Q1. A Carrier signal of $5\cos 2\pi \times 10^6 t$ is Amplitude modulated by a message signal of $2\cos 6\pi \times 10^3 t$ with $\mu = 0.5$. If antenna Resistance is given by 10Ω.

Find: AM Bandwidth, Carrier Power, total Power, Sideband Power, efficiency and plot frequency spectrum.

Sol

Given

$c(t) = 5\cos 2\pi \times 10^6 t$

$A_c = 5V$, $f_c = 10^6 Hz = 1000\,kHz$

$m(t) = 2\cos 6\pi \times 10^3 t$

$A_m = 2V$, $f_m = 3\,kHz$

$2\pi f_m = 6\pi \times 10^3$

$f_m = \frac{\cancel{6}^{3}\cancel{\pi} \times 10^3}{\cancel{2\pi}}$

$\mu = 0.5$, $R = 10\Omega$

(i) AM B.W $= 2 \times f_m = 2 \times 3\,kHz = 6\,kHz$

(ii) Carrier Power $P_c = \frac{A_c^2}{2R} = \frac{5^2}{2 \times 10} = \frac{25}{20} = 1.25W$

(iii) $P_t = P_c\left[1 + \frac{\mu^2}{2}\right]$

$= 1.25\left[1 + \frac{(0.5)^2}{2}\right]$

$= 1.25\left[1 + \frac{0.25}{2}\right]$

$= 1.41\,W$

(iv) $P_t = P_c + P_{SB}$

$P_{SB} = P_t - P_c = 1.41 - 1.25$

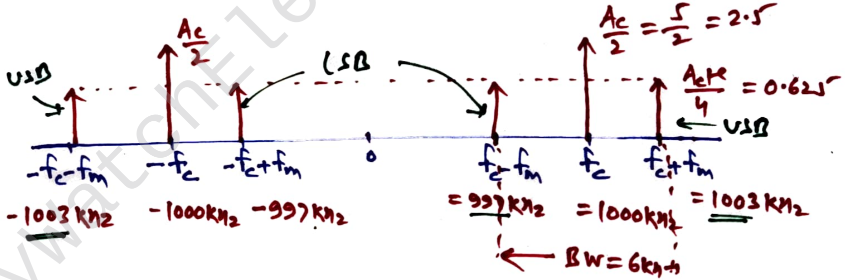
$= 0.156\,W$

$= 156\,mW$

$$P_{USB} = P_{LSB} = \frac{0.156}{2} = 78\ mW$$

(v) $$\eta = \frac{\mu^2}{2+\mu^2} = \frac{(0.5)^2}{2+(0.5)^2} = 0.11$$

$$\%\eta = 11\%$$



Q1. An AM signal is given by

$$S_{AM}(t) = 5\cos 1600\pi t + 10\cos 2000\pi t + 5\cos 2400\pi t$$

find all the Parameters of AM.

Sol we know that

$$S_{AM}(t) = A_c \cos 2\pi f_c t + \frac{A_c\mu}{2}\cos 2\pi(f_c+f_m)t + \frac{A_c\mu}{2}\cos 2\pi(f_c-f_m)t$$

$A_c = 10V$, $f_c = 1000Hz$ $\quad 2\pi f_c = 2000\pi$, $f_c = 1000Hz$, $\quad f_c+f_m = \cancel{240}$ $1200Hz$ $\quad 2\pi(f_c+f_m) = 2400\pi$, $f_c+f_m = 1200\ Hz$

$\frac{A_c\mu}{2} = 5 \Rightarrow \mu = \frac{10}{10} = 1$ $\qquad f_c - f_m = 800Hz$, $f_m = 200\ Hz$

(i) AM B.W $= 2\times f_m = 2\times 200Hz = 400Hz$

(ii) $P_c = \frac{A_c^2}{2R} = \frac{100}{2\times 1} = 50W$

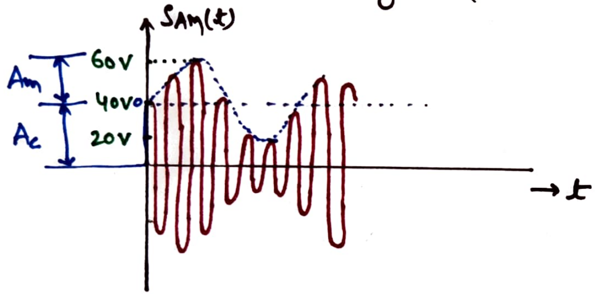
(iii) $P_t = P_c\left[1+\frac{\mu^2}{2}\right] = 1.5\times 50 = 75W$

(iv) $P_{SB} = P_t - P_c = 75-50 = 25W$

(V) $\eta = \frac{\mu^2}{2+\mu^2} = \frac{1}{3} = 0.33$

$\%\eta = 33.33\%$

Q2. Find the Percentage modulation for the given AM signal



Sol

$$\mu = \frac{A_{max} - A_{min}}{A_{max} + A_{min}} = \frac{60-20}{60+20} = \frac{40}{80} = \frac{1}{2} = 0.5$$

50% modulation

$$\mu = \frac{A_m}{A_c} = \frac{60-40}{40} = \frac{20}{40} = \frac{1}{2} = 0.5$$

50% modulation

Multi-tone AM :-

If message signal contains more than one frequency the resulting modulated signal is known as multi-tone AM.

Assume $m(t) = A_{m_1} \cos 2\pi f_{m_1} t + A_{m_2} \cos 2\pi f_{m_2} t$

We know that

$$S_{AM}(t) = A_c[1 + K_a m(t)] \cos 2\pi f_c t$$

$$= A_c[1 + K_a A_{m_1} \cos 2\pi f_{m_1} t + K_a A_{m_2} \cos 2\pi f_{m_2} t] \cos 2\pi f_c t$$

Assume $K_a A_{m_1} = \mu_1$

$K_a A_{m_2} = \mu_2$

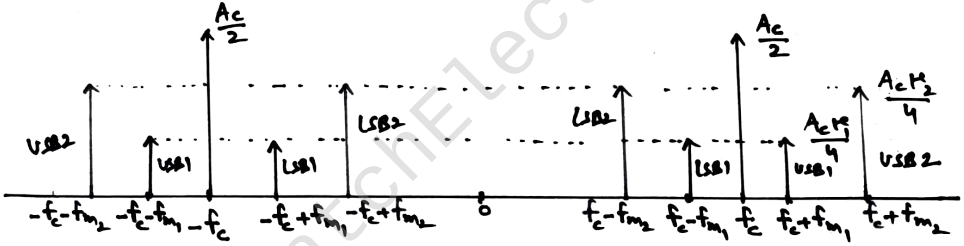
$$S_{AM}(t) = A_c[1 + \mu_1 \cos 2\pi f_{m_1} t + \mu_2 \cos 2\pi f_{m_2} t] \cos 2\pi f_c t$$

$$S_{AM}(t) = A_c \cos 2\pi f_c t + A_c \mu_1 \cos 2\pi f_c t \cos 2\pi f_{m_1} t + A_c \mu_2 \cos 2\pi f_c t \cos 2\pi f_{m_2} t$$

$$= A_c \cos 2\pi f_c t + \frac{A_c \mu_1}{2} \cos 2\pi \underbrace{(f_c + f_{m_1})}_{USB-1} t + \frac{A_c \mu_1}{2} \cos 2\pi \underbrace{(f_c - f_{m_1})}_{LSB-1} t$$

$$+ \frac{A_c \mu_2}{2} \cos 2\pi \underbrace{(f_c + f_{m_2})}_{USB-2} t + \frac{A_c \mu_2}{2} \cos 2\pi \underbrace{(f_c - f_{m_2})}_{LSB-2} t$$

Assume $f_{m_2} > f_{m_1}$
&
$\mu_2 > \mu_1$



$$\text{AM Bandwidth} = (\cancel{f_c} + f_{m_2}) - (\cancel{f_c} - f_{m_2})$$
$$= 2 f_{m_2}$$
$$= 2 f_{max}$$
$$= 2 \times \text{highest frequency of message signal}$$

Power of AM:-

$$P_t = P_c + P_{USB} + P_{LSB}$$
$$= P_c + P_{USB_1} + P_{USB2} + P_{LSB1} + P_{LSB2}$$

$$P_c = \frac{A_c^2}{2R}, \quad P_{USB_1} = P_{LSB_1} = \frac{A_c^2 \mu_1^2}{8R}$$
$$P_{USB2} = P_{LSB2} = \frac{A_c^2 \mu_2^2}{8R}$$

$$P_t = \frac{A_c^2}{2R} + \frac{A_c^2 \mu_1^2}{4R} + \frac{A_c^2 \mu_2^2}{4R}$$
$$= \frac{A_c^2}{2R}\left[1 + \frac{\mu_1^2 + \mu_2^2}{2}\right]$$

$$P_t = P_c\left[1 + \frac{\mu_t^2}{2}\right]$$

$$\mu_t = \sqrt{\mu_1^2 + \mu_2^2}$$

Total modulation index

Modulation Efficiency:-

$$\eta = \frac{\mu_t^2}{2 + \mu_t^2}$$

Current Relationship:

$$I_t = I_c \sqrt{1 + \frac{\mu_t^2}{2}}$$

Voltage Relationship:

$$V_t = V_c \sqrt{1 + \frac{\mu_t^2}{2}}$$

Q1. An AM signal is given by

$$s(t) = [10 + 6\cos 2\pi \times 10^4 t + 8\cos 4\pi \times 10^4 t]\cos 2\pi \times 10^6 t$$

(i) Find all the parameters of AM.

(ii) Find the frequency components in the given AM signal and plot the spectrum.

Sol. we know the standard form for multitone AM signal

$$s(t) = A_c[1 + \mu_1 \cos 2\pi f_{m_1} t + \mu_2 \cos 2\pi f_{m_2} t]\cos 2\pi f_c t$$

$$s(t) = 10[1 + 0.6\cos 2\pi \times 10^4 t + 0.8\cos 4\pi \times 10^4 t]\cos 2\pi \times 10^6 t$$

By comparing above two eqn.

$A_c = 10\,V$, $\mu_1 = 0.6$, $\mu_2 = 0.8$, $\mu_t = \sqrt{\mu_1^2 + \mu_2^2} = \sqrt{0.36 + 0.64} = 1$

$f_{m_1} = 10^4\,Hz = 10\,kHz$, $f_{m_2} = 2\times 10^4\,Hz = 20\,kHz$, $f_c = 10^6\,Hz = 1000\,kHz$

(i) AM $B.W = 2 \times f_{max} = 2 \times 20\ kHz = 40\ kHz$

$$P_c = \frac{A_c^2}{2R} = \frac{100}{2} = 50\ W$$

$$P_t = P_c\left[1 + \frac{\mu_t^2}{2}\right] = 1.5 P_c = 75 W$$

we know

$$P_t = P_c + P_{SB}$$

$$P_{SB} = P_t - P_c = 75 - 50 = 25 W$$

$$P_{USB} = P_{LSB} = \frac{25}{2} = 12.5 W$$

$$P_{USB_1} = \frac{A_c^2 \mu_1^2}{8R} = \frac{100 \times (0.6)^2}{8} = 4.5 W$$

$$P_{USB_2} = \frac{A_c^2 \mu_2^2}{8R} = \frac{100 \times (0.8)^2}{8} = 8 W$$

$$\eta = \frac{\mu_t^2}{2 + \mu_t^2} = 0.33$$

$$\% \eta = 33.33\%$$

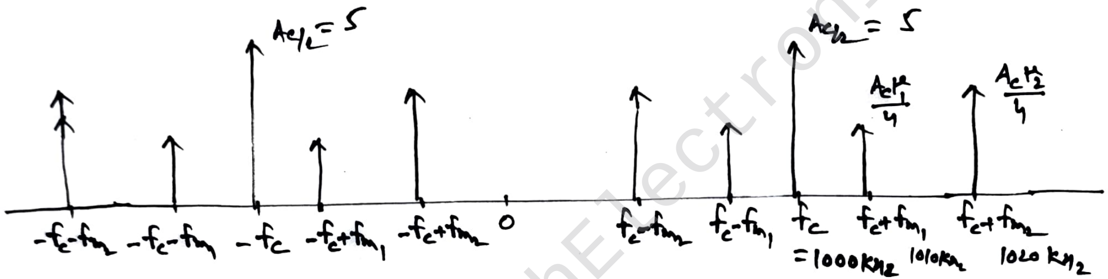
(ii) frequency components

$f_c = 1000\ kHz$, $f_c + f_{m_1} = 1010\ kHz$

$f_c - f_{m_1} = 990\ kHz$

$f_c + f_{m_2} = 1020\ kHz$

$f_c - f_{m_2} = 980\ kHz$



$A_c/2 = 5$
$A_c/2 = 5$
$\frac{A_c \mu_1}{4}$
$\frac{A_c \mu_2}{4}$
$-f_c-f_{m_2}$
$-f_c-f_{m_1}$
$-f_c$
$-f_c+f_{m_1}$
$-f_c+f_{m_2}$
0
$f_c-f_{m_2}$
$f_c-f_{m_1}$
f_c
$f_c+f_{m_1}$
$f_c+f_{m_2}$
=1000 kHz
1010 kHz
1020 kHz

AM Modulators:-

The circuits which are used for the Generation of Amplitude modulated waves are known as AM modulators.

1. Square law modulator
2. Switching modulator

1. Square Law Modulator:-

In this circuit a square law device like diode is used therefore it is known as square law modulator.

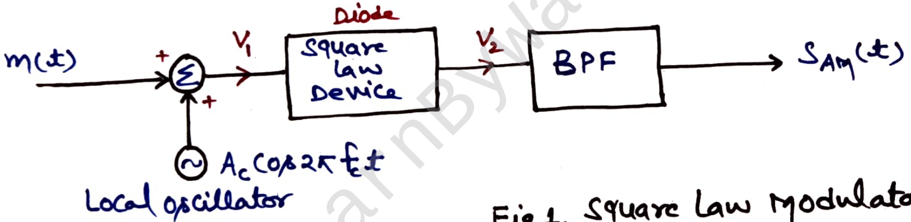
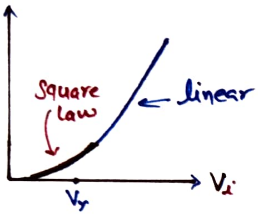
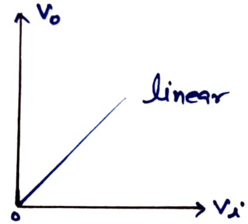
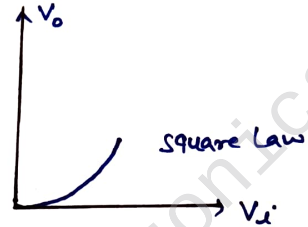


Fig 1. Square law modulator







$$V_o = a_1 V_i + a_2 V_i^2 + a_3 V_i^3 + \cdots$$

<u>Diode characteristic</u>

At low voltages : square law char.

At high voltages : Linear char.

→ We know that the strength of practical message signal is much small and if we generate carrier with little strength the diode can be operate in square law characteristic region.

→ Now consider the sum of message signal and carrier signal is represented by V_1 and its peak voltage is near to V_γ.

$$V_1 = m(t) + c(t)$$
$$= m(t) + A_c \cos 2\pi f_c t \quad \text{——— (1)}$$

The output of square law device is given by

$$V_2 = \underline{a_1 V_1 + a_2 V_1^2} + a_3 V_1^3 + \cdots$$

Consider only first two terms

$\cos^2 A = \frac{1+\cos 2A}{2}$ ($2f_c$)

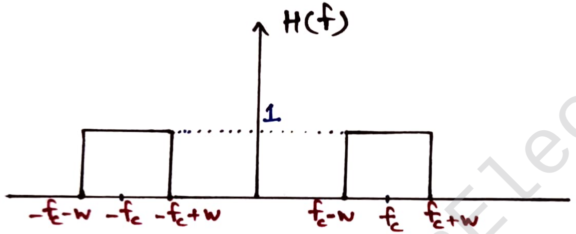
$$V_2 = a_1 V_1 + a_2 V_1^2$$
$$= a_1 (m(t) + A_c \cos 2\pi f_c t) + a_2 (m(t) + A_c \cos 2\pi f_c t)^2$$
$$= a_1 (m(t) + A_c \cos 2\pi f_c t) + a_2 (m^2(t) + A_c^2 \cos^2 2\pi f_c t + 2 A_c m(t) \cos 2\pi f_c t)$$

——— (2)

4

The output of diode (i.e V_2) is applied in BPF.

The Response of BPF should be as shown in figure



Band Pass filter output will be the final modulated signal $S_{AM}(t)$

$$(BPF)_{o/p} = S_{AM}(t) = a_1 A_c \cos 2\pi f_c t + 2 a_2 A_c m(t) \cos 2\pi f_c t$$

$$S_{AM}(t) = a_1 A_c \left[1 + \frac{2a_2}{a_1} m(t)\right] \cos 2\pi f_c t$$

$$\boxed{S_{AM}(t) = A_c' [1 + k_a m(t)] \cos 2\pi f_c t}$$

where: $A_c' = a_1 A_c$

$k_a = \frac{2a_2}{a_1}$ (Amplitude sensitivity)

Demodulation of AM :-

→ Demodulation of AM signals can be performed with the help of following demodulators.

1. Square law demodulator
2. Envelope detector or Diode detector

} $\mu \le 1$

3. Synchronous detector or coherent detector } Any value of μ

1. Square Law Demodulator :-

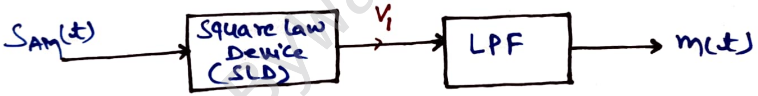


Fig. Block diagram of square law demodulator

We know General expression for AM signal

$$S_{AM}(t) = A_c[1 + k_a m(t)]\cos 2\pi f_c t = A_c \cos 2\pi f_c t + A_c k_a m(t)\cos 2\pi f_c t$$

Let square law device output is represented by V_1

$$V_1 = a_1 S_{AM}(t) + a_2 S_{AM}^2(t)$$

$$= a_1\left[A_c\cos 2\pi f_c t + A_c k_a m(t)\cos 2\pi f_c t\right] + a_2\left[A_c\cos 2\pi f_c t + A_c k_a m(t)\cos 2\pi f_c t\right]^2$$

$$= a_1\left[A_c\cos 2\pi f_c t + A_c k_a m(t)\cos 2\pi f_c t\right] + a_2\left[A_c^2\cos^2 2\pi f_c t + A_c^2 k_a^2 m^2(t)\cos^2 2\pi f_c t + 2A_c^2 k_a m(t)\cos^2 2\pi f_c t\right]$$

$$V_1 = a_1 A_c \cos 2\pi f_c t + a_1 A_c k_a m(t)\cos 2\pi f_c t + a_2 A_c^2\left[\frac{1+\cos 4\pi f_c t}{2}\right] + a_2 A_c^2 k_a^2 m^2(t)\left[\frac{1+\cos 4\pi f_c t}{2}\right] + a_2\cdot 2\cdot A_c^2 k_a m(t)\cdot\left[\frac{1+\cos 4\pi f_c t}{2}\right]$$

$$(LPF)_{o/p} = \underbrace{\frac{a_2 A_c^2 K_a^2 m^2(t)}{2}}_{\text{Noise}} + \underbrace{\frac{\cancel{2} a_2 A_c^2 K_a m(t)}{\cancel{2}}}_{\text{Signal}}$$

→ If $\frac{S}{N} >>> 1$, then $m(t)$ can be completely reconstructed.

→ If $\frac{S}{N} < 1$, then $m(t)$ can not be completly reconstructed.

From above equation

$$\frac{S}{N} = \frac{\cancel{a_2} \cancel{A_c^2} \cancel{K_a} \cancel{m(t)}}{\frac{\cancel{a_2} \cancel{A_c^2} K_a^{\cancel{2}} m^{\cancel{2}}(t)}{2}} = \frac{2}{K_a m(t)}$$

for $m(t) = A_m \cos 2\pi f_m t$

$$\frac{S}{N} = \frac{2}{K_a A_m \cos 2\pi f_m t} = \frac{2}{\mu \cos 2\pi f_m t}$$

for $\cos 2\pi f_m t = 1$, $\boxed{\frac{S}{N} = \frac{2}{\mu}}$

→ $\frac{S}{N}$ should be high for perfect reconstruction that means μ should be very low.

$$\boxed{\frac{S}{N} = \frac{2}{\mu}}$$

Ex. $\frac{S}{N} = 10$, μ should be 0.2

$$\eta = \frac{\mu^2}{2+\mu^2} = \frac{0.2^2}{2+0.2^2} \simeq 2\%$$

that means 98% Power Wasted

→ If μ is low, efficiency will be very low.

