UNIT-3

Q) Solve by Newton's method –

(a) minimize $f(x_1, x_2) = 100(x_2 - x_1^2)^2 + (1-x_1)^2$ from the starting point $\{-1.2, 1.0\}$

Sol: Here $X_i = \begin{Bmatrix} -1.2 \\ 1.0 \end{Bmatrix}$

$f(x_1, x_2) = 100[x_2^2 + x_1^4 - 2x_2x_1^2] + [1 + x_1^2 - 2x_1]$

$= 100x_1^4 + 100x_2^2 - 200x_2x_1^2 + x_1^2 - 2x_1 + 1$

I st iteration

$\nabla f_1 = \begin{bmatrix} \frac{\partial f}{\partial x_1} \\ \frac{\partial f}{\partial x_2} \end{bmatrix}_{X_1} = \begin{bmatrix} 400x_1^3 - 200x_2x_1 + 2x_1 - 2 \\ 200x_2 - 200x_1^2 \end{bmatrix}_{(-1.2, 1)}$

$200 - 200(1.2)^2$

$= \begin{bmatrix} -215.6 \\ -88 \end{bmatrix} \neq 0$

$[J_1] = \begin{bmatrix} 1200x_1^2 - 400x_2 + 2 & -400x_1 \\ -400x_1 & 200 \end{bmatrix}_{(-1.2, 1)} = \begin{bmatrix} 1330 & +480 \\ +480 & 200 \end{bmatrix}$

$\begin{matrix} \frac{\partial^2 f}{\partial x_1^2} & \frac{\partial^2 f}{\partial x_1 \partial x_2} \\ \frac{\partial^2 f}{} & \frac{\partial^2 f}{\partial x_2^2} \end{matrix}$

$[J_1]^{-1} = \frac{1}{25600} \begin{bmatrix} 200 & -480 \\ -480 & 1300 \end{bmatrix}$

$X_{i+1} = X_i - [J_i]^{-1} \nabla f_i$

$X_2 = X_1 - [J_1]^{-1} \nabla f_1$

$= \begin{bmatrix} -1.2 \\ 1.0 \end{bmatrix} - \begin{bmatrix} 0.0056 & -0.0134 \\ -0.0134 & 0.0365 \end{bmatrix} \begin{bmatrix} -215.6 \\ -88 \end{bmatrix}$

$= \begin{bmatrix} -1.2 \\ 1.0 \end{bmatrix} - \begin{bmatrix} -0.02816 \\ -0.3229 \end{bmatrix} \Rightarrow \begin{bmatrix} -1.17184 \\ 1.32294 \end{bmatrix}$

Scanned by CamScanner

2nd iteration

$$\nabla f_2 = [\nabla f_1]_{x_2}$$

$$= \begin{bmatrix} -1966.93 \\ 498.948 \end{bmatrix} \neq 0$$

$$J_2 = \begin{bmatrix} 1591.6431 & 51.5609 \\ 51.5609 & 200 \end{bmatrix}$$

$$[J_2]^{-1} = \begin{bmatrix} 6.335\times10^{-4} & -1.633\times10^{-4} \\ -1.633\times10^{-4} & 8.042\times10^{-3} \end{bmatrix}$$

$$X_3 = X_2 - [J_2]^{-1} \nabla f_2$$

$$= \begin{bmatrix} -1.17184 \\ 1.3229 \end{bmatrix} - \begin{bmatrix} J_2 \end{bmatrix}^{-1} \begin{bmatrix} -1966.93 \\ 498.948 \end{bmatrix}$$

$$= \begin{bmatrix} -1.17184 \\ 1.3229 \end{bmatrix} - \begin{bmatrix} -1.32782 \\ 2.83689 \end{bmatrix}$$

$$= \begin{bmatrix} 0.15568 \\ -1.51399 \end{bmatrix}$$

Scanned by CamScanner

3rd iteration

$$\nabla f_3 = [\nabla f_2]_{X_3}$$

$$= \begin{bmatrix} 31.824 \\ -38.155 \end{bmatrix}_{(0.15568,\,-1.51399)} \neq 0$$

$$J_3 = \begin{bmatrix} 236.67 & -62.272 \\ -62.272 & 200 \end{bmatrix} \neq 0$$

$$J_3^{-1} =$$

$$X_4 = X_3 - [J_3]^{-1}\,\nabla f_3$$

$$= \begin{bmatrix} 0.15568 \\ -1.51399 \end{bmatrix} - \begin{bmatrix} 4.6023\times10^{-3} & 1.432\times10^{-3} \\ 1.432\times10^{-3} & 5.44617\times10^{-3} \end{bmatrix} \begin{bmatrix} 31.824 \\ -38.155 \end{bmatrix}$$

$$= \begin{bmatrix} 0.15568 \\ -1.51399 \end{bmatrix} - \begin{bmatrix} 0.0917 \\ -0.1613 \end{bmatrix}$$

$$= \begin{bmatrix} 0.0639 \\ -1.3526 \end{bmatrix}$$

$$\nabla f_4 = [\nabla f_3]_{X_4}$$

$$= \begin{bmatrix} 2.035 \\ -271.3366 \end{bmatrix}$$

Scanned by CamScanner

① Minimize $f(x_1, x_2) = 4x_1^2 + 3x_2^2 - 5x_1x_2 - 8x_1$ from the starting point $\begin{Bmatrix} -1.2 \\ 1.0 \end{Bmatrix}$

Sol:- Here $X_1 = \begin{Bmatrix} -1.2 \\ 1.0 \end{Bmatrix}$

$$f(x_1, x_2) = 4x_1^2 + 3x_2^2 - 5x_1x_2 - 8x_1$$

1st iteration

$$\nabla f_1 = \begin{bmatrix} \frac{\partial f}{\partial x_1} \\ \frac{\partial f}{\partial x_2} \end{bmatrix} = \begin{bmatrix} 8x_1 - 5x_2 - 8 \\ 6x_2 - 5x_1 \end{bmatrix}_{X_1}$$

$$= \begin{bmatrix} -22.6 \\ 12 \end{bmatrix}$$

$$J_1 = \begin{bmatrix} 8 & -5 \\ -5 & 6 \end{bmatrix}_{X_1} \qquad [J_1]^{-1} = \frac{1}{23}\begin{bmatrix} 6 & 5 \\ 5 & 8 \end{bmatrix}$$

$$[J_1]^{-1} = \begin{bmatrix} 0.2608 & 0.2173 \\ 0.2173 & 0.3478 \end{bmatrix}$$

$$X_2 = X_1 - [J_1]^{-1} \nabla f_1$$

$$= \begin{bmatrix} -1.2 \\ 1.0 \end{bmatrix} - \begin{bmatrix} J_1^{-1} \end{bmatrix} \begin{bmatrix} -22.6 \\ 12 \end{bmatrix}$$

$$\begin{bmatrix} -1.2 \\ 1.0 \end{bmatrix} - \begin{bmatrix} -3.286 \\ -0.7373 \end{bmatrix}$$

$$= \begin{bmatrix} -0.8714 \\ 1.7373 \end{bmatrix}$$

Scanned by CamScanner

2nd iteration:

$$\nabla f_2 = [\nabla f]_{x_2}$$

$$= \begin{bmatrix} -23.6577 \\ 14.7808 \end{bmatrix} \neq 0$$

$$J_2 = \begin{bmatrix} 8 & -5 \\ -5 & 6 \end{bmatrix} \qquad J_2^{-1} = \begin{bmatrix} 0.2608 & 0.2173 \\ 0.2173 & 0.3478 \end{bmatrix}$$

$$x_3 = x_2 - [J_2]^{-1} \nabla f_2$$

$$= \begin{bmatrix} -0.8714 \\ 1.7373 \end{bmatrix} - \begin{bmatrix} 0.2608 & 0.2173 \\ 0.2173 & 0.3478 \end{bmatrix} \begin{bmatrix} -23.6577 \\ 14.7808 \end{bmatrix}$$

$$= \begin{bmatrix} -0.8714 \\ 1.7373 \end{bmatrix} - \begin{bmatrix} -2.958 \\ -5.5\ 92 \times 10^{-5} \end{bmatrix}$$

$$x_3 = \begin{bmatrix} 2.0866 \\ +1.737 \end{bmatrix}$$

3rd iteration $[\nabla f]_{x_3} = \begin{bmatrix} 0.0078 \\ -0.011 \end{bmatrix}$

(c) minimize $f(x_1, x_2) = x_1^4 - 2x_1^2 x_2 + x_1^2 + x_2^2 - 2x_1 + x_2 + 1$ with starting point $\begin{Bmatrix} 1.5 \\ -1.0 \end{Bmatrix}$ up to 2 iterations.

Sol:- $X_1 = \begin{Bmatrix} 1.5 \\ -1.0 \end{Bmatrix}$

$$\nabla f_1 = \begin{bmatrix} \frac{\partial f}{\partial x_1} \\ \frac{\partial f}{\partial x_2} \end{bmatrix}_{X_1} = \begin{bmatrix} 4x_1^3 - 4x_1 x_2 + 2x_1 - 2 \\ -2x_1^2 + 2x_2 + 1 \end{bmatrix}_{(1.5,\,-1.0)}$$

$$= \begin{bmatrix} 20.5 \\ -5.5 \end{bmatrix}$$

Scanned by CamScanner

$$J_1 = \begin{bmatrix} 12x_1^2 - 4x_2 + 2 & -4x_1 \\ -4x_1 & 2 \end{bmatrix}_{(1.5,\,-1.0)} = \begin{bmatrix} 33 & -6 \\ -6 & 2 \end{bmatrix}$$

x_1

$$[J_1]^{-1} = \begin{bmatrix} \frac{33}{30} & \cancel{\frac{6}{30}} \\ \cancel{\frac{6}{30}} & \frac{2}{30} \end{bmatrix} \quad \begin{bmatrix} \frac{1}{15} & -\frac{1}{5} \\ -\frac{1}{5} & \frac{11}{10} \end{bmatrix}$$

$$x_2 = x_1 - [J_1]^{-1}[\nabla f_1]$$

$$= \begin{bmatrix} 1.5 \\ -1.0 \end{bmatrix} - \begin{bmatrix} 0.0666 & -0.2 \\ -0.2 & 1.1 \end{bmatrix} \begin{bmatrix} 20.5 \\ -3.5 \end{bmatrix}$$

$$= \begin{bmatrix} 1.5 \\ -1.0 \end{bmatrix} - \begin{bmatrix} 2.466 \\ -10.15 \end{bmatrix} = \begin{bmatrix} 0.966 \\ 9.15 \end{bmatrix}$$

<u>2nd iteration</u>

$$\nabla f_2 = [\nabla f]_{x_2} = \begin{bmatrix} -31.817 \\ -148.145 \end{bmatrix}$$

$$J_2 = \begin{bmatrix} -11{,}553.277 & 127.268 \\ 127.268 & 2 \end{bmatrix}$$

$$J_2^{-1} = \begin{bmatrix} -5.08\times10^{-5} & 3.2380\times10^{-3} \\ 3.2380\times10^{-3} & 0.2939 \end{bmatrix}$$

$$x_3 = x_2 - [J_2]^{-1}[\nabla f_2]$$

$$= \begin{bmatrix} 0.966 \\ 9.15 \end{bmatrix} - \begin{bmatrix} J_2^{-1} \end{bmatrix} \begin{bmatrix} -31.817 \\ -148.145 \end{bmatrix}$$

Scanned by CamScanner

$$\begin{bmatrix} 0.966 \\ 9.15 \end{bmatrix} - \begin{bmatrix} -0.478 \\ -43.65 \end{bmatrix}$$

$$\begin{array}{r} 0.966 \\ 0.478 \\ \hline 1.444 \end{array}$$

$$= \begin{bmatrix} 1.444 \\ 52.8 \end{bmatrix}$$

(d) minimize $f(x_1, x_2) = (x_1 + 2x_2 - 7)^2 + (2x_1 + x_2 - 5)^2$ with starting point $\begin{Bmatrix} -1.5 \\ 1.5 \end{Bmatrix}$ upto 2 iteration.

Sol. $X_1 = \begin{Bmatrix} -1.5 \\ 1.5 \end{Bmatrix}$

$$f(x_1, x_2) = x_1^2 + 4x_2^2 + 49 + 4x_1x_2 - 28x_2 - 14x_1 + 4x_1^2 + x_2^2 + 25 + 4x_1x_2 - 20x_1 - 10x_2$$

$$= 5x_1^2 + 5x_2^2 + 8x_1x_2 - 38x_2 - 34x_1 + 74$$

1st iteration

$$\nabla f_1 = \begin{bmatrix} \frac{\partial f}{\partial x_1} \\ \frac{\partial f}{\partial x_2} \end{bmatrix}_{(x_1, x_2)} = \begin{bmatrix} 10x_1 + 8x_2 - 34 \\ 10x_2 + 8x_1 - 38 \end{bmatrix}_{(-1.5,\, 1.5)}$$

$$\Rightarrow \begin{bmatrix} -37 \\ -35 \end{bmatrix}$$

$$J_1 = \begin{bmatrix} 10 & 8 \\ 8 & 10 \end{bmatrix} \quad [J_1^{-1}] = \frac{1}{36} \begin{bmatrix} 10 & -8 \\ -8 & 10 \end{bmatrix}$$

$$= \begin{bmatrix} 0.277 & -0.222 \\ -0.222 & 0.277 \end{bmatrix}$$

$$X_2 = X_1 - [J_1]^{-1} \nabla f_1$$

$$= \begin{bmatrix} -1.5 \\ 1.5 \end{bmatrix} - \begin{bmatrix} J_1^{-1} \end{bmatrix} \begin{bmatrix} -37 \\ -35 \end{bmatrix}$$

$$= \begin{bmatrix} -1.5 \\ 1.5 \end{bmatrix} - \begin{bmatrix} -2.504 \\ -1.481 \end{bmatrix} = \begin{bmatrix} 1.004 \\ 1.981 \end{bmatrix}$$

Scanned by CamScanner

2nd iteration

$$\nabla f_2 = [\nabla f]_{x_2}$$

$$= \begin{bmatrix} -8.112 \\ -10.158 \end{bmatrix}$$

$$x_3 = [x_2] - [J_2]^{-1} [\nabla f_2]$$

$$= \begin{bmatrix} 1.004 \\ 1.981 \end{bmatrix} - \begin{bmatrix} J_2 \end{bmatrix}^{-1} \begin{bmatrix} -8.112 \\ -10.158 \end{bmatrix}$$

$$= \begin{bmatrix} 1.004 \\ 1.981 \end{bmatrix} - \begin{bmatrix} -2.0215 \\ -1.0200 \end{bmatrix}$$

$$= \begin{bmatrix} 3.045 \\ 3.001 \end{bmatrix}$$

$$\begin{array}{r} 1.981 \\ 1.020 \\ \hline 3.001 \end{array}$$

(c) minimize $f(x_1, x_2) = (10x_1 + 6x_2 - 9)^2 + (6x_1 + 10x_2 - 11)^2$ with starting point $\begin{Bmatrix} -1.0 \\ 1.0 \end{Bmatrix}$

Sol: $X_1 = \begin{Bmatrix} -1.0 \\ 1.0 \end{Bmatrix}$

$$f(x_1, x_2) = 100x_1^2 + 36x_2^2 + 81 + \cancel{120x_1x_2} - 108x_2 - 180x_1 + 36x_1^2 + 100x_2^2 + 121 - \cancel{120x_1x_2} - 220x_2 - 132x_1$$

$$\Rightarrow 136x_1^2 + 136x_2^2 + 202 - 328x_2 - 312x_1$$

$$\nabla f_1 = \begin{bmatrix} \frac{\partial f}{\partial x_1} \\ \frac{\partial f}{\partial x_2} \end{bmatrix}_{X_1} = \begin{bmatrix} 272x_1 - 312 \\ 272x_2 - 328 \end{bmatrix}_{(-1,1)} = \begin{bmatrix} -584 \\ -56 \end{bmatrix}$$

Scanned by CamScanner

$$J_1 = \begin{bmatrix} 272 & 0 \\ 0 & 272 \end{bmatrix} = [J_1]^{-1} = \begin{bmatrix} \frac{1}{272} & 0 \\ 0 & \frac{1}{272} \end{bmatrix}$$

$$x_2 = x_1 - [J_1]^{-1} \nabla f_1$$

$$= \begin{bmatrix} -1.0 \\ 1.0 \end{bmatrix} - \begin{bmatrix} \frac{1}{272} & 0 \\ 0 & \frac{1}{272} \end{bmatrix} \cdot \begin{bmatrix} -584 \\ -56 \end{bmatrix}$$

$$= \begin{bmatrix} -1.0 \\ 1.0 \end{bmatrix} - \begin{bmatrix} 2.147 \\ -0.205 \end{bmatrix}$$

$$= \begin{bmatrix} -3.147 \\ 1.205 \end{bmatrix}$$

2nd iteration

$$\nabla f_2 = \begin{bmatrix} -1167.984 \\ -0.24 \end{bmatrix}$$

$$x_3 = x_2 - [J_2]^{-1} \nabla f_2$$

$$= \begin{bmatrix} -3.147 \\ 1.205 \end{bmatrix} - \begin{bmatrix} \frac{1}{272} & 0 \\ 0 & \frac{1}{272} \end{bmatrix} \begin{bmatrix} -1167.984 \\ -0.24 \end{bmatrix}$$

$$= \begin{bmatrix} -3.147 \\ 1.205 \end{bmatrix} - \begin{bmatrix} -4.294 \\ 8.82 \times 10^{-4} \end{bmatrix}$$

$$= \begin{bmatrix} 1.147 \\ +1.204 \end{bmatrix}$$

Scanned by CamScanner

(b) Solve by univariate method—

(1) minimize $f(x_1, x_2) = 2x_1^2 + x_2^2$ from $\{1,2\}$.

Sol: Iteration 1

$X_1 = \{1,2\}$ $\quad 2+4$ $\quad S_1 = (1,0)$

$S_1 = \{1,0\}$ $\quad \varepsilon = 0.01$ $\quad \varepsilon = 0.01$

$f_1 = f(X_1) = f(1,2) = 6.$

$f_1^+ = f(X_1 + \varepsilon S_1) = f((1,2) + 0.01(1,0)) = f(1.01, 2)$

$= 6.0402$ $\quad 1+0.01\times1,\ 2+0.01\times0$

$f_1^- = f(X_1 - \varepsilon S_1) = f((1,2) - 0.01(1,0)) = f(0.99, 2)$

$= 5.9602$

$f_1^- < f_1 \Rightarrow -S$ is the correct direction.

$x_i \mp$

$f(X_1 - \lambda_1 S_1) = f((1,2) - \lambda_1(1,0))$ $\quad x_{i+1} = x_i \mp \lambda_i S$

$= f(1-\lambda_1, 2)$.

$2(1-\lambda_1)^2 + 4$

$2(1+\lambda_1^2 - 2\lambda_1) + 4$

$2\lambda_1^2 - 4\lambda_1 + 6$

$\cancel{f(-\lambda_1, 0) = 2\lambda_1^2}$ $f(1-\lambda_1, 2) = 2\lambda_1^2 - 4\lambda_1 + 6$

$$\frac{\partial f}{\partial \lambda_1} = 4\lambda_1 - 4 = 0 \Rightarrow \lambda_1^* = \frac{4}{4} = 1$$

Iteration 2

$X_2 = X_1 - \lambda_1^* S_1.$

$= (1,2) - 1(1,0).$ $\quad S_2 = (0,1)$

$= (0,2)$

$f_2(X_2) = 4$

$f(X_2 + \varepsilon S_2) = f_2^+ = f[(0,2) + 0.01(0,1)]$

$= f(0, 2.01) = 4.0401$

Scanned by CamScanner

$f(X_2 - \varepsilon S_2) = f_2^- = f[(0,2) - 0.01[0,2]]$

$= f(0, 1.98) = 3.9204$

$f_2^- < f_2 \quad \therefore -S_1$ is the correct direction.

$f(X_2 - \lambda_2 S_2) = f[(0,2) - \lambda_2(0,1)]$

$= f(0, 2-\lambda_2)$

$f(X_2 - \lambda_2 S_2) = (2-\lambda_2)^2$

$\frac{\partial f}{\partial \lambda_2} = 2(2-\lambda_2) = 0 \Rightarrow \lambda_2^* = 2$

(b) minimize $f(x_1, x_2) = x_1 - x_2 + 2x_1^2 + x_2^2 + 2x_1x_2$ from the starting point $(0,0)$ using $\varepsilon = 0.01$.

$-\frac{9.8 \times 10^3}{1000}$

0.0098

Sol. Iteration I $\quad X_1 = \{0,0\}$
$S_1 = \{1,0\}$

$f_1 = f(X_1) = f(0,0) = 0$

$f_1^+ = f(X_1 + \varepsilon S_1) = f((0,0) + 0.01(1,0))$

$= f(0.01, 0) = 0.0102$

$f_1^- = f(X_1 - \varepsilon S_1) = f[(0,0) - 0.01(1,0)]$

$= f(-0.01, 0) = -0.0098$

$f_1^- < f_1 \quad \therefore -S_1$ is the correct direction.

$f(X_1 - \lambda_1 S_1) = f((0,0) - \lambda_1(1,0)) = f(-\lambda_1, 0)$

$f(-\lambda_1, 0) = -\lambda_1 + 2\lambda_1^2.$

$\frac{\partial f}{\partial \lambda_1} = 4\lambda_1 - 1 \Rightarrow \lambda_1^* = \frac{1}{4}$

Scanned by CamScanner

Iteration 2

$$x_2 = x_1 - \lambda_1^* S_1$$
$$= (0,0) - \frac{1}{4}(1,0)$$
$$= \left(-\frac{1}{4}, 0\right) = (-0.25, 0)$$

$$f_2 = f(x_2) = -0.1250$$

$$f_2^+ = f(x_2 + \varepsilon S_2) = f[(-0.25, 0) + 0.01(0,1)]$$
$$= f(-0.25, 0.01)$$
$$= -0.1399$$

$$f_2^- = f(x_2 - \varepsilon S_2) = f[(-0.25, 0) - 0.01(0,1)]$$
$$= f(-0.25, -0.01)$$
$$= -0.1099$$

$f_2^+ < f_2$

$\therefore$ $+S_2$ is the correct direction

$$f(x_2 + \lambda_2 S_2) = f[(-0.25, 0) + \lambda_2(0,1)]$$
$$= f(-0.25, \lambda_2)$$

$$f(-0.25, \lambda_2) = -0.125 \cancel{+} \lambda_2 + \lambda_2^2 - 0.50\lambda_2$$
$$\Rightarrow \lambda_2^2 - 1.5\lambda_2 - 0.125$$

$$\frac{\partial f}{\partial \lambda_2} = 0 \Rightarrow 2\lambda_2 = 1.5$$
$$\lambda_2^* = \frac{3}{4} = 0.75$$

Scanned by CamScanner

② minimize $f(x_1, x_2, x_3) = x_1^2 + 3x_2^2 + 5x_3^2 - 2x_1x_2 + 3x_2x_3 - 8x_2$ with point $\begin{Bmatrix} 1 \\ 2 \\ 3 \end{Bmatrix}$ with $\epsilon = 0.01$.

Sol: Iteration 1 $\quad X_1 = (1, 2, 3)$

$S_1 = (1, 0, 0)$

$f_1 = f(X_1) = 65 - 16 \Rightarrow 49$

$f_1^+ = f(X_1 + \epsilon S_1) = f((1,2,3) + 0.01(1,0,0))$

$= f(1.01, 2, 3) = 48.9$

$f_1^- = f(X_1 - \epsilon S_1) = f((1,2,3) - 0.01(1,0,0))$

$= f(0.99, 2, 3)$

$= 49.09$

$f_1^+ < f_1 \quad \Rightarrow + S_1$ is the correct direction

$\therefore f(X_1 + \lambda_1 S_1) = f_1((1,2,3) + \lambda_1(1,0,0))$

$= f(1+\lambda_1, 2, 3)$

$= (1+\lambda_1)^2 + 12 + 45 - 2(1+\lambda_1)2 + 18 - 7(1+\lambda_1)$
-16

$= 1 + \lambda_1^2 + 2\lambda_1 + 57 + 2 - 11(1+\lambda_1) =$

$= \lambda_1^2 + 2\lambda_1 + 60 - 11 - 11\lambda_1 \Rightarrow \lambda_1^2 - 9\lambda_1 + 49 = 0$

$\frac{\partial f}{\partial \lambda_1} = 2\lambda_1^* - 9 \Rightarrow \lambda_1^* = \frac{9}{2}$

Iteration 2 $\quad S_2 = (0, 1, 0)$

$X_2 = X_1 + \lambda_1^* S_1$

$= (1, 2, 3) + \frac{9}{2}(1, 0, 0)$

$= (5.5, 2, 3)$

$f_2 = f(X_2) = 28.75$

Scanned by CamScanner

$f_2^+ = f(X_2 + \varepsilon S_2) = f((5.5, 2, 3) + 0.01(0,1,0))$

$= f(5.5, 2.01, 3)$

$= 28.7703$

$f_2^- = f(X_2 - \varepsilon S_2) = f((5.5, 2, 3) - 0.01(0,1,0))$

$= f(5.5, 1.99, 3)$

$= 28.7303$

$f_2^- < f_2$

$\therefore -S_2$ is the new correct direction

$f(X_2 - \lambda_2 S_2) = f((5.5, 2, 3) - \lambda_2(0,1,0))$

$= f((5.5, 2-\lambda_2, 3))$

$\Rightarrow (5.5)^2 + 3(2-\lambda_2)^2 + 5\times 9 - 2(5.5)(2-\lambda_2) + 3(2-\lambda_2)(3)$
$- 7\times 5.5 - 8(2-\lambda_2)$

$\Rightarrow 36.75 + 3(4+\lambda_2^2 - 4\lambda_2) - 11(2-\lambda_2) + 9(2-\lambda_2)$
$- 8(2-\lambda_2)$

$\Rightarrow 36.75 + 12 + \underline{3\lambda_2^2} - \underline{12\lambda_2} - 22 + \underline{11\lambda_2} + 18 - \underline{9\lambda_2} - 16 + \underline{8\lambda_2}$

$\Rightarrow 3\lambda_2^2 - 2\lambda_2 + 28.75 = 0$

$\frac{\partial f}{\partial \lambda_2} \Rightarrow 6\lambda_2 - 2 \Rightarrow 0 \qquad \lambda_2^* = \frac{1}{3}$

<u>3rd Iteration</u> $\qquad X_3 = X_2 - \lambda_2^* S_2$

$= (5.5, 2, 3) - \frac{1}{3}(0,1,0)$

$= (5.5, 1.66, 3)$

Scanned by CamScanner

$f_3(X_3) = 28.4168$

$f_3^+ = f(X_3 + \varepsilon S_3) = f[(5.5, 1.66, 3) + 0.01(0,0,1)]$

$= f(5.5, 1.66, 3.01)$

$= 28.1671$

$f_3^- = f(X_3 - \varepsilon S_3) = f[(5.5, 1.66, 3) - 0.01(0,0,1)]$

$= f(5.5, 1.66, 2.99)$

$= 28.6675$

$f_3^+ < f_3$

$\therefore +S_3$ is the new correct direction.

$f(X_3 + \lambda_3 S_3) = f[(5.5, 1.66, 3) + \lambda_3 (0,0,1)]$

$= f(5.5, 1.66, 3+\lambda_3)$

$\Rightarrow (5.5)^2 + 3(1.66)^2 + 5 \times (3+\lambda_3)^2 - 2 \times 5.5 \times 1.66 + 3 \times 1.66 \times (3+\lambda_3)$

$- 7 \times 5.5 - 8 \times 1.66$

$\Rightarrow -31.52 + 5(9 + \lambda_3^2 + 6\lambda_3) + 4.98(3 + \lambda_3)$

$\Rightarrow 5\lambda_3^2 + 34.98\lambda_3 + 28.42 =$

$\frac{\partial f}{\partial \lambda_3} = 10\lambda_3 = -34.98$

$\Rightarrow \lambda_3^* = -3.498$

(d)

Scanned by CamScanner

(a) minimize $f(x_1, x_2) = 4x_1^2 + 3x_2^2 - 5x_1x_2 - 8x_1$ with starting point $\begin{Bmatrix} 2 \\ -2 \end{Bmatrix}$ given $\epsilon = 0.01$.

Sol: Iteration 1 $\quad X_1 = \langle 2, -2 \rangle$

$S_1 = \langle 1, 0 \rangle$

$f_1 = f(X_1) = 32$

$f_1^+ = f(X_1 + \epsilon S_1) = f((2,-2) + 0.01(1,0))$

$= f(2.01, -2)$

$= 32.1804$

$f_1^- = f(X_1 - \epsilon S_1) = f((2,-2) - 0.01(1,0))$

$= f(1.99, -2)$

$= 31.18204$

$f_1^+ < f_1 \Rightarrow +S_1$ is the new correct direction

$f(X_1 + \lambda_1 S_1) = f((2,-2) + \lambda_1(1,0))$

$= f(2+\lambda_1, -2)$

$f(2+\lambda_1, -2) = 4(2+\lambda_1)^2 + 3(4) + 5 \times 2 \times (2+\lambda_1) - 8(2+\lambda_1)$

$= 4(4+\lambda_1^2+4\lambda_1) + 12 + 10(2+\lambda_1) - 8(2+\lambda_1)$

$\Rightarrow 4\lambda_1^2 + 18\lambda_1 + 32.$

$\frac{\partial f}{\partial \lambda_1} = 8\lambda_1 + 18 = 0 \quad \Rightarrow \lambda_1^* = \frac{9}{4} = 2.25$

Iteration 2 $\quad S_2 = \langle 0, 1 \rangle$

$X_2 = X_1 + \lambda_1^* S_1 = (4.25, -2)$

$f(X_2) = 92.75$

$$f(X_2 + \varepsilon S_2) = f((4.25, -2) + 0.01(0,1))$$
$$= f(4.25, -1.99)$$
$$= 92.4178$$

$$f(X_2 - \varepsilon S_2) = f((4.25, -2) + 0.01(0,1))$$
$$= f(4.25, -2.01)$$
$$= \cancel{68.8477}\ 93.08$$

$f_2^{+} < f_2$

$\therefore +S_2$ is the new direction

$$f(X_2 - \lambda_2 S_2) = f((4.25, -2) - \lambda_2(0,1))$$
$$= f(4.25, -2 - \lambda_2)$$

$$f(4.25, -2-\lambda_2) = 4 \times (4.25)^2 + 3(-2-\lambda_2)^2 + 5(4.25)(\lambda_2 + 2) - 8 \times 4.25$$
$$= 38.25 + 3(\lambda_2^2 + 4 + 4\lambda_2) + 21.25\lambda_2 + 42.5$$
$$= 3\lambda_2^2 + 33.75\lambda_2 + 92.75 = 0$$

$$\frac{\partial f}{\partial \lambda_2} = 6\lambda_2 + 33.75 = 0$$

$$\lambda_2^{*} = -5.541$$

$$X_3 = [(4.25, -2) + 5.541(0,1)]$$
$$= (4.25, 3.541)$$

(Q-3) Solve by Steep-Descent method—

(a) minimize $f(x_1, x_2) = 2x_1^2 + x_2^2$ by using starting point as $\{1, 2\}$.

Sol:

Iteration 1 $\nabla f_1 = \nabla f(x_1) = \begin{bmatrix} \frac{\partial f}{\partial x_1} \\ \frac{\partial f}{\partial x_2} \end{bmatrix}_{(1,2)}$

$S_1 = -\boxed{\nabla f_1}$

$= \begin{bmatrix} 4x_1 \\ 2x_2 \end{bmatrix}_{1,2} = \begin{bmatrix} 4 \\ 4 \end{bmatrix}$

$\boxed{S_1 = -\nabla f_1}$

$f(x_1 + \lambda_1 S_1) = f\left(x_1 + \lambda_1 \begin{bmatrix} -4 \\ -4 \end{bmatrix}\right)$

$= f\left((1,2) + \lambda_1 \begin{pmatrix} -4 \\ -4 \end{pmatrix}\right)$

$= f(1 - 4\lambda_1, 2 - 4\lambda_1)$

$f(1-4\lambda_1, 2-4\lambda_1) = 2(1-4\lambda_1)^2 + (2-4\lambda_1)^2$

$= 2[1 + 16\lambda_1^2 - 8\lambda_1] + 4 + 16\lambda_1^2 - 16\lambda_1$

$= 32\lambda_1^2 - 16\lambda_1 + 2 + 16\lambda_1^2 - 16\lambda_1 + 4$

$= 48\lambda_1^2 - 32\lambda_1 + 6$

$\frac{\partial f}{\partial \lambda_1} = 96\lambda_1 = 32$

$\lambda_1^* = \frac{\cancel{32} \cancel{16} 2}{\cancel{96} \cancel{48} 6} = \frac{1}{3}$

$X_2 = (1 - 4\lambda_1, 2 - 4\lambda_1) = \left(1 - \frac{4}{3}, 2 - \frac{4}{3}\right)$

$= \left(\frac{-1}{3}, \frac{2}{3}\right)$

Scanned by CamScanner

Iteration 2

$$S_2 = -\nabla f_2 = -\nabla f(X_2)$$

$$= \begin{bmatrix} 4x_1 \\ 2x_2 \end{bmatrix}_{(-1/3, 2/3)} = -\begin{bmatrix} -4/3 \\ 4/3 \end{bmatrix} = \begin{bmatrix} 4/3 \\ -4/3 \end{bmatrix}$$

$$f(X_2 + \lambda_2 S_2) = f\left(\left(-1/3, 2/3\right) + \lambda_2 \begin{pmatrix} 4/3 \\ -4/3 \end{pmatrix}\right)$$

$$= f\left(\left(-\frac{1}{3} + \frac{4}{3}\lambda_2\right), \frac{2}{3} - \frac{4}{3}\lambda_2\right)$$

$$f(X_2+\lambda_2 S_2) = 2\left(-\frac{1}{3} + \frac{4}{3}\lambda_2\right)^2 + \left(\frac{2}{3} - \frac{4}{3}\lambda_2\right)^2$$

$$= 2\left[\frac{1}{9} + \frac{16}{9}\lambda_2^2 - \frac{8}{9}\lambda_2\right) + \left(\frac{4}{9} - \frac{16}{9}\lambda_2^2 - \frac{16}{9}\lambda_2\right)$$

$$= \frac{16}{9}\lambda_2^2 - \frac{32}{9}\lambda_2 + \frac{6}{9}$$

$$\frac{\partial f}{\partial \lambda_2} = \frac{32}{9}\lambda_2 - \frac{32}{9} = 0$$

☆ Check until ∇f_i becomes 0

$$\therefore \lambda_2^* = 1$$

$$X_3 = \left(1, -\frac{2}{3}\right)$$

(b) minimize $f(x_1, x_2) = 6x_1^2 + 2x_2^2 - 6x_1x_2 - x_1 - x_2$ by using starting point $\{1, 2\}$.

Sol: Iteration 1 $\nabla f(x_1) = \begin{bmatrix} \frac{\partial f}{\partial x_1} \\ \frac{\partial f}{\partial x_2} \end{bmatrix}_{(1,2)}$

$$= \begin{bmatrix} 12x_1 - 6x_2 - 1 \\ 4x_2 - 6x_1 - 1 \end{bmatrix}_{(1,2)} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$S_1 = -\nabla f_1$

$f(x_1 + \lambda_1 S_1) = f\left(x_1 + \lambda_1 \begin{pmatrix} 1 \\ -1 \end{pmatrix}\right)$

$= f\left((1,2) + \lambda_1 (1,-1)\right)$

$= f(1+\lambda_1, 2-\lambda_1)$

$f(1+\lambda_1, 2-\lambda_1) = 6(1+\lambda_1)^2 + 2(2-\lambda_1)^2 - 6(1+\lambda_1)(2-\lambda_1) - (1+\lambda_1)$
$- (2-\lambda_1)$

$= 6(1+\lambda_1^2+2\lambda_1) + 2(4+\lambda_1^2 - 4\lambda_1) - 6(2+\lambda_1-\lambda_1^2)$
$- 1 - \cancel{\lambda_1} - 2 + \cancel{\lambda_1}$

$= 6 + 6\lambda_1^2 + 12\lambda_1 + 8 + 2\lambda_1^2 - 8\lambda_1 - 12 - 6\lambda_1 + 6\lambda_1^2 - 3$

$\Rightarrow 14\lambda_1^2 \cancel{+} 2\lambda_1 - 1$

$\frac{\partial f}{\partial \lambda_1} = 28\lambda_1 = 2 \quad \Rightarrow \lambda_1^* = \frac{1}{14}$

$x_2 = (1.0714, 1.9285)$

Iteration 2 $\quad S_2 = -\nabla f_2 = -\begin{bmatrix} 12x_1 - 6x_2 - 1 \\ 4x_2 - 6x_1 - 1 \end{bmatrix}_{x_2}$

$= \begin{bmatrix} -0.2858 \\ -0.2856 \end{bmatrix}$

$f(x_2 + \lambda_2 S_2) = f(x_2 + \lambda_2 S_2)$

$= f(1.0714 \cancel{+} 0.2858\lambda_2, 1.9285 - 0.2856\lambda_2)$

Scanned by CamScanner

$$\Rightarrow 6(1.0714-0.2858\lambda_2)^2+2(1.9285-0.2856\lambda_2)^2-6(1.0714-0.2858\lambda_2)$$
$$(1.9285-0.2856\lambda_2)-(1.0714-0.2858\lambda_2)-(1.9285-0.2856\lambda_2)$$

$$\Rightarrow 6(1.147+0.0816\lambda_2^2-0.6124)+2(3.7191+0.081\lambda_2^2-1.1015\lambda_2^2)$$
$$-(6.4284-1.7148\lambda_2)(1.9285-0.2856\lambda_2)-2.999-0.8714$$

$$\Rightarrow 2.1589\lambda_2^2+5.141\lambda_2+0.123=0$$

$$\frac{\partial f}{\partial \lambda_2}= \quad \lambda_2^* = -\frac{5.141}{2.158} = -2.382$$

$$\lambda_2^* = -2.382$$

$$x_3 = (1.7521, 2.608)$$

P.O

(c) $f(x_1,x_2) = x_1 - x_2 + 2x_1^2 + 2x_1x_2 + x_2^2$ with starting point $\{0, 1\}$

Sol: Iteration 1 $\quad S_1 = -\nabla f_1 = -\begin{bmatrix} \partial f/\partial x_1 \\ \frac{\partial f}{\partial x_2} \end{bmatrix}_{(0,1)}$

$$= -\begin{bmatrix} 1+4x_1+2x_2 \\ -1+2x_1+2x_2 \end{bmatrix}_{(0,1)}$$

$$= -\begin{bmatrix} 3 \\ 1 \end{bmatrix} = \begin{bmatrix} -3 \\ -1 \end{bmatrix}$$

$$f(x_1+\lambda_1 S_1) = f((0,1)+\lambda_1(-3,-1)) = f(-3\lambda_1, 1-\lambda_1)$$

$$f(-3\lambda_1, 1-\lambda_1) = -3\lambda_1 + (1-\lambda_1) + 2(9\lambda_1^2) + 2(-3\lambda_1)(1-\lambda_1) + (1-\lambda_1)^2$$
$$= -3\lambda_1 + 1 - \lambda_1 + 18\lambda_1^2 - 6\lambda_1 + 6\lambda_1^2 + 1 + \lambda_1^2 - 2\lambda_1$$
$$= 25\lambda_1^2 - 12\lambda_1 + 2 =$$

$$\frac{\partial f}{\partial \lambda_1} = 50\lambda_1 = 12 \Rightarrow \boxed{\lambda_1^* = \frac{6}{25}}$$

Scanned by CamScanner

$$X_2 = \left(-\frac{3\times 6}{25},\ 1-\frac{6}{25}\right)$$

$$= (-0.72,\ 0.76)$$

<u>Iteration 2</u>

$$S_2 = -\nabla f_2$$

$$= -\begin{bmatrix} 1+4x_1+2x_2 \\ -1+2x_1+2x_2 \end{bmatrix}_{(-0.72,\ 0.76)}$$

$$= \begin{bmatrix} 0.36 \\ 0.92 \end{bmatrix}$$

$$f(X_2+\lambda_2 S_2) = f(X_2 + \lambda_2(S_2))$$

$$= f\left((-0.72, 0.76) + \lambda_2(0.36, 0.92)\right)$$

$$= f\left(-0.72+\lambda_2 0.36,\ 0.76+0.92\lambda_2\right)$$

$$= -0.72+0.36\lambda_2 - 0.76 - 0.92\lambda_2 + 2(0.5184 + 0.1296\lambda_2^2 - 0.5184\lambda_2) + (0.5776 + 0.8464\lambda_2^2 + 1.3984\lambda_2) + (-0.5472 - 0.3888\lambda_2 + 0.3312\lambda_2^2)$$

$$\Rightarrow 1.4368\lambda_2^2 - 0.5872\lambda_2 - 0.4128$$

$$\frac{df}{d\lambda_2} \Rightarrow 2.8736\lambda_2 = 0.5872$$

$$\lambda_2^* = 0.2043$$

$$X_3 = (-0.646,\ 0.947)$$

(d) minimize $f(x_1, x_2) = -3x_1 - 2x_2 + 2x_1^2 + 2x_1x_2 + 1.5x_2^2$ with starting point $\{1, -1\}$

Sol: Iteration 1

$$S_1 = -\nabla f_1 = -\nabla f(x_1)$$

$$= -\begin{bmatrix} -3 + 4x_1 + 2x_2 \\ -2 + 2x_1 - 10x_2 \end{bmatrix}_{(1,-1)}$$

$$= -\begin{bmatrix} -3+4-2 \\ -2+2+10 \end{bmatrix} = -\begin{bmatrix} -1 \\ 10 \end{bmatrix} = \begin{bmatrix} 1 \\ -10 \end{bmatrix}$$

$$f(x_1 + \lambda_1 S_1) = f\left((1,-1) + \lambda_1 (1, -10)\right)$$

$$= f\left((1+\lambda_1, -1 \cancel{+} 10\lambda_1)\right)$$

$$f(1+\lambda_1, -1+10\lambda_1) = -3(1+\lambda_1) - 2(-1+10\lambda_1) + 2(1+\lambda_1)^2 + 2(1+\lambda_1)(10\lambda_1 - 1) + 1.5(10\lambda_1 - 1)^2$$

$10\lambda_1 - 1 + 10\lambda_1 - 1$ $100\lambda_1^2 + 1 - 20\lambda_1$ $10\lambda_1 - 1 + 10\lambda_1^2 \cancel{+} \lambda_1$ $10\lambda_1^2 + 9\lambda_1 - 1$

$$\Rightarrow -3 - 3\lambda_1 + 2 - 20\lambda_1 + 2\lambda_1^2 + 2 + 4\lambda_1 + 150\lambda_1^2 + 1.5 - 30\lambda_1 + -2 + 18\lambda_1 + 20\lambda_1^2 = 0$$

$$\Rightarrow 172\lambda_1^2 + (-31\lambda_1) + (4.3) = 0$$

$$\frac{\partial f}{\partial \lambda_1} = 344\lambda_1 - 31 = 0$$

$$\boxed{\lambda_1^* = \frac{31}{344}} \boxed{= 0.090}$$

$-1 + 0.9$

Iteration 2

$$\cancel{X}_2 = (1.09, -0.1)$$

$$S_2 = -\nabla f_2 = -\begin{bmatrix} 1.16 \\ 1.18 \end{bmatrix}$$

$$f(x_2 + \lambda_2 S_2) \left(1.09 - 1.16\lambda_2, -0.1 - 1.18\lambda_2\right)$$

$$\cancel{\lambda_2^* = \frac{27}{69}} =$$

Scanned by CamScanner

$-3(1.09 - 1.16\lambda_2) - 2(-0.1 - 1.18\lambda_2) + 2(1.09 - 1.16\lambda_2)^2 +$
$2(1.09 - 1.16\lambda_2)(-0.1 - 1.18\lambda_2) + 1.5(-0.1 - 1.18\lambda_2)^2$

$= 2.6912\lambda_2^2 + 2.7376\lambda_2^2 + 2.0886\lambda_2^2 +$
$3.48\lambda_2 + 2.36\lambda_2 - 5.0576\lambda_2 + (-0.2524)\lambda_2$
$+ (-0.354)\lambda_2$

$= 7.5174\lambda_2^2 + 0.1761\lambda_2 = 0$

$\Rightarrow \lambda_2 = -0.0117$

(Q-4) Solve by random search —

(a) minimize $f(x_1, x_2) = 12x_1^2 - 8x_1x_2 + 0.2x_2^2 - 0.5x_1 - 2x_2$ in range $-5 \le x_1 \le 5$ & $-10 \le x_2 \le 10$ upto 10 iteration.

$l_1 \quad u_1 \quad l_2 \quad u_2$

Sol: $f(x_1, x_2) = 12x_1^2 - 8x_1x_2 + 0.2x_2^2 - 0.5x_1 - 2x_2$

Let us take a random no. b/w (0,1) be 0.4806

Iteration 1

$l_1 + (\ ) (u_1 - l_1)$ 0.4, 0.6

$X = \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} -5 + 0.4(5+5) \\ -10 + 0.6(10+10) \end{Bmatrix}$

$= \begin{Bmatrix} -1 \\ 2 \end{Bmatrix}$

$l_1 + (\ )(u_1 - l_1)$
$l_2 + (\ )(u_2 - l_2)$

$f(-1, 2) = 25.3$

Iteration 2 0.33, 0.34

$X = \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} -5 + 0.33 \times 10 \\ -10 + 0.6 \times 20 \end{Bmatrix} = \begin{Bmatrix} -1.7 \\ -8.8 \end{Bmatrix}$

$= -51.062$

Scanned by CamScanner

Iteration 8   0.25, 0.26

|

|

|

(b) minimize $f(x_1, x_2) = 15x_1^2 - 18x_1x_2 + 0.6x_2^2 - 1.67x_1 - 7x_2$ in range $-2 \le x_1 \le 2$ and $-4 \le x_2 \le 4$ is using random search method upto 10 iterations given set of values —

$((x_1, x_2) = (0.50, 0.60), (0.25, 0.26), (0.98, 0.97), (0.45, 0.46),$
$(0.234, 0.235), (0.63, 0.64), (0.534, 0.544),$
$(0.712, 0.713), (0.434, 0.435), (0.782, 0.783)\})$

0.1   0.01   0.001   0.1, 0.01, 0.001

Sol:- Iteration 1

$$x_1 = \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} -2 + 0.50(4) \\ -4 + 0.60(8) \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0.8 \end{Bmatrix}$$

$f(0, 0.8) = \cancel{0}$ -5.216

Iteration 2

$$x_2 = \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} -2 + 0.25(4) \\ -4 + 0.26(8) \end{Bmatrix} = \begin{Bmatrix} 0 \\ -1.92 \end{Bmatrix}$$

$f(0, -1.92) = 15.65184$

Iteration 3

$$x_3 = \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} -2 + 0.98 \times 4 \\ -4 + 0.97 \times 8 \end{Bmatrix} = \begin{Bmatrix} 1.92 \\ 3.76 \end{Bmatrix}$$

$f(1.92, 3.76) = -95.693$

|

|

|

(c) Similarly

(4)(5) Derive the geometric dual of —

(1) $f(x) = 20x_2x_3x_4^4 + 20x_1^2x_3^{-1} + 5x_2x_3^2$ subject to

$5x_2^{-5}x_3^{-1} \le 1$, $10x_1^{-1}x_2^3x_4^{-1} \le 1$, $x_i \ge 0$, $i = 1$ to 4

Sol:

$n = 4$

$N_0 = 3$

$N_1 = 1$

$N_2 = 1$

$m = 2$

$N = N_0 + N_1 + N_2 = 5$

$d = N - n - 1 = 5 - 4 - 1 = 0$

∴ Degree of Difficulty $= 0$

So Dual ~~problem can be stated as~~ —

Find max $v(\lambda) = \begin{Bmatrix} \lambda_{01} \\ \lambda_{02} \\ \lambda_{03} \\ \lambda_{04} \\ \lambda_{05} \end{Bmatrix}$ to maximize. subm

$$v(\lambda) = \prod_{k=0}^{2} \prod_{j=1}^{N_k} \left(\frac{c_{kj}}{\lambda_{kj}} \sum_{l=1}^{N_k} \lambda_{kl} \right)^{\lambda_{kj}}$$

$$= \prod_{j=1}^{N_0} \left(\frac{c_{0j}}{\lambda_{0j}} \sum_{l=1}^{N_0} \lambda_{0l} \right)^{\lambda_{0j}} \times \prod_{j=1}^{N_1} \left(\frac{c_{1j}}{\lambda_{0j}} \sum_{l=1}^{N_0} \lambda_{1l} \right)^{\lambda_{1j}}$$

$$\times \prod_{j=1}^{N_2} \left(\frac{c_{2j}}{\lambda_{2j}} \sum_{l=1}^{N_2} \lambda_{2l} \right)^{\lambda_{2j}}$$

$$= \prod_{j=1}^{3} \left(\frac{c_{0j}}{\lambda_{0j}} \sum_{l=1}^{3} \lambda_{0l} \right)^{\lambda_{0j}} \times \prod_{j=1}^{1} \left(\frac{c_{1j}}{\lambda_{0j}} \sum_{l=1}^{1} \lambda_{1l} \right)^{\lambda_{1j}}$$

$(\lambda_{01} + \lambda_{02} + \lambda_{03})$

$$\times \prod_{j=1}^{1} \left(\frac{c_{2j}}{\lambda_{0j}} \sum_{l=1}^{1} \lambda_{2l} \right)^{\lambda_{2j}}$$

Scanned by CamScanner

$$\Rightarrow \left(\frac{C_{01}}{\lambda_{01}}(\lambda_{01}+\lambda_{02}+\lambda_{03})\right)^{\lambda_{01}} \left(\frac{C_{02}}{\lambda_{02}}(\lambda_{01}+\lambda_{02}+\lambda_{03})\right)^{\lambda_{02}}$$

$$\left(\frac{C_{03}}{\lambda_{03}}(\lambda_{01}+\lambda_{02}+\lambda_{03})\right)^{\lambda_{03}} \times \left(\frac{C_{11}}{\lambda_{01}}\lambda_{11}\right)^{\lambda_{11}} \times$$

$$\left(\frac{C_{21}}{\lambda_{a}}\lambda_{21}\right)^{\lambda_{21}} \qquad \left(\frac{5\times5\times5}{3}\right)^{3/5} \times \left(\frac{10\times5\times5}{4}\right)^{4/5}$$

we know that,

Normality condition is given by

$$\sum_{j=1}^{N_0} \lambda_{0j} = 1$$

$$\Rightarrow \sum_{j=1}^{3} \lambda_{0j} = 1 \Rightarrow \boxed{\lambda_{01}+\lambda_{02}+\lambda_{03}=1} \quad \text{—(1)}$$

Orthogonal constraint are—

$$\sum_{k=0}^{m} \sum_{j=1}^{N_k} a_{kij}\lambda_{kj} = 0 \qquad \boxed{i=1,2\ldots n}$$

$$= \sum_{k=0}^{2} \sum_{j=1}^{N_k} a_{kij}\lambda_{kj} = 0$$

$$\Rightarrow \sum_{j=1}^{N_0} a_{0ij}\lambda_{0j} + \sum_{j=1}^{N_1} a_{1ij}\lambda_{1j} + \sum_{j=1}^{N_2} a_{2ij}\lambda_{2j} = 0$$

$$\Rightarrow a_{0i1}\lambda_{01} + a_{0i2}\lambda_{02} + a_{0i3}\lambda_{03} + a_{1i1}\lambda_{11} + a_{2i1}\lambda_{21} = 0$$

$i = 1,2,3,4$

for $i=1$

$$a_{011}\lambda_{01} + a_{012}\lambda_{02} + a_{013}\lambda_{03} + a_{111}\lambda_{11} + a_{211}\lambda_{21} = 0 \quad \text{—(2)}$$

for $i=2$:

$$a_{021}\lambda_{01} + a_{022}\lambda_{02} + a_{023}\lambda_{03} + a_{121}\lambda_{11} + a_{221}\lambda_{21} = 0 \quad (3)$$

Scanned by CamScanner

For $i=3$

$$a_{031}\lambda_{01} + a_{032}\lambda_{02} + a_{033}\lambda_{03} + a_{131}\lambda_{11} + a_{231}\lambda_{21} = 0$$

For $i=4$

$$a_{041}\lambda_{01} + a_{042}\lambda_{02} + a_{043}\lambda_{03} + a_{141}\lambda_{11} + a_{241}\lambda_{21} = 0$$

From (2)

~~$a_{011} =$~~

~~$a_{021} =$~~

$$\begin{array}{ccccc} \lambda_{01} & \lambda_{02} & \lambda_{03} & \lambda_{11} & \lambda_{21} \end{array}$$
$$\begin{bmatrix} 0 & 2 & 0 & 0 & -1 \\ 1 & 0 & 1 & -5 & 3 \\ 1 & -1 & 2 & -1 & 0 \\ 4 & 0 & 0 & 0 & -1 \end{bmatrix}$$

$$\Rightarrow \boxed{\lambda_{01} + \lambda_{02} + \lambda_{03} = 1} \quad \text{—— (1)}$$

$$2\lambda_{02} - \lambda_{21} = 0 \quad \text{—— (2)}$$

$$\lambda_{01} + \lambda_{03} - 5\lambda_{11} + 3\lambda_{21} = 0 \quad \text{—— (3)}$$

$$\lambda_{01} - \lambda_{02} + 2\lambda_{03} - \lambda_{11} = 0 \quad \text{—— (4)}$$

$$4\lambda_{01} - \lambda_{21} = 0 \quad \text{—— (5)}$$

$$\therefore 2\lambda_{02} = 4\lambda_{01}$$

$$\boxed{\lambda_{02} = 2\lambda_{01}}$$

$$= \lambda_{01} + 2\lambda_{01} + \lambda_{03} = 1$$

$$\boxed{3\lambda_{01} + \lambda_{03} = 1}$$

(3) − 5×(4)

$$\Rightarrow -4\lambda_{01} + 5\lambda_{03} + 5\lambda_{02} + 15\lambda_{21} - 10\lambda_{03} = 0$$

$$\Rightarrow -4\lambda_{01} - 5\lambda_{03} + 5\lambda_{02} + 15\lambda_{21} = 0$$

Scanned by CamScanner

$6\lambda_{01} - 5\lambda_{03} + 18\lambda_{21} = 0$

$\Rightarrow \frac{\lambda_{21}}{4} + \frac{\lambda_{21}}{2} + \lambda_{03} = 1$

$\cancel{\lambda_{03}} = 3\lambda_{21} = 4 - 4\lambda_{03}$

$\lambda_{21} = \frac{4 - 4\lambda_{03}}{3}$

$\Rightarrow \lambda_{01} + \lambda_{03} + 3\lambda_{21} = 5(\lambda_{01} - \lambda_{02} + 2\lambda_{03})$

$\Rightarrow 4\lambda_{01} - 5\lambda_{02} - \lambda_{03} + 10\lambda_{03} - 3\lambda_{21} = 0$

$\Rightarrow 4\lambda_{01} - 5\lambda_{02} + 9\lambda_{03} = 3\lambda_{21}$

$\Rightarrow 4\lambda_{01} - 5\lambda_{02} + 9\lambda_{03} = 3(4\lambda_{01})$

$\Rightarrow 8\lambda_{01} + 5\lambda_{02} + 9\lambda_{03} = 0$

$\Rightarrow 18\lambda_{01} + 9\lambda_{03} = 0$

$\boxed{2\lambda_{01} = \lambda_{03}}$

$\boxed{\lambda_{01} = 1/5}$ $\boxed{\lambda_{02} = \frac{2}{5}}$ $\boxed{\lambda_{03} = \frac{2}{5}}$

$\boxed{\lambda_{21} = \frac{4}{5}}$ $\boxed{\lambda_{11} = \frac{3}{5}}$

$$= \begin{bmatrix} \lambda_{01} \\ \lambda_{02} \\ \lambda_{03} \\ \lambda_{11} \\ \lambda_{21} \end{bmatrix} = \begin{bmatrix} 1/5 \\ 2/5 \\ 2/5 \\ 3/5 \\ 4/5 \end{bmatrix}$$ — Put in (A)

V2 16012431-SS

(b) Minimize subject to —

$f(x) = 2x_1 x_2 + 2x_1 x_2^{-1} x_3 + 4x_1^{-1} x_2^{2} x_3^{-1/2}$ subject to $\sqrt{3} x_2^{-1} + 3x_1^{-1} x_3^{-1/2} \le 1$ $\quad x_i \ge 0 \quad i = 1,2,3.$

Sol:

$n = 3$ (no. of variables)

$N_0 = 3$ (No. of terms in objective fn)

$N_1 = 2$ (Terms in first constraint)

$m = 1$ (Total no. of constraints).

$N = N_0 + N_1 = 5$

Union — 0 degree, 1 degree

& $N - n - 1 = 5 - 3 - 1 \ne 0$

Degree of difficulty $= 1$

(c) Minimize $f(x) = x_1 x_2 + 2x_1^{-1} x_3 + 5x_3 + 10 x_2^{-1}$, $x_i \ge 0$, $i = 1,2,3$ by Geometric programming method.

Sol: $f(x) = u_1(x) + u_2(x) + u_3(x) + u_4(x)$

$c_1 = 1 \quad c_2 = 2 \quad c_3 = 5 \quad c_4 = 10$

$$f(x) = \sum_{j=1}^{N} c_j x_1^{a_{1j}} \cdot x_2^{a_{2j}} \cdots x_n^{a_{nj}}$$

$n = 3$, $N = 4$

$\Rightarrow N = 4 \Rightarrow$ No. of terms. $n = 3$ (no. of variables)

$\Rightarrow c_1 x_1^{a_{11}} x_2^{a_{21}} x_3^{a_{31}} + c_2 x_1^{a_{12}} x_2^{a_{22}} x_3^{a_{32}} +$

$c_3 x_1^{a_{13}} x_2^{a_{23}} x_3^{a_{33}} +$

$c_4 x_1^{a_{14}} x_2^{a_{24}} x_3^{a_{34}}$

OR

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ \cancel{a_{41}} & \cancel{a_{42}} & \cancel{a_{43}} & \cancel{a_{44}} \end{bmatrix} = \begin{array}{cccc} A_1 & A_2 & A_3 & A_4 \\ \end{array} \begin{bmatrix} 1 & -1 & 0 & 0 \\ 1 & 0 & 0 & -1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$

Scanned by CamScanner

Orthogonal conditions are —

$$\sum_{j=1}^{N} a_{kj} \Delta_j^* = 0 \Rightarrow \sum_{j=1}^{4} a_{kj} \Delta_j^* = 0 \qquad k = 1, 2 \ldots n$$

$$\Rightarrow a_{k1}\Delta_1^* + a_{k2}\Delta_2^* + a_{k3}\Delta_3^* + a_{k4}\Delta_4^* = 0$$

For $k=1$

$$a_{11}\Delta_1^* + a_{12}\Delta_2^* + a_{13}\Delta_3^* + a_{14}\Delta_4^* = 0$$

For $k=2$

$$a_{21}\Delta_1^* + a_{22}\Delta_2^* + a_{23}\Delta_3^* + a_{24}\Delta_4^* = 0$$

For $k=3$.

$$a_{31}\Delta_1^* + a_{32}\Delta_2^* + a_{33}\Delta_3^* + a_{34}\Delta_4^* = 0 \quad (n)$$

And

Orthonormal condition is

$$\sum_{j=1}^{N} \Delta_j^* = 1$$

$$\Rightarrow \sum_{j=1}^{4} \Delta_j^* = 1$$

S
4

$$\Rightarrow \boxed{\Delta_1^* + \Delta_2^* + \Delta_3^* + \Delta_4^* = 1}.$$

~~Δ_{11}^*~~

$$\left.\begin{aligned} \Delta_1^* - \Delta_2^* &= 0 \\ \Delta_1^* - \Delta_4^* &= 0 \end{aligned}\right\} \quad \Delta_1^* = \Delta_2^* = \Delta_4^*$$

$$\Delta_2^* + \Delta_3^* = 0 \qquad 1-3$$

$$3\Delta_2^* + \Delta_3^* = 1$$

$$\Delta_2^* + \Delta_3^* = 0$$

$$2\Delta_2^* = 1 \Rightarrow \boxed{\Delta_2^* = \frac{1}{2}}$$

Scanned by CamScanner

$$\Rightarrow \left\{ \begin{array}{l} \Delta_1^* = \Delta_2^* = \Delta_4^* = \frac{1}{2} \\ \Delta_3^* = -1/2 \end{array} \right\}$$

$(-\sqrt{10})^{5/2}$

$$J^* = \left(\frac{C_1}{\Delta_1^*}\right)^{\Delta_1^*} \left(\frac{C_2}{\Delta_2^*}\right)^{\Delta_2^*} \left(\frac{C_3}{\Delta_3^*}\right)^{\Delta_3^*} \left(\frac{C_4}{\Delta_4^*}\right)^{\Delta_4^*}$$

$$= (1\times2)^{1/2} (2\times2)^{1/2} (-8\times2)^{-1/2} (10\times2)^{1/2}$$

???

⑥ (a) Derive the geometrical dual of —

minimize $\quad J(x) = x_1 x_2^{-2} x_3^{-1} + 2x_1^{-1} x_2^{-3} x_4 + 10 x_1 x_3 x_4$

subject to $\quad 3x_1^{-1} x_3 x_4^{-2} + 4x_3 x_4 \le 1$

$\quad 5x_1^{-1} x_2^{-2} x_3 \le 1 \quad x_i \ge 0 \quad i = 1, 2.$

Sol:

$n = 4$

$N_0 = 3$

$N_1 = 2$

$N_2 = 1$

$m = 2$

$N = N_0 + N_1 + N_2 = 6$

$N - n - 1 = 6 - 4 - 1 = \underline{\underline{1}}$

Scanned by CamScanner

(e) minimize $f(x) = x_1 x_2 x_3^{-2} + 2x_1^{-1} x_2^{-1} x_3 + 5x_2 + 3x_1 x_2^{-1}$, $x_i \geq 0$, $i = 1,2,3$ by geometric programming method.

Sol:- $f(x) = u_1(x) + u_2(x) + u_3(x) + u_4(x)$

$C_1 = 1 \quad C_2 = 2 \quad C_3 = 5 \quad C_4 = 3$

$$f(x) = \sum_{j=1}^{N} c_j x_1^{a_{1j}} x_2^{a_{2j}} \cdots x_n^{a_{nj}}$$

$$= \sum_{j=1}^{4} c_j x_1^{a_{1j}} x_2^{a_{2j}} x_3^{a_{3j}}$$

$$= C_1 x_1^{a_{11}} x_2^{a_{21}} x_3^{a_{31}} + C_2 x_1^{a_{12}} x_2^{a_{22}} x_3^{a_{32}} + C_3 x_1^{a_{13}} x_2^{a_{23}} x_3^{a_{33}} + C_4 x_1^{a_{14}} x_2^{a_{24}} x_3^{a_{34}}$$

OR

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 & 1 \\ 1 & -1 & 1 & -1 \\ -2 & 1 & 0 & 0 \end{bmatrix}$$

orthogonal conditions are

$$\sum_{j=1}^{N} a_{kj} \Delta_j^* = 0 \Rightarrow \sum_{j=1}^{4} a_{kj} \Delta_j^* = 0$$

$$\Rightarrow a_{k1}\Delta_1^* + a_{k2}\Delta_2^* + a_{k3}\Delta_3^* + a_{k4}\Delta_4^* = 0$$

for $k=1$

$$a_{11}\Delta_1^* + a_{12}\Delta_2^* + a_{13}\Delta_3^* + a_{14}\Delta_4^* = 0$$

for $k=2$

$$a_{21}\Delta_1^* + a_{22}\Delta_2^* + a_{23}\Delta_3^* + a_{24}\Delta_4^* = 0$$

for $k=3$

$$a_{31}\Delta_1^* + a_{32}\Delta_2^* + a_{33}\Delta_4^* + a_{34}\Delta_4^* = 0$$

& by orthonormal condition

$$\Delta_1^* + \Delta_2^* + \Delta_3^* + \Delta_4^* = 1 \quad \text{--- (1)}$$

Scanned by CamScanner

$\Delta_1^* - \Delta_2^* + \Delta_4^* = 0$

$\Delta_1^* - \Delta_2^* + \Delta_3^* - \Delta_4^* = 0$

$-2\Delta_1^* + \Delta_2^* = 0$

$\Delta_2^* = 2\Delta_1^*$ $\qquad\qquad 2\Delta_1^* = \Delta_3$

$\Rightarrow \quad -\Delta_1^* + \Delta_4^* = 0$

$\therefore \Delta_4^* = \Delta_1^*$

$\Delta_1^* + \Delta_2^* + \Delta_3^* + \Delta_4^* = 1$

$\Rightarrow \quad 2\Delta_1^* + 2\Delta_1^* + \Delta_3^* = 1$

$\Rightarrow \quad 4\Delta_1^* + \Delta_3^* = 1$

$\Rightarrow \quad \boxed{\Delta_1^* = \frac{1}{6}} \qquad \boxed{\Delta_2^* = \frac{2}{6} = \frac{1}{3}}$

$\boxed{\Delta_3^* = \frac{1}{3}} \qquad \boxed{\Delta_4^* = \frac{1}{6}}$

$f^* = (6)^{1/6} (6)^{1/3} (15)^{1/3} (18)^{1/6}$

$\Rightarrow 1.348 \times 1.817 \times 2.466 \times 1.6188$

$= 9.777$

We know that $\boxed{U_j^* = \Delta_j^* f^*}$

$U_1^* = \Delta_1^* f^*$

$U_2^* = \Delta_2^* f^*$

$U_3^* = \Delta_3^* f^*$

$U_4^* = \Delta_4^* f^*$

Scanned by CamScanner

$$x_1 x_2 x_3^{-2} = \frac{1}{6} \times 9.777$$

$$\Rightarrow \boxed{\frac{x_1 x_2}{x_3^2} = 1.6295}$$

$$\Rightarrow \frac{2x_3}{x_1 x_2} = \cancel{1.6295} \quad 3.259 =$$

$$\Rightarrow \boxed{\frac{x_3}{x_1 x_2} = 1.6295}$$

$$\Rightarrow 5x_2 = \frac{1}{6} \times 9.777$$

$$\cancel{x_2 = 0.3259} \quad \cancel{x_2 = 0.6518} \quad \boxed{x_2 = 1.3036}$$

$$\Rightarrow \frac{3x_1}{x_2} = 1.6295$$

$$\boxed{\frac{x_1}{x_2} = 0.543}$$

$$\boxed{x_1 \Rightarrow 0.708}$$

$$\boxed{x_3 = 1.153}$$

~~(e) minimize $= x_1 x_2 x_3^{-2} + 2x_1^{-1} x_2^{-1} x_3 + 5x_2 + 3x_1 x_2^{-2}$~~

(f) minimize $f(x) = x_1^{-2} x_2^{-1} + \frac{1}{4} x_1^2 x_2^{-1} x_3^{-1} + x_1^{-1} x_3^2 x_4$ subject to $\frac{3}{4} x_1 x_2 + \frac{3}{8} x_2 x_3 x_4^{-3} \leq 1$ $\quad x_i \geq 0 \quad i = 1,2,3.$

Sol: $\quad n = 4 \quad N_0 = 3 \quad N_1 = 2 \quad m = 1$

$N = N_0 + N_1 = 5$

$N - n - 1 = 0 \Rightarrow \therefore$ 0 degree of difficulty.

So dual problem may be stated as —

Scanned by CamScanner

$$\lambda = \begin{Bmatrix} \lambda_{01} \\ \lambda_{02} \\ \lambda_{03} \\ \lambda_{04} \\ \lambda_{05} \end{Bmatrix}$$

$$V(\lambda) = \prod_{K=0}^{1} \prod_{j=1}^{N_K} \left(\frac{C_{Kj}}{\lambda_{Kj}} \sum_{l=1}^{N_K} \lambda_{Kl} \right)^{\lambda_{Kj}}$$

$$= \prod_{j=1}^{N_0} \left(\frac{C_{0j}}{\lambda_{0j}} \sum_{l=1}^{N_0} \lambda_{0l} \right)^{\lambda_{0j}} \times \prod_{j=1}^{N_1} \left(\frac{C_{1j}}{\lambda_{1j}} \sum_{l=1}^{N_1} \lambda_{1l} \right)^{\lambda_{1j}}$$

$$= \prod_{j=1}^{3} \left(\frac{C_{0j}}{\lambda_{0j}} \sum_{l=1}^{3} \lambda_{0l} \right)^{\lambda_{0j}} \times \prod_{j=1}^{1} \left(\frac{C_{1j}}{\lambda_{1j}} \sum_{l=1}^{2} \lambda_{1l} \right)^{\lambda_{1j}}$$

$$= \left(\frac{C_{01}}{\lambda_{01}} (\lambda_{01}+\lambda_{02}+\lambda_{03}) \right)^{\lambda_{01}} \left(\frac{C_{02}}{\lambda_{02}} (\lambda_{01}+\lambda_{02}+\lambda_{03}) \right)^{\lambda_{02}}$$

$$\left(\frac{C_{03}}{\lambda_{03}} (\lambda_{01}+\lambda_{02}+\lambda_{03}) \right)^{\lambda_{03}} \times \left(\frac{C_{11}}{\lambda_{11}} (\lambda_{10}+\lambda_{20}) \right)^{\lambda}$$

we know that,

$$\sum_{j=1}^{N_0} \lambda_{0j} = 1 \quad \text{(Normality condition)}$$

$$= \sum_{j=1}^{3} \lambda_{0j} = 1 \Rightarrow \boxed{\lambda_{01}+\lambda_{02}+\lambda_{03}=1}$$

<u>Orthogonal constraints</u>

$$\cancel{\prod_{K=0}^{1} \prod_{j=1}^{N_K} \left(\frac{C_{Kj}}{\lambda_{Kj}} \sum_{l=1}^{N_K} \lambda_{Kl} \right)}$$

$$\sum_{k=0}^{m}\sum_{j=1}^{N_k} a_{kij}\,\lambda_{kj} = 0 \qquad i = 1,2,3 \ldots n$$

$$\sum_{k=0}^{1}\sum_{j=1}^{N_k} a_{oij}\,\lambda_{oj} = 0$$

$$= \sum_{j=1}^{N_0=3} a_{oi1}\,\lambda_{oj} + \sum_{j=1}^{N_1=2} a_{oij}\,\lambda_{1j} = 0$$

$$\Rightarrow a_{oi1}\lambda_{01} + a_{oi1}\lambda_{02} + a_{oi1}\lambda_{03} + a_{oi1}\lambda_{11} + a_{oi2}\lambda_{12} = 0$$

$i = 1,2,3$

<u>For $i=1$</u>

$$a_{011}\lambda_{01} + a_{011}\lambda_{02} + a_{011}\lambda_{03} + a_{011}\lambda_{11} + a_{012}\lambda_{12} = 0$$

<u>For $i=2$</u>

$$a_{021}\lambda_{01} + a_{021}\lambda_{02} + a_{021}\lambda_{03} + a_{021}\lambda_{11} + a_{022}\lambda_{12} = 0$$

<u>For $i=3$</u>

$$a_{031}\lambda_{01} + a_{031}\lambda_{02} + a_{031}\lambda_{03} + a_{031}\lambda_{11} + a_{032}\lambda_{12} = 0$$

$$\begin{array}{c} \begin{array}{ccccc} \lambda_{01} & \lambda_{02} & \lambda_{03} & \lambda_{11} & \lambda_{21} \end{array} \\ \begin{bmatrix} -2 & +2 & -1 & 1 & 0 \\ -1 & -1 & 0 & 1 & 1 \\ 0 & -1 & 2 & 0 & 1 \\ 0 & 0 & 1 & 0 & -3 \end{bmatrix} \end{array}$$

$$\boxed{\lambda_{01} + \lambda_{02} + \lambda_{03} = 1} \quad \text{— (1)}$$

$$\Rightarrow \quad -2\lambda_{01} + 2\lambda_{02} - \lambda_{03} + \lambda_{11} = 0 \quad \text{— (2)}$$

$$-\lambda_{01} - \lambda_{02} + \lambda_{11} + \lambda_{21} = 0 \quad \text{— (3)}$$

$$-\lambda_{02} + 2\lambda_{03} + \lambda_{21} = 0 \quad \text{— (4)}$$

$$\lambda_{03} - 3\lambda_{21} = 0 \quad \text{— (5)}$$

$$\Rightarrow \quad \lambda_{03} = 3\lambda_{21} \qquad \Rightarrow \boxed{\lambda_{21} = \frac{\lambda_{03}}{3}}$$

Scanned by CamScanner

Put in (4)

$$-\lambda_{02} + 2\lambda_{03} + \frac{\lambda_{03}}{3} = 0$$

$$= -3\lambda_{02} + 3\lambda_{03} = 0$$

$$\boxed{\lambda_{03} = \lambda_{02}}$$

~~$-\lambda_{01} - \lambda_{02} + \lambda_{21}$~~ $=$

$$-2\lambda_{01} - 2\lambda_{02} - \lambda_{03} = -\lambda_{01} - \lambda_{02} + \frac{\lambda_{03}}{3}$$

$$\Rightarrow \quad \lambda_{01} + \lambda_{02} + \frac{4\lambda_{03}}{3} = 0$$

$$\Rightarrow \boxed{3\lambda_{01} + 3\lambda_{02} + 4\lambda_{03} = 0}$$

$$\Rightarrow \quad 3\lambda_{01} + 7\lambda_{02} = 0$$

$$\Rightarrow \boxed{\lambda_{01} = -\frac{7}{3}\lambda_{02}}$$

$\Rightarrow \quad 2 \times (2) + (4)$

$$\Rightarrow -4\lambda_{01} - 4\lambda_{02} - \cancel{2\lambda_{03}} + 2\lambda_{11} + \lambda_{02} + \cancel{2\lambda_{03}} + \lambda_{21} = 0$$

$$\Rightarrow -4\lambda_{01} - 3\lambda_{02} + 2\lambda_{11} + \frac{\lambda_{03}}{3} = 0$$

$$\Rightarrow -12\lambda_{01} - 9\lambda_{02} + 6\lambda_{11} + \lambda_{03} = 0$$

$$= -12\lambda_{01} - 9\lambda_{02} + \lambda_{03} + 6\left(\lambda_{01} + \lambda_{02} - \frac{\lambda_{03}}{3}\right) = 0$$

$$\Rightarrow -6\lambda_{01} - 3\lambda_{02} - \lambda_{03} = 0$$

$$\Rightarrow \boxed{6\lambda_{01} + 3\lambda_{02} + \lambda_{03} = 0}$$

$-6+14-6$

$\lambda_{01} = +3 \qquad \lambda_{02} = -7 \qquad \lambda_{03} = +3$

$\lambda_{11} = -2 \qquad$ Ans $\rightarrow$

Scanned by CamScanner

$$\Lambda = \begin{Bmatrix} 3 \\ -7 \\ 3 \\ -2 \\ -2 \end{Bmatrix}$$

$V(A)$ is answer.

Q(h) minimize $z = 7x_1 x_2^{-1} + 3x_2 x_3^{-2} + 5x_1^{-3} x_2 x_3 + x_1 x_2 x_3$ & $x_i \geq 0$ $i = 1,2,3$ by geometric programming method.

Sol= $f(x) = u_1(x) + u_2(x) + u_3(x) + u_4(x)$

$c_1 = 7$ $c_2 = 3$ $c_3 = 5$ $c_4 = 1$

$$f(x) = \sum_{j=1}^{N} c_j x_1^{a_{1j}} x_2^{a_{2j}} \cdots x_n^{a_{nj}}$$

$N = 4$ $n = 3$

$$f(x) \Rightarrow c_1 x_1^{a_{11}} x_2^{a_{21}} x_3^{a_{31}} + c_2 x_1^{a_{12}} x_2^{a_{22}} x_3^{a_{32}} + c_3 x_1^{a_{13}} x_2^{a_{23}} x_3^{a_{33}} + c_4 x_1^{a_{14}} x_2^{a_{24}} x_3^{a_{34}}$$

$$\text{OR} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix} = \begin{bmatrix} 1 & 0 & -3 & 1 \\ -1 & 1 & 1 & 1 \\ 0 & -2 & 1 & 1 \end{bmatrix}$$

Orthogonal conditions are

$$\sum_{j=1}^{4} a_{kj} \Delta_j^* = 0 \Rightarrow$$

$$a_{k1} \Delta_1^* + a_{k2} \Delta_2^* + a_{k3} \Delta_3^* + a_{k4} \Delta_4^* = 0$$

For $k=1$

Scanned by CamScanner

$a_{11}\Delta_1^* + a_{12}\Delta_2^* + a_{13}\Delta_3^* + a_{14}\Delta_4^* = 0$

For $K=2$

$a_{21}\Delta_1^* + a_{22}\Delta_2^* + a_{23}\Delta_3^* + a_{24}\Delta_4^* = 0$

for $K=3$

$a_{31}\Delta_1^* + a_{32}\Delta_2^* + a_{33}\Delta_3^* + a_{34}\Delta_4^* = 0$

and By orthonormal cond

$$\Delta_1^* + \Delta_2^* + \Delta_3^* + \Delta_4^* = 1$$

$$\Delta_1^* - 3\Delta_3^* \; \Delta_4^* = 0$$

$$-\Delta_1^* + \Delta_2^* + \Delta_3^* + \Delta_4^* = 0$$

$$-2\Delta_2^* + \Delta_3^* + \Delta_4^* = 0$$

$$\Rightarrow \cancel{2}\Delta_2^* + 2\Delta_3^* + 2\Delta_4^* = 1$$

$$-\cancel{2}\Delta_2^* + \Delta_3^* + \Delta_4^* = 0$$

$$3\Delta_3^* + 3\Delta_4^* = 1$$

$$\frac{5+3}{24}$$

& $\quad -2\Delta_3^* + 2\Delta_4^* + \Delta_2^* = 0$

$$= -4\Delta_3^* + 4\Delta_4^* + \cancel{2\Delta_2^*} = 0$$

$$\Delta_3^* + \Delta_4^* - \cancel{2\Delta_2^*} = 0$$

$$\frac{1}{8} + \frac{5}{24} + \frac{1}{6}$$

$$= \frac{3+5+4}{24}$$

$$\frac{1}{12}$$

$$-3\Delta_3^* + 5\Delta_4^* = 0$$

$$3\Delta_3^* = 5\Delta_4^*$$

$\Rightarrow$ $\boxed{\Delta_4^* = \frac{1}{8}}$ $\boxed{\Delta_3^* = \frac{5}{24}}$ $\boxed{\Delta_2^* = \frac{1}{6}}$

$\Delta_1^* = \frac{11}{12}$

$\cancel{y^* = (7\times 12)^{1/8} (3\times}$

$$y^* = \left(\frac{7\times 12}{11}\right)^{\frac{11}{12}} \left(3\times 6\right)^{\frac{1}{6}} \left(\frac{5\times 24}{8}\right)^{\frac{5}{24}} \left(8\right)^{\frac{1}{8}}$$

$$= 6.446 \times 1.618 \times 1.955 \times 1.296$$

$$= 26.442$$

<u>we know that</u>

$$u_j^* = \Delta_j^* y^*$$

$$u_1^* = \Delta_1^* y^*$$

$$= \frac{7x_1}{x_2} = \frac{11}{12} \times 26.442$$

$$\Rightarrow \boxed{\frac{x_1}{x_2} = 3.462}$$

$$u_2^* = \Delta_2^* y^*$$

$$\Rightarrow \frac{3x_2}{x_3^2} = \frac{1}{6} \times 26.442$$

$$\boxed{\frac{x_2}{x_3^2} = 1.469}$$

$$u_3^* = \Delta_3^* y^*$$

$$\Rightarrow \frac{5x_2x_3}{x_1^3} = \frac{5}{24} \times 26.442$$

$$\boxed{\frac{x_2x_3}{x_1^3} = 1.101}$$

$$u_4^* = \Delta_4^* y^*$$

$$\Rightarrow x_1x_2x_3 = \frac{1}{8} \times 26.442$$

$$\boxed{x_1x_2x_3 = 3.305}$$

$$\cancel{\Rightarrow x_1 4 =}\ x_2 = 1.469x_3^2$$

$$x_1 = 3.462 \times 1.469x_3^2$$

$$\Rightarrow 3.462 \times 1.469\, x_3^2 \times 1.469x_3^2 \times x_3 = 3.305$$

$$x_3^5 = 0.4423$$

$$\boxed{x_3^* = 0.849}$$

$$\boxed{x_2^* = 1.060}$$

$$\boxed{x_1^* = 3.672}$$

(i) Derive the geometric dual of — minimize $f(x_1,x_2) = x_1^{-3} x_2 + x_1^{3/2} x_3^{-1}$ subject to

(3)

$\frac{7}{3} x_1^{3} x_2^{-1} + 6 x_1^{-1} x_3^{-1/2} \le 1$

Sol:

$n = 3$

$N_0 = 2$

$N_1 = 2$

$m = 1$

$N = N_0 + N_1 = 4$

& $N - n - 1 \Rightarrow 4 - 3 - 1 = \underline{0}$

$\therefore$ 0 degree of difficulty.

$\therefore$ Dual problem may be stated as

Find $\lambda = \begin{Bmatrix} \lambda_{01} \\ \lambda_{02} \\ \lambda_{03} \\ \lambda_{04} \end{Bmatrix}$ to maximize.

$$v(\lambda) = \prod_{k=0}^{1} \prod_{j=1}^{N_k} \left(\frac{c_{kj}}{\lambda_{kj}} \sum_{l=1}^{N_k} \lambda_{kl} \right)^{\lambda_{kj}}$$

$$= \prod_{j=1}^{N_0} \left(\frac{c_{0j}}{\lambda_{0j}} \sum_{l=1}^{N_0} \lambda_{0l} \right)^{\lambda_{0j}} \times \prod_{j=1}^{N_1} \left(\frac{c_{1j}}{\lambda_{1j}} \sum_{l=1}^{N_1} \lambda_{1l} \right)^{\lambda_{1j}}$$

$$\Rightarrow \left(\frac{c_{01}}{\lambda_{01}} (\lambda_{01} + \lambda_{02}) \right)^{\lambda_{01}} \times \left(\frac{c_{02}}{\lambda_{02}} (\lambda_{01} + \lambda_{02}) \right)^{\lambda_{02}} \times$$

$$\left(\frac{c_{11}}{\lambda_{11}} (\lambda_{11} + \lambda_{12}) \right)^{\lambda_{11}} \times \left(\frac{c_{12}}{\lambda_{12}} (\lambda_{11} + \lambda_{12}) \right)^{\lambda_{12}}$$

we know that

$$\boxed{\lambda_{01} + \lambda_{02} = 1} \quad \text{———} \quad (1)$$

Scanned by CamScanner

and By orthogonal constraint

$$\sum_{K=0}^{m} \sum_{j=1}^{N_K} a_{Kij} \Lambda_{Kj} = 0 \qquad i = 1, 2, \dots n$$

$$= \sum_{K=0}^{1} \sum_{j=1}^{N_K} a_{Kij} \Lambda_{Kj} = 0 \qquad i = 1, 2, 3.$$

$$\Rightarrow \sum_{j=1}^{N_0} a_{0ij} \Lambda_{0j} + \sum_{j=1}^{N_1} a_{1ij} \Lambda_{1j} = 0$$

$$\Rightarrow a_{0i1} \Lambda_{01} + a_{0i2} \Lambda_{02} + a_{1i1} \Lambda_{11} + a_{1i2} \Lambda_{12} = 0$$

For $i = 1$

$$a_{011} \Lambda_{01} + a_{012} \Lambda_{02} + a_{111} \Lambda_{11} + a_{112} \Lambda_{12} = 0$$

For $i = 2$

$$a_{021} \Lambda_{01} + a_{022} \Lambda_{02} + a_{121} \Lambda_{11} + a_{122} \Lambda_{12} = 0$$

For $i = 3$

$$a_{031} \Lambda_{01} + a_{032} \Lambda_{02} + a_{131} \Lambda_{11} + a_{132} \Lambda_{12} = 0$$

$$\begin{array}{cccc} \Lambda_{01} & \Lambda_{02} & \Lambda_{11} & \Lambda_{12} \end{array}$$
$$\begin{bmatrix} -3 & 3/2 & 3 & -1 \\ 1 & 0 & -1 & 0 \\ 0 & -1 & 0 & -1/2 \end{bmatrix}$$

$$-3\Lambda_{01} + \frac{3}{2} \Lambda_{02} + 3\Lambda_{11} - \Lambda_{12} = 0$$

$$\Lambda_{01} - \Lambda_{11} = 0 \quad \Rightarrow \boxed{\Lambda_{01} = \Lambda_{11}}$$

$$-\Lambda_{02} - \frac{1}{2} \Lambda_{12} = 0$$

$$\boxed{\Lambda_{02} = \frac{1}{2} \Lambda_{12}}$$

$$= -\cancel{3\Lambda_{01}} + \frac{3}{2} \Lambda_{02} + \cancel{3\Lambda_{01}} - 2\Lambda_{02} = 0$$

$$\Leftarrow \boxed{\Lambda_{12} = \boxed{\Lambda_{02} = 0}} \qquad \boxed{\Lambda_{01} = 1} = \Lambda_{11}$$

Scanned by CamScanner

$$\lambda = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

$v = 1$ Answer.

(i) Derive the geometric dual of problem :-

$$f(x) = 10x_1x_2 + 2x_1x_2^{-2}x_3^{-1} + 5x_1^{-2}x_2^{2}x_3^{-1/2}$$

subject to $\frac{7}{5}x_1^3x_2^{-1} + 6x_1^{-1}x_3^{-1/2} \le 1$

Sol :- $n = 3 \qquad N_0 = 3$

$m = 1 \qquad N_1 = 2$

$N = N_0 + N_1 = 5$

$N - n - 1 = 5 - 3 - 1 = 1 \neq 0$

(ii) Derive geometric dual —

$$f(x) = x_1^{-3/4}x_2 + x_1^{3/2}x_2^{-2}x_3^{-1/3} + x_1x_2^{-3}x_3^{-1}$$

subject to $\frac{7}{5}x_1^3x_2^{-1} + 6x_1^{-1}x_3^{-1/2} \le 1$

Solo:- $n = 3$

$N_0 = 3$

$N_1 = 2$

$m = 1$

$N = 5$

$N - n - 1 = 5 - 3 - 1 \neq 0$

Scanned by CamScanner

Degree of Difficulty 1

Q) $f(x) = 2x_1x_2 + 2x_1 x_2^{-1} x_3^{-1} + 4x_1^{-1} x_2^{2} x_3^{-1/2}$ subje to $\sqrt{3}\, x_2^{-1} + 3x_1^{-1} x_3^{-1/2} \le 1$ $\quad x_i \ge 0$

Sol:

$N_0 = 3$ $\quad c_{01} = 2$ $\quad c_{02} = 2$

$n = 3$ $\quad c_{03} = 4$ $\quad c_{11} = \sqrt{3}$

$N_1 = 2$ $\quad c_{12} = 3$

$m = 1$

$N = N_0 + N_1 = 5$

$N - n - 1 \Rightarrow 1$

Dual problem may be stated as –

maximize

$$v(\lambda) = \prod_{k=0}^{m} \prod_{j=1}^{N_k} \left(\frac{c_{kj}}{\lambda_{kj}} \sum_{l=1}^{N_k} \lambda_{kl} \right)^{\lambda_{kj}}$$

subject to

$$\sum_{j=1}^{N_0} \lambda_{0j} = 1$$

$$\sum_{k=0}^{m} \sum_{j=1}^{N_k} a_{kij} \lambda_{kj} = 0 \qquad i = 1, 2 \ldots n$$

$$\sum_{j=1}^{N_k} \lambda_{kj} \ge 0 \qquad k = 1, 2 \ldots m$$

$$\Rightarrow v(\lambda) = \prod_{k=0}^{1} \prod_{j=1}^{N_k} \left(\frac{c_{kj}}{\lambda_{kj}} \sum_{l=1}^{N_k} \lambda_{kl} \right)^{\lambda_{kj}}$$

$$= \prod_{j=1}^{N_0 = 3} \left(\frac{c_{0j}}{\lambda_{0j}} \sum_{l=1}^{N_0} \lambda_{0l} \right)^{\lambda_{0j}} \times \prod_{j=1}^{N_1 = 2} \left(\frac{c_{1j}}{\lambda_{1j}} \sum_{l=1}^{N_1} \lambda_{1l} \right)$$

Scanned by CamScanner

$$\Rightarrow \left(\frac{C_{01}}{\lambda_{01}}(\lambda_{01}+\lambda_{02}+\lambda_{03})\right)^{\lambda_{01}} \times \left(\frac{C_{02}}{\lambda_{02}}(\lambda_{01}+\lambda_{02}+\lambda_{03})\right)^{\lambda_{02}} \times$$

$$\left(\frac{C_{03}}{\lambda_{03}}(\lambda_{01}+\lambda_{02}+\lambda_{03})\right)^{\lambda_{03}} \times \left(\frac{C_{11}}{\lambda_{11}}(\lambda_{11}+\lambda_{12})\right)^{\lambda_{11}}$$

$$\times \left(\frac{C_{12}}{\lambda_{12}}(\lambda_{11}+\lambda_{12})\right)^{\lambda_{12}} \text{———} (A)$$

subject to

$$\lambda_{01}+\lambda_{02}+\lambda_{03}=1 \text{———} (1)$$

&

$$\sum_{k=0}^{1}\sum_{j=1}^{N_K} a_{kij}\lambda_{kj}=0 \qquad i=1,2,3\ldots n$$

$$\Rightarrow \sum_{j=1}^{N_0} a_{0ij}\lambda_{0j} + \sum_{j=1}^{N_1} a_{1ij}\lambda_{1j}=0$$

$$\Rightarrow \sum_{j=1}^{3} a_{0ij}\lambda_{0j} + \sum_{j=1}^{2} a_{1ij}\lambda_{1j}=0$$

$$\Rightarrow a_{0i1}\lambda_{01}+a_{0i2}\lambda_{02}+a_{0i3}\lambda_{03}+a_{1i1}\lambda_{11}+$$
$$a_{1i2}\lambda_{12}=0$$

for $i=1$

$$a_{011}\lambda_{01}+a_{012}\lambda_{02}+a_{013}\lambda_{03}+a_{111}\lambda_{11}+a_{112}\lambda_{12}=0$$

for $i=2$

$$a_{021}\lambda_{01}+a_{022}\lambda_{02}+a_{023}\lambda_{03}+a_{121}\lambda_{11}+a_{122}\lambda_{12}=0$$

for $i=3$

$$a_{031}\lambda_{01}+a_{032}\lambda_{02}+a_{033}\lambda_{03}+a_{131}\lambda_{11}+a_{132}\lambda_{12}=0$$

Scanned by CamScanner

& $\sum_{j=1}^{N_k} \Lambda_{kj} \geq 0$ $\quad k = 1, 2 - - m$

$\sum_{j=1}^{N_k} \Lambda_{kj} \geq 0$

$= \sum_{j=1}^{2} \Lambda_{kj} \geq 0$

$\Rightarrow \boxed{\Lambda_{11} + \Lambda_{12} \geq 0}$

$$\begin{array}{ccccc} \Lambda_{01} & \Lambda_{02} & \Lambda_{03} & \Lambda_{11} & \Lambda_{12} \end{array}$$

$$\begin{bmatrix} 1 & 1 & -1 & 0 & -1 \\ 1 & -1 & 2 & -1 & 0 \\ 0 & -1 & -\frac{1}{2} & -1 & 0 \end{bmatrix}$$

$\boxed{\Lambda_{01} + \Lambda_{02} + \Lambda_{03} = 1}$

$\Rightarrow \boxed{\Lambda_{01} + \Lambda_{02} - \Lambda_{03} - \Lambda_{12} = 0}$

$\Lambda_{01} + \Lambda_{02} + 2\Lambda_{03} - \Lambda_{11} = 0$

$-\Lambda_{02} - \frac{1}{2}\Lambda_{03} - \Lambda_{11} = 0$

$\Lambda_{01} + \Lambda_{02} = 1 - \Lambda_{03}$ —— ①

$\Lambda_{01} + \Lambda_{02} - \Lambda_{12} = \Lambda_{03}$ — ②

$-\Lambda_{01} + \Lambda_{02} + \Lambda_{11} = 2\Lambda_{03}$ — ③

$-2\Lambda_{02} - 2\Lambda_{11} = \Lambda_{03}$ — ④

$\boxed{\Lambda_{12} = 1 - 2\Lambda_{03}}$ //

$$\Lambda_{01} + \Lambda_{02} = -\Lambda_{03} + 1$$
$$-\Lambda_{01} + \Lambda_{02} + \Lambda_{11} = 2\Lambda_{03}$$

$$2\Lambda_{02} + \Lambda_{11} = \Lambda_{03} + 1$$

$$-2\Lambda_{02} - 2\Lambda_{11} = \Lambda_{03}$$

$$-\Lambda_{11} = 2\Lambda_{03} + 1$$

$$\boxed{\Lambda_{11} = -2\Lambda_{03} - 1}$$

$1 - 2\Lambda_{03} - 2\Lambda_{03} - 1$

$$-2\Lambda_{02} = \Lambda_{03} + 2(-2\Lambda_{03} - 1)$$
$$= -3\Lambda_{03} - 2$$

$$\boxed{\Lambda_{02} = \frac{3}{2}\Lambda_{03} + 1}$$

$$\Lambda_{01} = 1 - \Lambda_{03} - 1 - \frac{3}{2}\Lambda_{03}$$

$$\boxed{\Lambda_{01} = -\frac{5}{2}\Lambda_{03}}$$

Put in (A)

$$\max V = \left(\frac{2 \times 2}{-5\Lambda_{03}}\right)^{-\frac{5}{2}\Lambda_{03}} \left(\frac{2 \times 2}{3\Lambda_{03} + 2}\right)^{\frac{3\Lambda_{03}+2}{2}} \left(\frac{4}{\Lambda_{03}}\right)^{\Lambda_{03}}$$

$$\left(\frac{\sqrt{3} \times 4\Lambda_{03}}{+2\Lambda_{03} + 1}\right)^{(-2\Lambda_{03} - 1)} \left(\frac{3 \times 4\Lambda_{03}}{-1 + 2\Lambda_{03}}\right)^{(1 - 2\Lambda_{03})}$$

Taking log both sides.

Scanned by CamScanner

$$\log \max V = -\frac{5}{2}\Lambda_{03}\left(\log\left(-\frac{4}{5}\Lambda_{03}\right)\right) + \frac{3\Lambda_{03}+2}{2}\log\left(\frac{4}{3\Lambda_{03}+2}\right)$$

$$+ \Lambda_{03}\log\left(\frac{4}{\Lambda_{03}}\right) + \left(-2\Lambda_{03}-1\right)\log\left(\frac{4\sqrt{3}}{2\Lambda_{03}+1}\right)$$

$-\frac{5}{2}\Lambda_{03}[$

$$+ \left(1-2\Lambda_{03}\right)\log\left(\frac{12\Lambda_{03}}{2\Lambda_{03}-1}\right)$$

$$\frac{1}{\max V}\frac{\partial(\max V)}{\partial\Lambda_{03}} = \left[-\frac{5}{2}\log\left(-\frac{4}{5}\Lambda_{03}\right)\right] + \left[-\frac{5}{2}\right]$$

$$\cancel{+}\ \frac{3}{2}\log\left(\frac{4}{3\Lambda_{03}+2}\right) - \left(\frac{3}{2}\frac{\Lambda_{03}\times\cancel{3}}{\cancel{3}\Lambda_{03}}\right) +$$

$$+\log\left(\frac{4}{\Lambda_{03}}\right) + \left(\frac{\cancel{\Lambda_{03}\times\Lambda_{03}}}{\cancel{4}}\times\frac{-\cancel{4}}{\cancel{\Lambda_{03}^2}}\right) - 2\log\left(\frac{4\sqrt{3}}{\cancel{2\Lambda_{03}+1}}\right)$$

$$+ \cancel{\frac{(-2\Lambda_{03}-1)(2\Lambda_{03}+1)}{4\sqrt{3}}}\ \ +2\log(2\Lambda_{03}+1) + \frac{2(2\Lambda_{03}+1)}{2\Lambda_{03}+1}$$

$$\cancel{+}\ 2\log 12\Lambda_{03} + \frac{\cancel{12}(1-2\Lambda_{03})}{\cancel{12}\Lambda_{03}} +$$

$$\cancel{-2\log(2\Lambda_{03}-1) - 2} = 0$$

$$= -\frac{5}{2}\log\left(-\frac{4}{5}\Lambda_{03}\right) - \frac{5}{2} + \frac{3}{2}\log 4 - \frac{3}{2}\log(3\Lambda_{03}+2)$$

$$-\frac{3}{2} + \log 4 - \log\left(\Lambda_{03}\right) - 1 - 2\log(4\sqrt{3})$$

$$+\cancel{2\log(2\Lambda_{03}+1)}\ \cancel{+}\ 2\log 12\Lambda_{03} - 2 + \frac{1-2\Lambda_{03}}{\Lambda_{03}} + \frac{2}{2\ldots}$$

$$\Rightarrow \frac{-5}{4} + (\log 4)\frac{2}{2} - \frac{3}{2} - 1 - 2 = \frac{5}{2}\log\left(\frac{-4}{5}\lambda_{03}\right) - \frac{3}{2}\log(3\lambda_{03}+2)$$

$$+ \log(\lambda_{03}) + 2\log 12\lambda_{03} +$$

$$\frac{1-2\lambda_{03}}{\lambda_{03}} + \frac{2(2\lambda_{03}+1-1)}{2\lambda_{03}+1}\Big)$$

$$\Rightarrow -4.8469 = \cancel{\frac{5}{2}\log\left(\frac{-4}{5}\right)} + \log\left(\frac{-4}{5}\lambda_{03}\right)^{2.5} - \log(3\lambda_{03}+2)^{1.5}$$

$$+ \log\lambda_{03} + 2\log 12 + \log(\lambda_{03})^{2}$$

$$+ \frac{1}{\lambda_{03}} \cancel{-2} + \cancel{2} - \frac{2}{2\lambda_{03}+1}$$

$$\Rightarrow -4.8469 = \log\left[\frac{\left(-\frac{4}{5}\lambda_{03}\right)^{2.5}}{(3\lambda_{03}+2)^{1.5}} \times \lambda_{03} \times \lambda_{03}^{2}\right] + \frac{1}{\lambda_{03}} - \frac{2}{2\lambda_{03}+1}$$

-2.158

Guess λ_{03}