Patents and Innovation in Software

Tom Augspurger and Caleb Floyd

March 12, 2013

This Project

- ▶ Investigate the state of intellectual property protection.
 - ► Economic and social implications
- ► Search for theories that would allow us to analyze IP protection in the software industry.
 - ► Find commonalities in competing theories.
 - ► Search for existing empirical results.
 - ► Investigate the existence of the open source phenomenon.

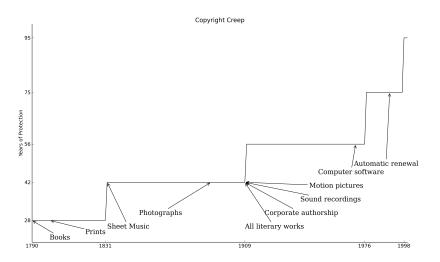
Where are we?

- ► Some theories are amenable; some aren't.
- ▶ One example where the open source phenomenon fits.
- ► One example where it doesn't.

Outline

- ► Copyright Law and Patents.
- ▶ Boldrin and Levine (2008).
- ► Acemoglu and Akcigit (2012).

The Copyright Creep



Ethics for the Information Age (2ND Edition) Micheal J. Quinn

- ► CTEA of 1998
 - ► Created prior to 1978: 95 year protection.
 - ▶ Created after 1978: lifetime of the author plus 70 years.
 - ► Challenged on grounds of:
 - ► The Copyright Clause "limited Times"
 - ► The First Amendment
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- ▶ However, a physical machine or process which makes use of a mathematical algorithm is different from an invention which claims the algorithm in the abstract.
- ► Software is effectively patentable.

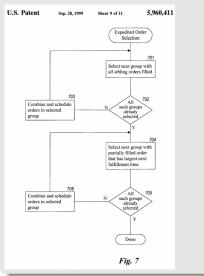
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Amazon One-Click Patent

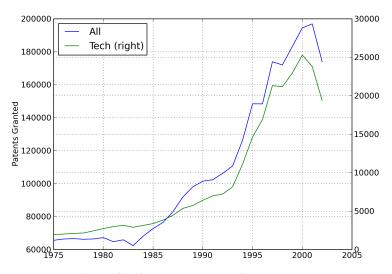
A method and system for placing an order to purchase an item via the Internet. The order is placed by a purchaser at a client system and received by a server system. The server system receives purchaser information including identification of the purchaser, payment information, and shipment information from the client system. The server system then assigns a client identifier to the client system and associates the assigned client identifier with the received purchaser information. The server system sends to the client system the assigned client identifier and an HTML document identifying the item and including an order button. The client system receives and stores the assigned client identifier and receives and displays the HTML document. In response to the selection of the order button, the client system sends to the server system a request to purchase the identified item. The server system receives the request and combines the purchaser information associated with the client identifier of the client system to generate an order to purchase the item in accordance with the billing and shipment information whereby the purchaser effects the ordering of the product by selection of the order button.



Some Pictures

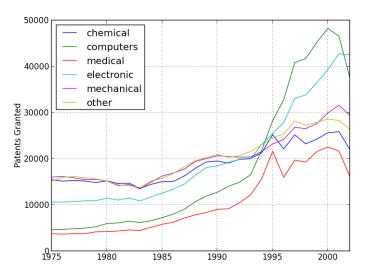
- ▶ NBER Patent Data File. Hall, Jaffe, Trajtenberg (2001).
- ▶ Patents granted from 1963 1999. (Since updated).
- ▶ Date, industry, citations, and company.

Growth in Patent Applications



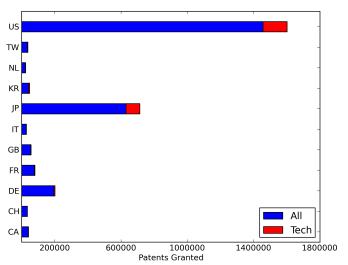
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By Industry



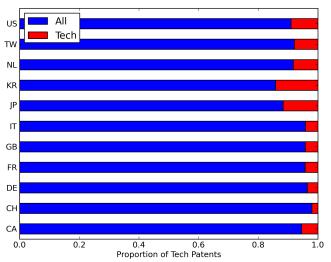
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Patents by Country



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- ► Canon Ink Jet printers.

- ► Control over product performance.
- ▶ Hobbyists and enthusiasts.
- ▶ Display of skill or resume padding.
 - ▶ Hann et. al (2004)
 - ▶ Lerner and Tirole (2002)
- ► Competitive rents (Boldrin & Levine).
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The evidence (and the common sense of anyone involved with OS software) shows that the source of competitive rents is the complementary sale of expertise.

