Unit-2: 3- Phase Systems

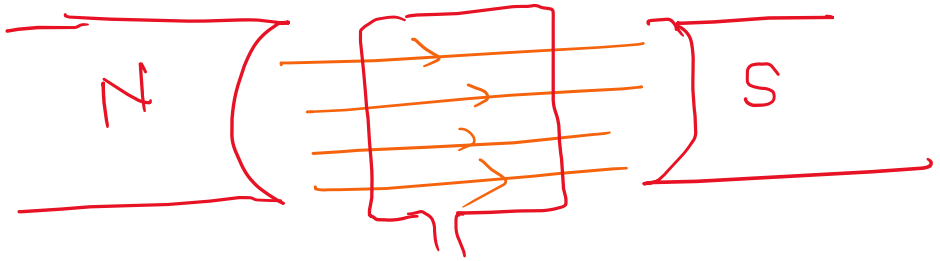
Dr. Awadhesh Kumar

Assistant Professor, EED

MMMUT Gorakhpur

3-ϕ System Advantages :

1-ϕ	3-ϕ
1) 1-ϕ, pulsating,	3-ϕ, Rotating
2) not s.s.	self starting of m/c
3) pf ↓	pf ↑
4) P	$\frac{3}{4}$th ↓ Cu
5) Rating ↑ phases	1.5 times more

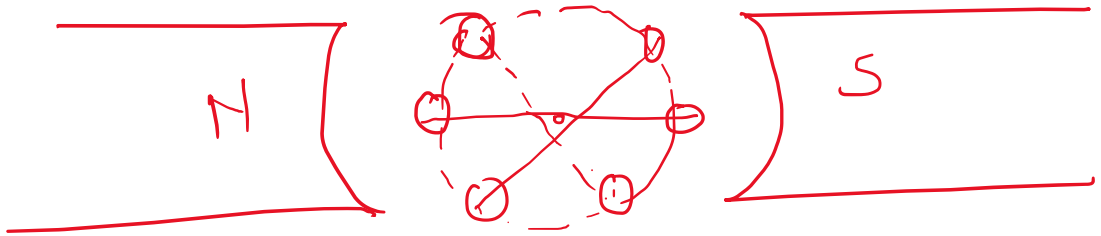


RCR

$\theta \rightarrow w \rightarrow t$

$e = -\frac{d\phi}{dt}$

$20^{\circ} \rightarrow t$



Field system B → Stator

Armature C → Rotor
Coil

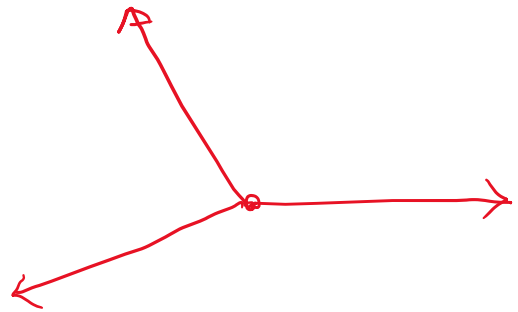
speed

120° apart

R Y B

C_1 C_2 C_3

max emf



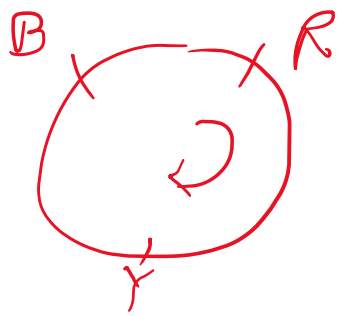
Phase sequence

RYB

BRY

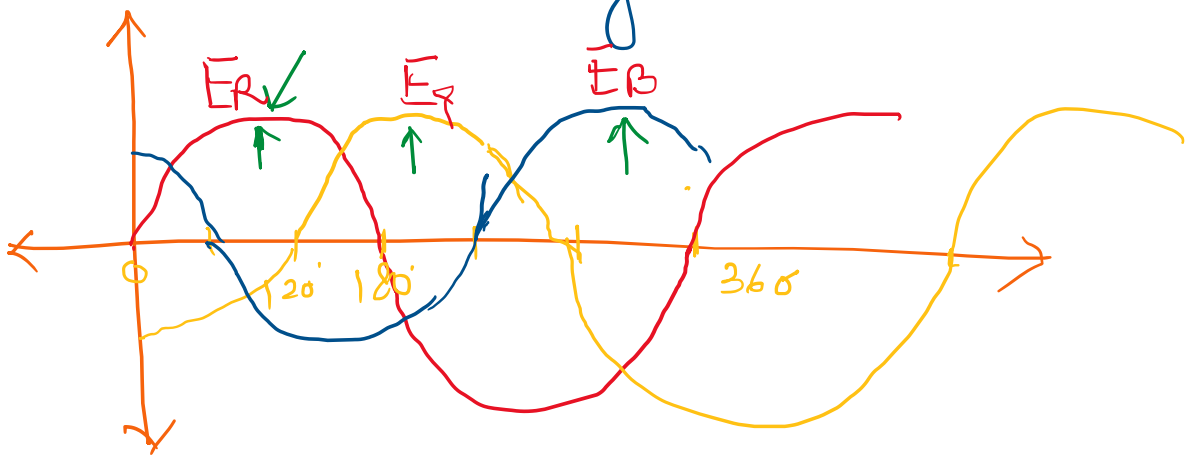
YBR

$\frac{360}{3} = 120^{\circ}$



wave diagram

RYB



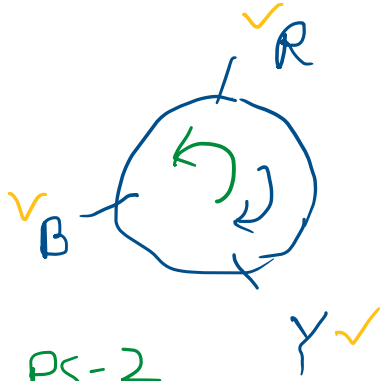
$E_R = V_m \sin\omega t \equiv V_m \angle 0°$