→ But for efficient power distribution η should be high so that square law demodulator is not preffered for AM demodulation.

Envelope Detector:-

Envelope detector is used for the demodulation of AM signals.

It is very simple and cheaper.

Envelope detector (ED) extract +ve envelope of the applied signal and produces at the output.

Envelope detector can be used for $\mu \leq 1$, It is failed for $\mu > 1$.

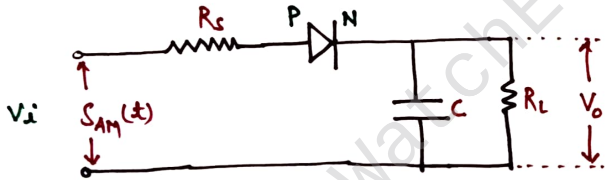


Fig1. Envelope Detector

Envelope detector can be designed with the help of diode & Capacitor as shown in figures.

Assume diode is <u>ideal</u>

P > N ⟶ F.B ⟶ S.C (short circuit)

P < N ⟶ R.B ⟶ O.C (open circuit)

Assume the input of ED is a sinusoidal wave as shown in fig2.

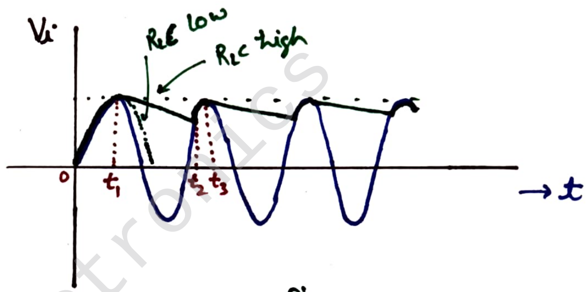
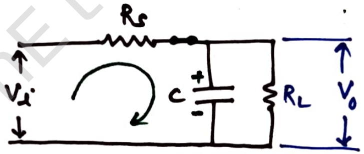


Fig.2

Now the working of ED can be explained as follows –



Case (i) when $t = 0^+$ (Greater than zero)

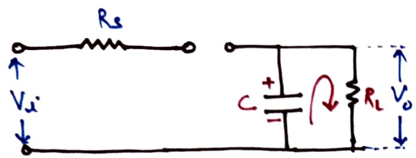
P > N ⟶ F.B ⟶ S.C

Time constant

R_SC should be small, so that capacitor charges rapidly towards input.

Case (ii) when $t = t_1^+$

P < N ⟶ R.B ⟶ O.C



→ Time constant $R_L C$ should be high, so that capacitor discharges slowly.

→ If capacitor discharges slowly the output signal can follow the envelope of input signal.

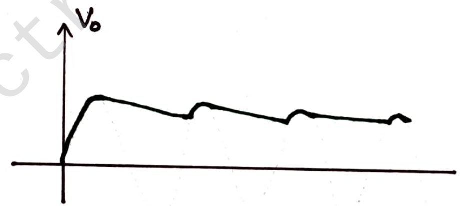
Case (iii) when $t = t_2^+$ to t_3

P > N → F.B → S.C

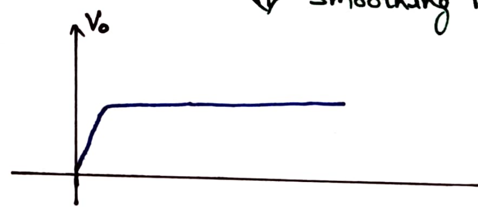
Case (iv) when $t = t_3^+$

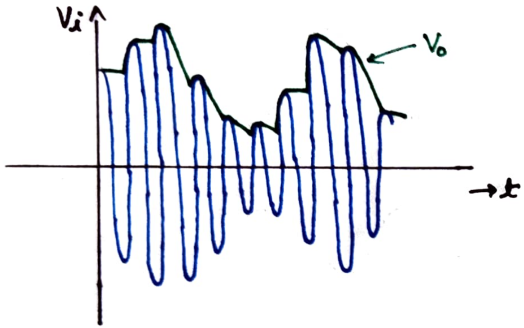
Capacitor discharges

Same procedure follow for further input.



⇓ Smoothing filter





→ For proper envelope detection $R_s C$ should be small and $R_L C$ should be high.

NOTE:

	ED input	ED output
(i)	$V_m \cos 2\pi f_c t$	V_m
(ii)	$A_c[1+k_a m(t)] \cos 2\pi f_c t$	$A_c[1+k_a m(t)]$

$$A_c[1+k_a m(t)]$$

$$= \underbrace{A_c + A_c k_a m(t)}$$

$\Downarrow$ Passing through Capacitor

$A_c k_a m(t)$

Importance of Amplitude sensitivity (k_a):-

We know that

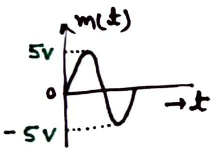
$$S_{AM}(t) = A_c\left[1 + \underline{k_a}\, m(t)\right] \cos 2\pi f_c t$$

for envelope detector

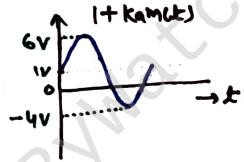
$A_c[1 + k_a m(t)]$ should be +ve that means

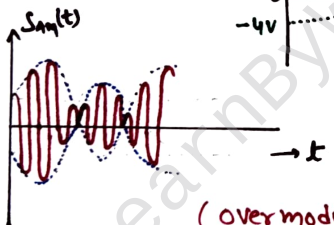
$1 + k_a m(t)$ should be +ve.

Case (i)



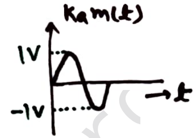
Assume $k_a = 1$

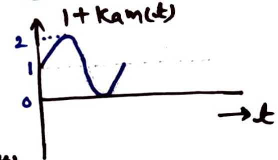


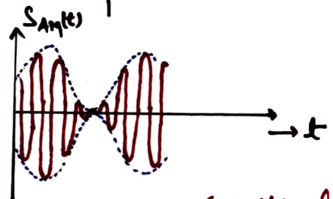


(over modulated)

Case (ii) Assume $k_a = \frac{1}{5}$

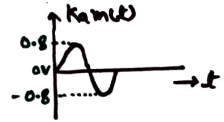


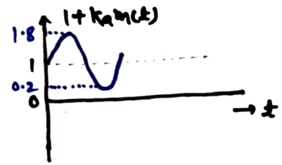


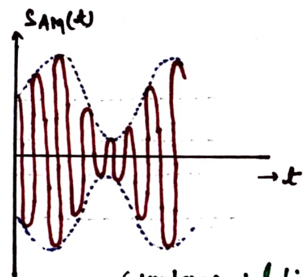


(Critical Modulation)

Case (iii) Assume $k_a = \frac{1}{6}$







(Under Modulation)

→ After observing three cases we can say that-

Ka is used for normalization of message signal for proper Envelope detection.

Advantages of AM:-

1. Demodulation is simple.
2. AM can be used for long distance communication.

Drawbacks of AM:-

1. Transmitter power wasted.
2. AM needs high channel bandwidth.
3. A.M Transmission is highly noisy.
4. Affected by Quadrature Null Effect (QNE).

Synchronous Detector or Coherent detector:-

Synchronous detector is used for the demodulation of AM signal. or we can say to recover message signal m(t).

For perfact reconstruction of message signal, Local oscillator output should be perfactly synchronous in both frequency and phase with respect to transmitter carrier.

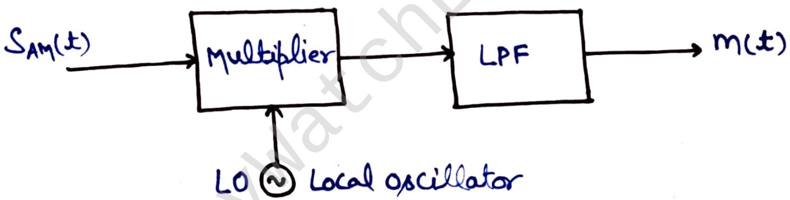


Fig. Coherent detector

$$(LPF)_{o/p} = \frac{A_c^2 k_a m(t)}{2} \longrightarrow \boxed{\text{Amplifier}} \longrightarrow m(t)$$

Cascaded Amplifier

Case(ii) Assume LO output $= A_c \cos[2\pi f_c t + \phi]$ (No phase synchronization)

$$(Mul)_{o/p} = [A_c \cos 2\pi f_c t + A_c k_a m(t) \cos 2\pi f_c t] \cdot [A_c \cos(2\pi f_c t + \phi)]$$

$$= A_c^2 \cos(2\pi f_c t + \phi) \cdot \cos 2\pi f_c t + A_c^2 k_a m(t) \cos(2\pi f_c t + \phi) \cdot \cos 2\pi f_c t$$

$$= \frac{A_c^2}{2} \cos(4\pi f_c t + \phi) + \frac{A_c^2}{2} \cos\phi + \frac{A_c^2 k_a m(t)}{2} \cos(4\pi f_c t + \phi) + \frac{A_c^2 k_a m(t)}{2} \cos\phi$$

$$(LPF)_{o/p} = \frac{A_c^2 k_a m(t)}{2} \cos\phi$$

If $\phi = 0^\circ$ $\quad (LPF)_{o/p} = \dfrac{A_c^2 k_a m(t)}{2}$

If $\phi = 60^\circ$ $\quad (LPF)_{o/p} = \dfrac{A_c^2 k_a m(t)}{4} \longrightarrow \boxed{\text{Amplifier}} \longrightarrow m(t)$

→ Frequency synchronization can be maintained easily but to maintain phase synchronization additional circuitary is used which makes synchronous detector very complex.

$$s_{AM}(t) = A_c[1+k_a m(t)]\cos 2\pi f_c t$$

$$= A_c\cos 2\pi f_c t + A_c k_a m(t)\cos 2\pi f_c t$$

→ Now consider two cases for local oscillator output

Case (i) Assume LO output = $A_c\cos 2\pi f_c t$ (Perfect synchronization)

$$(Mul)_{o/p} = s_{AM}(t)\cdot(LO)_{o/p}$$

$$= A_c^2\cos^2 2\pi f_c t + A_c^2 k_a m(t)\cos^2 2\pi f_c t$$

$$= A_c^2\left[\frac{1+\cos 4\pi f_c t}{2}\right] + A_c^2 k_a m(t)\left[\frac{1+\cos 4\pi f_c t}{2}\right]$$

but if $\phi = 90^\circ$

$$(LPF)_{O/P} = \frac{A_c^2 k_a m(t)}{2} \cos 90^\circ = 0$$

This is known as Quadrature Null Effect (QNE)

→ For reconstruction of message signal ϕ should be constant. To maintain ϕ to be constant additional circuitary have to be used which makes synchronous detector or coherent detector very complex.

DSB-SC : (Double Sideband - Suppressed Carrier)

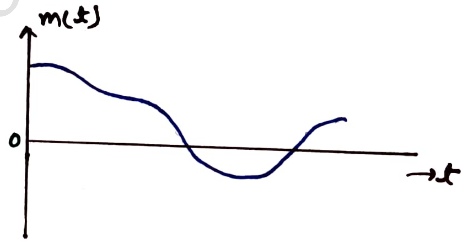
Assume message signal $m(t)$ and carrier signal $c(t)$.

$$c(t) = A_c \cos 2\pi f_c t$$

General Expression of DSB-SC or simply DSB is given by

$$S_{DSB}(t) = m(t) \cdot c(t)$$

$$\boxed{S_{DSB}(t) = A_c\, m(t) \cos 2\pi f_c t} \quad \text{--- (1)}$$



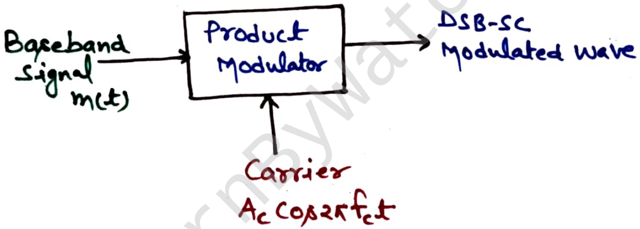
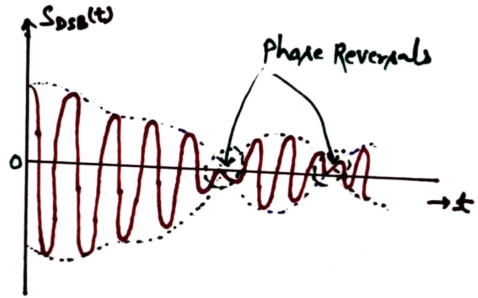


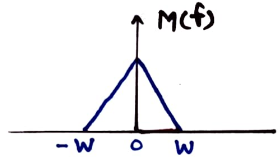
Fig. Block diagram of Product Modulator



To plot frequency spectrum, Take fourier Transform of eqn (1).

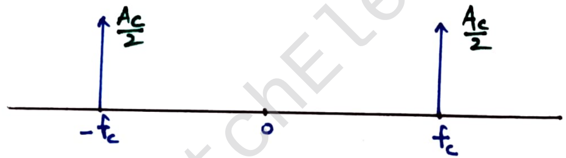
$$S(f) = \frac{A_c}{2}\left[M(f-f_c) + M(f+f_c)\right] \quad \text{---} \quad (2)$$

$m(t) \longleftrightarrow$



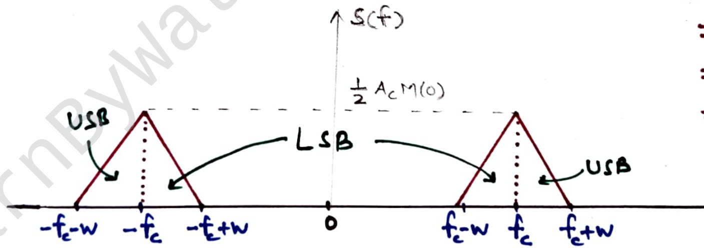
$c(t) \longleftrightarrow$

$= A_c \cos 2\pi f_c t$



$s_{DSB}(t) \longleftrightarrow$

$= A_c m(t) \cos 2\pi f_c t$



DSB Bandwidth

$= \cancel{f_c} + W - \cancel{f_c} + W$

$= 2W$

$= 2 \times$ msg signal B.W

Single tone DSB-SC :-

Assume $m(t) = A_m \cos 2\pi f_m t$ (single freq^n)

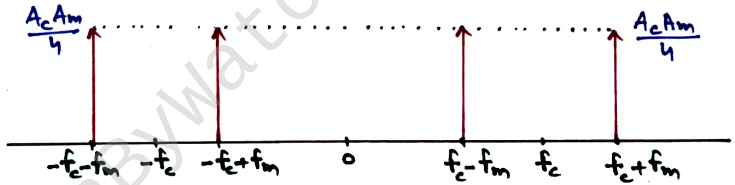
$c(t) = A_c \cos 2\pi f_c t$

$$S_{DSB}(t) = m(t)\cdot c(t)$$

$$= A_c A_m \cos 2\pi f_c t \cos 2\pi f_m t$$

$$S_{DSB}(t) = \frac{A_c A_m}{2}\cos 2\pi (f_c+f_m)t + \frac{A_c A_m}{2}\cos 2\pi (f_c-f_m)t$$

Frequency spectrum:



$$\text{DSB B.W} = \cancel{f_c}+f_m-(\cancel{f_c}-f_m)$$

$$= 2f_m = 2\times \text{msg signal } \cancel{\text{B.W}} \text{ frequency}$$

Total Power of DSB-SC :-

In case of DSB-SC only side-bands are present, therefore

$$P_t = P_{SB} = P_{USB} + P_{LSB}$$

$$P_{USB} = \frac{\left(\frac{A_c A_m}{2}\right)^2}{2R} = \frac{A_c^2 A_m^2}{8R} = P_{LSB}$$

$$\boxed{P_t = \frac{A_c^2 A_m^2}{4R}}$$

Modulation Efficiency (η) :-

We know that Efficiency is given by: $\eta = \frac{\text{Sideband Power}}{\text{Total Power}} = \frac{P_{SB}}{P_t} = 1$

$$\% \eta = 100\%$$

(It means 100% Power taken by Sidebands)

Q1. A carrier signal of $10\sqrt{3}\cos 2\pi \times 10^6 t$ is DSB modulated by the message signal of $\sqrt{2}\cos 2\pi \times 10^4 t$. Find B.W, total power, side band power and Efficiency.

Sol

$A_c = 10\sqrt{3}$, $f_c = 10^6 Hz = 1000 KHz$

$A_m = \sqrt{2}$, $f_m = 10^4 Hz = 10 KHz$

$B.W = 2 \times f_m = 2 \times 10 = 20 KHz$

$$P_t = \frac{A_c^2 A_m^2}{4R} = \frac{(100 \times 3) \times 2}{4 \times 1} = \frac{600}{4} = 150 W$$

$$P_{USB} = P_{LSB} = \frac{150}{2} = 75 W$$

$\% \eta = 100\%$

Q2. A Carrier of $15 \cos 2\pi \times 10^6 t$ is DSB modulated by the message signal $\cos 2\pi \times 10^4 t + 5 \cos 4\pi \times 10^4 t + 10 \cos 6\pi \times 10^4 t$.

(i) Find Bandwidth, total power and Efficiency

(ii) Frequency component present in the DSB signal

Sol

Given.

$A_c = 15V$, $f_c = 1000\ kHz$

$A_{m_1} = 1V$, $A_{m_2} = 5V$, $A_{m_3} = 10V$

$f_{m_1} = 10\ kHz$, $f_{m_2} = 20\ kHz$, $f_{m_3} = 30\ kHz$

(i)

$$B.W = 2 \times f_{max} = 2 \times 30 = 60\ kHz$$

$$P_t = \frac{A_c^2 [A_{m_1}^2 + A_{m_2}^2 + A_{m_3}^2]}{4R} = \frac{225 \times [1 + 25 + 100]}{4 \times 1} = 7087.5W$$

$$\%\eta = 100\%$$

(ii) frequency components

$f_c \pm f_{m_1}$, $f_c \pm f_{m_2}$

1010 kHz & 990 kHz, 1020 kHz & 980 kHz

$f_c \pm f_{m_3}$

1030 kHz & 970 kHz

Ring Modulator :-

Ring modulator can be designed with the help of 4-diodes connected in the form of a Ring to generate DSB-SC signal as shown in fig 1.

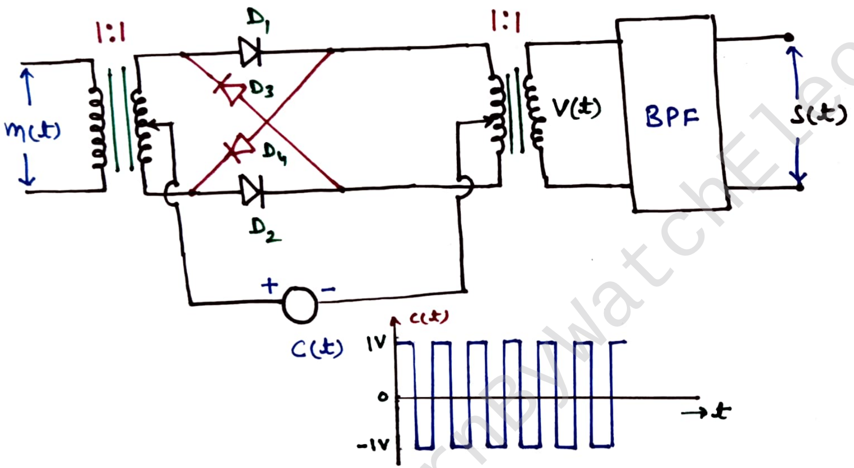


Fig.1. Ring Modulator

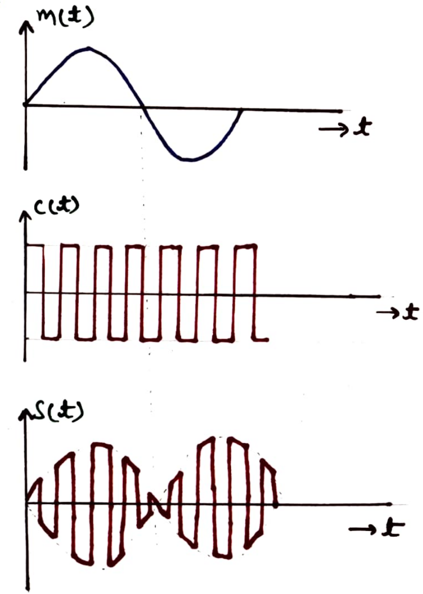


Fig. 2

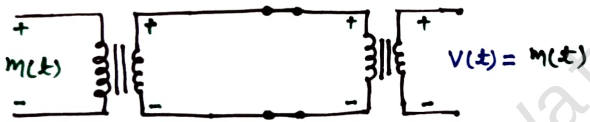
→ The carrier c(t) is taken as squarewave signal to make the analysis easy.

