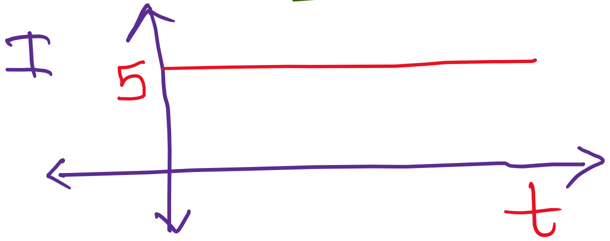
Unit-2 AC ckt Analysis (1-ϕ ckts)

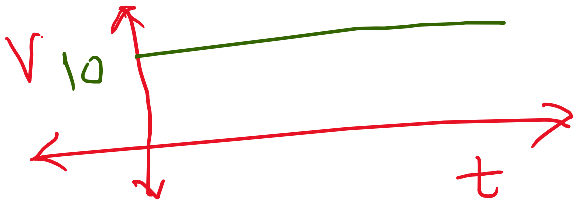
DC ckts : Eqs.(DC) , R

AC ckts : Eqs (AC) , R, L, C   R, L, C, R-L, R-C, R-L-C

DC

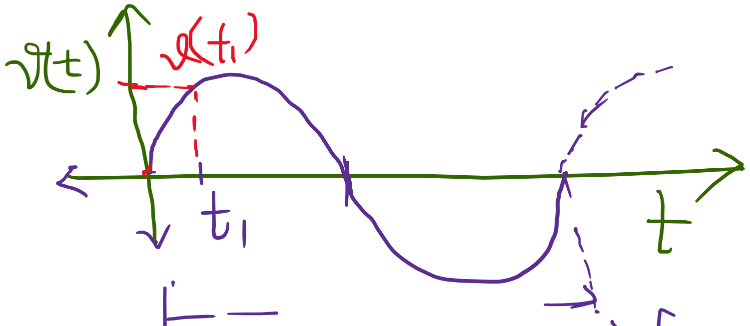


I = 5 A



V = 10V

AC



$v(t) = V_0 \sin \omega t$

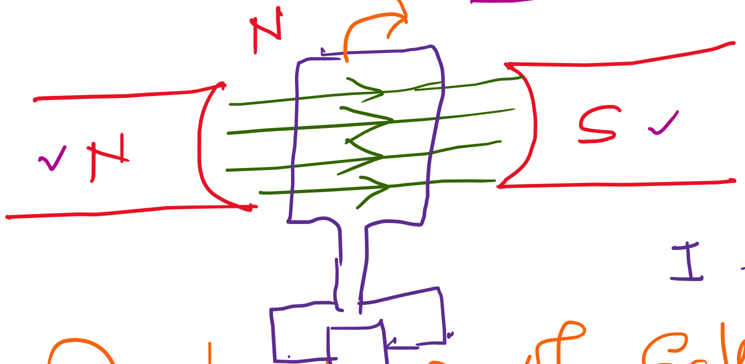
v(t) varies with time

half of the cycle '+'ve
other " '−'ve } alternating

v(t) = ?

v(t) → instantaneous voltage

$\theta \quad \dot{\theta} = \omega$



low

I flow

- B : mag field system
- C : coil/conductor
- R : Relative motion

both field & cond^r

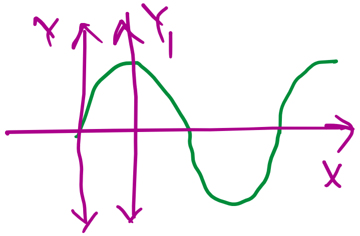
Faraday's Law of Electromagnetic Induction :

