Examples based on dual simplex method

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Find the solution of the problem:

Max Z = $-4x_1 - 6x_2 - 18\,x_3$

Subject to

$x_1 + 3\,x_3 \geq 3$

$x_2 + 2\,x_3 \geq 5,$

$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0.$

Ans.

Problem write as

Max Z = $-4x_1 - 6x_2 - 18\,x_3$ subject to

$-x_1 - 3\,x_3 \leq -3$

$-x_2 - 2\,x_3 \leq -5,\quad$ with $\;x_1 \geq 0, x_2 \geq 0, x_3 \geq 0.$

Also, above problem can be written as

Max Z = $-4x_1 - 6x_2 - 18\,x_3$ subject to

$-x_1 - 3\,x_3 + s_1 = -3$

$-x_2 - 2\,x_3 + s_2 = -5,\quad$ with $\;x_1 \geq 0, x_2 \geq 0, x_3 \geq 0.$

For solution, write as-

	c_j		-4	-6	-18	0	0
Basic Variables	c_B	X_B	x_1	x_2	x_3	s_1	s_2
s_1	0	-3	-1	0	-3	1	0
s_2	0	-5	0	-1	-2	0	1
	Z = 0		4	6	18	0	0
			$\uparrow\Delta_1$	$\uparrow\Delta_2$	$\uparrow\Delta_3$		

s_1	0	-3	-1	0	-3	1	0
x_2	-6	5	0	1	2	0	-1
			4	0	6	0	6
x_3	-18	1	$\frac{1}{3}$	0	1	$-\frac{1}{3}$	0
x_2	-6	3	$-2/3$	1	0	$-1/3$	-1
			2	0	0	2	6

For outgoing vector

$= \min_{B.} \{x_B\}$

$= \min\{-3, -5\}$

$= -5 \Rightarrow s_2$ is outgoing vector

Entering vector

$\max\left\{ \frac{\Delta j}{a_{rj}},\ a_{rj} < 0 \right\}$

$r = 2$

$\max\left\{ \frac{\Delta j}{a_{2j}},\ a_{2j} < 0 \right\}$

$= \max\left\{ \frac{4}{-},\ \frac{6}{-1},\ \frac{18}{-2} \right\}$

s_1 is outgoing vector (2nd iteration)

$\max\{\ 4 \quad 6$

$\max\left\{ \frac{4}{-1}, \frac{6}{-3} \right\}$

$= \max\{-4, -2\} = -2$

$\Rightarrow x_3$ is entering vector in the basis

as value of all basic variables are +ve, so, get the soln.

$x_3 = 1, \; x_2 = 3, \; x_1 = 0,$

$\max z = -18 \times 1 + (-6) \times 3$
$= -36$ Ans

$= \max\left\{ \frac{4}{—}, \frac{6}{-1}, \frac{18}{-2} \right\}$

$= \max\{-6, -9\}$

$= -6 = \frac{\Delta_2}{a_{22}}$

$\Rightarrow x_2$ is entering vector in the basis.

Find the soln by dual simplex method.

$\min z = 2x_1 + x_2$ s/to

$3x_1 + x_2 \geq 3$

$4x_1 + 3x_2 \geq 6$

$x_1 + 2x_2 \geq 3$

$x_1 \geq 0, x_2 \geq 0$

$\max z' = -2x_1 - x_2$, $z' = -z$, s/to

$-3x_1 - x_2 \leq -3$

$-4x_1 - 3x_2 \leq -6$

$-x_1 - 2x_2 \leq -3$, $x_1 \geq 0, x_2 \geq 0$

	cj		-2	-1	0	0	0
Basic Variables	C_B	X_B	x_1	x_2 ↓	s_1	s_2	s_3
s_1	0	-3	-3	-1	1	0	0
→ s_2	0	-6	-4	$\boxed{-3}$	0	1	0
s_3	0	-3	-1	-2	0	0	1
	$z' = 0$		2	1	0	0	
→ s_1	0	-1	$\boxed{-5/3}$ ↓	0	1	-1/3	0
x_2	-1	2	4/3	1	0	-1/3	0
s_3	0	1	5/3	0	0	-2/3	1

s_2 is leaving vector

$\max\left\{ \frac{2}{-4}, \frac{1}{-3} \right\}$

$\max\left\{ -\frac{1}{2}, -\frac{1}{3} \right\}$

$= -\frac{1}{3}$

$s_1 \rightarrow$ outgoing vector

x_2	-1	2	$4/3$	1	0	$-1/3$		$s_1 \to$ outgoing vector
s_3	0	1	$5/3$	0	0	$-2/3$	1	
	$z' =$		$2/3$	0	0	$1/3$	0	
x_1	-2	$\frac{3}{5}$	1	0	$-\frac{3}{5}$	$\frac{1}{5}$	0	$\max\left\{\frac{2/3}{-5/3}, \frac{1/3}{-1/3}\right\}$
x_2	-1	$\frac{6}{5}$	0	1	$\frac{4}{5}$	$-\frac{3}{5}$	0	$\max\left\{-2/5, -1\right\}$
s_3	0	0	0	0	1	-1	1	
	$z' = -\frac{12}{5}$		0	0	$2/5$	$1/5$	0	$= -2/5$

$\Rightarrow x_1$ is entering vector.

$$x_1 = \frac{3}{5}, \quad x_2 = \frac{6}{5}, \quad \max z = -2 \times \frac{3}{5} - 1 \times \frac{6}{5}$$

$$= \frac{-6-6}{5} = -\frac{12}{5}$$

$$z' = -\frac{12}{5}, \text{ so } z = -z' = \frac{12}{5} \quad \underline{\text{Ans}}$$