EXERCISE 3

TIME DOMAIN SPECIFICATIONS OF 1ST AND 2ND ORDER SYSTEMS

DATE: ***Reg. No. :***

LAB PREREQUISITES:

Exercise 1 to 2

PREREQUISITE KNOWLEDGE:

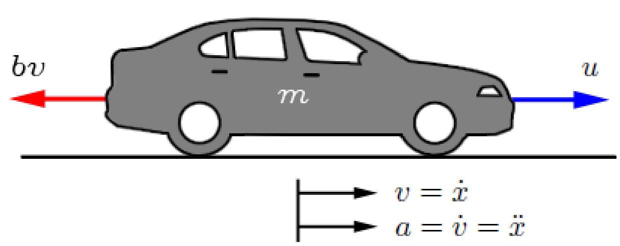
Fundamentals of MATLAB programming

OBJECTIVES:

The objective of this exercise will be to study the performance characteristics of first and second order systems using MATLAB and SIMLINK

PRELAB:

Automatic cruise control is an excellent example of a feedback control system found in many modern vehicles. The purpose of the cruise control system is to maintain a constant vehicle speed despite external disturbances, such as changes in wind or road grade. This is accomplished by measuring the vehicle speed, comparing it to the desired or reference speed, and automatically adjusting the throttle according to a control law.



We consider here a simple model of the vehicle dynamics, shown in the free-body diagram (FBD) above. The vehicle, of mass m, is acted on by a control force, u. The force u represents the force generated at the road/tire interface. For this simplified model we will assume that we can control this force directly and will neglect the dynamics of the powertrain, tires, etc., that go into generating the force. The resistive forces, bv, due to rolling resistance and wind drag, are assumed to vary linearly with the vehicle velocity, v, and act in the direction opposite the vehicle's motion.With these assumptions we are left with a first-order mass-damper system. Summing forces in the x-direction and applying Newton's 2nd law, we arrive at the following system equation:

$$m\dot{v} + bv = u$$

Use the following values for the parameters:

- (m) vehicle mass 1000 kg
- (b) damping coefficient 50 N.s/m
- (u) nominal control force 500 N

Derive the transfer function of the system where the velocity of the vehicle is the controlled (output) variable.

On a graph sheet, obtain the time domain response of the system for a **step input force** of 500N. Take sufficient number of data points.

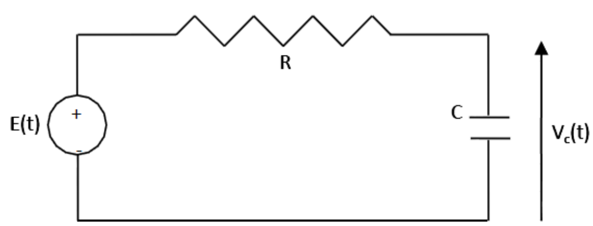
Obtain the following parameters. Show all necessary calculations.

- steady state value of the system
- rise time
- settling time
- time constant
- DC gain

READINGS:

Overview of First Order Systems:

An electrical RC-circuit is the simplest example of a first order system. It comprises of a resistor and capacitor connected in series to a voltage supply as shown below on the figure. Assume the values **a.** R=2KΩ and C=0.01F **b.** R=2.5KΩ and C=0.003F



If the capacitor is initially uncharged at zero voltage when the circuit is switched on, it starts to charge due to the current 'i' through the resistor until the voltage across it reaches the supply voltage. As soon as this happens, the current stops flowing or decays to zero, and the circuit becomes like an open circuit. However, if the supply voltage is removed, and the circuit is closed, the capacitor will discharge the energy it stored again through the resistor. The time it takes the capacitor to charge depends on the time constant of the system, which is defined as the time taken by the voltage across the capacitor to rise to approximately 63% of the supply voltage. For a given RC-circuit, this time constant is τ=RC. Hence its magnitude depends on the values of the circuit components. The RC circuit will always behave in this way, no matter what the values of the components. That is, the voltage across the capacitor will never increase indefinitely. In this respect we will say that the system is passive and because of this property it is stable.

