

# Analog CMOS Integrated Circuit Design Cheat Sheet

By Xiao Ma (mxlol233@outlook.com)[https://github.com/TuringKi/Analog-CMOS-Integrated-Circuit-Design-Cheat-Sheet]

## Model of MOS Transistors

Process parameters ( $n, V_{TH}, KP, V_E$ ):

$$t_{OX} = \frac{L_{min}}{50} \quad (1)$$

$$t_{si} = \sqrt{\frac{2\epsilon_{si}(\Phi - V_{BD})}{qN_B}} \quad (2)$$

$$C_{OX} = \frac{\epsilon_{OX}}{t_{OX}} \quad (3)$$

$$C_D = \frac{\epsilon_{si}}{t_{si}} \quad (4)$$

$$KP = \mu C_{OX} \quad (5)$$

$$\beta = KP \frac{W}{L} \quad (6)$$

$$Q_{dep} = \sqrt{4q\epsilon_{si}|\Phi_F|N_{sub}} \quad (7)$$

$$V_{TH0} = \Phi_{MS} + 2\Phi_F + \frac{Q_{dep}}{C_{OX}} \quad (8)$$

Changing the Gate voltage  $V_{GS}$  will thus change the conductivity of the channel and hence the current  $I_{DS}$ . In a similar way, changing the Bulk voltage  $V_{BS}$  will thus also change the conductivity of the channel and will thus change the current  $I_{DS}$  as well. The gate gives the MOST operation, whereas the bulk gives JFET operation.

Indeed, a Junction FET is by definition a FET in which the current is controlled by a junction capacitance.

All MOST devices are thus parallel combinations of MOSTs and JFETs.

The Bulk voltage works like a "back-gate", which is also called as "body effect":

$$V_{TH} = V_{TH0} + \gamma(\sqrt{|2\Phi_F| + V_{BS}} - \sqrt{|2\Phi_F|}) \quad (9)$$

$$n = \frac{\gamma}{\sqrt{|2\Phi_F| + V_{BS}}} = 1 + \frac{C_D}{C_{OX}} \quad (10)$$

In linear region:

$$I_{DS} = \beta[(V_{GS} - V_{TH})V_{DS} - \frac{1}{2}V_{DS}^2] \quad (11)$$

$$R_{on} = \frac{1}{\beta(V_{GS} - V_{TH})} \quad (12)$$

Channel-Length modulation in saturation region:

$$K' = \frac{KP}{2n} \quad (13)$$

$$\lambda = \frac{1}{V_E L} \quad (14)$$

$$I_{DS} = K' \frac{W}{L} (V_{GS} - V_{TH})^2 (1 + \lambda V_{DS}) \quad (15)$$

$$r_o = \frac{\partial V_{DS}}{\partial I_{DS}} \approx \frac{1}{\lambda I_{DS}} = \frac{V_E L}{I_{DS}} \quad (16)$$

Saturation region has three distinctive regions: weak-inversion (exponential region), strong-inversion, and velocity saturation.

## Value Examples In 0.35μm Process Nodes

Names	Symbols	Values
dielectric constant of sub-silicon	$\epsilon_{si}$	1 pF/cm
dielectric constant of gate-oxide	$\epsilon_{OX}$	0.34 pF/cm
electron charge	$q$	$1.6 \times 10^{-19}$ C
minium channel length	$L_{min}$	0.35 μm
width of gate-oxide	$t_{OX}$	0.1 nm
width of depletion layer	$t_{si}$	7 nm
junction built-in voltage	$\Phi$	0.6 V
drain-bulk voltage	$V_{BD}$	-3.3V
gate-oxide capacitance	$C_{OX}$	0.5 μF/cm <sup>2</sup>
depletion layer capacitance	$C_D$	0.1 μF/cm <sup>2</sup>
bulk doping level	$N_B$	$4 \times 10^{17}$ cm <sup>-3</sup>
P type mobility rate	$\mu_p$	250 cm <sup>2</sup> /Vs
N type mobility rate	$\mu_n$	600 cm <sup>2</sup> /Vs
N type KP	$KP_n$	300 μA/V <sup>2</sup>
	$n$	1.2 ... 1.5
	$ 2\Phi_F $	0.6 V
	$\gamma$	0.5 ... 0.8 V <sup>1/2</sup>
N type $K'$	$K'_n$	100 μA/V <sup>2</sup>
P type $K'$	$K'_p$	40 μA/V <sup>2</sup>
	$V_{GSTt}$	70 mV
	$v_{sat}$	10 <sup>7</sup> cm/s
	$\theta L$	0.2 μm/V

