Model-Free Adaptive Iterative Learning Containment Control for Nonlinear Multiagent Systems Under Sparse Sensor Attacks

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Abstract

In this work, the model-free adaptive iterative learning control approach is used to study the containment control problem of nonlinear multiagent systems under sparse sensor attacks. Firstly, the nonlinear dynamics of each agent is transformed into an approximate linear model along the iterative axis by using the compact form dynamic linearization (CFDL) method. Secondly, the detection mechanism and adaptive switching strategy are designed for the multi-source mixed attacks on sensors, which greatly alleviates the negative impact of sparse sensor attacks on the system. Subsequently, the distributed model-free adaptive iterative learning control algorithm is presented to ensure that the output of each follower is constrained within the convex hull formed by the output of leaders. Then, the convergence of the proposed control algorithm is analyzed and extended to the switching topology. Finally, the control effect is verified by several simulation examples.

Containment control, sparse sensor attacks, model-free adaptive iterative learning control, nonlinear multiagent systems.

I. Introduction

During the last few decades, with the increase of task complexity and data volume, the intelligent individual has been gradually difficult to complete the needs of the system, so the multiagent systems came into being. Due to excellent collaboration ability and distributed decision-making mechanism, multiagent systems have been widely used in autonomous vehicles [?], robot collaboration [?], Internet of Things [?]- [?] and distributed sensor networks [?]. Containment control is considered to be an important problem in multiagent systems cooperation. The objective of containment control is to find a suitable strategy so that all followers can converge into a convex hull composed of multiple leaders. So far, a great deal of research has been done on containment problems with different system models [?]- [?].

The above work is carried out on the basis of a common premise. In other words, the system model is available. However, with the progress of science and technology, the accurate system model becomes difficult to obtain, which makes the research of the problem into a bottleneck. At this time, some scholars introduced model-free adaptive control methods. The method only relies on the real-time measurement data of the controlled system, and does not rely on any mathematical model information of the controlled system. In [?]- [?]. In the above literature, model-free adaptive control shows good control effect and robustness. Unfortunately, model-free adaptive control only works in the time domain.

However, in engineering practice, many systems perform the same control tasks repeatedly in a finite time interval. The time domain control method does not have the ability to learn from the past repeated operations, that is, the control error is also repeated. In contrast, the iterative learning control method can reduce the system error by using the memory device to store the past duplicate data to correct the current control input. Therefore, combining the characteristics of model-free adaptive control and iterative learning control, the design and analysis methods of model-free adaptive iterative learning control are proposed.

In the present work, via ISMC technology, the fault-tolerant problem is studied for a class of linear continuous systems based on simplified deadzone model with respect to actuator dead-zone failure and unmatched external perturbations. First of all, the deadzone is modeled as a combination of a line and a bounded disturbance-like term. The nonlinear control law is then designed to guarantee that sliding mode can be achieved. And the bounded disturbance-like term which enters the state equation at the same point as the control input can be rejected completely. What's more, a novel bounded real lemma with allowing extra degree of freedom for design is presented by using the well-known Projection Lemma. Further, based on this new lemma, the state feedback controller given by linear matrix inequality (LMI) is presented such that the resulting closed-loop system in the sliding mode is asymptotically stable.It is worth noting that the deadzone failure can be completely compensated and the system is more robust against unmatched disturbances than using H_∞ control alone. At last, a numerical example is presented to show the effectiveness of the proposed scheme.

The notations used throughout this paper are fairly standard. R^n denotes the n-dimensional Euclidean space, $R^{m\times n}$ is the set of all $m \times n$ real matrices. In addition, we use '*' as an ellipsis for the terms that are introduced by symmetry. I means the unit matrix with appropriate dimensions, I_n is the identity matrix of order n. $\otimes$ is the Kronecker product of two matrices. The direct sum of matrices $A_i, i = 1, 2, \cdots, n$ will be denoted as $A_1 \oplus A_2 \oplus \cdots \oplus A_n \triangleq diag\{A_1, A_2, \cdots, A_n\}$. In addition, for a matrix M, $He\{M\}$ denotes $M + M^T$.

