Patents and Innovation in Software

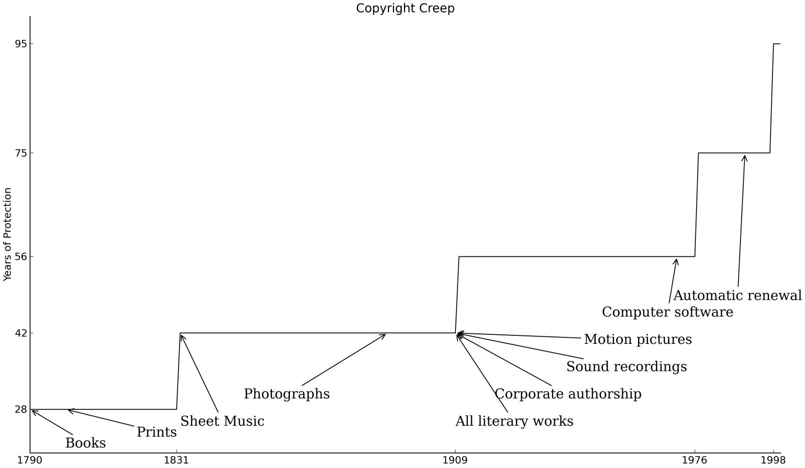
Tom Augspurger and Caleb Floyd

March 12, 2013

Outline

- Copyright Law and Patents.
- Boldrin and Levine (2008.)
- Acemoglu and Akcigit (2012.)

The Copyright Creep



Copyright Term Extension Act

- CTEA of 1998
  - Created prior to 1978: 95 year protection.
  - Created after 1978: lifetime of the author plus 70 years.
  - Challenged on grounds of:
    - The Copyright Clause – "limited Times"
    - The First Amendment
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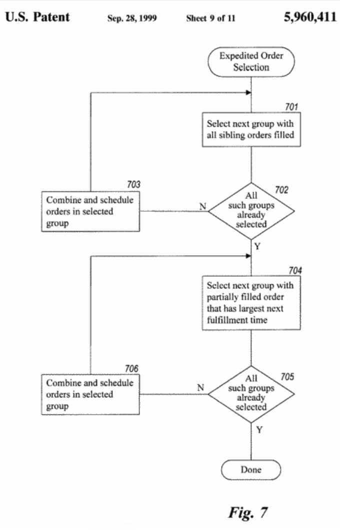
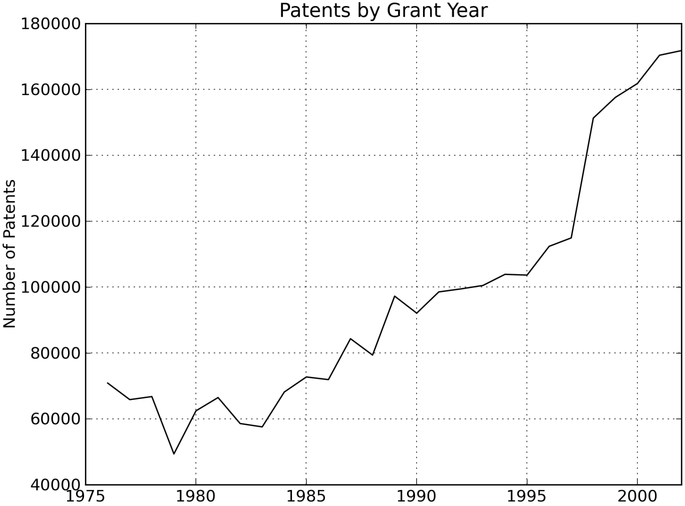
Amazon One-Click Patent

A method and system for placing an order to purchase an item via the Internet. The order is placed by a purchaser at a client system and received by a server system. The server system receives purchaser information including identification of the purchaser, payment information, and shipment information from the client system. The server system then assigns a client identifier to the client system and associates the assigned client identifier with the received purchaser information. The server system sends to the client system the assigned client identifier and an HTML document identifying the item and including an order button. The client system receives and stores the assigned client identifier and receives and displays the HTML document. In response to the selection of the order button, the client system sends to the server system a request to purchase the identified item. The server system receives the request and combines the purchaser information associated with the client identifier of the client system to generate an order to purchase the item in accordance with the billing and shipment information whereby the purchaser effects the ordering of the product by selection of the order button.



