Nonlinear Dynamics and Chaos I

Exercice Set 2 - Questions

Question 1

Consider the nonlinear oscillator

$$\ddot{x} + \omega_0^2 x = \varepsilon M x^2,$$

where $\varepsilon M x^2$ represents a small nonlinear forcing term ($0 \leq \varepsilon \ll 1, M > 0$).
Using Lindstedt's method, find an $\mathcal{O}(\varepsilon)$ approximation for nonlinear motions as a function of their initial position, with zero initial velocity.

Question 2

Consider the forced *van der Pol equation*

$$\ddot{x} + \varepsilon(x^2 - 1)\dot{x} + x = F\cos(\omega t),$$

which arises in models of self-excited oscillation, such as those of a valve generator with a cubic valve characteristic. Here $F, \omega > 0$ are parameters, and $0 \leq \varepsilon \ll 1$.

(i) For small values of ε, find an approximation for an **exactly** $2\pi/\omega$-periodic solution of the equation. The error of your approximation should be $\mathcal{O}(\varepsilon)$.

(ii) For $\varepsilon = 0.1$, $\omega = 2$, and $F = 1$, verify your prediction numerically by solving the equation numerically. Plot your numerical solution along with your analytic prediction computed in (i).

Note: For chaotic dynamics in the forced van der Pol equation, see Section 2.1 of *Guckenheimer & Holmes*.