Study on Energy Stored in Thin Layers of MEMS Ultrasonic Resonance Separator

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Abstract **The particles suspending in liquid suffer acoustical radiation force from ultrasonic field. The force may cause particles movement and has special advantage to separate particles from liquid in Micro Electro Mechanical Systems (MEMS). It is important to improve the energy stored in liquid layer so as to increase separation ability for decreasing drive demand in MEMS. Resonance may hugely increase the magnitude and generally be employed in micro separator. The summits of admittance spectrum show the resonance frequencies. It may come from resonance in one layer or multi layers in micro separator. This paper studies the difference of energy stored in layers at each rein frequency. It provides the way to select the most effect separation frequency other than admittance spectrum.**

Key words **Micro Electro Mechanical Systems (MEMS); ultrasonic; separation; fluid**

There are multi conventional ways to remove particles suspending in liquid such as sedimentation, flocculation, centrifugation, and filtration. They work on respective physical principles. Sedimentation and flocculation are based on gravity for difference of density between particles and host liquid. Centrifugation hugely increases the effect from difference of density with controlled centrifugal acceleration other than gravity. Filtration works on limit in size. Electric or magnetic field may be used for special particles also. Sometimes adsorbability is also employed to remove particles in liquid. Actually fluid theory shows that particles are given an addition time-averaged radiation force other than vibration from sound wave in liquid. It shows a conceivable way to move particles other than ways mentioned above.

1 Separating Particles in Fluid Based on the Time-Averaged Radiation Force Caused by Ultrasonic

The sound radiation force depends on mechanical properties of particles. King gave the result of rigid small ball in ideal non-viscous fluid with progressive and standing wave[1]:

$$\langle F_p \rangle = 4\pi k^4 a^6 \langle \bar{E}_p \rangle k_p(\lambda,\sigma) \qquad (1)$$

$$\langle F_s \rangle = 4\pi ka3 \langle \bar{E}_s \rangle K_s(\lambda,\sigma)\sin(2kx) \qquad (2)$$

where $\langle F_p \rangle$ and $\langle F_s \rangle$ are the time-averaged radiation force from progressive and standing waves, respectively; a is the radius of particle; $k=\omega/\upsilon$ is the wave number in host liquid; $ka \ll 1$; $\langle \bar{F}_s \rangle$ and $\langle \bar{F}_p \rangle$ are time-averaged density of energy in liquid from standing and progressive wave, respectively; and K_p and K_s are the acoustic contrast factors:

$$K_p(\lambda,\sigma) = \frac{1}{(1+2\lambda)^2}[(\lambda - \frac{1+2\lambda}{3\lambda\sigma^2})^2 + \frac{2}{9}(1-\lambda)^2] \qquad (3)$$

$$K_s(\lambda,\sigma) = \frac{1}{3}[\frac{5\lambda-2}{2\lambda+1} - \frac{1}{\lambda\sigma^2}] \qquad (4)$$

where $\lambda=\rho_0/\rho$, $\sigma=\upsilon_0/\upsilon=k/k_0$, ρ_0, k_0 and υ_0 are the density, wave number and sound speed of particles, and ρ, k and υ refer to those of host fluid.

King's result was later expended to compressible and other shape ball and viscous fluid by more researchers. Yosioka studied the calculations of the acoustic radiation pressure on compressible spheres suspended freely in a non-viscous fluid for a plane sound wave[2]. Westervelt derived a general expression for the force owing to radiation pressure acting on an object of any shape and having an arbitrary normal boundary impedance[3]. Subsequently more effort has been spend on sound attenuation and secondary force. Good agreement between theory and experiment was found. Eventually King's calculation is enough accurate for low concentration and $ka \ll 1$ in low

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viscosity host fluid. The radiation force from standing wave is far more than from progressive wave. Furthermore, it will hugely increase the energy density stored in standing wave with resonance. Usually the radiation pressure on particles from sound wave is weak. General ultrasonic separators work with standing wave.

The radiation pressure from standing wave makes particles to move to the sound pressure node. Moreover, the particle objective position lies somewhere steadily. Then the mixture-portion with concentrated particles can be collected with some way and the clean liquid with fewer particles is collected from other place. A latent way is displayed to separate particles suspending in fluid with standing ultrasonic[4]. The separation works on density and sound speed ratio between particles and hosting fluid. It is very suitable for neutral particles in density or charge or magnet, such as cells. In order to insulate the ultrasonic transducer from liquid there usually are four layers in typical ultrasonic resonance separator, including transducer, matching, liquid, and reflection layers with thicknesses defined as t_T, t_M, t_L, and t_R, respectively, as shown in Fig.1. There is enough difference in sound impedance between reflection layer and air baking to cause almost full reflection and resonance in layers. Matching and reflection layers also keep liquid in resonance chamber.

