 \, {}^5C_2}{{}^{11}C_3}$$

$$|E| = \binom{6}{1}\binom{5}{2}$$

e.g.

10 married couples, a total of 20 people. Choose 5 people at random. What is the probability that these 5 are not related to each other.

$$|S| = \binom{20}{5}$$

$C_1$ $C_2$ $C_3$ - $C_4$ $C_5$ $C_6$ $C_7$ $C_{10}$ $C_8$ $C_9$

Choose 5 couples from here.

$\binom{10}{5}$ $m_2 w_2 \rightarrow 1$, $m_5 w_5 \rightarrow 2$, $m_7 w_7 \rightarrow 3$, $m_8 w_8 \rightarrow 4$, $m_9 w_9 \rightarrow 5$

$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$

$\binom{10}{5} * 2^5 = |E|$

$$p = \frac{{}^{10}C_5 * 2^5}{{}^{20}C_5} = \frac{|E|}{|S|}$$

What is the probability of getting a straight in a hand of poker? (4)

no. of straight flushes

$$= 40$$

| | | | | |
|---|---|---|---|---|
| A | 2 | 3 | 4 | 5 |
| 2 | 3 | 4 | 5 | 6 |
| ⋮ | | | | |
| 10 | J | Q | K | A |

$10 * 4$ suits

$$\frac{40}{52C_5}$$

no. of straights.

$$\frac{10 * [4^5 - 4]}{52C_5} = p \text{ of getting a straight}$$

p( flush )

all of same suit.

1. Choose a suit.

$4_{C_1} = 4$

2. Choose any 5 cards

~~$13_5$~~ $13_{C_5}$ ways

$$\frac{4 * {}^{13}C_5 - 40}{{}^{52}C_5}$$

# Full House

(6)

2 denominations: ${}_{13}C_2 * 2$

$\left[ (8,8,8,K,K) \right]_2$

$(8,8,K,K,K)$

$8: {}_4C_3$

$K: {}_4C_2$

$$\frac{{}_{13}C_2 * 2 * {}_4C_2 * {}_4C_3}{{}_{52}C_5}$$

Full House, Straight, Flush, Straight Flush : H.W.

e.g. Birthday Paradox

There are $n$ people in a room. What is the probability that 2 or more people share the same birthday?

365 days in a year.

| $n$ | $p$ |
|---|---|
| 366 | 1 |
| 23 | 0.5 |
| 70 | 0.999 |