Reflection of plane wave at Oblique incidence:

Plane containing propagation of incident wave and unit normal vector to the boundary is plane of incidence.

Types of Oblique incidence:
- → Parallel Polarisation
- → Perpendicular Polarisation

If Electric field of uniform plane wave is parallel to plane of incidence, it is called Parallel polarisation.

If the Electric field of uniform plane wave is perpendicular to the plane of incidence, it is called Perpendicular polarisation.

$\nabla \cdot \vec{E} = 0$ — (1)

$\nabla \cdot \vec{H} = 0$ — (2)

$\nabla \times \vec{E} = -j\omega\mu H$ — (3)

$\nabla \times \vec{H} = (\sigma + j\omega\varepsilon)\vec{E}$

$\nabla \times \vec{H} = j\omega\varepsilon\vec{E}$ — (4)

→ Oblique incidence is considered only for $\sigma = 0$

→ Maxwells equation ~~for~~ in terms of propagation vector:

∇ is replaced by $\vec{K}$

$\vec{K} \cdot \vec{E} = 0$

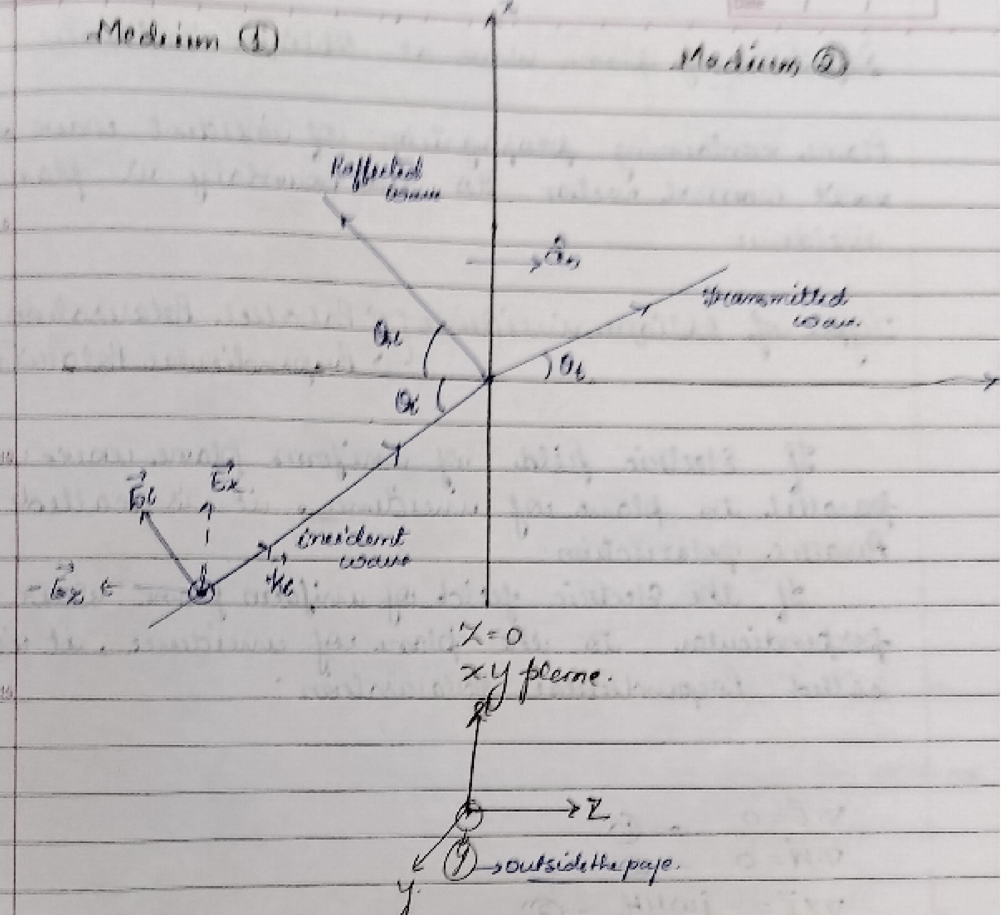
$\vec{K} \cdot \vec{H} = 0$

$\vec{K} \times \vec{E} = \omega\mu\vec{H}$

$\vec{K} \times \vec{H} = -\omega\varepsilon\vec{E}$

E, H, K are mutually orthogonal.