For the RC-circuit as shown in Fig. 1, the equation governing its behavior is given by

$$\frac{dv_c(t)}{dt} + \frac{1}{RC} v_c(t) = \frac{1}{RC} E \quad \text{where } v_c(0) = v_0$$

where v_c (t)is the voltage across the capacitor, R is the resistance and C is the capacitance. The constant τ=RC is the time constant of the system and is defined as the time required by the system output i.e. v_c (t)to rise to 63% of its final value (which is E). Hence the above equation can be expressed in terms of the time constant as:

$$\tau \frac{dv_c(t)}{dt} + v_c(t) = E \quad \text{where } v_c(0) = v_0$$

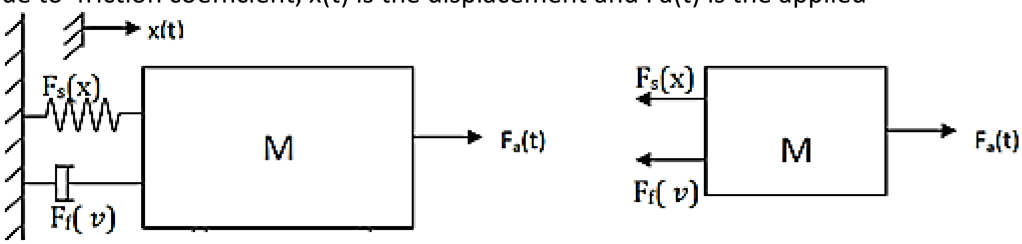
Obtaining the transfer function of the above differential equation, we get

$$\frac{V_C(s)}{E(s)} = \frac{1}{\tau s + 1}$$

where τ is time constant of the system and the system is known as the first order system. The performance measures of a first order system are its time constant and its steady state.

Overview of Second Order Systems:

Consider the following Mass-Spring-Damper system shown in the figure. Where Fs(x) is the spring force, Ff($\dot{x}$) is the force due to friction coefficient, x(t) is the displacement and Fa(t) is the applied



Where

$$a = \frac{dv(t)}{dt} = \frac{d^2x(t)}{dt^2} \text{ is the acceleration,}$$
$$v = \frac{dx(t)}{dt} \text{ is the speed,}$$

and

$$x(t) \text{ is the displacement.}$$

force:

Then the transfer function representation of the system is given by

$$TF = \frac{Output}{Input} = \frac{F(s)}{X(s)} = \frac{1}{(Ms^2 + Bs + K)}$$

The above system is known as a second order system.

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

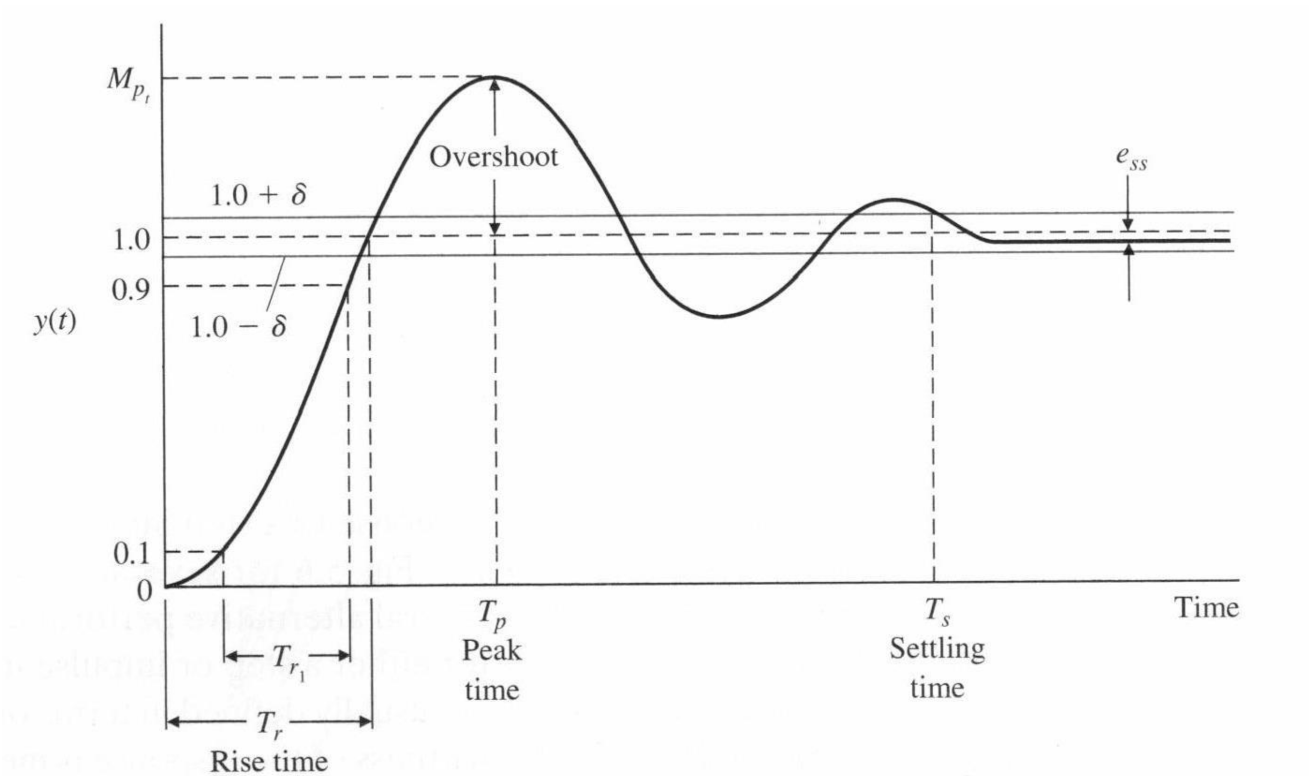
The generalized notation for a second order system described above can be written as

