Analog CMOS Integrated Circuit Design Cheat Sheet

By Xiao Ma (mxlol233@outlook.com)[https://github.com/TuringKi/Analog-CMOS-Integrated-Circuit-Design-Cheat-Sheet]

Model of MOS Transistors

Process parameters (n,V_{TH},KP, V_E):

$$t_{OX} = \frac{L_{min}}{50} \tag{1}$$

$$t_{si} = \sqrt{\frac{2\epsilon_{si}(\Phi - V_{BD})}{qN_B}} \tag{2}$$

$$C_{OX} = \frac{\epsilon_{OX}}{t_{OX}} \tag{3}$$

$$C_D = \frac{\epsilon_{si}}{t_{si}} \tag{4}$$

$$KP = \mu C_{OX} \tag{5}$$

$$\beta = KP\frac{W}{L} \tag{6}$$

$$Q_{dep} = \sqrt{4q\epsilon_{si}|\Phi_F|N_{sub}} \tag{7}$$

$$V_{TH0} = \Phi_{MS} + 2\Phi_F + \frac{Q_{dep}}{C_{OX}} \tag{8}$$

Changing the Gate voltage V_{GS} will thus change the conductivity of the channel and hence the current I_{DS}. In a similar way, changing the Bulk voltage V_{BS} will thus also change the conductivity of the channel and will thus change the current I_{DS} as well. The gate gives the MOST operation, whereas the bulk gives JFET operation.
Indeed, a Junction FET is by definition a FET in which the current is controlled by a junction capacitance.
All MOST devices are thus parallel combinations of MOSTs and JFETs.
The Bulk voltage works like a "back-gate", which is also called as "body effect":

$$V_{TH} = V_{TH0} + \gamma(\sqrt{|2\Phi_F| + V_{BS}} - \sqrt{|2\Phi_F|}) \tag{9}$$

$$n = \frac{\gamma}{\sqrt{|2\Phi_F| + V_{BS}}} = 1 + \frac{C_D}{C_{OX}} \tag{10}$$

In linear region:

$$I_{DS} = \beta[(V_{GS} - V_{TH})V_{DS} - \frac{1}{2}V_{DS}^2] \tag{11}$$

$$R_{on} = \frac{1}{\beta(V_{GS} - V_{TH})} \tag{12}$$

Channel-Length modulation in saturation region:

$$K' = \frac{KP}{2n} \tag{13}$$

$$\lambda = \frac{1}{V_E L} \tag{14}$$

$$I_{DS} = K'\frac{W}{L}(V_{GS} - V_{TH})^2(1 + \lambda V_{DS}) \tag{15}$$

$$r_o = \frac{\partial V_{DS}}{\partial I_{DS}} \approx \frac{1}{\lambda I_{DS}} = \frac{V_E L}{I_{DS}} \tag{16}$$

Saturation region has three distinctive regions: weak-inversion (exponential region), strong-inversion, and velocity saturation.

Value Examples In $0.35\mu m$ Process Nodes

Names	Symbols	Values
dielectric constant of sub-silicon	ϵ_{si}	1 pF/cm
dielectric constant of gate-oxide	ϵ_{OX}	0.34 pF/cm
electron charge	q	1.6×10^{-19} C
minium channel length	L_{min}	0.35 μm
width of gate-oxide	t_{OX}	0.1 nm
width of depletion layer	t_{si}	7 nm
junction built-in voltage	Φ	0.6 V
drain-bulk voltage	V_{BD}	-3.3V
gate-oxide capacitance	C_{OX}	0.5 μF/cm^2
depletion layer capacitance	C_D	0.1 μF/cm^2
bulk doping level	N_B	4×10^{17} cm^{-3}
P type mobility rate	μ_p	250 cm^2/Vs
N type mobility rate	μ_n	600 cm^2/Vs
N type KP	KP_n	300 μA/V^2
	n	$1.2 \cdots 1.5$
	$\lvert 2\Phi_F \rvert$	0.6 V
	γ	$0.5 \cdots 0.8$ V$^{\frac{1}{2}}$
N type K'	K'_n	100 μA/V^2
P type K'	K'_p	40 μA/V^2
	V_{GSTt}	70 mV
	v_{sat}	10^7 cm/s
	θL	0.2 μm/V

Weak-Inversion

$$I_{DS} = I_{D0}\frac{W}{L}e^{\frac{V_{GS}}{n\frac{KT}{q}}} \tag{17}$$

$$g_m = \frac{\partial I_{DS}}{\partial V_{GS}} = \frac{I_{DS}}{n\frac{KT}{q}} \tag{18}$$

strong-inversion

Ignore channel-length modulation:

$$I_{DS} = K'\frac{W}{L}(V_{GS} - V_{TH})^2 \tag{19}$$

$$g_m = \frac{2I_{DS}}{V_{GS} - V_{TH}} \tag{20}$$

Transition Point Between Weak-Inversion and Strong-Inversion

The voltage and current at transition point between weak-inversion and strong-inversion:

$$V_{GSt} = 2n\frac{KT}{q} + V_{TH} \tag{21}$$

$$I_{DSt} \approx K'\frac{W}{L}(2n\frac{KT}{q})^2 \tag{22}$$

EKV model, a smooth model for weak-inversion and strong-inversion regions:

$$I_{DS} = K'\frac{W}{L}(V_{GS} - V_{TH})^2[ln(1 + e^{\frac{V_{GS}}{V_{GSt}}})]^2 \tag{23}$$

