Models of Market Microstructure

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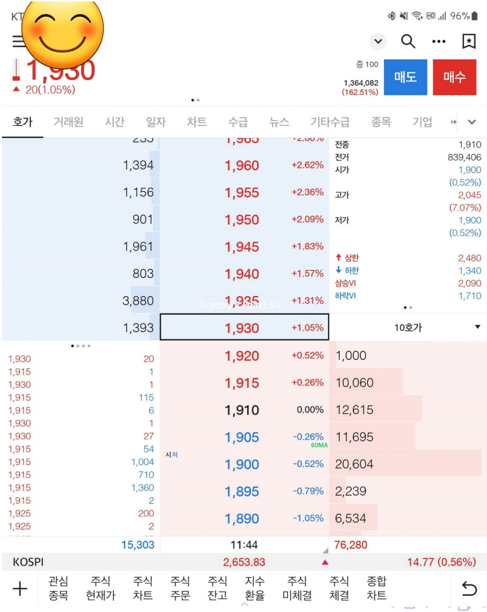
Trading and Price Formation

- This line of the literature analyzes the formation of prices in financial markets in a setting where traders possess pieces of information and trade on it.
- Depending on the market microstructure, the price gets to reflect the information of traders with certain degree of precision.
- This line of the literature is distinct from traditional asset pricing, where prices are assumed efficient (reflecting all available information) and are set by risk-return considerations.

Trading and Price Formation



Grossman and Stiglitz, 1988

Key Ingredients

- Two types of traders: Informed, Noise
- Each trader submits a demand curve for the asset
- Agents are price takers, but they learn from the price
- The demand curve depends on the information the price reveals about the value of the asset.
- There is an exogenous supply for the asset.
- In equilibrium, market clears.
- Agents choose whether to pay the cost of becoming informed.

Grossman and Stiglitz, 1988

The Model

- There is a safe asset yielding a return R and a risky asset yielding a random return u:

$$u = \theta + \epsilon$$

  - θ can be thoought of as the fundamental of the asset. It can be observed at a cost c. ϵ is independent noise.
- In equilibrium, a proportion λ of individuals choose to become informed. Their demand for the risky asset is a function of the price P and of the fundamental θ.

The Model

- $1 - \lambda$ of agents choose to remain uninformed. Their demand for the risky asset is a function of the price P, which serves two roles:
  - Directly affects their payoff by determining how much they pay.
  - Indirectly affects their expected payoff by revealing information about the fundamental θ.
- There is an exogenous supply of the asset x.
- The equilibrium price is set such that market clears. For each level of informed trading, we then get a price as a function of supply and fundamentals: $P_\lambda(\theta, x)$

The Agents Problem

- Trader i chooses the amount of the safe asset(M_i) and risky asset(X_i) to maximize his expected utility from final wealth W_{1i}
  - (Utility) $V(W_1 i) = -e^{-aW_{1i}}$
  - $W_{1i} = RM_i + uX_i$
  - $M_i + PX_i = W_{0i} = \bar{M}_i + P\bar{X}_i$
  - Trader i has $\bar{M}_i, \bar{X}_i$ endowments of each asset
- Conditional on his information, the agent maximizes:

$$E(V(W_{1i})) = -exp(a[E(W_{1i})] - \frac{a^2}{2} Var(W_{1i}))$$

- This yields the general conditional demand function:

$$X_i = \frac{E(u|I) - RP}{aVar(u|I)}$$

Equilibrium

- To solve for an equilibrium, a linear price function is assumed:

$$P_\lambda(\theta, x) = \alpha_{1\lambda} + \alpha_{2\lambda}\theta + \alpha_{3\lambda}x$$

- Which can be expressed as:

$$P_\lambda(\theta, x) = \alpha_{1\lambda} + \alpha_{2\lambda}(\theta - \frac{a\sigma_\epsilon^2}{\lambda}x)$$

