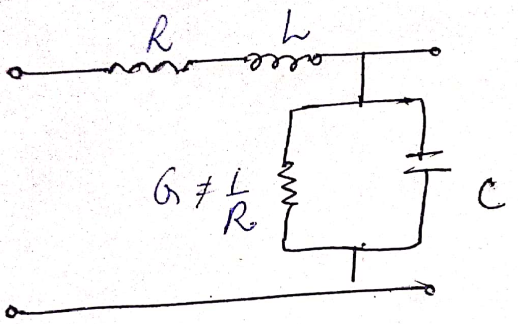
• Transmission Lines - Equations of Voltage and Current on TX Line
- Propagation Constant and Characteristic Impedance
- Reflection Co-efficient and VSWR

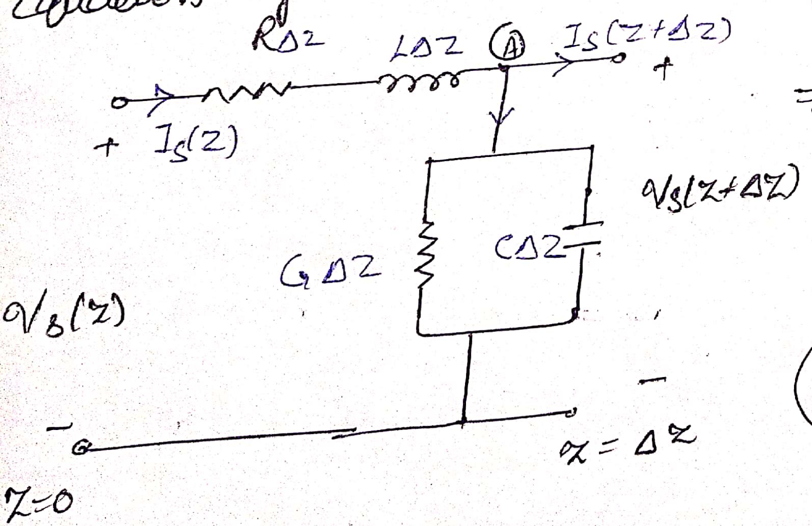
Equivalent diagram:



Primary Constant:
R (Ω/m)
L (H/m)
G (℧/m)
C (F/m)

→

Equation of Transmission line:



Applying KVL:

⇒ $V_s(z) - I_s(z) R\Delta z - j\omega L \Delta z\, I_s(z)$
$- V_s(z+\Delta z) = 0$

⇒ $V_s(z) - V_s(z+\Delta z) = I_s(z)\Delta z\,(R+j\omega L)$

⇒ $\frac{V_s(z) - V_s(z+\Delta z)}{\Delta z} = I_s(z)(R+j\omega L)$

Taking lim as Δz is very small

$\lim_{\Delta z \to 0} \frac{V_s(z) - V_s(z+\Delta z)}{\Delta z} = \lim_{\Delta z \to 0} I_s(z)(R+j\omega L)$

⇒ $\frac{dV_s(z)}{dz} = I_s(z)(R+j\omega L)$ ①

$\Rightarrow \frac{d^2 V_s}{dz^2} = \frac{d I_s(z)}{dz}(R+j\omega L) - ②$

Applying nodal at A

$-I_s(z) + I_s(z+\Delta z) + V_s(z+\Delta z)(G+j\omega C)\Delta z = 0$

$\Rightarrow I_s(z) - I_s(z+\Delta z) = V_s(z+\Delta z)(G+j\omega C)\Delta z$

$\Rightarrow \frac{I_s(z) - I_s(z+\Delta z)}{\Delta z} = V_s(z+\Delta z)(G+j\omega C)$

$\Rightarrow \lim_{\Delta z \to 0} \frac{I_s(z) - I_s(z+\Delta z)}{\Delta z} = \lim_{\Delta z \to 0} V_s(z+\Delta z)(G+j\omega C)$

$\Rightarrow \frac{d I_s(z)}{dz} = V_s(z)(G+j\omega C) - ③$

$\Rightarrow \frac{d^2 I_s(z)}{dz^2} = \frac{d V_s(z)}{dz}(G+j\omega C) - ④$

$\frac{18}{30}$

$\frac{12+1+1}{30}$

$\frac{14}{32} \times 43.75\%$

$\frac{18}{2}$

$\frac{d^2 V_s(z)}{dz^2} = V_s(z)(G+j\omega C)(R+j\omega L)$

$\boxed{\frac{d^2 V_s(z)}{dz^2} - \gamma^2 V_s(z) = 0}$ Voltage Equation (using eq ② and ③)

