Transportation Problem

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The Transportation Problem- The Transportation problem is to transport various amounts of a single homogeneous commodity, that are initially stored at various origins, to different destinations in such a way that the total transportation cost is a minimum.

For example - A tyre manufaturing concern has m factories located in different cities. The Total supply potential of manufactured product is

Factories	Market	Transport cost per ton in Rs W	X	Y	Z	Availability
	A	c_{11}	c_{12}	c_{13}	c_{14}	b_1
	B	c_{21}	c_{22}	c_{23}	c_{24}	b_2
	C	c_{31}	c_{32}	c_{33}	c_{34}	b_3
Requirement		d_1	d_2	d_3	d_4	

L.P.P. of this Transportation problem is

Minimize $Z = c_{11}x_{11} + c_{12}x_{12} + \cdots + c_{34}x_{34}$ → objective function

subject to

$$a_{11}x_{11} + a_{12}x_{12} + \cdots + a_{14}x_{14} \le b_1$$

$$\vdots$$

$$a_{31}x_{31} + a_{32}x_{32} + \cdots + a_{34}x_{34} \le b_3$$

$$a_{11}x_{11} + a_{21}x_{21} + a_{31}x_{31} + a_{41}x_{41} \ge d_1$$

$$\vdots$$

$$a_{14}x_{14} + a_{24}x_{24} + a_{34}x_{34} + a_{44}x_{44} \ge d_4$$

$$x_{ij} \ge 0.$$

So, T. Pro. is

minimize $Z = \sum_{j=1}^{n} \sum_{i=1}^{m} c_{ij} x_{ij}$ subject

$\sum^{m} \ldots x_{ij} \le b_i$

$j=1 \quad i=1$

$$\sum_{i=1}^{m} a_{ij}\, x_{ij} \le b_i$$

$$\sum_{j=1}^{n} a_{ij}\, x_{ji} \geq d_j$$

$$x_{ij} \geq 0.$$

here assumed that $\Sigma a_i = \Sigma b_j$ ✓

$\Sigma a_i \neq \Sigma b_j$

Feasible Sol^n:- A set of non-negative individual allocation ($x_{ij} \geq 0$) which simultaneously removes deficiencies is called the feasible sol^n

B.F.S.- A F.S. to a m-origins, n-destinations problem is said to be a basic if the no. of +ve allocations are m+n-1.

if the no. of allocations in a basic sol^n are less than m+n-1, it is called degenate sol^n. (otherwise non-degenerate sol^n).

Optimum Sol^n- A feasible sol^n is said to be optimum if it minimize the total transportation cost.

Note: A necessary and sufficient condition for the existence of feasible sol^n of a T.P. is

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$$

Note 2- The number of basic variables in a T.P. are

Note 2 – The number of basic variables in a T.P. are at most m+n−1.

Existence of an optimal solution

There always exists an optimal solution to a balanced transportation problem

Methods for initial basic feasible solution

By North - West Corner Rule(Steping Stone Method)

Example-1

Warehouse → Factory ↓	W_1	W_2	W_3	W_4	Factory Capacity
F_1	19	30	50	10	7
F_2	70	30	40	60	9
F_3	40	8	70	20	18
Warehouse Requirement	5	8	7	14	34

Ans

	W_1	W_2	W_3	W_4	Availability
F_1	19 [5]	30 [2]	50	10	~~7~~ ~~2~~ 0
F_2	70	30 [6]	[3] 40	60	~~9~~ ~~3~~ 0
F_3	40	8	[4] 70	20 [14]	~~18~~ ~~14~~ 0
Requirements	~~5~~ 0	~~8~~ ~~6~~ 0	~~7~~ ~~4~~ 0	~~14~~ 0	

min{2, 8} = 2

min{6, 9} = 6

min{3, 7} = 3

	W_1	W_2	W_3	W_4	Availability
F_1	19 [5]	30 [2]			7
F_2		30 [6]	[3] 40		9
F_3			[4] 70	[14] 20	18
Requirements	5	8	7	14	

Basic solⁿ is $= 19\times5 + 30\times2 + 30\times6 + 3\times40 + 4\times70 + 14\times20 = 1015.$

