Tutorial 9

1. The ideal size of a first-year class at a particular college is 150 students. The college, knowing from past experience that, on the average, only 30 percent of those accepted for admission will actually attend, uses a policy of approving the applications of 450 students. Compute the probability that more than 150 first-year students attend this college.

2. Let X be an exponential random variable with parameter λ. Calculate (a) $E[X]$ and (b) $\mathrm{Var}(X)$.

3. Let X be a continuous nonnegative random variable with density function f, and let $Y = X^n$. Find f_Y, the probability density function of Y.

4. The following table uses 1992 data concerning the percentages of male and female full-time workers whose annual salaries fall into different ranges:

Earnings range	Percentage of females	Percentage of males
≤ 9999	8.6	4.4
$10,000$–$19,999$	38.0	21.1
$20,000$–$24,999$	19.4	15.8
$25,000$–$49,999$	29.2	41.5
$\geq 50,000$	4.8	17.2

   Suppose that random samples of 200 male and 200 female full-time workers are chosen. Approximate the probability that

   (a) at least 70 of the women earn $\$25,000$ or more;

   (b) at most 60 percent of the men earn $\$25,000$ or more;

   (c) at least three-fourths of the men and at least half the women earn $\$20,000$ or more.

5. At a certain bank, the amount of time that a customer spends being served by a teller is an exponential random variable with mean 5 minutes. If there is a customer in service when you enter the bank, what is the probability that he or she will still be with the teller after an additional 4 minutes?

6. A roulette wheel has 38 slots, numbered 0, 00, and 1 through 36. If you bet 1 on a specified number then you either win 35 if the roulette ball lands on that number or lose 1 if it does not. If you continually make such bets, approximate the probability that

   (a) you are winning after 34 bets;

   (b) you are winning after 1000 bets;

   (c) you are winning after 100,000 bets.

   Assume that each roll of the roulette ball is equally likely to land on any of the 38 numbers.