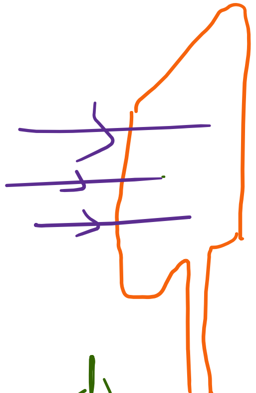
1st law : $e = \frac{d\phi(t)}{dt}$

emf

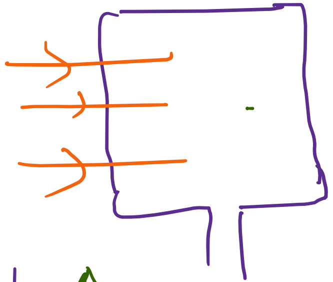
Voltage

ϕ. flux





ϕ_{max}



$\phi_{min} = 0$

$\pi/2$

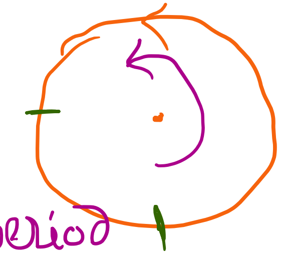
$\cos\theta \rightarrow \sin\theta$

$\theta = \omega t$

$\omega = \frac{\theta}{t}$

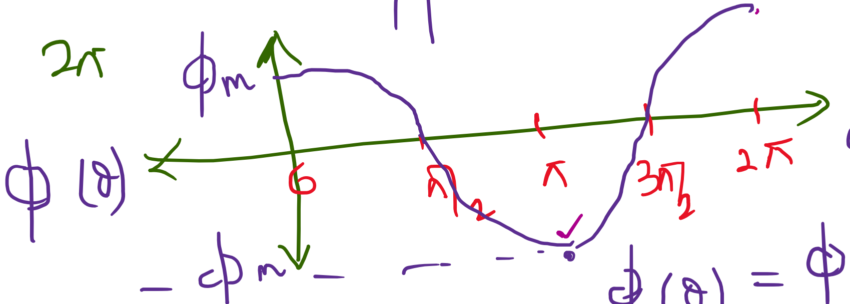
$\omega = \frac{2\pi}{T}$

$\theta = 2\pi$



T, time period

ϕ	ϕ_{max}	0	$-\phi_{max}$	0	ϕ_{max}
θ	0	$\pi/2$	π	$3\pi/2$	2π



$\phi(\theta) = \phi_m \cos\omega t$

$\omega = \frac{2\pi}{T}$

$\boxed{f = \frac{1}{T}}$

$\omega = 2\pi f$

F. 2^{nd} Law :

Lenz's law

$\theta \rightarrow$ angular displacement

$\omega \rightarrow$ angular speed

$T \rightarrow$ Time period

$f \rightarrow$ frequency

time period T ~~sec~~ $\rightarrow$ 1 rev

1 sec $\rightarrow$ $\frac{1}{T}$ rev

$f = \frac{1}{T}$

$N \rightarrow$ no. of turns

~~total~~ $\phi = N\phi$

$\omega t = \theta$

$$\boxed{e = -\frac{d\phi}{dt}}$$

$\phi(t) = \phi_m \cos \omega t$

$$e = -\frac{d(N\phi(t))}{dt} = -\frac{d}{dt}\{N\phi_m \cos \omega t\}$$

$$e = +N\phi_m\omega \cdot \sin\omega t$$

$\sin \omega t = 1$

$$e_{max} = N\phi_m\omega = N\phi_m 2\pi f$$
$$= 2\pi N\phi_m \cdot f$$

AC

$\boxed{v(t) = V_0 \sin(\omega t - \phi)}$

$$\boxed{e(t) = e_m \sin\omega t}$$

$v(t) = V_0 \sin(\omega t - \phi)$

inst volt — max volt

$\omega \rightarrow$

$\omega = 2\pi f = \frac{2\pi}{T}$

$f = \frac{1}{T}$

10 V DC | 10 V ← V_0 AC

7 V AC

$\theta = \omega t$

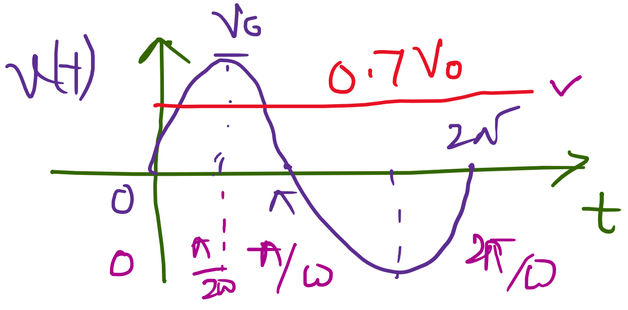
$t = \theta/\omega$

θ — t

$t \propto \theta$

$i(t) = I_0 \sin(\omega t - \phi)$

Average value $= \frac{1}{T}\int_0^T v(t)\,dt$



Effective value $\equiv$ RMS value $= \sqrt{\frac{1}{T}\int_0^T v^2(t)\,dt}$

for sin wave $V_{rms} = \frac{V_0}{\sqrt{2}} = 0.707\, V_0$

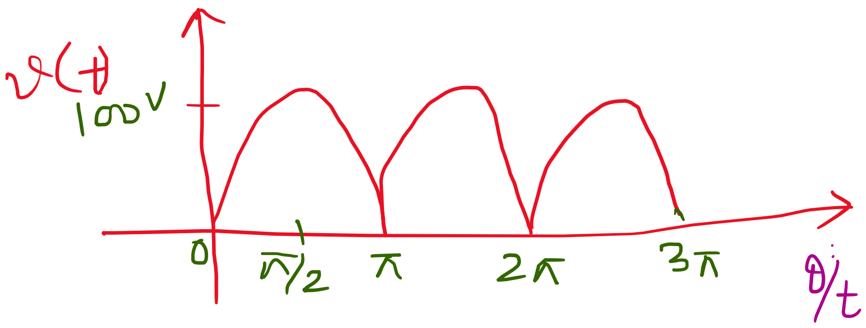
70% of V_0

$v_{av} = \frac{1}{\pi}\int_0^\pi v(t)\,dt$

$v_{rms}^2 = \frac{1}{\pi}\int_0^\pi v^2(t)\,dt$

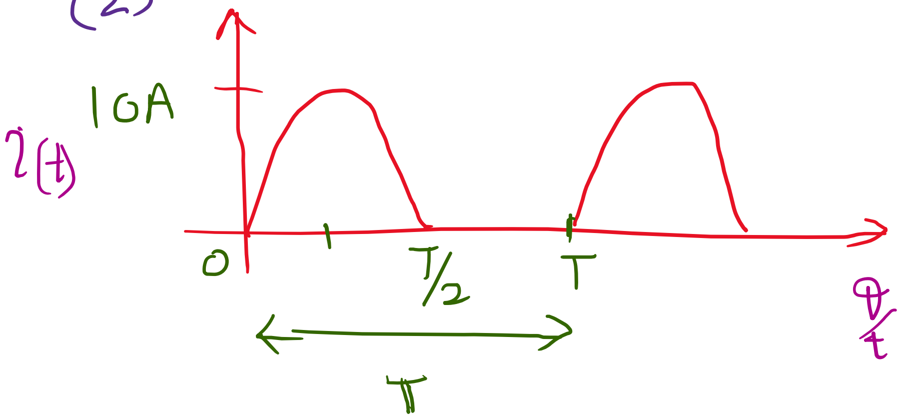
7A/DC   10A/AC

Prob (1)



T = π

Prob (2)



(1) RMS Value

(2) Average Value

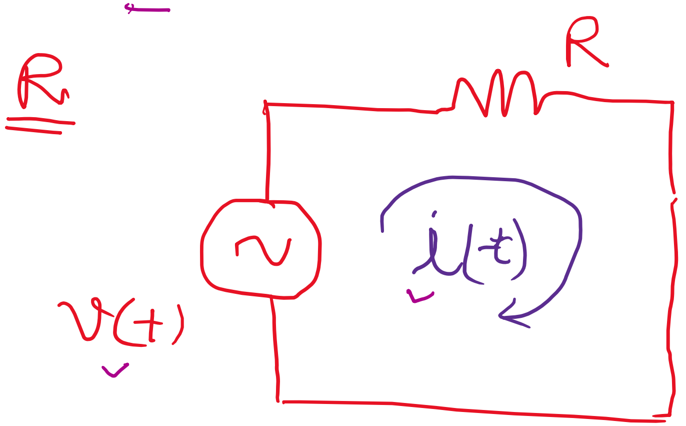
(3) Form factor = $\frac{\text{RMS}}{\text{Average}}$

(4) Peak factor = $\frac{\text{Max}}{\text{RMS}}$

1-ϕ AC circuits :

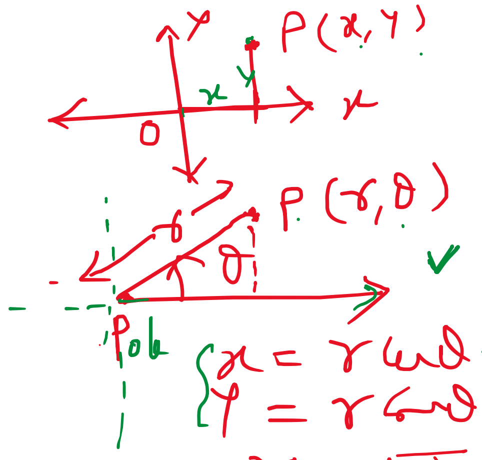
$$v(t) = V_0 \sin \omega t$$

R



$V_{rms} \equiv V$ effective

$I_{rms} = I$ "



$\begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases}$

$\begin{cases} r = \sqrt{x^2+y^2} \\ \theta = \tan^{-1}\left(\frac{y}{x}\right) \end{cases}$

DC $\rightarrow$ $V = I \cdot R$

AC $\rightarrow$ $v(t) = R \cdot i(t)$

$$i(t) = \frac{v(t)}{R} = \left(\frac{V_0}{R}\right)\sin \omega t = I_0 \sin \omega t$$

$$\left\{ I_0 = \frac{V_0}{R} \right\}$$

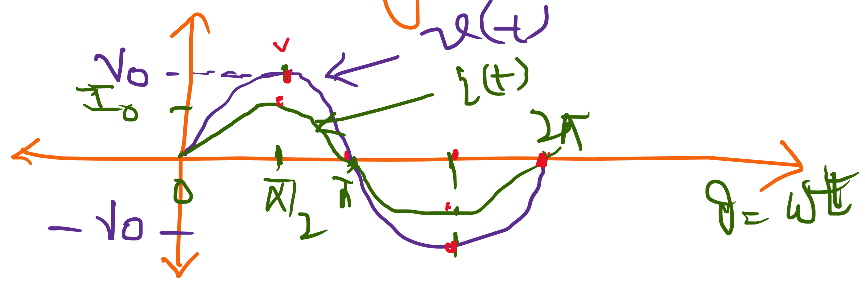
$v(t) = V_0 \sin(\omega t) \equiv V_0 \angle 0^\circ$

$i(t) = I_0 \sin \omega t \equiv I_0 \angle 0^\circ$

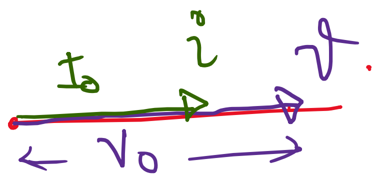
v & i are in same phase

$$v \,\hat{}\, i = 0^\circ$$

Wave diagram



Phasor diagram



Power in Resistive ckt :

$V_{rms} = V$ ✓
$I_{rms} = I$ ✓

$p(t) = v(t) \cdot i(t)$

instantaneous power ✓

$p(t) = V_0 \sin\omega t \cdot I_0 \sin\omega t$

$= V_0 I_0 \cdot \sin^2\omega t$

$= \frac{V_0 I_0}{2} \cdot 2\sin^2\omega t$

$p(t) = \left(\frac{V_0}{\sqrt{2}}\right)\left(\frac{I_0}{\sqrt{2}}\right)(1 - \cos 2\omega t)$

$$\boxed{p(t) = V \cdot I\,(1 - \cos 2\omega t)}$$

pulsating/alternating with double the supply f.

