Patents and Innovation in Software

Tom Augspurger and Caleb Floyd

March 12, 2013

This Project

- Investigate the state of intellectual property protection.
  - Economic and social implications
- Search for theories that would allow us to analyze IP protection in the software industry.
  - Find commonalities in competing theories.
  - Search for existing empirical results.
  - Investigate the open source phenomenon.

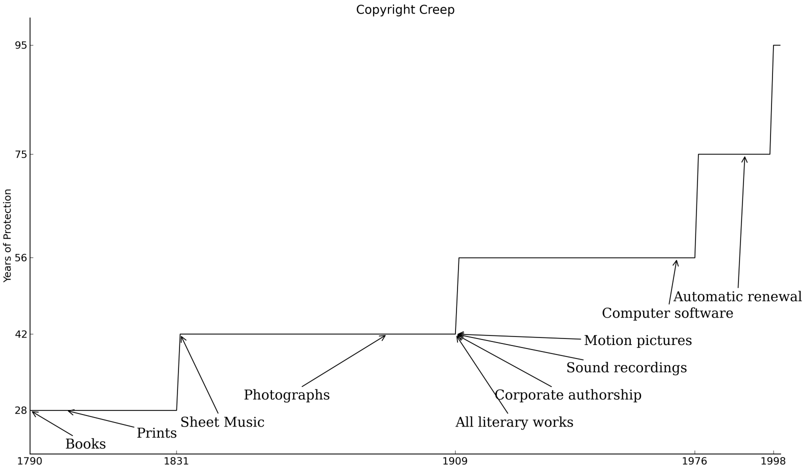
Where are we?

- Some theories are amenable; some aren't.
- One example where the open source phenomenon fits.
- One example where it doesn't.

Outline

- Copyright Law and Patents.
- Boldrin and Levine (2008).
- Acemoglu and Akcigit (2012).

The Copyright Creep



Ethics for the Information Age (2ND Edition) Micheal J. Quinn

Copyright Term Extension Act

- CTEA of 1998
  - Created prior to 1978: 95 year protection.
  - Created after 1978: lifetime of the author plus 70 years.
  - Challenged on grounds of:
    - The Copyright Clause – "limited Times"
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Amazon One-Click Patent

A method and system for placing an order to purchase an item via the Internet. The order is placed by a purchaser at a client system and received by a server system. The server system receives purchaser information including identification of the purchaser, payment information, and shipment information from the client system. The server system then assigns a client identifier to the client system and associates the assigned client identifier with the received purchaser information. The server system sends to the client system the assigned client identifier and an HTML document identifying the item and including an order button. The client system receives and stores the assigned client identifier and receives and displays the HTML document. In response to the selection of the order button, the client system sends to the server system a request to purchase the identified item. The server system receives the request and combines the purchaser information associated with the client identifier of the client system to generate an order to purchase the item in accordance with the billing and shipment information whereby the purchaser effects the ordering of the product by selection of the order button.

U.S. Patent Sep. 28, 1999 Sheet 9 of 11 **5,960,411**

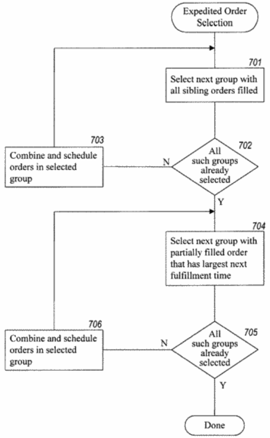
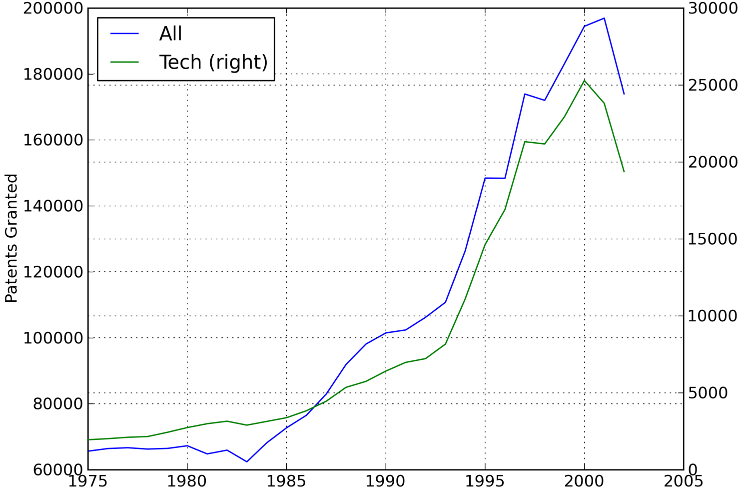


Fig. 7

Some Pictures

- NBER Patent Data File. Hall, Jaffe, Trajtenberg (2001).
- Patents *granted* from 1963 - 1999. (Since updated).
- Date, industry, citations, and company.

