Università degli Studi di Padova

Quantum Information and Computing

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Exercise #07

Physics of Data

Theory

We consider the quantum system formed by N spin-1/2 particles. The problem given in an Hamiltonian represented by a $2^N X\ 2^N$ matrix:

H = $\lambda \sum_{i=1}^{N} \sigma_i^z - \sum_{i=1}^{N-1} \sigma_i^x \sigma_{i+1}^x$

Where we set J = 1. The notation simplifies the one of a tensor product that:

$\sigma_i^z = \mathbb{1}_1 \otimes \dots \mathbb{1}_{i-1} \otimes \sigma_i^z \otimes \mathbb{1}_{i+1} \otimes \dots \mathbb{1}_N$

$\sigma_i^x \sigma_{i+1}^x = \mathbb{1}_1 \otimes \dots \mathbb{1}_{i-1} \otimes \sigma_i^x \otimes \sigma_{i+1}^x \otimes \mathbb{1}_{i+2} \otimes \dots \mathbb{1}_N$

λ is the interaction strength parameter and the σs are the Pauli matrices which do not commute:

$[\sigma_i^j, \sigma_i^k] = 2i\varepsilon_{ijk}\, \sigma_i^l$

The system is built on a one-dimensional lattice with nearest neighbor interactions; an external magnetic field perpendicular to the x-axis causes an energetic bias.
The Hamiltonian presents a spin-flip symmetry.

Theory

The model can be exactly solved for all coupling constants: we observe 3 regimes.
Let $\Delta\mathcal{E}$ be the energy gap between the lowest excited state(s) and the ground state.

1. Ordered phase. For $|\lambda| < 1$, the ground state breaks the spin-flip symmetry and is thus two-fold degenerate;
2. Disordered phase. For $|\lambda| < 1$, the ground state preserves the spin-flip symmetry, and is non-degenerate;
3. Gapless phase. When $|\lambda| = 1 \equiv \lambda c$, the system undergoes a quantum phase transition.

Since we can only deal with a small number of spins N, the solutions are more precise in the thermodynamic limit, $N \to \infty$.

$$\varepsilon_0^{MF} = \begin{cases} -1 - \lambda^2/4 & \text{if } \lambda \in [-2,2] \\ -|\lambda| & \text{otherwise} \end{cases}$$

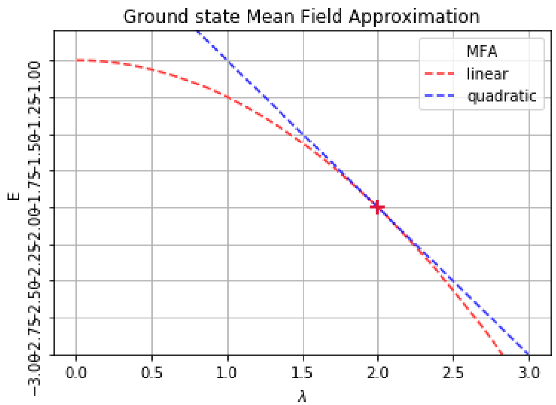
Code development

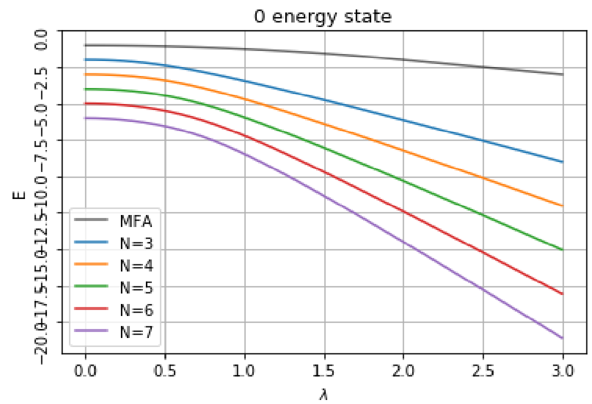
$$H = \lambda \sum_{i=1}^{N} \sigma_i^z - \sum_{i=1}^{N-1} \sigma_i^x \sigma_{i+1}^x$$

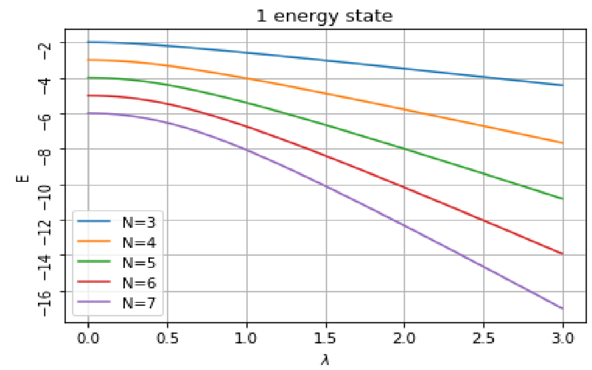
- We set the function into 2 terms A and B, where A = $\lambda \sum_{i=1}^{N} \sigma_i^z$ and B = $\sum_{i=1}^{N-1} \sigma_i^x \sigma_{i+1}^x$