$E_Y = V_m \sin(\omega t - 120°) = V_m \angle -120°$

$E_B = V_m \sin(\omega t - 240°) = V_m \angle -240°$

$V_m \sin(\omega t + 120°) = V_m \angle 120°$

PS-1



RYB

YBR

BRY

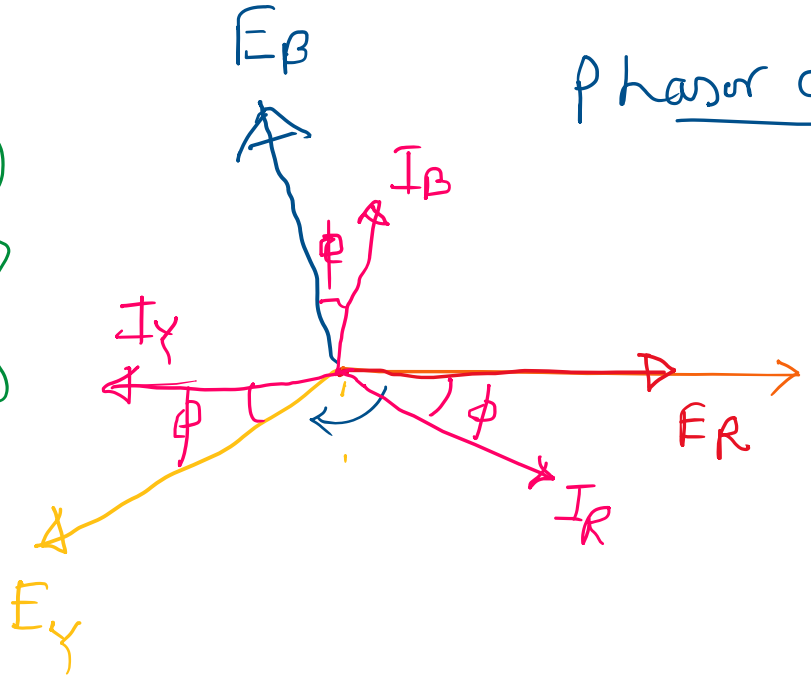
PS-2

RBY

BYR

YRB

Phasor diagram



Balanced Load

3-ϕ, Logging Load

R phase

$V_m \angle 0°$ $I_m \angle -\phi$

Y

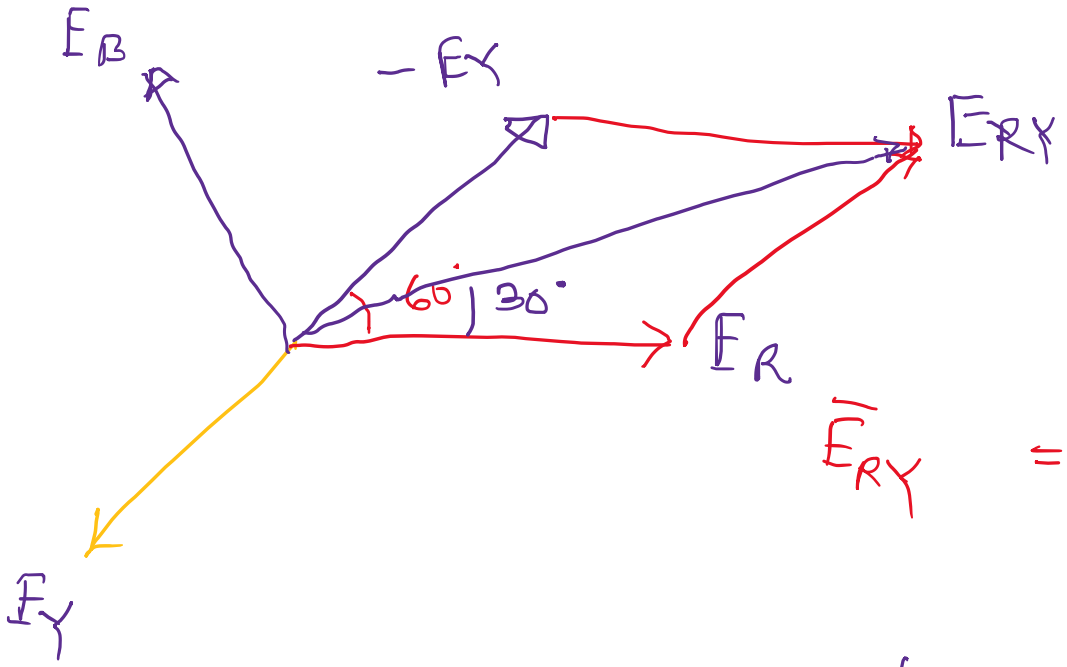
$V_m \angle -120°$ $I_m \angle -120° - \phi$

