Additional Discussions

Inflection Points, Coinciding Roots & Turning Points

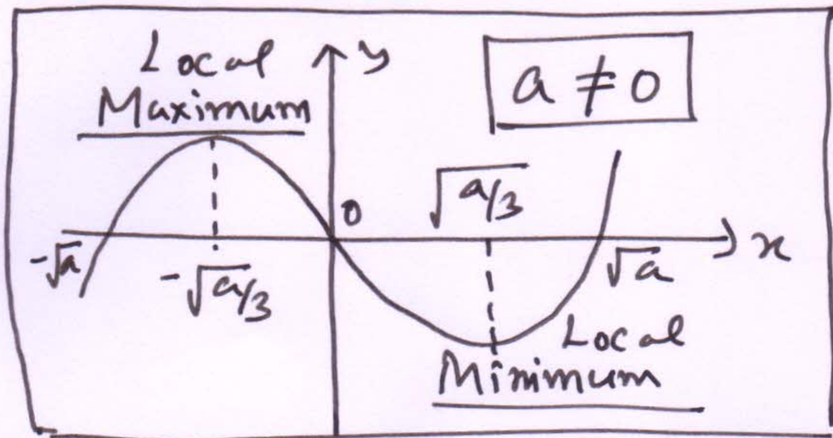
1/. $\boxed{y = f(x) = -ax + x^3}$ $\boxed{a>0}$. When $y = 0$

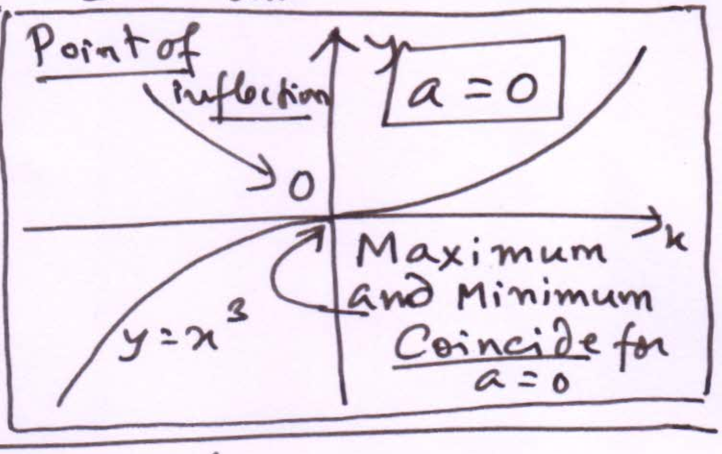
$\Rightarrow \boxed{x = 0}$, & $\boxed{x = \pm\sqrt{a}}$.

$\Rightarrow \boxed{\frac{dy}{dx} = -a + 3x^2} \Rightarrow \boxed{\frac{dy}{dx} = 0}$ will be at $\boxed{x = \pm\sqrt{\frac{a}{3}}}$

$\Rightarrow \boxed{\frac{d^2y}{dx^2} = 6x}$ i) When $x = \sqrt{\frac{a}{3}}$, $\frac{d^2y}{dx^2} > 0 \Rightarrow$ Minimum

ii) When $x = -\sqrt{\frac{a}{3}}$, $\frac{d^2y}{dx^2} < 0 \Rightarrow$ Maximum





2/. $\boxed{y = f(x) = -ax^2 + x^4}$ $\boxed{a>0}$. When $y = 0$, $\Rightarrow \boxed{x^2 = 0}$, $\boxed{x = \pm\sqrt{a}}$

Two roots coincide at $x = 0$

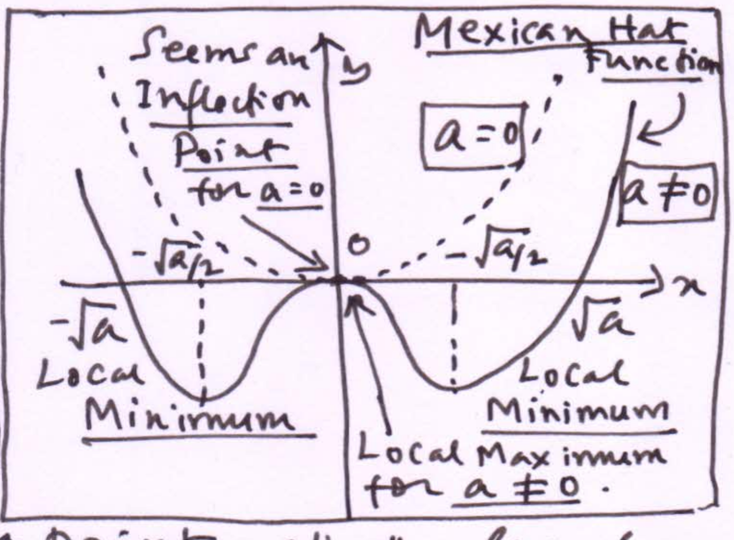
$\boxed{\frac{dy}{dx} = f'(x) = -2ax + 4x^3}$ If $\boxed{f'(x) = 0} \Rightarrow \boxed{x = 0,\ x = \pm\sqrt{a/2}}$