Secondly, an attack defense scheme is designed for the multi-source mixed attacks occurring in the sensor channel. The scheme is composed of detection mechanism and adaptive switching strategy, which can reduce the negative effects of sparse sensor attacks by switching channel in time after detecting abnormal behavior.

II. System Description and Problem Statement

A. Deadzone Model

Deadzone is a static input-output relationship which for a range of input values gives no output. Once the output appears, the slope between the input and the output is constant. The analytical expression of the deadzone characteristic is

$$y_i^l(t+1,k) = f_i^l(y_i^l(t,k), \cdots, y_i^l(t-n_y,k), u_i(t,k), \cdots, u_i(t-n_u,k)) \tag{1}$$

$$\Delta y_i^l(t+1,k) = \phi_i(t,k)\Delta u_i(t,k) \tag{2}$$

$$e_i^l(t,k) = max\left\{y_{n+1}(t) - y_i^l(t,k), \cdots, y_{n+p}(t) - y_i^l(t,k)\right\} \tag{3}$$

$$g_i^l(t,k) = \begin{cases} 1, & \text{if } \left|e_i^l(t,k)\right| \geq \hbar \\ 0, & \text{if } \left|e_i^l(t,k)\right| < \hbar \end{cases} \tag{4}$$

$$\tilde{y}_i(t,k) = \left[\imath_i^1(t), \imath_i^2(t), \cdots, \imath_i^x(t)\right] \begin{bmatrix} y_i^1(t,k) \\ y_i^2(t,k) \\ \vdots \\ y_i^x(t,k) \end{bmatrix} \tag{5}$$

$$\imath_i^l(t) = \begin{cases} 1, & \mathcal{Q}_i^1(t) \neq 0 \ \cap \cdots \cap \mathcal{Q}_i^{l-1}(t) \neq 0 \cap \mathcal{Q}_i^l(t) = 0 \\ 0, & others \end{cases} \tag{6}$$

$$\mathcal{Q}_i^l(t) = \sum_{t=1}^{T} g_i^l(t,k) \tag{7}$$

$$\xi_i(t,k) = \sum_{j\in\mathbb{N}_j} a_{ij}(\tilde{y}_j(t,k) - \tilde{y}_i(t,k)) + \sum_{q=n+1}^{n+p} a_{iq}(y_q(t) - \tilde{y}_i(t,k)) \tag{8}$$

$$\begin{aligned}\hat{\phi}_i(t,k) =& \hat{\phi}_i(t,k-1) + \frac{\eta\Delta u_i(t,k-1)}{\mu + |\Delta u_i(t,k-1)|^2} \\ & \times (\Delta\tilde{y}_i(t+1,k-1) - \hat{\phi}_i(t,k-1)\Delta u_i(t,k-1))\end{aligned} \tag{9}$$

$$\hat{\phi}_i(t,k) = \hat{\phi}_i(t,1),\ if \left|\hat{\phi}_i(t,k)\right| \le \varsigma\ or\ \ |\Delta u_i(t,k-1)| \le \varsigma\ or\ sign(\hat{\phi}_i(t,k)) \ne sign(\hat{\phi}_i(t,1)) \tag{10}$$

$$u_i(t,k) = u_i(t,k-1) + \frac{\rho\hat{\phi}_i(t,k)}{\lambda + \left|\hat{\phi}_i(t,k)\right|^2}\xi_i(t+1,k-1) \tag{11}$$

The key features of the deadzone in the control problems investigated in this paper are

A1 The deadzone output $\Phi(u(t))$ is not available for measurement.

A2 The deadzone slopes in positive and negative region are same, i.e. $m_r = m_l = m$.