→ The strength of c(t) is high therefore diode working is considered on the basis of c(t).

→ The working of ring modulator can be explain in two cases.
Assume diodes are ideal.

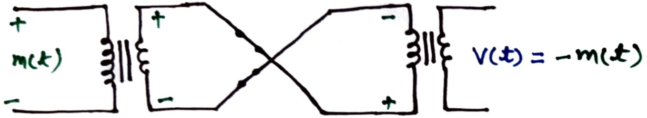
Case (i) when c(t) is +ve

Diodes D_1 and D_2 will be in forward bias and act as short circuit.
Diodes D_3 and D_4 will be in Reverse bias and act as open circuit.



Case (ii) when c(t) is −ve

Diodes D_3 and D_4 will be ~~in~~ forward bias and act as short circuit.
Diodes D_1 and D_2 will be Reverse bias and act as open circuit.



From the above two cases we can say

$$V(t) = m(t) \cdot c(t)$$

but the fourier series expansion of $c(t)$ shows that it contains lot of odd harmonics.

$$c(t) = \frac{4}{\pi} \sum_{n=1,3,5\ldots} \frac{(-1)^{\frac{n-1}{2}}}{n} \cos(n 2\pi f_c t)$$

→ when $V(t)$ is passed through BPF tuned to f_c, the output will be the desired DSB-SC signal, which can be given by

$$\boxed{s(t) = \frac{4}{\pi} m(t) \cos(2\pi f_c t)}$$

Quadrature Carrier Multiplexing:-

→ The quadrature null effect of the coherent detector may be put to good use in the construction of Quadrature carrier multiplexing or Quadrature Amplitude Modulation (QAM).

→ This scheme enables two DSB-SC modulated waves (Resulting from the application of two physically independent message signals) to occupy the same channel bandwidth and yet it allows for the seperation of the two message signals at the Receiver output.

→ It is therefore a bandwidth conservation scheme.

→ A block diagram for the Quadrature carrier multiplexing system Transmitter is shown in fig(a). and for Receiver is shown in fig(b).

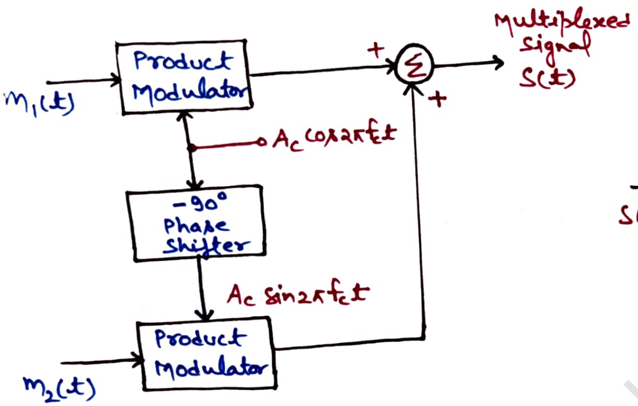


Fig (a) QAM Transmitter

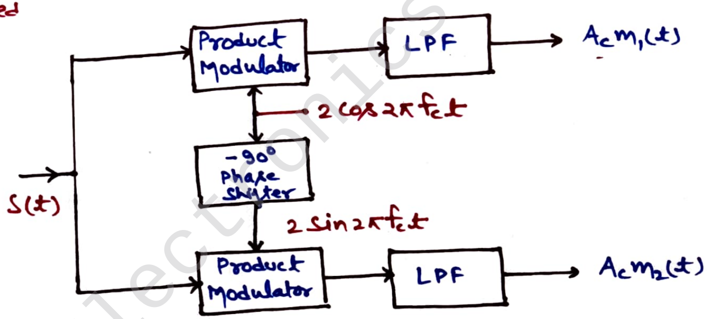


Fig (b) QAM Receiver

→ The transmitted signal s(t) consist of the sum of two product modulator outputs

$$s(t) = A_c m_1(t) \cos 2\pi f_c t + A_c m_2(t) \sin 2\pi f_c t$$

→ At QAM Receiver

The multiplexed signal $s(t)$ is applied simultaneously to two seperate coherent detectors that are supplied with two local carriers of the same frequency but differing in phase by $-90°$.

→ The output of the top detector is $A_c m_1(t)$, whereas the output of the bottom detector is $A_c m_2(t)$.

→ For the system to operate satisfactorily, it is important to maintain the correct phase and frequency relationships between local oscillators used in the transmitter and receiver parts of the system.

→ To maintain this synchronization, we may send a Pilot signal outside the Passband of the modulated signal.

→ Pilot signal typically consist of a low-Power sinusoidal tone whose frequency and phase are related to the carrier wave $c(t)$.

Costas Loop Receiver :-

→ In coherent detection, at Receiving end a carrier is required that should be phase coherent with the transmitted carrier.

→ It can be possible if we transmit a carrier component with the modulated signal.

→ But DSB-SC signal has no such component.

→ Costas Loop has the capability to generate a coherent carrier at the receiver and therefore used for the demodulation of DSB-SC signals.

→ Fig 1. shows the block diagram for costas loop.

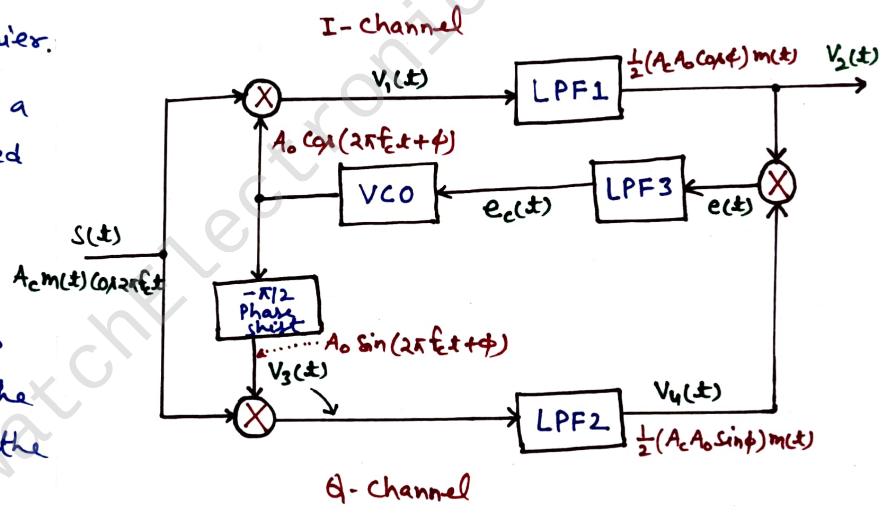


Fig 1. Costas Loop

The voltage controlled oscillator (VCO) produces a periodic waveform whose frequency is controlled by the input voltage $e_c(t)$.

The output frequency of the VCO, when $e_c(t)=0$ is known as free Running frequency of VCO.

Case (i) Let us assume that the frequency and phase of the VCO output are same as that of the incoming carrier, then

i.e $(VCO)_{o/p} = A_0 \cos 2\pi f_c t$

$$V_1(t) = s(t)\cdot (VCO)_{o/p}$$

$$= A_c m(t) \cos 2\pi f_c t \cdot A_0 \cos 2\pi f_c t$$

$$= A_c A_0 m(t) \cdot \cos^2 2\pi f_c t$$

$$V_1(t) = A_c A_0 m(t) \cdot \left[\frac{1+\cos 4\pi f_c t}{2}\right]$$

$$(LPF1)_{o/p} = \frac{A_0 A_c m(t)}{2} = V_2(t)$$

$V_2(t) \propto m(t)$, desired signal

Similarly

$$V_3(t) = A_c m(t) \cos 2\pi f_c t \cdot A_0 \sin 2\pi f_c t$$

$$= A_c A_0 m(t) \sin 4\pi f_c t$$

$$(LPF2)_{o/p} = 0 = V_4(t)$$

Case (ii) Assume $(VCO)_{o/p} = A_0 \cos(2\pi f_c t + \phi)$

(No Phase Synchronization)

$$V_1(t) = A_c m(t) \cos 2\pi f_c t \cdot A_0 \cos(2\pi f_c t + \phi)$$

$$= \frac{A_c A_0 m(t)}{2} \cos(4\pi f_c t + \phi) + \frac{A_c A_0 m(t)}{2} \cos\phi$$

$$(LPF1)_{o/p} = \frac{A_c A_0 m(t)}{2} \cos\phi = V_2(t)$$

Similarly

$$(LPF2)_{O/P} = \frac{A_c A_o m(t)}{2} \sin\phi = V_4(t)$$

In this case $V_4(t)$ is not zero, therefore some error signal will be generated.

$$e(t) = V_2(t) \cdot V_4(t)$$

$$= \frac{1}{2} A_c A_o m(t) \cos\phi \cdot \frac{1}{2} A_c A_o m(t) \sin\phi$$

$$= \frac{1}{4} [A_c A_o m(t)]^2 \cos\phi \sin\phi$$

$$e(t) = \frac{1}{8} [A_c A_o m(t)]^2 \sin 2\phi$$

→ $e(t)$ is applied to LPF3, which has very narrow passband.

Hence $e_c(t) = C_o \sin 2\phi$

where C_o is the DC value of $\frac{1}{8}[A_c A_o m(t)]^2$.

This DC control voltage ensures that the VCO output is coherent with the carrier used for modulation.

Single Side band (SSB) Modulation :-

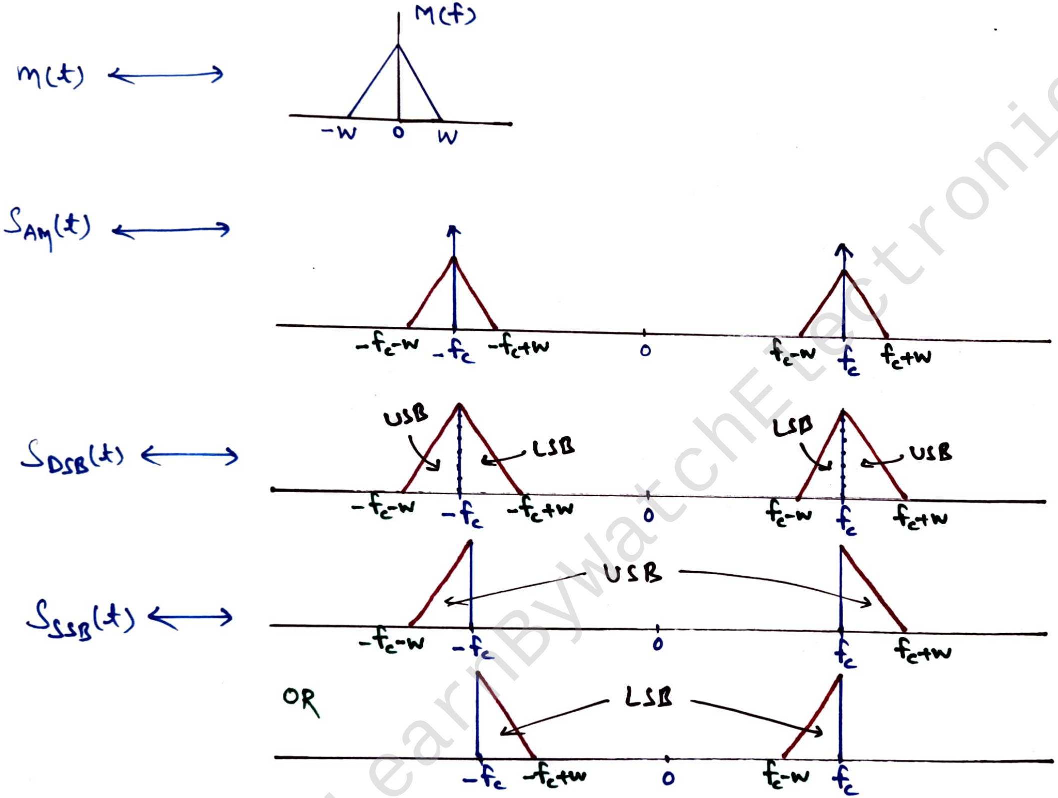
→ Two main Parameters are considered while designing a communication system that are: 1. Transmission Power
2. Transmission Bandwidth

→ In case of standard AM or DSB-C or DSB-FC, both are very high.

→ In case of DSB-SC Transmission Power is less required than AM but Transmission bandwidth is same as that of AM.

→ In case of SSB, only one sideband will be Transmitted (because both the sidebands contain same information).

→ Therefore Transmission Power as well as Transmission bandwidth Requirment will be reduced as compared to AM & DSB-SC.



AM B·W = 2W

DSB B·W = 2W

SSB B·W = W

= msg signal B/W

Single tone SSB :-

Assume $m(t) = A_m \cos 2\pi f_m t$

We know that

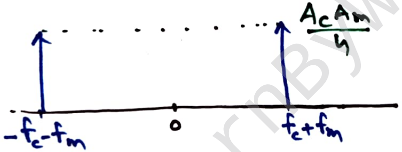
$$S_{AM}(t) = A_c \cos 2\pi f_c t + \frac{A_c \mu}{2} \cos 2\pi (f_c + f_m) t + \frac{A_c \mu}{2} \cos 2\pi (f_c - f_m) t$$

$$S_{DSB}(t) = \frac{A_c A_m}{2} \cos 2\pi (f_c + f_m) t + \frac{A_c A_m}{2} \cos 2\pi (f_c - f_m) t$$

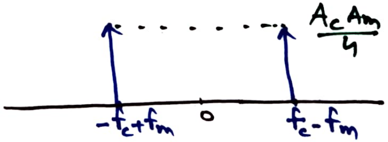
$$S_{SSB}(t) = \frac{A_c A_m}{2} \cos 2\pi (f_c \pm f_m) t$$

$+$: USB

$-$: LSB



USB



LSB

Power of SSB :-

Total Power $P_t = P_{SB} = P_{USB}$ or P_{LSB}

$$P_t = \frac{\left(\frac{A_c A_m}{2}\right)^2}{2R} = \frac{A_c^2 A_m^2}{8R}$$

Efficiency (η): $\eta = \frac{P_{SB}}{P_t} = 1$, $\% \eta = 100\%$

General Expression of SSB :-

We know that $S_{SSB}(t) = \frac{A_c A_m}{2} \cos 2\pi (f_c \pm f_m) t$

+ : USB
− : LSB

$$= \frac{A_c A_m}{2} \cos(2\pi f_c t \pm 2\pi f_m t)$$

$$S_{SSB}(t) = \frac{A_c A_m}{2} \cos 2\pi f_c t \, \cos 2\pi f_m t \mp \frac{A_c A_m}{2} \sin 2\pi f_c t \, \sin 2\pi f_m t$$

$-$: USB
$+$: LSB

We have $m(t) = A_m \cos 2\pi f_m t$

and $90°$ phase shift of $m(t) = A_m \sin 2\pi f_m t$

$= \hat{m}(t)$

$\hat{m}(t)$ is the hilbert transformation of $m(t)$

After putting $m(t)$ & $\hat{m}(t)$ in place of $A_m \cos 2\pi f_m t$ & $A_m \sin 2\pi f_m t$ above equation becomes

$$\boxed{S_{SSB}(t) = \frac{A_c\, m(t)}{2} \cos 2\pi f_c t \mp \frac{A_c\, \hat{m}(t)}{2} \sin 2\pi f_c t}$$

$-$: USB
$+$: LSB

Generation of SSB or SSB Modulators :-

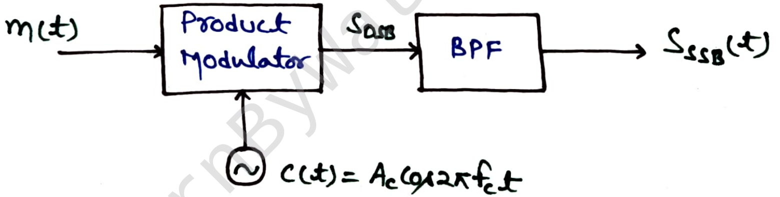
SSB modulation can be Performed using two methods -

① Frequency discrimination method

② Phase discrimination method.

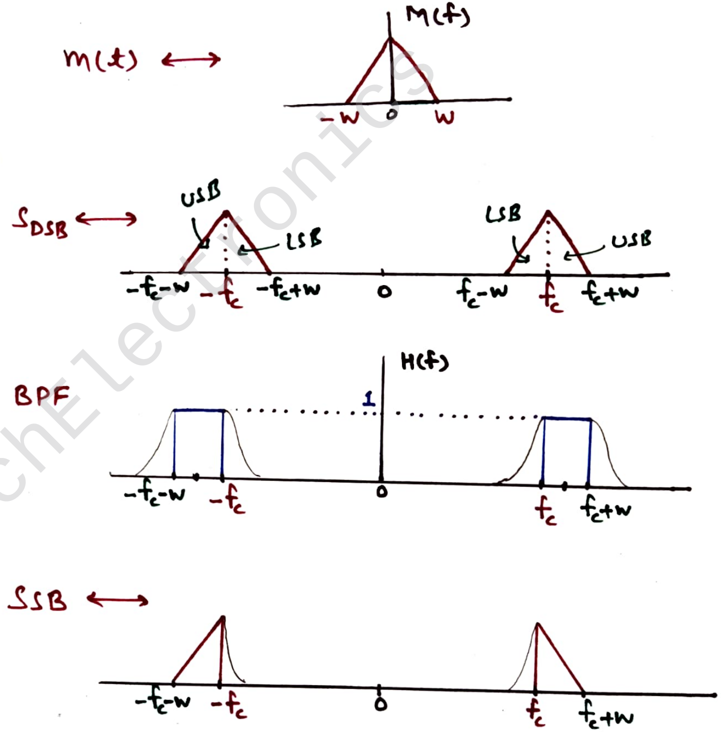
① Frequency discrimination method :

In this method DSB signal is passed through proper band pass filter (BPF) to generate SSB signal.



→ The center frequency of BPF decide whether USB signal is generated or LSB signal is generated.

→ Suppose we want to transmit the USB, then using an <u>ideal</u> Band pass filter (BPF) with center frequency $f_c + \frac{W}{2}$, we can obtain desired result as shown in spectrum.



Drawback of frequency Discrimination method:

→ Since ideal BPF is not available. In the resulting SSB signal some additional frequency component will be present.

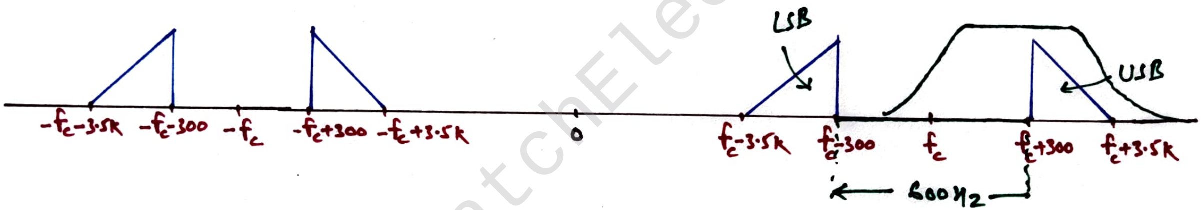
→ Because of this drawback SSB modulation is limited for the transmission of voice signals only.

→ For voice signal, in the resulting DSB-SC signal, wide band gap of 600Hz exist between USB & LSB so that filtering using Practical BPF is possible hence voice signal can be transmitted by SSB.

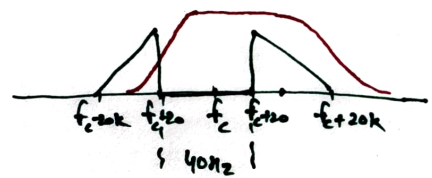
Voice signal
(300Hz to 3.5KHz)

$m(t) \longleftrightarrow$

$S_{DSB} \longleftrightarrow$



Audio
(20Hz - 20KHz)



Phase Discrimination method :-

→ This method is used to generate Single tone SSB.

→ For the generation of multi-tone SSB wide band 90° phase shifter is required which is practically not realizable.

→ Therefore for generation of multi-tone SSB frequency discrimination method is used.