DC Power

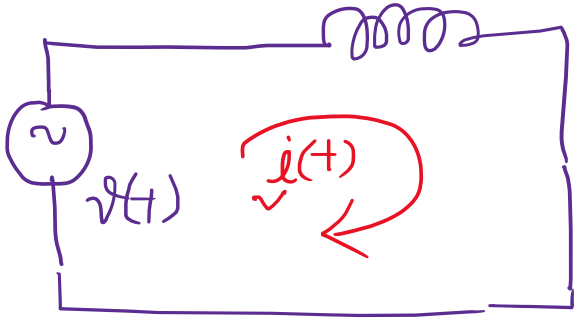
$P = V \cdot I$ ✓
$= \frac{V^2}{R}$ ✓
$= I^2 R$ ✓

$P_{av} = \frac{1}{T}\int_0^T p(t)\, dt = VI$

$$\boxed{P = V \cdot I} = V_{rms} I_{rms}$$

L

L (H)



$$\begin{cases} v = L\dfrac{di}{dt} \checkmark \\ i = \dfrac{1}{L}\int v\,dt \end{cases}$$

$R \neq f(f)$

$v(t) = V_0 \sin\omega t$ ✓

$i(t) = \frac{1}{L}\int v(t)\,dt$

$= \frac{1}{L}\int V_0 \sin\omega t\,dt$

$= \frac{V_0}{L}\left(-\frac{\cos\omega t}{\omega}\right)$

$$\boxed{(A)\ i_L(t) = \left(\frac{\overset{\text{Volts}}{V_0}}{\underbrace{\omega L}_{\Omega}}\right)\cdot \sin(\omega t - \pi/2)}$$

$\frac{V_0}{\omega L} = I_0$

$(A)\ i_R(t) = \frac{\overset{\text{Volts}}{V_0}}{\underset{\Omega}{R}} \sin\omega t$

$\rightarrow v(t) = V_0 \sin\omega t \equiv V_0\angle 0°$ ✓

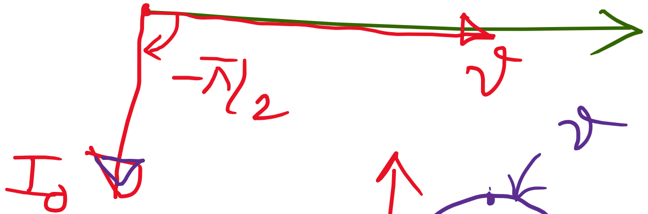
$\rightarrow i(t) = I_0 \sin(\omega t - \pi/2) \equiv \vec{I_0}\angle -\pi/2$ ✓

✓ $X_L = \omega L = 2\pi f L \rightarrow X_L \propto f$

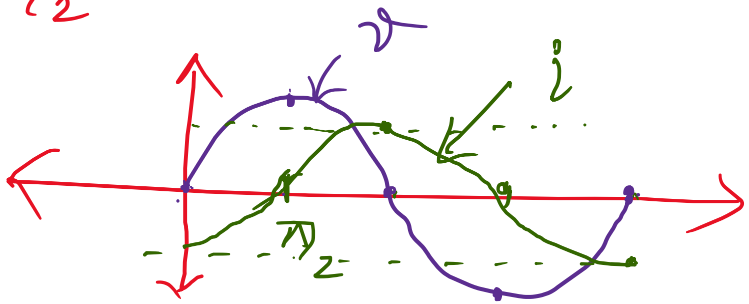
$I_0 = \frac{V_0}{X_L} = \frac{V_0}{\omega L}$ ✓

Phasor

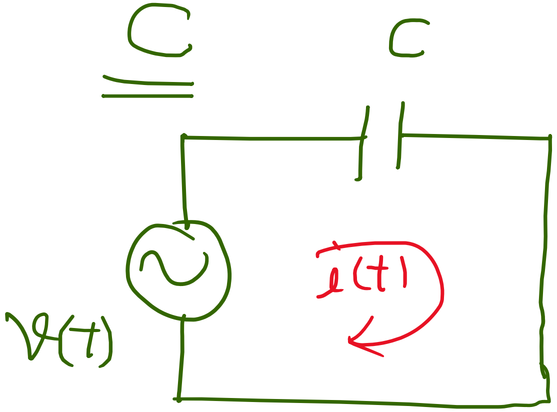
i lags the v by $\pi/2$



Wave



C



$$i(t) = C\frac{dv(t)}{dt}$$

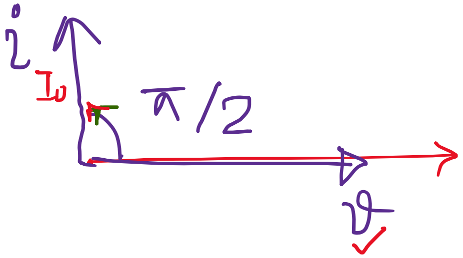
$$= C\frac{d(V_0 \sin\omega t)}{dt}$$

$$= C V_0 \omega \cos\omega t$$

$$i(t) = \frac{V_0}{\left(\frac{1}{\omega C}\right)} \sin(\omega t + \pi/2) = \frac{V_0}{X_c} \sin(\omega t + \pi/2)$$

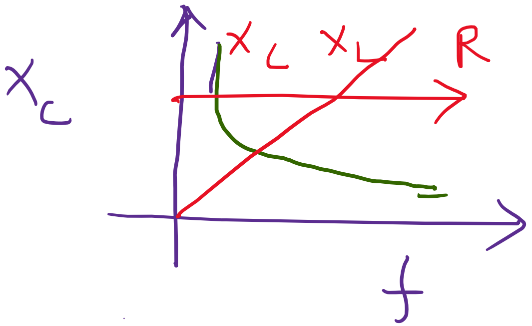
$$v(t) = V_0 \sin\omega t \equiv V_0 \angle 0$$

$$i(t) = \frac{V_0}{(1/\omega C)} \sin(\omega t + \pi/2) \equiv I_0 \angle \pi/2$$



i leads v by $\pi/2$ in a pure cap.

