EXERCISE 10

PD COMPENSATED QUBE SERVO DC MOTOR

Date ***Reg. No. :***

LAB PREREQUISITES:

Exercise 1-9

PREREQUISITE KNOWLEDGE:

Fundamentals of MATLAB programming and Simulink.

OBJECTIVES:

To design a PD compensator for the Qube-Servo 2 model to achieve a peak time of 0.15s and a % overshoot of 2.5%.

Servo Model:

The QUBE-Servo 2 voltage-to-position transfer function is

$$P(s) = \frac{\Theta_m(s)}{V_m(s)} = \frac{K}{s(\tau s + 1)}, \qquad (1)$$

where K = 26:5 rad/(V-s) is the model steady-state gain, τ = 0:155 s is the model time constant, $\Theta m(s) = L[\vartheta m(t)]$ is the motor / disk position, and $Vm(s) = L[vm(t)]$ is the applied motor voltage.

PID CONTROL:

The proportional, integral, and derivative control can be expressed mathematically as follows

$$u(t) = k_p e(t) + k_i \int_0^t e(\tau)d\tau + k_d \frac{de(t)}{dt}. \qquad (2)$$

The corresponding block diagram is given in Figure 1. The control action is a sum of three terms referred to as proportional (P), integral (I) and derivative (D) control gain. The controller Equation 2 can also be described by the transfer function

$$C(s) = k_p + \frac{k_i}{s} + k_d s. \qquad (3)$$

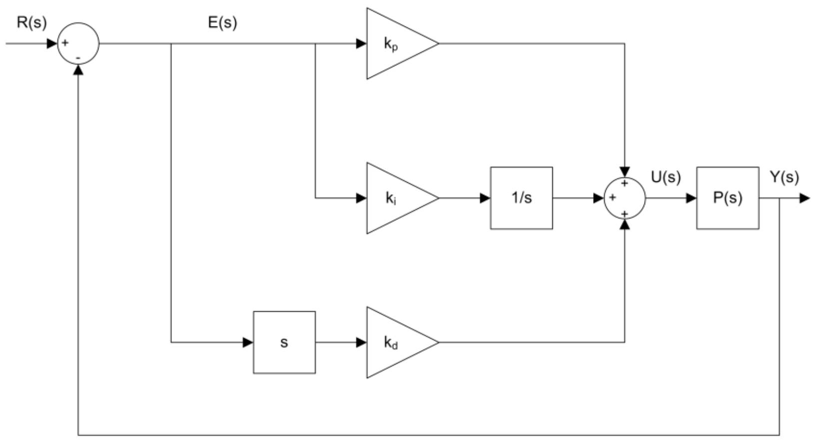

Figure.1. Block diagram of PID control

The functionality of the PID controller can be summarized as follows. The proportional term is based on the present error, the integral term depends on past errors, and the derivative term is a prediction of future errors. The PID controller described by Equation 2 or 3 is an ideal PID controller. However, attempts to implement such a controller may not lead to a good system response for real-world system. The main reason for this is that measured signals always include measurement noise. Therefore, differentiating a measured (noisy) signal will result in large fluctuations, thus will result in large fluctuations in the control signal

PV POSITION CONTROL:

The integral term will not be used to control the servo position. A variation of the classic PD control will be used: the proportional-velocity control illustrated in Figure 2. Unlike the standard PD, only the negative velocity is fed back (i.e. not the velocity of the *error*) and a low-pass filter will be used in-line with the derivative term to suppress measurement noise. The combination of a first order low-pass filter and the derivative term results in a high-pass filter *H*(*s*) which will be used instead of a direct derivative.