$$C(s) = \frac{\omega_n^2}{s\left(s^2 + 2\zeta\omega_n s + \omega_n^2\right)}$$

for which the transient output, as obtained from the Laplace transform table

$$c(t) = 1 - \frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \sin(\omega_n \sqrt{1-\zeta^2} t + \cos^{-1}(\zeta))$$

where0 < ζ <1. The transient response of the system changes for different values of damping ratio, ζ . Standard performance measures for a second order feedback system are defined in terms of step response of a system. Where, the response of the second order system is shown below.



The performance measures could be described as follows:

Rise Time: The time for a system to respond to a step input and attains a response equal to a percentage of the magnitude of the input. The 0-100% rise time, T_r, measures the time to 100% of the magnitude of the input. Alternatively, T_{r1}, measures the time from 10% to 90% of the response to the step input.

Peak Time: The time for a system to respond to a step input and rise to peak response.

Overshoot: The amount by which the system output response proceeds beyond the desired response. It is calculated as

$$\text{P.O.}=\frac{M_{p_t}-f\upsilon}{f\upsilon}\times 100\%$$

where M_{Pt} is the peak value of the time response, and *fv* is the final value of the response.

Settling Time: The time required for the system's output to settle within a certain percentage of the input amplitude (which is usually taken as 2%). Then, settling time, Ts, is calculated as

$$T_s=\frac{4}{\zeta\omega_n}$$

LTI Viewer

The LTI (Linear Time Invariant) Viewer is an interactive graphical user interface (GUI) for analyzing the time and frequency responses of linear systems and comparing such systems. In particular, some of the graphs the LTI Viewer can create are step and impulse responses, Bode, Nyquist, Nichols, pole-zero plots, and responses with arbitrary inputs. In addition, the values of critical measurements on these plots can be displayed with a click of the mouse. The following table shows the critical measurements that are available for each plot.

-	Peak time or Peak frequency	Settling time	Rise time	Steady state value	Gain/phase margins; zero dB/1800 frequencies	Pole-zero value
Step	✓	✓	✓	✓	-	-
Impulse	✓	✓	-	-	-	-
Bode	✓	-	-	-	✓	-
Nyquist	✓	-	-	-	✓	-
Nichols	✓	-	-	-	✓	-
Pole-zero	-	-	-	-	-	✓

The following steps may be followed to use the LTI Viewer

- Create LTI transfer functions
- Access the LTI Viewer
- Select LTI transfer functions for the LTI Viewer from **Import** menu.
- Select the LTI objects for the next response plot
- Select the plot type by right clicking anywhere in the plot and selecting **Plot Types**
- Select the characteristics by right clicking anywhere in the plot and selecting **Characteristics**

PROGRAMS, OBSERVATIONS AND INFERENCES

FIRST ORDER SYSTEMS

1. Find the step response for the above mentioned cruise and electrical first order systems with given set of values by using the transfer function.
2. Find the time constant value for two systems at which the system reaches the 63 % of its final output.
3. Obtain the steady state value of the system, rise time and settling time.
4. Perform first three tasks using LTI Viewer.

