Optimization Techniques Paper Code – BMS-09 Lecture – 04(Unit -1)

Topic-Multiple Variables Optimization – Kuhn Tucker Condition



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Unit-01

Classical Optimization Techniques: Single variable optimization, Multi-variable with no constraints. Non-linear programming: One Dimensional Minimization methods. Elimination methods: Fibonacci method, Golden Section method

Unit-02

Unit-02

Linear Programming: Constrained Optimization Techniques:

Simplex method, Solution of System of Linear Simultaneous equations, Revised Simplex method, Transportation problems, Karmarkar's method, Duality Theorems, Dual Simplex method, Decomposition principle.

MULTIVARIABLE OPTIMIZATION WITH INEQUALITY CONSTRAINTS

This section is concerned with the solution of the following problem:



Minimize $f(\mathbf{X})$

Subject to

$$y = 1, 2, ..., m$$

$$y = 0, \quad j = 1, 2, ..., m$$

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$$y = 0, \quad$$

Kuhn-Tucker Conditions

Kuhn–Tucker Conditions for above problem is the conditions to be satisfied at a constrained minimum point, X^* , the problem stated in Eq. (1) can be expressed as

$$\frac{\partial f}{\partial x_i} + \sum_{j \in J_1} \lambda_j \frac{\partial g_j}{\partial x_i} = 0, \qquad i = 1, 2, \dots, n$$
 (2)

$$\lambda_j > 0, \qquad j \in J_1 \tag{3}$$

These are called Kuhn— $Tucker\ conditions$ after the mathematicians who derived them as the necessary conditions to be satisfied at a relative minimum of f(X).

or the above statement can be written as (the set of active constraints is not known)

(i)
$$\frac{\partial f}{\partial x_i} + \sum_{j=1}^m \lambda_j \frac{\partial g_j}{\partial x_i} = 0, \qquad i = 1, 2, \dots, n \quad \text{and variables}$$

$$\partial x_i = \sum_{j=1}^{N} \lambda_j g_j = 0, \quad j = 1, 2, \dots, m$$

$$\lambda_j g_j = 0, \quad j = 1, 2, \dots, m$$

$$\beta_j \leq 0, \quad j = 1, 2, \dots, m$$

$$(j) g_j \leq 0, j = 1, 2, \dots, m$$

$$(iv)$$
 $\lambda_j \geq 0, \qquad j = 1, 2, \ldots, m$

Subject to constaints $g_i(x) > 0$, then K.T. as

$$(11) g_j(x) > 0$$

© Max f(x) subject to constraints $g_j(x) \leq 0$

then k.T. are

- (i) same in above
 - (iv) 3,50 (iv) 4,50

- D Max f(x) subject to constraints are

 9; (x) ≥0

 Then R.T. are,
 - 1) same as above
 - (1) fame as above
 - (II) 9, (X) >, O
 - (1) 4°, 50

Minimize
$$f(x_1, x_1) = x_1^2 + x_2^2 + u_0 x_1 + v_0 x_2 + v_0 x_2$$

 $g_1 \cong x_1 - 50 > 0 , x_1, x_2 > 0$

$$L = \chi_1^2 + \chi_2^2 + 40\chi_1 + 20\chi_2 + \lambda_1(\chi_1 - 50)$$
 - (1)

$$\frac{\partial L}{\partial x_1} = 2x_1 + 40 + d_1 = 0$$
 2(a)

$$\frac{\partial L}{\partial x} = 2x + 20 = 0$$
 2(b)

$$J_1(x_1-50)=0$$

$$g_1 = x_1 - 50 > 0$$
 - g
 $\lambda_1 \le 0$ - g

From g $\lambda_1(x_1 - 50) = 0$
 $g = x_1 - 50 = 0$
 $g = x_1 - 10 = 0$
 $g = x_1 -$

COTE II:
$$x_1 - 50 = 0 = 0$$
 $x_1 = 50$

(2) = $x_1 = -100$

(2) $x_2 = -10$

Extra poid is $(50, -10)$, $x_1 = -140$

from (3) $x_1 = -140$

(5) $x_2 = -10$

Extra poid is $(50, -10)$, $x_1 = -140$

from (3) $x_1 = -140$

(6) $x_2 = -140$

(7) $x_1 = -140$

(8) $x_2 = -140$

(9) $x_1 = -140$

(9) $x_2 = -140$

(9) $x_1 = -140$

(9) $x_2 = -140$

(10) $x_1 = -140$

(11) $x_2 = -140$

(11) $x_1 = -140$

(12) $x_2 = -140$

(13) $x_1 = -140$

(14) $x_1 = -140$

(15) $x_1 = -140$

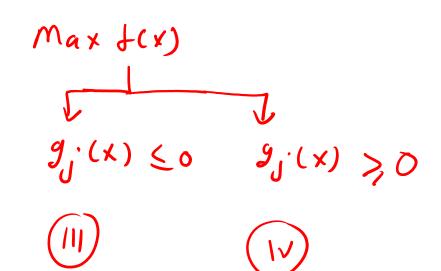
(16) $x_2 = -140$

(17) $x_1 = -140$

(18) $x_1 = -140$

(19) $x_1 = -140$

(19)



Q2 Optimize the problems.

Minimize $f(x_1,x_1,x_2) = x_1^2 + x_2^2 + 4y_1^2 + 4y_2^2 + 4y_1^2 + 4y_2^2 + 4y_1^2 + 4y_2^2 + 4y_1^2 + 4y_1$

$$g_{\lambda} = \chi_1 + \chi_2 - |\omega\rangle 0$$

$$g_3 = x_1 + x_2 + x_3 - 150 > 0$$
.