Patents and Innovation in Software

Caleb Floyd and Tom Augspurger

March 10, 2013

Outline

- Copyright Law and Patents.
- Boldrin and Levine (2008.)
- Acemoglu and Akcigit (2012.)

Copyright Term Extension Act

- CTEA of 1998
  - Created prior to 1978: 95 year protection.
  - Created after 1978: lifetime of the author plus 70 years.
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    - The Copyright Clause – "limited Times"
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Diamond v. Diehr (1981)

- Prior to 1981 software was effectively not patentable.
- Mathematical formulas in the abstract are not eligible for patent protection.
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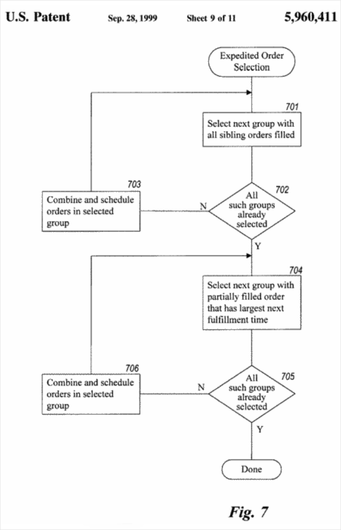
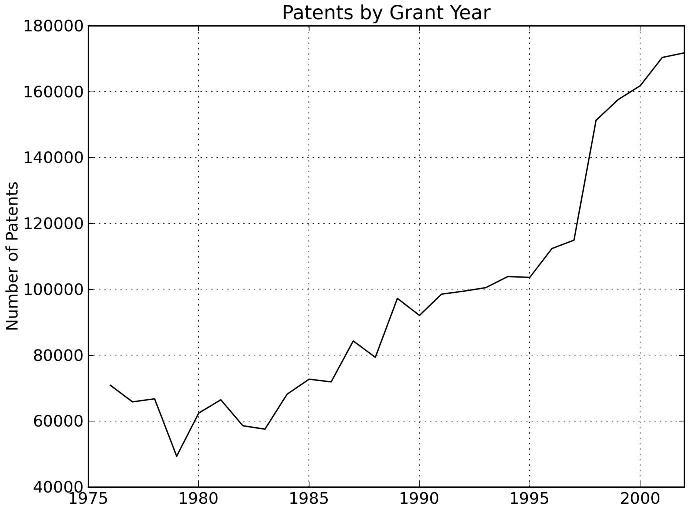
Amazon One-Click Patent

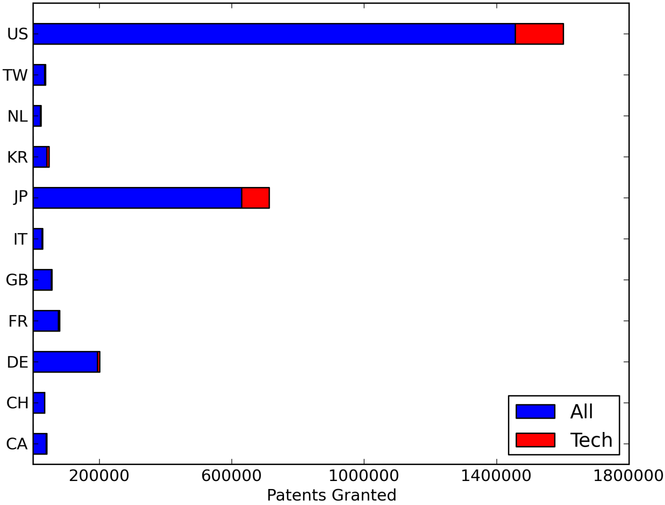
A method and system for placing an order to purchase an item via the Internet. The order is placed by a purchaser at a client system and received by a server system. The server system receives purchaser information including identification of the purchaser, payment information, and shipment information from the client system. The server system then assigns a client identifier to the client system and associates the assigned client identifier with the received purchaser information. The server system sends to the client system the assigned client identifier and an HTML document identifying the item and including an order button. The client system receives and stores the assigned client identifier and receives and displays the HTML document. In response to the selection of the order button, the client system sends to the server system a request to purchase the identified item. The server system receives the request and combines the purchaser information associated with the client identifier of the client system to generate an order to purchase the item in accordance with the billing and shipment information whereby the purchaser effects the ordering of the product by selection of the order button.