$$I_0 = \frac{V_0}{X_c}$$



capacitive reactance

$$X_c = \frac{1}{\omega C}$$

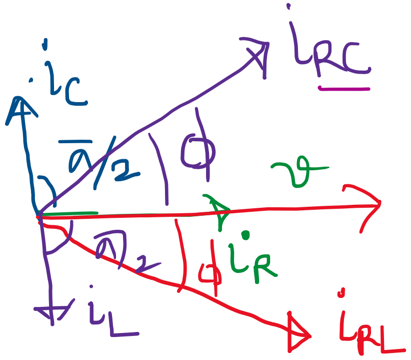
$$(\Omega)$$

$$X_c = \frac{1}{(2\pi C) f}$$

$$X_c \propto \frac{1}{f}$$

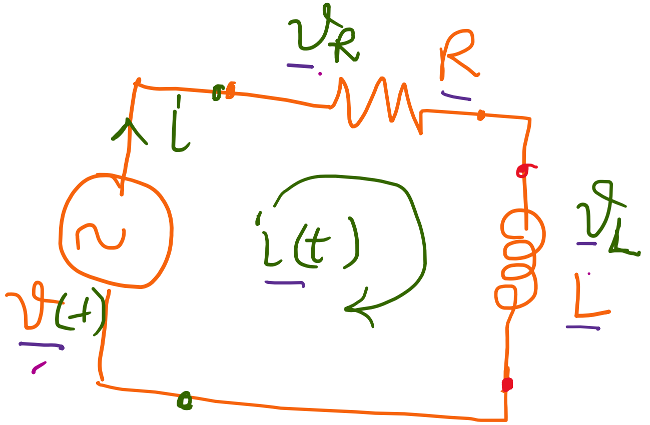
$$X_c f = \text{const}$$

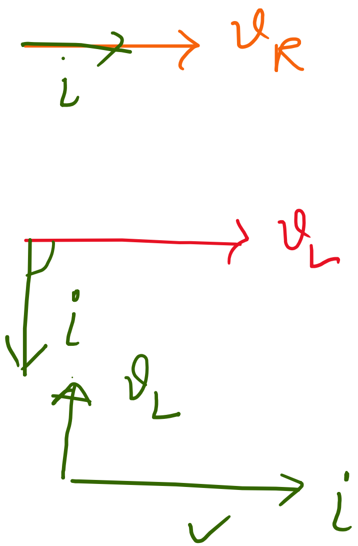
$$\boxed{x \cdot y = C}$$



$$\phi = 0 \longleftrightarrow \pi/2$$

R-L circuit

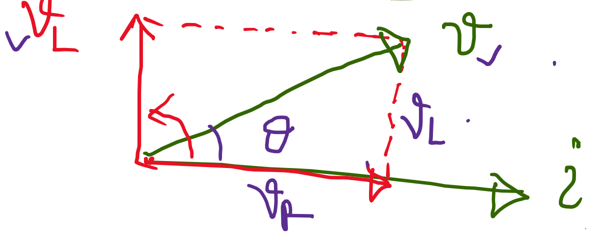




$v(t) = V_0 \sin \omega t$ ✓

$i_R(t) = \frac{V_0}{R} \sin \omega t$ ✓

$i_L(t) = \frac{V_0}{X_L} \sin(\omega t - \pi/2)$ ✓ $\quad X_L = \omega L$



$\bar{v}_R + \bar{v}_L = \bar{v}$

$v_R = i.R$

$v_L = i(jX_L)$

impedance

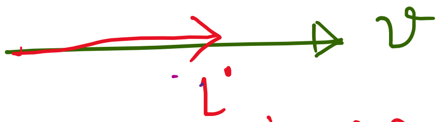
$v = iR + ijX_L$ ✓

$v = i(R + jX_L)$ ✓

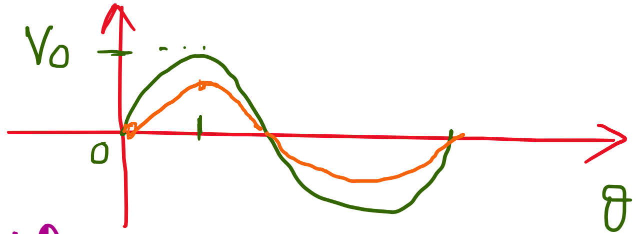
$Z = v/i = R + jX_L$

$v = V_0 \sin \omega t$

$\underline{R}$



v & i are in phase



$\frac{V_0}{R} = I_0$

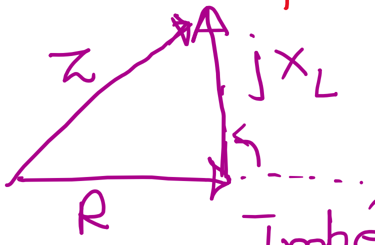
$\dot{\theta} = \frac{d\theta}{dt} = \omega$

$\omega . t = \theta$

$t = \frac{\theta}{\omega}$

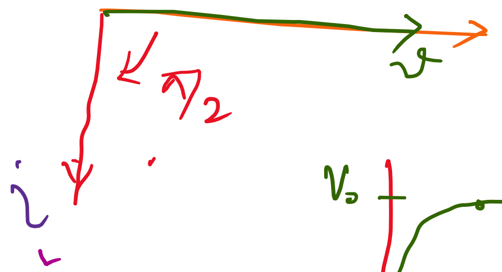
$t \rightarrow$

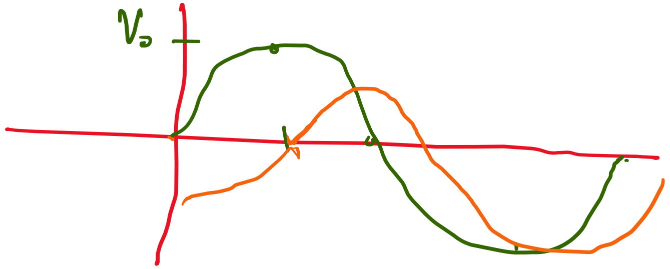
$\theta \rightarrow$



Impedance Δ

$\underline{L}$

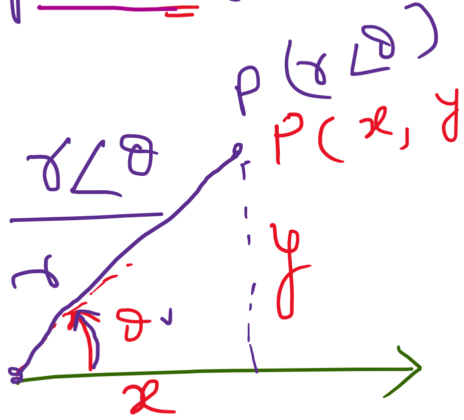




operator j (Rotation $\pi/2$) maths

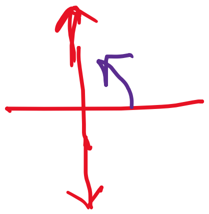
$\underbrace{\sqrt{-1} = i}$
$\parallel$
j ✓

$$z = x + iy$$
$$= r\cos\theta + j\,r\sin\theta$$
$$= r(\cos\theta + j\sin\theta)$$
$$= r e^{j\theta}$$
$$= r\angle\theta$$



$P(x, y) = x + jy$
$x = r\cos\theta$
$y = r\sin\theta$

$r = \sqrt{x^2+y^2}$
$\theta = \tan^{-1}(y/x)$

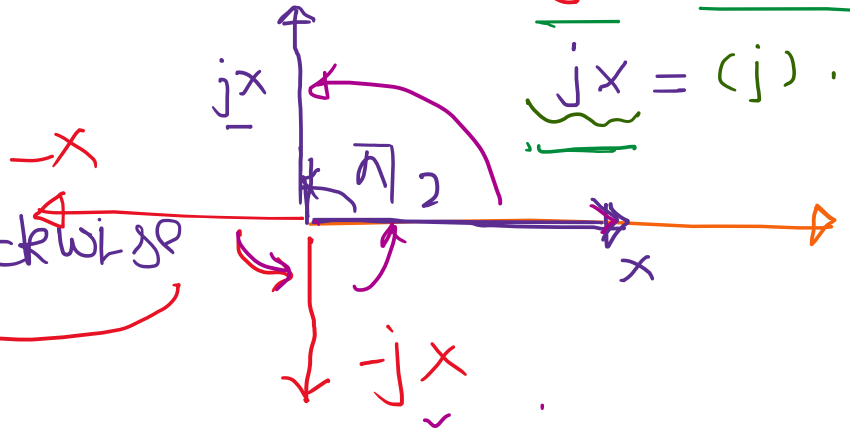


Real Nos
$$\left\{\begin{aligned} &✓X = X + j\cdot 0 = ✓X\angle 0 ✓ \quad (r_1\angle\theta_1) \\ &✓j = 0 + j\cdot 1 = 1\angle \pi/2 ✓ \quad (r_2\angle\theta_2) \end{aligned}\right.$$

$z_1 \; P_1(r_1\angle\theta_1) \quad \cdot P_2(r_2\angle\theta_2) \; z_2$

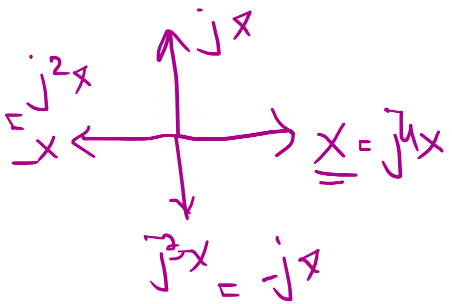
$$z_1 z_2 = r_1 r_2 \angle \theta_1 + \theta_2$$

$$jX = (j)\cdot(X) = (1\angle\pi/2)\cdot(X\angle 0)$$
$$= X\angle\pi/2$$



$$\underbrace{\pi/2 \text{ Rotator in anticlockwise}}$$

$$✓ j\cdot(jX) = j^2 X = -X$$
$$j(-X) = -jX$$



$P(x, y)$, $z = x + jy = (x, y)$, $r\angle\theta$, $re^{j\theta}$, $r\cos\theta + j\sin\theta$

R-L

$$Z = R + jX_L$$

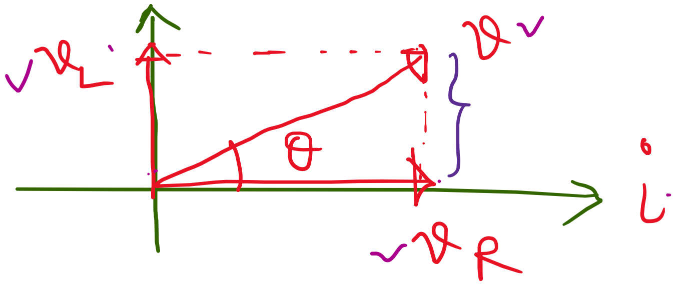
$$|Z| = \sqrt{R^2 + X_L^2}$$

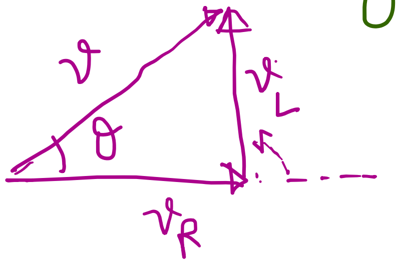
$$v = V_0 \sin \omega t$$

$$I_0 = \frac{V_0}{|Z|}$$

$$i = I_0 \sin(\omega t - \theta)$$

$$\theta = \tan^{-1}\left(\frac{v_L}{v_R}\right) = \tan^{-1}\left(\frac{X_L}{R}\right)$$

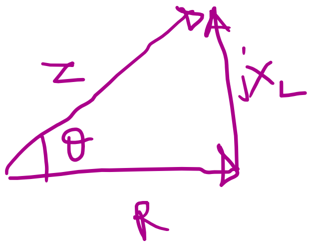




$$\bar{v} = \bar{v}_R + \bar{v}_L$$

$$\tan\theta = \frac{v_L}{v_R} = \frac{i X_L}{i R} = \frac{X_L}{R}$$

$$\theta = \tan^{-1}\left(\frac{X_L}{R}\right)$$



$R = |Z|\cos\theta$, $X_L = |Z|\sin\theta$

Numerical

R, L, $v(t) = V_0 \sin \omega t$

$\omega = 2\pi f$ $\quad f$

$X_L = \omega L \quad (\Omega)$

i will lag by θ

$Z = R + jX_L$

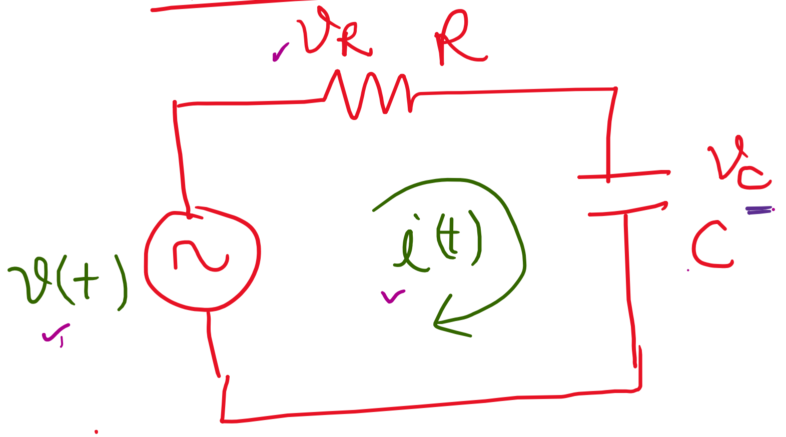
$|Z| = \sqrt{R^2 + X_L^2}$

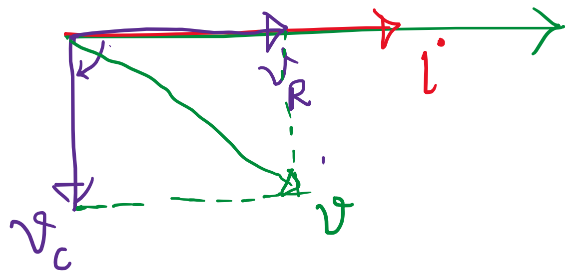
$\theta = \tan^{-1}\left(\frac{V_L}{V_R}\right) = \tan^{-1}\left(\frac{X_L}{R}\right)$

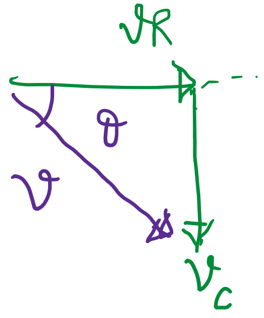
$$I_0 = \frac{V_0}{|Z|} = \frac{V_0}{\sqrt{R^2 + X_L^2}}$$

$$i(t) = I_0 \sin(\omega t - \theta)$$

R-C circuit







i leads v by θ

$$\theta = \tan^{-1}\left(\frac{v_C}{v_R}\right) = \tan^{-1}\left(\frac{X_C}{R}\right)$$

$$I_0 = \frac{V_0}{|Z|} = \frac{V_0}{\sqrt{R^2+X_C^2}}$$

$$\bar{v} = \bar{v}_R + \bar{v}_C$$

$$v = iR + i(-jX_C)$$

$$v = i(R - jX_C)$$

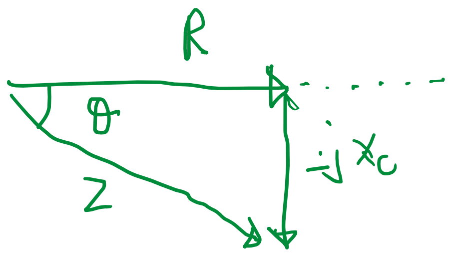
$$\frac{v}{i} = Z = R - jX_C$$

$$|Z| = \sqrt{R^2+X_C^2}$$

$$v(t) = V_0 \sin \omega t$$

$$X_C = \frac{1}{\omega C}$$

$$i(t) = I_0 \sin(\omega t + \theta)$$



$$\tan\theta = \frac{X_C}{R}$$

Numerical Problems :

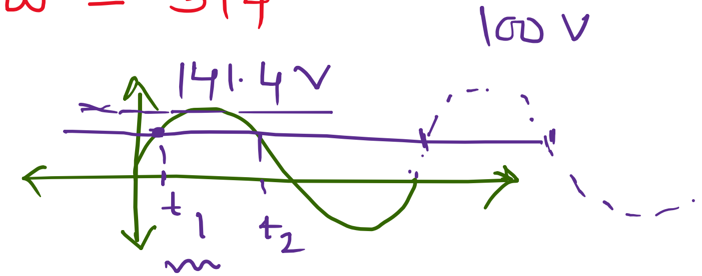
1) $v = 141.4 \sin 314t$, Find (i) f. (ii) RMS Voltage (iii) Average Voltage (iv) the instantaneous voltage at $t = 3ms$ (vi) FF. (vii) P.F. (v) Time taken to for voltage to reach 100V for the first time passing through zero.

$v = 141.4 \sin 314t$ .... Volts

$v = V_0 \sin \omega t$

$V_0 = 141.4$ Volts

$\omega = 314$



(i) $\omega = 2\pi f = 314$

$f = \frac{314}{2\pi} = 50\,Hz$.

(ii) RMS voltage

$V_{rms} = \frac{V_0}{\sqrt{2}} = \frac{141.4}{\sqrt{2}} = 100\ V$

(iii) Average Value

$V_{avg} = \frac{2V_0}{\pi}$

$= \frac{2 \times 141.4}{\pi}$

$= 90.07\ V$

$$v(t) = 141.4 \sin \underbrace{314t}$$

$v(t) = ?$ $\quad t = 3ms.$

$$= 141.4 \sin\left(\underbrace{314 \times 3 \times 10^{-3} \times \frac{180}{\pi}}_{\text{degree}}\right)$$

$= \_\_\_\_$

ωt

rad/sec × sec

⇒ rad → $\_\_°\_\_$

$\pi^c \longrightarrow 180°$

$1^c \longrightarrow \frac{180}{\pi}$

$$\underbrace{3ms = \_\_\_\_ \text{ rad} = \_\_\_\_ \text{ degree}}$$

$v(t) = 100 \quad t = ?$

$v(t) = 141.4 \sin 314t$

$100 = 141.4 \sin 314t$

$\sin 314t = \left(\frac{100}{141.4}\right) \longrightarrow$

$314t = \sin^{-1}(\quad) \Rightarrow \quad t = \_\_\_\_$

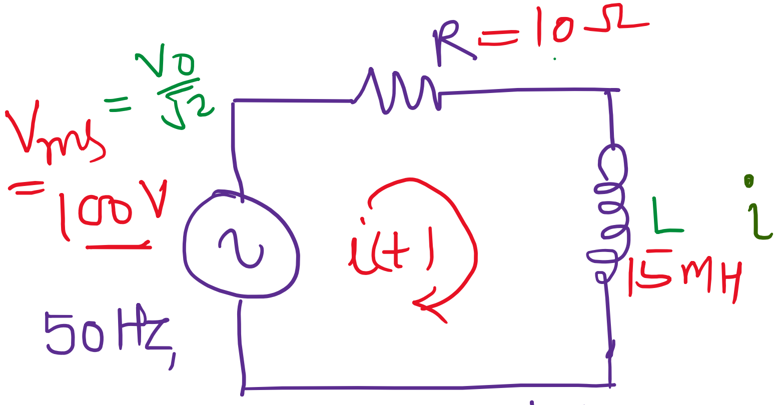
$\sin\theta = \sin\alpha$

$\theta = 2n\pi \pm \alpha$

$n = 0, 1, \ldots$

2) $R = 10\,\Omega$ $\quad L = 15\text{ mH}$, $\quad f = 50\text{ Hz}$, $\quad 100\text{ V}$ RMS supply