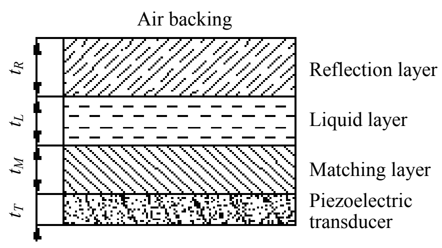


Fig.1 Layers in ultrasonic separator

The summits of admittance spectrum depend on the thickness of layers[5-7]. The energy stored in standing wave will be easy increased with frequency. But it maybe leads to another negative result that ultimate positions of particles movement become closely so as to difficultly collect particles. The influence will be ignored for small size separator with only one node plane in liquid layer.

2 Single Ultrasound Node MEMS Separator with Multi Summits in Spectrum

Recently, Micro Electro Mechanical Systems (MEMS) has shown enormously advantages in many fields. There is attractive potential use in fluid with MEMS for treating biotechnology and measurement. Separation with ultrasonic seems become more useful way for MEMS than conventional ways such as filter, sedimentation, flocculation or centrifugation because of the restriction in space and operation. Micro separator with four layers is also suitable for MEMS manufacture as device in ordinary size. There are convenient ways to deal with layers in separator. PZT (Piezoelectric lead Zirconate Titanate) even is easier placed multi positions in MEMS than assembly configuration.

The force on suspending particles from standing wave is given in Eq.(2). It depends on the distribution of ultrasonic in liquid layer. The force drives particles to pressure node. It is obviously effective and convenience to collect particles at only the constant position in laminar flow other than multi positions. Hawkes described a stainless steel separator with liquid layer thickness of half wavelength. The concentration flow is drawn out from a strip straddling on centre plane as shown in Fig.2[8].

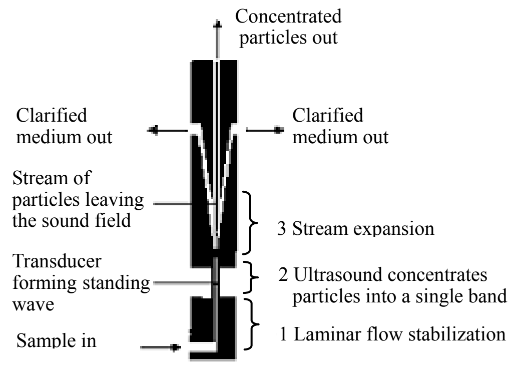


Fig.2 Separator with laminar

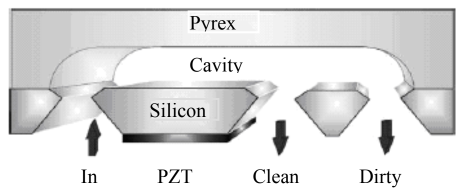
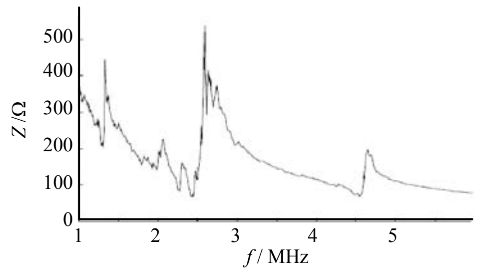


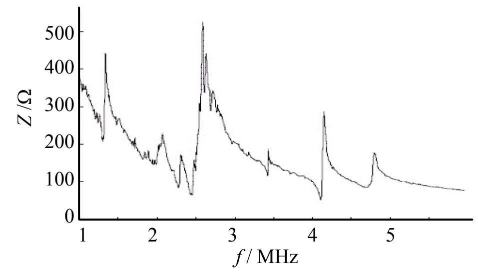
Fig.3 Micro separator

Harris expended this half wavelength thickness separator to micro separator based on MEMS processing. The micro cavity was created in a Pyres wafer with a 30 min etch in 48% buffered hydrofluoric acid using a chrome/gild mask. The ports were created in silicon wafer with standard wet KOH etching leaving chamfers. The silicon and Pyrex wafer were anodically banded together. The clean flow lying aside was drawn out from one side as shown in Fig.3[9].

