Analog CMOS Integrated Circuit Design Cheat Sheet

By Xiao Ma (mxlol233@outlook.com)[https://github.com/TuringKi/Analog-CMOS-Integrated-Circuit-Design-Cheat-Sheet]

Model of MOS Transistors

Process parameters (n,V_{TH},KP, V_E):

$$t_{OX} = \frac{L_{min}}{50} \tag{1}$$

$$t_{si} = \sqrt{\frac{2\epsilon_{si}(\Phi - V_{BD})}{qN_B}} \tag{2}$$

$$C_{OX} = \frac{\epsilon_{OX}}{t_{OX}} \tag{3}$$

$$C_D = \frac{\epsilon_{si}}{t_{si}} \tag{4}$$

$$KP = \mu C_{OX} \tag{5}$$

$$\beta = KP\frac{W}{L} \tag{6}$$

$$Q_{dep} = \sqrt{4q\epsilon_{si}|\Phi_F|N_{sub}} \tag{7}$$

$$V_{TH0} = \Phi_{MS} + 2\Phi_F + \frac{Q_{dep}}{C_{OX}} \tag{8}$$

Changing the Gate voltage V_{GS} will thus change the conductivity of the channel and hence the current I_{DS}. In a similar way, changing the Bulk voltage V_{BS} will thus also change the conductivity of the channel and will thus change the current I_{DS} as well. The gate gives the MOST operation, whereas the bulk gives JFET operation.
Indeed, a Junction FET is by definition a FET in which the current is controlled by a junction capacitance.
All MOST devices are thus parallel combinations of MOSTs and JFETs.
The Bulk voltage works like a "back-gate", which is also called as "body effect":

$$V_{TH} = V_{TH0} + \gamma(\sqrt{|2\Phi_F| + V_{BS}} - \sqrt{|2\Phi_F|}) \tag{9}$$

$$n = \frac{\gamma}{\sqrt{|2\Phi_F| + V_{BS}}} = 1 + \frac{C_D}{C_{OX}} \tag{10}$$

In linear region:

$$I_{DS} = \beta[(V_{GS} - V_{TH})V_{DS} - \frac{1}{2}V_{DS}^2] \tag{11}$$

$$R_{on} = \frac{1}{\beta(V_{GS} - V_{TH})} \tag{12}$$

Channel-Length modulation in saturation region:

$$K' = \frac{KP}{2n} \tag{13}$$

$$\lambda = \frac{1}{V_E L} \tag{14}$$

$$I_{DS} = K'\frac{W}{L}(V_{GS} - V_{TH})^2(1 + \lambda V_{DS}) \tag{15}$$

$$r_o = \frac{\partial V_{DS}}{\partial I_{DS}} \approx \frac{1}{\lambda I_{DS}} = \frac{V_E L}{I_{DS}} \tag{16}$$

Saturation region has three distinctive regions: weak-inversion (exponential region), strong-inversion, and velocity saturation.

Value Examples In $0.35\mu m$ Process Nodes

Names	Symbols	Values
dielectric constant of sub-silicon	ϵ_{si}	1 pF/cm
dielectric constant of gate-oxide	ϵ_{OX}	0.34 pF/cm
electron charge	q	1.6×10^{-19} C
minium channel length	L_{min}	0.35 μm
width of gate-oxide	t_{OX}	0.1 nm
width of depletion layer	t_{si}	7 nm
junction built-in voltage	Φ	0.6 V
drain-bulk voltage	V_{BD}	-3.3V
gate-oxide capacitance	C_{OX}	0.5 μF/cm^2
depletion layer capacitance	C_D	0.1 μF/cm^2
bulk doping level	N_B	4×10^{17} cm^{-3}
P type mobility rate	μ_p	250 cm^2/Vs
N type mobility rate	μ_n	600 cm^2/Vs
N type KP	KP_n	300 μA/V^2
	n	$1.2 \cdots 1.5$
	$\|2\Phi_F\|$	0.6 V
	γ	$0.5 \cdots 0.8$ V$^{\frac{1}{2}}$
N type K'	K'_n	100 μA/V^2
P type K'	K'_p	40 μA/V^2
	V_{GSTt}	70 mV
	v_{sat}	10^7 cm/s
	θL	0.2 μm/V

