Copyright Term Extension Act

- CTEA of 1998
  - Created prior to 1978: 95 year protection
  - Created after 1978: lifetime of the author plus 70 years
  - Challenged on grounds of
    - Copyright Clause – "limited Times"
    - The First Amendment
    - The public trust doctrine
  - Upheld in *Eldred v. Ashcroft* by SCOTUS (January 15th, 2003)

Diamond v. Diehr (1981)

- Prior to 1981 software was effectively not patentable
- Mathematical formulas in the abstract are not eligible for patent protection
- However, a physical machine or process which makes use of a mathematical algorithm is different from an invention which claims the algorithm in the abstract
- Hence software is deemed patentable as it's an implementation of an algorythm

Amazon One-Click Patent

A method and system for placing an order to purchase an item via the Internet. The order is placed by a purchaser at a client system and received by a server system. The server system receives purchaser information including identification of the purchaser, payment information, and shipment information from the client system. The server system then assigns a client identifier to the client system and associates the assigned client identifier with the received purchaser information. The server system sends to the client system the assigned client identifier and an HTML document identifying the item and including an order button. The client system receives and stores the assigned client identifier and receives and displays the HTML document. In response to the selection of the order button, the client system sends to the server system a request to purchase the identified item. The server system receives the request and combines the purchaser information associated with the client identifier of the client system to generate an order to purchase the item in accordance with the billing and shipment information whereby the purchaser effects the ordering of the product by selection of the order button.

Amazon.png

Why does open source coexist?

- control over product performance
- hobbyists/enthusiasts
- display of skill/resume padding
  - Hall et. al
- competitive rents (Boldrin & Levine)
  - Which model version fits?
  - What can we say about the implications?

The evidence (and the common sense of anyone involved with OS software) shows that the source of competitive rents is the complementary sale of expertise.

...only small rents can be obtained through the sale of copies. [Purchasers] also have a demand for services, ranging from support and consulting to customization. They naturally prefer to hire the creators of the programs who in the process of writing the software have developed specialized expertise that is not easily matched by imitators.

- Boldrine & Levine (2009)

Intro

- Daron Acemogllu and Ufuk Akcigit (2012) - Intellectual Property Rights Policy, Competition And Innovation.
- Optimal *state-dependent* Intellectual Property Rights policy in a dynamic environment.
- IPR depends on technology gap in an industry (state-dependence).
- Standard tradeoff between monopoly distortions and motivation.
- Novel motivation for leaders.

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Previous Research

- *Static* tradeoff between R&D incentive and monopoly distortions. Mixed conclusions.
- Mechanism design approach. Menu of patents and fees.
- Step-by-step innovation (Aghion, Harris, and Vickers 1997) — Higher growth from stiffer competition.

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Consumers' Preferences

- Single final good. Continuum of 1 individuals.

$$\mathbb{E}_t \int_t^{\infty} exp(-\rho(s-t)) \ln C(s) ds$$

  where ρ is the discount factor.

- Also supply 1 unit of labor inelastically.
- Also own balanced portfolio of intermediate goods producers.

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Technology-Final Good

- Output of final good: $Y(t) = C(t)$.
- Production of $Y(t)$:

$$\ln Y(t) = \int_0^1 \ln y(j,t) dj$$

where $y(j,t)$ is the quantity of intermediate good j used.
- Perfect substitutes between intermediate varieties.

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Technology-Intermediate Good

- Each industry $j \in [0,1]$ has two firms competing. Firms denoted by i (leader) and $-i$ (follower).
- Intermediate goods produced according to:

$$y(j,t) = q_i(j,t) l_i(j,t)$$

where q_i is a technology level and l_i is labor used.
- Yields marginal cost:

$$MC_i(j,t) = \frac{w(t)}{q_i(j,t)}$$

- Limit pricing:

$$p(j,t) = \frac{w(t)}{q_{-i}(j,t)}$$

where $-i$ denotes the follower (less advanced technology).
- Cobb-Douglas production of final good implies:

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Technology-Innovation

- Innovation follows Poisson process with flow rate:

$$x_i(j,t) = F(h_i(j,t))$$

  where $h_i(j,t)$ is the number of workers employed in R&D. Also define $G(x_i(j,t)) \equiv F^{-1}(x_i(j,t))$ (R&D employment).
- Successful innovation **by the leader** increments technology by factor $\lambda > 1$.
- If the follower innovates, he catches up with the leader (Not patent infringing).
- Technology levels are ladder rungs: $q_i(j,t) = \lambda^{n_{ij}(t)}$, with $n_{ij}(t)$ giving the rung for firm i in industry j.
- Mainly concerned with the technology gap: $n_j(t) = n_{ij}(t) - n_{-ij}(t)$