...only small rents can be obtained through the sale of copies. [Purchasers] also have a demand for services, ranging from support and consulting to customization. They naturally prefer to hire the creators of the programs who in the process of writing the software have developed specialized expertise that is not easily matched by imitators.

- Boldrin & Levine (2009)

Boldrin & Levine

Boldrin & Levine: alternate notation

Table: Alternate Notation

BL		New
δ	\longrightarrow	β
β	\longrightarrow	λ
ζ	\longrightarrow	$1 - \delta$

- ▶ Distinguish between productive input and consumption good: $\{k, c\}$.
- $ightharpoonup c_t = F(k_t^c, l_t^c), x_t = G(k_t^k, l_t^k).$
- ▶ Agent solves $\sum_{t=0}^{\infty} \beta^t [u(c_t) wL_t]$:
 - $\triangleright \lambda k_t$ units available tomorrow: $k_{t+1} = \lambda k_t + x_t$.
 - $\triangleright \lambda > 1$ gives us the 24/7 case

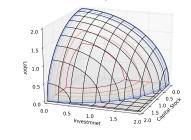
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► Given $\{k_t, x_t, L_t\}$, the solution $c_t = T(k_t, x_t, L_t)$ traces a production possibility frontier



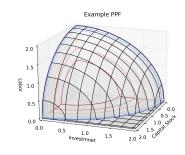
Example PPF

- $ightharpoonup L_t$ solves $\max_{L_t} u[T(k_t, x_t, L_t)] wL_t$
- ► The problem restated:

$$\nu(k_0) = \max_{\{k_t\}_{t=1}^{\infty}} \sum_{t=0}^{\infty} \beta^t V(k_t, k_{t+1} - \lambda k_t)$$
s.t.
$$\lambda k_t + \overline{x}(k_t) \ge k_{t+1} \ge \lambda k_t$$

▶ As before, $q_0 = \nu'(k_0) > 0$ yields positive competitive rents

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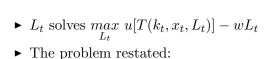


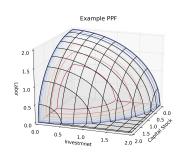
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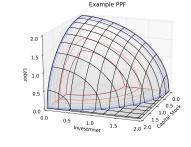


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- ► Investment: $x_t = G(L_t)$ (labor is chosen according to $L_t = g(x_t)$)
- ▶ Consumption (services): $c_t = f(h_t)$
- ▶ The innovator starts with h_0
 - As soon as this occurs, others can begin accumulating productive capacity (expertise in the software)
 - $h_{t+1} = x_t + (1 \delta) * h_t$

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- ▶ Planners Problem:

$$\nu(h_t) = \max_{x_t \ge 0} \{ u(c_t) - wg(x_t) + \beta \nu(h_{t+1}) \}$$

▶ First order condition:

$$wg'(x_t) = \beta \nu'(h_{t+1})$$

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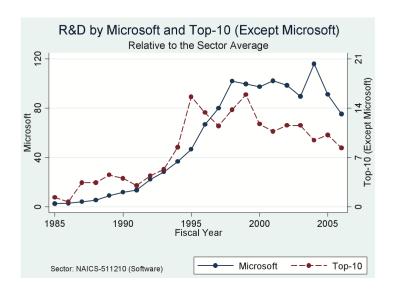
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Preview

- ► Consumers eating final good. Supplying Labor.
- ► Final goods producer aggregates.
- ► Intermediate producers hiring labor and innovating.
- ► State-dependent patent policy will motivate monopolists.
- ► Describe equilibrium vaguely.
- ▶ Numerical example to compare optimal policies.

Motivation



Consumers' Preferences

► Single final good. Continuum of 1 individuals.

$$\mathbb{E}_t \int_t^\infty exp(-\rho(s-t)) \ln C(s) ds$$

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- ▶ Leader innovation: technology \uparrow by factor $\lambda > 1$.
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Patent Policy

- ▶ Patents expire at Poisson rate: $\eta_{n_j}(t)$.
- \blacktriangleright Law of motion for technology gap in industry j:

$$\eta_j(t + \Delta t) = \begin{cases} \eta_j(t) + 1 & \text{prob } x_i(j, t) \Delta t + o(\Delta t) \\ 0 & \text{prob } x_{-i}(j, t) \Delta t + \eta_{n_{j(t)}} \Delta t + o(\Delta t) \\ \eta_j(t) & \text{with the remainder} \end{cases}$$

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Equilibrium

- ▶ $\mu(t) \equiv \mu_n(t)_{n=0}^{\infty}$ is a distribution of *industries* over *technology gaps*.
- ▶ Loosely define an Allocation as a sequence of decisions for leaders and followers, sequence of wage rates, and a sequence of distributions over gaps.
- ▶ Loosely define an EQUILIBRIUM as a sequence of decisions, wages, and output such that markets clear, firms' expected profits are maximized, and R&D policies are best responses.