Growth in Patent Applications



Patents by Country



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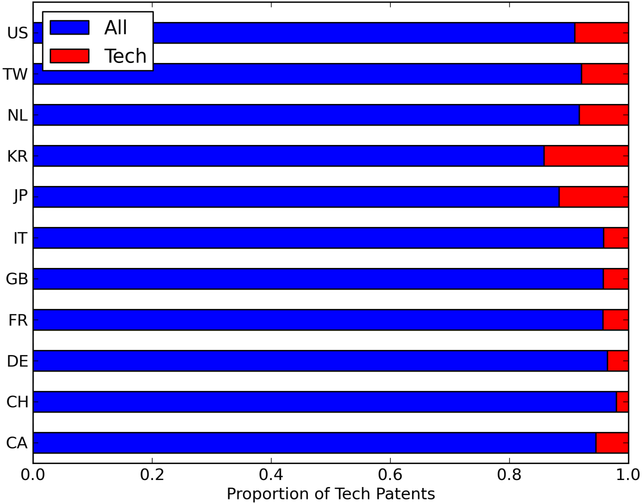

Most Cited Patent

- February 2, 1988: Patent No. 4,723,129.
- Bubble jet recording method and apparatus in which a heating element generates bubbles in a liquid flow path to project droplets.

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- Bubble jet recording method and apparatus in which a heating element generates bubbles in a liquid flow path to project droplets.
- Canon Ink Jet printers.

The evidence (and the common sense of anyone involved with OS software) shows that the source of competitive rents is the complementary sale of expertise.

...only small rents can be obtained through the sale of copies. [Purchasers] also have a demand for services, ranging from support and consulting to customization. They naturally prefer to hire the creators of the programs who in the process of writing the software have developed specialized expertise that is not easily matched by imitators.

- Boldrin & Levine (2009)

Why does open source coexist?

- Control over product performance.
- Hobbyists and enthusiasts.
- Display of skill or resume padding.
  - Hall et. al
- Competitive rents (Boldrin & Levine).
  - Which model version fits?
  - What can we say about the implications?

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Boldrin & Levine

Boldrin & Levine: alternate notation

Table : Alternate Notation

BL		New
δ	$\longrightarrow$	β
β	$\longrightarrow$	λ
ζ	$\longrightarrow$	$1 - \delta$

Boldrin & Levine: General Model Revisited

- Distinguish between productive input and consumption good: $\{k, c\}$.
- $c_t = F(k_t^c, l_t^c)$, $x_t = G(k_t^k, l_t^k)$.
- Agent solves $\sum_{t=0}^{\infty} \beta^t [u(c_t) - wL_t]$:
  - λk_t units available tomorrow without allocating resources for production: $k_{t+1} = \lambda k_t + x_t$.
  - $\lambda > 1$ gives us the 24/7 case.

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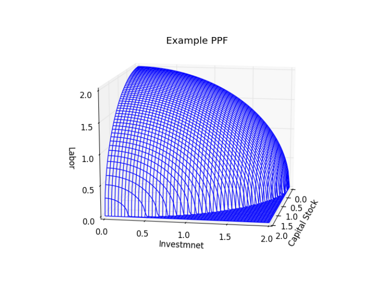
Boldrin & Levine: General Model Revisited

- Given $\{k_t, x_t, L_t\}$, the solution $c_t = T(k_t, x_t, L_t)$ traces a production possibility frontier



