BEC 28-secB-22

Angle Modulation

Assume carrier signal $c(t) = A_c \cos(w_c t + \phi(t))$

Radians Radians

Expression of angle modulated signal

$$s(t) = A_c \cos(\theta(t))$$

Total angle

$$\theta(t) = w_c t + \phi(t)$$
$$= 2\pi f_c t + \phi(t)$$

⇒ If total angle i.e. $\theta(t)$ varies with message signal variation then corresponds to Angle modulation.

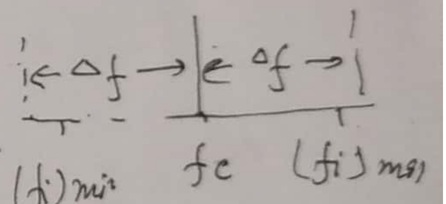
⇒ If Angle modulation occurs because of depends of f_c on m then called as frequency modulation

⇒ If Angle modulation occurs due to dependence of ϕ on m then called as phase modulation.

Instantaneous angular frequency / Instantaneous angular velocity

$$\frac{d\theta(t)}{dt} = w_i = w_c + \frac{d\phi(t)}{dt}$$

Instantaneous frequency $\boxed{f_i = f_c + \frac{1}{2\pi}\frac{d\phi(t)}{dt}}$



$$f_i - f_c = \frac{1}{2\pi}\frac{d\phi(t)}{dt}$$

Δf = frequency deviation

$$\boxed{\Delta f = f_i - f_c = \frac{1}{2\pi}\frac{d\phi(t)}{dt}}$$

Inst. freq. in terms of freq. dev.

$$f_i = f_c + \Delta f$$

fixed

$(f_i)_{max} = f_c + (\Delta f)_{max}$ $\Rightarrow f_c = \frac{f_{i\,max} + f_{i\,min}}{2}$

$(f_i)_{min} = f_c - (\Delta f)_{max}$

Carrier Swing $2\Delta f_{max} = (f_{i\,max}) - f_{i\,min}$.

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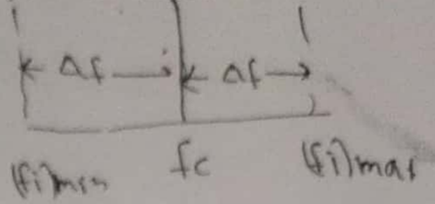
$$\boxed{f_c = \qquad 2}$$

carrier swing
$$\boxed{2\Delta f_{max} = (f_i)_{max} - (f_i)_{min}}$$



phase Modulation → ~~phase of the carrier is varied in accordance with m(t)~~

Assume carrier signal ~~before~~ after phase modulation

$$\left[\begin{aligned} c(t) &= A_c \cos(2\pi f_c t + \phi(t)) \\ &= A_c \cos(\omega_c t + \phi(t)) \end{aligned}\right]$$

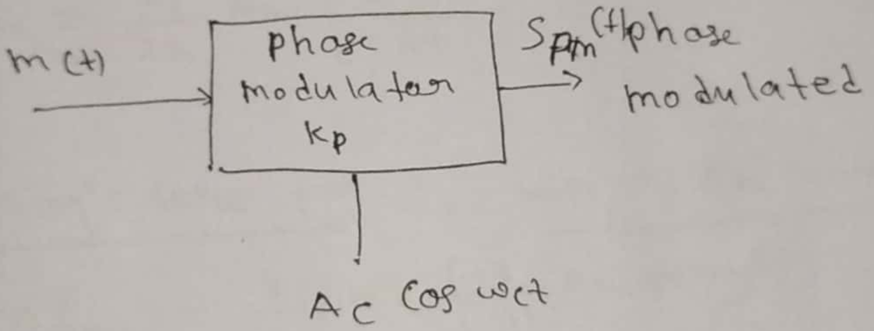
phase of the carrier is varied in accordance with $m(t)$

$\phi(t) \propto m(t)$

rad ← $\phi(t) = K_p\, m(t)$ → Volt

K_p → rad/volt

K_p is phase sensitivity of phase modulator $\left(\frac{rad}{volt}\right)$



→ No modulation → No phase modulation

→ if $m(t) = 0 \Rightarrow \phi(t) = 0$ no modulation

⇒ K_p specifies amount of phase shift in the carrier signal for 1 volt change in message sig.

