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# Chapter 12

## The Space Link

### 12.1 Introduction

This chapter describes how the link-power budget calculations are made. These calculations basically relate two quantities, the transmit power and the receive power, and show in detail how the difference between these two powers is accounted for.

Link-budget calculations are usually made using decibel or decilog quantities. These are explained in App. G. In this text [square] brackets are used to denote decibel quantities using the basic power definition. Where no ambiguity arises regarding the units, the abbreviation dB is used. For example, Boltzmann's constant is given as  $-228.6$  dB, although, strictly speaking, this should be given as  $-228.6$  decilog relative to  $1$  J/K. Where it is desirable to show the reference unit, this is indicated in the abbreviation, for example, dBHz means decibels relative to  $1$  Hz.

### 12.2 Equivalent Isotropic Radiated Power

A key parameter in link-budget calculations is the *equivalent isotropic radiated power*, conventionally denoted as EIRP. From Eqs. (6.4) and (6.5), the maximum power flux density at some distance  $r$  from a transmitting antenna of gain  $G$  is

$$\Psi_M = \frac{GP_S}{4\pi r^2} \quad (12.1)$$

An isotropic radiator with an input power equal to  $GP_S$  would produce the same flux density. Hence, this product is referred to as the EIRP, or

$$\text{EIRP} = GP_S \quad (12.2)$$

EIRP is often expressed in decibels relative to 1 W, or dBW. Let  $P_s$  be in watts; then

$$[\text{EIRP}] = [P_s] + [G] \text{ dBW} \quad (12.3)$$

where  $[P_s]$  is also in dBW and  $[G]$  is in dB.

**Example 12.1** A satellite downlink at 12 GHz operates with a transmit power of 6 W and an antenna gain of 48.2 dB. Calculate the EIRP in dBW.

**Solution**

$$\begin{aligned} [\text{EIRP}] &= 10 \log \left( \frac{6\text{W}}{1\text{W}} \right) + 48.2 \\ &= \underline{56 \text{ dBW}} \end{aligned}$$

For a paraboloidal antenna, the isotropic power gain is given by Eq. (6.32). This equation may be rewritten in terms of frequency, since this is the quantity which is usually known.

$$G = \eta(10.472fD)^2 \quad (12.4)$$

where  $f$  is the carrier frequency in GHz,  $D$  is the reflector diameter in m, and  $\eta$  is the aperture efficiency. A typical value for aperture efficiency is 0.55, although values as high as 0.73 have been specified (Andrew Antenna, 1985).

With the diameter  $D$  in feet and all other quantities as before, the equation for power gain becomes

$$G = \eta(3.192fD)^2 \quad (12.5)$$

**Example 12.2** Calculate the gain in decibels of a 3-m paraboloidal antenna operating at a frequency of 12 GHz. Assume an aperture efficiency of 0.55.

**Solution**

$$G = 0.55 \times (10.472 \times 12 \times 3)^2 \cong 78168$$

Hence,

$$[G] = 10 \log 78168 = 48.9 \text{ dB}$$

## 12.3 Transmission Losses

The  $[\text{EIRP}]$  may be thought of as the power input to one end of the transmission link, and the problem is to find the power received at the other end. Losses will occur along the way, some of which are constant.

Other losses can only be estimated from statistical data, and some of these are dependent on weather conditions, especially on rainfall.

The first step in the calculations is to determine the losses for *clear-weather* or *clear-sky conditions*. These calculations take into account the losses, including those calculated on a statistical basis, which do not vary significantly with time. Losses which are weather-related, and other losses which fluctuate with time, are then allowed for by introducing appropriate *fade margins* into the transmission equation.

### 12.3.1 Free-space transmission

As a first step in the loss calculations, the power loss resulting from the spreading of the signal in space must be determined. This calculation is similar for the uplink and the downlink of a satellite circuit. Using Eqs. (12.1) and (12.2) gives the power-flux density at the receiving antenna as

$$\Psi_M = \frac{\text{EIRP}}{4\pi r^2} \quad (12.6)$$

The power delivered to a matched receiver is this power-flux density multiplied by the effective aperture of the receiving antenna, given by Eq. (6.15). The received power is therefore

$$\begin{aligned} P_R &= \Psi_M A_{\text{eff}} \\ &= \frac{\text{EIRP}}{4\pi r^2} \frac{\lambda^2 G_R}{4\pi} \\ &= (\text{EIRP})(G_R) \left( \frac{\lambda}{4\pi r} \right)^2 \end{aligned} \quad (12.7)$$

Recall that  $r$  is the distance, or range, between the transmit and receive antennas and  $G_R$  is the isotropic power gain of the receiving antenna. The subscript  $R$  is used to identify the receiving antenna.

