Asset Pricing: Overview

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Basic Definitions and Notations

- A financial asset with price p_t gives rights to obtain payoff x_{t+1}. Then, the gross return of investing in that asset is given by

$$R_{t+1} \equiv \frac{x_{t+1}}{p_t}$$

.

- Net return is

$$r_{t+1} = R_{t+1} - 1$$

Basic Definitions and Notations

- Let $x_{t+1} = d_{t+1} + p_{t+1}$, then

$$\begin{aligned} r_{t+1} &= \frac{d_{t+1} + p_{t+1}}{p_t} - 1 \\ &= \left(\frac{d_{t+1}}{p_t}\right) + \left(\frac{p_{t+1} - p_t}{p_t}\right) \\ &= \text{Income yield} + \text{capital gains(loss)} \end{aligned}$$

- Income yeild = Payouts made to investors
- capital gain = change in price

Basic Definitions and Notations

- This relationship can be applied to various instruments.

	Price (p_t)	Payoff (x_{t+1})
Stock	p_t	$p_{t+1} + d_{t+1}$
One-period bond	p_t	1
Option	C	$\max(S_T - K, 0)$

Compounded Returns

- What is the average annual return of an investment strategy which earned a return of 10% in year 1 and a return of 20% in year 2?
- If \$1 is invested initially, then its value will be $\$(1 + x_{t+1})$ at the end of year 1.
- If money is neither withdrawn nor added, then $\$(1 + x_{t+1})$ will be invested at year 2.
- Thus, the average annual return r^* is given by

$$(1 + r^*)(1 + r^*) = (1 + r_1)(1 + r_2).$$

Expected Returns, Realized Returns

- At the decision-making period, there are some unknown variables. In this case we can only predict by calculating the expected return.

$$E_t\left(r_{t+1}\right) = E_t\left(\frac{d_{t+1} + p_{t+1}}{p_t} - 1\right) = \frac{E_t\left(d_{t+1}\right) + E_t\left(p_{t+1}\right)}{p_t} - 1$$

- When we know what happened in the past, we can simply calculate the realized return.

$$r_{t+1} = \frac{d_{t+1} + p_{t+1}}{p_t} - 1$$

Some Statistics - Mean, Variance, and Covariance

- The mean (expected value) of a random Variable X is
  - (discrete) $E(X) = \sum p_i x_i$
  - (continuous) $E(X) = \int x f(x) dx$
- The variance of a random Variable X is
  - (discrete) $Var(X) = \sum p_i \left(x_i - E(X)\right)^2$
  - (continuous) $Var(X) = \int \left(x_i - E(X)\right)^2 f(x) dx$
- The covariance of two random variables X and Y is

$$Cov\left(X, Y\right) = E\left[\left(X - E(X)\right)\left(Y - E(Y)\right)\right]$$

- The correlation of X and Y is

$$Corr\left(X, Y\right) = \frac{Cov\left(X, Y\right)}{\sqrt{Var(X)}\sqrt{Var(Y)}}$$

Some Statistics - Sample Mean and Variance

- Population means and variances are not known. Therefore, they must be estimated from sample data.
- The typical estimates of population mean and variance are sample mean and sample variance.
- (Sample mean) $\bar{x} = \frac{1}{T} \sum_{i=1}^{n} x_i$
- (Sample variance) $s^2 = \frac{1}{T-1} \sum_{i=1}^{n} (x_i - \bar{x})^2$

Some Statistics - Example

- Let r_p be the return of a portfolio with weights w_i in each security s_i.
- What is the expected return and variance of portfolio return?
- Expected return $E(r_p) = \sum_{i=1}^{n} w_i E(r_i)$
- Variance $Var(r_p) = Var\left(\sum_{i=1}^{n} w_i r_i\right) = \sum_{i=1}^{n} \sum_{j=1}^{n} w_i w_j Cov\left(r_i, r_j\right)$

EMH

- The efficient-market hypothesis states that asset prices reflect all available information.
- The majority of financial models are in the same line with EMH.
- How and why? What is the main implication of EMH on asset pricing?

