PRINCIPLES OF
COMMUNICATION SYSTEMS

NOTES FOR 4TH SEMESTER ELECTRONICS & COMMUNICATION

SUBJECT CODE: 15EC45

INSTITUTIONALMISSION AND VISION

Vision

- Development of academically excellent,culturally vibrant,socially responsible and globally competent human resources.

Mission

- To keep pace with advancements in knowledge and make the students competitive and capable at the global level.
- To create an environment for the students to acquire the right physical, intellectual, emotional and moral foundations and shine as torch bearers of tomorrow's society.
- To strive to attain ever-higher benchmarks of educational excellence.

DEPARTMENTAL MISSION AND VISION

Vision

To develop highly skilled and globally competent professionals in the field of Electronics and Communication Engineering to meet industrial and social requirements with ethical responsibility.

Mission

- To provide State-of-art technical education in Electronics and Communication at undergraduate and post-graduate levels to meet the needs of the profession and society.
- To adopt the best educational methods and achieve excellence in teaching-learning and research.
- To develop talented and committed human resource, by providing an opportunity for innovation, creativity and entrepreneurial leadership with high standards of professional ethics, transparency and accountability.
- To function collaboratively with technical Institutes/Universities/Industries and offer opportunities for long-term interaction with academia and industry.
- To facilitate effective interactions among faculty and students, and promote networking with alumni, industries, institutions and other stake-holders.

Program outcomes (POs)

Engineering Graduates will be able to:

PO1. **Engineering knowledge**: Apply the knowledge of mathematics, science, engineering fundamentals, and an engineering specialization to the solution of complex engineering problems.

PO2. **Problem analysis**: Identify, formulate, review research literature, and analyze complex engineering problems reaching substantiated conclusions using first principles of mathematics, natural sciences, and engineering sciences.

PO3. **Design/development of solutions**: Design solutions for complex engineering problems and design system components or processes that meet the specified needs with appropriate consideration for the public health and safety, and the cultural, societal, and environmental considerations.

PO4. **Conduct investigations of complex problems**: Use research-based knowledge and research methods including design of experiments, analysis and interpretation of data, and synthesis of the information to provide valid conclusions.

PO5. **Modern tool usage**: Create, select, and apply appropriate techniques, resources, and modern engineering and IT tools including prediction and modeling to complex engineering activities with an understanding of the limitations.

PO6. **The engineer and society**: Apply reasoning informed by the contextual knowledge to assess societal, health, safety, legal and cultural issues and the consequent responsibilities relevant to the professional engineering practice.

PO7. **Environment and sustainability**: Understand the impact of the professional engineering solutions in societal and environmental contexts, and demonstrate the knowledge of, and need for sustainable development.

PO8. **Ethics**: Apply ethical principles and commit to professional ethics and responsibilities and norms of the engineering practice.

PO9. **Individual and team work**: Function effectively as an individual, and as a member or leader in diverse teams, and in multidisciplinary settings.

PO10. **Communication**: Communicate effectively on complex engineering activities with the engineering community and with society at large, such as, being able to comprehend and write effective reports and design documentation, make effective presentations, and give and receive clear instructions.

PO11. **Project management and finance**: Demonstrate knowledge and understanding of the engineering and management principles and apply these to one's own work, as a member and leader in a team, to manage projects and in multidisciplinary environments.

PO12. Life-long learning: Recognize the need for, and have the preparation and ability to engage in independent and life-long learning in the broadest context of technological change.

Program Specific Outcomes (PSOs)

At the end of graduation the student will be able,

- To comprehend the fundamental ideas in Electronics and Communication Engineering and apply them to identify, formulate and effectively solve complex engineering problems using latest tools and techniques.
- To work successfully as an individual pioneer, team member and as a leader in assorted groups, having the capacity to grasp any requirement and compose viable solutions.
- To be articulate, write cogent reports and make proficient presentations while yearning for continuous self improvement.
- To exhibit honesty, integrity and conduct oneself responsibly, ethically and legally; holding the safety and welfare of the society paramount.

Program Educational Objectives (PEOs)

- Graduates will have a successful professional career and will be able to pursue higher education and research globally in the field of Electronics and Communication Engineering thereby engaging in lifelong learning.
- Graduates will be able to analyse, design and create innovative products by adapting to the current and emerging technologies while developing a conscience for environmental/ societal impact.
- Graduates with strong character backed with professional attitude and ethical values will have the ability to work as a member and as a leader in a team.
- Graduates with effective communication skills and multidisciplinary approach will be able to redefine problems beyond boundaries and develop solutions to complex problems of today's society.

PRINCIPLES OF COMMUNICATION SYSTEMS [As per Choice Based Credit System (CBCS) scheme] SEMESTER – IV (EC/TC)			
Subject Code	15EC45	IA Marks	20
Number of Lecture Hours/Week	04	Exam Marks	80
Total Number of Lecture Hours	50	Exam Hours	03
CREDITS – 04			

Course objectives: This course will enable students to:

- Design simple systems for generating and demodulating AM, DSB, SSB and VSB signals
- Understand the concepts in Angle modulation for the design of communication systems
- Design simple systems for generating and demodulating frequency modulated signals
- Learn the concepts of random process and various types of noise.
- Evaluate the performance of the communication system in presence of noise.
- Analyze pulse modulation and sampling techniques

Modules	Teaching Hours	Revised **Bloom's** Taxonomy (RBT) Level
Module – 1		
AMPLITUDE MODULATION: Introduction, Amplitude Modulation: Time & Frequency – Domain description, Switching modulator, Envelop detector. DOUBLE SIDE BAND-SUPPRESSED CARRIER MODULATION: Time and Frequency – Domain description, Ring modulator, Coherent detection, Costas Receiver, Quadrature Carrier Multiplexing. SINGLE SIDE–BAND AND VESTIGIAL SIDEBAND METHODS OF MODULATION: SSB Modulation, VSB Modulation, Frequency Translation, Frequency- Division Multiplexing, Theme Example: VSB Transmission of Analog and Digital Television (Chapter 3 of Text).	10 Hours	L1, L2, L3
Module – 2		
ANGLE MODULATION: Basic definitions, Frequency Modulation: Narrow Band FM, Wide Band FM, Transmission bandwidth of FM Signals, Generation of FM Signals, Demodulation of FM Signals, FM Stereo Multiplexing, Phase–Locked Loop: Nonlinear model of PLL, Linear model of PLL, Nonlinear Effects in FM Systems. The Superheterodyne Receiver (refer Chapter 4 of Text).	10 Hours	L1, L2, L3

Module – 3		
RANDOM VARIABLES & PROCESS: Introduction, Probability, Conditional Probability, Random variables, Several Random Variables. Statistical Averages: Function of a random variable, Moments, Random Processes, Mean, Correlation and Covariance function: Properties of autocorrelation function, Cross–correlation functions (refer Chapter 5 of Text). NOISE: Shot Noise, Thermal noise, White Noise, Noise Equivalent Bandwidth (refer Chapter 5 of Text), Noise Figure (refer Section 6.7 of Text).	10 Hours	L1, L2, L3
Module – 4		
NOISE IN ANALOG MODULATION: Introduction, Receiver Model, Noise in DSB-SC receivers, Noise in AM receivers, Threshold effect, Noise in FM receivers, Capture effect, FM threshold effect, FM threshold reduction, Pre-emphasis and De-emphasis in FM (refer Chapter 6 of Text).	10 Hours	L1, L2, L3
Module – 5		
DIGITAL REPRESENTATION OF ANALOG SIGNALS: Introduction, Why Digitize Analog Sources?, The Sampling process, Pulse Amplitude Modulation, Time Division Multiplexing, Pulse-Position Modulation, Generation of PPM Waves, Detection of PPM Waves, The Quantization Process, Quantization Noise, Pulse–Code Modulation: Sampling, Quantization, Encoding, Regeneration, Decoding, Filtering, Multiplexing (refer Chapter 7 of Text), Application to Vocoder (refer Section 6.8 of Reference Book 1).	10 Hours	L1, L2, L3

Course Outcomes: At the end of the course, students will be able to:

- Determine the performance of analog modulation schemes in time and frequency domains.
- Determine the performance of systems for generation and detection of modulated analog signals.
- Characterize analog signals in time domain as random processes and in frequency domain using Fourier transforms.
- Characterize the influence of channel on analog modulated signals
- Determine the performance of analog communication systems.
- Understand the characteristics of pulse amplitude modulation, pulse position modulation and pulse code modulation systems.

Graduating Attributes (as per NBA)

- Engineering Knowledge
- Problem Analysis
- Design / development of solutions (partly)

Question paper pattern:

- The question paper will have ten questions.
- Each full Question consisting of 16 marks.
- There will be 2 full questions (with a maximum of four sub questions) from each module.
- Each full question will have sub questions covering all the topics under a module.
- The students will have to answer 5 full questions, selecting one full question from each module.

Text Book:

Communication Systems, Simon Haykins & Moher, 5th Edition, John Willey, India Pvt. Ltd, 2010, ISBN 978 – 81 – 265 – 2151 – 7.

Reference Books:

1. Modern Digital and Analog Communication Systems, B. P. Lathi, Oxford University Press., 4th edition.
2. An Introduction to Analog and Digital Communication, Simon Haykins, John Wiley India Pvt. Ltd., 2008, ISBN 978–81–265–3653–5.
3. Principles of Communication Systems, H.Taub & D.L.Schilling, TMH, 2011.
4. Communication Systems, Harold P.E, Stern Samy and A Mahmond, Pearson Edition, 2004.
5. Communication Systems: Analog and Digital, R.P.Singh and S.Sapre: TMH 2nd edition, 2007.

MODULE-1

AMPLITUDE MODULATION: Introduction, Amplitude Modulation: Time & Frequency – Domain description, Switching modulator, Envelop detector.

DOUBLE SIDE BAND-SUPPRESSED CARRIER MODULATION: Time and Frequency – Domain description, Ring modulator, Coherent detection, Costas Receiver, Quadrature Carrier Multiplexing.

SINGLE SIDE–BAND AND VESTIGIAL SIDEBAND METHODS OF MODULATION: SSB Modulation, VSB Modulation, Frequency Translation, Frequency- Division Multiplexing, Theme Example: VSB Transmission of Analog and Digital Television.

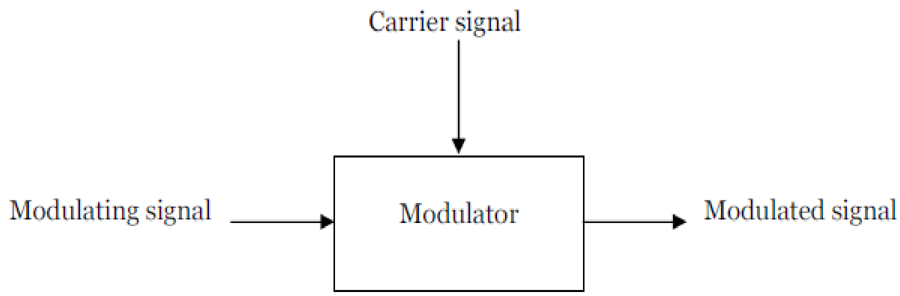
1.1 Objectives:

- Forms of amplitude modulation techniques that include conventional AM- Representation, Generation and detection are discussed.
- Double sideband Representation, Generation and detection are discussed. It covers theoretical and practical aspects of AM.
- Study about Single side band suppression and VSB Modulation and demodulation techniques.

1.2. Introduction:

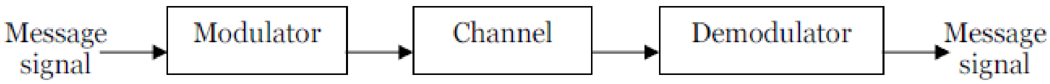
A large number of information sources are analog sources such as speech, images, and videos. Today, they are transmitted as analog signal transmission, especially in audio and video broadcast. The transmission of an analog signal is either by modulation of the amplitude, the phase, or the frequency of a sinusoidal carrier.

Modulation is the process of putting information onto a high frequency carrier for transmission (frequency translation). Modulation occurs at the transmitting end of the system.



Block diagram of modulation process

At the transmitter, modulation process occurs when the transmission takes place at the high frequency carrier, which has been modified to carry the lower frequency information. At the receiver, demodulation takes place. Once this information is received, the lower frequency information must be removed from the high-frequency carrier.



Block diagram of modulation and demodulation process

There are several strong reasons why the modulation is important in analog communication system:

(a) The frequency of the human voice range from about 20 to 30 kHz. If every one transmitted those frequencies directly as radio waves, interference would cause them to be inefficient. (so, we need a higher frequency to carry the baseband frequency)

(b) To overcome hardware limitation because transmitting such lower frequencies require antennas with miles in wavelength

(c) Modulation is to reduce noise which result in the optimization of signal to noise ratio, SNR

(d) To minimize the effects of interference

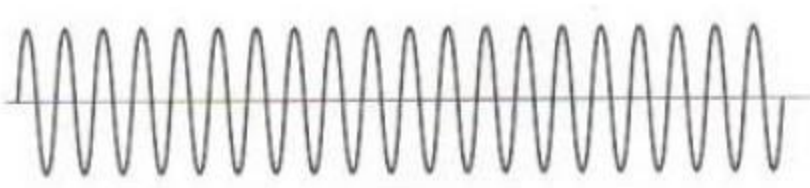
1.3. Types of modulation:

In analog communication systems, we use the sinusoidal signal as the frequency carrier. And as the sinusoidal wave can be represented in three parameters; amplitude, frequency and phase, these parameters may be varied for the purpose of transmitting information giving respectively the modulation methods:

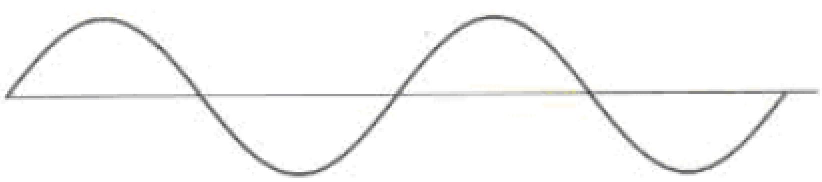
(a) Amplitude Modulation (AM) - the amplitude of the carrier waveform varies with the information signal

(b) Frequency Modulation (FM) - the frequency of the carrier waveform varies with the information signal

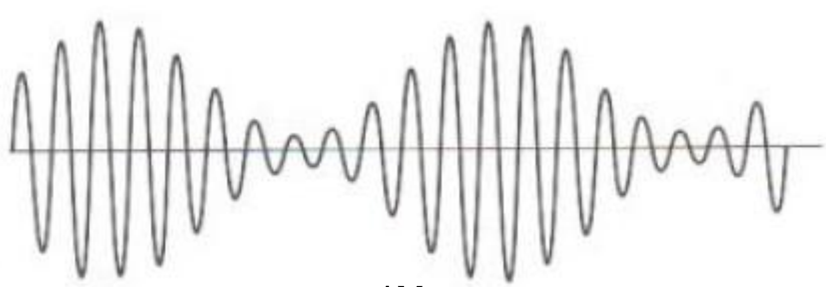
(c) Phase Modulation (PM) - the phase of the carrier waveform varies with the information signal



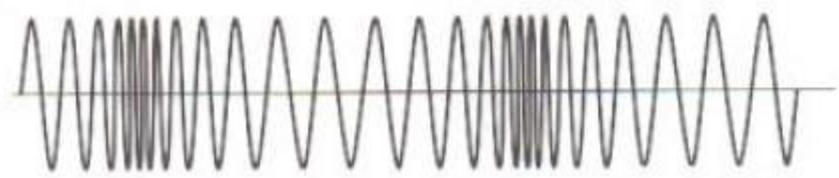
Carrier wave



Modulating wave



AM wave



FM wave

1.4. Amplitude Modulation:

In this type of modulation the amplitude of a sinusoidal carrier is varied according to the transmitted message signal. Let m(t)be the message signal we would like to transmit, ka be the amplitude sensitivity (modulation index), and c(t) = Accos(2πfct)be the sinusoidal carrier signal, where Ac is the amplitude of the carrier and fc is the carrier frequency.

The transmitted AM signal waveform is described by

$$s_{AM}(t) = A_c\left[1 + k_a m(t)\right]\cos(2\pi f_c t)$$

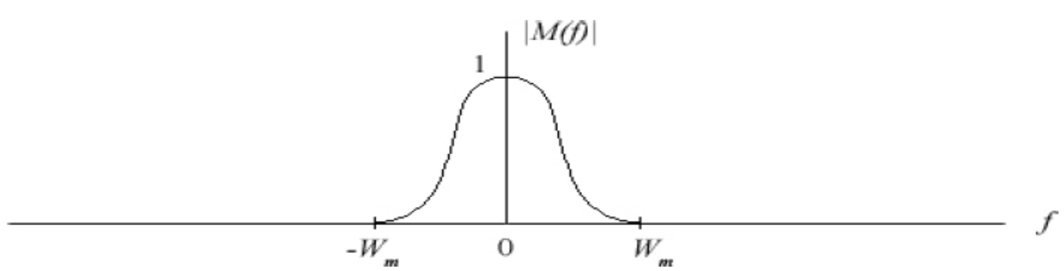
Requirements:

1. $|k_a m(t)| < 1, \forall t$
2. $f_c >> W_m$ where W_m is the message bandwidth

Taking the Fourier transform of the modulated waveform, we get

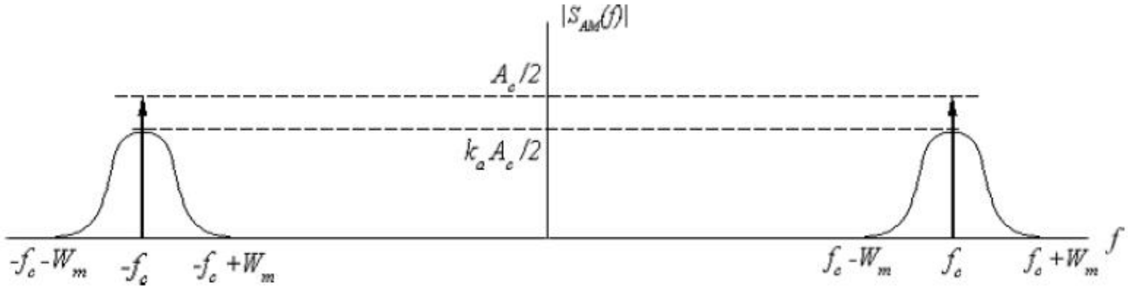
$$\begin{aligned} S_{AM}(f) &= F\{s_{AM}(t)\} \\ &= F\{A_c\cos(2\pi f_c t) + A_c k_a m(t)\cos(2\pi f_c t)\} \\ &= \frac{A_c}{2}[\delta(f - f_c) + \delta(f + f_c)] + \frac{k_a A_c}{2}[M(f - f_c) + M(f + f_c)] \end{aligned}$$

Let $|M(f)|$ be described by



Magnitude spectrum of $m(t)$

Then the magnitude spectrum of $s_{AM}(t)$ is



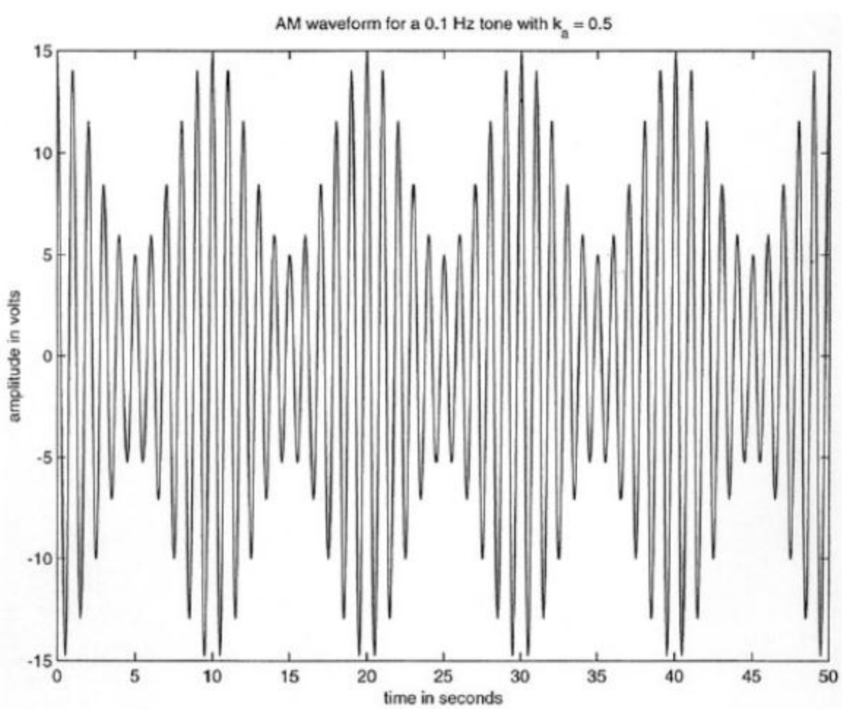
Magnitude spectrum of $s_{AM}(t)$

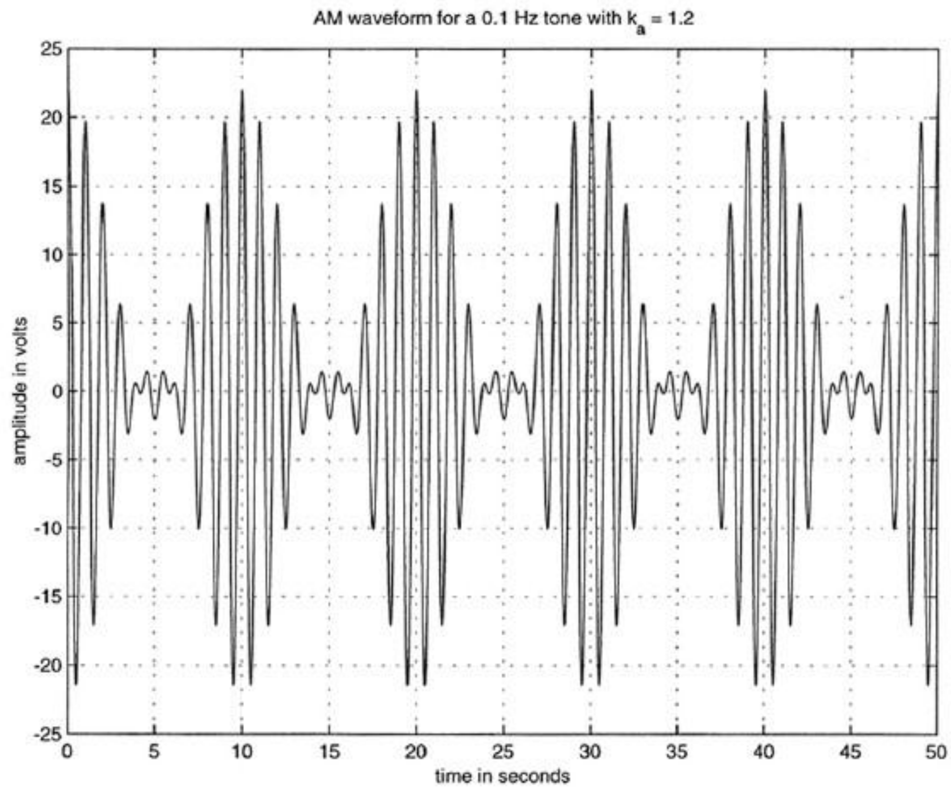
Observations:

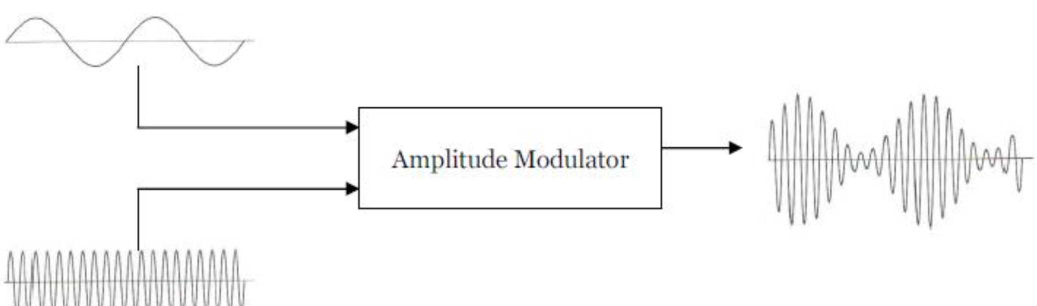
1. The transmission bandwidth is $B_{AM} = 2W_m$
2. Carrier signal is transmitted explicitly (delta functions are present in frequency spectrum)

Let the message signal be the tone $m(t) = \cos(0.2\pi t)$, then

$$s_{AM}(t) = A_c\left[1 + k_a \cos(0.2\pi t)\right]\cos(2\pi f_c t)$$







AM block diagram

Note:

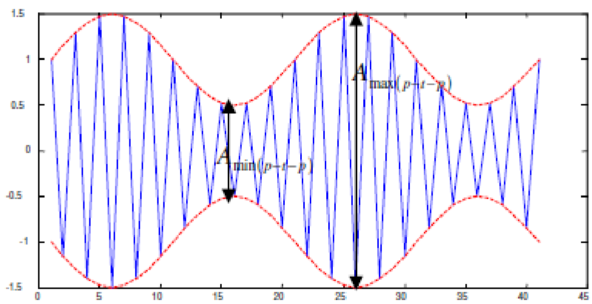
Modulation Index:

The degree of modulation is an important parameter and is known as the modulation index. It is the ratio of the peak amplitude of the modulation signal, A_m to the peak amplitude of the carrier signal, A_c.

$$m = \frac{A_m}{A_c}$$

The modulation index, m is also referred as percent modulation, modulation factor and depth of modulation. It is a number lying between 0 and 1 and is typically expressed as a percentage. The modulation index can be determined by measuring the actual values of the modulation voltage and the carrier voltage and computing the ratio.

In practice, the modulation index of an AM signal can be computed from A_{max} and $A_{min.}$ as below:



$A_{\max}$ = half the peak-to-peak value of the AM signal

$A_{\min}$ = half the peak-to-peak value of the AM signal

A_m = half the difference of $A_{\max}$ and $A_{\min}$.

A_c = half the sum of $A_{\max}$ and $A_{\min}$.

The values for $A_{\max}$ and $A_{\min}$ can be obtained directly from the oscilloscope.

The evaluation of the modulation index m can be achieved by invoking the following expression:

$$m = \frac{\frac{1}{2}(A_{\max} - A_{\min})}{\frac{1}{2}(A_{\max} + A_{\min})}$$

$$= \frac{A_m}{A_c}$$

Modulation index can determine the behavior of modulation index:

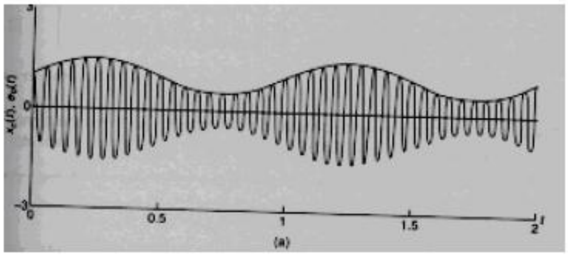
(a) under modulation

(b) ideal modulation

(c) over modulation

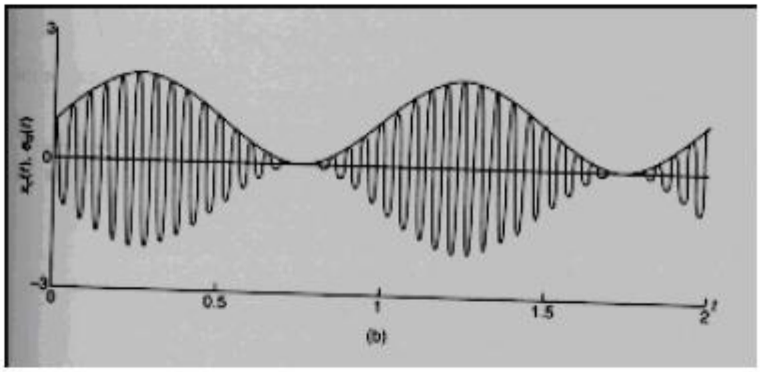
Under Modulation:

When $m < 1$, we call this as **under modulation** By ensuring the amplitude of $s_m(t)$ to be less than the carrier amplitude, message signal can comfortably be retrieved from the envelope waveform of $s(t)$.



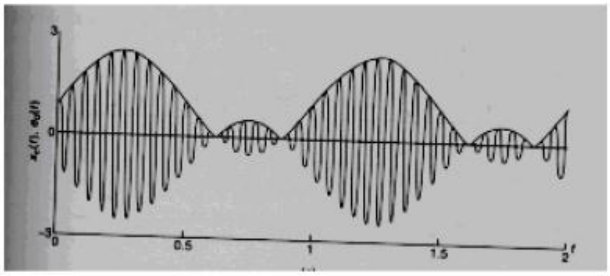
Ideal Modulation:

When $m = 1$, this is the best modulation where to ensure successful retrieval of the original transmitted information at the receiver end. The ideal condition for amplitude modulation (AM) is when $m = 1$ also means $A_m = A_c$; this will give rise to the generation of the maximum message signal outputs at the receiver without distortion.



Over Modulation:

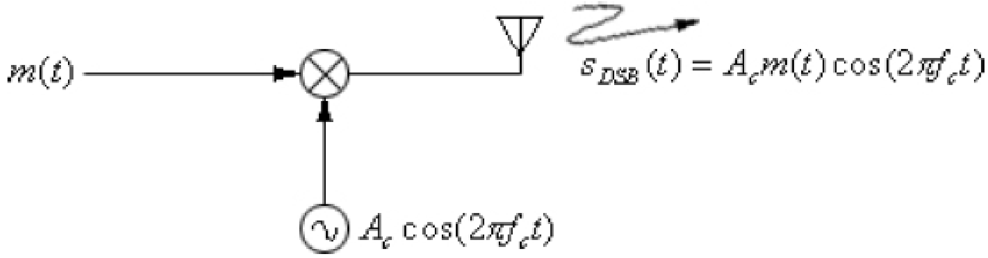
When $m > 1$, we call this as **over modulation.** If the amplitude of the modulating signal is higher than the carrier amplitude, this will cause severe distortion to the modulated signal.