Total basic Solⁿ $= m+n-1 = 3+4-1 = 6$

Second Method: The Row Minimum Method

Warehouse → Factory ↓	W_1	W_2	W_3	W_4	Factory Capacity
F_1	19	30	50	10	7
F_2	70	30	40	60	9
F_3	40	8	70	20	18
Warehouse	5	8	7	14	34

Warehouse → Factory ↓	W_1	W_2	W_3	W_4	Factory Capacity
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F_3	40	8	70	20	18
Warehouse Requirement	5	8	7	14	34

	W_1	W_2	W_3	W_4	Availability
F_1	19	30	50	$\boxed{7}$ 10	~~7~~ 0
F_2	70	$\boxed{8}$ 30	$\boxed{1}$ 40	60	~~9~~ ~~1~~ 0
F_3	$\boxed{5}$ 40	8	$\boxed{6}$ 70	$\boxed{7}$ 20	~~18~~ ~~11~~ ~~6~~ 0
Requirements	~~5~~ 0	~~8~~ 0	~~7~~ ~~6~~ 0	~~14~~ ~~7~~ 0	

Check the minimum value in row wise

min {19, 30, 50, 10} = 10

now {7, 14} = 7

	W_1	W_2	W_3	W_4	Availability
F_1				$\boxed{7}$ 10	7
F_2		$\boxed{8}$ 30	$\boxed{1}$ 40		9
F_3	$\boxed{5}$ 40		$\boxed{6}$ 70	$\boxed{7}$ 20	18
Requirements	5	8	7	14	

x_{ij}

$m+n-1 = 3+4-1 = 6$

Ans

$$\min Z = \sum c_{ij} x_{ij} = 7\times 10 + 8\times 30 + 1\times 40 + 5\times 40 + 6\times 70 + 7\times 20$$

$$= 1110 \text{ Rs.}$$

Vogel's Approximation Method(Unit Cost Penalty Method)

Warehouse → Factory ↓	W_1	W_2	W_3	W_4	Factory Capacity
F_1	19	30	50	10	7
F_2	70	30	40	60	9
F_3	40	8	70	20	18
Warehouse Requirement	5	8	7	14	34

Try to find the penalty cost:

1. This can be find by using the difference between the smallest and next smallest costs in each row and column respectively.
2. Now choose the maximum penalty.

	W_1	W_2	W_3	W_4	Availability	(I)	(II)	(III)	(IV)	(V)
F_1	$\boxed{5}$ 19	30	50	$\boxed{2}$ 10	~~7~~ ~~2~~ 0	9	9	40	40	
F_2	70	30 $\boxed{7}$	40	$\boxed{2}$ 60	~~9~~ ~~7~~ 0	10	20	20	20	20
F_3	40 $\boxed{8}$	8	70	$\boxed{10}$ 20	~~18~~ ~~10~~ 0	12	20	50	—	

	W_1	W_2	W_3	W_4	Availability					
F_2	70	30	$\boxed{7}$ 40	$\boxed{2}$ 60	~~9~~ ~~7~~ 0	10	20	20	20	20
F_3	40	$\boxed{8}$ 8	70	$\boxed{10}$ 20	~~18~~ ~~10~~ 0	12	20	50	—	
Requirements	~~5~~ 0	~~8~~ 0	~~7~~ 0	~~14~~ ~~4~~ ~~2~~ 0						

(i)	21	22↑	10	10
(ii)	21↑	—	10	10
(iii)	—	—	10	10
(iv)			10	50↑
(v)			40	60↑

	W_1	W_2	W_3	W_4	Availability
F_1	$\boxed{5}$ 19			$\boxed{2}$ 10	7
F_2			$\boxed{7}$ 40	$\boxed{2}$ 60	9
F_3		$\boxed{8}$ 8		$\boxed{10}$ 20	18
Requirements	5	8	7	14	

$m+n-1 = 6$

Ans $5 \times 19 + 8 \times 8 + 7 \times 40 + 2 \times 10 + 2 \times 60 + 10 \times 20$
$= 779$

The End