Nonlinear Dynamics and Chaos I.
Problem set 6

1. Use the harmonic balance method with a single-harmonic to find an approximation for small-amplitude periodic responses of the damped-forced pendulum equation

$$\ddot{x} + \delta\dot{x} + \sin x = \gamma \sin \omega t,$$

where the damping coefficient δ and the forcing amplitude γ are small positive parameters. Plot the response amplitude as a function of the forcing for different forcing values and for $\delta = 0.1$. Determine and plot the backbone curve associated with the forced response.

2. Fix $\omega = 2$ and determine the range of γ/δ numerically for which the above pendulum has a heteroclinic tangle. *Hint: Determine the heteroclinic pair of solutions of the above pendulum for $\delta = \gamma = 0$ then find numerically the transverse zeros of the Melnikov functions computed for the separatrices. Sketch qualitatively the topology of the resulting heteroclinic tangle.*