**151-0530-00L, Spring, 2020**

**Nonlinear Dynamics and Chaos II**

**Homework Assignment 3**

Due: Wednesday, April 8;
Please submit by email to Dr. Shobhit Jain <shjain@ethz.ch>

1. Show that if $\lambda$ is an eigenvalue of a linearized Hamiltonian system, then so is $-\lambda$. (*Hint:* For any symmetric matrix $A$ and any nonsingular matrix $B$, the eigenvalues of $BA$ and $B^{-1}(BA)B$ coincide.)

2. A *gradient system* is a dynamical system of the form

$$\dot{x} = -DV(x), \qquad x \in \mathbb{R}^n,$$

with some smooth function $V$. Apart from the absence of the symplectic matrix $J$, gradient systems appear similar to Hamiltonian systems. In fact, they are rather different, as the following exercises show.

(a) Show that the eigenvalues of a linearized gradient system are always real.

(b) Find a condition for $V$ under which a fixed point of a gradient system is asymptotically stable.

(c) Show that gradient systems cannot have periodic orbits.

(d) Given a smooth function $f(x)$, propose a numerical method for finding the local minima and maxima of $f$.

3. Let $f : X \to Y$ be a smooth function, where $X \subset \mathbb{R}^n$ is a manifold. Prove that the graph of $f$,

$$\text{graph}(f) = \{(x, y) \in X \times Y : \; y = f(x)\}$$

is always a manifold.

4. Show that the tangent space of a manifold at any point is independent of the local parametrization used in its construction (i.e., another local parametrization would give the same tangent space).

5. Prove that the tangent bundle of a manifold is a manifold by constructing an explicit local parametrization.