Unit - I ACM

Algebraic and Transcendental Equation:

Algebraic eq^n:

$y = f(x)$

$y = 2x^2 + 3x + 1$

$y = a_0 x^n + a_1 x^{n-1} + \dots a_n, \quad a_0 \neq 0$

Here, $a_0 = 2, \quad a_1 = 3, \quad a_2 = 1.$

So $y = 2x^2 + 3x + 1 \rightarrow$ Algebraic eq^n

An algebraic eq^n having finite no. of terms than eq^n is called algebraic eqn.

Transiendental eq^n

Eq^n having special functions. such as:

$f(x) = x - \sin x$

$f(x) = e^x$

$f(x) = \sin x - \cos x$

$\sin x, \cos x, \tan x$
$\sin^{-1} x, \cos^{-1} x, \sec^{-1} x$
$e^x, \log x$

In Transindental eq^n in term of polynomia are infinite no. of terms.

$y = e^x = \left(1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots \infty\right).$

In Algebraic eq^n no. of roots or sol^n is excatly equal to the degree of the polynomial.

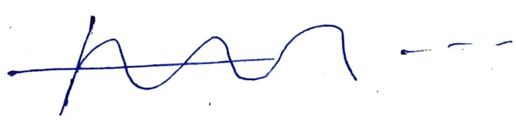
But in Transcendental eq^n this is not true. i.e.

(i) $y = e^x \neq 0 \quad \forall x$

So, no any sol^n.

(ii) $y = 1 - e^x$, at $x = 0$, $y = 0 \rightarrow$ one sol^n

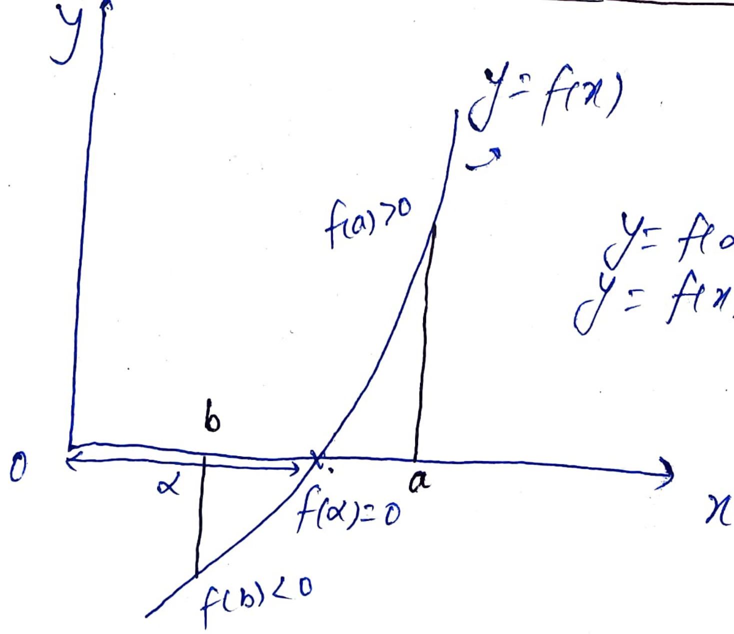
(iii) $y = \sin x$



infinite sol^n

* In case of Transcendental eq^n may be there is a no sol^n may be finite no. of sol^n may be infinite no. of sol^n.

Graphically sol^n of an equation:



$y = f(\alpha) = 0$
$y = f(x) = 0$

$\Rightarrow$

Ex: x_1, x_2, x_3 are the roots of eqn. then eqn will be:. And Let suppose value of roots are given

$$(x-x_1)(x-x_2)(x-x_3)=0$$

$$a'x^3+b'x^2+c'x+d'=0$$

$ax^2+bx+c=0$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$ax^3+bx^2+cx+d=0$

for the roots of cubic eqn. 1st find at what value of x the given cubic eqn become zero.

