Assignment 7 (April 16)   Deadline: April 18

Part-1

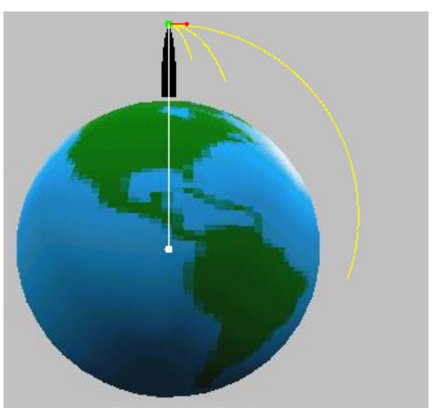
Gravitation (planetary motion) : computational investigation

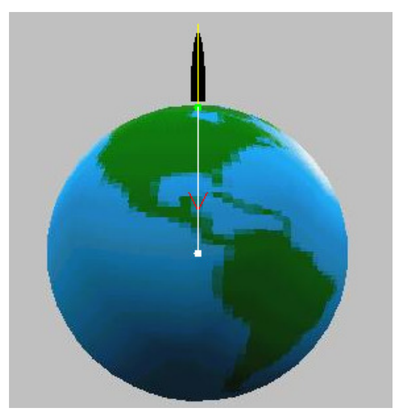
We have done some previous assignments, taking gravitation into account.

For example projectile motion….you can extend the same idea and you can easily understand satellite/planetary orbits!!

Any projectile launched from the surface of Earth is an Earth satellite (**only for a short time**). For example, a ball thrown with a given initial velocity and it sails in a modest orbit that soon intersects Earth not far from its point of launch.

And then you keep increasing the initial velocity, it would travel farther. Further increasing the speed would result in ever larger, rounder elliptical paths and more distant impact points (yellow curves – figure below). Finally, at one particular launch speed the ball would glide out just above Earth's surface all the way around to the other side without ever striking the ground. At a greater speed the orbit of the ball will be a circle.**(Refer figure below: left with finite initial "vx" and "vy"** velocity (red arrow)**, right with only finite "vy")**





1. Model: hypothetical solar system

Simplest situation; two body problem – **a sun and a single planet**. Goal – investigation of some of the properties of this model of solar system as discussed in class today. If planet is earth, goal is to calculate the position of Earth as a function of time.

The force in this problem? Assume a x-y co-ordinate system to describe the motion of earth in orbit around the sun. Sun is at the origin, Earth is located at → (x,y)s. Start with circular motion (assumption).

Write down the equations of motion in x,y taking components of the force. As usual 2 First order ODEs need to be converted into difference equations. (imp: direction of force !!)

Circular motion, so force must be equated to ?→as discussed in class.

Characteristics time and length scales ?. Choose time step accordingly. (Astronomical units!!; distance between earth and Sun, 1 year to complete the circular path).

(hint: Velocity of earth can be taken as 2*pi , if you use astronomical units (radius=1 AU, and time 1 year), you should arrive at following differential equations):

$$
\begin{aligned}
v_{x,i+1} &= v_{x,i} - \frac{4\pi^2 x_i}{r_i^3}\,\Delta t \\
x_{i+1} &= x_i + v_{x,i+1}\,\Delta t \\
v_{y,i+1} &= v_{y,i} - \frac{4\pi^2 y_i}{r_i^3}\,\Delta t \\
y_{i+1} &= y_i + v_{y,i+1}\,\Delta t\ ,
\end{aligned}
$$

Make a table listing orbital data for the planets in our solar system (Mass, Radius (distance from the sun), time period and eccentricity), take only 7 - leave mercury and pluto. Eccentricity tells the nature of the orbit; initially can be taken a fixed value such as ".02" for all (that tells why we left mercury and pluto).

Proper Choice of initial conditions is important as discussed during the class: (x,y,vx,vy).

Try with different initial velocities, and investigate the nature of the orbit.Plot the orbit in the x-y plane (use AU for x and y axes).

Use the model to calculate the period for the 7 planets and calculate the quantity (T^2/a^3), where "a" is the radius of the orbit (assuming circular motion) for each planet. Does it confirm Kepler's third law.

Time step choice is important: try .0001, .001 , .01, .1 years and see. What happens if you choose larger time steps (report)?

Part 2

2. Orbits

Introduction: For systems where one mass is significantly greater than the others, the motion of objects around the central mass can be simplified to motion around a fixed point. The type of motion can be subdivided into two main categories - orbits that are bounded and orbits that are unbounded.

For bounded orbits, two categories exist - circular and elliptical orbits. In both cases the orbiting body returns to something very close to the initial conditions. As discussed in class, the circular orbit exists for initial velocity vector pointing tangential to the line connecting the orbited and orbiting body with magnitude that balances 'centrifugal' acceleration (or acceleration of circular motion) with the acceleration caused by the mass of the orbited body. We need to look into the equations discussed today.For these conditions to be met the initial velocity magnitude of the orbiting body will be $v=(GM/R)^{1/2}$, where M is the orbited body, R is the distance between them, and G is the fundamental constant of gravity.

For the orbit to be elliptical, the same initial conditions apply exceptfor the initial velocity magnitude - this value must be less than or greater than that of a circular orbit, up to the limiting value of the velocity of a parabolic orbit, where the kinetic energy of motion is equal to the gravitational energy of gravity

$KE=PE \implies mv^2/2 = GMm/R \implies v(\text{parabolic})=(2GM/R)^{1/2}$.

Velocities greater than that of parabolic orbits are *hyperbolic*, and never return to their initial conditions. A parabolic orbit will return to its initial conditions when time = infinity.

We can now consider the elliptical orbits of the planets (Kepler's first law), and we assume that the Sun is stationary at one focus of the ellipse. Mass of sun is huge (how much compared to other planets? Already in the table you prepared, also look at the mass ratios) so that motion of the sun can be ignored.

Important force: force of attraction due to gravity. Direction of force?

Important points to be verified and reported using your computational implementation:

For elliptical orbits, the force due to gravity changes magnitude since the separation changes.

At the same time, what happens to the kinetic energy of the system as a function of time? (As the separation between the "Sun" and the "planet" changes, the gravitational potential energy of the system changes too. While the kinetic energy of the system changes, the sum of the kinetic energy and the potential energy of the system must remain a constant throughout the motion of the objects. Simulate it and make a plot to show the above.

Investigate the direction of velocity and acceleration in both the case (circular and eliptical) as well as magnitude. (you can use the radial and tangential directions to describe the motion of the planet)

Notice that between some points (along the orbit) the planet is speeding up, and between some other points the planet is slowing down. Report on it.

In the absence of a net external torque acting on a system, a particle's angular momentum remains constant. Does a particle sweep out equal areas in equal times (with respect to origin)? During equal time intervals, the radius vector from the sun to a planet sweeps out equal areas. What does this tell you about the angular momentum of the planets? What does this tell you about the motion of the planets/ planet's orbit?

Starting at t = 0, run the simulation for 2 years (not real time, simulation time!). How much area has been swept out by the planet in this time interval? What about from 2 to 4 years? Does it matter where you are in the orbit? Report your computational observations and comment on conservation of angular momentum. (Here the only force on the planet is gravity, and gravity cannot create a torque because the radius arm and the force are always in the same direction.)