Unit 4

Mathematical reasoning is a critical skill that enables us to analyse a given hypothesis without any reference to a particular context or meaning. In layman's words, when a scientific inquiry or statement is examined, the reasoning is not based on an individual's opinion. Derivations and proofs require a factual and scientific basis.

Mathematical critical thinking and logical reasoning are important skills that are required to solve maths reasoning questions.

 Basic terns that are used in a mathematical reasoning-

1) Statement-

Any sentence in mathematics which follows the following rules is a statement.

- A sentence needs to be either true or false but not to be considered a mathematically accepted statement.
- Any sentence which is either imperative or interrogative or exclamatory cannot be considered a mathematically validated statement.
- A Sentence containing one or many variables is termed an open statement. An open statement can become a statement if the variables present in the sentence are replaced by definite values.

Example: The distance from the centre of a circle to any point on the circumference of the circle is equal.

2) Conjunction and Disjunction-

Whenever statements are joined to make a new statement and all the conditions need to be fulfilled, it is a Conjunction. 'And', 'with' are commonly used to join such statements.

Whenever statements are joined to make a new statement and only one of the conditions needs to be fulfilled, it is a Disjunction. 'Or', 'But' are commonly used to join such statements.

The conjunction is true only if the original statements are found to be true.
The conjunction is false if the original statement or statements are found to be false.
The conjunction is true if only one statement is found to be true.
The conjunction is false if none of the original statements are found to be true.

Example: Square is a polygon, and a parallelogram can also be a square.

These are a few mathematical terminologies that will help you comprehend and apply mathematical reasoning. These terms will also help you solve and understand reasoning questions.

Types of mathematical reasoning-

A) Inductive reasoning-

Inductive reasoning is based on observations and not any hypothesis. If any phenomena are observed for n number of times, it can be generalized. This generalization is based on observation and therefore it may be false. Inductive reasoning is a logical guess which can be backed up by using valid reasons.

This type of reasoning is not used in geometry, for instance, one may observe a few right triangles and conclude all triangles to be right triangles. Therefore, other mathematical tools are used to prove geometrical results. An example of inductive reasoning will help elucidate the concept.

Example of Inductive Reasoning:

Statement: I picked a ball from the bag, and it happens to be a red ball. I picked a second red ball. A third ball from the bag is also red. Therefore, all the balls in the bag are red.

Reasoning: All the balls picked up from the bag are red. Therefore, we can say all the balls are red. This is an example of inductive reasoning where existing data is analysed to come to a general conclusion.

B) Deductive reasoning-

Deductive reasoning is based on the exact opposite principles of induction. Unlike Inductive reasoning, Deductive reasoning is not based on simple generalizations. A Hypothesis is required or a statement that has to be true under specified conditions for deductive reasoning to be valid. In the case of Inductive reasoning, the conclusion may be false but Deductive reasoning is true in all cases.

Therefore, Deductive reading is used for geometrical and mathematical proofs. The following example will simplify the concepts discussed in this section.

Example of Deductive Reasoning:

Statement: The sum of angles in a triangle is always equal to 180° and ABC is a Triangle.

Reasoning: Here in the given statement, we are considering two hypotheses, where the sum of angles in a triangle is said to be 180° and ABC is a triangle. Based on the given hypotheses we deduce that the sum of angles of ABC is 180°.

<p style="text-align:center">C) <mark>Abductive reasoning-</mark></p>

Abductive reasoning is a modified version of Inductive Reasoning and takes a more practical approach. In the case of inductive reasoning, the data or observation is complete but in real situations, most of the data is not available at the time of making a decision. So based on the data and its availability, the conclusion may vary, and reasoning may change.

Statement: The heights of four students studying in a class were found to be 160cm, 162cm, 163 cm, 167 cm respectively. The measuring scale available had the least count of 1cm.

Reasoning: As per the data and hypotheses available at the time of observation, the average height comes out to be 163cm. But once a new measuring scale was installed the least count was found to be 0.1 cm and the recorded height of students changed. This also impacted the Average height which came to be 63.8 cm.

As discussed in this section, reasoning techniques are categorized in three major sections. An understanding of Inductive, Abductive and Deductive reasoning will help you solve any reasoning question. It is important to identify the reasoning technique which has to be used to solve a question from examination point of view.

What is mathematically reasoning statement?
A Mathematical statement is a statement written in such a way that it can either be true or false but can never be both true and false simultaneously.

## <mark>Types of reasoning statement-</mark>

There are three main types of reasoning statements:

- o   <mark>Simple Statements</mark>
- o   <mark>Compound Statements</mark>
- o   <mark>If-Then Statement</mark>s

### <mark>simple statement</mark>

Simple statements are those types of statements that are direct and do not cover any modifier. These statements are somewhat comfortable to work on and do not need much reasoning. In other words, a statement is said to be simple if it cannot be split down into two or more statements. Some examples of the simple statement are:

- o   The sunsets in the west.
- o   8 is an even number.
- o   A rectangle has two sides equal.
- o   Delhi is the capital of India.

### <mark>compound statement</mark>

With the aid of certain connectives, we can combine different statements. Such a statement made up of two or more statements is identified as a compound statement. In other words, a compound statement is one that is made up of two or more simple statements. The connectives can be "and", "or", etc. Some examples of the compound statement are:

**Example 1:**

"The number 13 is both an odd and prime number" can be split into two statements "13 is an odd number" and "13 is a prime number" therefore it is a compound statement.