Grossman and Stiglitz, 1988

Equilibrium

- $P_\lambda(\theta, x) = \alpha_{1\lambda} + \alpha_{2\lambda}(\theta - \frac{a\sigma_\epsilon^2}{\lambda}x)$
- The price is more informative about the fundamental θ when:
  - There are more informed traders(high λ)
  - When traders are less risk averse(low a)
  - When there is less noise in the payoff of the risky asset(low σ_ϵ^2)
- A trader decides to become informed if the difference between the expected utility of informed and uninformed traders is at least as high as the cost of information acquisition(c)
  - The paper shows that there is a unique internal($\lambda \in (0, 1)$) equilibrium, since the benefit from informed trading is decreasing in the proportion of informed traders.

Bayesian-Nash Equilibrium models

- The rational-expectations equilibrium cannot be easily interpretend in the context of a game or a real-world financial market.
- Traders take the price as given and do not consider their effect on the price, while at the same time they realized that other traders impact the price and thus that they can learn from it.
- As an alternative to the rational-expectation models, Bayesian-Nash equilibrium type models were proposed.
    - Kyle(1985)
    - Glosten and Milgrom(1985)

Kyle, 1985

- The model also derives equilibrium security prices when traders have asymmetric information.
- Agents consider their actions: They are aware that their actions have an impact on price.
- There are three types of agents in the market:
  - The Market Maker
  - Informed trader
  - Noise trader
- Market liquidity(measured by price impact or bid-ask spreads) are a product of the trading behavior of agents.

Kyle, 1985

The Model

- There is a risky asset with payoff $\tilde{v} \sim N(p_0, \sum_0)$
- A noise trader trades quantity $\tilde{u} \sim N(0, \sigma_u^2)$ ($\tilde{u}$ and $\tilde{v}$ are independent)
- An informed trader is trading an endogenous quantity $\tilde{x} = X(\tilde{v})$
- The observed price(set by the market maker) is $\tilde{p}$.
    - The market maker observes the total order flow $\tilde{y} \equiv \tilde{x} + \tilde{u}$(but not), and takes the position $-\tilde{y}$ to clear the market.
    - Thus, the price is a function of the total order flow: $\tilde{p} = P(\tilde{y})$
- The profit of the informed trader is given by: $\tilde{\pi} = (\tilde{v} - \tilde{p})\tilde{x}$

Kyle, 1985

Equilibrium

- An equilibrium is a set of X and P that satisfies two conditions:
  - Profit maximization: $E\{\tilde{\pi}(X, P)|\tilde{v} = v\} \geq E\{\tilde{\pi}(X', P)|\tilde{v} = v\}$
  - Market Efficiency: $\tilde{p}(X, P) = E\{\tilde{v}|\tilde{x} + \tilde{u}\}$
- Essentially, the agent chooses an optimal strategy, taking into account the pricing rule and his effect on the price.

Equilibrium

- To solve for an equilibrium, a linear equilibrium is assumed:
  - $P(y) = \mu + \lambda y$
  - $X(v) = \alpha + \beta v$
- The solution of the system is,
  - $\lambda = \frac{1}{2}(\frac{\sigma_u^2}{\sum_0})^{-1/2}$,
  - $\beta = \frac{1}{2}(\frac{\sigma_u^2}{\sum_0})^{1/2}$,
  - $\mu = p_0$,
  - $\alpha = -\beta p_0$.

Liquidity and Informativeness

- The parameter of most interest in the model is λ, which is usually referred to as Kyle's Lambda.
  - It captures price impact: by how much the price moves with the order flow
  - This is commonly used as a measure of the illiquidity of financial markets.
  - λ increases in the amount of uncertainty about the fundamental $\tilde{v}$ and decrease in the amount of noise.
  - This is how the market maker protects himself against losing money to an informed trader.
- The informed trader internalizes the effect that he has on the price, and wants to trade less aggressively when this effect is large(β is inversely related to λ).
- Given the agent's strategy, more noise trading reduces informativeness, but also encourages the trader to trade more aggressively.