Financial Econometrics

Chapter 5: Predicting Asset Returns

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References

- Cochrane, J.H. (2001) *Asset Pricing*, Princeton University Press.
- Cochrane, J.H. (2006) The Dog That Did Not Bark: A Defense of Return Predictability, mimeo.
- Campbell, J.Y., A.W. Lo and A.C. MacKinlay (1997) *The Economics of Financial Markets*, Princeton University Press.

Conventional views on the predictability of asset returns

- Stock returns are close to unpredictable and nearly identically distributed.
  Prices are random walk represented by

  $$\ln(P_t) = \ln(P_{t-1}) + u_t; \;\; u_t \sim iid\left(0, \sigma^2\right)$$

  or log returns follow iid process.

Conventional views on the predictability of asset returns

- Bond returns are nearly unpredictable.
  If long–term bond yields are higher than short–term yields, it means that short–term interest rates are expected to rise in the future. Therefore, owning either long–term or short–term bond will provide the same return in the future.

Conventional views on the predictability of asset returns

- Foreign exchange returns are not predictable.
  Holding foreign or domestic bonds yield the same rate of return.
- Stock market volatility (varaince) does not change over time. (returns are iid process)
- Professional managers do not outperform simple indices and passive portfolios once one corrects for risk.

These views mean that asset markets are informationally efficient.

New, emerging views on the predictability of asset markets

- Variables including the dividend/price ratio and term premium can predict substantial amounts of stock return variates. But daily, weekly and monthly stock returns are still close to unpredictable.
- Bond returns are predictable.
  A steeply upward sloping yield curve means that expected returns on long–term bonds are higher than on short–term bonds for the next year.

New, emerging views on the predictability of asset markets

- Foreign exchange returns are predictable.
  See, e.g., Mark, N.C. (1995) Exchange Rates and Fundamentals: Evidence on Long-horizon Predictability, American Economic Review.
- Stock market volatility changes through time.
- Some funds seem to outperform simple indices even after controlling for risks.

Still, markets are believed to be reasonably efficient.

Long–horizon stock return regressions

- Model

$$R_{t+k}\left[k\right] - R_{t+k,f} = \alpha + \beta\left(D_t/P_t\right) + u_t$$

$$\begin{aligned} D_t &: \text{dividend,} \\ P_t &: \text{price,} \\ R_{t,f} &: \text{risk–free return.} \end{aligned}$$

If $\beta > 0$, high dividend/price ratio predicts high return.

Long–horizon stock return regressions

- Some empirical results for the long-horizon stock return regression are reported in Cochrane (2001, p.390).

Horizon k (years)	$\hat{\beta}$	$s.e.$	R^2
1	5.3	(2.0)	0.15
2	10	(3.1)	0.23
3	15	(4.0)	0.37
5	33	(5.8)	0.60

Sample: 1947–1996. Value weighted NYSE-treasury bill was regressed on the percent value weighted dividend/price ratio.

Long–horizon stock return regressions

- Other variables such as the term spread between long– and short–term bonds, the default spread (corporate less T-bill yield), the T–bill rate and the earnings/dividend ratio, the book/market ratio have been used with some success. See Cochrane (2001, p.391).

Long–horizon stock return regressions

Long–horizon return forecasts

Horizon k (years)	cay	$d-p$	$d-e$	$rrel$	R^2
1	6.7				0.18
1		0.14	0.08		0.04
1				-4.5	0.10
1	5.4	0.07	-0.05	-3.8	0.23
6	12.4				0.16
6		0.95	0.68		0.39
6				-5.10	0.03
6	5.9	0.89	0.65	1.36	0.42

Long–horizon stock return regressions

Return	:	log real excess returns on the S&P composite index $\left(r_t^r - r_{f,t}^r\right)$
cay	:	consumption to wealth ratio
d	:	log of the sum of the past four quarters of dividends
e	:	log of a single quarter's earnings per share
p	:	log of the index
rrel	:	T-bill rate minus its 12 month backward moving average
$d-p$	:	dividend yield
$d-e$	:	payout ratio
sample	:	1952:IV–1998:III
original source	:	Lettau and Ludvigson, Journal of Finance, 2001

Long–horizon stock return regressions

At one–year horizon, *cay* and *rrel* are more important. At six–year horizon, $d - p$ and $d - e$ becomes more important.