→ Parallel Polarisation



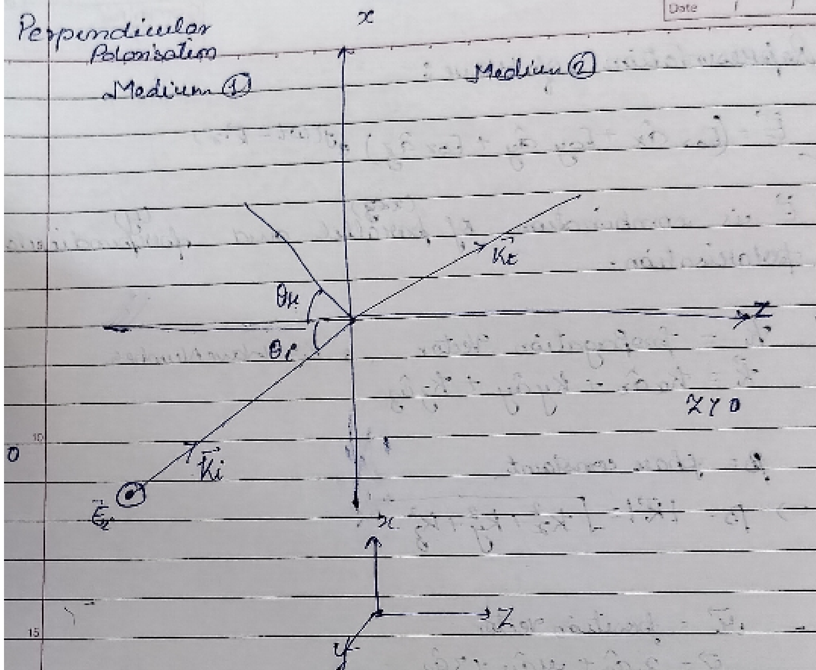
Plane of incidence → xz
Interface /Boundary → xy

→ According to right hand thumb rule fore finger is representing the dirⁿ of $\vec{G}$.

→ $\vec{E}$, $\vec{H}$ and direction of plane wave propagation are perpendicular to each other.

→ Two components of $\vec{E}$, $\vec{E}_x$ and $\vec{E}_z$ are parallel to x and z plane. ⇒ Parallel Polarisation.

Perpendicular Polarisation



→ According to right hand thumb rule, if thumb is representing direction of $\vec{E}$, then $\vec{E}$ will be outward of page (i.e. in +y dirⁿ).

→ Plane of incidence → xz plane.
Boundary / Interface → z=0 plane (xy plane)

Representation of Wave:

$$\vec{E} = (E_{0x}\,\hat{a}_x + E_{0y}\,\hat{a}_y + E_{0z}\,\hat{a}_z)\, e^{j(\omega t - \vec{k}\cdot\vec{r})}$$

$\vec{E}$ is combination of parallel $(x\,\&\,z)$ and perpendicular (y) polarisation.

$\vec{k}$ = propagation vector , k = Wave Number

$$\vec{k} = k_x\hat{a}_x + k_y\hat{a}_y + k_z\hat{a}_z$$

β = phase constant

$$\Rightarrow \beta = |\vec{k}| = \sqrt{k_x^2 + k_y^2 + k_z^2}$$

$\vec{r}$ = position vector

$$\vec{r} = x\,\hat{a}_x + y\,\hat{a}_y + z\,\hat{a}_z$$

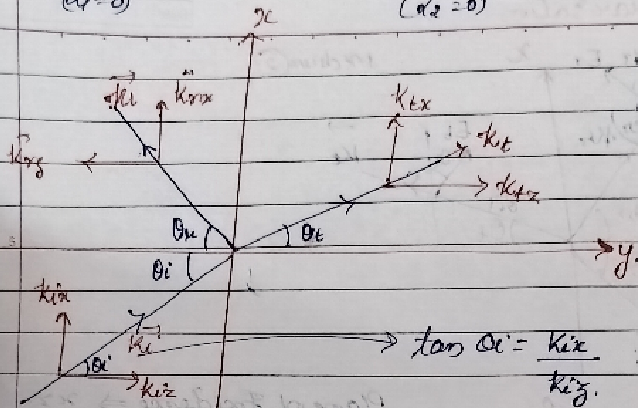
$$\Rightarrow \vec{k}\cdot\vec{r} = k_x\cdot x + k_y\cdot y + k_z\cdot z$$