→ We know the general expression for SSB signal

$$S_{SSB}(t) = \frac{A_c m(t)}{2} \cos 2\pi f_c t \mp \frac{A_c \hat{m}(t)}{2} \sin 2\pi f_c t$$

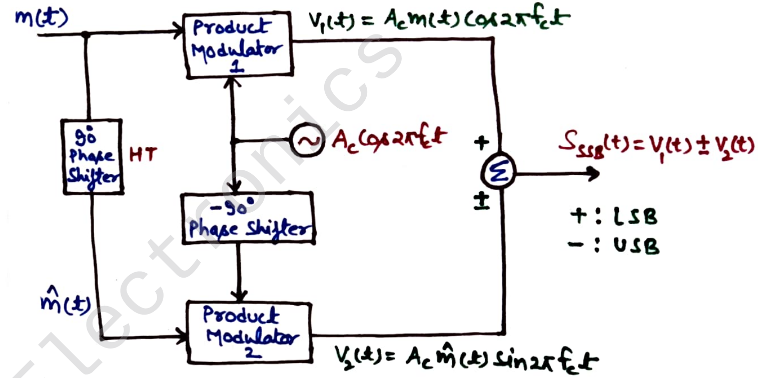


Fig. 1.

→ To implement above expression phase discriminator method requires, two product modulators, two $\frac{\pi}{2}$ phase shifters and an adder. One of the phase shifter is actually a hilbert transformer (HT) which provides 90° phase shift for all the component in M(f).

→ The design of hilbert transformer is not too difficult using digital filter design techniques.

→ Therefore Phase discrimination or Phase shift method of SSB generation is better suited for digital implementation.

Demodulation of SSB :-

Demodulation of SSB signals can be done with the help of Synchronous detector.

Synchronous detector:-

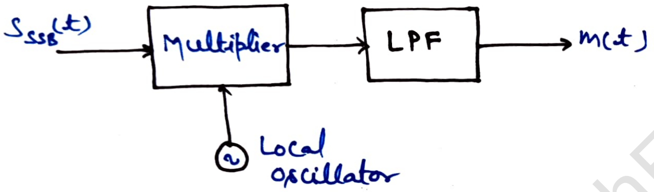


Fig. Block diagram of synchronous detector

We know the standard expression for SSB signal

$$S_{SSB}(t) = \frac{A_c m(t)}{2} \cos 2\pi f_c t \mp \frac{A_c \hat{m}(t)}{2} \sin 2\pi f_c t$$

$-$: USB
$+$: LSB

Case (i) Assume $(LO)_{O/P} = A_c \cos 2\pi f_c t$ (Perfect synchronization)

$$(Mul)_{O/P} = S_{SSB}(t) \cdot (LO)_{O/P}$$

$$= \frac{A_c^2 m(t)}{2} \cos^2 2\pi f_c t \mp \frac{A_c^2 \hat{m}(t)}{4} \sin 4\pi f_c t$$

$$= \frac{A_c^2 m(t)}{2} \left[\frac{1 + \cancel{\cos 4\pi f_c t}}{2} \right] \mp \frac{A_c^2 \hat{m}(t)}{4} \cancel{\sin 4\pi f_c t}$$

$$(LPF)_{O/P} = \frac{A_c^2 m(t)}{4} \longrightarrow \boxed{Amp} \longrightarrow m(t)$$

Case (ii) Assume $(LO)_{O/P} = A_c \cos(2\pi f_c t + \phi)$ (No phase synchronization)

$$(Mul)_{O/P} = S_{SSB}(t) \cdot (LO)_{O/P}$$

$$= \left(\frac{A_c m(t)}{2} \cos 2\pi f_c t \mp \frac{A_c \hat{m}(t)}{2} \sin 2\pi f_c t \right) \cdot A_c \cos(2\pi f_c t + \phi)$$

$$(\text{Mul})_{o/p} = \frac{A_c^2 m(t)}{4}\cos(4\pi f_c t+\phi) + \frac{A_c^2 m(t)}{4}\cos\phi \mp \frac{A_c^2 \hat{m}(t)}{4}\sin(4\pi f_c t+\phi) \pm \frac{A_c^2 \hat{m}(t)}{2}\sin\phi$$

$$(\text{LPF})_{o/p} = \frac{A_c^2 m(t)}{4}\cos\phi \pm \frac{A_c^2 \hat{m}(t)}{4}\sin\phi$$

If $\phi = 0$

$$(\text{LPF})_{o/p} = \frac{A_c^2 m(t)}{4}$$

If $\phi = 90^\circ$

$$(\text{LPF})_{o/p} = \pm\frac{A_c^2 \hat{m}(t)}{4}$$

(No Quadrature Null Effect)

Vestigial Side band (VSB) Modulation:-

→ To Produce SSB signal from DSB signal ideal filters should be used but Practically ideal filters are not physible.

→ SSB modulation have some drawbacks like-

→ Generation of an SSB signal is difficult.

→ Selective filtering is to be done to get the original signal back.

→ Phase shifter should be exactly tuned to 90°.

→ To overcome these drawbacks, VSB modulation is used.

→ VSB Can be viewed as a compromise between SSB and DSB-SC.

→ In VSB modulation, one of the sideband is Partially Suppressed and a vestige of the other sideband is transmitted to compensate for that suppression.

→ The resulting signal has a B.W > the B.W of msg signal but < the B.W of DSB signal.

→ The transmission Bandwidth of VSB modulation is

$$B_T = W + f_v$$

where : W is the message bandwidth
and f_v is the width of the vestigial sideband

Generation of VSB AM :

→ Generate a DSB-SC signal

→ Pass the DSB-SC signal through a BPF with frequency response H(f)

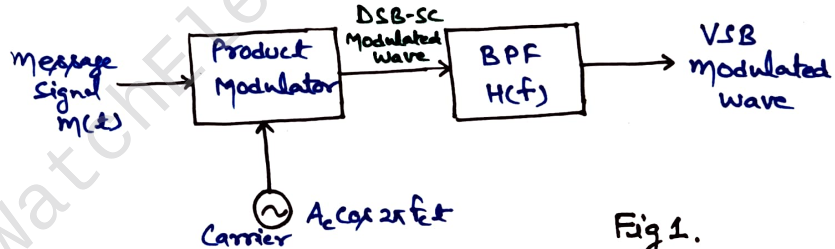


Fig 1.

→ Assuming that a vestige of the lower sideband is transmitted then the frequency response H(f) of the BPF takes the form as shown in fig 2.

→ Inside the transition interval
$|f_c - f_v| \leq |f| \leq |f_c + f_v|$
the following two conditions are satisfied.

(i) The sum of the values of the magnitude Response $|H(f)|$ at any two frequencies equally displaced above and below f_c is Unity.

(ii) The Phase Response $\arg(H(f))$ is linear. That is, $H(f)$ satisfies the condition

$$H(f-f_c) + H(f+f_c) = 1 \qquad \text{for } -W \leq f \leq W$$

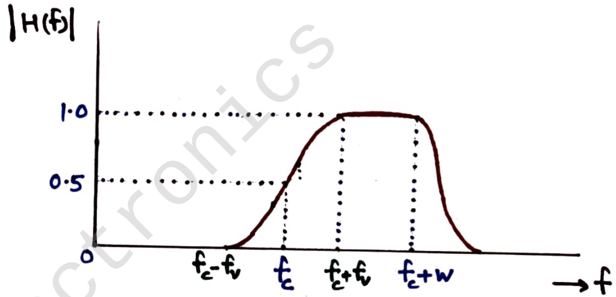
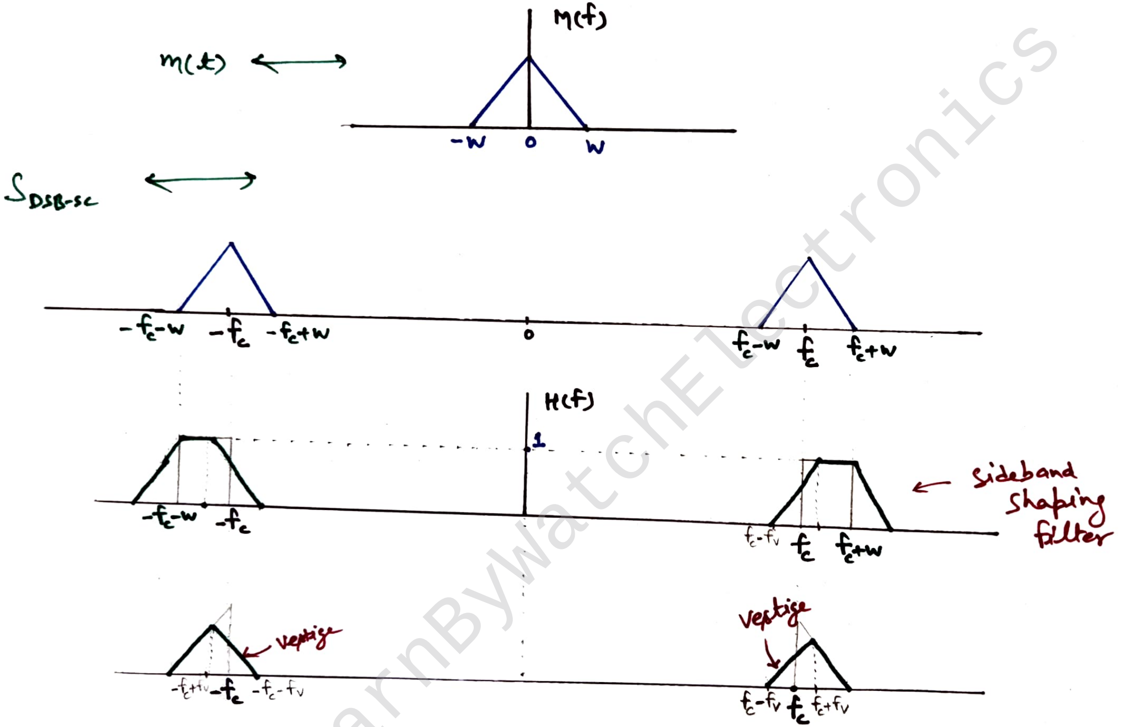


Fig.2. Magnitude Response of VSB filter (only +ve frequency Portion is shown)


M(f)
m(t)
-W
0
W
S DSB-SC
-fc-W
-fc
-fc+W
0
fc-W
fc
fc+W
H(f)
1
-fc-W
-fc
fc-fv
fc
fc+W
Sideband shaping filter
vestige
-fc+fv
-fc
-fc-fv
vestige
fc-fv
fc
fc+fv

Frequency Division Multiplexing (FDM) :-

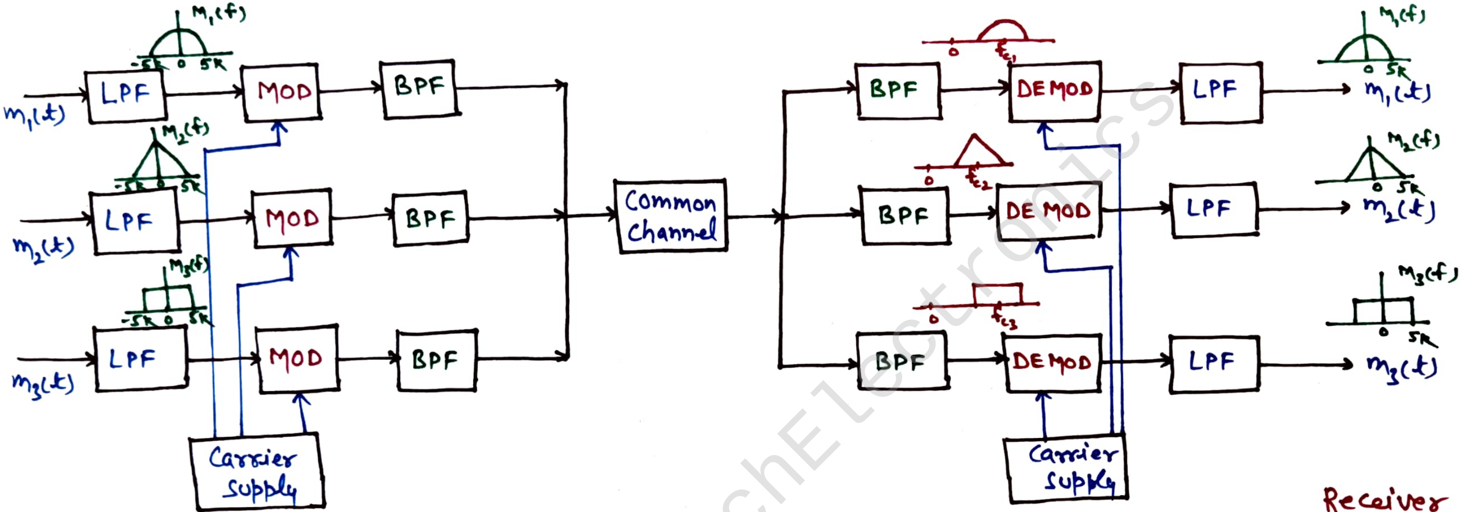
Multiplexing is the process of combining several message signals for their Simultaneous transmission over the same channel.

→ In FDM continuous wave (CW) modulation is used to translate each message signal to reside in a specific frequency slot inside the Passband of the channel by assigning it a distinct carrier frequency.

→ At the Receiver, a bank of band Pass filters is used to seperate the different modulated signals and prepare them individually for demodulation as shown in fig 1.

→ Low Pass filters (LPF's) are used to band limit the given message signals.

→ The significant frequencies are present at lower level therefore we always use Low Pass filter (LPF).

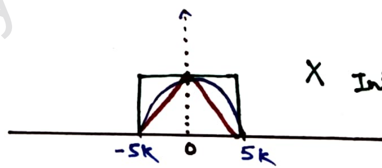


Receiver

Fig 1. Block diagram of FDM (for three signals)

Transmitter

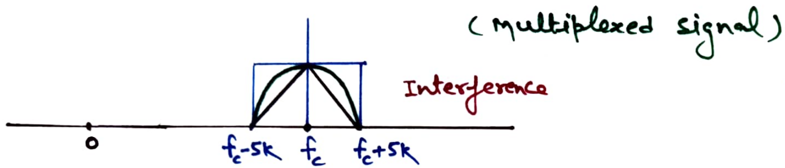
Case (i) No modulation



(Spectrum of multiplexed signal)

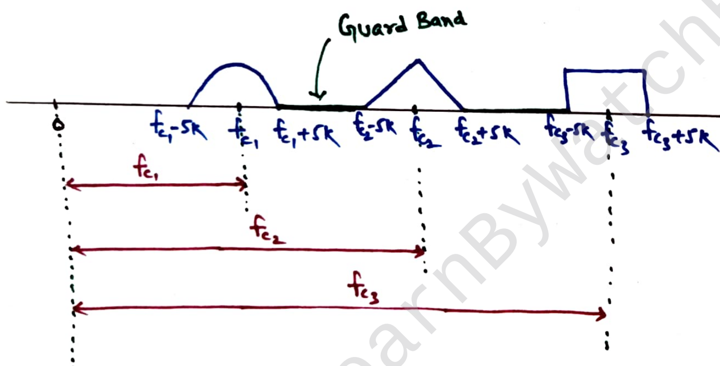
X Interference

Case (ii) Same carrier frequency f_c for all modulators



→ Therefore different frequencies are required to avoid interference.

Case (iii) Consider different carrier frequencies ($f_{c_1}, f_{c_2}, f_{c_3}$)



$$f_{c_2} >> f_{c_1} + 10K$$

$$f_{c_3} >> f_{c_2} + 10K$$

Angle Modulation :-

In angle modulation, total angle of the carrier signal is varied in accordance to message signal Amplitude Variations.

$$c(t) = A_c \cos(2\pi f_c t + \phi) \quad \leftarrow \text{Radians}$$

$$= A_c \cos[\theta(t)]$$

Where : $\theta(t) = 2\pi f_c t + \phi$ ← Total Angle

If angle modulation occurs due to dependence of f_c on $m(t)$ then it is called as Frequency Modulation

If angle modulation occurs due to dependence of ϕ on $m(t)$, then it is called as Phase Modulation.

Phase modulation :-

Carrier before modulation : $c(t) = A_c \cos 2\pi f_c t$

Carrier After phase modulation

$$S_{pm}(t) = A_c \cos[2\pi f_c t + \phi(t)]$$

$$\underset{\text{Rad}}{\phi(t)} = \underset{\frac{\text{Rad}}{\text{Volt}}}{k_p}\, \underset{\text{volt}}{m(t)}$$

Where : k_p : Phase sensitivity of Phase modulator

k_p specifies the change in phase for 1V Amplitude change in message signal.

for $m(t) = 0$: No modulation $\rightarrow \phi(t) = 0$

Frequency modulation :-

frequency of the carrier before modulation : 'f_c'

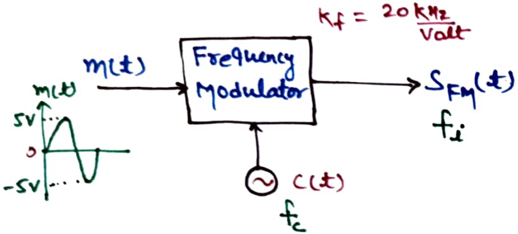
Frequency of carrier After frequency modulation : 'f_i'

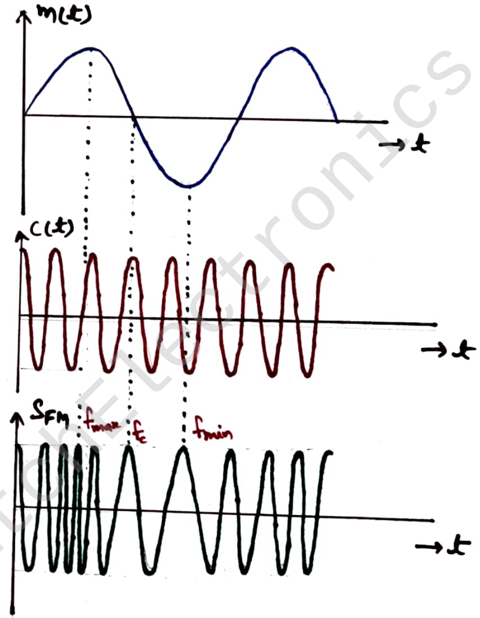
$$\underset{\text{Hz}}{\underset{\text{Instantaneous frequency}}{f_i}} = \underset{\text{Hz}}{f_c} + \underset{\frac{\text{Hz}}{\text{Volt}}}{k_f}\, \underset{\text{Volt}}{m(t)}$$

Where : k_f : frequency sensitivity of frequency modulator

k_f specifies the amount of frequency change of the carrier signal per 1 volt change in the message signal.

If $m(t) = 0$: No modulation : $f_i = f_c$





$$f_i = f_c + k_f\, m(t)$$

$m(t) = 0 \quad , \quad f_i = f_c$

$m(t) = 5V \quad , \quad f_i = f_c + 100\ kHz$

$m(t) = -5V \quad , \quad f_i = f_c - 100\ kHz$

So we can say, when

$m(t) = 0 \quad , \quad f_i = f_c$

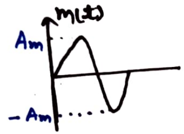
$m(t) = +ve \quad , \quad f_i > f_c$

$m(t) = -ve \quad , \quad f_i < f_c$

In FM, message signal voltage variations are converted as corresponding carrier signal frequency variations. So Frequency Modulation is also called as Voltage to Frequency conversion.

