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BASIC ELECTRONICS

(GEC101)

Unit-3

Operational Amplifiers

Introduction to Operational Amplifiers

- The operational amplifier, most commonly referred as '**op-amp**' was introduced in 1940s. The first operational amplifier was designed in 1948 using vacuum tubes.
- In those days, it was used in the analog computers to perform a variety of mathematical operations such as addition, subtraction, multiplication etc. Due to its use in performing mathematical operations it has been given a name operational amplifier. Due to the use of vacuum tubes, the early op-amps were bulky, power consuming and expensive.
- Robert J. Widlar at Fairchild brought out the popular 741 integrated circuit (IC) op-amp between 1964 to 1968. The IC version of op-amp uses BJTs and FETs which are fabricated along with the other supporting components, on a single semiconductor chip or wafer which is of a pinhead size.
- With the help of IC op-amp, the circuit design becomes very simple. The variety of useful circuits can be built without the necessity of knowing about the complex internal circuitry.
- Moreover, IC op-amps are inexpensive, take up less space and consume less power. The IC op-amp has become an integral part of almost every electronic circuit which uses linear integrated circuit. The modern linear IC op-amp works at **lower voltages**. It is so **low in cost** that millions are now in use, annually.
- Because of their **low cost**, **small size**, **versatility**, **flexibility**, and **dependability**, op-amps are used in the fields of process control, communications, computers, power and signal sources, displays and measuring systems.
- The op-amp is basically an excellent high gain d.c. amplifier.

Review Question

1. *What is operational amplifier ?*

Op-amp Symbol and Terminals

- The symbol for an op-amp along with its various terminals, is shown in the Fig.1.

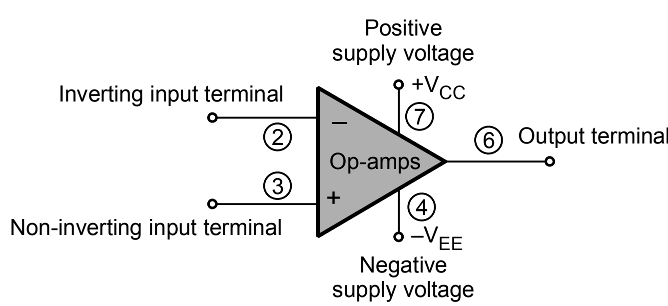

Fig.1 Op-amp symbol

- The op-amp is indicated basically by a triangle which points in the direction of the signal flow.

All the op-amps have minimum following five terminals :

i. The positive supply voltage terminal V_{CC} or + V.

ii. The negative supply voltage terminal $-V_{EE}$ or – V.

iii. The output terminal.

iv. The inverting input terminal, marked as negative.

v. The non-inverting input terminal, marked as positive.

- The input at inverting input terminal results in opposite polarity (antiphase) output. While the input at non-inverting input terminal results in the same polarity (phase) output. This is shown in the Fig. 2 (a) and (b).

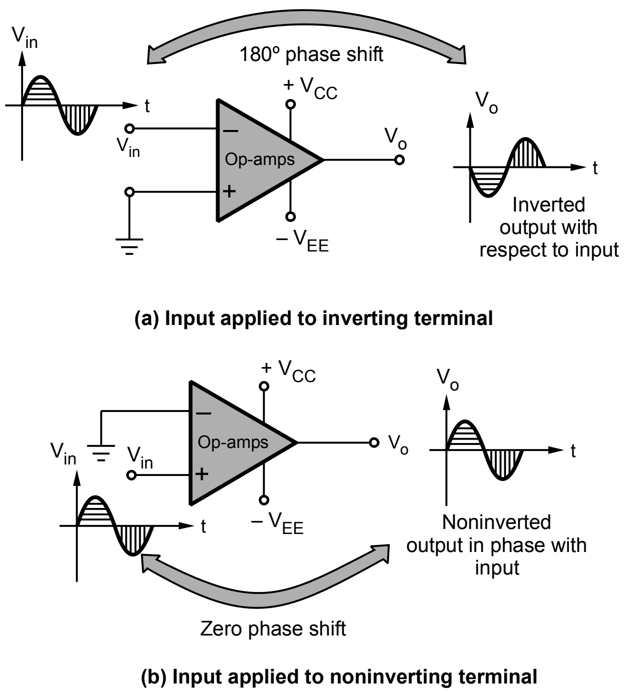

Fig. 2

- The input and output are in antiphase means having 180° phase difference in between them while inphase input and output means having 0° phase difference in between them.

- The op-amp works on a dual supply. A **dual supply** consists of two supply voltages both d.c., whose middle point is generally the ground terminal.
- The dual supply is generally balanced i.e. the voltages of the positive supply $+V_{CC}$ and that of the negative supply $-V_{EE}$ are same in magnitude. The typical commercially used power supply voltages are ± 15 V.
- But if the two voltage magnitudes are not same in a dual supply it is called as **unbalanced dual supply**.
- Practically in most of the op-amp circuits balanced dual supply is used. The other popular balanced dual supply voltages are ± 9 V, ± 12 V, ± 22 V etc.

Review Question

1. *With the op-amp symbol explain the importance of inverting and noninverting terminals.*

Ideal Op-amp

- The ideal op-amp is basically an amplifier which amplifies the difference between the two input signals. Hence it is called the **differential amplifier** or **difference amplifier.**
- Consider an ideal differential amplifier shown in the Fig. 1.

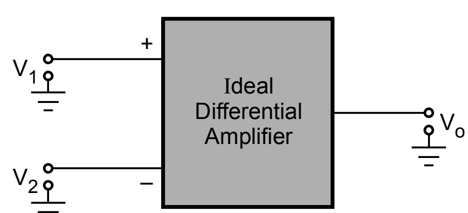

Fig.1 Ideal differential amplifier

- V_1 and V_2 are the two input signals while V_o is the single ended output. Each signal is measured with respect to the ground.
- In an ideal differential amplifier, the output voltage V_o is proportional to the difference between the two input signals. Hence we can write,

$$V_o \propto (V_1 - V_2) \quad \text{...(1)}$$

Differential Gain A_d

- From the equation (6.25.1) we can write,

$$V_o = A_d (V_1 - V_2)$$

where A_d is the constant of proportionality.