Growth in Patent Applications



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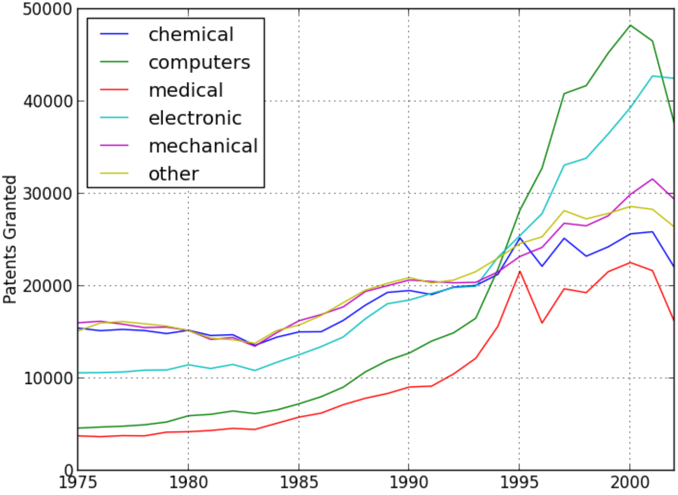
By Industry



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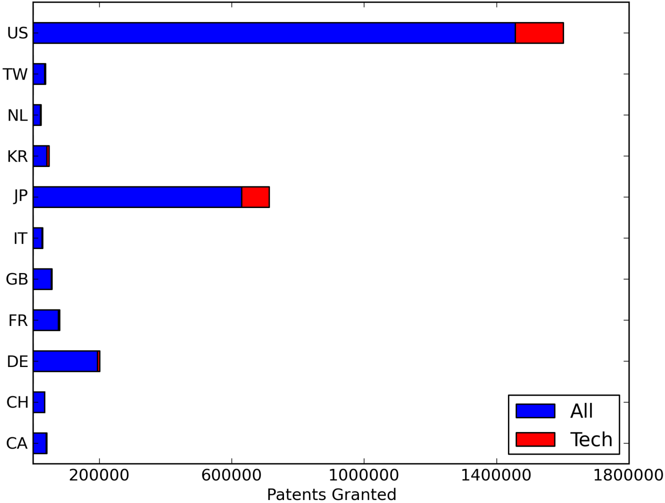
Patents by Country



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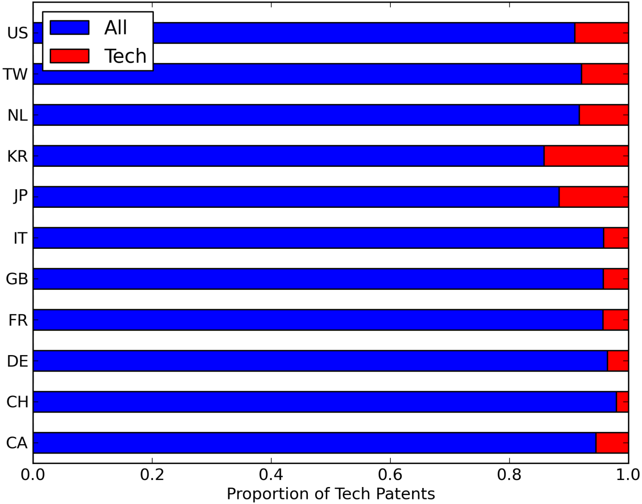

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- Canon Ink Jet printers.

Why does open source coexist?

- Control over product performance.
- Hobbyists and enthusiasts.
- Display of skill or resume padding.
  - Lerner and Tirole (2002)
  - Hann et. al (2004)
- Competitive rents (Boldrin & Levine).
  - Which model version fits?
  - What can we say about the implications?

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The evidence (and the common sense of anyone involved with OS software) shows that the source of competitive rents is the complementary sale of expertise.