A3 The deadzone parameters b_r,b_l and m are unknown, but their signs are known: $b_r > 0, b_l < 0, m > 0$.

A4 The deadzone parameters b_r,b_l and m are bounded, i.e. there exist known constants $b_{r\min}$, $b_{r\max}$, $b_{l\min}$, $b_{l\max}$, $m_{\min}$, $m_{\max}$ such that$b_r \in [b_{r\min}, b_{r\max}], b_l \in [b_{l\min}, b_{l\max}], m \in [m_{\min}, m_{\max}]$.

Remark 1. *Assumption(A1) is common in practical systems, such as servomotors and servo valves. If $\Phi(u(t))$ is measurable, the control of deadzone will be relatively easy and will not be discussed in this paper. Assumption(A2) is generally adopted in the literature (see, for example, [?], [?]) and can commonly be met in the industrial systems. Assumptions (A3) and (A4) are physically satisfied in real plants.*

From a practical point of view, we can re-define model (**??**) as a simplified deadzone model [**?**] which makes the design of controller simpler [**?**].

$$\Phi(u(t)) = mu(t) + d(u(t)) \tag{12}$$

where m is called the general slope of the deadzone, $\Phi(u(t))$ can be calculated from (**??**) and (**??**) as,

$$d(u(t)) = \begin{cases} -mb_r & for\ \ u(t) \ge b_r \\ -mu(t) & for\ \ b_l < u(t) < b_r \\ -mb_l & for\ u(t) \le b_l \end{cases} \tag{13}$$

From Assumptions (A2) and (A4), one can conclude that $d(u(t))$ is bounded, and satisfies $|d(u(t))| \le \bar{d}$, where $\bar{d}$ is the upper-bound, which can be chosen as $\bar{d}$ = $\max\{m_{\max}b_{r\max},\ -m_{\max}b_{l\min}\}$, where $b_{l\min}$ carries a negative value.

B. System Model

Consider a linear time-invariant continuous system in the form of

$$\dot{x}(t) \quad = \quad Ax(t) + B\Phi(u(t)) + Ew(t) \tag{14}$$

where $x(t) \in R^n$ is the state vector, $w(t) \in R^r$ is an exogenous disturbance in $\mathcal{L}_2[0,\infty)$, $u(t) \in R^m$ is the control input, and$z(t) \in R^q$ is the regulated output, respectively. The system matrices A, B, E, C, and D are known constant matrices of appropriate dimensions.

Then, the resulting model of continuous-time system in (**??**) considering the deadzone model (**??**) and (**??**) is given by

$$\dot{x}(t) \quad = \quad Ax(t) + Bmu(t) + Bd(u(t)) + Ew(t) \tag{15}$$

Remark 2. *It is obvious that there is no consideration about the deadzone failure when $m = 1$, $d(u(t)) = 0$.*

C. Problem Statement

The problem of interest in this paper is to develop a sliding mode controller $u(t)$ such that the deadzone failure can be compensated completely and the closed-loop system in the sliding mode(**??**) is asymptotically stable, and is more robust against unmatched disturbances than using H_∞ control alone.

To facilitate control system design, the following lemmas are presented and will be used in the later developments.

Lemma 1. *(Projection Lemma) [?], [?] Given a symmetric matrix $\Phi \in R^{n\times n}$ and two matrices T_1, T_2 of column dimension n, there exists a matrix X that satisfies*

$$\Phi + He\{T_1^T X^T T_2\} < 0,$$

if and only if the following projection inequalities with respect to X are satisfied.

$$N_{T_1}^T \Phi N_{T_1} < 0, N_{T_2}^T \Phi N_{T_2} < 0,$$

where N_{T_1} and N_{T_2} denote arbitrary bases of the null-spaces of T_1 and T_2, respectively.

