Simplex method example

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Solve by simplex method

Max $z = 3x_1 + 2x_2 + x_3$ s/to

$x_1 + 2x_2 + x_3 \leq 430$

$3x_1 + 2x_3 \leq 460$

$x_1 + 4x_2 \leq 420$, $x_1 \geq 0$, $x_2 \geq 0$, $x_3 \geq 0$.

$4 < 6$

$4 + 2 = 6$ → Slack variable

$6 > 2$

$6 - 4 = 2$ → Surplus variable

slack variables and surplus variables

Max $z = 3x_1 + 2x_2 + x_3 + 0s_1 + 0s_2 + 0s_3$ s/to,

$x_1 + 2x_2 + x_3 + s_1 = 430$

$3x_1 + 2x_3 + s_2 = 460$

$x_1 + 4x_2 + s_3 = 420$

to start

at $x_1 = x_2 = x_3 = 0$

$s_1 = 430$, $s_2 = 460$,

$s_3 = 420$

$s_1 \geq 0$, $s_2 \geq 0$, $s_3 \geq 0$

C_B → cost coefficient of Basic variables

	$z \rightarrow$		3	2	1	0	0	0	
Basic variables	C_B	X_B (Soln.)	x_1	x_2	x_3	s_1	s_2	s_3	minimum ratio
s_1	0	430	1	2	1	1	0	0	$430/1 = 430$ → minimum
s_2	0	460	3	0	2	0	1	0	$460/3 = 159.3$ →
s_3	0	420	1	4	0	0	0	1	$420/1 = 420$
Zet evaluation $z = 0$			-3	-2	-1	0	0	0	

$z_1 = C_B x_1 - c_1$

$z = C_B X_B = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} [430 \quad 460 \quad 420] = 0$

$z_1 = C_B x_1 - c_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix} - 3$ → [column vector of coefficient of x_1]

$= 0 - 3 = -3$

column

$z_2 = C_B x_2 - c_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 2 \\ 0 \\ 4 \end{bmatrix} - 2 = -2$

C_B → coefficient of Basic variables in the Objective function.

For entering variable in term of Basic variable

take minimum of net evaluation (only negative net)

if we denote Δ_j is the net evaluation of j^{th} column. and defined as $\Delta_j = C_B x_j - C_j$

so, minimum $\{-3, -2, -1\} = -3$

which is the correspond to x_1

$\Rightarrow x_1$ is the entering variable in term of Basic variable.

now, For outgoing vector, take a minimum ratio as

minimum $\left\{\frac{X_B}{x_j}\right\}$ =

$j = 1 \Rightarrow$ as x_1 is entering variable.

		$Z \rightarrow$	3	2	1	0	0	0	
Basic variable	C_B	X_B (Soln.) ↓	x_1	x_2	x_3	s_1	s_2	s_3	minimum ratio
s_1	0	430	1	2	1	1	0	0	$430/1 = 430$ → minimum
s_2	0	460	[3]	0	2	0	1	0	$460/3 = 159.3$ →
s_3	0	420	1	4	0	0	0	1	$420/1 = 420$
net evaluation		$Z = 0$	-3	-2	-1	0	0	0	
s_1	0	$\frac{830}{3}$	0	2	1/3	1	-1/3	0	$\frac{830}{3\times 2} = 138$
x_1	3	$\frac{460}{3}$	1	0	2/3	0	1/3	0	—
s_3	0	$\frac{800}{3}$	0	[4]	-2/3	0	-1/3	1	$\frac{800}{3\times 4} = 66$ ←
			0	-2	1	0	1	0	
s_1	0	$430/3$	0	↑ 0	0	1	$\frac{1}{6}$	-1/2	
x_1	3	$\frac{460}{3}$	1	0	2/3	0	$\frac{1}{3}$	0	

x_1	3	$\frac{460}{3}$	1	0	$2/3$	0	$\frac{1}{3}$	0
x_2	2	$\frac{800}{12}$	0	1	$-1/6$	0	$-\frac{1}{12}$	$\frac{1}{4}$
			0	0	$2/3$	0	$5/6$	$1/2$

as all net evaluations (Δ_j) are zero or positive, so we stop here.

$x_1 = \frac{460}{3}$

$x_2 = \frac{800}{12} = \frac{200}{3}$

$x_3 = 0$

$f = 3 \times \frac{460}{3} + 2 \times \frac{200}{3}$ Ans

optimize the problem:

Maximize $Z = -2x_1 - x_2 + 5x_3$ s.t.

$x_1 - 2x_2 + x_3 \leq 8$

$-3x_1 + 2x_2 \leq 18$

$2x_1 + x_2 - 2x_3 \leq 4$, $x_1 \geq 0$, $x_2 \geq 0$, $x_3 \geq 0$.

Soln →

Max $Z = -2x_1 - x_2 + 5x_3 + 0s_1 + 0s_2 + 0s_3$

$x_1 - 2x_2 + x_3 + s_1 = 8$

$-3x_1 + 2x_2 + s_2 = 18$

$2x_1 + x_2 - 2x_3 + s_3 = 4$, $s_1 \geq 0$, $s_2 \geq 0$, $s_3 \geq 0$.

For starting the soln, $x_1 = 0$, $x_2 = 0$, $x_3 = 0$

get $s_1 = 8$, $s_2 = 18$, $s_3 = 4$, here s_1, s_2, s_3 are basic variables

	c_j		-2	-1	5	0	0	0	
B.V.	c_B	x_B	x_1	x_2	x_3	s_1	s_2	s_3	Minimum Ratio
s_1	0	8	1	-2	[1]	1	0	0	$8/1 = 8 \leftarrow$
s_2	0	18	-3	2	0	0	1	0	—

y_1	0	8	1	-2	[1]	1	0	0	8/1	= 8 ←
y_2	0	18	-3	2	0	0	1	0	—	
y_3	0	4	2	1	-2	0	0	1	—	
	Net evaluation		2	1	-5 ↑	0	0	0		
x_3	5	8	1	-2	1	1	0	0	—	
y_2	0	18	-3	[2]	0	0	1	0	18/2 = 9 ←	
y_3	0	20	4	-3	0	2	0	1	—	
			7	-9 ↑	0	5	0	0		
x_3	5	26	-2	0	1	1	1	0		
x_2	-1	9	-3/2	1	0	0	1/2	0		
y_3	0	47	-1/2	0	0	2	3/2	1		
			-13/2 ↑	0	0	5	9/2	0		

This is entering vector

as Δ_1 is negative and corresponding column, all elements are negative.

$\Rightarrow$ this prob. has unbounded soln.