The right-hand side of Eq. (12.7) is separated into three terms associated with the transmitter, receiver, and free space, respectively. In decibel notation, the equation becomes

$$[P_R] = [\text{EIRP}] + [G_R] - 10 \log \left( \frac{4\pi r}{\lambda} \right)^2 \quad (12.8)$$

The received power in dBW is therefore given as the sum of the transmitted EIRP in dBW plus the receiver antenna gain in dB minus a third term, which represents the free-space loss in decibels. The free-space loss component in decibels is given by

$$[\text{FSL}] = 10 \log \left( \frac{4\pi r}{\lambda} \right)^2 \quad (12.9)$$

Normally, the frequency rather than wavelength will be known, and the substitution  $\lambda = c/f$  can be made, where  $c = 10^8$  m/s. With frequency in megahertz and distance in kilometers, it is left as an exercise for the student to show that the free-space loss is given by

$$[\text{FSL}] = 32.4 + 20 \log r + 20 \log f \quad (12.10)$$

Equation (12.8) can then be written as

$$[P_R] = [\text{EIRP}] + [G_R] - [\text{FSL}] \quad (12.11)$$

The received power  $[P_R]$  will be in dBW when the  $[\text{EIRP}]$  is in dBW, and  $[\text{FSL}]$  in dB. Equation (12.9) is applicable to both the uplink and the downlink of a satellite circuit, as will be shown in more detail shortly.

**Example 12.3** The range between a ground station and a satellite is 42,000 km. Calculate the free-space loss at a frequency of 6 GHz.

**Solution**

$$[\text{FSL}] = 32.4 + 20 \log 42,000 + 20 \log 6000 = \underline{200.4 \text{ dB}}$$

This is a very large loss. Suppose that the  $[\text{EIRP}]$  is 56 dBW (as calculated in Example 12.1 for a radiated power of 6 W) and the receive antenna gain is 50 dB. The receive power would be  $56 + 50 - 200.4 = -94.4$  dBW. This is 355 pW. It also may be expressed as  $-64.4$  dBm, which is 64.4 dB below the 1-mW reference level.

Equation (12.11) shows that the received power is increased by increasing antenna gain as expected, and Eq. (6.32) shows that antenna gain is inversely proportional to the square of the wavelength. Hence, it might be thought that increasing the frequency of operation (and therefore decreasing wavelength) would increase the received power. However, Eq. (12.9) shows that the free-space loss is also inversely proportional to the square of the wavelength, so these two effects cancel. It follows, therefore, that for a constant EIRP, the received power is independent of frequency of operation.

If the transmit power is a specified constant, rather than the EIRP, then the received power will increase with increasing frequency for given antenna dish sizes at the transmitter and receiver. It is left as an exercise for the student to show that under these conditions the received power is directly proportional to the square of the frequency.

### 12.3.2 Feeder losses

Losses will occur in the connection between the receive antenna and the receiver proper. Such losses will occur in the connecting waveguides, filters, and couplers. These will be denoted by RFL, or  $[\text{RFL}]$  dB, for *receiver*

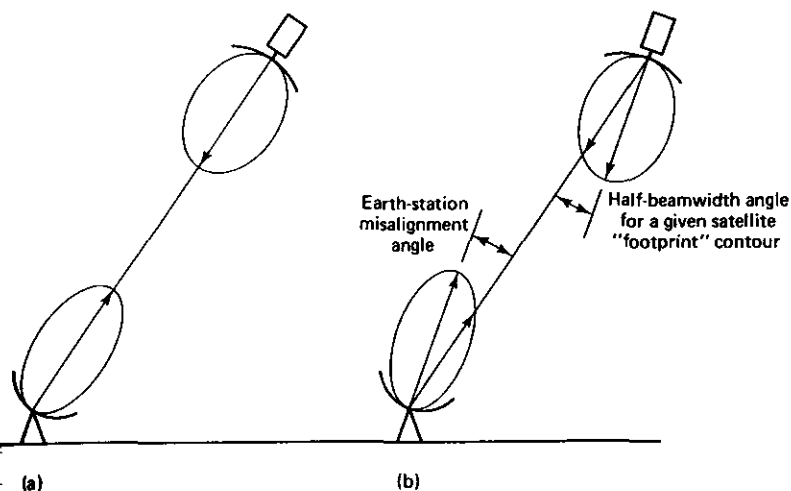
feeder losses. The [RFL] values are added to [FSL] in Eq. (12.11). Similar losses will occur in the filters, couplers, and waveguides connecting the transmit antenna to the *high-power amplifier* (HPA) output. However, provided that the EIRP is stated, Eq. (12.11) can be used without knowing the transmitter feeder losses. These are needed only when it is desired to relate EIRP to the HPA output, as described in Secs. 12.7.4 and 12.8.2.