Lemma 2. *For a prescribed scalar $\gamma > 0$, the system with the transfer function as $T(s) = \bar{C}(sI - \bar{A})^{-1}\bar{E}$ is asymptotically stable, and $\|T(s)\|_\infty < \gamma$ if and only if there exists matrix $P = P^T > 0$ such that*

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \tag{16}$$

$$\begin{bmatrix} PA_{cl} + A_{cl}^T P & PB_{cl} & C_{cl}^T \\ * & -\gamma^2 I & D_{cl}^T \\ * & * & -I \end{bmatrix} < 0. \tag{17}$$

There is no doubt that the standard bounded real lemma is given by a strict matrix inequality in Lemma **??**. However, there is a drawback of the above lemma. The construct products between the Lyapunov matrix and the system's dynamic matrices will introduce some conservativeness. Taking into account the above drawback, a novel bounded real lemma is obtained by using Projection Lemma.

III. Main Result

Consider a sliding mode controller as follows

$$u(x,t) \quad = \quad u_0(x,t) + u_1(x,t) \tag{18}$$

The nominal control $u_0(x,t)$ is responsible for the performance of the nominal system; $u_1(x,t)$ is a discontinuous control action that rejects the perturbations by ensuring the sliding motion. Usually,the discontinuous control and the linear control are selected as

$$u_1(x,t) \quad = \quad -\rho(x,t)(GB)^{-1}\tfrac{s(x,t)}{\|s(x,t)\|} \tag{19}$$

and

$$u_0(x,t) \quad = \quad Kx(t) \tag{20}$$

respectively.

Now the integral sliding surface is chosen to be

$$s(x,t) \quad = \quad G[x(t)-x(t_0)-\textstyle\int_{t_0}^{t}(Ax(\tau)+Bm_{min}u_0(\tau))\,dt] \tag{21}$$

where $s \in R^m$, $u_0(t) \in R^m$ designed by state feedback is a control input to guarantee unmatched disturbance attenuation once the system is in the sliding mode, and G is chosen as B^+ to avoid amplification of the unmatched disturbances [**?**].

Differentiating (**??**) with respect to time and substituting $\dot{x}(t)$ from (**??**) and $u(t)$ from (**??**)obtains

$$\dot{s}(x,t) \quad = \quad GB((m-m_{min})u_0+mu_l+d(t))+GEw(t)$$

then

$$mu_{leq} = -m_{min}u_0 - d(t) - (GB)^{-1}GEw(t)$$

we can obtain the sliding mode dynamics

$$\begin{aligned} \dot{x}(t) \quad &= \quad Ax(t)+(I-B(GB)^{-1}G)Ew(t)+Bm_{min}u_0(t) \\ z(t) \quad &= \quad Cx(t)+Du_0(t) \end{aligned} \tag{22}$$

Remark 3. *The novelty of the paper lies in that the way to treat the deadzone failure. The bounded disturbance-like term $d(u(t))$ which enters the state equation at the same point as the control input can be rejected completely by ISMC technology, and the linear part of the deadzone will be compensated by state feedback controller in the following design.*

Consider the system (**??**) associated with the control law (**??**), the closed-loop system in the sliding mode can be expressed as follows

$$\begin{aligned} \dot{x}(t) \quad &= \quad A_{cl}x(t)+B_{cl}w(t) \\ z(t) \quad &= \quad C_{cl}x(t) \end{aligned} \tag{23}$$

where

$$A_{cl} = A + Bm_{min}K, B_{cl} = (I - B(GB)^{-1}G)E, C_{cl} = C + DK.$$

The transfer function matrix of the closed-loop system in (**??**) in the sliding mode is given by

$$T(s) = C_{cl}(sI - A_{cl})^{-1}B_{cl}. \tag{24}$$

Now we will focus on obtaining the state feedback H_∞ controller design conditions on the sliding surface and the achievement conditions.

First, we utilize the following improved bounded real lemma [**?**], which is crucial to our derivation.