1.5. Double side band large carrier:

To conserve transmitted power, let us suppress the carrier, i.e., let the transmitted waveform be described by

$$s_{DSB}(t) = A_c m(t)\cos(2\pi f_c t).$$



This is called double side-band suppressed carrier (DSB-SC) modulation.

Also known as full AM. In Amplitude Modulation, the baseband or the information signal is modulated to the carrier signal to produce the modulated sine wave.

Consider the carrier signal,

$s_c(t) = A_c \cos(\omega_c t)$ where $\omega_c = 2\pi f_c$

The modulating signal (information signal),

$s_m(t) = A_m \cos(\omega_m t)$

Then, the amplitude-modulated can be expressed as

$$s(t) = [A_c + s_m(t)]\cos(\omega_c t)$$
$$= [A_c + A_m \cos(\omega_m t)]\cos(\omega_c t)$$

The amplitude term of the AM signal $s(t)$ is

$$A = (A_c + A_m \cos(\omega_m t))$$
$$= (A_c + mA_c \cos(\omega_m t))$$
$$= A_c(1 + m\cos(\omega_m t))$$

where notation m in expression above is termed the modulation index. Simply a measurement for the degree of modulation and bears the relationship of the ratio of A_m to A_c,

$$m = \frac{A_m}{A_c}$$

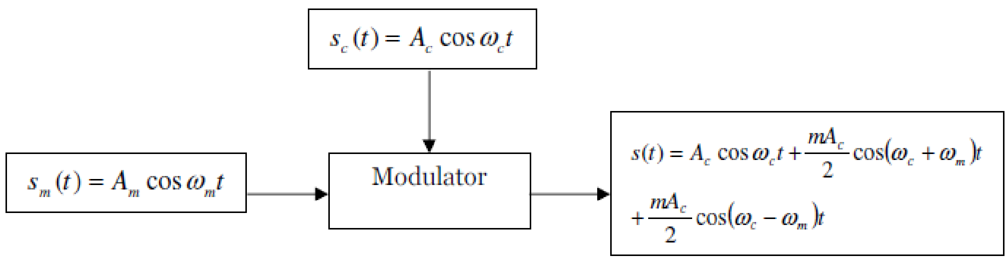
Therefore the full AM signal may be written as

$$s(t) = A_c(1 + m\cos(\omega_m t))\cos(\omega_c t)$$

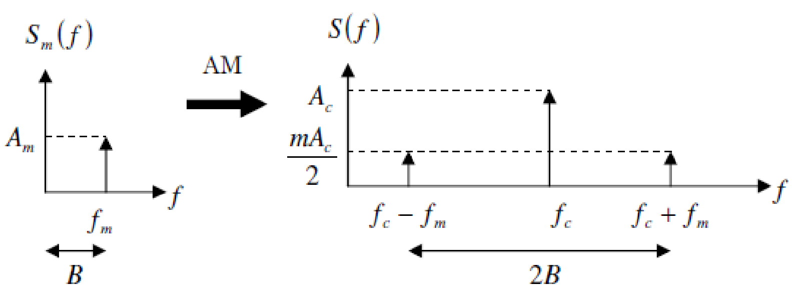
or

$$s(t) = A_c \cos\omega_c t + \frac{mA_c}{2}\cos(\omega_c + \omega_m)t + \frac{mA_c}{2}\cos(\omega_c - \omega_m)t$$

using: $\cos A \cos B = 1/2[\cos(A+B) + \cos(A-B)]$



The frequency description of the AM signal (i.e. frequency spectrum of AM) – DSB-LC:



From the above analysis, we found that the frequency spectrum of AM waveform DSB-LC:

- A component of carrier frequency, f_c
- An upper sideband (USB), whose highest frequency component is at $f_c + f_m$
- A lower sideband (LSB), whose highest frequency component is at $f_c - f_m$
- The bandwidth of the modulated waveform is twice the information signal bandwidth

- Because of the two sidebands in the frequency spectrum with carrier frequency, thus it is often called Double Sideband with Large Carrier (DSB-LC)

1.6. Double Side band Suppressed carrier:

Consider the carrier,

$s_c(t) = A_c \cos(w_c t)$ where $\omega_c = 2\pi f_c$

The modulating signal (information signal),

$s_m(t) = A_m \cos(\omega_m t\ )$ where $\omega_m = 2\pi f_m$

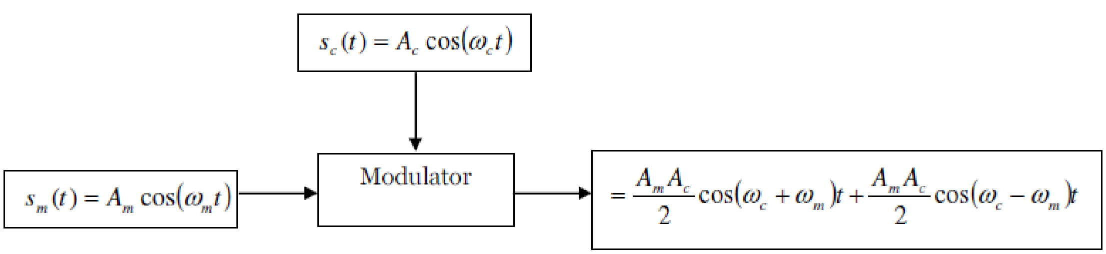
Then, the amplitude-modulated can be expressed as

$s_m(t) = A_c \cos(\omega_c t\ ) A_m \cos(\omega_m t)$

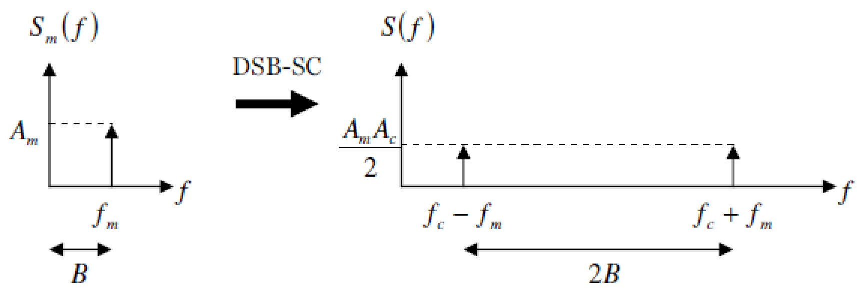
As noted earlier, where there are two sidebands in the frequency spectrum, USB and LSB, and it is called as Double-sided band (DSB).

But the carrier component in full AM or DSB-LC does not convey any information, it may be removed or suppressed during the modulation process to attain a higher power efficiency, hence Double Side Band Suppressed Carrier (DSB-SC) Modulation.

$$= \frac{A_m A_c}{2} \cos(\omega_c + \omega_m)t + \frac{A_m A_c}{2} \cos(\omega_c - \omega_m)t$$



The frequency description of the AM signal (i.e. frequency spectrum of AM) – DSB-SC:



Note: Notice that there is no carrier frequency (band).

From the above analysis, we found that the frequency spectrum of AM waveform – DSB-SC:

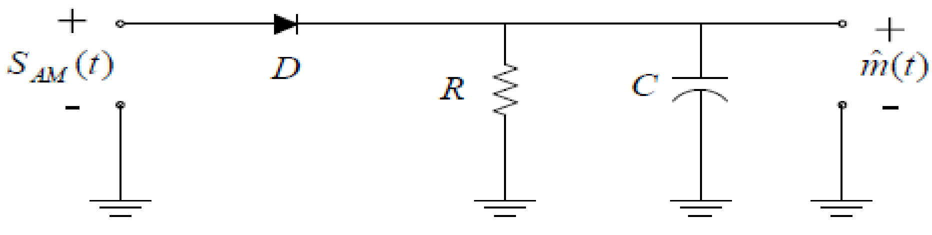
- No component of carrier frequency, f_c
- An upper sideband (USB), whose highest frequency component is at $f_c + f_m$
- A lower sideband (LSB), whose highest frequency component is at $f_c - f_m$
- The bandwidth is twice the modulating signal bandwidth
- Because of the two sidebands in the frequency spectrum without carrier frequency, thus it is often called Double Sideband with Suppressed Carrier (DSB-SC)

1.6. Receivers for AM and DSB:

Receivers can be classified into coherent and non-coherent categories.
Definition: If a receiver requires knowledge of the carrier frequency and phase to extract the message signal from the modulated waveform, then it is called coherent.

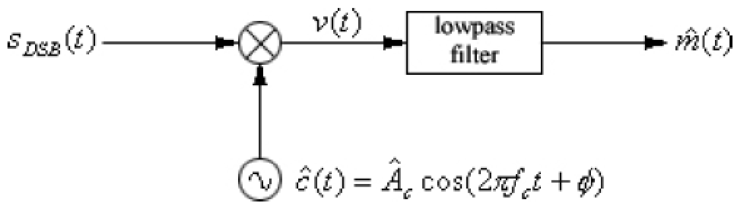
Definition: If a receiver does not require knowledge of the phase (only rough knowledge of the carrier frequency) to extract the message signal from the modulated waveform, then it is called non-coherent.

Non-coherent demodulator (receiver) for standard AM



Peak envelop Detector

Coherent demodulator for DSB-SC: Consider the following demodulator



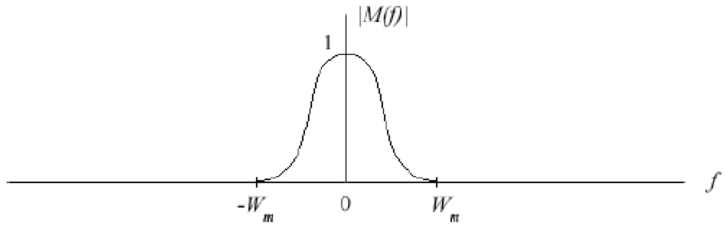
At the output of the mixer,

$$v(t) = \hat{A}_c \cos(2\pi f_c t + \phi) s_{DSB}(t)$$
$$= \hat{A}_c \cos(2\pi f_c t + \phi) \cdot A_c m(t) \cos(2\pi f_c t)$$
$$= \frac{\hat{A}_c A_c}{2} m(t)\left[\cos(4\pi f_c t + \phi) + \cos\phi\right]$$

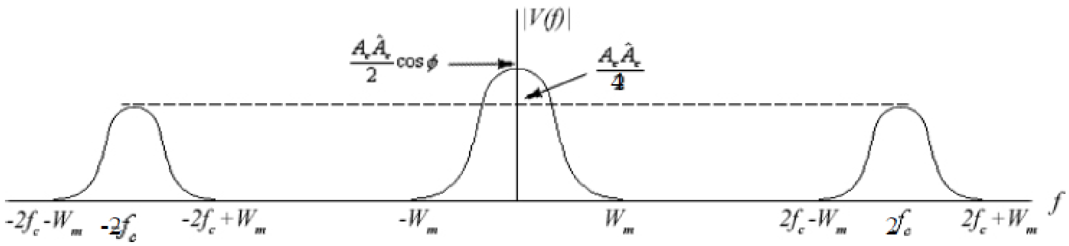
and

$$V(f) = F\{v(t)\}$$
$$= \frac{A_c \hat{A}_c}{4}\left[M(f - 2f_c)e^{j\phi} + M(f + 2f_c)e^{-j\phi}\right] + \frac{A_c \hat{A}_c}{2} M(f)\cos\varphi$$

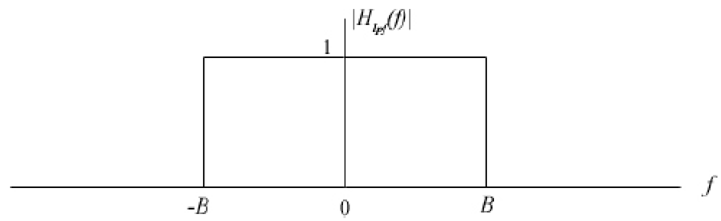
Let the message signal have the following magnitude spectrum



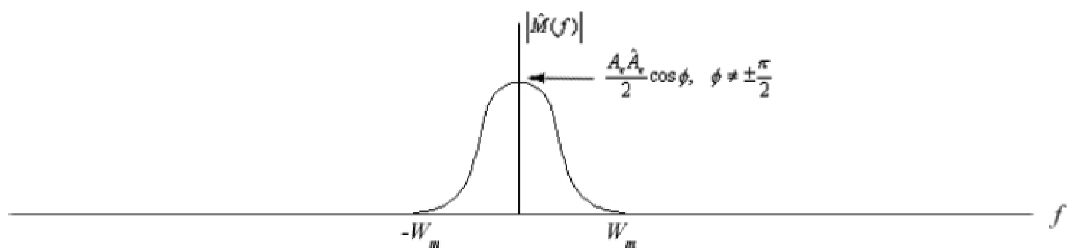
Then, if $f_c > W_m$,



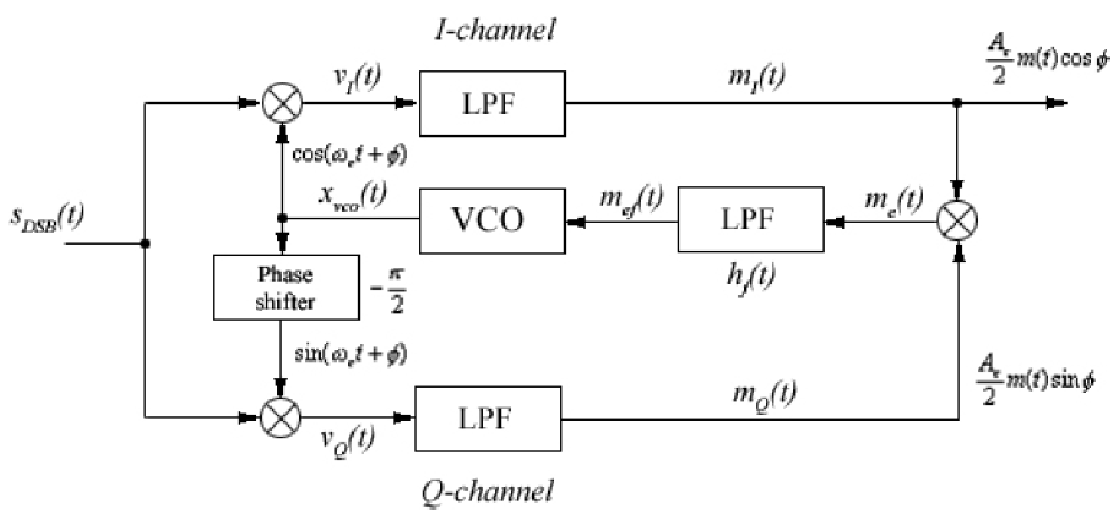
Suppose $H_{lpf}(f)$ is such that



Then, if $W_m \le B < 2f_c - W_m$, $\hat{M}(f) = \frac{A_c \hat{A}_c}{2} M(f) \cos\phi$



<u>Coherent Costas loop receiver for DSB-SC</u> :



I-channel:

After downconverwsion,

$$v_I(t) = A_c m(t)\cos(\omega_c t)\cdot\cos(\omega_c t+\phi)$$
$$= \frac{A_c}{2} m(t)\left[\cos(2\omega_c t+\phi)+\cos\phi\right]$$

At the output of the lowpass filter, with |H(0)| = 1,

$$m_I(t) = \frac{A_c}{2}\cos\phi\cdot m(t)$$

Q-channel:

$$v_Q(t) = \frac{A_c}{2} m(t)\left[\sin(2\omega_c t+\phi)+\sin\phi\right]$$
$$m_Q(t) = \frac{A_c}{2}\sin\phi\cdot m(t)$$

Feedback path:

At the output of the multiplier,

$$m_e(t) = \frac{A_c^2}{4} m^2(t)\sin\phi\cos\phi$$
$$= \frac{A_c^2}{8} m^2(t)\sin 2\phi$$

At the output of the lowpass filter,

$$m_{ef}(t) = \int_{-\infty}^{\infty} m_e(\tau)h_f(t-\tau)d\tau$$

The purpose of $h_f(t)$ is to smooth out fast time variations of $m_e(t)$

The output of the VCO is described by

$$x_{VCO}(t) = \cos\left(\omega_c t+\phi(t)\right)$$

Where ω_c is the VCO's reference frequency and $\phi(t) = k_v \int_0^t m_{ef}(\tau)d\tau,$ is the residual phase angle due to the tracking error. The constant k_v is the frequency sensitivity of the VCO in rad/s/volt (it depends on the circuit implementation).

The instantaneous frequency in radians/sec of the VCO's output is given by

$$\frac{d[\omega_c t + \phi(t)]}{dt} = \omega_c + k_v m_{ef}(t),$$

Clearly, if $\phi(t)$ were small, then the instantaneous frequency would be close to ω_c and the output of the I-path would also be proportional to $m(t)$.

1.7. Single-side band modulation:

Standard amplitude modulation and DSBSC are wasteful of bandwidth beause they both require a transmission bandwidth equal twice the message bandwidth

Thus the channel needs to provide only the same bandwidth as the message signal. When only one side band is transmitted the modulation is reffered to as single side band modulation.

1.7.1. Frequency domain description:

Precise frequency domain description of a single side band modulation wave depends on which side band is transmitted.

Consider a messge signal M(f) limited to the band $-W \leq f \leq W$. the spectrum of DSBSC modulated wave obtained by multiplying m(t) with the carrier wave is shown.

The upper band is represented is mirror image of the lower side band.

The transmission band requirement of SSB is one half that required for DSBSC or AM modulation

The principle disadvantage of SSB is complexity , cost of its implementation.

1.7.2 Frequency discrimination description method for generating an SSB modulated wave:

Two conditions must be satisfied

The message signal m(t) has little or no frequency content

The highest frequency component W of the message signal m(t) is much lesser than carrier frequency

Then under these conditions the desired side band will appear in a non overlapping interval in the spectrum in such a way that it may be selected by an appropriate filter

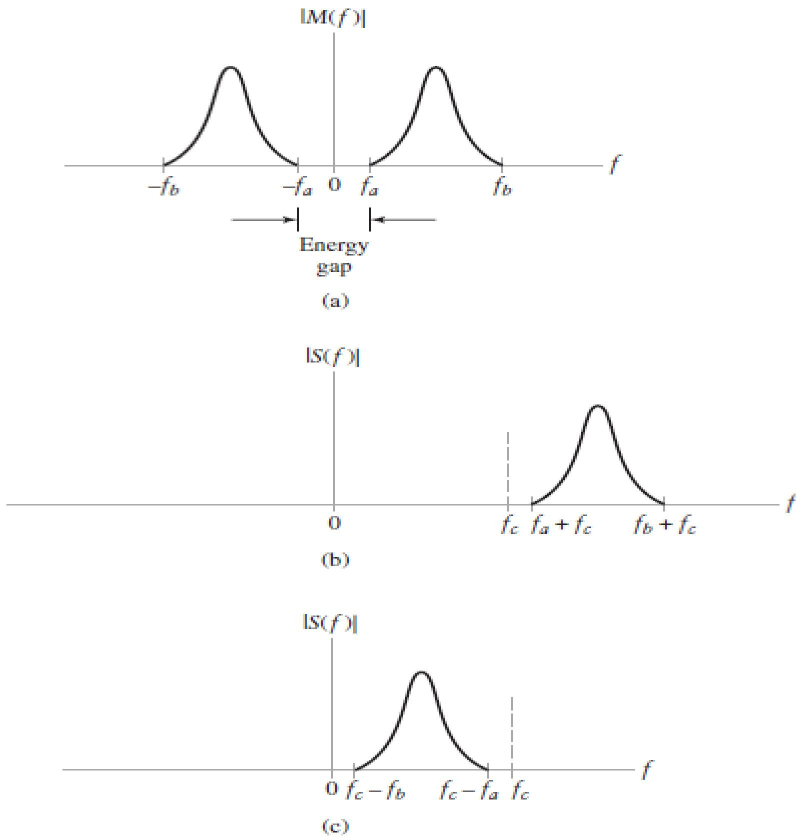
The most severe requirement of this method of SSB generation usually arises from the unwanted side band

In designing the band pass filter in the SSB modulation scheme we must satisfy the following two conditions

The pass band of the filter occupies the same frequency range as the spectrum of the desired
SSB modulated wave.

The width of the guard band of the filter, separating the pass band from the stop band where the unwanted side band of the input lies is the twice the lowest frequency component of the message signal

The frequency separation between the sidebands of this DSBSC modulated wave is effectively twice the first carrier frequency f_1, thereby permitting the second filter to remove the unwanted sideband.



1.8 .VSB Modulated Wave

Vestigial sideband (VSB) modulation distinguishes itself from SSB modulation in two practical respects:

1. Instead of completely removing a sideband, a trace or vestige of that sideband is transmitted; hence, the name “vestigial sideband.”
2. Instead of transmitting the other sideband in full, almost the whole of this second band is also transmitted.

1.9. Frequency Domain Description:

Specifically, the transmitted vestige of the lower sideband compensates for the amount removed from the upper sideband. The transmission bandwidth required by the VSB modulated wave is given by: - B = W + f

Where, W is message bandwidth, f is the bandwidth of the vestigial sideband.

VSB has the virtue of conserving bandwidth like SSB, while retaining the low frequency baseband characteristics of double sideband modulation.
It is basically used in transmission of tv signals where good phase characteristics and transmission of low frequency components is important.

Transmits USB or LSB and vestige of other sideband
Reduces bandwidth by roughly a factor of 2
Generated using standard AM or DSBSC modulation, then filtering
Standard AM or DSBSC demodulation
VSB used for image transmission in TV signals

1.10. Generation of VSB modulate wave:

To generate VSB modulated wave, we pass a DSBSC modulated wave through a sideband shaping filter.

The design of the filter depends on the desired spectrum of the VSB modulated wave. The relation between transfer function H(f)of the filter and the spectrum S(f) of the VSB modulated wave is given by –

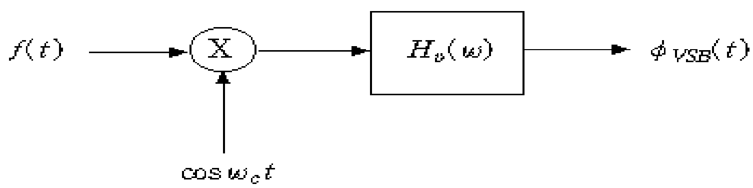
$$S(f)= Ac/2[M(f-fc) + M(f+fc)]H(f),$$ where M(f) is message spectrum.

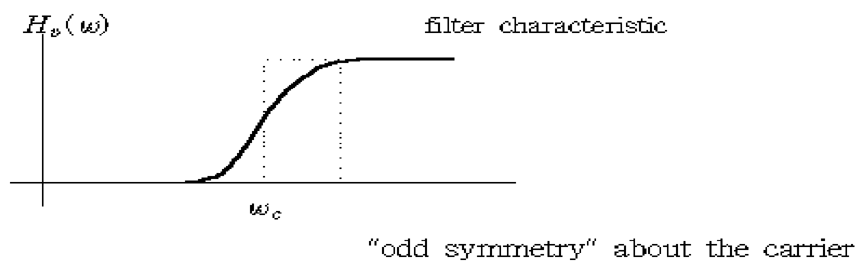
To determine the specifications of the filter transfer function H(f) so that S(f) defines the spectrum of the s(t), we pass s(t) through a coherent detector.
Thus, multiplying s(t) by a locally generated sine wave cos(2 π fc t), which is synchronous with the carrier wave Ac cos(2 π fc t), we get v(t)= cos(2 π fc t)s(t).
The relation in frequency domain gives the Fourier transform of v(t) as

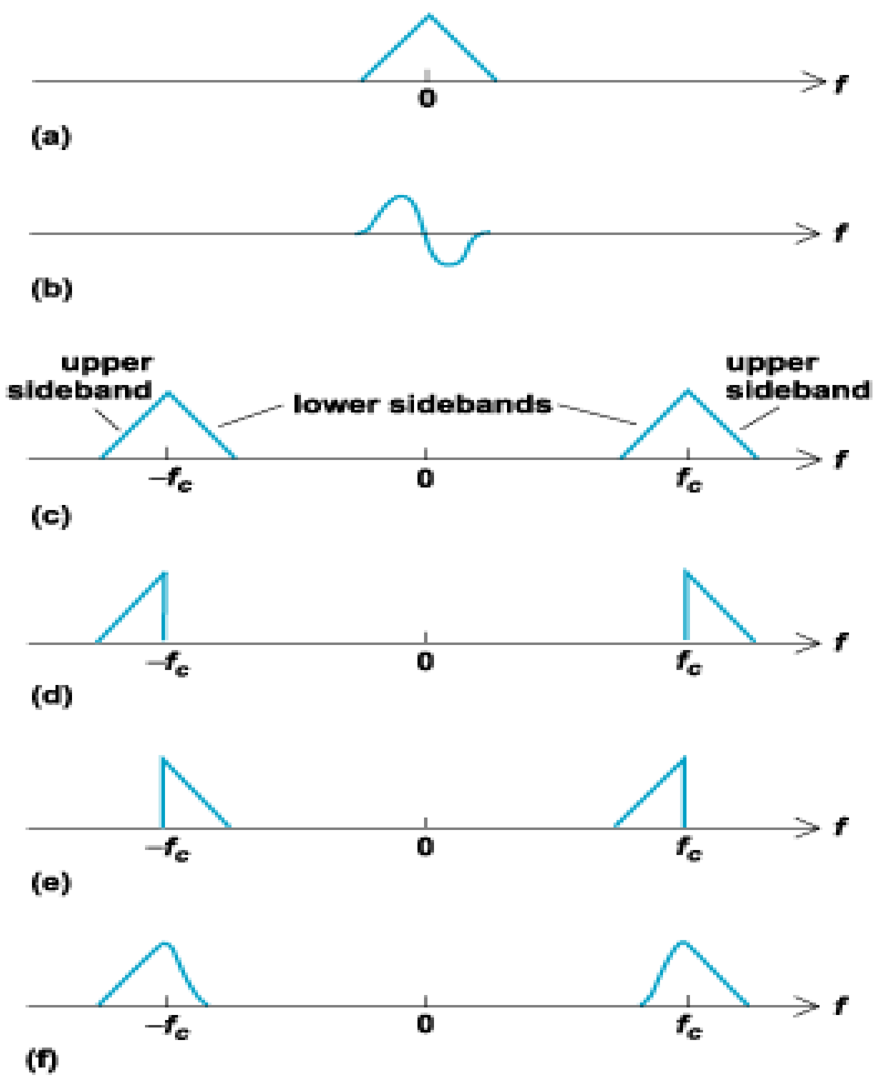
$$V(f) = 0.5[S(f-fc) + S(f+fc)]$$

The final spectrum is given by : -

$$Vo(f)=Ac/4\ M(f)\ [H (f - fc) + H (f + fc)]$$


f(t)
X
H_v(ω)
φ_VSB(t)
cos ω_c t


H_v(ω)
filter characteristic
ω_c
"odd symmetry" about the carrier


f
0
(a)
(b)
upper sideband
lower sidebands
upper sideband
−f_c
f_c
(c)
(d)
(e)
(f)

1.11. Envelope Detection of VSB wave plus Carrier:

The modified modulator wave applied to the envelope detector input as

$$s(t) = Ac[1+0.5kam(t)]\cos(2\pi fct) - 0.5kaAcmQ(t)\sin(2\pi fct)$$

The envelope detector output denoted by a(t) is given as – a(t)= Ac{[a+0.5kam(t)]^2 + [0.5kamQ(t)]^2}^0.5

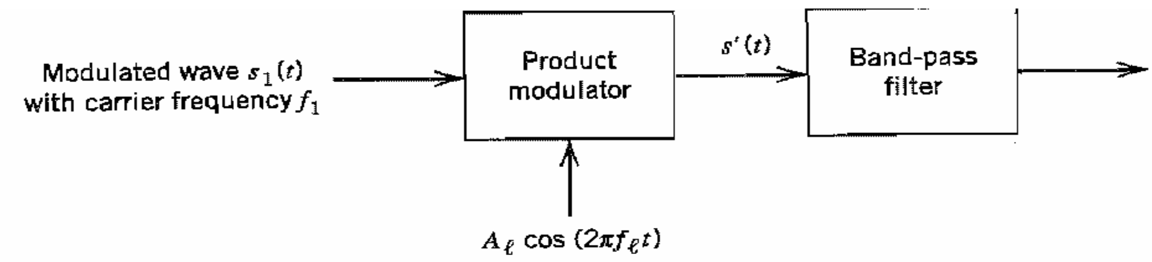
The distortion can be reduced either by reducing the % modulation to reduce ka or by increasing the width of the vestigial sideband to reduce mQ(t).

1.12. Comparison of Amplitude Modulation Techniques:

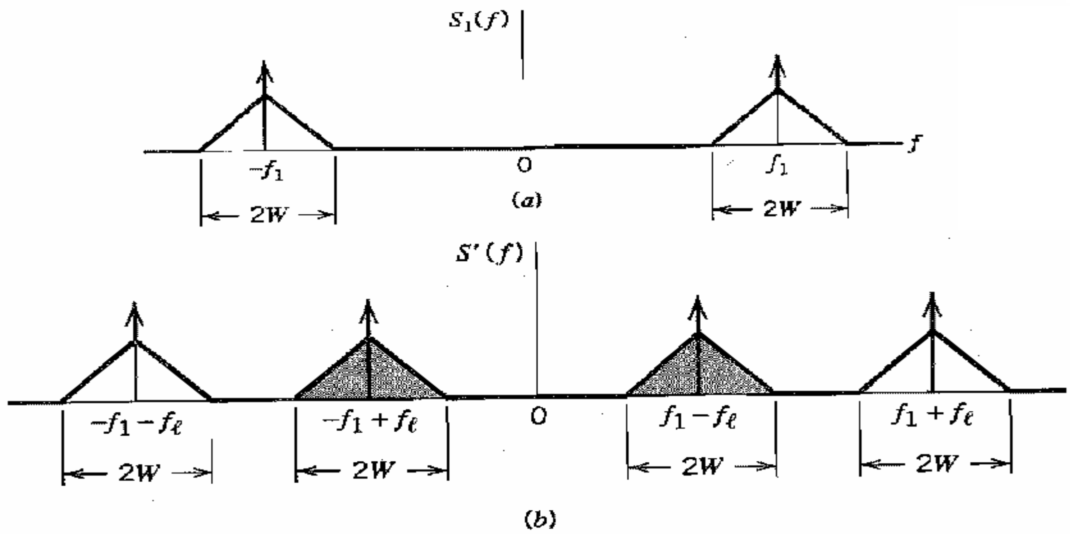
- In standard AM systems the sidebands are transmitted in full, accompanied by the carrier. Accordingly, demodulation is accomplished by using an envelope detector or square law detector. On the other hand in a suppressed carrier system the receiver is more complex because additional circuitry must be provided for purpose of carrier recovery.
- Suppressed carrier systems require less power to transmit as compared to AM systems thus making them less expensive.