Weak-Inversion

$$I_{DS} = I_{D0}\frac{W}{L}e^{\frac{V_{GS}}{n\frac{KT}{q}}} \tag{17}$$

$$g_m = \frac{\partial I_{DS}}{\partial V_{GS}} = \frac{I_{DS}}{n\frac{KT}{q}} \tag{18}$$

strong-inversion

Ignore channel-length modulation:

$$I_{DS} = K'\frac{W}{L}(V_{GS} - V_{TH})^2 \tag{19}$$

$$g_m = \frac{2I_{DS}}{V_{GS} - V_{TH}} \tag{20}$$

Transition Point Between Weak-Inversion and Strong-Inversion

The voltage and current at transition point between weak-inversion and strong-inversion:

$$V_{GSt} = 2n\frac{KT}{q} + V_{TH} \tag{21}$$

$$I_{DSt} \approx K'\frac{W}{L}(2n\frac{KT}{q})^2 \tag{22}$$

EKV model, a smooth model for weak-inversion and strong-inversion regions:

$$I_{DS} = K'\frac{W}{L}(V_{GS} - V_{TH})^2[ln(1 + e^{\frac{V_{GS}}{V_{GSt}}})]^2 \tag{23}$$

Let:

$$v = \frac{V_{GS}}{V_{GSt}} \tag{24}$$

$$i = \frac{I_{DS}}{I_{DSt}} = [ln(1 + e^v)]^2 \tag{25}$$

then,

$$v = ln(e^{\sqrt{i}} - 1) \tag{26}$$

$$V_{GS} - V_{TH} = V_{GSTt}ln(e^{\sqrt{i}} - 1) \tag{27}$$

where:

$$V_{GSTt} = V_{GSt} - V_{TH} = 2n\frac{KT}{q} \tag{28}$$

When $v = 1$, $i = 1$, we also have:

$$I_{DSt} = K'\frac{W}{L}(V_{GSt} - V_{TH})^2 \tag{29}$$

Velocity Saturation

$$I_{DS} = WC_{OX}v_{sat}(V_{GS} - V_{TH}) \tag{30}$$

$$g_m = WC_{OX}v_{sat} \tag{31}$$

Transition Point Between Strong-Inversion and Velocity Saturation

A smooth model for strong-inversion and velocity saturation regions:

$$I_{DS} = \frac{K'\frac{W}{L}(V_{GS} - V_{TH})^2}{1 + \theta(V_{GS} - V_{TH})} \tag{32}$$

where:

$$\theta = \frac{\mu}{2n}\frac{1}{v_{sat}L} \tag{33}$$

θL is constant:

$$\theta L = \frac{\mu}{2n}\frac{1}{v_{sat}} \tag{34}$$

$$g_{m,sat} = WC_{OX}v_{sat} = \frac{K'W}{\theta L} \tag{35}$$

The voltage and current at transition point between strong-inversion and velocity saturation:

$$V_{GSt} = \frac{1}{\theta} + V_{TH} = 2nL\frac{v_{sat}}{\mu} + V_{TH} \tag{36}$$

$$I_{DSt} = K'WL(2n\frac{v_{sat}}{\mu})^2 \tag{37}$$

1st Order Time-constants and Transfer-Constants (TTCs)

A system with N frequency-dependent elements, th transfer function in complex-frequency form:

$$H_s = \frac{a_0 + a_1 s}{1 + b_1 s} \tag{38}$$

where:

$$a_0 = H^0 \tag{39}$$

$$b_1 = \sum_{i=1}^{N}\tau_i^0 \tag{40}$$

$$a_1 = \sum_{i=1}^{N}\tau_i^0 H^i \tag{41}$$

where H^0 is the low-frequency gain:

$$H^0 : C_1, C_2, \cdots, C_N = 0 \tag{42}$$

or:

$$H^0 : L_1, L_2, \cdots, L_N = 0 \tag{43}$$

For capacitor:

$$\tau_i^0 = C_i R_i^0 \tag{44}$$

or for inductor:

$$\tau_i^0 = \frac{L_i}{R_i^0} \tag{45}$$

where R_i^0 is the resistance seen by the capacitor C_i looking into port i with all other reactive elements connected to the other ports at their zero value (hence the superscript), namely open-circuited capacitors (and short circuited inductors):

$$R_i^0 : C_1, C_2, C_{i-1}, C_{i+1}, \cdots, C_N = 0 \tag{46}$$

or:

$$R_i^0 : L_1, L_2, \cdots, L_{i-1}, L_{i+1}, \cdots, L_N = 0 \tag{47}$$

Bandwidth estimation by 1st-order TTCs:

$$\omega_h \approx \frac{1}{b_1 - \frac{a_1}{a_0}} = \frac{1}{\sum_{i=1}^{N}\tau_i^0(1 - \frac{H^i}{H^0})} \tag{48}$$

2nd Order Time-constants and Transfer-Constants (TTCs)

A system with 2 frequency-dependent elements, th transfer function:

$$H_s = \frac{a_0 + a_1 s + a_2 s^2}{1 + b_1 s + b_2 s^2} \tag{49}$$

where:

$$a_0 = H^0 \tag{50}$$
$$b_1 = \tau_1^0 + \tau_2^0 \tag{51}$$
$$a_1 = \tau_1^0 H^1 + \tau_2^0 H^2 \tag{52}$$
$$b_2 = \tau_1^0 \tau_2^1 = \tau_2^0 \tau_1^2 \tag{53}$$
$$a_2 = \tau_1^0 \tau_2^1 H^{12} = \tau_2^0 \tau_1^2 H^{12} \tag{54}$$

in which, H^1 (H^2) evaluated with the frequency-dependent element at the port 1(2) at its infinite value (i.e., shorted capacitors and open inductors):

$$H^1 : C_1 = \infty, C_2 = 0 \tag{55}$$
$$H^2 : C_1 = 0, C_2 = \infty \tag{56}$$

or:

$$H^1 : L_1 = \infty, L_2 = 0 \tag{57}$$
$$H^2 : L_1 = 0, L_2 = \infty \tag{58}$$

For capacitor:

$$\tau_1^0 = C_1 R_1^0 \tag{59}$$
$$\tau_2^0 = C_2 R_2^0 \tag{60}$$
$$\tau_1^2 = C_1 R_1^2 \tag{61}$$
$$\tau_2^1 = C_2 R_2^1 \tag{62}$$

or for inductor:

$$\tau_1^0 = \frac{L_1}{R_1^0} \tag{63}$$
$$\tau_2^0 = \frac{L_2}{R_2^0} \tag{64}$$
$$\tau_1^2 = \frac{L_1}{R_1^2} \tag{65}$$
$$\tau_2^1 = \frac{L_2}{R_2^1} \tag{66}$$

in which, R_1^2 is the resistance seen by C_1 (the subscript) when C_2 (the superscript) is infinite valued (shorted):

$$R_1^2 : C_2 = \infty \tag{67}$$
$$R_2^1 : C_1 = \infty \tag{68}$$

or:

$$R_1^2 : L_2 = \infty \tag{69}$$
$$R_2^1 : L_1 = \infty \tag{70}$$

General Time-constants and Transfer-Constants (TTCs)

A system with N frequency-dependent elements, th transfer function:

$$H_s = \frac{a_0 + a_1 s + a_2 s^2 \cdots + a_n s^n + \cdots}{1 + b_1 s + b_2 s^2 \cdots + b_n s^n + \cdots} \tag{71}$$

where:

$$a_0 = H^0 \tag{72}$$
$$b_1 = \sum_{i=1}^{N} \tau_i^0 \tag{73}$$
$$a_1 = \sum_{i=1}^{N} \tau_i^0 H^i \tag{74}$$
$$b_2 = \sum_{i=1}^{i<j} \sum_{j=i+1}^{j \le N} \tau_i^0 \tau_j^i \tag{75}$$
$$a_2 = \sum_{i=1}^{i<j} \sum_{j=i+1}^{j \le N} \tau_i^0 \tau_j^i H^{ij} \tag{76}$$
$$b_n = \sum_{i=1}^{i<j} \sum_{j=i+1}^{j<k} \sum_{k=j+1\cdots}^{k<\ldots \le N} \tau_i^0 \tau_j^i \tau_k^{ij} \cdots \tag{77}$$
$$a_n = \sum_{i=1}^{i<j} \sum_{j=i+1}^{j<k} \sum_{k=j+1\cdots}^{k<\ldots \le N} \tau_i^0 \tau_j^i \tau_k^{ij} \cdots H^{ijk\ldots} \tag{78}$$

and for capacitor:

$$\tau_i^0 = C_i R_i^0 \tag{79}$$
$$\tau_i^{jk\cdots} = C_i R_i^{jk\cdots} \tag{80}$$

or for inductor:

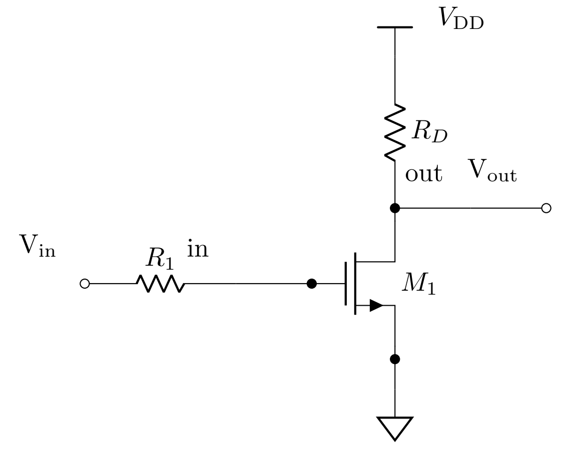
$$\tau_i^0 = \frac{L_i}{R_i^0} \tag{81}$$
$$\tau_i^{jk\cdots} = \frac{L_i}{R_i^{jk\cdots}} \tag{82}$$

In the form of zeros and poles:

$$H_s = a_0 \frac{(1 - \frac{s}{z_1})(1 - \frac{s}{z_1}) \cdots (1 - \frac{s}{z_m})}{(1 - \frac{s}{p_1})(1 - \frac{s}{p_1}) \cdots (1 - \frac{s}{p_n})} \tag{83}$$

Single Stage Amplifier: Common Source (CS), with Resistive Load

Common Source, with resistance R_D:



Transfer function of voltage gain in low-frequency:

$$A_v^0 = \frac{V_{out}}{V_{in}} = -g_m R_D \tag{84}$$

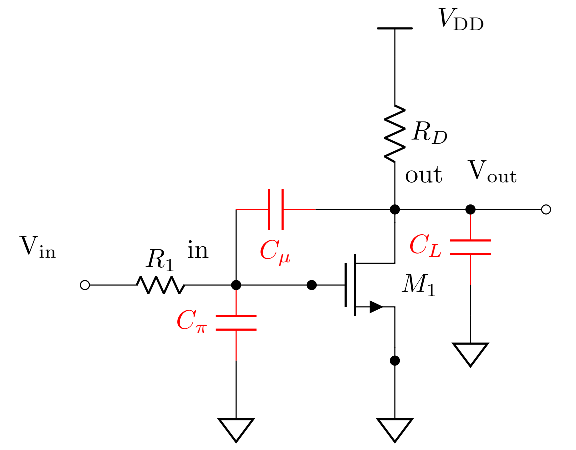
consider channel-length modulation:

$$A_v^0 = \frac{V_{out}}{V_{in}} = -g_m R_D || r_o = -g_m \frac{R_D r_o}{R_D + r_o} \tag{85}$$

The equivalent resistance at point *out* look down $M1$:

$$R_{out} = r_o \tag{86}$$

Consider high-frequency gain with 3 capacitors c_π, c_μ, c_L:



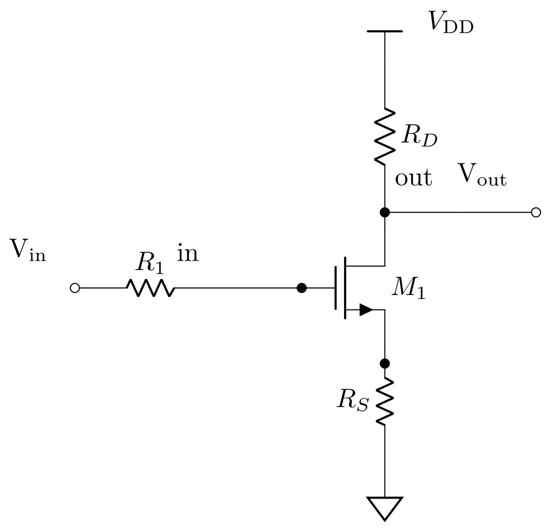
$$\tau_\pi^0 = C_\pi R_1 \tag{87}$$
$$\tau_\mu^0 = C_\mu (R_{left} + R_{right} + G_m R_{left} R_{right}) = C_\mu (R_1 + R_D + g_m R_1 R_D) \tag{88}$$
$$\tau_L^0 = C_L R_D \tag{89}$$
$$\tau_\mu^\pi = C_\mu R_D \tag{90}$$
$$\tau_\pi^L = C_\pi R_1 \tag{91}$$
$$\tau_\mu^L = C_\mu R_1 \tag{92}$$
$$\tau_\mu^{\pi L} = \tau_L^{\pi\mu} = 0 \tag{93}$$
$$A_v^\pi = H^L = 0 \tag{94}$$
$$A_v^\mu = \frac{r_m || R_D}{R_1 + r_m || R_D} = \frac{R_D}{R_1 + R_D + g_m R_1 R_D} \tag{95}$$
$$A_v^{\pi\mu} = A_v^{L\mu} = A_v^{L\pi} = 0 \tag{96}$$
$$a_0 = A_v^0 \tag{97}$$
$$b_1 = \tau_\pi^0 + \tau_\mu^0 + \tau_L^0 = R_1[C_\pi + C_\mu(1 + g_m R_D)] + R_D(C_\mu + C_L) \tag{98}$$
$$a_1 = \tau_\pi^0 A_v^\pi + \tau_\mu^0 A_v^\mu + \tau_L^0 A_v^L = C_\mu R_D \tag{99}$$
$$b_2 = \tau_\pi^0 \tau_\mu^\pi + \tau_L^0 \tau_\mu^L + \tau_L^0 \tau_\pi^L = R_1 R_D (C_\mu C_\pi + C_\mu C_L + C_\pi C_L) = R_{left} R_{right} (\Delta C)^2 \tag{100}$$
$$a_2 = \tau_\pi^0 \tau_\mu^\pi A_v^{\pi\mu} + \tau_L^0 \tau_\mu^L A_v^{L\mu} + \tau_L^0 \tau_\pi^L A_v^{L\pi} = 0 \tag{101}$$
$$b_3 = \tau_\pi^0 \tau_\mu^\pi \tau_L^{\pi\mu} = 0 \tag{102}$$
$$a_3 = \tau_\pi^0 \tau_\mu^\pi \tau_L^{\pi\mu} A_v^{\pi\mu L} = 0 \tag{103}$$