Theorem 1. *For a prescribed scalar $\gamma > 0$, the sliding dynamics on $s(x,t) = 0$ with the transfer function as $T(s) = \bar{C}(sI - \bar{A})^{-1}\bar{E}$ is asymptotically stable, and satisfies the norm constraint $\|T(s)\|_\infty < \gamma$ if and only if there exist matrices $Q = Q^T > 0$, Z and a sufficiently small scalar $\epsilon > 0$ such that*

$$\begin{bmatrix} Q - He\{Z\} & Z^T(I + \epsilon A_{cl}) & 0 & \sqrt{\epsilon} Z^T B_{cl} \\ * & -Q & \sqrt{\epsilon} C_{cl}^T & 0 \\ * & * & -\gamma^2 I & D_{cl} \\ * & * & * & -I \end{bmatrix} < 0. \tag{25}$$

Proof. By suitable block-row-column exchanges of (**??**), it is easy to obtain

$$\begin{bmatrix} -I & \sqrt{\epsilon} B_{cl}^T Z & 0 & D_{cl}^T \\ * & Q - Z - Z^T & Z^T(I + \epsilon A_{cl}) & 0 \\ * & * & -Q & \sqrt{\epsilon} C_{cl}^T \\ * & * & * & -\gamma^2 I \end{bmatrix} < 0. \tag{26}$$

Obviously, the matrix inequality (**??**) can be equivalently rewritten as

$$\begin{bmatrix} -I & 0 & 0 & D_{cl}^T \\ * & Q & 0 & 0 \\ * & * & -Q & \sqrt{\epsilon} C_{cl}^T \\ * & * & * & -\gamma^2 I \end{bmatrix} + He\left\{ \begin{bmatrix} 0 \\ I \\ 0 \\ 0 \end{bmatrix} Z^T \begin{bmatrix} \sqrt{\epsilon} B_{cl}^T \\ -I \\ I + \epsilon A_{cl}^T \\ 0 \end{bmatrix}^T \right\} < 0 \tag{27}$$

Noting that explicit bases of the null-spaces of $\begin{bmatrix} 0 & I & 0 & 0 \end{bmatrix}$ and $\begin{bmatrix} \sqrt{\epsilon} B_{cl} & -I & I + \epsilon A_{cl} & 0 \end{bmatrix}$ are given as $N_1 = \begin{bmatrix} I & 0 & 0 \\ 0 & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix}$ and $N_2 = \begin{bmatrix} I & 0 & 0 \\ \sqrt{\epsilon} B_{cl} & I + \epsilon A_{cl} & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix}$.

Then, canceling the matrix Z from (**??**) by Lemma **??** leads to

$$N_1^T \Psi N_1 = \begin{bmatrix} -I & 0 & D_{cl}^T \\ * & -Q & \sqrt{\epsilon} C_{cl}^T \\ * & * & -\gamma^2 I \end{bmatrix} < 0,$$

$$N_2^T \Psi N_2 = \begin{bmatrix} -I + \epsilon B_{cl}^T Q B_{cl} & \sqrt{\epsilon} B_{cl}^T Q \mathcal{A}^T & D_{cl}^T \\ * & \mathcal{A} Q \mathcal{A}^T - Q & \sqrt{\epsilon} C_{cl}^T \\ * & * & -\gamma^2 I \end{bmatrix} < 0, \tag{28}$$

where $\Psi = \begin{bmatrix} -I & 0 & 0 & D_{cl}^T \\ * & Q & 0 & 0 \\ * & * & -Q & \sqrt{\epsilon} C_{cl}^T \\ * & * & * & -\gamma^2 I \end{bmatrix}$, $\mathcal{A} = I + \epsilon A_{cl}^T$.