- SSB modulation requires minimum transmitter power and maximum transmission band with for conveying a signal from one point to other thus SSB modulation is preferred.
- VSB modulation requires a transmission band with that is intermediate then that of SSB or DSBSC.
- DSBSC modulation, SSB modulation, and VSB modulation are examples of linear modulation. The output of linear modulator can be expressed in the canonical form given by
- $s(t)= s1(t)\cos(2\pi fct) - sQ(t)\sin(2\pi fct)$.
- In SSB and VSB modulation schemes the quadrature component is only to interfere with the in phase component so that power can be eliminated in one of the sidebands.
- The band pass representation can also be used to describe quadrature amplitude modulation.The complex envelope of the linearly modulated wave s(t) equals s(t)=s1(t)+jsQ(t).

1.13. Frequency Translations:



Block diagram of mixer.



(a) Spectrum of modulated signal $s_1(t)$ at the mixer input.
(b) Spectrum of the corresponding signal $s'(t)$ at the output of the product modulator in the mixer.

1.14. Frequency Division Multiplexing:

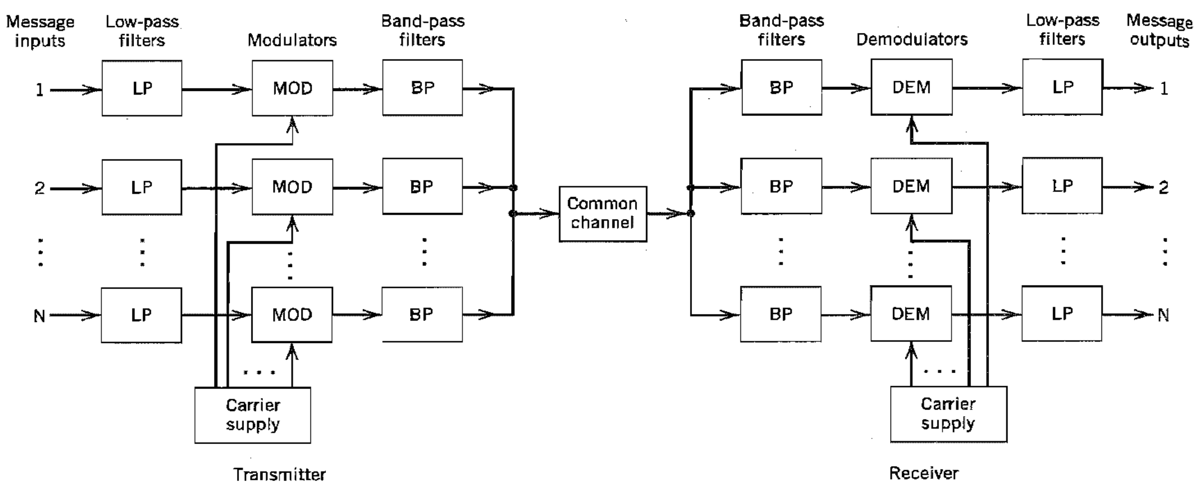


FIGURE 2.18 Block diagram of FDM system.

Multiplexing is a scheme, whereby a number of independent signals can be combined into a composite signal suitable for transmission over a common channel. Voice frequencies transmitted over telephone systems, for example, range from 300 to 3100 Hz. To transmit a number of these signals over the same channel (e.g. cable), the signals must be kept apart so that they do not interfere with each other, and thus they can be separated at the receiving end. This is accomplished by separating the signals either in frequency or in time. The technique of separating the signals in frequency is referred to as frequency-division multiplexing (FDM), whereas the technique of separating the signals in time is called time-division multiplexing (TDM).

Following each signal input, we have shown a low- pass filter, which is designed to remove high-frequency components that do not contribute significantly to signal representation but are capable of disturbing other message signals that share the common channel. These low-pass filters may be omitted only if the input signals are sufficiently band-limited initially. The filtered signals are applied to modulators that shift the frequency ranges of the signals so as to occupy mutually exclusive frequency intervals. The necessary carrier frequencies needed to perform these frequency translations are obtained from a carrier supply. For the modulation, we may use any one of the methods described in previous sections of this chapter. However, in telephony, the most widely used method of modulation in frequency-division multiplexing is single sideband modulation, which, in the case of voice signals, requires a bandwidth that is approximately equal to that of the original voice signal. In practice, each voice input is usually assigned a bandwidth of 4 kHz. The band-pass filters following the modulators are used to restrict the band of each modulated wave to its prescribed range. The resulting band-pass filter outputs are next combined in parallel to form the input to the common channel. At the receiving terminal, a bank of band-pass filters, with their inputs connected in parallel, is used to separate the message signals on a frequency-occupancy basis. Finally, the original message signals are recovered by individual demodulators. Note that the FDM system shown in Figure.

EXAMPLE: **Modulation steps in a 60-channel FDM system**

The practical implementation of an FDM system usually involves many steps of modulation and demodulation, as illustrated in Figure The first multiplexing step combines 12 voice inputs into a basic group, which is formed by having the nth input modulate a carrier at frequency f_C=60+4nkHz, where n =1, 2,..12. The lower sidebands are then selected by band-pass filtering and combined to form a group of 12 lower sidebands (one for each voice input). Thus the basic group occupies the frequency band 60–108 kHz. The next step in the FDM hierarchy involves the combination of five basic groups into a super group. This is accomplished by using the nth group to modulate a carrier of frequency f_C=372+48n kHz, where n=1, 2, .. , 5. Here again the lower sidebands are selected by filtering and then combined to form a super group occupying the band 312–552 kHz. Thus, a super group is designed to accommodate 60 independent voice inputs. The reason for forming the super group in this manner is that economical filters of the required characteristics are available only over a limited frequency range. In a similar manner, super groups are combined into master groups, and master- groups are combined into very large groups.

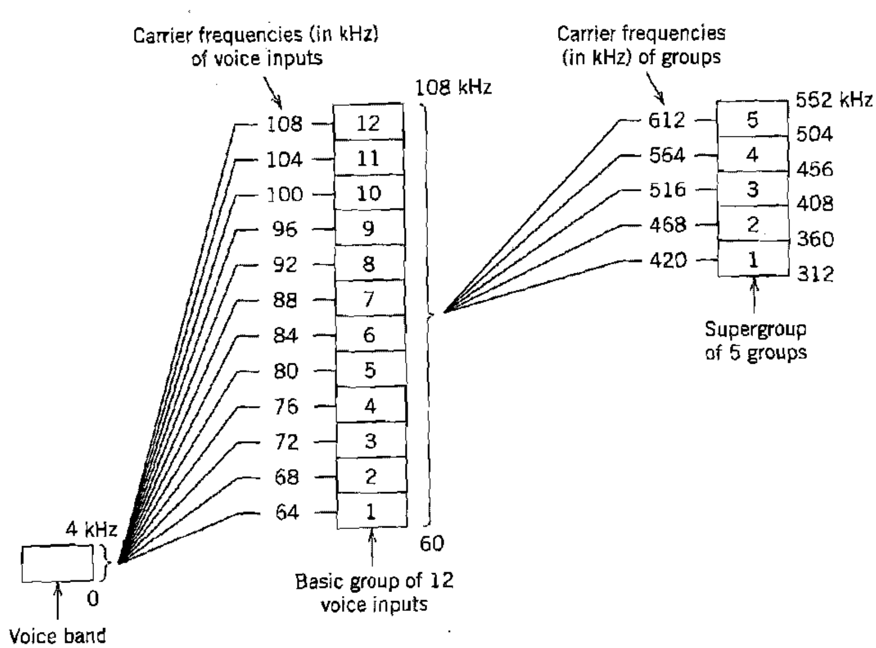


FIGURE 2.19 Illustrating the modulation steps in an FDM system.

1.15. Outcomes

- Earn the basic principles and engineering issues involved in analog communication systems.
- Appreciate the uses of modulation techniques.
- A detailed study in terms of bandwidth, spectral forms and other factors.
- Merits and demerits of SSB and VSB.
- Introduction to FDM scheme.

1.16. Further Readings

- **Modern digital and analog Communication systems** B. P. Lathi, Oxford University Press., 4th ed, 2010
- **Communication Systems**, Harold P.E, Stern Samy and A Mahmond, Pearson Edn, 2004.
- **Communication Systems**: Singh and Sapre**:** Analog and digital TMH 2nd , Ed 2007.
- www.youtube.com/watch?v=kVQ7mr2TU2U
- nptel.ac.in/courses/117105085/

1.17. Recommended Questions

1. Explain the need for Modulation?
2. Explain the generation of AM wave using switching modulator with equations, waveforms and spectrum before and after filtering process?
3. Explain the time domain & frequency domain representation of AM wave?
4. Show that square law device can be used to detect AM wave?
6. What is Quadrature null effect? How it can be eliminate?
7. Explain the generation of DSB-SC using Ring modulator?
8. With a neat diagram explain Quadrature carrier multiplexing?
9. Explain the advantage of SSB over DSBSC.
10. Compare the different AM schemes.
11. Explain the FDM scheme.

MODULE-2

ANGLE MODULATION: Basic definitions, Frequency Modulation: Narrow Band FM, Wide Band FM, Transmission bandwidth of FM Signals, Generation of FM Signals, Demodulation of FM Signals, FM Stereo Multiplexing, Phase–Locked Loop: Nonlinear model of PLL, Linear model of PLL, Nonlinear Effects in FM Systems. The Super heterodyne Receiver.

2.1 Objectives:

- Understand the concepts in Angle modulation for the design of communication systems.
- Difference between Narrow band FM and Wide band FM and the generation techniques are studied.
- Design simple systems for generating and demodulating frequency modulated signals.

2.2 Introduction:

Angle modulation is a method of analog modulation in which either the phase or frequency of the carrier wave is varied according to the message signal. In this method of modulation the amplitude of the carrier wave is maintained constant.

- ***Angle Modulation is a method of modulation in which either Frequency or Phase of the carrier wave is varied according to the message signal.***

In general form, an angle modulated signal can represented as

$$s(t) = A_c \cos[\theta(t)] \qquad ...(2.1)$$

Where A_c is the amplitude of the carrier wave and $\theta(t)$ is the angle of the modulated carrier and also the function of the message signal.

The instantaneous frequency of the angle modulated signal, s(t) is given by

$$f_i(t) = \frac{1}{2\pi}\frac{d\theta(t)}{dt} \qquad ...(2.2)$$

The modulated signal, s(t) is normally considered as a rotating phasor of length A_c and angle $\theta(t)$. The angular velocity of such a phasor is $d\theta(t)/dt$, measured in radians per second.

An un-modulated carrier has the angle $\theta(t)$ defined as

$$\theta(t) = 2\pi f_c t + \phi_c \qquad .....(2.3)$$

Where f_c is the carrier signal frequency and $_c$ is the value of $\theta(t)$ at t = 0.

The angle modulated signal has the angle, $\theta(t)$ defined by

$$\theta(t) = 2\pi f_c t + \phi(t) \qquad .....(2.4)$$

There are two commonly used methods of angle modulation:

1. Frequency Modulation, and
2. Phase Modulation.

Phase Modulation (PM):

In phase modulation the angle is varied linearly with the message signal m(t) as :

$$\theta(t) = 2\pi\ f_c t + k_p\, m(t) \qquad .....(2.5)$$

where k_p is the phase sensitivity of the modulator in radians per volt.

Thus the phase modulated signal is defined as

$$s(t) = A_c \cos 2\ f_c t + k_p m(t) \qquad ...(2.6)$$

Frequency Modulation (FM):

In frequency modulation the instantaneous frequency $f_i(t)$ is varied linearly with message signal, m(t) as:

$$f_i(t) = f_c + k_f m(t) \qquad ....(2.7)$$

where k_f is the frequency sensitivity of the modulator in hertz per volt.

The instantaneous angle can now be defined as

$$\theta(t) = 2\pi\ f_c t + 2\pi\, k_f \int_0^t m(t)dt \qquad ....(28)$$

and thus the frequency modulated signal is given by

$$s(t) = A_c \cos(2\pi\ f_c t + 2\pi\, k_f \int_0^t m(t)dt \qquad ....(2.9)$$

The PM and FM waveforms for the sinusoidal message signal are shown in the fig-2.1.

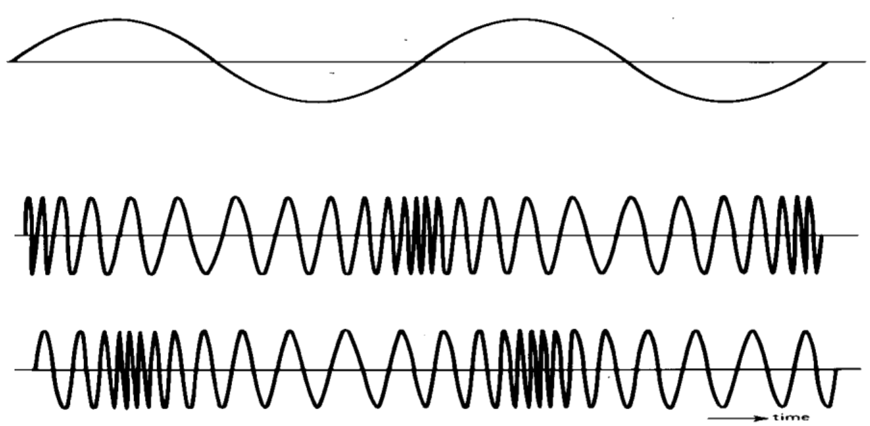


Fig: 2.1 – PM and FM Waveforms with a message signal

Example 2.1:

Find the instantaneous frequency of the following waveforms:

(a) $S_1(t) = Ac \ \text{Cos}\ [100\pi\, t + 0.25\, \pi\ ]$

(b) $S_2(t) = Ac \ \text{Cos}\ [100\pi\, t + \sin(20\, \pi\, t)\ ]$

(c) $S_3(t) = Ac \ \text{Cos}\ [100\pi\, t + (\pi\, t^2)\ ]$

Solution: Using equations (2.1) and (2.2):

(a) $f_i(t) = 50$ Hz; Instantaneous frequency is constant.

(b) $f_i(t) = 50 + 10\cos(20\,\pi\, t)$; Maximum value is 60 Hz and minimum value is 40 Hz.

Hence, instantaneous frequency oscillates between 40 Hz and 60 Hz.

(c) $f_i(t) = (50 + t)$

The instantaneous frequency is 50 Hz at t=0 and varies linearly at 1 Hz/sec.

2.2 Relation between Frequency Modulation and Phase Modulation:

A frequency modulated signal can be generated using a phase modulator by first integrating m(t) and using it as an input to a phase modulator. This is possible by considering FM signal as phase modulated signal in which the modulating wave is integral of m(t) in place of m(t). This is shown in the fig-2.2(a). Similarly, a PM signal can be generated by first differentiating m(t) and then using the resultant signal as the input to a FM modulator, as shown in fig-2.2(b).

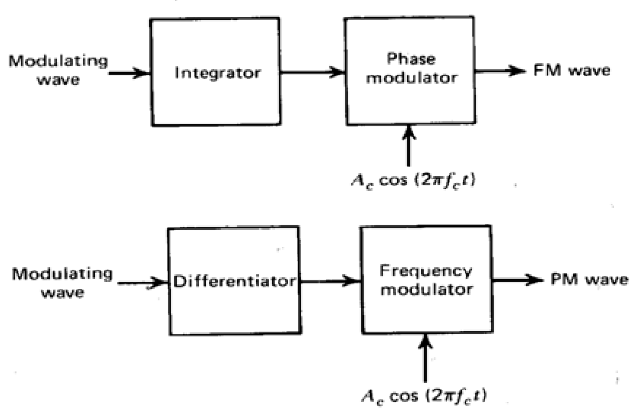


Fig: 2.2 – Scheme for generation of FM and PM Waveforms

2.3 Single-Tone Frequency Modulation:

Consider a sinusoidal modulating signal defined as:

$$m(t) = A_m \, Cos(\, 2\pi \, f_m \, t) \qquad \text{…. (2.10)}$$

Substituting for m(t) in equation (2.9), the instantaneous frequency of the FM signal is

$$f_i(t) = f_c + k_f A_m \cos(2\pi f_m t) \;=\; f_c + \Delta f \cos(2\pi f_m t)$$

where Δf is called the frequency deviation given by $\Delta f = k_f A_m$ ..... (2.11a)

and the instantaneous angle is

$$\begin{aligned}\theta(t) &= 2\pi \int_0^t f_i(t)dt \\ &= 2\pi f_c t + \frac{\Delta f}{f_m}\sin(\,2\pi f_m t) \\ &= 2\pi f_c t + \beta \sin(\,2\pi f_m t) \qquad \text{....}(2.11\text{b})\end{aligned}$$

$$\text{where } \beta = \frac{\Delta f}{f_m}; \text{ modulation index}$$

The resultant FM signal is

$$s(t) = A_c \cos[2\pi f_c t + \beta \sin(2\pi f_m t)] \qquad \text{....}(2.12)$$

The frequency deviation factor indicates the amount of frequency change in the FM signal from the carrier frequency f_c on either side of it. Thus FM signal will have the frequency components between (f_c - Δf) to (f_c +Δf). The modulation index, β represents the phase deviation of the FM signal and is measured in radians. Depending on the value of β, FM signal can be classified into two types:

1. Narrow band FM (β << 1) and
2. Wide band FM (β >> 1).

Example-2.2: A sinusoidal wave of amplitude 10volts and frequency of 1 kHz is applied to an FM generator that has a frequency sensitivity constant of 40 Hz/volt. Determine the frequency deviation and modulating index.

Solution: Message signal amplitude, A_m = 10 volts, Frequency f_m = 1000 Hz and the frequency sensitivity, k_f = 40 Hz/volt.

Frequency deviation, $\Delta f = k_f A_m = 400$ Hz

Modulation index, $\beta = \Delta f / f_m = 0.4$, (indicates a narrow band FM).

Example-2.3: A modulating signal m(t) =10 Cos(10000πt) modulates a carrier signal, A_c Cos($2\pi f_c t$). Find the frequency deviation and modulation index of the resulting FM signal. Use k_f = 5kHz/volt.

Solution: Message signal amplitude, A_m = 10 volts, Frequency f_m = 5000 Hz and the frequency sensitivity, k_f = 5 kHz/volt.

Frequency deviation, $\Delta f = k_f A_m = 50$ kHz

Modulation index, $\beta = \Delta f / f_m = 10$, (indicates a wide band FM).

Frequency Domain Representation of Narrow Band FM signal:

Expanding the equation (2.12) using trigonometric identities,

$$\begin{aligned} s(t) &= A_c \cos\left[2\pi f_c t + \beta \sin(2\pi f_m t)\right] \\ &= A_c \cos\left(2\pi f_c t\right)\cos\left[\beta \sin(2\pi f_m t)\right] - A_c \sin\left(2\pi f_c t\right)\sin\left[\beta \sin(2\pi f_m t)\right] \end{aligned}$$

For NBFM, (β << 1), we can approximate,

$$\cos\left[\beta \sin(2\pi f_m t)\right] \approx 1 \ \ and \ \ \sin\left[\beta \sin(2\pi f_m t)\right] \approx \beta \sin(2\pi f_m t)$$

Hence, $$s(t) = A_c \cos(2\pi f_c t) - A_c \beta \sin(2\pi f_c t)\, \sin(2\pi f_m t) \qquad .....(2.13)$$

Using trigonometric relations;

$$s(t) = A_c \cos(2\pi f_c t) + \frac{A_c \beta}{2}\left[\cos(2\pi (f_c + f_m)t) - \cos(2\pi (f_c - f_m)t\right] \quad ..(2.14)$$

The above equation represents the NBFM signal. This representation is similar to an AM signal, except that the lower side frequency has negative sign. The magnitude spectrum of NBFM signal is shown in fig-2.3, which is similar to AM signal spectrum. The bandwidth of the NBFM signal is $2f_m$, which is same as AM signal.

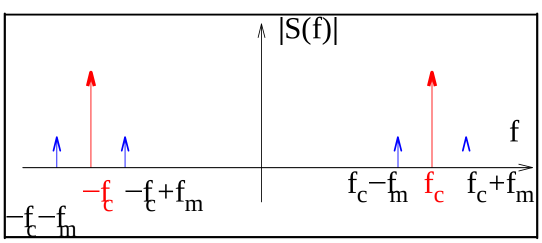


Fig: 2.3 - Magnitude Spectrum of NBFM Waveform.

2.4 Frequency Domain Representation of Wide-Band FM signals:

The FM wave for sinusoidal modulation is given by

$$\begin{aligned} s(t) &= A_c \cos[2\pi f_c t + \beta \sin(2\pi f_m t)] \\ &= A_c \cos(2\pi f_c t)\cos[\beta \sin(2\pi f_m t)] - A_c \sin(2\pi f_c t)\sin[\beta \sin(2\pi f_m t)] \end{aligned}$$

The FM wave can be expressed in terms of complex envelope as:

$$\begin{aligned} s(t) &= \mathrm{Re}[A_c \exp(j2\pi f_c t + j\beta \sin(2\pi f_m t))] \\ &= \mathrm{Re}[\tilde{s}(t)\exp(j2\pi f_c t)] \qquad \ldots (2.15) \end{aligned}$$

The complex envelope of the FM wave

$$\tilde{s}(t) = A_c \exp[j\beta \sin(2\pi f_m t)] \ \textit{and} \ \tilde{s}(t): \text{periodic function with } f_m$$

The complex envelope is a periodic function of time, with a fundamental frequency equal to the modulation frequency f_m. The complex envelope can be expanded in the form of complex series:

$$\tilde{s}(t) = \sum_{n=-\infty}^{\infty} c_n \exp[j2\pi n f_m t] \qquad \ldots (2.16)$$

The complex Fourier coefficient, c_n equals,

$$\begin{aligned} c_n &= f_m \int_{-\frac{1}{2f_m}}^{\frac{1}{2f_m}} \tilde{s}(t) \exp(-j2\pi n f_m t)dt \\ &= f_m A_c \int_{-\frac{1}{2f_m}}^{\frac{1}{2f_m}} \exp[j\beta \sin(2\pi f_m t) - j2\pi n f_m t]dt \qquad \ldots (2.17) \end{aligned}$$

Substituting x = ($2\pi f_m t$), in the above equation we can rewrite

$$c_n = \frac{A_c}{2\pi}\int_{-\pi}^{\pi} \exp(j(\beta \sin x - nx))dx \qquad \ldots (2.18)$$

The n^th order Bessel function of the first kind is defined as

$$J_n(\beta) = \frac{1}{2\pi}\int_{-\pi}^{\pi} \exp(j(\beta \sin x - nx))dx \qquad \ldots (2.19)$$

Comparing equations (2.18) and (2.19), we get $C_n = A_c J_n(\beta)$
Substituting in (2.16), the complex envelope is

$$\tilde{s}(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta)\exp(j2\pi n f_m t) \qquad \ldots (2.20)$$

Substituting in (2.15), the FM signal can be written as

$$s(t) = A_c \,\mathrm{Re}\left[\sum_{n=-\infty}^{\infty} J_n(\beta) \exp\left[j2\pi\left(f_c + nf_m\right)t\right] \right]$$

$$\mathrm{s(t)} = A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \cos\left[2\pi\left(f_c + nf_m\right)t\right] \qquad ...(2.21)$$

The above equation is the Fourier series representation of the single tone FM wave. Applying the Fourier transform to (2.21),

$$S(f) = \frac{A_c}{2} \sum_{n=-\infty}^{\infty} J_n(\beta)\left[\delta\left(f - f_c - nf_m\right) + \delta\left(f + f_c + nf_m\right)\right] \qquad ...(2.22)$$

The spectrum S(f) is shown in fig-2.4. The above equation indicates the following:

(i) FM signal has infinite number of side bands at frequencies ($f_c \pm nf_m$).

(ii) Relative amplitudes of all the spectral lines depends on the value of $J_n(\beta)$.

(iii) The number of significant side bands depends on the modulation index (β). With (β<<1), only $J_0(\beta)$ and $J_1(\beta)$ are significant. But for (β>>1), many sidebands exists.

(iv) The average power of an FM wave is $P = 0.5A_c^2$ (based on Bessel function property).

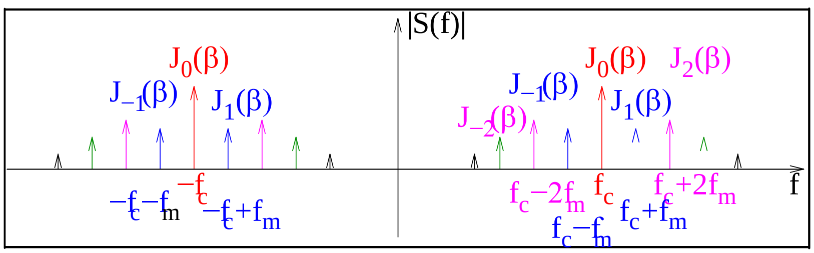


Fig: 2.4 - Magnitude Spectrum of Wide Band FM Wave.

Bessel's Function:

Bessel function is an useful function to represent the FM wave spectrum. The general plots of Bessel functions are shown in fig-2.5 and table (2.1) gives the values for Bessel function coefficients. Some of the useful properties of Bessel functions are given below:

(a) $J_n(\beta) = (-1)^n J_{-n}(\beta)$ for all n (2.23a)

(b) $J_{n+1}(\beta) + J_{n-1}(\beta) = \dfrac{2n}{\beta} J_n(\beta)$ (2.23b)

(c) $\sum_{n=-\infty}^{\infty} J_n^{\,2}(\beta) = 1$ (2.23c)

(d) For smaller values of β, $J_0(\beta) \cong 1,\ J_1(\beta) \cong \frac{\beta}{2}$ *and* $J_n(\beta) \cong 0,\ \textit{for}\ n > 2$

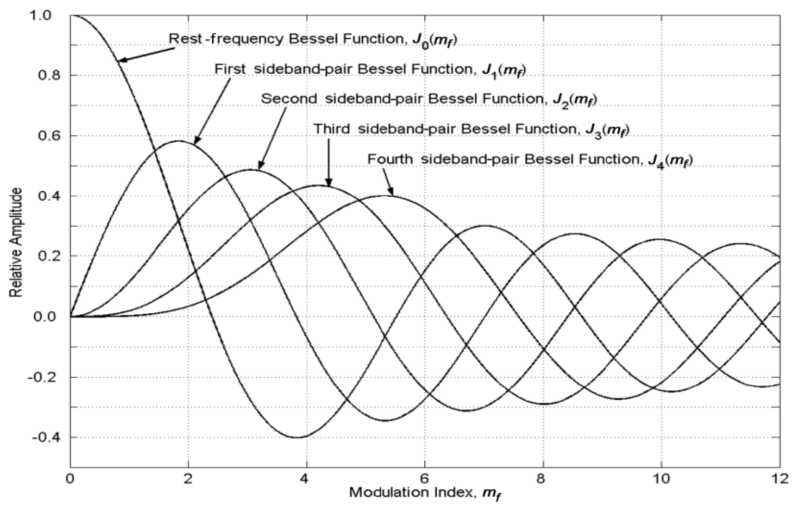


Fig: 2.5 – Plots of Bessel functions

x	*Bessel-function order, n*																
m_f	J_0	J_1	J_2	J_3	J_4	J_5	J_6	J_7	J_8	J_9	J_{10}	J_{11}	J_{12}	J_{13}	J_{14}	J_{15}	J_{16}
0.00	1.00	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
0.25	0.98	0.12	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
0.5	0.94	0.24	0.03	—	—	—	—	—	—	—	—	—	—	—	—	—	—
1.0	0.77	0.44	0.11	0.02	—	—	—	—	—	—	—	—	—	—	—	—	—
1.5	0.51	0.56	0.23	0.06	0.01	—	—	—	—	—	—	—	—	—	—	—	—
2.0	0.22	0.58	0.35	0.13	0.03	—	—	—	—	—	—	—	—	—	—	—	—
2.41	0	0.52	0.43	0.20	0.06	0.02	—	—	—	—	—	—	—	—	—	—	—
2.5	−.05	0.50	0.45	0.22	0.07	0.02	0.01	—	—	—	—	—	—	—	—	—	—
3.0	−.26	0.34	0.49	0.31	0.13	0.04	0.01	—	—	—	—	—	—	—	—	—	—
4.0	−.40	−.07	0.36	0.43	0.28	0.13	0.05	0.02	—	—	—	—	—	—	—	—	—
5.0	−.18	−.33	0.05	0.36	0.39	0.26	0.13	0.05	0.02	—	—	—	—	—	—	—	—
5.53	0	−.34	−.13	0.25	0.40	0.32	0.19	0.09	0.03	0.01	—	—	—	—	—	—	—
6.0	0.15	−.28	−.24	0.11	0.36	0.36	0.25	0.13	0.06	0.02	—	—	—	—	—	—	—
7.0	0.30	0.00	−.30	−.17	0.16	0.35	0.34	0.23	0.13	0.06	0.02	—	—	—	—	—	—
8.0	0.17	0.23	−.11	−.29	−.10	0.19	0.34	0.32	0.22	0.13	0.06	0.03	—	—	—	—	—
8.65	0	0.27	0.06	−.24	−.23	0.03	0.26	0.34	0.28	0.18	0.10	0.05	0.02	—	—	—	—
9.0	−.09	0.25	0.14	−.18	−.27	−.06	0.20	0.33	0.31	0.21	0.12	0.06	0.03	0.01	—	—	—
10.0	−.25	0.04	0.25	0.06	−.22	−.23	−.01	0.22	0.32	0.29	0.21	0.12	0.06	0.03	0.01	—	—
12.0	0.05	−.22	−.08	0.20	0.18	−.07	−.24	−.17	0.05	0.23	0.30	0.27	0.20	0.12	0.07	0.03	0.01

<u>Table: 2.1</u>

The Spectrum of FM signals for three different values of β are shown in the fig-2.6. In this spectrum the amplitude of the carrier component is kept as a unity constant. The variation in the amplitudes of all the frequency components is indicated.