$Z = |Z| \angle \theta$

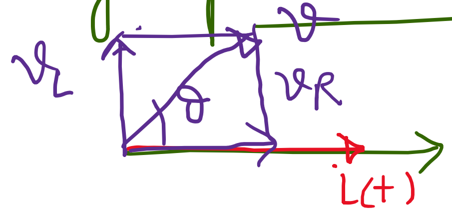


$i(t) = ?$

Z

I_0, I_{rms}

i lag by θ with v



$v(t) = \sqrt{2} \times 100 \sin 314t$ ✓

$R = 10\,\Omega$

$\omega = 2\pi f = 2 \times 3.14 \times 50 = 314\text{ rad/s}$

$X_L = \omega L = 314 \times 15 \times 10^{-3} = 4.7\,\Omega$

$Z = R + jX_L = 10 + j4.7\,\Omega$

$|Z| = \sqrt{R^2 + X_L^2} = \sqrt{10^2 + 4.7^2} = \ldots$

$I_{rms} = \frac{V_{rms}}{|Z|} = \frac{100}{} = \ldots$

✓ $I_0 = \frac{V_0}{|Z|} = \frac{\sqrt{2} \times 100}{|Z|} = \ldots$

$\theta = \tan^{-1}\left(\frac{v_L}{v_R}\right)$

$\theta = \tan^{-1}\left(\frac{X_L}{R}\right)$

✓ $\theta = \ldots$

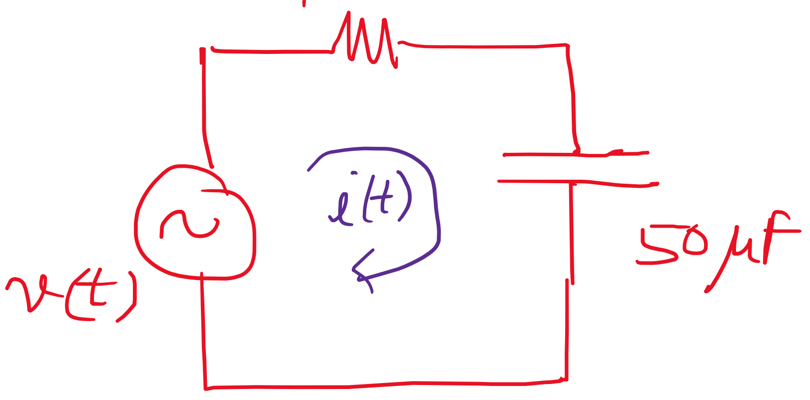
$i(t) = I_0 \sin(\omega t - \theta)$

$\underbrace{i(t) = I_0 \sin(314t - \theta)}$ Ans

3)

RC

$i(t) = ?$



$v(t) = 100 \sin(628t + 30°)$

$R = 10\ \Omega$

$\omega = 628$ rad/sec

$f = 100$ Hz

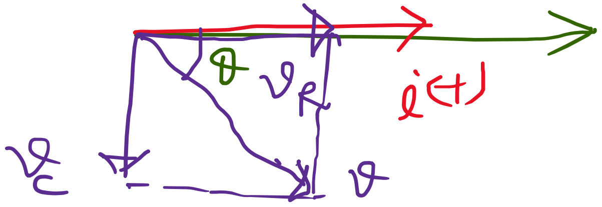
$X_c = \frac{1}{\omega C} = \frac{1}{628 \times 50 \times 10^{-6}} = \ldots$

$Z = R - jX_c$

$|Z| = \sqrt{R^2 + X_c^2}$

$\theta = \tan^{-1}\left(\frac{X_c}{R}\right) = \ldots$

$I_0 = \frac{V_0}{|Z|} = \ldots$



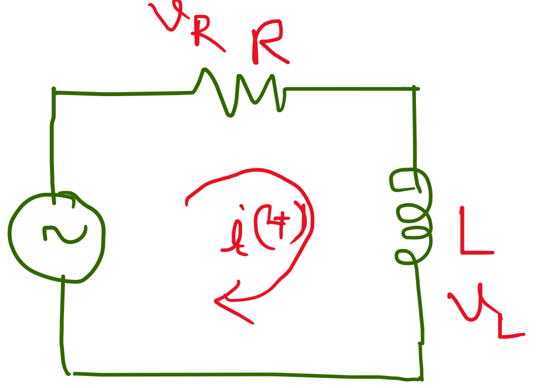
i(t) leads the voltage v(t) by $\underline{\underline{\theta}}$

$$\underbrace{i(t) = I_0 \sin(628t + 30 + \theta)}$$

4) R-L

$v = 283 \sin 314t = 283\angle 0$

$i(t) = 4 \sin(314t - 45^\circ) = 4\angle -45^\circ$



$\theta = v \wedge i$

$= 45^\circ$

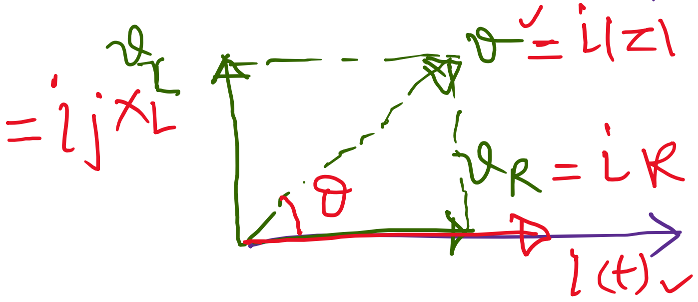
(i) $R = 70.75 \times 0.767 =$ ___

(ii) $L = 70.75 \times 0.707 / 314 =$ ___

(iii) $pf = \cos\theta = \frac{R}{|Z|} = \frac{1}{\sqrt{2}} = 0.767$

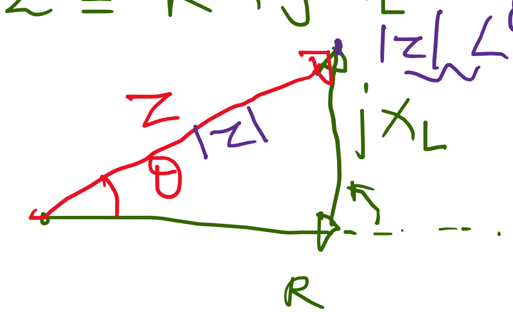
$I_0 = \frac{V_0}{|Z|}$

freq, $f = \frac{\omega}{2\pi} = \frac{314}{2\pi} = 50\text{Hz}$

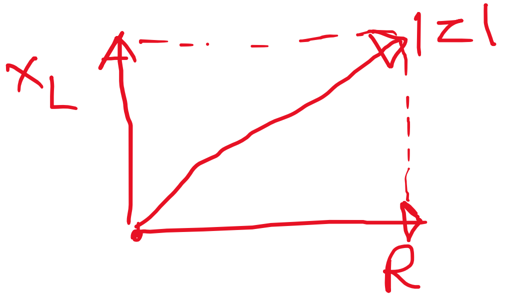


$|Z| = \frac{V_0}{I_0} = \frac{V_{rms}}{I_{rms}} = \frac{283}{4} = 70.75\ \Omega$

$Z = R + jX_L$



Impedance triangle



$\tan\theta = \frac{X_L}{R}$

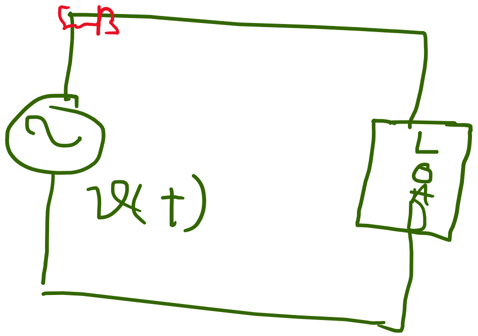
$\omega L =$

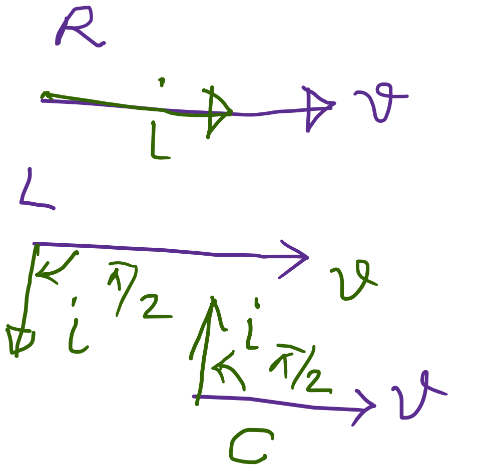
$R = |Z| \cos\theta$, $X_L = |Z| \sin\theta = 70.75 \times 0.767$

