Lecture 5

Informed Search

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Reading for This Class:
Chapter 3, Russell and Norvig

Review

- **Last Class**
  - **Implementation of Blind Search strategies (never looks ahead to goal)**
    - **Breadth-first search**
    - **Depth-first search**
    - **Uniform-cost search**
  - **Analysis of Blind Search Strategy**

- **This Class**
  - **Heuristic Search**

- **Next Class**
  - **Local Search**

Heuristic Search

- **Informed Search (Heuristic Search): use <u>problem-specific knowledge</u> beyond the definition of the problem itself to look ahead to goal**

- **General approach of heuristic search:**
  - **Best-first Search**
    - **Idea: use an evaluation function f(n) for each node**
    - **f(n) is constructed as a cost estimate, and the node with the lowest f(n) is expanded first**
    - **Implemented using a priority queue based on f(n) not g(n)**
    - **Goal-test when node is expanded**

  - **The choice of f(n) determines the search strategy**
    - **Greedy best-first search**
    - **A* search**

Heuristic Function

- **Path-cost Function**
  - g(n) = path cost from root node to node n

- **Heuristic Function**
  - h(n) = an estimated cost from node n to goal
  - nonnegative, problem-specific functions, with one constraint: if n is a goal node, then h(n)=0
  - If h(n1) < h(n2), we guess it is cheaper to reach the goal from n1 than n2

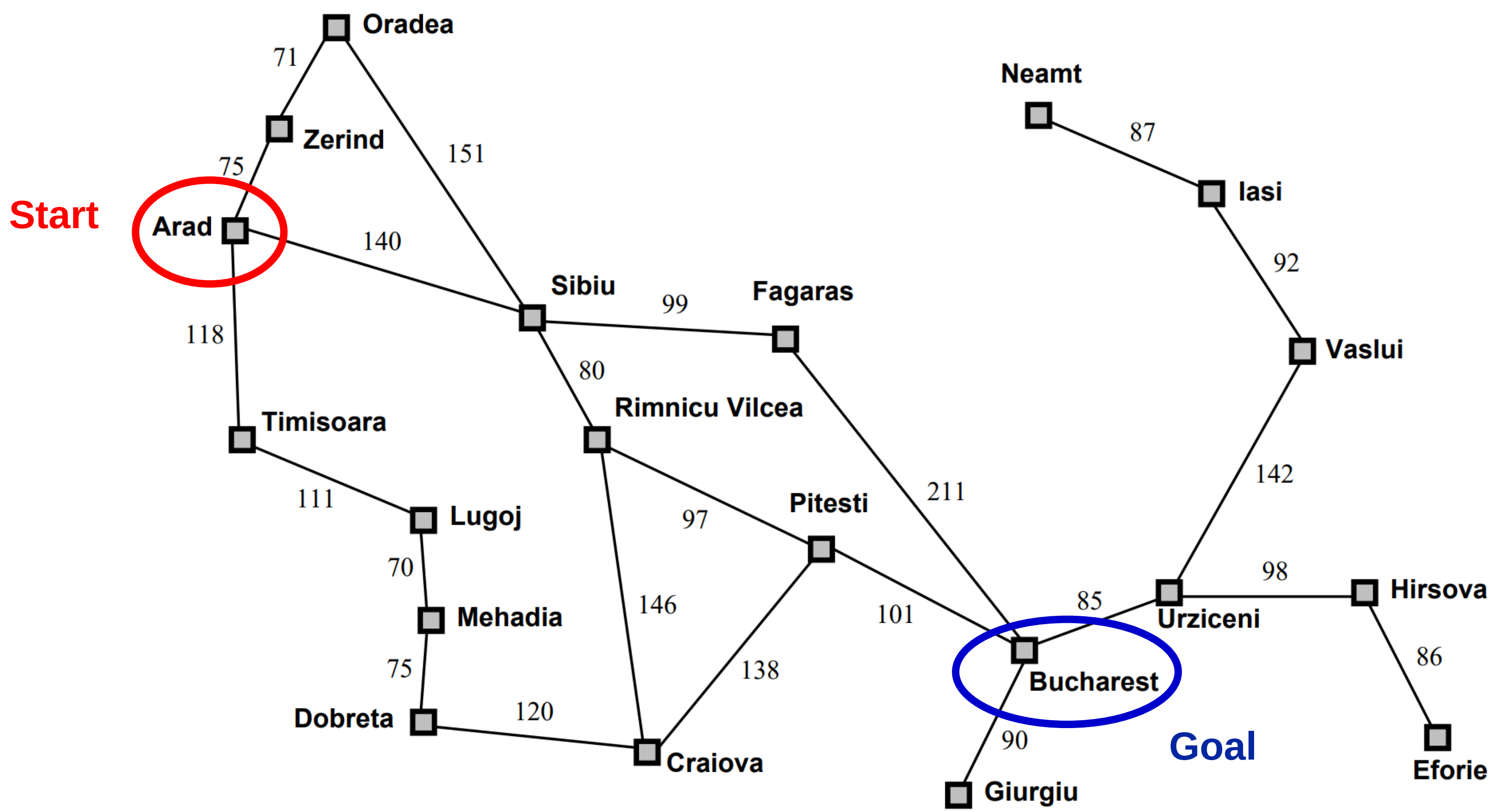
- **Evaluation Function**
  - f(n) = h(n), resulting in greedy best-first search
  - f(n) = g(n) + h(n), resulting in A* search

Greedy Best-First Search

- **Strategy**
  - Ignore the cost to get to node n, g(n)
  - Try to expand the node that is closest to the goal
  - The node is likely to lead to a solution quickly
  - Evaluation function is the heuristic function
    - $f(n) = h(n)$