Growth in Patent Applications



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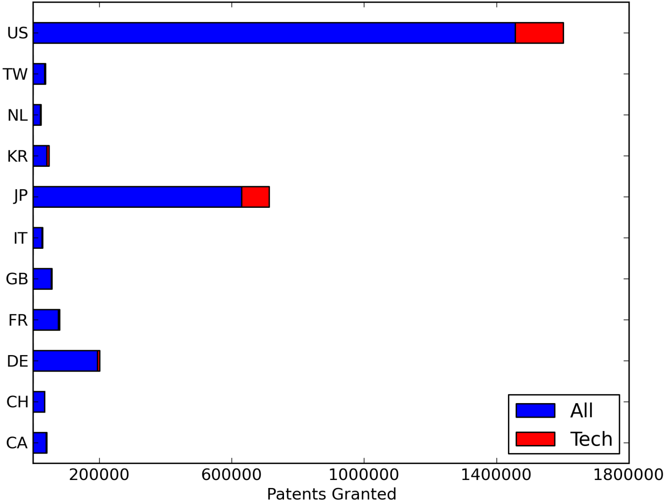
Patents by Country



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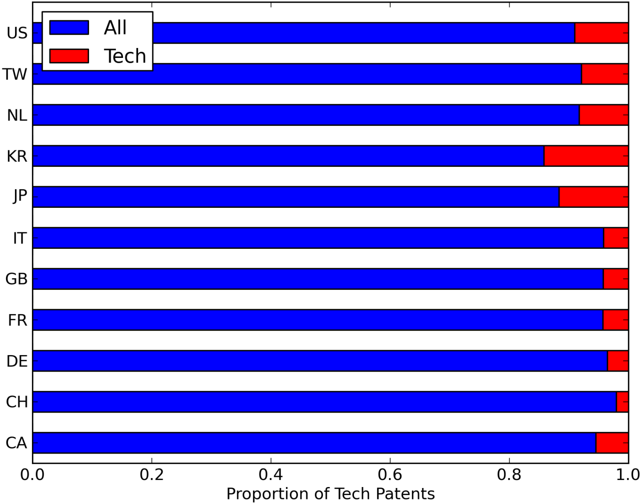

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- Canon Ink Jet printers.

Why does open source coexist?

- Control over product performance.
- Hobbyists and enthusiasts.
- Display of skill or resume padding.
  - Hann et. al (2004)
- Competitive rents (Boldrin & Levine).
  - Which model version fits?
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The evidence (and the common sense of anyone involved with OS software) shows that the source of competitive rents is the complementary sale of expertise.

...only small rents can be obtained through the sale of copies. [Purchasers] also have a demand for services, ranging from support and consulting to customization. They naturally prefer to hire the creators of the programs who in the process of writing the software have developed specialized expertise that is not easily matched by imitators.

- Boldrin & Levine (2009)

Boldrin & Levine

Boldrin & Levine: alternate notation

Table : Alternate Notation

BL		New
δ	$\longrightarrow$	β
β	$\longrightarrow$	λ
ζ	$\longrightarrow$	$1-\delta$

Boldrin & Levine: General Model Revisited

- Distinguish between productive input and consumption good: $\{k, c\}$.
- $c_t = F(k_t^c, l_t^c)$, $x_t = G(k_t^k, l_t^k)$.

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- Agent solves $\sum_{t=0}^{\infty} \beta^t [u(c_t) - wL_t]$:
  - λk_t units available tomorrow: $k_{t+1} = \lambda k_t + x_t$.
  - $\lambda > 1$ gives us the 24/7 case.

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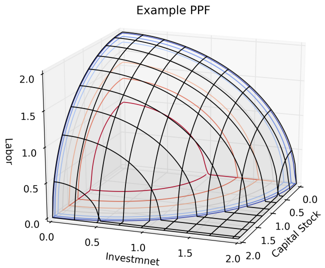
- Given $\{k_t, x_t, L_t\}$, the solution $c_t = T(k_t, x_t, L_t)$ traces a production possibility frontier



- L_t solves $\max_{L_t} u[T(k_t, x_t, L_t)] - wL_t$
- The problem restated:

$$\nu(k_0) = \max_{\{k_t\}_{t=1}^{\infty}} \sum_{t=0}^{\infty} \beta^t V(k_t, k_{t+1} - \lambda k_t)$$

$$s.t. \qquad \lambda k_t + \overline{x}(k_t) \geq k_{t+1} \geq \lambda k_t$$

- As before, $q_0 = \nu'(k_0) > 0$ yields positive competitive rents

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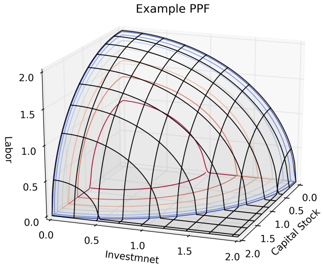