Let:

$$v = \frac{V_{GS}}{V_{GSt}} \tag{24}$$

$$i = \frac{I_{DS}}{I_{DSt}} = [ln(1 + e^v)]^2 \tag{25}$$

then,

$$v = ln(e^{\sqrt{i}} - 1) \tag{26}$$

$$V_{GS} - V_{TH} = V_{GSTt}ln(e^{\sqrt{i}} - 1) \tag{27}$$

where:

$$V_{GSTt} = V_{GSt} - V_{TH} = 2n\frac{KT}{q} \tag{28}$$

When $v = 1$, $i = 1$, we also have:

$$I_{DSt} = K'\frac{W}{L}(V_{GSt} - V_{TH})^2 \tag{29}$$

Velocity Saturation

$$I_{DS} = WC_{OX}v_{sat}(V_{GS} - V_{TH}) \tag{30}$$

$$g_m = WC_{OX}v_{sat} \tag{31}$$

Transition Point Between Strong-Inversion and Velocity Saturation

A smooth model for strong-inversion and velocity saturation regions:

$$I_{DS} = \frac{K'\frac{W}{L}(V_{GS} - V_{TH})^2}{1 + \theta(V_{GS} - V_{TH})} \tag{32}$$

where:

$$\theta = \frac{\mu}{2n}\frac{1}{v_{sat}L} \tag{33}$$

θL is constant:

$$\theta L = \frac{\mu}{2n}\frac{1}{v_{sat}} \tag{34}$$

$$g_{m,sat} = WC_{OX}v_{sat} = \frac{K'W}{\theta L} \tag{35}$$

The voltage and current at transition point between strong-inversion and velocity saturation:

$$V_{GSt} = \frac{1}{\theta} + V_{TH} = 2nL\frac{v_{sat}}{\mu} + V_{TH} \tag{36}$$

$$I_{DSt} = K'WL(2n\frac{v_{sat}}{\mu})^2 \tag{37}$$

1st Order Time-constants and Transfer-Constants (TTCs)

A system with N frequency-dependent elements, th transfer function in complex-frequency form:

$$H_s = \frac{a_0 + a_1 s}{1 + b_1 s} \tag{38}$$

where:

$$a_0 = H^0 \tag{39}$$

$$b_1 = \sum_{i=1}^{N}\tau_i^0 \tag{40}$$

$$a_1 = \sum_{i=1}^{N}\tau_i^0 H^i \tag{41}$$

where H^0 is the low-frequency gain:

$$H^0 : C_1, C_2, \cdots, C_N = 0 \tag{42}$$

or:

$$H^0 : L_1, L_2, \cdots, L_N = 0 \tag{43}$$

For capacitor:

$$\tau_i^0 = C_i R_i^0 \tag{44}$$

or for inductor:

$$\tau_i^0 = \frac{L_i}{R_i^0} \tag{45}$$

where R_i^0 is the resistance seen by the capacitor C_i looking into port i with all other reactive elements connected to the other ports at their zero value (hence the superscript), namely open-circuited capacitors (and short circuited inductors):

$$R_i^0 : C_1, C_2, C_{i-1}, C_{i+1}, \cdots, C_N = 0 \tag{46}$$

or:

$$R_i^0 : L_1, L_2, \cdots, L_{i-1}, L_{i+1}, \cdots, L_N = 0 \tag{47}$$

Hight bandwidth frequency estimation by 1st-order TTCs:

$$\omega_h \approx \frac{1}{b_1 - \frac{a_1}{a_0}} = \frac{1}{\sum_{i=1}^{N}\tau_i^0(1 - \frac{H^i}{H^0})} \tag{48}$$

Meanwhile, define:

$$R_i^\infty : C_1, C_2, C_{i-1}, C_{i+1}, \cdots, C_N = \infty \tag{49}$$

or:

$$R_i^\infty : L_1, L_2, \cdots, L_{i-1}, L_{i+1}, \cdots, L_N = \infty \tag{50}$$

$$\tau_i^\infty = C_i R_i^\infty \tag{51}$$

or for inductor:

$$\tau_i^\infty = \frac{L_i}{R_i^\infty} \tag{52}$$

Low bandwidth frequency estimation:

$$\omega_l \approx \sum_{i=0}^{N}\frac{1}{\tau_i^\infty} \tag{53}$$

2nd Order Time-constants and Transfer-Constants (TTCs)

A system with 2 frequency-dependent elements, th transfer function:

$$H_s = \frac{a_0 + a_1 s + a_2 s^2}{1 + b_1 s + b_2 s^2} \tag{54}$$

where:

$$a_0 = H^0 \tag{55}$$

$$b_1 = \tau_1^0 + \tau_2^0 \tag{56}$$

$$a_1 = \tau_1^0 H^1 + \tau_2^0 H^2 \tag{57}$$

$$b_2 = \tau_1^0 \tau_2^1 = \tau_2^0 \tau_1^2 \tag{58}$$

$$a_2 = \tau_1^0 \tau_2^1 H^{12} = \tau_2^0 \tau_1^2 H^{12} \tag{59}$$

in which, H^1 (H^2) evaluated with the frequency-dependent element at the port 1(2) at its infinite value (i.e., shorted capacitors and open inductors):

$$H^1 : C_1 = \infty, C_2 = 0 \tag{60}$$

$$H^2 : C_1 = 0, C_2 = \infty \tag{61}$$

or:

$$H^1 : L_1 = \infty, L_2 = 0 \tag{62}$$

$$H^2 : L_1 = 0, L_2 = \infty \tag{63}$$

For capacitor:

$$\tau_1^0 = C_1 R_1^0 \tag{64}$$

$$\tau_2^0 = C_2 R_2^0 \tag{65}$$

$$\tau_1^2 = C_1 R_1^2 \tag{66}$$

$$\tau_2^1 = C_2 R_2^1 \tag{67}$$

or for inductor:

$$\tau_1^0 = \frac{L_1}{R_1^0} \tag{68}$$

$$\tau_2^0 = \frac{L_2}{R_2^0} \tag{69}$$

$$\tau_1^2 = \frac{L_1}{R_1^2} \tag{70}$$

$$\tau_2^1 = \frac{L_2}{R_2^1} \tag{71}$$

in which, R_1^2 is the resistance seen by C_1 (the subscript) when C_2 (the superscript) is infinite valued (shorted):

$$R_1^2 : C_2 = \infty \tag{72}$$

$$R_2^1 : C_1 = \infty \tag{73}$$

or:

$$R_1^2 : L_2 = \infty \tag{74}$$

$$R_2^1 : L_1 = \infty \tag{75}$$

Bandwith-gain Product And Optimal Stages

Considering a common source amplifier with one load capacitor C_L and one drain resistor R_D, its low frequency voltage gain is:

$$A_v^0 = -g_m R_D \tag{76}$$

From 1-st order TTCs, we have the high frequency voltage gain:

$$A_v(s) = \frac{-g_m R_D}{1 + C_L R_D s} \tag{77}$$

and high bandwidth frequency:

$$\omega_h = \frac{1}{C_L R_D} \tag{78}$$

Define the bandwidth-gain product:

$$\omega_u \equiv |A_v^0| \omega_h = \frac{g_m}{C_L} \tag{79}$$

The magnitude of the total gain for N amplifiers connected in series with the same gain is:

$$|A_{v,total}(s)| = (\frac{-g_m R_D}{1 + C_L R_D s})^N \tag{80}$$

Let $|A_{v,total}(s)|$ equation to $\frac{1}{2}(-g_m R_D)^N$, which is $-2dB$ points, get the high bandwidth frequency:

$$\omega_{h,total} = \omega_h \sqrt{2^{\frac{1}{N}} - 1} = \frac{\omega_u}{|A_v^0|} \sqrt{2^{\frac{1}{N}} - 1} = \frac{\omega_u}{|A_{v,total}^0|^{\frac{1}{N}}} \sqrt{2^{\frac{1}{N}} - 1} \tag{81}$$

the ratio between N stages $\omega_{h,total}$ and one stage $\omega_h^{'}$ with same gain:

$$\frac{\omega_{h,total}}{\omega_h^{'}} = \frac{\frac{\omega_u}{|A_{v,total}^0|^{\frac{1}{N}}} \sqrt{2^{\frac{1}{N}} - 1}}{\frac{\omega_u}{|A_{v,total}^0|}} = |A_{v,total}^0|^{1-\frac{1}{N}} \sqrt{2^{\frac{1}{N}} - 1} \tag{82}$$

The stages number N_{opt} where the $\frac{\omega_{h,total}}{\omega_h^{'}}$ is maximum:

$$N_{opt} \approx 1.85 ln(|A_{v,total}^0|) \tag{83}$$

The optimal gain for each of the stages:

$$A_{v,opt}^0 = e - 1 \tag{84}$$

General Time-constants and Transfer-Constants (TTCs)

A system with N frequency-dependent elements, th transfer function:

$$H_s = \frac{a_0 + a_1 s + a_2 s^2 \cdots + a_n s^n + \cdots}{1 + b_1 s + b_2 s^2 \cdots + b_n s^n + \cdots} \tag{85}$$

where:

$$a_0 = H^0 \tag{86}$$

$$b_1 = \sum_{i=1}^{N} \tau_i^0 \tag{87}$$

$$a_1 = \sum_{i=1}^{N} \tau_i^0 H^i \tag{88}$$

$$b_2 = \sum_{i=1}^{i<j} \sum_{j=i+1}^{j \leq N} \tau_i^0 \tau_j^i \tag{89}$$

$$a_2 = \sum_{i=1}^{i<j} \sum_{j=i+1}^{j \leq N} \tau_i^0 \tau_j^i H^{ij} \tag{90}$$

$$b_n = \sum_{i=1}^{i<j} \sum_{j=i+1}^{j<k} \sum_{k=j+1\cdots}^{k<\ldots\leq N} \tau_i^0 \tau_j^i \tau_k^{ij} \cdots \tag{91}$$

$$a_n = \sum_{i=1}^{i<j} \sum_{j=i+1}^{j<k} \sum_{k=j+1\cdots}^{k<\ldots\leq N} \tau_i^0 \tau_j^i \tau_k^{ij} \cdots H^{ijk\ldots} \tag{92}$$

and for capacitor:

$$\tau_i^0 = C_i R_i^0 \tag{93}$$

$$\tau_i^{jk\cdots} = C_i R_i^{jk\cdots} \tag{94}$$

or for inductor:

$$\tau_i^0 = \frac{L_i}{R_i^0} \tag{95}$$

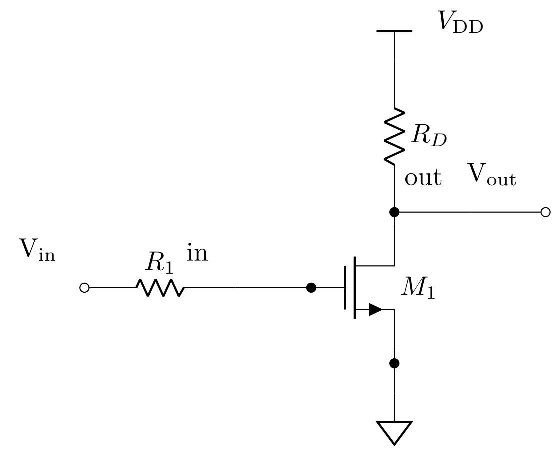
$$\tau_i^{jk\cdots} = \frac{L_i}{R_i^{jk\cdots}} \tag{96}$$