$\frac{d^2 I_s(z)}{dz^2} = I_s(z)(R+j\omega L)(G+j\omega C)$

$\boxed{\frac{d^2 I_s(z)}{dz^2} - \gamma^2 I_s(z) = 0}$ Current Equation (using ④ and ①)

Solution : $\boxed{\begin{aligned} V_s(z) &= V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z} \\ I_s(z) &= I_0^+ e^{-\gamma z} + I_0^- e^{\gamma z} \end{aligned}}$

$\Downarrow$

$\gamma = \alpha + j\beta = \sqrt{(R+j\omega L)(G+j\omega C)}$

$\gamma \rightarrow$ Propagation Constant

→ The Wavelength $\lambda = \frac{2\pi}{\beta}$

→ Wave Velocity $u = \frac{\omega}{\beta} = f\lambda$

→ **Characteristic Impedance - (Z_0)**

It is defined as ratio of forward travelling voltage wave and forward travelling current wave at any point of transmission line

$$\boxed{Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}} = \frac{V_0^+}{I_0^+} = -\frac{V_0^-}{I_0^-}}$$

$$Z_0 = R_0 + jX_0$$

$$\because -\gamma V_0^+ = -(R+j\omega L) I_0^+$$

$$-\gamma V_0^- = -(R+j\omega L) I_0^-$$

$$\Rightarrow \therefore Z_0 = \frac{V_0^+}{I_0^+} = \frac{-V_0^-}{I_0^-} = \frac{R+j\omega L}{\gamma} = \frac{\gamma}{G+j\omega C}$$

$$Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}}$$

→ **Lossless Transmission Line:**

$$\boxed{R = G = 0}$$

$$\gamma = \alpha + j\beta = \sqrt{(R+j\omega L)(G+j\omega C)}$$

$$\Rightarrow \alpha + j\beta = \sqrt{j\omega L \times j\omega C}$$

$$\Rightarrow \alpha + j\beta = j\omega\sqrt{LC}$$

$$\Rightarrow \boxed{\alpha = 0 \text{ and } \beta = \omega\sqrt{LC}}$$

- A transmission line is said to be lossless if the conductors of the line are perfect ($\sigma \simeq \infty$) and the dielectric medium separating them is lossless ($\sigma \simeq 0$).

$$\Rightarrow \boxed{v_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}}}$$

$$\Rightarrow Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}} = \sqrt{\frac{0+j\omega L}{0+j\omega C}}$$

$$\Rightarrow \boxed{Z_0 = \sqrt{\frac{L}{C}}} \Rightarrow \boxed{v_p = \frac{1/C}{1/C\sqrt{1/LC}} = \frac{1/C}{1/\sqrt{\frac{LC}{C^2}}} = \frac{1}{CZ_0}}$$

→ **Distortionless Transmission Line:**

- A distortionless line is one in which the attenuation constant α is frequency independent while the phase constant β is linearly dependent on frequency.

$$\boxed{\frac{R}{G} = \frac{L}{C}}$$

$$\gamma = \alpha + j\beta = \sqrt{(R+j\omega L)(G+j\omega C)}$$

$$\Rightarrow \alpha + j\beta = \sqrt{RG\left(1+j\frac{\omega L}{R}\right)\left(1+\frac{j\omega C}{G}\right)}$$

$$\Rightarrow = \sqrt{RG}\sqrt{\left(1+j\frac{\omega L}{R}\right)\left(1+j\frac{\omega L}{R}\right)} = \sqrt{RG}\left(1+j\frac{\omega L}{R}\right)$$

$$\Rightarrow \alpha + j\beta = \sqrt{RG} + j\omega\sqrt{RG\frac{L^2}{R^2}}$$

$$\Rightarrow \alpha + j\beta = \sqrt{RG} + j\omega\sqrt{L\left(\frac{LG}{R}\right)}$$