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Patent Policy

- Patents expire at Poisson rate: $\eta_{n_j}(t)$
- Preserves stationarity of the value functions.
- Law of motion for technology gap in industry j:

$$\eta_j(t+\Delta t) = \begin{cases} \eta_j(t)+1 & \text{prob } x_i(j,t)\Delta t + o(\Delta t) \\ 0 & \text{prob } x_{-i}(j,t)\Delta t + \eta_{n_{j(t)}}\Delta t + o(\Delta t) \\ \eta_j(t) & \text{with the remainder} \end{cases}$$

- Write the patent policy as $\boldsymbol{\eta} : \mathbb{N} \to \mathbb{R}$.

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Equilibrium

- $\boldsymbol{\mu}(t) \equiv \mu_n(t)_{n=0}^{\infty}$ is a distribution of *industries* over *technology gaps.*
- Define an Allocation as a sequence of decisions for leaders and followers, sequence of wage rates, and a sequence of distributions over gaps.
- Define an Equilibrium as a sequence of decisions, wages, and output such that markets clear, firms' expected profits are maximized, and R&D policies are best responses.

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Labor Market

- Three sources of demand: Production of intermediaries, and R&D by each firm.
- Combine demand for intermediates: $y(j,t) = q_i(j,t) l_i(j,t)$, and $y(j,t) = \frac{q_{-i}(j,t)}{w(t)} Y(t)$ to get

$$l_n(t) = \frac{\lambda^{-n} Y(t)}{w(t)}$$

and so

$$1 \geq \sum_{n=0}^{\infty} \mu_n(t) \left[\frac{1}{\omega(t)\lambda^n} + G(x_n(t)) + G(x_{-n}(t)) \right]$$

where $\omega(t)$ is labor's share of income.

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Firm's Value Function

- Net present value when leading by n:

$$V_n(t) = \mathbb{E}_t \int_t^{\infty} exp(-r(s-t))[\Pi(s) - w(s)G(\hat{x}(s))]\, ds$$

- As a "normalized" value function ($v_n(t) = V_n(t)/Y(t)$):

$$pv_n = \max_{x_n \geq 0} \; (1-\lambda^{-n}) - \omega^* G(x_n) + x_n[v_{n+1} - v_n] + [x^*_{-n} + \eta_n][v_0 - v_n]$$

- Instantaneous operating profits: $(1-\lambda^{-n})$.
- R&D costs: $\omega^*(t)G(x_n(t))$.
- With probability $x_n(t)$ you innovate.
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- Follower's value function:

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Optimization

In equilibrium, R&D policies must follow:

$$x_n^* = max\{G'^{-1}\left(\frac{[v_{n+1} - v_n]}{\omega^*}\right), 0\}$$

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Relationship between v_n and x_n: Say patent protection weakens:

- Disincentive Effect: $\downarrow v_{n+1} \Rightarrow \downarrow v_{n+1} - v_n \Rightarrow \downarrow x_n^*$.
- Incentive Effect: $\downarrow v_{n+1} \Rightarrow \uparrow v_{n+2} - v_{n+1} \Rightarrow \uparrow x_{n+1}^*$.
- Composition Effect: x_n is decreasing in n. More firms in close competition. Higher aggregate R&D.
- Level Effect: Reduce prices and markups. Less duplicative R&D.

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- Disincentive Effect: $\downarrow v_{n+1} \Rightarrow \downarrow v_{n+1} - v_n \Rightarrow \downarrow x_n^*$.
- Incentive Effect: $\downarrow v_{n+1} \Rightarrow \uparrow v_{n+2} - v_{n+1} \Rightarrow \uparrow x_{n+1}^*$.
- Composition Effect: x_n is decreasing in n. More firms in close competition. Higher aggregate R&D.
- Level Effect: Reduce prices and markups. Less duplicative R&D.

Optimization

In equilibrium, R&D policies must follow:

$$x_n^* = max\{G'^{-1}\left(\frac{[v_{n+1} - v_n]}{\omega^*}\right), 0\}$$

$$x_{-n}^* = max\{G'^{-1}\left(\frac{[v_0 - v_{-n}]}{\omega^*}\right), 0\}$$

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Uniform Policy

Problem is identical for all followers.

Given some assumptions (positive R&D, non-zero profits) ...