In the form of zeros and poles:

$$H_s = a_0 \frac{(1 - \frac{s}{z_1})(1 - \frac{s}{z_1}) \cdots (1 - \frac{s}{z_m})}{(1 - \frac{s}{p_1})(1 - \frac{s}{p_1}) \cdots (1 - \frac{s}{p_n})} \tag{97}$$

Single Stage Amplifier: Common Source (CS), with Resistive Load

Common Source, with resistance R_D:



Transfer function of voltage gain in low-frequency:

$$A_v^0 = \frac{V_{out}}{V_{in}} = -g_m R_D \tag{98}$$

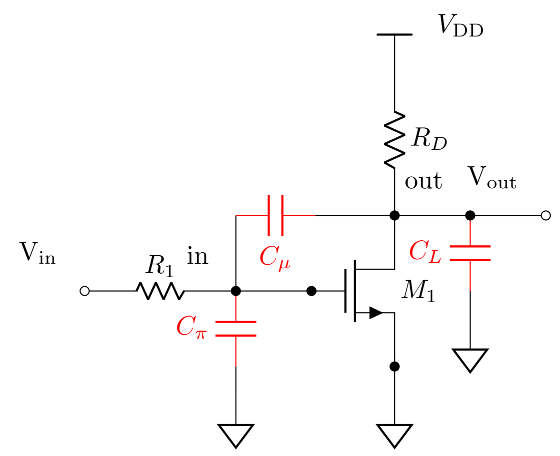
consider channel-length modulation:

$$A_v^0 = \frac{V_{out}}{V_{in}} = -g_m R_D || r_o = -g_m \frac{R_D r_o}{R_D + r_o} \tag{99}$$

The equivalent resistance at point *out* look down $M1$:

$$R_{out} = r_o \tag{100}$$

Consider high-frequency gain with 3 capacitors c_π, c_μ, c_L:



$$\tau_\pi^0 = C_\pi R_1 \tag{101}$$

$$\tau_\mu^0 = C_\mu (R_{left} + R_{right} + G_m R_{left} R_{right}) = C_\mu (R_1 + R_D + g_m R_1 R_D) \tag{102}$$

$$\tau_L^0 = C_L R_D \tag{103}$$

$$\tau_\mu^\pi = C_\mu R_D \tag{104}$$

$$\tau_\pi^L = C_\pi R_1 \tag{105}$$

$$\tau_\mu^L = C_\mu R_1 \tag{106}$$

$$\tau_\mu^{\pi L} = \tau_L^{\pi\mu} = 0 \tag{107}$$

$$A_v^\pi = H^L = 0 \tag{108}$$

$$A_v^\mu = \frac{r_m || R_D}{R_1 + r_m || R_D} = \frac{R_D}{R_1 + R_D + g_m R_1 R_D} \tag{109}$$

$$A_v^{\pi\mu} = A_v^{L\mu} = A_v^{L\pi} = 0 \tag{110}$$

$$a_0 = A_v^0 \tag{111}$$

$$b_1 = \tau_\pi^0 + \tau_\mu^0 + \tau_L^0 = R_1 [C_\pi + C_\mu (1 + g_m R_D)] + R_D (C_\mu + C_L) \tag{112}$$

$$a_1 = \tau_\pi^0 A_v^\pi + \tau_\mu^0 A_v^\mu + \tau_L^0 A_v^L = C_\mu R_D \tag{113}$$

$$b_2 = \tau_\pi^0 \tau_\mu^\pi + \tau_L^0 \tau_\mu^L + \tau_L^0 \tau_\pi^L = R_1 R_D (C_\mu C_\pi + C_\mu C_L + C_\pi C_L) = R_{left} R_{right} (\Delta C)^2 \tag{114}$$

$$a_2 = \tau_\pi^0 \tau_\mu^\pi A_v^{\pi\mu} + \tau_L^0 \tau_\mu^L A_v^{L\mu} + \tau_L^0 \tau_\pi^L A_v^{L\pi} = 0 \tag{115}$$

$$b_3 = \tau_\pi^0 \tau_\mu^\pi \tau_L^{\pi\mu} = 0 \tag{116}$$

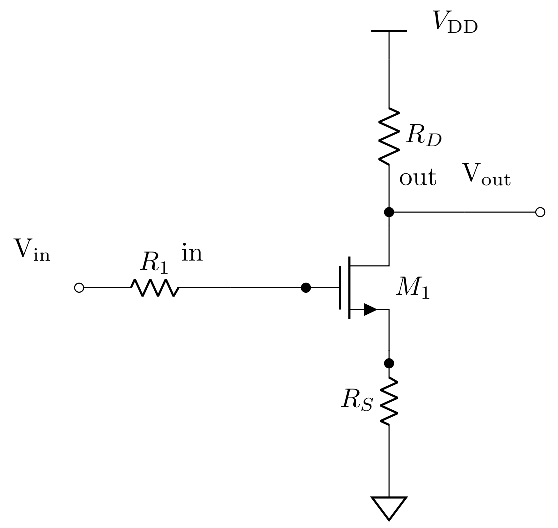
$$a_3 = \tau_\pi^0 \tau_\mu^\pi \tau_L^{\pi\mu} A_v^{\pi\mu L} = 0 \tag{117}$$

Finally, we have:

$$A_v(s) = \frac{A_v^0 (1 - r_m C_\mu s)}{1 + [R_1 [C_\pi + C_\mu (1 + g_m R_D)] + R_D (C_\mu + C_L)] s + R_1 R_D (C_\mu C_\pi + C_\mu C_L + C_\pi C_L) s^2} \tag{118}$$

Single Stage Amplifier: Common Source (CS), with Source Degeneration

Common Source, with resistance R_D and R_S:



The low frequency gain:

$$A_v^0 = -\frac{g_m R_D}{1+g_m R_S} \tag{119}$$

The equivalent transconductance:

$$G_m = \frac{g_m}{1+g_m R_S} \tag{120}$$

consider channel-length modulation and body effect:

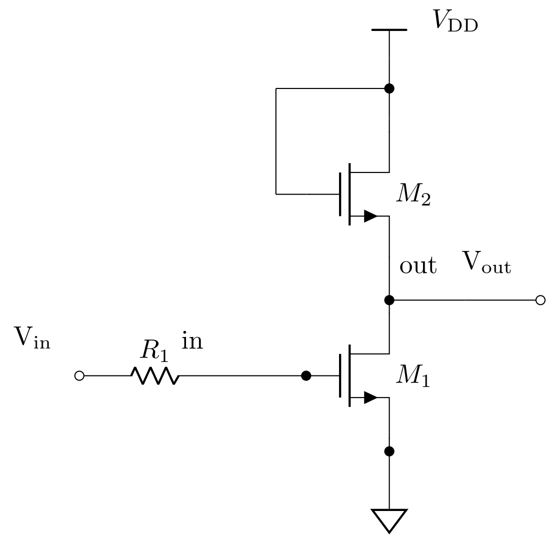
$$G_m = \frac{g_m r_o}{R_S + [1+(g_m+g_{mb})R_s]r_o} \tag{121}$$

$$R_{out} = r_o + [1+(g_m+g_{mb})r_o]R_S \approx g_m r_o R_S \tag{122}$$

$$A_v^0 = -G_m(R_{out}||R_D) = -\frac{g_m r_o}{R_S+[1+(g_m+g_{mb})R_s]r_o}\{[r_o+[1+(g_m+g_{mb})r_o]R_S]||R_D\} \tag{123}$$

Single Stage Amplifier: Common Source (CS), with Diode-Connected Load

Common Source, with diode-connected transistor $M2$:



Consider body effect and channel-length modulation, the equivalent resistance at point *out* look into $M2$:

$$R_{out,up} = \frac{1}{g_{m2}+g_{mb2}}||r_o \tag{124}$$

With negligible channel-length modulation, we have the low frequency gain:

$$A_v^0 = -g_{m1}\frac{1}{g_{m2}+g_{mb2}} = -\frac{g_{m1}}{g_{m2}}\frac{1}{1+\eta} \tag{125}$$

where:

$$\frac{1}{1+\eta} = \frac{g_{mb2}}{g_{m2}} \tag{126}$$

Since I_{DS} is same at $M1$ and $M2$ (from drain to source), we have:

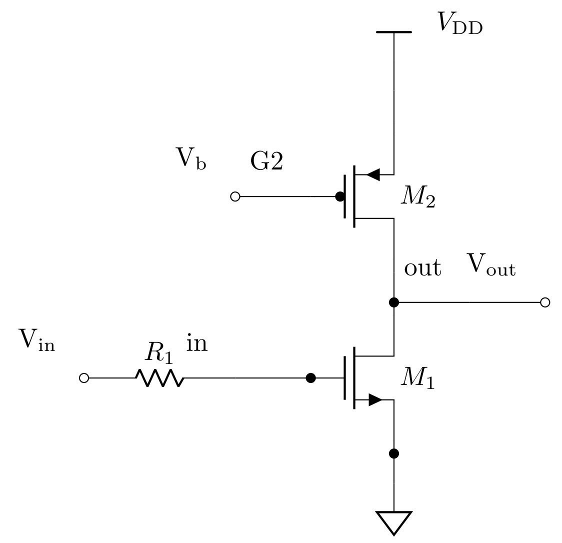
$$A_v^0 = -\sqrt{\frac{W_1/L_1}{W_2/L_2}}\frac{1}{1+\eta} \tag{127}$$