- Why does long–horizon regression provide higher coefficient estimate? Consider the predictive regression

$$\begin{aligned} r_{t+1} &= a x_t + \varepsilon_{t+1}; \\ x_{t+1} &= \rho x_t + \delta_{t+1}. \end{aligned}$$

Long–horizon stock return regressions

Then

$$
\begin{aligned}
r_{t+1} + r_{t+2} &= a\left(1+\rho\right)x_t + a\delta_{t+1} + \varepsilon_{t+1} + \varepsilon_{t+2}; \\
r_{t+1} + r_{t+2} + r_{t+3} &= a\left(1+\rho+\rho^2\right)x_t + a\rho\delta_{t+1} + a\delta_{t+2} + \varepsilon_{t+1} + \varepsilon_{t+2} + \\
&\vdots
\end{aligned}
$$

When ρ is close to 1, the coefficient of x_t increases with horizon.

Explaining high stock price

- When prices are high relative to dividends,
  1. dividends are expected to rise in the future $(D_t \uparrow)$
  2. returns are expected to become low in the future $(P_t \downarrow)$
  3. price/dividend ratio grows explosively

  Virtually all variation in price/dividend ratios has reflected varying expected excess returns.

Explaining high stock price

- Price can be expressed in terms of future dividends and returns. Consider the identity (cf. Campbell and Shiller (1988), Review of Financial Studies)

$$1 = (1 + R_{t+1})^{-1} (1 + R_{t+1}) = (1 + R_{t+1})^{-1} \frac{P_{t+1} + D_{t+1}}{P_t}$$

and hence

$$\begin{aligned} \frac{P_t}{D_t} &= (1 + R_{t+1})^{-1} \frac{P_{t+1} + D_{t+1}}{P_t} \times \frac{P_t}{D_t} \\ &= (1 + R_{t+1})^{-1} \left(1 + \frac{P_{t+1}}{D_{t+1}}\right) \frac{D_{t+1}}{D_t}. \end{aligned}$$

Explaining high stock price

- Taking logs of

$$\frac{P_t}{D_t} = (1 + R_{t+1})^{-1} \left(1 + \frac{P_{t+1}}{D_{t+1}}\right) \frac{D_{t+1}}{D_t},$$

we have

$$p_t - d_t = -r_{t+1} + \Delta d_{t+1} + \ln\left(1 + e^{p_{t+1} - d_{t+1}}\right),$$

where $r_{t+1} = \ln(\frac{P_{t+1}+D_{t+1}}{P_t}) \approx p_{t+1} - p_t$. The Taylor expansion of $\ln\left(1 + e^{p_{t+1}-d_{t+1}}\right)$ around $P/D = \exp(p - d)$ gives[1]

$$\ln\left(1 + e^{p_{t+1}-d_{t+1}}\right) \approx \ln\left(1 + \frac{P}{D}\right) + \frac{P/D}{1 + P/D}\left[p_{t+1} - d_{t+1} - (p - d)\right]$$

[1] $\ln(1 + e^x) \approx \ln(1 + e^{x_o}) + \frac{e^{x_o}}{1+e^{x_o}}(x - x_o)$.

Using this, we obtain

$$p_t - d_t \approx -r_{t+1} + \Delta d_{t+1} + k + \rho \left(p_{t+1} - d_{t+1} \right).$$

Iterate this relation forward, then

$$p_t - d_t \approx \text{const} + \sum_{j=1}^{\infty} \rho^{j-1} \left(\Delta d_{t+j} - r_{t+j} \right).$$

Taking conditional expectations, we obtain an ex ante relation

$$p_t - d_t \approx \text{const} + E_t \sum_{j=1}^{\infty} \rho^{j-1} \left(\Delta d_{t+j} - r_{t+j} \right).$$

This shows that a high price/dividend ratio must be followed by high dividend growth or low returns.

Explaining high stock price

- Assume $E\left(\Delta d_{t+j} - r_{t+j}\right) = 0$. Then,

$$
\begin{aligned}
Var\left(p_t - d_t\right) &= E\left[p_t - d_t - E\left(p_t - d_t\right)\right]^2 \\
&= E\left[p_t - d_t - E\left(p_t - d_t\right)\right]\left[\sum_{j=1}^{\infty} \rho^{j-1}\left(\Delta d_{t+j} - r_{t+j}\right)\right] \\
&= Cov\left(p_t - d_t,\ \sum_{j=1}^{\infty} \rho^{j-1} \Delta d_{t+j}\right) \\
&\quad - Cov\left(p_t - d_t,\ \sum_{j=1}^{\infty} \rho^{j-1} r_{t+j}\right),
\end{aligned}
$$

positive variance of the price/dividend ratio implies that the dividend growth and/or the return should be correlated with the ratio. Almost all variation in price/dividend ratio is due to changing return forecasts.

Explaining high stock price

Almost all variation in price/dividend ratio is due to changing return forecasts as shown in the following table (from Cochrane, 2001, p.398).