For β = 1, the amplitude of the carrier component is more than the side band frequencies as shown in fig-2.6a. The amplitude level of the side band frequencies is decreasing. The dominant components are ($f_c \pm f_m$) and ($f_c \pm 2f_m$). The amplitude of the frequency components ($f_c \pm nf_m$) for n>2 are negligible.

For β = 2, the amplitude of the carrier component is considered as unity. The spectrum is shown in fig-2.6b. The amplitude level of the side band frequencies is varying. The amplitude levels of the components ($f_c \pm f_m$) and ($f_c \pm 2f_m$) are more than carrier frequency component; whereas the amplitude of the component ($f_c \pm 3f_m$) is lower than the carrier amplitude. The amplitude of frequency components ($f_c \pm nf_m$) for n>3 are negligible.

The spectrum for β = 5, is shown in fig-2.6c. The amplitude of the carrier component is considered as unity. The amplitude level of the side band frequencies is varying. The amplitude levels of the components ($f_c \pm f_m$), ($f_c \pm 3f_m$), ($f_c \pm 4f_m$) and ($f_c \pm 5f_m$), are more than carrier frequency component; whereas the amplitude of the component ($f_c \pm 2f_m$) is lower than the carrier amplitude. The amplitude of frequency components ($f_c \pm nf_m$) for n>8 are negligible.

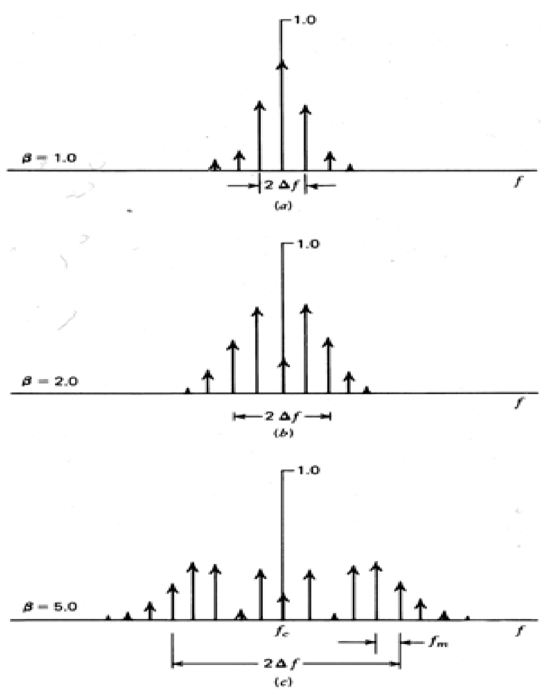


Fig: 2.6 – Plots of Spectrum for different values of modulation index.
(Amplitude of carrier component is constant at unity)

Example-2.4:

An FM transmitter has a power output of 10 W. If the index of modulation is 1.0, determine the power in the various frequency components of the signal.

Solution: The various frequency components of the FM signal are

f_c, ($f_c \pm f_m$), ($f_c \pm 2f_m$), ($f_c \pm 3f_m$), and so on.

The power associated with the above frequency components are: (Refer (2.21))

$(J_0)^2$, $(J_1)^2$, $(J_2)^2$, and $(J_3)^2$ respectively.

From the Bessel function Table, for $\beta = 1$;

$J_0 = 0.77$, $J_1 = 0.44$, $J_2 = 0.11$, and $J_3 = 0.02$

Let $P = 0.5(A_c)^2 = 10$ W.

Power associated with f_c component is $P_0 = P\,(J_0)^2 = 10\,(0.77)^2 = 2.929$ W.

Similarly, $P_1 = P\,(J_1)^2 = 10\,(0.44)^2 = 1.936$ W.

$P_2 = P\,(J_2)^2 = 10\,(0.11)^2 = 0.121$ W.

$P_3 = P\,(J_3)^2 = 10\,(0.02)^2 = 0.004$ W.

Note: Total power in the FM wave,

$$P_{total} = P_0 + 2P_1 + 2P_2 + 2P_3$$

$$= 2.929 + 2(1.936) + 2(.121) + 2(.004) = 10.051 \text{ W}$$

Example-2.5:

A 100 MHz un-modulated carrier delivers 100 Watts of power to a load. The carrier is frequency modulated by a 2 kHz modulating signal causing a maximum frequency deviation of 8 kHz. This FM signal is coupled to a load through an ideal Band Pass filter with 100MHz as center frequency and a variable bandwidth. Determine the power delivered to the load when the filter bandwidth is:

(a) 2.2 kHz (b) 10.5 kHz (c) 15 kHz (d) 21 kHz

Ans: Modulation index**,** $\beta = 8\text{ k} / 2\text{ k} = 4$;

From the Bessel function Table- 2.1; for $\beta = 4$;

$J_0 = -0.4$, $J_1 = -0.07$, $J_2 = 0.36$, $J_3 = 0.43$, $J_4 = 0.28$, $J_5 = 0.13$, $J_6 = 0.05$, $J_7 = 0.02$

Let $P = 0.5(A_c)^2 = 100$ W and

$P_0 = P\,(J_0)^2 = 100\,(-0.4)^{2} = 16$ Watts.

$P_1 = P\,(J_1)^2 = 100\,(-0.07)^2 = 0.490$ W.

$P_2 = P\,(J_2)^2 = 100\,(0.36)^2 = 12.960$ W.

$P_3 = P\,(J_3)^2 = 100\,(0.43)^2 = 18.490$ W.

$P_4 = P\,(J_4)^2 = 100\,(0.28)^2 = 7.840$ W.

$P_5 = P\,(J_5)^2 = 100\,(0.13)^2 = 1.690$ W.

$P_6 = P\,(J_6)^2 = 100\,(0.05)^2 = 0.250$ W.

(a) Filter Bandwidth = 2.2 kHz

The output of band pass filter will contain only one frequency component f_c.

Power delivered to the load, $P_d = P_0 = 16$ Watts.

(b) Filter Bandwidth = 10.5 kHz

The output of band pass filter will contain the following frequency components:

f_c, $(f_c \pm f_m)$, and $(f_c \pm 2f_m)$

Power delivered to the load, $P_d = P_0 + 2P_1 + 2P_2 = 42.9$ Watts.

(c) Filter Bandwidth = 15 kHz

The output of band pass filter will contain the following frequency components:

f_c, $(f_c \pm f_m)$, $(f_c \pm 2f_m)$, and $(f_c \pm 3f_m)$,

Power delivered to the load, $P_d = P_0 + 2P_1 + 2P_2 + 2P_3 = 79.9$ Watts.

(d) Filter Bandwidth = 21 kHz

The output of band pass filter will contain the following frequency components:

f_c, $(f_c \pm f_m)$, $(f_c \pm 2f_m)$, $(f_c \pm 3f_m)$, $(f_c \pm 4f_m)$, and $(f_c \pm 5f_m)$,

Power delivered to the load, $P_d = P_0 + 2P_1 + 2P_2 + 2P_3 + 2P_4 + 2P_5 = 98.94$ Watts.

Example-2.6:

A carrier wave is frequency modulated using a sinusoidal signal of frequency f_m and amplitude A_m. In a certain experiment conducted with f_m=1 kHz and increasing A_m, starting from zero, it is found that the carrier component of the FM wave is reduced to

zero for the first time when A_m=2 volts. What is the frequency sensitivity of the modulator? What is the value of A_m for which the carrier component is reduced to zero for the second time?

Ans: The carrier component will be zero when its coefficient, $J_0(\beta)$ is zero.

From Table 2.1: $J_0(x) = 0$ for x= 2.44, 2.53, 8.62.

$\beta = \Delta f / f_m = k_f A_m / f_m$ and $k_f = \beta f_m / A_m$ = (2.40)(1000) / 2 = 1.22 kHz/V

Frequency Sensitivity, k_f = 1.22 kHz/V

The carrier component will become zero for second time when β = 2.53.

Therefore, $A_m = \beta f_m / k_f$ = 2.53 (1000) / 1220 = 4.53 volts

2.5 Transmission Bandwidth of FM waves:

An FM wave consists of infinite number of side bands so that the bandwidth is theoretically infinite. But, in practice, the FM wave is effectively limited to a finite number of side band frequencies compatible with a small amount of distortion. There are many ways to find the bandwidth of the FM wave.

1. Carson's Rule: In single–tone modulation, for the smaller values of modulation index the bandwidth is approximated as $2f_m$. For the higher values of modulation index, the bandwidth is considered as slightly greater than the total deviation 2Δf. Thus the Bandwidth for sinusoidal modulation is defined as:

$B_T \cong 2\Delta f + 2 f_m = 2\Delta$

$= 2(\beta + 1) f_m$

$$B_T \cong 2\Delta f + 2 f_m = 2\Delta f\left(1 + \frac{1}{\beta}\right) = 2(\beta + 1) f_m \qquad (2.24)$$

For non-sinusoidal modulation, a factor called Deviation ratio (D) is considered. The deviation ratio is defined as the ratio of maximum frequency deviation to the bandwidth of message signal.

Deviation ratio , D = (Δf / W), where W is the bandwidth of the message signal and the corresponding bandwidth of the FM signal is,

$$B_T = 2(D + 1) W \qquad \text{... (2.25)}$$

2. Universal Curve : An accurate method of bandwidth assessment is done by retaining the maximum number of significant side frequencies with amplitudes greater than 1% of the un-modulated carrier wave. Thus the bandwidth is defined as "*the 99 percent bandwidth of an FM wave as the separation between the two frequencies beyond which none of the side-band frequencies is greater than 1% of the carrier amplitude obtained when the modulation is removed".*

Transmission Bandwidth - $\mathbf{BW = 2\, n_{max}\, f_m}$, (2.26)

where $\mathbf{f_m}$ is the modulation frequency and 'n' is the number of pairs of side-frequencies such that $|J_n(\beta)| > \mathbf{0.01}$. The value of $\mathbf{n_{max}}$ varies with modulation index and can be determined from the Bessel coefficients. The table 2.2 shows the number of significant side frequencies for different values of modulation index.

The transmission bandwidth calculated using this method can be expressed in the form of a universal curve which is normalised with respect to the frequency deviation and plotted it versus the modulation index. (Refer fig-2.7).

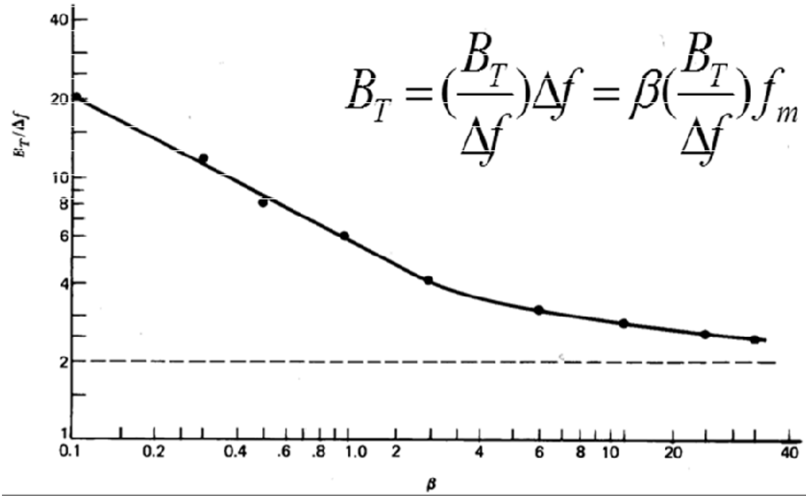
Table 2.2

Number of Significant Side Frequencies of a Wide-band FM Signal for Varying Modulation Index

Modulation Index β	Number of Significant Side Frequencies $2n_{max}$
0.1	2
0.3	4
0.5	4
1.0	6
2.0	8
5.0	16
10.0	28
20.0	50
30.0	70

From the universal curve, for a given message signal frequency and modulation index the ratio (B/ Δf) is obtained from the curve. Then the bandwidth is calculated as:

$$B_T = \left(\frac{B_T}{\Delta f}\right)\Delta f = \beta\left(\frac{B_T}{\Delta f}\right) f_m \qquad \ldots(2.27)$$



Universal Curve for evaluating the bandwidth of FM wave

Fig: 2.7 – Universal Curve

Example-2.7:

Find the bandwidth of a single tone modulated FM signal described by

$S(t)=10 \ \cos[2\pi10^8t + 6 \sin(2\pi10^3t)]$.

Solution: Comparing the given s(t) with equation-(2.12) we get

Modulation index, $\beta = 6$ and Message signal frequency, $f_m = 1000$ Hz.

By *Carson's rule* (equation - 2.24),

Transmission Bandwidth, $B_T = 2(\beta + 1) f_m$

$$B_T = 2(7)1000 = 14000 \text{ Hz} = 14 \text{ kHz}$$

Example-2.8:

Q. A carrier wave of frequency 91 MHz is frequency modulated by a sine wave of amplitude 10 Volts and 15 kHz. The frequency sensitivity of the modulator is 3 kHz/V.
(a) Determine the approximate bandwidth of FM wave using Carson's Rule.
(b) Repeat part (a), assuming that the amplitude of the modulating wave is doubled.
(c) Repeat part (a), assuming that the frequency of the modulating wave is doubled.

Solution: (a) Modulation Index, $\beta = \Delta f / f_m = k_f A_m / f_m = 3x10/15 = 2$

By Carson's rule; Bandwidth, $B_T = 2(\beta + 1)\, f_m = 90$ kHz

(b) When the amplitude, Am is doubled,

New Modulation Index, $\beta = \Delta f / f_m = k_f A_m / f_m = 3x20/15 = 4$

Bandwidth, $B_T = 2(\beta+1)f_m = 150$ kHz

(c) when the frequency of the message signal, fm is doubled

New Modulation Index, $\beta = 3x10/30 = 1$

Bandwidth, $B_T = 2(\beta+1)f_m = 120$ kHz.

Example-2.9:

Q. Determine the bandwidth of an FM signal, if the maximum value of the frequency deviation Δf is fixed at 75kHz for commercial FM broadcasting by radio and modulation frequency is W= 15 kHz.

Solution: Frequency deviation, $D = (\Delta f / W) = 5$

Transmission Bandwidth, $B_T = 2(D + 1)\, W = 12x15$ kHz = 180 kHz

Example-2.10:

Q. Consider an FM signal obtained from a modulating signal frequency of 2000 Hz and maximum Amplitude of 5 volts. The frequency sensitivity of modulator is 2 kHz/V. Find the bandwidth of the FM signal considering only the significant side band frequencies.

Solution: Frequency Deviation = 10 kHz

Modulation Index, $\beta = \Delta f / f_m = k_f A_m / f_m = 5$;

From table –(2.2) ; $2n_{max} = 16$ for $\beta = 5$,

Bandwidth, $B_T = 2\, n_{max}\, f_m = 16x2$ kHz = 32 kHz.

Example-2.11: A carrier wave of frequency 91 MHz is frequency modulated by a sine wave of amplitude 10 Volts and 15 kHz. The frequency sensitivity of the modulator is 3 kHz/V. Determine the bandwidth by transmitting only those side frequencies with amplitudes that exceed 1% of the unmodulated carrier wave amplitude. Use *universal curve* for this calculation.

Solution:

Frequency Deviation, Δf = 30 kHz

Modulation Index, β = 3x10/15 = 2

From the *Universal curve*; for $\beta = 2$; $(B / \Delta f) = 4.3$

Bandwidth, $B = 4.3\ \Delta f = 129$ kHz

2.6 Generation of FM Waves:

There are two basic methods of generating FM waves: *indirect method and direct method*. In indirect method a NBFM wave is generated first and frequency multiplication is next used to increase the frequency deviation to the desired level. In direct method, the carrier frequency is directly varied in accordance with the message signal. To understand the indirect method it is required to know the generation of NBFM waves and the working of frequency multipliers.

Generation of WBFM using Direct Method:

In direct method of FM generation, the instantaneous frequency of the carrier wave is directly varied in accordance with the message signal by means of an voltage controlled oscillator. The frequency determining network in the oscillator is chosen with high quality factor (Q-factor) and the oscillator is controlled by the incremental variation of the reactive components in the tank circuit of the oscillator. A *Hartley Oscillator* can be used for this purpose.

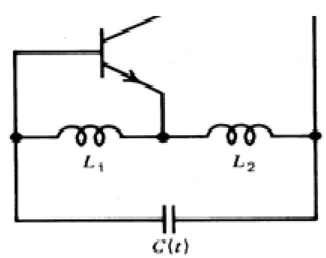


Fig: 2.11 – Hartley Oscillator (tank circuit) for generation of WBFM wave.

The portion of the tank circuit in the oscillator is shown in fig:2.11. The capacitive component of the tank circuit consists of a fixed capacitor shunted by a voltage-variable capacitor. The resulting capacitance is represented by C(t) in the figure. The voltage variable capacitor commonly called as varactor or varicap, is one whose capacitance depends on the voltage applied across its electrodes. The *varactor diode* in the reverse bias condition can be used as a voltage variable capacitor. The larger the voltage applied across the diode, the smaller the transition capacitance of the diode.

The frequency of oscillation of the Hartley oscillator is given by:

$$f_i(t) = \frac{1}{2\pi\sqrt{(L_1 + L_2)c(t)}} \qquad ...(2.30)$$

Where the L_1 and L_2 are the inductances in the tank circuit and the total capacitance, c(t) is the fixed capacitor and voltage variable capacitor and given by:

$$c(t) = c_0 + \Delta c\cos(2\pi f_m t) \qquad ...(2.31)$$

Let the un-modulated frequency of oscillation be f_0. The instantaneous frequency $f_i(t)$ is defined as:

$$f_i(t) = f_0\left[1 + \frac{\Delta c}{c_0}\cos(2\pi f_m t)\right]^{-\frac{1}{2}} \qquad ...(2.32)$$

$$\text{where } f_0 = \frac{1}{2\pi\sqrt{(L_1 + L_2)c_0}} \qquad ...($$

$$\therefore f_i(t) = f_0\left[1 + \frac{\Delta c}{c_0}\cos(2\pi f_m t)\right]^{-\frac{1}{2}}$$

$$\cong f_0\left[1 - \frac{\Delta c}{2c_0}\cos(2\pi f_m t)\right]$$

Thus the instantaneous frequency $f_i(t)$ is defined as:

$$\therefore f_i(t) \cong f_0 + \Delta f\cos(2\pi f\ t) \qquad ...(2.34)$$

The term, Δf represents the frequency deviation and the relation with Δc is given by:

$$\left(\frac{\Delta c}{2c_0} = -\frac{\Delta f}{f_0}\right) \qquad ... \qquad (2.35)$$

Thus the output of the oscillator will be an FM wave. But the direct method of generation has the disadvantage that the carrier frequency will not be stable as it is not generated from a highly stable oscillator.

Generally, in FM transmitter the frequency stability of the modulator is achieved by the use of an auxiliary stabilization circuit as shown in the fig.(2.12).

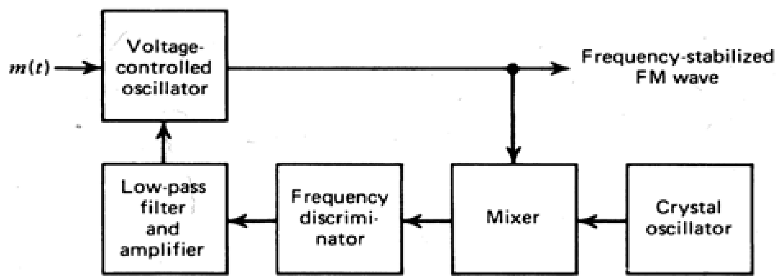


Fig: 2.12 – Frequency stabilized FM modulator.

The output of the FM generator is applied to a mixer together with the output of crystal controlled oscillator and the difference is obtained. The mixer output is applied to a frequency discriminator, which gives an output voltage proportional to the instantaneous frequency of the FM wave applied to its input. The discriminator is filtered by a low pass filter and then amplified to provide a dc voltage. This dc voltage is applied to a voltage controlled oscillator (VCO) to modify the frequency of the oscillator of the FM generator. The deviations in the transmitter carrier frequency from its assigned value will cause a change in the dc voltage in a way such that it restores the carrier frequency to its required value.

2.7 Advantages and disadvantages of FM over AM:

Advantages of FM over AM are:

1. Less radiated power.
2. Low distortion due to improved signal to noise ratio (about 25dB) w.r.t. to man made interference.
3. Smaller geographical interference between neighbouring stations.
4. Well defined service areas for given transmitter power.

Disadvantages of FM:

1. Much more Bandwidth (as much as 20 times as much).
2. More complicated receiver and transmitter.

2.8 Applications:

Some of the applications of the FM modulation are listed below:

I. FM Radio,(88-108 MHz band, 75 kHz,)

II. TV sound broadcast, 25 kHz,

III. 2-way mobile radio, 5 kHz / 2.5 kHz.

Additional Examples:

Example 2.13: An FM wave is defined below.

$S(t) = 12 \sin(6x10^8 \pi t + 5 \sin 1250 \pi t)$

Find the carrier and modulating frequencies, the modulating index, and the maximum deviation of the FM wave. Also find the bandwidth of the FM wave. What power will the FM wave dissipate in a 10 ohm resistor?

Solution: From equation 2.12, we have

$$s(t) = A_c \cos\left[2\pi f_c t + \beta \sin(2\pi f_m t)\right]$$

Comparing with the given FM wave,

Carrier frequency = $3x10^8$ Hz = 300 MHz

Modulating signal frequency, f_m = 625 Hz

Modulation Index, β = 5 ;

Maximum frequency deviation, $\Delta f = \beta f_m$ = 3125 Hz.

Using Carson's rule, Bandwidth = 2(3125 + 625) = 7500 Hz

Power dissipated across resistor = P,

$$P = \frac{(A_c)^2}{2R} = \frac{144}{20} = 7.2W$$

Example 2.14: Consider an FM signal with :

Δf = 10 kHz, f_m = 10 kHz, A_c = 10 V, f_c = 500 kHz

Compute and draw the spectrum for FM signal.

Solution:

Modulation index**,** β = 10 k / 10 k = 1;

From Bessel function Table- 2.1;

for β = 1; the coefficients are $J_0 = 0.77$, $J_1 = 0.44$, $J_2 = 0.11$, $J_3 = 0.02$.

The spectrum is defined as

$$S(f) = \frac{A_c}{2} \sum_{n=-\infty}^{\infty} J_n \left[\delta\left(f - f_c - nf_m\right) + \delta\left(f + f_c + nf_m\right) \right]$$

The single sided spectrum is shown in fig:Ex-2.14.

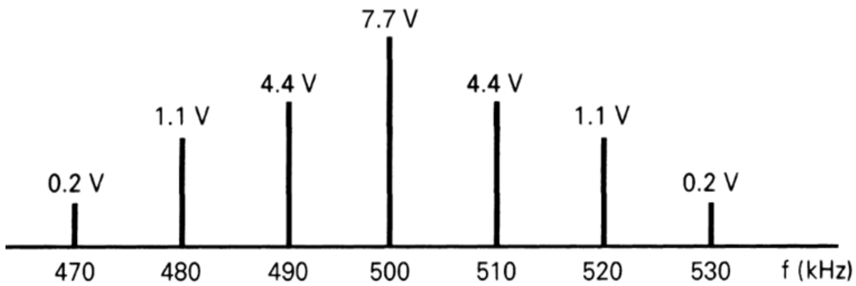


Fig: Ex-2.14 – Frequency Spectrum (for example 2.14)

2.9 Demodulation of FM waves:

Frequency demodulation is the process that enables us to recover the original modulating signal from a frequency modulated signal. Frequency Demodulator produces an output signal with amplitude directly proportional to the instantaneous frequency of FM wave.

Frequency demodulators are broadly classified into two categories:

(i) Direct method – examples: frequency discriminators and zero crossing detectors.

(ii) Indirect method – example: phase locked loop.

The direct methods use the direct application of the definition of instantaneous frequency.

The indirect method depends on the use of feed back to track variations in the instantaneous frequency of the input signal.

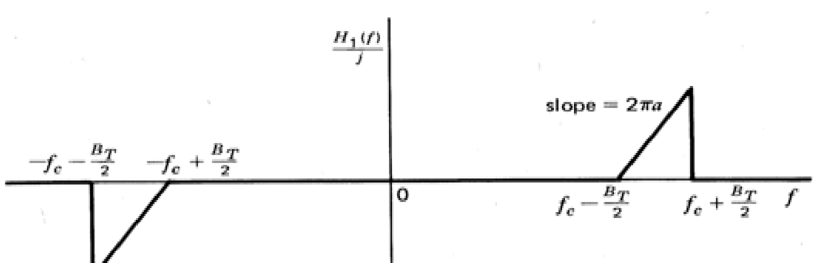
Balanced Frequency Discriminator: The frequency discriminator consists of slope circuits and envelope detectors. An ideal slope circuit is characterized by the transfer function that is purely imaginary, varying linearly with frequency inside a prescribed frequency interval. The transfer function defined by the equation 2.36 and is shown in figure 2.15a.

$$s(t) = A_c \cos\left[2\pi f_c t + 2\pi k_f \int_0^t m(\tau)d\tau\right], \quad \text{where } f_i(t) = f_c + k_f m(t) \qquad (2.36$$

Let the slope circuit be simply differentiator:

$$s_1(t) = -A_c\left[2\pi f_c + 2\pi k_f m(t)\right]\sin\left[2\pi f_c t + 2\pi k_f \int_0^t m(\tau)d\tau\right]$$

$$s_o(t) \approx -A_c\left[2\pi f_c + 2\pi k_f m(t)\right]$$



(a)

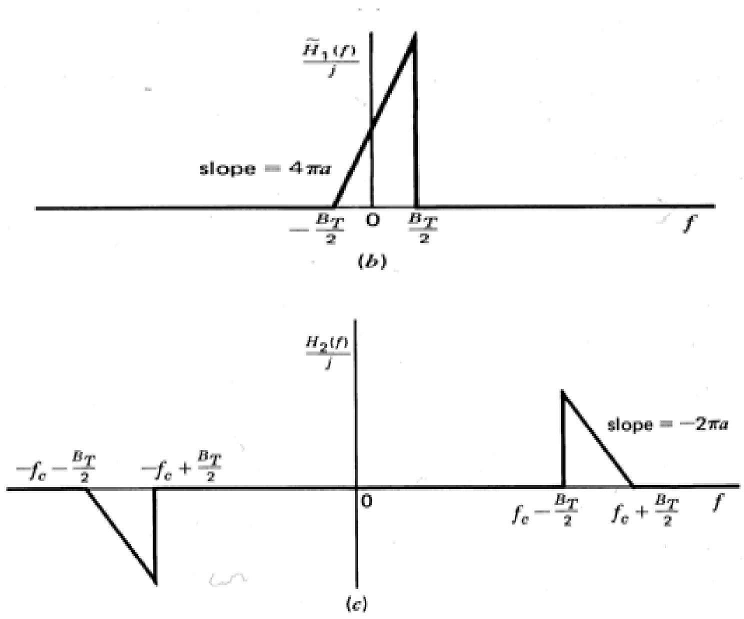


Fig:2.15 – (a) Frequency response of ideal slope circuit, $H_1(f)$.

(b) Frequency response of complex low pass filter equivalent

(c) Frequency response of ideal slope circuit complementary to part(a).

Consider an FM signal s(t) having spectrum from ($f_c - B_T/2$) to ($f_c + B_T/2$) and zero outside this range. Let $s_1(t)$ be the output of the slope circuit. Any slope circuit can be considered as an equivalent low pass filter driven with the complex envelope of input, FM wave.

Let $\tilde{H}_1(f)$ is complex transfer function of the slope circuit. This function is related to $H_1(f)$ by

$$\tilde{H}_1(f - f_c) = H_1(f),\ for\ f > 0 \quad ...(2.37)$$

Using equations 2.37 and 2.38, we get

$$\tilde{H}_1(f) = \begin{cases} j2\pi a\left(f + \frac{B_T}{2}\right) & , \quad -\frac{B_T}{2} \le f \le \frac{B_T}{2} \\ 0 & , \quad \text{otherwise} \end{cases} \quad ...(2.38)$$

This is depicted in the figure 2.15b.

Let s(t) be the FM wave, defined in equation(2.36) and its complex envelope be $\tilde{s}(t)$ given by:

complex envelope of *s(t)*

$$\tilde{s}(t) = A_c \exp\left[j2\pi k_f \int_0^t m(t)dt \right] \qquad (2.39)$$

Let $\tilde{s}_1(t)$ be the complex envelope of the response of the slope circuit defined by fig-2.a.

The Fourier transform of $\tilde{s}_1(t)$ is

$$\therefore \tilde{S}_1(f) = \tilde{H}_1(f)\tilde{S}(f)$$

$$= \begin{cases} j2\pi a\left(f + \frac{B_T}{2}\right) \tilde{S}(f) \,, & -\frac{B_T}{2} \le f \le \frac{B_T}{2} \\ 0 \,, & \text{otherwise} \end{cases} \qquad (2.40)$$

Using the differentiation in time domain property of Fourier transform;

$$\therefore \tilde{s}_1(t) = a\left[\frac{d\tilde{s}(t)}{dt} + j\pi B_T \tilde{s}(t) \right]$$

$$= j\pi B_T a A_c \left[1 + \frac{2k_f}{B_T} m(t) \right] \exp\left[j2\pi k_f \int_0^t m(t)dt \right] \qquad (2.41)$$

Therefore the response of the slope circuit is defined as :

$$s_1(t) = \mathrm{Re}\left[\tilde{s}_1(t)\exp(j2\pi f_c t)\right]$$

$$= \pi B_T a A_c \left[1 + \frac{2k_f}{B_T} m(t) \right] \cos\left[2\pi f_c t + 2\pi k_f \int_0^t m(t)dt + \frac{\pi}{2} \right] \qquad (2.42)$$

The signal, $s_1(t)$ is a hybrid modulated wave in which both the amplitude and frequency of the carrier wave vary the message signal, m(t).

If $\left| \frac{2k_f}{B_T} m(t) \right| < 1$ for all t, resulting output of envelope detector is

$$|\tilde{s}_1(t)| = \pi B_T a A_c \left[1 + (\frac{k_f}{B_T} m^2 t) \right] \qquad (2.43)$$

The bias term ($\pi B_T\, a\, A_c$) is proportional to the slope 'a' of the transfer function of the slope circuit. The bias may be removed by subtracting from the envelope detector output from the output of a second envelope detector preceded by a complementary slope circuit with a transfer function $H_2(f)$ as described in fig:6.4c.

$$\tilde{H}_2(f) = -\tilde{H}_1(-f) \qquad (2.44)$$

Let $s_2(t)$ be the output of the complementary slope circuit produced by the incoming FM wave s(t). The envelope of the circuit is

$$|\tilde{s}_2(t)| = \pi B_T a A_c \left[1 - \frac{2k_f}{B_T} m(t)\right] \qquad (2.45)$$

The difference between the two envelopes in equations is

$$s_0(t) = |\tilde{s}_1(t)| - |\tilde{s}_2(t)| = 4\pi k_f a A_c m(t) \qquad (2.46)$$

Thus an ideal frequency discriminator can be modelled as a pair of slope circuits followed by envelope detectors and a summer as shown in fig-6.5 which is called balanced frequency discriminator.

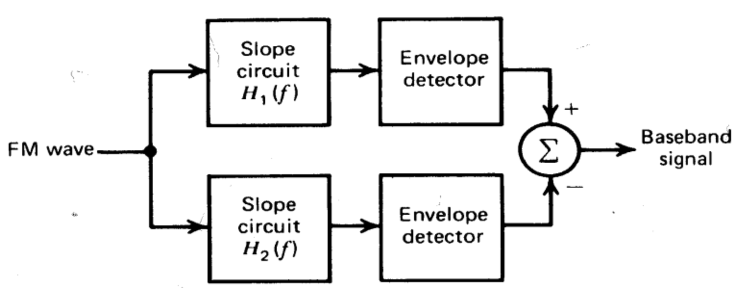


Fig: 2.16 –Idealized model of Balanced frequency discriminator.

The idealized model can be closely realized using the circuit shown in fig-2.16 which consists of two resonant circuits. The upper and lower resonant filters are tuned to frequencies above and below the un-modulated carrier frequency, f_c. The amplitude responses of the tuned filters and the total response are shown in the fig-2.18.

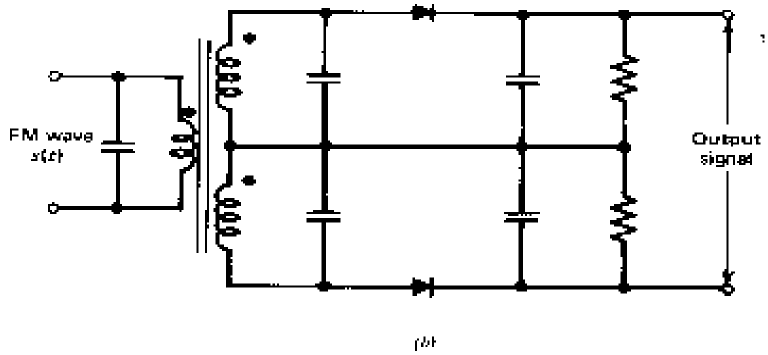


Fig: 2.17 –Balanced frequency discriminator.

The overall performance of the balanced frequency discriminator will be good only when both the filters have high Q-factor and a proper frequency separation between the tuning frequencies of the two filters. However there will be distortions present in the output due to the following factors:

1. Spectrum of FM wave is not exactly zero for the frequencies outside the range.
2. Tuned filters are not strictly band-limited
3. RC filters in the envelope detector introduce distortions
4. Tuned filter characteristics are not linear over the whole frequency band.

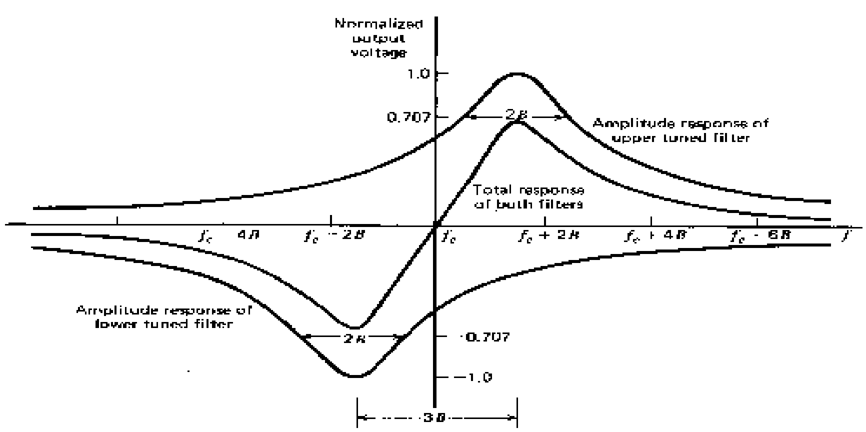


Fig: 2.18 – Frequency response.

2.10 Phase-Locked Loop (PLL):

The PLL is a negative feedback system that consists of three major components: a multiplier, a loop filter and a voltage controlled oscillator (VCO) connected in the form of a loop as shown in the fig-2.19.The VCO is a sine wave generator whose frequency is determined by the voltage applied to it.

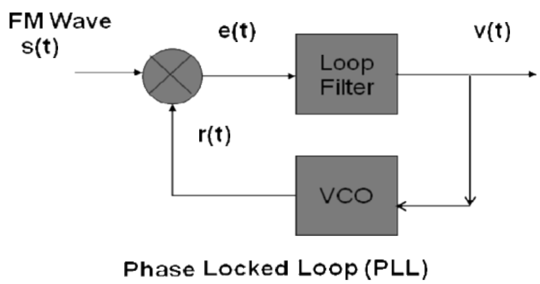


Fig: 2.19 – Phase Locked Loop

Initially the VCO is adjusted such that control voltage is zero, two conditions are satisfied:

1. Frequency of VCO is set precisely at the carrier frequency, fc and
2. VCO output has a 90^o phase shift w.r.t. to the un-modulated carrier wave.

Let the input signal applied to the PLL be s(t):

$$s(t) = A_c \sin[2\ f_c t +\ _1(t)]$$

$$where\ \ \phi_1\ t = \pi k_f \int_0^t m\ t\ dt\ (\ )\ 2\ (\ )\ . \qquad \ldots\ (2.47)$$

Let the VCO output be r(t):

$$r(t) = A_v \cos[2\ f_c t +\ _2(t)]\ . \qquad \ldots(2.48)$$

$$where\ \ \phi_2(t) = 2\pi k_v \int_0^t v(t)dt \qquad \ldots.(2.49)$$

where k_v is the frequency sensitivity of VCO in Hz/volt.

Incoming FM signal and VCO output are applied to the multiplier. The Multiplier output

$e(t) = 2\, k_m\, s(t)\, r(t)$, produces two components: (where k_m – multiplier gain in volt^{-1})

1. A high frequency component:

$$k_m A_c A_v \sin[\, 4\pi f_c t + \phi_1(t) + \phi_2(t)\,]$$

2. A low frequency component:

$$k_m A_c A_v \sin[\, \phi_1(t) - \phi_2(t)\,]$$

The high frequency component is eliminated by the loop filter (low pass type) and VCO.

Then the input to the Loop filter is:

$$e(t) = k_m A_c A_v \sin[\, \phi_1(t) - \phi_2(t)\,] \qquad (2.50)$$

$$e(t) = k_m A_c A_v \sin \quad _e(t)] \; [\qquad ....(2.51)$$

$$\text{The Phase error : } \phi_e(t) = \phi_1(t) - \phi_2(t)$$
$$= \phi_1(t) - 2\pi k_v \int_0^t v(t)\,dt \qquad ....(2.52)$$

The loop filter operates on the input e(t) to produce the output: $v(t) = e(t) * h(t);$

where h(t) is impulse response of the loop filter.

$$v(t) = \int_{-\infty}^{\infty} e(\tau) h(t-\tau)\, d\tau \quad ...\ 6.21 \qquad (2.53)$$

Differentiating the equation 2.52; we get

$$\frac{d \; _e(t)}{dt} = \frac{d \; _1(t)}{dt} - 2\pi k_v\, v(t)$$

Using 2.53 and 2.50; and rearranging

$$\frac{d\phi_e(t)}{dt} = \frac{d\phi_1(t)}{dt} - 2\pi K_0 \int_{-\infty}^{\infty} \sin[\phi_e(\tau)] h(t-\tau) d\tau \quad ...(2.54)$$

where $K_0 = k_m k_v A_c A_v$: loop - gain parameter

The equation (2.54) represents a Non-linear model of a phase locked loop and is shown in fig-2.20. The nonlinear model resembles the basic PLL structure. The Multiplier is replaced by a subtractor and a sinusoidal nonlinearity, and the VCO by an integrator. But the nonlinear model is very difficult to analyze.

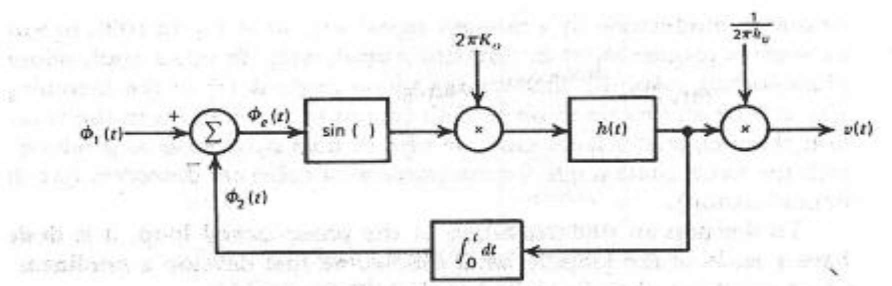


Fig: 2.20 – Non-linear model of a Phase Locked Loop

Linear Model of Phase-Locked Loop :

The nonlinear model is a complicated model and is difficult to analyze. Hence a simpler version of this model called a linear model is obtained.

An assumption that phase error, $\phi_e(t)$ is small leads to an approximation:

$$\sin[\phi_e(t)] = \phi_e(t) \quad ...... \quad (2.55)$$

and substituting in2.50; we get

$$e(t) = k_m A_c A_v \phi_e(t) \quad .... \quad (2.56)$$

Thus the nonlinear model in fig-2.20 can be written in a simplified form as shown in fig:2.21.

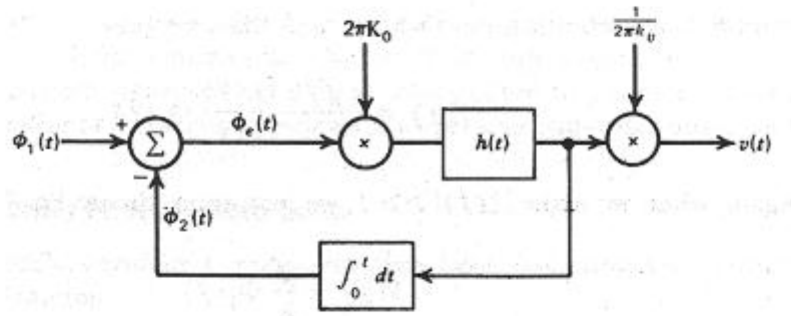


Fig: 2.21 –Linear model of a Phase Locked Loop

Substituting equation leads to

$$\frac{d\phi_e(t)}{dt} = \frac{d\phi_1(t)}{dt} - 2\pi K_0 \int_{-\infty}^{\infty} [\phi_e(\tau)] h(t-\tau)\, d\tau$$

$$\frac{d\phi_e(t)}{dt} + 2\pi K_0 [\phi_e(t) * h(t)] = \frac{d\phi_1(t)}{dt} \qquad ...(2.57)$$

Applying Fourier transform to equation 2.57;

$$j2\pi f\, \phi_e(f) + 2\pi K_0\, \phi_e(f)\, H(f) = j2\pi f\, \phi_1(f)$$

$$\Phi_e(f) = \frac{jf}{jf + K_0 H(f)} \Phi_1(f) \qquad ....(2.58)$$

Let *L(f)* be open-loop transfer function of PLL.

$$L(f) = K_0 \frac{H(f)}{jf} \qquad ......(2.59)$$

Then $\phi_e(f)$ can be written as

$$\Phi_e(f) = \frac{1}{1+L(f)} \Phi_1(f) \qquad ......(2.60)$$

The output of the linear model in fig 2.21, v(t) can be defined as

$$v(t) = \frac{K_0}{k_v}\Phi_e(t) * h(t)$$

Applying Fourier transform :

$$V(f) = \frac{K_0}{k_v}H(f)\,\Phi_e(f) = \frac{jf}{k_v}L(f)\Phi_e(f)$$

$$V(f) = \frac{jf}{k_v}\frac{L(f)}{1+L(f)}\Phi_1(f)$$

As the loop transfer function, *L(f)* approaches large value, $\phi_e(f)$ approaches zero. The phase of VCO becomes asymptotically equal to the phase of the incoming wave and *phase-lock* is established.

For $|L(f)| >> 1, \quad V(f) \cong \frac{jf}{k_v}\Phi_1(f)$

For $|L(f)| >> 1, \quad V(f) \cong \frac{jf}{k_v}\Phi_1(f)$

Applying inverse Fourier transform to the above equation:

$$v(t) \cong \frac{1}{2\ \ k_v}\frac{d\ \ _1(t)}{dt}$$

Substituting for $\phi_1(t)$

$$v(t) \cong \frac{k_f}{k_v}m(t)$$

Thus the output of the linearized model is proportional to the message signal.

2.11 FM Stereo Multiplexing:

Stereo multiplexing is a form of frequency division multiplexing designed to transmit two separate signals via the same carrier. It is widely used in the FM radio broadcasting to send two different elements of a program. For example the different elements can be sections of orchestra, a vocalist and an accompanist. This gives a spatial dimension to its perception for the listener at the receiving end.

The two important factors that influence the FM stereo transmission are:

1. The transmission has to operate within the allocated FM broadcast channels.
2. It has to compatible with the monophonic receivers.

The FM stereo transmitter consists of a multiplexing system. The block diagram of the multiplexer is shown in fig-2.22.

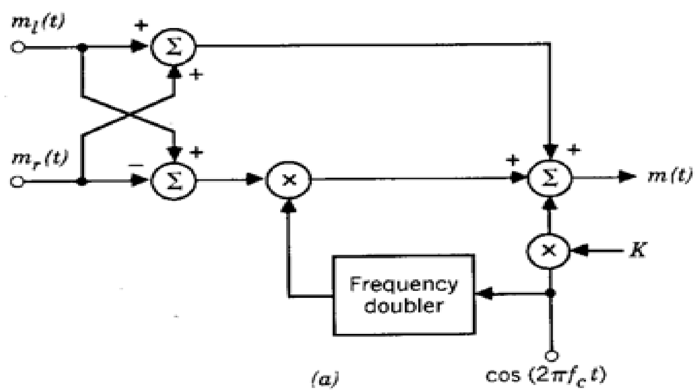


Fig: 2.22 –Multiplexer system in transmitter of FM stereo.

Let $m_l(t)$ and $m_r(t)$ denote the two signals from the two different microphones at the transmitter end of the system. They are applied to a matrixer that generates the sum signal and the difference signal. The sum signal $[m_l(t)+m_r(t)]$ is used in the base band form only. The difference signal $[m_l(t) - m_r(t)]$ along with a 38 kHz sub-carrier are applied to a product modulator to generate a DSBSC modulated wave. The sub- carrier is generated from a frequency doubler using 19 kHz oscillator. The three signals: sum signal, difference signal and a pilot carrier signal of frequency 19 kHz are combined/added to obtain the multiplexed signal. The multiplexed signal can be defined as:

$$m(t) = [\, m_l(t) + m_r(t) \,] + [\, m_l(t) - m_r(t) \,] \cos(4\pi f_c t) + K \cos(2\pi f_c t)$$

where f_c=19 kHz and K is a constant, chosen to maintain the pilot between 8% and 10% of the peak frequency deviation. The spectrum of the multiplexed signal is shown in the fig2.23.

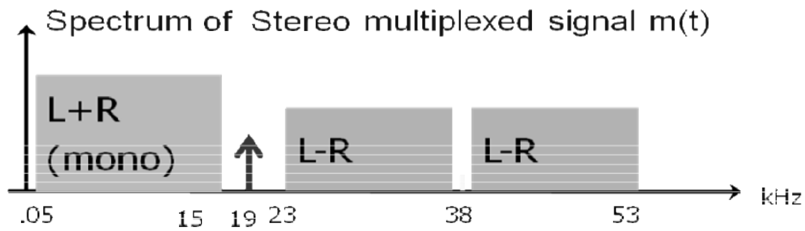


Fig: 2.23 - Spectrum of Stereo multiplexed signal, m(t).

The multiplexed signal is used as a modulating signal for the FM modulator to produce an FM signal for transmission.

The FM stereo receiver consists of demodulator and de-multiplexer. The de-multiplexer is shown in fig.2.24. The de-multiplexer consists of three filters: a low pass filter and two band pass filters. The base band low pass filter recovers the sum signal in the band 0-15 kHz, the band pass filter recovers the modulated difference signal in the band 23-53 kHz and a narrow band pass filter is used to recover the pilot carrier 19 kHz. The sub-carrier signal of frequency 38 kHz is obtained by frequency doubling the pilot carrier. The difference signal is then obtained from sub-carrier and modulated difference signal using a coherent detector. The matrixer (add/subtractor) reconstructs the left signal, $m_l(t)$ and the right signal, $m_r(t)$.

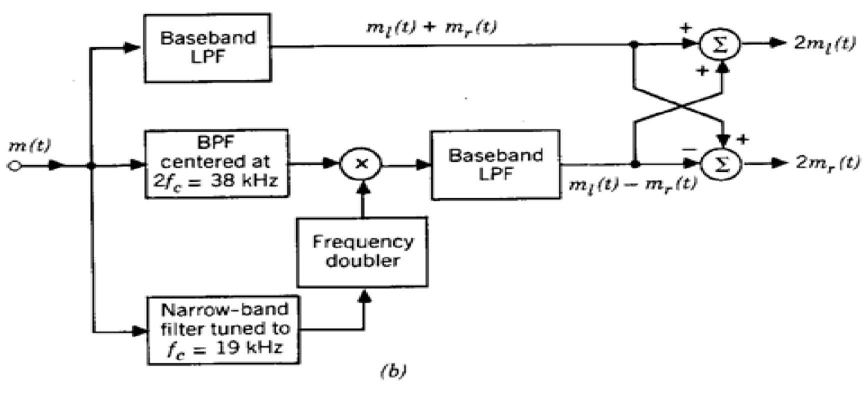


Fig: 2.24 : De-multiplexer system in receiver of FM stereo.

2.12 Super Heterodyne Receiver:

A receiver based on super heterodyne principle is shown in the fig.6.16. The RF amplifier boosts the signal strength of received weak signal. The mixer and local oscillator together performs the down conversion of FM signal to IF range. The discriminator circuit performs the demodulation process to detect the message signal.

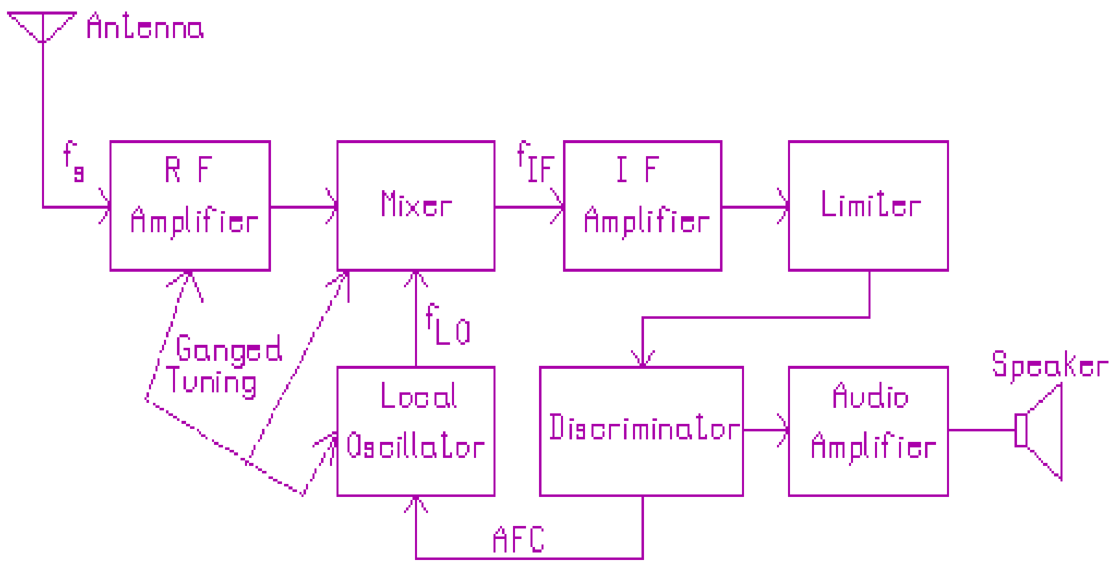


Fig: 2.25: A simple FM receiver

Amplitude Limiter: An FM wave gets distorted when it is transmitted through the channel. The FM wave at the input of the receiver will not have constant amplitude because of the channel imperfections. Hence at the receiver it is necessary to remove the amplitude variations prior to the demodulation. This is carried out by an amplitude limiter circuit in the receiver.

An amplitude limiter circuit consists of a hard limiter and a band pass filter. The hard limiter will convert distorted FM wave into an FM square wave. The band pass filter will block all the harmonics in FM square wave to obtain an undistorted FM wave.

De-emphasis: At the FM receiver, the signal after demodulation must be *de-emphasized* by a filter with complementary characteristics as the pre-emphasis filter to restore the relative amplitudes of the modulating signal. The de-emphasis network is a simple R-C network similar to a low pass filter.

2.13 Outcomes:

- Merits and demerits of Frequency modulation & How to use Narrow band FM and Wide band FM.
- Direct method of generation of FM.
- Balanced slope detector of detection of FM.
- The application of FM in FM stereo multiplexing are explained.

2.14 Further Readings

TEXT BOOK:

1. Communication Systems, Simon Haykins, 5th Edition, John Willey, India Pvt. Ltd, 2009

2. An Introduction to Analog and Digital Communication, Simon Haykins, John Wiley India Pvt. Ltd., 2008

REFERENCE BOOKS:

1. Modern digital and analog Communication systems B. P. Lathi, Oxford University Press., 4th ed, 2010

2. Communication Systems, Harold P.E, Stern Samy and A Mahmond, Pearson Edn, 2004.

3. Communication Systems: Singh and Sapre: Analog and digital TMH 2nd , Ed 2007

2.15 Recommended Questions

1. What do you understand by narrowband FM?
2. Define frequency modulation.
3. Define modulation index of frequency modulation.
4. What do you meant by multitone modulation?
5. Define phase modulation.
6. What are the types of Frequency Modulation?
7. What is the basic difference between an AM signal and a narrowband FM signal?
8. What are the two methods of producing an FM wave?
9. Compare WBFM and NBFM.
10. List the properties of the Bessel function.
11. Give the average power of an FM signal.
12. What is the use of crystal controlled oscillator?

MODULE-3

RANDOM VARIABLES & PROCESS: Introduction, Probability, Conditional Probability, Random variables, Several Random Variables. Statistical Averages: Function of a random variable, Moments, Random Processes, Mean, Correlation and Covariance function: Properties of autocorrelation function, Cross–correlation functions.

NOISE: Shot Noise, Thermal noise, White Noise, Noise Equivalent Bandwidth, Noise Figure.

3.1 Objectives

- Understand the concept of probability and different interpretations of probability.
- Develop their understanding of random processes particularly as they apply to communication systems.
- Understand the modeling of physical systems using the tools of multivariate random processes.
- Understand and characterize the output of linear systems excited by random processes
- Different types of noises and their classification are explained & The students will be to understand the effects of noise over a system.

3.2 Radom Variable

To define a random variable let us consider that

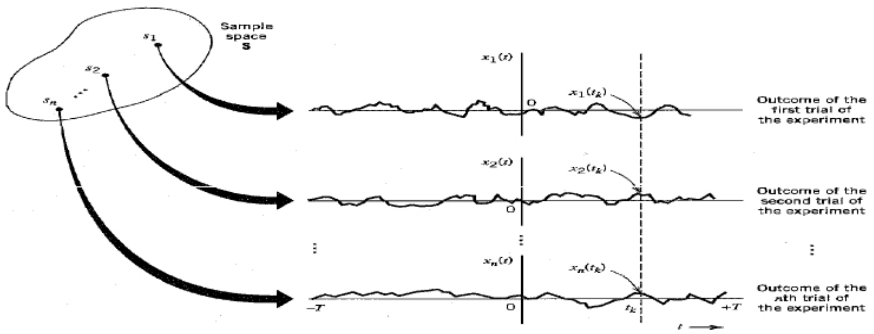


FIGURE 1.1 An ensemble of sample functions.

x1(t) is an outcome of experiment 1
x2(t) is the outcome of experiment 2
xn(t) is the outcome of experiment n

Each sample point in S is associated with a sample function x(t). X(t; s) is a random process is an ensemble of all time functions together with a probability rule. X(t; sj) is a realization or sample function of the random process.Probability rules assign probability to any meaningful event associated with an observation An observation is a sample function of the random process.

{x1(tk); x2(tk); :::; xn(tk)g = f X(tk; s1);X(tk; s2); :::;X(tk; sn)} ----(3.1)

X(tk; sj) constitutes a random variable. Outcome of an experiment mapped to a real number. An oscillator with a frequencyω0 with a tolerance of 1%.The oscillator can take values betweenω0 (1±0.01). Each realization of the oscillator can take any value between (ω0) (0.99) to (ω0) (1.01). The frequency of the oscillator can thus be characterized by a random variable.

3.3 Statistical Averages

Statistical averages are important in the measurement of quantities that are obscured by random variations. As an example consider the problem of measuring a voltage level with a noisy instrument. Suppose that the unknown voltage has value a and that the instrument has an uncertainty x. The observed value may be y = a + x. Suppose that n independent measurements are made under identical conditions, meaning that neither the unknown value of the voltage nor the statistics of the instrument noise

change during the process. Let us call the n measurements y_i, $1 \le i \le n$.

Under our model of the process, it must be the case that $y_i = a + x_i$.

Now form the quantity

$$(n) = 1/n \Sigma y_i \qquad \text{where } i=1\ldots n \qquad \text{---(3.2)}$$

This is the empirical average of the observed values. It is important to note that (n) is a random variable because it is a numerical value that is the outcome of a random experiment. That means that it will not have a single certain value. We expect to obtain a different value if we repeat the experiment and obtain n new measurements. We also expect that the result depends upon the value of n, and have the sense that larger values of n should give better results.

3.4 Types of Random Variables

There are two types of random variables. They are:

- Discrete Random Variable: An RV that can take on only a finite or countably infinite set of outcomes.
- Continuous Random Variable: An RV that can take on any value along a continuum (but may be reported "discretely").Random Variables are denoted by upper case letters (*Y*).Individual outcomes for RV are denoted by lower case letters(y).

Discrete Random Variable

Discrete Random Variable are the ones that takes on a countable number of values this means you

can sit down and list all possible outcomes without missing any, although it might take you an infinite amount of time.

X = values on the roll of two dice: X has to be either 2, 3, 4, …, or 12.

Y = number of accidents on the UTA campus during a week: Y has to be 0, 1, 2, 3, 4, 5, 6, 7, 8, ……………"real big number"

Probability Distribution:

Table, Graph, or Formula that describes values a random variable can take on, and its corresponding probability (discrete RV) or density (continuous RV).

1. Discrete Probability Distribution: Assigns probabilities (masses) to the individual outcomes
2. Continuous Probability Distribution: Assigns density at individual points, probability of ranges can be obtained by integrating density function
3. Discrete Probabilities denoted by: $p(y) = P(Y=y)$
4. Continuous Densities denoted by: $f(y)$
5. Cumulative Distribution Function: $F(y) = P(Y \leq y)$

For a discrete random variable, we have a probability mass function (pmf).The pmf looks like a bunch of spikes, and probabilities are represented by the heights of the spikes. For a continuous random variable, we have a probability density function (pdf). The pdf looks like a curve, and probabilities are represented by areas under the curve.

Continuous Random Variable

Continuous Random Variable is usually measurement data [time, weight, distance, etc] the one that takes on an uncountable number of values this means you can never list all possible outcomes even if you had an infinite amount of time.
X = time it takes you to drive home from class: X > 0, might be 30.1 minutes measured to the nearest tenth but in reality the actual time is 30.10000001………………… minutes?)

A continuous random variable has an infinite number of possible values &the probability of any one particular value is zero.

3.5 Random Process

A (one-dimensional) random process is a (scalar) function y(t), where t is usually time, for which the future evolution is not determined uniquely by any set of initial data| or at least by any set that is knowable to you and me. In other words, \random process" is just a fancy phrase that means \unpredictable function". Random processes y take on a continuum of values ranging over some interval, often but not always $-\infty$ to $+\infty$. The generalization to y's with discrete (e.g., integral) values is straightforward.

Examples of random processes are:

(i) the total energy E(t) in a cell of gas that is in contact with a heat bath; (ii) the temperature T(t) at the corner of Main Street and Center Street in Logan, Utah;

(iii) the earth-longitude _(t) of a specific oxygen molecule in the earth's atmosphere.

One can also deal with random processes that are vector or tensor functions of time. Ensembles of random processes. Since the precise time evolution of a random process is not predictable, if one wishes to make predictions one can do so only probabilistically. The foundation for probabilistic predictions is an ensemble of random processes |i.e., a collection of a huge number of random processes each of which behaves in its own, unpredictable way. The probability density function describes the general distribution of the magnitude of the random process, but it gives no information on the time or frequency content of the process. Ensemble averaging and Time averaging can be used to obtain the process properties

Ensemble averaging

Properties of the process are obtained by averaging over a collection or 'ensemble' of sample records using values at corresponding times

Time averaging

Properties are obtained by averaging over a single record in time

Stationary random processes:

A random process is said to be stationary if its statistical characterization is independent of the observation interval over which the process was initiated. Ensemble averages do not vary with time. An ensemble of random processes is said to be stationary if and only if its probability distributions p_n depend only on time differences, not on absolute time:

$$p_n(y_n; t_n + \tau; \ldots; y_2; t_2 + \tau; y_1; t_1 + \tau) = p_n(y_n; t_n; \ldots; y_2; t_2; y_1; t_1): \quad \text{---(3.3)}$$

If this property holds for the absolute probabilities p_n. Most stationary random processes can be treated as ergodic. A random process is ergodic if every member of the process carries with it the complete statistics of the whole process. Then its ensemble averages will equal appropriate time averages. Of necessity, an ergodic process must be stationary, but not all stationary processes are ergodic.