### 12.3.3 Antenna misalignment losses

When a satellite link is established, the ideal situation is to have the earth station and satellite antennas aligned for maximum gain, as shown in Fig. 12.1a. There are two possible sources of off-axis loss, one at the satellite and one at the earth station, as shown in Fig. 12.1b.

The off-axis loss at the satellite is taken into account by designing the link for operation on the actual satellite antenna contour; this is described in more detail in later sections. The off-axis loss at the earth station is referred to as the *antenna pointing loss*. Antenna pointing losses are usually only a few tenths of a decibel; typical values are given in Table 12.1.

In addition to pointing losses, losses may result at the antenna from misalignment of the polarization direction (these are in addition to the polarization losses described in Chap. 5). The polarization misalignment losses are usually small, and it will be assumed that the antenna misalignment losses, denoted by [AML], include both pointing and polarization losses resulting from antenna misalignment. It should be noted



12.1 (a) Satellite and earth-station antennas aligned for maximum gain; (b) earth station situated on a given satellite "footprint," and earth-station antenna misaligned.

**TABLE 12.1 Atmospheric Absorption Loss and Satellite Pointing Loss for Cities and Communities in the Province of Ontario**

Location	Atmospheric absorption dB, summer	Satellite antenna pointing loss, dB	
		$\frac{1}{4}$ Canada coverage	$\frac{1}{2}$ Canada coverage
Cat Lake	0.2	0.5	0.5
Fort Severn	0.2	0.9	0.9
Geraldton	0.2	0.2	0.1
Kingston	0.2	0.5	0.4
London	0.2	0.3	0.6
North Bay	0.2	0.3	0.2
Ogoki	0.2	0.4	0.3
Ottawa	0.2	0.6	0.2
Sault Ste. Marie	0.2	0.1	0.3
Sioux Lookout	0.2	0.4	0.3
Sudbury	0.2	0.3	0.2
Thunder Bay	0.2	0.3	0.2
Timmins	0.2	0.5	0.2
Toronto	0.2	0.3	0.4
Windsor	0.2	0.5	0.8

SOURCE: Telesat Canada Design Workbook.

that the antenna misalignment losses have to be estimated from statistical data, based on the errors actually observed for a large number of earth stations, and of course, the separate antenna misalignment losses for the uplink and the downlink must be taken into account.

#### 12.3.4 Fixed atmospheric and ionospheric losses

Atmospheric gases result in losses by absorption, as described in Sec. 4.2 and by Eq. (4.1). These losses usually amount to a fraction of a decibel, and in subsequent calculations, the decibel value will be denoted by [AA]. Values obtained for some locations in the Province of Ontario, Canada, are shown in Table 12.1. Also, as discussed in Sec. 5.5, the ionosphere introduces a depolarization loss given by Eq. (5.19), and in subsequent calculations, the decibel value for this will be denoted by [PL].

#### 12.4 The Link-Power Budget Equation

As mentioned at the beginning of Sec. 12.3, the [EIRP] can be considered as the input power to a transmission link. Now that the losses for the link have been identified, the power at the receiver, which is the power output of the link, may be calculated simply as [EIRP] - [LOSSES] +  $[G_R]$ , where the last quantity is the receiver antenna gain. Note carefully that decibel addition must be used.

The major source of loss in any ground-satellite link is the free-space spreading loss [FSL], as shown in Sec. 12.3.1, where Eq. (12.13) is the basic link-power budget equation taking into account this loss only. However, the other losses also must be taken into account, and these are simply added to [FSL]. The losses for clear-sky conditions are

$$[\text{LOSSES}] = [\text{FSL}] + [\text{RFL}] + [\text{AML}] + [\text{AA}] + [\text{PL}] \quad (12.12)$$

The decibel equation for the received power is then

$$[P_R] = [\text{EIRP}] + [G_R] - [\text{LOSSES}] \quad (12.13)$$

where  $[P_R]$  = received power, dBW

$[\text{EIRP}]$  = equivalent isotropic radiated power, dBW

$[\text{FSL}]$  = free-space spreading loss, dB

$[\text{RFL}]$  = receiver feeder loss, dB

$[\text{AML}]$  = antenna misalignment loss, dB

$[\text{AA}]$  = atmospheric absorption loss, dB

$[\text{PL}]$  = polarization mismatch loss, dB

**Example 12.4** A satellite link operating at 14 GHz has receiver feeder losses of 1.5 dB and a free-space loss of 207 dB. The atmospheric absorption loss is 0.5 dB, and the antenna pointing loss is 0.5 dB. Depolarization losses may be neglected. Calculate the total link loss for clear-sky conditions.