Let $x=\alpha \rightarrow$ eqn will be zero.

$$(x-\alpha)\underbrace{(a'x^2+b'x+c')}=0$$

$\rightarrow$ Solve it & find other two roots

Condition

① $f(x)$ is continuous on $[a,b]$

② $f(a)\cdot f(b)<0$

i.e. $f(x)$ intersect x-axis at least one point

their exists:-

$\Rightarrow$ one real roots

or $\Rightarrow$ odd no. of real root.

Methods of find solution of Transcendental Equation:

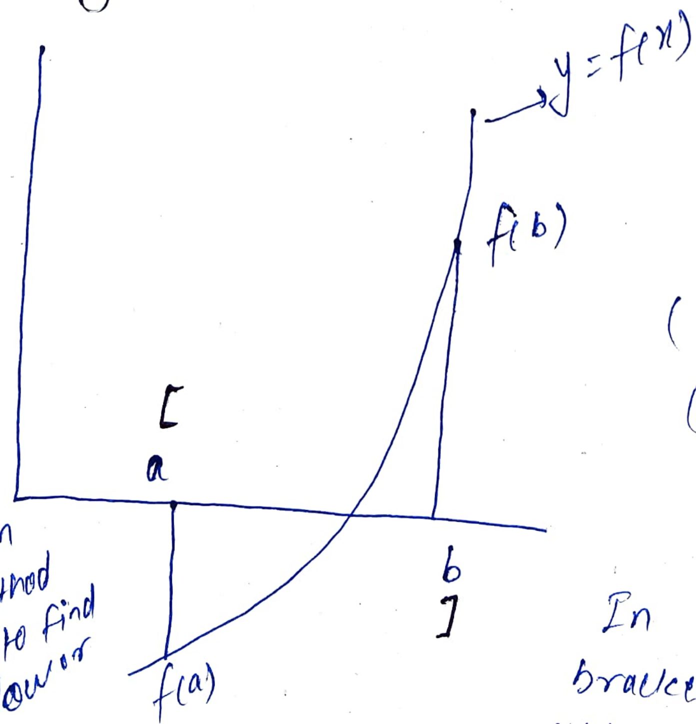
① **Bracketing method**

- (i) Bisection method
- (ii) Regula falsi method
  or
  false position method

② **Iterative method:** (Non-Bracketing method)

- (i) Newton Raphson method
- (ii) fixed point method / Iterative method
- (iii) Secant method.

① **Bracketing method:**



$(-\infty, \infty)$

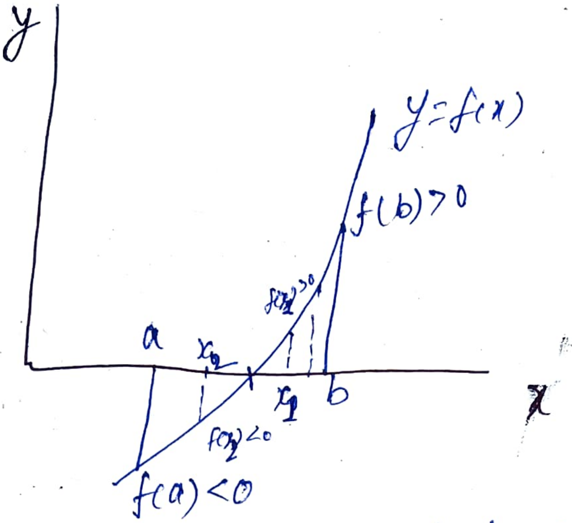
(a, b)

In case of non bracketing method we no need to find value ~~of~~ at lower bracket.

In case of non-bracketing method: we not need to find these bracket. we start from any orbitory point

(a) ① Bisection method:-

$f(a) \times f(b) < 0$



Let $f(x)$ be a continuous b/w a & b & Let $f(a)$ be $-ve$ and $f(b)$ be $+ve$

Then the first approximation of true root is

$x_1 = \frac{a+b}{2}$, if $f(x_1) = 0$, then x_1 is root of $f(x)=0$

Otherwise the root lies b/w a & x_1 or x_1 & b accordingly

$x_0 = \frac{a+b}{2}$

$f(x_0) = 0$

$f(x_0) > 0$

$f(x_0) < 0$

$x_1 = \frac{x_0+b}{2}$

$x_2 = \frac{x_0+x_1}{2}$

$\vdots$

x_4

$\vdots$

x

as $f(x_1)$ is positive or negative. then we bisect the interval as before & continue the process until the root is found accuracy.