$\Rightarrow$ Boundary Conditions that must be satisfied for all x and y

- $\omega_i = \omega_r = \omega_t = \omega \Rightarrow$ frequency matching Condn
- $k_{ix} = k_{rx} = k_{tx} = k_x$
- $k_{iy} = k_{ry} = k_{ty} = k_y$

$\left.\begin{matrix} k_{ix} = k_{rx} = k_{tx} = k_x \\ k_{iy} = k_{ry} = k_{ty} = k_y \end{matrix}\right\} \Rightarrow$ Phase matching Condn

Medium 1 ($\alpha_1 = 0$) Medium 2 ($\alpha_2 = 0$)



$\tan\theta_i = \dfrac{k_{ix}}{k_{iz}}$

$k_{ix} = k_{rx}$

$|\vec{k_i}| \sin\theta_i = |\vec{k_r}| \sin\theta_r$

$\beta_1 \sin\theta_i = \beta_1 \sin\theta_r$

$\boxed{\theta_i = \theta_r}$ Law of Reflection

$k_{ix} = k_{tx}$

$|\vec{k_i}| \sin\theta_i = |\vec{k_t}| \sin\theta_t$

$\boxed{\beta_1 \sin\theta_i = \beta_2 \sin\theta_2}$

$\because \beta_1 \sin\theta_i = \beta_2 \sin\theta_t$

$\cancel{\omega}\sqrt{\mu_1\epsilon_1} \sin\theta_i = \cancel{\omega}\sqrt{\mu_2\epsilon_2} \sin\theta_t$

mult. by c both side

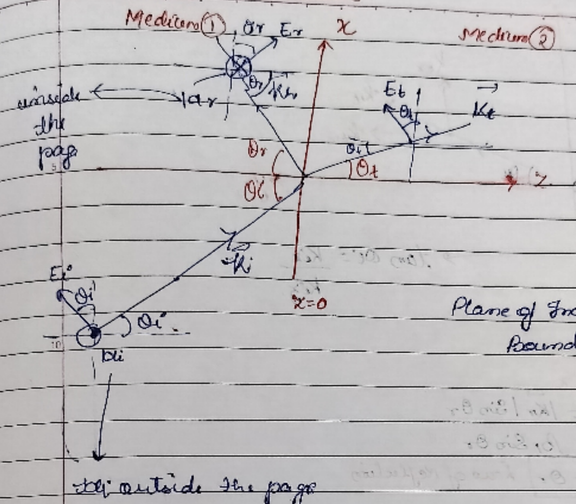
$\Rightarrow c\sqrt{\mu_1\epsilon_1} \sin\theta_i = \sqrt{\mu_2\epsilon_2} \sin\theta_t$

$\boxed{n_1 \sin\theta_i = n_2 \sin\theta_t}$ Snell's Law

ce η = Refractive Index

$\eta = \dfrac{\text{Velocity in vaccum}}{\text{Velocity in medium}} = \dfrac{c}{v_p} = c\sqrt{\mu\epsilon}$

Parallel Polarisation:



Plane of Incidence ⇒ xz.

Boundary ⇒ $z=0$

$\cancel{\hat{a}_i = \hat{a}_k \times \hat{a}_E}$

⇒ $\vec{k}_i$ = incident propagation vector

$\beta_1 = |\vec{k}_1|$

$\vec{k}_t$ ⇒ transmitted propagation vector

$\beta_2 = |\vec{k}_t|$

⇒ $\vec{E} = E_0 \hat{a}_E e^{j(\omega t - \vec{k}\cdot\vec{r})}$

$\vec{E}_s = E_0 \hat{a}_E e^{-\vec{k}\cdot\vec{r}}$ ⟶ After suppressing time vector

