CAPM

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April 7, 2022

Review

- Let's recall the generalized case: N risky assets and one riskfree asset.
- The optimization problem was

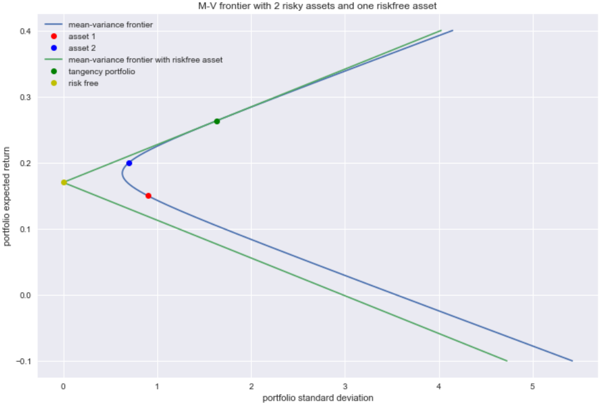
$$\min_{w} w^{'} \sum w$$

s.t

$$w^{'} (E - r_f \mathbb{I}) = \mu - r_f$$

(Case 3: Example)

$$\mu_1 = 0.15,\ \sigma_1 = 0.9,\ \sigma_{12} = 0.2,\ \mu_2 = 0.2,\ \sigma_2 = 0.7,\ r_f = 0.17$$



Interpreting F.O.C

- Are there more implications?
- Let's make things a bit easier.
- N risky assets and one riskfree asset $\rightarrow$ 2 risky assets and one riskfree asset.

$$\min_{w} Var\left(r_p\right) = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\sigma_{12}$$

s.t

$$w_0 r_f + w_1\mu_1 + w_2\mu_2 = \mu_p,$$

$$w_0 + w_1 + w_2 = 1$$

Interpreting F.O.C

- F.O.Cs are

$$(w_1): \; 2w_1\sigma_1^2 + 2w_2\sigma_{12} = \lambda(\mu_1 - r_f)$$
$$(w_2): \; 2w_2\sigma_1^2 + 2w_1\sigma_{12} = \lambda(\mu_2 - r_f)$$

- By rearranging them, we obtain

$$\frac{\mu_1 - r_f}{Cov(r_1, r_p)} = \frac{\mu_2 - r_f}{Cov(r_2, r_p)} = \frac{\mu_p - r_f}{Var(r_p)}$$

- At equilibrium, excess return relative to risk are equalized for all assets in the mean variance frontier.
- Risk that matters comes from the covariance between assets. Thus, the risk of adding an asset to the portfolio are generated from the asset's covariance with the portfolio.
- $\mu_i - r_f$ and $Cov(r_i, r_p)$ can be understood as the marginal benefit and cost of adding asset i to the portfolio, respectively.

From Markowitz to CAPM

- Lets get a step closer to CAPM.
- We restart from

$$\frac{\mu_1 - r_f}{Cov\left(r_1, r_p\right)} = \frac{\mu_2 - r_f}{Cov\left(r_2, r_p\right)} = \frac{\mu_p - r_f}{Var\left(r_p\right)}.$$

- By rearranging this, we get

$$E\left(R_i\right) = R_f + \frac{Cov\left(r_i, r_p\right)}{Var\left(r_p\right)}\left[E\left(R_p\right) - R_F\right]$$

From Markowitz to CAPM

- Let $\beta_{i,p} = \frac{Cov(r_i, r_p)}{Var(r_p)}$, then

$$E\left(R_i\right) = R_f + \beta_{i,p}\left[E\left(R_p\right) - R_F\right].$$

- This relation holds for every optimal portfolio. In other words, it holds for any portfolio p on the mean-variance efficient frontier. Thus, we can replace p with the tangency portfolio.

$$E\left(R_i\right) = R_f + \beta_{i,T}\left[E\left(R_T\right) - R_F\right].$$

- How can we interprete $\beta_{i,T}$?

From Markowitz to CAPM

- Even though it looks like an asset pricing model, it still has two problems.
  - There is no concept of market equilibrium.
  - How can we identify the tangency portfolio?
- CAPM solve these problems.

