IVa If $f(x) = P(x)$ (a polynomial)

$$P.I. = \frac{1}{f(D)} \cdot P(x)$$

$$= \frac{1}{(1-g(D))} \cdot P(x)$$

$$= (1-g(D))^{-1} \cdot P(x)$$

$$= (1+g(D)+(g(D))^2 + \cdots)\, P(x)$$

eg: $(D-2)^2 y = x^2$

$$\Rightarrow C.F = (c_1 x + c_2) e^{2x}$$

$$P.I. = \frac{1}{(D-2)^2} \cdot x^2$$

$$= \frac{1}{D^2-4D+4} \cdot x^2$$

$$= \frac{1}{4}\left(\frac{1}{1-(D-D^2/4)}\right) \cdot x^2$$

$$= \frac{1}{4}\left(1-(D-D^2/4)\right)^{-1} \cdot x^2$$

$$= \frac{1}{4}\left(1 + \left(D-\frac{D^2}{4}\right) + \left(D-\frac{D^2}{4}\right)^2 + \cdots\right) x^2$$

$$\cancel{= \frac{1}{4}\left(x^2 + \left(D + \frac{D^4}{16} - \frac{2D^3}{4}\right) + (\right.}$$

$$= \frac{1}{4}\left(x^2 + 2x - \frac{1}{2} + 2\right) \qquad \boxed{C.S. = C.F + P.I}$$

$$= \frac{1}{4}\left(x^2 + 2x + 3/2\right)$$

V   If $f(x) = e^{ax} \cdot \phi(x)$

$$P.I = \frac{1}{F(D)} e^{ax} \cdot \phi(x)$$

$$= e^{ax} \frac{1}{F(D+a)} \cdot \phi(x) \qquad D \to D+a$$

e.g. P.I. for $(D^2 - 2D + 4)y = e^x \cos x$

$$P.I. = \frac{1}{D^2 - 2D + 4} \cdot e^x \cos x$$

$$= e^x \frac{1}{(D+1)^2 - 2(D+1) + 4} \cos x$$

$$= e^x \frac{1}{D^2 + \cancel{2D} + 1 - \cancel{2D} - 2 + 4} \cos x$$

$$= e^x \frac{1}{D^2 + 3} \cos x \qquad D^2 \to -1$$

$$= e^x \frac{1}{-1+3} \cos x$$

$$= \frac{1}{2} e^x \cos x \text{ Ans}$$

e.g. C.F. for $(D^4 + 2D^2 + 1)y = x^2 \cos x$

auxiliary eqn is

$$m^4 + 2m^2 + 1 = 0$$

$$(m^2 + 1)^2 = 0$$

$$m = \pm i, \pm i$$

$$C.F. = (c_1x+c_2)\cos x + (c_3x+c_4)\sin x$$

$$P.I. = \frac{1}{D^4+2D^2+1} x^2\cos x$$

$$= R.P\left(\frac{1}{D^4+2D^2+1} e^{ix} x^2\right)$$

$(\because e^{ix} = \cos x + i\sin x)$

$\Rightarrow \cos x = R.P.(e^{ix})$

$$= R.P.\left(e^{ix}\frac{1}{(D+i)^4+2(D+i)^2+1}x^2\right)$$

$$= R.P.\left(e^{ix}\frac{1}{(D^4-2D^2-2D-1)+4D^2i}x^2\right)$$

$$= R.P.\left(e^{ix}\cdot -\left(\frac{1}{1+2D+2D^2-D^4-4D^2i}x^2\right)\right)$$

$$= -R.P.\left(e^{ix}\frac{1}{1-(D^4-2D^2-2D+4D^2i)}x^2\right)$$

$$= -R.P.\left(e^{ix}\left(1-(D^4-2D^2-2D+4D^2i)\right)^{-1}x^2\right)$$

$$= -R.P.\left(e^{ix}\left(1+(D^4-2D^2-2D+4D^2i)+(D^4-2D^2-2D+4D^2i)^2+\dots\right)x^2\right)$$