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Incident Wave:

$$\vec{E_{is}} = [E_{io}\cos\theta_i\,\hat{a}_x + E_{io}\sin\theta_i(-\hat{a}_z)]\,e^{-j[x\beta_1\sin\theta_i + z\beta_1\cos\theta_i]} \quad \text{--- (1)}$$

$$\vec{k_i} = |\vec{k_i}|\cos\theta_i\,\hat{a}_z + |\vec{k_i}|\sin\theta_i\,\hat{a}_x$$

$$\vec{k_i} = \beta_1\cos\theta_i\,\hat{a}_z + \beta_1\sin\theta_i\,\hat{a}_x$$

$$\vec{r} = x\hat{a}_x + y\hat{a}_y + z\hat{a}_z$$

$$\vec{k_i}\cdot\vec{r} = \beta_1\sin\theta_i\,x + \beta_1\cos\theta_i\,z$$

$$\vec{H_{is}} = H_{io}\hat{a}_y\,e^{-j[x\beta_1\sin\theta_i + z\beta_1\cos\theta_i]}$$

$$= \frac{E_{io}}{\eta_1}\hat{a}_y\,e^{-j[x\beta_1\sin\theta_i + z\beta_1\cos\theta_i]} \quad \text{--- (2)}$$

Reflected Wave:

$$\vec{E_{rs}} = [E_{ro}\cos\theta_r\cdot\hat{a}_x + E_{ro}\sin\theta_r\,\hat{a}_z]\,e^{-j[x\beta_1\sin\theta_r - z\beta_1\cos\theta_r]} \quad \text{--- (3)}$$

$$\vec{k_r} = |\vec{k_r}|\cos\theta_r(-\hat{a}_z) + |\vec{k_r}|\sin\theta_r$$

$$\vec{H_{rs}} = H_{ro}(-\hat{a}_y)\cdot e^{-j[x\beta_1\sin\theta_r - z\beta_1\cos\theta_r]}$$

$$= \frac{E_{ro}}{\eta_1}(-\hat{a}_y)\cdot e^{-j[x\beta_1\sin\theta_r - z\beta_1\cos\theta_r]} \quad \text{--- (4)}$$

Transmitted Wave:

$$\vec{E_{ts}} = [E_{to}\cos\theta_t\cdot\hat{a}_x + E_{to}\sin\theta_t(-\hat{a}_z)]\cdot e^{-j[z\beta_2\cos\theta_t + x\beta_2\sin\theta_t]} \quad \text{--- (5)}$$

$$\vec{H_{ts}} = \frac{E_{to}}{\eta_2}\hat{a}_y\,e^{-j[z\beta_2\cos\theta_t + x\beta_2\sin\theta_t]} \quad \text{--- (6)}$$

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Boundary Conditions:

$E_{t1} = E_{t2}$ at $z=0$ and $H_{t1} = H_{t2}$ at $z=0$

⇒ All the x, y components are tangential components and z components are normal components.

$\vec{E}_{t1} = \vec{E}_{is}(z=0) + \vec{E}_{rs}(z=0) \Longrightarrow$ for medium (1)

$\vec{E}_{t2} = \vec{E}_{ts}(z=0) \Rightarrow$ for medium (2)

$$\Rightarrow E_{t1} = E_{t2}$$

$$E_{io}\cos\theta_i \, e^{-jx\beta_1\sin\theta_i} + E_{ro}\cos\theta_r \, e^{-jx\beta_1\sin\theta_r} = E_{to}\cos\theta_t \, e^{-jx\beta_2 \ldots}$$

⇒ We know ⇒ $\left\{ \begin{array}{l} \theta_i = \theta_r \\ \beta_1\sin\theta_i = \beta_2\sin\theta_t \end{array} \right\}$

$$\Rightarrow E_{io}\cos\theta_i \cdot \cancel{e^{-jx\beta_1\sin\theta_i}} + E_{ro}\cos\theta_i \, \cancel{e^{-jx\beta_1\sin\theta_i}} = E_{to}\cos\theta_t \cdot \cancel{e^{-jx\beta_2 \ldots}}$$

$$\Rightarrow E_{io}\cos\theta_i + E_{ro}\cos\theta_i = E_{to}\cos\theta_t$$

$$\Rightarrow E_{io} + E_{ro} = E_{to}\frac{\cos\theta_t}{\cos\theta_i}$$

$$\Rightarrow \boxed{1 + \frac{E_{ro}}{E_{io}} = \frac{E_{to}}{E_{io}}\frac{\cos\theta_t}{\cos\theta_i}} \quad \text{— (7)}$$