$$\Rightarrow \alpha + j\beta = \sqrt{RG} + j\omega\sqrt{LC}$$

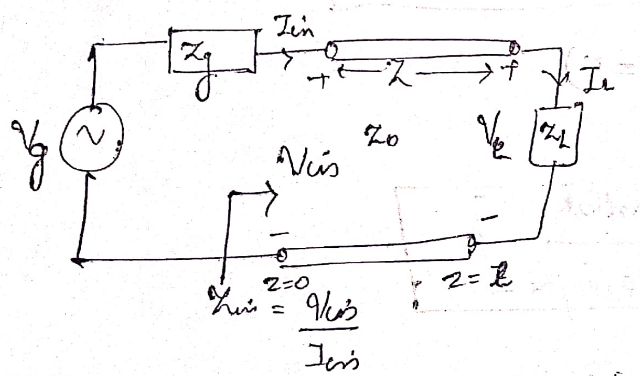
$$\boxed{\alpha \doteq \sqrt{RG} \text{ and } \beta = \omega\sqrt{LC}}$$

$$\rightarrow \boxed{v_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}}}$$

$$\rightarrow Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}} = \sqrt{\frac{R\left(1+j\frac{\omega L}{R}\right)}{G\left(1+\frac{j\omega C}{G}\right)}} = \sqrt{\frac{R\left(1+j\omega\frac{L}{R}\right)}{G\left(1+j\omega\frac{L}{R}\right)}}$$

$$\boxed{Z_0 = \sqrt{\frac{R}{G}} = \sqrt{\frac{L}{C}}}$$

→ **Input Impedance of Transmission line.**



$$V_s(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z} \quad \text{—(1)}$$

$$I_s(z) = I_0^+ e^{-\gamma z} + I_0^- e^{\gamma z}$$

$$I_s(z) = \frac{V_0^+}{Z_0} e^{-\gamma z} - \frac{V_0^-}{Z_0} e^{\gamma z} \quad \text{—(2)}$$

At $z=0$, $V_s(0) = V_{in} = V_0^+ + V_0^- \quad$ —(3)

$$I_s(0) = I_{in} = \frac{V_0^+}{Z_0} - \frac{V_0^-}{Z_0}$$

$$I_{in} Z_0 = V_0^+ - V_0^- \quad \text{—(4)}$$

At $z=l$, $V_s(l) = V_L = V_0^+ e^{-\gamma l} + V_0^- e^{\gamma l} \quad$ —(5)

$$I_s(l) = I_L = \frac{V_0^+}{Z_0} e^{-\gamma l} - \frac{V_0^-}{Z_0} e^{\gamma l} \quad \text{—(6)}$$

(Add Eq (5) and (6)) $\Rightarrow V_L + I_L Z_0 = 2V_0^+ e^{-\gamma l}$

$$V_0^+ = \left(\frac{V_L + I_L Z_0}{2}\right) e^{\gamma l} = \left(\frac{I_L Z_L + I_L Z_0}{2}\right) e^{\gamma l}$$

(sub 5 and 6) $\Rightarrow V_L - I_L Z_0 = 2V_0^- e^{\gamma l}$

$$V_0^- = \frac{V_L - I_L Z_0}{2} e^{-\gamma l}$$

$$\boxed{V_0^- = \frac{I_L(Z_L - Z_0)}{2} e^{-\gamma l}} \text{ and } \boxed{V_0^+ = \frac{I_L(Z_L + Z_0)}{2} e^{\gamma l}}$$

$$Z_{in} = \frac{V_{in}}{I_{in}} = \frac{V_0^+ + V_0^-}{\frac{V_0^+}{Z_0} - \frac{V_0^-}{Z_0}} = Z_0\left[\frac{V_0^+ + V_0^-}{V_0^+ - V_0^-}\right]$$

$$\Rightarrow Z_{in} = Z_0\left[\frac{\frac{I_{in}}{2}(Z_L+Z_0)e^{\gamma l} + \frac{I_{in}}{2}(Z_L - Z_0)e^{-\gamma l}}{\frac{I_{in}}{2}(Z_L+Z_0)e^{\gamma l} - \frac{I_{in}}{2}(Z_L-Z_0)e^{-\gamma l}}\right]$$

$$= Z_0\left[\frac{Z_L\left(\frac{e^{\gamma l}+e^{-\gamma l}}{2}\right) + Z_0\left(\frac{e^{\gamma l}-e^{-\gamma l}}{2}\right)}{Z_L\left(\frac{e^{\gamma l}-e^{-\gamma l}}{2}\right) + Z_0\left(\frac{e^{\gamma l}+e^{-\gamma l}}{2}\right)}\right]$$

$$\Rightarrow \cancel{Z_{in} = \frac{V_{in}}{I_{in}}}$$

$$\Rightarrow Z_{in} = Z_0\left[\frac{Z_L\cosh(\gamma l) + Z_0\sinh(\gamma l)}{Z_L\sinh(\gamma l) + Z_0\cosh(\gamma l)}\right]$$

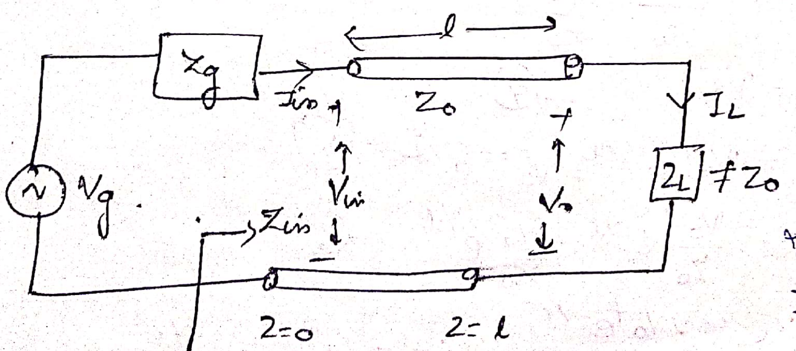
$$\boxed{Z_{in} = Z_0\left[\frac{Z_L + Z_0\tanh\gamma l}{Z_0 + Z_L\tanh\gamma l}\right]}$$

For Lossless; $\gamma = \alpha + j\beta$ $(\alpha = 0)$

$$\gamma = j\beta$$

$$\boxed{Z_{in} = Z_0\left[\frac{Z_L + jZ_0\cdot\tan\beta l}{Z_0 + jZ_L\tanh\beta l}\right]}$$

⇒ Reflection Coefficient at load:



$$V_s(z) = V_0^+ e^{-\gamma l} + V_0^- e^{\gamma l}$$

$$I_s(z) = \frac{V_0^+}{Z_0}e^{-\gamma z} - \frac{V_0^-}{Z_0}e^{\gamma z}$$

$$V_s(z=l) = V_0^+ e^{-\gamma l} + V_0^- e^{\gamma l} = V_L \quad \text{(A)}$$

$$I_s(z=l) = I_L Z_0 = V_0^+ e^{-\gamma l} - V_0^- e^{\gamma l} \quad \text{(B)}$$

Reflection Co-efficient = Reflected Wave / ~~Transmitted Wave~~ Incident Wave

$$= V_0^- e^{\gamma l}$$

$$\boxed{\Gamma_V = \frac{V_0^-}{V_0^+} e^{2\gamma l}}$$ ⇒ Voltage Reflection Co-efficient

Adding eqⁿ (A) and eqⁿ (B)

$$V_L + I_L Z_0 = 2V_0^+ e^{-\gamma l}$$

$$V_0^+ = \left(\frac{V_L + I_L Z_0}{2}\right) e^{\gamma l}$$

$$V_0^+ = \left(\frac{I_L Z_L + I_L Z_0}{2}\right) e^{\gamma l}$$

Similarly $V_0^- \left(\frac{I_L Z_L - I_L Z_0}{2}\right) e^{-\gamma l}$

The Voltage Reflection Co-efficient at any point on the line is the ratio of the reflected voltage wave to that of the incident wave.

$$\Gamma_V = \frac{(I_L Z_L - I_L Z_0) e^{-\gamma l/2}}{(I_L Z_L + I_L Z_0) e^{\gamma l/2}} e^{2\gamma l}$$

$$\boxed{\Gamma_V = \frac{Z_L - Z_0}{Z_L + Z_0}}$$ At load

$$\Gamma_I = \frac{-\frac{V_0^-}{Z_0} e^{\gamma l}}{\frac{V_0^+}{Z_0} e^{-\gamma l}} \Rightarrow \text{Current Reflection Co-efficient}$$

$$\Gamma_I = -\frac{V_0^-}{V_0^+} e^{2\gamma l} = -\Gamma_V$$

$$\Rightarrow \boxed{\Gamma_I = -\left(\frac{Z_L - Z_0}{Z_L + Z_0}\right)}$$

$$\boxed{\Gamma_I = -\Gamma_V}$$

→ The current reflection coefficient at any point on the line is the negative of the voltage reflection co-efficient at the point.