$$= -R.P.\left(e^{ix}\left(x^2+(-4-4x+8i)+4D^2x^2\right)\right)$$

$$= -R.P.\left(e^{ix}\left(x^2+\cancel{+}4-4x+8i+8\right)\right)$$

$$= -R.P.\left(e^{ix}\left(x^2-4x+4+8i\right)\right)$$

$$= R.P.\left((\cos x+i\sin x)\left((4x-4-x^2)+8i\right)\right)$$

$$= \cos x(4x-4-x^2) - 8\sin x$$

$$= -(x^2-4x+4)\cos x - 8\sin x$$

$(D+i)^2 = D^2+2Di-1$

$((D+i)^2)^2 = (D^2-1+2Di)^2$

$= D^4-2D+1-4D^2+2(D^2-1)2Di$

$= (D^4-4D^2-2D+1)+(4D^3-4D)i$

$(D+i)^4+2(D+i)^2+1$

$= (D^4-4D^2-2D+1$
$2D^2-2+4Di+1)$
$+(4D^3-4D)i$

$= (D^4-2D^2-2D-1)$
$+(4D^3+4D-4D)i$

$= (D^4-2D^2-2D-1)$
$+(4D^3)i$

Simultaneous d.e. :-

If two or more dependent variables are funct's of a single independent variable, the equations involving their derivatives are called simultaneous equation, e.g.

$$\frac{dx}{dt} + 4y = t$$

$$\frac{dy}{dt} + 2x = e^{t}$$

$$\left[\begin{array}{l} x = x(t) \\ y = y(t) \end{array} \right.$$

The method of solving these eq^n is based on the process of ellimination, as we solve algebraic simultaneous eq^n.

ex The eq^n of motion of a particle are given as

$$\cancel{\frac{dx}{dt} + wy = 0} \qquad \frac{dx}{dt} + wy = 0$$

$$\cancel{\frac{dy}{dt} + wx = 0} \qquad \frac{dy}{dt} - wy = 0$$

Find the path of the particle and show that it is a circle.

Sol^n $\frac{d}{dt} \equiv D$

$$Dx + wy = 0 \quad \text{—— (1)}$$

$$-wx + Dy = 0 \quad \text{—— (2)}$$

(1) $\times w$ ~~+~~ (2) $\times D$

$$w\cancel{D}x + w^2 y = 0$$

$$-w\cancel{D}x + D^2 y = 0$$

$$(D^2 + w^2)y = 0$$

variables are
able, the equations
ed simultaneous

$$\begin{cases} x = x(t) \\ y = y(t) \end{cases}$$

based on the
e algebraic

ticle are given as

$wy = 0$

$wy = 0$

and show that

$m = \pm i\omega$ (roots of auxiliary eqn

$$y = A\cos\omega t + B\sin\omega t \quad -(3)$$

$$\Rightarrow Dy = -A\omega\sin\omega t + B\omega\cos\omega t$$

on putting the value of Dy in (2)

$$x = -A\sin\omega t + B\cos\omega t \quad -(4)$$

squaring & adding (3) & (4) we have

$$\boxed{x^2 + y^2 = A^2 + B^2}$$

This is the eqn of circle.

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$$\frac{dx}{dt} - y = 1 \quad -(*)$$

$$\frac{dy}{Dt} - x = 1$$

$$\Rightarrow \quad Dx - y = 0 \quad -(1) \qquad D \cong \frac{d}{dt}$$

$$-x + Dy = 1 \quad -(2)$$

(1) $\times$ D & (2) $\times$ D

$$D^2x - x = D1 + 1$$

$$(D^2 - 1)x = 1$$

$$C.F. = \cancel{(c_1t + c_2)e^t} \qquad c_1e^t + c_2e^{-t}$$

$$P.I. = \frac{1}{D^2-1} \cdot e^{0t} = -1$$

$$C.S.(x) = \cancel{(c_1t + c_2)e^t - 1} \qquad c_1e^t + c_2e^{-t} - 1$$

from (*)

$$y = \frac{dx}{dt} - 1$$

$$= c_1 e^{t} - c_2 e^{-t} - 1$$

$$\left.\begin{aligned} x &= c_1 e^{t} + c_2 e^{-t} - 1 \\ y &= c_1 e^{t} - c_2 e^{-t} - 1 \end{aligned}\right\} \text{m}$$