

Semiconductor Physics

25.1 INTRODUCTION

Among all the classes of solids semiconductors are the most interesting and useful substances. These materials exhibit a wide spectrum of phenomena, covering the entire range from the strictly metallic to the insulator, having extremely versatile in terms of applications. The variety of physical devices employing semiconductors is truly impressive. Although semiconductors have been studied for a long time since the 1920's but they become more popular after the invention of transistor during 1940's by Shocklery, Bardeen and Brattain. With the invention of transistors and development of other related devices, semiconductors have become the most actively studied substances in solid state physics. During these studies we have learned much about the basic properties of these solids, and how to utilize them in designing still newer and more efficient devices.

This chapter is devoted to the basic physical properties of semiconducting materials such as qualitative discussion of band theory of solids with special reference to the band structure of pure semiconductors, classification of solids on the basis of band theory, charge density in semiconductor, effect of temperature and doping on the band structure and carrier concentration, extrinsic semiconductors, idea of effective mass, density of state expression of density of states in valence band and conduction band, fermi distribution function and importance of fermi level, carrier concentration in valence and conduction band of intrinsic and extrinsic semiconductors, position of fermi level in intrinsic and extrinsic semiconductors, mobility of electron and holes in valence band conduction band, expression for conductivity, resistivity, expression for conductivity of intrinsic semiconductors and experimental determination of band gap in intrinsic semiconductors.

25.2 ENERGY BANDS IN SOLIDS

In an isolated atom, there are discrete energy levels as $1s, 2s, 3s, \dots$ which are occupied by the electrons of the atom, as shown in Fig. 25.1(a). If all the atoms of a solid are assumed to be isolated from one another, then they have completely identical electronic schemes of their energy levels. When these isolated atoms are brought together to form a solid, various interactions occur between the neighbouring atoms. During this interaction, the higher energy levels are affected considerably, and consequently, the energy levels of the outer shells are slightly altered without violating Pauli's exclusion principle. Hence, the single energy levels of atoms split into large number of energy levels as shown in Fig. 25.1(b). Since, there are

2 ● Engineering Physics

many atoms in a solid, so the separation between the split energy levels is very small. Hence, the split energy levels are almost continuous and are said to form an energy band.

There are energy bands in a solid corresponding to the energy levels in an atom. An electron in a solid can have only those discrete energies that lie within these energy bands. These energy bands are known as the allowed energy bands, which are generally separated by some energy gap known as the forbidden energy bands or not allowed energy bands. Energy band occupied by the valence electrons is known as the valence band, and the energy band which is empty or occupied by the conduction electrons is known as the conduction band.

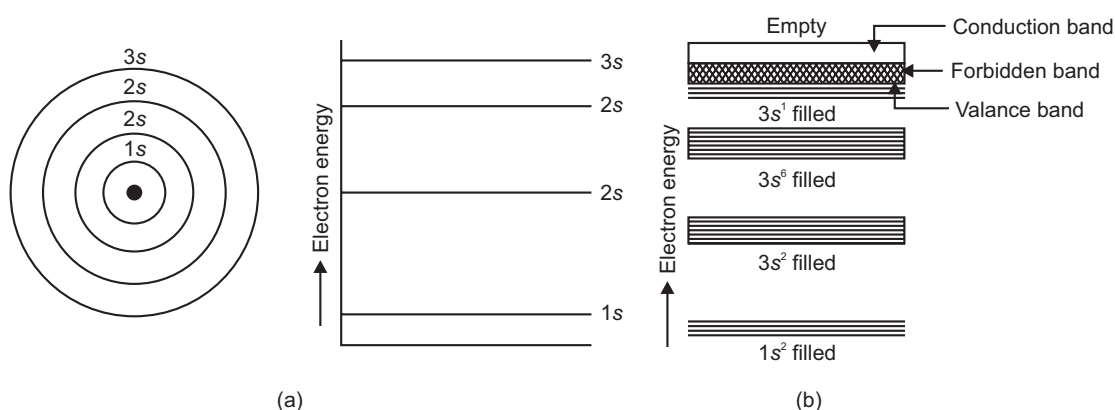


Fig. 25.1 (a) Electrons of an isolated atom in discrete energy levels and (b) energy bands formed due to the interaction between different sodium atoms in a solid

In order to clarify the concept of valence band and conduction band. Let us take an example of sodium crystal. This crystal has many closely packed sodium atoms, which is usually referred to as crystal lattice. Each sodium atom has eleven electrons with electronic configuration $1s^2, 2s^2, 2p^6$, and $3s^1$. In this electronic configuration, all the subshells are filled except $3s^1$. The outermost electron $3s^1$ of this atom is called valence electron. When more number of atoms are brought together to form a solid, then the outer energy levels are split to form an energy band (i.e., large number of split energy levels closely spaced). In this band, there are two regions: one region is occupied by the valence electrons known as, valence band, whereas the other region is either empty or occupied by the conduction electrons (called conduction band) as shown in Fig. 25.1(b). Valence band is also known as the highest occupied band, whereas the conduction band is the lowest unoccupied band. The forbidden energy gap between the conduction band and the valence band is known as the energy band gap.

25.3 CLASSIFICATION OF SOLIDS ON THE BASIS OF ENERGY BAND DIAGRAM

On the basis of the forbidden energy gap, solid are broadly classified into following three categories:

- (i) conductors (metals), (ii) insulators, (iii) semiconductors,

The electrical properties of the materials can be explained by the concept of energy bands. Depending on the nature of band occupation by electrons and the width of forbidden bands, some important characteristics of above said class of solids are discussed below.

25.3.1 Conductors (Metals)

In case of conductors, there is no forbidden gap between the valence band and the conduction band. It is observed that the valence band overlaps with the conduction band in metals as shown in Fig. 25.2. There are sufficient number of free electrons, available for electrical conduction. Since, conduction band and valence band overlap in case of metals, the electrons from valence band freely enter the conduction band. Most important point in conductors is that due to the absence of forbidden gap between the conduction band and the valence band, there is no chance of the presence of holes. Hence, the total current in conductors is simply due to the flow of electrons only.

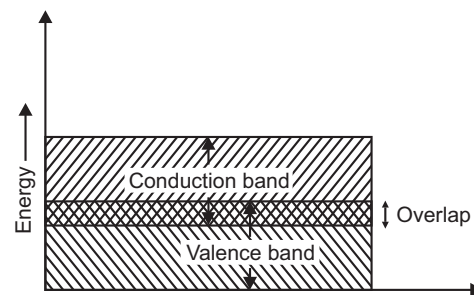


Fig. 25.2 Conduction band and valence band conductors

For further discussion, let us consider the example of sodium, which has electronic configuration as $1s^2, 2s^2, 2p^6, 3s^1$. From this configuration, it is clear that $1s, 2s, 2p$ bands in sodium are completely filled, which the outermost ($3s$) band has only one electron and can accommodate one more electron. Thus, the outer valence band is half-filled. Under the influence of external electric field, electrons of valence band acquire sufficient kinetic energy to move towards the conduction band, where they can move freely to participate in the process of conduction.

Thus, we can conclude that in conductors, there is overlapping between valence and conduction bands. Hence, valence band itself behaves as the conduction band. Due to this reason, metals in general have high electrical conductivity.

25.3.2 Insulators

A class of solids behaves as insulator if it satisfies the following two conditions.

- (i) it has even number of valence electrons per atom and
- (ii) the valence band the conduction band are separated by a large energy gap compared with kT .

The band diagram of insulators is given in Fig. 25.3(a). Actually, due to the large band gap, electrons cannot jump from the valence band to the conduction band. In insulators, the valence electrons are bound tightly to their parent atoms. For example, in case of glass, the valence band is completely filled at 0 K and energy band gap between the valence band and the conduction band is of the order of 10 eV. An external electric field cannot provide this amount of energy to an electron in solid. Therefore, such solids behave as insulators because no free electrons are available to conduct electricity. Hence, we can conclude that the insulators are those solids which have full valence band an empty conduction band with a large energy band gap between the valence band and the conduction band.

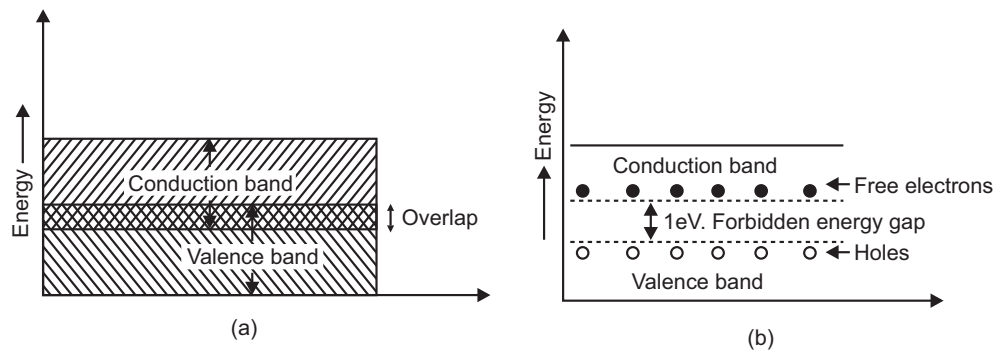


Fig. 25.3 Conduction band and valence band in **insulators**

25.3.3 Semiconductors

In case of semiconductors, the energy band gap (forbidden gap) between the filled valence band and the empty conduction band is small as compared to the insulators and more as compared to the conductors. Usually, this energy band gap is of the order of 1 eV. Energy band structure of semiconductors is shown to Fig. 25.3(b). Germanium and silicon are the common examples of semiconductors. In germanium the forbidden energy gap is 0.7 eV, whereas in case of silicon, it is 1.1 eV. Due to the small energy band gap between the valence band and the conduction band, the electrons of valence band can be thermally excited to the conduction band, where they are able to conduct small current on application of electric field to the crystal. At 0 K temperature, a semiconductor behaves as an insulator, but at room temperature a semiconductor has the following peculiar properties:

- (i) A partially filled conduction band
- (ii) A partially filled valence band
- (iii) A very narrow energy band gap (\cong eV)

25.4 THEORY OF BAND FORMATION IN SOLIDS

According to the Bohr's quantum mechanical theory for isolated atom electron have only discrete energy levels considering the simplest atom (hydrogen atom) the energy of n th orbit of an isolated atom can be given as

$$E_n = -\frac{q^4 m_0}{8 \epsilon_0^2 h^2 n^2} \quad (25.1)$$

where

- m_0 = mass of free electron
- q = charge of electron
- ϵ_0 = Permittivity of free space
- h = Plank's constant
- n = Principal quantum number

Energy expressed in equation (25.1) is usually measured in electron volt which is abbreviated as eV.

Putting the values of different parameters in equation (25.1) we get

$$E_n = -\frac{13.6}{n^2} \quad (25.2)$$

where $n = 1, 2, 3 \dots$ and negative sign indicate that electrons are bound in their respective energy levels.

According to the detailed study it is observed that for higher principal quantum number ($n \geq 2$) energy levels split according to their angular momentum quantum number ($l = 0, 1, 2, \dots, n - 1$).

When two identical atoms are brought closer, the doubly degenerate energy level split into two closely spaced energy levels, as shown in Fig. 25.4. The splitting of energy level is the consequence of interaction of wave functions of electrons of individual energy level.

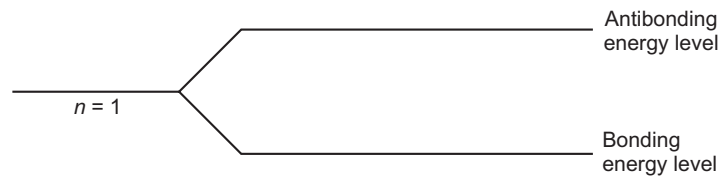


Fig. 25.4 Splitting of single energy level into doubly degenerate energy level.

Similar to above discussion when N isolated atoms are brought together to form a solid, the orbits of the outer electrons of different atoms overlap and interact with other. This interaction including those forces of attraction and repulsion between atoms, causes a shift in the energy levels. Due to this interaction instead of two levels, N separate but closely levels are formed. For the large value of N result is an essentially continuous band of energy. Depending on the interatomic spacing for the crystal the band of N levels can extend over a few eV. The splitting of degenerate state into a band of allowed energies is shown in Fig. 25.5, where a is the equilibrium interatomic distance.

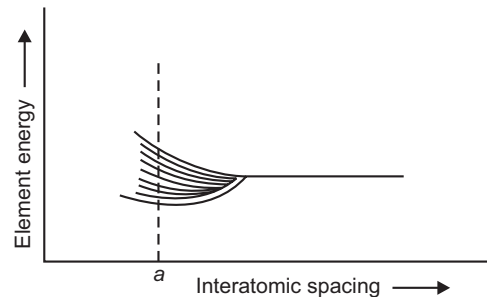


Fig. 25.5 Splitting of degenerate state into a band of allowed energies.

25.5 BAND FORMATION IN PURE SEMICONDUCTOR (SILICON)

In case of semiconducting materials the actual band splitting is much more complicated. In order to illustrate the formation of band in pure semiconductor let us consider the case of silicon. Isolated atom of silicon have 14 electrons which has been shown in Fig. 25.6. Out of these 14 electrons, 10 occupy deep lying energy levels whose orbital radius is much smaller than the interatomic separation in the crystal. The four remaining valence electrons of outer energy level ($n = 3$) are loosely bound and can be involved in the chemical reactions or in the process of conduction under the influence of external

impetus. Thus we need to consider only the outer shell (for $n = 3$) for the valence electrons, because rest two inner shells are completely full and tightly bound to the nucleus. The $3s$ subshell (i.e. for $n = 3$, and $l = 0$) has two allowed quantum states per atom. This subshell contains two valence electrons at $T = 0$ K. The $3p$ subshell (for $n = 3$ and $l = 1$) has six allowed quantum state per atom and contains remaining two valence electrons.

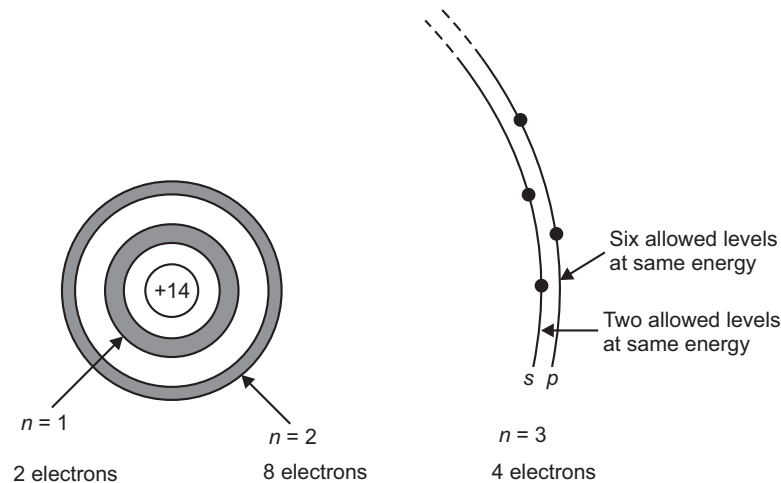


Fig. 25.6 Isolated atom of silicon with 14 electrons.

The schematic diagram of the formation of band in a silicon crystal with N isolated silicon atoms is shown in Fig. 25.7.

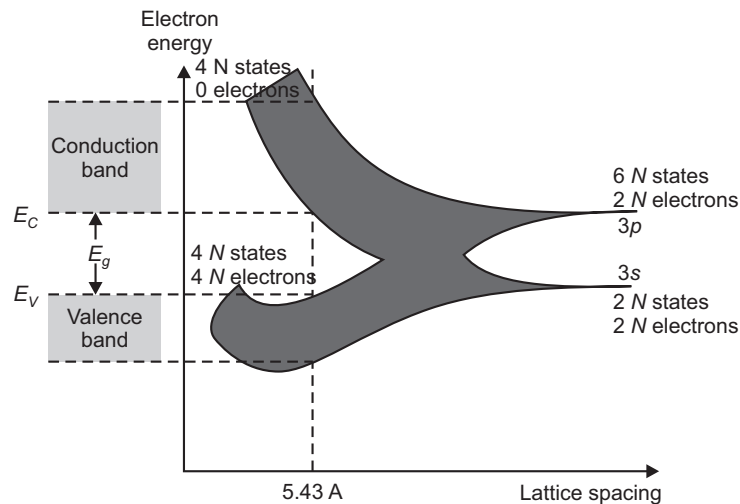


Fig. 25.7 Schematic diagram of the formation of bands in silicon crystal with N atoms.

As the interaction distance decreases, the $3s$ and $3p$ subshell of N silicon interact and overlap to form bands. When $3s$ and $2p$ bands grow, they merge into a single band and this band (band of $3s$ - $3p$ levels) contains $8N$

available states. At the equilibrium interatomic spacing of silicon, the band splits into two bands separated by an energy gap E_g . The upper band contains $4N$ states called conduction band. The lower band called valence band, contains rest $4N$ states as it is shown in Fig. 25.7. At the temperature $T = 0$ K, the electron occupy the lowest energy states, so that all the states in the lower band (the valence band) will be full and all states in the upper band (conduction band) will be empty. The lowest energy level of conduction band (bottom of conduction band) is called E_C and highest energy level (top of the valence band) is known as E_V .

The energy band gap between the bottom of the conduction band top of the valence band ($E_C - E_V$) is known as E_g as it is indicated in the left of Fig. 25.7. Physically, E_g is the energy required to break a band in the semiconductor to free the electron to the conduction band which leave a hole in the valence band.

25.6 IDEA OF EFFECTIVE MASS OF ELECTRON

It is observed that a moving electron in the crystal lattice of a solid material faces interaction with the atoms which affect its movement. Due to this reason the movement of electron is not described by Newton's law of motion. In order to describe the motion of electron according to the Newton's law of motion the concept of effective mass is introduced. Effective mass of electron may be different due to the different circumstances.

Let us consider on electron inside a crystal in a state k with the electronic wave function $\exp(ik)$. For a wave packet the group velocity can be given as

$$v = \frac{dw}{dk} = \frac{1}{\hbar} = \frac{d(\hbar w)}{dk} = \frac{1}{\hbar} \left(\frac{dE}{dk} \right) \quad (25.3)$$

Under the influence of external electric field, the force experienced by the electron is

$$F = \frac{dp}{dt} = m \frac{dv}{dt} \quad (25.4)$$

where p is the momentum of electron.

Now putting the value of v in equation (25.4) from equation (25.3) and using the relation $p = \hbar k$ we get,

$$\begin{aligned} \frac{\hbar}{m} \frac{dk}{dt} &= \frac{d}{dt} \left[\frac{1}{\hbar} \frac{dE}{dk} \right] \\ \text{or} \quad \frac{\hbar}{m} \frac{dk}{dt} &= \frac{1}{\hbar} \frac{d}{dt} \left[\frac{dE}{dk} \right] \\ &= \frac{1}{\hbar} \frac{d}{dk} \left(\frac{dE}{dk} \right) \frac{dk}{dt} \end{aligned}$$

$$\text{or} \quad \frac{1}{m} = \frac{1}{\hbar^2} \left(\frac{d^2 E}{dk^2} \right)$$

Now replacing m by m^* in above equation we get

$$m^* = \frac{\hbar^2}{\frac{d^2 E}{dk^2}} \quad (25.5)$$

Equation (25.5) gives the idea of effective mass of an electron moving in the crystal.

Effective mass of electron has a special importance in solid state electronics because the results of free electron theory can be applied in band theory of solids by replacing the rest mass m of electron by effective

mass of electron. It is used in transport calculations, such as transport of electrons under the influence of fields or carrier gradients in different semiconductors. Also used to calculate the density of states.

25.7 BAND STRUCTURE IN SEMICONDUCTING MATERIALS WITH THE CONCEPT OF EFFECTIVE MASS

Semiconductors are the solids in which the highest occupied energy band, the valence band, is completely full at $T = 0^\circ\text{K}$, but in which the gap above this band is also small, so that electrons may be excited thermally at room temperature from the valence band to the next-higher band, which is known as the conduction band. Usually the number of excited electrons is appreciable (at room temperature) whenever the energy gap E_g is less than 2 eV. The material may then be classified as a semiconductor.

When electrons are excited across the gap, they reach to the bottom of conduction band (CB), thus the population of electron increase, and the top of valence band (VB) populated by holes. Due to above process, both bands are now only partially filled, and would carry a current if an electric field is applied. Although the conductivity of semiconductor is very small but it is sufficiently large for many practical purposes.

Only the conduction band and valence bands are of much interest to us here, because only these two bands contribute to the current. Other bands below valence band are completely filled and those above conduction band are completely empty, thus these group of bands contribute no current; hence they may be ignored so far as semiconducting properties are concerned.

Simple band structure of a semiconductor is shown in Fig. 25.8. The energy of the conduction band can be given as

$$E_c(k) = \frac{\hbar^2 k^2}{2m_e^*} + E_g - \frac{\hbar^2 k^2}{2m_e^*} \quad \dots(25.6)$$

where k is the wave vector and m_e^* is the effective mass of the electron. E_g is the energy band gap. The zero energy level lie at the top of valence band.

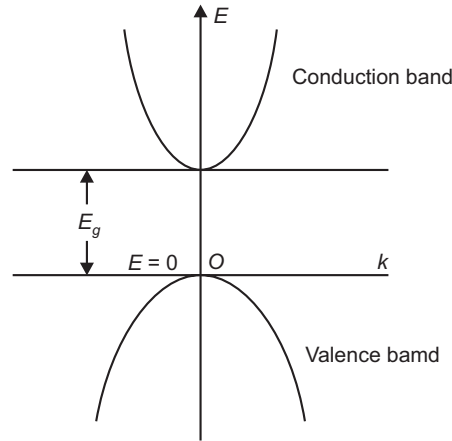


Fig. 25.8 Band structure in semiconductor

The energy of the valence band can be given as

$$E_V(k) = \frac{\hbar^2 k^2}{2m_h^*} \quad (25.7)$$

where m_h^* is the effective mass of the hole. As it is clear from above equations the primary band structure parameters are the electron and hole masses m_e and m_h and the band gap E_g . These parameters for some semiconductors are given in table 25.1

Table 25.1 Parameters for band structure of semiconductors at room temperature

Group	Crystal	E_g , eV	Effective mass, m/m_o	
			Electrons	Holes
IV	C	5.3	$m_l = 0.97, m_t = 0.91$	0.5, 0.16
	Si	1.1	$m_l = 1.6, m_t = 0.08$	0.3, 0.04
	Ge	0.7		
	Sn	0.08		
III-V	Ga As	1.4	0.07	0.09
	Ga P	2.3	0.12	0.50
	Gasb	0.7	0.20	0.39
	In As	0.4	0.03	0.02
	In P	1.3	0.07	0.69
	In Sb	0.2	0.01	0.18

Note

m_l and m_t are the longitudinal and transverse masses, respectively, of ellipsoidal energy.

The conduction and valence bands in semiconductors are related to the atomic sates. In order to understand it, let us take the example of hydrogen molecule, when two hydrogen atoms are brought together to form a molecule the atomic $1s$ state splits into two states: a low-energy bonding state and a high-energy antibonding state. In solid hydrogen, these states broaden into bonding and antibonding energy bands, respectively. Similarly, the valence and conduction bands in semiconductors respectively, the bonding and antibonding bands of the corresponding atomic valence states. Thus the valence band and conduction band in si, for example, result from the bonding and antibonding stats of the hybrid $3s^1 3p^3$. Similar explanations can be given to the bands in Ge, C and other semiconductors.

25.8 DENSITY OF STATES

Density of states is very important for the better understanding of electronic processes, particularly is transport phenomena. The density of states for electrons in a band yields the number of states in a certain energy range. It is denoted by $D(E)$ and defined as

$$D(E) dE = \text{The number of electron states per unit volume in the energy range } (E, E + dE) \quad (25.8)$$

In order to evaluate $D(E)$ we use the definition given in equation (25.8). Let us draw a shell in k -space which inner and outer surfaces are determined by the energy contours $E(k) = E$ and $E(k) = E + dE$, respectively, as shown in Fig. 25.9.

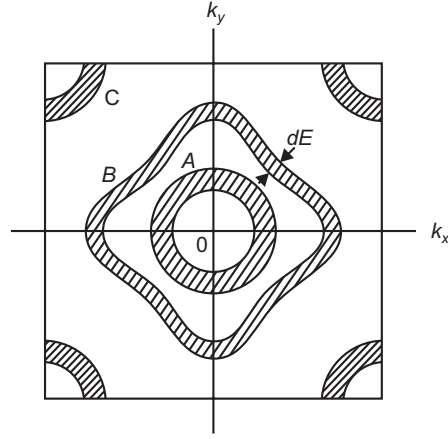


Fig. 25.9 Shells in k -space used to evaluate $D(E)$

The number of k values lying inside this shell then gives the number of states. When this number of states divided by the thickness of the shell dE , yields the desired function $D(E)$.

From the above discussions it is clear that $D(E)$ is closely related to the shape of the energy contours, and hence the band structure. The complexities of this structure are reflected in terms of the form taken by $D(E)$.

In order to evaluate $D(E)$ first we consider the case in which the dispersion relation for electron energy has the standard form,

$$E = \frac{\hbar^2 k^2}{2m^*} \quad (25.9)$$

As we know that the dispersion relation depicted in equation (25.9) are true for those states lying close to the bottom of the band near the origin of the Brillouin zone. The energy contours corresponding to equation (25.9) are clearly concentric spheres surrounding the origin. Thus the resulting density of states will be spherical in shape as illustrated by shell A in Fig. (25.9). The volume of this spherical shell can be given as $4\pi k^2 dk$, where k is the radius and dk is the thickness of the shell. The number of allowed k values per unit volume of k -space is $1/(2\pi)^3$, thus the number of states lying in the shell in the energy range E and $E + dE$ can be given as.

$$\text{Number of states} = \frac{1}{(2\pi)^3} 4\pi k^2 dk \quad (25.10)$$

The right hand side of equation (25.10) can be written in terms of energy rather than in terms of k , using equation (25.9). Now equation (25.10) can be expressed as.

$$\text{Number of states} = \frac{1}{4\pi^2} \left(\frac{2m^*}{\hbar^2} \right)^{3/2} dE^{1/2} \quad (25.11)$$

Comparing equation (25.11) with the definition of $D(E)$ expressed in equation (25.8). We get

$$D(E) = \frac{1}{4\pi^2} \left(\frac{2m^*}{\hbar^2} \right)^{3/2} dE^{1/2} \quad (25.12)$$

Considering the spin degeneracy each k state may accommodate two electrons of opposite spins thus the exact value of $D(E)$ can be obtained by multiplying right hand side of equation (25.12) by two. Hence $D(E)$ can be given as

$$D(E) = \frac{1}{2\pi^2} \left(\frac{2m^*}{\hbar^2} \right)^{3/2} dE^{1/2} \quad (25.13)$$

From the equation (25.13) it is clear that $D(E) \propto E^{1/2}$ which means that $D(E)$ has parabolic shape as shown in Fig. (25.7). $D(E)$ increases with E because from Fig. (25.10), for larger energy, radius is large thus the volume of the shell consequently the larger number of states will be within it. Again $D(E) \propto (m^*)^{3/2}$ which indicates that for larger mass greater density of states will be there.

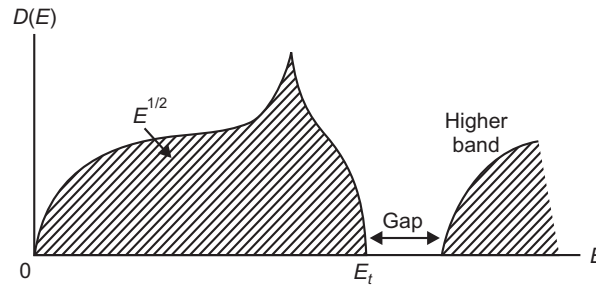


Fig. 25.10 The density of state

25.8.1 Density of States for Valence Band and Conduction Band

In order to discuss the density of states for valence and conduction band we need to understand what happens with the density of states at higher energy level? We can get the proper answer of this question from Fig. (25.9) where it is clear that at sufficiently large energies, the shell starts to intersect the boundaries of the zone, e.g. shell C in Fig. (25.9) in which case the volume of the shell begins to shrink with a concomitant decrease in the number of states. Thus the density of states decreases continuously with increase in energy and vanishes completely when the shell lies entirely outside the zone shown in Fig. (25.10).

The energy at which $D(E)$ vanishes is marked as the top of valence band. The density of states remains zero for certain energy range which marks the energy gap, until a new energy band appears, with its own density of states. The limit at which $D(E)$ vanishes is known as the top of valence band and the limit up to which it remains as such till the arrival of new energy band is known as the bottom of the conduction band.

The expression for the density of states in the energy range lying close to the top of the band can be derived if the band can be represented by negative effective mass as it is in case of lattice waves. Now using the same procedure as in case of lattice waves we can derive the general expression for $D(E)$, close to the top of the band. This expression can be given as

$$D(E) = \frac{1}{2\pi^2} \left(\frac{2|m^*|}{\hbar^2} \right)^{3/2} (E_t - E)^{1/2} \quad (25.14)$$

which E_t is the top of the valence band and $E < E_t$. Equation (25.14) is the general equation for density of states for conduction and valence band. We can write the expression of $D(E)$ for conduction band and valence band with choice of suitable value of E_t .

Now the density of state for conduction band can be given as

$$D_e(E) = \frac{1}{2\pi^2} \left(\frac{2m_e^*}{\hbar^2} \right)^{3/2} (E - E_c)^{1/2} \quad (25.15)$$

Where E_c is the energy of the bottom of conduction band. Similarly the density of state for valence band can be given as

$$D_h(E) = \frac{1}{2\pi^2} \left(\frac{2m_h^*}{\hbar^2} \right)^{3/2} (E_v - E)^{1/2} \quad (25.16)$$

Where E_v is the energy of the top of valence band.

25.9 FERMİ DISTRIBUTION FUNCTION

Fermi surface is closely related to the free electron model, the significance of this surface in solid-state physics has been derived from the fact that only those electrons lying near it participate in the process of conduction. In order to explain the presence of charge carriers in conduction band and valence band a Fermi distribution function is introduced. This function is represented by $F(E)$, which obey Fermi-Dirac statistics. This function is applicable for those particles which must obey following conditions:

- (i) The particles should be indistinguishable from each other i.e. there should be no restriction between different ways in which particles are chosen
- (ii) Each sublevel or cell may contain 0 or one particle
- (iii) The sum of energies of all the particles in the different quantum groups taken together constitute the total energy of the system.

These conditions are fully satisfied by the charge carrier in the conduction band i.e. electrons. Thus, the Fermi-Dirac distribution function can be applied to deal with the charge carriers in the conduction and valence band. Fermi-Dirac distribution function can be defined as.

$$F(E) = \frac{1}{e^{(E - E_F) / k_B T} + 1} \quad (25.17)$$

This function gives the probability that an energy level E is occupied by an electron when the system is at the temperature T . E_F is a energy reference level known as Fermi energy level. Fermi-Dirac function $F(E)$ is plotted versus E as shown in Fig. (25.11).

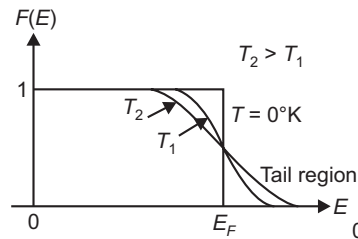


Fig. 25.11 The Fermi-Dirac distribution function

From the above plot it is clear that, as the temperature rises, the unoccupied region below the Fermi level E_F becomes longer which implies that the occupation of high energy states increases with increase

in temperature as a most common conclusion is that increasing the temperature raises the overall energy of the system. It is also seen that $F(E) = \frac{1}{2}$ at the Fermi level ($E = E_F$) regardless of the temperature.

That is, the probability that the Fermi level is occupied is always equal to one half.

In semiconducting materials the tail region of the FD distribution is of particular interest. In this region the inequality $(E - E_F) \gg k_B T$ holds true, and one may therefore neglect the term unity in the denominator of equation 25.17. Thus the FD distribution reduces to the form

$$F(E) = e^{E_F/k_B T} e^{-E/k_B T} \quad (25.18)$$

which is well known classical or Maxwell-Boltzman distribution, law, this simple distribution law is sufficient for the discussion of electron statistics in semiconductors.

25.10 FERMİ LEVEL (E_F) AND ITS IMPORTANCE

As it is discussed in the see 25.9 that energy distribution of electrons in a solid is governed by the Fermi-Dirac statistics. The probability of occupying any electronic state E by an electron is given by Fermi Dirac distribution function as:

$$F(E) = \frac{1}{1 + \exp\left(\frac{E - E_F}{k_B T}\right)} \quad (25.19)$$

which k_B is the Boltzman's constant, T is absolute temperature and E_F is the reference energy, called Fermi level. $F(E)$ is usually known as Fermi function. Let us consider two cases at $T = 0$ K.

Case I $E < E_F$, i.e. the desired energy level is below the Fermi level and hence (25.19) reduced to

$$F(E) = 1$$

From above condition it is clear that the probability of occupancy of any energy level below the Fermi level by carriers is 100% at $T = 0$ K, i.e. all the states below the Fermi level is completely filled by charge carriers at absolute zero temperature.

Case II $E > E_F$: At this condition equation (25.19) reduced to $F(E) = 0$ at $T = 0$ K from which it is clear that all the energy levels above the Fermi level is completely unoccupied by the charge carriers at $T = 0$ K.

Thus, Fermi level can be defined as the reference level above which all the levels are unoccupied by the charge carriers and below it all the levels are completely occupied by charge carriers at $T = 0$ K.

In order to define Fermi energy in another way let us consider $E = E_F$ at any temperature then Fermi-Dirac distribution function is given as

$$F(E = E_F) = \frac{1}{2} = 50\%$$

Thus the Fermi energy is the energy at which the probability of occupation is 50% at any temperature. If the energy is at least several $k_B T$ above the Fermi level then equation (25.19) can be given as.

$$F(E) \simeq e^{-\frac{(E - E_F)}{k_B T}} \quad (25.20)$$

Equation (25.20) gives the idea of the probability that an electron occupies a state located at energy E . Similarly, if the energy is at least several $k_B T$ below the Fermi level then equation (25.19) can be given as

$$F(E) \simeq 1 - e^{-\frac{(E_F - E)}{k_B T}} \quad (25.21)$$

Which gives the idea about the probability of occupied state located at energy E .

25.11 CARRIER CONCENTRATION IN INTRINSIC SEMICONDUCTOR

In semiconductor materials, electrons and holes are usually referred as free carriers or simply carriers, because these particles are responsible for carrying electric current in semiconductors. Thus the number of charge carriers are very important in semiconductors because they determine the electrical conductivity. In order to determine the number of charge carriers basic results of statistical mechanics is used. In this regard most important result is Fermi-Dirac (FD) distribution function as given in equation (25.19) i.e.,

$$F(E) = \frac{1}{\exp((E - E_F) / k_B T) + 1}$$

This function gives the probability that an energy level E is occupied by an electron when the system is at temperature T .

The carrier concentration in conduction band and valence band can be calculated by using the suitable values of respective density of states and limit of energy levels in these bands.

25.11.1 Concentration of electrons in the Conduction Band

In order to calculate the concentration of electrons in conduction band, let us consider the number of states in the energy range $(E, E + dE)$ is equal to $D_e(E)dE$, where $D_e(E)$ is the density of electron states. Since each of these states has an occupation probability $F(E)$, the number of electrons actually found in this energy range is equal to $F(E) D_e(E) dE$. The concentration of electrons throughout the conduction band can be obtained by the integration over the band i.e.

$$n = \int_{E_{C1}}^{E_{C2}} F(E) D_e(E) dE \quad (25.22)$$

where E_{C1} and E_{C2} are the bottom and top of the band, respectively as shown in Fig. 25.12(a)

It is clear from the fig 25.12(b) that the entire conduction band falls in the tail region. Thus we may use the Maxwell-Boltzmann function for $F(E)$. The expression for energy density appropriate to the standard band form can be given as

$$D_e(E) = \frac{1}{2\pi^2} \left(\frac{2m_e}{\hbar^2} \right)^{3/2} (E - E_g)^{1/2} \quad (25.23)$$

where it is considered that zero-energy level is lying at the top of the valence band. Thus $D_e(E)$ vanishes for $E < E_g$, and is finite only for $E_g < E$, as shown in fig 25.12(c).

Substituting the values of $D_e(E)$, $F(E)$ in equation (25.22) we get

$$n = \frac{1}{2\pi^2} \left(\frac{2m_e}{\hbar^2} \right)^{3/2} e^{E_F/k_B T} \int_{E_g}^{\infty} (E - E_g)^{1/2} e^{-E/k_B T} dE \quad (25.24)$$

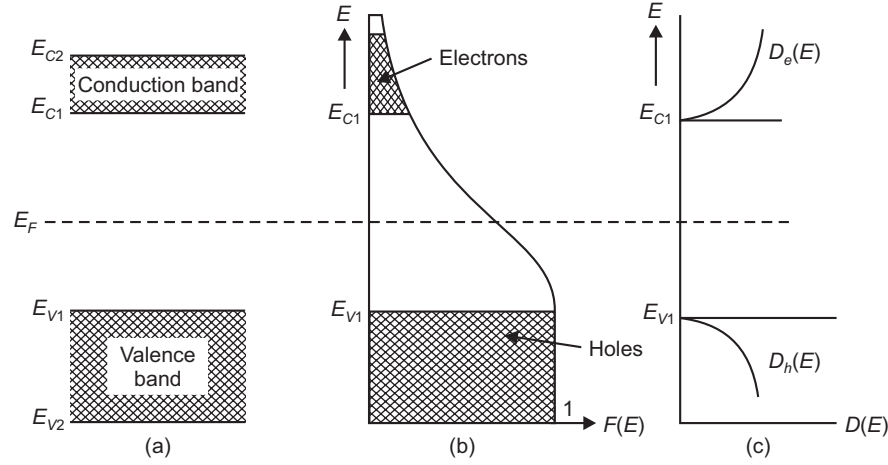


Fig. 25.12 (a) Conduction and valence band (b) Distribution function (c) Density of state for electrons and holes: $D_e(E)$ and $D_h(E)$.

In the above calculation top of conduction band has considered at infinity. Since the integrand decreases exponentially at high energies. The error introduced by changing this limit from E_{C2} to ∞ is almost negligible. By changing the variables and using the following result in equation (25.24)

$$\int_0^{\infty} x^{\frac{1}{2}} e^{-x} dx = \frac{\pi^{1/2}}{2}$$

We get

$$n = 2 \left(\frac{m_e k_B T}{2\pi\hbar^2} \right)^{3/2} e^{E_F/k_B T} e^{-E_g/k_B T} \quad (25.25)$$

This equation (25.25) gives the concentration of electron in conduction band.

25.11.2 Concentration of Holes in the Valence Band

Similar idea as we used for the calculation of electron concentration in conduction band, we will use to calculate the concentration of holes in valence band. The probability that a hole occupies a level E in valence band is equal to $1-F(E)$ since $F(E)$ is the probability of electron occupation, so the probability of hole occupation F_h can be given as

$$F_h = 1 - F(E) \quad (25.26)$$

Since in this case the involved energy range is much lower than E_F , so, the Fermi, Dirac function can be given as

$$F_h = 1 - \frac{1}{e^{(E-E_F)/k_B T} + 1} = \frac{1}{e^{(E_F-E)/k_B T} + 1} \approx e^{-E_F/k_B T} e^{E/k_B T} \quad (25.27)$$

In the above expression approximation has been done in view of the inequality $(E_F - E) \gg k_B T$. The validity of this inequality is also apparent in Fig. 25.11 (b), which shows that $E_F - E$ is of the order of $E_g/2$ which is much larger than $k_B T$ at room temperature. The density of states for the holes can be given as

$$D_h(E) = \frac{1}{2\pi^2} \left(\frac{2m_h}{\hbar^2} \right)^{3/2} (-E)^{1/2} \quad (25.27)$$

Here $(-E)$ in above equation is positive, because the zero energy level is at the top of the valence band, and the energy is measured positive upward and negative downward from this level.

Thus the hole concentration is given as

$$p = \int_{-\infty}^0 F_h(E) D_h(E) dE \quad (25.28)$$

Substituting the values of $F_h(E)$ and $D_h(E)$ in eq. (25.28) and evaluating the integral as in case of electron concentration we get

$$p = 2 \left(\frac{m_h k_B T}{2\pi\hbar^2} \right)^{3/2} e^{-E_F/k_B T} \quad (25.29)$$

Equation (25.29) gives the concentration of holes in the valence band.

25.11.3 Temperature Dependence of Electron Concentration in Conduction Band

The concentration of electrons in the conduction band is in fact equal to the holes in valence band. It is due to the fact that the electrons in the conduction band are due to the excitations from the valence band across the energy gap. Thus, for each electron, excited hole is created in the valence band. Therefore

$$n = p \quad (25.30)$$

Substituting the values of n and p in equation (25.30) we get an equation involving the only unknown, E_F , the solution of this equation is

$$E_F = \frac{1}{2} E_g + \frac{3}{4} k_B T \log \left(\frac{m_h}{m_e} \right) \quad (25.31)$$

Since $k_B T \ll E_g$, thus the first term of above equation will dominate and the fermi energy level will lie close to the middle of energy gap. This is consistent with earlier assertions that both the bottom of CB and top of the VB are far from the fermi level.

Now the concentration of electron can be evaluated explicitly by using equation (25.31) in equation (25.25), which gives us

$$n = 2 \left(\frac{k_B T}{2\pi\hbar^2} \right)^{3/2} (m_e m_h)^{3/4} e^{-E_g/2k_B T} \quad (25.32)$$

The most important feature of this expression is that n increases very rapidly-exponentially with temperature, particularly by virtue of the exponential factor. Thus it may be concluded that as the temperature is raised, a vastly greater number of electrons are excited across the gap.

A plot of $\log n$ versus $1/T$ has been shown in following Fig. 25.13. This curve is a straight line having its slope equal to $(-E_g/2k_B)$. The $[T]^{3/2}$ dependence in equation (25.32) is negligible in comparison with exponential dependence.

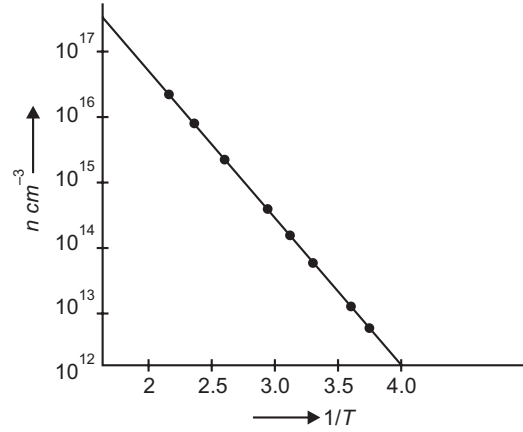


Fig. 25.13 Electron concentration n versus $1/T$ in Ge.

We can estimate the numerical value of n by substituting the values $E_g = 1$ eV, $m_e = m_h = m_o$ and $T = 300^\circ\text{K}$. Using these values we can find $n = 10^{15}$ electrons/cm³ a typical value of carrier concentration in semiconductors.

25.11.4 Effect of Doping on Band Structure and Carrier Concentration

In case of intrinsic semiconductor usually the electron and hole concentration is very less than the effective density of states in the conduction band or the valence band; respectively. But when small amount of impurities are doped in intrinsic semiconductor then its conductivity increases many fold. It is observed that for small amount of donor impurities doped in silicon and gallium arsenide, there is enough thermal energy to supply the energy E_D to ionize all donor impurities at room temperature. Due to ionization of donor impurities same number of electrons will be there in conduction band. In case of complete ionization electron density can be given as

$$n = N_d \quad (25.33)$$

where, N_d is the donor concentration. Due to complete ionization of donor impurities the donor level E_D is introduced just below the bottom of conduction band. Thus the equal number of electrons (which are mobile) and donor ions (which are immobile) will be introduced as shown in Fig. 25.14(a).

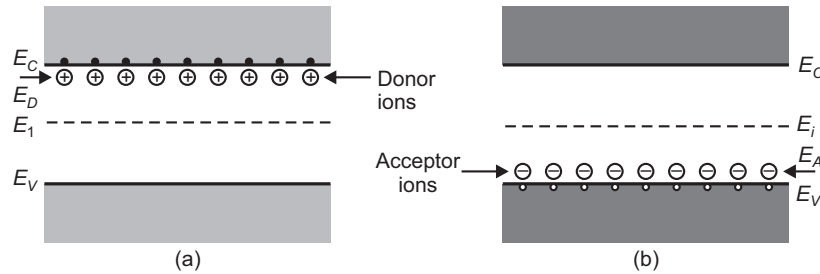


Fig. 25.14 (a) Effect of donor impurities in extrinsic semiconductor (b) Effect of acceptor impurities in extrinsic semiconductor

Due to the donor concentration N_d and effective density of states N_c , fermi level can be given as

$$E_C - E_F = k_B T \log \left(\frac{N_c}{N_d} \right) \quad (25.34)$$

Similarly if small amount of acceptor impurities are doped in semiconductor then at the condition of complete ionization the concentration of holes is given as

$$p = N_a \quad (25.35)$$

where N_a is the acceptor concentration. Under the effect of above doping fermi level can be given as

$$E_F - E_V = k_B T \log (N_v/N_a) \quad (25.36)$$

where N_v is the effective density of state. The effect of doping in terms of acceptor ions is shown in Fig. 25.14(b).

25.12 SEMICONDUCTOR STATISTICS (ESTIMATION OF CARRIER CONCENTRATION)

Usually in semiconductors both donors and acceptors are available. Electrons in the conduction band become available due to two main reasons, either by interband thermal excitation or by thermal ionization of the donors. Similarly holes in the valence band may be created either by interband excitation, or by thermal excitation of electrons from the valence into the acceptor level. In addition to the above processes, electron may fall from the donor level to the acceptor level. Above processes are shown in Fig. 25.15.

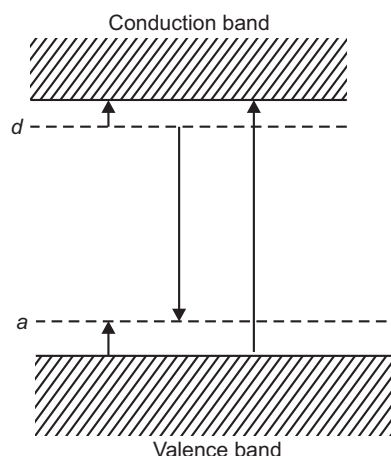


Fig. 25.15 Various electronic processes in the semiconductor material.

In the calculation for finding the concentrations of charge carriers (both electron holes) if we consider all the processes into the account, it become quite complicated. In order to avoid these complication we shall consider a few special cases which are often encountered in practice. Depending on the physical parameters, two regions may be distinguished as intrinsic and extrinsic regions.

25.12.1 Carrier Concentration Statistics for Intrinsic Region

Carrier concentration in intrinsic region is determined mainly by thermally induced interband transitions. Thus we have good approximation.

$$n = p \quad (25.37)$$

i.e. number of electrons are equal to the holes.

Now we can write the expression for carrier concentration as given in equation (25.25), namely

$$n = p = n_i = 2 \left(\frac{k_B T}{2\pi\hbar^2} \right)^{3/2} e^{-E_g/2k_B T} \quad (25.38)$$

where n_i is known as intrinsic concentration, obtained in intrinsic region when the impurity doping is small. For donors and acceptors concentrations N_d and N_a , validity of intrinsic condition is given as

$$n_i \gg (N_d - N_a) \quad (25.39)$$

In order to understand above condition let us consider that there are N_d electrons at the donor level, but of these a number N_a may fall into the acceptors, leaving only $N_d - N_a$ electrons to be excited from the donor level into the conduction band. According to the condition given in equation (25.39), the ionization of all remaining impurities is not sufficient to appreciably affect the number of electrons excited thermally from the valence band. At this condition semiconductor may be treated as a pure sample, and the effect of impurities are negligible.

From the expression (25.38) it is clear that n_i increases rapidly with temperature i.e. the intrinsic condition becomes more favourable at higher temperature. All semiconductors, in fact, become intrinsic at sufficiently high temperatures, unless the doping is unusually high.

25.12.2 Carrier Concentration Statistics for Extrinsic Region

Often it is observed that the intrinsic condition is not satisfied in semiconductors. For the common dopings encountered about 10^{15} cm^{-3} , the number of carriers supplied by the impurities is large enough to change the intrinsic concentration appreciably at room temperature. The contribution of impurities, in fact, frequently exceeds those carriers that are supplied by interband excitation. At these conditions the sample becomes in extrinsic region.

There are two types of extrinsic region semiconductors. First are those in which donor concentration greatly exceeds the acceptor concentration, i.e. $N_d \gg N_a$. For such materials the number of electrons can be evaluated quite easily, because the donor ionization energy is very small, thus all the donors are essentially ionized and their electrons go into the conduction band. Therefore, to a good approximation we can write

$$n = N_d \quad (25.40)$$

In this case the concentration of holes is small. In order to calculate it we use equation (25.25) and (25.29) because these equations are valid even in the case of doped samples. Using these two equations we can write

$$n \cdot p = 4 \left(\frac{k_B T}{2\pi\hbar^2} \right)^3 (m_e m_h)^{3/2} e^{-E_g/k_B T} \quad (25.41)$$

From the above equation (25.41) it is clear that fermi level (E_F) disappeared from right hand side, thus the product np is independent of E_F , and hence of the amount and type of doping, the product $n \cdot p$ depends only on the temperature.

Comparing equation (25.38) and equation (25.41) we can write

$$n \cdot p = n_i^2 \quad (25.42)$$

From equation (25.42) it is clear that if there is no change in temperature, the product $n \cdot p$ is a constant and independent of doping. It means if the electron concentration is increased, by varying the doping, the hole concentration decreases, and vice-versa. In case if doping is mainly due to donor type impurities then $n \simeq n_d$ as given in equation (25.40), using equation (25.40) and equation (25.42) we can write as

$$p = \frac{n_i^2}{N_d} \quad (25.43)$$

For extrinsic region, $n_i \ll N_d$, and hence $p \ll N_d = n$. In this way we conclude that the concentration of electrons is much larger than that of holes. Thus the sample of semiconducting materials in which the condition $n \gg p$ is satisfied are known as n -type semiconductor.

In other type of extrinsic region semiconductors $N_a \gg N_d$ i.e. the doping is mainly by acceptors. Using earlier argument (as in case of n -type semiconductor) for acceptor impurities we can write

$$p \simeq N_a \quad (25.44)$$

i.e. all the acceptors are ionized. In this case the electron concentration, which is small, is given by

$$n = \frac{n_i^2}{N_a} \quad (25.45)$$

such materials are known as p -type semiconductors which are characterised by the dominance of holes (acceptors).

For the ionization of almost all donors we assume sufficiently high temperature which is certainly true at room temperature. But when temperature is reduced continuously then a point is reached at which the thermal energy becomes too small to cause electron excitation. At this condition electrons falls from the conduction band into the donor level, and conductivity of sample reduced dramatically. Such condition is referred to as freeze-out, in which the electron are now “frozen” at their impurity sites. The temperature at which freeze-out condition occurs, can be calculated by the equation $E_d \simeq k_B T$, this temperature is about 100°K. The variation of the electron concentration with temperature in n -type sample is given in Fig. 25.16 with reference to the regions of freeze-out, extrinsic and intrinsic respectively.

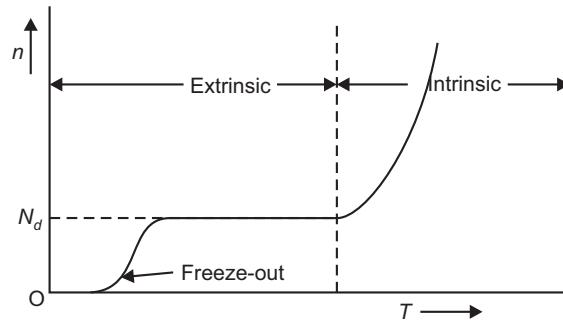


Fig. 25.16 Variation of electron concentration n with temperature in an n -type semiconductor.

25.12.3 Effect of Carrier Concentration on Band Structure

A semiconductor is said to be intrinsic or degenerate if electron and hole concentration is assumed to be much lower than effective density (i.e. $n_i \ll N_V$). Usually the number of carrier concentration of electrons and holes are same in number (i.e. $n_i = p_i$) and Fermi energy level lie exactly in middle of valence band and conduction band in intrinsic semiconductors. The schematic diagram of density of states, Fermi-Dirac distribution function and carrier concentration is shown in Fig. 25.17(a).

Consider a n -type of extrinsic semiconductor material ($N_d \doteq n_o \gg n_i$), in which electrons are the majority carriers and holes are minority carriers at thermal equilibrium means $n_o \gg p$. Here n_o is majority charge carriers in n -type and p_o is the majority charge carriers in p -type semiconductors. Similar to n -type semiconductors in p -type semiconductor ($N_a = p_o \gg n_i$) where holes are the majority charge carriers ($p_o \gg n_o$). According to the law of mass action the product $n_o p_o$ at equilibrium is constant and independent of the type of dopant.

Now for n -type semiconductors from equation (25.45) we can write:

$$p_o = \frac{n_i^2}{N_o}$$

Similarly using equation (25.43) for p -type semiconductors

$$n_o = \frac{n_i^2}{p_o}$$

The useful relation of E_F in terms of E_i and n_i for n -type semiconductors can be given as

$$E_F - E_i = k_B T \log \left(\frac{N_d}{n_i} \right)$$

Similarly for p -type semiconductors we can write

$$E_i - E_F = k_B T \log \left(\frac{N_d}{n_i} \right)$$

The schematic diagram for density of states Fermi-Dirac distribution function carriers concentration for extrinsic type semiconductors is shown in Fig. 25.17(b).

Above equation of fermi level predict that the distance between the fermi level E_F and intrinsic fermi level is a logarithmic function of doping concentration.

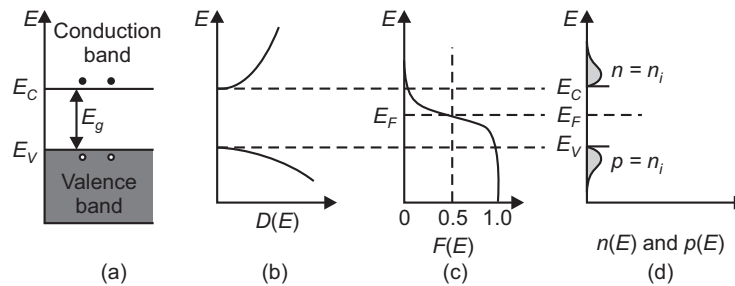


Fig. 25.17(a) Intrinsic semiconductors **(a)** Schematic band diagram **(b)** Density of state **(c)** Fermi distribution function **(d)** Carrier concentration

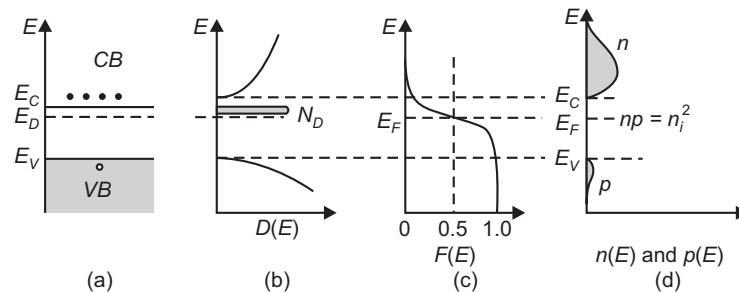


Fig. 25.17(b) Extrinsic semiconductors **(a)** Schematic band diagram **(b)** Density of state **(c)** Fermi distribution function **(d)** Carrier concentration

25.13 MOBILITY OF ELECTRON AND HOLES

In order to describe the current conduction mechanism in intrinsic semiconductors, let us take the example of germanium. The electronic configuration of germanium is $1s^2, 2s^2, 2p^6, 3s^2, 3p^6, 3d^{10}, 4s^2, 4p^2$, which shows that the outermost orbit contains four electrons. Thus, a germanium atom has four valence electrons, i.e., it is a tetravalent element. Each of the four valence electrons in a germanium atom is shared with the valence electrons of four adjacent germanium atoms making four electron pairs as shown in Fig. 25.18(a). Hence, all valence electrons of each germanium atom are used to form covalent bond with their neighbouring atoms and no electron is free to participate in the process of conduction, particularly at 0 K. In this way, pure germanium behaves as an insulator at 0 K. As the temperature increases, valence electrons acquire sufficient energy to break their covalent bond and become free to participate in the conduction process. Such electrons are known as *free electrons* or *conduction electrons*.

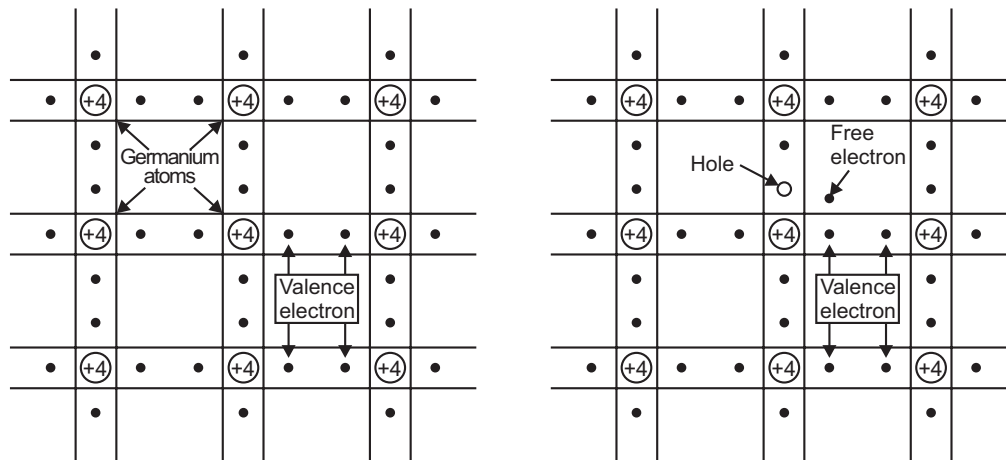


Fig. 25.18 **(a)** Two-dimensional representation of germanium crystal and **(b)** two-dimensional representation of germanium with an electron-hole pair

When electrons escape from the covalent bond, each electron leaves behind an empty space or electron deficiency known as *hole* as shown in Fig 25.18(b). In the conduction process, holes also act as current

carriers. Thus, an *electron–hole pair* is generated on breaking a covalent bond. Since a hole is a strong centre of attraction for an electron (because holes behave as positively charged particles), the hole attracts an electron from the neighbouring atom to fill it. When the electron jumps into the hole position to reconstruct the broken covalent bond, a hole is created at the initial position of the jumping electron. Thus, the motion of electron is also regarded as the motion of hole in the opposite direction. Thus electrons and holes move in opposite directions under the influence of external electric field. In this way, the current constituted in semiconductors is not only due to electrons but also due to both electrons and holes.

25.14 EXTRINSIC SEMICONDUCTORS

The electrical conductivity of intrinsic semiconductors can be increased by adding small impurity (generally, 1 impurity atom for 10^8 atoms of semiconductor). The process of adding impurities to semiconductors is known as *doping* and the impurity which is added is known as *dopant*. The semiconductor added to the impurity atoms is known as *doped* or *extrinsic semiconductor*. The added impurity may be pentavalent or trivalent. A few suitable pentavalent impurities are phosphorus, arsenic, antimony, etc., whereas trivalent elements are boron, aluminium, gallium, etc. Depending on the added impurity elements in pure semiconductors, the extrinsic semiconductors are of the following two types:

- (i) Donor or *N-type semiconductor*
- (ii) Acceptor or *P-type semiconductor*

25.14.1 Donor or N-type Semiconductor

When a pentavalent atom of group V (having five valence electrons) such as phosphorus, arsenic, or antimony is added to a pure semiconductor, then the resulting extrinsic semiconductor is known as the donor or *N-type semiconductor*.

If a small amount of phosphorus is added to the germanium crystal, then four of its five valence electrons form covalent bonds with the four neighbouring germanium atoms. The fifth valence electron remains loosely bound to its nucleus as shown in Fig. 25.19(a). Thus, each impurity atom donates a free

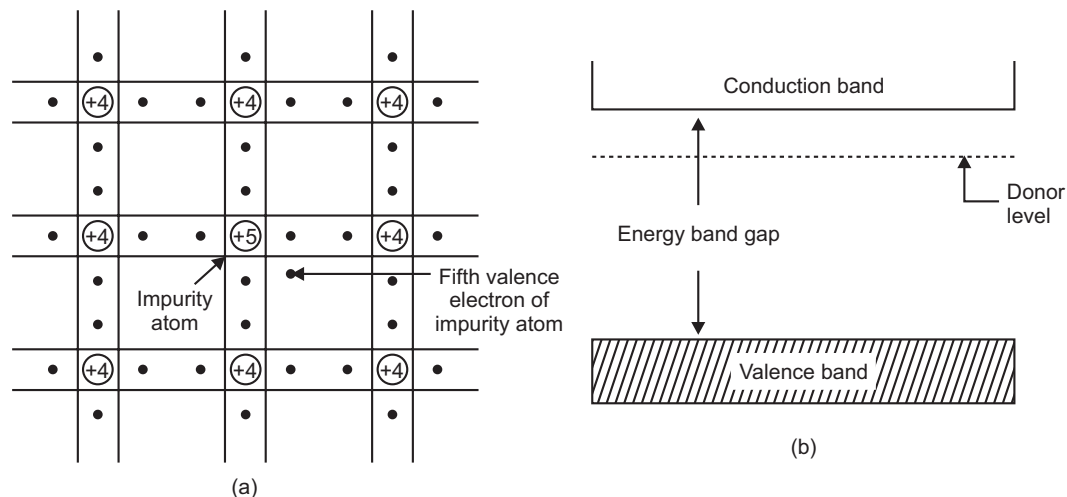


Fig. 25.19 (a) Pentavalent impurity in Ge crystal and (b) energy band diagram of N-type semiconductor

electron to the semiconductor. Due to this tendency of impurity atom, it is known as the donor impurity. Such an extrinsic semiconductor which has donor impurity is called *N*-type semiconductor because it has negative charge carriers (electrons).

The loosely bound electron of impurity atom requires small but definite amount of energy to detach itself from its parent nucleus to participate in the conduction process. This amount of energy is very small, which can be easily provided by the thermal agitation inside the crystal. The energy level corresponding to the loosely bound electron (fifth valence electron) lies in the band gap just below the conduction band and is known as the donor level as shown in Fig. 25.19(b).

In *N*-type semiconductors, electrons are the majority charge carriers but some holes are also present which are thermally generated at room temperature. These holes are known as minority charge carriers because these are very less as compared to the electrons.

25.14.2 Acceptor or P-type Semiconductor

When a trivalent atom of III group (having three valence electrons) such as boron, aluminium, gallium, etc., is added to a pure semiconductor, then the resulting extrinsic semiconductor is known as the acceptor or *p*-type semiconductor.

If a small amount of indium is added to the germanium crystal, then three valence electrons of indium form three covalent bonds with the three neighbouring germanium atoms, while the fourth bond is not completed due to the deficiency of one electron. This deficiency of an electron is known as a hole [Fig. 25.20(a)], which behaves as a positively charged particle. Since holes are the centre of attraction for the electrons, therefore, holes attract electrons from nearby covalent bonds. When an electron from a covalent bond jumps into a hole, the electron leaves an empty space (hole) at its initial position. This new hole is again filled by another electron from a nearby bond, giving rise to another hole and so on. Thus, a hole moves freely throughout the crystal lattice.

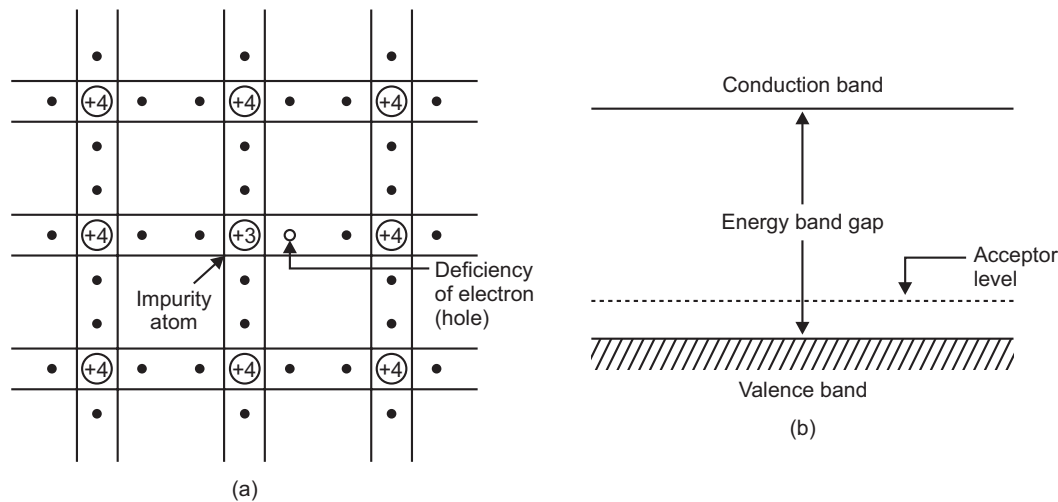


Fig. 25.20 (a) Trivalent impurity in Ge crystal and (b) energy band diagram of P-type semiconductor

The trivalent impurity atom has a tendency to accept one electron from a neighbouring germanium atom to complete the fourth covalent bond. Therefore, it is called as acceptor impurity. Holes participate in the conduction process through electrons. The energy required during this process is very small

which is supplied by thermal agitation. The energy level corresponding to the hole is located just above the valence band, which is known as the acceptor level, as shown in Fig. 25.20(b). The semiconductor containing acceptor type impurity has large number of positive charge carriers (holes), and therefore, it is known as *P*-type extrinsic semiconductor.

25.15 ENERGY BAND DIAGRAM AND POSITION OF FERMI LEVEL IN AN INTRINSIC SEMICONDUCTOR

As per earlier discussions it is clear that in an intrinsic semiconductor, the number of electrons in the conduction band is equal to the number of holes in the valence band. The concentration of electrons in the conduction band of intrinsic semiconductor is given by the equation (25.25) with slight modification regarding the consideration of limit for the occurrence of conduction band. Considering E_C as the energy value of bottom level of conduction band equation (25.25) and be written as

$$n = 2 \left(\frac{2\pi m_e k_B T}{h^2} \right)^{3/2} \exp \left[\frac{-(E_C - E_F)}{k_B T} \right] \quad (25.46)$$

Similarly the expression for hole can be given with slight modification in equation (25.29) with the consideration of top level energy value of valence band is E_V . Thus the expression for holes can be given as

$$p = 2 \left(\frac{2\pi m_h k_B T}{h^2} \right)^{3/2} \exp \left[\frac{-(E_F - E_V)}{k_B T} \right] \quad (25.47)$$

where m_e and m_h are the effective masses of electron and holes respectively at the absolute temperature T .

For intrinsic semiconductors $p = n$

so we have

$$\begin{aligned} \left(\frac{2\pi m_e k_B T}{h^2} \right)^{3/2} \exp \left[\frac{-(E_C - E_F)}{k_B T} \right] &= 2 \left(\frac{2\pi m_h k_B T}{h^2} \right)^{E_C - E_F} \exp \left[\frac{-(E_F - E_V)}{k_B T} \right] \\ \text{or } m_e^{3/2} \exp \left[\frac{-(E_C - E_F)}{k_B T} \right] &= m_h^{3/2} \exp \left[\frac{-(E_F - E_V)}{k_B T} \right] \\ \text{or } \exp \left[\frac{(E_F - E_V)}{k_B T} - \frac{(E_C - E_F)}{k_B T} \right] &= \left(\frac{m_h}{m_e} \right)^{3/2} \\ \text{or } \exp \left[\frac{2E_F - E_V - E_C}{k_B T} \right] &= \left(\frac{m_h}{m_e} \right)^{3/2} \end{aligned} \quad (25.48)$$

Taking the logarithm on both sides of above equation (25.48) we get

$$\begin{aligned} \left[\frac{2E_F - E_V - E_C}{k_B T} \right] &= \log \left(\frac{m_h}{m_e} \right)^{3/2} \\ \text{or } 2E_F - E_V - E_C &= \frac{3}{2} k_B T \log \left(\frac{m_h}{m_e} \right) \\ \text{or } 2E_F &= E_V + E_C + \frac{3}{2} k_B T \log \left(\frac{m_h}{m_e} \right) \end{aligned}$$

or
$$E_F = \left(\frac{E_V + E_C}{2} \right) + \frac{3}{4} k_B T \log \left(\frac{m_h}{m_e} \right)$$

Generally $m_e \simeq m_h$ so above equation takes the form

$$E_F = \frac{(E_V + E_C)}{2} \quad (25.49)$$

This expression shows that for an intrinsic semiconductor the fermi level E_F lies exactly in the centre of forbidden energy gap E_g between the conduction band E_C and valence band E_V . Energy band diagram and fermi level for intrinsic semiconductor is shown in Fig. 25.21.

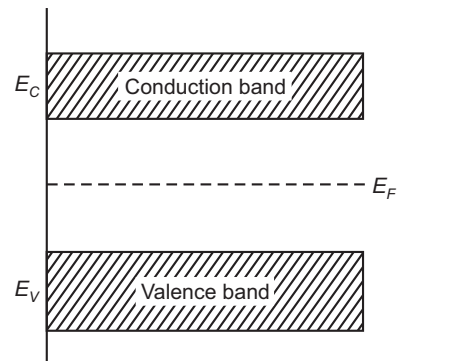


Fig. 25.21 Energy band diagram and Fermi level in intrinsic semiconductor.

25.16 ENERGY BAND DIAGRAM AND POSITION OF FERMİ LEVEL IN EXTRINSIC SEMICONDUCTOR

The addition of donor impurities contributes electron energy levels high in the band gap of semiconductor, so that electrons can be easily excited into the conduction band. Due to this reason effective fermi level shifts to a point about halfway between the donor level and the conduction band as shown in Fig. 25.22. Under the influence of externally applied voltage electrons can be elevated to the conduction band. Majority charge carriers are electrons here so these materials are said to be *N*-type semiconductor.

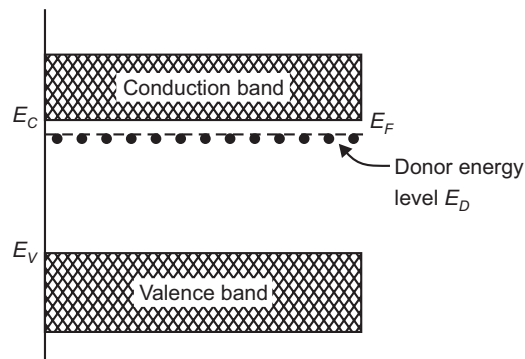


Fig. 25.22 Energy Band diagram of *N*-type semiconductor

For donor impurities, using equation (25.40) we can write

$$N_d = n$$

$$\text{or } N_d = 2 \left(\frac{2\pi m_e k_B T}{h^2} \right)^{3/2} \exp \left[\frac{-(E_C - E_F)}{k_B T} \right] \quad (25.50)$$

The temperature dependence of concentration of donor atoms shows that it changes significantly only for exponential term on Right hand side of above equation (25.50). Thus we can write

$$N_d = N_c \exp \left[\frac{-(E_C - E_F)}{k_B T} \right]$$

$$\text{or } \frac{N_d}{N_c} = \exp \left[\frac{-(E_C - E_F)}{k_B T} \right] \quad (25.51)$$

where $N_c = 2 \left[\frac{2\pi m_e k_B T}{h^2} \right]^{3/2} = \text{constant}$

Taking logarithm on both sides of above equation (25.51) we get

$$\log \left(\frac{N_c}{N_d} \right) = \left[\frac{(E_C - E_F)}{k_B T} \right]$$

$$\text{or } k_B T \log \left(\frac{N_c}{N_d} \right) = E_C - E_F$$

$$\text{or } E_F = E_C - k_B T \log \left(\frac{N_c}{N_d} \right) \quad (25.52)$$

From this equation it is clear that fermi level in *n*-type semiconductors is slightly below the bottom of conduction band as it is shown in Fig. 25.22.

In case of the addition of acceptor impurities contributes hole level low the semiconductor band gap so that electrons can be easily excited from valence band into these levels, leaving mobile holes in the valence band. This shifts the effective Fermi level to a point about halfway between the acceptor levels and the valence band as shown in Fig. 25.23. Under the influence of externally applied voltage, electrons can be elevated from valence band to the holes in the band gap. Since electron can be exchange between the holes, the holes are said to be mobile.

At normal temperature, using equation (25.37) we can write

$$N_a = p$$

$$\text{or } N_a = 2 \left(\frac{2\pi m_h k_B T}{h^2} \right)^{3/2} \exp \left[\frac{-(E_F - E_V)}{k_B T} \right] \quad (25.53)$$

From the above expression it is clear that the temperature dependence of concentration of acceptor atoms shows that it changes significantly only for exponential term on right hand side of above equation (25.53). Thus the above equation can be written as

$$N_a = N_c \exp \left[\frac{-(E_F - E_V)}{k_B T} \right]$$

$$\text{or} \quad \frac{N_a}{N_c} = \exp\left[\frac{E_V - E_F}{k_B T}\right] \quad (25.54)$$

$$\text{where} \quad N_V = 2 \left(\frac{2\pi m_h k_B T}{h^2} \right)^{3/2} = \text{constant}$$

Taking logarithm on both sides of above equation (25.54) we get

$$\log\left(\frac{N_a}{N_v}\right) = \left[\frac{(E_V - E_F)}{k_B T}\right]$$

$$\text{or} \quad k_B T \log\left(\frac{N_a}{N_c}\right) = E_V - E_F$$

$$\text{or} \quad = E_V + k_B T \log\left(\frac{N_V}{N_a}\right) \quad (25.55)$$

This expression shows that the Fermi level in *P*-type semiconductor lies slightly above the top of valence band as shown in Fig. 25.23.

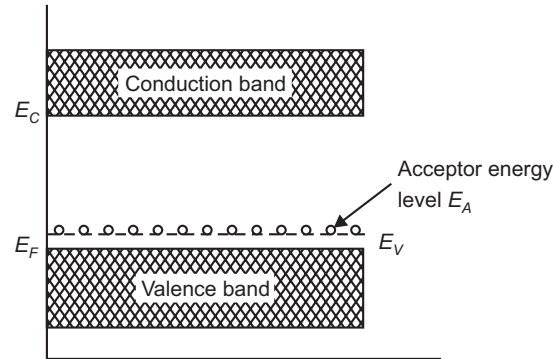


Fig. 25.23 Energy band diagram of *p*-type semiconductor

The position of Fermi level in *N*-type and *P*-type semiconductors can be understood by the expression of Fermi level energy too which is given by

$$E_F = \frac{h^2}{2m} \left[\frac{3\rho}{8\pi} \right]^{2/3} \quad (25.56)$$

where ρ is the density of charge carriers.

25.17 MOBILITY

In semiconductor physics mobility is an important transport coefficient, which is defined as the electron drift velocity per unit applied electric field strength. The electron drift velocity under influence of applied electric field E can be given as

$$v_e = -\frac{e\tau_e}{m_e} E \quad (25.57)$$

where negative sign is due to the negative charge on the electron. According to above definition electron mobility μ_e can be given as v_d/E . Therefore

$$\mu_e = \frac{V_d}{E} = \frac{e\tau_e}{m_e} \quad (25.58)$$

Mobility can also be defined as a measure of the rapidity or swiftness of the motion of the electron in the field. For the longer lifetime (τ_e) of the electron with smaller effective mass, the mobility will be high. Mobility of electrons and holes in some common semiconductors at room temperature is given in following table 25.2. Mobility of electrons and holes in valence band and conduction band is described in Section 25.13 of this chapter.

Table 25.2 Mobilities for various semiconductors at room temperature

S. No.	Crystal	Mobility μ ($\text{cm}^2/\text{volt-s}$)	
		Electron	Hole
1.	C	1800	1600
2.	Si	1350	475
3.	Ge	3900	1900
4.	GaAs	8500	400
5.	GaP	110	75
6.	GaSb	4000	1400
7.	InAs	33000	460
8.	InP	4600	150
9.	InSb	80000	750
10.	CdS	340	18
11.	CdSe	600	—
12.	CdTe	300	65
13.	ZnS	120	5
14.	ZnSe	530	16
15.	ZnTe	530	900

25.18 ELECTRICAL CONDUCTIVITY

The electrical conductivity in semiconductors is very important, because in these materials electrons and holes both contribute to electrical current. But in order to understand the conduction mechanism in semiconductor in simplified way let us consider a sample having only one type of charge carriers such as *N*-type semiconductor.

When an electric field is applied to *N*-type sample electrons drift opposite to the field and carry a net electric current. The electrons represented by an effective mass m_e and may be treated according to the free electron model. Now the electrical conductivity can be given as

$$\sigma_e = \frac{ne^2\tau_e}{m_e} \quad (25.59)$$

where τ_e is the lifetime of the electron. In order to calculate the numerical value of σ_e , we consider $n = 10^{15} \text{ cm}^{-3}$ or 10^{21} m^{-3} , $\tau_e = 10^{-12} \text{ s}$, and $m_e = 0.1 m_o$. With these values we get $\sigma_e \approx 1 (\text{ohm-m})^{-1}$, which is a typical figure in semiconductors. Although this value of conductivity of semiconductors is very less in comparison to metals i.e. of the order of $10^7 (\text{ohm-m})^{-1}$, but the conductivity of semiconductor is still sufficiently large for practical applications.

The small value of σ_e in semiconductors in comparison of metals is due the less concentration of conducting electrons in semiconductors. We can realise it knowing the fact that in metals electron concentration is 10^{28} m^{-3} while in semiconductors it is 10^{21} m^{-3} . The ratio of these figures substantially accounts for the relative values of conductivities.

25.19 EXPRESSION FOR CONDUCTIVITY OF SEMICONDUCTOR MATERIALS

The conduction mechanism in a semiconductor is different from a conductor. In case of a conductor, conductivity is given by only one type of the charge carriers (i.e., electrons), while in other case, it is given by both electrons and holes. If n is the number of free electrons per unit volume, with its mobility μ_e and charge e , then the conductivity due to the electrons is given as

$$\sigma_n = ne\mu_e \quad (25.60)$$

Similarly, if p is the number of holes per unit volume, with its mobility μ_h and charge e , then the conductivity due to holes is given as

$$\sigma_p = pe\mu_h \quad (25.61)$$

Now, the total conductivity of the semiconductor contributed by both electrons and holes will be given as

$$\sigma = \sigma_n + \sigma_p = e(n\mu_e + p\mu_h) \quad (25.62)$$

For intrinsic semiconductors, $n = p = n_i$. Thus,

$$\sigma = en_i(\mu_e + \mu_h) \quad (25.63)$$

Conductivity of N-type Semiconductors

For N-type semiconductors, the expression of conductivity is given as

$$\sigma = e(n\mu_e + p\mu_h)$$

where electrons (n) are the majority charge carriers and holes (p) are the minority charge carriers. Since $n \gg p$, so the previous expression can be written as

$$\sigma = en\mu_e$$

where n is charge carriers due to the donor impurity, which can be represented as n_d . Hence, the above expression will become

$$\sigma = en_d\mu_e \quad (25.64)$$

Conductivity of P-type Semiconductors

For P-type semiconductors, the expression of conductivity is given as

$$\sigma = e(p\mu_h + n\mu_e)$$

In case of P-type semiconductors, majority charge carriers are due to the acceptor type impurity. Therefore,

$$p\mu_h = n_a\mu_h \gg n\mu_e$$

Thus, the above expression is given as

$$\sigma = en_d \mu_h \quad (25.65)$$

25.20 RESISTIVITY

In general the reciprocal of the conductivity is defined as the resistivity of a material. It is an intensive material property and is independent of size and shape of the material. In fact resistivity is a function of carrier concentration and mobility. Mathematically the resistivity of a material (ρ) is expressed as

$$\rho = \frac{1}{\sigma} \Omega\text{-cm} \quad (25.66)$$

where σ is the conductivity of the material.

Resistivity of Semiconducting Material

From the equation (25.60) we know that the conductivity of intrinsic semiconducting material due to electron is

$$\sigma_n = ne\mu_e$$

Since resistivity is defined as the reciprocal of conductivity so, the resistivity in intrinsic semiconductor for the electrons is given as

$$\rho_n = \frac{1}{\sigma_n} = \frac{1}{ne\mu_e} \quad (25.67)$$

Similarly resistivity for holes can be given as

$$\rho_h = \frac{1}{\sigma_h} = \frac{1}{pe\mu_h} \quad (25.67)$$

Now the total resistivity in an intrinsic semiconductor is given as

$$\rho = \rho_n + \rho_h = \frac{1}{\sigma_n} + \frac{1}{\sigma_h} = \frac{1}{e} \left[\frac{1}{n\mu_e} + \frac{1}{p\mu_h} \right] \quad (25.67)$$

Resistivity in N-type Semiconductors

In N-type semiconductor electrons are the majority charge carriers and holes (p) are the minority charge carriers. Since $n \gg p$, so equation (25.63) can be reduced as

$$\sigma = e n \mu_e$$

where n is charge carriers due to donor impurity, which can be represented as n_d . Hence the above expression will become

$$\sigma_N = e n_d \mu_e$$

Now the resistivity for N-type semiconductors can be given as

$$\rho_N = \frac{1}{\sigma_N} = \frac{1}{en_d \mu_e} \quad (25.68)$$

Resistivity in P-type Semiconductors

For P-type semiconductors, the expression of conductivity can be given as

$$\sigma_P = e n_a \mu_h$$

where n_a is the number of charge carriers due to the acceptor type impurity.

Now the resistivity for P -type semiconductors can be given as

$$\rho_P = \frac{1}{\sigma_p} = \frac{1}{en_a\mu_h} \quad (25.69)$$

25.21 EXPERIMENTAL DETERMINATION OF BAND GAP OF INTRINSIC SEMICONDUCTOR

In order to determine the band gap of an intrinsic semiconductor four probe method is most commonly used. This is a simple and reliable experimental technique used for determining band gap. In this method four probe assembly with oven, semiconducting sample, current source and voltmeter is required.

Four probe set up: In this arrangement four spring type contacts are used to avoid contact resistance. This four probe arrangement consist of four spring type collinear equally spaced probes coated with zinc as shown in Fig. 25.24. The probes are mounted in a teflon bush for good electrical insulation. The outer probes (1, 4) are used to pass current I , through the specimen for known conductivity sample. The current I is usually low, of the order of milliampere and is measured by milliammeter of 0–10 mA range. The inner probes (2,3) measure voltage V by digital electronic millivoltmeter specially designed for this purpose. The whole arrangement is mounted on a suitable stand and leads are provided for current and voltage arrangements. At the top of the four probe arrangement stand a hole is provided to insert a thermometer to measure the oven temperature. A good quality thermometer of generally 0–200°C range is used to measure the oven temperature. A temperature variation from room temperature to 175°C is studied in a small oven with built in supply inside the set up. The supply has three different output voltages for the oven to change the rate of heating.

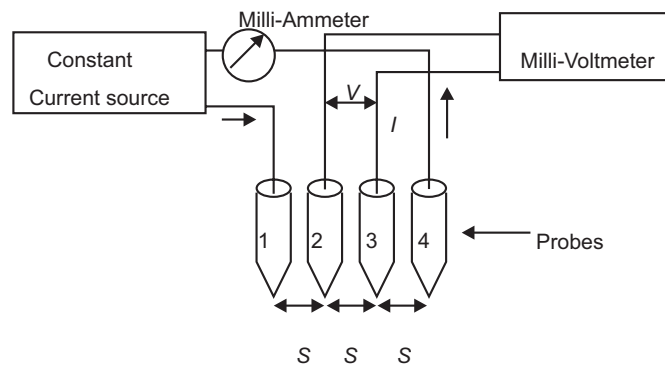


Fig. 25.24 Four probe arrangement used for measuring energy band gap

Theory and Formula used: For the current I in the outer probes and potential difference V across inner probes, then the resistivity of the sample can be given as

$$\rho_o = \frac{V}{I} \times 2\pi S \quad (25.70)$$

where s is the distance between any two successive probes. If it is 0.159 then

$$2\pi S = 1 \quad \text{and} \quad \rho_o = \frac{V}{I}$$

For very small thickness compared to the probe distance equation (25.70) is not applicable directly, but a correction factor for it is required. Now using the correction factor for a thin slice non-conducting bottom surface, the resistivity may be expressed as

$$\rho = \frac{\rho_o}{F(W/S)} \quad (25.71)$$

where W is the thickness of the semiconducting material and S is the probe spacing. The value of $F(W/S)$ can be obtained by given data sheet.

We know that the forbidden energy band gap E_g of semiconductor in terms of resistivity can be given as

$$\log_e \rho = \frac{E_g}{2k_B T}$$

$$\text{or} \quad E_g = 2k_B \frac{2.3026 \log_{10} \rho}{(1/T)} \quad (25.72)$$

where k_B is the Boltzmann's constant equals to 8.6×10^{-5} eV/kg and $\log_e \rho = 2.3026 \log_{10} \rho$. Thus, the slope of the graph between $\log_{10} \rho$ and $10^3/T$ gives

$$\frac{\log_{10} \rho}{1/T} = \frac{E_g}{2k_B}$$

$$\text{or} \quad E_g = 2k_B \times 2.3026 \left(\frac{\log_{10} \rho}{1/T} \right) \quad (25.73)$$

Thus plotting a graph between $\log_{10} \rho$ and $\frac{1}{T} \times 10^3$ and calculating its slope of the graph we can determine the energy band gap.

Data Required: Following observations are required for above calculations:

Thickness of the crystal $W = \dots\dots$ cm

Distance between the probes, S = cm

Value of current, $I = \dots\dots$ mA

<i>S. No.</i>	<i>Temperature of the sample</i>		<i>Voltage (V)</i>	<i>Resistivity (ρ)</i>	<i>$\log_{10}(\rho)$</i>	$\frac{1}{T^{\circ}K} \times 10^3$
	<i>In $^{\circ}C$</i>	<i>In Kelvin K</i>				

Solved Examples

Example 25.1

Find the thermal equilibrium electron and hole concentration at $T = 300$ K and position of Fermi level w.r.t. E_C in silicon semiconductor for 1×10^{15} electrons/cm³.

Given that: $E_g = 1.12$ eV, $n_i = 1.5 \times 10^{10}$ /cm³, $n_o = 10^{15}$ /cm³, $k_B T = 26$ meV

Solution

We know that

$$\begin{aligned} p_o &= \frac{n_i^2}{n_o} \\ &= \frac{2.25 \times 10^{20}}{10^{15}} = 2.25 \times 10^5/\text{cm}^3 \end{aligned}$$

From above value of p_o it is clear that $n_o \gg p_o$.

Now using, $n_o = n_i e^{(E_F - E_i)/k_B T}$

$$10^{15} = 1.5 \times 10^{10} e^{(E_F - E_i)/k_B T}$$

$$\begin{aligned} (E_F - E_i) &= 11.107 k_B T \\ &= 0.29 \text{ eV} \end{aligned}$$

Since $E_C - E_i = E_g/2 = 0.56$ eV

$$\begin{aligned} \text{Therefore } E_C - E_F &= (E_C - E_i) - (E_F - E_i) \\ &= 0.56 \text{ eV} - 0.29 \text{ eV} \\ &= 0.27 \text{ eV} \end{aligned}$$

Example 25.2

Calculate the probability that an energy state in the conduction band at $E = E_C + k_B T$ is occupied by an electron. Given that $T = 300$ K, $E_C - E_F = 0.20$ eV, $N_C = 2.8 \times 10^{19}$ /cm³.

Solution

Probability of occupying energy state at $(E_C + k_B T)$ by an electron can be given as

$$\begin{aligned} F(E_C + k_B T) &= \frac{1}{1 + \exp\left(\frac{E_C + k_B T - E_F}{k_B T}\right)} \simeq \exp\left[-\frac{E_C + k_B T - E_F}{k_B T}\right] \\ &\simeq \exp\left[-\left(\frac{0.20 \text{ eV} + 0.026 \text{ eV}}{0.026 \text{ eV}}\right)\right] \simeq 1.63 \times 10^{-4} \\ n_o &= N_C \exp\left[-\frac{(E_C - E_F)}{k_B T}\right] \end{aligned}$$

$$\begin{aligned}
&= 2.8 \times 10^{19} \exp \left[- \left(\frac{0.2 \text{ eV}}{0.026 \text{ eV}} \right) \right] \\
&\simeq 1.24 \times 10^{16} / \text{cm}^3
\end{aligned}$$

From above result it is clear that the probability of occupying any energy level above the conduction band edge by an electron is very small.

Example 25.3

Consider a silicon semiconductor material at $T = 300 \text{ K}$, in which Fermi level is 0.25 eV above the valence band edge. Find the thermal equilibrium concentration of electrons and holes.

Given that E_g for Si at $T = 300 \text{ K}$, is 1.12 eV , $E_C - E_F = 0.87 \text{ eV}$

$$N_V = 1.04 \times 10^{19} / \text{cm}^3 \text{ and } N_C = 2.8 \times 10^{19} / \text{cm}^3 \text{ at } T = 300 \text{ K}$$

Solution

When Fermi level is closer to valence band edge material will be P -type, thus the concentration of majority charge carriers are

$$p_o = N_V e^{-(E_F - E_V)/k_B T}$$

$$\text{Now } p_o = (1.04 \times 10^{19}) \exp \left[- \frac{0.25}{0.026} \right] = 6.68 \times 10^{14} / \text{cm}^3$$

$$\text{and } n_o = (2.8 \times 10^{19}) \exp \left[- \frac{0.87}{0.26} \right] \simeq 7.23 \times 10^{14} / \text{cm}^3$$

Example 25.4

Find the equilibrium electron and hole concentration and locate the Fermi level w.r.t. intrinsic level in Si at $T = 300 \text{ K}$. Given that $N_d = 8 \times 10^{16} / \text{cm}^3$, $n_i = 1.5 \times 10^{10} / \text{cm}^3$ and $N_a = 2 \times 10^{16} / \text{cm}^3$.

Solution

Since the order of N_d and N_a are of same order, so the given sample is not strong extrinsic material.

According to the given data, $N_d > N_a$ so the sample will be N -type material.

$$\begin{aligned}
\text{Electron concentration in } N\text{-type silicon} &= n_{no} = (N_d - N_a) \\
&= 6 \times 10^{16} / \text{cm}^3
\end{aligned}$$

$$\text{Minority hole concentration } p_{no} = \frac{n_i^2}{n_{no}} = 3750 / \text{cm}^3$$

$$\begin{aligned}
n_{no} &= n_i \exp [(E_F - E_i)/k_B T] \\
\Rightarrow 6 \times 10^{16} &= 1.5 \times 10^{10} \exp [(E_F - E_i)/k_B T] \\
6.10^{16} &= 1.5 \times 10^{10} \exp [(E_F - E_i)/k_B T]
\end{aligned}$$

$$\text{or } 4 \times 10^6 = \exp [(E_F - E_i)/k_B T]$$

$$\text{or } (E_F - E_i) = 0.395 \text{ eV}$$

Example 25.5

A sample of silicon is doped every where with a background concentration of $N_A = 4 \times 10^{16}/\text{cm}^3$. Then $10^{17}/\text{cm}^3$ donors are added. Find the thermal equilibrium concentration of electrons and holes in the original and final materials and draw the energy band diagram for each.

Solution

Originally material is P-type semiconductor having $p_o = 4 \times 10^{16}/\text{cm}^3$

$$n_o = \frac{n_i^2}{p_o} = \frac{2.25 \times 10^{20}}{4 \times 10^{16}} = 0.5625 \times 10^4/\text{cm}^3$$

$$= 5.625 \times 10^3/\text{cm}^3$$

$$E_F - E_V = k_B T \log \left(\frac{N_v}{p_o} \right) = 0.17 \text{ eV}$$

This value is greater than $2.3 k_B T$, thus the material is non-degenerate. After further doping

$$N_A = 4 \times 10^{16}/\text{cm}^3$$

$$N_d = 10^{17}/\text{cm}^3$$

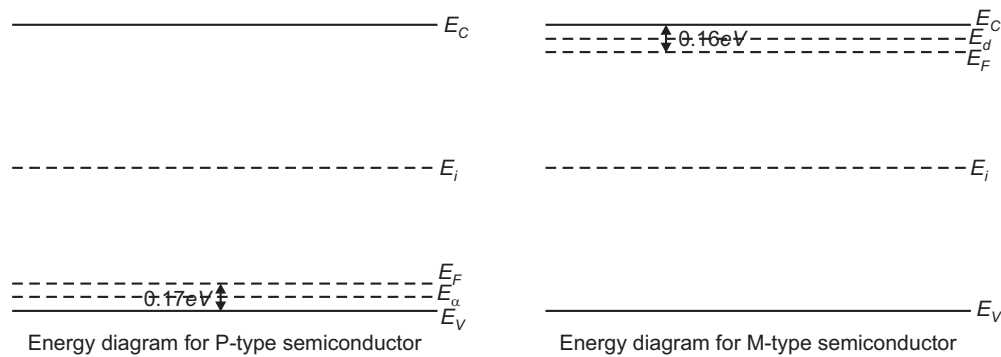
$N_d > N_A$ hence, resulting material is a N-type

$$n_{no} = (N_d - N_A) = 6 \times 10^{16}/\text{cm}^3$$

$$p_{no} = \frac{n_i^2}{n_{no}} = \frac{2.25 \times 10^{20}}{6 \times 10^{16}} = 0.375 \times 10^4$$

$$= 3.75 \times 10^3/\text{cm}^3$$

$$E_C - E_F = k_B T \log \left(\frac{N_c}{N_d} \right) = 0.16 \text{ eV}$$

**Example 25.6**

Calculate the density of impurity atoms that must be added to an intrinsic silicon crystal to convert it to (i) $10^{-1} \Omega\text{-m}$ P-type silicon and (ii) $10^{-1} \Omega\text{-m}$ N-type silicon. The electron and hole mobilities for silicon are $\mu_e = 0.135 \text{ m}^2/\text{V-s}$ and $\mu_h = 0.048 \text{ m}^2/\text{V-s}$ respectively.

Solution

- (i) For
- P*
- type semiconductor, the electrical resistivity
- ρ_p
- can be expressed as

$$\rho_p = \frac{1}{\sigma_p} = \frac{1}{en_e\mu_h} = \frac{1}{eN_a\mu_h}$$

Thus the required density of acceptor atom can be given as

$$N_a = \frac{1}{e\rho_p\mu_h} = \frac{1}{1.6 \times 10^{-19} \times 0.048 \times 10^{-1}} \\ = 1.30 \times 10^{21} \text{ m}^{-3}$$

- (ii) For
- N*
- type semiconductor, the electrical resistivity
- ρ_n
- can be given as

$$\rho_n = \frac{1}{\sigma_n} = \frac{1}{en_e\mu_e} = \frac{1}{eN_d\mu_e}$$

Thus the required density of donor atom can be given as

$$N_d = \frac{1}{e\rho_n\mu_e} = \frac{1}{1.6 \times 10^{-19} \times 0.135 \times 10^{-1}} \\ = 4.63 \times 10^{21} \text{ m}^{-3}$$

Example 25.7

A sample of intrinsic silicon has 0.13 and 0.15 m²/V-s electron and hole mobilities respectively at 300 K. If the density of electrons and holes are each equal to $1.5 \times 10^{16} \text{ m}^{-3}$ at 300 K then.

- Calculate the electrical conductivity
- Find the electrical conductivity for addition of 1 donor impurity atom in 10^9 silicon atoms
- Find the electrical conductivity for addition of 1 acceptor impurity atom in 10^9 silicon atoms

Solution

- (i) For intrinsic semiconductor, the electrical conductivity
- σ
- can be given as

$$\sigma = en_i(\mu_h + \mu_e)$$

putting the given values in above expression we get

$$\sigma = 1.6 \times 10^{-19} \times 1.5 \times 10^{16} \times (0.13 + 0.05) \\ = 4.32 \times 10^{-4} \Omega^{-1}\text{m}^{-1}$$

- (ii) When 1 donor impurity atom in
- 10^9
- silicon atoms is added then the density of electron or ionised donor atoms can be given as

$$n_e = N_d = \frac{5 \times 10^{28}}{10^9} = 5 \times 10^{19} \text{ m}^{-3}$$

Now, the density of holes in

$$n_h = \frac{n_i^2}{N_d} = \frac{(1.5 \times 10^{16})^2}{5 \times 10^{19}} = 4.5 \times 10^{13} \text{ m}^{-3}$$

Since $n_e \gg n_h$ the sample behaves like *N*-type semiconductor, so the electrical conductivity can be given as

$$\begin{aligned}\sigma_n &= en_e \mu_e = 1.6 \times 10^{-19} \times 5 \times 10^{19} \times 0.13 \\ &= 1.04 \Omega^{-1} \text{ m}^{-1}\end{aligned}$$

- (iii) When 1 acceptor impurity is added in 10^9 silicon atoms then the density of holes or ionised acceptor atoms can be given as

$$n_h = N_a = \frac{5 \times 10^{28}}{10^9} = 5 \times 10^{19} \text{ m}^{-3}$$

Now the density of electrons can be given as

$$n_e = \frac{n_i^2}{N_a} = \frac{(1.5 \times 10^{16})^2}{5 \times 10^{19}} = 4.5 \times 10^{13} \text{ m}^{-3}$$

From the above result it is clear that $n_e \ll n_h$ so the sample will behave like *P*-type semiconductor.

Now the electrical conductivity can be given as

$$\begin{aligned}\sigma_p &= en_h \mu_h = 1.6 \times 10^{-19} \times 5 \times 10^{19} \times 0.05 \\ &= 0.4 \Omega^{-1} \text{ m}^{-1}\end{aligned}$$

Example 25.8

Show that the minimum conductivity of a semiconductor sample occurs when

$$n_o = n_i \sqrt{\frac{\mu_h}{\mu_e}}$$

Solution

Using the expression

$$\sigma = e(\mu_e n + \mu_h p)$$

At thermal equilibrium

$$\sigma = e \left(\mu_e n_o + \mu_h \frac{n_i^2}{n_o} \right)$$

for minimum conductivity

$$\frac{d\sigma}{dn_o} = 0$$

$$\text{i.e.} \quad \frac{d\sigma}{dn_o} = e \left(\mu_e - \mu_h \frac{n_i^2}{n_o^2} \right) = 0$$

$$\text{or} \quad \mu_e = \mu_h \frac{n_i^2}{n_o^2}$$

$$\text{or} \quad n_o^2 = n_i^2 \frac{\mu_h}{\mu_e}$$

$$\text{or} \quad n_o = n_i \sqrt{\frac{\mu_h}{\mu_e}}$$

25.22 FORMULAE AND HIGHLIGHTS

1. Energy band occupied by valence electrons is known as valence band, and the energy band which are empty or occupied by the conduction electrons is known as conduction band.
2. In case of semiconductors the energy band gap is of the order of 1 eV.
3. In fact energy band gap is the energy required to break a bond in the semiconductor to free the electron to the conduction band which leave a hole in valence band.
4. Effective mass of an electron can be given as

$$m^* = \hbar^2 / \frac{d^2 E}{dk^2}$$

5. The energy of the conduction band in terms of effective mass can be given as

$$E_C(k) = E_g + \frac{\hbar^2 k^2}{2m_e^*}$$

6. Energy of the valence band can be given as

$$E_V(k) = -\frac{\hbar^2 k^2}{2m_h^*}$$

7. Density of states is defined as

$D(E) dE$ = The number of electron states per unit value in the energy range $(E, E + dE)$

8. General expression of density of states can be given as

$$D(E) = \frac{1}{2\pi^2} \left(\frac{2m^*}{\hbar^2} \right)^{3/2} E^{1/2}$$

9. Expression for density of states for conduction band can be given as

$$D_e(E) = \frac{1}{2\pi^2} \left(\frac{2m_e^*}{\hbar^2} \right)^{3/2} (E - E_C)^{1/2}$$

10. Expression for density of states for valence band can be given as

$$D_h(E) = \frac{1}{2\pi^2} \left(\frac{2m_h^*}{\hbar^2} \right)^{3/2} (E_V - E)^{1/2}$$

11. Fermi-Dirac distribution function is given as

$$F(E) = \frac{1}{e^{(E - E_F)/k_B T} + 1}$$

12. For $E < E_F$ at $T = 0 K$

$$F(E) = 1$$

and for $E > E_F$ at $T = 0 K$

$$F(E) = 0$$

13. If E is at least several $k_B T$ above the Fermi level then Fermi Dirac distribution function $F(E)$ can be expressed as

$$F(E) \simeq e^{-\frac{(E - E_F)}{k_B T}}$$

14. For E is at least several $k_B T$ below the Fermi level then Fermi-Dirac distribution function $F(E)$ can be expressed as

$$F(E) \simeq 1 - e^{-\frac{(E_F - E)}{k_B T}}$$

15. Concentration of electrons in the conduction band can be given as

$$n = 2 \left(\frac{m_e k_B T}{2\pi \hbar^2} \right)^{3/2} e^{\frac{(E_F - E_g)}{k_B T}}$$

16. Concentration of holes in the valen band can be given as

$$p = 2 \left(\frac{m_h k_B T}{2\pi \hbar^2} \right)^{3/2} e^{-E_F/k_B T}$$

17. As the temperature is raised, a vastly greater number of electrons are excited across the gap.

18. For N -type semiconductors

$$p = \frac{n_i^2}{N_d}$$

and for P -type semiconductors

$$n = \frac{n_i^2}{N_d}$$

19. When the material of Vth group is doped in IVth group intrinsic semiconductors then N -type extrinsic semiconductors are obtained.
20. When the material of IIIrd group is doped in IVth group intrinsic semiconductor then p -type extrinsic semiconductors are formed.
21. In N -type semiconductors a donor energy level is introduced just below the botton of conduction band.
22. In P -type semiconductors an acceptor energy level is introduced just above the valence band.
23. Position of Fermi energy level in intrinsic type semiconductors can be given as

$$E_F = \frac{E_V + E_C}{2}$$

24. The position of Fermi energy level in N -type extrinsic semiconductors can be given as

$$E_F = E_c - k_B T \log \left(\frac{N_c}{N_d} \right)$$

25. The position of Fermi energy level in P -type extrinsic semiconductors can be given as

$$E_F = E_V - k_B T \log \left(\frac{N_v}{N_a} \right)$$

26. Mobility of electron can be expressed as

$$\mu_e = \frac{V_e}{E} = \frac{e\tau_e}{m_e}$$

27. Conductivity of electron is expressed as

$$\sigma_e = \frac{ne^2\tau_e}{m_e}$$

28. Conductivity of intrinsic semiconductor can be given as

$$\sigma = en_i(n\mu_e + p\mu_h)$$

29. Conductivity of *N*-type extrinsic semiconductor is expressed as

$$\sigma = en_d\mu_e$$

30. Conductivity of *P*-type extrinsic semiconductor is expressed as

$$\sigma = en_a\mu_p$$

31. Resistivity of intrinsic semiconductor can be given as

$$\rho = \frac{1}{\sigma_n} + \frac{1}{\sigma_p} = \frac{1}{e} \left[\frac{1}{n\mu_e} + \frac{1}{p\mu_h} \right]$$

32. Resistivity of *N*-type extrinsic semiconductor can be given as

$$\rho_N = \frac{1}{en_d\mu_e}$$

33. Resistivity of *P*-type extrinsic semiconductor can be given as

$$\rho_P = \frac{1}{en_a\mu_h}$$

34. Energy band gap of intrinsic semiconductor can be expressed as

$$E_g = 2k_B \frac{2.3026 \log_{10} \rho}{(1/T)}$$

Exercises

Section A

Theoretical Questions

1. Explain the concept of band formation in solids? How the discrete energy level of isolated atoms form energy bands.
2. What is the band theory of solids? Classify conductors, insulators and semiconductors on the basis of band theory of solids.
3. Discuss the band formation in pure semiconductor (silicon) using schematic diagram.
4. What is the concept of effective mass of electron? Derive an expression for the effective mass of electron.

5. Obtain the expression of energy for conduction band in semiconductor with the concept of effective mass.
6. What do you mean by density of energy states in metals? obtain the expression for density of states for conduction and valence band.
7. Explain Fermi-Dirac distribution function. Plot this function for various temperature including 0K.
8. What do you understand by "Fermi level"? Explain its importance in semiconductor.
9. Obtain the expression for carrier concentration in conduction band of intrinsic semiconductor.
10. Obtain the expression for carrier concentration in valence band of intrinsic semiconductor.
11. Discuss the temperature dependence of electron concentration in conduction band of semiconductor material.
12. Explain the effect of doping on the band structure and carrier concentration in semiconducting materials.
13. Using the concept of semiconductor statistics estimate the carrier concentration in intrinsic and extrinsic region of a semiconducting material.
14. What is Fermi-level? Derive an expression for the Fermi energy in terms of the number of electrons per unit volume.
15. Explain the effect of carrier concentration on the band structure of semiconducting materials.
16. Define *N*-type and *P*-type semiconductors. Explain the mechanism of current flow in these semiconductors.
17. Show that the Fermi energy level lies exactly in the centre of the forbidden energy gap between conduction band and the valence band in intrinsic semiconductor.
18. Show that the Fermi energy level E_F lies slightly below the bottom of the conduction band in an *N*-type semiconductor.
19. Show that the Fermi energy level E_F lies slightly above the top of the valence band in a *P*-type semiconductor.
20. What is mobility? How it varies with life time of electron and its effective mass?
21. Define electrical conductivity obtain the expression of conductivity for intrinsic and extrinsic semiconductors.
22. Define resistivity? Obtain the expression of resistivity for intrinsic and extrinsic semiconductors.
23. Describe four probe method for determining the band gap of intrinsic semiconductors.

Section B Numerical Problems

1. Find the intrinsic carrier concentration for silicon semiconductor material at room temperature ($T = 300$ K).

Given that, Effective mass of electron $= m_e^* = 1.09 m_o$

Effective mass of hole $= m_h^* = 1.15 m_o$

Mass of free electron $= m_o = 9.1 \times 10^{-31}$ kg

Energy band gap for silicon = 1.2 eV

(Ans. $1.08 \times 10^{10}/\text{cm}^3$)

Hint: We know that

$$n_i = 2 \left(\frac{2\pi k_B T}{h^2} \right)^{3/2} (m_e^* m_h^*)^{3/4} e^{-E_g/k_B T}$$

putting the values required in above expression

$$n_i = 2 \left(\frac{2\pi \times 0.026}{h^2} \right)^{3/2} (1.09 \times 1.15)^{3/4} (m_0^2)^{3/4} e^{-\frac{1.12}{2 \times 0.026}}$$

or $n_i = 1.08 \times 10^{10}/\text{cm}^3$

2. Calculate the probability that an energy state in valence band at $E = E_V - k_B T$ is empty of an electron and calculate the thermal equilibrium hole concentration in silicon semiconductor at $T = 350$ K. Given: $E_F - E_V = 0.25$ eV, $N_v = 1.04 \times 10^{19}/\text{cm}^3$ (Ans. 9.34×10^5 , $3.3 \times 10^{15}/\text{cm}^3$)

Hint: $N_v(T) = 2 \left(\frac{2\pi k_B T}{h^2} \right)^{3/2}$

$$N_v(T = 350 \text{ K}) = 1.04 \times 10^{19} \left(\frac{350}{360} \right)^{3/2} = 1.31 \times 10^{19} / \text{cm}^3$$

$$k_B T = (0.0259) \left(\frac{350}{360} \right) = 0.0302 \text{ eV}$$

The probability that an energy state in the valence band at $E = E_V - k_B T$ is occupied by hole

$$\Rightarrow 1 - F(E_V - k_B T) \simeq \exp \left[-\frac{E_F - E_V + k_B T}{k_B T} \right] \simeq 9.34 \times 10^{-5}$$

Now $p_o = N_v \exp \left[-\frac{(E_F - E_V)}{k_B T} \right]$
 $= 3.33 \times 10^{15}/\text{cm}^3$

3. Find the electron and hole concentrations in GaAs doped with $N_a = 10^{16}/\text{cm}^3$, and locate the position of Fermi level. Given $E_g = 1.43$ eV at $T = 300$ K, $N_c = 4.4 \times 10^{17}/\text{cm}^3$, $N_v = 8.3 \times 10^8/\text{cm}^3$. (Ans. $4.8 \times 10^{-4}/\text{cm}^3$, $10^{16}/\text{cm}^3$, $E_F - E_V = 0.17$ eV)

Hint: $N_i = \sqrt{N_c N_v} e^{-E_g/2k_B T}$
 $= 2.2 \times 10^6/\text{cm}^3$

Since $N_a \gg n_i$ therefore,

$$p_o = n_a = 10^{16}/\text{cm}^3$$

$$E_F - E_V = k_B T \log \left(\frac{N_v}{N_a} \right)$$

$$= 0.17 \text{ eV}$$

Electron concentration $n_o = \frac{n_i^2}{p_o} = \frac{(2.2 \times 10^6)^2}{10^{16}}$
 $= 4.8 \times 10^{-4}/\text{cm}^3$

4. Find the energy by which intrinsic Fermi level E_i is offset from midgap for Si at room temperature.
Given: $k_B T = 0.026$ eV, $m_h^* = 1.15 m_o$, $m_e^* = 1.09 m_o$ (Ans. 1.05 meV)

Hint: $E_i = (E_{\text{midgap}}) + \frac{3}{4} k_B T \log \left(\frac{m_h^*}{m_e^*} \right)$

$$E_i - E_{\text{midgap}} = \frac{3}{4} k_B T \log \left(\frac{1.15}{1.09} \right) = 1.05 \text{ meV}$$

5. Evaluate the temperature at which there is one percent probability that a state with an energy 0.5 electron volt above the Fermi energy, will be occupied by an electron. (Ans. $T = 1264$ K)

Hint: $F(E) = \frac{1}{1 + \exp[(E - E_F) / k_B T]}$

$$E = E_F + 0.5$$

$$F(E) = \frac{1}{100}$$

$$0.01 = \frac{1}{1 + e^x} \text{ where } x = \frac{0.5}{k_B T}$$

$$k_B T = \frac{0.5}{2.303 \times \log_{10} 99} = 0.109 \text{ eV}$$

$$T = \frac{0.109 \times 1.6 \times 10^{-19}}{1.38 \times 10^{-23}}$$

$$T = 1264 \text{ K}$$

6. A sample of intrinsic germanium has 0.36 and 0.17 m²/V-s electron and hole mobilities respectively. If the density of electrons and holes are each equal to $2.5 \times 10^{18} \text{ m}^{-3}$, find the electrical conductivity and resistivity of the sample. (Ans. $\sigma = 0.212 \Omega^{-1} \text{ m}^{-1}$, $\rho = 4.7 \Omega \cdot \text{m}$)

Hint: For intrinsic semiconductor, the electrical conductivity σ can be expressed as

$$\sigma = en_i (\mu_h + \mu_e)$$

$$\sigma = 1.6 \times 10^{-19} \times 2.5 \times 10^{18} \times (0.36 + 0.17) = 0.212 \Omega^{-1} \text{ m}^{-1}$$

Also, the electrical resistivity ρ is

$$\rho = \frac{1}{\sigma} = \frac{1}{0.212} = 4.7 \Omega \cdot \text{m}$$

7. Calculate the mean free time and mean free path of an electron having a mobility of 1000 cm²/V-s at room temperature. Assuming $m_e^* = 0.26 m_o$ and $v_{th} = 10^5$ m/s. (Ans. 14.8 nm)

$$\tau_n = \frac{\mu_n m_e^*}{e} = \frac{1000 \times 10^{-4} \times 0.26 \times 10^{-19} \times 9.1 \times 10^{-12}}{1.6 \times 10^{-19}} = 1.48 \times 10^{-13} \text{ s}$$

mean free path $l = \tau_n v_{th}$

$$= 1.48 \times 10^{-13} \times 10^7 = 14.8 \text{ nm}$$

Section C
Multiple Choice Questions

1. Valence band and conduction band overlap each other in
 - (a) conductor
 - (b) semiconductor
 - (c) insulator
 - (d) none of these
2. The forbidden energy gap of carbon in diamond structure is
 - (a) 0.7 eV
 - (b) 1.1 eV
 - (c) 6 eV
 - (d) none of these
3. Fermi energy level lie exactly in the centre of the forbidden energy gap E_g between the valence band and conduction band.
 - (a) *N*-type extrinsic semiconductor
 - (b) *P*-type extrinsic semiconductor
 - (c) intrinsic semiconductor
 - (d) none of these
4. Fermi energy level lies slightly above the top of valence band in
 - (a) *N*-type extrinsic semiconductor
 - (b) *P*-type extrinsic semiconductor
 - (c) intrinsic semiconductor
 - (d) none of these
5. If e , and μ respectively represent the charge, mobility and concentration of electrons respectively then the electrical conductivity of material can be given as
 - (a) n/μ_e
 - (b) μ_e/n
 - (c) ne
 - (d) $ne\mu$
6. The general expression for the Fermi energy of a metal at OK is
 - (a) $3.65 \times 10^{-19} \times n^{2/3}$ electron volt
 - (b) $3.65 \times 10^{19} \times n^2$ eV
 - (c) $3.65 \times 10^{19} \times n^{1/2}$ eV
 - (d) $3.65 \times 10^{19} \times n^{3/2}$ eV
7. If the Fermi energy of a metal is 1.4 electron volt, the Fermi temperature of the metal is approximately
 - (a) 1.6×10^3 K
 - (b) 1.6×10^4 K
 - (c) 1.6×10^5 K
 - (d) 1.6×10^6 K
8. At the temperature $T = 0$ in a semiconductor all the states in the valence band are:
 - (a) completely filled
 - (b) partially filled
 - (c) completely empty
 - (d) none of these
9. Effective mass of electron can be given as
 - (a) $m^* = \hbar^2 / \frac{d^2 E}{dk^2}$
 - (b) $m^* = \hbar^2 \left(\frac{d^2 E}{dk^2} \right)$
 - (c) $m^* = \frac{d^2 E / dk^2}{\hbar^2}$
 - (d) $m^* = \frac{1}{\hbar^2} \left(\frac{d^2 E}{dk^2} \right)$

10. Energy of the conduction band is given as

$$\begin{array}{ll} \text{(a)} E_C(k) = E_g + \frac{\hbar^2 k^2}{2m_e^*} & \text{(b)} E_C(k) = E_g - \frac{\hbar^2 k^2}{2m_e^*} \\ \text{(c)} E_C(k) = \frac{\hbar^2 k^2}{2m^*} & \text{(d)} m^* = \frac{\hbar^2 k^2}{2m^*} - k \end{array}$$

11. Expression of density of energy states can be given

$$\begin{array}{ll} \text{(a)} \frac{1}{2\pi^2} \left(\frac{2m^*}{\hbar^2} \right)^{3/2} E & \text{(b)} \frac{1}{2\pi^2} \left(\frac{2m^*}{\hbar^2} \right) E^2 \\ \text{(c)} \frac{1}{2\pi^2} \left(\frac{\hbar^2}{2m^*} \right)^{3/2} E & \text{(d)} \frac{1}{2\pi^2} \left(\frac{2m^*}{\hbar^2} \right)^{3/2} E^{1/2} \end{array}$$

12. At the condition $E < E_F$ and $T = 0\text{K}$ Fermi distribution function will be reduced to

$$\begin{array}{ll} \text{(a)} F(E) = 1/2 & \text{(b)} F(E) = 1 \\ \text{(c)} F(E) = 2 & \text{(d)} F(E) = 0 \end{array}$$

13. At the condition $E < E_F$ and $T = 0\text{K}$ Fermi distribution function will be reduced to

$$\begin{array}{ll} \text{(a)} F(E) = 1 & \text{(b)} F(E) = 1/2 \\ \text{(c)} F(E) = 0 & \text{(d)} F(E) = -1 \end{array}$$

14. Carrier concentration of electron in conduction band of intrinsic semiconductor can be given as

$$\begin{array}{ll} \text{(a)} n = \left(\frac{m_e k_B T}{2\pi \hbar^2} \right)^{3/2} e^{\frac{(E_F - E_g)}{k_B T}} & \\ \text{(b)} n = \left(\frac{m k_B T}{2\pi \hbar^2} \right) \cdot e^{-E_g/k_B T} & \\ \text{(c)} n = 2 \left(\frac{m_e k_B T}{2\pi \hbar^2} \right)^{3/2} e^{\frac{(E_F - E_g)}{k_B T}} & \\ \text{(d)} \text{none of these} & \end{array}$$

15. A plot of $\log n$ versus $1/T$ is a straight line having its slope equal to

$$\begin{array}{ll} \text{(a)} E_g/2k_B & \text{(b)} -E_g/2k_B \\ \text{(c)} 2E_g/k_B & \text{(d)} E_g/k_B \end{array}$$

16. All semiconductors become intrinsic at

$$\begin{array}{ll} \text{(a)} \text{sufficiently high temperature} & \\ \text{(b)} \text{sufficiently low temperature} & \\ \text{(c)} \text{zero temperature} & \\ \text{(d)} \text{none of these} & \end{array}$$

17. In *N*-type semiconductors majority charge carriers are
 (a) electrons (b) Holes
 (c) electrons and holes both (d) none of these
18. Mathematically the mobility of electron is expressed as
 (a) $\mu_e = e\tau_e m_e$ (b) $\frac{m_e}{e\tau_e}$
 (c) $\frac{\tau_e}{em_e}$ (d) $\frac{e\tau_e}{m_e}$
19. Conductivity of electron is given as
 (a) $\sigma_e = \frac{n_e^2 \tau_e}{m_e}$ (b) $\sigma_e = \frac{m_e n \tau_e}{e^2}$
 (c) $\sigma_e = \frac{m_e}{ne^2 \tau_e}$ (d) $\frac{m_e \tau_e}{ne^2}$
20. Expression for the conductivity of semi conductor material is given as
 (a) $\sigma = en_i \mu_e \mu_h$ (b) $\sigma = \frac{en_i}{\mu_e \mu_h}$
 (c) $\sigma = en_i (\mu_e + \mu_h)$ (d) none of these
21. Resistivity of *N*-type semiconductors can be given as
 (a) $\rho_N = en_d \mu_e$ (b) $\rho_N = \frac{e}{n_d \mu_e}$
 (c) $\rho_N = -\frac{\mu_e}{en_d}$ (d) $\rho_N = \frac{1}{en_d \mu_e}$
22. In four probe method for determining the energy band gap the slope of the graph between $\log_{10} \rho$ and $\frac{1}{T} \times 10^3$ gives the value of
 (a) $E_g/k_B T$ (b) $E_g/2k_B$
 (c) $2E_g k_B$ (d) $\frac{2k_B}{E_g}$

Answers

- | | | | | | |
|---------|---------|---------|---------|---------|---------|
| 1. (a) | 2. (c) | 3. (c) | 4. (b) | 5. (d) | 6. (a) |
| 7. (b) | 8. (a) | 9. (a) | 10. (a) | 11. (d) | 12. (b) |
| 13. (c) | 14. (c) | 15. (b) | 16. (a) | 17. (a) | 18. (d) |
| 19. (a) | 20. (c) | 21. (d) | 22. (b) | | |

Section D
Fill in the Blanks

1. Semiconductor have energy band gap of the order of eV.
2. Depending on the for the crystal the band of N levels can extend over few eV.
3. Energy gap between the bottom of the conduction band and top of valence band is known as
4. In semiconducting material valence band is at $T = 0\text{K}$.
5. Density of states is defined as the number of per unit volume in the energy range $(E, E + dE)$.
6. Fermi distribution function gives the that an energy level E is occupied by an electron when the system is at the temperature T .
7. Fermi Dirac function gives the value of $1/2$ at
8. In case of semiconductors a plot of $\log n$ versus $\frac{1}{T}$ gives a
9. When trivalent atom of III group added to pure semiconductor then extrinsic semiconductor known as
10. In case of intrinsic semiconductors the Fermi energy level lie conduction band and valence band.
11. Mobility is defined as per unit electric field.
12. Reciprocal of conductivity is defined as of a material.

Answers

- | | |
|----------------------------|------------------------|
| 1. 1 eV | 2. Interatomic spacing |
| 3. E_g (Energy band gap) | 4. Full |
| 5. Electron states | 6. Probability |
| 7. $E = E_F$ | 8. Straight line |
| 9. N -type semiconductor | 10. Exactly between |
| 11. Drift velocity | 12. Resistivity |