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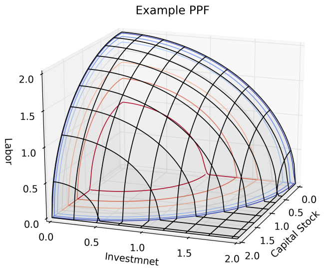

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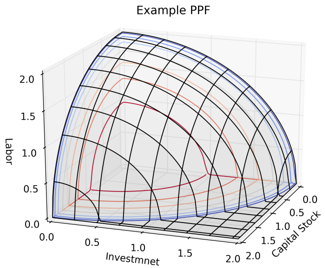

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Open source innovation and selling expertise

- Investment: $x_t = G(L_t)$
  (labor is chosen according to $L_t = g(x_t)$)
- Consumption (services): $c_t = f(h_t)$
- The innovator starts with h_0
  - As soon as this occurs, others can begin accumulating productive capacity (expertise in the software)
  - $h_{t+1} = x_t + (1 - \delta) * h_t$

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- Planners Problem:
  $\nu(h_t) = \underset{x_t \geq 0}{max}\{u(c_t) - wg(x_t) + \beta\nu(h_{t+1})\}$
- First order condition:
  $wg'(x_t) = \beta\nu'(h_{t+1})$
- This can be decentralized with prices $\{p_t, q_t\}$ for services and capital
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- Rearranging:

$$q_0 = \sum_{t=0}^{\infty} (\beta(1-\delta))^t u'(c_t) f'(c_t)$$

- The open source innovation is viable as long as $q_0 k_0 > C$
- Perhaps more elucidating:

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- Standard tradeoff between monopoly distortions and motivation.
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- Mechanism design approach. Menu of patents and fees.
- Step-by-step innovation (Aghion, Harris, and Vickers 1997) — Higher growth from stiffer competition.

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Consumers' Preferences

- Single final good. Continuum of 1 individuals.

$$\mathbb{E}_t \int_t^\infty exp(-\rho(s-t))\ln C(s)ds$$

where ρ is the discount factor.
- Supply 1 unit of labor.
- Own balanced portfolio of intermediate goods producers.

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Technology-Final Good

- Output of final good: $Y(t) = C(t)$.
- Production of $Y(t)$:

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where $y(j,t)$ is the quantity of intermediate good j used.
- Perfect substitutes between intermediate varieties.

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Technology-Intermediate Good

- Each industry $j \in [0,1]$ has two firms. Firms denoted by i (leader) and $-i$ (follower).
- Output:

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$$x_i(j,t) = F(h_i(j,t))$$

  where $h_i(j,t)$ is the number of workers in R&D. Also define $G(x_i(j,t)) \equiv F^{-1}(x_i(j,t))$ (R&D employment).
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- Patents expire at Poisson rate: $\eta_{n_j}(t)$.
- Law of motion for technology gap in industry j:

$$\eta_j(t+\Delta t) = \begin{cases} \eta_j(t)+1 & \text{prob } x_i(j,t)\Delta t + o(\Delta t) \\ 0 & \text{prob } x_{-i}(j,t)\Delta t + \eta_{n_{j(t)}}\Delta t + o(\Delta t) \\ \eta_j(t) & \text{with the remainder} \end{cases}$$

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- $\boldsymbol{\mu}(t) \equiv \mu_n(t)_{n=0}^{\infty}$ is a distribution of *industries* over *technology gaps*.
- Loosely define an ALLOCATION as a sequence of decisions for leaders and followers, sequence of wage rates, and a sequence of distributions over gaps.
- Loosely define an EQUILIBRIUM as a sequence of decisions, wages, and output such that markets clear, firms' expected profits are maximized, and R&D policies are best responses.

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Labor Market

- Three sources of demand: Production of intermediaries, and R&D by each firm.
- Combine the demand for intermediates: $y(j,t) = \frac{q_{-i}(j,t)}{w(t)} Y(t)$ with the production function to get:

$$l_n(t) = \frac{\lambda^{-n} Y(t)}{w(t)}$$

and so

$$1 \geq \sum_{n=0}^{\infty} \mu_n(t) \Big[\frac{1}{\omega(t)\lambda^n} + G(x_n(t)) + G(x_{-n}(t)) \Big]$$

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Firm's Value Function

- Net present value when leading by n:

$$V_n(t) = \mathbb{E}_t \int_t^\infty exp(-r(s-t))[\Pi(s) - w(s)G(\hat{x}(s))]\, ds$$

- "Normalized" value function ($v_n(t) = V_n(t)/Y(t)$):

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Given some assumptions (positive R&D, non-zero profits) ...