Particles are moving to the node plane as mixture flowing. Fluid drag force, acoustical radiation force and buoyancy result in the particles movement profile. The micro separator ability depends on frequency because energy stored in liquid layer leads to the acoustical radiation force intensively varying with frequency. There are multi summits at the admittance spectrum for both type devices. They stand at different amplitudes. But some of them don't precisely lie on frequencies that bring integer times of half wavelength in liquid layer. Sometimes there even are higher peaks occurring beyond liquid layer resonance frequencies. It becomes important to select the most effective separation frequency to decrease drive demand in MEMS. There is inherence difficulty to measure sound response straight in MEMS channel. The method to select driving frequency for maximizing energy stored in liquid layer based on input characteristic is useful. There distinctly are two new peaks on impedance spectrum for full device more than empty as shown in Fig.4[10]. The level of new peaks is even lower than other peaks. It gives the clue to identify resonation in the liquid layer or multi layers different from in PZT and the silicon layer.



(a) Input impedance for the empty device



(b) Input impedance for the fluid filled device

Fig.4 Impedance spectrums for (a) and (b)

3 Study on Energy Stored in Layers with Network Theory

3.1 Sound Equivalent Network of Layer with Electrical Analog

The velocity u in the analytical solution to wave equation for one dimension is

$$u = \frac{F_m}{\rho A \upsilon} e^{\mathrm{i}(\omega t - kx)} \tag{5}$$

where F_m, k, υ, A, and ρ are force amplitude, wave number, sound velocity, area, and density, respectively. The relation between the force F and the velocity u is $F = Z_a u$, similar to current in long wire $V = Z_c I$, where $Z_a = \rho A \upsilon$ is the specific acoustic impedance, ρ represents mass as inductance L acting on current, $\upsilon = 1/\sqrt{\rho c}$ relates to the elastic constant c as capacitance C acting on voltage. Undoubtedly, such the relation may be simply simulated as electrical network and provides a convenient way to process acoustic parameters. Electrical analogs of sound parameters, force and velocity, are voltage and current. Then special acoustic impedance also is introduced like the electrical impedance. Therefore the network theory may be quoted to process acoustic element. The resonance micro separator may be simplified to works on one dimension model because the lateral size is far bigger than thickness. Moreover the transducer PZT vibrates in thickness. Every thin layer represents acoustic impedance in one dimension. For every layer to transmit sound the acoustic equivalent network is described as two ports circuit as shown in Fig.5[11].

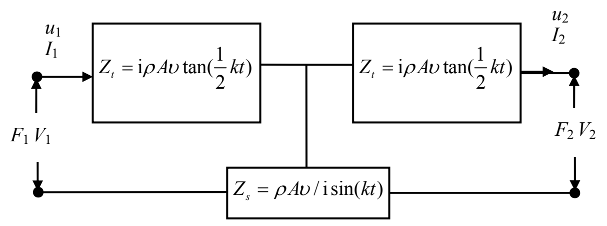


Fig.5 Sound equivalent network of layer with electrical analogy

Here t is thickness. Special acoustic impedances Z_t, and Z_s relate to the thickness. They are employed to describe the final acoustic effect of layer but not represent any alone acoustic element. They bring phase shift. The damp is ignored for thin layer. F_1, u_1, F_2, u_2 represent the sound pressure and velocities at surfaces of the layer with the following relations:

$$Z_1 = \frac{F_1}{u_1} = \frac{V_1}{I_1}, \quad Z_2 = \frac{F_2}{u_2} = \frac{V_2}{I_2} \tag{6}$$

The layer behaves the total equivalent impedance Z_1 to the previous layer, that is

$$Z_1 = Z_t + \frac{(Z_t + Z_2) Z_s}{Z_t + Z_2 + Z_s} \tag{7}$$

For a layer without loss, Eq.(7) can be written in trigonometric function form:

$$Z_1 = \rho A \upsilon \frac{Z_2 \cos kt + \mathrm{i}\rho A\upsilon \sin kt}{\rho A\upsilon \cos kt + \mathrm{i}Z_2 \sin kt} \tag{8}$$

