UNIT- III &IV OPTIMIZATION TECHNIQUES

1. Minimize $f(x_1,x_2) = x_1^4 - 2\,x_1^2x_2 + x_1^2 + x_2^2 - 2\,x_1 + 3x_2 + 10$ with stating point $\begin{Bmatrix} 1.5 \\ -1.5 \end{Bmatrix}$ by Newton's method up to two iterations.
2. Minimize $f(x_1,x_2) = x_1^4 - 2\,x_1^2x_2 + x_1^2 + x_2^2 - 2\,x_1 + x_2 + 1$ with stating point $\begin{Bmatrix} 1.5 \\ -1.0 \end{Bmatrix}$ by Newton's method up to two iterations.
3. Minimize $f(x_1,x_2) = 3x_1^4 - 4\,x_1^2x_2 + 2x_1^2 + 2x_2^2 - 2\,x_1 + x_2 + 1$ with stating point $\begin{Bmatrix} 1.5 \\ -1.0 \end{Bmatrix}$ by Newton's method up to two iterations.
4. Minimize $f(x_1,x_2) = (\,10\,x_1 + 6\,x_2 - 9)^2 + (6\,x_1 + 10\,x_2 - 11)^2$ with starting point $\begin{Bmatrix} -1.0 \\ 1.0 \end{Bmatrix}$ by Newton's method up to two iterations.
5. Solve by Newton's method to *Minimize* $f(x_1,x_2) = (x_1 + 2x_2 - 7)^2 + (2x_1 + x_2 - 5)^2$ with the starting point $\begin{Bmatrix} -1.0 \\ 1.0 \end{Bmatrix}$.
6. Minimize $f(x_1,x_2) = (\,10\,x_1 + 6\,x_2 - 9)^2 + (6\,x_1 + 10\,x_2 - 11)^2$ with starting point $\begin{Bmatrix} -2.0 \\ 2.0 \end{Bmatrix}$ by Newton's method up to two iterations.
7. Minimize $f(x_1,x_2) = 2x_1 - 3x_2 + 2x_1^2 + 2x_1x_2 + x_2^2$ with starting point $\begin{Bmatrix} 1 \\ -1 \end{Bmatrix}$ by Steepest Descent method up to two iterations.
8. Minimize $f(x_1,x_2) = -3x_1 - 2x_2 + 2x_1^2 + 2x_1x_2 + \left(\frac{3}{2}\right)x_2^2$ with starting point $\begin{Bmatrix} 1 \\ -1 \end{Bmatrix}$ by Steepest Descent method up to two iterations.
9. Minimize $f(x_1,x_2) = 6x_1^{\,2} + 2x_2^{\,2} - 6x_1x_2 - x_1 - x_2$ by using the Steepest Descent method with starting point $\{1, 2\}$.
10. Minimize $f(x_1,x_2) = 4x_1^2 + 3\,x_2^2 - 5\,x_1x_2 - 8\,x_1$ with starting point $\begin{Bmatrix} 1.5 \\ -1.5 \end{Bmatrix}$ by Univariate method up to two iterations given that $\varepsilon = 0.01$.
11. Minimize $f(x_1,x_2) = 4x_1^2 + 3\,x_2^2 - 5\,x_1x_2 - 8\,x_1$ with starting point $\begin{Bmatrix} 2.5 \\ -2.5 \end{Bmatrix}$ by Univariate method up to two iterations given that $\varepsilon = 0.01$.
12. Minimize $f(x_1,x_2) = 2x_1^3 - 8x_1^2\,x_2 + \left(\frac{1}{5}\right)x_2^2 - 5\,x_1 - 7\,Sin^{-1}\left(\frac{x_1}{x_2}\right)$ in the range $-5 \le x_1 \le 5$ and $-10 \le x_2 \le 10$ by using random search method up to 6 iterations given that set of values as $\{(r_1,r_2) = (.50, 0.60), (.25, .26), (.98,\ .97), (.45, .46),\ (.234, .235), (.63, .64)\}$.
13. Minimize $f(x_1,x_2) = 15x_1^2 - 18x_1x_2 + \left(\frac{13}{15}\right)x_2^2 - \left(\frac{5}{3}\right)x_1 - x_2\,tan^{-1}\left(\frac{1}{x_1}\right)$ in the range $-3 \le x_1 \le 4$ and $-5 \le x_2 \le 6$ by using random search method up to 10 iterations given the set of values as $\{(r_1,r_2) = (0.50, 0.60), (.25, .26), (.98,\ .97), (.45, .46), (.234, .235),\ (.63, .64),\ (.543, .544), (.712,\ 0.713), (.434,\ .435), (.782,\ .783)\}$.
14. Minimize $f(x_1,x_2,x_3) = 2x_1^2 + 5\,x_2^2 + 7\,x_3^2 - 2\,x_1x_2 + 3\ x_2x_3 - 7\,x_1 - 8\,x_2$ with starting point $\begin{Bmatrix} 1 \\ 3 \\ 4 \end{Bmatrix}$ by Univariate method up to three iterations given that $\varepsilon = 0.01$.
15. Minimize $f(x_1,x_2) = x_1 - 2x_2 + 5x_1^2 + 10\,x_1x_2 + 3x_2^2$ with starting point $\begin{Bmatrix} -2 \\ 1 \end{Bmatrix}$ by Steepest Descent method up to two iterations.