**Example 2:**

- o   Even numbers are divisible by 2.
- o   2 is an even number.

The above two statements can be combined together as a compound statement:

Even numbers are divisible by 2 and 2 is also an even number.

Note: The simple statements which create a compound statement are defined as component statements.

According to mathematical reasoning, if we encounter an if-then statement. i.e., 'if a then b', then by proving that a is true, b can be proved to be true or if we prove that b is false, then a is also false.

If we encounter a statement which says 'a if and only if b', then we can give reason for such a statement by showing that if a is true, then b is also true and if b is true, then a is also true.

**Example:**

a: 8 is multiple of 64.

b: 8 is a factor of 64.

Since one of the given statements i.e., a is true, therefore, a or b is true.

The number and letter series refers to the series that arranges both numbers and letters in sequential order so that every term in that specific series is calculated with particular specific rules. These rules are taken from the mathematical functions and applied to get the right answer. This series is the logical preparation of English alphabets and numbers in a particular mathematical pattern.

 Some tips for cracking the letter and number series are to break the whole complex series into simple ones, focus on the change in series with successive patterns and eliminate the wrong choices that help the candidates for solving complex number-letter series in the examination. These series must follow a pattern of division and multiplication of the given numbers and solve the missing series.

**Types of Number Series-**

**1. Addition Series:** In this type of number series reasoning, specific numbers based on some pattern are added to get the next number.

**2. Subtraction Series:** In this type of number series reasoning, specific numbers based on some pattern are subtracted to get the next number.

**3. Multiplication Series:** In this type of number series reasoning, a particular type of number pattern is multiplied to get the next number.

**4. Division Series:** In this type of number series reasoning, a particular type of number pattern is divided to get the next number.

**5. Square Series:** In this type of number series reasoning, each number is a perfect square of a particular number pattern.

**6. Cube Series:** In this type of number series reasoning, each number is a perfect cube of a particular number pattern.

**7. Fibonacci Series:** In this type of number series reasoning, the next number is the addition of two previous numbers.

**8. Alternating Series:** In this type of number series reasoning, multiple number patterns are used alternatively to form a series.

**9. Mixed Operator Series:** In this type of number series reasoning, multiple operators are applied to get the next number in the series.

**10. Arranging Number:** In this type of number series reasoning, candidates need to rearrange numbers, as specified, and then answer the given questions.

Types of letter (alphabetic) series-

There are three types of alphabetic series-

*1. Positional series*

In the positional series, the terms of the series form a certain pattern as per the position of the letters in the English alphabet.

**Example Question:** Find the next term in the series: A, C, E, G, ?

**Solution:** As we can see, the first, second, third letters of the series are moved two places forward to obtain the successive terms of the series.

So the missing term would be two letter ahead of 'G' which is 'I'. Therefore 'I' is the missing term in the series.

**Common mistakes made:** While attempting questions of alphabetical shifting, students try to save up time by trying to find the answer through mental calculations which, can be very inefficient and time-consuming. Hence, it is highly recommended to write down the alphabets first and then search for the pattern.

*2. Continuous pattern series*

These types of questions consist of a series of letters, which follow a certain pattern within which some letters are missing. Hence, the task of the examinee is to find those omitted letters such that the series start following a prominent pattern.

**Example Question:** Complete the series _a_ba_b_ab

1. abba
2. baab
3. bbab
4. aaab

**Solution:** Our 1st step is to find any possible pattern between the consecutive letter which in this case is that 'a' is followed by a 'b'; and also 'b' is followed by an 'a'.

So from that we can say that the first blank should a 'b'. Now the 3rd blank could be a 'a' so that we can have 'baab' or a 'b' so that we have the pattern 'bbabb'.

So the plausible answers would be 'baab' or 'bbbb'. Since 'bbbb' is not given, that means the answer would be 2.

*3. <mark>Alpha-Numeric series</mark>*

These types of questions are a mixed form of numeric and alphabetical series. Here, the terms of the given series are a combination of letters and numerals, which move according to a set pattern.

**Example Question:** Find the next term in the alpha-numeric series:

 A2Z, C3X, E5V, G7T, I11R, K13P, ?

**Solution:** The patterns followed by the letters are as follows

<mark>Codes</mark>-

Coded relationship contains problems that involve interpreting a given relationship-string, which is coded in a particular fashion and then matching it with the relationship mentioned in the question.

<mark>types of coding</mark>

**1)<mark>Letter/Number coding</mark>**

In this type of coding, we deal with questions, in which the letters of a word are replaced by certain other letters or numbers according to a specific pattern/rule to form a code. You are required to detect the coding pattern / rule and answer the questions that are asked, based on that coding pattern / rule.

**2) <mark>Direct coding</mark>**

In direct coding, the code letters/numbers/symbols occur in the same sequence as the corresponding letters occur in the words. This is basically a direct substitution method.

**3) <mark>Fictitious language coding</mark>**

Here, coded form of two or more sentences is given and you are required to find the code of a particular word or message.

To analyse such codes, any two messages bearing a common word/number are picked up. The common code will represent that particular word/number.

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Proceeding similarly by picking up all possible combinations of two, the entire message can be decoded and the codes for every individual word/number can be found.

In some cases, there is no common word/number in the message given. In those cases, the words/numbers are coded separately. You have to identify the correct pattern of coding and find the code for the word or message given in the question.

**4) <mark>Substitution coding</mark>**

In this type of coding, some particular words are coded with certain substituted word and on the basis of substituted word, the code of given word is derived.