CSE 569 Homework #1

Total 3 points

Due on the Blackboard on Friday, Sept. 23, 11:59PM.

Notes:

1. The exam questions will be similar to these homework problems (and in particular, Q2 through Q5 were taken from past exams). Therefore, <u>no sample exams will be posted, since these problems serve as samples already.</u>
2. Submission of homework must be electronic. Most problems can be solved by hand. You can write down your solutions and then either type your work or take pictures of your hand-written sheets, and then upload your work onto the Blackboard.
3. If you have any question on the homework problems, you should post your question on the Blackboard discussion board (under the Homework 1 Q & A), instead of sending emails to the instructor or TAs. The instructor will answer your question there. This will help to avoid repeated questions, and also help the entire class to stay on the same page whenever any clarification/correction is made.

Q1. <u>Part A</u>. Consider the following *game*: Someone shows you three hats and tells you that the there is a prize in one of them. He asks you to choose one of the hats. You choose one hat and tell him which one you chose. He then lifts one of the hats you didn't choose and there is nothing under that hat. He then tells you that you can either stay with the hat you have originally chosen or switch to the other remaining hat. What should you do? Explain your answer.

<u>Part B</u>. (*Use this to help ensure your Part A is correct*). Design a computer-based experiment (i.e., write a computer program) to simulate the above game to verify your answer, by playing the game many times to obtain an averaged performance.

Q2. Consider two multivariate normally-distributed classes,

$$p(\mathbf{x} \mid \omega_i) = \frac{1}{(2\pi)^{d/2}|\Sigma_i|^{1/2}} \exp\left[-\frac{1}{2}(\mathbf{x}-\boldsymbol{\mu}_i)^t \Sigma_i^{-1} (\mathbf{x}-\boldsymbol{\mu}_i)\right], \text{ for } i = 1, 2$$

where the notations are as defined on Slide 28 of Notes 02. We have seen in the class that, when the covariance matrices are equal, i.e., $\mathbf{\Sigma_1 = \Sigma_2 = \Sigma}$, the decision boundary is linear for minimum-error-rate classification. Now let's assume the following risk values: $\lambda_{12}=2$, $\lambda_{21}=1$, $\lambda_{11}=\lambda_{22}=0$. Derive the equation for the decision boundary in this case (we still assume $\mathbf{\Sigma_1 = \Sigma_2 = \Sigma}$).

Q3. Consider a 1-dimensional 2-class classification problem with class-conditionals as follows:

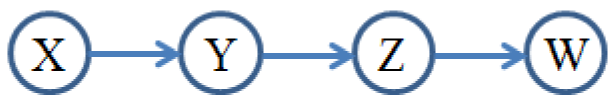
$$p(x\,|\,\omega_1) = \begin{cases} \dfrac{-x+1}{2}, & x \in [-1, 1] \\ 0, & otherwise \end{cases} \qquad\qquad p(x\,|\,\omega_2) = \begin{cases} \dfrac{x+1}{2}, & x \in [-1, 1] \\ 0, & otherwise \end{cases}$$

(a) If the priors are equal, i.e., $P(\omega_1)= P(\omega_2)=0.5$, what is the Bayesian decision boundary for doing minimum error rate classification? What is the Bayes error in this case?

(b) If the priors are given as $P(\omega_1)=0.7$, $P(\omega_2)=0.3$, what is the Bayesian decision boundary for doing minimum error rate classification? What is the Bayes error in this case?

(c) Again, assuming equal priors, i.e., $P(\omega_1)= P(\omega_2)=0.5$, but the misclassification costs are asymmetric, with the cost functions given as: $\lambda_{12}=2$, $\lambda_{21}=1$, $\lambda_{11}=\lambda_{22}=0$. What is the Bayesian decision boundary in this case, and what is the corresponding Bayes risk?

Q4. Consider the following simple Bayesian network, where all the nodes are assumed to be binary random variables, i.e., X=x_0 or x_1 with certain probabilities, and similar notations will be used for Y, Z, and W.



This Bayesian network is fully specified if we are given the following (conditional) probabilities:
(for notational simplicity, we write $P(x_1)$ to mean $P(X=x_1)$, and so on)

$P(x_1) = 0.60$;
$P(y_1 \,|\, x_1) = 0.40$, $P(y_1 \,|\, x_0) = 0.30$;
$P(z_1 \,|\, y_1) = 0.25$, $P(z_1 \,|\, y_0) = 0.60$;
$P(w_1 \,|\, z_1) = 0.45$, $P(w_1 \,|\, z_0) = 0.30$;

(a) Suppose that X is measured and its value is x_1, compute the probability that we will obverse W having a value w_0, i.e., $P(w_0 \,|\, x_1)$.
(b) Suppose that W is measured and its value is w_1, compute the probability that we will obverse X having a value x_0, i.e., $P(x_0 \,|\, w_1)$.

Q5. True-or-False: For a two-class classification problem using the minimum-error-rate rule, in general the decision boundary can take any form. However, if the underlying class-conditionals are Gaussian densities, then the decision boundary is linear (hyperplanes).

[ ] True [ ] False

Brief explanation of your answer: