151-0530-00L, Spring, 2020

Nonlinear Dynamics and Chaos II

Homework Assignment 6

Due: Wednesday, May 27;
Please submit by email to Dr. Shobhit Jain <shjain@ethz.ch>

1. Use the Lyapunov-Perron integral-equation approach discussed in class to prove the existence of a local strong stable manifold $W^{ss}_{loc}(0)$ for two-dimensional, autonomous dynamical systems. In particular, consider a system of the form

$$\dot{x} = f(x), \quad x \in \mathbb{R}^2, \quad f(0) = 0, \quad \mathrm{Spect}[Df(0)] = \{-\lambda_{ss}, -\lambda_s\},$$

where $\lambda_{ss} > \gamma > \lambda_s > 0$, and work out the details of the existence and uniqueness argument for W^{ss} in a way similar to what we covered in class for the stable manifold of a saddle point in $\mathbb{R}^2$. *Hint*: follow the outline given in class for this problem, but also fill in the details.

2. Consider the planar dynamical system

$$\begin{aligned} \dot{x} &= -x, \\ \dot{y} &= -\sqrt{14}y + x^2 + x^3 + x^4. \end{aligned} \tag{1}$$

(a) Use the existence theorem for autonomous SSM to conclude the existence of an analytic slow SSM. What is the minimal smoothness category is which this SSM is already unique? (b) Find the exact expression for the slow SSM via Taylor-expansion. (c) Solve the nonlinear system (1) explicitly for arbitrary initial conditions and show that among these solutions, the SSM you have identified is indeed the unique smoothest invariant manifold tangent to the slow subspace at the origin. How smooth are the other invariant manifolds tangent to the slow subspace? (d) Plot a few general solutions of (1) as well as the SSM you have identified. (e) Obtain an exact reduced-order model for the system (1).