Variance decomposition of value weighted NYSE price/dividend ratio

	Dividends	Returns
Real	-34	138
s.e.	10	32
Nominal	30	85
s.e.	41	19

Entries in the second and fourth rows are the estimates of $100 \times Cov\left(p_t - d_t, \sum_{j=1}^{\infty} \rho^{j-1} \Delta d_{t+j}\right) / Var\left(p_t - d_t\right)$ and $-100 \times Cov\left(p_t - d_t, \sum_{j=1}^{\infty} \rho^{j-1} r_{t+j}\right) / Var\left(p_t - d_t\right)$, respectively.

Explaining high stock price

Again from Cochrane (2001, p.390), regressing D_{t+k}/D_t on D_t/P_t yields the following results.

Horizon k (years)	$\hat{\beta}$	$s.e.$	R^2
1	2.0	1.1	0.06
2	2.5	2.1	0.06
3	2.4	2.1	0.06
5	4.7	2.4	0.12

These results shows that the relation between the dividend growth and price/dividend ratio is weak.

Evidence of predictability

See Tables 2.4 and 2.8 of Campbell, Lo and Mackinlay (1997).

1. Strong evidence of positive autocorrelation at lag one for index returns.
2. Weak negative autocorrelations are observed for individual securities.
3. Autocorrelations across stocks are quite strong.

Mean reversion

- Long–run regression
  Fama and French (1988, JPE) estimated the model

$$r_{t+k}\left[k\right] = a + b_k r_t\left[k\right] + \varepsilon_{t+k}.$$

  Negative and significant b coefficients were found: a string of good past returns forecasts bad future returns.

Mean reversion

- Variance ratio

  If r_t are uncorrelated,

$$\begin{aligned} Var\left(r_{t+k}\left[k\right]\right) &= Var\left(r_{t+1}+r_{t+2}+\cdots+r_{t+k}\right) \\ &= kVar\left(r_{t+1}\right). \end{aligned}$$

  The variance ratio statistic is defined by

$$V_k = \frac{Var\left(r_{t+k}\left[k\right]\right)}{kVar\left(r_{t+1}\right)}.$$

  - This should be close to one if r_t are uncorrelated.
  - If the variance ratio is less than one, it implies that stocks are safer for "long-run investors" who can tolerate ups and downs of the market.
  - Porterba and Summers (1988, JFE) found variance ratios below one, suggesting that the returns are correlated and that negative correlations exist.

Mean reversion: Empirics

From Cochrane (2001, p.412),

Mean reversion using logs, 1926-1996 (log value-weighted NYSE return - log T-bill return used)

	Horizon k (years)					
	1	2	3	5	7	10
$\frac{\sigma(k-period\ \ excess\ \ return)}{\sqrt{k}}$	19.8	20.6	19.7	18.2	16.5	16.3
b_k	0.08	-0.15	-0.22	-0.04	0.24	0.08

Mean reversion: Empirics

- Evidence of mean reversion at periods 2, 3 and 5. But it disappears at longer horizons.
- Variance ratio at year 10 is $(16.3/19.8)^2 = 0.68$ (< 1).

Relative mean reversion in international stock markets

- Balvers, Wu and Gilliland (2000) "Mean Reversion across National Stock Markets and Parametric Contrarian Investment Strategies". Journal of Finance.

Relative mean reversion in international stock markets

- p_{it} : log of the total return index of the stock market in country i at the end of period t.
- Evolution of p_{it} is described by a mean-reverting process

$$p_{i,t+1} - p_{i,t} = a_i + \lambda(p^*_{i,t+1} - p_{it}) + \varepsilon_{i,t+1}.$$

p^*_{it} : an unobserved fundamental value of the index.

Relative mean reversion in international stock markets

- Paramter λ is the speed of mean reversion and is assumed to be the same across countries.
- If $0 < \lambda < 1$, deviations of p_{it} from its fundamental or trend value $p^*_{i,t+1}$ will be reversed over time.
- But if $\lambda = 0$, the log price follows a unit root process so that there is no mean reversion.

Relative mean reversion in international stock markets

- Assume
$$p^*_{it} = p^*_{r,t} + z_i + \eta_{i,t}$$
where $p^*_{r,t}$ is a reference country's fundamental value of the index and z_i a constant.
- Then,
$$r_{i,t+1} - r_{r,t+1} = \alpha_i - \lambda(p_{i,t} - p_{r,t}) + \omega_{i,t+1},$$
where $r_{i,t+1} \equiv p_{i,t+1} - p_{i,t}$.
- No mean reversion (i.e., $\lambda = 0$) corresponds to the presence of a unit root in $p_{i,t} - p_{r,t}$.
- Balvers, Wu and Gilliland report evidence of mean reversion in relative stock index prices using stock index data of 18 nations during the period 1969-1996.