$$\Rightarrow \boxed{1 + \Gamma_{11} = \tau_{11}\frac{\cos\theta_t}{\cos\theta_i}} \quad \text{— (C)}$$

$$\Rightarrow H_{t1} = H_{t2}$$

$$\frac{E_{io}}{\eta_1}e^{-jx\beta_1\sin\theta_i} - \frac{E_{ro}}{\eta_1}e^{-jx\beta_1\sin\theta_r} = \frac{E_{to}}{\eta_2}e^{-jx\beta_2\sin\theta_2}$$

$$\Rightarrow \frac{E_{io}}{\eta_1}\cancel{e^{-jx\beta_1\sin\theta_i}} - \frac{E_{ro}}{\eta_1}\cancel{e^{-jx\beta_1\sin\theta_i}} = \frac{E_{to}}{\eta_2}\cancel{e^{-jx\beta_1\sin\theta_i}}$$

$$\Rightarrow E_{io} - E_{ro} = E_{to}\frac{\eta_1}{\eta_2}$$

$$\Rightarrow \boxed{\frac{\cancel{E_{io}}}{1} - \frac{E_{ro}}{E_{io}} = \frac{E_{to}}{E_{io}}\frac{\eta_1}{\eta_2}} \Rightarrow \boxed{1 - \frac{E_{ro}}{E_{io}} = \frac{E_{to}}{E_{io}}\frac{\eta_1}{\eta_2}}$$

$$\Rightarrow \boxed{1 - \Gamma_{11} = \frac{\eta_1}{\eta_2}\tau_{11}} \quad \text{— (D)}$$

Using C & D:

$$2 = \left(\frac{\cos\theta_t}{\cos\theta_i} + \frac{\eta_1}{\eta_2}\right)\tau_{11}$$

$$2 = \left(\frac{\eta_2\cos\theta_t + \eta_1\cos\theta_i}{\eta_2\cos\theta_i}\right)\tau_{11}$$

$$\boxed{\tau_{11} = \frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_i}}$$

$$1 - \Gamma_{11} = \frac{\eta_1}{\eta_2}\left(\frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_i}\right)$$

$$\Rightarrow \boxed{\Gamma_{11} = \frac{\eta_2\cos\theta_t - \eta_1\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_i}} \quad \text{— (E)}$$

Brewster Angle:- Angle of incidence for which reflection co-efficient is zero is called Brewster angle.

If $\theta_i = \theta_{B11} \Rightarrow \Gamma_{11} = 0$

from eqn (E)

$$\Gamma_{11} = \frac{\eta_2\cos\theta_t - \eta_1\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_i} = 0$$

$$\Rightarrow \eta_2\cos\theta_t - \eta_1\cos\theta_i = 0$$

$$\boxed{\eta_2\cos\theta_t = \eta_1\cos\theta_i}$$

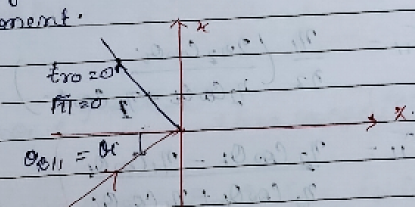
$$\Rightarrow \boxed{\theta_{B11} = \sin^{-1}\sqrt{\frac{1 - \frac{\mu_2\varepsilon_1}{\mu_1\varepsilon_2}}{1 - (\varepsilon_1/\varepsilon_2)^2}}}$$

Significance of Brewster Angle:-

It shows that by proper choice of angle of incidence the reflection from any material for a parallel polarised wave can be cancelled.

If a wave has electric field intensity which has component parallel and perpendicular to the plane of incidence and if wave incident on a material interface at brewster angle, the reflection of parallel polarised component is cancelled but not that of perpendicularly polarised.

The reflected wave consist of perpendicular polarised component.



If $\theta_{B\parallel} = \theta_i$ ⇒ there is no reflected wave.

⇒ But if still there is reflected wave then it is perpendicularly polarised wave.

→ For any general wave, polarised or unpolarised the reflected wave will be linearly perpendicularly polarised to the plane of incidence if angle of incidence is brewster angle.