The air backing may be considered as zero sound pressure relative to solid, i.e. F=0. The circuit should be short for air layer, Z_2=0. For the last layer near to air backing the impedance Z_{short} is expressed as

$$Z_{\text{short}} = Z_t + \frac{Z_t Z_s}{Z_t + Z_s} = \mathrm{i}\rho A\upsilon \tan kt \tag{9}$$

It describes reflection of the layer without loss. With thickness $kt = \pi, 2\pi, \cdots, n\pi$ the layer presents a pressure release boundary to previous layer as if short again. With thickness $kt = 0.5\pi, 1.5\pi, \cdots, (n-0.5)\pi$ it acts as full reflection with pressure rigid boundary. The loss in material actually exists from the inner viscoelastic damping. The damping is simply and reasonably considered as linear result of velocity. The relation of strain T and stress S may be expressed as $T = cS + \eta \partial S/\partial t$. The viscous factor η may be included into elastic constant c then complex form $\overline{c}$ is shown as below:

$$\overline{c} = c + \mathrm{i}\omega\eta \tag{10}$$

The sound velocity υ is transferred into complex $\overline{\upsilon}$:

$$\overline{\upsilon} = \sqrt{\overline{c}/\rho} = \upsilon(1 + \mathrm{i}\mu\omega) \tag{11}$$

The complex sound velocity $\overline{\upsilon}$ brings real impedance in Z_t and Z_s. It presents the energy loss in the layer. The acoustical material quality factor Q relates to the damp:

$$Q = \frac{c}{\omega\eta} \tag{12}$$

3.2 The Equivalent Circuit of Micro Separator

The micro ultrasonic separator includes four layers besides PZT. The PZT transfers electrical energy from circuit into vibration in thickness. It certainly behaves different from the thin layers without emitting sound. In order to process the PZT with simulated electrical network, Mason presented an exact equivalent circuit with three ports[12], which was separated into an electrical port and two acoustic ports. He introduced an ideal electromechanical transformer φ and an imaginary negative capacitance $-C$ at the electrical port. Based on the equivalent circuit of PZT, the equivalent circuit of micro separator is shown in Fig.6. The air backings behind PZT and reflection layer are treated as short connection in acoustic impedance. The dashed is employed to denote boundaries between layers. There is each sound pressure P and velocity u at every boundary. The superscript P, M, L, R of Z_t and Z_s present equivalent impedances related to the PZT, matching layer, liquid layer, and reflection layer, respectively.

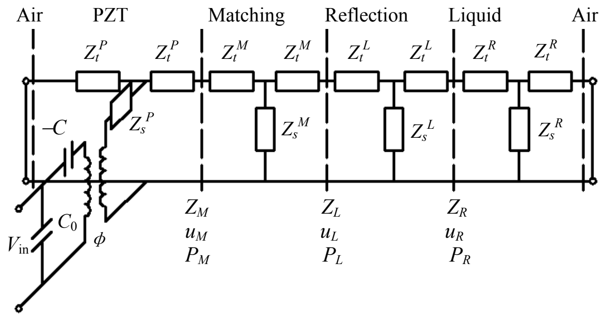


Fig.6 Equivalent network of micro separator

3.3 Resonation in Layers

Based on Fig.5 and Eq.(8), any layer with thickness $kt = \pi, 2\pi, \cdots, n\pi$ transfers the impedance at next boundary to previous surface as if there is not the layer. However there still is acoustic pressure inside. The layer is resonating and does nothing in acoustic pressure transmitting between layers.

3.3.1 Resonance in Fluid Layer

The admittance spectrum of electrical port is related to acoustic impedance of every layer. The acoustic impedance Z_M, Z_L and Z_R present impedances of the matching layer, liquid layer and reflection layer, each includes all impedance of the next layers acting on the previous layers, respectively. When the thickness of liquid layer is half wavelength with driving frequency f=f_L, liquid layer gives a minimum effect to the acoustic impedance Z_L:

$$Z_L = Z_R = \mathrm{i}\rho_R A\upsilon_R \tan k_R t_R \tag{13}$$

where the subscript R represents variables related to the reflection layer. The condition above presents resonance of liquid layer and leads to a trough at the impedance spectrum from the electrical port of the PZT.