$\boxed{\frac{d^2y}{dx^2} = f''(x) = -2a + 12x^2}$

i.) When $x = 0$, $\frac{d^2y}{dx^2} = -2a < 0$ (Maximum)

ii.) When $x = \pm\sqrt{\frac{a}{2}}$, $\frac{d^2y}{dx^2} = 4a > 0$ (Minima)

When $a = 0$, $\boxed{\frac{d^2y}{dx^2} = \frac{dy}{dx} = y = 0}$



$\Rightarrow$ $x = 0$ Appears to be an inflection point with the local maximum and two minima coinciding. In actual fact one minimum survives. (Spontaneous symmetry breaking).

Roots of Unity on the Argand Plane

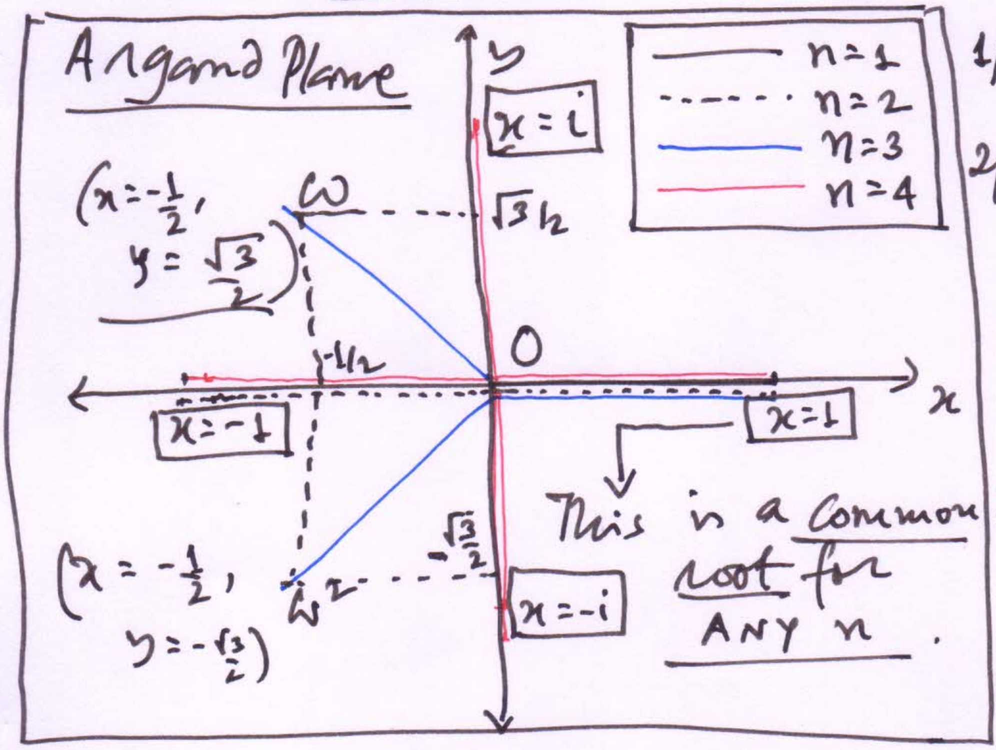
$\boxed{x^n = 1}$ ⇒ There will be n roots of unity.

i.) $n = 1$, $\boxed{x - 1 = 0}$ ⇒ One root at $\boxed{x = 1}$.

ii.) $n = 2$, $\boxed{x^2 - 1 = 0}$ ⇒ Two roots at $\boxed{x = \pm 1}$.

iii.) $n = 3$, $\boxed{x^3 - 1 = 0}$ ⇒ Three roots at $\boxed{x = 1, \omega, \omega^2}$.

iv.) $n = 4$, $\boxed{x^4 - 1 = 0}$ ⇒ Four roots at $\boxed{x = \pm 1, \pm i}$



1/. $\boxed{|x| = 1}$ → Magnitude of roots

2/. The direction will be divided evenly along $\frac{2\pi}{n}$ angles on the plane.

3/. If n is odd, there is only one real root, $x = 1$.

4/. If n is even, two real roots, $x = \pm 1$.

In the Euler notation, if there are n roots of unity, then each root is $\boxed{x_k = \cos\left(\frac{2\pi k}{n}\right) + i \sin\left(\frac{2\pi k}{n}\right)}$ with $\boxed{k = 1, 2, \ldots, n}$. The last root is $x_n = 1$.

If $\boxed{k = 0, 1, \ldots, n-1}$, then $\boxed{x_0 = 1}$, the first root.

Since $\boxed{e^{i\theta} = \cos\theta + i\sin\theta}$, $\boxed{x_k = e^{i\frac{2\pi k}{n}}}$.

This gives exact roots of unity for $\boxed{x^n = 1}$ upto any degree n.