Finding the number of digits in 2^n

January 26, 2023

Introduction

In order to find the number of digits in numbers of the form 2^n, we can employ logarithms. We set out in the following sections how to do this.

1 The formula

The key here is to write 2^n in the form 10^m, where the value of m can give the number of digits in the answer in base 10. We can see how this works if we take an example; $10^4.5 \approx 31,623$. This number has five digits which is found by rounding the power up to the nearest integer. In the special case where power is already an integer we can add one to find the number of digits i.e. 10^4 has five digits. So starting with

$$2^n = 10^m, \tag{1}$$

we need to rearrange this to find the value of m. To do this we take the logarithm of base 2 of both sides giving

$$\log_2(2^n) = \log_2(10^m), \tag{2}$$

rearranging we get

$$n = m \log_2(10), \tag{3}$$

from this we can easily find m and hence the number of digits.

2 Alternative Bases

In order to make the code run more quickly we can apply the same idea but in a higher base. If we set our base to be b, for our equation we have

$$n = m \log_2(b). \tag{4}$$

A final rearranged form looks like this

$$m = \frac{n}{\log_2(b)}, \tag{5}$$

where, n is the power of two we are calculating, b is the base we are working in and m the number of digits.

Conclusion

We have found a handy formula (5), that allows one to find the number of digits in numbers of the form 2^n, in any base. This is very useful for this code as it enables an array of the correct length to be initialised at the start. It also allows the number of elements in the array that have been given values to be known at any point in the calculation.