Non-stationary random processes:

Non-stationary random processes arise when one is studying a system whose evolution is influenced by some sort of clock that cares about absolute time. For example, the speeds v(t) of the oxygen molecules in downtown Logan, Utah make up an ensemble of random processes regulated in part by the rotation of the earth and the orbital motion of the earth around the sun; and the influence of these clocks makes v(t) be a non-stationary random process. By contrast, stationary random processes arise in the absence of any regulating clocks. An example is the speeds v (t) of oxygen molecules in a room kept at constant temperature.

Worked Examples

Problem 3.1

Write down the sample space for an experiment of tossing three coins simultaneously.

Solution:

If the three coins are tossed simultaneously then for each coin there can be either a HEAD (H) or TAIL(T) as an outcome. Therefore the sample space which consists of all the probable out comes is as follows:

S = {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}

Problem 3.2

In a simple experiment to rolling a Die, find the probability of event that an odd number is thrown.

Solution:

- Sample of the experiment, S = {1,2,3,4,5}
- The event that an odd number is thrown Ao={1,3,5}
- For an honest die, Every outcome 1,2,3,4,5,6 is equally likely and all the outcomes are mutually exclusive that means when any one is present others just cannot be present.
  There for probability of each outcome=P(1)=P(2)=….P(6)=1/6
  Now we have to find the probability of Ao
  P(Ao)= P(1) U P(3) U P(5)
  As all the outcomes are mutually exclusive,
  P(Ao) = P(1)+P(3)+P(5) = 3/6 = ½

3.6 NOISE

Classification of Noise

Noise may be put into following two categories.

1. **External noises**, i.e. noise whose sources are external.

   External noise may be classified into the following three types:

   - ***Atmospheric noises***
   - ***Extraterrestrial noises***
   - ***Man-made noises or industrial noises.***

2. **Internal noise in communication**, i.e. noises which get, generated within the receiver or communication system.

   Internal noise may be put into the following four categories.

   - ***Thermal noise or white noise or Johnson noise***
   - ***Shot noise.***
   - ***Transit time noise***
   - ***Miscellaneous internal noise.***

External noise cannot be reduced except by changing the location of the receiver or the entire system. Internal noise on the other hand can be easily evaluated Mathematically and can be reduced to a great extent by proper design. As already said, because of the fact that internal noise can be reduced to a great extent, study of noise characteristics is a very important part of the communication engineering.

Explanation of External Noise

Atmospheric Noise

Atmospheric noise or static is caused by lighting discharges in thunderstorms and other natural electrical disturbances occurring in the atmosphere. These electrical impulses are random in nature. Hence the energy is spread over the complete frequency spectrum used for radio communication.

Atmospheric noise accordingly consists of spurious radio signals with components spread over a wide frequency range. These spurious radio waves constituting the noise get propagated over the earth in the same fashion as the desired radio waves of the same frequency. Accordingly at a given receiving point, the receiving antenna picks up not only the signal but also the static from all the thunderstorms, local or remote.

The field strength of atmospheric noise varies approximately inversely with the frequency. Thus large atmospheric noise is generated in low and medium frequency (broadcast) bands while very little noise is generated in the VHF and UHF bands. Further VHF and UHF components of noise are limited to the line-of-sight (less than about 80 Km) propagation. For these two-reasons, the atmospheric noise becomes less severe at Frequencies exceeding about 30 MHz.

Extraterrestrial Noise

There are numerous types of extraterrestrial noise or space noises depending on their sources. However, these may be put into following two subgroups.

1. ***Solar noise***
2. ***Cosmic noise***

Solar Noise

This is the electrical noise emanating from the sun. Under quite conditions, there is a steady radiation of noise from the sun. This results because sun is a large body at **a** very high temperature (exceeding 6000°C on the surface), and radiates electrical energy in the form of noise over a very wide frequency spectrum including the spectrum used for radio communication. The intensity produced by the sun varies with time. In fact, the sun has a repeating 11-Year noise cycle. During the peak of the cycle, the sun produces some amount of noise that causes tremendous radio signal interference, making many frequencies unusable for communications. During other years. the noise is at a minimum level.

Cosmic noise

Distant stars are also suns and have high temperatures. These stars, therefore, radiate noise in the same way as our sun. The noise received from these distant stars is thermal noise (or black body noise) and is distributing almost uniformly over the entire sky. We also receive noise from the center of our own galaxy (The Milky Way) from other distant galaxies and from other virtual point sources such as quasars and pulsars.

Man-Made Noise (Industrial Noise)

By man-made noise or industrial- noise is meant the electrical noise produced by such sources as automobiles and aircraft ignition, electrical motors and switch gears, leakage from high voltage lines, fluorescent lights, and numerous other heavy electrical machines. Such noises are produced by the arc discharge taking place during operation of these machines. Such man-made noise is most intensive in industrial and densely populated areas. Man-made noise in such areas far exceeds all other sources of noise in the frequency range extending from about 1 MHz to 600 MHz

Explanation of Internal Noise in communication

Thermal Noise

Conductors contain a large number of ***'free"*** electrons and ***"ions"*** strongly bound by molecular forces. The ions vibrate randomly about their normal (average) positions, however, this vibration being a function of the temperature. Continuous collisions between the electrons and the vibrating ions take place. Thus there is a continuous transfer of energy between the ions and electrons. This is the source of resistance in a conductor. The movement of free electrons constitutes a current which is purely random in nature and over a long time averages zero. There is a random motion of the electrons which give rise to noise voltage called thermal noise.

Thus noise generated in any resistance due ***to random motion of electrons i5 called thermal noise or white or Johnson noise.***

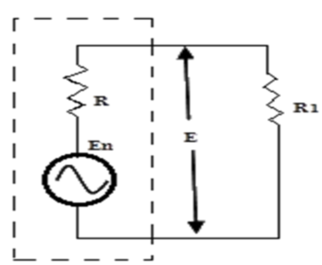
3.7 NOISE EQUIVALENT BANDWIDTH

The analysis of thermal noise is based on the Kinetic theory. It shows that the temperature of particles is a way of expressing its internal kinetic energy. Thus *"Temperature"* of a body can be said to be equivalent to the statistical rms value of the velocity of motion of the particles in the body. At -273°C (or zero degree Kelvin) the kinetic energy of the particles of a body becomes zero .Thus we can relate the noise power generated by a resistor to be proportional to its absolute temperature. Noise power is also proportional to the bandwidth over which it is measured. From the above discussion we can write down.

$P_n \propto TB$
$P_n = KTB$ ------ (1)

Where Pn = Maximum noise power output of a resistor.
K = Boltzmann's constant = 1.38 x10-23 joules I Kelvin.
T = Absolute temperature.
B = Bandwidth over which noise is measured.

From equation (1), an equivalent circuit can be drawn as shown in below figure



$$P_n = \frac{E^2}{R_1} = \frac{E^2}{R} = \frac{(\frac{E_n}{2})^2}{R} = \frac{E_n^2}{4R}$$

$$[\therefore from\ Fig\ if\ R = R_2, E_n = \frac{E}{2}]$$

$$E_{n2} = 4RP_n$$

$$E_{n2} = 4R\ KTB$$

$$E_n = \sqrt{4KTRB} ---------2$$

From equation (2), we see that the square of the rms noise voltage is proportional to the absolute temperature of le resistor, the value of the resistor, and the bandwidth over which it is measured. En is quite independent of the Frequency.

Example

R.F. amplifier is saving an input resistor of 8Kr and works in the frequency range of 12 to 15.5 MHz Calculate the rms noise voltage at the input to this amplifier at an ambient temperature of 17^{o}C?

Solution:

$$E_n = \sqrt{4KTRB}$$

$$E_n = \sqrt{4 \times 1.38 \times 10^{-23} \times (17+273) \times (15.5-12) \times 10^6 \times 8 \times 10^3}$$

$$E_n = \sqrt{4 \times 1.38 \times 290 \times 3.5 \times 1 \times 10^{-14}}$$

$$= 21.17\ \mu v$$

Shot Noise

The most common type of noise is referred to as shot noise which is produced by the random arrival of 'electrons or holes at the output element, at the plate in a tube, or at the collector or drain in a transistor. Shot noise is also produced by the random movement of electrons or holes across a PN junction.

Even through current flow is established by external bias voltages, there will still be some random movement of electrons or holes due to discontinuities in the device. An example of such a discontinuity is the contact between the copper lead and the semiconductor materials. The interface between the two creates a discontinuity that causes random movement of the current carriers.

Transit Time Noise

Another kind of noise that occurs in transistors is called transit time noise.

Transit time is (he duration of time that it takes for a current carrier such as a hole or current to move from the input to the output.

The devices themselves are very tiny, so the distances involved are minimal. Yet the time it takes for the current carriers to move even a short distance is finite. At low frequencies this time is negligible. But when the frequency of operation is high and the signal being processed is the magnitude as the transit time, then problem can occur. The transit time shows up as a kind of random noise within the device, and this is directly proportional to the frequency of operation.

MISCELLANEOUS INTERNAL NOISES

Flicker Noise

Flicker noise or modulation noise is the one appearing in transistors operating at low audio frequencies. Flicker noise is proportional to the emitter current and junction temperature. However, this noise is inversely proportional to the frequency. Hence it may be neglected at frequencies above about 500 Hz and it, Therefore, possess no serious problem.

Transistor Thermal Noise

Within the transistor, thermal noise is caused by the emitter, base and collector internal resistances. Out of these three regions, the base region contributes maximum thermal noise.

Partition Noise

Partition noise occurs whenever current has to divide between two or more paths, and results from the random fluctuations in the division. It would be expected, therefore, that a diode would be less noisy than a transistor (all other factors being equal) If the third electrode draws current (i.e.., the base current). It is for this reason that the inputs of microwave receivers are often taken directly to diode mixers.

Signal to Noise Ratio.

Noise is usually expressed as a power because the received signal is also expressed in terms of power. By Knowing the signal to noise powers the signal to noise ratio can be computed. Rather than express the signal to noise ratio as simply a number, you will usually see it expressed in terms of decibels.

$$Signal\ TO\ Noise\ Ratio = 10\log\frac{Signal\ power}{Noise\ Power} = 10\log\frac{P_s}{P_n}$$

A receiver has an input signal power of l.2µW. The noise power is 0.80µW. The signal to noise ratio is

Signal to Noise Ratio = 10 Log (1.2/0.8)

= 10 log 1.5
= 10 (0.176)
= 1.76 Db

3.8 NOISE FIGURE

Noise Figure

Noise Figure **F** is designed as the ratio of the signal-to-noise power at the input to the signal to noise power at the output.

The device under consideration can be the entire receiver or a single amplifier stage. The noise figure **F** also called the **noise factor** can be computed with the expression

F = Signal to Noise power Input/Signal to noise power output

You can express the noise figure as a number, more often you will see it expressed in decibels.

3.9 Outcomes

- To compute the autocorrelation function of a random process.
- To determine whether a random process is stationary (if possible) or wide-sense stationary (WSS).
- To determine the power spectral density (PSD) of WSS random processes.
- Analyze the different noises affecting a two port network and a cascade of networks.
- The students will be to understand the effects of noise over a system.

3.10 Further Readings

- Communication Systems, Simon Haykins, 5th Edition, John Willey, India Pvt. Ltd, 2009.

- An Introduction to Analog and Digital Communication, Simon John Haykins,Wiley India Pvt.Ltd., 2008

- www.youtube.com/watch?v=kVQ7mr2TU2U

- nptel.ac.in/courses/117185

3.11 Recommended Questions

1. Define mean, Auto correlation and auto co variance of a random process
2. Define power spectral density and explain its properties. Explain cross correlation and Auto correlation.
3. State the properties of Gaussian process?
4. Define conditional probability, Random variable and mean?
5. Explain Joint probability of the events A & B?
6. Explain Conditional probability of the events A & B?
7. Explain the properties Gaussian process?
8. Compare auto correlation and cross correlation functions?
9. What is Noise?
10. Define Noise figure and noise factor?
11. Derive the expression for noise equivalent bandwidth?
12. Define Shot noise & Thermal Noise?
13. Define noise equivalent bandwidth? Explain noise figure measurement?

MODULE-4

NOISE IN ANALOG MODULATION: Introduction, Receiver Model, Noise in DSB-SC receivers, Noise in AM receivers, Threshold effect, Noise in FM receivers, Capture effect, FM threshold effect, FM threshold reduction, Pre-emphasis and De- emphasis in FM.

4.0 Objectives:

- Different types of noises and their classification are explained.
- The students will be to understand the effects of noise over a system.

4.1 INTRODUCTION

In this chapter we study the noise performance of analog (continuous wave) modulation system. We begin the study by describing signal to noise ratios that provide basis for evaluating the noise performance of an analog communication receiver. Signal to Noise ratios: The output signal to noise ratio is a measure of describing the fidelity with which the demodulation process in the receiver recovers the original message from the received modulated signal in the presence of noise. It is defined as the ratio of the average power of the message signal to the average power of the noise, both measured at the receiver output.

4.2 RECEIVER MODEL

Signal to Noise ratios:

The output signal to noise ratio is a measure of describing the fidelity with which the demodulation process in the receiver recovers the original message from the received modulated signal in the presence of noise.

It is defined as the ratio of the average power of the message signal to the average power of the noise, both measured at the receiver output.

The output signal to noise ratio is unambiguous as long as the recovered message and noise at the demodulator output is additive. This requirement is satisfied exactly by linear receivers such as coherent detectors and approximately by using non-linear receivers such as envelope detector and frequency discrimination used in FM, provided that input noise power is small compared with average carrier power.

The calculation of the output signal to noise ratio involves the use of an **idealized receiver model** and its details depends on the **channel noise** and the **type of demodulation** used in the receiver. Therefore, (SNR)o does not provide sufficient information when it is necessary to compare the output signal to noise ratios of different analog modulation – demodulation schemes. To make such comparison, baseband transmission model is used. The baseband transmission model is shown in fig

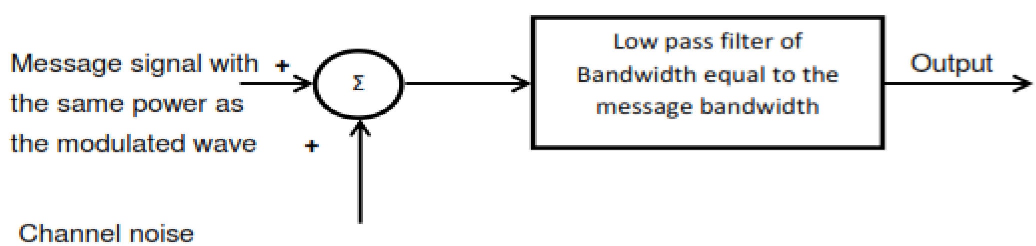


Fig 4.1baseband transmission model.

The baseband transmission model assumes:

1. The transmitted or modulated message signal power is fixed
2. The baseband low pass filter passes the message signal, and rejects out of band noise.

According to the baseband transmission model, we may define the channel signal to noise ratio, referred to the receiver input as

$$(SNR)_C = \frac{\text{average power of the modulated signal}}{\text{Avg power of noise measured in message bandwidth}}$$

It is independent of the type of modulation or demodulation used.

$$\text{Figure of merit} = \frac{(SNR)o}{(SNR)c} \quad \text{-----------------------------------8.3}$$

This ratio may be viewed as a frame of reference for comparing different modulation systems and we may normalize the noise performance of a specific modulation – demodulation system by dividing the output signal to noise ratio of the system by
channel signal to noise ratio. This ratio is known as figure of merit for the system.

4.3 NOISE IN DSBSC RECEIVER

DSBSC RECEIVER:

Consider a DSBSC wave defined by

s(t) = Accos2   fct m(t)

where Accos2   fct is the carrier wave and m(t) is the message signal.

The average power of the DSBSC modulated wave s(t) is Ac2P/2,

Where Ac is the amplitude of the carrier and P is the average power of the message signal m(t).

With a noise power spectral density of N0/2, defined for both positive and negative frequencies,

The average noise power in the message bandwidth W is equal to WN0.

Therefore, the channel signal to noise ratio of the system is

Next, we have to determine the output signal to noise ratio.

$$(\mathrm{SNR})_c = \frac{A^2 c P}{2WN0} \text{---8.7}$$

The total signal at the coherent detector input may be expressed as:

x(t) = s(t) + n(t)

= Accos2   fct m(t) + nI(t)cos2   fct-nQ(t)sin2   fct   -------------------4.8

Where nI(t) and nQ(t) are the in-phase and quadrature components of n(t), with respect to the carrier cos2   fct.

The product modulator output of the coherent detector is

v(t) =x(t) cos2   fct

=1/2 Ac m(t) +1/2 nI(t)+1/2[Ac m(t) + nI(t)]cos4   fct - 1/2 AcnQ(t)sin4   fct   -----4.9

The low pass filter in the coherent detector removes the high frequency

components of v(t), yielding a receiver output

y(t) = 1/2 Ac m(t) +1/2 nI(t)   ---4.10

Above equation(4.10) indicates that

1. The message m(t) and in-phase noise component nI(t)of the narrow band noise n(t) appear additively at the receiver output.

2. The quadrature component nQ(t) of the noise n(t) is completely rejected by the coherent detector.

The message signal component at the receiver output equals 1/2 Ac m(t). Hence, the average power of message signal at the receiver output is equal to Ac2P/4, where P is the average power of the original message signal m(t).

The noise component at the receiver output is 1/2 nI(t). Hence, the power spectral density of the output noise equals ¼ times nI(t). To calculate the average power of the noise at the receiver output, we first determine the power spectral density of the in-phase noise component nI(t).

The power spectral density SN(f) of the narrow band noise n(t) is of the form shown in fig(4.2) below.

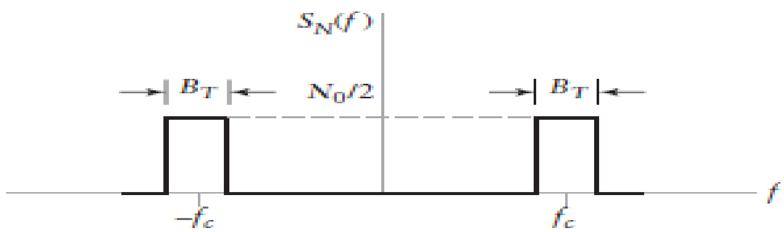


Fig 4.2 Power spectral density of narrow band noise n(t) at IF filter output

Hence, the power spectral density of nI(t)as shown in fig (4.4) below.

SNI(f)=SNQ(f)

Power spectral density of in-phase and quadrature component of narrow band noise n(t).Evaluating the area under the curve of power spectral density of fig ,above and multiplying the result by ¼, we find that the average noise power at the receiver output is equal to WN0/2. Then, the output signal to noise ratio for DSBSC modulation is given by

$$(SNR)_o = \frac{A^2cP}{2WN0} \text{--} 8.11$$

The figure of merit is

$$FOM = \frac{(SNR)o}{(SNR)c} = 1 \quad \text{--} 8.12$$

4.4 NOISE IN SSB RECEIVER

SSB Receivers:

Let us consider the incoming signal is an SSB wave and assume that only the lower side band is transmitted, then we may express the modulated wave as

$$s(t) = \frac{A_c}{2} m(t) \cos(2\pi f_c t) + \frac{A_c}{2} \hat{m}(t) \sin(2\pi f_c t)$$

where

m(t) is the Hilbert transform of message

We may make following observations with respect to in-phase and quadrature component of eqn 4

1. The two components are uncorrelated with each other. Therefore, their power spectral densities are additive.
2. The Hilbert transform (t) is obtained by passing m(t) through a linear filter with transfer function –jsgn(f). The squared magnitude of this transfer function is equal to one for all frequencies. Therefore, m(t) and (t) have the same average power.

Next, procedure is similar to DSBSC receiver.

The average power of s(t) is given by Ac2P/4, contributed by in-phase and quadrature component of SSB wave s(t).

i.e., Average power of in-phase and quadrature component is Ac2P/4.

The average noise power in the message bandwidth W is WN0

Then, channel signal to noise ratio is

$$(SNR)_c = \frac{A^2cP}{4WN0} \text{--} 8.14$$

Next is to find (SNR)o,

The transmission bandwidth B = W. The mid band frequency of the power spectral density SN(f) of the narrow-band noise n(t) is differ from the carrier frequency fc by W/2. Therefore, we may express n(t) as

$$n(t) = n_I(t)\cos 2\pi(f_c - W/2)t - n_Q(t)\sin 2\pi(f_c - W/2)t \quad \text{------------4.15}$$

The coherent detector output

$$y(t) = 1/4\, A_c\, m(t) + 1/2\, n_I(t)\cos \pi Wt + 1/2\, n_Q(t)\sin \pi Wt \quad \text{--------------------4.16}$$

Equation 4.16 consists of both in-phase and quadrature component of narrow band noise n(t) and the quadrature component t) of the modulated signal is completely eliminated.

The message component in the receiver output is 1/4 Ac m(t) so that the average power of the recovered message is $A_c^2P/16$.

The noise component in the receiver output is $1/2\, n_I(t)\cos \pi Wt + 1/2\, n_Q(t)\sin \pi Wt$.

The average power of the output noise is WN0/4. (WN0/8 + WN0/8)

The (SNR)o is

$$(SNR)_o = \frac{A^2{}_c P}{4WN_0} \text{--8.17}$$

Then FOM is

$$FOM = \frac{(SNR)o}{(SNR)c} = 1 \quad \text{--8.18}$$

The FOM of AM receiver using envelope detection is always less than unity.

When compared to DSBSC and SSB the noise performance of AM is always inferior.

4.5 NOISE IN AM RECEIVER

AM RECEIVER MODEL:

It is customary to model channel noise as a sample function of white noise process whose mean is zero and power spectral density is constant. The channel noise is denoted by w(t), and its power spectral density is No/2 defined for both negative and positive frequencies. Where No is the average noise power per unit bandwidth measured at the front end of the receiver. The received signal consists of an amplitude modulated signal s(t) corrupted by the channel noise w(t). To limit the effect of noise on signal, we may pass the received signal through a band pass filter of bandwidth wide enough to accommodate s(t).

AM receiver is of super-heterodyne type, this filtering is performed in two sections: RF section and IF section. Fig 4.3 below shows the idealized receiver model for amplitude modulation.

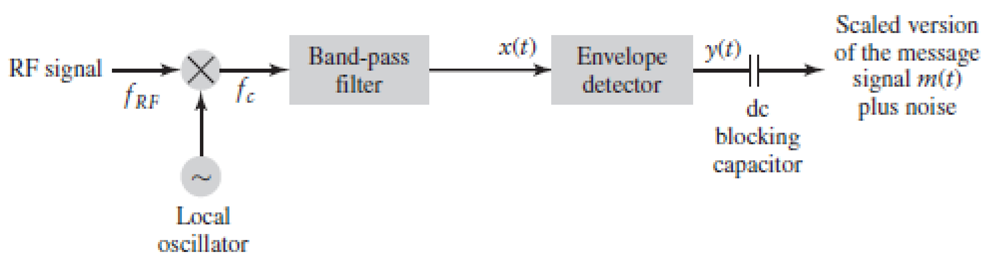


Fig 4.3 Idealized receiver model for amplitude modulation

IF filter accounts for the combinations of two effects:

1. The filtering effect of the actual IF section in the super heterodyne AM receiver
2. The filtering effect of actual RF section in the receiver translated down to the IF band.

i.e., the IF section provides most of the amplification and selectivity in the receive

The IF filter bandwidth is just wide enough to accommodate the bandwidth of modulated signal s(t) and is tuned so that its mid band frequency is equal to the carrier frequency except SSB modulation. The ideal band pass characteristics of IF filter is shown in fig below,

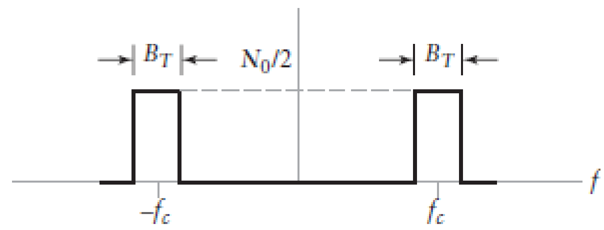


Fig 4.4 Idealized characteristic of IF filter.

fc – Mid band frequency of the IF filter

B – transmission Bandwidth of the modulated signal s(t)

The composite signal x(t), at the IF filter o/p is defined by

x(t) = s(t) + n(t) ---4.4

Where n(t) is a band-limited version of the white noise w(t).

n(t) is the sample function of a noise process N(t) with the following power spectral density:

The band limited noise n(t) may be considered as narrow band noise.

The modulated signal s(t) is a band pass signal , its exact description is depends on the type of modulation used.

The time domain representation of narrow band noise can be expressed in two different ways,

1. Represented in terms of in-phase and quadrature components.

   This method is well suited for the noise analysis of AM receivers using coherent detector and also for envelope detection.
2. Represented in terms of envelope and phase.

   This method is well suited for the noise analysis of FM receivers.

Signal to noise ratios for coherent reception:

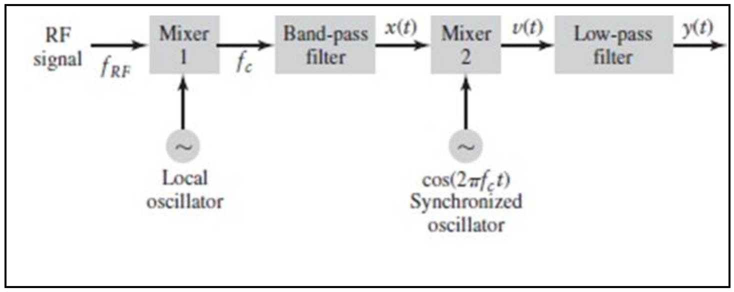


Fig 4.4 Model of DSBSC receiver using coherent detection.

We begin the noise analysis by evaluating (SNR)o, (SNR)c for an AM receiver using coherent detection.In put signal is either DSBSC or SSB modulated wave and pass it through IF filter. The use of coherent detection requires multiplication of IF filter output x(t) by Locally generated sinusoidal wave cos2 fct and then, low pass filtering the product. For convenience, we assume that amplitude of locally generated signal is unity. For satisfactory operation local oscillator must be synchronized to carrier both in frequency and phase, we assume that perfect synchronization.

Coherent detection has the unique feature that for any signal-to-noise ratio, an output strictly proportional to the original message signal always present. That is the output message component is unmutilated and the noise component always appears additively with the message irrespective of the signal-to-noise ratio.

Exampe1. **Single-Tone Modulation**

Consider the message m(t) is a single- tone or single frequency signal as

$$m(t) = A_m \cos 2\pi f_m t$$

where Am – Amplitude of the message and

fm – frequency of the message.

Then, the AM wave is given by

$$s(t) = A_c[\, 1+ \mu \cos 2\pi f_m t]\cos 2\pi f_c t$$

where $\mu = A_m k_a$ is the modulation factor.

The average power of the modulating wave m(t) is

P = ½ A2m

Therefore, Eqn 4.25 becomes

$$\text{FOM} = \frac{(\text{SNR})o}{(\text{SNR})c} = \frac{\frac{1}{2}k_a^2 A_m^2}{1+\frac{1}{2}k_a^2 A_m^2}$$

$$\text{FOM} = \frac{(\text{SNR})o}{(\text{SNR})c} = \frac{\mu^2}{1+\mu^2} \text{ ------------------------------------8.26}$$

When $\mu = 1$, FOM is 1/3.

This means that, AM system must transmit three times as much average power as a suppressed carrier system in order to achieve the same quality of noise performance.

4.6 THRESHOLD EFFECT

When the carrier-to-noise ratio at the receiver input of a standard AM is small compared to unity, the noise term dominates and the performance of the envelope detector changes completely.

In this case it is convenient to represent the narrow band noise n(t) in terms of

its envelope r(t) and phase Ψ(t), as given by

n(t) = r(t) cos [2πfct + Ψ(t)] --4.29

The phasor diagram of the detector input x(t) = s(t) + n(t) is shown in fig below,

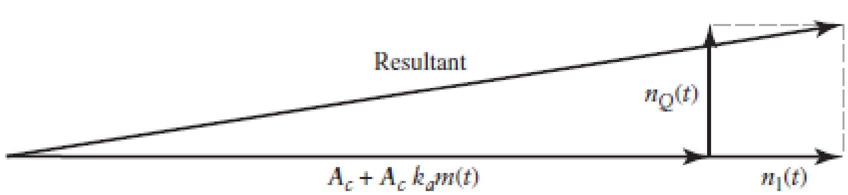


Fig 4.10 Phasor diagram for AM wave plus narrow-band noise for the case of low carrier-to-noise ratio

In the fig, we have used the noise as a reference, because it is a dominant term. To the noise phasor, we added a phasor representing the signal term $A_c[1+k_a m(t)]$, with the angle between them is $\Psi(t)$.

From the fig it is observed that carrier amplitude is small compared to the noise envelope r(t). Then, we may approximate the envelope detector output as

$$y(t) = r(t) + A_c \cos[\Psi(t)] + A_c k_a m(t) \cos[\Psi(t)] \quad \text{------------------------4.30}$$

The above relation shows that, the detector output is not proportional to the

message signal m(t). The last term contains the message signal m(t) multiplied by noise in the form of cos [Ψ(t)]. The phase Ψ(t) of a noise uniformly distributed over 2π radians, and it can have values between 0 to 2π with equal probability.