**Solution** The total link loss is the sum of all the losses:

$$\begin{aligned} [\text{LOSSES}] &= [\text{FSL}] + [\text{RFL}] + [\text{AA}] + [\text{AML}] \\ &= 207 + 1.5 + 0.5 + 0.5 \\ &= \underline{209.5 \text{ dB}} \end{aligned}$$

## 12.5 System Noise

It is shown in Sec. 12.3 that the receiver power in a satellite link is very small, on the order of picowatts. This by itself would be no problem because amplification could be used to bring the signal strength up to an acceptable level. However, electrical noise is always present at the input, and unless the signal is significantly greater than the noise, amplification will be of no help because it will amplify signal and noise to the same extent. In fact, the situation will be worsened by the noise added by the amplifier.

The major source of electrical noise in equipment is that which arises from the random thermal motion of electrons in various resistive and active devices in the receiver. Thermal noise is also generated in the

lossy components of antennas, and thermal-like noise is picked up by the antennas as radiation.

The available noise power from a thermal noise source is given by

$$P_N = kT_N B_N \quad (12.14)$$

Here,  $T_N$  is known as the equivalent noise temperature,  $B_N$  is the equivalent noise bandwidth, and  $k = 1.38 \times 10^{-23}$  J/K is Boltzmann's constant. With the temperature in kelvins and bandwidth in hertz, the noise power will be in watts. The noise power bandwidth is always wider than the -3-dB bandwidth determined from the amplitude-frequency response curve, and a useful rule of thumb is that the noise bandwidth is equal to 1.12 times the -3-dB bandwidth, or  $B_N \approx 1.12 \times B_{-3\text{dB}}$ . The bandwidths here are in hertz (or a multiple such as MHz).

The main characteristic of thermal noise is that it has a *flat frequency spectrum*; that is, the noise power per unit bandwidth is a constant. The noise power per unit bandwidth is termed the *noise power spectral density*. Denoting this by  $N_0$ , then from Eq. (12.14),

$$N_0 = \frac{P_N}{B_N} = kT_N \text{ J} \quad (12.15)$$

The noise temperature is directly related to the physical temperature of the noise source but is not always equal to it. This is discussed more fully in the following sections. The noise temperatures of various sources which are connected together can be added directly to give the total noise.

**Example 12.5** An antenna has a noise temperature of 35 K and is matched into a receiver which has a noise temperature of 100 K. Calculate (a) the noise power density and (b) the noise power for a bandwidth of 36 MHz.

**Solution**

$$(a) N_0 = (35 + 100) \times 1.38 \times 10^{-23} = \underline{1.86 \times 10^{-21} \text{ J}}$$

$$(b) P_N = 1.86 \times 10^{-21} \times 36 \times 10^6 = \underline{0.067 \text{ pW}}$$

In addition to these thermal noise sources, intermodulation distortion in high-power amplifiers (see Sec. 12.7.3) can result in signal products which appear as noise and in fact is referred to as *intermodulation noise*. This is discussed in Sec. 12.10.

### 12.5.1 Antenna noise

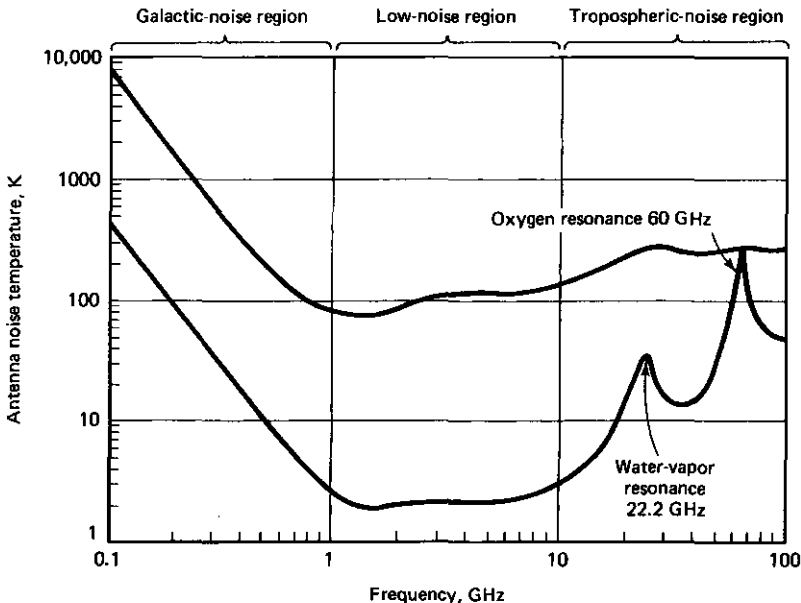
Antennas operating in the receiving mode introduce noise into the satellite circuit. Noise therefore will be introduced by the satellite receive antenna and the ground station receive antenna. Although the physical



origins of the noise in either case are similar, the magnitudes of the effects differ significantly.