By some simple algebraic manipulations and using Schur complement lemma, the inequality (**??**) can also be rewritten as

$$\begin{bmatrix} -I + \gamma^{-2} D_{cl}^T D_{cl} & \sqrt{\epsilon} B_{cl}^T Q \mathcal{A}^T + \gamma^{-2} \sqrt{\epsilon} D_{cl}^T C_{cl} & \sqrt{\epsilon} B_{cl}^T Q \\ * & \mathcal{A} Q \mathcal{A}^T - Q + \gamma^{-2} \epsilon C_{cl}^T C_{cl} & 0 \\ * & * & -Q \end{bmatrix} < 0, \tag{29}$$

which implies that the following inequalities hold.

$$\Sigma = \gamma^2 I - D_{cl}^T D_{cl} > 0,$$

$$\begin{bmatrix} W_1 + \gamma^2 W_2^T \Sigma^{-1} W_2 & 0 \\ * & -Q \end{bmatrix} + O(\epsilon) < 0$$

where $W_1 = A_{cl}^T Q + QA_{cl} + \gamma^{-2} C_{cl}^T C_{cl}, W_2 = B_{cl}^T Q + \gamma^{-2} D_{cl}^T C_{cl}^T$.

Obviously, when ϵ is sufficiently small, (**??**) holds.

$$\begin{bmatrix} W_1 + \gamma^2 W_2^T \Sigma^{-1} W_2 & 0 \\ * & -Q \end{bmatrix} < 0. \tag{30}$$

Then, by using the Schur complement lemma and some algebraic manipulations, the result in (**??**) is given.

$$\begin{bmatrix} A_{cl}^T Q + QA_{cl} & QB_{cl} & C_{cl}^T \\ * & -I & D_{cl}^T \\ * & * & -\gamma^2 I \end{bmatrix} < 0. \tag{31}$$

Let $J_1 = \begin{bmatrix} I & 0 & 0 \\ 0 & 0 & I \\ 0 & I & 0 \end{bmatrix}$ and $P = Q^{-1}$. Then, multiplying (**??**) on the right by J_1^T and on the left by J_1, (**??**) is obtained by some simple matrix manipulations.

$$\begin{bmatrix} PA_{cl}^T + A_{cl} P & PC_{cl}^T & B_{cl} \\ * & -\gamma^2 I & D_{cl} \\ * & * & -I \end{bmatrix} < 0. \tag{32}$$

Further, based on duality theory, for the transfer function $C_{cl}(sI - A_{cl})^{-1} B_{cl} + D_{cl}$ and its dual $B_{cl}^T (sI - A_{cl}^T)^{-1} C_{cl}^T + D_{cl}^T$, the H_∞ norms are equivalent. Then, (**??**) can be obtained.

This completes the proof. □

Remark 4. *By the introduction of additional matrix and a small sufficient scalar, additional degrees of freedom can be obtained as in [?], which may make the condition be of use in the solution of many difficult control synthesis problems in a potentially less conservative framework, and the linear part of the deadzone can be effectively compensated.*

Theorem 2. *For a prescribed scalar $\gamma > 0$and a sufficiently small scalar $\epsilon > 0$, the closed-loop sliding dynamics system in (**??**) is asymptotically stable, and satisfies the norm constraint $\|T(s)\|_\infty < \gamma$ if and only if there exist matrices $0 < \tilde{Q}_s = Q_s^T = \begin{bmatrix} \tilde{Q}_{sij} \end{bmatrix}_{2\times 2} \in R^{2n\times 2n}$with $\tilde{Q}_{sij}$ having appropriate dimensions, and matrices M and G such that the following LMI hold:*

$$\begin{bmatrix} \tilde{Q}_s - He\{G\} & G + \epsilon \Gamma_a & 0 & \sqrt{\epsilon} B_{cl} \\ * & -\tilde{Q}_s & \sqrt{\epsilon}\Gamma_c^T & 0 \\ * & * & -\gamma^2 I & 0 \\ * & * & * & -I \end{bmatrix} < 0, \tag{33}$$

where

$$\Gamma_a = A_{cl} G + Bm_{min} M, \Gamma_c = CG + DM,$$

In addition, the constraints $\|T(s)\|_\infty < \gamma$ is satisfied simultaneously for the closed-loop system with the stabilizing state feedback gain matrix