B

$V_m \angle -240°$ $I_m \angle -240° - \phi$

RYB

Phase Voltage



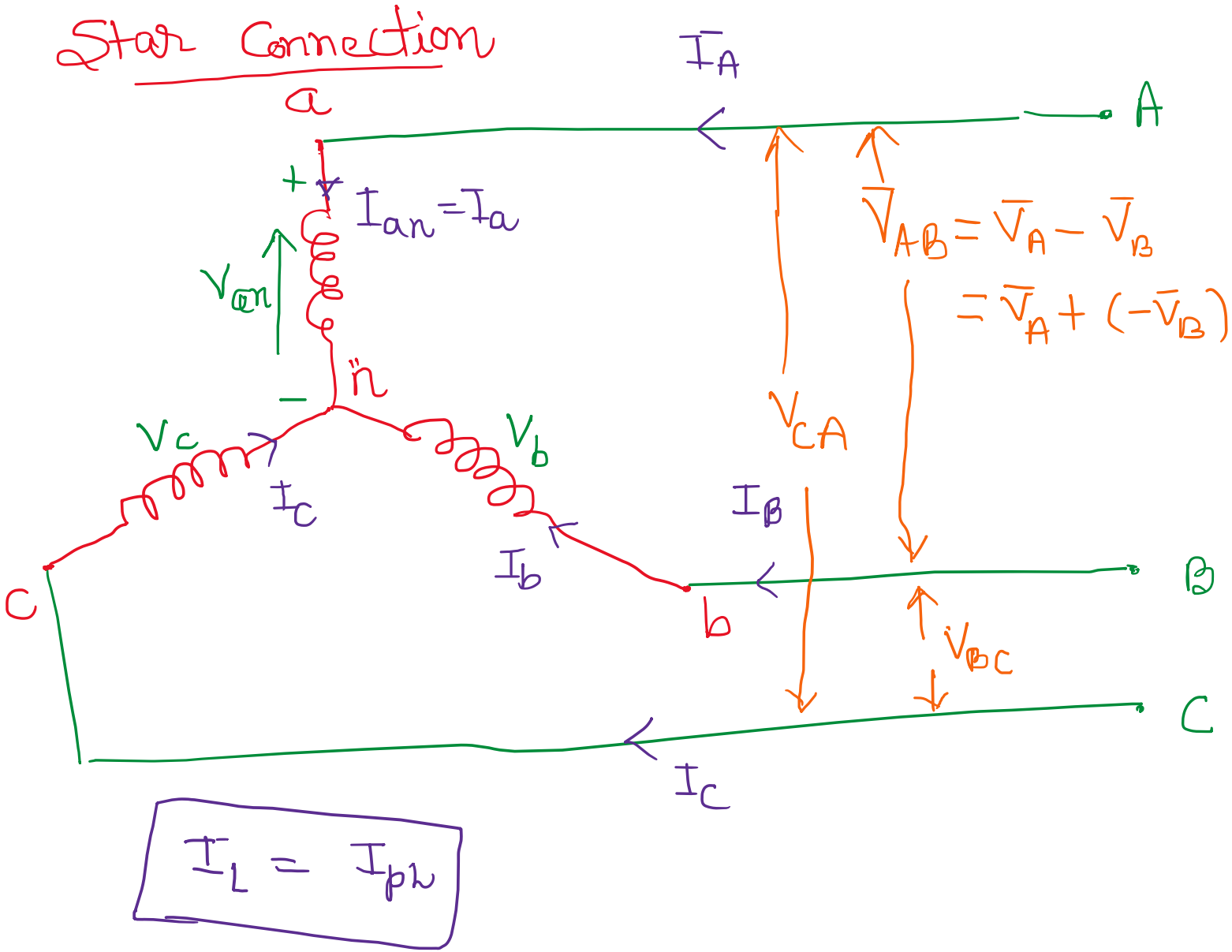
$$\bar{E}_{RY} = \bar{E}_R + (-\bar{E}_Y)$$

$$\bar{E}_R - \bar{E}_Y$$

$$V_{AB} = V_A - V_B$$

$$E_{RY} = E_R - E_Y$$

Star Connection



$$I_L = I_{ph}$$

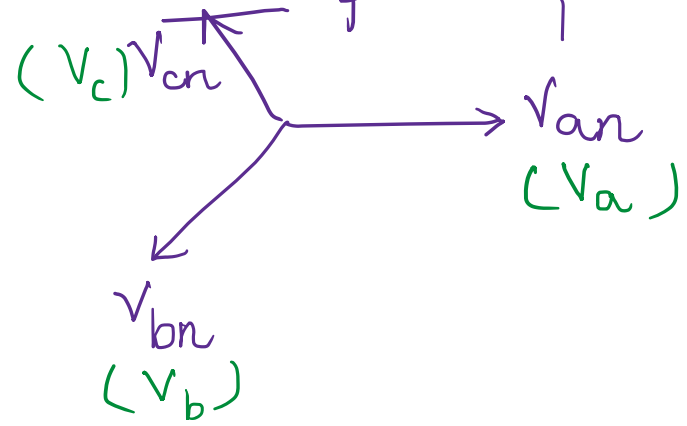
3-ϕ, 3-wire

3-ϕ, 4 wire

Line & phase

$$|V_{an}| = |V_{bn}| = |V_{cn}| = V_{ph}$$

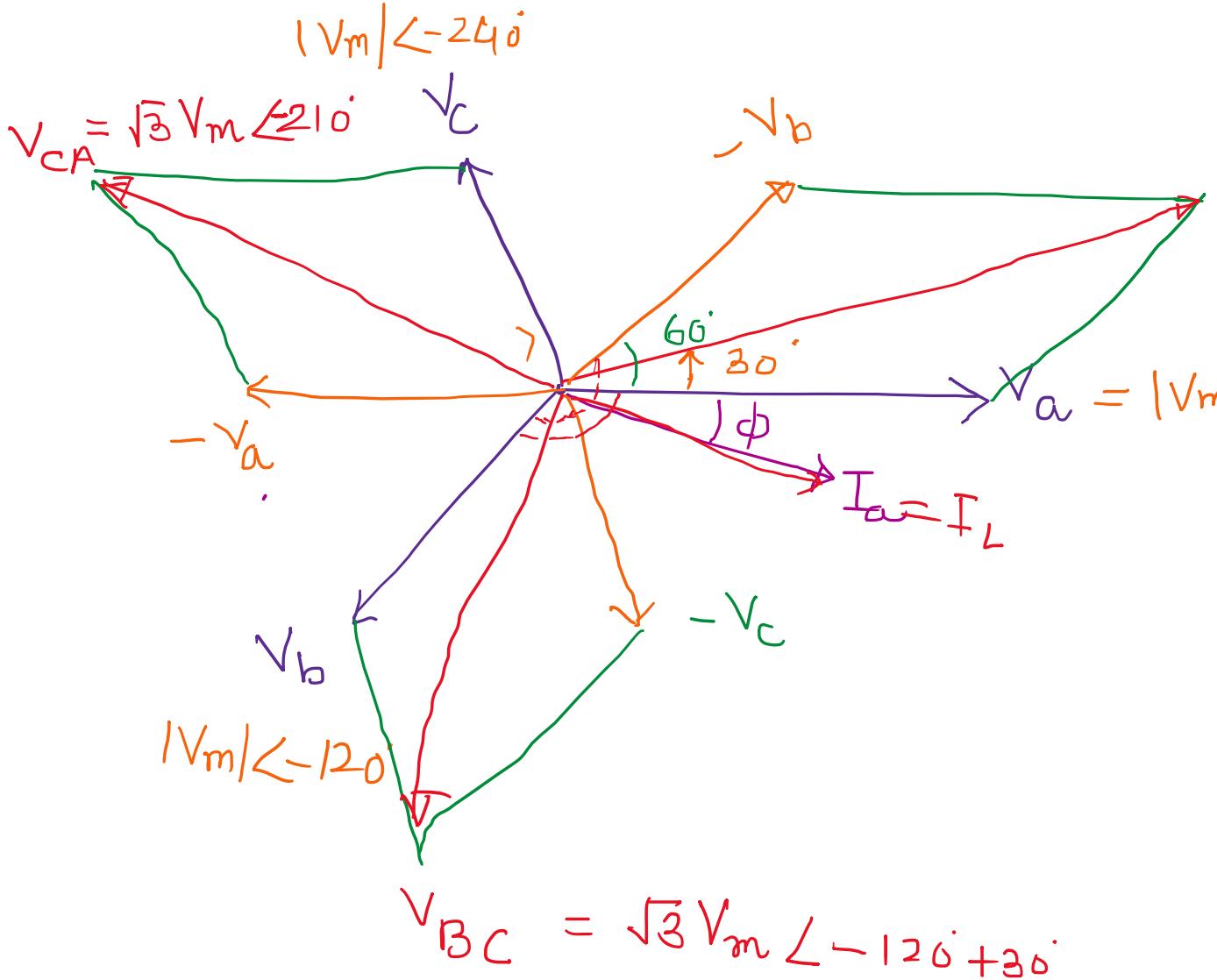
abc phase seq



$$|V_{AB}| = |V_{BC}| = |V_{CA}| = V_L$$

$$|I_A| = |I_B| = |I_C| = I_L$$

$$|I_a| = |I_b| = |I_c| = I_{ph}$$



abc

$V_{AB} = \sqrt{3}|V_m| \angle 30° = \sqrt{3} V_m \sin(\omega t + 30°)$

$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$

$= \sqrt{V_m^2 + V_m^2 + 2 V_m V_m \cos 60°}$

$= \sqrt{3} V_m$

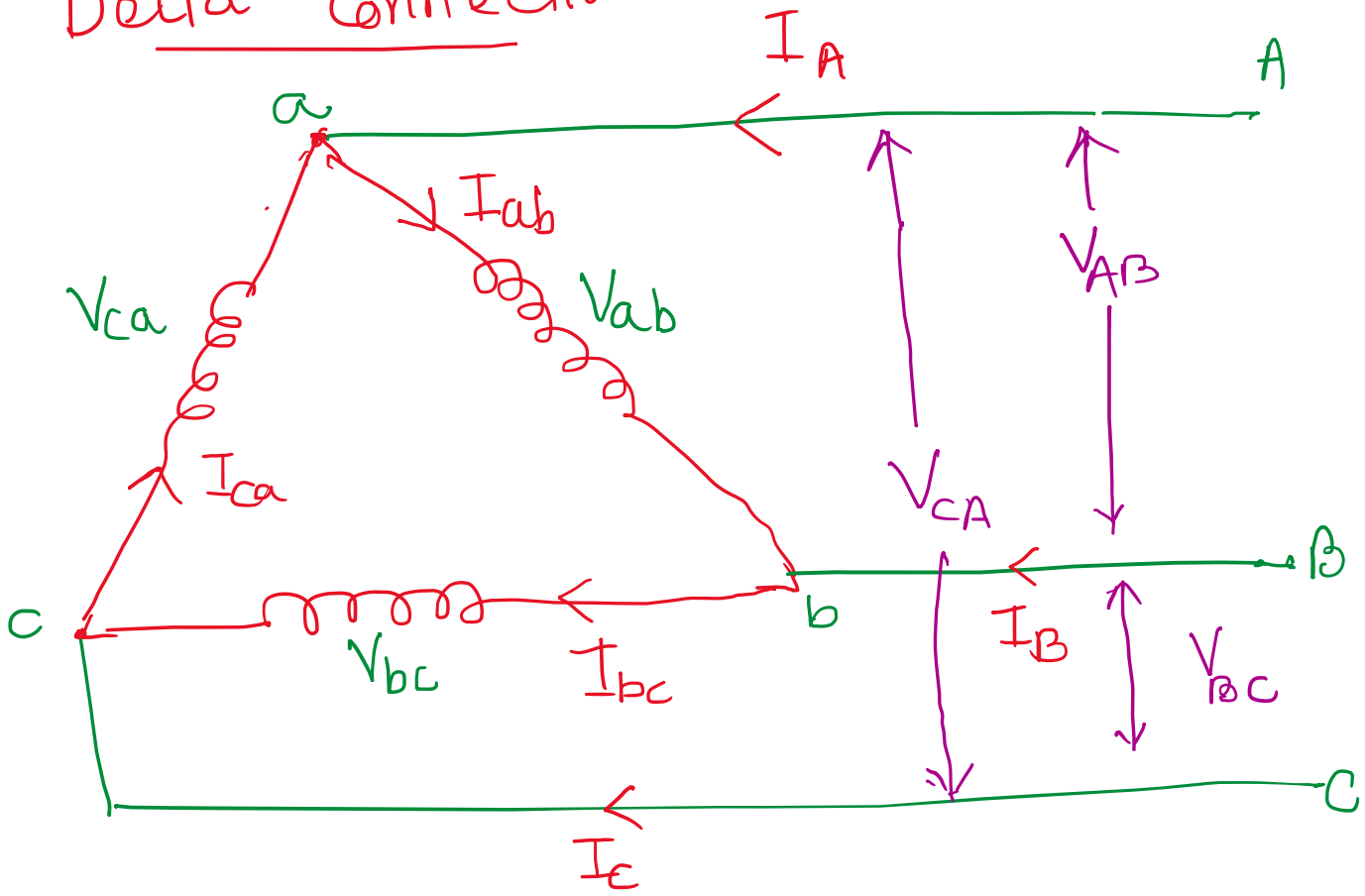
3-ϕ Lagging load

$V_L \wedge V_{ph} = 30°$

$V_L = \sqrt{3} V_{ph}$
$I_L = I_{ph}$

Delta Connection

3 ϕ, 3-wire



V_{ab} V_{bc} V_{ca} → phase

V_{AB} V_{BC} V_{CA} → Line

$V_{ab} = V_{AB}$

$$\boxed{V_{ph} = V_L}$$

node a

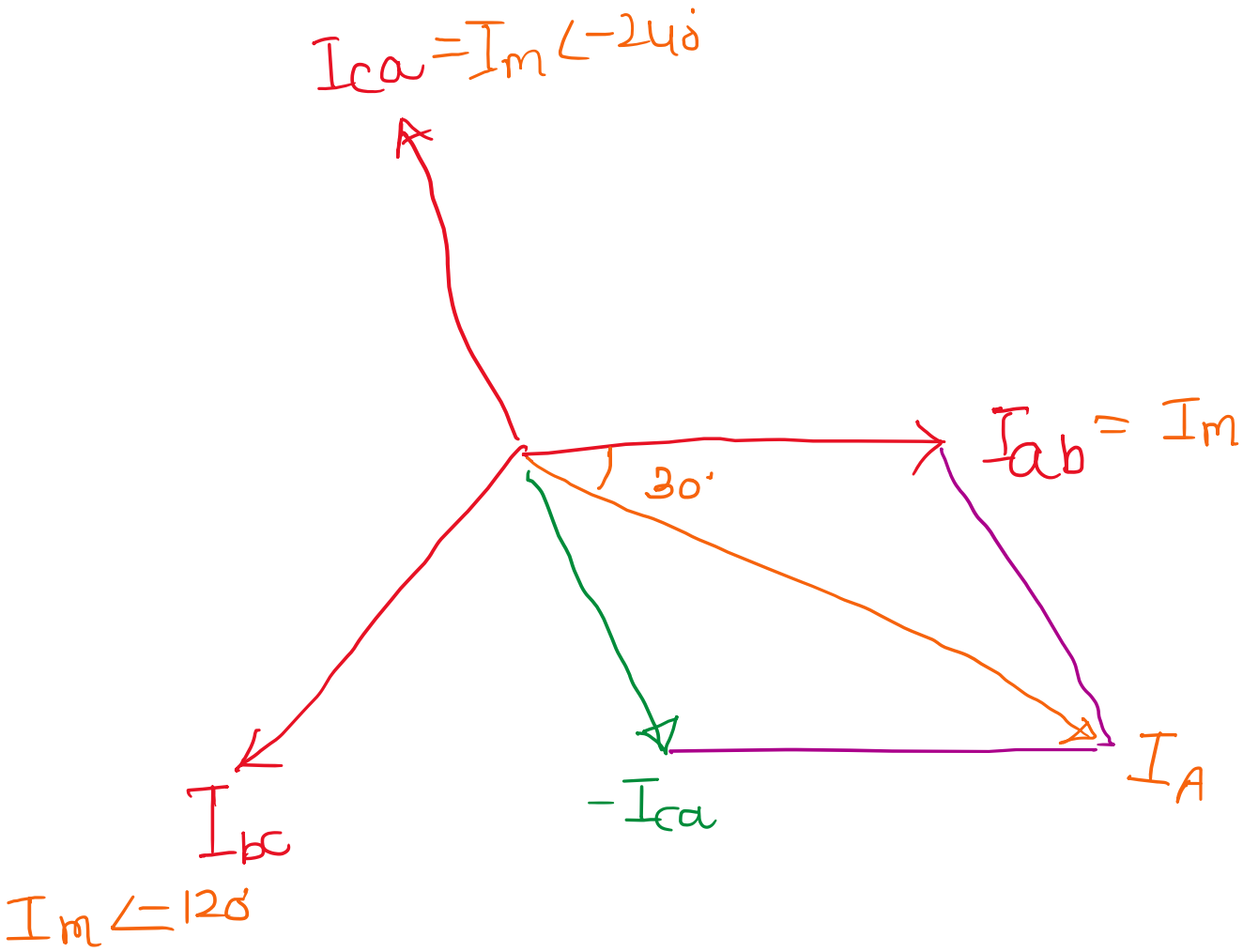
" b

" c

$$\bar{I}_A = \bar{I}_{ab} - \bar{I}_{ca} \quad \text{—— (1)}$$

$$\bar{I}_B = \bar{I}_{bc} - \bar{I}_{ab} \quad \text{—— (2)}$$

$$\bar{I}_C = \bar{I}_{ca} - \bar{I}_{bc} \quad \text{—— (3)}$$



$$\bar{I}_A = \bar{I}_{ab} - \bar{I}_{ca}$$
$$= \bar{I}_{ab} + (-\bar{I}_{ca})$$

$$I_A \wedge I_{ab} = -30°$$

$$I_A = \sqrt{I_m^2 + I_m^2 + 2 I_m \cdot I_m \cdot \cos 60°}$$

$$= \sqrt{3}\, I_m$$

$$V_L = V_{ph}$$

$$I_L = \sqrt{3}\, I_{ph}$$

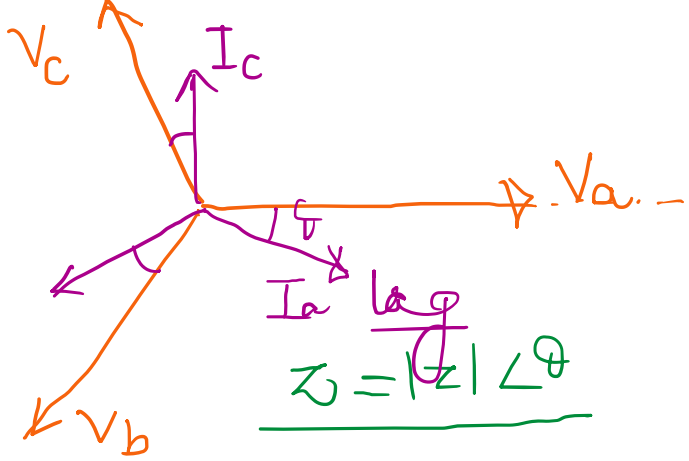
3-ϕ Power :-

abc

$V_a = V_p' \sin \omega t$

$V_b = V_p' \sin(\omega t - 120^\circ)$

$V_c = V_p' \sin(\omega t - 240^\circ)$



$Z = |Z| \angle \theta$

$|V_a| = |V_b| = |V_c| = V_p'$ max V

$|I_a| = |I_b| = |I_c| = I_p'$ max value

$I_a = I_p' \sin(\omega t - \theta)$

$I_b = I_p' \sin(\omega t - 120^\circ - \theta)$

$I_c = I_p' \sin(\omega t - 240^\circ - \theta)$

$p(t) = V_a I_a + V_b I_b + V_c I_c$

$= V_p' I_p' \sin\omega t \sin(\omega t - \theta) + V_p' I_p' \sin(\omega t - 120) \sin(\omega t - 120^\circ - \theta) +$

$+ V_p' I_p' \sin(\omega t - 240^\circ) \sin(\omega t - 240^\circ - \theta)$

$= \frac{V_p' I_p'}{2} [\, 2 \sin\omega t \sin(\omega t - \theta) + \_\_\_\_\_ + \_\_\_\_ \,]$

$[\, \cos\theta - \cos(2\omega t - \theta) + \cos\theta - \_\_ + \cos\theta - \_\_ \,]$

$$= \frac{V_p'}{\sqrt{2}} \frac{I_p'}{\sqrt{2}} \cdot 3\cos\theta$$

$$= V_p I_p \, 3\cos\theta$$

$$p(t) = 3 V_p I_p \cos\theta$$

$V_p \rightarrow$ } RMS value of phase voltage & current
$I_p \rightarrow$ }

$$\begin{cases} P = 3 V_p I_p \cos\theta & \text{KW} \\ Q = 3 V_p I_p \sin\theta & \text{KVAR} \\ S = 3 V_p I_p & \text{KVA} \end{cases}$$

1 − ϕ

$$p(t) = VI\cos\theta - VI\cos(2\omega t - \theta)$$

$$P = VI\cos\theta$$ active

$$Q = VI\sin\theta$$ Reactive

$$S = VI$$ Apparent

Y

$$P = 3 \frac{V_L}{\sqrt{3}} I_L \cos\theta = \sqrt{3} V_L I_L \cos\theta$$

Δ

$$P = 3 V_L \frac{I_L}{\sqrt{3}} \cdot \cos\theta = \sqrt{3} V_L I_L \cos\theta$$

$$Q = \sqrt{3} V_L I_L \sin\theta$$

$$S = \sqrt{3} V_L I_L$$

Y $\begin{cases} V_L = \sqrt{3} V_p \\ I_L = I_p \end{cases}$

Δ $\begin{cases} V_L = V_{ph} \\ I_L = \sqrt{3} I_p \end{cases}$

$Z = R + jX$

$$Z = \frac{V_{ph}\angle\theta_1}{I_{ph}\angle\theta_2}$$

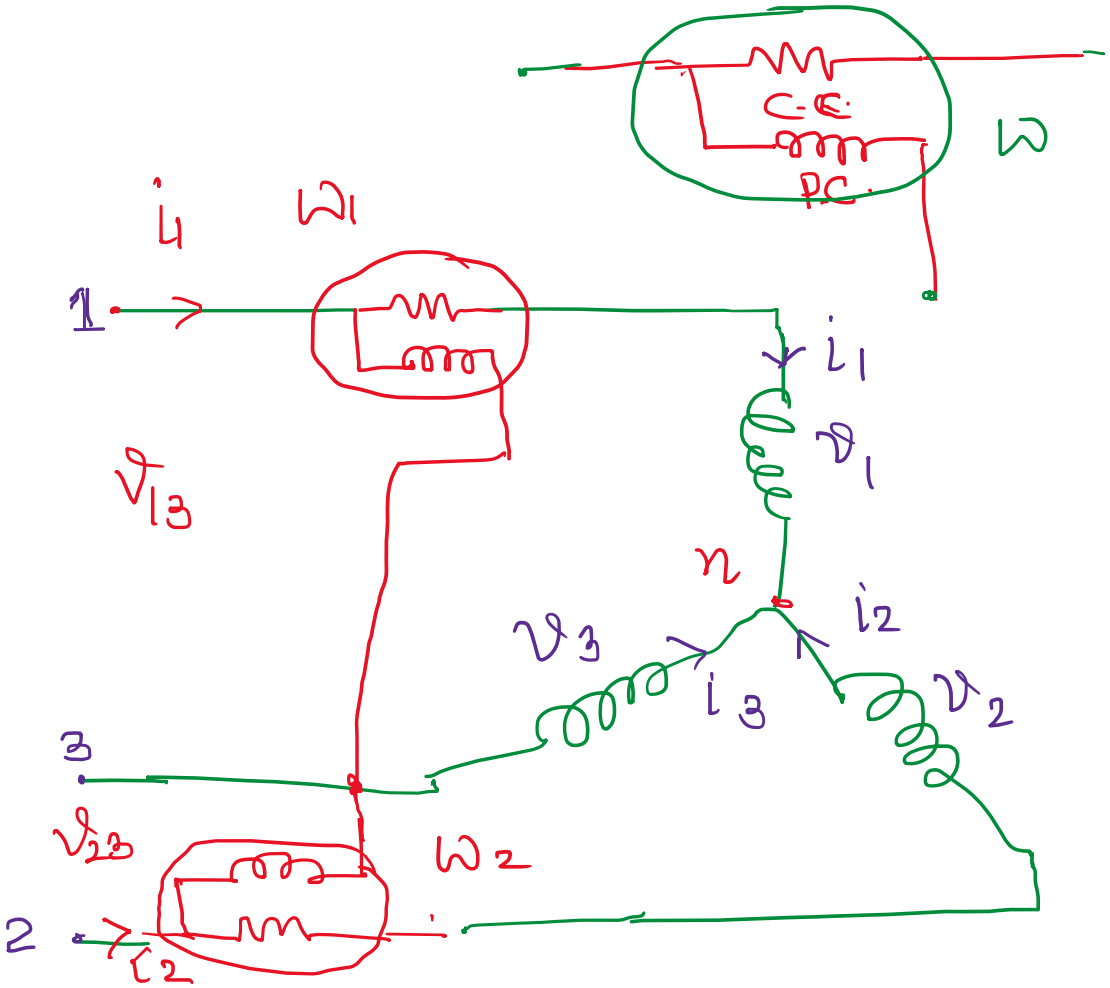
$$Z\angle\theta = \frac{V_p}{I_p}\angle\theta_1 - \theta_2$$

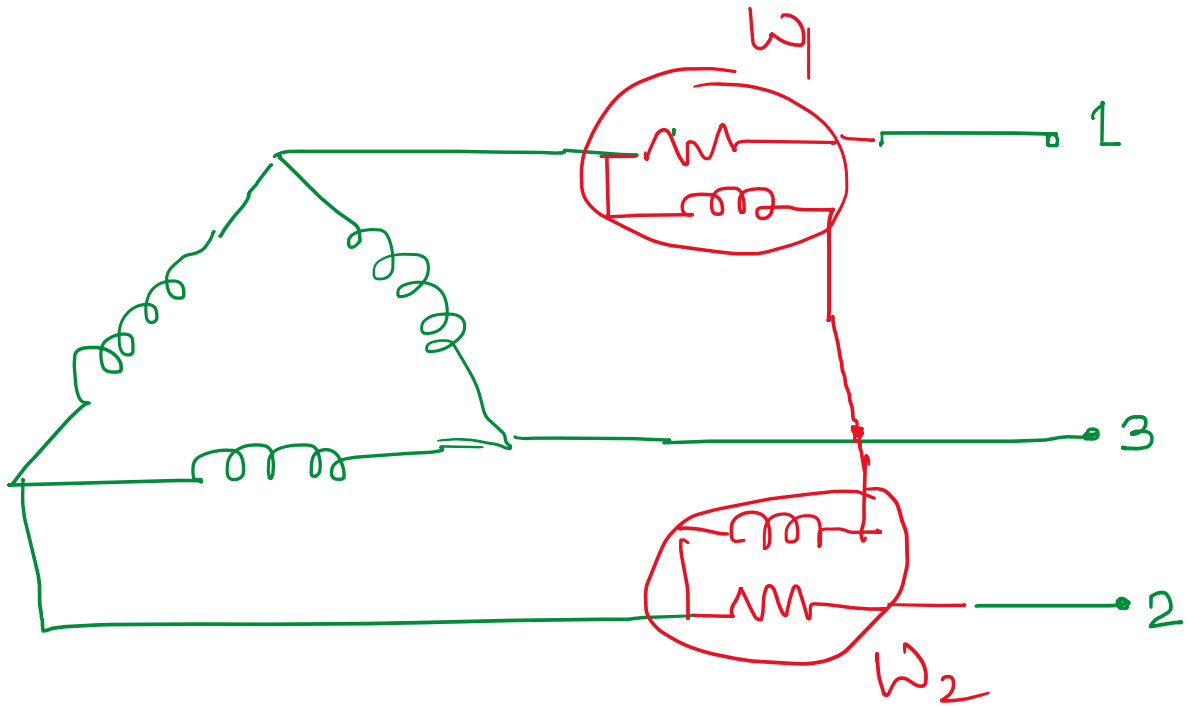
$\theta = \phi$

Measurement of 3-ϕ Power:

By 2-wattmeter method:

Wattmeter measures $P = VI\cos\phi$





$$W_1 = v_{13} i_1$$

$$W_2 = v_{23} i_2$$

$$v_{13} = (v_1 - v_3)$$

$$v_{23} = (v_2 - v_3)$$

$$W_1 + W_2 = v_{13} i_1 + v_{23} i_2$$

$$= (v_1 - v_3) i_1 + (v_2 - v_3) i_2$$

$$= v_1 i_1 + v_2 i_2 - v_3 (i_1 + i_2)$$

$$= v_1 i_1 + v_2 i_2 + v_3 i_3$$

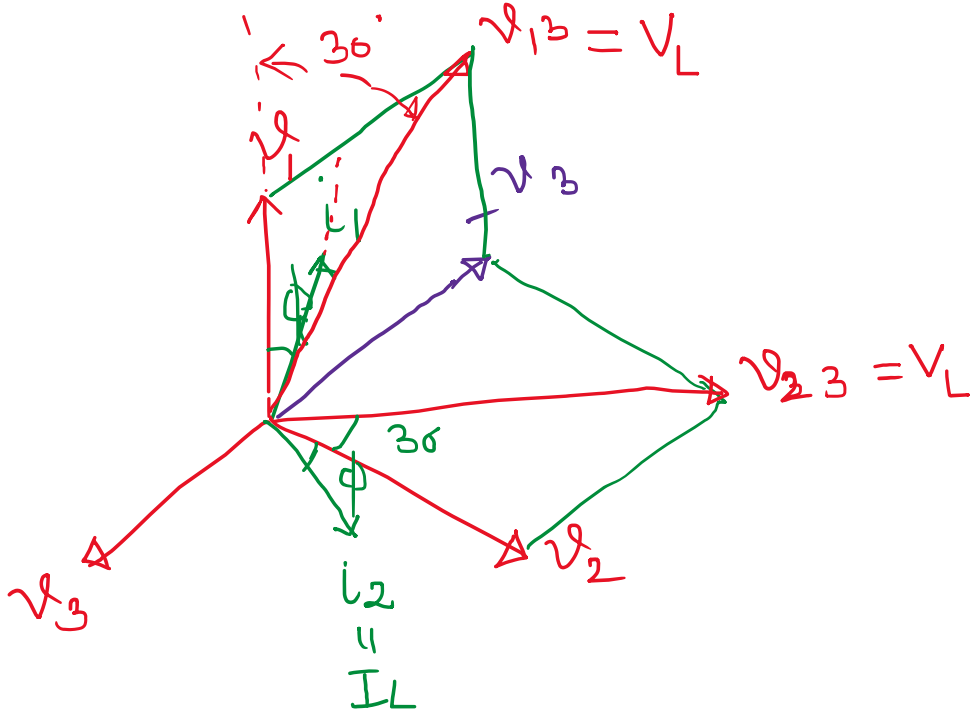
$$i_1 + i_2 + i_3 = 0$$

$$\Downarrow$$

$$i_1 + i_2 = -i_3$$

sum of two wattmeter readings = 3-ϕ total Power

$$\boxed{W_1 + W_2 = P_1 + P_2 + P_3}$$



Lagging load

$v_1 \hat{} i_1 = \phi$

$v_{13} = \bar{v}_1 - \bar{v}_3$

$= \bar{v}_1 + (-\bar{v}_3)$

$v_{13} \hat{} i_1 = 30 - \phi$

$i_1 = I_L$

$W_1 = v_{13} i_1$

$W_1 = V_L I_L \cos(30 - \phi)$

$W_2 = v_{23} i_2$

$W_2 = V_L I_L \cos(30 + \phi)$

$$W_1 + W_2 = V_L I_L \left[\cos(30 - \phi) + \cos(30 + \phi) \right]$$

$$= V_L I_L \, 2 \cos 30 \cos\phi$$

$$\boxed{W_1 + W_2 = \sqrt{3} V_L I_L \cos\phi} \quad \text{—— (1)}$$

$$W_1 - W_2 = V_L I_L \left[\cos(30 - \phi) - \cos(30 + \phi) \right]$$

$$\boxed{W_1 - W_2 = V_L I_L \, 2 \sin 30 \sin\phi = V_L I_L \sin\phi} \quad \text{—— (2)}$$

$$W_1 + W_2 = \sqrt{3} V_L I_L \cos\phi \quad \text{— (1)}$$

$$W_1 - W_2 = V_L I_L \sin\phi \quad \text{— (2)}$$

$$\frac{\tan\phi}{\sqrt{3}} = \frac{W_1 - W_2}{W_1 + W_2}$$

$$\tan\phi = \sqrt{3}\left(\frac{W_1 - W_2}{W_1 + W_2}\right)$$

$$\phi = \tan^{-1}\sqrt{3}\left(\frac{W_1 - W_2}{W_1 + W_2}\right)$$

$$pf = \cos\phi = \cos\tan^{-1}\sqrt{3}\left(\frac{W_1 - W_2}{W_1 + W_2}\right)$$

① $W_1 = W_2$ $\quad$ upf

② $W_2 = 0$ $\quad$ W_1 = finite

$\phi < 60$

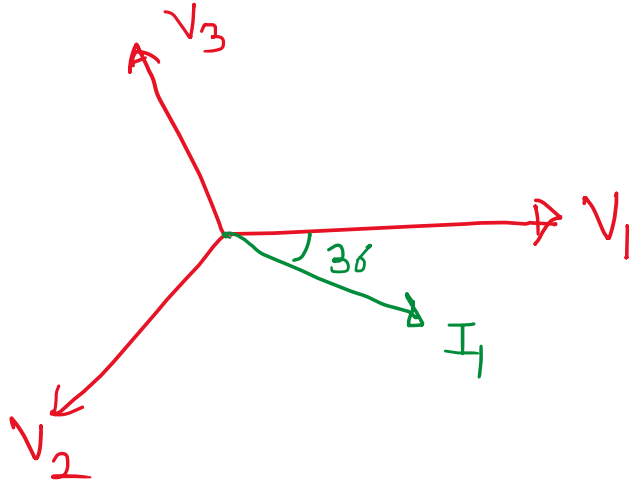
$\cos\phi = \frac{1}{2}$ $\quad$ lag

Prob: bal Y, 3-ϕ, 50Hz, 415V (V_L) supply, $20\angle -30^\circ$ A (I_{ph}) Lagging

(i) Total Power supplied:

(ii) the overall pf

(iii) Phasor diagram



1-2-3

$V_L = 415V$

$V_p = \frac{V_L}{\sqrt{3}} = \frac{415}{\sqrt{3}} = 239.6\,V$ $(V_L = \sqrt{3}\,V_{ph})$

$I_L = I_p = 20\,A$ $(I_L = I_{ph})$

$\phi = V_1 \hat{} I_1 = -30^\circ$

(i) $P = \sqrt{3} V_L I_L \cos\phi = \sqrt{3} \times 415 \times 20 \cos 30^\circ$

(ii) $\cos\phi = \cos 30^\circ = 0.866$ Lagging

bal Δ

Prob 1 $Z = 12 + j9\ \Omega$ / phase, 3-ϕ, 400 V = V_L

$= R + jX_L$

$|Z| = \sqrt{R^2 + X_L^2} = 15\ \Omega$

1) I_L

2) pf

3) Power drawn, $P = \sqrt{3} V_L I_L \cos\phi$

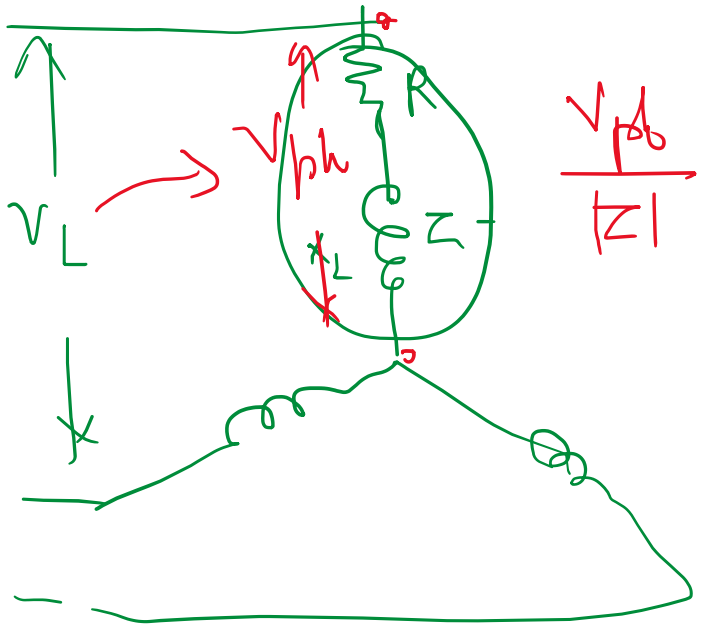
4) Reactive volt amp $Q = \sqrt{3} V_L I_L \sin\phi$

5) Volt Amp, $S = \sqrt{3} V_L I_L$

$I_{ph} = \frac{V_{ph}}{|Z|}$.

$V_{ph} = V_L$

$I_L = \sqrt{3} I_{ph}$



$\frac{V_{ph}}{|Z|}$ $\frac{V_{ph}\angle\theta}{Z\angle\phi}$

$Z = R$ $R\angle 0^\circ$

$= R + jX$ $|Z|\angle\phi$

$= R - jX$ $|Z|\angle -\phi$

$= jX$ $X\angle 90^\circ$

$= -jX$ $X\angle -90^\circ$

$\frac{V_{ph}\angle\theta}{jX}$

$= \frac{V_{ph}\angle\theta}{X\angle 90^\circ}$

$= (V_p/X)\angle\theta - 90^\circ$