The antenna noise can be broadly classified into two groups: noise originating from antenna losses and *sky noise*. Sky noise is a term used to describe the microwave radiation which is present throughout the universe and which appears to originate from matter in any form at finite temperatures. Such radiation in fact covers a wider spectrum than just the microwave spectrum. The equivalent noise temperature of the sky, as seen by an earth-station antenna, is shown in Fig. 12.2. The lower graph is for the antenna pointing directly overhead, while the upper graph is for the antenna pointing just above the horizon. The increased noise in the latter case results from the thermal radiation of the earth, and this in fact sets a lower limit of about  $5^\circ$  at C band and  $10^\circ$  at Ku band on the elevation angle which may be used with ground-based antennas.

The graphs show that at the low-frequency end of the spectrum, the noise decreases with increasing frequency. Where the antenna is zenith-pointing, the noise temperature falls to about 3 K at frequencies between



**Figure 12.2** Irreducible noise temperature of an ideal, ground-based antenna. The antenna is assumed to have a very narrow beam without sidelobes or electrical losses. Below 1 GHz, the maximum values are for the beam pointed at the galactic poles. At higher frequencies, the maximum values are for the beam just above the horizon and the minimum values for zenith pointing. The low-noise region between 1 and 10 GHz is most amenable to application of special, low-noise antennas. (From Philip F. Panter, "Communications Systems Design," McGraw-Hill Book Company, New York, 1972. With permission.)

about 1 and 10 GHz. This represents the residual background radiation in the universe. Above about 10 GHz, two peaks in temperature are observed, resulting from resonant losses in the earth's atmosphere. These are seen to coincide with the peaks in atmospheric absorption loss shown in Fig. 4.2.

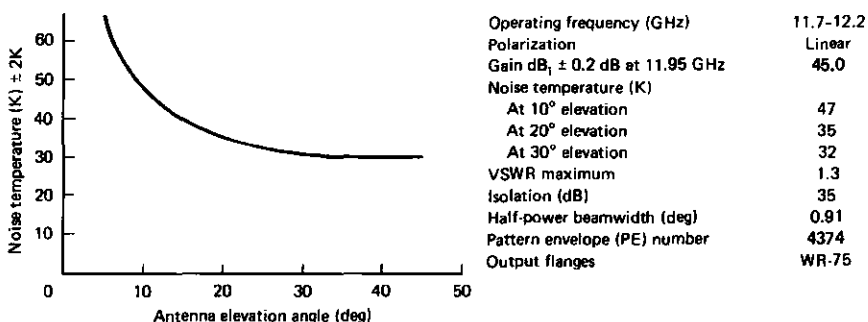
Any absorptive loss mechanism generates thermal noise, there being a direct connection between the loss and the effective noise temperature, as shown in Sec. 12.5.5. Rainfall introduces attenuation, and therefore, it degrades transmissions in two ways: It attenuates the signal, and it introduces noise. The detrimental effects of rain are much worse at Ku-band frequencies than at C band, and the downlink rain-fade margin, discussed in Sec. 12.9.2, must also allow for the increased noise generated.

Figure 12.2 applies to ground-based antennas. Satellite antennas are generally pointed toward the earth, and therefore, they receive the full thermal radiation from it. In this case the equivalent noise temperature of the antenna, excluding antenna losses, is approximately 290 K.

Antenna losses add to the noise received as radiation, and the total antenna noise temperature is the sum of the equivalent noise temperatures of all these sources. For large ground-based C-band antennas, the total antenna noise temperature is typically about 60 K, and for the Ku band, about 80 K under clear-sky conditions. These values do not apply to any specific situation and are quoted merely to give some idea of the magnitudes involved. Figure 12.3 shows the noise temperature as a function of angle of elevation for a 1.8-m antenna operating in the Ku band.

### 12.5.2 Amplifier noise temperature

Consider first the noise representation of the antenna and the *low noise amplifier* (LNA) shown in Fig. 12.4a. The available power gain of the amplifier is denoted as  $G$ , and the noise power output, as  $P_{no}$ . For the



**Figure 12.3** Antenna noise temperature as a function of elevation for 1.8-m antenna characteristics. (Andrew Bulletin 1206; courtesy of Andrew Antenna Company, Limited.)

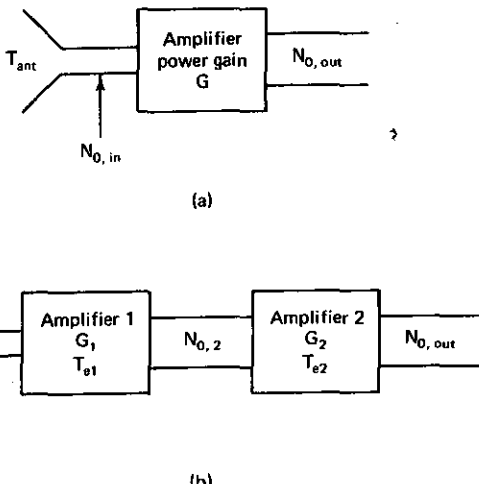


Figure 12.4 Circuit used in finding equivalent noise temperature of (a) an amplifier and (b) two amplifiers in cascade.

