Laplace transformation ⇒

Laplace transformation is an integral transform

$$g(\alpha) = \int_a^b f(t)\, K(\alpha,t)\cdot dt$$

$g(\alpha)$ ↓ o/p, $f(t)$ ↓ i/p, $K(\alpha,t)$ ↓ integral Karnal

General transform

Laplace transform is the general case of Fourier transform.

$$x(t) \rightleftharpoons X(\omega) = \int_{-\infty}^{\infty} x(t)\cdot \underbrace{e^{-j\omega t}}_{I.K}\, dt$$

$X(\omega)$ ↓ o/p, $x(t)$ ↓ i/p

fourier transform

ω ⇒ real varriable

Laplace

$$f(s) = \int_{-\infty}^{\infty} f(t)\cdot e^{-st}\cdot dt$$

$f(s)$ ↓ o/p, $f(t)$ ↓ i/p, e^{-st} ↓ I.K

$$\boxed{s = \sigma + j\omega}$$

s ⇒ Complex varriable

σ ⇒ damping factor

ω ⇒ freq.

Bilatral L.T ⇒

$$\boxed{F(s) = \int_{-\infty}^{\infty} f(t) \cdot e^{-st} \cdot dt}$$

Boath sided transformation.

Unilatral - L.T ⇒

✓ $f(t)$ $t \geq 0$

$$F(s) = \int_{0}^{\infty} f(t) \cdot e^{-st} \cdot dt$$

Invarse Lapalace transform ⇒

$$f(t) = \frac{1}{2\pi j} \int_{\sigma - j\omega}^{\sigma + j\omega} F(s) \cdot e^{st} \cdot ds$$

Do not use these formula.

Relation b/w Laplace to Fourier transform ⇒

$$L.T \xrightarrow{s = j\omega} F.T$$

$f(t) \rightleftharpoons F(\omega)$ → fourier transform pare

$f(t) \rightleftharpoons f(s)$ → Laplace transform pare.

$$F = \int_{-\infty}^{\infty} f(t) \cdot e^{-st} \cdot dt$$

$$s = \sigma + j\omega$$

$$F(s) = \int_{-\infty}^{\infty} f(t).e^{-st}.dt$$ $f(t) \rightarrow$ time Domain Signal

$$F(s) = \int_{-\infty}^{\infty} f(t).e^{-(\sigma+j\omega)}.dt$$

$$f(s) = \int_{-\infty}^{\infty} \underset{\substack{\downarrow \\ i/p}}{f(t)}.e^{-\sigma t}.\underbrace{e^{-j\omega}.dt}_{J/R}$$

$$F(s) = F\{f(t).e^{-\sigma t}\}$$

L.T is Forier transfor $f(t).e^{-\sigma t}$.

if $\sigma = 0$ or $\boxed{s = j\omega}$

$$F(s) = F\{f(t)\} = F(\omega)$$

<u>Condition for existance</u> of L.T.

Bilatral L.T of $f(t)$ = F.T of $f(t).e^{-\sigma t}$

$$F(s) = \int_{-\infty}^{\infty} f(t).e^{-\sigma t}.e^{-j\omega t}.dt$$

$$F(s) = \int_{-\infty}^{\infty} f_1(t).e^{-j\omega t}.dt$$

for existance of $F(s)$ $f_1(t)$ should be absolutly intigrable.

A function will be absoalutly intigrable if

$$\int_{-\infty}^{\infty} |f_1(t)|\, dt < \infty$$

$$\because \quad f_1(t) = f(t) \cdot e^{-\sigma t}$$

so the condition for existence of L.T is

$$\text{FO}: \quad \boxed{\int_{-\infty}^{\infty} \left| f(t) \cdot e^{-\sigma t} \cdot e^{-j\omega t} \right| dt < \infty}$$

$\downarrow$

we will get range of σ

with range σ we can get Region of conversion

Ex - find the ROC of signal $f(t) = \exp(2t) \cdot u(t)$

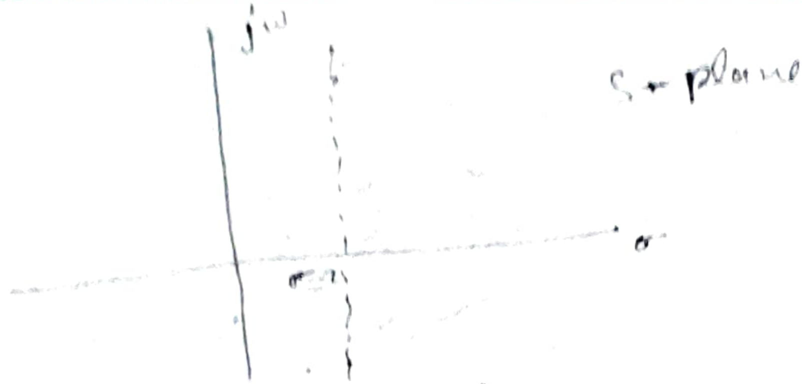
$$f(t) = e^{2t} u(t)$$

$$\int_{-\infty}^{\infty} \left| e^{2t} u(t) \cdot e^{-\sigma t} \right| dt$$

$$\int_{0}^{\infty} \left| e^{(2-\sigma)t} \right| dt < \infty$$

if $2 - \sigma < 0$

$2 < \sigma \Rightarrow \boxed{\sigma > 2}$



?
ROC

And laplace transformation exist in this region.

Region of convergence & its properties :-

ROC :- it is the range of complex variable 's' in s-plane for which laplace transform is finite or convergent.

Properties :-

① ROC does not include any poles

$$F(s) = \frac{1}{s+2}$$

to find pole

$$s+2=0$$

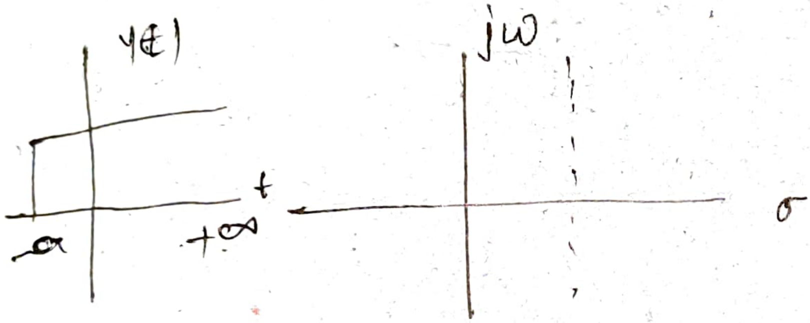
$$\boxed{s=-2}$$

at $s=-2$

$$F(s) = \infty$$

Laplace transform is finite in ROC so the point $s=-2$ will not be included in ROC.

(2). For Right sided signals, ROC is right side to the rightmost pole;



right sided signal are those signal which are extend from a finite point to $+\infty$.

(3) For left sided signals, ROC is left side to the left most pole.

(4). For the absolute integrability of a signal or the stability of a system, ROC should include imaginary axis.

(5) for both sided signal, ROC is a strip in s-plane.

(5). for finite duration signal, ROC is entire s-plane, Excluding $s=0$ &/or $+\infty$ &/or $-\infty$

#

Q→ check the stability of LTI system and comment about the existence of $h(t)$

1:- $h(t) \rightleftharpoons H(s)$, ROC: $\sigma > 2$

properties of Laplace transform :

① Linearity →

$f_1(t) \rightleftharpoons F_1(s)$ ROC = R_1

$f_2(t) \rightleftharpoons F_2(s)$ ROC = R_2

$\alpha f_1(t) \rightleftharpoons \alpha F_1(s)$ ROC = R_1

$\beta f_2(t) \rightleftharpoons \beta F_2(s)$ ROC = R_2

$\alpha f_1(t) + \beta f_2(t) \rightleftharpoons \alpha F_1(s) + \beta F_2(s)$,

ROC = $R_1 \cap R_2$

② proof

$$F(s) = \int_{-\infty}^{\infty} f(t) \cdot e^{-st} \, dt$$

$$F(s) = \int_{-\infty}^{\infty} [\alpha f_1(t) + \beta f_2(t)] e^{-st} \, dt$$

$$F(s) = \alpha \underbrace{\int_{-\infty}^{\infty} f_1(t) \cdot e^{-st} \, dt}_{f_1(s)} + \beta \underbrace{\int_{-\infty}^{\infty} f_2(t) \cdot e^{-st} \, dt}_{f_2(s)}$$

③. Conjugation $f(t) \rightleftharpoons F(s)$

$$\boxed{f^*(t) \rightleftharpoons F^*(s^*)}$$

Proof →

$$F(s) = \int_{-\infty}^{\infty} f(t) \cdot e^{-st} \cdot dt$$

$$F^*(s) = \int_{-\infty}^{\infty} f^*(t) \cdot e^{-s^* t} \cdot dt$$

replace $s \rightarrow s^*$

$$F^*(s^*) = \int_{-\infty}^{\infty} f^*(t) \cdot e^{-(s^*)^* t} \cdot dt$$

$$\underset{\text{O/P}}{F^*(s^*)} = \int_{-\infty}^{\infty} \underset{\text{I/P}}{f^*(t)} \cdot \underbrace{e^{-st}}_{\text{IK}} \cdot dt$$

Time Reversal →

$$f(t) \rightleftharpoons F(s) \quad ROC = R$$

$$\boxed{f(-t) \rightleftharpoons F(-s) \quad ROC = -R}$$

Proof

$$\underbrace{F(s)}_{\text{O/P}} = \int_{-\infty}^{\infty} \underbrace{f(t)}_{\text{I/P}} \cdot \underbrace{e^{-st}}_{\text{IK}} \cdot dt$$

$t \rightarrow -t$

$$F(s) = \int_{-\infty}^{\infty} f(-t) \cdot e^{-st} \cdot dt$$

$$f(t) \rightarrow f(-t) \rightleftharpoons F'(s) = ?$$

$$F'(s) = \int_{-\infty}^{\infty} f(-t) \cdot e^{-st} \cdot dt$$

$$-t = \tau \Rightarrow t = -\tau \Rightarrow dt = -d\tau$$

$$\begin{cases} t = -\infty , & \tau = \infty \\ t = \infty , & \tau = -\infty \end{cases}$$

$$F'(s) = -\int_{\infty}^{-\infty} f(\tau) \cdot e^{+s\tau} \cdot d\tau$$

$$F'(s) = -\int_{+\infty}^{-\infty} f(\tau) \cdot e^{+s\tau} \cdot d\tau$$

$$F'(s) = \int_{-\infty}^{\infty} f(\tau) \cdot e^{+s\tau} \cdot d\tau$$

$$F'(s) = \int_{-\infty}^{\infty} f(\tau) \cdot e^{-(-s)\tau} \cdot d\tau$$

$$= F'(-s)$$

Time scaling $\Rightarrow$

$$f(t) \rightleftharpoons F(s), \quad ROC = R$$

$$\boxed{f(at), a \neq 0 \rightleftharpoons \frac{1}{|a|} F\left(\frac{s}{a}\right), \quad ROC = |a| R}$$

5. time shifting $\Rightarrow$

$$f(t) \rightleftharpoons F(s), \quad ROC = R$$

$$\boxed{f(t \pm t_0) \rightleftharpoons F(s) \cdot e^{\pm s t_0}, \quad ROC = R.}$$

note $\quad -t_0 \Rightarrow$ Right shift

$\quad +t_0 \Rightarrow$ left shift

6 $\Rightarrow$ frq. shifting $\Rightarrow$ [shifting in s-domain]

$$f(t) \rightleftharpoons F(s), \quad ROC = R$$

$$e^{\pm s_0 t} \cdot f(t) \rightleftharpoons F(s \mp s_0), \quad ROC = R + Re[s_0]$$

NOTE $\Rightarrow$ $\quad F(s) \rightarrow$ pole/zero at $s = a$

$F(s - s_0) \rightarrow$ pole/zero at $s = a + s_0$

7* Convolution in time

$f_1(t) \rightleftharpoons F_1(s)$ ROC $= R_1$

$f_2(t) \rightleftharpoons F_2(s)$ ROC $= R_2$

$$\boxed{f_1(t) * f_2(t) \rightleftharpoons F_1(s) \cdot F_2(s), \quad R_1 \cap R_2 = \text{ROC}}$$

(8). Multiplication in time:-

(Convolution in frequency)

$$f_1(t) \cdot f_2(t) \rightleftharpoons \frac{1}{2\pi j}\left[F_1(s) * F_2(s)\right],$$

$$\text{ROC} = R_1 \cap R_2$$

(9). Differentiation in Time

for Bilateral L.T.

$f(t) \rightleftharpoons F(s)$, ROC $= R$

$\dfrac{\partial^n f(t)}{\partial t^n} \rightleftharpoons s^n \cdot F(s)$, ROC $= R$

Unilateral T.F.

$$\frac{\partial^n f(t)}{\partial t^n} \rightleftharpoons s^n F(s) - s^{n-1} f(0^-) - s^{n-2} f'(0^-) - s^{n-3} f''(0^-) \text{------}$$

$$f(0^-) = f(t) \Big|_{t=0^-}$$

$$f'(0^-) = \frac{\partial f(t)}{\partial t} \Big|_{t=0^-}$$

$$f''(0^-) = \frac{\partial^2 f(t)}{\partial t} \Big|_{t=0^-}$$

(10). Intigration in time.

$f(t) \rightleftharpoons F(s)$, ROC = R

Bi-latral L.T.

$-\infty$ to t

Let $t \to \tau$

$$\boxed{g(t) = \int_{-\infty}^{t} f(\tau)\cdot d\tau \rightleftharpoons \frac{F(s)}{s} \ ; \ ROC = R \cap Re\{s\} > 0}$$

for unilatral.

$$\boxed{\int_{-\infty}^{t} f(\tau)\cdot d\tau \rightleftharpoons \frac{F(s)}{s} + \frac{\int_{-\infty}^{0} f(\tau)\cdot d\tau}{s}}$$

(11). Differentiation in freq.

$f(t) \rightleftharpoons F(s)$, ROC = R

$t^n \cdot f(t) \rightleftharpoons (-1)^n \cdot \frac{d^n F(s)}{ds^n}$; ROC = R

(12). intigration in freq

$f(t) \rightleftharpoons F(s)$

$$\boxed{\frac{f(t)}{t} \rightleftharpoons \int_{s}^{\infty} F(s)\cdot ds}$$