Finally, we have:

$$A_v(s) = \frac{A_v^0 (1 - r_m C_\mu s)}{1 + [R_1[C_\pi + C_\mu(1 + g_m R_D)] + R_D(C_\mu + C_L)]s + R_1 R_D (C_\mu C_\pi + C_\mu C_L + C_\pi C_L) s^2} \tag{104}$$

Single Stage Amplifier: Common Source (CS), with Source Degeneration

Common Source, with resistance R_D and R_S:



The low frequency gain:

$$A_v^0 = -\frac{g_m R_D}{1+g_m R_S} \tag{105}$$

The equivalent transconductance:

$$G_m = \frac{g_m}{1+g_m R_S} \tag{106}$$

consider channel-length modulation and body effect:

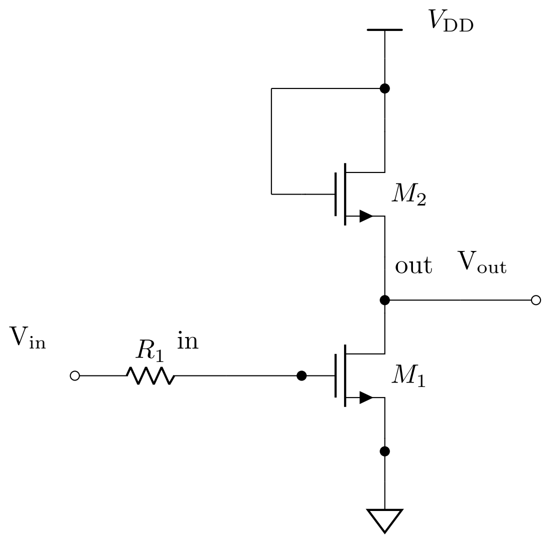
$$G_m = \frac{g_m r_o}{R_S+[1+(g_m+g_{mb})R_s]r_o} \tag{107}$$

$$R_{out} = r_o + [1+(g_m+g_{mb})r_o]R_S \approx g_m r_o R_S \tag{108}$$

$$A_v^0 = -G_m(R_{out}||R_D) = -\frac{g_m r_o}{R_S+[1+(g_m+g_{mb})R_s]r_o}\{[r_o+[1+(g_m+g_{mb})r_o]R_S]||R_D\} \tag{109}$$

Single Stage Amplifier: Common Source (CS), with Diode-Connected Load

Common Source, with diode-connected transistor $M2$:



Consider body effect and channel-length modulation, the equivalent resistance at point *out* look into $M2$:

$$R_{out,up} = \frac{1}{g_{m2}+g_{mb2}}||r_o \tag{110}$$

With negligible channel-length modulation, we have the low frequency gain:

$$A_v^0 = -g_{m1}\frac{1}{g_{m2}+g_{mb2}} = -\frac{g_{m1}}{g_{m2}}\frac{1}{1+\eta} \tag{111}$$

where:

$$\frac{1}{1+\eta} = \frac{g_{mb2}}{g_{m2}} \tag{112}$$

Since I_{DS} is same at $M1$ and $M2$ (from drain to source), we have:

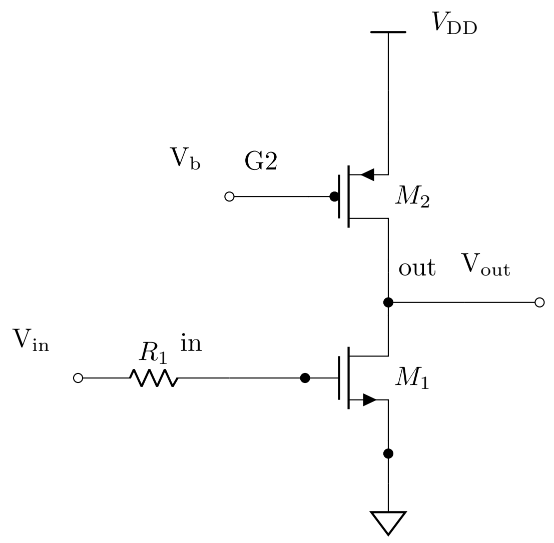
$$A_v^0 = -\sqrt{\frac{W_1/L_1}{W_2/L_2}}\frac{1}{1+\eta} \tag{113}$$

If we replace nmos $M2$ with pmos, the body effect of $M2$ will disappear:

$$A_v^0 = -\sqrt{\frac{\mu_n W_1/L_1}{\mu_p W_2/L_2}} \tag{114}$$

Single Stage Amplifier: Common Source (CS), with Current-Source Load

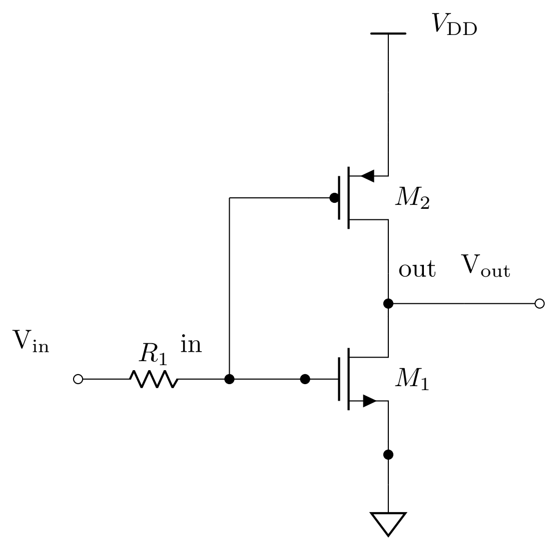
$M2$ works as a current source:



The low frequency gain:

$$A_v^0 = -g_{m1}(r_{o1}||r_{o2}) \tag{115}$$

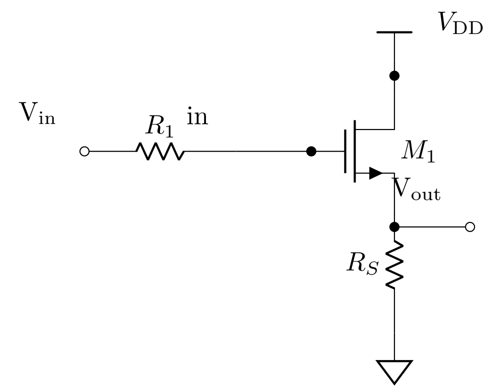
Single Stage Amplifier: Common Source (CS), with Active Load



The low frequency gain:

$$A_v^0 = -(g_{m1}+g_{m2})(r_{o1}||r_{o2}) \tag{116}$$

Single Stage Amplifier: Common Drain (CD) or Source Follower



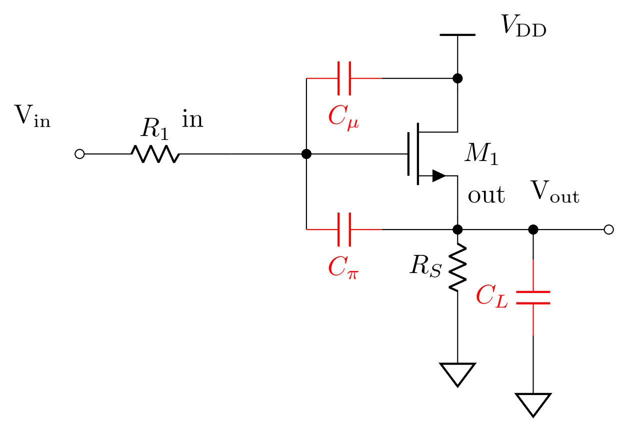
The low frequency gain:

$$A_v^0 = \frac{g_m R_S}{1+(g_m+g_{mb})R_S} \tag{117}$$

The equivalent resistance at point *out* look up into $M1$:

$$R_{out,up} = \frac{1}{g_m+g_{mb}}||r_o \tag{118}$$

Consider high-frequency gain with 3 capacitors:



We have:

$$\tau_\mu^0 = C_\mu R_1 \tag{119}$$

$$\tau_\pi^0 = C_\pi \frac{R_1+R_S}{1+g_m R_S} \tag{120}$$

$$\tau_L^0 = C_L(r_m||R_S) \tag{121}$$

$$\tau_L^\pi = C_L(R_1||R_S) \tag{122}$$

$$\tau_\mu^L = C_L R_1 \tag{123}$$

$$A_v^\mu = A_v^L = 0 \tag{124}$$

$$A_v^\pi = \frac{R_S}{R_1+R_S} \tag{125}$$

$$b_1 = C_\mu R_1 + C_\pi \frac{R_1+R_S}{1+g_m R_S} + C_L(r_m||R_S) \tag{126}$$

$$a_1 = C_\pi \frac{1}{1+g_m R_S} \tag{127}$$

$$b_2 = \frac{R_1 R_S}{1+g_m R_S}(C_\pi C_\mu + C_\pi C_L) + C_L C_\mu (r_m||R_S)R_1 \tag{128}$$

$$a_2 = b_3 = a_3 = 0 \tag{129}$$

We ignore body effect:

$$a_0 = \frac{g_m R_S}{1+g_m R_S} \tag{130}$$

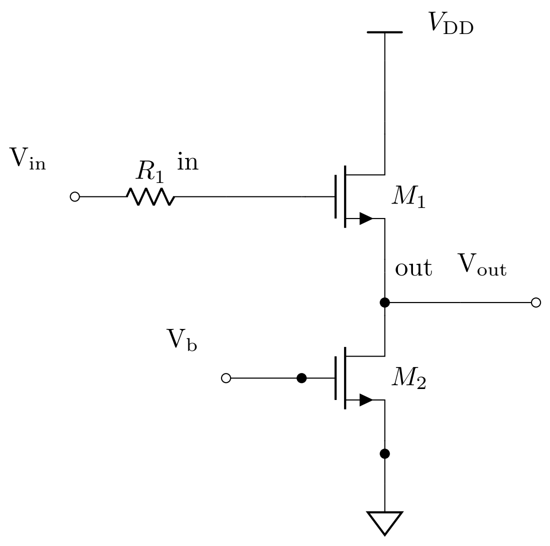
Now we have the high frequency voltage gain in s domain:

$$A_v(s) = \frac{g_m R_S + C_\pi s}{1+g_m R_S + [(C_\mu g_m R_1 + C_\pi + C_L)R_S + (C_\mu + C_\pi)R_1]s + R_1 R_S \Delta C^2 s^2} \tag{131}$$

where,

$$\Delta C^2 = C_\mu C_\pi + C_\pi C_L + C_\mu C_L \tag{132}$$

Single Stage Amplifier: Common Drain (CD) or Source Follower, With Transistor Load



The low frequency gain:

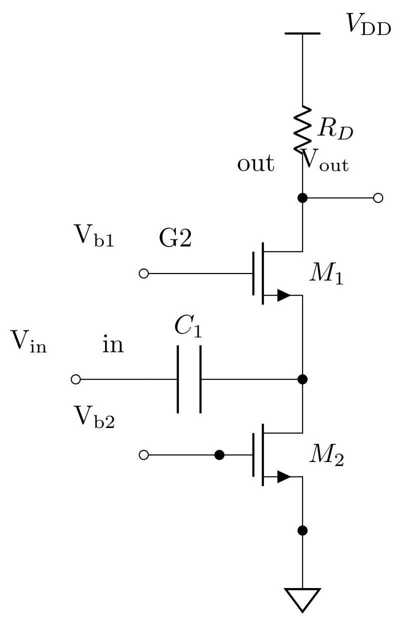
$$A_v^0 = \frac{R_{eq}}{R_{eq} + \frac{1}{g_{m1}}} \tag{133}$$

where:

$$R_{eq} = \frac{1}{g_{mb1}} || r_{o1} || r_{o2} \tag{134}$$

Single Stage Amplifier: Common Gate (CG)