Example: Route Finding

- **Suppose we start from Arad and our goal is to be in Bucharest**
- **Need to design a heuristic function**



Example: Route Finding Heuristic Function

- **Heuristic Function**
  - Using the straight-line distance h_{SLD}
  - h_{SLD} cannot computed from the problem description itself and is from experience
  - We require $h_{SLD}(In(Bucharest)) = 0$

Arad	366	**Mehadia**	241
Bucharest	0	**Neamt**	234
Craiova	160	**Oradea**	380
Drobeta	242	**Pitesti**	100
Eforie	161	**Rimnicu Vilcea**	193
Fagaras	176	**Sibiu**	253
Giurgiu	77	**Timisoara**	329
Hirsova	151	**Urziceni**	80
Iasi	226	**Vaslui**	199
Lugoj	244	**Zerind**	374

Values of h_{SLD}—straight-line distances to Bucharest.

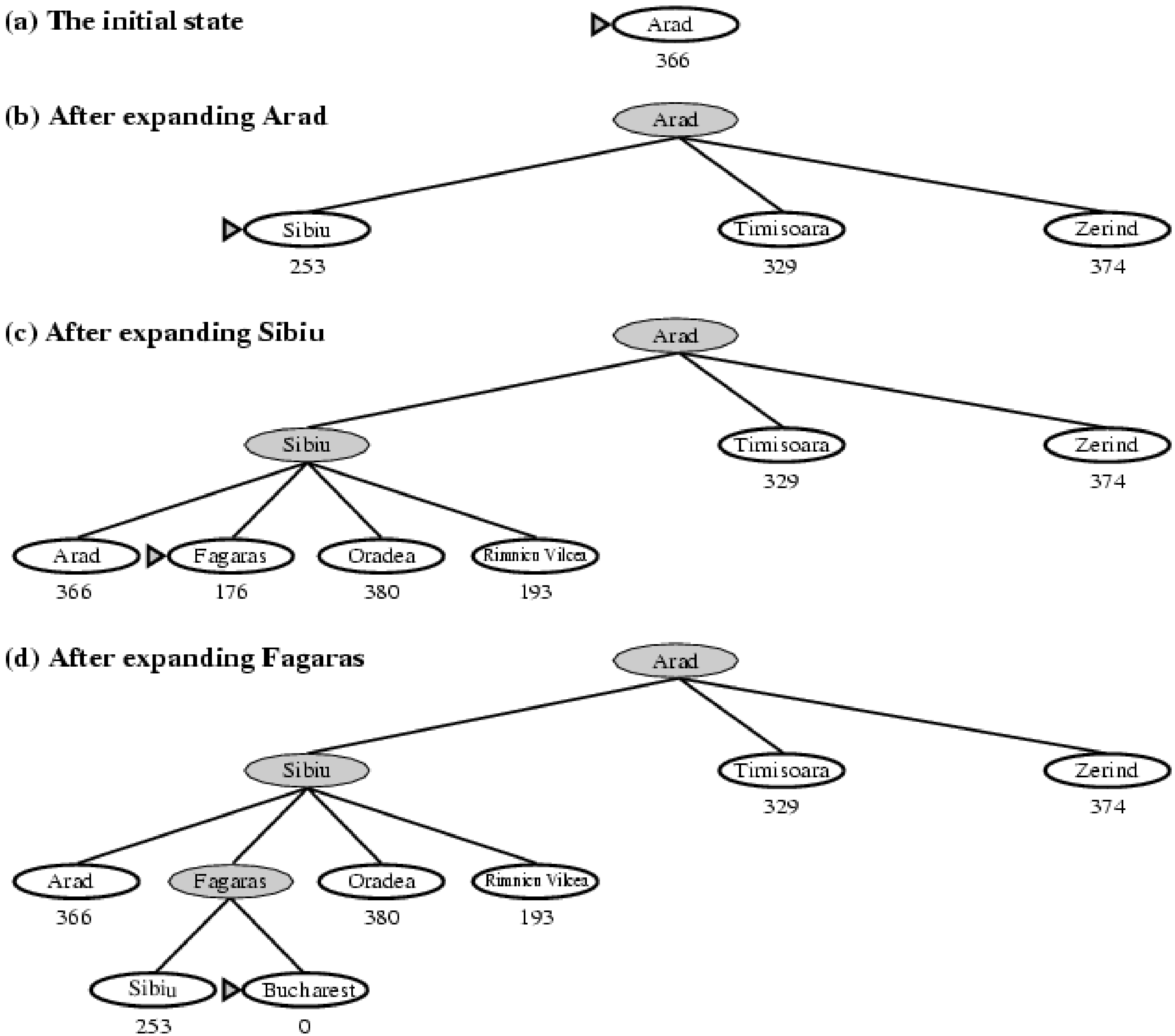
Stages in a Greedy Best-First Tree Search