Power in 1-ϕ AC ckts :

$$v(t) = V_0 \sin \omega t \equiv V_0 \angle 0^\circ$$

$$i(t) = I_0 \sin(\omega t - \phi) \equiv I_0 \angle \phi$$





$$p(t) = v(t) \cdot i(t)$$

inst. power

$$= V_0 \sin \omega t \cdot I_0 \sin(\omega t - \phi)$$

$$= \frac{V_0 I_0}{2} \left\{ 2 \underbrace{\sin \omega t}_{A} \underbrace{\sin(\omega t - \phi)}_{B} \right\}$$

$$= \frac{V_0}{\sqrt{2}} \cdot \frac{I_0}{\sqrt{2}} \left[\cos \phi - \cos(2\omega t - \phi) \right]$$

$$= V_{rms} I_{rms} \cos \phi - V_{rms} I_{rms} \cos(2\omega t - \phi)$$

$$p(t) = \underbrace{V.I \cos \phi} - VI \cos(2\omega t - \phi)$$

$$i_R(t) = I_0 \sin \omega t \equiv I_0 \angle 0^\circ$$

$$i_L(t) = I_0 \sin(\omega t - \pi/2) \equiv I_0 \angle \pi/2$$

$$i_C(t) = I_0 \sin(\omega t + \pi/2) \equiv I_0 \angle \pi/2$$

RL

$$i_{RL}(t) = I_0 \sin(\omega t - \theta)$$

RC

$$i_{RC}(t) = I_0 \sin(\omega t + \theta)$$

$p(t) = VI\cos\phi - VI\sin(2\omega t - \phi)$

$\omega \rightarrow 2\pi f$

$2\omega \rightarrow 2\pi(2f)$

Instantaneous power in 1-ϕ AC ckt is pulsating with double the supply frequency.

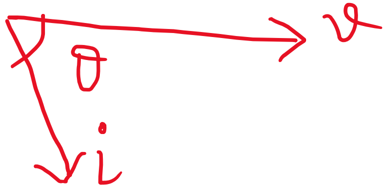
$P = P_{avg} = \frac{1}{T}\int_0^T p(t)\,dt = \frac{1}{T}\int_0^T VI\cos\phi\,dt + 0 = VI\cos\phi$

$\boxed{P = VI\cos\phi}$ true power / active power / Real power / Watt power

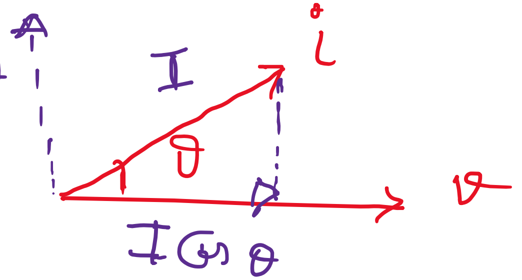
Watts.

Lagging load (R-L) i will lag v by θ

Cap / Leading load



I sinθ (in quadrature to v)



I cosθ (in phase with v)

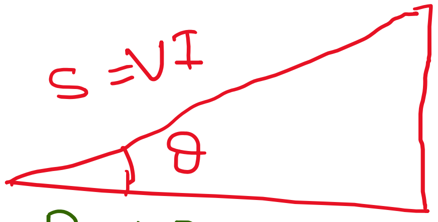
$P = V \cdot I \cos\theta$ W, active/true KW, MW Real, Actual

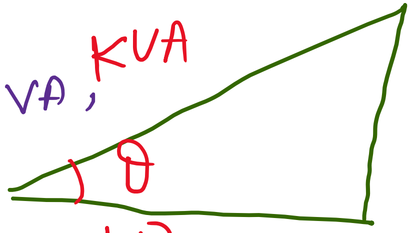
$Q = V \cdot I \sin\theta$ VAR, KVAR, MVAR Reactive power (Volt amp reactive) Quality of power

$S = V \cdot I$ apparent power. VA, KVA, MVA

$\xrightarrow{i \quad v}$

Power $\triangle$





upf → Resistive
$\theta = 0$
$\cos\theta = 1$

$0 \le \cos\theta \le 1$

Power factor

1.

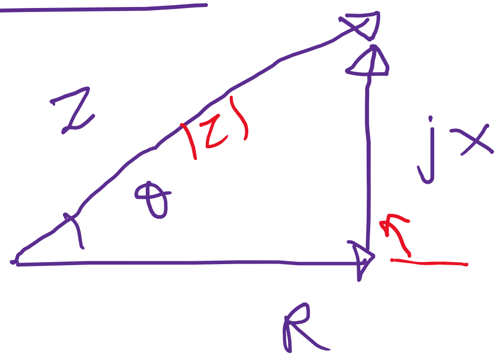
$Pf = \cos\theta = \dfrac{P}{VI} = \dfrac{R}{|Z|}$

$$\boxed{I = \frac{P}{V \cos\theta}}$$

$\cos\theta \downarrow \ \theta \uparrow$

$I = \dfrac{K}{\cos\theta}$; $I \propto \dfrac{1}{\cos\theta}$

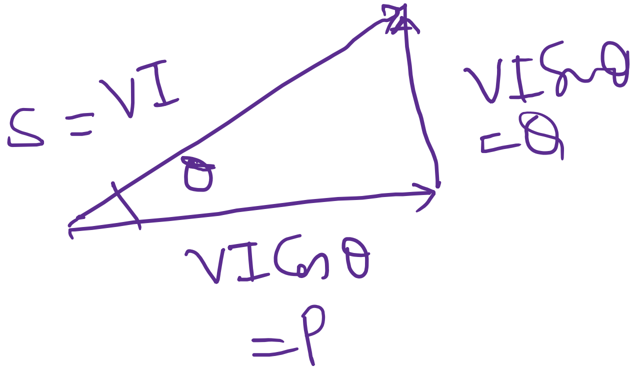
* Impedance Δ



$$\left\{\begin{array}{l} R = |Z| \cos\theta \\ X = |Z| \sin\theta \end{array}\right.$$

$$|Z| = \sqrt{R^2 + X^2}$$

* Power Δ



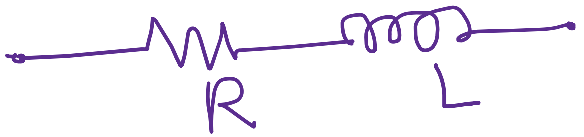
$$\left\{\begin{array}{l} \# \; S = VI \\ \# \; P = VI\cos\theta \\ \# \; Q = VI\sin\theta \end{array}\right.$$

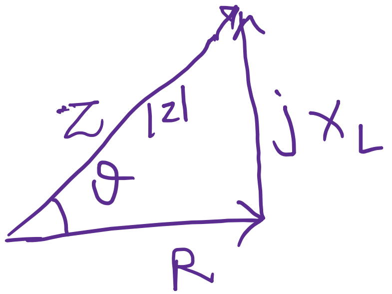
$$\text{Reactive pf} = \sin\theta = \frac{Q}{VI}$$