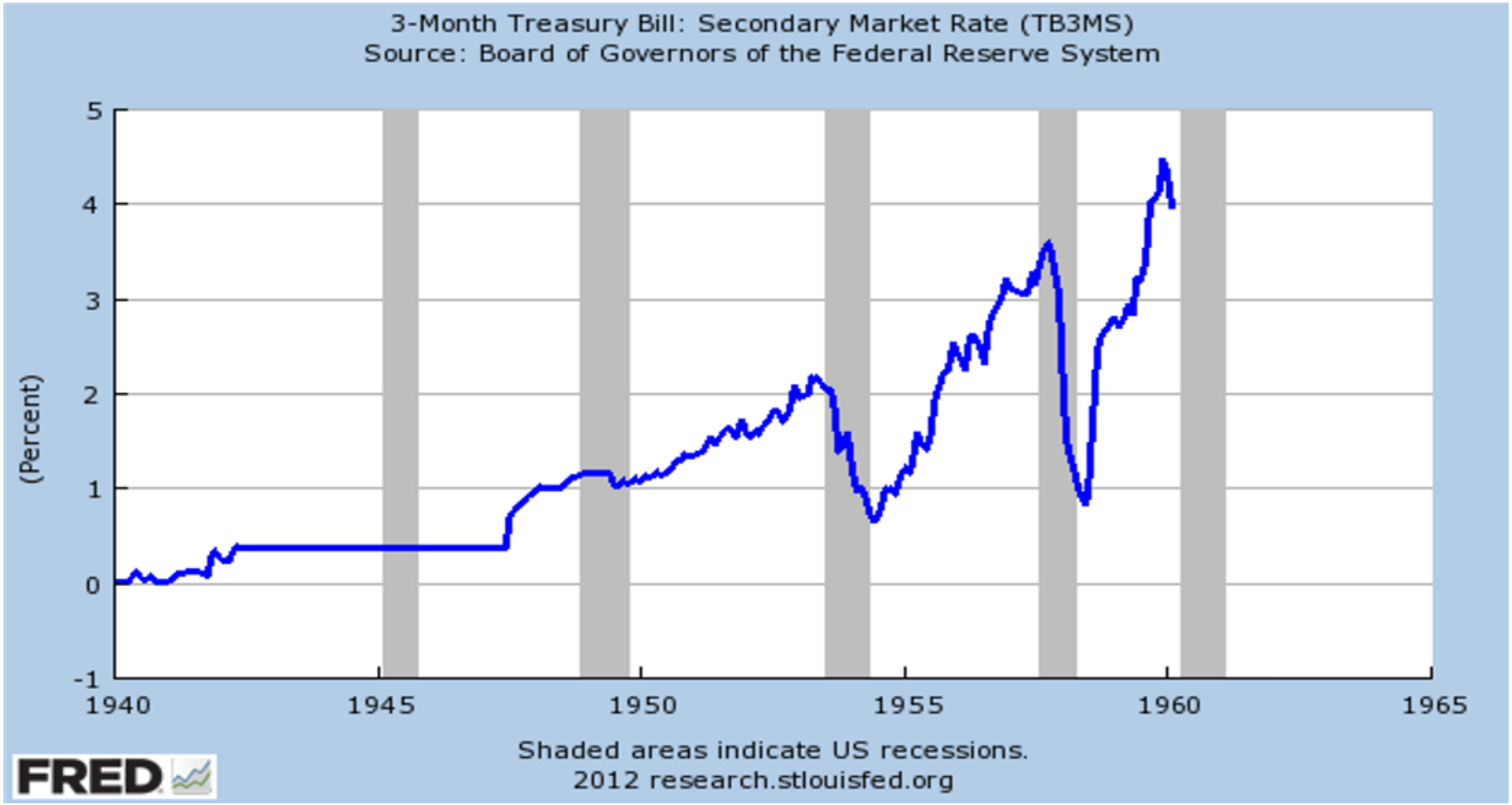
The role of expectations

- Expectations play a critical role in financial markets. To list a few examples,
- Markets' expectations about future inflation affects
  - Market interest rates (and thus, bond prices)
  - Central bank activities
- Markets' expectations about future economic activity affects
  - Stock prices
  - Commodity prices (e.g., oil)

How are market expectations formed? - Adaptive model

- Before the 1960s, most economists assumed that market participants formed adaptive expectations about the future, or that
  - Market expectations about a variable were based primarily on past values of that variable.
  - These expectations changed slowly over time.
  - This approach reflected the relatively stable economy of the early post World War 2 period.

How are market expectations formed? - Adaptive model



Problems with the adaptive model

- A particular variable could easily be affected by many other variables (not just the variable itself). Thus, financial market participants are likely to use all relevant data in forming an expectation about a variable.
- By the 1970s, the economy began to experience sudden and dramatic swings.
- As a result, economists realized that expectations could change very quickly.

How are market expectations formed? - Rational expectations model

- A second approach to financial market expectations, a.k.a rational expectations, took hold in the 1960s.
  - According to the rational expectations model, market participants form expectations using all available information (not just past information and not just the variable itself).
  - The model also assumes that new information is constantly being introduced to the market.
- The rational expectations model, in turn, became a bridge to the EMH (Efficient Markets Hypothesis).
- Basically, EMH assumes that asset prices reflect all available information that directly impact the future cash flow of a security.

EMH

- Eugene Fama (1965) :
  "In an efficient market, competition among many intelligent participants leads to a situation where, at any point in time, the actual prices of securities already reflects the effects of information based on"
  - events that have already occured
  - events as of now
  - events that market expects to take place in the future
- Thus, financial asset prices reflect the best knowledge of the past, the present and predictions of the future.
- Competition as source of efficiency.
- Random walk theory: Since new information is random, all future price changes are independent from previous price changes; thus, future stock prices cannot be predicted.

Three forms of EMH

- Weak-form EMH: Stock prices already reflect all information contained in the history of trading.
- Semistrong-form EMH: Stock prices already reflect all public information.
- Strong-form EMH: Stock prices already reflect all relevant information, including private information.

Implications

- According to EMH, stock prices only react to new information.
- Since information arrival is unpredictable, stock prices are also unpredictable.
- As a result, stock returns can only be described in the terms of risk-return relationship.

Implications