Frontier queue:
Arad 366

Frontier queue:
Sibiu 253
Timisoara 329
Zerind 374

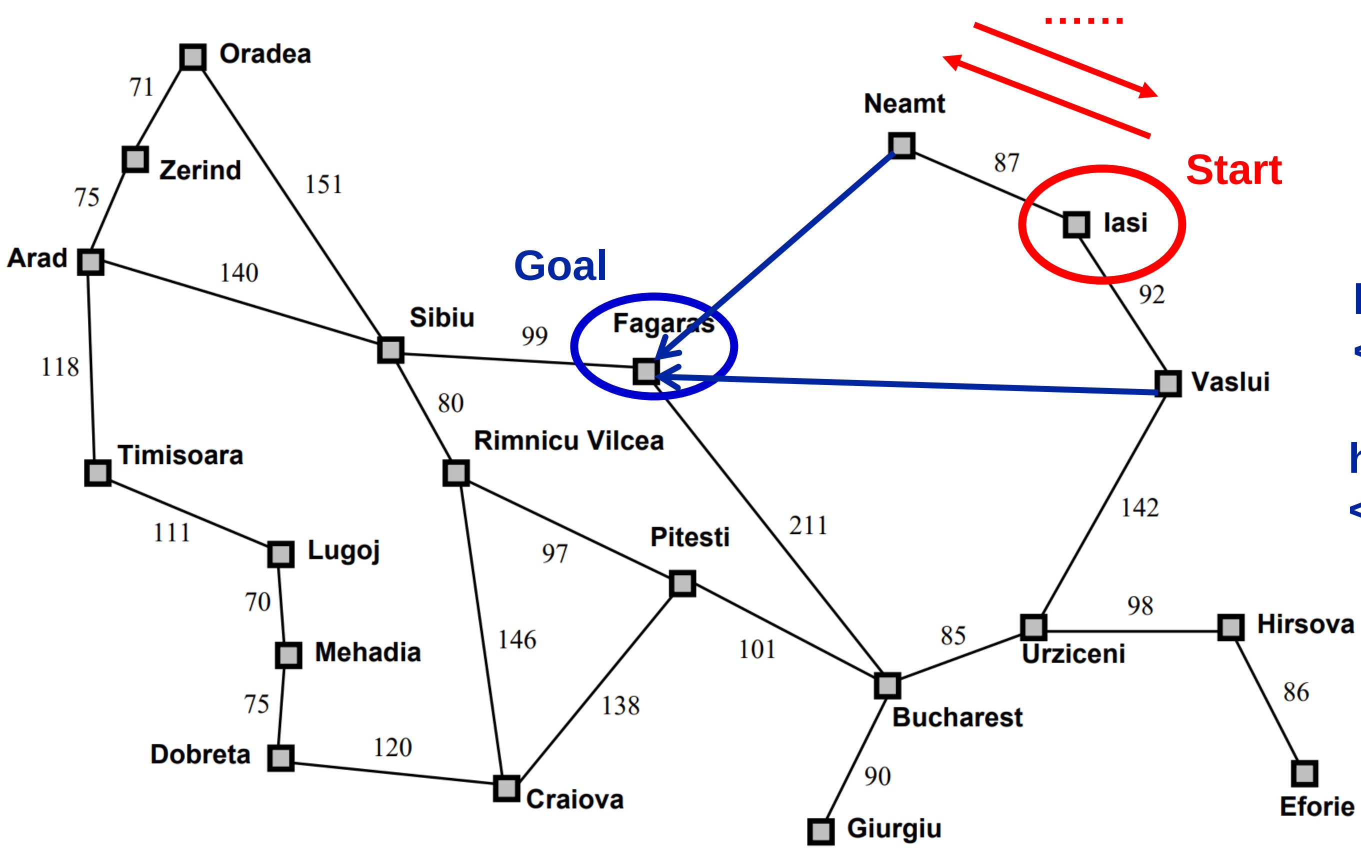
Frontier queue:
Fagaras 176
Rimnicu Vilcea 193
Timisoara 329
Arad 366
Zerind 374
Oradea 380

Frontier queue:
Bucharest 0
Rimnicu Vilcea 193
Sibiu 253
Timisoara 329
Arad 366
Zerind 374
Oradea 380



Analysis of Greedy Best-first Search

- Complete?
  - No, tree search can get stuck in loops, E.g., going from Iasi to Fagaras
  - Graph search is complete in finite space



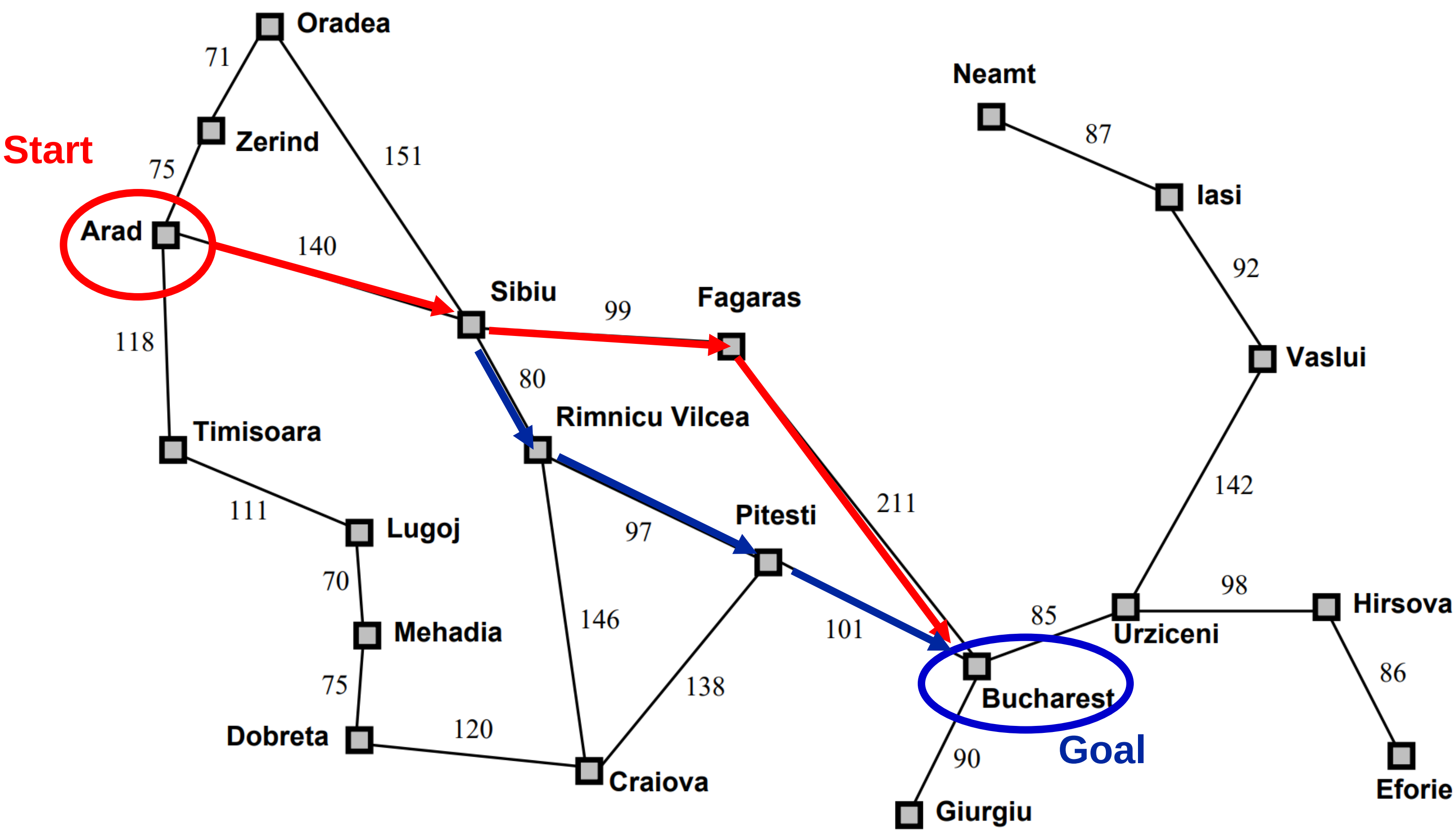
$h_{SLD}(In(Neamt)) < h_{SLD}(In(Vaslusi))$