- The A_d is the gain with which differential amplifier amplifies the difference between two input signals. Hence it is called **differential gain. Thus,** A_d **= Differential gain.**
- The difference between the two inputs $(V_1 - V_2)$ is generally called **difference voltage** and denoted as V_d.

$$\therefore \quad \boxed{V_o = A_d V_d}$$

- Hence the differential gain can be expressed as,

$$\boxed{A_d = \frac{V_o}{V_d}}$$

- Generally the differential gain is expressed in its decibel (dB) value as,

$$\boxed{A_d = 20 \text{ Log}_{10} (A_d) \text{ in dB}}$$

Common Mode Gain A_c

- If we apply two input voltages which are equal in all the respects to the differential amplifier i.e. $V_1 = V_2$ then ideally the output voltage must be equal to zero.
- But the output voltage of the practical differential amplifier not only depends on the difference voltage but also depends on the average common level of the two inputs.
- Such an average level of the two input signals is called **common mode signal** denoted as V_c.

$$\therefore \quad \boxed{V_c = \frac{V_1 + V_2}{2}}$$

- Practically, the differential amplifier produces the output voltage proportional to such common mode signal, also.
- The gain with which op-amp amplifies the common mode signal to produce the output is called **common mode gain** of an op-amp denoted as A_c.

$$\therefore \quad \boxed{V_o = A_c V_c}$$

- Thus there exists some finite output for $V_1 = V_2$ due to such common mode gain A_c, in case of practical op-amps.
- So the total output of any differential amplifier can be expressed as,

$$\therefore \quad \boxed{V_o = A_d V_d + A_c V_c} \quad \text{... Total output}$$

- **For an ideal op-amp, the differential gain A_d must be infinite while the common mode gain must be zero.** This ensures zero output for $V_1 = V_2$.

Common Mode Rejection Ratio (CMRR)

- The ability of an op-amp to reject a common mode signal is expressed by a ratio called **common mode rejection ratio** denoted as CMRR or ρ.
- It is defined as the ratio of the differential voltage gain A_d to common mode voltage gain A_c.

$$\therefore \quad \boxed{\text{CMRR} = \rho = \left|\frac{A_d}{A_c}\right|}$$

- **Ideally the common mode voltage gain is zero, hence the ideal value of CMRR is infinite.**
- For a practical differential amplifier A_d is large and A_c is small hence the value of CMRR is also very large.
- Many a times, CMRR is also expressed in dB, as

$$\therefore \quad \boxed{\text{CMRR in dB} = 20 \log \left|\frac{A_d}{A_c}\right| \quad \text{dB}}$$

Review Questions

1. *Explain the function of differential amplifier.*
2. *Define the differential gain of a basic differential amplifier.*
3. *What is difference voltage ? How differential gain is expressed in decibels ?*
4. *Define common mode gain of a differential amplifier. What is its ideal value ?*
5. *Define CMRR of an op-amp. What is its ideal value ?*

Voltage Levels and Saturating Property of Op-amp

- The supply voltages of the op-amp are $+V_{CC}$ and $-V_{EE}$. These supply voltage levels decide the maximum output voltage levels of the op-amp.
- **Practically the op-amp output saturates at the voltages slightly less than the supply voltages $+V_{CC}$ and $-V_{EE}$.**
- Thus the output voltage of the op-amp can be driven to within 1 V of $+V_{CC}$ and $-V_{EE}$ before the output saturation takes place. This is called **saturating property** of the op-amp.
- Practically maximum output swing of the op-amp is 0.9 times the supply voltages.
- Thus the op-amp using $\pm$ 15 V supply has maximum output voltage swing as $0.9 \times (\pm 15\text{ V})$ i.e. $\pm$ 13.5 V.
- Ideally it is considered that the output voltage of the op-amp saturates at $+V_{CC}$ and $-V_{EE}$ levels.
- The output voltage levels are shown in the Fig. 1.

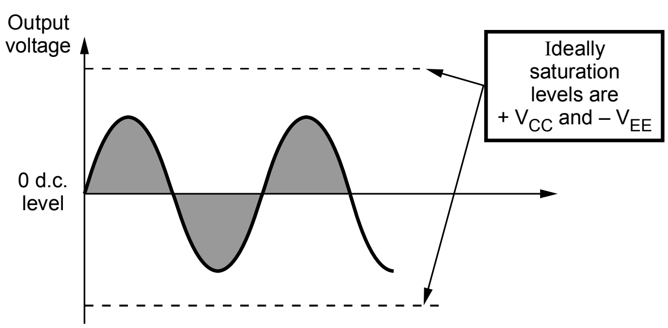

Fig. 1 Output levels with zero d.c. level

- The output of an op-amp saturates if tries to increase more than its supply voltages.
- If the output amplitude tries to increase more than the supply voltages, then the clipping occurs as shown in the Fig. 2 due to saturation.

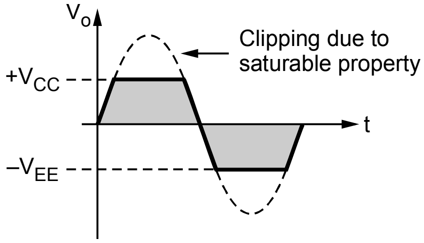

Fig 2

> **Review Question**
> 1. *What is saturating property of op-amp ?*

Ideal Op-amp Characteristics

- The Fig. 1 shows an ideal op-amp. It has two input signals V_1 and V_2 applied to non-inverting and inverting terminals, respectively.

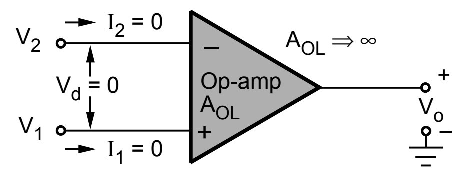

Fig. 1 Ideal op-amp

- The various characteristics of an ideal op-amp are :

a) Infinite voltage gain : ($A_{OL} = \infty$)

- It is denoted as A_{OL}. It is the differential open loop gain and is infinite for an ideal op-amp.

b) Infinite input impedance : ($R_{in} = \infty$)

- The input impedance is denoted as R_{in} and is infinite for an ideal op-amp. This ensures that no current can flow into an ideal op-amp.

c) Zero output impedance : ($R_o = 0$)

- The output impedance is denoted as R_o and is zero for an ideal op-amp. This ensures that the output voltage of the op-amp remains same, irrespective of the value of the load resistance connected.

d) Zero offset voltage : ($V_{ios} = 0$)

- The presence of the small output voltage though $V_1 = V_2 = 0$ is called an offset voltage. It is zero for an ideal op-amp. This ensures zero output for zero input signal voltage.

e) Infinite bandwidth : (BW = ∞)

- The range of frequency over which the amplifier performance is satisfactory is called its **bandwidth**. The bandwidth of an ideal op-amp is infinite. This means the operating frequency range is from 0 to ∞. This ensures that the gain of the op-amp will be constant over the frequency range from d.c. (zero frequency) to infinite frequency. So op-amp can amplify d.c. as well as a.c. signals.

f) Infinite CMRR : ($\rho = \infty$)

- The ratio of differential gain and common mode gain is defined as CMRR. Thus infinite CMRR of an ideal op-amp ensures zero common mode gain. Due to this common mode noise output voltage is zero for an ideal op-amp.

g) Infinite slew rate : (S = ∞)

- This ensures that the changes in the output voltage occur simultaneously with the changes in the input voltage.
- The slew rate is important parameter of op-amp. When the input voltage applied is step type which changes instantaneously then the output also must change rapidly as input changes. If output does not change with the same rate as input then there occurs distortion in the output. Such a distortion is not desirable.
- **Infinite slew rate indicates that output changes simultaneously with the changes in the input voltage**.
- The parameter slew rate is actually defined as the maximum rate of change of output voltage with time and expressed in V/µs.

$$\boxed{\text{Slew rate} = S = \left.\frac{dV_o}{dt}\right|_{\text{maximum}}}$$

- The slew rate is measured with unity gain circuits.
- Its ideal value is infinite for the op-amp.

h) No effect of temperature :

- The characteristics of op-amp do not change with temperature.

i) Power Supply Rejection Ratio : (PSRR = 0)

- The power supply rejection ratio is defined as the ratio of the change in input off set voltage due to the change in supply voltage producing it, keeping other power supply voltage constant. It is also called **power supply sensitivity or supply voltage rejection ratio (SVRR).**
- It is expressed in mV/V or µV/V and its ideal value is zero.
- These ideal characteristics of op-amp are summarized in the Table 1.

Characteristics	Symbol	Values
Open loop voltage gain	A_{OL}	∞
Input impedance	R_{in}	∞
Output impedance	R_o	0
Offset voltage	V_{ios}	0
Bandwidth	B.W.	∞
C.M.R.R	ρ	∞
Slew rate	S	∞
Power supply rejection ratio	PSRR	0

Table 6.27.1 Ideal op-amp characteristics

Review Questions

1. *Explain the ideal op-amp characteristics.*
2. *Explain the following ideal characteristics of op-amp : 1) CMRR 2) Slew rate 3) Band width.*
3. *Define : i) BW ii) PSRR iii) CMRR for an op-amp.*

Open Loop Configuration of Op-amp

- The simplest possible way to use an op-amp is in the open loop mode.
- The Fig. 1 shows an op-amp in the open loop condition.

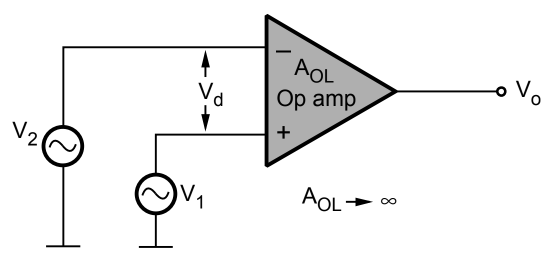

Fig.1 Open loop operation of an op-amp

- The d.c. supply voltages applied to the op-amp are V_{CC} and $-V_{EE}$ and the output varies linearily only between V_{CC} and $-V_{EE}$.

- Since open loop gain A_{OL} is very large, the output voltage V_0 is either at its positive saturation voltage $(+V_{sat})$ or negative saturation voltage $(-V_{sat})$ for very small values of V_d.
- This is shown in the Fig. 2.

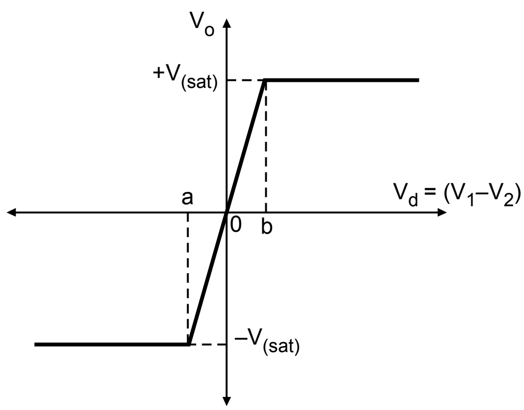

Fig. 2 Voltage transfer characteristics

- Thus very small noise voltage present at the input also gets amplified due to its high open loop gain and op-amp gets saturated.
- It can be seen from the Fig. 2, only for small range of input signal (from point a to b), it behaves linearly.
- This range is very small and practically due to high open loop gain, op-amp either shows $+V_{sat}$ or $-V_{sat}$ level.
- This indicates the inability of op-amp to work as a linear small signal amplifier in the open loop mode.
- Hence, the op-amp is generally not used in the open loop configuration.
- Such an open loop behaviour of the op-amp finds some rare applications like voltage comparator, zero crossing detector etc.

Review Questions

1. *Why op-amp can not be used in open loop configuration for linear applications ?*
2. *Why is it necessary to reduce the gain of op-amp from its open loop value ?*

Op-amp with Negative Feedback

- The utility of op-amp increases considerably if it is used in a closed loop mode
- The closed loop mode is possible using feedback. The feedback allows to feed some part of the output back to the input.
- In linear applications the op-amp is always used with negative feedback. The feedback helps to control gain which otherwise drives op-amp into saturation.
- The negative feedback is possible by adding a resistor as shown in the Fig. 1, called **feedback resistor**.

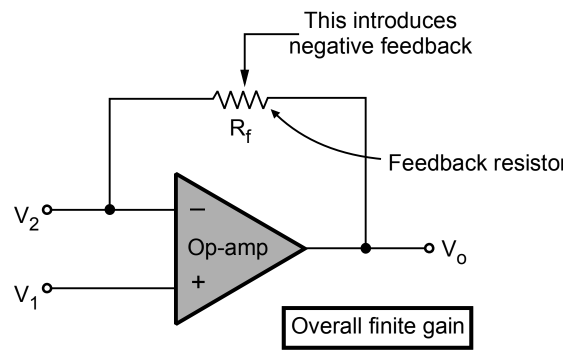

Fig.1 Op-amp with negative feedback

- The feedback is said to be negative as the feedback resistor connects the output to the inverting input terminal.
- The gain resulting with feedback is called **closed loop gain** of the op-amp.
- Most of the linear circuits use op-amp in a closed loop mode with negative feedback with R_f. This is because, due to reduced gain, the output is not driven into the saturation and the circuit behaves in a linear manner.

Review Question

1. *How op-amp is used in the closed loop configuration ?*

Realistic Simplifying Assumptions

- We can make two assumptions which are realistic and simplify the analysis of op-amp circuits to a great extent.

1. Zero Input Current :

- The current drawn by either of the input terminals (inverting and noninverting) is zero.

2. Virtual Ground :

- This means the differential input voltage V_d between the noninverting and inverting input terminals is essentially zero.
- Even if output voltage is few volts, due to large open loop gain of op-amp, the difference voltage V_d at the input terminals is almost zero.

$$V_o = V_d A_{OL}$$

i.e. $$V_d = \frac{V_o}{A_{OL}}$$

= Almost zero as A_{OL} is very very high

$$\therefore \quad V_d = (V_1 - V_2) = 0$$

i.e. $$V_1 = V_2 \qquad \dots (1)$$

- Thus we can say that under linear range of operation there is virtually short circuit between the two input terminals, in the sense that their voltages are same.
- The Fig. 1 shows the concept of the virtual ground. The thick line indicates the virtual short circuit between the input terminals.

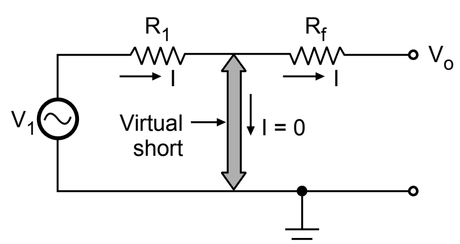

Fig. 1 Concept of virtual ground

- Now if the non-inverting terminal is grounded, by the concept of virtual short, the inverting terminal is also at ground potential, though there is no physical connection between the inverting terminal and the ground. This is the **principle of virtual ground**.

- The steps of analysis based on these assumptions are,

Step 1 : Input current of the ideal op-amp is always zero. Using this, the current distribution in the circuit is obtained.

Step 2 : The input terminals of the op-amp are always at the same potential. Thus if one is grounded, the other can be treated to be virtually grounded. From this, the expressions for various branch currents can be obtained.

Step 3 : Analyzing the various expressions obtained, eliminating unwanted variables, the output expression in terms of input and circuit parameters can be obtained.

Review Questions

1. *Define concept of virtual ground with respect to op-amp.*
2. *State the realistic assumptions related to op-amp and state their use.*

Op-amp Applications

- The countless simple circuits using one or more operational amplifiers, some external resistors and the capacitors can be constructed. Such op-amp applications are classified as linear and nonlinear type of applications.
- In the linear applications, output voltage varies linearily with respect to the input voltage. The negative feedback is the base of linear applications.
- Some of the linear applications are voltage follower, differential amplifier, instrumentation amplifier, inverting amplifier, non-inverting amplifier etc.
- The nonlinear input to output characteristics is the feature of nonlinear applications. The typical nonlinear applications are precision rectifiers, comparators, clampers, limiters, Schmitt trigger circuit etc.

Review Question

1. *What is an op-amp ? Mention some of its applications.*

Ideal Inverting Amplifier

- An amplifier which provides a phase shift of 180º between input and output is called **inverting amplifier**.
- The basic circuit diagram of an inverting amplifier using op-amp is shown in the Fig. 1 (a).

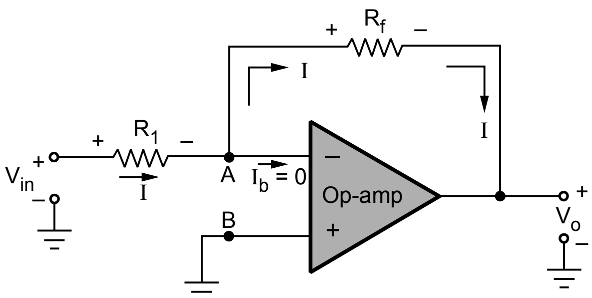

Fig. 1 (a) Inverting amplifier

- By the concept of virtual ground the two input terminals are always at the same potential.
- As node B is grounded, node A is also at ground potential, from the concept of virtual ground, so $V_A = 0$

$$\therefore \quad I = \frac{V_{in} - V_A}{R_1}$$

$$= \frac{V_{in}}{R_1} \quad (\text{as } V_A = 0) \qquad \dots (1)$$

- The op-amp input current is always zero hence entire current I passes through the resistance R_f.
- Now from the output side, considering the direction of current I we can write,

$$I = \frac{V_A - V_o}{R_f}$$

$$= \frac{-V_o}{R_f} \quad (\text{as } V_A = 0) \qquad \dots (2)$$

- Equating (1) and (2) we get, $\frac{V_{in}}{R_1} = -\frac{V_o}{R_f}$

$$\therefore \quad \boxed{A_V = \frac{V_o}{V_{in}} = -\frac{R_f}{R_1} \text{ (Gain with feedback)}} \qquad \dots (3)$$

- The $\frac{R_f}{R_1}$ is the gain of the amplifier while negative sign indicates that the polarity of output is opposite to that of input. Hence it is called **inverting amplifier.**
- The waveforms of inverting amplifier are shown in the Fig. 1 (b).

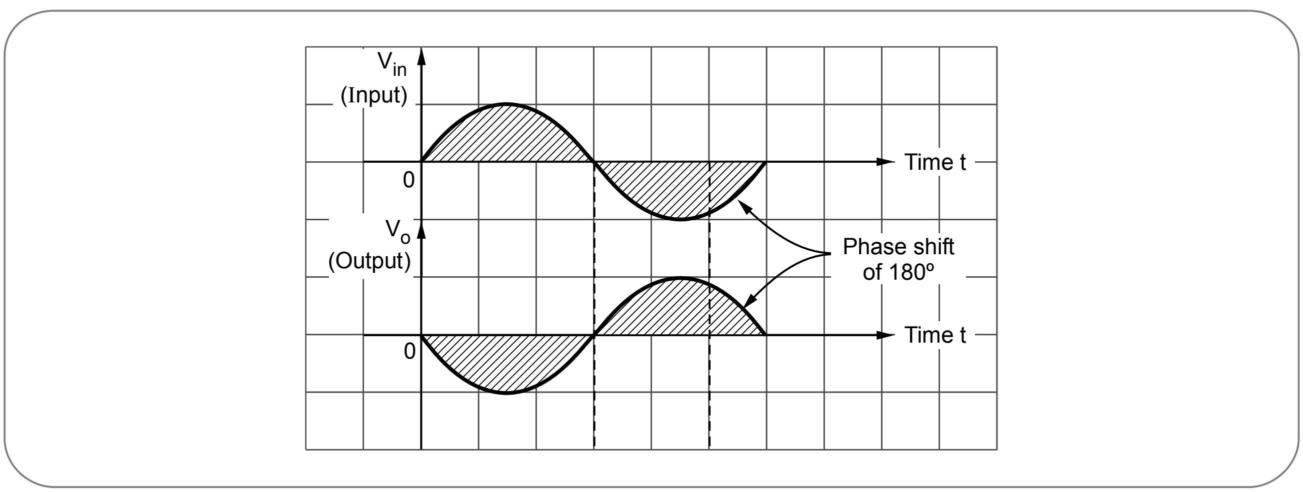

Fig. 1 (b) Waveforms of inverting amplifier

Observations :

1. The output is inverted with respect to input, which is indicated by minus sign.
2. The voltage gain is independent of open loop gain of the op-amp, which is assumed to be large.
3. The voltage gain depends on the ratio of the two resistances. Hence selecting R_f and R_1, the required value of gain can be easily obtained.
4. If $R_f > R_1$, the gain is greater than 1.