The acoustic pressure at boundary of every layer relates to the distribution of impedance of layers. The energy stored in liquid layer depends on the velocity or pressure of liquid layer boundary, and on the quality factor Q_L of liquid:

$$Q_L = \frac{c_L}{k_L \upsilon_L \eta_L} \tag{14}$$

where c_L, k_L and η_L represent liquid elastic constant, wave number and viscous factor, respectively. Usually Q_L is some less than theoretical value because gassy or attenuation. The energy stored in liquid layer is shown as below:

$$\langle \overline{E}_S \rangle = 2Q_L \rho_L A t_L u_L^2 \tag{15}$$

3.3.2 Resonance in Fluid Layer and Reflection Layer

Based on Eq.(8), the acoustic impedance Z_L at boundary between the liquid and silicon layer relates to the thickness and the impedance Z_R of reflection layer:

$$Z_L = \rho_L A \upsilon_L \frac{Z_R \cos k_L t_L + \mathrm{i}\rho_L A \upsilon_L \sin k_L t_L}{\rho_L A \upsilon_L \cos k_L t_L + \mathrm{i} Z_R \sin k_L t_L} \tag{16}$$

The subscript L represents variables relating to liquid layer. The loss in material is ignored. The resonance occurs when the imaginary of impedance Z_L equals zero with driving frequency f=f_0, i.e.

$$\rho_R \upsilon_R \tan k_R t_R + \rho_L \upsilon_L \tan k_L t_L = 0 \tag{17}$$

The resonance brings more notable summit in admittance spectrum than alone resonance in liquid layer as shown in Fig.4. The loss is described with complex sound velocity $\overline{\upsilon}$. The quality factor relates to impedance Z_L. The impedance is written in complex form:

$$\overline{Z}_L = \rho_L A \overline{\upsilon}_L \frac{\overline{Z}_R \cos k_L t_L + \mathrm{i}\rho_L A \overline{\upsilon}_L \sin k_L t_L}{\rho_L A \overline{\upsilon}_L \cos k_L t_L + \mathrm{i}\overline{Z}_R \sin k_L t_L} \tag{18}$$

Then the combination impedance is approximatively transferred as two orders network with quality factor Q_0:

$$\overline{Z}_{L0} = R_0 \left[1 + \mathrm{i}Q_0 \left(\frac{f}{f_0} - \frac{f_0}{f} \right) \right] \tag{19}$$

The impedance leads to the distribution of velocity and pressure in layers. The energy stored in liquid layer is

$$\langle \overline{E}_S \rangle = Q_0^2 A t_L R_0 u_L^2 \tag{20}$$

3.3.3 Resonance Related to Maximum Impedance on Fluid Layer Boundary

The very high impedance at the transducer boundary is desired to enhance sound pressure on the boundary between the liquid layer and the matching layer. Then there is another resonance in fluid and reflection layers. The resonation occurs when the driving frequency f=f_1 satisfies the following formula:

$$\tan k_L t_L = \frac{\rho_L \upsilon_L}{\rho_R \upsilon_R \tan k_R t_R} \tag{21}$$

Based on Eq.(18), the approximate combination impedance of liquid and reflection layers is represented in complex with quality factor Q_1 including the loss given in Eq.(16):

$$\overline{Z}_{L1} = R_1 \left[1 + \mathrm{i}Q_1 \left(\frac{f}{f_1} - \frac{f_1}{f} \right) \right] \tag{22}$$

The resonance related to maximizing impedance brings the distribution of velocity and pressure in layers. The energy stored in liquid is expressed as below:

$$\langle \overline{E}_S \rangle = Q_1^2 A t_L \frac{P_L^2}{Z_1} \tag{23}$$

3.3.4 Experiment Result with a MEMS Separator

The MEMS separator is shown in Fig.7 with 182.3 µm liquid layer and 514 µm silicon layer. The 163.8 µm thick film PZT layers are sandwiched with gold electrodes printed and polarized in a field. The thickness of Pyrex is 1 572 µm.

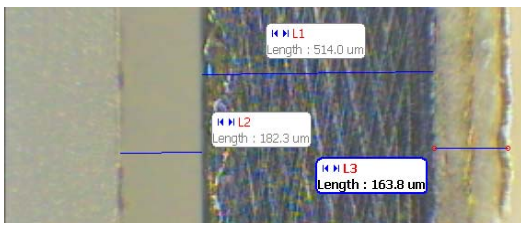


Fig.7 Section of a separator

There is a 50 Ω output resistance in the amplifier with constant amplitude. The input voltage of the separator varies with frequency shown in Fig.8. There are three obvious falls at 3.37 MHz, 3.87 MHz and 4.43 MHz in full device different from the empty chamber. These troughs related to frequency come from resonation in liquid layer and reflective Pyrex layer.