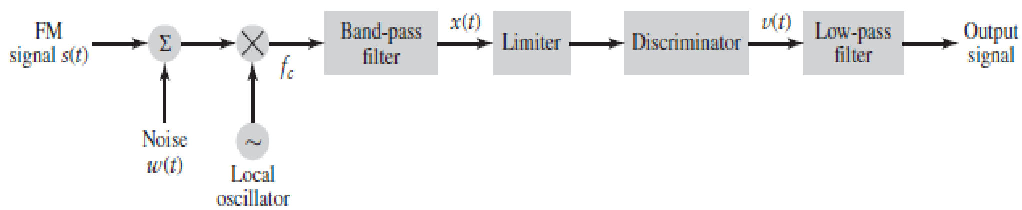
Therefore, we have a complete loss of information. The loss of a message in an envelope detector that operates at a low carrier to noise ratio is referred to as threshold effect.

It means a value of the carrier to noise ratio below which the noise performance of a detector deteriorates much more rapidly than high carrier to noise rario as in eqn4.24((SNR)o).

4.7 NOISE IN FM RECEIVER

FM Receiver Model:

To study the noise performance of a FM receiver idealized FM receiver model is required. Fig below shows the idealized FM receiver model,



The noise w(t) is modelled as white noise of zero mean and power spectral density N0/2. The received FM signal s(t) has a carrier frequency fc and transmission bandwidth B, so that only a negligible amount of power lies outside the frequency

band fc- B/2 ≤ |f| ≤ fc+ B/2. The FM transmission bandwidth B is in excess of twice the message bandwidth W by an amount that depends on the deviation ratio of the incoming FM wave.

The IF filter in fig above represents the combined filtering effect of RF sectionand IF sections of an FM receiver of the super-heterodyne type. The filter has a midband frequency fc and bandwidth B, passes the FM signal without any distortion. The bandwidth B is small compared with the mid band frequency fc. The transfer function of the IF filter is shown in fig 4.12 above. The filtered narrow band noise n(t) is represented in terms of its in-phase and quadrature components.

The limiter in fig is used to remove any amplitude variations at the IF output. The discriminator is assumed to be ideal and its output proportional to the deviation in the instantaneous frequency of the carrier away from fc. The base band low pass filter is ideal and its bandwidth is equal to message bandwidth.

Noise in FM Reception:

For the noise analysis of FM receivers, the narrow-band noise n(t) at the

output of the IF filter is expressed as

n(t) = r(t) cos [2πfct + Ψ(t)] ---4.31

The envelope r(t) and phase Ψ(t) are defined in terms of the in-phase

component nI(t) and quadrature component nQ(t) as follows, r(t) = [n2I(t)+ n2Q(t)]1/2 ---4.32

and

Ψ(t) = tan -1 (nQ(t) /nI(t)) --4.33

Assume that the FM signal at the IF filter output is given by

$s(t) = A_c\cos [2\pi f_c t + 2\pi k_f \int_0^t m(t)dt]$ ---------------------------8.34

where A_c -- carrier amplitude

f_c – carrier frequency

k_f–frequency sensitivity

m(t) – message

For convenience, we define

$\Phi(t) = 2\pi k_f \int_0^t m(t)dt$ ---8.35

Then, we may express s(t) as

s(t) = Accos [2πfct + Φ(t)] ---4.36

The IF filter output is

x(t) = s(t) + n(t)

= Accos [2πfct + Φ(t)] + r(t) cos [2πfct + Ψ(t)] ------4.37

Fig below shows the phasor diagram of x(t),

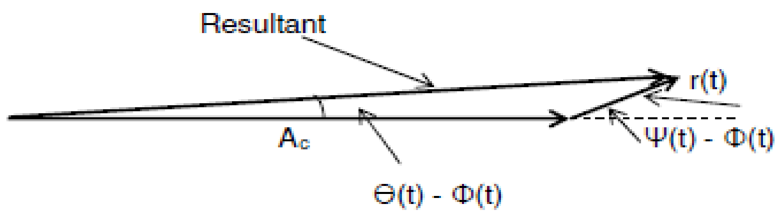


Fig 8.13 Phasor diagram for FM wave plus narrow band noise for the case of high carrier-to-noise ratio

The relative phase ϴ(t) of the resultant is obtained from the fig above is

$$\Theta(t) = \Phi(t) + \tan^{-1}\frac{r(t)\sin[\Psi(t)-\Phi(t)]}{A_c+r(t)\cos[\Psi(t)-\Phi(t)]}$$ ---------------------------------------8.38

The envelope of the x(t) is not considered, because any envelope variation is limited by the limiter.

Now, our aim is to determine error in the instantaneous frequency of the carrier wave caused by the presence of narrow band noise. The discriminator output is proportional to $\dot{\theta}(t)$, where $\dot{\theta}(t)$ is the derivative of $\Theta(t)$ with respect to time.

We assume that the carrier-to-noise ratio measured at the discriminator input is large compared with unity. Then, the expression for relative phase is given by

$$\Theta(t) = \Phi(t) + r(t)/A_c \sin[\Psi(t) - \Phi(t)] \quad \text{------------------------------}8.39$$

The first term in eqn 8.39 is proportional to message m(t) as in eqn 8.35. Hence, using eqn 8.35 and 8.39, the discriminator output is given by

$$v(t) = \frac{1}{2\pi}\frac{d\Theta(t)}{dt}$$

$$v(t) = k_f m(t) + n_d(t) \quad \text{--}8.40$$

where the noise term $n_d(t)$ is defined by

$$n_d(t) = \frac{1}{2\pi Ac}\frac{d\,\{r(t)\sin[\Psi(t) - \Phi(t)]\}}{dt} \quad \text{-------------------}8.41$$

Thus, we may state that under the condition of high carrier to noise ratio, the calculation of the average output noise power in an FM receiver depends only on the carrier amplitude Acand the quadrature noise component nQ(t).

Recall that, differentiation of a function with respect to time corresponds to multiplication of its Fourier Transform by j2πf. It means, we may obtain the noise process nd(t) by passing nQ(t) through a linear filter of transfer function equal to

$$\frac{j2\pi f}{2\pi Ac} = \frac{jf}{Ac} \quad \text{-----------------------------------}8.45$$

Then, the power spectral density of SNd(f) is related to the power spectral density SNQ(f) of nQ(t) as,

$$S_{Nd}(f) = \frac{f^2}{A^2c}S_{NQ}(f) \quad \text{---------------------}8.46$$

4.8 PRE-EMPHASIS AND DE-EMPHASIS

Pre-emphasis

Pre-emphasis refers to boosting the relative amplitudes of the modulating voltage for higher audio frequencies from 2 to approximately 15 KHz.

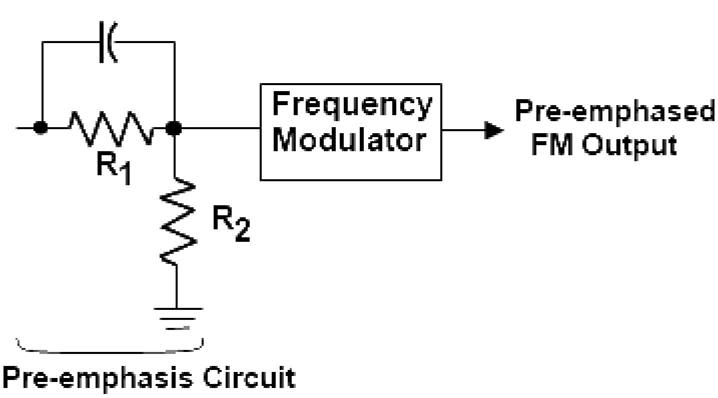
De-emphasis

De-emphasis means attenuating those frequencies by the amount by which they are boosted.

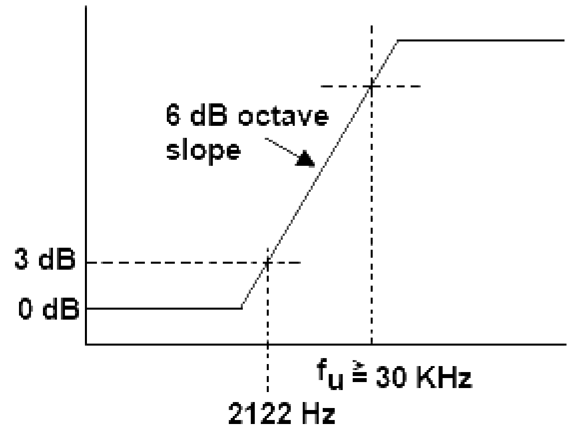
However pre-emphasis is done at the transmitter and the de-emphasis is done in the receiver. The purpose is to improve the signal-to-noise ratio for FM reception. A time constant of 75μs is specified in the RC or L/Z network for pre-emphasis and de-emphasis.

Pre-emphasis circuit

At the transmitter, the modulating signal is passed through a simple network which amplifies the high frequency, components more than the low-frequency components. The simplest form of such a circuit is a simple high pass filter of the type shown in fig (a). Specification dictate a time constant of 75 microseconds (μs) where t = RC. Any combination of resistor and capacitor (or resistor and inductor) giving this time constant will be satisfactory. Such a circuit has a cutoff frequency fco of 2122 Hz. This means that frequencies higher than 2122 Hz will he linearly enhanced. The output amplitude increases with frequency at a rate of 6 dB per octave. The pre-emphasis curve is shown in Fig (b). This pre-emphasis circuit increases the energy content of the higher-frequency signals so that they will tend to become stronger than the high frequency noise components. This improves the signal to noise ratio and increases intelligibility and fidelity.



(a) Pre-emphasis Circuit



(b) Pre-emphasis Curve

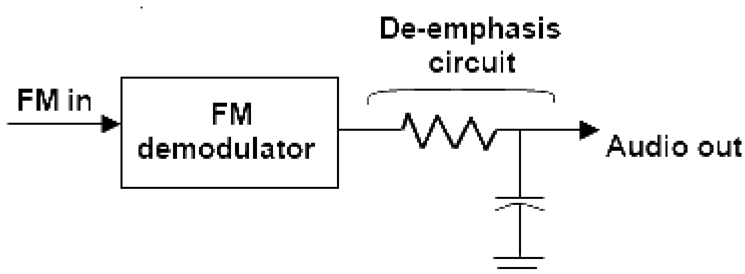
The pre-emphasis circuit also has an upper break frequency fu where the signal enhancement flattens out.

See Fig (b). This upper break frequency is computed with the expression.

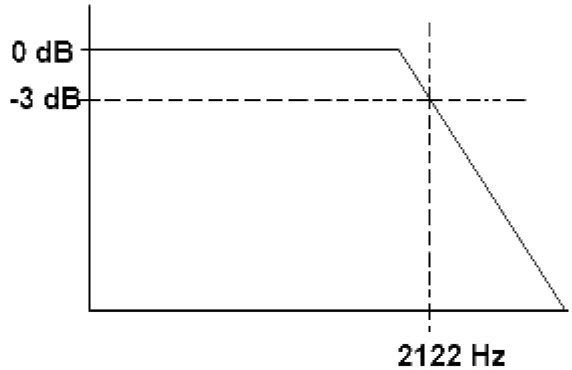
fu = R1 +(R2/2πR1R1C)

It is usually set at some very high value beyond the audio range. An fu of greater than 30KHz is typical.

De-emphasis Circuit

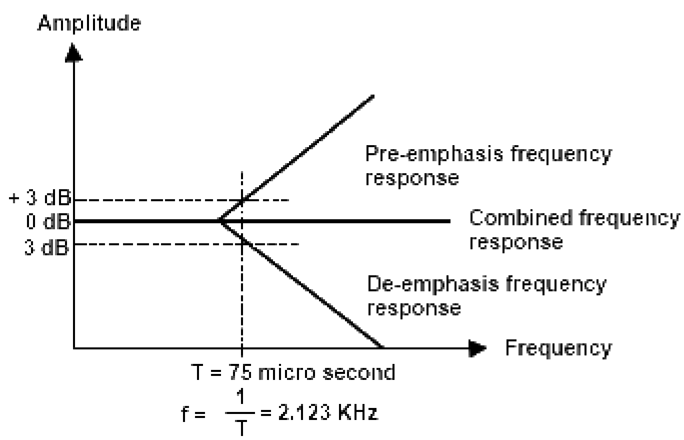


(c) De-emphasis circuit



(d) De-emphasis Curve

To return the frequency response to its normal level, a de-emphasis circuit is used at the receiver. This is a simple low-pass filter with a constant of 75 πs. See figure (c). It features a cutoff of 2122 Hz and causes signals above this frequency to be attenuated at the rate of 6bB per octave. The response curve is shown in Fig (d). As a result, the pre-emphasis at the transmitter is exactly offset by the de-emphasis circuit in the receiver, providing a normal frequency response. The combined effect of pre-emphasis and de-emphasis is to increase the high-frequency components during transmission so that they will be stronger and not masked by noise.



Combined frequency response

4.9 OUTCOME

- Analyze the different noises affecting a two port network and a cascade of networks.
- The students will be to understand the effects of noise over a system

4.10 QUESTIONS

1. Derive the expression for o/p SNR of an AM receiver using an envelope detector?
2. Explain Capture effect and Threshold effect?
3. Explain Pre-emphasis?
4. Explain De-emphasis?
5. Derive the expression for o/p SNR of an FM receiver using coherent detector?
6. Derive the expression for FOM of DSB-SCreceiver using coherent detector?

4.11 Further Readings

TEXT BOOK:

1. Communication Systems, Simon Haykins, 5th Edition, John Willey, India Pvt. Ltd, 2009

2. An Introduction to Analog and Digital Communication, Simon Haykins, John Wiley India Pvt. Ltd., 2008

REFERENCE BOOKS:

1. Modern digital and analog Communication systems B. P. Lathi, Oxford University Press., 4th ed, 2010
2. Communication Systems, Harold P.E, Stern Samy and A Mahmond, Pearson Edn, 2004.
3. Communication Systems: Singh and Sapre: Analog and digital TMH 2nd , Ed 2007.

MODULE-5

DIGITAL REPRESENTATION OF ANALOG SIGNALS: Introduction, Why Digitize Analog Sources?, The Sampling process, Pulse Amplitude Modulation, Time Division Multiplexing, Pulse-Position Modulation, Generation of PPM Waves, Detection of PPM Waves, The Quantization Process, Quantization Noise, Pulse– Code Modulation: Sampling, Quantization, Encoding, Regeneration, Decoding, Filtering, Multiplexing. Application to Vocoder.

5.0 Objectives:

- Introduction to sampling process.
- Explanation of PCM and Quantization.
- Introduction to TDM Scheme.

5.1 Introduction

In continuous-wave (CW) modulation some parameter of a sinusoidal carrier wave is varied continuously in accordance with the message signal. This is in direct contrast to pulse modulation. In pulse modulation, some parameter of a pulse train is varied in accordance with the message signal.

In this context, we may distinguish two families of pulse modulation, analog pulse modulation and digital pulse modulation, depending on how the modulation is performed. In analog pulse modulation, a periodic pulse train is used as the carrier wave, and some characteristic feature of each pulse (e.g., amplitude, duration, or position) is varied in a continuous manner in accordance with the corresponding sample value of the message signal. Thus, in analog pulse modulation, information is transmitted basically in analog form, but the transmission takes place at discrete times. In digital pulse modulation, on the other hand, the message signal is represented in a form that is discrete in both time and amplitude, thereby permitting its transmission in digital form as a sequence of coded pulses. Simply put, digital pulse modulation has no CW counterpart.

The use of coded pulses for the transmission of analog information-bearing signals represents a basic ingredient in the application of digital communications. This chapter may therefore be viewed as the transition from analog to digital communications in our study of the principles of communication systems.

5.2 Sampling Process

In the sampling process, an analog signal is converted into a corresponding sequence of samples that are usually spaced uniformly in time. Clearly, for such a procedure to have practical utility, it is necessary that we choose the sampling rate properly, so that the sequence of samples uniquely defines the original analog signal.

Consider an arbitrary signal g1t2 of finite energy, which is specified for all time t. A seg- ment of the signal g1t2 is shown in Fig. 5.1(a). Suppose that we sample the signal g1t2 instantaneously and at a uniform rate, once every T_S seconds. Consequently, we obtain an infinite sequence of samples spaced T_S seconds apart and denoted by $g\{nT_s\}$, where n takes on all possible integer values, both positive and negative. We refer to T_S as the sampling period or sampling interval and to its reciprocal $f_S=1>T_S$ as the sampling rate. This ideal form of sampling is called instantaneous sampling.

Let $g_\delta(t)$ denote the signal obtained by individually weighting the elements of a periodic sequence of Dirac delta functions spaced T_s seconds apart by the sequence of numbers $\{g(nT_s)\}$, as shown by (see Fig. 5.1(b))

$$g_\delta(t) = \sum_{n=-\infty}^{\infty} g(nT_s)\delta(t - nT_s) \tag{5.1}$$

We refer to $g_\delta(t)$ as the *instantaneously (ideal) sampled signal*. The term $\delta(t - nT_s)$ represents a delta function positioned at time $t = nT_s$. From the definition of the delta function presented in Section 2.4, recall that such an idealized function has unit area. We may therefore view the multiplying factor $g(nT_s)$ in Eq. (5.1) as a "mass" assigned to the delta function $\delta(t - nT_s)$. A delta function weighted in this manner is closely approximated by a rectangular pulse of duration Δt and amplitude $g(nT_s)/\Delta t$; the smaller we make Δt, the better the approximation will be.

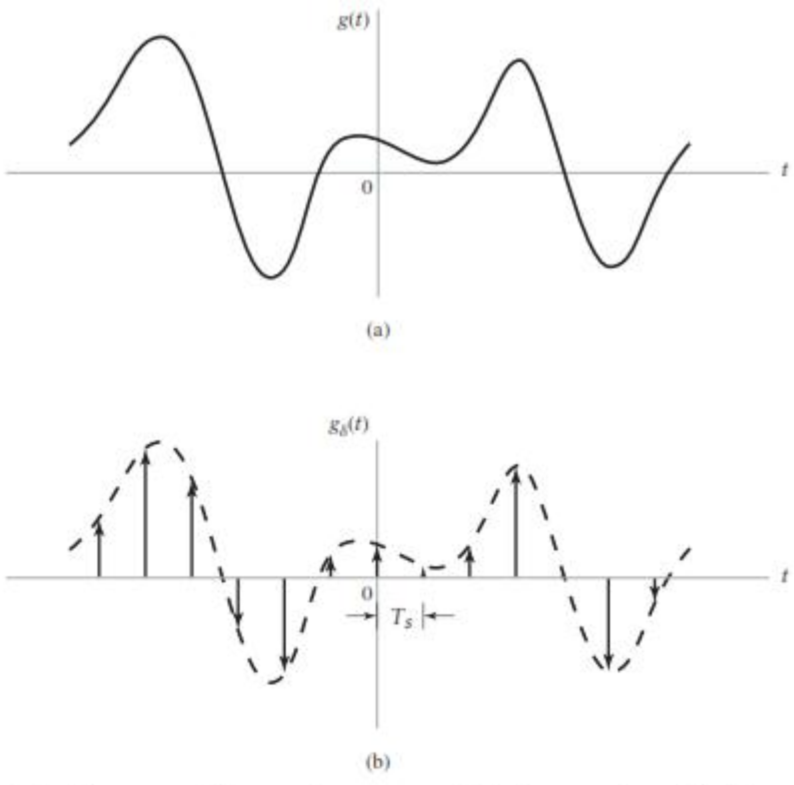


FIGURE 5.1 Illustration of the sampling process. (a) Analog waveform $g(t)$. (b) Instantaneously sampled representation of $g(t)$.

Sampling Theorem

The relations of Eq. (5.2) apply to any continuous-time signal $g(t)$ of finite energy. Suppose, however, that the signal $g(t)$ is *strictly band-limited*, with no frequency components higher than W hertz. That is, the Fourier transform $G(f)$ of the signal $g(t)$ has the property that $G(f)$ is zero for $|f| \geq W$, as illustrated in Fig. 5.2(a); the shape of the spectrum shown in this figure is intended for the purpose of illustration only. Suppose also that we choose the sampling period $T_s = 1/2W$, which, as we shall see, is the maximum permissible value. Then the corresponding spectrum $G_\delta(f)$ of the sampled signal $g_\delta(t)$ is as shown in Fig. 5.2(b). Putting $T_s = 1/2W$ in Eq. (5.2) and using $G_\delta(f)$ to denote the Fourier transform of $g_\delta(t)$, we may write

$$G_\delta(f) = \sum_{n=-\infty}^{\infty} g\left(\frac{n}{2W}\right) \exp\left(-\frac{j\pi nf}{W}\right) \tag{5.3}$$

Equation (5.3) defines the Fourier transform $G_\delta(f)$ of the sequence $\{g(n/2W)\}_{n=-\infty}^{\infty}$, which is obtained by uniform sampling of a continuous-time signal $g(t)$ at the special rate $(1/T_s) = f_s = 2W$. The formula obtained by using the sampling period $T_s = 1/2W$, shown in Eq. (5.3), is called the *discrete-time Fourier transform*[1] of the sequence $\{g(nT_s)\}_{n=-\infty}^{\infty}$.

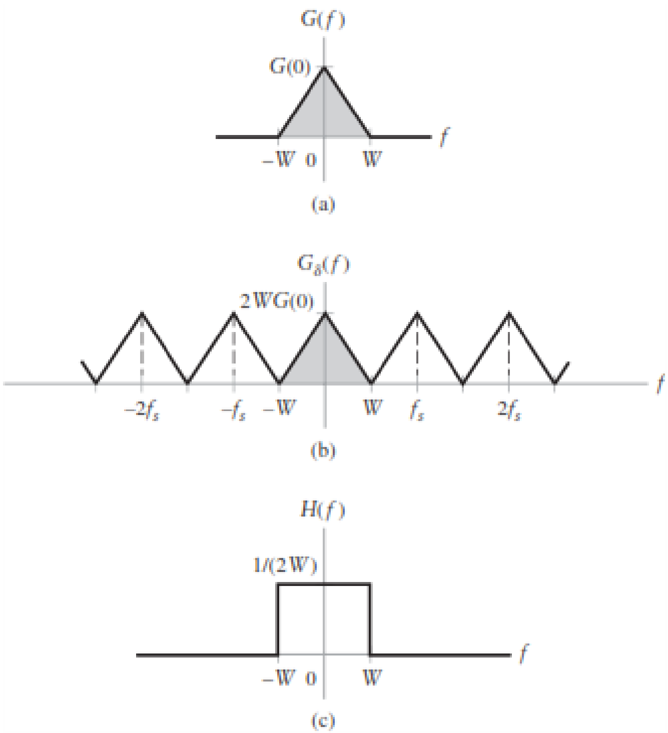


FIGURE 5.2 (a) Spectrum of a strictly band-limited signal $g(t)$. (b) Spectrum of instantaneously sampled version of $g(t)$ for a sampling period $T_s = 1/2W$. (c) Frequency response of ideal low-pass filter aimed at recovering the original message signal $g(t)$ from its uniformly sampled version.

5.3 Pulse Amplitude Modulation

In pulse-amplitude modulation (PAM), the amplitudes of regularly spaced pulses are varied in proportion to the corresponding sample values of a continuous message signal; the pulses can be of a rectangular form or some other appropriate shape. Pulse-amplitude modulation as defined here is somewhat similar to natural sampling, where the message signal is multiplied by a periodic train of rectangular pulses. In natural sampling, however, the top of each modulated rectangular pulse is permitted to vary with the message signal, whereas in PAM it is maintained flat.

The waveform of a PAM signal is illustrated in Fig. 5.5. The dashed curve in this figure depicts the waveform of the message signal m1t2, and the sequence of amplitude modulated rectangular pulses shown as solid lines represents the corresponding PAM signal s(t). There are two operations involved in the generation of the PAM signal:

1. Instantaneous sampling of the message signal m(t) every T_S seconds, where the sampling rate $f_S = 1>T_S$ is chosen in accordance with the sampling theorem.

2. Lengthening the duration of each sample, so that it occupies some finite value T.

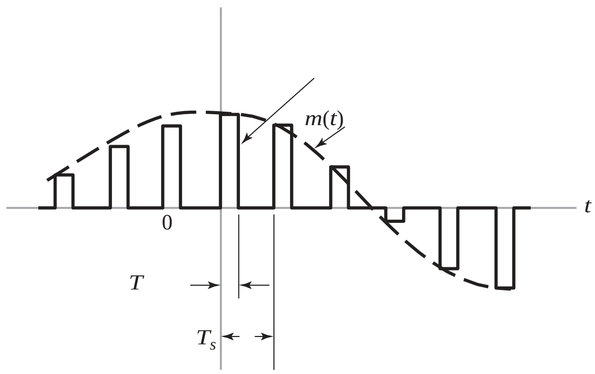


FIGURE 5.5 Flat-top sampling of a message signal.

5.4 Pulse-Position Modulation

In pulse-amplitude modulation, pulse amplitude is the variable parameter. Pulse duration is the next logical parameter available for modulation. In pulse-duration modulation (PDM), the samples of the message signal are used to vary the duration of the individual pulses. This form of modulation is also referred to as pulse-width modulation or pulse-length modulation. The modulating signal may vary the time of occurrence of the leading edge, the trailing edge, or both edges of the pulse. In Fig. 5.8(c) the trailing edge of each pulse is varied in accordance with the message signal, assumed to be sinusoidal as shown in Fig. 5.8(a). The periodic pulse carrier is shown in Fig. 5.8(b).

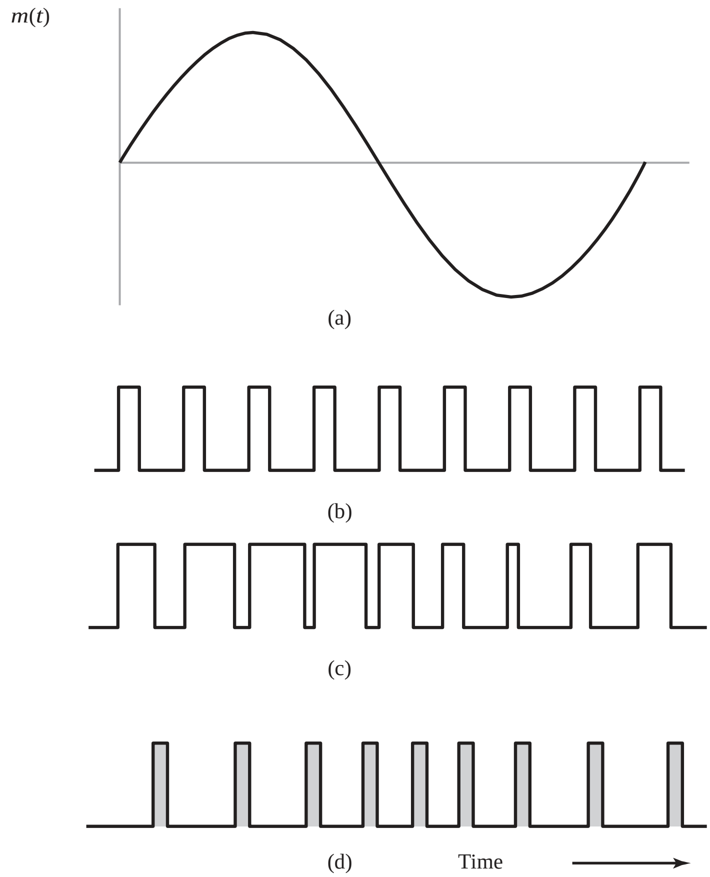


FIGURE 5.8 Illustration of two different forms of pulse-time modulation for the case of a sinusoidal modulating wave. (*a*) Modulating wave. (*b*) Pulse carrier. (*c*) PDM wave. (*d*) PPM wave.

5.5 PCM [Pulse Code Modulation]

PCM is an important method of analog –to-digital conversion. In this modulation the analog signal is converted into an electrical waveform of two or more levels. A simple two level waveform is shown in fig 5.9.

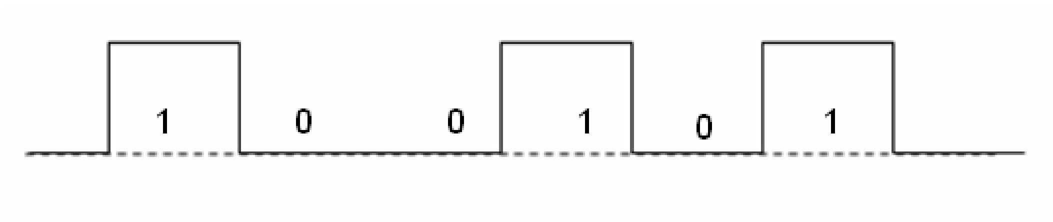


Fig:5.9 A simple binary PCM waveform

The PCM system block diagram is shown in fig 5.10 The essential operations in the transmitter of a PCM system are Sampling, Quantizing and Coding. The Quantizing and encoding operations are usually performed by the same circuit, normally referred to as analog to digital converter.

The essential operations in the receiver are regeneration, decoding and demodulation of the quantized samples. Regenerative repeaters are used to reconstruct the transmitted sequence of coded pulses in order to combat the accumulated effects of signal distortion and noise.

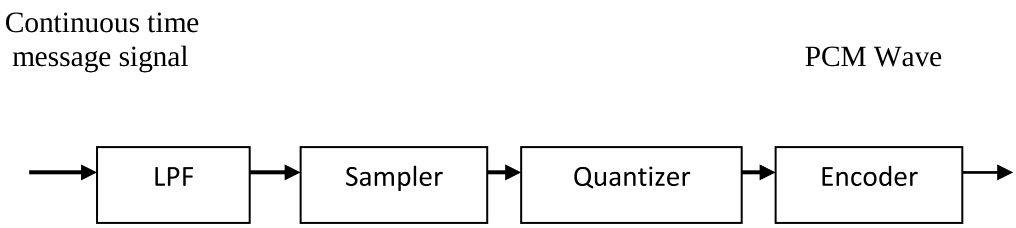
PCM Transmitter:

Basic Blocks:

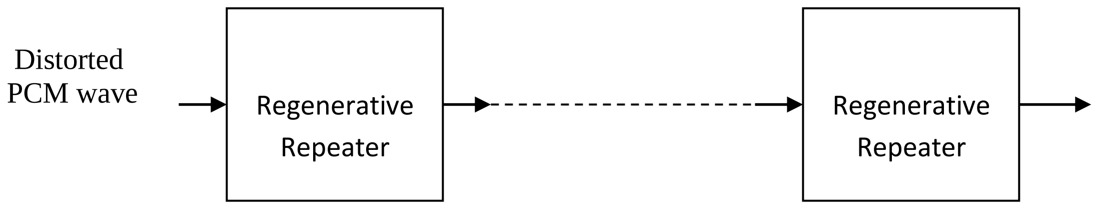
1. Anti aliasing Filter
2. Sampler
3. Quantizer
4. Encoder

An anti-aliasing filter is basically a filter used to ensure that the input signal to sampler is free from the unwanted frequency components.
For most of the applications these are low-pass filters. It removes the frequency components of the signal which are above the cutoff frequency of the filter. The cutoff frequency of the filter is chosen such it is very close to the highest frequency component of the signal.