   If $R_f < R_1$, the gain is less than 1.

   If $R_f = R_1$, the gain is unity.

   Thus the output voltage can be greater than, less than or equal to the input voltage, in magnitude.
5. If the ratio of R_f and R_1 is K which is other than one, the circuit is called **scale changer** while for $R_f/R_1 = 1$ it is called **phase inverter**.
6. The closed loop gain is denoted as A_{VF} or A_{VCL} i.e. gain with feedback.

Ex. 1 *An op-amp is used in inverting mode with $R_1 = 1\ k\Omega$, $R_f = 15\ k\Omega$, $V_{CC} = \pm 15\ V$. Calculate the output voltage for the following inputs -*
i) $V_{in} = 150\ mV$ ii) $V_{in} = 2\ V$

Sol. : $R_1 = 1\ k\Omega$, $R_f = 15\ k\Omega$, $V_{CC} = \pm 15\ V$

In the inverting mode,

$$A = -\frac{R_f}{R_1} = -\frac{15}{1} = -15 = \frac{V_o}{V_{in}}$$

i) $V_{in} = 150\ mV$

$\therefore$ $V_o = A\ V_{in} = -15 \times 150 \times 10^{-3}$

$= \mathbf{-0.225\ V}$

ii) $V_{in} = 2\ V$

$\therefore$ $V_o = A\ V_{in} = -15 \times 2 = -30\ V$

But output saturates at $\pm$ 15 V hence in this case the output will be saturated as -15 V and **– 30 V is not practically possible**.

$\therefore$ $V_o =$ **– 15 V saturated output**

Review Questions

1. *Prove that for an inverting amplifier,*

$$A_V = -\frac{R_f}{R_1}$$

2. *Why an inverting amplifier is called scale changer ? Explain with the help of neat circuit diagram and derivation of expression*

Ideal Non-Inverting Amplifier

- An amplifier which amplifies the input without producing any phase shift between input and output is called **Non-inverting amplifier**.
- The basic circuit diagram of a noninverting amplifier using op-amp is shown in the Fig. 1 (a).

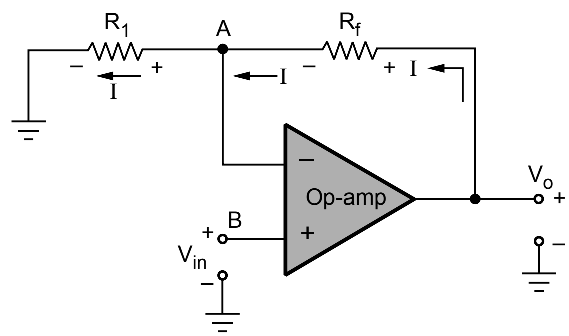

Fig. 1 (a) Non-inverting amplifier

- The input is applied to the noninverting input terminal of the op-amp.
- The node B is at potential V_{in}, hence the potential of point A is same as B which is V_{in}, from the concept of virtual ground.

$\therefore$ $V_A = V_B = V_{in}$

- From the output side we can write,

$$I = \frac{V_o - V_A}{R_f} = \frac{V_o - V_{in}}{R_f}$$

$$(\text{As } V_A = V_{in}) \quad \ldots (1)$$

- At the inverting terminal,

$$I = \frac{V_A - 0}{R_1}$$

$$= \frac{V_{in}}{R_1} \quad (\text{As } V_A = V_{in}) \quad \ldots (2)$$

- Entire current passes through R_1 as input current of op-amp is zero.

- Equating equations (6.33.1) and (6.33.2),

$$\frac{V_o - V_{in}}{R_f} = \frac{V_{in}}{R_1}$$

$$\therefore \quad \frac{V_o}{R_f} = \frac{V_{in}}{R_f} + \frac{V_{in}}{R_1} = V_{in}\left[\frac{(R_1 + R_f)}{R_1 \, R_f}\right]$$

$$\therefore \quad \frac{V_o}{V_{in}} = \frac{(R_1 + R_f)\,R_f}{R_1 \, R_f} = \frac{R_1 + R_f}{R_1}$$

$$\boxed{A_V = \frac{V_o}{V_{in}} = 1 + \frac{R_f}{R_1}} \quad \ldots (3)$$

- The positive sign indicates that there is no phase shift between input and output.
- The waveforms of the non-inverting amplifier are shown in the Fig. 1 (b).

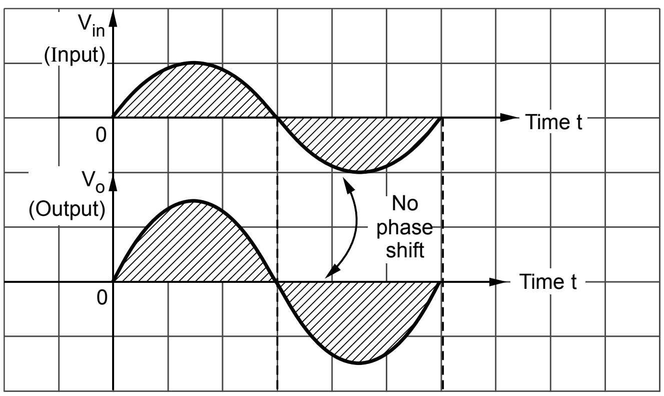

Fig. 1 (b) Waveforms of non-inverting amplifier

Important Point Regarding Non-inverting Amplifier :

- In non-inverting amplifier the input may not be applied directly to the non-inverting terminal as considered while deriving the output expression but it may be applied through some circuit.
- Let V_{in} is the input voltage applied to the non-inverting amplifier through some resistive network such that the voltage available at the non-inverting input terminal is V_B which is different than V_{in}. Then the **non-inverting amplifier always amplifies voltage available to its non-inverting terminal by the factor** $\left(1 + \frac{R_f}{R_1}\right)$**. Hence** $V_o = \left(1 + \frac{R_f}{R_1}\right) V_B$.

The Table 1 provides the comparison of the ideal inverting and noninverting amplifier op-amp circuits.

Sr. No.	Ideal inverting amplifier	Ideal noninverting amplifier
1.	Voltage gain = $-R_f / R_1$	Voltage gain = $1 + (R_f / R_1)$
2.	The output is inverted with respect to input.	No phase shift between input and output.
3.	The voltage gain can be adjusted as greater than, equal to or less than one.	The voltage gain is always greater than one.
4.	The input impedance is R_1 .	The input impedance is extremely large.