Assume $m(t) = A_m \cos 2\pi f_m t$ or $A_m \sin 2\pi f_m t$

$$f_i = f_c + k_f m(t)$$



Maximum frequency of the resulting FM signal

$$f_{max} = f_c + k_f A_m$$

Minimum frequency of the resulting FM signal

$$f_{min} = f_c - k_f A_m$$

Maximum frequency deviation

$$\Delta f = \max[k_f m(t)]$$

$$\Delta f = k_f A_m \text{ Hz}$$

$$f_{max} = f_c + \Delta f$$

$$f_{min} = f_c - \Delta f$$

Total frequency swing of the FM signal

$$= f_{max} - f_{min}$$

$$= \cancel{f_c} + \Delta f - \cancel{f_c} + \Delta f$$

$$= 2\Delta f$$

General Expression of FM signal :-

Carrier signal $c(t) = A_c \cos(2\pi f_c t + \phi)$

$= A_c \cos[\theta(t)]$

where : $\theta(t) = 2\pi f_c t + \phi$ ← constant

Differentiating both sides of $\theta(t)$ w.r.to t

$$\frac{d\theta(t)}{dt} = 2\pi f_c$$

If instantaneous frequency is f_i, then above expression can be written as-

$$\frac{d\theta_i(t)}{dt} = 2\pi f_i \quad \text{—— (1)}$$

Integrating eqn (1)

$$\theta_i(t) = 2\pi \int f_i \, dt$$

$$= 2\pi \int_0^t [f_c + k_f m(t)] dt$$

$$\theta_i(t) = 2\pi f_c t + 2\pi k_f \int_0^t m(t) dt \quad \text{—— (2)}$$

We know the general expression for FM signal

$$S_{FM}(t) = A_c \cos[\theta_i(t)] \quad \text{—— (3)}$$

Put eqn (2) in eqn (3)

$$\boxed{S_{FM}(t) = A_c \cos\left[2\pi f_c t + 2\pi k_f \int m(t) dt\right]}$$

Single tone FM :-

We know the General expression for FM signal

$$S_{FM}(t) = A_c \cos\left[2\pi f_c t + 2\pi k_f \int m(t)\, dt\right]$$

Assume $m(t) = A_m \cos 2\pi f_m t$

Now the above becomes

$$S_{FM}(t) = A_c \cos\left[2\pi f_c t + \cancel{2\pi} k_f A_m \frac{\sin 2\pi f_m t}{\cancel{2\pi} f_m}\right]$$

$$= A_c \cos\left[2\pi f_c t + \frac{k_f A_m}{f_m} \sin 2\pi f_m t\right]$$

$$\boxed{S_{FM}(t) = A_c \cos\left[2\pi f_c t + \beta \sin 2\pi f_m t\right]}$$

where : $\beta = \dfrac{k_f A_m}{f_m} = \dfrac{\Delta f}{f_m}$

Modulation index

Narrowband FM :-

→ Depending on the value of modulation index (β), FM is classified as -

FM

Narrowband FM (NBFM) ($\beta \leq 1$)

Wideband FM (WBFM) ($\beta > 1$)

General expression for single tone FM is given by

$$S_{FM}(t) = A_c \cos[2\pi f_c t + \beta \sin 2\pi f_m t]$$

$$S_{FM}(t) = A_c \cos 2\pi f_c t \cos(\underbrace{\beta \sin 2\pi f_m t}_{\theta}) - A_c \sin 2\pi f_c t \sin(\underbrace{\beta \sin 2\pi f_m t}_{\theta})$$

for NBFM, $\beta \leq 1$ (small).

For small values of θ

$\cos\theta \simeq 1$ $\quad\quad$ $\cos\theta \cong 1$

$\sin\theta \simeq \theta$ $\quad\quad$ $\sin\theta = \theta$

$$S_{NBFM}(t) \cong A_c \cos 2\pi f_c t \cdot 1 - A_c \sin 2\pi f_c t \cdot \beta \sin 2\pi f_m t$$

$$S_{NBFM}(t) \cong A_c \cos 2\pi f_c t - A_c\beta \sin 2\pi f_c t \cdot \sin 2\pi f_m t$$

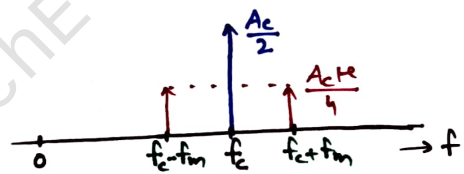
$$S_{NBFM}(t) = A_c \cos 2\pi f_c t - \frac{A_c\beta}{2} \cos 2\pi (f_c - f_m) t + \frac{A_c\beta}{2} \cos 2\pi (f_c + f_m) t$$

We know the expression of standard AM

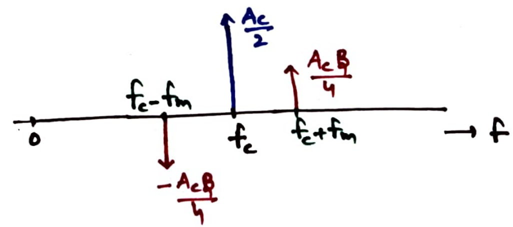
$$S_{AM}(t) = A_c \cos 2\pi f_c t + \frac{A_c\mu}{2} \cos 2\pi (f_c - f_m) t + \frac{A_c\mu}{2} \cos 2\pi (f_c + f_m) t$$

Expression of AM & NBFM will be same except 180° phase shift at LSB frequency component

$S_{AM}(t) \longleftrightarrow$



$S_{NBFM}(t) \longleftrightarrow$



Power of NBFM :—

$$P_t = P_c + P_{USB} + P_{LSB}$$

$$P_c = \frac{A_c^2}{2R} \quad , \quad P_{USB} = P_{LSB} = \frac{A_c^2 \beta^2}{8R}$$

$$\frac{\left(\frac{A_c \beta}{2}\right)^2}{2R} = \frac{A_c^2 \beta^2}{8R}$$

$$P_t = \frac{A_c^2}{2R} + \frac{A_c^2 \beta^2}{4R} = \frac{A_c^2}{2R}\left[1 + \frac{\beta^2}{2}\right]$$

$$P_t = P_c\left[1 + \frac{\beta^2}{2}\right]$$

→ The Bandwidth and Power Requirments of NBFM are almost similar to AM.

→ Because of its much similarity with AM, NBFM is given least Practical significance as compared to WBFM.

Narrowband FM :-

→ Depending on the value of modulation index (β), FM is classified as -

FM

- Narrowband FM (NBFM) ($\beta \leq 1$)
- Wideband FM (WBFM) ($\beta > 1$)

General expression for single tone FM is given by

$$S_{FM}(t) = A_c \cos[2\pi f_c t + \beta \sin 2\pi f_m t]$$

$$S_{FM}(t) = A_c \cos 2\pi f_c t \cos(\underbrace{\beta \sin 2\pi f_m t}_{\theta}) - A_c \sin 2\pi f_c t \sin(\underbrace{\beta \sin 2\pi f_m t}_{\theta})$$

for NBFM, $\beta \leq 1$ (small).

For small values of θ

$\cos\theta \simeq 1$ $\qquad$ $\cos\theta \cong 1$

$\sin\theta \simeq \theta$ $\qquad$ $\sin\theta = \theta$

$$S_{NBFM}(t) \simeq A_c \cos 2\pi f_c t \cdot 1 - A_c \sin 2\pi f_c t \cdot \beta \sin 2\pi f_m t$$

$$S_{NBFM}(t) \cong A_c \cos 2\pi f_c t - A_c\beta \sin 2\pi f_c t \cdot \sin 2\pi f_m t$$

$$S_{NBFM}(t) = A_c \cos 2\pi f_c t - \frac{A_c\beta}{2} \cos 2\pi (f_c - f_m)t + \frac{A_c\beta}{2} \cos 2\pi (f_c + f_m)t$$

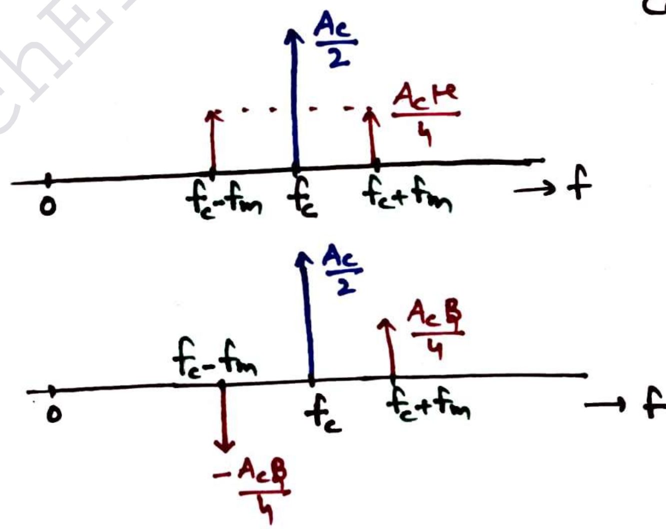
We know the expression of standard AM

$$S_{AM}(t) = A_c \cos 2\pi f_c t + \frac{A_c\mu}{2} \cos 2\pi (f_c - f_m)t + \frac{A_c\mu}{2} \cos 2\pi (f_c + f_m)t$$

Expression of AM & NBFM will be same except $180°$ phase shift at LSB frequency component

$S_{AM}(t) \longleftrightarrow$

$S_{NBFM}(t) \longleftrightarrow$



Power of NBFM :—

$$P_t = P_c + P_{USB} + P_{LSB}$$

$$P_c = \frac{A_c^2}{2R} \quad , \quad P_{USB} = P_{LSB} = \frac{A_c^2 \beta^2}{8R}$$

$$\frac{\left(\frac{A_c \beta}{2}\right)^2}{2R} = \frac{A_c^2 \beta^2}{8R}$$

$$P_t = \frac{A_c^2}{2R} + \frac{A_c^2 \beta^2}{4R} = \frac{A_c^2}{2R}\left[1 + \frac{\beta^2}{2}\right]$$

$$P_t = P_c\left[1 + \frac{\beta^2}{2}\right]$$

→ The Bandwidth and Power Requirments of NBFM are almost similar to AM.

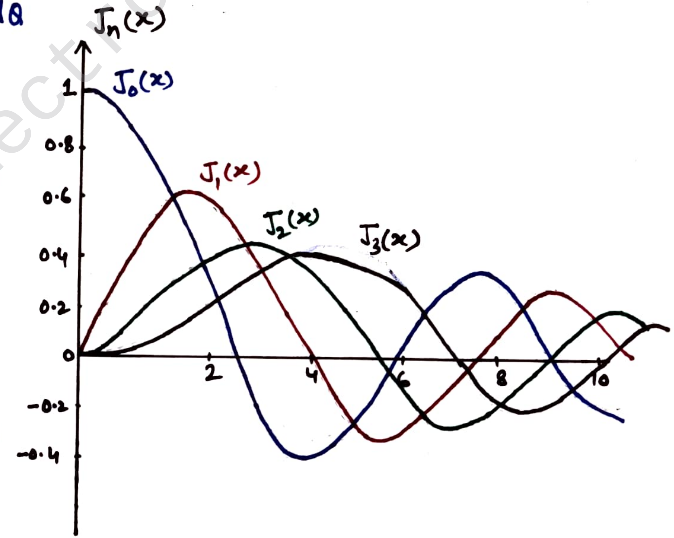
→ Because of its much similarity with AM, NBFM is given least Practical significance as compared to WBFM.

Wideband Frequency Modulation (WBFM) :-

→ Modulation index $\beta > 1$

Bessel function :-

$$J_n(x) = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{j(x\sin\theta - n\theta)}\, d\theta$$



Properties :-

① $J_n(x) \downarrow$ as $n \uparrow$ (Except some exceptional cases)

$$J_0(x) > J_1(x) > J_2(x) \cdots$$

② $J_{-n}(x) = (-1)^n J_n(x)$

$$J_{-n}(x) = J_n(x), \text{ when } n = \text{even}$$

$$J_{-n}(x) = -J_n(x), \text{ when } n = \text{odd}$$

③ $\displaystyle\sum_{n=-\infty}^{\infty} J_n^2(x) = 1$

④ $J_n(x)$ always results in real quantity.

General Expression of WBFM:-

$$S_{FM}(t) = A_c \cos[\underbrace{2\pi f_c t + \beta \sin 2\pi f_m t}_{\theta}]$$

$$\cos\theta = Re[e^{j\theta}] \qquad \{\because e^{j\theta} = \cos\theta + j\sin\theta$$

$$S_{FM}(t) = A_c Re[e^{j(2\pi f_c t + \beta \sin 2\pi f_m t)}]$$

$$= A_c Re[e^{j2\pi f_c t} \cdot e^{j\beta \sin 2\pi f_m t}] \quad \text{—— (1)}$$

In eq (1) let $f(t) = e^{j\beta \sin 2\pi f_m t}$, is a periodic function with $T = \frac{1}{f_m}$

We know exponential fourier series

$$f(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t}, \quad \omega_0 = \frac{2\pi}{T} = 2\pi f_m$$

$$C_n = \frac{1}{T}\int_{-T/2}^{T/2} f(t)\, e^{-jn\omega_0 t}\, dt$$

$$e^{j\beta \sin 2\pi f_m t} = \sum_{n=-\infty}^{\infty} C_n e^{jn 2\pi f_m t} \quad \text{—— (2)}$$

$$C_n = \frac{1}{1/f_m}\int_{-\frac{1}{2f_m}}^{\frac{1}{2f_m}} e^{j\beta \sin 2\pi f_m t} \cdot e^{-jn2\pi f_m t}\, dt$$

$$= f_m \int_{-\frac{1}{2f_m}}^{\frac{1}{2f_m}} e^{j(\beta \sin 2\pi f_m t - n 2\pi f_m t)}\, dt$$

We know that

$$J_n(x) = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{j(x \sin\theta - n\theta)}\, d\theta$$

Assume

$$2\pi f_m t = \theta$$

$$2\pi f_m dt = d\theta$$

$$dt = \frac{d\theta}{2\pi f_m}$$

for

$t = -\frac{1}{2f_m}$, $\theta = \cancel{2}\pi \cancel{f_m} \times \frac{-1}{\cancel{2}\cancel{f_m}} = -\pi$

$t = \frac{1}{2f_m}$, $\theta = \cancel{2}\pi \cancel{f_m} \times \frac{1}{\cancel{2}\cancel{f_m}} = \pi$

Now the expression of C_n becomes

$$C_n = \cancel{f_m} \int_{-\pi}^{\pi} e^{j(\beta \sin\theta - n\theta)} \cdot \frac{d\theta}{2\pi \cancel{f_m}}$$

$$= \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{j(\beta \sin\theta - n\theta)} \cdot d\theta$$

$$= J_n(\beta)$$

Substitute C_n in eqn (2)

$$e^{j\beta \sin 2\pi f_m t} = \sum_{n=-\infty}^{\infty} J_n(\beta)\, e^{jn 2\pi f_m t}$$

Now substitute above expression in eqn (1)

$$S_{FM}(t) = A_c \, Re\left[e^{j2\pi f_c t} \cdot \sum_{n=-\infty}^{\infty} J_n(\beta)\, e^{jn2\pi f_m t}\right]$$

$$= A_c \, Re\left[\sum_{n=-\infty}^{\infty} J_n(\beta)\, e^{j2\pi(f_c + n f_m)t}\right]$$

$$\boxed{S_{FM}(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \cos 2\pi(f_c + n f_m)t}$$

General expression for single tone FM.

$$S_{WBFM}(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \cos 2\pi(f_c + n f_m)t, \quad \beta > 1$$

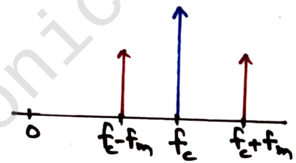
Practical Bandwidth of WBFM :-
(Carson's Rule)

Carson's Rule tells about how many significant sidebands are present in an FM signal.

Since WBFM signal corresponds to band-unlimited signal therefore before transmission it should be bandlimited by retaining all of its lower order significant frequency components and eliminating higher order frequency components.

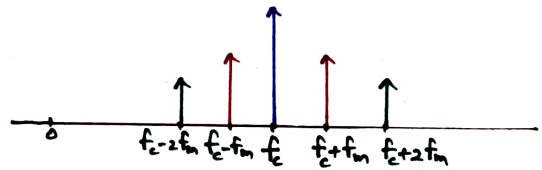
Assume WBFM contains significant sideband upto 1st order

After band limiting
(Pass through a BPF)



$B \cdot W = 2f_m$

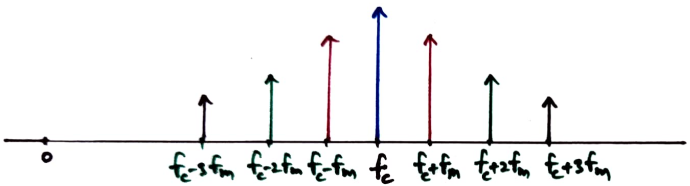
Assume significant side band upto 2nd order



$\cancel{f_c}+2f_m - \cancel{f_c}+2f_m$

$B \cdot W = 4f_m$
$= 2(2f_m)$

Assume significant sidebands upto $\underline{\underline{3}}^{rd}$ order



$$B.W = 6f_m = \underline{\underline{3}}\times(2f_m)$$

According to Carson, WBFM contains significant sidebands upto '$\underline{\beta+1}$' order.

$$\boxed{B.W = (\beta+1)\,2f_m}$$

$$BW = \left(\frac{\Delta f}{f_m}+1\right)2f_m$$

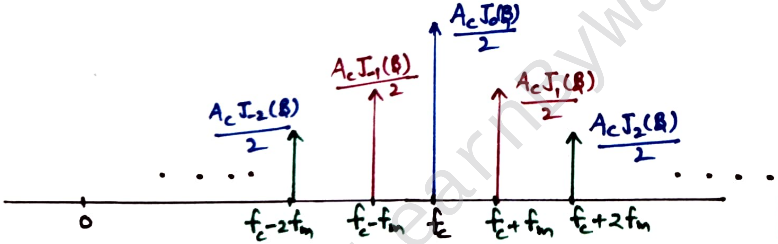
$$\boxed{B.W = 2(\Delta f + f_m)}$$

Spectrum of WBFM :-

We know the expression for WBFM

$$S_{WBFM} = A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \cos 2\pi (f_c + n f_m) t \quad ; \beta > 1$$

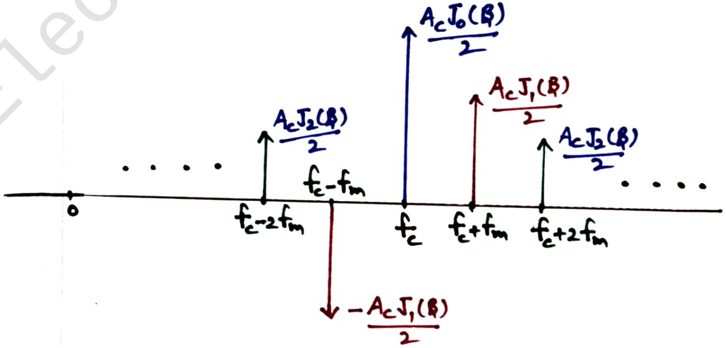
$$S_{WBFM} = A_c J_0(\beta) \cos 2\pi f_c t + A_c J_1(\beta) \cos 2\pi (f_c + f_m) t$$
$$+ A_c J_{-1}(\beta) \cos 2\pi (f_c - f_m) t$$
$$+ A_c J_2(\beta) \cos 2\pi (f_c + 2f_m) t$$
$$+ A_c J_{-2}(\beta) \cos 2\pi (f_c - 2f_m) t$$
$$+ \cdots\cdots$$



By the Property of Bessel function

$$J_{-n}(x) = J_n(x) \quad ; n \text{ even}$$
$$J_{-n}(x) = -J_n(x) \quad ; n \text{ odd}$$

$$J_0(\beta) > J_1(\beta) > J_2(\beta) \cdots\cdots$$



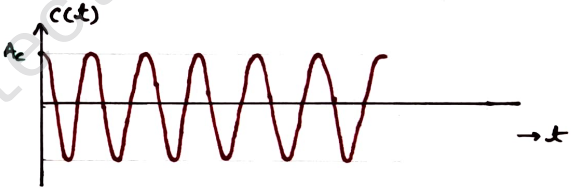
[Considered only +ve side of spectrum]

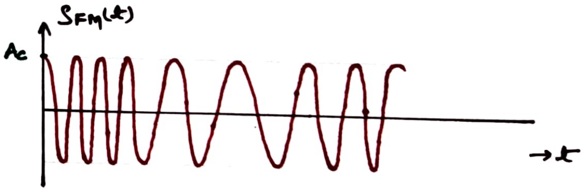
$$P_t = \cdots\cdots + \frac{A_c^2 J_{-2}^2(\beta)}{2R} + \frac{A_c^2 J_{-1}^2(\beta)}{2R} + \frac{A_c^2 J_0^2(\beta)}{2R} + \frac{A_c^2 J_1^2(\beta)}{2R} + \frac{A_c^2 J_2^2(\beta)}{2R} + \cdots$$

$$P_t = \frac{A_c^2}{2R}\left[\cdots + J_{-2}^2(\beta) + J_{-1}^2(\beta) + J_0^2(\beta) + J_1^2(\beta) + J_2^2(\beta) + \cdots\right]$$

$$P_t = \frac{A_c^2}{2R} \underbrace{\sum_{n=-\infty}^{\infty} J_n^2(\beta)}_{1}$$

$$\boxed{P_t = \frac{A_c^2}{2R}}$$





→ WBFM contains carrier frequency component, infinite number of USB and LSB.