$h_{SLD}(In(Iasi)) < h_{SLD}(In(Vaslusi))$

Consider the problem of getting from Iasi to Fagaras

Analysis of Greedy Best-first Search

- **Optimal? No (A -> S -> R -> P -> B)**



Analysis of Greedy Best-first Search

- Requirement of Greedy Best-first Search
  - We need additional information to design the heuristic function
    - E.g., straight line distance in our route-finding example
- Performance
  - Complete?
    - No, tree search version can get stuck in loops
    - Yes for graph search in finite space
  - Not optimal: can have a better solution
  - Time complexity: $O(b^m)$ in the worst case
    - but a good heuristic can give dramatic improvement
  - Space complexity: $O(b^m)$ in the worst case
    - keep all nodes in memory
- Conclusion
  - Good Heuristic is the Key
    - The complexity can be reduced substantially
  - Bad Heuristic can make the problem worse

Greedy Algorithms

- **Step-by-step algorithms**
- **Sometimes works well for optimization problems**
- **A greedy algorithm works in phases. At each phase:**
  - **You take the best you can get right now, without regard for future consequences**
  - **You hope that by choosing a *local* optimum at each step, you will end up at a *global* optimum**

Uniform Cost Search vs. Greedy Best First Search

- **Same Strategy**
  - Implementation
    - A priority queue
- **Difference**
  - Uniform Cost Search
    - Cares for the path cost g(n)
    - Expand nodes with the lowest path cost
  - Greedy Best First Search
    - Cares for the heuristic h(n)
    - Expand nodes with the lowest heuristic value
- **Thinking**
  - Can we combine the two search strategies?
  - A* Search

A* Search

- **Goal**
  - Minimizing the total estimated solution cost
- **Evaluation function**
  - f(n)
    - Estimated cost of the cheapest solution through n
    - f(n)=g(n)+h(n)
    - A* search sorts frontier by f(n)
  - g(n)
    - History
    - Gives the path cost from the start node to node n
    - Uniform cost search sorts frontier by g(n)
  - h(n)
    - Estimated cost of the cheapest path from node n to the goal
    - Heuristic function
    - Greedy best first search sorts frontier by h(n)

Example: Route Finding
A* Search

- **Heuristic Function**
  - Using the straight-line distance
- **Path Cost Function**
  - Using the path cost from the graph