- L_t solves $\max_{L_t} u[T(k_t, x_t, L_t)] - wL_t$
- The problem restated:

$$\nu(k_0) = \max_{\{k_t\}_{t=1}^{\infty}} \sum_{t=0}^{\infty} \beta^t V(k_t, k_{t+1} - \lambda k_t)$$

$$s.t. \qquad \lambda k_t + \overline{x}(k_t) \geq k_{t+1} \geq \lambda k_t$$

- As before, $q_0 = \nu'(k_0) > 0$ yields positive competitive rents

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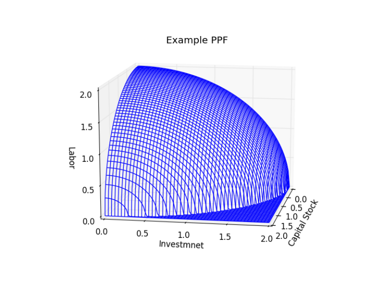

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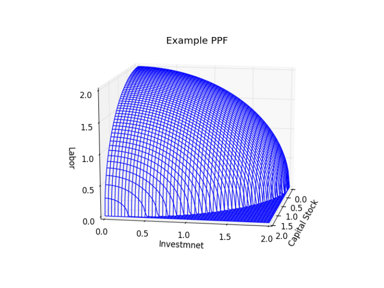

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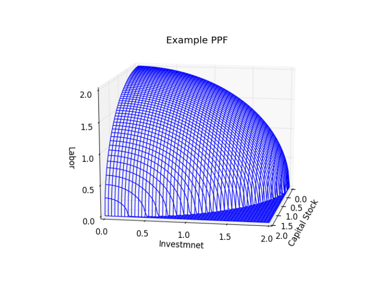

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Open source innovation and selling expertise

- Additional productive capacity only requires labor to be produced: $x_t = G(L_t)$ (labor is chosen according to $L_t = g(x_t)$)
- Consumption (services) is produced from productive capacity $c_t = f(h_t)$
- The innovator comes into the market with productive capacity of h_0
  - As soon as this occurs, others can begin accumulating productive capacity (expertise in the software)
  - $h_{t+1} = x_t + (1 - \delta) * h_t$

Open source innovation and selling expertise

- Consumer utility same as the general case
- Planners Problem:
  $\nu(h_t) = \max_{x_t \geq 0}\{u(c_t) - wg(x_t) + \beta\nu(h_{t+1})\}$
- First order condition:
  $wg'(x_t) = \beta\nu'(h_{t+1})$
- This can be decentralized with prices p_t, q_t for services and capital
  - $p_t = u'(c_t)$
  - $q_t = \nu'(h_t) = u'(c_t)f'(h_t) + \beta(1-\delta)\nu'(h_{t+1})$

Open source innovation and selling expertise

- Rearranging we get:

$$q_0 = \sum_{t=0}^{\infty} (\beta(1-\delta))^t u'(c_t) f'(c_t)$$

- The open source innovation is viable as long as $q_0 k_0 > C$
- Perhaps more elucidating:
$$q_0 = \underbrace{u'(c_0) f'(h_0)}_{\text{first mover advantage}} + \underbrace{(1-\delta) w g'(x_0)}_{\text{cost of imitation}}$$

Intro

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- IPR depends on technology gap in an industry (state-dependence).
- Standard tradeoff between monopoly distortions and motivation.
- Novel motivation for leaders.

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Previous Research

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- Mechanism design approach. Menu of patents and fees.
- Step-by-step innovation (Aghion, Harris, and Vickers 1997) — Higher growth from stiffer competition.

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Consumers' Preferences

- Single final good. Continuum of 1 individuals.

$$\mathbb{E}_t \int_t^{\infty} exp(-\rho(s-t)) \ln C(s) ds$$

where ρ is the discount factor.

- Also supply 1 unit of labor inelastically.
- Also own balanced portfolio of intermediate goods producers.