$$K = MG^{-1}. \tag{34}$$

Proof. Following similar lines as in [**?**], to make the problem tractable, first construct the matrix Z^T as

$$Z^T = \begin{bmatrix} Y & N \\ N & -N \end{bmatrix}. \tag{35}$$

Also introduce the transformation matrix $\mathcal{J}_1 = (I_2 \otimes E) \oplus I \oplus I$ with $E = \begin{bmatrix} I & I \\ 0 & I \end{bmatrix}$. Then, pre- and post-multiply (**??**) by $\mathcal{J}_1$ and $\mathcal{J}_1^T$, respectively, and delete the second and the forth rows and columns, it follows that

$$\begin{bmatrix} (1,1) & (1,2) & 0 & \sqrt{\epsilon}(Y+N)B_{cl} \\ * & (2,2) & \sqrt{\epsilon}(C+DK)^T & 0 \\ * & * & -\gamma^2 I & 0 \\ * & * & * & -I \end{bmatrix} < 0 \tag{36}$$

where

$$\begin{aligned} (1,1) &= Q_{s11} + He\{Q_{s12}\} + Q_{s22} - He\{Y+N\}, \\ (1,2) &= Y + N + \epsilon(Y+N)(A_\delta + Bm_{min}K), \\ (2,2) &= -Q_{s11} + He\{Q_{s12}\} + Q_{s22}. \end{aligned}$$

Further, define $G^T = (Y+N)^{-1}$, $\tilde{Q}_s = G^T(Q_{11} + He\{Q_{s12}\} + Q_{s22})G$ and $M = C_cG$, then, pre- and post-multiply (**??**) by $(I_3 \otimes G^T) \oplus I \oplus I$ and $(I_3 \otimes G) \oplus I \oplus I$, respectively, (**??**) is equivalent to (**??**). This completes the proof. □

Theorem 3. *For a prescribed sufficiently small constant $\mu > 0$, the linear continuous system (**??**) subjected to Assumptions 1-4 with multiple inputs containing actuator dead zone, is asymptotically converge to the sliding mode (**??**), if the nonlinear control law is chosen as (**??**)*

$$u_1(x,t) = -\rho(t)(GB)^{-1}\frac{s(x,t)}{\|s(x,t)\|} \tag{37}$$

where

$$\rho(x,t) = \frac{1}{m_{min}}(\mu + \|GE\|\|\bar{d}(x,t)\| + \|GB\|\|\bar{w}(x,t)\|)$$

Proof. Let us choose a *Lyapunov* functional candidate

$$V(t) = \frac{1}{2}s^2$$

Calculating the difference of $V(t)$ along the system (**??**), we have

$$\begin{aligned} \dot{V}(t) = s^T\dot{s}(x,t) &= s^T(GB((m - m_{min})u_0 + mu_l + d(t)) + GEw(t)) \\ &\leq \|s\|(-m_{min}\rho + \|GE\|\|\bar{d}(x,t)\| + \|GB\|\|\bar{w}(x,t)\|) \\ &< 0 \end{aligned}$$

This completes the proof. □

Remark 5. *The chattering phenomenon of nonlinear controller design will be serious. We can employ continuous boundary layer approximation of discontinuous control presented in [?], [?] to eliminate it. But the key point in this paper is not here, so we do not focus on it.*

Then, based on Theorem 2 and Theorem 3, the following algorithm is presented for the design of integral sliding mode controller:

Algorithm 1.

Step 1. Minimize γ^2 subject to the LMI constraints (**??**). Denoting the optimal solution as M, G, and γ, then, the state-feedback controller gain can be obtained by $K = MG^{-1}$.

Step 2. Substitute K into (**??**). Then, the integral sliding surface can be given by $s(x,t) = B^{+}[x(t) - x(t_0) - \int_{t_0}^{t_f}((A + BMG^{-1})x(\tau))\, dt]$.