If we replace nmos $M2$ with pmos, the body effect of $M2$ will disappear:

$$A_v^0 = -\sqrt{\frac{\mu_n W_1/L_1}{\mu_p W_2/L_2}} \tag{128}$$

Single Stage Amplifier: Common Source (CS), with Current-Source Load

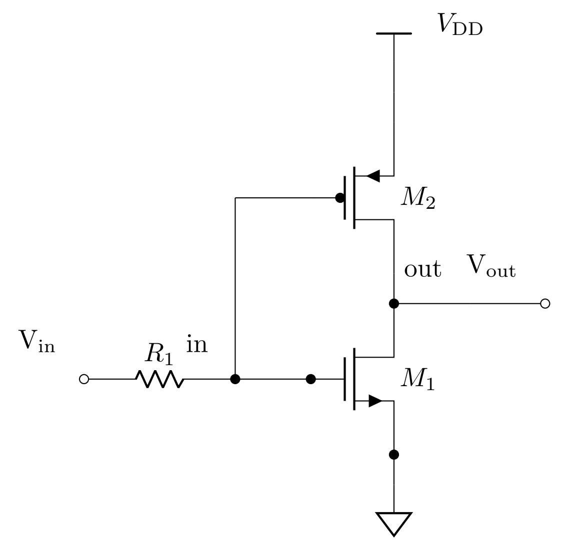
$M2$ works as a current source:



The low frequency gain:

$$A_v^0 = -g_{m1}(r_{o1}||r_{o2}) \tag{129}$$

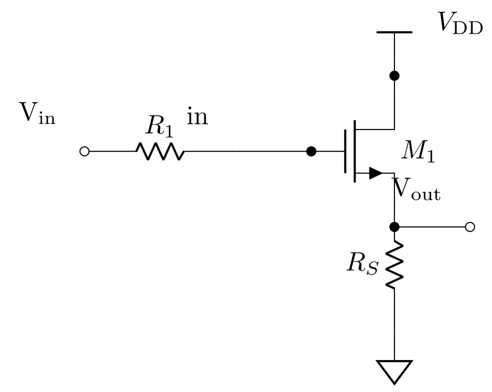
Single Stage Amplifier: Common Source (CS), with Active Load



The low frequency gain:

$$A_v^0 = -(g_{m1}+g_{m2})(r_{o1}||r_{o2}) \tag{130}$$

Single Stage Amplifier: Common Drain (CD) or Source Follower



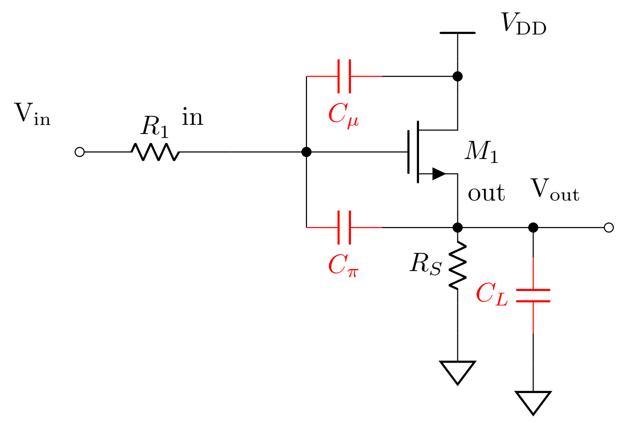
The low frequency gain:

$$A_v^0 = \frac{g_m R_S}{1+(g_m+g_{mb})R_S} \tag{131}$$

The equivalent resistance at point *out* look up into $M1$:

$$R_{out,up} = \frac{1}{g_m+g_{mb}}||r_o \tag{132}$$

Consider high-frequency gain with 3 capacitors:



We have:

$$\tau_\mu^0 = C_\mu R_1 \tag{133}$$

$$\tau_\pi^0 = C_\pi \frac{R_1+R_S}{1+g_m R_S} \tag{134}$$

$$\tau_L^0 = C_L(r_m||R_S) \tag{135}$$

$$\tau_L^\pi = C_L(R_1||R_S) \tag{136}$$

$$\tau_\mu^L = C_L R_1 \tag{137}$$

$$A_v^\mu = A_v^L = 0 \tag{138}$$

$$A_v^\pi = \frac{R_S}{R_1+R_S} \tag{139}$$

$$b_1 = C_\mu R_1 + C_\pi\frac{R_1+R_S}{1+g_m R_S} + C_L(r_m||R_S) \tag{140}$$

$$a_1 = C_\pi\frac{1}{1+g_m R_S} \tag{141}$$

$$b_2 = \frac{R_1 R_S}{1+g_m R_S}(C_\pi C_\mu + C_\pi C_L) + C_L C_\mu (r_m||R_S)R_1 \tag{142}$$

$$a_2 = b_3 = a_3 = 0 \tag{143}$$

We ignore body effect:

$$a_0 = \frac{g_m R_S}{1+g_m R_S} \tag{144}$$

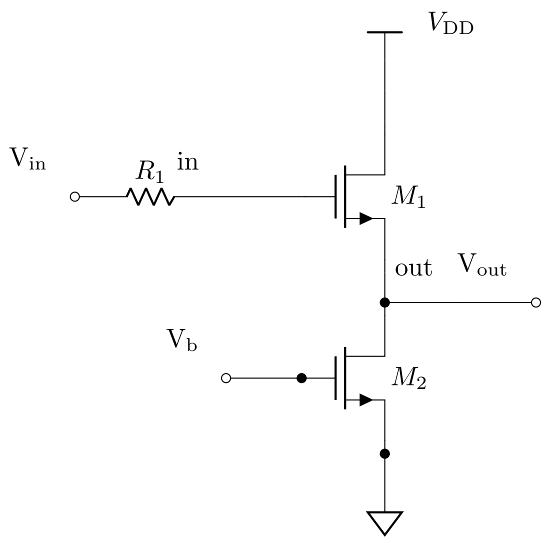
Now we have the high frequency voltage gain in s domain:

$$A_v(s) = \frac{g_m R_S + C_\pi s}{1+g_m R_S+[(C_\mu g_m R_1 + C_\pi + C_L)R_S + (C_\mu + C_\pi)R_1]s + R_1 R_S \Delta C^2 s^2} \tag{145}$$

where,

$$\Delta C^2 = C_\mu C_\pi + C_\pi C_L + C_\mu C_L \tag{146}$$

Single Stage Amplifier: Common Drain (CD) or Source Follower, With Transistor Load