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Technology-Final Good

- Output of final good: $Y(t) = C(t)$.
- Production of $Y(t)$:

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- Perfect substitutes between intermediate varieties.

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- Each industry $j \in [0, 1]$ has two firms. Firms denoted by i (leader) and $-i$ (follower).
- Intermediate goods produced according to:

$$y(j,t) = q_i(j,t)l_i(j,t)$$

where q_i is a technology level and l_i is labor used.
- Limit pricing:

$$p(j,t) = \frac{w(t)}{q_{-i}(j,t)}$$

- Cobb-Douglas production of final good implies:

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Technology-Innovation

- Innovation follows Poisson process with flow rate:

$$x_i(j,t) = F(h_i(j,t))$$

  where $h_i(j,t)$ is the number of workers in R&D. Also define $G(x_i(j,t)) \equiv F^{-1}(x_i(j,t))$ (R&D employment).
- Successful innovation **by the leader** increments technology by factor $\lambda > 1$.
- Follower innovation: quick catch-up (not patent infringing).
- Technology levels are ladder rungs: $q_i(j,t) = \lambda^{n_{ij}(t)}$, with $n_{ij}(t)$ giving the rung for firm i in industry j.
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- Technology levels are ladder rungs: $q_i(j,t) = \lambda^{n_{ij}(t)}$, with $n_{ij}(t)$ giving the rung for firm i in industry j.
- Technology gap: $n_j(t) = n_{ij}(t) - n_{-ij}(t)$

Patent Policy

- Patents expire at Poisson rate: $\eta_{n_j}(t)$.
- Law of motion for technology gap in industry j:

$$\eta_j(t+\Delta t) = \begin{cases} \eta_j(t)+1 & \text{prob } x_i(j,t)\Delta t + o(\Delta t) \\ 0 & \text{prob } x_{-i}(j,t)\Delta t + \eta_{n_{j(t)}}\Delta t + o(\Delta t) \\ \eta_j(t) & \text{with the remainder} \end{cases}$$

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- $\boldsymbol{\mu}(t) \equiv \mu_n(t)_{n=0}^{\infty}$ is a distribution of *industries* over *technology gaps.*
- Loosely define an ALLOCATION as a sequence of decisions for leaders and followers, sequence of wage rates, and a sequence of distributions over gaps.
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Labor Market

- Three sources of demand: Production of intermediaries, and R&D by each firm.
- Combine demand for intermediates: $y(j,t) = q_i(j,t)l_i(j,t)$, and $y(j,t) = \frac{q_{-i}(j,t)}{w(t)}Y(t)$ to get

$$l_n(t) = \frac{\lambda^{-n}Y(t)}{w(t)}$$

and so

$$1 \geq \sum_{n=0}^{\infty} \mu_n(t)\Big[\frac{1}{\omega(t)\lambda^n} + G(x_n(t)) + G(x_{-n}(t))\Big]$$

where $\omega(t)$ is labor's share of income.

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Firm's Value Function

- Net present value when leading by n:

$$V_n(t) = \mathbb{E}_t \int_t^{\infty} exp(-r(s-t))[\Pi(s) - w(s)G(\hat{x}(s))]\, ds$$

- As a "normalized" value function ($v_n(t) = V_n(t)/Y(t)$):

$$pv_n = \max_{x_n \geq 0} \; (1-\lambda^{-n}) - \omega^* G(x_n) + x_n[v_{n+1} - v_n] + [x^*_{-n} + \eta_n][v_0 - v_n]$$

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Problem is identical for all followers.
Given some assumptions (positive R&D, non-zero profits) ...