21 | Saturday

23 | Monday

Standing Wave Ratio (SWR)

$$S = \frac{V_{max}}{V_{min}} = \frac{I_{max}}{I_{min}} = \frac{1+|\Gamma_L|}{1-|\Gamma_L|}$$

$$\boxed{S = \frac{1+|\Gamma_L|}{1-|\Gamma_L|}}$$

SWR is defined as the ratio of the maximum magnitude of the standing wave to minimum magnitude of the standing wave.

22 | Sunday

Voltage Standing Wave Ratio – It is a measure of the impedance matching between the transmission line and antenna.

$$VSWR = S_V = \frac{|V_{max}|}{|V_{min}|} = \frac{1+|\Gamma_V|}{1-|\Gamma_V|}$$

$$\boxed{S_V = \frac{1+|\Gamma_V|}{1-|\Gamma_V|}}$$

March'18						
S	M	T	W	T	F	S
				01	02	03
04	05	06	07	08	09	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	31

April'18						
S	M	T	W	T	F	S
01	02	03	04	05	06	07
08	09	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30					

May'18						
S	M	T	W	T	F	S
		01	02	03	04	05
06	07	08	09	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30	31		

Current Standing Wave Ratio :

$$ISWR = S_I = \frac{|I_{max}|}{|I_{min}|} = \frac{1+|\Gamma_I|}{1-|\Gamma_I|}$$

$$\boxed{S_I = \frac{1+|\Gamma_I|}{1-|\Gamma_I|}}$$

Range of $\Gamma \Rightarrow -1 \leq \Gamma \leq 1$

Range of $S \Rightarrow 1 < S \leq \infty$

Also $\boxed{\Gamma = \frac{S-1}{S+1}}$

Q. Airline has $Z_0 = 70\Omega$ and phase constant $(\beta) = 3\frac{rad}{m}$, and $f = 100MHz$.

→ means air filled b/w conductors.

Lossless

$$Z_0 = 70 = \sqrt{\frac{L}{C}}$$

$$\beta = 3 = \omega\sqrt{LC}$$

$$210 = \omega L$$

$$L = \frac{210}{2\pi f} = \quad -(1)$$

$$\frac{70}{3} = \sqrt{\frac{L}{C}} \times \frac{1}{\omega\sqrt{LC}} = \frac{1}{\omega C}$$

$$C = \frac{3}{70 \times 10^8} \quad -(2)$$

Q. A distortionless line has $Z_0 = 60\Omega$, $\alpha = 20$ mNp/m and $v = 0.6c$. Find R, C. ($f = 100MHz$)

$$\sqrt{\frac{R}{G}} = Z_0 = \sqrt{\frac{L}{C}} = 60 \quad -(1)$$

$$v = \frac{1}{\sqrt{LC}} = 0.6 \times 3 \times 10^8 \quad -(3)$$

$$\alpha = \sqrt{RG} = 20 \times 10^{-3} \quad -(2)$$

March'18						
S	M	T	W	T	F	S
				01	02	03
04	05	06	07	08	09	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	31

April'18						
S	M	T	W	T	F	S
01	02	03	04	05	06	07
08	09	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30					

May'18						
S	M	T	W	T	F	S
		01	02	03	04	05
06	07	08	09	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30	31		

April 2018

अप्रैल

28 | Saturday शनि

$$\cancel{\frac{v}{Z_0} = \frac{1}{\sqrt{LC}}}$$

$$vZ_0 = \sqrt{\frac{L}{C}} \times \frac{1}{\sqrt{LC}} = \frac{1}{C}$$

$$C = \frac{1}{vZ_0} = \frac{1}{60 \times 0.6 \times 3 \times 10^8}$$

$$\cancel{\sqrt{RG} \times \sqrt{\frac{R}{G}} = 60 \times 20 \times 10^{-3}}$$

$$R = 1200 \times 10^{-3} = 1.2\,\Omega/m$$

29 | Sunday रवि

$$\lambda = \frac{v}{f} = \frac{0.6 \times 3 \times 10^8}{100 \times 10^6}$$

$$\boxed{\lambda = 1.8m}$$

30 | Monday सोम

June'18

S	M	T	W	T	F	S
					01	02
03	04	05	06	07	08	09

July'18

S	M	T	W	T	F	S
01	02	03	04	05	06	07
08	09	10	11	12	13	14

August'18

S	M	T	W	T	F
			01	02	03
05	06	07	08	09	10