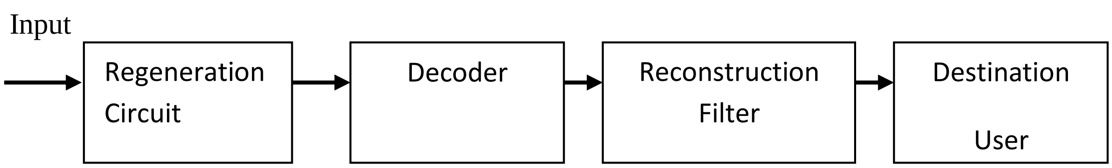
Sampler unit samples the input signal and these samples are then fed to the Quantizer which outputs the quantized values for each of the samples. The quantizer output is fed to an encoder which generates the binary code for every sample. The quantizer and encoder together is called as analog to digital converter.



(a) TRANSMITTER



(b) Transmission Path



(c) RECEIVER

Fig: 5.10 - PCM System : Basic Block Diagram

5.6 Quantization Process

The process of transforming Sampled amplitude values of a message signal into a discrete amplitude value is referred to as Quantization.

The quantization Process has a two-fold effect: the peak-to-peak range of the input sample values is subdivided into a finite set of decision levels or decision thresholds that are aligned with the risers of the staircase, and the output is assigned a discrete value selected from a finite set of representation levels that are aligned with the treads of the staircase.

A quantizer is memory less in that the quantizer output is determined only by the value of a corresponding input sample, independently of earlier analog samples applied to the input.

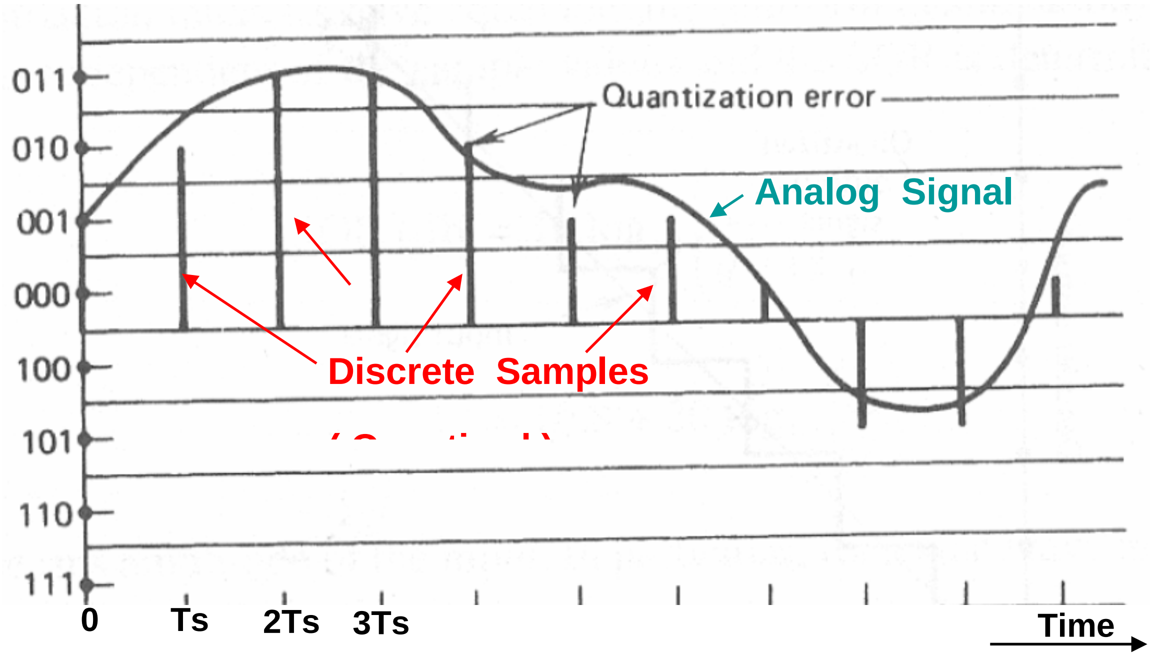


Fig:5.11 Typical Quantization process.

Types of Quantizers:

- Uniform Quantizer
- Non- Uniform Quantizer

In Uniform type, the quantization levels are uniformly spaced, whereas in non-uniform type the spacing between the levels will be unequal and mostly the relation is logarithmic.

Types of Uniform Quantizers: (based on I/P - O/P Characteristics)
Mid-Rise type Quantizer
Mid-Tread type Quantizer

In the stair case like graph, the origin lies the middle of the tread portion in Mid –Tread type where as the origin lies in the middle of the rise portion in the Mid-Rise type.

Mid – tread type: Quantization levels – odd number.
Mid – Rise type: Quantization levels – even number.

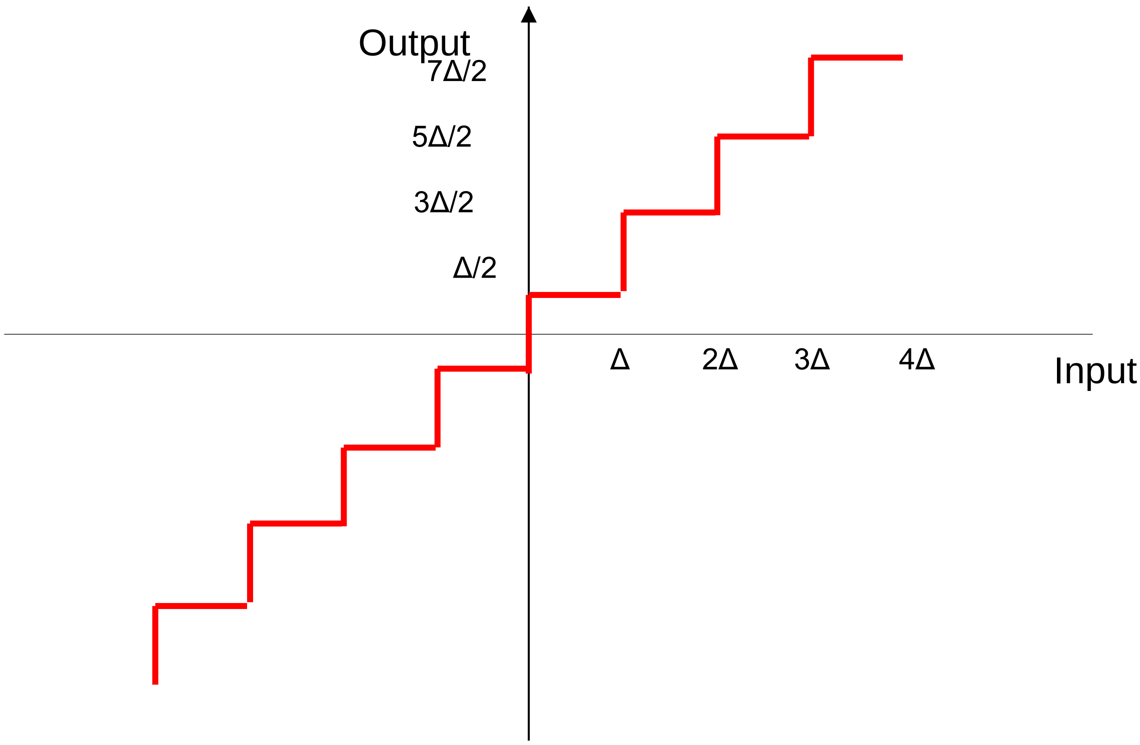


Fig:5.12 Input-Output Characteristics of a Mid-Rise type Quantizer

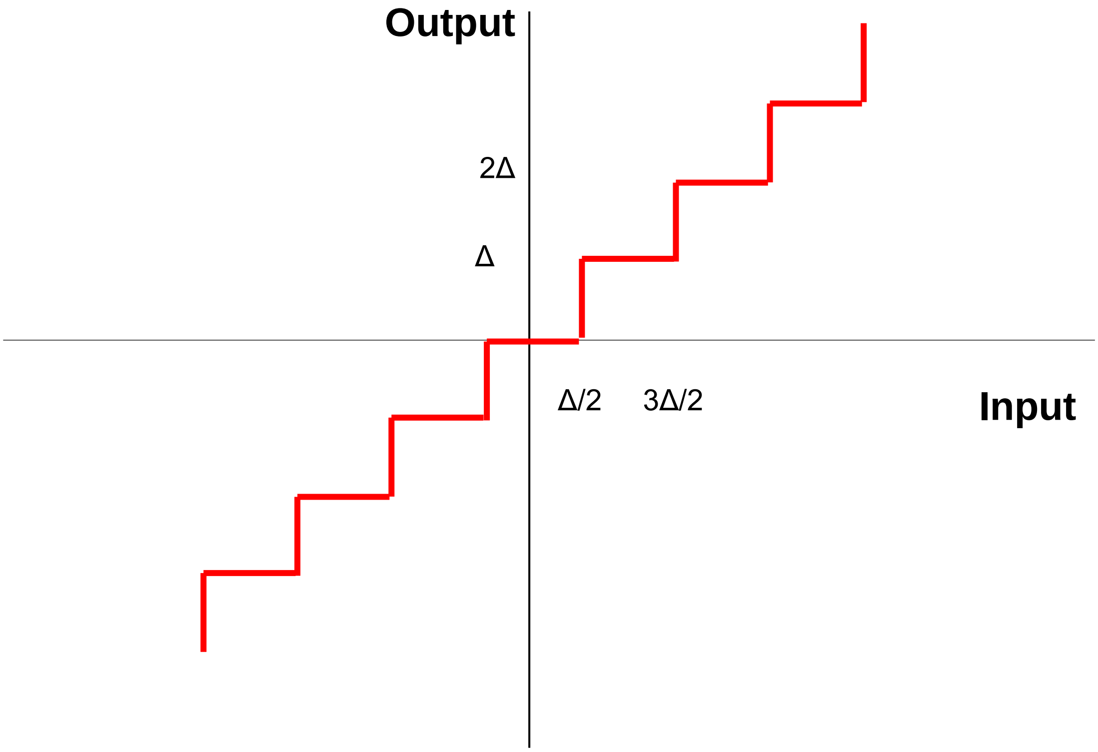


Fig:5.13 Input-Output Characteristics of a Mid-Tread type Quantizer

Compression Laws.

Two Commonly used logarithmic compression laws are called μ - law and A – law. μ-law:

In this companding, the compressor characteristics is defined by equation 3.29. The normalized form of compressor characteristics is shown in the figure 5.14. The μ-law is used for PCM telephone systems in the USA, Canada and Japan. A practical value for μ is 255.

$$\frac{c(|x|)}{x_{\max}} = \frac{\ln(1+\mu|x|/x_{\max})}{\ln(1+\mu)} \qquad 0 \le \frac{|x|}{x_{\max}} \le 1$$

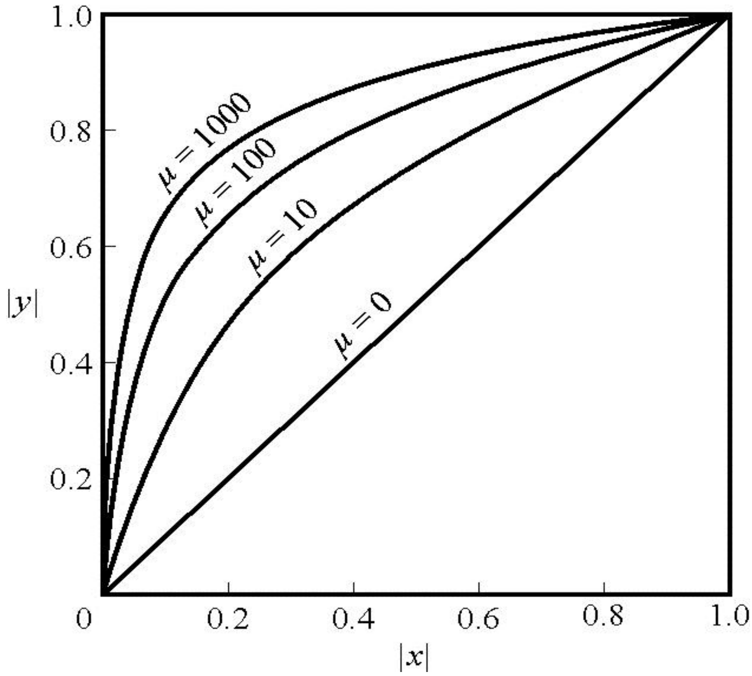


Fig: 5.14 Compression characteristics of μ-law

A-law: In A-law companding the compressor characteristics is defined by equation 3.30. The normalized form of A-law compressor characteristics is shown in the figure 5.15. The A-law is used for PCM telephone systems in Europe. A practical value for A is 100.

$$\frac{c(|x|)}{x_{\max}} = \begin{cases} \dfrac{A|x|/x_{\max}}{1+\ln A} & 0 \le \dfrac{|x|}{x_{\max}} \le \dfrac{1}{A} \\[2ex] \dfrac{1+\ln\left(A|x|/x_{\max}\right)}{1+\ln A} & \dfrac{1}{A} \le \dfrac{|x|}{x_{\max}} \le 1 \end{cases}$$

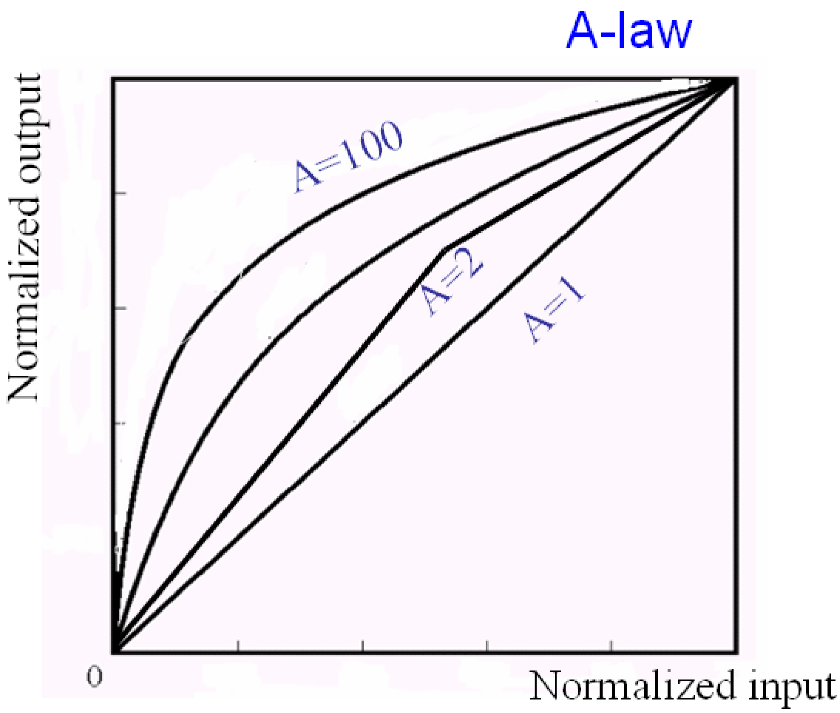


Fig. 5.15: A-law compression Characteristics.

Advantages of Non Uniform Quantizer

- Reduced Quantization noise
- High average SNR

5.7 Delta Modulation (DM)

Delta Modulation is a special case of DPCM. In DPCM scheme if the base band signal is sampled at a rate much higher than the Nyquist rate purposely to increase the correlation between adjacent samples of the signal, so as to permit the use of a simple quantizing strategy for constructing the encoded signal, Delta modulation (DM) is precisely such as scheme. Delta Modulation is the one-bit (or two-level) versions of DPCM.

DM provides a staircase approximation to the over sampled version of an input base band signal. The difference between the input and the approximation is quantized into only two levels, namely, $\pm\delta$ corresponding to positive and negative differences, respectively, Thus, if the approximation falls below the signal at any sampling epoch, it is increased by δ. Provided that the signal does not change too rapidly from sample to sample, we find that the stair case approximation remains within $\pm\delta$ of the input signal. The symbol δ denotes the absolute value of the two representation levels of the one-bit quantizer used in the DM. These two levels are indicated in the transfer characteristic of Fig 3.14. The step size Δ of the quantizer is related to δ by

$$\Delta = 2\delta$$

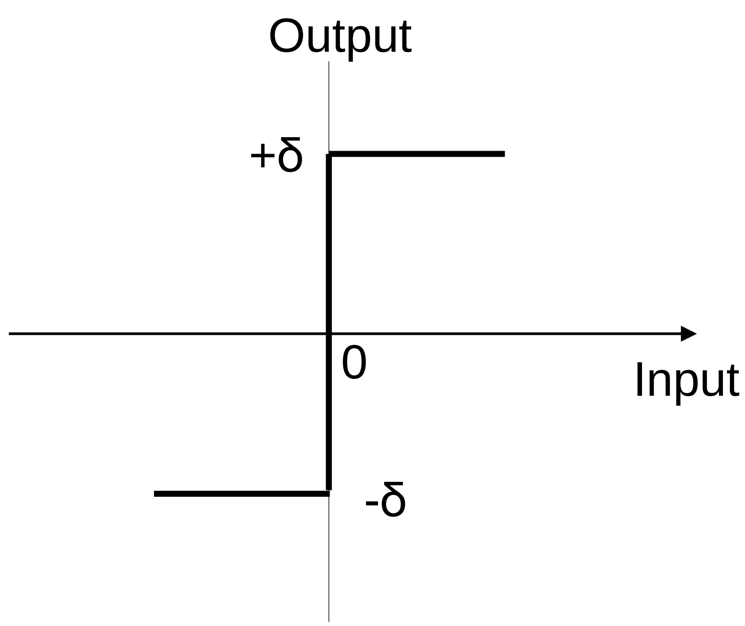


Fig-5.16: Input-Output characteristics of the delta modulator.

Let the input signal be x(t) and the staircase approximation to it is u(t). Then, the basic principle of delta modulation may be formalized in the following set of relations

$$e(nT_s) = x(nT_s) - x^{\wedge}(nT_s)$$
$$e(nTs) = x(nTs) - u(nTs - Ts)$$
$$b(nT_s) = \delta \operatorname{sgn}[e(nT_s)] \quad and$$
$$u(nT_s) = u(nT_s - T_s) + b(nT_s)$$

where T_s is the sampling period; $e(nT_s)$ is a prediction error representing the difference between the present sample value $x(nT_s)$ of the input signal and the latest approximation to it, namely $\hat{x}(nT_s) = u(nT_s - T_s)$.The binary quantity, $b(nT_s)$ is the one-bit word transmitted by the DM system.

The transmitter of DM system is shown in the figure3.15. It consists of a summer, a two-level quantizer, and an accumulator. Then, from the equations of we obtain the output as,

$$u(nTs) = \delta \sum_{i=1}^{n} \operatorname{sgn}[e(iTs)] = \sum_{i=1}^{n} b(iTs)$$

At each sampling instant, the accumulator increments the approximation to the input signal by ±δ, depending on the binary output of the modulator.

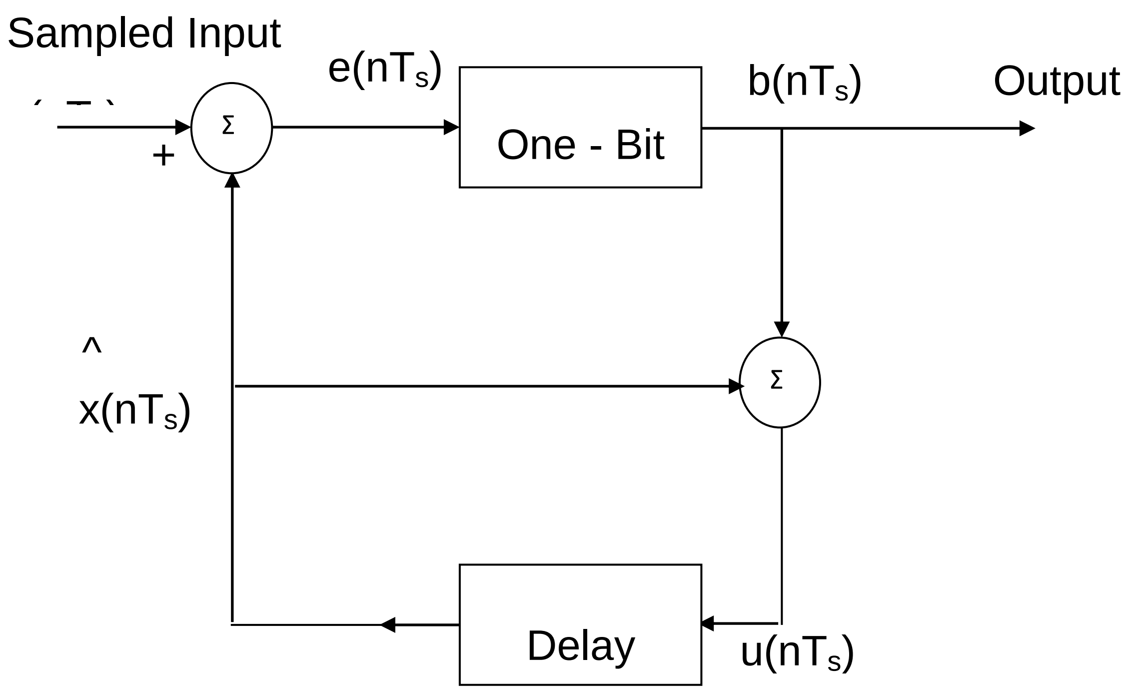


Fig 5.17 - Block diagram for Transmitter of a DM system

In the receiver, shown in fig, the stair case approximation u(t) is reconstructed by passing the incoming sequence of positive and negative pulses through an accumulator in a manner similar to that used in the transmitter. The out-of –band quantization noise in the high frequency staircase waveform u(t) is rejected by passing it through a low-pass filter with a band-width equal to the original signal bandwidth.

Delta modulation offers two unique features:

No need for Word Framing because of one-bit code word.
Simple design for both Transmitter and Receiver

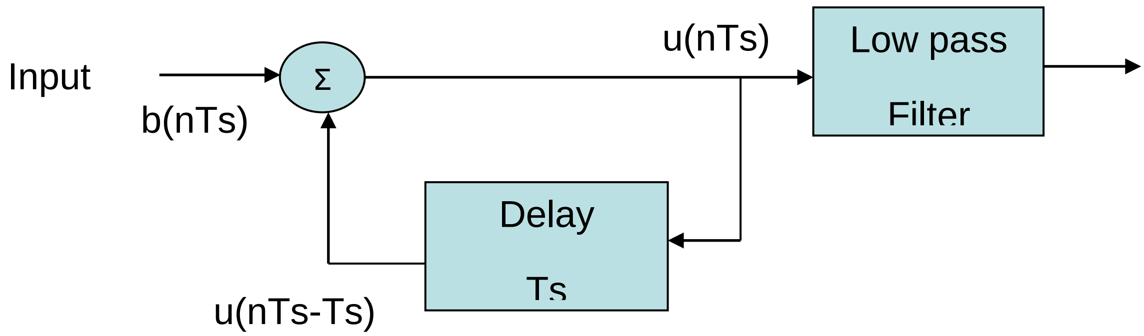


Fig 5.18 - Block diagram for Receiver of a DM system

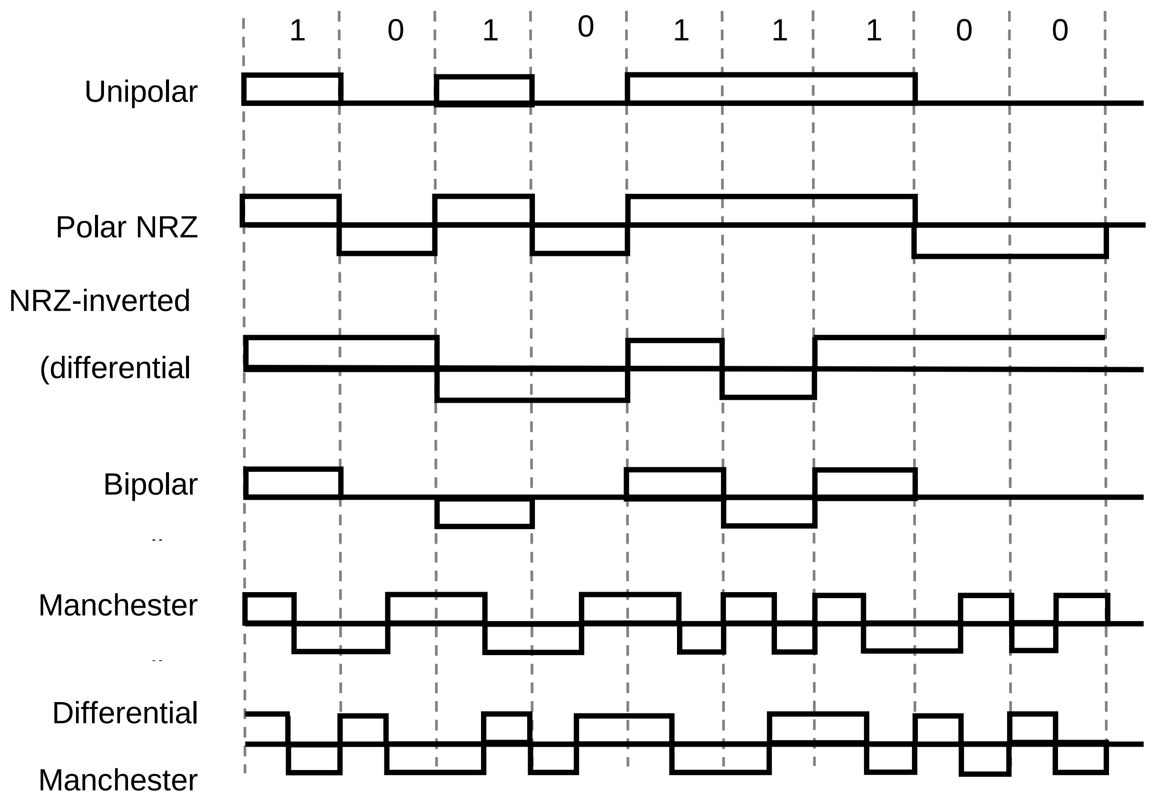
5.8 Line Codes

In base band transmission best way is to map digits or symbols into pulse waveform.

This waveform is generally termed as Line codes.

RZ: Return to Zero [pulse for half the duration of T_b]

NRZ Return to Zero[pulse for full duration of T_b]



5.9 Time-Division Multiplexing

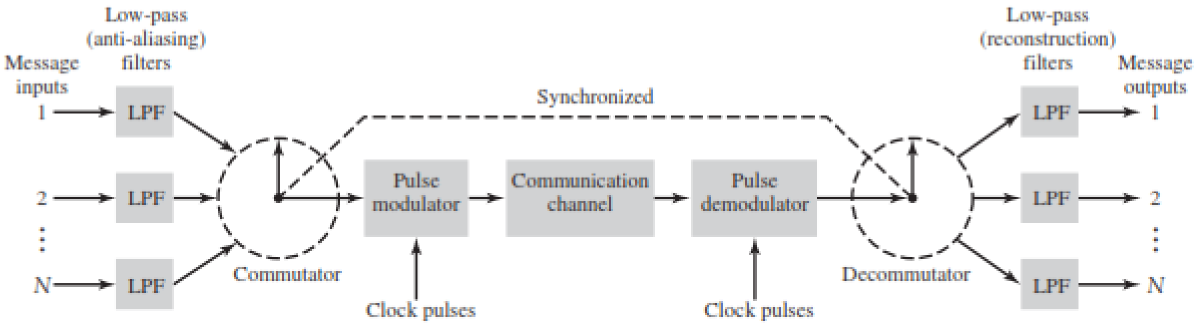


FIGURE 5.21 Block diagram of TDM system.

It's often practical to combine a set of low-bit-rate streams, each with a fixed and pre-defined bit rate, into a single high-speed bit stream that can be transmitted over a single channel. This technique is called time division multiplexing (TDM) and has many applications, including wireline telephone systems and some cellular telephone systems. The main reason to use TDM is to take advantage of existing transmission lines. It would be very expensive if each low-bit-rate stream were assigned a costly physical channel (say, an entire fiber optic line) that extended over a long distance.

Choosing the proper size for the time slots involves a trade-off between efficiency and delay. If the time slots are too small (say, one bit long) then the multiplexer must be fast enough and powerful enough to be constantly switching between sources (and the demultiplexer must be fast enough and powerful enough to be constantly switching between users). If the time slots are larger than one bit, data from each source must be stored (buffered) while other sources are using the channel. This storage will produce delay. If the time slots are too large, then a significant delay will be introduced between each source and its user. Some applications, such as teleconferencing and videoconferencing, cannot tolerate long delays.

5.10 Outcomes:

- The students will able to learn the importance of sampling process.
- PCM Scheme and different types of quantization.
- Importance of TDM.

5.11 Further Readings

TEXT BOOK:

- 1. Communication Systems, Simon Haykins, 5th Edition, John Willey, India Pvt. Ltd, 2009
- 2. An Introduction to Analog and Digital Communication, Simon Haykins, John Wiley India
- Pvt. Ltd., 2008

REFERENCE BOOKS:

- 1. Modern digital and analog Communication systems B. P. Lathi, Oxford University Press., 4th ed, 2010
- 2. Communication Systems, Harold P.E, Stern Samy and A Mahmond, Pearson Edn, 2004.
- 3. Communication Systems: Singh and Sapre: Analog and digital TMH 2nd , Ed 2007

5.12 Recommended Questions

1. State and prove sampling theorem.
2. Mention merits and demerits of digital communication
3. What is PAM? Explain its working and give practical application of it.
4. Explain time division multiplexing along with its advantages.
5. With neat block diagram explain DM and quantization noise in DM