Optimization Techniques
Paper Code – BMS-09
Lecture – 04(Unit -1)
Topic-Multiple Variables Optimization – Kuhn Tucker Condition

Dr. Ram Keval

Mathematics and Scientific Computing Deptt.

M. M. M. University of Technology Gorakhpur

Unit-01

Classical Optimization Techniques: Single variable optimization, Multi-variable with no constraints. Non-linear programming: One Dimensional Minimization methods. Elimination methods: Fibonacci method, Golden Section method

Unit-02

Unit-02

Linear Programming: Constrained Optimization Techniques: Simplex method, Solution of System of Linear Simultaneous equations, Revised Simplex method, Transportation problems, Karmarkar's method, Duality Theorems, Dual Simplex method, Decomposition principle.

MULTIVARIABLE OPTIMIZATION WITH INEQUALITY CONSTRAINTS

This section is concerned with the solution of the following problem:

a

$$\text{Minimize } f(\mathbf{X})$$

Subject to

$$g_j(\mathbf{X}) \leq 0, \quad j = 1, 2, \ldots, m \tag{1}$$

$g_j(x) + s_j = 0$

→ slack variable

max $f(x)$

s/b $g_j(x) = 0$

$L = f(x) + \sum \lambda_j g_j$

Kuhn–Tucker Conditions

Kuhn–Tucker Conditions for above problem is the conditions to be satisfied at a constrained minimum point, $\mathbf{X}^*$, the problem stated in Eq. (1) can be expressed as

$$\frac{\partial f}{\partial x_i} + \sum_{j \in J_1} \lambda_j \frac{\partial g_j}{\partial x_i} = 0, \qquad i = 1, 2, \ldots, n \qquad (2)$$

$$\lambda_j > 0, \qquad j \in J_1 \qquad (3)$$

These are called *Kuhn–Tucker conditions* after the mathematicians who derived them as the necessary conditions to be satisfied at a relative minimum of *f (**X**)*.

*or the above statement can be written as (*the set of active constraints is not known *)*

(i) $$\frac{\partial f}{\partial x_i} + \sum_{j=1}^{m} \lambda_j \frac{\partial g_j}{\partial x_i} = 0, \qquad i = 1, 2, \ldots, n$$ → no. of variables

(ii) $$\lambda_j g_j = 0,^{\ddagger} \qquad j = 1, 2, \ldots, m$$ → stands for no. of constraint.

(iii) $$g_j \leq 0, \qquad j = 1, 2, \ldots, m$$

(iv) $$\lambda_j \geq 0, \qquad j = 1, 2, \ldots, m$$

(4)

(b) min $f(x)$

Subject to constraints $g_j(x) \geq 0$, then K.T. as

(i)

(ii)

(iii) $g_j(x) \geq 0$

(iv) $\lambda_j \leq 0$

(c) Max $f(x)$ subject to constraints

$$g_j(x) \le 0$$

then K.T. are

(i) same in above

(ii) " "

(iii) $g_j \le 0$

(iv) $\lambda_j \le 0$

(d) Max $f(x)$ subject to constraints are

$$g_j(x) \geq 0$$

Then K.T. are.

(i) same as above

(ii) same as above

(iii) $g_j(x) \geq 0$

(iv) $\lambda_j \geq 0$

Solve the Proble with the help of K. T.

Minimize $f(x_1, x_2) = x_1^2 + x_2^2 + 40x_1 + 20x_2$ s/to

$g_1 \cong x_1 - 50 \geq 0$, $x_1, x_2 \geq 0$

For maxima / minima, 1st construct Lagrange fn as

$$L = x_1^2 + x_2^2 + 40x_1 + 20x_2 + \lambda_1 (x_1 - 50) \quad -(1)$$

$$\frac{\partial L}{\partial x_1} = 2x_1 + 40 + \lambda_1 = 0 \qquad 2(a)$$

$$\frac{\partial L}{\partial x_2} = 2x_2 + 20 = 0 \qquad 2(b)$$

$$\lambda_1 (x_1 - 50) = 0 \qquad (3)$$

$g_1 \equiv x_1 - 50 \geq 0$ — (4)

$\lambda_1 \leq 0$ — (5)

From (3) $\lambda_1(x_1 - 50) = 0$

$\Rightarrow \lambda_1 = 0$ or $x_1 - 50 = 0$

Case I : $\lambda_1 = 0 \Rightarrow 2x_1 + 40 = 0 \Rightarrow x_1 = -20$

from 2(b), $2x_1 + 20 = 0 \Rightarrow x_1 = -10$

but (4), $-10 - 50 = -60 \geq 0$ which is not true.

$\Rightarrow$ Case I is not valid or we cannot get soln from this case.

case II : $x_1 - 50 = 0 \Rightarrow x_1 = 50$

2a $\Rightarrow d_1 = -140$

2b $x_2 = -10$

Extra point is $(50, -10)$, $d_1^o = -140$

from 4 $g_1 = x_1 - 50 \geq 0 \Rightarrow 50 - 50 \geq 0$, true.

from 5 $d_1 \leq 0 \Rightarrow d_1 = -140 \leq 0$, true

$\Rightarrow$ this is the soln

$f(x_1^*, x_2^*) = (50)^2 + (-10)^2 + 40 \times 50 + 20 \times (-10)$

$=$ Ans

$\text{Min } f(x)$

$g_j(x) \leq 0$ or $g_j(x) \geq 0$

I II

$\text{Max } f(x)$

$g_j(x) \leq 0$ $g_j(x) \geq 0$

III IV

Q 2 Optimize the problems.

minimize $f(x_1, x_2, x_3) = x_1^2 + x_2^2 + x_3^2 + 40x_1 + 20x_2$

subject to constraints

$$g_1 = x_1 - 50 \geq 0$$

$$g_2 = x_1 + x_2 - 100 \geq 0$$

$$g_3 = x_1 + x_2 + x_3 - 150 \geq 0.$$