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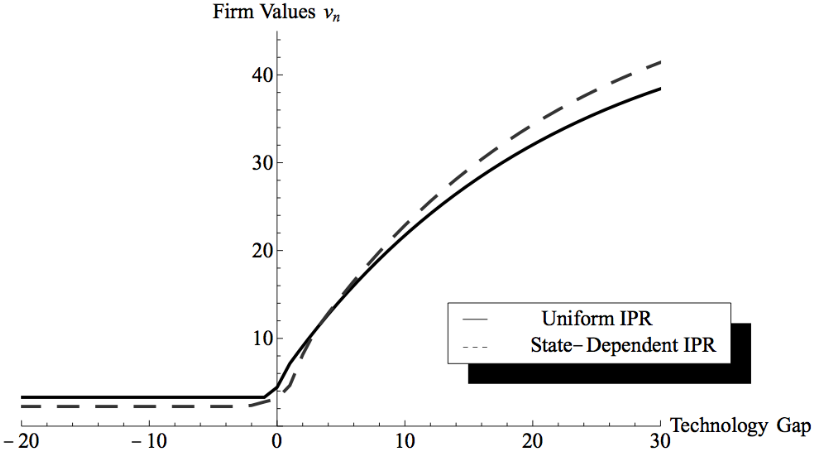

FIGURE 2. Value functions.

Full IPR

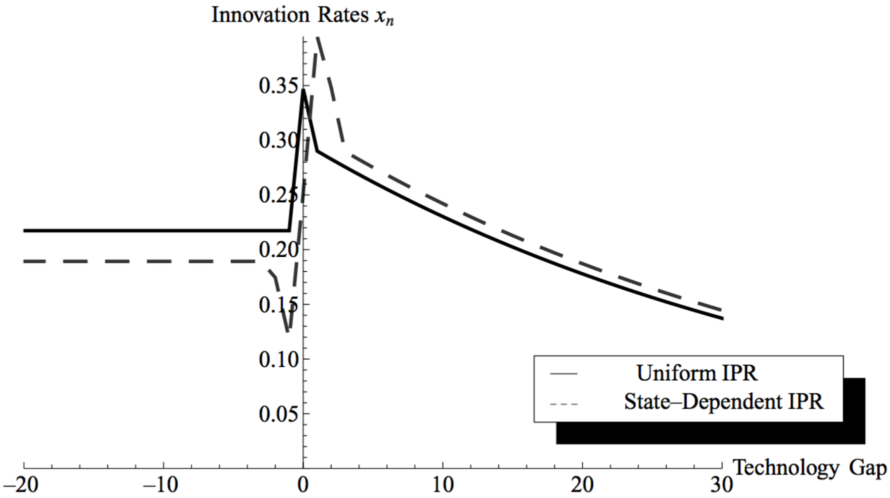

FIGURE 3. R&D efforts.

Full IPR

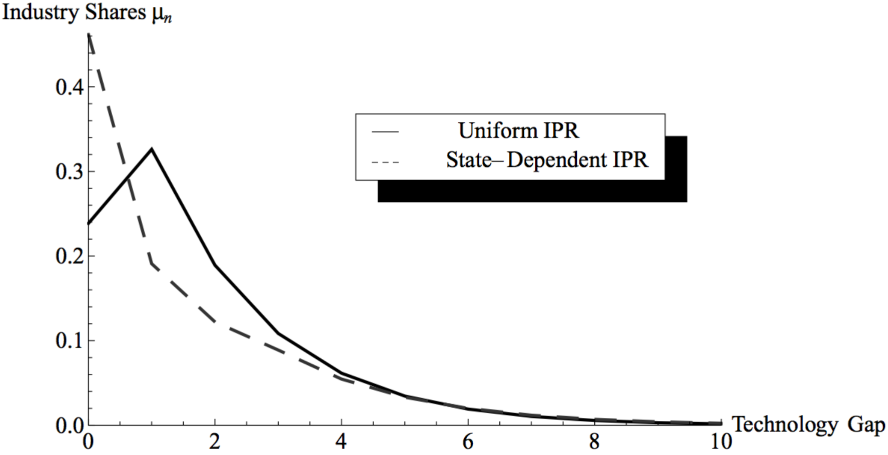

FIGURE 4. Industry shares.

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Comparision

Conclusion

- In the open source spirit: https://github.com/TomAugspurger/software