moment we will work with the noise power per unit bandwidth, which is simply noise energy in joules as shown by Eq. (12.15). The input noise energy coming from the antenna is

$$N_{0,\text{ant}} = kT_{\text{ant}} \quad (12.16)$$

The output noise energy  $N_{0,\text{out}}$  will be  $GN_{0,\text{ant}}$  plus the contribution made by the amplifier. Now all the amplifier noise, wherever it occurs in the amplifier, may be *referred to the input* in terms of an equivalent input noise temperature for the amplifier  $T_e$ . This allows the output noise to be written as

$$N_{0,\text{out}} = Gk(T_{\text{ant}} + T_e) \quad (12.17)$$

The total noise referred to the input is simply  $N_{0,\text{out}}/G$ , or

$$N_{0,\text{in}} = k(T_{\text{ant}} + T_e) \quad (12.18)$$

$T_e$  can be obtained by measurement, a typical value being in the range 35 to 100 K. Typical values for  $T_{\text{ant}}$  are given in Sec. 12.5.1.

### 12.5.3 Amplifiers in cascade

The cascade connection is shown in Fig. 12.4b. For this arrangement, the overall gain is

$$G = G_1 G_2 \quad (12.19)$$

The noise energy of amplifier 2 referred to its own input is simply  $kT_{e2}$ . The noise input to amplifier 2 from the preceding stages is  $G_1k(T_{\text{ant}} + T_{e1})$ , and thus the total noise energy *referred to amplifier 2 input* is

$$N_{0,2} = G_1k(T_{\text{ant}} + T_{e1}) + kT_{e2} \quad (12.20)$$

This noise energy may be referred to amplifier 1 input by dividing by the available power gain of amplifier 1:

$$\begin{aligned} N_{0,1} &= \frac{N_{0,2}}{G_1} \\ &= k\left(T_{\text{ant}} + T_{e1} + \frac{T_{e2}}{G_1}\right) \end{aligned} \quad (12.21)$$

A system noise temperature may now be defined as  $T_S$  by

$$N_{0,1} = kT_S \quad (12.22)$$

and hence it will be seen that  $T_S$  is given by

$$T_S = T_{\text{ant}} + T_{e1} + \frac{T_{e2}}{G_1} \quad (12.23)$$

This is a very important result. It shows that the noise temperature of the second stage is divided by the power gain of the first stage when referred to the input. Therefore, in order to keep the overall system noise as low as possible, the first stage (usually an LNA) should have high power gain as well as low noise temperature.

This result may be generalized to any number of stages in cascade, giving

$$T_S = T_{\text{ant}} + T_{e1} + \frac{T_{e2}}{G_1} + \frac{T_{e3}}{G_1G_2} + \dots \quad (12.24)$$

### 12.5.4 Noise factor

An alternative way of representing amplifier noise is by means of its *noise factor*,  $F$ . In defining the noise factor of an amplifier, the source is taken to be at *room temperature*, denoted by  $T_0$ , usually taken as 290 K. The input noise from such a source is  $kT_0$ , and the output noise from the amplifier is

$$N_{0,\text{out}} = FGkT_0 \quad (12.25)$$

Here,  $G$  is the available power gain of the amplifier as before, and  $F$  is its noise factor.

A simple relationship between noise temperature and noise factor can be derived. Let  $T_e$  be the noise temperature of the amplifier, and let the source be at room temperature as required by the definition of  $F$ . This means that  $T_{\text{ant}} = T_0$ . Since the same noise output must be available whatever the representation, it follows that

$$Gk(T_0 + T_e) = FGkT_0$$

or

$$T_e = (F - 1) T_0 \quad (12.26)$$

This shows the direct equivalence between noise factor and noise temperature. As a matter of convenience, in a practical satellite receiving system, noise temperature is specified for low-noise amplifiers and converters, while noise factor is specified for the main receiver unit.

The *noise figure* is simply  $F$  expressed in decibels:

$$\text{Noise figure} = [F] = 10 \log F \quad (12.27)$$

**Example 12.6** An LNA is connected to a receiver which has a noise figure of 12 dB. The gain of the LNA is 40 dB, and its noise temperature is 120 K. Calculate the overall noise temperature referred to the LNA input.

**Solution** 12 dB is a power ratio of 15.85:1, and therefore,

$$T_{e2} = (15.85 - 1) \times 290 = 4306 \text{ K}$$

A gain of 40 dB is a power ratio of  $10^4$ :1, and therefore,

$$\begin{aligned} T_{\text{in}} &= 120 + \frac{4306}{10^4} \\ &= \underline{120.43 \text{ K}} \end{aligned}$$

In Example 12.6 it will be seen that the decibel quantities must be converted to power ratios. Also, even though the main receiver has a very high noise temperature, its effect is made negligible by the high gain of the LNA.