Table 1

Ex. 1 *An op-amp is used in noninverting mode with* $R_1 = 1\ k\Omega$, $R_f = 12\ k\Omega$, $V_{CC} = \pm 15\ V$. *Calculate output voltage for i)* $V_{in} = 250\ mV$ *ii)* $V_{in} = 3\ V$

Sol. : $R_1 = 1\ k\Omega$, $R_f = 12\ k\Omega$, $V_{CC} = \pm 15\ V$

For noninverting mode,

$$A = \left(1 + \frac{R_f}{R_1}\right) = \left(1 + \frac{12}{1}\right) = \mathbf{13} = \frac{V_o}{V_{in}}$$

i) $V_{in} = 250\ mV$

$\therefore \quad V_o = A\ V_{in} = 13 \times 250 \times 10^{-3} = \mathbf{3.25\ V}$

ii) $V_{in} = 3\ V$

$\therefore \quad V_o = A\ V_{in} = 13 \times 3 = \mathbf{39\ V}$

But op-amp output saturates at $\pm V_{CC}$ i.e. ± 15 V. Thus practically **output saturates at + 15 V** and **39 V output is not practically possible**.

Review Question

1. *Draw non-inverting amplifier using op-amp and derive expression for its output voltage.*

Integrators

- Consider the op-amp integrator circuit as shown in the Fig. 6.34.1.
- The node B is grounded. The node A is also at the ground potential from the concept of virtual ground.

$\therefore \qquad V_A = 0 = V_B$

- As input current of op-amp is zero, the entire current I flowing through R_1, also flows through C_f, as shown in the Fig. 1.

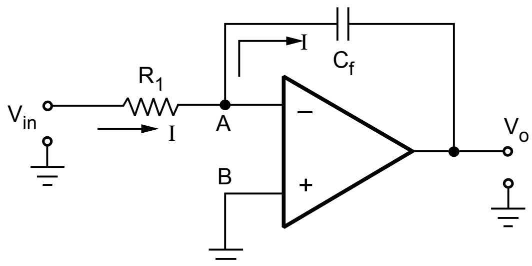

Fig. 1 Op-amp integrator

From input side we can write,

$$I = \frac{V_{in} - V_A}{R_1} = \frac{V_{in}}{R_1} \qquad \dots (1)$$

From output side we can write,

$$I = C_f \frac{d(V_A - V_o)}{dt}$$

i.e.
$$I = -C_f \frac{dV_o}{dt} \qquad \dots (2)$$

Equating the two equations (1) and (2),

$$\frac{V_{in}}{R_1} = -C_f \frac{dV_o}{dt} \qquad \dots (3)$$

Integrating both sides,

$$\int_0^t \frac{V_{in}}{R_1} dt = -C_f \int \frac{dV_o}{dt} \cdot dt$$

i.e.
$$\int_0^t \frac{V_{in}}{R_1} dt = -C_f V_o \qquad \dots (4)$$

$$\therefore \quad \boxed{V_o = -\frac{1}{R_1 C_f} \int_0^t V_{in}\, dt + V_o(0)} \qquad \dots (5)$$

where $V_o(0)$ is the constant of integration, indicating the initial output voltage.

- The equation (5) shows that the output is $-1/R_1C_f$ times the integral of input and R_1C_f is called **time constant** of the integrator.
- The negative sign indicates that there is a phase shift of 180° between input and output.
- Sometimes a resistance R_{comp}=R_1 is connected to the noninverting terminal to provide the bias compensation. This is shown in the Fig. 2.

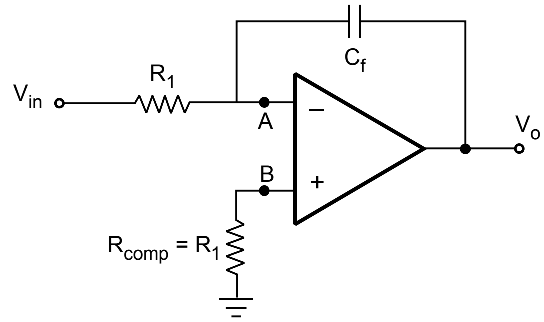

Fig. 2 Integrator with bias compensation

- As the input current of op-amp is zero, the node B is still can be treated at ground potential in this circuit.

> **Key Point :** Hence the above analysis is equally applicable to the integrator circuit with bias compensation. And the output is the perfect integration of the input.

Input and Output Waveforms

- Let us see the output waveforms, for various input signals. For simplicity of understanding, assume that the time constant $R_1C_f = 1$ and the initial voltage $V_o(0) = 0$ V.

i) Step input signal

- Let the input waveform is of step type, with a magnitude of A units as shown in the Fig. 3.

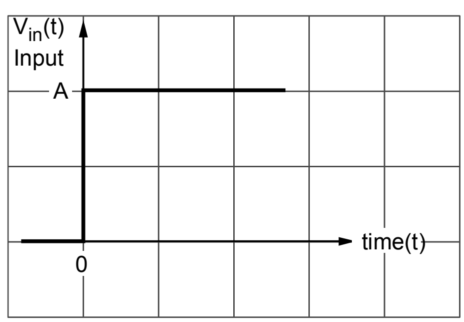

Fig. 3 Step input signal

- Mathematically the step input can be expressed as,

$$\boxed{\begin{aligned} V_{in}(t) &= A \text{ for } t \geq 0 \\ \text{And } &= 0 \text{ for } t < 0 \end{aligned}} \qquad \dots (6)$$

From equation (5), with $R_1C_f = 1$ and $V_o(0) = 0$ we can write,

$$\therefore \quad V_o(t) = -\int_0^t V_{in}(t)\,dt = -\int_0^t A\,dt$$

$$= -A\int_0^t dt = -A\,[t]_o^t = -At \quad \ldots(7)$$

- Thus output waveform is a straight line with a slope of –A where A is magnitude of the step input. The output waveform is shown in the Fig. 4.

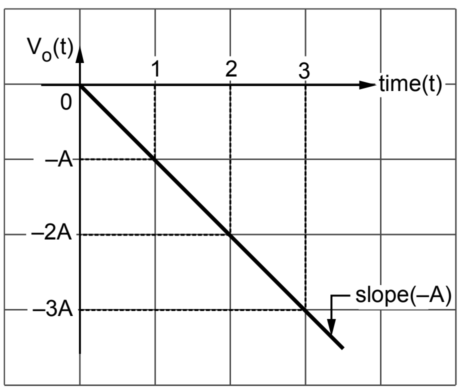

Fig. 4 Output waveform for step input

ii) Square wave input signal

- Let the input waveform is a square wave as shown in the Fig. 5.

