Nonlinear Dynamics & Chaos I

Exercice Set 5 Questions

Question 1

Consider the quadratic *Duffing equation*

$$\dot{u} = v,$$
$$\dot{v} = \beta u - u^2 - \delta v,$$

where $\delta > 0$, and $0 \leq |\beta| \ll 1$.

(a) Construct a β-dependent center manifold up to quadratic order near the origin for small β values.

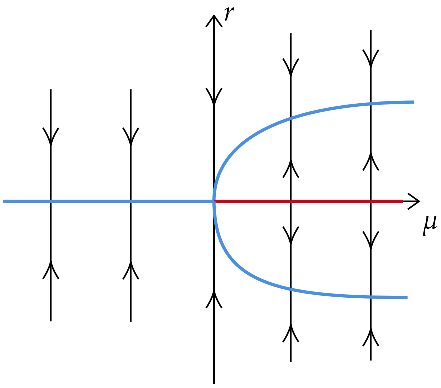
(b) Construct a stability diagram for the reduced system on the center manifold using β as a bifurcation parameter.

Question 2

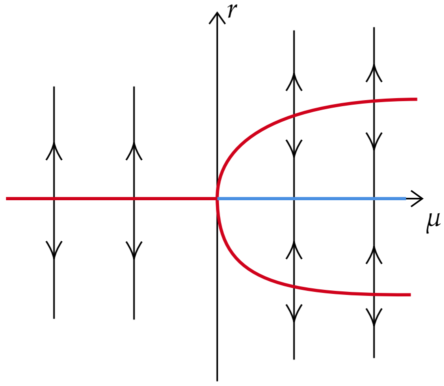
Assume that a dynamical system, depending on a parameter μ, undergoes a <u>subcritical</u> Hopf bifurcation at $\mu = 0$. Let

$$\begin{cases} \dot{r} = r(d_0\mu + a_0 r^2) \\ \dot{\theta} = \omega + e_0 r^2 + b_0 \mu \end{cases}$$

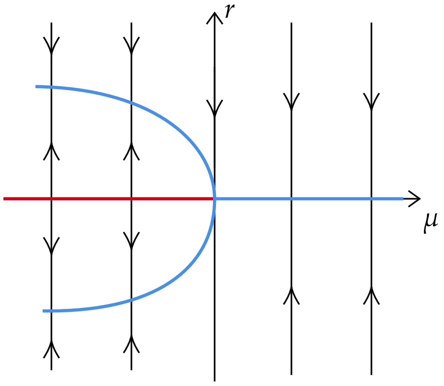
be the truncated normal form on the center manifold W^c_μ in polar coordinates. Which figure represents the correct bifurcation diagram for this system?



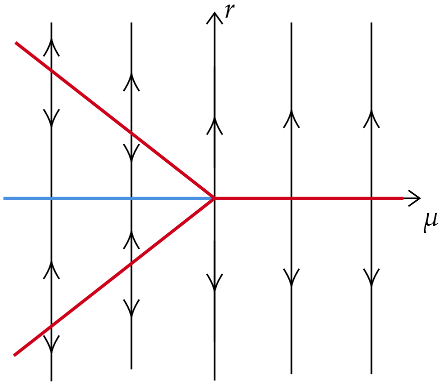
(a)



(b)



(c)



(d)

Question 3

Assume that the dynamical system $\dot{\mathbf{x}} = f(\mathbf{x}, \mu)$, $(\mathbf{x} \in \mathbb{R},\ \mu \in \mathbb{R})$ undergoes a codimension 1 bifurcation at $y = 0$. If $f(-x, \mu) = -f(x, \mu)$, what type of bifurcation is possible at $\mu = 0$?

(a) Saddle-node

(b) Transcritical

(c) Pitchfork

(d) None

Question 4

Consider the dynamical system
$$\dot{x} = A(\mu)x + f(x;\mu)$$
where $x \in \mathbb{R},\ f(x,0) = -f(-x,0),\ \forall x \in \mathbb{R},\ \mu \in \mathbb{R},\ f \in C^1$. Which of the following statements are true?

(a) This system cannot have a saddle-node bifurcation at $\mu = 0$.

(b) This system will have either a Hopf bifurcation or a transcritical bifurcation at $\mu = 0$.

(c) This system has a hyperbolic fixed point at $x = 0$, and hence cannot have a bifurcation at $\mu = 0$.

(d) None of the above

Question 5

Consider a dynamical system
$$\dot{x} = A(\mu^2)x + f(x,\mu)$$
where $x \in \mathbb{R}^2,\ A \in \mathbb{R}^{2\times 2},\ \mu \in \mathbb{R},\ f(x,\mu) = \mathcal{O}(|x|^2),\ \nabla \cdot f(x) < 0$ for $|x| \ll 1$ where the 2×2 matrix depends on μ^2. Assume that A(0) has a purely imaginary pair of eigenvalues.
Which of the following statements are true?

(a) This system has a subcritical Hopf bifurcation at $\mu = 0$.

(b) This system has a supercritical Hopf bifurcation at $\mu = 0$.

(c) The $x = 0$ fixed point does not undergo a Hopf bifurcation.

(d) The $x = 0$ fixed point undergoes a Hopf bifurcation, but its type cannot be determined from the information given.