## Weak-Inversion

$$I_{DS} = I_{D0} \frac{W}{L} e^{\frac{V_{GS}}{n \frac{KT}{q}}} \quad (17)$$

$$g_m = \frac{\partial I_{DS}}{\partial V_{GS}} = \frac{I_{DS}}{n \frac{KT}{q}} \quad (18)$$

## strong-inversion

Ignore channel-length modulation:

$$I_{DS} = K' \frac{W}{L} (V_{GS} - V_{TH})^2 \quad (19)$$

$$g_m = \frac{2I_{DS}}{V_{GS} - V_{TH}} \quad (20)$$

## Transition Point Between Weak-Inversion and Strong-Inversion

The voltage and current at transition point between weak-inversion and strong-inversion:

$$V_{GS_t} = 2n \frac{KT}{q} + V_{TH} \quad (21)$$

$$I_{DS_t} \approx K' \frac{W}{L} (2n \frac{KT}{q})^2 \quad (22)$$

EKV model, a smooth model for weak-inversion and strong-inversion regions:

$$I_{DS} = K' \frac{W}{L} (V_{GS} - V_{TH})^2 [\ln(1 + e^{\frac{V_{GS}}{V_{GS_t}}})]^2 \quad (23)$$

Let:

$$v = \frac{V_{GS}}{V_{GS_t}} \quad (24)$$

$$i = \frac{I_{DS}}{I_{DS_t}} = [\ln(1 + e^v)]^2 \quad (25)$$

then,

$$v = \ln(e^{\sqrt{i}} - 1) \quad (26)$$

$$V_{GS} - V_{TH} = V_{GST_t} \ln(e^{\sqrt{i}} - 1) \quad (27)$$

where:

$$V_{GST_t} = V_{GS_t} - V_{TH} = 2n \frac{KT}{q} \quad (28)$$

When  $v = 1$ ,  $i = 1$ , we also have:

$$I_{DS_t} = K' \frac{W}{L} (V_{GS_t} - V_{TH})^2 \quad (29)$$

## Velocity Saturation

$$I_{DS} = WC_{OX} v_{sat} (V_{GS} - V_{TH}) \quad (30)$$

$$g_m = WC_{OX} v_{sat} \quad (31)$$

## Transition Point Between Strong-Inversion and Velocity Saturation

A smooth model for strong-inversion and velocity saturation regions:

$$I_{DS} = \frac{K' \frac{W}{L} (V_{GS} - V_{TH})^2}{1 + \theta(V_{GS} - V_{TH})} \quad (32)$$

where:

$$\theta = \frac{\mu}{2n} \frac{1}{v_{sat} L} \quad (33)$$

$\theta L$  is constant:

$$\theta L = \frac{\mu}{2n} \frac{1}{v_{sat}} \quad (34)$$

$$g_{m,sat} = WC_{OX} v_{sat} = \frac{K' W}{\theta L} \quad (35)$$

The voltage and current at transition point between strong-inversion and velocity saturation:

$$V_{GS_t} = \frac{1}{\theta} + V_{TH} = 2nL \frac{v_{sat}}{\mu} + V_{TH} \quad (36)$$

$$I_{DS_t} = K' W L (2n \frac{v_{sat}}{\mu})^2 \quad (37)$$

## 1st Order Time-constants and Transfer-Constants (TTCs)

A system with  $N$  frequency-dependent elements, th transfer function in complex-frequency form:

$$H_s = \frac{a_0 + a_1 s}{1 + b_1 s} \quad (38)$$

where:

$$a_0 = H^0 \quad (39)$$

$$b_1 = \sum_{i=1}^N \tau_i^0 \quad (40)$$

$$a_1 = \sum_{i=1}^N \tau_i^0 H^i \quad (41)$$

where  $H^0$  is the low-frequency gain:

$$H^0 : C_1, C_2, \dots, C_N = 0 \quad (42)$$

or:

$$H^0 : L_1, L_2, \dots, L_N = 0 \quad (43)$$

For capacitor:

$$\tau_i^0 = C_i R_i^0 \quad (44)$$

or for inductor:

$$\tau_i^0 = \frac{L_i}{R_i^0} \quad (45)$$

where  $R_i^0$  is the resistance seen by the capacitor  $C_i$  looking into port i with all other reactive elements connected to the other ports at their zero value (hence the superscript), namely open-circuited capacitors (and short circuited inductors):

$$R_i^0 : C_1, C_2, C_{i-1}, C_{i+1}, \dots, C_N = 0 \quad (46)$$

or:

$$R_i^0 : L_1, L_2, \dots, L_{i-1}, L_{i+1}, \dots, L_N = 0 \quad (47)$$

Bandwidth estimation by 1st-order TTCs:

$$\omega_h \approx \frac{1}{b_1 - \frac{a_1}{a_0}} = \frac{1}{\sum_{i=1}^N \tau_i^0 (1 - \frac{H^i}{H^0})} \quad (48)$$

### 2nd Order Time-constants and Transfer-Constants (TTCs)

A system with 2 frequency-dependent elements, th transfer function:

$$H_s = \frac{a_0 + a_1 s + a_2 s^2}{1 + b_1 s + b_2 s^2} \quad (49)$$

where:

$$a_0 = H^0 \quad (50)$$

$$b_1 = \tau_1^0 + \tau_2^0 \quad (51)$$

$$a_1 = \tau_1^0 H^1 + \tau_2^0 H^2 \quad (52)$$

$$b_2 = \tau_1^0 \tau_2^1 = \tau_2^0 \tau_1^2 \quad (53)$$

$$a_2 = \tau_1^0 \tau_2^1 H^{12} = \tau_2^0 \tau_1^2 H^{12} \quad (54)$$

in which,  $H^1$  ( $H^2$ ) evaluated with the frequency-dependent element at the port 1(2) at its infinite value (i.e., shorted capacitors and open inductors):

$$H^1 : C_1 = \infty, C_2 = 0 \quad (55)$$

$$H^2 : C_1 = 0, C_2 = \infty \quad (56)$$

or:

$$H^1 : L_1 = \infty, L_2 = 0 \quad (57)$$

$$H^2 : L_1 = 0, L_2 = \infty \quad (58)$$

For capacitor:

$$\tau_1^0 = C_1 R_1^0 \quad (59)$$

$$\tau_2^0 = C_2 R_2^0 \quad (60)$$

$$\tau_1^2 = C_1 R_1^2 \quad (61)$$

$$\tau_2^1 = C_2 R_2^1 \quad (62)$$

or for inductor:

$$\tau_1^0 = \frac{L_1}{R_1^0} \quad (63)$$

$$\tau_2^0 = \frac{L_2}{R_2^0} \quad (64)$$

$$\tau_1^2 = \frac{L_1}{R_1^2} \quad (65)$$

$$\tau_2^1 = \frac{L_2}{R_2^1} \quad (66)$$

in which,  $R_1^2$  is the resistance seen by  $C_1$  (the subscript) when  $C_2$  (the superscript) is infinite valued (shorted):

$$R_1^2 : C_2 = \infty \quad (67)$$

$$R_2^1 : C_1 = \infty \quad (68)$$

or:

$$R_1^2 : L_2 = \infty \quad (69)$$

$$R_2^1 : L_1 = \infty \quad (70)$$

### General Time-constants and Transfer-Constants (TTCs)

A system with  $N$  frequency-dependent elements, th transfer function:

$$H_s = \frac{a_0 + a_1 s + a_2 s^2 \cdots + a_n s^n + \cdots}{1 + b_1 s + b_2 s^2 \cdots + b_n s^n + \cdots} \quad (71)$$

where:

$$a_0 = H^0 \quad (72)$$

$$b_1 = \sum_{i=1}^N \tau_i^0 \quad (73)$$

$$a_1 = \sum_{i=1}^N \tau_i^0 H^i \quad (74)$$

$$b_2 = \sum_{i=1}^{i < j} \sum_{j=i+1}^{j \leq N} \tau_i^0 \tau_j^i \quad (75)$$

$$a_2 = \sum_{i=1}^{i < j} \sum_{j=i+1}^{j \leq N} \tau_i^0 \tau_j^i H^{ij} \quad (76)$$

$$b_n = \sum_{i=1}^{i < j} \sum_{j=i+1}^{j < k} \sum_{k=j+1}^{k \leq N} \tau_i^0 \tau_j^i \tau_k^{ij} \cdots \quad (77)$$

$$a_n = \sum_{i=1}^{i < j} \sum_{j=i+1}^{j < k} \sum_{k=j+1}^{k \leq N} \tau_i^0 \tau_j^i \tau_k^{ij} \cdots H^{ijk \cdots} \quad (78)$$

and for capacitor:

$$\tau_i^0 = C_i R_i^0 \quad (79)$$

$$\tau_i^{jk \cdots} = C_i R_i^{jk \cdots} \quad (80)$$

or for inductor:

$$\tau_i^0 = \frac{L_i}{R_i^0} \quad (81)$$

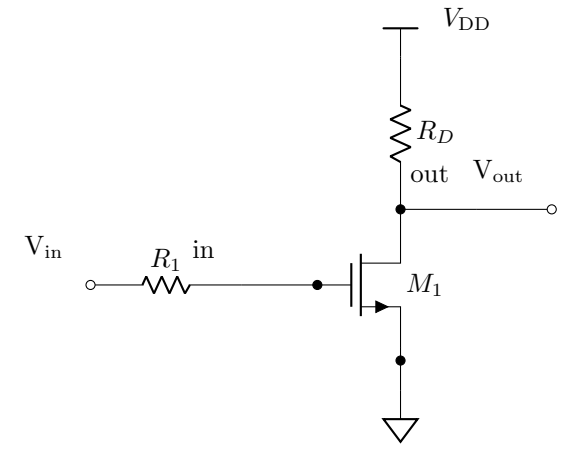
$$\tau_i^{jk \cdots} = \frac{L_i}{R_i^{jk \cdots}} \quad (82)$$

In the form of zeros and poles:

$$H_s = a_0 \frac{(1 - \frac{s}{z_1})(1 - \frac{s}{z_2}) \cdots (1 - \frac{s}{z_m})}{(1 - \frac{s}{p_1})(1 - \frac{s}{p_2}) \cdots (1 - \frac{s}{p_n})} \quad (83)$$

### Single Stage Amplifier: Common Source (CS), with Resistive Load

Common Source, with resistance  $R_D$ :



Transfer function of voltage gain in low-frequency:

$$A_v^0 = \frac{V_{out}}{V_{in}} = -g_m R_D \quad (84)$$

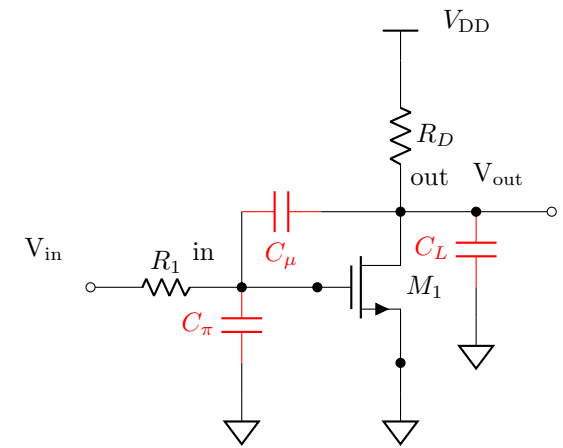
consider channel-length modulation:

$$A_v^0 = \frac{V_{out}}{V_{in}} = -g_m R_D || r_o = -g_m \frac{R_D r_o}{R_D + r_o} \quad (85)$$