Step 3. Set the control as $u(x,t) = MG^{-1}x(t) - \rho\frac{s(x,t)}{\|s(x,t)\|}$,where $\rho = \frac{1}{m_{min}}(\mu + \|GE\|\bar{d} + \|GB\|\bar{w})$.

$\lim\limits_{T_f\to\infty}\int_{T_0}^{T_f} = 1$

$\sum\limits_{i=1}^{2^m}$

IV. Numerical Simulations

In this section, a numerical example is proposed to show the effectiveness of the proposed control strategy. For this purpose, consider the continuous-time system (**??**) with

$$A = \begin{bmatrix} 0.05 & -0.05 & -0.23 & -0.99 \\ 1.67 & -0.78 & 0.77 & -1.53 \\ -1.46 & 0.60 & -2.97 & -1.06 \\ -1.78 & 1.49 & 1.69 & 0 \end{bmatrix}, B = \begin{bmatrix} 1.27 & 0.18 \\ 0.63 & 2 \\ -1.83 & 1.39 \\ 0.52 & 1.92 \end{bmatrix}, E = \begin{bmatrix} -1.3 & -0.1 \\ -1.87 & 0 \\ 1.79 & -0.4 \\ -0.95 & 1.76 \end{bmatrix},$$

$$C = \begin{bmatrix} -1.97 & 1.04 & 1.26 & 0.16 \end{bmatrix}, D_{21} = \begin{bmatrix} 0.4 & 1.28 \end{bmatrix}. \tag{38}$$

It need to point out that we always set $\epsilon = 0.02$, and $m = 1.2$ in this example.

In the simulation, the bounds of the parameters of the dead-zone are $m_{\min} = 0.85$, $m_{\max} = 1.25$, $b_{r\min} = 0.1, b_{r\max} = 0.6, b_{l\min} = -0.7, b_{l\max} = -0.1$, then we can calculate the $\bar{d}(u(t)) = 0.875$.

Minimizing γ subject to (**??**) in Theorem **??**, the H_∞ state feedback controller's gain matrix K is obtained as follows by using (**??**),

$$K = \begin{bmatrix} -6.6234 & -1.8059 & 3.0005 & 0.7187 \\ 3.6089 & -0.2482 & -1.9220 & -0.3496 \end{bmatrix}$$

and the performance indexes γ is obtained as $4.2995e-007$. Then, from Theorem **??**, we get nonlinear control law parameter $\rho = 1.1301$.

To illustrate the simulation results of the control objectives, with the zero initial state, the disturbance $w^T(t) = \begin{bmatrix} w_1^T(t) & w_2^T(t) \end{bmatrix}$ added to the system is assumed to be:

$$w_1(t) = w_2(t) = \begin{cases} 0.2\,cos(t) & 20 \le t \le 30 \\ 0 & otherwise. \end{cases} \tag{39}$$

From Fig.1, the simulation results show that the system is more robust than using H_∞ technology alone when the unmatched disturbance is added to the system. It can also be concluded that the system states can be affected severely when the deadzone fault occurs using traditional integral sliding mode controller, while the system is on the sliding mode all the time if the deadzone failure occurs using the proposed scheme.

From Fig.2, the closed loop system is on the integral sliding surface whether the deadzone or unmatched disturbances occurs using proposed scheme. While the system will be out of the sliding surface using traditional scheme.

V. Conclusion

In this paper, robust integral sliding mode control has been proposed for multi inputs systems with nonsymmetric deadzone and unmatched disturbances. Considering unmatched perturbations, the state feedback controller in combination with ISM surface has been proposed to further robustify than using H_∞ alone. As far as the case of deadzone is concerned, the integral sliding mode controller ensures that, even in the presence of uncertainties, the deadzone can be compensated completely. Finally, simulation studies are presented to illustrate the effectiveness of the proposed control.

Acknowledgement

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