### 12.5.5 Noise temperature of absorptive networks

An *absorptive network* is one which contains resistive elements. These introduce losses by absorbing energy from the signal and converting it to heat. Resistive attenuators, transmission lines, and waveguides are all examples of absorptive networks, and even rainfall, which absorbs energy from radio signals passing through it, can be considered a form

of absorptive network. Because an absorptive network contains resistance, it generates thermal noise.

Consider an absorptive network, which has a power loss  $L$ . The power loss is simply the ratio of input power to output power and will always be greater than unity. Let the network be matched at both ends, to a terminating resistor,  $R_T$ , at one end and an antenna at the other, as shown in Fig. 12.5, and let the system be at some ambient temperature  $T_x$ . The noise energy transferred from  $R_T$  into the network is  $kT_x$ . Let the network noise be represented at the output terminals (the terminals connected to the antenna in this instance) by an equivalent noise temperature  $T_{NW,0}$ . Then the noise energy radiated by the antenna is

$$N_{\text{rad}} = \frac{kT_x}{L} + kT_{NW,0} \quad (12.28)$$

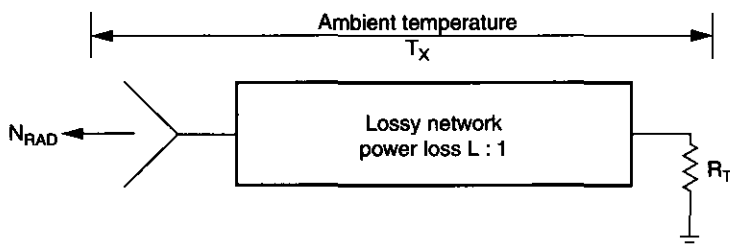
Because the antenna is matched to a resistive source at temperature  $T_x$ , the available noise energy which is fed into the antenna and radiated is  $N_{\text{rad}} = kT_x$ . Keep in mind that the antenna resistance to which the network is matched is fictitious, in the sense that it represents radiated power, but it does not generate noise power. This expression for  $N_{\text{rad}}$  can be substituted into Eq. (12.28) to give

$$T_{NW,0} = T_x \left( 1 - \frac{1}{L} \right) \quad (12.29)$$

This is the equivalent noise temperature of the network referred to the output terminals of the network. The equivalent noise at the output can be transferred to the input on dividing by the network power gain, which by definition is  $1/L$ . Thus, the equivalent noise temperature of the network referred to the network input is

$$T_{NW,i} = T_x(L - 1) \quad (12.30)$$

Since the network is bilateral, Eqs. (12.29) and (12.30) apply for signal flow in either direction. Thus, Eq. (12.30) gives the equivalent noise



**Figure 12.5** Network matched at both ends, to a terminating resistor  $R_T$  at one end and an antenna at the other.

temperature of a lossy network referred to the input at the antenna when the antenna is used in receiving mode.

If the lossy network should happen to be at room temperature, that is,  $T_x = T_0$ , then a comparison of Eqs. (12.26) and (12.30) shows that

$$F = L \quad (12.31)$$

This shows that at room temperature the noise factor of a lossy network is equal to its power loss.

### 12.5.6 Overall system noise temperature

Figure 12.6a shows a typical receiving system. Applying the results of the previous sections yields, for the system noise temperature referred to the input,

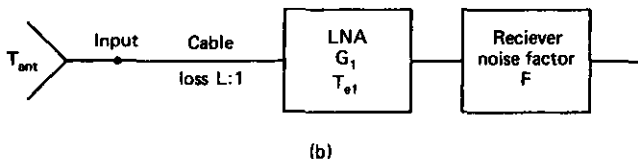
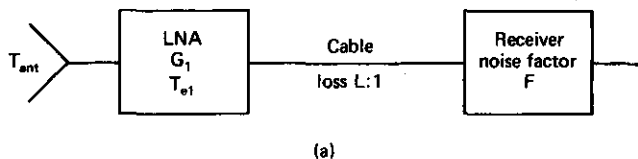
$$T_S = T_{\text{ant}} + T_{e1} + \frac{(L-1)T_0}{G_1} + \frac{L(F-1)T_0}{G_1} \quad (12.32)$$

The significance of the individual terms is illustrated in the following examples.

**Example 12.7** For the system shown in Fig. 12.6a, the receiver noise figure is 12 dB, the cable loss is 5 dB, the LNA gain is 50 dB, and its noise temperature 150 K. The antenna noise temperature is 35 K. Calculate the noise temperature referred to the input.