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Labor Market

- ► Three sources of demand: Production of intermediaries, and R&D by each firm.
- ▶ Combine the demand for intermediates: $y(j,t) = \frac{q_{-i}(j,t)}{q_{n}(t)}Y(t)$ with

$$l_n(t) = \frac{\lambda^{-n}Y(t)}{w(t)}$$

$$1 \ge \sum_{n=0}^{\infty} \mu_n(t) \left[\frac{1}{\omega(t)\lambda^n} + G(x_n(t)) + G(x_{-n}(t)) \right]$$

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where $\omega(t)$ is labor's share of income.

▶ Net present value when leading by n:

$$V_n(t) = \mathbb{E}_t \int_t^\infty exp(-r(s-t))[\Pi(s) - w(s)G(\hat{x}(s))] ds$$

$$pv_n = \max_{x_n \ge 0} (1 - \lambda^{-n}) - \omega^* G(x_n) + x_n [v_{n+1} - v_n] + [x_{-n}^* + \eta_n] [v_0 - v_n]$$

- ▶ Instantaneous operating profits: $(1 \lambda^{-n})$.
- ▶ R&D costs: $\omega^*(t)G(x_n(t))$.
- ▶ With probability $x_n(t)$ you innovate.
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Uniform Policy

Problem is identical for all followers.

Given some assumptions (positive R&D, non-zero profits) . . .

- ▶ $v_{-1} \le v_0$; $\{v_n\}_{n=0}^{\infty}$ is bounded and strictly increasing.
- $x_0^* > x_1^*, x_0^* \ge x_{-1}^*, \text{ and } x_{n+1}^* \le x_n^* \ \forall n \in \mathbb{N}.$
- ▶ Tied firms innovate the most.
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- ▶ Turns out that if $\eta_n = \eta \ \forall n, \ \eta^* = 0$ (Patents never expire).
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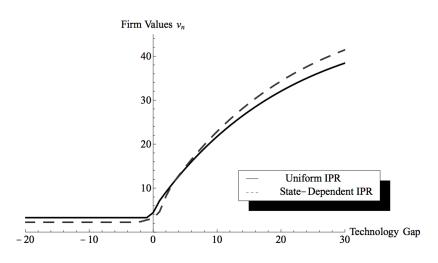


FIGURE 2. Value functions.

Full IPR

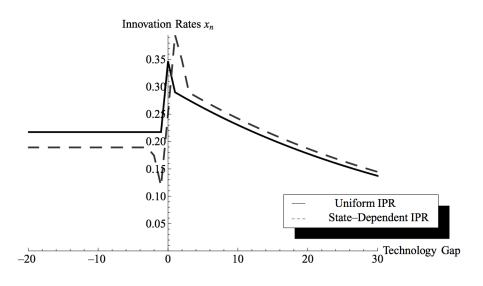


FIGURE 3. R&D efforts.

Full IPR

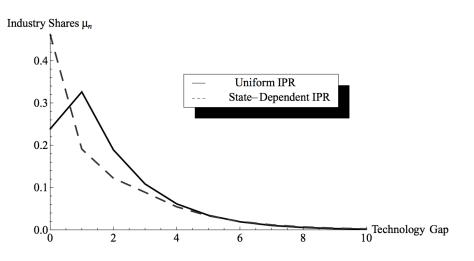


FIGURE 4. Industry shares.

Summary

- ► State-Dependent patent policy to motive all producers to innovate.
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Comparison

- ► Ask different questions: Are patents necessary for innovation? vs. What the optimal patent policy in this framework?
- ▶ Uniform vs. state-dependent.
- ► Patent policy recommendations:
 - ▶ Boldrin-Levine: Not necessary (Acemoglu agrees in his book)
 - ▶ Acemoglu-Akcigit: Optimal policy is infinite if uniform, increasing otherwise.

Conclusion

- - In the open source spirit:
 - ► https://github.com/TomAugspurger/software
 - $\blacktriangleright \ \, \rm https://github.com/CalebFloyd/software$