GRAPH

$G(V, E)$

$V = \{V_1, V_2, V_3 ---- V_n\}$

&

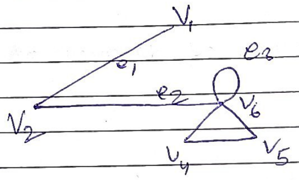
$E = \{e_1, e_2, e_3 --- e_n\}$

$e = (V_1, V_2)$

A graph is the diagram formed by the vertices joined by lines called edges. It is denoted by G where set of vertex

$e = (V_1, V_2)$

Ex



$V = \{V_1, V_2, V_3, V_4, V_5, V_6\}$

$E = \{e_1, e_2, e_3, e_4, e_5, e_6\}$

Parallel edges (e_2 and $e_?$)

Edges that have two same vertices are called parallel edges

Loop (e_3)

Empty graphs

~~Null graphs~~ graph with no edges.

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Null Graph

No vertices in this graph

Trivial

Graph with only one vertex is trivial

Adjacent Vertices and Edges

Number of edges connected with vertices V is called degree of V. (d(V))

Every contributes two degree

Degree of graph = sum of degrees of all vertices

→ Two types of degrees

i. Indegree ii. Outdegree (descrete graph)

Walk

A walk is alternating sequence b/w vertices and edges

No of edges involve in walk is called length of walk.

If Intial and terminal vertices consides then walk is close otherweise open

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Walk may be vertices as well as edges.

Trail:

A walk is trail if all edges are diffrent
Trail may be open or closed

Circuit

Closed trail is called circuit.

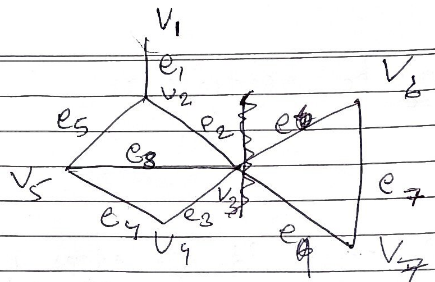
Path

Walk is called path if all vertices and edges are distenct

Cycle

A path in which initial and terminal vertices consides.

	Repeted		Terminal
	Vertices	Eedges	
Walk (open)	✓	✓	✗
(close)	✓	✓	✓
Trail	~~✗~~ ✓	✗	✗
circuit	✓	✗	✓
path	✗	✗	✗
Cycle	int = terminal	✗	✓

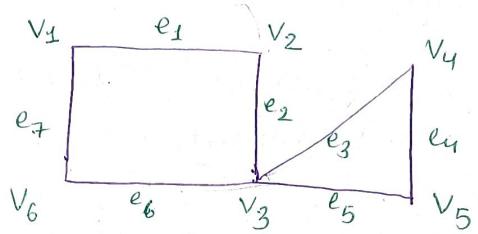


(1) Walk of length.

* Eulerian Graph

→ A connected graph G is Eulerian if there exist a closed trail containing every edge only once.

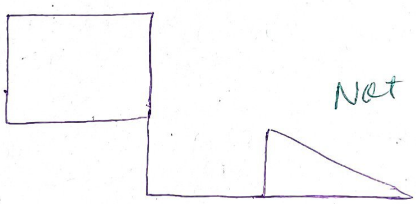
(i)



Euler's graph

$V_1 e_1 V_2 e_2 V_3 e_3 V_4 e_4 V_5 e_5 V_3 e_6 V_6 e_7 V_1$

(ii)



Not eulerian graph.

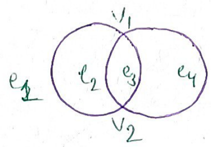
→ A Connected graph G is euler graph if ~~and~~ only all the vertices of G are of even degree

→ A Connected graph G is eulerian if and only if its edges can be decompose into Cycles

→ ~~If W is a walk from vertex u to vertex v then W contain an odd no. of paths.~~

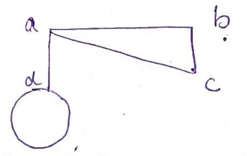
* Euler's Circuit

→ A Circuit is Euler if it travels each edge exactly once.



* Bridge

→ Edge in graph that disconnects the graph is called ~~to~~ Bridge.

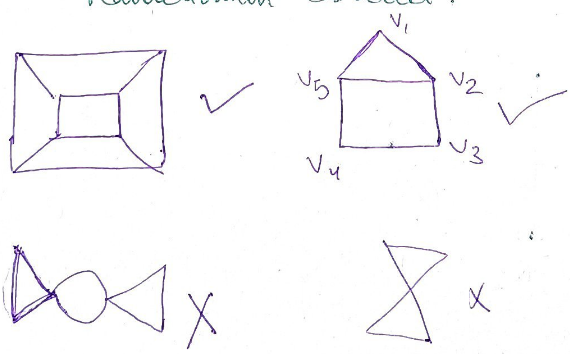


Bridge = {a, d}.

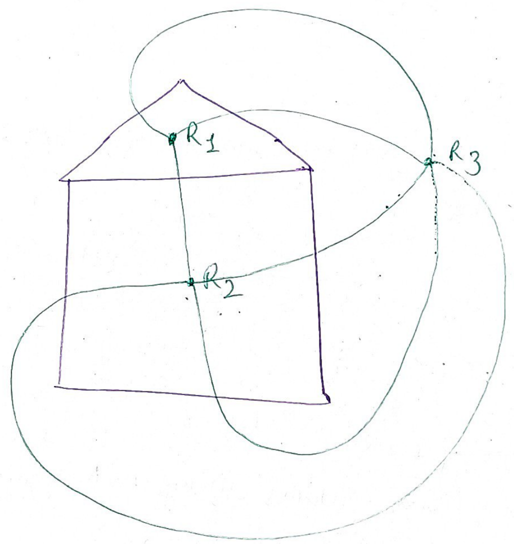
* Hamiltonian Path

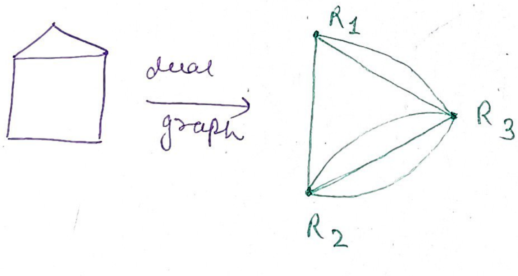
→ A path which contains every vertex exactly once

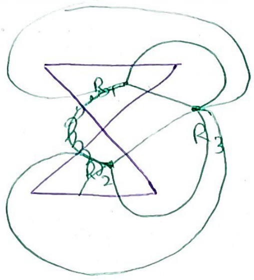
→ A graph G is called Hamiltonian if it contains Hamiltonian Circuit.

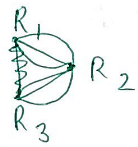


→ Let G be ~~an~~ a linear graph with n vertices. if sum of degree of each pair of vertices is (n-1) or greater then their exist a Hamiltonian path in G.









* Graph Colouring

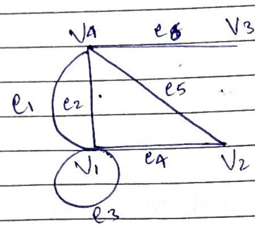
→ Process of ~~alloting~~ assigning the ~~tot~~ colour to the vertices in a graph. Such that no two adjacent vertices recieves same colour. is known as graph Colouring.

* Chromatic Number

→ Minimum no. of Colours required to Colour a graph is called Chromatic Number.

~~Adjecde~~
~~Adjecti~~

Adjecenecy:



	V_1	V_2	V_3	V_4
V_1	2	1	0	2
V_2	1	0	0	1
V_3	0	0	0	1
V_4	2	1	1	0

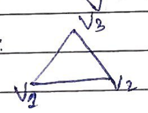
Incidency:

	e_1	e_2	e_3	e_4	e_5	e_6
V_1	1	1	1	1	0	0
V_2	0	0	0	1	1	0
V_3	0	0	0	0	0	1
V_4	1	1	0	0	1	1

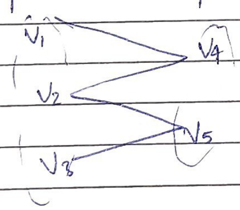
Chromatic no. (χ):

Complete graph (K_n):

K_3:

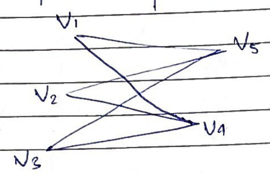


Bipartite Graph:



(3, 2)

Complete Bipartite Graph ($K_{m,n}$) , (3,2)



Properties : (n = vertices

1) $\chi(K_n) = n$

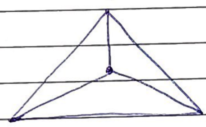
2) $\chi(K_{m,n}) = 2$

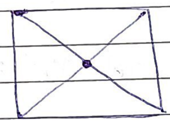
3) $\chi(C_n) = 2$, n is even
$= 3$, n is odd

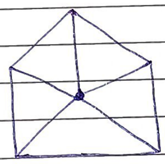
C = cyclic

4) $\chi(W_{n+1})$:
$= 3$, n is even
$= 4$, n is odd

Wheel Graph :







Chromatic Polynomial: ~~CC~~ $P(\lambda) = P(G, \lambda)$

$P(\lambda)$ which gives no. of different ways to colour a graph.

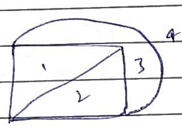
Properties:

1) If graph G has G_1, G_2, G_3 ----- G_n component then
$P(G,\lambda) = P(G_1,\lambda_1)\, P(G,\lambda_2),\, P(G,\lambda_3)$ ---- $P(G,\lambda_n)$.

2) Chromatic polynomial of K_n is
$P_n(\lambda) = \lambda(\lambda-1)(\lambda-2)$ ------ $(\lambda-n-1)$.

• Embedding of a planar graph:

The graph which can be draw on plane without intersecting is known as planar graph.



→ regions = 4

• Euler's Formula:

$$\boxed{n - e + r = 2}$$

n = no. of vertices
e = no. of edges
r = no. of regions

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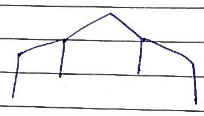
TREE:

→ Tree is a connected graph having a no cycling neither loop.

→ Its edges are called branches.

Properties of Tree:

1) One and only one path b/w every pair of vertices.
<=> graph is tree.



2) If G is ~~a tree with n vertices then there is exactly~~ a connected with n vertices and (n-1) edges then G is tree.

3) ~~A~~ graph G is tree if and only if it is minimumly connected.

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