→ The actual Bandwidth of WBFM is Infinite.

→ For WBFM strength of higher order sidebands goes on decreasing and finally becomes zero.

→ For WBFM lower order sidebands are said to be significant and higher order sidebands are insignificant.

Power of FM:-

$$P_t = \cdots\cdots + P_{LSB_2} + P_{LSB_1} + P_c + P_{USB_1} + P_{USB_2} + \cdots\cdots$$

$$P_c = \frac{A_c^2 J_0^2(\beta)}{2R}, \quad P_{USB_1} = \frac{A_c^2 J_1^2(\beta)}{2R}, \quad P_{LSB_1} = \frac{A_c^2 J_{-1}^2(\beta)}{2R}$$

$$P_{USB_2} = \frac{A_c^2 J_2^2(\beta)}{2R}, \quad P_{LSB_2} = \frac{A_c^2 J_{-2}^2(\beta)}{2R}$$

Generation of FM :- or FM Modulator

Basically there are two methods for generating FM signals.

1. Direct method or Parameter Variation method or Hartley oscillator method.
2. In-direct method or Armstrong method.

1. Direct method for Generation of FM signal :-

→ In this method, Frequency modulation is done by using a voltage controlled oscillator (VCO).

→ VCO is an oscillator whose frequency can be controlled by external voltage.

→ In a VCO, the oscillation frequency changes linearly with control voltage.

→ Thus in a direct FM generation system, the instantaneous frequency of carrier wave is varied in accordance with baseband signal $m(t)$ Amplitude variations by using VCO.

→ An FM generator which uses Hartley oscillator is shown in figure.

→ The frequency of oscillation of Hartley oscillator is given by

$$f = \frac{1}{2\pi\sqrt{(L_1+L_2)C}}$$

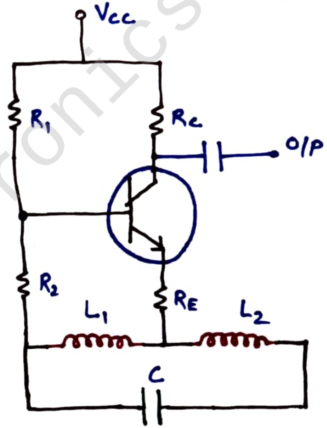


Fig.1. Hartley oscillator

→ The frequency determining network consist of two inductors L_1 and L_2 and a capacitor C. The capacitor is assumed to consist of a fixed capacitor shunted by a voltage variable capacitor such as Varactor diode.

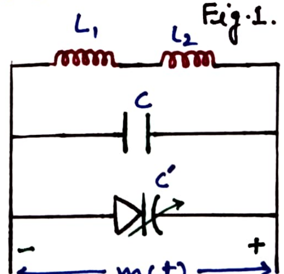
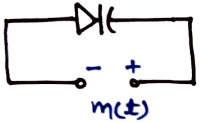


Fig 2.

$$c' \propto \frac{1}{W}$$

$$f_i = \frac{1}{2\pi\sqrt{(L_1+L_2)(C+C')}}$$

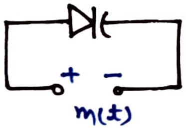
Case (i) $m(t) = +ve$



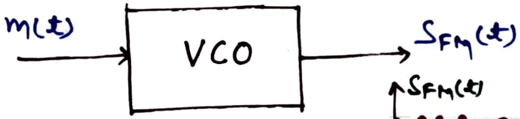
$$f_i = \frac{1}{2\pi\sqrt{(L_1+L_2)(C+C')}}$$

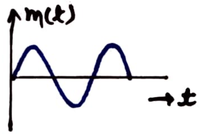
Reverse bias $\uparrow \rightarrow W\uparrow \rightarrow C'\downarrow \rightarrow f_i\uparrow$

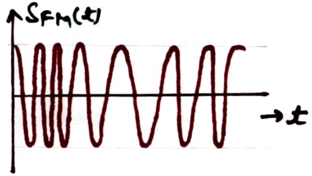
Case (ii) $m(t) = -ve$



Forward bias $\uparrow \rightarrow W\downarrow \rightarrow C'\uparrow \rightarrow f_i\downarrow$







(2) In-direct method of frequency modulation :- or Armstrong method

The indirect method of generating WBFM was first proposed by E.H Armstrong so this method is popularly known as Armstrong method.

In this method a Narrowband FM wave is first produced by modulating the carrier wave by baseband signal $m(t)$. Then the resulting NBFM signal is frequency multiplied to obtain wideband FM signal as shown in figure.

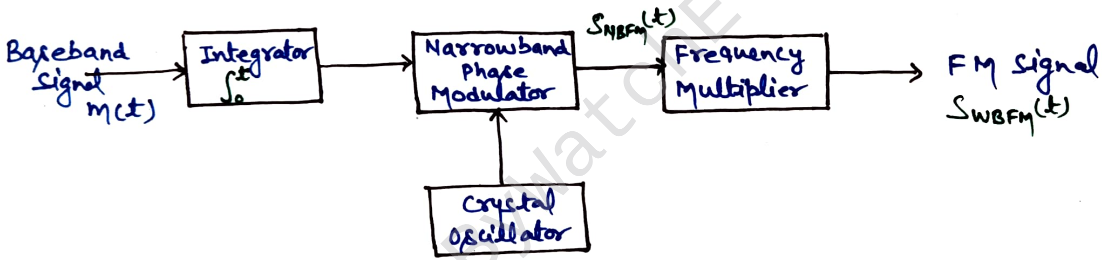


Fig. Block diagram of In-direct method of FM generation

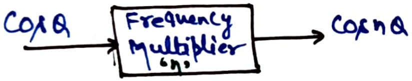
The output of the phase modulator is the Narrowband frequency modulated (NBFM) signal which is denoted by $S_{NBFM}(t)$.

Thus for a sinusoidal modulating signal

$$S_{NBFM}(t) = A_c \cos\left[2\pi f_1 t + \beta \sin 2\pi f_m t\right]$$

Where : f_1 : frequency of oscillator

β : modulation index which is kept less than 1.



When NBFM is passed through frequency multiplier, we get WBFM.

$$(\text{Freq mul})_{O/P} = S_{WBFM}(t) = A_c \cos\left[2\pi n f_1 t + n\beta \sin 2\pi f_m t\right]$$

$$S_{WBFM}(t) = A_c \cos\left[2\pi f_c t + \beta' \sin 2\pi f_m t\right]$$

Where: $\beta' = n\beta > 1$

$n f_1 = f_c$

Demodulation of FM :-

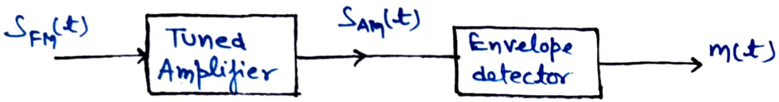
1. Frequency discrimination method
   (i) Slope detector
   (ii) Balanced slope detector

2. Phase Discrimination method
   (i) Foster-Seelay Detector
   (ii) Ratio detector
   (iii) PLL detector

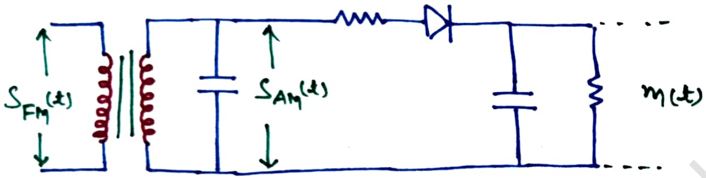
→ Generally for FM demodulation PLL demodulator is preffered because except PLL all other demodulators require transformer for their construction.

Slope detector :-

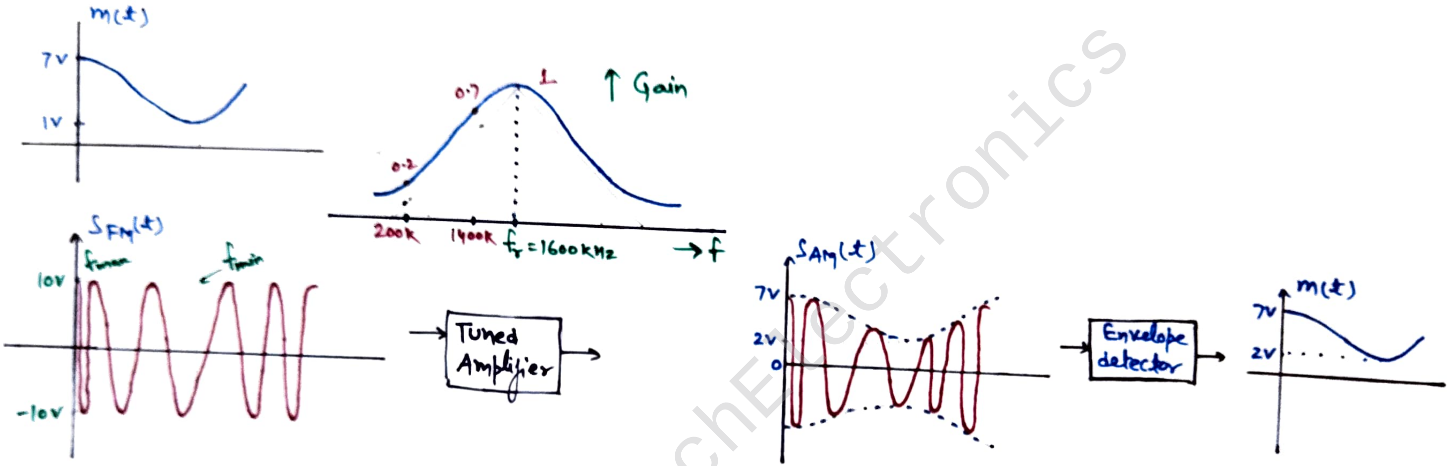
→ Slope detector can be constructed with the help of a Tuned Amplifier and an envelope detector as shown in figure.



$f_r > f_{max}$



→ Tuned Amplifier is used as frequency to voltage converter so we can say that frequency modulated signal is converted in to Amplitude modulated signal. Now we know that AM signal can be easily demodulated with the help of envelope detector. Therefore we get message signal that was frequency modulated.



Assume $f_{max} = 1400$ KHz , $f_r > f_{max}$

$f_{min} = 200$ KHz $\quad f_r = 1600$ KHz

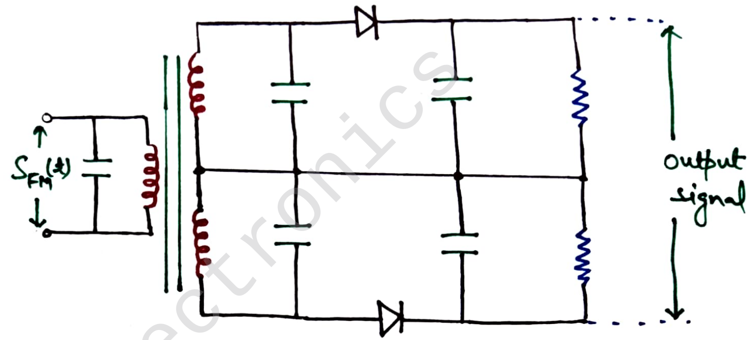
→ Since gain-frequency characteristic of tuned Amplifier is non-linear in nature therefore some non-linearity is introduced in frequency to voltage conversion. Therefore message signal can not be perfectly reconstructed and an error is introduced known as slope error.

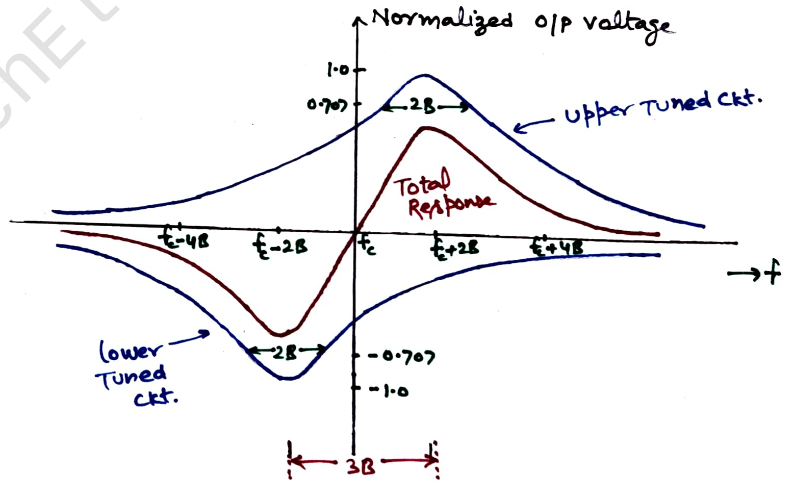
Balanced Slope detector :-

Balanced slope detector is designed with the help of two slope detector that are connected in such a way, the resulting slope error will be minimum as possible.

→ The upper and lower Resonant circuits are tuned to frequencies above and below the un-modulated Carrier frequency f_c.

→ As the $f_i(t)$ changes, the Amplitude Variations tend in opposite direction, by taking the difference of these variations we can obtain the resultant frequency to voltage characteristic known as S-Curve.

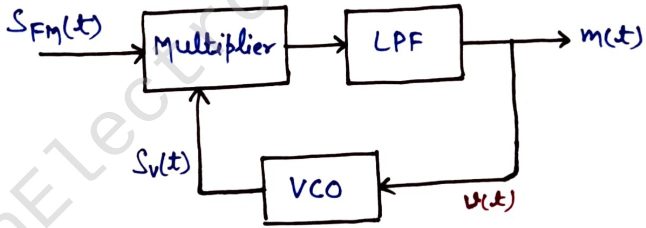




Phase-Locked Loop (PLL) Demodulator:-

→ PLL is used for the demodulation of FM signals.

→ The input of voltage controlled oscillator (VCO) will be the message signal m(t), which is taken as feedback from output.



→ As the message signal Amplitude changes correspondingly the frequency of VCO output will be changed so that frequency synchronization can be maintained w.r to FM signal which has to be demodulated.

$$S_{FM}(t) = A_c \cos\left[2\pi f_1 t + \underbrace{2\pi k_f \int m(t)\, dt}_{\phi_1(t)}\right]$$

$$(VCO)_{O/P} = S_v(t) = A_v \sin\left[2\pi f_2 t + \underbrace{2\pi k_v \int v(t)\, dt}_{\phi_2(t)}\right]$$

[∵ VCO output should have 90° Phase shift with respect to transmitted Carrier

For reconstruction of m(t)

(i) f_1 should be made equal to f_2, then PLL is said to be in LOCK MODE.

(ii) $\phi_1(t)$ should be made equal to $\phi_2(t)$, then PLL is said to be in CAPTURE MODE.

→ VCO forms -ve feedback for the PLL demodulator because of this -ve feedback within short time f_1 is made equal to f_2 and PLL is said to be in LOCK MODE.

i.e. $f_1 = f_2 = f_c$

$$S_{FM}(t) = A_c \cos[2\pi f_c t + \phi_1(t)]$$

$$S_V(t) = A_V \sin[2\pi f_c t + \phi_2(t)]$$

$$(Mul)_{O/P} = S_{FM}(t) \cdot S_V(t)$$

$$= \frac{A_c A_V}{2} \sin[4\pi f_c t + \phi_1(t) + \phi_2(t)] - \frac{A_c A_V}{2} \sin[\phi_1(t) - \phi_2(t)]$$

$\phi_1(t) - \phi_2(t) = \phi_e(t)$ (Phase error)

(Should be minimum)

$$(Mul)_{o/p} = \frac{A_c A_v}{2} \sin\left[4\pi f_c t + \phi_1(t) + \phi_2(t)\right] - \frac{A_c A_v}{2} \sin \phi_e(t)$$

The first term of above equation contains very high frequency so it is not allowed by the LPF.

and in 2^{nd} term, since $\phi_e(t)$ is very small

$$\sin \phi_e(t) \cong \phi_e(t)$$

$$v(t) = \phi_e(t) * h(t)$$

$$V(f) = \phi_e(f) \cdot H(f) \quad \text{—— (A)}$$

We know

$$\phi_e(t) = \phi_1(t) - \phi_2(t)$$

$$\phi_e(t) = \phi_1(t) - 2\pi K_v \int v(t)\, dt$$

$$\phi_e(t) = \phi_1(t) - 2\pi K_v \int \phi_e(t) * h(t)\, dt$$

Differentiating both sides w.r.to t.

$$\frac{d}{dt}\phi_e(t) = \frac{d}{dt}\phi_1(t) - 2\pi K_v[\phi_e(t) * h(t)]$$

Taking Fourier Transform

$$j\cancel{2\pi}f\,\phi_e(f) = j\cancel{2\pi}f\,\phi_1(f) - \cancel{2\pi}K_v\,\phi_e(f)\cdot H(f)$$

$$\phi_e(f)[jf + K_v H(f)] = jf\,\phi_1(f)$$

$$\downarrow \phi_e(f) = \frac{jf\,\phi_1(f)}{jf + K_v H(f)\uparrow}$$

From above expression it is clear that

$\phi_e(f)$ will be small, if $H(f)$ is large.

and if $\phi_e(t)$ is small

$$\phi_1(t) \cong \phi_2(t)$$

If $H(f) = \infty$ (Not Possible)

$$\phi_e(f) = 0$$

$$\phi_1(t) = \phi_2(t)$$

Hence by maintaining Passband gain of LPF to be very high value $\phi_e(t)$ becomes very small.

$$\phi_e(f) = \frac{\cancel{jf}\,\phi_1(f)}{\cancel{jf}\left[1 + \frac{K_v}{jf}H(f)\right]}$$

$$1 + \frac{K_v}{jf}H(f) \cong \frac{K_v}{jf}H(f)$$ ($\because$ $H(f)$ is very High)

$$\phi_e(f) = \frac{\phi_1(f)}{\frac{K_v}{jf}H(f)}$$

from eqⁿ (A)

$$V(f) = \phi_e(f) \cdot H(f)$$

$$= \frac{\phi_1(f)}{\frac{K_v}{jf} \cdot \cancel{H(f)}} \cdot \cancel{H(f)}$$

$$V(f) = \frac{jf\,\phi_1(f)}{K_v} \times \frac{2\pi}{2\pi}$$

Taking Inverse fourier Transform

$$V(t) = \frac{1}{2\pi K_v} \cdot \frac{d}{dt}\,\phi_1(t)$$

$$= \frac{1}{\cancel{2\pi} K_v} \cdot \frac{\cancel{d}}{\cancel{dt}}\left[\cancel{2\pi} K_f \cancel{\int} m(t)\,dt\right]$$

$$v(t) = \frac{K_f}{K_v} \cdot m(t)$$

$$v(t) \propto m(t)$$

NOTE:

→ Negative feedback formed by VCO is responsible for LOCK MODE.
and Low Pass filter (LPF) is responsible for CAPTURE MODE.

→ PLL Demodulation is based on differentiation method.

