Analog CMOS Integrated Circuit Design Cheat Sheet

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Model of MOS Transistors

Process parameters (n,V_{TH},KP, V_E):

$$t_{OX} = \frac{L_{min}}{50} \tag{1}$$

$$t_{si} = \sqrt{\frac{2\epsilon_{si}(\Phi - V_{BD})}{qN_B}} \tag{2}$$

$$C_{OX} = \frac{\epsilon_{OX}}{t_{OX}} \tag{3}$$

$$C_D = \frac{\epsilon_{si}}{t_{si}} \tag{4}$$

$$KP = \mu C_{OX} \tag{5}$$

$$\beta = KP\frac{W}{L} \tag{6}$$

$$Q_{dep} = \sqrt{4q\epsilon_{si}|\Phi_F|N_{sub}} \tag{7}$$

$$V_{TH0} = \Phi_{MS} + 2\Phi_F + \frac{Q_{dep}}{C_{OX}} \tag{8}$$

$$V_{TH} = V_{TH0} + \gamma(\sqrt{|2\Phi_F| + V_{BS}} - \sqrt{|2\Phi_F|}) \tag{9}$$

$$n = \frac{\gamma}{\sqrt{|2\Phi_F| + V_{BS}}} = 1 + \frac{C_D}{C_{OX}} \tag{10}$$

In linear region:

$$I_{DS} = \beta[(V_{GS} - V_{TH})V_{DS} - \frac{1}{2}V_{DS}^2] \tag{11}$$

$$R_{on} = \frac{1}{\beta(V_{GS} - V_{TH})} \tag{12}$$

Channel-Length modulation in saturation region:

$$K' = \frac{KP}{2n} \tag{13}$$

$$\lambda = \frac{1}{V_E L} \tag{14}$$

$$I_{DS} = K'\frac{W}{L}(V_{GS} - V_{TH})^2(1 + \lambda V_{DS}) \tag{15}$$

$$r_o = \frac{\partial V_{DS}}{\partial I_{DS}} \approx \frac{1}{\lambda I_{DS}} = \frac{V_E L}{I_{DS}} \tag{16}$$

Saturation region has three distinctive regions: weak-inversion (exponential region), strong-inversion, and velocity saturation.

Value Examples In $0.35\mu m$ Process Nodes

Names	**Symbols**	**Values**
dielectric constant of sub-silicon	ϵ_{si}	1 pF/cm
dielectric constant of gate-oxide	ϵ_{OX}	0.34 pF/cm
electron charge	q	1.6×10^{-19} C
minium channel length	L_{min}	0.35 μm
width of gate-oxide	t_{OX}	0.1 nm
width of depletion layer	t_{si}	7 nm
junction built-in voltage	Φ	0.6 V
drain-bulk voltage	V_{BD}	-3.3V
gate-oxide capacitance	C_{OX}	0.5 μF/cm^2
depletion layer capacitance	C_D	0.1 μF/cm^2
bulk doping level	N_B	4×10^{17} cm^{-3}
P type mobility rate	μ_p	250 cm^2/Vs
N type mobility rate	μ_n	600 cm^2/Vs
N type KP	KP_n	300 μA/V^2
	n	$1.2\cdots1.5$
	$\lvert 2\Phi_F\rvert$	0.6 V
	γ	$0.5\cdots0.8$ V$^{\frac{1}{2}}$
N type K'	K'_n	100 μA/V^2
P type K'	K'_p	40 μA/V^2
	V_{GSTt}	70 mV
	v_{sat}	10^7 cm/s
	θL	0.2 μm/V

Weak-Inversion

$$I_{DS} = I_{D0}\frac{W}{L}e^{\frac{V_{GS}}{n\frac{KT}{q}}} \tag{17}$$

$$g_m = \frac{\partial I_{DS}}{\partial V_{GS}} = \frac{I_{DS}}{n\frac{KT}{q}} \tag{18}$$

strong-inversion

Ignore channel-length modulation:

$$I_{DS} = K'\frac{W}{L}(V_{GS} - V_{TH})^2 \tag{19}$$

$$g_m = \frac{2I_{DS}}{V_{GS} - V_{TH}} \tag{20}$$

Transition Point Between Weak-Inversion and Strong-Inversion

The voltage and current at transition point between weak-inversion and strong-inversion:

$$V_{GSt} = 2n\frac{KT}{q} + V_{TH} \tag{21}$$

$$I_{DSt} \approx K'\frac{W}{L}(2n\frac{KT}{q})^2 \tag{22}$$

EKV model, a smooth model for weak-inversion and strong-inversion regions:

$$I_{DS} = K'\frac{W}{L}(V_{GS} - V_{TH})^2[ln(1 + e^{\frac{V_{GS}}{V_{GSt}}})]^2 \tag{23}$$