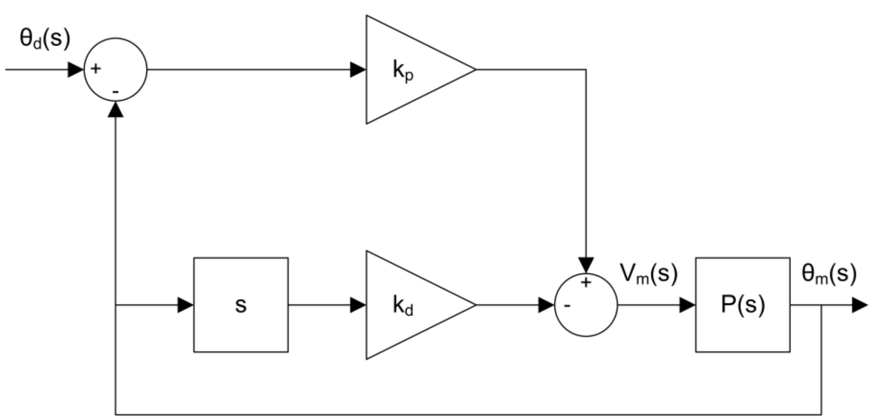

Figure.2. Block diagram of PV control

The back-emf (electromotive) voltage $e_b(t)$ depends on the speed of the motor shaft, w_m, and the back-emf constant of the motor, k_m. It opposes the current flow. The back emf voltage is given by:

$$e_b(t) = k_m \omega_m(t)$$

Table 1. Qube servo 2 system parameters

The proportional-velocity (PV) control has the following structure

$$u = k_p\left(r(t) - y(t)\right) - k_d\dot{y}(t), \qquad (4)$$

where *kp* is the proportional gain, *kd* is the derivative (velocity) gain, *r* = *ϑd*(*t*) is the setpoint or reference motor /load angle, *y* = *ϑm*(*t*) is the measured load shaft angle, and *u* = *Vm*(*t*) is the control input (applied motor voltage). The closed-loop transfer function of the QUBE-Servo 2 is denoted *Y* (*s*)/*R*(*s*) = Θ*m*(*s*)/Θ*d*(*s*). Assume all initial conditions are zero, i.e. *ϑm*(0-) = 0 and *ϑm*_ (0-) = 0, taking the Laplace transform of Equation 4 yields

U(*s*) = *kp*(*R*(*s*) - *Y* (*s*)) - *kdsY* (*s*);
which can be substituted into Equation 1 to result in

$$Y(s) = \frac{K}{s(\tau s + 1)}\left(k_p\left(R(s) - Y(s)\right) - k_d s Y(s)\right).$$

Solving for *Y* (*s*)/*R*(*s*), we obtain the closed-loop expression

$$\frac{Y(s)}{R(s)} = \frac{K k_p}{\tau s^2 + (1 + K k_d)s + K k_p}. \qquad (5)$$

This is a second-order transfer function. Recall the standard second-order transfer function

$$\frac{Y(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}. \qquad (6)$$

PROGRAMS, OBSERVATIONS AND INFERENCES

1. **Calculate the control gains needed to satisfy these requirements of peak time and percentage overshoot, Using the QUBE-Servo 2 model parameters K and τ given above in Background section of this lab.**
2. **Implement a PV controller with a low pass filter of 100/(s+100)**
3. **Implement a PV control loop on the QUBE-Servo 2 model transfer function that relates Voltage *Vm*(*s*) to Position θ*m*(*s*).**
4. **Build and run the QUARC controller. Determine the responses – (i) Position (rad) (ii) Voltage**
5. **Keep the derivative gain at 0 and vary *kp* between 1 and 4. What does the proportional gain do when controlling servo position?**
6. **Set *kp* from calculation and vary the derivative gain *kd* between 0 and 0:15 V/(rad/s). What is its effect on the position response?**

RESULTS & INFERENCES:

Evaluation Component	Maximum Marks	Marks Obtained
Pre-lab Tasks	10	
In-Lab Tasks	20	
Post-lab Tasks	10	
Bonus Tasks	10	
Signature of Faculty with Date		

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