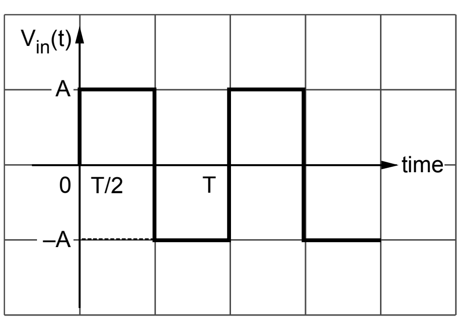

Fig. 5 Square wave input signal

- It can be observed that the square wave is made up of steps i.e. a step of A between time period of 0 to T/2 while a step of – A units between a time period of T/2 to T and so on.

Mathematically it can be expressed as,

$$\boxed{\begin{aligned} V_{in}(t) &= A, \quad 0 < t < T/2 \\ &= -A, \quad T/2 < t < T \end{aligned}} \quad \ldots(8)$$

- This is the expression for the input signal for one period.
- For the period 0 to T/2 output will be straight line with slope – A as discussed for step input. From t = T/2 till t = T, the slope of the straight line will become – (–A) i.e. + A.
- So the output can be expressed mathematically for one period as,

$$\boxed{\begin{aligned} V_o(t) &= -At \quad 0 < t < T/2 \\ &= +At \quad T/2 < t < T \end{aligned}} \quad \ldots(9)$$

The output waveform is shown in the Fig. 6.

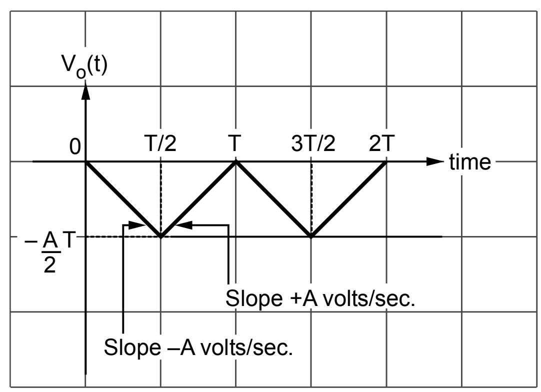

Fig. 6 Output waveform for square wave input

iii) Sine wave input signal

- Let the input waveform is purely sinusoidal with a frequency of ω rad/sec.
- Mathematically it can be expressed as,

$$\boxed{V_{in}(t) = V_m \sin \omega t} \quad \ldots(10)$$

where V_m is the amplitude of the sine wave and T be the period of the waveform.

- To find the output waveform, use the equation (6.34.5) with $R_1C_f = 1$ and $V_o(0) = 0$ V.

$$\therefore \quad V_o(t) = -\int V_{in}\,dt = -\int V_m \sin \omega t\,dt$$

$$= -V_m\left[\frac{1}{\omega}(-\cos \omega t)\right]$$

$$\therefore \qquad \boxed{V_o(t) = -\frac{V_m}{\omega}(-\cos\omega t)} \qquad \ldots (11)$$

- Thus it can be seen that the output of an integrator is a cosine waveform for a sine input.
- Due to inverting integrator, the output waveform is as shown in the Fig.7.

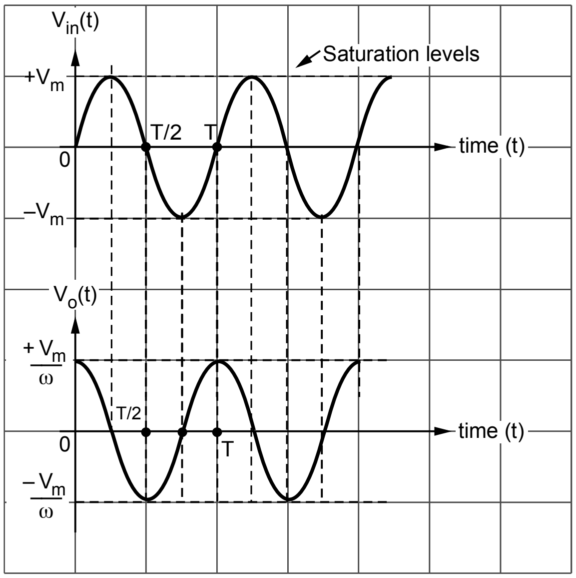

Fig.7 Sine wave input and cosine output

Applications of Practical Integrator

The integrator circuits are most commonly used in the following applications :

a) In the analog computers.

b) In solving the differential equations.

c) In analog to digital converters.

d) Various signal wave shaping circuits.

e) In ramp generators.

Review Questions

1. *Draw the circuit diagram of the op-amp integrator and derive the expression for its output voltage.*
2. *Draw the output waveforms of an integrator for i) Step ii) Square and iiii) Sine wave inputs*
3. *List the applications of an integrator.*

Differentiators

- The op-amp differentiator circuit is shown in the Fig. 1.

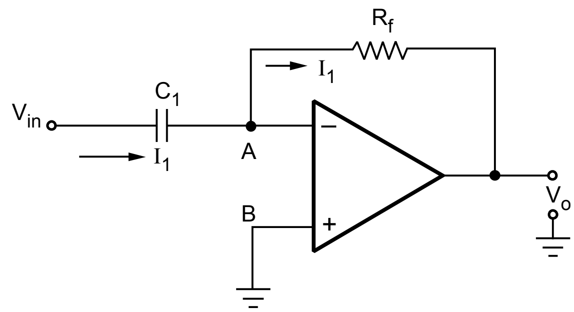

Fig. 1 Op-amp differentiator

- The node B is grounded. The node A is also at the ground potential hence $V_A = 0$.
- As input current of op-amp is zero, entire current I_1 flows through the resistance R_f.
- From the input side we can write,

$$I_1 = C_1 \frac{d(V_{in} - V_A)}{dt}$$

$$= C_1 \frac{dV_{in}}{dt} \qquad \ldots (1)$$

From the output side we can write,

$$I = \frac{(V_A - V_o)}{R_f} = -\frac{V_o}{R_f} \qquad \ldots (2)$$

Equating the two equations,

$$C_1 \frac{dV_{in}}{dt} = -\frac{V_o}{R_f} \qquad \ldots (3)$$

$$\boxed{V_o = -C_1 R_f \frac{dV_{in}}{dt}} \qquad \ldots (4)$$

- The equation shows that the output is C_1R_f times the differentiation of the input and product C_1R_f is called **time constant** of the differentiator.
- The negative sign indicates that there is a phase shift of 180° between input and output. The main advantage of such an active differentiator is the small time constant required for differentiation.
- In practice a resistance $R_{comp} = R_f$ is connected to the noninverting terminal to provide the bias compensation. This is shown in the Fig. 2.