- $v_{-1} \leq v_0$; $\{v_n\}_{n=0}^{\infty}$ is bounded and strictly increasing.
- $x_0^* > x_1^*, x_0^* \geq x_{-1}^*$, and $x_{n+1}^* \leq x_n^*$ $\forall n \in \mathbb{N}$.
- Tied firms innovate the most.
- Past that innovation is decreasing in the gap.

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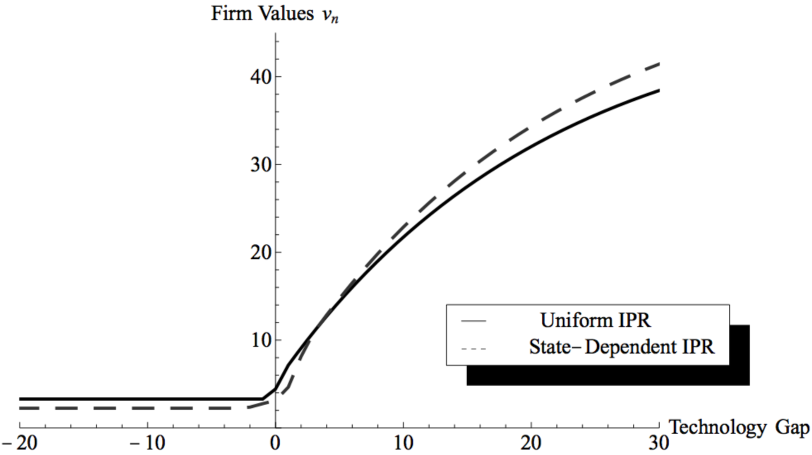

FIGURE 2. Value functions.

Full IPR

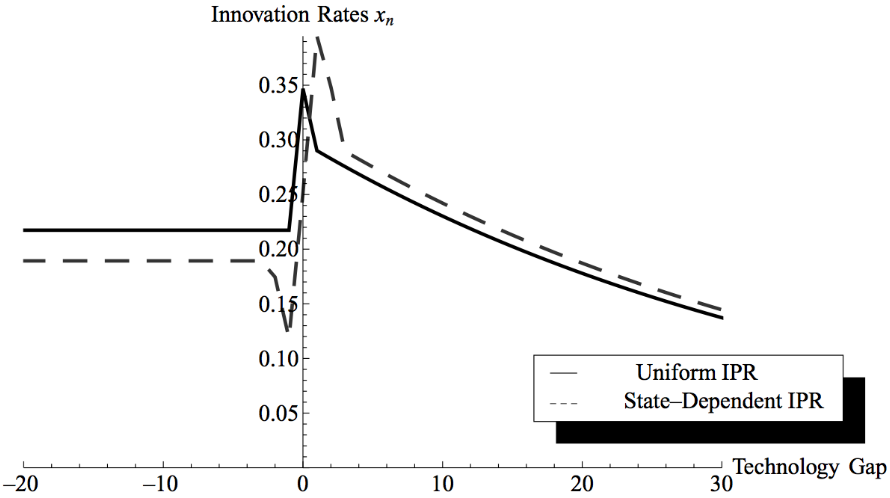

FIGURE 3. R&D efforts.

Full IPR

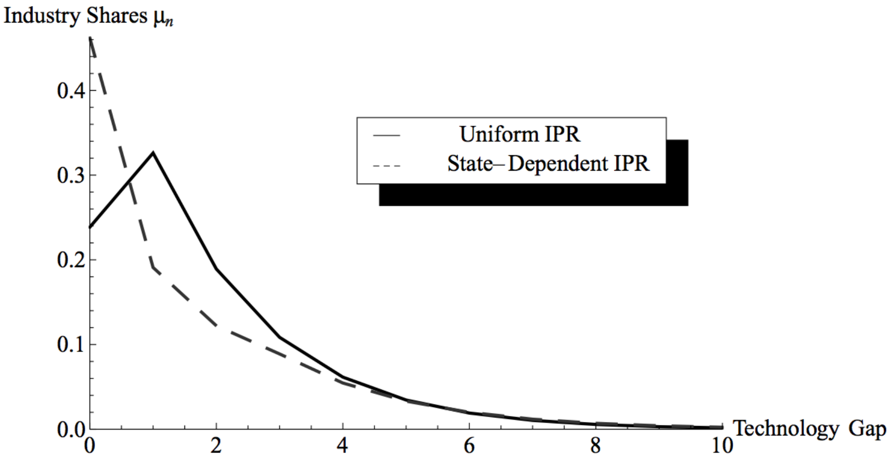

FIGURE 4. Industry shares.

Software and Open Source

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- State-Dependent patent policy to motive all producers to innovate.
- Found that stronger protection should be given to those further ahead.