Q. Find the real root of the eqⁿ :-

$$f(x) = x^3 - x - 1 = 0$$

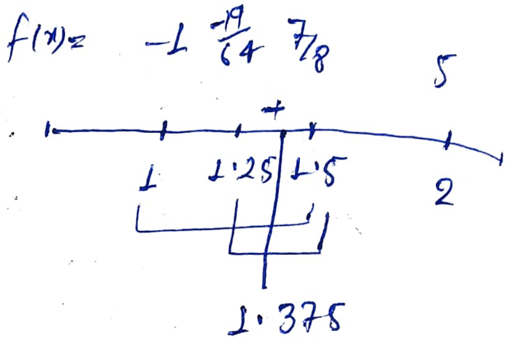
Solⁿ :-

$f(-1) = -1 + 1 - 1 = -1$

$f(0) = 0 - 0 - 1 = -1$

$-ve \rightarrow f(1) = 1 - 1 - 1 = -1$

$+ve \rightarrow f(2) = 8 - 2 - 1 = 5$



$$x_1 = \frac{1+2}{2} = 1.5$$

$$f(1.5) = (1.5)^2 - (1.5) - 1 = 7/8$$

$$x_2 = \frac{1+1.5}{2} = 1.25$$

$$f(1.25) = (1.25)^2 - (1.25) - 1.25 = -\frac{19}{64}$$

$$x_3 = \frac{1.25 + 1.5}{2} = 1.375$$

$$f(1.375) = \underline{+ve}$$

$$x_4 = \frac{1.25 + 1.375}{2} = 1.3125$$

Q. find the real root of the eqⁿ $x^3-2x-5=0$

Sol → $f(x) = x^3 - 2x - 5 = 0$

$f(0) = -5$

$f(1) = 1-2-5 = -6$

-ve → $f(2) = 8-4-5 = -1$

+ve → $f(3) = 27-6-5 = 16$

-ve	+ve	+ve
2	2.5	3

$x = \frac{2+3}{2} = 2.5$

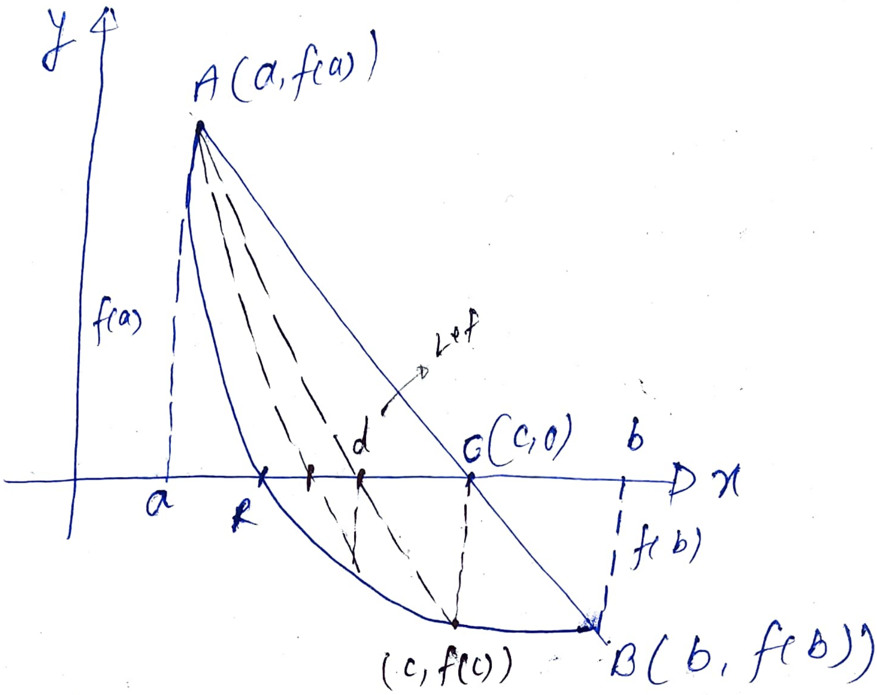
$f(2.5) = (2.5)^3 - 2(2.5) - 5$

$f(2.5) = 5.625 > 0$

x	⊖ a	⊕ b	$x = \frac{a+b}{2}$	f(x)
1	2	3	2.5	5.625 > 0
2	2	2.5	2.25	1.89067 > 0
3	2	2.25	2.125	0.3457 > 0
4	2	2.125	2.0625	-0.3513 < 0
5	2.0625	2.125	2.09375	-0.0089 < 0
6	2.09375	2.125	2.10938	0.1668 > 0
7	2.09375	2.10938	2.10156	0.07856 > 0
8	2.09375	2.10156	2.09766	0.03471 > 0
9	2.09375	2.09766	2.09570	0.01286 > 0
10	2.09375	2.09570	2.09473	0.00195 > 0
11	2.09375	2.09473	2.09424	0.0034 < 0
12	2.09424	2.09473		

2.094

False position method (Regula-falsi method)



Slop of AB = Slop of AC

$$\frac{f(b)-f(a)}{b-a} = \frac{0-f(a)}{c-a}$$

$$\text{Slop} = \frac{y_2-y_1}{x_2-x_1}$$

$$c-a = \frac{-f(a)(b-a)}{f(b)-f(a)}$$