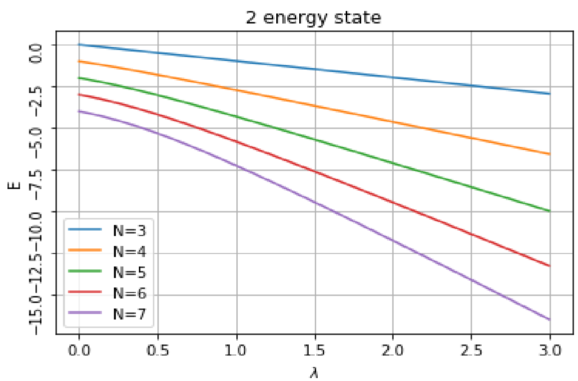
```fortran
function ising_init_H(N,lambda) result(H)
  integer   :: N
  double precision :: lambda
  double complex, dimension(:,:), allocatable :: H, int_A, int_B

  integer :: ii,jj,kk,ll

  allocate(H(2**N,2**N))
  H = 0.0 * H

  ! External field part: \lambda \sum_i^N \sigma_z^i
  do ii = 1, N, 1
    do jj = 1, 2**N, 1
      H(jj,jj) = H(jj,jj) + -2*(modulo( (jj-1)/int(2**(N-ii)),2) ) +1
    end do
  end do
  H = lambda * H ! Adding the magnetization field factor

  ! Interaction part -\sum_i^{N-1}\sigma_x^{i+1}\sigma_x^i
  do ii = 1, N-1, 1
    allocate(int_A(2**N,2**N))
    allocate(int_B(2**N,2**N))
    int_A = int_A * 0.0
    int_B = int_B * 0.0
    do kk = 0,2**(ii-1)-1,1
      do jj=1,2**(N-ii),1
        int_A(kk*(2**(N-ii+1))  + 2**(N-ii)+jj, kk*(2**(N-ii+1))  + jj) = 1
        int_A(kk*(2**(N-ii+1))  + jj, kk*(2**(N-ii+1))  + 2**(N-ii)+jj) = 1
      end do
    end do
    do kk = 0,2**(ii)-1,1
      do jj=1,2**(N-ii-1),1
        int_B(kk*(2**(N-ii)) + 2**(N-ii-1)+jj, kk*(2**(N-ii)) + jj) = 1
        int_B(kk*(2**(N-ii)) + jj, kk*(2**(N-ii)) + 2**(N-ii-1)+jj) = 1
      end do
    end do
    if(.False. .eqv. .True.) then
    print*, "mata"
    do jj = 1, ubound(int_A, 1)
      print*, "|", real(int_A(jj, :)), "|"
    end do
    print*, "matB"
    do jj = 1, ubound(int_B, 1)
      print*, "|", real(int_B(jj, :)), "|"
    end do
    end if
    H = H - matmul(int_B,int_A)
```

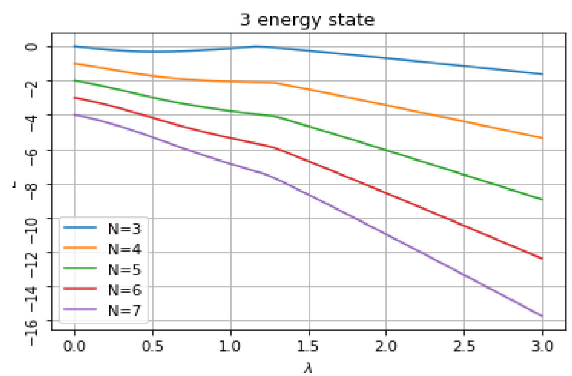
Result

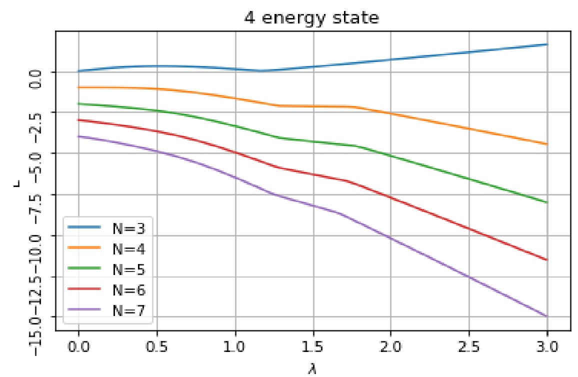

Ground state Mean Field Approximation
MFA
linear
quadratic
E
λ


0 energy state
MFA
N=3
N=4
N=5
N=6
N=7
E
λ


1 energy state
N=3
N=4
N=5
N=6
N=7
E
λ


2 energy state
N=3
N=4
N=5
N=6
N=7
E
λ


3 energy state
N=3
N=4
N=5
N=6
N=7
λ


4 energy state
N=3
N=4
N=5
N=6
N=7
λ

Thanks for the attention