Expression of PM signal, $\phi(t) \propto m(t)$

→ $S_{PM}(t) = A_c \cos[\omega_c t + K_p m(t)]$

for ~~sig~~ single tone sinusoidal modulating signal

$m(t) = A_m \cos \omega_m t$

$S_{PM}(t) = A_c \cos(\omega_c t + \underbrace{K_p A_m}_{\beta} \cos \omega_m t)$

$\beta \rightarrow$ modulation index for angle modulation

$$\boxed{\beta = K_p A_m}$$

$$f_i = f_c + \frac{1}{2\pi} \frac{d\phi(t)}{dt}$$

for PM signal, $\phi(t) = K_p m(t)$

$$f_i = f_c + \frac{1}{2\pi} \frac{d\phi(t)}{dt}$$

$$f_i = f_c + \frac{K_p}{2\pi} \left[\frac{dm(t)}{dt}\right]$$ — slope of $m(t)$

→ Frequency Deviation for PM

$$\Delta f = \frac{K_p}{2\pi} \left[\frac{dm(t)}{dt}\right]$$

$$\Delta f_{max} = \frac{K_p}{2\pi} \max\left[\frac{dm(t)}{dt}\right]$$

→ for single tone PM, $m(t) = A_m \cos \omega_m t$

$$\Delta f_{max} = \frac{K_p}{2\pi} \max\left[\frac{d}{dt}(A_m \cos \omega_m t)\right]$$

$$= \frac{K_p}{2\pi} A_m \omega_m \left[-(\sin \omega_m t)\right]_{max}$$

$$= \frac{K_p A_m \omega_m}{2\pi}$$

$$\boxed{\Delta f_{max} = \underbrace{K_p A_m}_{\beta} f_m}$$

Modulation Index :-

$$\boxed{\beta = \frac{\Delta f}{f_m}}$$

Carrier swing $= 2 \Delta f_{max}$

$= 2 K_p A_m f_m$ (Hz)

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freq Deviations:-

$\Delta f = \beta f_m$

$$\boxed{\Delta f = \beta f_m}$$

$$\boxed{\beta = \frac{\Delta f}{f_m}}$$

→ phase deviation →

Difference between phase of modulated carrier and phase of un-modulated carrier

→ unmodulated carrier $c(t) = A_c \cos(\omega_c t + 0^\circ)$

→ modulated carrier $S_{pm}(t) = A_c \cos(\omega_c t + \underbrace{K_p m(t)}_{\phi(t)\ \text{modulated}})$

$$\Delta\phi = \phi(t)_{mod} - \phi(t)_{unmod}$$

$$= K_p m(t) - 0$$

$$\boxed{\Delta\phi = K_p m(t)}$$

$$\Delta\phi_{max} = \max[K_p m(t)]$$

→ for single tone sinusoidal signal

$$\Delta\phi_{max} = \max[K_p A_m \cos\omega_m t]$$

$$\cancel{\Delta\phi_{max} = K_p A_m}$$

Phase deviation:-

$$\boxed{\Delta\phi_{max} = K_p A_m = \beta}$$

$$\beta = \frac{\Delta f}{f_m}$$

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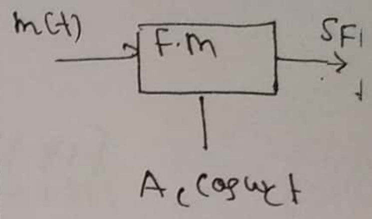
Frequency Modulation

→ Frequency of unmodulated carrier signal is f_c

→ frequency of the carrier signal after freq modulation

f_i → Instantaneous frequency



~~$\Delta f \propto K_f\, m(t)$~~

→ $\Delta f \propto m(t)$

$$\Delta f = K_f\, m(t)$$

⇒ $f_i = f_c + K_f\, m(t)$

K_f → frequency sensitivity

$$K_f = \frac{\Delta f}{m(t)} \quad \left(\frac{Hz}{Volt}\right)$$

→ K_f specifies amount of frequency change in carrier signal per 1V change in the message signal.

Expression for F.M.

$$S_{FM}(t) = A_c \cos(w_c t + \phi(t))$$

$$\Delta f = \frac{1}{2\pi}\frac{d\phi(t)}{dt} = f_i - f_c$$

$$K_f\, m(t) = \frac{1}{2\pi}\frac{d\phi(t)}{dt}$$

$$\int K_f\, m(t)\, \cancel{2\pi}\, dt = \int d\phi(t)$$

$$\phi(t) = 2\pi K_f \int m(t)\, dt$$

$$S_{FM}(t) = A_c \cos\left[w_c t + 2\pi K_f \int m(t)\, dt\right]$$

$(\Delta f)_{max} = \max K_f\, m(t)$

$(f_i)_{max} = f_c + (\Delta f)$

$(f_i)_{min} = f_c - (\Delta f)$

for single tone sinusoids

$(\Delta f)_{max} = K_f A_m$

$(f_i)_{max} = f_c + K$

$(f_i)_{min} = f_c -$

Carrier sw

$2(\Delta f)_{max} = 2$

If unit of K_f is in rad/volt-sec then

$$S_{FM}(t) = A_c \cos\left[w_c t + \cancel{2\pi} K_f \int m(t)\, dt\right]$$

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for single tone sinusoid message signal

$$s_{FM}(t) = A_c \cos\left[\omega_c t + 2\pi k_f \int A_m \cos\omega_m t \, dt\right]$$

$$= A_c \cos\left[\omega_c t + \frac{2\pi k_f A_m}{\omega_m} \sin\omega_m t\right] \quad \omega_m = 2\pi f_m$$

$$= A_c \cos\left[\omega_c t + \frac{k_f A_m}{f_m} \sin\omega_m t\right]$$ — modulation in

$$s_{FM}(t) = A_c \cos\left[\omega_c t + \beta \sin\omega_m t\right]$$

$$\beta = \frac{k_f A_m}{f_m} = \frac{\Delta f_{max}}{f_m}$$

phase deviation

$$\Delta\phi = \phi_{mod} - \phi_{unmod}$$

$$= 2\pi k_f \int m(t)\, dt - 0$$

$$= 2\pi k_f \int m(t)\, dt$$

$$\Delta\phi_{max} = 2\pi k_f \max\left[\int m(t)\, dt\right]$$

for single tone sinusoids

$$\Delta\phi_{max} = 2\pi k_f \max\left[\int A_m \cos\omega_m t \, dt\right]$$

$$= \frac{2\pi k_f A_m \max}{\omega_m}\left[\sin\omega_m t\right]$$

$$= \frac{2\pi k_f A_m}{\omega_m} = \frac{k_f A_m}{f_m}$$

$$\boxed{\Delta\phi_{max} = \frac{k_f A_m}{f_m} = \beta} = \boxed{\frac{\Delta f_{max}}{f_m}}$$

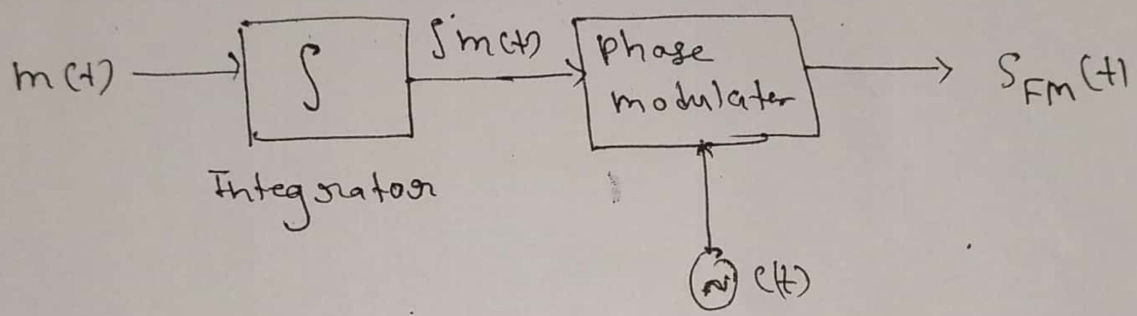
Relationship b/w phase modulation (PM) and Frequency modulation (FM)

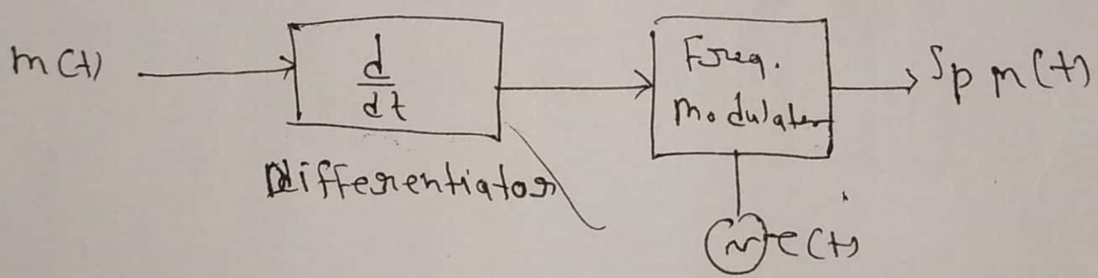
~~$S_{pm}(t) = A_c[\cos w_c t + k_p m(t)]$~~

~~$S_{FM}(t) = A_c[$~~

$$S_{PM}(t) = A_c \cos[w_c t + k_p m(t)]$$

$$S_{FM}(t) = A_c \cos[w_c t + 2\pi k_f \int_0^t m(t)\,dt]$$





$$FM[m(t)] = PM[\int m(t)]$$

$$PM[m(t)] = FM[\tfrac{d}{dt} m(t)]$$

Frequency Modulation

→ Freq of unmodulated signal is → f_c

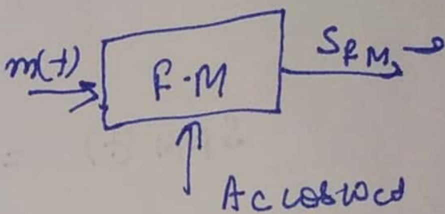
→ Freq of the carrier after freq modulation.

→ Instante → $f_i = f_c + \Delta f$. $= \Delta f = f_i - f_c$.

$\Delta f \propto m(t)$

freq Deviation

$$\Delta f = K_f\, m(t)$$



$$\Rightarrow \quad f_i = f_c + K_f\, m(t)$$

K_f → freq sensitivity

$$K_f = \frac{\Delta f}{m(t)} \left(\frac{Hz}{Volt}\right)$$

$$S_{FM}(t) = A_c \cos(\omega_c t + \phi(t))$$

$$f_i - f_c = \Delta f = \frac{1}{2\pi}\frac{d\phi}{dt}.$$

$$K_f\, m(t) = \frac{1}{2\pi}\frac{d\phi}{dt}.$$

Integrating both side

$$\int K_f\, m(t) = \frac{1}{2\pi}\int \frac{d\phi}{dt}$$

$$\phi(t) = 2\pi K_f \int m(t)\, dt$$

$$S_{FM}(t) = A_c \cos\left[\omega_c (t) + 2\pi K_f \int m(t)\, dt\right]$$

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for single tone $m(t) = A_m \cos \omega_m(t)$

$$S_{FM}(t) = A_c \cos\left[\omega_c(t) + 2\pi K_f \int A_m \cos \omega_m(t)\right]$$

$$= A_c \cos\left[\omega_c(t) + \frac{2\pi K_f A_m}{\omega_m} \sin \omega_m(t)\right]$$

$\omega = 2\pi f_m$

$$S_{FM}(t) = A_c \cos\left[\omega_c(t) + \frac{K_f A_m}{f_m} \sin \omega_m t\right]$$

→ modulation Index

$$S_{FM}(t) = A_c \cos\left[\omega_c(t) + \beta \sin \omega_m(t)\right]$$

where $\boxed{\beta = \frac{K_f A_m}{f_m} = \frac{\Delta f_{max}}{f_m}}$

Phase deviation: $\Delta\phi = \phi_{mod} - \phi_{unmodulated}$

$$= 2\pi K_f \int m(t)\, dt - 0$$

$$= 2\pi K_f \int m(t)\, dt$$

$$\Delta\phi_{max} = 2\pi K_f \max\left[\int m(t)\, dt\right]$$

for single tone sinusoidal $m(t) = A_m \cos \omega_m(t)$

$$\Delta\phi_{max} = 2\pi K_f \max\left[\int A_m \cos \omega_m(t)\, dt\right]$$

$$= 2\pi K_f A_m \max\left[\sin \omega_m t\right]$$

$$= \frac{2\pi K_f A_m}{\omega_m} = \frac{K_f A_m}{f_m}$$

$\Delta\phi_{max} = K_f A_m / \ldots = \beta = \Delta f_{max} / \ldots$

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