...only small rents can be obtained through the sale of copies. [Purchasers] also have a demand for services, ranging from support and consulting to customization. They naturally prefer to hire the creators of the programs who in the process of writing the software have developed specialized expertise that is not easily matched by imitators.

- Boldrin & Levine (2009)

Boldrin & Levine

Boldrin & Levine: alternate notation

Table : Alternate Notation

BL		New
δ	$\longrightarrow$	β
β	$\longrightarrow$	λ
ζ	$\longrightarrow$	$1-\delta$

Boldrin & Levine: General Model Revisited

- Distinguish between productive input and consumption good: $\{k, c\}$.
- $c_t = F(k_t^c, l_t^c)$, $x_t = G(k_t^k, l_t^k)$.
- Agent solves $\sum_{t=0}^{\infty} \beta^t [u(c_t) - wL_t]$:
  - λk_t units available tomorrow: $k_{t+1} = \lambda k_t + x_t$.
  - $\lambda > 1$ gives us the 24/7 case.

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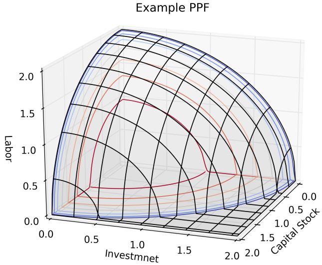
Boldrin & Levine: General Model Revisited

- Given $\{k_t, x_t, L_t\}$, the solution $c_t = T(k_t, x_t, L_t)$ traces a production possibility frontier



- L_t solves $\max\limits_{L_t} u[T(k_t, x_t, L_t)] - wL_t$
- The problem restated:

$$\nu(k_0) = \max_{\{k_t\}_{t=1}^{\infty}} \sum_{t=0}^{\infty} \beta^t V(k_t, k_{t+1} - \lambda k_t)$$

$$s.t. \qquad \lambda k_t + \overline{x}(k_t) \geq k_{t+1} \geq \lambda k_t$$

- As before, $q_0 = \nu'(k_0) > 0$ yields positive competitive rents

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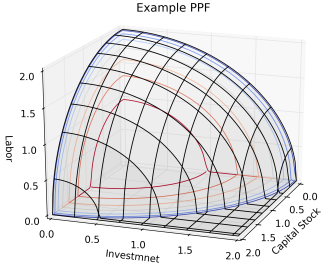

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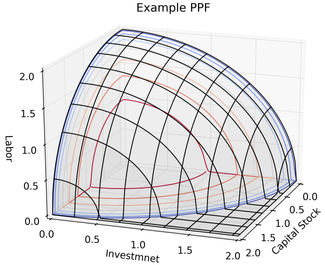

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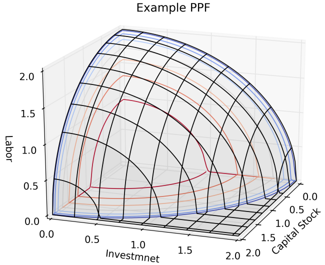

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Special Case

Pills and Reverse Engineering is offered as a special case of the general model.

- This special case fits their idea of competitive rents in open source.
- c_t = services, h_t = expertise

Open Source Innovation and Selling Expertise

- Investment: $x_t = G(L_t)$
  (labor is chosen according to $L_t = g(x_t)$)
- Consumption (services): $c_t = f(h_t)$
- Law of motion:
  $$h_{t+1} = x_t + (1 - \delta)h_t$$
- Innovator starts with h_0
  - Instantly, others can begin accumulating productive capacity (expertise in the the software)

$$x_0^* = G(L_0^*)$$

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  $$\nu(h_t) = \max_{x_t \geq 0}\{u(c_t) - wg(x_t) + \beta\nu(h_{t+1})\}$$
- First order condition:
  $$wg'(x_t) = \beta\nu'(h_{t+1})$$
- This can be decentralized with prices $\{p_t, q_t\}$ for services and capital
  - $p_t = u'(c_t)$
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Open source innovation and selling expertise

- Rearranging:

$$q_0 = \sum_{t=0}^{\infty} (\beta(1-\delta))^t u'(c_t) f'(c_t)$$

- The open source innovation is viable as long as $q_0 k_0 > C$
- Perhaps more elucidating:

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- Optimal *state-dependent* Intellectual Property Rights policy in a dynamic environment.
- Standard tradeoff between monopoly distortions and motivation.
- Novel motivation for leaders.