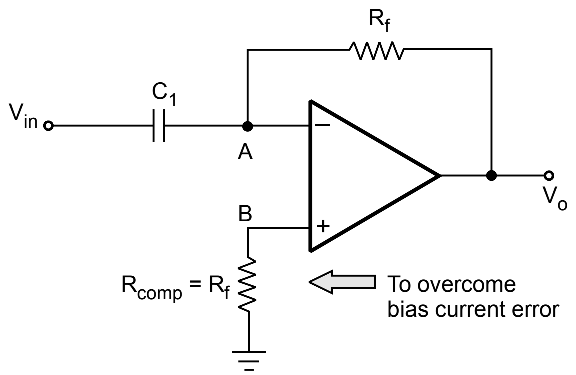

Fig. 2 Differentiation with bias compensation

Input and Output Waveforms

Let us study the output waveforms, for various input signals.

For simplicity of understanding, assume that the values of R_f and C_1 are selected to have time constant ($R_f C_1$) as unity.

i) Step input signal

- Let the input waveform is of step type with a magnitude of A units. Mathematically it is expressed as,

$$V_{in}(t) = A \quad \text{for} \quad t \geq 0 \quad \text{and} \quad = 0 \text{ for } t < 0 \qquad \dots (5)$$

Now mathematically, the output of the differentiator must be,

$$V_o(t) = -\frac{d\,V_{in}}{dt} = -\frac{d\,(A)}{dt} = 0 \qquad \text{(as A is constant)} \qquad \dots (6)$$

- Actually the step input takes a finite time to rise from 0 to A volts.
- Due to this finite time, the differentiator output is not zero but appears in the form of a spike at t = 0.
- As the circuit acts as an inverting differentiator, the negative going spike or impulse appears at t = 0 and after that output remains zero.
- Both input and output waveforms of the differentiator with a step input, are shown in the Fig. 3.

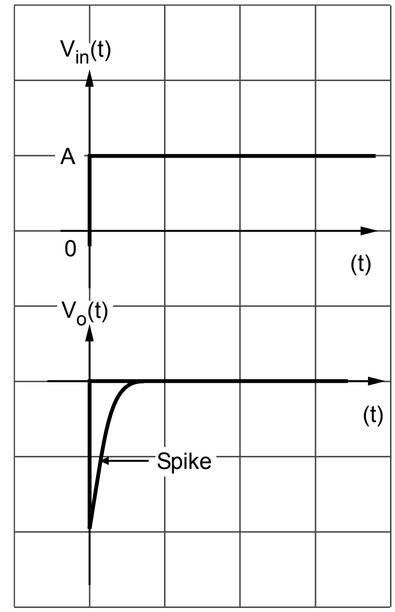

Fig. 3 Input and output for step input

ii) Square wave input signal

- The square wave is made of steps i.e. step of A volts from t = 0 to t = T/2, while a step of –A volts from t = T/2 to t = T and so on.
- Mathematically it can be expressed as,

$$V_{in}(t) = A \qquad 0 < t < T/2 \qquad \dots (7)$$
$$= -A \qquad T/2 < t < T$$

- The differentiator behaves similar to its behaviour to step input.
- For positive going impulse, the output shows negative going impulse and for negative going input, the output shows positive going impulse.
- Hence the total output for the square wave input is in the form of train of impulses or spikes.
- The input and output waveforms are shown in the Fig. 4.

iii) Sine wave input

- Let the input waveform be purely sinusoidal with a frequency of ω rad/sec. Mathematically it can be expressed as,

$$\boxed{V_{in}(t) = V_m \sin \omega t} \qquad \dots (8)$$

where V_m is the amplitude of the sine wave and T is the period of the waveform.

$$V_o(t) = \frac{-d\,V_{in}(t)}{dt}$$

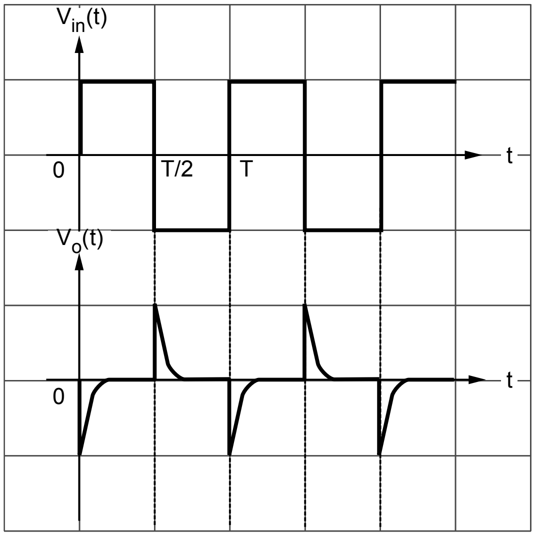

Fig. 4 Input and output for square wave input

for $\quad R_f C_1 = 1 = -\frac{d}{dt}(V_m \sin \omega t)$

$$\therefore \quad \boxed{V_o(t) = -V_m \cdot \omega \cos \omega t} \quad \dots (9)$$

So at $\quad t = 0, \quad V_o(t) = -V_m \omega$

while $\quad t = \frac{T}{2}, \quad V_o(t) = +V_m \omega$

- Thus the output of the differentiator is a consine waveform, for a sine wave input. The input and output waveform is shown in the Fig. 5.

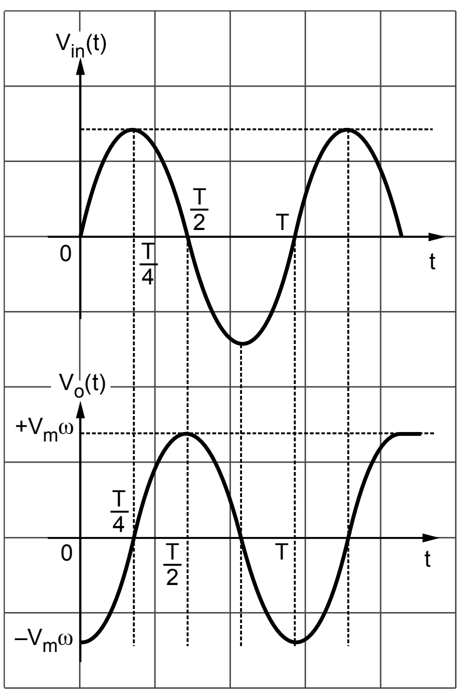

Fig. 5 Input and output for sine wave input

Applications of Practical Differentiator

- The practical differentiator circuits are most commonly used in :
  i) In the wave shaping circuits to detect the high frequency components in the input signal.
  ii) As a rate of change detector in the FM demodulators.
- The differentiator circuit is avoided in the analog computers.

Review Questions

1. *Draw the circuit diagram of the op-amp differentiator and derive the expression for its output voltage.*
2. *Draw the output waveforms of op-amp differentiatior if its input waveform is :*
   a) Step b) Square c) Sine.
3. *List the applications of differentiators.*