$$c = a - \frac{f(a)(b-a)}{f(b)-f(a)}$$

$$\boxed{c = \frac{a f(b) - b f(a)}{f(b)-f(a)}}$$

~~Supp~~ at c Let f(c) = −ve value

working process

① find the interval [a, b] s.t. $f(a)\cdot f(b) < 0$

② find $c = \frac{a f(b) - b f(a)}{f(b)-f(a)}$

③ $f(a)\cdot f(c) < 0$ roots lies in [a, c].
$f(b)\cdot f(c) < 0$. roots lie in [c, b].

④ Repeat step ① & ②.

Q. Find a real root of $x^3-2x-5=0$ using the method false position four iterations

Sol^n $f(x) = x^3-2x-5=0$

$f(2) = 8-4-5 = -1$ (-ve)

$f(3) = 27-6-5 = 16$ (+ve)

1st Iteration

$a = 2, \quad b = 3$

$f(a) = -1, \quad f(b) = 16$

$$c = \frac{32+3}{16+1} = \frac{35}{17} = 2.0588$$

$f(c) = f(2.0588) = -0.3908 < 0$

2nd Iteration

Now, $a = 2.0588 \quad b = 3$

$f(a) = -0.3908 \quad f(b) = 16$

$$c = \frac{16(2.0588) + 3(0.3908)}{16 + 0.3908}$$

$c = 2.0812$

$f(c) = f(2.0812) = -0.1479 < 0$

3rd iteration

$a = 2.0812$ $\quad b = 3$

$f(a) = -0.1479$ $\quad f(b) = 16$

$$c = \frac{16(2.0812) + 3(0.1479)}{16 + 0.1479}$$

$c = 2.0896$

$f(c) = f(2.0896) = -0.0551$

4th iteration

$a = 2.0896$ $\quad b = 3$

$f(a) = -0.0551$ $\quad f(b) = 16$

$$c = \frac{16(2.0896) + 3(0.0551)}{16 + 0.0551} = \underline{2.0927}$$

~~$f(c) = f(2.0927) =$~~

Hence, the required root is $= \underline{\underline{2.0927}}$ Ans

Q. Using false-position method find a real root of the equation $x \log_{10} x - 1.2 = 0$ in 3 steps

Sol^n $f(1) = 0 - 1.2 = -1.2 < 0$

✓ $f(2) = -0.6 < 0$ (-ve)

✓ $f(3) = 0.23 > 0.$ (+ve)

$f(2.5) = -ve$
$f(2.6) = -ve$
✓ $f(2.7) = -ve$
✓ $f(2.8) = +ve$

Ist

$a = 2.7$ $b = 2.8$

$f(a) = -0.0353$ $f(b) = 0.052$

$$c = \frac{(2.7)(0.052) + (2.8)(0.0353)}{0.052 + 0.0353}$$

$$c = 2.7404$$

$f(c) = f(2.7404) = -0.00021 < 0$

IInd

$a = 2.7404$ $b = 2.8$

$f(a) = -0.00021$ $f(b) = 0.052$

$$c = \frac{(0.052)(2.7404) + (2.8)(0.00021)}{0.052 + 0.00021} = 2.7406$$

$f(c) = f(2.7406) = -0.00004 < 0$

$a = 2.7406$ $\qquad$ $b = 2.8$

$f(a) = -0.00004$ $\qquad$ $f(b) = 0.052$

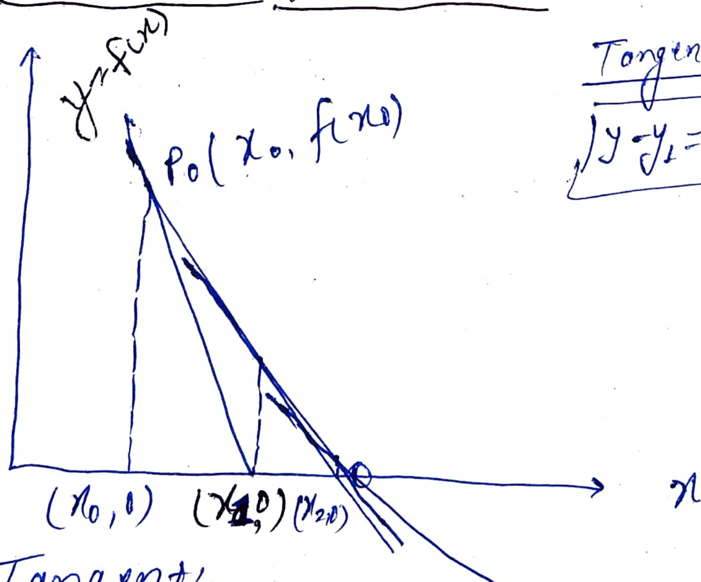
III rd

$$c = \frac{(0.052)(2.7406) + (2.8)(0.00004)}{0.052 + 0.00004}$$

$\boxed{c = 2.7406}$ Ans

Solution of Algebraic, Trancedental Eqn

Newton Raphson methode.



Tangent

$\boxed{y - y_1 = \frac{dy}{dx}(x - x_1)}$

Eqn of Tangent:

$y - f(x_0) = f'(x_0)(x - x_0)$

$0 - f(x_0) = f'(x_0)(x_1 - x_0)$

$x_1 - x_0 = \frac{-f(x_0)}{f'(x_0)}$

$$\boxed{x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}}$$

$$\boxed{x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}}$$

$f'(x_n) \neq 0$

Q. Find by Newton Raphson method a root of eqn $x^3 - 3x - 5 = 0$

Sol:
$f(x) = x^3 - 3x - 5$

$f(1) = 1 - 3 - 5 = -7$

$f(2) = 8 - 6 - 5 = -3$ → this closer to zero.

so, $x_0 = 2$

$f(3) = 27 - 9 - 5 = 16$

$\boxed{x_0 = 2}$

if $f(2) = -3$, $f(3) = +3$ } suppose then we take any value for x_0. most preferable -ve

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$f(x) = x^3 - 3x - 5$

$f'(x) = 3x^2 - 3$

$$x_{n+1} = x_n - \frac{x_n^3 - 3x_n - 5}{3x_n^2 - 3}$$

put $n = 0$.

$$x_1 = x_0 - \frac{x_0^3 - 3x_0 - 5}{3x_0^2 - 3} = 2 - \frac{(-3)}{9}$$

$x_1 = 2 + \frac{1}{3} = 2.3333$

$\boxed{x_1 = 2.3333}$

put $n = 1$

$$x_{1+1} = x_1 - \frac{x_1^3 - 3x_1 - 5}{3x_1^2 - 5} = (2.333) - \frac{(2.333)^3 - 3(2.333) - 5}{3(2.333)^2 - 3}$$

$$\boxed{x_2 = 2.2805}$$

put $n = 2$

$$x_3 = x_2 - \frac{x_2^3 - 3x_2 - 5}{3x_2^2 - 5} = (2.2805) - \frac{(2.2805)^3 - 3(2.2805) - 5}{3\cdot(2.2805) - 3}$$

$$\boxed{x_3 = 2.2790}$$

put $n = 3$

$$x_4 = x_3 - \frac{x_3^3 - 3x_3 - 5}{3x_3^2 - 5}$$

$$= (2.2790) - \frac{(2.2790)^3 - 3(2.2790) - 5}{3(2.2790)^2 - 5}$$

$\boxed{x_4 = 2.2790} = x_3$. (root repeat)

So, required root $= \underline{2.2790}$ Ans

Q. Use Newton Raphson method to find a real root of $\cos x - xe^x = 0$. Corrected to four decimal place

Sol/ⁿ $f(x) = \cos x - xe^x$

$f(0) = \cancel{1} - 0 = \cancel{1}$

$f(1) = \cos 1 - 1 \cdot e^1 = -2.1779$

$f'(x) = -\sin x - e^x - xe^x$

$= -\sin x - e^x(x+1)$.

0.5

$\boxed{x_0 = 0.5}$

0 इस और 1 के value से Ans आ जायेगा।

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_{n+1} = x_n + \frac{\cos x_n - x_n e^{x_n}}{\sin x_n + e^{x_n}(x_n+1)}.$$

put $n=0$

$$x_1 = 0.5 + \frac{\cos(0.5) - (0.5)e^{0.5}}{\sin(0.5) + e^{0.5}(0.5+1)}$$

$x_1 = 0.51802$

put $n = 1$

$$x_2 = x_1 + \frac{\cos x_1 - x_1 e^{x_1}}{\sin x_1 + e^{x_1}(x_1+1)}$$

$$= 0.51802 + \frac{\cos(0.51802) - (0.51802)\, e^{0.51802}}{\sin(0.51802) + e^{0.51802}(0.51802+1)}$$

$x_2 = 0.518$ Ans

$x_1 \approx x_2$ GCL

Q Apply Newton Raphson method to solve the eq^n:- $2(x-3) = \log_{10} x$

Sol:

$f(x) = 2(x-3) - \log_{10} x$

$f(x) = 2x - 6 - \log_{10} x$

$f(3) = 6 - 6 - \log_{10} 3 = -0.47712$

$f(4) = 8 - 6 - \log_{10} 4 = 1.39794$

$x_0 = 3.5$ $\quad \frac{3+4}{2} = 3.5$

$\log_{10} x = 0.4343 \ln x$ or $0.4343 \log_e x$

$f(x) = 2x - 6 - 0.4343 \log_e x$

$$f'(x) = 2 - \frac{0.4343}{x}$$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_{n+1} = x_n - \frac{2x_n - 6 - 0.4343 \log_e x}{2 - \frac{0.4343}{x_n}}$$

put $\underline{n=0}$

$$x_1 = x_0 - \frac{2x_0^2 - 6x_0 - 0.4343 x_0 \log_e x_0}{2x_0 - 0.4343}$$

$$\boxed{x_1 = 3.25696}$$

put $\underline{n=1}$

$x_2 = 3.256366$

put $\underline{n=2}$

$x_3 = 3.256$

$\underline{x_2 = x_3}$

Ans = $\underline{3.256366}$

Gauss-Seidal Iterative method.