DERIVATION OF TRANSFER FUNCTION FOR FIRST ORDER SYSTEMS

PROGRAMS FOR FIRST ORDER SYSTEMS

MECHANICAL SYSTEM

```
clc;
clear all;
close all;
F=input('Enter the value for input force F= ');
M=input('Enter the value for Mass M=');
B=input('Enter the value for Damping coefficient B=');
num=[F];
den=[M B];
sys=tf(num,den)
ltiview(sys);
```

ELECTRICAL SYSTEM

```
clc;
clear all;
close all;
V=input('Enter the value for input voltage V=');
R=input('Enter the value for resistance R= ');
C=input('Enter the value for capacitance C=');
s=tf('s');
sys=V/((s*R*C)+1)
ltiview(sys);
```

SECOND ORDER SYSTEMS

1. Find all time domain specifications of the second order systems which are derived in the Exercise number 2 using Matlab codes.

TRANSFER FUNCTION OF SECOND ORDER SYSTEMS (from experiment-2)

MECHANICAL

ELECTRICAL

DERIVATION OF TRANSFER FUNCTION FOR RLC CIRCUIT

PROGRAMS FOR SECOND ORDER SYSTEM

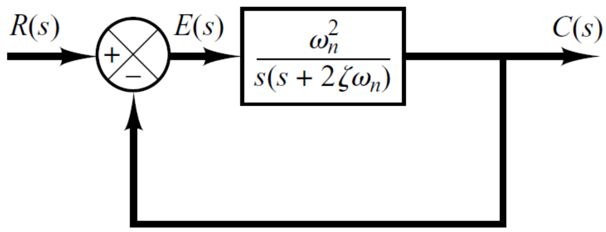
MECHANICAL

```
clc;
clear all;
close all;
F=input('Enter the value for input force F= ');
M=input('Enter the value for Mass M=');
B=input('Enter the value for Damper B=');
K=input('Enter the value for Spring constant K=');
num=[F];
den=[M B K];
sys=tf(num,den)
ltiview(sys);
```

ELECTRICAL RLC CIRCUIT

```
clc;
clear all;
close all;
V=input('Enter the value for input voltage V=');
R=input('Enter the value for resistance R= ');
C=input('Enter the value for capacitance C=');
L=input('Enter the value for inductance L=');
s=tf('s');
sys=V/(((s^2)*L*C)+(s*R*C)+1)
subplot(2,1,1);
impulse(sys);
title('Impulse response for RLC circuit');
ylabel('output voltage');
subplot(2,1,2);
step(sys);
title('Step response for RLC circuit');
ylabel('output voltage');
ltiview(sys);
```

2. **Effect of damping ratio ζ on performance measures**: For a single-loop second order feedback system given below



Using MATLAB find the step response of the system for values of ω_n = 1 and ζ = 0.1, 0.4, 0.7, 1.0 and 2.0.Plot all the results in the same figure window and fill the following table.

Damping Ratio ζ	Rise time	Peak Time	% Overshoot	Settling time	Steady state value
0.1					
0.4					
0.7					
1.0					
2.0					

PROGRAM

```
clc;
clear all;
close all;
z=input('Enter the value for zeta z=');
w=input('Enter the value for omega n w=');
num=[w*w];
den=[1 2*w*z w*w];
sys=tf(num,den)
ltiview(sys);
```

RESULTS & INFERENCES:

Evaluation Component	Maximum Marks	Marks Obtained
Pre-lab Tasks	10	
In-Lab Tasks	20	
Post-lab Tasks	10	
Bonus Tasks	10	
Signature of Faculty with Date		

(This page must be the last page of the exercise)

PRE-LAB TASKS:

1. **Derive the transfer function of first order systems**
   a. **Mechanical system**
   b. **Electrical system**
2. **Derive the transfer function of second order systems**
   a. **Mechanical system**
   b. **Electrical system**

IN-LAB TASKS

Task 1: Create m file for first order system to obtain the time domain specifications for both mechanical and electrical system.

Task 2: Create m file for the second order systems and obtain the time domain specifications.

Task 3: Tabulate the Effect of damping ratio ζ on performance measures for the second order system.

ADDITIONAL TASKS/LEARNING FOR BONUS MARKS: