Optimization Techniques
Paper Code – BMS-09
Lecture – 01(Unit -1)
Topic-Single Variable Optimization

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Unit-01

Classical Optimization Techniques: Single variable optimization, Multi-variable with no constraints. Non-linear programming: One Dimensional Minimization methods. Elimination methods: Fibonacci method, Golden Section method

Unit-02

Unit-02

Linear Programming: Constrained Optimization Techniques: Simplex method, Solution of System of Linear Simultaneous equations, Revised Simplex method, Transportation problems, Karmarkar's method, Duality Theorems, Dual Simplex method, Decomposition principle.

Books & References

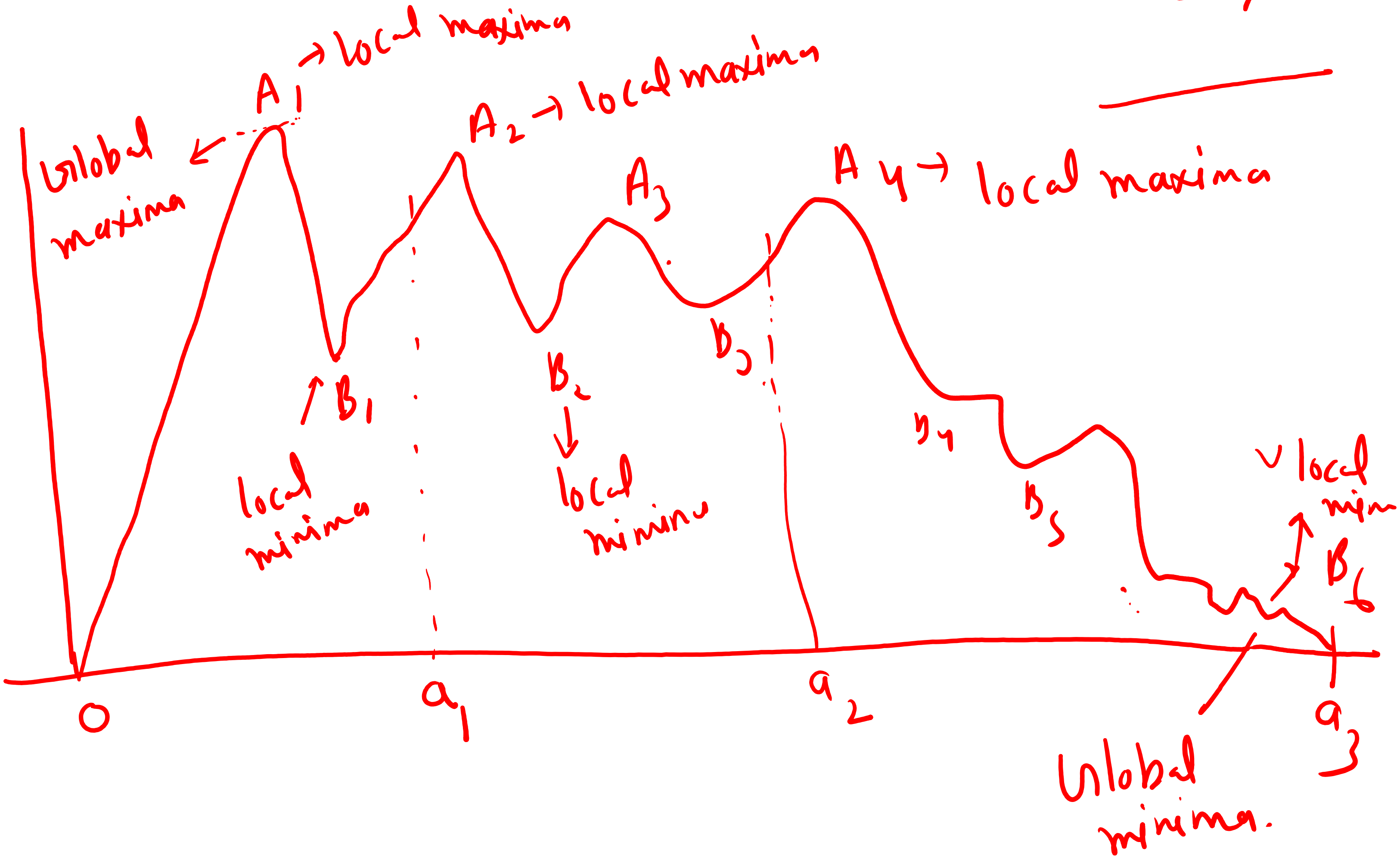
1. S.S. Rao; Engineering Optimization, New Age International
2. E.J. Haug and J.S. Arora; Applied Optimal Design, Wiley New York
3. Kalyanmoy Deb; Optimization for Engineering Design, Prentice Hall of India

Single variable Optimization

A function $f(x)$ of one variable, is said to have a relative or local ~~mini~~ minima at a point $x = x^*$ if $f(x^*) \leq f(x^* + h)$ for all sufficiently small +ve and -ve values of h.

Similarly, a point $x = x^*$ is said to have a relative maxima if $f(x^*) \geq f(x^* + h)$ for all sufficiently small +ve and -ve values of h (very close to zero).

A function $f(x)$ is said to have a global or absolute minimum at a point $x = x^*$ if $f(x^*) \leq f(x)$ for all x, in the domain over which $f(x)$ is defined.



Theorem 2.1 Necessary Condition If a function $f(x)$ is defined in the interval $a \leq x \leq b$ and has a relative minimum at $x = x^*$, where $a < x^* < b$, and if the derivative $df(x)/dx = f'(x)$ exists as a finite number at $x = x^*$, then $f'(x^*) = 0$.

Theorem 2.2 Sufficient Condition Let $f'(x^*) = f''(x^*) = \cdots = f^{(n-1)}(x^*) = 0$, but $f^{(n)}(x^*) \neq 0$. Then $f(x^*)$ is (i) a minimum value of $f(x)$ if $f^{(n)}(x^*) > 0$ and n is even; (ii) a maximum value of $f(x)$ if $f^{(n)}(x^*) < 0$ and n is even; (iii) neither a maximum nor a minimum if n is odd.

This theorem says that, $f(x)$, then

1st find $f'(x) = 0 \Rightarrow$ get stationary point, say, x^*

now, find $f''(x) = \begin{cases} > 0 \text{ at } x^* \Rightarrow \text{minima} \\ < 0 \text{ at } x^* \Rightarrow \text{maxima} \end{cases}$

if at x^*, $f''(x) = 0$, need to calculate $f'''(x)$

at x^*, $f'''(x^*) \neq 0$, we can say, this point is neither maxima nor minima.

again, if $f'''(x^*) = 0$, need to calculate $f^{IV}(x^*)$...

Determine the maximum and minimum values of the function

$$f(x) = 12x^5 - 45x^4 + 40x^3 + 5$$

Ans $f'(x) = 60x^4 - 180x^3 + 120x^2$

For stationary point, $f'(x) = 0$

$$\Rightarrow 60(x^4 - 3x^3 + 2x^2) = 0$$

$$\Rightarrow 60x^2(x^2 - 3x + 2) = 0$$

$$\Rightarrow 60x^2(x-1)(x-2) = 0$$

$$\Rightarrow x = 0, 1, 2$$

For maxima/minima, find second derivatives

$$f''(x) = 240x^3 - 540x^2 + 240x$$

$$= 60(4x^3 - 9x^2 + 4x)$$

at at point $x=1$, $f''(x) = 60(4-9+4) = -60 < 0$

so, $x=1$ is a maximum point and maximum value is

$f(x) = 12$ at $x=1$.

again at $x=2$, $f''(x) = 60(32-36+8)$

$= 240 > 0$

$\Rightarrow x=2$ is a minimum point and minimum value is, $f(x) = 12(2^5 - 45 \times 2^4 + 40 \times 2^3 + 5) = -11$

again at $x=0$, $f''(x) = 60(4 \cdot 0 - 9 \cdot 0 + 4 \cdot 0) = 0$

so, need to calculate, $f'''(x)$

$f'''(x) = 60(12x^2 - 18x + 4)$, so

at, $x=0$, $f'''(x) = 60(12 \cdot 0 - 18 \cdot 0 + 4)$

$f'''(x) = 240 \neq 0$, here, see n is odd $(n=3)$

so, $x=0$, point is a inflection point (neither maximum nor minimum).

Plot[$12x$^5 − $45x$^4 + $40x$^3 + 5, {x, −1,3}]