Let:

$$v = \frac{V_{GS}}{V_{GSt}} \tag{24}$$

$$i = \frac{I_{DS}}{I_{DSt}} = [ln(1 + e^v)]^2 \tag{25}$$

then,

$$v = ln(e^{\sqrt{i}} - 1) \tag{26}$$

$$V_{GS} - V_{TH} = V_{GSTt}ln(e^{\sqrt{i}} - 1) \tag{27}$$

where:

$$V_{GSTt} = V_{GSt} - V_{TH} = 2n\frac{KT}{q} \tag{28}$$

When $v = 1$, $i = 1$, we also have:

$$I_{DSt} = K'\frac{W}{L}(V_{GSt} - V_{TH})^2 \tag{29}$$

Velocity Saturation

$$I_{DS} = WC_{OX}v_{sat}(V_{GS} - V_{TH}) \tag{30}$$

$$g_m = WC_{OX}v_{sat} \tag{31}$$

Transition Point Between Strong-Inversion and Velocity Saturation

A smooth model for strong-inversion and velocity saturation regions:

$$I_{DS} = \frac{K'\frac{W}{L}(V_{GS} - V_{TH})^2}{1 + \theta(V_{GS} - V_{TH})} \tag{32}$$

where:

$$\theta = \frac{\mu}{2n}\frac{1}{v_{sat}L} \tag{33}$$

θL is constant:

$$\theta L = \frac{\mu}{2n}\frac{1}{v_{sat}} \tag{34}$$

$$g_{m,sat} = WC_{OX}v_{sat} = \frac{K'W}{\theta L} \tag{35}$$

The voltage and current at transition point between strong-inversion and velocity saturation:

$$V_{GSt} = \frac{1}{\theta} + V_{TH} = 2nL\frac{v_{sat}}{\mu} + V_{TH} \tag{36}$$

$$I_{DSt} = K'WL(2n\frac{v_{sat}}{\mu})^2 \tag{37}$$

2nd Order Time-constants and Transfer-Constants (TTCs)

A system with 2 frequency-dependent elements, th transfer function in complex-frequency form:

$$H_s = \frac{a_0 + a_1 s + a_2 s^2}{b_1 s + b_2 s^2} \tag{38}$$

where:

$$a_0 = H^0 \tag{39}$$

$$b_1 = \tau_1^0 + \tau_2^0 \tag{40}$$

$$a_1 = \tau_1^0 H^1 + \tau_2^0 H^2 \tag{41}$$

$$b_2 = \tau_1^0 \tau_2^1 = \tau_2^0 \tau_1^2 \tag{42}$$

$$a_2 = \tau_1^0 \tau_2^1 H^{12} = \tau_2^0 \tau_1^2 H^{12} \tag{43}$$

General Time-constants and Transfer-Constants (TTCs)

A system with N frequency-dependent elements, th transfer function:

$$H_s = \frac{a_0 + a_1 s + a_2 s^2 \cdots + a_n s^n + \cdots}{b_1 s + b_2 s^2 \cdots + b_n s^n + \cdots} \tag{44}$$

where:

$$a_0 = H^0 \tag{45}$$

$$b_1 = \sum_{i=1}^{N} \tau_i^0 \tag{46}$$

$$a_1 = \sum_{i=1}^{N} \tau_i^0 H^i \tag{47}$$

$$b_2 = \sum_{i=1}^{i<j} \sum_{j=i+1}^{j\leq N} \tau_i^0 \tau_j^i \tag{48}$$

$$a_2 = \sum_{i=1}^{i<j} \sum_{j=i+1}^{j\leq N} \tau_i^0 \tau_j^i H^{ij} \tag{49}$$

$$b_n = \sum_{i=1}^{i<j} \sum_{j=i+1}^{j<k} \sum_{k=j+1\cdots}^{k<\ldots\leq N} \tau_i^0 \tau_j^i \tau_k^{ij} \cdots \tag{50}$$

$$a_n = \sum_{i=1}^{i<j} \sum_{j=i+1}^{j<k} \sum_{k=j+1\cdots}^{k<\ldots\leq N} \tau_i^0 \tau_j^i \tau_k^{ij} \cdots H^{ijk\ldots} \tag{51}$$

and for capacitor,

$$\tau_i^0 = C_i R_i^0 \tag{52}$$

$$\tau_i^{jk\cdots} = C_i R_i^{jk\cdots} \tag{53}$$

or for inductor,

$$\tau_i^0 = \frac{L_i}{R_i^0} \tag{54}$$

$$\tau_i^{jk\cdots} = \frac{L_i}{R_i^{jk\cdots}} \tag{55}$$

In the form of zeros and poles:

$$H_s = a_0 \frac{(1-\frac{s}{z_1})(1-\frac{s}{z_1})\cdots(1-\frac{s}{z_m})}{(1-\frac{s}{p_1})(1-\frac{s}{p_1})\cdots(1-\frac{s}{p_n})} \tag{56}$$

Operational-Amp: Biasing Circuits