151-0530-00L, Spring, 2022

Nonlinear Dynamics and Chaos II

Homework Assignment 1

Due: Monday, March 28
Please email PDF file to Dr. Mattia Cenedese <mattiac@ethz.ch>

1. Show that the number of k-periodic orbits for the Bernoulli shift map on two symbols is

$$N(k) = \frac{1}{k}\left(2^k - \sum_{(i,k)} i\ N(i)\right),$$

where (i,k) means that the integer i divides the integer k.

2. Let A denote the transition matrix for a sub-shift $\sigma\colon \Sigma_A^N \mapsto \Sigma_A^N$ of finite type on N symbols.
   (a) Show that the number of fixed points of σ is equal to trace(A).
   (b) Show that the total number of *admissible* $k-$periodic points (i..e, $k-$periodic points whose minimal period may be less than k) is equal to trace(A^k).

3. Show that any two periodic orbits of the Bernoulli shift map are connected by infinitely many heteroclinic orbits.

4. Show that the Bernoulli shift map σ is topologically transitive on the symbol space Σ with respect to the metric $d(\cdot\ ,\cdot)$ defined in class. Specifically, show that for any two open sets $A, B \subset \Sigma$, there exists an integer N such that $\sigma^N(A) \cap B \neq \emptyset$. (*Hint*: Use the existence of a dense orbit for σ in Σ: there exists a symbol sequence $s^* \in \Sigma$ with the following property: for any $s \in \Sigma$ and for any $\delta > 0$, there exists an integer $N(s,\delta)$ such that $d(\sigma^{N(s,\delta)}(s^*), s) < \delta$.)

5. Show that the Bernoulli shift map σ has sensitive dependence on initial conditions on the symbol space Σ. Specifically, show that there exists a nonzero distance $\Delta > 0$, such that no matter how close two symbols s^* and $\bar{s}$ are in Σ, we have

$$d\left(\sigma^N(s^*), \sigma^N(\bar{s})\right) > \Delta$$

for some N.