**Solution** For the main receiver,  $F = 10^{1.2} = 15.85$ . For the cable,  $L = 10^{0.5} = 3.16$ . For the LNA,  $G = 10^5$ . Hence,

$$T_S = 35 + 150 + \frac{(3.16 - 1) \times 290}{10^5} + \frac{3.16 \times (15.85 - 1) \times 290}{10^5} \\ \cong \underline{185 \text{ K}}$$



**Figure 12.6** Connections used in examples illustrating overall noise temperature of system, Sec. 12.5.6.

temperature of a lossy network referred to the input at the antenna when the antenna is used in receiving mode.

If the lossy network should happen to be at room temperature, that is,  $T_x = T_0$ , then a comparison of Eqs. (12.26) and (12.30) shows that

$$F = L \quad (12.31)$$

This shows that at room temperature the noise factor of a lossy network is equal to its power loss.

### 12.5.6 Overall system noise temperature

Figure 12.6a shows a typical receiving system. Applying the results of the previous sections yields, for the system noise temperature referred to the input,

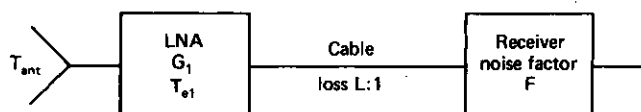
$$T_S = T_{\text{ant}} + T_{e1} + \frac{(L - 1)T_0}{G_1} + \frac{L(F - 1)T_0}{G_1} \quad (12.32)$$

The significance of the individual terms is illustrated in the following examples.

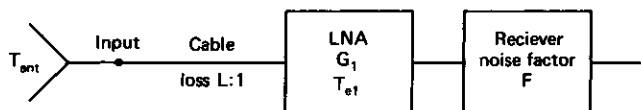
**Example 12.7** For the system shown in Fig. 12.6a, the receiver noise figure is 12 dB, the cable loss is 5 dB, the LNA gain is 50 dB, and its noise temperature 150 K. The antenna noise temperature is 35 K. Calculate the noise temperature referred to the input.

**Solution** For the main receiver,  $F = 10^{1.2} = 15.85$ . For the cable,  $L = 10^{0.5} = 3.16$ . For the LNA,  $G = 10^5$ . Hence,

$$T_S = 35 + 150 + \frac{(3.16 - 1) \times 290}{10^5} + \frac{3.16 \times (15.85 - 1) \times 290}{10^5} \\ \approx \underline{185 \text{ K}}$$



(a)



(b)

**Figure 12.6** Connections used in examples illustrating overall noise temperature of system, Sec. 12.5.6.



**Example 12.8** Repeat the calculation when the system of Fig. 12.6a is arranged as shown in Fig. 12.6b.

**Solution** In this case the cable precedes the LNA, and therefore, the equivalent noise temperature referred to the cable input is

$$T_s = 35 + (3.16 - 1) \times 290 + 3.16 \times 150 + \frac{3.16 \times (15.85 - 1) \times 290}{10^5}$$

$$= \underline{1136 \text{ K}}$$

Examples 12.7 and 12.8 illustrate the important point that the LNA must be placed ahead of the cable, which is why one sees amplifiers mounted right at the dish in satellite receive systems.

## 12.6 Carrier-to-Noise Ratio

A measure of the performance of a satellite link is the ratio of carrier power to noise power at the receiver input, and link-budget calculations are often concerned with determining this ratio. Conventionally, the ratio is denoted by  $C/N$  (or  $CNR$ ), which is equivalent to  $P_R/P_N$ . In terms of decibels,

$$\left[ \frac{C}{N} \right] = [P_R] - [P_N] \quad (12.33)$$

Equations (12.17) and (12.18) may be used for  $[P_R]$  and  $[P_N]$ , resulting in

$$\left[ \frac{C}{N} \right] = [\text{EIRP}] + [G_R] - [\text{LOSSES}] - [k] - [T_s] - [B_N] \quad (12.34)$$

The  $G/T$  ratio is a key parameter in specifying the receiving system performance. The antenna gain  $G_R$  and the system noise temperature  $T_s$  can be combined in Eq. (12.34) as

$$[G/T] = [G_R] - [T_s] \text{ dBK}^{-1} \quad (12.35)$$

Therefore, the link equation [Eq. (12.34)] becomes

$$\left[ \frac{C}{N} \right] = [\text{EIRP}] + \left[ \frac{G}{T} \right] - [\text{LOSSES}] - [k] - [B_N] \quad (12.36)$$

The ratio of carrier power to noise power density  $P_R/N_0$  may be the quantity actually required. Since  $P_N = kT_N B_N = N_0 B_N$ , then

$$\begin{aligned} \left[ \frac{C}{N} \right] &= \left[ \frac{C}{N_0 B_N} \right] \\ &= \left[ \frac{C}{N_0} \right] - [B_N] \end{aligned}$$