Phase Modulation :-

Carrier signal $c(t) = A_c \cos 2\pi f_c t$

Carrier signal After Phase modulation

$$S_{PM}(t) = A_c \cos[2\pi f_c t + \phi(t)]$$ ← Phase deviation

$$\phi(t) = K_p m(t)$$

K_p : Phase sensitivity of phase modulator

$$S_{PM}(t) = A_c \cos[2\pi f_c t + K_p m(t)]$$

Assume $m(t) = A_m \cos 2\pi f_m t$ or $A_m \sin 2\pi f_m t$

Maximum phase deviation $\Delta\phi = \max[\phi(t)]$

$$\Delta\phi = \max[K_p m(t)]$$

$$\boxed{\Delta\phi = K_p A_m} \text{ Rad.}$$

Single tone phase modulation -

Assume $m(t) = A_m \cos 2\pi f_m t$

we know that

$$S_{PM}(t) = A_c \cos[2\pi f_c t + K_p m(t)]$$

$$S_{PM}(t) = A_c \cos[2\pi f_c t + K_p A_m \cos 2\pi f_m t]$$

$$S_{PM}(t) = A_c \cos[2\pi f_c t + \beta \cos 2\pi f_m t]$$

where : $\beta = K_p A_m$

(Modulation index of PM)

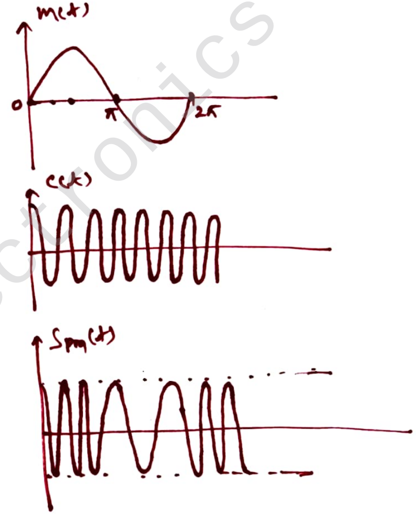
→ For phase modulation, maximum phase deviation $\Delta\phi$ and modulation index β both are equal and are independent of message signal frequency variations

$$\beta = K_p A_m = \Delta\phi$$

→ If we compare the General expressions of PM & FM, we observed that both are same except 90° phase shift at message frequency component.

$$S_{PM}(t) = A_c \cos[2\pi f_c t + \beta \cos 2\pi f_m t]$$

$$S_{FM}(t) = A_c \cos[2\pi f_c t + \beta \sin 2\pi f_m t]$$



→ The Bandwidth of PM is also equal to the Bandwidth of FM

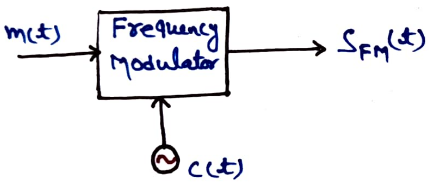
$$B.W = (\beta+1)2f_m = 2(\Delta f + f_m)$$

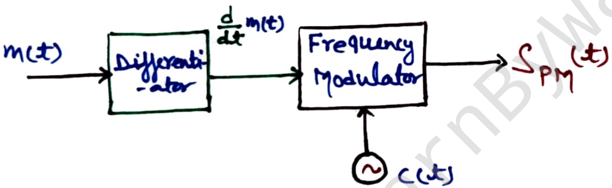
→ $$P_t = \frac{A_c^2}{2R}$$

Generation of PM from FM :-

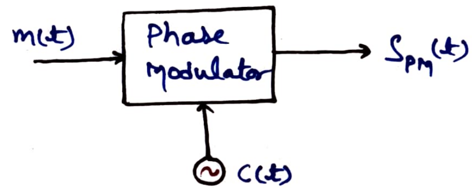
$$S_{PM}(t) = A_c \cos[2\pi f_c t + k_p m(t)]$$

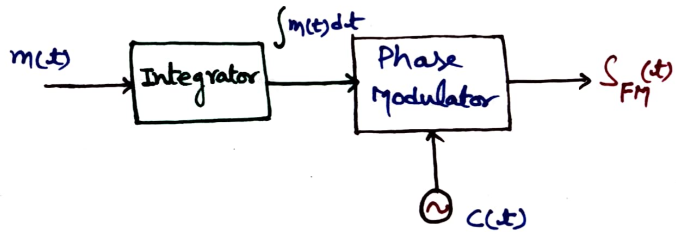
$$S_{FM}(t) = A_c \cos[2\pi f_c t + 2\pi k_f \int m(t)\,dt]$$





Generation of FM from PM :-





Radio Transmitters:-

Radio Frequency (RF) Transmitter is an electronic system that converts the information signal into RF signal capable of propagation over long distances.

Classification of Transmitters:-

→ The Radio Transmitters may be classified according to the type of modulation used.

→ The Analog transmitters include AM Transmitters, FM Transmitters and Pulse modulation Transmitter.

→ The Radio transmitter may also be classified according to the carrier frequency.

* Long wave Transmitters (30-300 KHz): Application in Aeronautical and marine navigation.
* Medium wave Transmitters (300-3000 KHz): Application in AM Broadcasting in 550-1650 KHz Range.
* Short wave Transmitters (3-30 MHz): Application in long distance Communication by the virtue of Ionospheric reflection.
* Very high frequency Transmitters (30-300 MHz): Application in FM Broadcasting in 88-108 MHz Range.
* Ultrahigh frequency Transmitters (300-3000 MHz): Application in television broadcasting, cellular Telephony and military services.
* Super high frequency Transmitters (3-30 GHz): Application in Radar and satellite Communication.

AM Transmitter :-

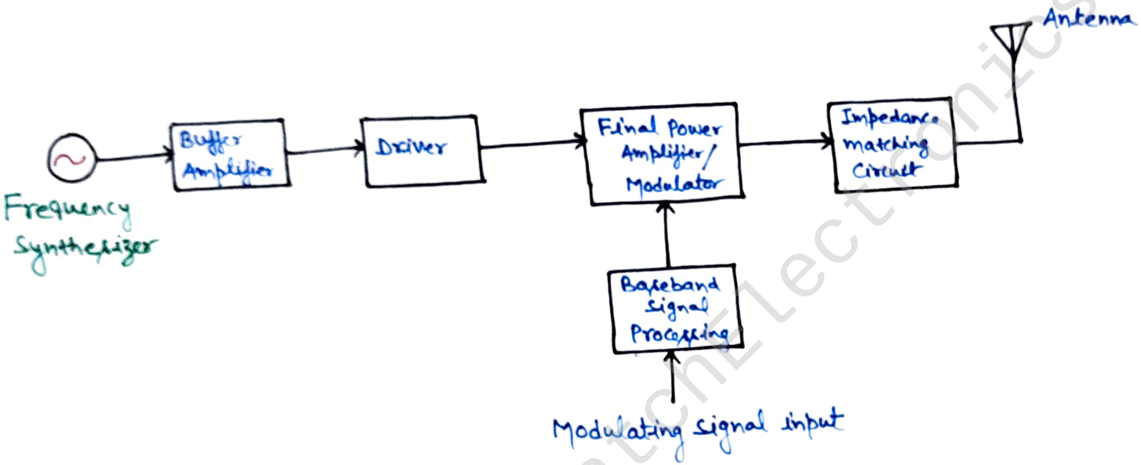


Fig 1. High-level AM Transmitter

→ Figure 1. Shows the block diagram of an AM Transmitter using high-level modulation.

→ In high-level AM Transmitter the carrier signal from the frequency synthesizer is first raised to a high power level before being modulated.

→ This is achieved by operating RF Power amplifiers (Driver) in class-c mode for maximum efficiency.

→ The Role of buffer Amplifier is to isolate the frequency synthesizer from the Power Amplifier Stages.

→ In case of the desired Carrier frequency is more than what the frequency synthesizer can produce, frequency multipliers can be inserted between buffer Amplifier and the driver Amplifiers.

→ The baseband signal Processing block perform filtering and Amplitude control of modulating or message signal in order to minimize the signal bandwidth and also to avoid overmodulation.

→ In high-level transmitter, the modulating signal varies the amplitude of the carrier by supplying to the final Power Amplifier.

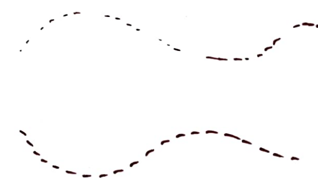
→ It is found in practice that a collector modulated class-c Amplifier tends to have better efficiency, low distortion and much better Power handling Capabilities than a base modulated Amplifier.

→ Because of these considerations broadcast AM Transmitters use high-level modulation.

→ In low-level modulation Transmitters, the carrier is modulated at a low power level and then raised to the desired level by using linear class-A Amplifier or class-B Push-Pull Amplifiers.

→ The low-level modulation has the advantage that the carrier can be fully modulated using a relatively small amount of the modulating signal.

→ However the disadvantage is that once the carrier is modulated, a linear chain of Amplifiers has to be used in order to preserve the modulation envelope.



FM Transmitter :-

→ An FM signal can be generated either by the direct method or indirect (Armstrong) method.

→ Direct FM may be generated using a Varactor diode, a reactance modulator or using VCO.

→ In case of indirect FM generation method, a Narrowband FM signal is first obtained using an integrator and phase modulator then NBFM signal is passed through a chain of frequency multipliers to obtain wideband FM signal.

→ The generated FM signal is then routed through Power Amplifiers and an antenna impedance matching network for broadcasting.

→ For commercial FM Radio broadcasting carrier frequency Range is - 88 MHz - 108 MHz with transmission bandwidth of 200 kHz.

→ Figure 1. Shows FM Transmitter using indirect (Armstrong) method.

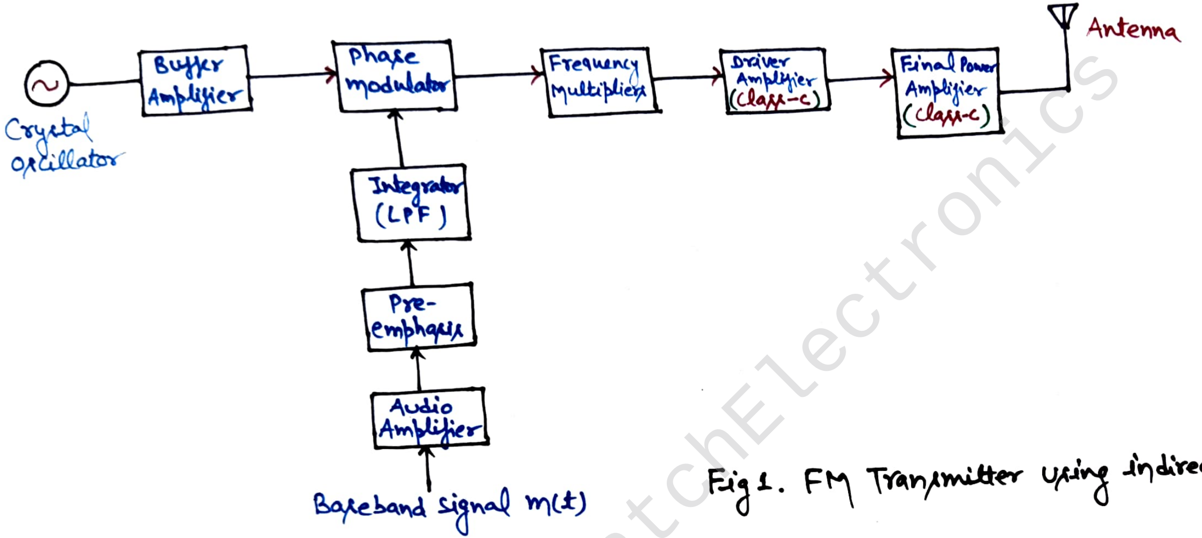


Fig 1. FM Transmitter using indirect (Armstrong) method

→ The carrier signal is obtained from a crystal oscillator that is isolated from the rest of the system by a buffer Amplifier.

→ The message signal is Amplified, Pre-emphasized, low Pass filtered (Integrated) and then applied to a phase modulator, the other input of the Phase modulator is carrier signal.

→ The carrier is modulated as per the Amplitude variations of the message signal.

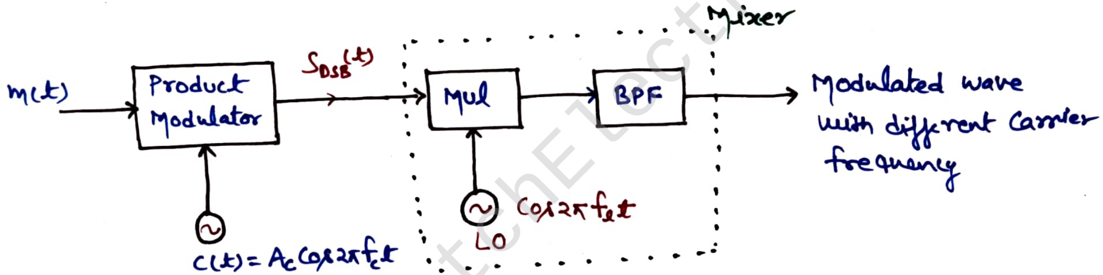
→ The desired carrier frequency and the desired frequency deviation of 75 KHz are achieved by using one or more stages of frequency multipliers.

→ After frequency multiplication the desired power level to derive the antenna is achieved by using a driver Amplifier and a power Amplifier both operating in the class-C mode for maximum efficiency.

Mixer :-

→ Mixer is a device that is used for frequency translation of the modulated signal.

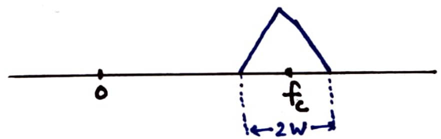
→ It consist of a multiplier followed by a band pass filter (BPF).



Assume

$m(t) \longleftrightarrow$

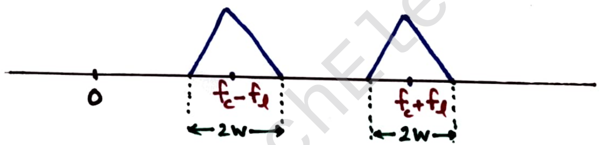
$S_{DSB}(t) \longleftrightarrow$



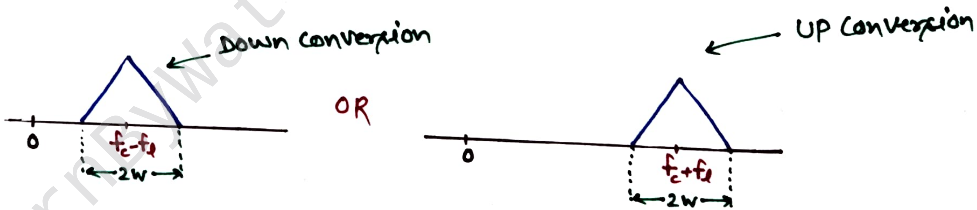
$$(Mul)_{o/p} = S_{DSB}(t) \cdot (LO)_{o/p} = A_c\, m(t) \cos 2\pi f_c t \cdot \cos 2\pi f_l t$$

Case (i) Assume $f_c > f_l$

$$(Mul)_{o/p} = \frac{A_c\, m(t)}{2} \cos 2\pi (f_c + f_l) t + \frac{A_c\, m(t)}{2} \cos 2\pi (f_c - f_l)$$

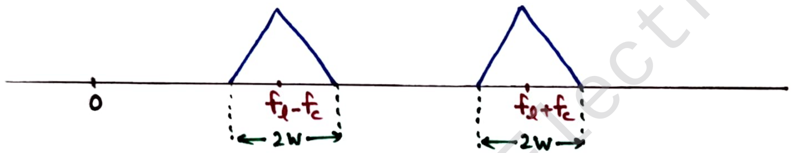


$(BPF)_{o/p} \longleftrightarrow$

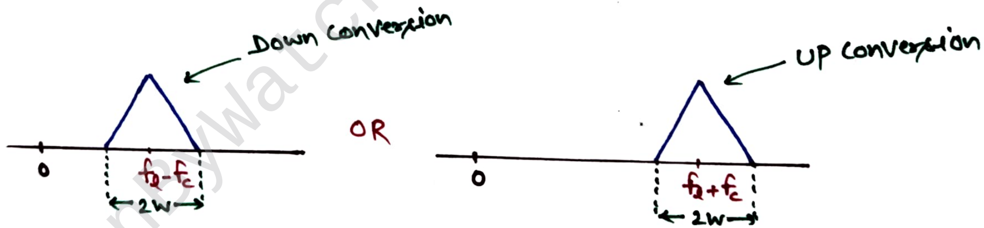


(ii) Assume $f_l > f_c$

$$(\text{Mul})_{O/P} = \frac{A_c m(t)}{2} \cos 2\pi (f_l + f_c)t + \frac{A_c m(t)}{2} \cos 2\pi (f_l - f_c)t$$



$(\text{BPF})_{O/P} \longleftrightarrow$



If $f_c = f_1$, $f_l = f_2$

UP conversion $= f_1 + f_2$, Down conversion $= |f_1 - f_2|$

Receivers:-

The main function of a Receiver is to select the desired station and reject all other stations.

→ Based on the construction Receivers are of two types

1. Tuned Radio Frequency (TRF) Receiver
2. Super Hetrodyne (SHD) Receiver.

→ Receivers are also classified based on Modulation Techniques used -

(i) AM Receiver → TRF, SHD

(ii) FM Receiver → TRF, SHD

→ As Per Guidelines of FCC (Federal communication commission) by US govt.

AM Tx:

Carrier frequency : 550 KHz - 1650 KHz

AM B.W : 10 KHz

Intermediate Frequency IF : 455 KHz

FM Tx:

Carrier freq^n : 88 MHz - 108 MHz

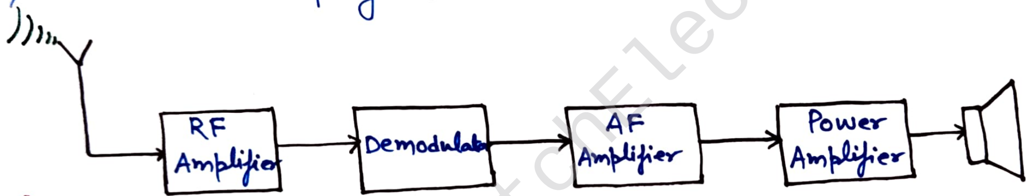
FM B.W : 200 KHz

IF : 10.7 MHz

AM Receivers :-

1 Tuned Radio frequency (TRF) Receiver :-

The main function of a Receiver is proper selection and rejection, for a TRF Receiver selection & Rejection is carried by RF Amplifier which is a Tuned Amplifier.



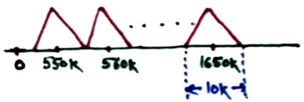


Fig 1. Block diagram of AM TRF Receiver

→ Tuned Amplifier Can be designed with the help of Transformer followed by a capacitor As Shown in fig.2.

Tuned Amplifier characteristic

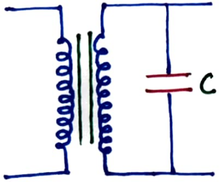
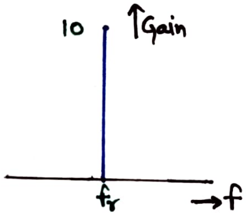
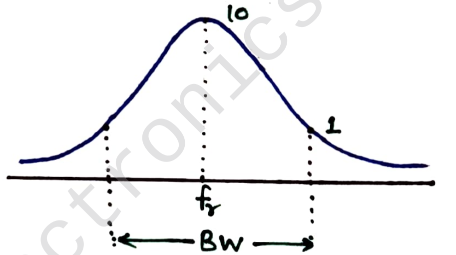


Fig 2. Tuned Amplifier

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$



Ideal Tuned Amplifier

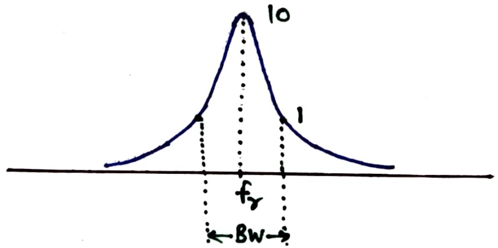


General Tuned Amplifier

$$B \cdot W = \frac{f_r}{Q}$$

$$B \cdot W \propto \frac{1}{Q}$$

→ For general Amplifier Quality factor (Q) is 160 to 180.

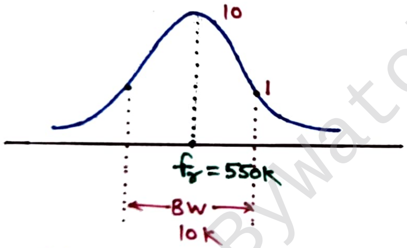


Practical (designed with care)

(i) Assume Receiver is tuned to 550K Station

Carrier frequency of selected or Tuned Station
$f_s = 550K$

→ To select this station Tuned Amplifier should be resonate at the same frequency, Therefore
$f_r = 550K$



$$f_r = \frac{1}{2\pi\sqrt{LC}} = 550K$$

$$Q = \frac{1}{2\pi}\sqrt{\frac{L}{C}}$$

$$B.W = \frac{f_r}{Q}$$

To get B.W of 10 KHz, Q should be 55.

$$B.W = \frac{550}{55} = 10K$$

(ii) Assume Receiver is tuned to 1050 KHz Station

$f_r = 1050K$

To get B.W of 10K, Q should be 105.

If achieved Q is 125

$$B.W = \frac{1050}{125} = 8.4\ KHz$$

→ If B.W of Tuned Amplifier < 10 KHz
Desired freq^n component will be attenuated.

→ If B.W of Tuned Amplifier > 10 KHz
Undesired freq^n component will be allowed.

Receiver Characteristic Parameters:-

① Selectivity:-

It is the ability of the Receiver to allow only desired frequency components and to reject or attenuate all other undesired frequency components.

Selectivity mainly depends on characteristic of tuned Amplifier.

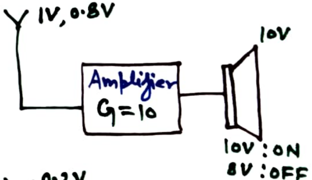
② Sensitivity:-

Sensitivity specifies minimum possible strength of the Received signal to be maintained to produce correspondingly faithfull output.

