The L.P.P. is defined as

Minimize $z = -6x_1 - 5x_2 - 3x_3 - 4x_4$ subject to

$x_1 + x_2 \le 5$

$3x_1 + 2x_2 \le 12$

$x_3 + 2x_4 \le 8$

$2x_3 + x_4 \le 10$

$x_1 + x_2 + x_3 + x_4 \le 7$

$2x_1 + x_2 + x_3 + 3x_4 \le 17$

→ this Prob. can be written as

$\min z = c_1^T X_1 + c_2^T X_2$: $c_1^T = [-6, -5]$

subject to : $c_2^T = [-3, -5]$

$A_1 X_1 + A_2 X_2 \le b_0$: $A_1 = \begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix}$

$B_1 X_1 \le b_1$: $A_2 = \begin{bmatrix} 1 & 1 \\ 1 & 3 \end{bmatrix}$

$B_2 X_2 \le b_2$

$B_1 = \begin{bmatrix} 1 & 1 \\ 3 & 2 \end{bmatrix}$

$X_1 = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$, $X_2 = \begin{bmatrix} x_3 \\ x_4 \end{bmatrix}$, $B_2 = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$

$b_0 = \begin{bmatrix} 7 \\ 17 \end{bmatrix}$, $b_1 = \begin{bmatrix} 5 \\ 12 \end{bmatrix}$, $b_2 = \begin{bmatrix} 8 \\ 10 \end{bmatrix}$.

$X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

Ans here 1st subproblem is

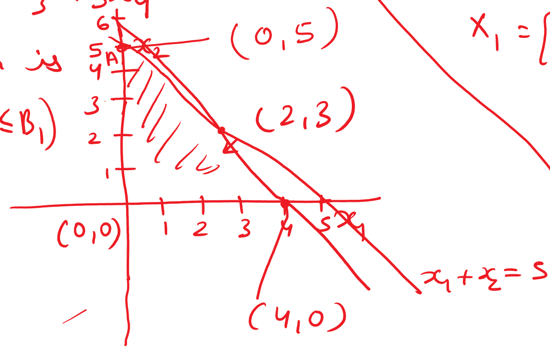
$x_1 + x_2 \le 5$

$3x_1 + 2x_2 \le 12$ → $(A_1 x_1 \le B_1)$

$2x_1 + 2x_2 \le 10$

$x_1 = 2$

$x_2 = 3$



So, Extrem points for first subproblem $(x_1^{(1)}, x_2^{(1)}, x_3^{(1)}, x_4^{(1)})$

$= \{(0,0), (4,0), (0,5), (2,3)\}$

So, write, $X_1 = \beta_{1,1} x_1^{(1)} + \beta_{1,2} x_2^{(1)} + \beta_{1,3} x_3^{(1)} + \beta_{1,4} x_4^{(1)}$

with $\beta_{1,1} + \beta_{1,2} + \beta_{1,3} + \beta_{1,4} = 1$

$\Rightarrow X_1 = \beta_{1,1}\begin{bmatrix} 0 \\ 0 \end{bmatrix} + \beta_{1,2}\begin{bmatrix} 4 \\ 0 \end{bmatrix} + \beta_{1,3}\begin{bmatrix} 0 \\ 5 \end{bmatrix} + \beta_{1,4}\begin{bmatrix} 2 \\ 3 \end{bmatrix}$

$X_1 = \begin{bmatrix} 4\beta_{1,2} + 2\beta_{1,4} \\ 5\beta_{1,3} + 3\beta_{1,4} \end{bmatrix}$, with $\beta_{1,1} + \beta_{1,2} + \beta_{1,3} + \beta_{1,4} = 1$.

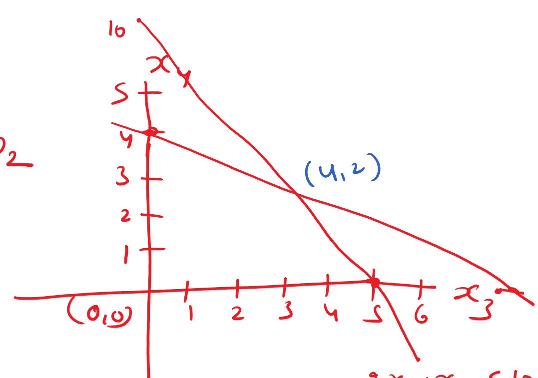
now, second sub problem is

$x_3 + 2x_4 \le 8$

$2x_3 + x_4 \le 10$ $\Rightarrow$ $B_2 X_2 \le b_2$

So, Extrem points $(x_1^{(2)}, x_2^{(2)}, x_3^{(2)}, x_4^{(2)})$

$= \{(0,0), (5,0), (0,4), (4,2)\}$



$X_2 = \beta\ x_1^{(2)} + \beta\ x_2^{(2)} + \beta\ x_3^{(2)} + \beta\ x_4^{(2)}$

$$x_2 = \beta_{2,1} x_1^{(2)} + \beta_{2,2} x_2^{(2)} + \beta_{2,3} x_3^{(2)} + \beta_{2,4} x_4^{(2)}$$

$$= \beta_{2,1}\begin{bmatrix}0\\0\end{bmatrix} + \beta_{2,2}\begin{bmatrix}5\\0\end{bmatrix} + \beta_{2,3}\begin{bmatrix}0\\4\end{bmatrix} + \beta_{2,4}\begin{bmatrix}4\\2\end{bmatrix}$$

$$= \begin{bmatrix}5\beta_{2,2} + 4\beta_{2,4}\\ 4\beta_{2,3} + 2\beta_{2,4}\end{bmatrix} \text{ with}$$

$$\beta_{2,1} + \beta_{2,2} + \beta_{2,3} + \beta_{2,4} = 1.$$

$$x_3 + 2x_4 = 8$$
$$4x_3 + 2x_4 = 20$$
$$-3x_3 = -12$$
$$x_3 = 4$$
$$4 + 2x_4 = 8$$
$$x_4 = 2$$

So, above problem can be write as

$$\text{Minimize } z = [-6, -5]\begin{bmatrix}4\beta_{1,2} + 2\beta_{1,4}\\ 5\beta_{1,3} + 3\beta_{1,4}\end{bmatrix} + [-3, -4]\begin{bmatrix}\beta_{2,2} + 4\beta_{2,4}\\ 4\beta_{2,3} + 2\beta_{2,4}\end{bmatrix}$$

subject to

$$\begin{bmatrix}1 & 1\\ 2 & 1\end{bmatrix}\begin{bmatrix}4\beta_{1,2} + 2\beta_{1,4}\\ 5\beta_{1,3} + 3\beta_{1,4}\end{bmatrix} + \begin{bmatrix}1 & 1\\ 1 & 3\end{bmatrix}\begin{bmatrix}\beta_{2,2} + 4\beta_{2,4}\\ 4\beta_{2,3} + 2\beta_{2,4}\end{bmatrix} \le \begin{bmatrix}7\\17\end{bmatrix}$$

$$\beta_{1,1} + \beta_{1,2} + \beta_{1,3} + \beta_{1,4} = 1$$
$$\beta_{2,1} + \beta_{2,2} + \beta_{2,3} + \beta_{2,4} = 1.$$

or