The equivalent resistance at point *out* look down  $M1$ :

$$R_{out} = r_o \quad (86)$$

Consider high-frequency gain with 3 capacitors  $c_\pi, c_\mu, c_L$ :



$$\tau_\pi^0 = C_\pi R_1 \quad (87)$$

$$\tau_\mu^0 = C_\mu (R_{left} + R_{right} + G_m R_{left} R_{right}) = C_\mu (R_1 + R_D + g_m R_1 R_D) \quad (88)$$

$$\tau_L^0 = C_L R_D \quad (89)$$

$$\tau_\pi^\pi = C_\mu R_D \quad (90)$$

$$\tau_\pi^L = C_\pi R_1 \quad (91)$$

$$\tau_\mu^L = C_\mu R_1 \quad (92)$$

$$\tau_\mu^{\pi L} = \tau_L^{\pi \mu} = 0 \quad (93)$$

$$H^\pi = H^L = 0 \quad (94)$$

$$H^\mu = \frac{r_m || R_D}{R_1 + r_m || R_D} = \frac{R_D}{R_1 + R_D + g_m R_1 R_D} \quad (95)$$

$$H^{\pi \mu} = H^{L \mu} = H^{L \pi} = 0 \quad (96)$$

$$a_0 = A_v^0 \quad (97)$$

$$b_1 = \tau_\pi^0 + \tau_\mu^0 + \tau_L^0 = R_1 [C_\pi + C_\mu (1 + g_m R_D)] + R_D (C_\mu + C_L) \quad (98)$$

$$a_1 = \tau_\pi^0 H^\pi + \tau_\mu^0 H^\mu + \tau_L^0 H^L = C_\mu R_D \quad (99)$$

$$b_2 = \tau_\pi^0 \tau_\mu^\pi + \tau_L^0 \tau_\mu^L + \tau_L^0 \tau_\pi^L = R_1 R_D (C_\mu C_\pi + C_\mu C_L + C_\pi C_L) = R_{left} R_{right} (\Delta C)^2 \quad (100)$$

$$a_2 = \tau_\pi^0 \tau_\mu^\pi H^{\pi \mu} + \tau_L^0 \tau_\mu^L H^{L \mu} + \tau_L^0 \tau_\pi^L H^{L \pi} = 0 \quad (101)$$

$$b_3 = \tau_\pi^0 \tau_\mu^\pi \tau_L^{\pi \mu} = 0 \quad (102)$$

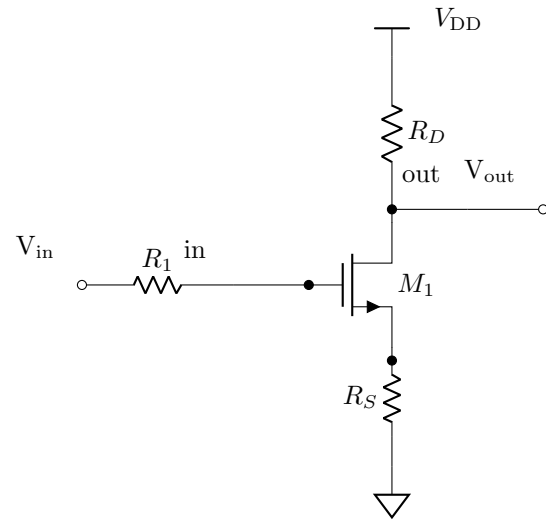
$$a_3 = \tau_\pi^0 \tau_\mu^\pi \tau_L^{\pi \mu} H^{\pi \mu L} = 0 \quad (103)$$

Finally, we have:

$$A_v(s) = \frac{A_v^0 (1 - r_m C_\mu s)}{1 + [R_1 [C_\pi + C_\mu (1 + g_m R_D)] + R_D (C_\mu + C_L)] s + R_1 R_D (C_\mu C_\pi + C_\mu C_L + C_\pi C_L) s^2} \quad (104)$$

### Single Stage Amplifier: Common Source (CS), with Source Degeneration

Common Source, with resistance  $R_D$  and  $R_S$ :



The low frequency gain:

$$A_v^0 = -\frac{g_m R_D}{1 + g_m R_S} \quad (105)$$

The equivalent transconductance:

$$G_m = \frac{g_m}{1 + g_m R_S} \quad (106)$$

consider channel-length modulation and body effect:

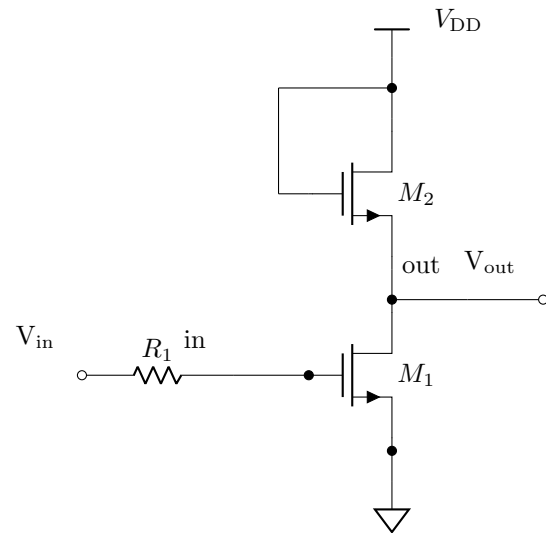
$$G_m = \frac{g_m r_o}{R_S + [1 + (g_m + g_{mb})r_o]r_o} \quad (107)$$

$$R_{out} = r_o + [1 + (g_m + g_{mb})r_o]R_S \approx g_m r_o R_S \quad (108)$$

$$A_v^0 = -G_m(R_{out} || R_D) = -\frac{g_m r_o}{R_S + [1 + (g_m + g_{mb})R_S]r_o} \{ [r_o + [1 + (g_m + g_{mb})r_o]R_S] || R_D \} \quad (109)$$

### Single Stage Amplifier: Common Source (CS), with Diode-Connected Load

Common Source, with diode-connected transistor  $M_2$ :



Consider body effect and channel-length modulation, the equivalent resistance at point *out* look into  $M_2$ :

$$R_{out,up} = \frac{1}{g_{m2} + g_{mb2}} || r_o \quad (110)$$

With negligible channel-length modulation, we have the low frequency gain:

$$A_v^0 = -g_{m1} \frac{1}{g_{m2} + g_{mb2}} = -\frac{g_{m1}}{g_{m2}} \frac{1}{1 + \eta} \quad (111)$$

where:

$$\frac{1}{1 + \eta} = \frac{g_{mb2}}{g_{m2}} \quad (112)$$

Since  $I_{DS}$  is same at  $M_1$  and  $M_2$  (from drain to source), we have:

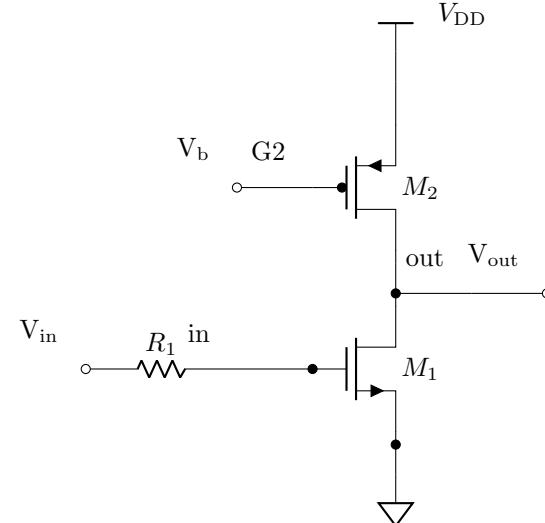
$$A_v^0 = -\sqrt{\frac{W_1/L_1}{W_2/L_2}} \frac{1}{1 + \eta} \quad (113)$$