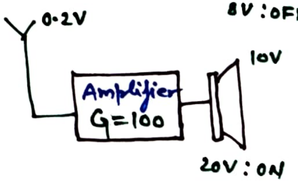
→ Sensitivity of a Receiver is mainly depends on Gain of the Amplifier stages.

Example:

Rx1:



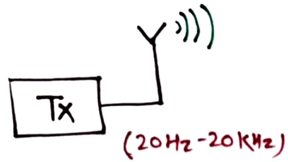
Rx2:

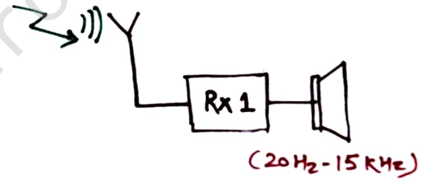


$Rx_2 > Rx_1$

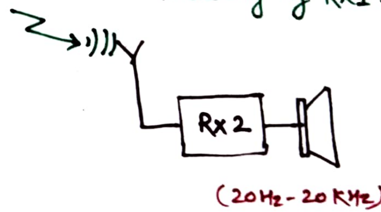
③ Fidelity :-

It specifies the ability of the receiver to produce all the frequency component of the transmitted Audio signal.





Fidelity of Rx1 is . low .



Fidelity of Rx2 is . High..

→ If the fidelity of the Receiver is high then clearity of reproduced audio signal will also be high.

→ In FM transmission of Audio signals, to improve fidelity, Pre-emphasis and De-emphasis circuits are used.

(2) Super Hetrodyne (SHD) Receiver :-

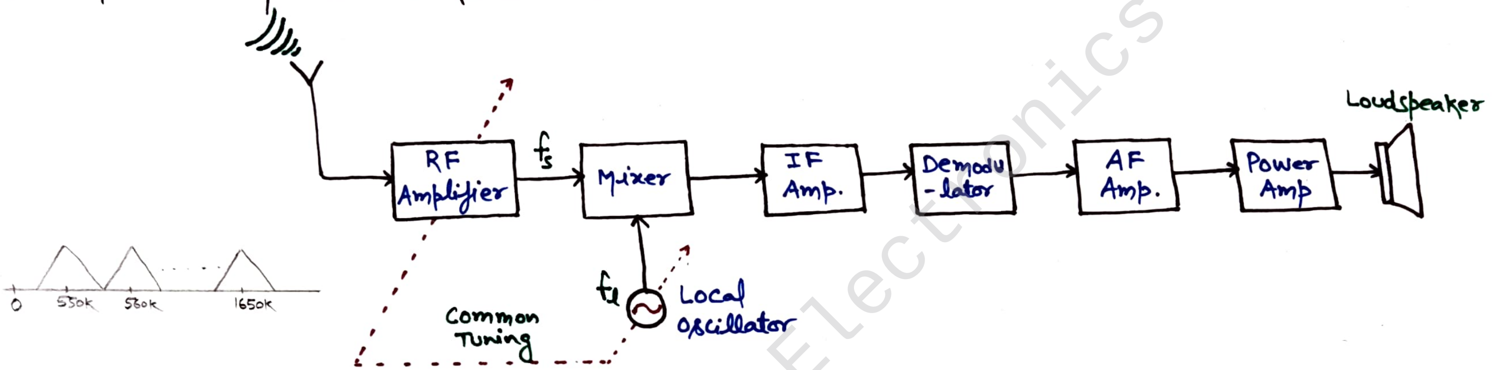


Fig 1. Block diagram of AM Receiver of The Superhetrodyne type.

→ Basically Superhetrodyne Receiver consist of a Radio frequency (RF) Amplifier, a mixer with local oscillator, Intermediate Frequency (IF) Amplifier, demodulator, Audio Frequency (AF) Amplifier and Power Amplifier.

→ The combination of mixer and local oscillator provides a hetrodyning function, by which the incoming signal is converted to a predefined fixed Intermediate frequency.

→ The value of Intermediate Frequency (IF) is lower than the incoming carrier frequency.

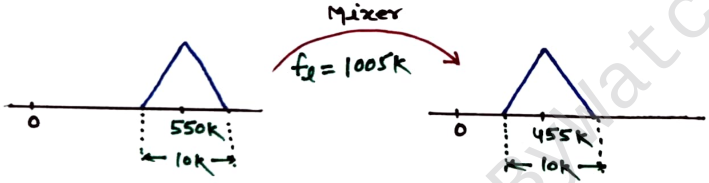
→ It means mixer performs down conversion.

$$IF = f_l - f_s = 455\ \text{KHz}$$

(Standard value)

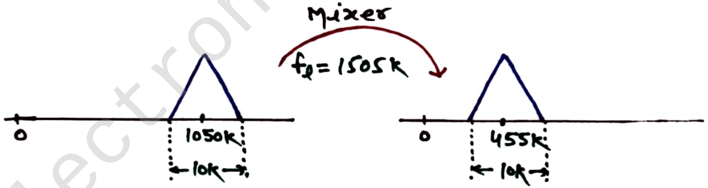
$$f_l = f_s + 455$$

Case (i) Assume Rx is tuned to 550 KHz station

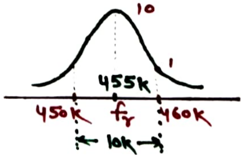


→ IF Amplifier will be a standard Tuned Amplifier, Tuned at 455 KHz

Case (ii) Assume Rx Tuned to 1050 KHz station



→ In Superhetrodyne Receiver tuning or station selection is achived by changing f_l of the mixer whereas in a TRF Receiver tuning is achived by changing f_r of the Tuned Amplifier.

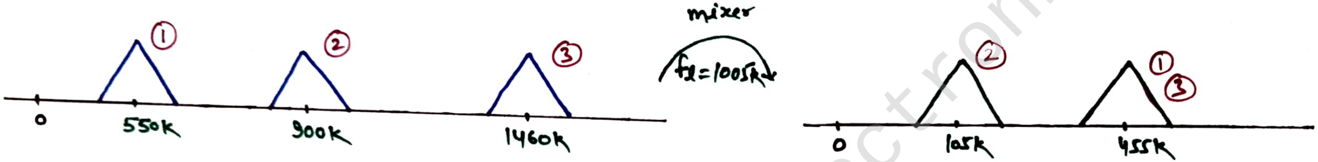


$f_r = IF = 455K$

$B.W = 10K$

$$Q = \frac{f_r}{B.W} = 45.5$$

Image frequency : (Drawback of SHD)



Assume Receiver is tuned to 550 kHz station

$$f_s = 550\,kHz$$

$$f_l = f_s + IF = 550 + 455 = 1005\ kHz$$

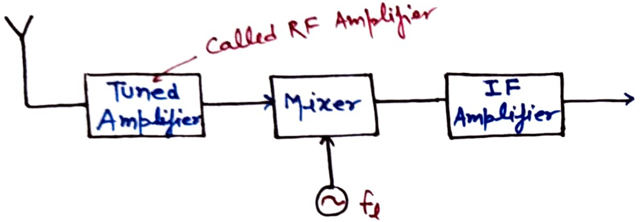
$$f_{si} = 550 + 2 \times 455 = 1460k$$

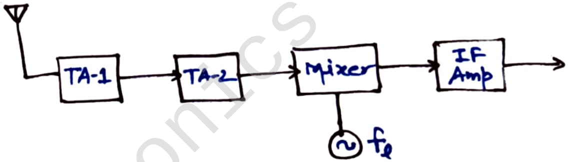
→ Image station frequency can be find with the help of following Relation

$$\boxed{f_{si} = f_s + 2 \cdot IF}$$

→ For proper Reception of desired station Image station should be suppressed.

→ To suppress Image station tuned Amplifier should be used before mixer.





→ Tuned Amplifier before mixer is a General tuned Amplifier which is for suppression of Image station.

→ IF Amplifier After mixer is a standard Amplifier and is for proper selection and Rejection.

Tuned Amp. Input

$f_s = 550K = 10V$

$f_{si} = 1460K = 10V$

$f_s = 550 \rightarrow 100V \longrightarrow 1000V$

$f_{si} = 1460K \rightarrow 1V \longrightarrow 0.01V$

→ To suppress Image station to the minimum possible extent, cascaded tuned Amplifier will be used before mixer.

Image Rejection Ratio (IRR) :-

Image Rejection Ratio specifies the effectiveness of tuned Amplifier in suppressing Image station.

$$IRR = \frac{\text{Gain offered by Tuned Amplifier to } f_s}{\text{Gain offered by Tuned Amplifier to } f_{si}}$$

$$\boxed{\alpha = \frac{G_{f_s}}{G_{f_{si}}}}$$

If a tuned Amplifier connected in Cascade then

$$\alpha = \alpha_1 \cdot \alpha_2 \cdot \alpha_3 \cdot \ldots$$

$$\boxed{\alpha = \sqrt{1 + P^2 Q^2}}$$

where: $P = \frac{f_{si}}{f_s} - \frac{f_s}{f_{si}}$

Q : Quality factor

Q1. A Receiver is tuned to 650 KHz station, IF = 550 KHz. Find local oscillator frequency and Image station frequency.

Sol

$f_s = 650$ KHz

$IF = 550$ KHz

$f_l = f_s + IF = 650 + 550 = 1200$ KHz

$f_{si} = f_s + 2IF = 650 + 2 \times 550$

$= 1750$ KHz

Q2. A Receiver is tuned to 550 KHz station and the local oscillator frequency is given by 1050 KHz.

(i) Find I.F and f_{si}

(ii) Find Image Rejection Ratio, if $Q = 45$

Sol

$f_s = 550$ KHz

$f_l = 1050$ KHz

(i) $I.F = f_l - f_s = 1050 - 550 = 500$ KHz

$f_{si} = f_s + 2IF = 550 + 2 \times 500 = 1550$ KHz

(ii) $Q = 45$

$\alpha = \sqrt{1 + \rho^2 Q^2}$

$\rho = \frac{f_{si}}{f_s} - \frac{f_s}{f_{si}}$

$= \frac{1550}{550} - \frac{550}{1550} = 2.463$

$\alpha = \sqrt{1 + (2.463)^2 \cdot (45)^2}$

$= 110.84$

$\simeq 111$

FM Receiver :-

→ FM Receiver is used for Receiving Frequency modulated signals.

→ In FM, message signal is stored in the form of frequency variations and these frequency variations are little affected by channel noise Therefore FM transmission is very much of Noise free.

→ For FM : Carrier frequency : 88 MHz – 108 MHz.
Bandwidth : 200 kHz
IF : 10.7 MHz

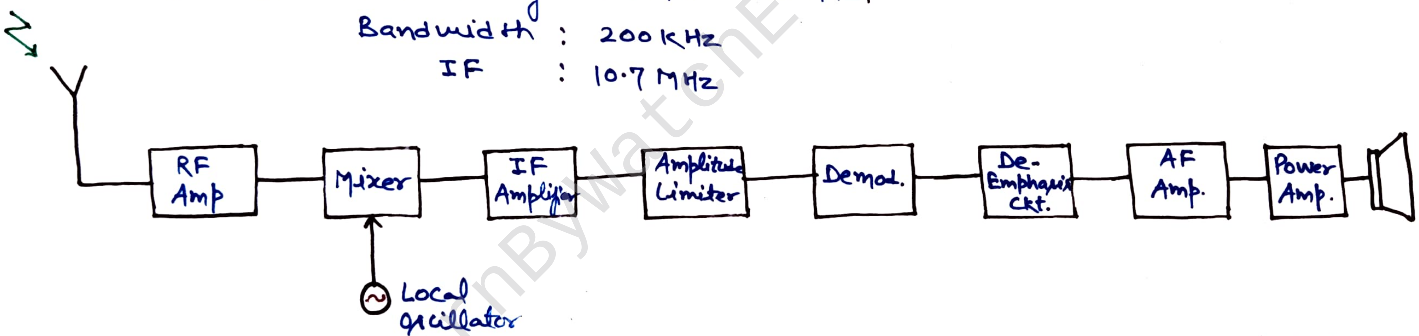
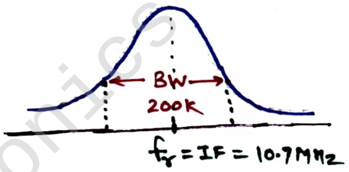
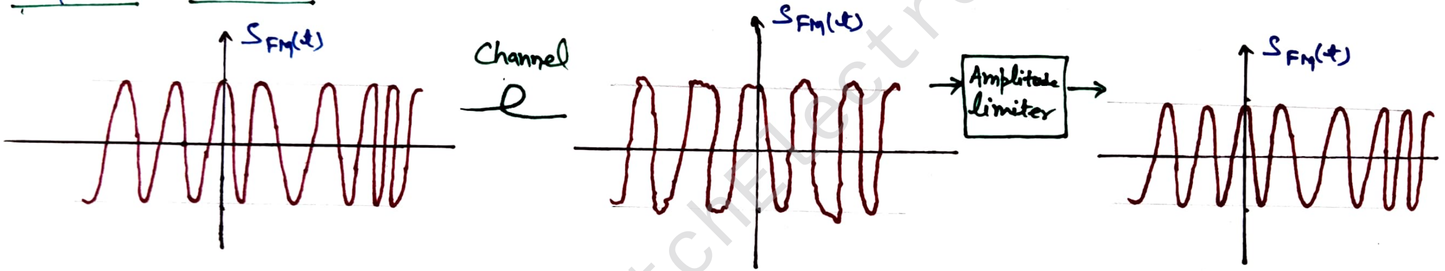


Fig. Block diagram of FM Receiver of superhetrodyne type

IF Amplifier will be a standard tuned Amplifier



Amplitude limiter -

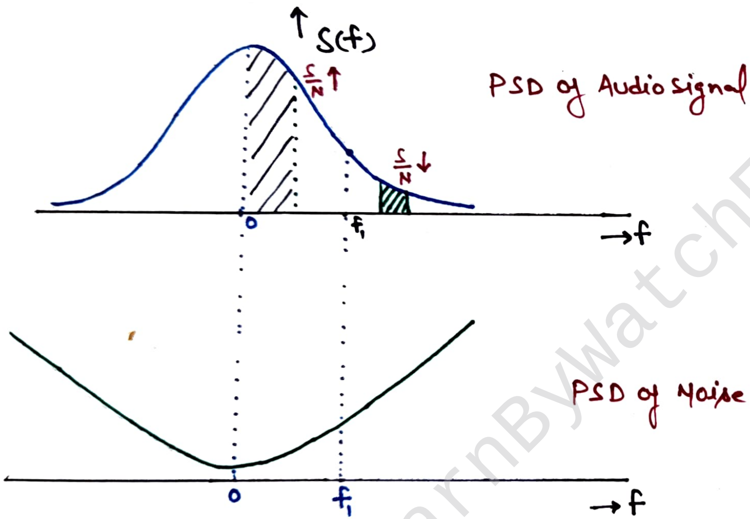


→ The operation of foster seley detector (FSD) is highly sensitive to Peak voltage fluctuations of FM signal therefore Amplitude limiter is essential.

→ Ratio Detector Provide high stablization towards voltage fluctuations of FM signal so Amplitude limiter is ~~essential~~. optional.

Pre-emphasis & De-emphasis :-

→ Pre-emphasis and De-emphasis are used to improve fidelity of FM transmission of Audio signals.



Up to frequency f_1 : $\frac{S}{N} >>> 1$, so the low frequency component can be Reproduced Comfortably.

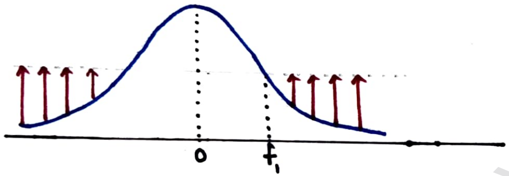
Above f_1 : $\frac{S}{N} << 1$, so the high frequency component can not be Reproduced Comfortably.

Pre-emphasis :-

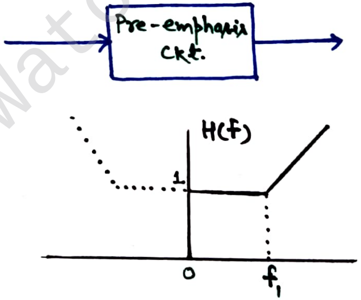
It is the Process of Artificial boosting of high frequency component of message signal to increase $\frac{S}{N}$ Ratio.

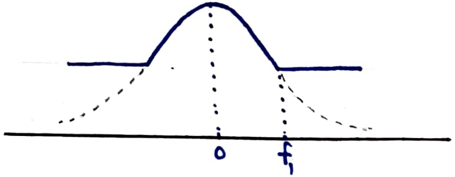
→ Pre-emphasis is done in the Transmitter before frequency modulation.

Before modulation :-



Lead Compensator Circuit

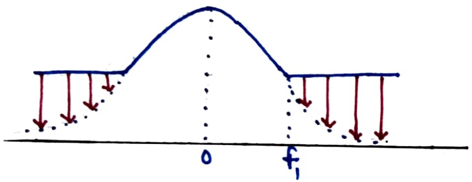




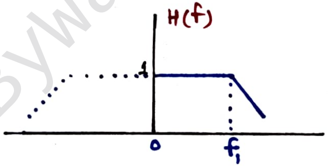
De-emphasis :-

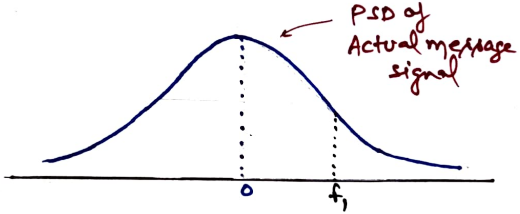
It is the process of decreasing the strength of high frequency component of message signal to get back the orginal transmitted message signal.

→ De-emphasis is performed in the Receiver after demodulation.

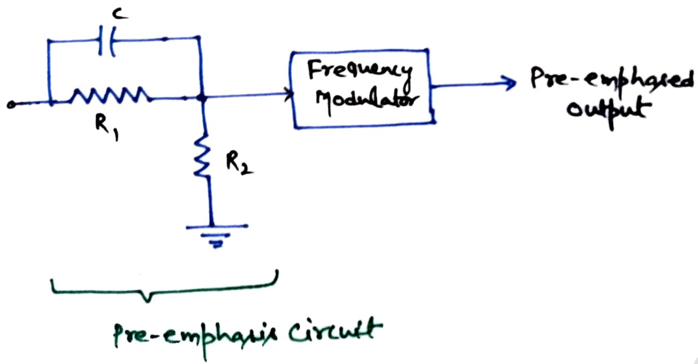


LPF ckt.

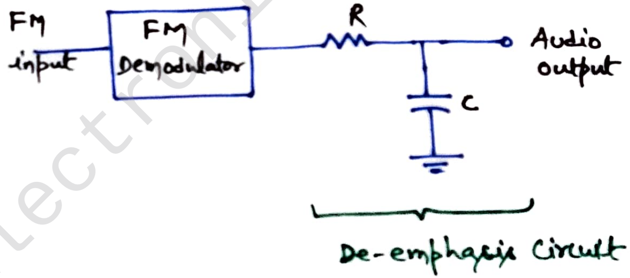




Pre-emphasis circuit :-

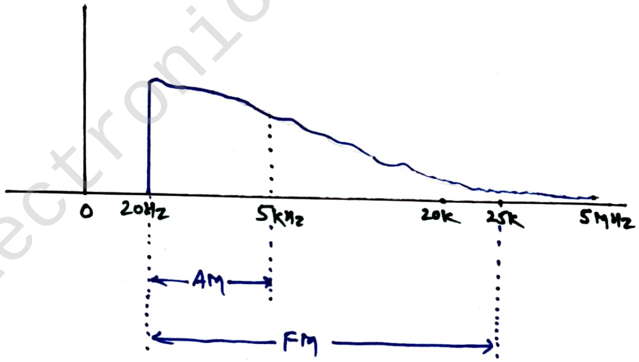


De-emphasis circuit :-



→ In AM Transmission low frequency component of Audio signal are considered for transmission, for these low freqn component $\frac{S}{N}$ is high therefore Pre-emphasis and De-emphasis not required for AM.

→ In Case of FM high frequency component of Audio signal are considered for transmission, for these high frequency component $\frac{S}{N}$ ratio is low therefore Pre-emphasis and De-emphasis are required.



FM B.W = $2(\Delta f + W)$ = 200kHz

Δf = 75 kHz (Standard)

W = 25kHz