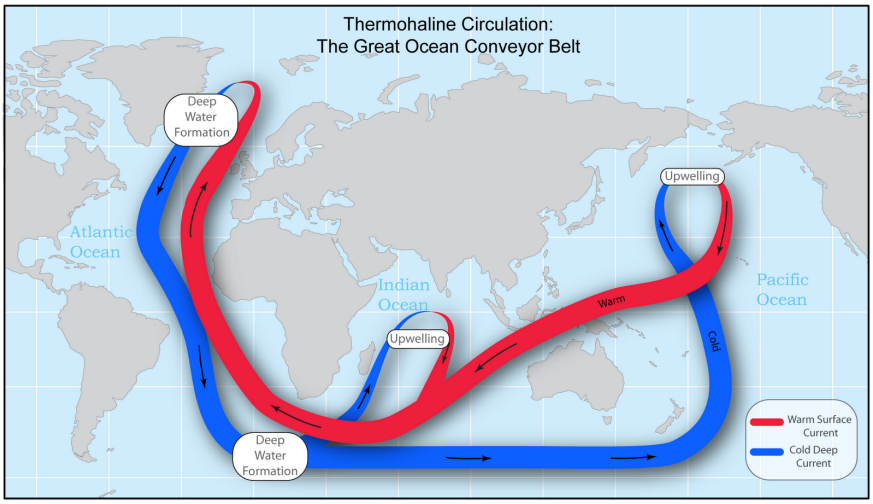
Written Exam for Nonlinear Dynamics and Chaos II, Spring Semester, 2013

August 20, 2013;9:00-10:00am, ML F36. *Use of class-notes (hand-written or printed copy) is allowed*

Thermohaline circulation (THC) is a part of the large-scale ocean circulation that is driven by global density gradients created by surface heat and freshwater fluxes. Stommel's box model (1961) is a qualitative description of the trends and equilibria in THC. This model couples the two fundamental drivers of TLC, temperature (*thermo*) and salt concentration (*-haline*), in a nonlinear fashion.



The non-dimensional **variables** of Stommel's model are:

$x(t)$**:** temperature difference between the tropics (lower latitudes) and the North-Atlantic (higher latitudes)

$y(t)$**:** salinity (i..e, salt concentration) difference between the above two regions of the ocean

The non-dimensional **parameters** of the model are:

τ_x**:** relaxation time to a constant temperature difference between northern and southern latitudes in the absence of coupling

τ_y**:** relaxation time to zero salinity difference between higher and lower latitudes in the absence of coupling. In practice, $\tau_x/\tau_y = \epsilon \ll 1$.

μ**:** measure of freshwater flux through clouds moving from lower to higher latitudes

η : nonlinear coupling parameter between temperature and salinity evolution

With this notation, Stommel's model can be written as

$$\dot{x} = -\frac{1}{\tau_x}(x-1) + \frac{1}{\tau_y} x\left[1+\eta^2(x-y)^2\right],$$

$$\dot{y} = \frac{\mu}{\tau_y} - \frac{1}{\tau_y} y\left[1+\eta^2(x-y)^2\right].$$

1. Show that Stommel's model has a globally attracting slow manifold that governs the asymptotic behavior of THC. Find a leading order approximation to this manifold. (*Hint*: rescale time by letting $s = t/\tau_y$.)
2. Compute the leading-order reduced flow on the slow manifold. Determine qualitatively the possible dynamical behaviors on the slow manifold as the parameters μ and η are varied.