If we replace nmos  $M_2$  with pmos, the body effect of  $M_2$  will disappear:

$$A_v^0 = -\sqrt{\frac{\mu_n W_1/L_1}{\mu_p W_2/L_2}} \quad (114)$$

### Single Stage Amplifier: Common Source (CS), with Current-Source Load

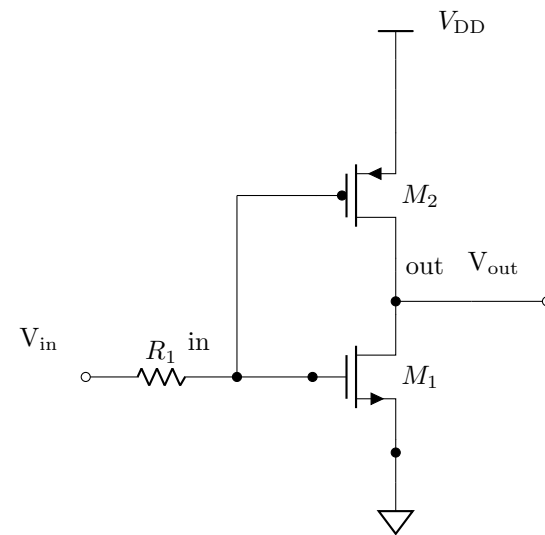
$M_2$  works as a current source:



The low frequency gain:

$$A_v^0 = -g_{m1}(r_{o1} || r_{o2}) \quad (115)$$

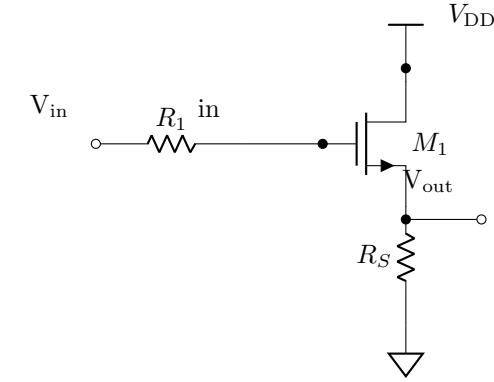
### Single Stage Amplifier: Common Source (CS), with Active Load



The low frequency gain:

$$A_v^0 = -(g_{m1} + g_{m2})(r_{o1} || r_{o2}) \quad (116)$$

### Single Stage Amplifier: Common Drain (CD) or Source Follower



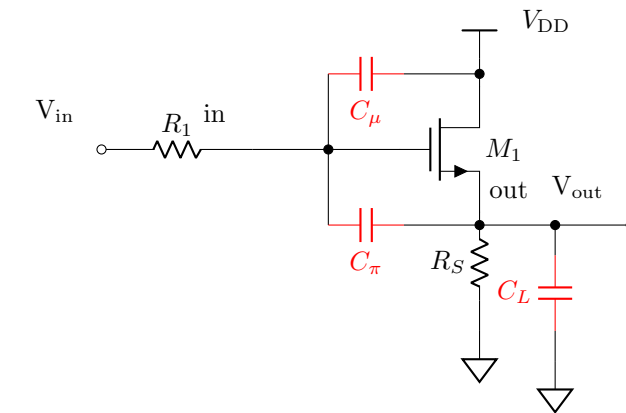
The low frequency gain:

$$A_v^0 = \frac{g_m R_S}{1 + (g_m + g_{mb})R_S} \quad (117)$$

The equivalent resistance at point *out* look up into  $M_1$ :

$$R_{out,up} = \frac{1}{g_m + g_{mb}} || r_o \quad (118)$$

Consider high-frequency gain with 3 capacitors:



We have:

$$\tau_\mu^0 = C_\mu R_1 \quad (119)$$

$$\tau_\pi^0 = C_\pi \frac{R_1 + R_S}{1 + g_m R_S} \quad (120)$$

$$\tau_L^0 = C_L(r_m || R_S) \quad (121)$$

$$\tau_L^\pi = C_L(R_1 || R_S) \quad (122)$$

$$\tau_\mu^L = C_L R_1 \quad (123)$$

$$H^\mu = H^L = 0 \quad (124)$$

$$H^\pi = \frac{R_S}{R_1 + R_S} \quad (125)$$

$$b_1 = C_\mu R_1 + C_\pi \frac{R_1 + R_S}{1 + g_m R_S} + C_L(r_m || R_S) \quad (126)$$

$$a_1 = C_\pi \frac{1}{1 + g_m R_S} \quad (127)$$

$$b_2 = \frac{R_1 R_S}{1 + g_m R_S} (C_\pi C_\mu + C_\pi C_L) + C_L C_\mu (r_m || R_S) R_1 \quad (128)$$

$$a_2 = b_3 = a_3 = 0 \quad (129)$$

We ignore body effect:

$$a_0 = \frac{g_m R_S}{1 + g_m R_S} \quad (130)$$

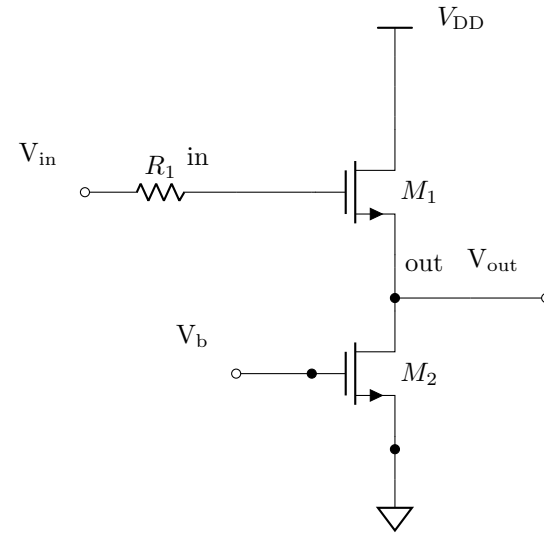
Now we have the high frequency voltage gain in  $s$  domain:

$$A_v(s) = \frac{g_m R_S + C_\pi s}{1 + g_m R_S + [(C_\mu g_m R_1 + C_\pi + C_L)R_S + (C_\mu + C_\pi)R_1]s + R_1 R_S \Delta C^2 s^2} \quad (131)$$

where,

$$\Delta C^2 = C_\mu C_\pi + C_\pi C_L + C_\mu C_L \quad (132)$$

#### Single Stage Amplifier: Common Drain (CD) or Source Follower, With Transistor Load



The low frequency gain:

$$A_v^0 = \frac{R_{eq}}{R_{eq} + \frac{1}{g_{m1}}} \quad (133)$$

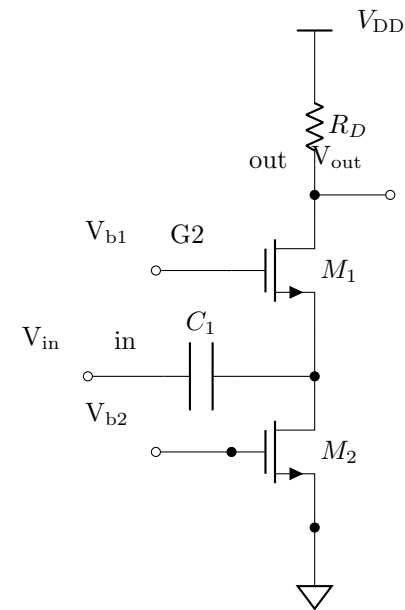
where:

$$R_{eq} = \frac{1}{g_{mb1}} || r_{o1} || r_{o2} \quad (134)$$

#### Single Stage Amplifier: Common Drain (CD) or Source Follower

#### Single Stage Amplifier: Common Gate (CG)

$M_2$  works as current source,  $C_1$  is coupled capacitor:

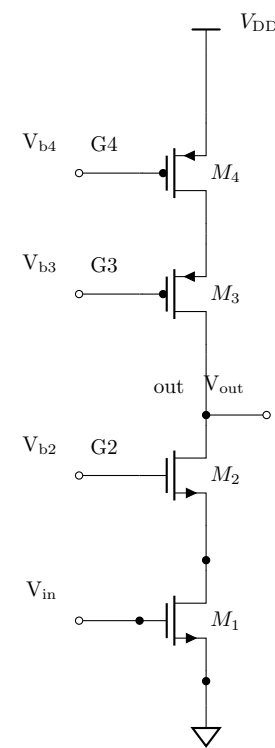


The low frequency voltage gain:

$$A_v^0 = g_m(1 + \eta)R_D \quad (135)$$

#### Single Stage Amplifier: Cascode

$M_3$  and  $M_4$  are formed the cascode current source:



The low frequency voltage gain:

$$A_v^0 \approx -g_{m1}(g_{m2}r_{o1}r_{o2} || g_{m3}r_{o3}r_{o4}) \quad (136)$$