Market Equlibrium

- The supply and demand of investors adjust the prices of all traded assets so that they will be included in the tangency portfolio.
  - Lets say that CJ CGV is not included.
  - Then the demand (in turn, price) for CJ CGV will decrease until it is attractive.
  - If the price for CJ CGV is attractive, then it will be included.
- When all the price adjustment stops, the market will be at equilibrium.
  - The market price of each asset will be at the level where demand equals supply.
  - The riskfree rate will be at the level where the total amount of money borrowed equals the total amount of money lent.
  - Net supply of riskfree rate is zero.

CAPM

- CAPM: "Market portfolio is tangency portfolio'

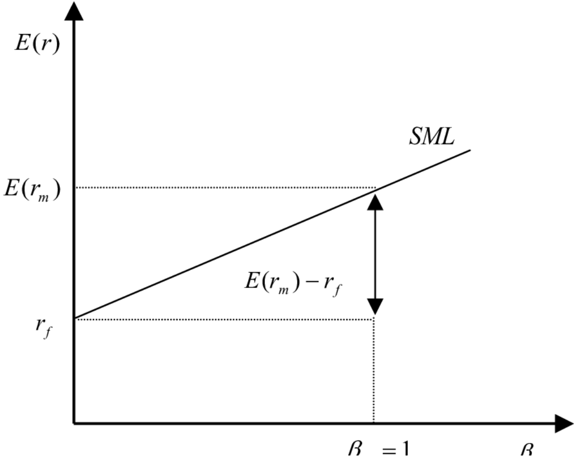
$$\begin{aligned} E(r_i) &= r_f + \beta_{i,M}\left[E\left(r_M\right) - r_f\right] \\ &= \text{riskfree rate} + \text{risk premium} \\ &= \text{riskfree rate} + \text{beta} \times \text{market price of risk} \end{aligned}$$

- Asset price is a function of how much the asset is exposed to systematic risk.

Security Market Line

- Security market line (SML): The relationship between expected return and beta.
- Given the risk of an asset (which is measured by its beta), the SML gives the required return necessary to compensate investors for risk.
- The slope of SML is the market price of risk: $E\left(r_M\right) - r_f$

(Case 3: Example)



CAPM Alpha

- Lets assume that we are fund managers. How can we evaluate our performance?
- Denote our portfolio as p, we got the following results.

$$R_p - R_f = 0.07 + 0.95^{**} \left(R_M - R_f \right)$$

- How can we interprete this result?

CAPM Alpha

- CAPM alpha (α): actual return - fair expected return
  (what does 'fair' mean?)
- Securities lying above (below) the SML are under(over)priced.
- Now, lets get back to our example.

$$R_p - R_f = 0.07 + 0.95^{**}\,(R_M - R_f)$$

- In the sense that portfolio p was build by us, CAPM alpha can also show how much a fund manager outperformed(or, underperformed) compared to benchmark.

More examples

- Assume $r_f = 5\%$ and $E(r_M) - 15\%$. A share of stock sells for \$50 today. It pays a dividend of \$1.5 per share at the end of the year. Its beta is 1.2. What do investors expect the stock to sell for at the end of the year?
- The expected rate of return for this stock(following CAPM) is

$$E(r_i) = 5 + 1.2(15 - 10) = 17\%$$

- Then, the expected price is

$$17 = \frac{P_{t+1} + 1.5}{50}$$

More Comments

- CAPM holds for portfolios as well as for individual assets.
- SML holds for all possible portfolios whereas CML holds only for efficient portfoilos.
- 'All portfolios, whether efficient or not, must lie on the SML. This is not true for the CML.' True or false?

Contributions and Limitations

- Market participants are using CAPM for remuneration or world portfolio composition or consulting for new industry.
- Identify systematic and non-system risk with a very simple model.
- However, empirical results supporting CAPM is limited or very scarce.
- Roll's critique: What is market portfolio?
  - One can never fully diversify a portfolio.
  - Even a 'market portfolio' is only a proxy for a fully-diversified portfolio.