Q. Solve the system of eqⁿ by using Gauss-Seidal iterative method.

$$2x_1 - x_2 + 0x_3 = 7 \qquad |2| > |-1| + 0$$

$$-x_1 + 2x_2 + x_3 = 1 \qquad |2| \geq |-1| + |-1|$$

$$0x_1 - x_2 + 2x_3 = 1 \qquad |2| > |0| + |-1|.$$

$$x_1 = \frac{1}{2}(7 + x_2)$$

$$x_2 = \frac{1}{2}(1 + x_1 + x_3)$$

$$x_3 = \frac{1}{2}(1 + x_2)$$

Initial Approximation:

Let initial iteration of x_1, x_2, x_3

$$x_1^{(0)} = 0, \quad x_2^{(0)} = 0, \quad x_3^{(0)} = 0$$

Ist iteration:

$$x_1^{(1)} = \frac{1}{2}(7 + 0) = 3.5$$

$$x_2^{(1)} = \frac{1}{2}(1 + 3.5 + 0) = 2.25$$ {we put updated value in x}

$$x_3^{(1)} = \frac{1}{2}(1 + 2.25) = 1.625$$

$\underline{II^{nd} \text{ iteration}}$

$$x_1^{(2)} = \frac{1}{2}\left(7 + x_2^{(1)}\right) = \frac{1}{2}(7 + 2.25) = \underline{4.625}$$

$$x_2^{(2)} = \frac{1}{2}\left(1 + x_1^{(2)} + x_3^{(1)}\right) = \frac{1}{2}(1 + 4.625 + 1.625)$$

$$= \underline{3.625}$$

$$x_3^{(2)} = \frac{1}{2}\left(1 + x_2^{(2)}\right) = \frac{1}{2}(1 + 3.625) = \underline{2.3125}$$

$\underline{III^{rd} \text{ iteration}}$

$$x_1^{(3)} = \frac{1}{2}\left(7 + x_2^{(2)}\right) = \frac{1}{2}(7 + 3.625) = \underline{5.3125}$$

$$x_2^{(3)} = \frac{1}{2}\left(1 + x_1^{(3)} + x_3^{(2)}\right) = \frac{1}{2}[1 + 5.3125 + 2.3125]$$

$$= \underline{4.3125}$$

$$x_3^{(3)} = \frac{1}{2}\left(1 + x_2^{3}\right) = \frac{1}{2}(1 + 4.3125) = \underline{2.65625}$$

$\underline{IV^{th} \text{ iteration}}$

$$x_1^{(4)} = \frac{1}{2}\left(7 + x_2^{(3)}\right) = \frac{1}{2}(7 + 4.3125) = \underline{5.65625}$$

$$x_2^{(4)} = \frac{1}{2}\left(1 + x_1^{(4)} + x_3^{(3)}\right) = \frac{1}{2}(1 + 5.65625 + 2.65625)$$

$$= \underline{4.65625}$$

$$x_3^{(4)} = \frac{1}{2}\left(1 + x_2^{(4)}\right) = \frac{1}{2}(1 + 4.65625) = \underline{2.828125}$$

V iteration

$x_1^{(5)} = 5.8281$ $\quad x_2^{(5)} = 4.8281$ $\quad x_3^{(5)} = 2.9106$

VI iteration

$x_1^{(6)} = 5.9140,$ $\quad x_2^{(6)} = 4.9140,$ $\quad x_3^{(6)} = 2.9570$

VII iteration

$x_1^{(7)} = 5.9570,$ $\quad x_2^{(7)} = 4.9570,$ $\quad x_3^{(7)} = 2.9785$

$x_1 = 5.9570 \simeq 6$

$x_2 = 4.9570 \simeq 5$

$x_3 = 2.9785 \simeq 3$

$\boxed{x_1 = 6, \quad x_2 = 5, \quad x_3 = 3}$ Ans

LU Decomposition Method

or

factorization method

or

Cholosky's method.

$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1$

$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = b_2$

$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3$

* $\boxed{AX = B} \Rightarrow LUX = B$ | $LY = B$

Let $UX = Y$ | $y = \frac{B}{L}$

$y = UX$

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = LU$$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \qquad U = \begin{bmatrix} U_{11} & U_{12} & U_{13} \\ 0 & U_{22} & U_{23} \\ 0 & 0 & U_{33} \end{bmatrix}$$

$$\boxed{A = LU}$$

* on computing elements of L & U:-

(i) First row of U i.e U_{11}, U_{12}, U_{13}.