Arad	366	Mehadia	241
Bucharest	0	Neamt	234
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Eforie	161	Rimnicu Vilcea	193
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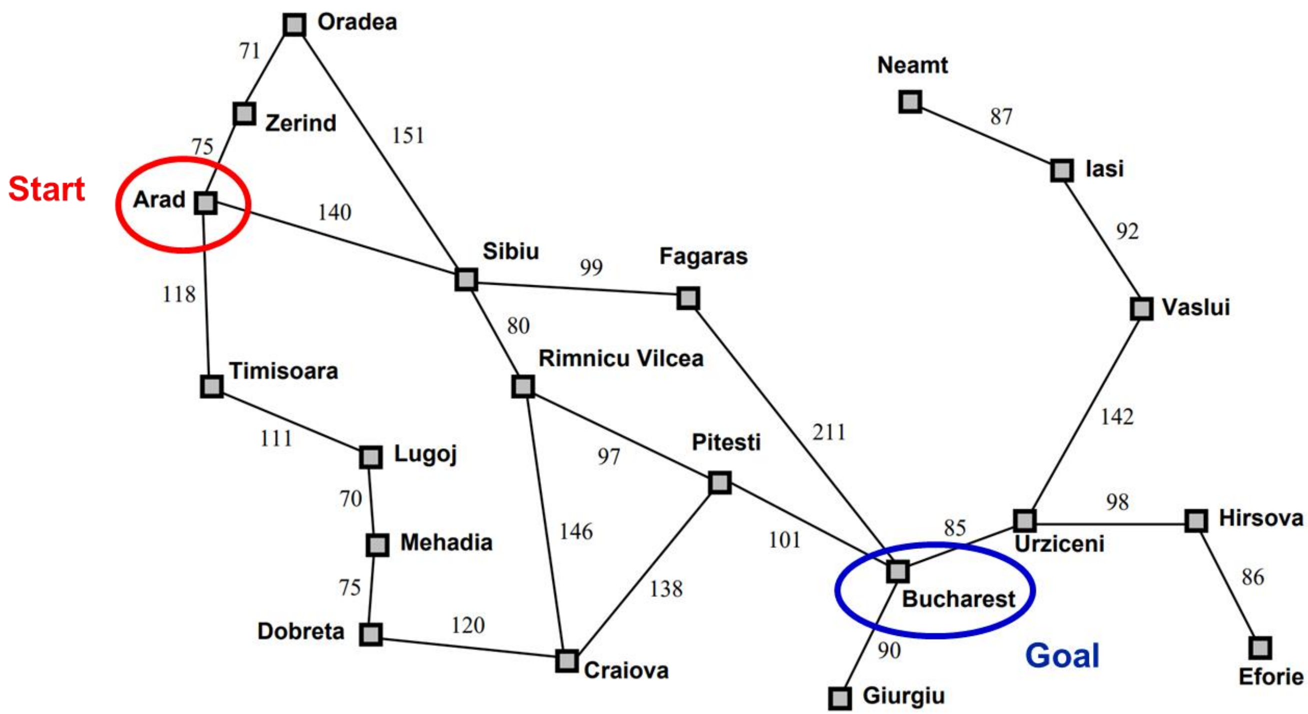