The low frequency gain:

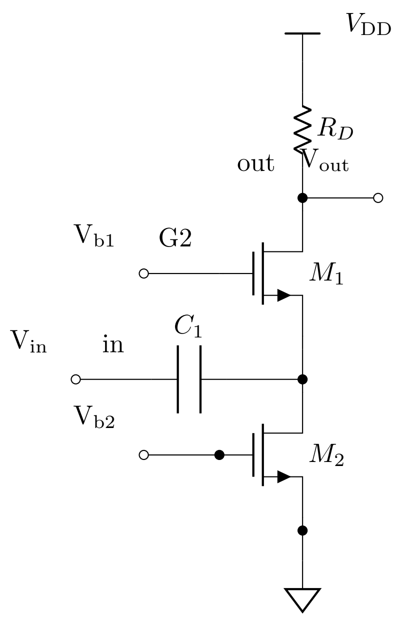
$$A_v^0 = \frac{R_{eq}}{R_{eq} + \frac{1}{g_{m1}}} \tag{147}$$

where:

$$R_{eq} = \frac{1}{g_{mb1}} || r_{o1} || r_{o2} \tag{148}$$

Single Stage Amplifier: Common Gate (CG)

M_2 works as current source, C_1 is coupled capacitor:

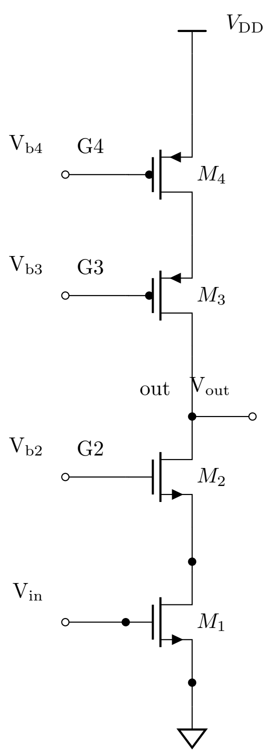


The low frequency voltage gain:

$$A_v^0 = g_m(1+\eta)R_D \tag{149}$$

Single Stage Amplifier: Cascode

M_3 and M_4 are formed the cascode current source:

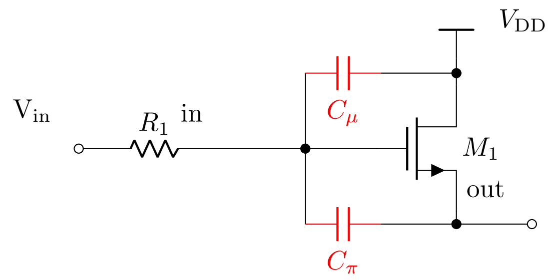


The low frequency voltage gain:

$$A_v^0 \approx -g_{m1}(g_{m2}r_{o1}r_{o2} || g_{m3}r_{o3}r_{o4}) \tag{150}$$

Input and Output Impedance At High Frequency of Source Follower

We consider the output impedance at *out* looking upon:



$$\tau_\mu = C_\mu R_1 \tag{151}$$

$$\tau_\pi = C_\pi r_m \tag{152}$$

$$\tau_\mu^\pi = C_\mu R_1 \tag{153}$$

$$Z_{out}^0 = r_m \tag{154}$$

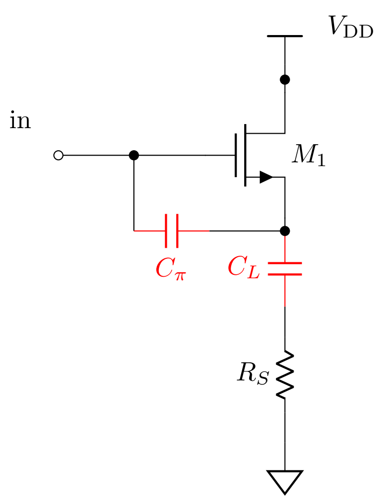
$$Z_{out}^\mu = r_m \tag{155}$$

$$Z_{out}^\pi = R_1 \tag{156}$$

$$Z_{out}^{\pi\mu} = 0 \tag{157}$$

$$Z_{out}(s) = r_m \frac{1 + (C_\mu + C_\pi)R_1 s}{1 + (C_\mu R_1 + C_\pi r_m)s + C_\pi C_\mu R_1 r_m s^2} \tag{158}$$

Consider the input impedance at *in* looking right.