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Previous Research

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- Mechanism design approach. Menu of patents and fees.
- Step-by-step innovation (Aghion, Harris, and Vickers 1997) — Higher growth from stiffer competition.

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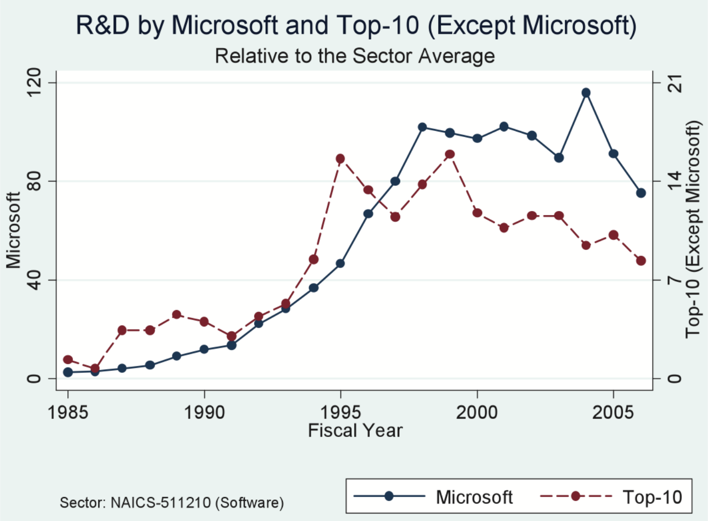
R&D by Microsoft and Top-10 (Except Microsoft)
Relative to the Sector Average
Microsoft
Top-10 (Except Microsoft)
0
40
80
120
0
7
14
21
1985
1990
1995
2000
2005
Fiscal Year
Sector: NAICS-511210 (Software)
Microsoft
Top-10

Preview

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- Final goods producer aggregates.
- Intermediate producers hiring labor and innovating.
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- Single final good. Continuum of 1 individuals.

$$\mathbb{E}_t \int_t^{\infty} exp(-\rho(s-t)) \ln C(s) ds$$

where ρ is the discount factor.

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- Production of $Y(t)$:

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$$x_i(j,t) = F(h_i(j,t))$$

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- Patents expire at Poisson rate: $\eta_{n_j}(t)$.
- Law of motion for technology gap in industry j:

$$\eta_j(t+\Delta t) = \begin{cases} \eta_j(t)+1 & \text{prob } x_i(j,t)\Delta t + o(\Delta t) \\ 0 & \text{prob } x_{-i}(j,t)\Delta t + \eta_{n_{j(t)}}\Delta t + o(\Delta t) \\ \eta_j(t) & \text{with the remainder} \end{cases}$$

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- $\boldsymbol{\mu}(t) \equiv \mu_n(t)_{n=0}^{\infty}$ is a distribution of *industries* over *technology gaps.*
- Loosely define an ALLOCATION as a sequence of decisions for leaders and followers, sequence of wage rates, and a sequence of distributions over gaps.
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Labor Market

- Three sources of demand: Production of intermediaries, and R&D by each firm.
- Combine the demand for intermediates: $y(j,t) = \frac{q_{-i}(j,t)}{w(t)} Y(t)$ with the production function to get:

$$l_n(t) = \frac{\lambda^{-n} Y(t)}{w(t)}$$

and so

$$1 \geq \sum_{n=0}^{\infty} \mu_n(t) \Big[\frac{1}{\omega(t)\lambda^n} + G(x_n(t)) + G(x_{-n}(t)) \Big]$$

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- Net present value when leading by n:

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- "Normalized"value function ($v_n(t) = V_n(t)/Y(t)$):

$$pv_n = \max_{x_n \geq 0} \; (1-\lambda^{-n}) - \omega^* G(x_n) + x_n[v_{n+1} - v_n] + [x^*_{-n} + \eta_n][v_0 - v_n]$$

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Problem is identical for all followers.
Given some assumptions (positive R&D, non-zero profits) ...

