Lecture - 7

There has been a murder. You are 60% sure that the butler committed the crime. A strand of brown hair is found. The butler also has brown hair. 20% of the population has brown hair. What is the probability that the ~~cook~~ butler committed the crime?

A: butler committed the crime

B: butler has brown hair.

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(B|A)P(A)}{P(B)}$$

$$B = (B \cap A) \cup (B \cap \bar{A})$$

(2)

$(A, \bar{A})$

mutually exclusive.

$$P(A|B) = \frac{P(B|A)\,P(A)}{P(B|A)\,P(A) + P(B|\bar{A})\,P(\bar{A})}$$

$$= \frac{1 * 0.6}{1 * 0.6 + 0.2 * 0.4}$$

$$= \frac{0.6}{0.6 + 0.08} = \frac{0.60}{0.68}$$

$$= \frac{60}{68} = \frac{15}{17} \approx 89\%$$

Independant Events

if $P(E|F) = P(E)$,

then E & F are independant.

$$P(E|F) = \frac{P(E \cap F)}{P(F)} = P(E)$$

$$P(E \cap F) = P(E)\,P(F)$$

$$= P(F)\,P(E)$$

$$P(F|E) = P(F)$$

$$P(A|B) := \frac{P(A \cap B)}{P(B)}$$

tossing 2 dice. (9)

A: sum is (6)

B: 1st dice shows 4

$P(A) = \frac{5}{36}$ | $P(A \cap B) = \frac{1}{36}$

$P(B) = \frac{6}{36}$

A: sum is 7 | (8)

B: 1st dice = 4

$P(A) = \frac{6}{36}$ | $P(A \cap B) = \frac{1}{36}$ (4,3)

$P(B) = \frac{6}{36}$

Independence of 3 events.

DEFINITION!

$$\begin{cases} P(AB) = P(A)P(B) \checkmark \\ P(BC) = P(B)P(C) \checkmark \\ P(CA) = P(C)P(A) \checkmark \\ P(ABC) = P(A)P(B)P(C) \times \end{cases}$$

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e.g. toss 2 coins.

A: 1st coin H

B: 2nd coin H

$P(A \cap B \cap C) = \frac{1}{4}$

C: Both show the same result $= \{HH, TT\}$ (not independent)

$P(A) = \frac{1}{2}$, $P(B) = \frac{1}{2}$, $P(C) = \frac{1}{2}$

$P(A \cap B) = \frac{1}{4}$, $P(B \cap C) = \frac{1}{4}$, $P(C \cap A) = \frac{1}{4}$

e.g. You toss 4 coins. All are Heads. Then you toss the 5th coin. What's the probability that it's a tail?

A: 1st 4 are Heads

B: 5th is a Tail

$$P(B|A) = \frac{P(B \cap A)}{P(A)} = \frac{1/32}{2/32}$$

$P(B) = \frac{1}{2}$

$P(A) = \frac{1}{2}$

A = {HHHHH, HHHHT}

B = {HHHHT}

4 heads have come, so I deserve a tail now.

$P(\text{at least one tail}) = 1 - P(\text{no tail}) = 1 - \frac{1}{32} = \frac{31}{32}$

eg: Toss a pair of dice.

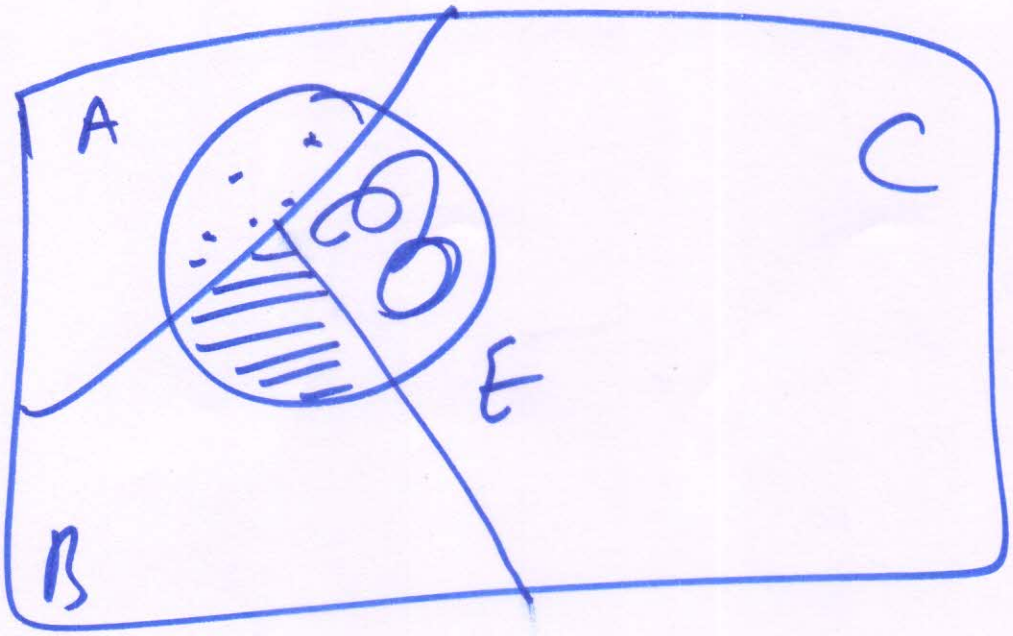
What is the probability that a total of 5 occurs before a total of 7?

E: a total of 5 occurs before a total of 7.

A: sum is 5
B: sum is 7
C: sum is neither 5 or 7 $\{2, 3, 4, 6, 8, 9, 10, 11, 12\}$

A, B, C are mutually exclusive & exhaustive.

$P(E) =$



$$E = (E \cap A) \cup (E \cap B) \cup (E \cap C)$$

$$P(E) = P(E \cap A) + P(E \cap B) + P(E \cap C) \neq P(E|A)P(A)$$

Nomen est omen!