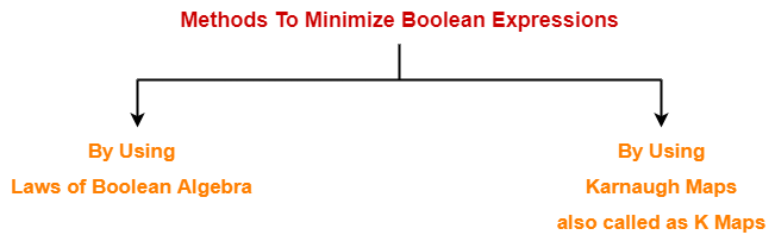


## K Maps | karnaugh Maps | Solved Examples

📁 Digital Design

### Minimization Of Boolean Expressions-

There are following two methods of minimizing or reducing the boolean expressions-



1. By using laws of Boolean Algebra
2. By using Karnaugh Maps also called as K Maps

In this article, we will discuss about Karnaugh Maps or K Maps.

### Karnaugh Map-

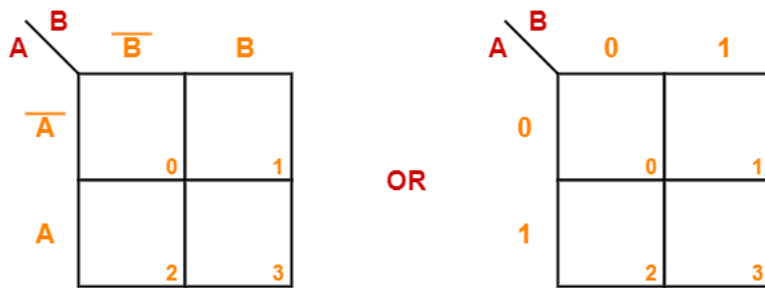
The Karnaugh Map also called as K Map is a graphical representation that provides a systematic method for simplifying the boolean expressions.

For a boolean expression consisting of n-variables, number of cells required in K Map =  $2^n$  cells.

## Two Variable K Map-

- Two variable K Map is drawn for a boolean expression consisting of two variables.
- The number of cells present in two variable K Map =  $2^2$  = 4 cells.
- So, for a boolean function consisting of two variables, we draw a 2 x 2 K Map.

Two variable K Map may be represented as-



**Two Variable K Map**

Here, A and B are the two variables of the given boolean function.

## Three Variable K Map-

- Three variable K Map is drawn for a boolean expression consisting of three variables.
- The number of cells present in three variable K Map =  $2^3$  = 8 cells.
- So, for a boolean function consisting of three variables, we draw a 2 x 4 K Map.

Three variable K Map may be represented as-

		BC			
		$\overline{B}\overline{C}$	$\overline{B}C$	$BC$	$B\overline{C}$
A	$\overline{A}$	0	1	3	2
	A	4	5	7	6

OR

		BC			
		00	01	11	10
A	0	0	1	3	2
	1	4	5	7	6

### Three Variable K Map

Here, A, B and C are the three variables of the given boolean function.

### Four Variable K Map-

- Four variable K Map is drawn for a boolean expression consisting of four variables.
- The number of cells present in four variable K Map =  $2^4$  = 16 cells.
- So, for a boolean function consisting of four variables, we draw a 4 x 4 K Map.

Four variable K Map may be represented as-

AB \ CD					
		$\overline{C}\overline{D}$	$\overline{C}D$	$CD$	$C\overline{D}$
$\overline{A}\overline{B}$		0	1	3	2
$\overline{A}B$		4	5	7	6
$AB$		12	13	15	14
$A\overline{B}$		8	9	11	10

OR

AB \ CD					
		00	01	11	10
00		0	1	3	2
01		4	5	7	6
11		12	13	15	14
10		8	9	11	10

Four Variable K Map

Here, A, B, C and D are the four variables of the given boolean function.

### Karnaugh Map Simplification Rules-

To minimize the given boolean function,

- We draw a K Map according to the number of variables it contains.
- We fill the K Map with 0's and 1's according to its function.
- Then, we minimize the function in accordance with the following rules.

### **Rule-01:**

- We can either group 0's with 0's or 1's with 1's but we can not group 0's and 1's together.
- X representing don't care can be grouped with 0's as well as 1's.

#### **NOTE**

There is no need of separately grouping X's i.e. they can be ignored if all 0's and 1's are already grouped.

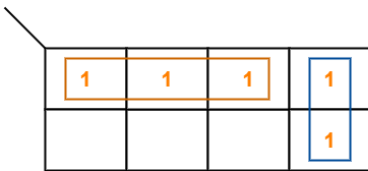
### **Rule-02:**

- Groups may overlap each other.

### **Rule-03:**

- We can only create a group whose number of cells can be represented in the power of 2.
- In other words, a group can only contain  $2^n$  i.e. 1, 2, 4, 8, 16 and so on number of cells.

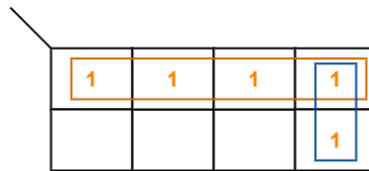
### **Example-**



A 2x4 grid with the following values: Row 1: 1, 1, 1, 1; Row 2: empty, empty, empty, 1. An orange box groups the first three cells of Row 1. A blue box groups the last two cells of Row 2.

1	1	1	1
			1

Incorrect

A 2x4 grid with the same values as the previous diagram. An orange box groups all four cells of Row 1. A blue box groups the last two cells of Row 2.

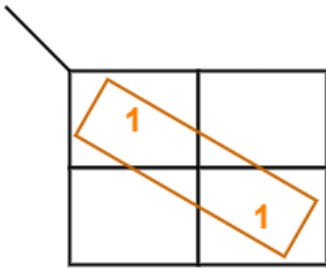
1	1	1	1
			1

Correct



## Rule-04:

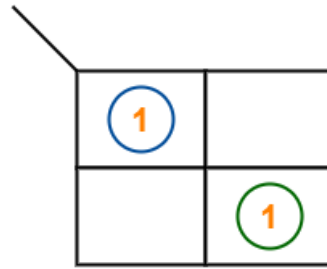
- Groups can be only either horizontal or vertical.
- We can not create groups of diagonal or any other shape.



A 2x2 grid with the following values: Row 1: 1, empty; Row 2: empty, 1. An orange box groups the two cells diagonally from top-left to bottom-right.

1	
	1

Incorrect

A 2x2 grid with the same values as the previous diagram. A blue circle groups the top-left cell, and a green circle groups the bottom-right cell.

1	
	1

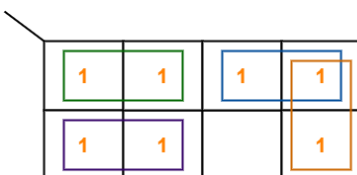
Correct



## Rule-05:

- Each group should be as large as possible.

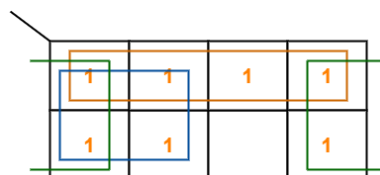
## Example-



A 2x4 grid with the following values: Row 1: 1, 1, 1, 1; Row 2: 1, 1, empty, 1. Four separate boxes (green, blue, purple, orange) each group a single '1' cell.

1	1	1	1
1	1		1

Incorrect

A 2x4 grid with the same values as the previous diagram. An orange box groups all four '1's in Row 1. A blue box groups the two '1's in Row 2. A green box groups the '1' in the bottom-right cell.

1	1	1	1
1	1		1

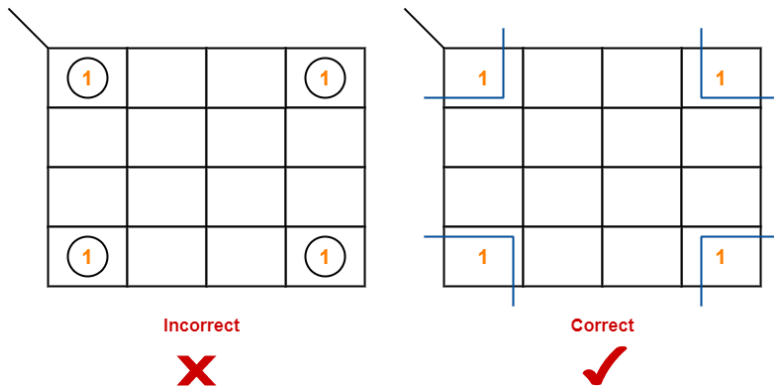
Correct



### Rule-06:

- Opposite grouping and corner grouping are allowed.
- The example of opposite grouping is shown illustrated in Rule-05.
- The example of corner grouping is shown below.

### Example-



### Rule-07:

- There should be as few groups as possible.

## PROBLEMS BASED ON KARNAUGH MAP-

### Problem-01:

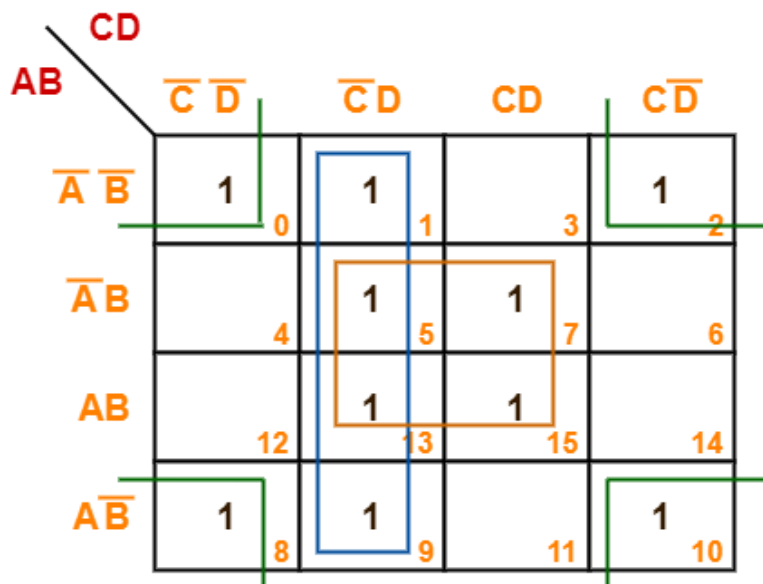
Minimize the following boolean function-

$$F(A, B, C, D) = \sum m(0, 1, 2, 5, 7, 8, 9, 10, 13, 15)$$

### Solution-

- Since the given boolean expression has 4 variables, so we draw a 4 x 4 K Map.
- We fill the cells of K Map in accordance with the given boolean function.
- Then, we form the groups in accordance with the above rules.

Then, we have-



Now,

$F(A, B, C, D)$

$$= (A'B + AB)(C'D + CD) + (A'B' + A'B + AB + AB')C'D + (A'B' + AB')(C'D' + CD')$$

$$= BD + C'D + B'D'$$

Thus, minimized boolean expression is-

$$F(A, B, C, D) = BD + C'D + B'D'$$

## **Problem-02:**

Minimize the following boolean function-



$$F(A, B, C, D) = \sum m(0, 1, 3, 5, 7, 8, 9, 11, 13, 15)$$

## Solution-

- Since the given boolean expression has 4 variables, so we draw a 4 x 4 K Map.
- We fill the cells of K Map in accordance with the given boolean function.
- Then, we form the groups in accordance with the above rules.

Then, we have-

		<b>CD</b>			
		<b><math>\bar{C}\bar{D}</math></b>	<b><math>\bar{C}D</math></b>	<b><math>CD</math></b>	<b><math>C\bar{D}</math></b>
<b>AB</b>	<b><math>\bar{A}\bar{B}</math></b>	1	1	1	
	<b><math>\bar{A}B</math></b>		1	1	
	<b><math>AB</math></b>		1	1	
	<b><math>A\bar{B}</math></b>	1	1	1	
		0	1	3	2
		4	5	7	6
		12	13	15	14
		8	9	11	10

Now,

$$F(A, B, C, D)$$

$$= (A'B' + A'B + AB + AB')(C'D + CD) + (A'B' + AB')(C'D' + C'D)$$

$$= D + B'C'$$

Thus, minimized boolean expression is-

$$F(A, B, C, D) = B'C' + D$$

### Problem-03:

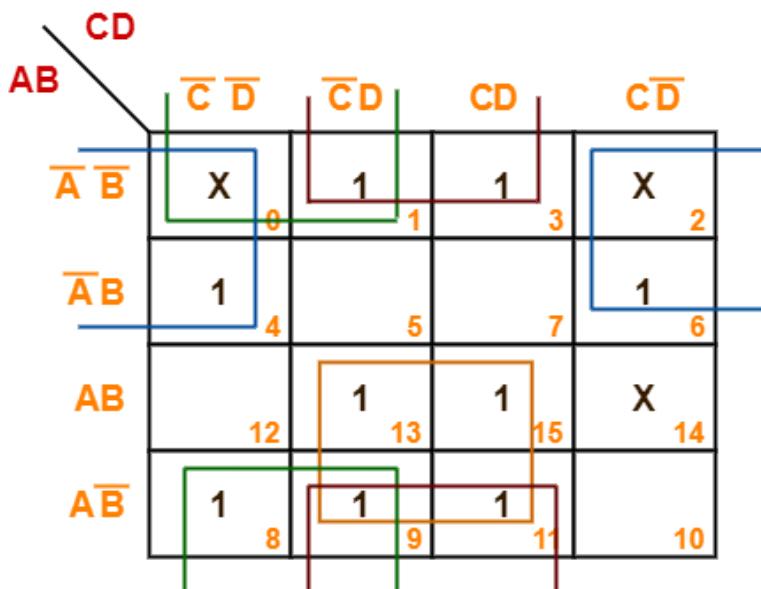
Minimize the following boolean function-

$$F(A, B, C, D) = \Sigma m(1, 3, 4, 6, 8, 9, 11, 13, 15) + \Sigma d(0, 2, 14)$$

### Solution-

- Since the given boolean expression has 4 variables, so we draw a 4 x 4 K Map.
- We fill the cells of K Map in accordance with the given boolean function.
- Then, we form the groups in accordance with the above rules.

Then, we have-



Now,

$$F(A, B, C, D)$$

$$= (AB + AB')(C'D + CD) + (A'B' + AB')(C'D + CD) + (A'B' + AB')(C'D' + C'D) + (A'B' + A'B)(C'D' + CD')$$

$$= AD + B'D + B'C' + A'D'$$

Thus, minimized boolean expression is-

$$F(A, B, C, D) = AD + B'D + B'C' + A'D'$$

### Problem-04:

Minimize the following boolean function-

$$F(A, B, C) = \sum m(0, 1, 6, 7) + \sum d(3, 5)$$

### Solution-

- Since the given boolean expression has 3 variables, so we draw a 2 x 4 K Map.
- We fill the cells of K Map in accordance with the given boolean function.
- Then, we form the groups in accordance with the above rules.

Then, we have-

		BC			
		$\overline{B}\overline{C}$	$\overline{B}C$	BC	$B\overline{C}$
$\overline{A}$	1	1	X		
A		X	1	1	

Now,

$$F(A, B, C)$$

$$= A'(B'C' + B'C) + A(BC + BC')$$

$$= A'B' + AB$$

Thus, minimized boolean expression is-

$$F(A, B, C) = AB + A'B'$$

### **NOTE-**

- It may be noted that there is no need of considering the quad group.
- This is because even if we consider that group, we will have to consider the other two duets.
- So, there is no use of considering that quad group.

### **Problem-05:**

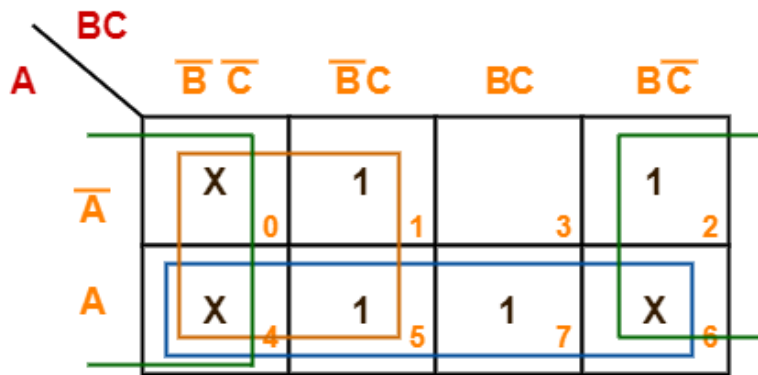
Minimize the following boolean function-

$$F(A, B, C) = \Sigma m(1, 2, 5, 7) + \Sigma d(0, 4, 6)$$

### **Solution-**

- Since the given boolean expression has 3 variables, so we draw a 2 x 4 K Map.
- We fill the cells of K Map in accordance with the given boolean function.
- Then, we form the groups in accordance with the above rules.

Then, we have-



Now,

$F(A, B, C)$

$$= (A + A')(B'C' + B'C) + A(B'C' + B'C + BC + BC') + (A + A')(B'C' + BC')$$

$$= B' + A + C'$$

Thus, minimized boolean expression is-

$$F(A, B, C) = A + B' + C'$$

## Problem-06:

Minimize the following boolean function-

$$F(A, B, C) = \sum m(0, 1, 6, 7) + \sum d(3, 4, 5)$$

## Solution-

- Since the given boolean expression has 3 variables, so we draw a 2 x 4 K Map.
- We fill the cells of K Map in accordance with the given boolean function.
- Then, we form the groups in accordance with the above rules.

Then, we have-

		BC			
A	$\bar{A}$	$\bar{B}\bar{C}$	$\bar{B}C$	$BC$	$B\bar{C}$
		1	1	X	
A	A	X	X	1	1
		4	5	7	6

Now,

$F(A, B, C)$

$$= (A + A')(B'C' + B'C) + A(B'C' + B'C + BC + BC')$$

$$= B' + A$$

Thus, minimized boolean expression is-

$$F(A, B, C) = A + B'$$

## Problem-07:

Minimize the following boolean function-

$$F(A, B, C, D) = \sum m(0, 2, 8, 10, 14) + \sum d(5, 15)$$

## Solution-

- Since the given boolean expression has 4 variables, so we draw a 4 x 4 K Map.
- We fill the cells of K Map in accordance with the given boolean function.
- Then, we form the groups in accordance with the above rules.

Then, we have-

		CD			
		$\overline{C}\overline{D}$	$\overline{C}D$	$CD$	$C\overline{D}$
AB	$\overline{A}\overline{B}$	1			1
	$\overline{A}B$		X		
AB	$A\overline{B}$			X	1
	$AB$				1

Now,

$F(A, B, C, D)$

$$= (AB + AB')CD' + (A'B' + AB')(C'D' + CD')$$

$$= ACD' + B'D'$$

Thus, minimized boolean expression is-

$$F(A, B, C, D) = ACD' + B'D'$$

## **Problem-08:**

Minimize the following boolean function-

$$F(A, B, C, D) = \sum m(3, 4, 5, 7, 9, 13, 14, 15)$$

## **Solution-**

- Since the given boolean expression has 4 variables, so we draw a 4 x 4 K Map.

- We fill the cells of K Map in accordance with the given boolean function.
- Then, we form the groups in accordance with the above rules.

Then, we have-

		CD			
		$\overline{C}\overline{D}$	$\overline{C}D$	$CD$	$C\overline{D}$
AB	$\overline{A}\overline{B}$			1	
	$\overline{A}B$	1	1	1	
	$AB$		1	1	1
	$A\overline{B}$		1		
		0	1	3	2
		4	5	7	6
		12	13	15	14
		8	9	11	10

Now,

$F(A, B, C, D)$

$$= A'B(C'D' + C'D) + (A'B' + A'B)(CD) + (AB + AB')(C'D) + AB(CD + C'D')$$

$$= A'BC' + A'CD + AC'D + ABC$$

Thus, minimized boolean expression is-

$$F(A, B, C, D) = A'BC' + A'CD + AC'D + ABC$$

It is important to note that we are not considering the quad group because we have to consider the duets anyhow.

## Problem-09:



Consider the following boolean function-

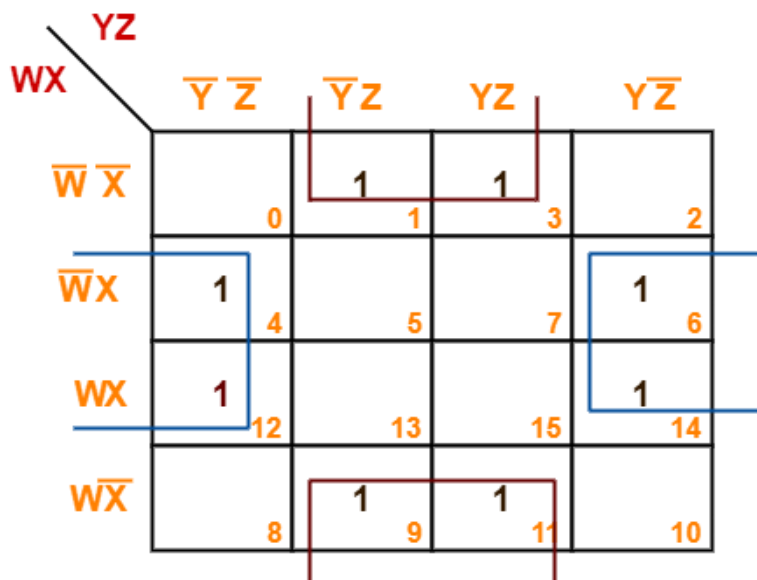
$$F(W, X, Y, Z) = \sum m(1, 3, 4, 6, 9, 11, 12, 14)$$

This function is independent \_\_\_\_\_ number of variables.  
Fill in the blank.

### Solution-

- Since the given boolean expression has 4 variables, so we draw a 4 x 4 K Map.
- We fill the cells of K Map in accordance with the given boolean function.
- Then, we form the groups in accordance with the above rules.

Then, we have-



Now,

$$F(W, X, Y, Z)$$

$$= (W'X + WX)(Y'Z' + YZ') + (W'X' + WX')(Y'Z + YZ)$$

$$= XZ' + X'Z$$

$$= X \oplus Z$$

Thus, minimized boolean expression is-

$$F(W, X, Y, Z) = X \oplus Z$$

Clearly, the given boolean function depends on only two variables X and Z.

Hence, it is independent of other two variables W and Y.

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