$$\tau_\pi = C_\pi r_m \tag{159}$$

$$\tau_L = C_L(r_m + R_S) \tag{160}$$

$$\tau_L^\pi = C_\mu R_S \tag{161}$$

We analysis the conductance Y_{in} first:

$$Y_{in}^0 = Y_{in}^\pi = Y_{in}^L = 0 \tag{162}$$

$$Y_{in}^0 = Y_{in}^\pi = Y_{in}^L = 0 \tag{163}$$

$$Y_{in}^{L\mu} = \frac{1}{R_S} \tag{164}$$

we have:

$$Z_{in}(s) = \frac{1}{Y_{in}(s)} = \frac{1 + [C_\pi r_m + C_L(r_m + R_S)]s + C_\mu C_L r_m R_S s^2}{C_\pi C_L r_m s^2} \tag{165}$$

when $R_S \to 0$:

$$Z_{in}(s) = \frac{1 + (C_\pi + C_L) r_m s}{C_\pi C_L r_m s^2} = \frac{1}{(C_L || C_\pi)s} + \frac{g_m}{C_L C_\pi s^2} \tag{166}$$

Feedback

The idea asymptotic transfer function:

$$H_\infty = H|_{k\to\infty} = \lim_{k\to\infty} \frac{kA}{1 + kAf} = \frac{1}{f} \tag{167}$$

In s domain:

$$H(s) = \frac{kA(s)}{1 + kA(s)f} \tag{168}$$

let $s = j\omega$, if $|A(j\omega)f| \gg 1$:

$$H(s) = \frac{1}{f} \tag{169}$$

$|A(j\omega)f| \ll 1$:

$$H(s) = A(s) \tag{170}$$

This indicates that the transfer function is very stable when the amplitude is large, and the transfer function is independent of feedback when the amplitude is small.
Given an open loop transfer function:

$$A(s) = \frac{A_0}{1 + \tau_1 s} \tag{171}$$

in the middle (not too large and not too small):

$$H(s) = \frac{A_0}{1 + A_0 f} \frac{1}{1 + \frac{\tau_1}{1 + A_0 f} s} \tag{172}$$

we have an equivalent τ_1':

$$\tau_1' = \frac{\tau_1}{1 + A_0 f} \tag{173}$$

Transfer function sensitivity:

$$\frac{\Delta H}{H} = \frac{\Delta A}{A} \frac{1}{1 + Af} \tag{174}$$

$$\frac{\Delta H}{H} = \frac{\Delta f}{f} \frac{Af}{1 + Af} \tag{175}$$

That is to say, when the gain A is large enough, the sensitivity of the transfer function is mainly determined by the feedback part.
Given an nonlinear transfer function g, and let:

$$y = g[A(x - fy)] \tag{176}$$

which means:

$$x = fy + \frac{g^{-1}(y)}{A} \tag{177}$$

if A is large enough,

$$\frac{y}{x} = \frac{1}{f} = H_\infty \tag{178}$$

that says, the feedback can make output and input to be linear.
For certain source device, we define the asymptotic transfer function:

$$H = H_\infty \frac{T}{1 + T} + H_0 \frac{1}{1 + T} \tag{179}$$

where T is the return ratio of this source, And

$$H_0 = H|_{k\to 0} \tag{180}$$

Blackman Formula:

$$Z = Z_0 \frac{1 + T_{sc}}{1 + T_{oc}} \tag{181}$$

Serial makes impedance up.
Shunt makes impedance down.
Loop gain:

$$\mathbf{T} = -\frac{u_y}{u_x} \tag{182}$$

for Unilateral:

$$\frac{1}{\mathbf{T}} = \frac{1}{\mathbf{T_v}} + \frac{1}{\mathbf{T_i}} \tag{183}$$

Bilateral:

$$\frac{1}{\mathbf{T}} = \frac{1}{\mathbf{T_v}} + \frac{1}{\mathbf{T_i}} + \frac{\mathbf{T_{i,revert}}}{\mathbf{T_{v,forward}}} \tag{184}$$

Stability and Frequency Compensation

For a feedback system, the close loop gain:

$$A_f = \frac{A}{1+Af} = \frac{A}{1+\mathbf{T}} \tag{185}$$

where $\mathbf{T}$ is the loop gain.
We assume the open loop gain A has no poles and zeros in RHP, then the stability of feedback system is determined by the roots of $1+\mathbf{T}=0$.
If one of the roots is located in RHP, the system is unstable.
Nyquist criteria: if the polar plot of $1+\mathbf{T}$ encircles the origin, then the polar plot of $\mathbf{T}$ encircles the point $(-1,0)$ because the latter is obtained by shifting the former to the left by one unit. Nyquist's theorem articulates this result as for a closed-loop system, $\frac{A}{1+\mathbf{T}}$, to be stable, the polar plot of $\mathbf{T}$ must not encircle the point $(-1,0)$ clockwise as s traverses a contour around the critical region clockwise.
The condition of feedback system is unstable:

$$\mathbf{T} = -1 \tag{186}$$

which implies:

$$\mathbf{T}(j\omega)|_{\omega=\theta} = e^{j\theta} \tag{187}$$

where $\theta=\pi$. Now we consider phase margin and gain margin.
Suppose θ_m is the phase margin about π when the amplitude is 1:

$$\mathbf{T} = e^{\theta_m - \pi} = -e^{\theta_m} \tag{188}$$

then we have:

$$A_f(j\omega) = \frac{1}{f}\frac{\mathbf{T}}{1+\mathbf{T}} = A_\infty \frac{e^{\theta_m}}{e^{\theta_m}-1} \tag{189}$$

the amplitude:

$$|A_f(j\omega)|^2 = \frac{A_\infty^2}{2(1-cos(\theta_m))} \approx \frac{A_\infty^2}{\theta_m^2} \tag{190}$$

$$|A_f(j\omega)| \approx \frac{A_\infty}{\theta_m} \tag{191}$$