(ii) First coloumn of L i.e l_{21}, l_{31},

(iii) Second row of U i.e U_{22}, U_{23}.

(iv) Second column of L i.e. l_{32}.

(v) Third row of U i.e. U_{33}

Q. Solve the following system of eqn. by LU decomposition method :-

$$x + 5y + z = 14$$
$$2x + y + 3z = 13$$
$$3x + y + 4z = 17$$

$$\boxed{AX = B}$$

$$\begin{bmatrix} 1 & 5 & 1 \\ 2 & 1 & 3 \\ 3 & 1 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 14 \\ 13 \\ 17 \end{bmatrix}$$

$$\boxed{A = LU}$$

$$\begin{bmatrix} 1 & 5 & 1 \\ 2 & 1 & 3 \\ 3 & 1 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \begin{bmatrix} U_{11} & U_{12} & U_{13} \\ 0 & U_{22} & U_{23} \\ 0 & 0 & U_{33} \end{bmatrix}$$

$$\begin{bmatrix} 1 & 5 & 1 \\ 2 & 1 & 3 \\ 3 & 1 & 4 \end{bmatrix} = \begin{bmatrix} U_{11} & U_{12} & U_{13} \\ l_{21}U_{11} & l_{21}U_{12} + U_{22} & l_{21}U_{13} + U_{23} \\ l_{31}U_{11} & l_{31}U_{12} + l_{32}U_{22} & l_{31}U_{13} + l_{32}U_{23} + U_{33} \end{bmatrix}$$

$\boxed{U_{11} = 1}$, $\boxed{U_{12} = 5}$ $\boxed{U_{13} = 1}$

$l_{21}U_{11} = 2$ $\quad$ $l_{21}U_{12} + U_{22} = 1$ $\quad$ $l_{21}U_{13} + U_{23} = 3$

$\boxed{l_{21} = 2}$ $\quad$ $2 \times 5 + U_{22} = 1$ $\quad$ $(2)(1) + U_{23} = 3$

$\boxed{U_{22} = -9}$ $\quad$ $\boxed{U_{23} = 1}$

Similarly.

compare each element & get all values

$\boxed{l_{32} = \frac{14}{9}}$

$\boxed{U_{33} = -5/9}$

$$A = LU$$

$$\begin{bmatrix} 1 & 5 & 1 \\ 2 & 1 & 3 \\ 3 & 1 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 14/9 & 1 \end{bmatrix} \begin{bmatrix} 1 & 8 & 1 \\ 0 & -9 & 1 \\ 0 & 0 & -5/9 \end{bmatrix}$$

$$AX = B$$

$$L\underline{UX} = B$$

Let $UX = Y$

$$LY = B$$

doolittle method

Lower

$$\begin{bmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ b & c & 1 \end{bmatrix}$$

Crout's method

Upper

$$\begin{bmatrix} 1 & a & b \\ 0 & 1 & c \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & \frac{14}{9} & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 14 \\ 13 \\ 17 \end{bmatrix}$$

$y_1 = 14$

$2y_1 + y_2 = 13$

$28 + y_2 = 13$

$y_2 = -15$

$3y_1 + \frac{14}{9} y_2 + y_3 = 17$

$3(14) + \frac{14}{9}(-15) + y_3 = 17$

$y_3 = -5/3$

$$UX = Y$$

$$\begin{bmatrix} 1 & 5 & 1 \\ 0 & -9 & 1 \\ 0 & 0 & -5/9 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 14 \\ -15 \\ -5/3 \end{bmatrix}$$

$x + 5y + 2z = 14$

$\boxed{x = 1}$

$-9y + z = 15$

$-9y + 3 = 15$

$\boxed{y = 2}$

$-5/9 \, z = -5/3$

$\boxed{z = 3}$