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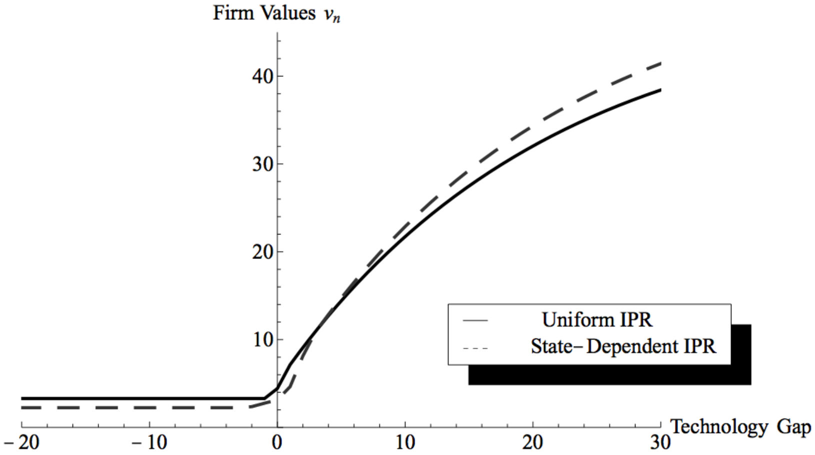

FIGURE 2. Value functions.

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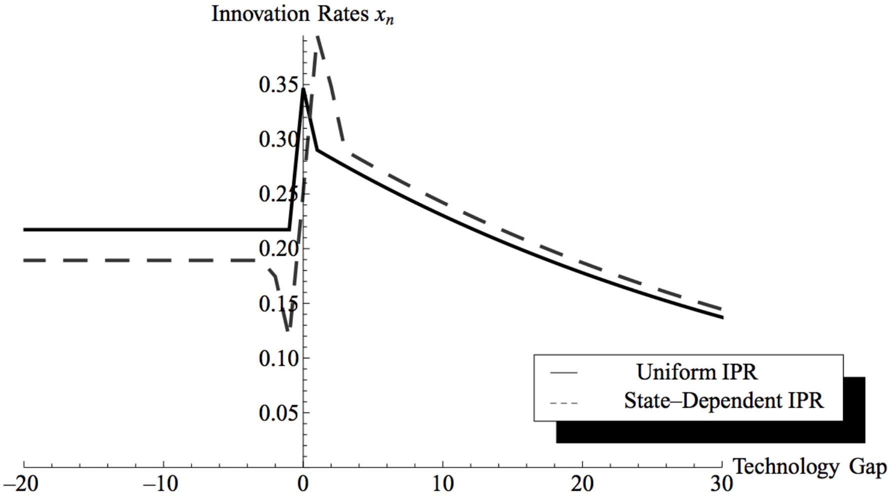

FIGURE 3. R&D efforts.

Comparing Optimal Policies

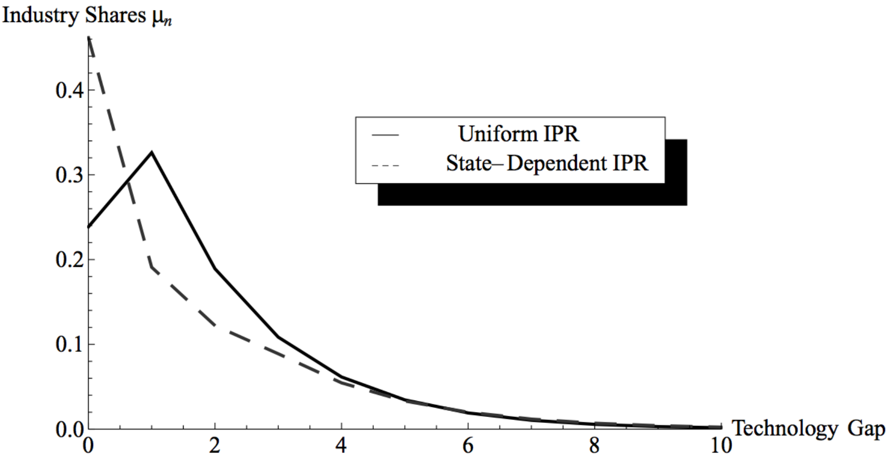

FIGURE 4. Industry shares.

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- Found that stronger protection should be given to those further ahead.

Comparison

- Ask different questions: Are patents necessary for innovation? vs. What the optimal patent policy in this framework?
- Uniform vs. state-dependent.
- Patent policy recommendations:
  - Boldrin-Levine: Not necessary (Acemoglu agrees in his book)
  - Acemoglu-Akcigit: Optimal policy is infinite if uniform, increasing otherwise.

Conclusion

- In the open source spirit, fork away!
  - https://github.com/TomAugspurger/software
  - https://github.com/CalebFloyd/software