Linear Homogeneous equation

$$a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = 0$$
$$a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = 0$$
$$\vdots$$
$$a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n = 0$$

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & & & \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & & & \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

(i) If $\rho(A) = n$ (no of variables)

(unique solⁿ)

(ii) If $\rho(A) = r < n$ (no of variables)

(Infinite solution)

There is no case for no solution of homogeneous linear equation.

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Q Investigate the consistency, and state the nature of the ~~solution~~/solutions.

(1)
$$x+2y+3z=0$$
$$3x+4y+4z=0$$
$$7x+10y+12z=0$$

(2)
$$4x+2y+z+3w=0$$
$$6x+3y+4z+7w=0$$
$$2x+y+w=0$$

(3)
$$4x-2y+6z=8$$
$$x+y-3z=-1$$
$$15x-3y+9z=21$$

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Q. Solve the equation
(i) $x+2y+3z=0,\ 3x+4y+4z=0,\ 7x+10y+12z=0$

Ⓐ this system can be written in matrix form as

$$\underbrace{\begin{bmatrix} 1 & 2 & 3 \\ 3 & 4 & 4 \\ 7 & 10 & 12 \end{bmatrix}}_{A}\underbrace{\begin{bmatrix} x \\ y \\ z \end{bmatrix}}_{X} = \underbrace{\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}}_{B}$$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 4 & 4 \\ 7 & 10 & 12 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 2 & 3 \\ 0 & -2 & -5 \\ 0 & -4 & -9 \end{bmatrix} \qquad \begin{array}{l} R_2 \to R_2 - 3R_1 \\ R_3 \to R_3 - 7R_1 \end{array}$$

$$\sim \begin{bmatrix} 1 & 2 & 3 \\ 0 & -2 & -5 \\ 0 & 0 & 1 \end{bmatrix} \qquad R_3 \sim R_3 - 2R_2$$

$\rho(A) = 3 \Rightarrow$ unique sol^n

$$\Rightarrow \begin{bmatrix} 1 & 2 & 3 \\ 0 & -2 & -5 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$x + 2y + 3z = 0$ — (i)
$-2y - 5z = 0$ — (ii)
$z = 0$ — (iii)

using (iii) in (ii), we have
$y = 0$ — (iv)

using (iv) & (iii) (i), we have
$x = 0$

$\Rightarrow \boxed{x = y = z = 0}$

eg $4x + 2y + z + 3w = 0$

$6x + 3y + 4z + 7w = 0$

$2x + y + w = 0$

the matrix representation is

$$\underbrace{\begin{bmatrix} 4 & 2 & 1 & 3 \\ 6 & 3 & 4 & 7 \\ 2 & 1 & 0 & 1 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}}_{X} = \underbrace{\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}}_{b}$$

$$A = \begin{bmatrix} 4 & 2 & 1 & 3 \\ 6 & 3 & 4 & 7 \\ 2 & 1 & 0 & 1 \end{bmatrix}$$

$$\sim \begin{bmatrix} 4 & 2 & 1 & 3 \\ 0 & 0 & \frac{5}{2} & \frac{5}{2} \\ 0 & 0 & -\frac{1}{2} & -\frac{1}{2} \end{bmatrix} \quad \begin{array}{l} R_2 \to R_2 - \frac{3}{2}R_1 \\ R_3 \to R_3 - \frac{1}{2}R_1 \end{array}$$

$$\sim \begin{bmatrix} 4 & 2 & 1 & 3 \\ 0 & 0 & \frac{5}{2} & \frac{5}{2} \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad R_3 \to R_3 + \frac{1}{5}R_2$$

$\rho(A) = 2 < 4$ (no of variable)

$\Rightarrow$ This system has infinite no. of solution.

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8) Investigate the consistency of the system

$4x-2y+6z=8$
$x+y-3z=-1$
$15x-3y+9z=21$

Matrix form

$$\begin{bmatrix} 4 & -2 & 6 \\ 1 & 1 & -3 \\ 15 & -3 & 9 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ -1 \\ 21 \end{bmatrix}$$

$$[A:b] = \left[\begin{array}{ccc:c} 4 & -2 & 6 & 8 \\ 1 & 1 & -3 & -1 \\ 15 & -3 & 9 & 21 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc:c} 2 & -1 & 3 & 4 \\ 1 & 1 & -3 & -1 \\ 5 & -1 & 3 & 7 \end{array}\right] \qquad \begin{matrix} R_1 \to \frac{1}{2}R_1 \\ R_3 \to \frac{1}{3}R_3 \end{matrix}$$

$$\sim \left[\begin{array}{ccc:c} 2 & -1 & 3 & 4 \\ 1 & 1 & -3 & -1 \\ 0 & 0 & 0 & 0 \end{array}\right] \qquad R_3 \to R_3 - 2R_1 - R_2$$

$$\sim \left[\begin{array}{ccc:c} 2 & -1 & 3 & 4 \\ 0 & \frac{3}{2} & -\frac{9}{2} & -3 \\ 0 & 0 & 0 & 0 \end{array}\right] \qquad R_2 \to R_2 - \frac{1}{2}R_1$$

$\rho(A) = 2 = \rho(A:b) = 2$ (consistent)
< 3 (no of variables)

$\Rightarrow$ infinite no. of solution.