M_2 works as current source, C_1 is coupled capacitor:

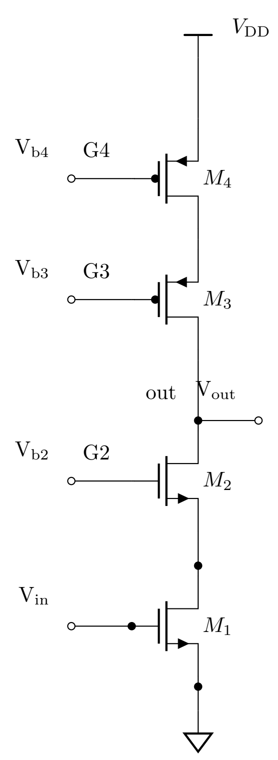


The low frequency voltage gain:

$$A_v^0 = g_m(1+\eta)R_D \tag{135}$$

Single Stage Amplifier: Cascode

M_3 and M_4 are formed the cascode current source:

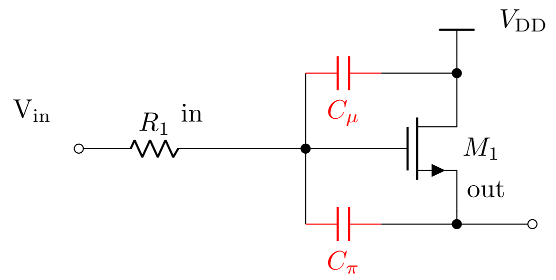


The low frequency voltage gain:

$$A_v^0 \approx -g_{m1}(g_{m2}r_{o1}r_{o2} || g_{m3}r_{o3}r_{o4}) \tag{136}$$

Input and Output Impedance At High Frequency of Source Follower

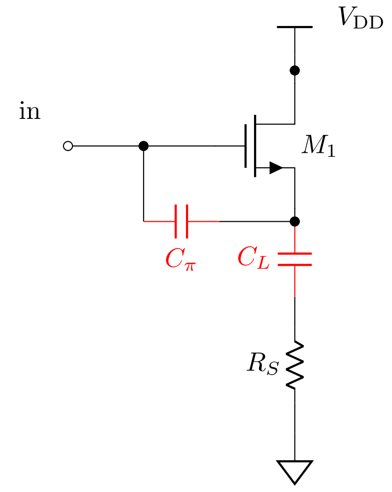
We consider the output impedance at *out* looking upon:



$$\tau_\mu = C_\mu R_1 \tag{137}$$
$$\tau_\pi = C_\pi r_m \tag{138}$$
$$\tau_\mu^\pi = C_\mu R_1 \tag{139}$$
$$Z_{out}^0 = r_m \tag{140}$$
$$Z_{out}^\mu = r_m \tag{141}$$
$$Z_{out}^\pi = R_1 \tag{142}$$
$$Z_{out}^{\pi\mu} = 0 \tag{143}$$

$$Z_{out}(s) = r_m \frac{1 + (C_\mu + C_\pi)R_1 s}{1 + (C_\mu R_1 + C_\pi r_m)s + C_\pi C_\mu R_1 r_m s^2} \tag{144}$$

Consider the input impedance at *in* looking right.



$$\tau_\pi = C_\pi r_m \tag{145}$$
$$\tau_L = C_L(r_m + R_S) \tag{146}$$
$$\tau_L^\pi = C_\mu R_S \tag{147}$$

We analysis the conductance Y_{in} first:

$$Y_{in}^0 = Y_{in}^\pi = Y_{in}^L = 0 \tag{148}$$
$$Y_{in}^0 = Y_{in}^\pi = Y_{in}^L = 0 \tag{149}$$
$$Y_{in}^{L\mu} = \frac{1}{R_S} \tag{150}$$

we have:

$$Z_{in}(s) = \frac{1}{Y_{in}(s)} = \frac{1 + [C_\pi r_m + C_L(r_m + R_S)]s + C_\mu C_L r_m R_S s^2}{C_\pi C_L r_m s^2} \tag{151}$$

when $R_S \to 0$:

$$Z_{in}(s) = \frac{1 + (C_\pi + C_L)r_m s}{C_\pi C_L r_m s^2} = \frac{1}{(C_L||C_\pi)s} + \frac{g_m}{C_L C_\pi s^2} \tag{152}$$