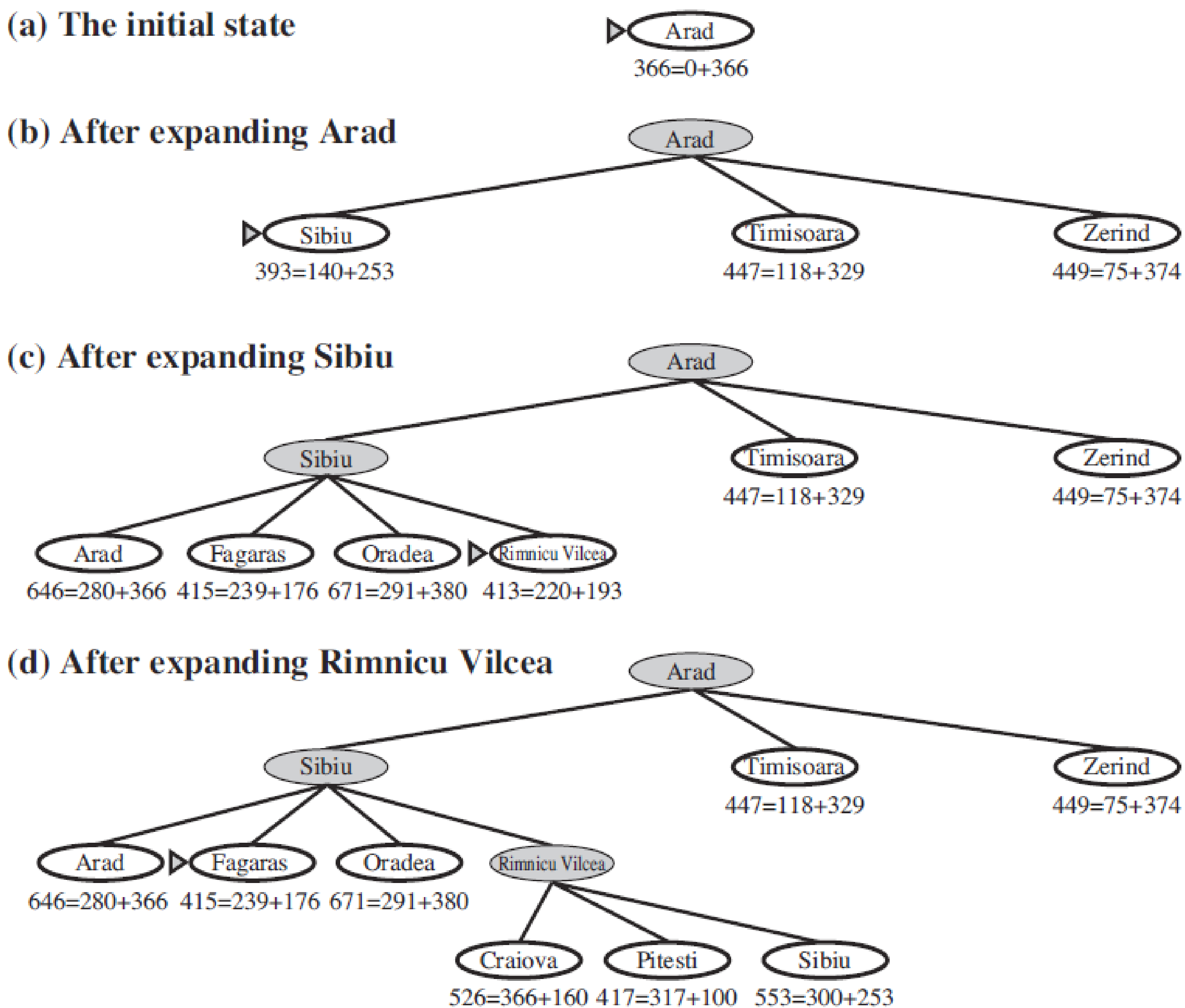
Progress in A* Tree Search

Frontier queue:
Arad 366

Frontier queue:
Sibiu 393
Timisoara 447
Zerind 449

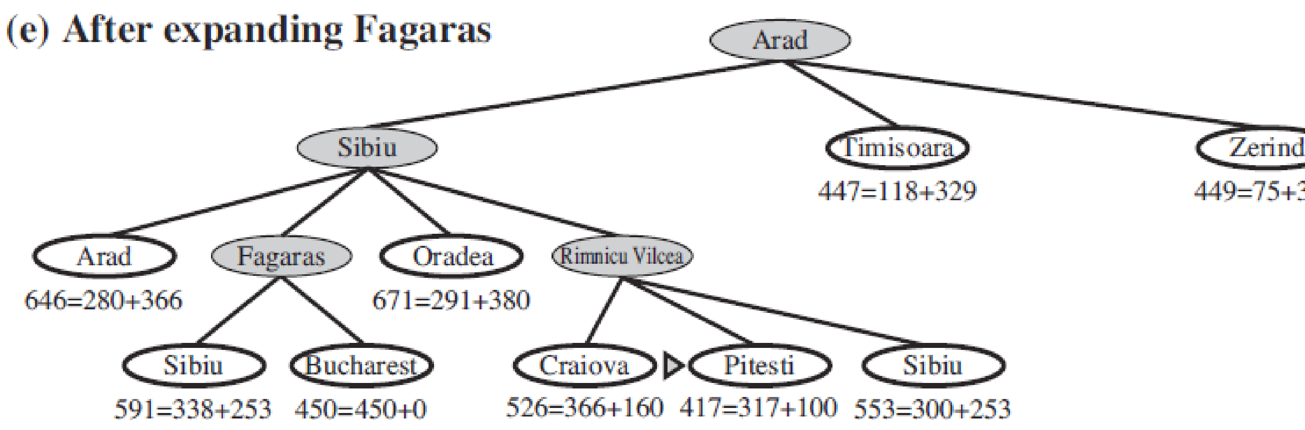
Frontier queue:
Rimricu Vicea 413
Fagaras 415
Timisoara 447
Zerind 449
Arad 646
Oradea 671

Frontier queue:
Fagaras 415
Pitesti 417
Timisoara 447
Zerind 449
Craiova 526
Sibiu 553
Arad 646
Oradea 671



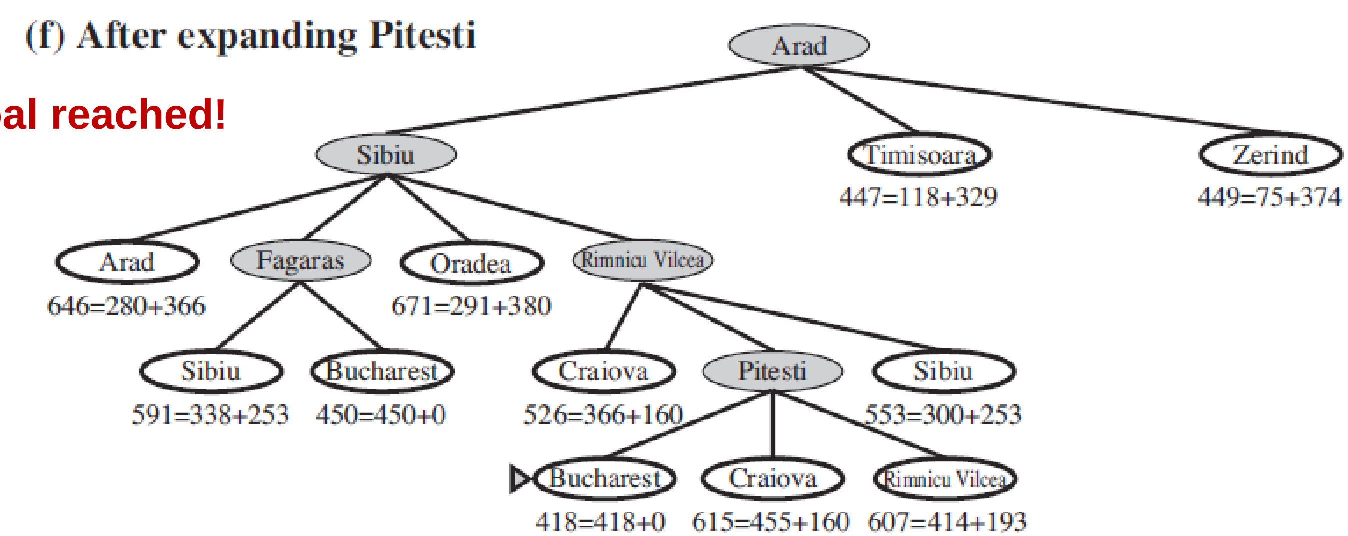
Progress in A* Tree Search

Frontier queue:
Pitesti 417
Timisoara 447
Zerind 449
Bucharest 450
Craiova 526
Sibiu 553
Sibiu 591
Arad 646
Oradea 671



Frontier queue:
Bucharest 418
Timisoara 447
Zerind 449
Bucharest 450
Craiova 526
Sibiu 553
Sibiu 591
Rimricu Vicea 607
Craiova 615
Arad 646
Oradea 671

Goal reached!



Another Example of A* Search
8-puzzle

- Develop a useful heuristic function
- A good heuristic function can reduce the search process

- 8-puzzle
- Size of reachable state space: 9!/2 = 181, 440 states
- Use heuristics

8-Puzzle

- **Function h(n) that estimates the cost of the cheapest path from node n to goal node.**
- **Example: 8-puzzle**

5		8
4	2	1
7	3	6

n

1	2	3
4	5	6
7	8	

goal

$h_1(n)$ =?

