Algebra of Matrices:-

Matrix:- A matrix A is a rectangular array of scalars ($a_{ij} \in \mathbb{R}, \mathbb{C}$), usually presented into the following form

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}_{m \times n}$$

→ The horizontal lines are row
→ The vertical lines are column

Various types of Matrices:-

(i) **Row matrix:-** If a matrix has only one row and any number of column, it is called a Row matrix

e.g. $A = [2 \;\; 3, \;\; 4]$

(ii) **column matrix:-** If a matrix has only one column and any number of row, it is called a column matrix.

e.g $A = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$

(iii) **Null Matrix or zero Matrix:-** A matrix in which ~~the number of rows is equal to the number of columns, is~~ all the elements are zero, is called a null or zero matrix

e.g $A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

(iv) **Square Matrix:-** A matrix, in which the number of rows is equal to the number of column is called a square matrix. $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$

(v) **Diagonal Matrix:-** A square matrix is called a diagonal matrix, if all its non diagonal elements are zero.

eg $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

Scalar Matrix :- A diagonal matrix in which all the diagonal elements are equal to a scalar say (k) is called a scalar matrix,

eg $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 4 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 \\ 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix}$

Unit or Identity Matrix :- A diagonal matrix is called identity matrix if all its elemts at diagonal are '1'. Generaly it is denoted by 'I'

$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Symmetric matrix :- A square matrix will be called symmetric if for all values of i and j, $a_{ij} = a_{ji}$ ~~i.e~~ A.
i.e. $A' = A$

e.g. $A = \begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix}$

Skew - Symmetric matrix :- A square matrix will be called skew symmetric if $a_{ij} = -a_{ji}$ i.e. $A' = A$

Note :- all the diagonal elements of zero. for skew symmetric matrix

e.g. $A = \begin{bmatrix} 0 & h & g \\ -h & 0 & f \\ -g & -f & 0 \end{bmatrix}$

Triangular matrix:- A square matrix, all of whose elements below the leading diagonal are zero, is called an upper triangular matrix. A square matrix, all of whose elements above the leading diagonals are zero, is called lower triangular matrix.

Transpose of a matrix:- If in a given matrix A, we interchange the rows and corresponding colums, the new matrix obtained is called a transpose of a matrix and it is denoted by A' or A^T

e.g. $A = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 0 & 5 \\ 6 & 7 & 8 \end{bmatrix}$, $A' = \begin{bmatrix} 2 & 1 & 6 \\ 3 & 0 & 7 \\ 4 & 5 & 8 \end{bmatrix}$

Orthogonal matrix:- A square matrix A is called orthogonal matrix if the product of the matrix A and the transpose matrix A' is an identity matrix e.g.

$$\boxed{A \cdot A' = I}$$

Note:- If $|A| = 1$, then it is called proper.

Conjugate of a matrix:-

$$A = \begin{bmatrix} 1+i & 2-3i & 4 \\ 7+2i & -i & 3-2i \end{bmatrix}$$

$$\overline{A} = \begin{bmatrix} 1-i & 2+3i & 4 \\ 7-2i & i & 3+2i \end{bmatrix}$$

Matrix A^θ, Transpose of conjugate of a matrix is ~~called~~ denoted by A^θ

$A = \begin{bmatrix} 1+i & 2-3i & 4 \\ 7+2i & -i & 3-2i \end{bmatrix}$, $\overline{A} = \begin{bmatrix} 1-i & 2+3i & 4 \\ 7-2i & i & 3+2i \end{bmatrix}$

$$A^\theta = (\overline{A})^T = \begin{bmatrix} 1-i & 7-2i \\ 2+3i & i \\ 4 & 3+2i \end{bmatrix}$$

Hermitian Matrix:— A square matrix A is called Hermitian matrix, if transpose of complex conjugate is equal to that matrix. i.e $\bar{A} = A$

eg: $A = \begin{bmatrix} 1 & i & 2+i \\ -i & 2 & 1-i \\ 2-i & 1+i & 2 \end{bmatrix}$, $\bar{A} = \begin{bmatrix} 1 & -i & 2-i \\ i & 2 & 1+i \\ 2+i & 1-i & 2 \end{bmatrix}$

$(\bar{A})^T = \begin{bmatrix} 1 & i & 2+i \\ -i & 2 & 1-i \\ 2-i & 1+i & 2 \end{bmatrix} = A$

Skew Hermitian Matrix:— A square matrix A is called skew Hermitian matrix if, $(\bar{A})^T = -A$

$$A = \begin{bmatrix} 0 & -2-3i \\ 2-3i & 0 \end{bmatrix}$$

$$\bar{A} = \begin{bmatrix} 0 & -2+3i \\ 2+3i & 0 \end{bmatrix}$$

$$(\bar{A})^T = \begin{bmatrix} 0 & 2+3i \\ -2+3i & 0 \end{bmatrix}$$

$$= -\begin{bmatrix} 0 & -2-3i \\ 2-3i & 0 \end{bmatrix}$$

$$= -A$$