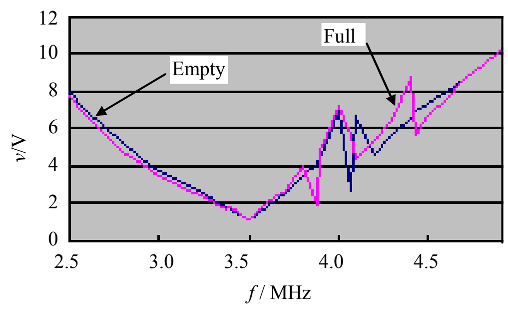


Fig.8 Input voltage of separator

The sound velocity is 1 480 m/s in water but 5 430 m/s in Pyrex. The wave numbers n in layers and the separation efficiencies e_{ff} of 1 µm latex particles are listed in Tab.1.

The thickness of Pyrex layer is approximate integer times of half-wavelength at 3.37 MHz. There is similar result in liquid layer at 4.43 MHz . The separation efficiency at 4.43 MHz is far higher than at other frequency because resonance exists in liquid layer and maximizes the stored energy.

Tab.1 Separation efficiency

f / MHz	In water		In Pyrex		e_{ff} / (%)
	k^{-1} /μm	n	k^{-1} /μm	n	
3.37	439	0.41	1611	0.98	33.4
3.87	382	0.47	1403	1.12	5.2
4.43	334	0.53	1225	1.28	74.4

4 Conclusions

The micro ultrasonic separator works on standing wave. The separation ability relates to node position of sound field in liquid layer. However it is important to increase sound energy stored in liquid. Based on the analog between sound variables of the 1-dimensional field with electrical network，the author presents the whole analogous network combining PZT, matching layer, liquid layer, and reflection layers. The analogous network may be used to calculate multi summits or troughs in admittance spectrum. The sound energy stored in liquid lies more relatively on quality factor than the impedance magnitude. There are different quality factors of resonances from the analogous network, such as resonance in the liquid layer only, resonance in the liquid layer and reflection layer, and resonance related to maximum impedance on fluid layer boundary. The result of separation shows huge difference between summits in admittance spectrum. The analogous network provides a convenient and clear way to maximize the ultrasonic energy stored in liquid.

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References

[1] King L V. On the acoustic radiation pressure on spheres[J]. Proc. R. Soc. London, 1934, Ser. A, 147: 212-240.

[2] Yasioka K, Kawasima Y. Acoustic radiation pressure on a compressible sphere[J]. Acoustica, 1955, 5:167-173.

[3] Westervelt P. The theory of steady forces caused by sound waves[J]. J. Acoust. Soc. Am., 1951, 23(4): 312-315.

[4] Groschl M. Ultrasonic separation of suspended particles-Part Ⅰ: Fundamental[J]. Acustica, 1998, 84(3): 432-447.

[5] Hill M, Shen Y, Hawkes J. Modelling of layered resonators for ultrasonic separation[J]. Ultrasonic, 2002, 40: 385-392.

[6] Hill M, Wood R. Modelling in the design of a flow-through ultrasonic separator[J]. Ultrasonics, 2000, 38: 662–665.

[7] Hill M. The selection of layer thicknesses to control acoustic radiation force profiles in layered resonators[J]. J. Acoust. Soc. Am., 2003, 114(5): 2654-2661.

[8] Hawkes J, Coakley W. Force field particle filter, combining ultrasound standing waves and laminar flow[J]. Sensors & Actuators B, 2001, 75: 213-222.

[9] Harris N, Hill M, Beeby S, et al. A silicon microfluidic ultrasonic separator[J]. Sensors and Actuators B, 2003, 95: 425-434.

[10] Harris N, Hill M. A dual frequency, ultrasonic, microengineering particle manipulator[J]. Ultrasonics, 2004, 42: 139-144.

[11] Sherrit S, Olazabal V, Sansinena J, et al, The Use of Piezoelectric Resonators for Characterization of Mechanical Properties of Polymers[Z]. In Proceeding of SPIE on Smart Structures Conference, San Diego, CA, 2003. 1-15.

[12] Mason W P. Electromechanical Transducers and Wave Filters[M]. Princeton NJ: Van Nostrand, 1948.

Brief Introduction to Author

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