16. Minimize $f(X) = x_1^{-2}\,x_2^{-1} + \frac{1}{4}\,x_1^2\,x_2^{-1}\,x_3^{-1} + x_1^{-1}x_3^2\,x_4$ subject to
$\frac{3}{4}\,x_1\,x_2 + \frac{3}{8}\,x_2\,x_3\,x_4^{-3} \le 1$ $x_i \ge 0,\ i = 1,2,3$ by geometric programming method.

17. Minimize $f(X) = x_1x_2x_3^{-3} + 17x_1^2x_2^{-3}x_3 + 34\,x_1^{-3}x_3 + 51\,x_1x_2$, $x_i \ge 0,\ i = 1,2,3$ by geometric programming method.

18. Minimize $\boldsymbol{x_1}$ subject to
$-4\,x_1^3 + 6\,x_2^2 \le 1$ $x_1 + x_2 \ge 1$ and $x_1 > 0, x_2 > 0$
by procedure of complementary geometric programming method.

19. Derive the geometric dual of the problem : $f(X) = x_1^{-\frac{3}{4}}x_2 + x_1^{\frac{3}{2}}\,x_2^{-2}x_3^{-\frac{1}{3}} + x_1\ x_2^{-3}x_3^{-1}$
subject to $\frac{7}{5}x_1^3x_2^{-1} + 6\,x_1^{-1}\,x_3^{-1/2} \le 1.$

20. Minimize $f(X) = 2x_1x_2 + 2x_1x_2^{-1}x_3 + 4x_1^{-1}x_2^{2}x_3^{-1/2}$ subject to $\sqrt{3}\ x_2^{-1} + 3x_1^{-1}x_3^{-1/2} \le 1$ and $x_i \ge 0,\ i = 1,2,3$ by geometric programming method.

21. What is posynomial? Explain properly the procedure to solve the unconstrained Geometric minimization problem. Write the geometric dual of the given problem: Minimize $f(X) =$
$x_1x_2^{-2}x_3^{-1} + 5\,x_1^{-1}x_2^{-3}\,x_3 + 2\,x_1\,x_3\,x_2 + 8x_1x_2^{-1/2} - x_1^{3/2}\,x_3$, $x_i \ge 0,\ i = 1,2,3.$

22. Derive the geometric dual of given problem: Min $f(X) = x_1^{-\frac{3}{4}}\,x_2 + x_1^{\frac{3}{2}}\,x_2^{-2}\,x_3^{-\frac{1}{3}} + x_1\ x_2^{-3}x_3^{-1}$
subject to $\frac{7}{5}x_1^3x_2^{-1} + 6\,x_1^{-1}\,x_3^{-1/2} \le 1.$

23. Write constrained Geometric minimization problem with *n* variables and *m* constrained and its Geometric dual. Also find the solution of given Geometric minimization problem
Minimize $f(X) = x_1^{-2} + \frac{1}{4}\,x_2^2x_3$ subject to $\frac{3}{4}x_1^2x_2^{-2} + \frac{3}{8}x_2x_3^{-2} \le 1,\ x_i \ge 0, i = 1,2,3.$

24. Minimize $f(X) = 10\,x_2x_3x_4^4 + 40x_1^2x_3^{-1} + 5x_2x_3^2$ subject to
$5\,x_2^{-5}x_3^{-1} \le 1,$
$10x_1^{-1}x_2^3\,x_4^{-1} \le 1,$ $x_i > 0,\ i = 1\ to\ 4$ by geometric programming method.

25. Minimize x_1 subject to $-4\,x_1^2 + 7\,x_2 \le 1$ $x_1 + x_2 \ge 1$ and $x_1 > 0, x_2 > 0$ by procedure of complementary geometric programming method.

26. Minimize $f(X) = 20\,x_2x_3x_4^4 + 20x_1^2x_3^{-1} + 5x_2x_3^2$ subject to
$5\,x_2^{-5}x_3^{-1} \le 1$ $10x_1^{-1}x_2^3\,x_4^{-1} \le 1$ $x_i > 0,\ i = 1\ to\ 4$
by geometric programming method.

27. Minimize $f(X) = x_1^{-2} + \frac{1}{4}\,x_2^2x_3$ subject to $\frac{3}{4}x_1^2x_2^{-2} + \frac{3}{8}x_2x_3^{-2} \le 1$ $x_i \ge 0, i = 1,2,3$
by geometric programming method.

28. Minimize $\boldsymbol{x_1}$ subject to
$-3\,x_1^2 + 7\,x_2 \le 1,$ $x_1 + x_2 \ge 1$ and $x_1 > 0, x_2 > 0,$
by procedure of complementary geometric programming method.

29. Derive the geometric dual of the problem : $(X) = 10\,x_1x_2 + 2\,x_1\,x_2^{-2}x_3^{-1} +$
$5\,x_1^{-2}\,x_2^2x_3^{-1/2}$ subject to $\frac{7}{5}x_1^3x_2^{-1} + 6\,x_1^{-1}\,x_3^{-1/2} \le 1.$

30. Derive the Geometric dual of the problem: Minimize $f(x_1, x_2) = x_1^{-3} x_2 + x_1^{3/2} x_3^{-1}$ subject to $x_1^{2} x_2^{-1} + \frac{1}{2} x_1^{-2} x_3^{3} \le 1$ and $x_1 > 0,\ x_2 > 0, x_3 > 0.$