8-Puzzle

- **Function h(n) that estimates the cost of the cheapest path from node n to goal node.**
- **Example: 8-puzzle**

5		8
4	2	1
7	3	6

n

1	2	3
4	5	6
7	8	

goal

$$h_1(n) = \text{number of misplaced tiles}$$
$$= 6$$

8-Puzzle

- **Function h(n) that estimates the cost of the cheapest path from node n to goal node.**
- **Example: 8-puzzle**

5		8
4	2	1
7	3	6

n

1	2	3
4	5	6
7	8	

goal

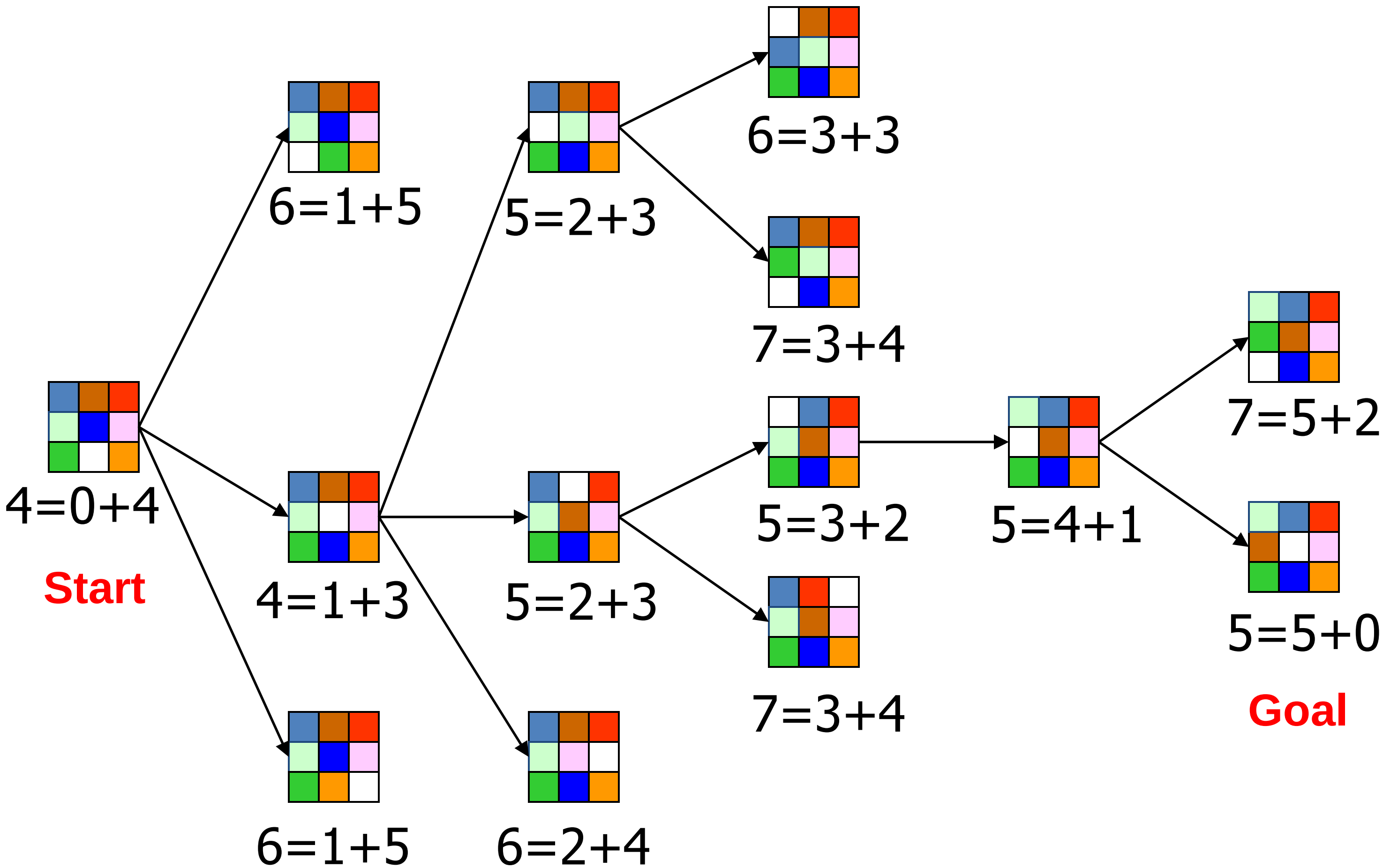
$h_2(n)$ = total Manhattan distance (sum of the distances of every numbered tile to its goal position)