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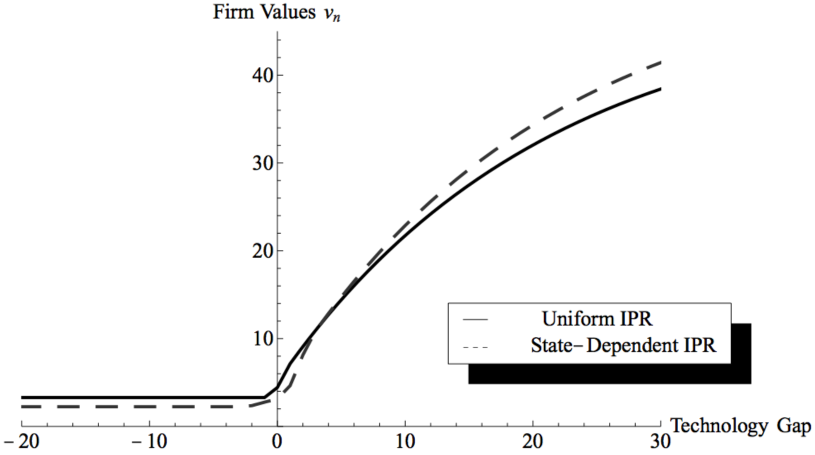

FIGURE 2. Value functions.

Full IPR

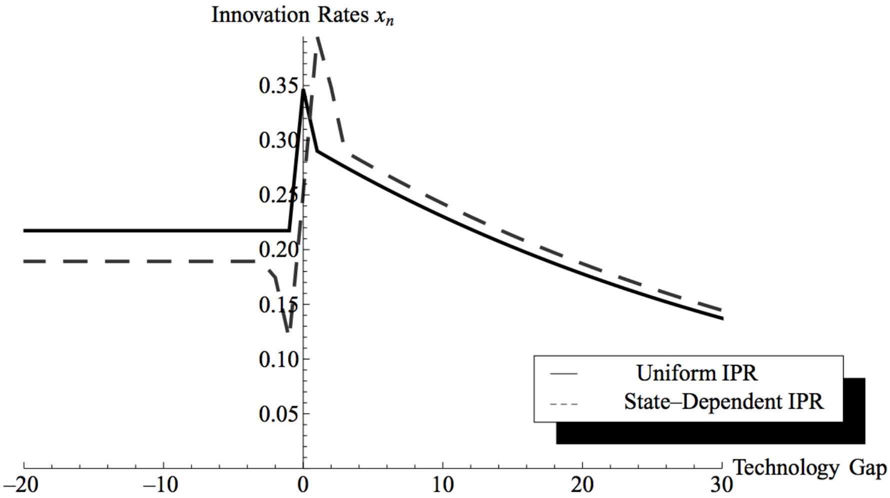

FIGURE 3. R&D efforts.

Full IPR

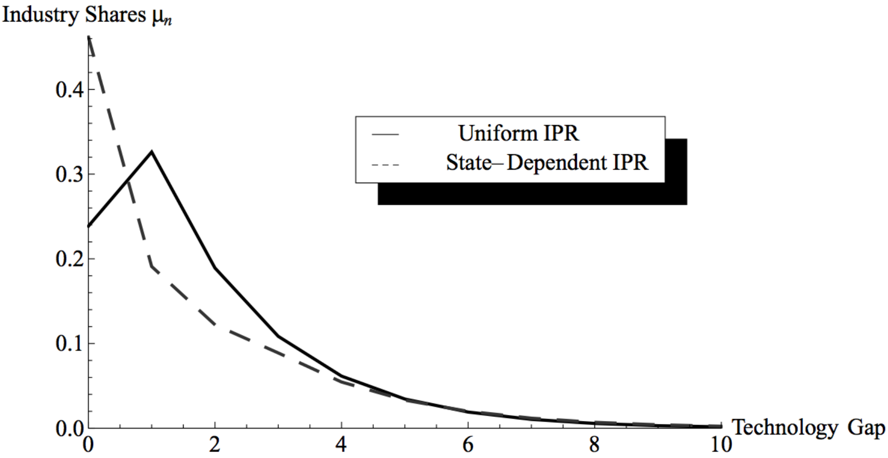

FIGURE 4. Industry shares.

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