$= 2 + 3 + 0 + 1 + 3 + 0 + 3 + 1$

$= 13$

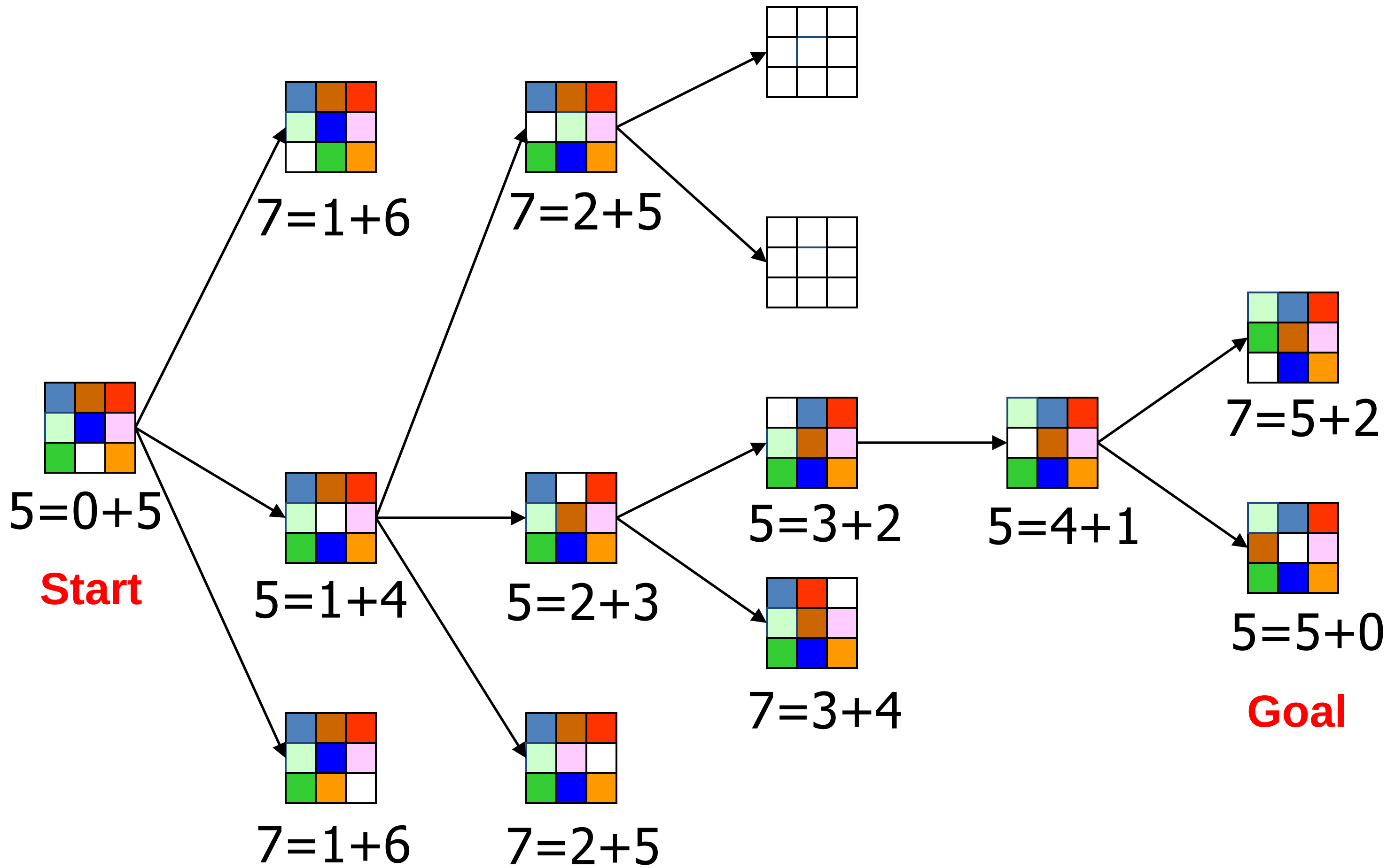
8-Puzzle in Graph Search

$f(n) = g(n) + h_1(n)$ with $h_1(n)$ = number of misplaced tiles



8-Puzzle in Graph Search

$f(n) = g(n) + h_2(n)$ with $h_2(n) = \Sigma$ distances of tiles to goal



Heuristic Function

- **Relaxed problem:**
  - **A problem with fewer restrictions on the actions than the original**
  - **The cost of an optimal solution to a relaxed problem is an admissible heuristic for the original problem**
  - **If the rules of the 8-puzzle are relaxed so that a tile can move anywhere, then $h_1(n)$ gives the shortest solution**
  - **If the rules are relaxed so that a tile can move to any adjacent square, then $h_2(n)$ gives the shortest solution**

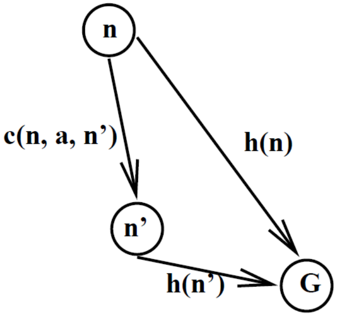
- **Define h(n) = cost of an exact solution to a relaxed problem (fewer restrictions on operator)**
  - **If $h_2(n) \geq h_1(n)$ for all n then h_2 dominates h_1**
  - **So h_2 is optimistic and more accurate than h_1**
  - **h_2 is therefore better for search because fewer nodes will be expanded**

- **Key point: the optimal solution cost of a relaxed problem is no greater than the optimal solution cost of the real problem (never overestimate)**

Analysis of A* Search

- **Complete?**
  - Yes, in finite state space
- **Optimal?**
  - Yes, with tree search if h(n) is admissible
  - Yes, with graph search if h(n) is both admissible and consistent
    - admissible heuristic
      - **Provided that h(n) never overestimates the cost to reach the goal**
      - **$h(n) \le h^*(n)$, where $h^*(n)$ is the true cost from node n to the goal**
      - **E.g., h_{SLD} never overestimates actual road distance**
    - consistent heuristic (triangle inequality)
      - **For every node n and every successor n' of n generated by any action a**
      - **the estimated cost of reaching the goal from n is no greater than the step cost of getting to n' plus the estimated cost of reaching the goal from n'**

$$h(n) \le c(n, a, n') + h(n')$$



- **Time complexity: $O(b^d)$ (worst case)**
  - but a good heuristic can give dramatic improvement
- **Space complexity: $O(b^d)$, keep all nodes in memory**

Analysis of A* Search

- **Drawback**
  - **Space requirement**
    - **Keep all generated nodes in memory**
    - **Easy to run out of memory**
    - **Not practical for many large-scale problems**

Summary

- Heuristics and Heuristic Functions
- Heuristic Algorithm
- Greedy Algorithm
- A* Algorithm

What I want you to do

- **Review Chapter 3**
- **Work on your assignment**