Q factor of a Coil

R-L

Coil





$$Z = R + jX_L$$
$$= R + j\omega L$$

$$pf = \cos\theta = \frac{R}{|Z|}$$
$$= \frac{R}{\sqrt{R^2 + X_L^2}}$$

for R is small compared to X_L

$$R < X_L$$
$$R^2 << X_L^2 \qquad |Z| = X_L$$

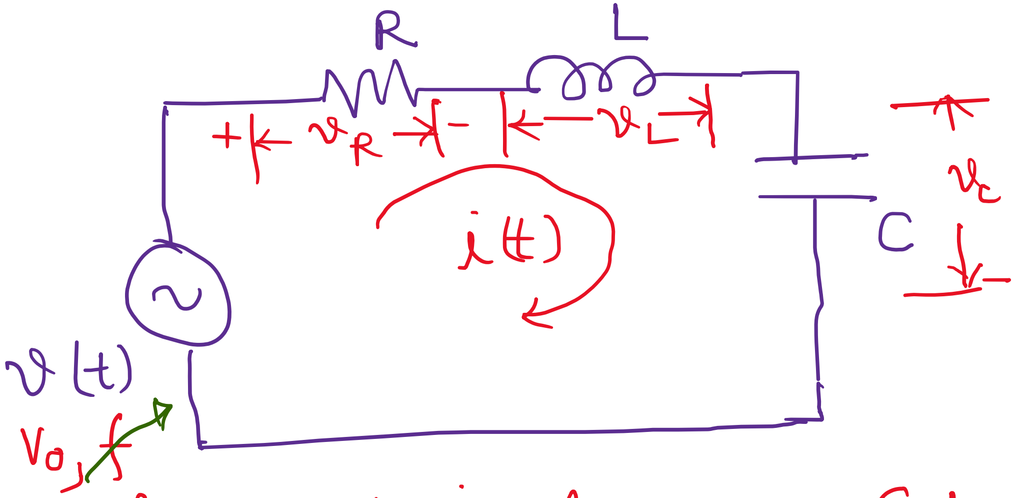
thus, $pf = \cos\theta$
$$= \frac{R}{X_L}$$

Q factor of a coil

$$= \frac{1}{\text{Power factor}}$$
$$= \frac{1}{\cos\theta}$$
$$= \frac{X_L}{R}$$

$$\boxed{Q = \frac{\omega L}{R}}$$

Series R-L-C circuit :



$v(t) = V_0 \sin \omega t$

$\bar{v} = \bar{v}_R + \underbrace{\bar{v}_L + \bar{v}_C}$

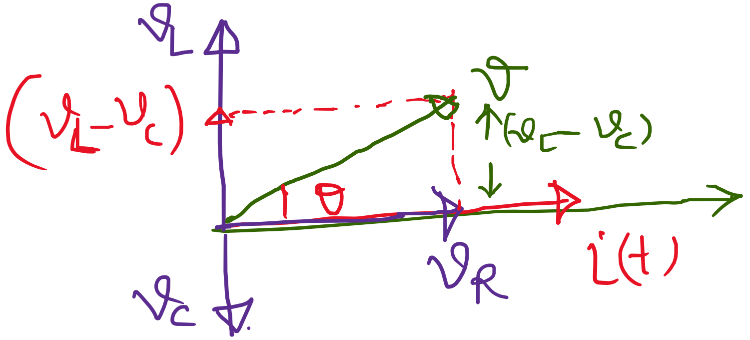
$X_L \to (X_L - X_C)$

RL   RLC

$i(t) = I_0 \sin(\omega t - \theta)$

$=$

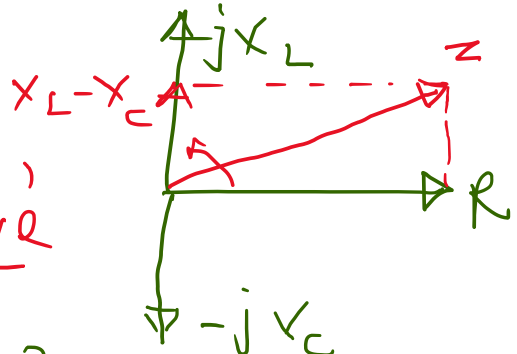
Phasor diagram



$v_R = iR$

$v_L = j\, i\, X_L = i(jX_L)$

$v_c = -j\, i\, X_c = i(-jX_c)$



Case 1) $v_L > v_c$; $X_L > X_c$

$(v_L - v_c)$ (Inductive) RL

$v = \sqrt{v_R^2 + (v_L - v_c)^2}$

$v = \sqrt{(iR)^2 + (iX_L - iX_c)^2}$

$v = i\sqrt{R^2 + (X_L - X_c)^2}$

$|Z| = v/i = \sqrt{R^2 + (X_L - X_c)^2}$

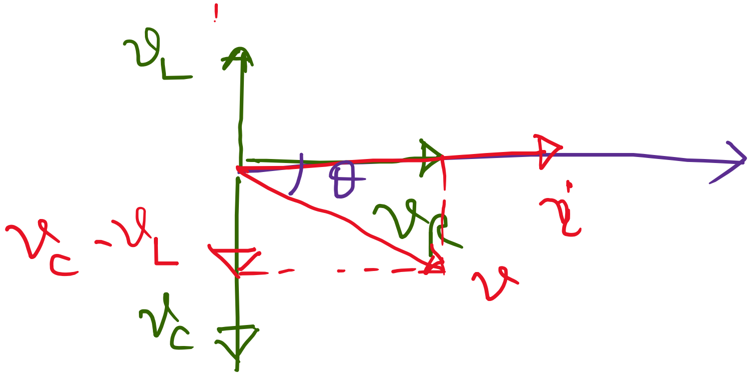
$Z = R + j(X_L - X_c)$

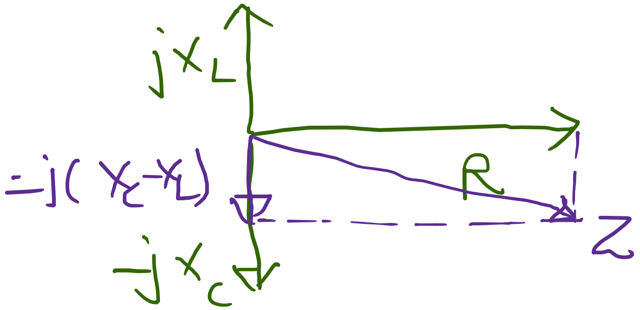
$I_0 = \dfrac{v}{|Z|}$

$\theta = \tan^{-1}\left(\dfrac{v_L - v_c}{v_R}\right)$

$= \tan^{-1}\left(\dfrac{X_L - X_c}{R}\right)$

Case 2 : $v_C > v_L$; $X_C > X_L$; Capacitive, $\frac{RC}{\checkmark}$ $X_C \longrightarrow (X_C - X_L)$





$$Z = R + jX_L - jX_C$$

$$Z = R - j(X_C - X_L)$$

$$Z = R - jX$$

$$\bar{v} = \bar{v}_R + \underbrace{\bar{v}_L + \bar{v}_C}$$

$$|v| = \sqrt{v_R^2 + (v_C - v_L)^2}$$

$$= i\sqrt{R^2 + (X_C - X_L)^2}$$

$$|Z| = \frac{|v|}{i} = \sqrt{R^2 + (X_C - X_L)^2}$$

$$\checkmark\ I_0 = \frac{V_0}{|Z|}$$

$$v(t) = V_0 \sin\omega t$$

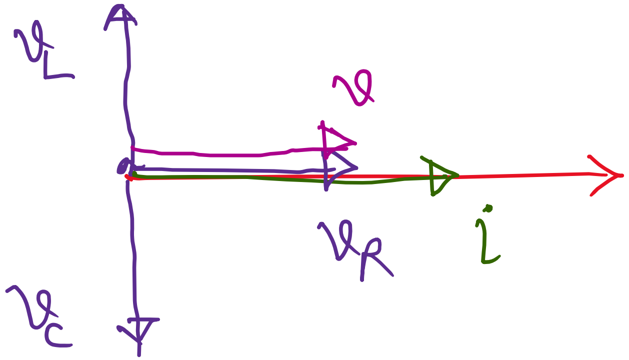
$$\checkmark\ \theta = \tan^{-1}\left(\frac{X_C - X_L}{R}\right)$$

$$\cos\theta = \frac{R}{|Z|} = \frac{R}{\sqrt{R^2 + (X_C - X_L^2)}}$$

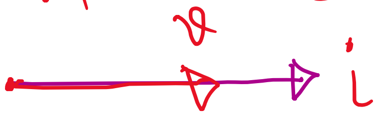
$$\underline{i(t) = I_0 \sin(\omega t + \theta)}$$

Case III : $\underbrace{v_L = v_c}$; $\underbrace{X_L = X_c}$ "series ckt"

$X_{net} = X_L - X_c = 0$ 'Resistive'



$\Rightarrow$



$\theta = v \hat{} i = 0^\circ$

$\cos\theta = \cos 0^\circ = 1$ upf

$$I_0 = \frac{V_0}{|Z|} = \frac{V_0}{R}$$

$$I_0 = \frac{V_0}{\sqrt{R^2 + (X_L \sim X_c)^2}}$$

$Z \rightarrow Z_{min}$

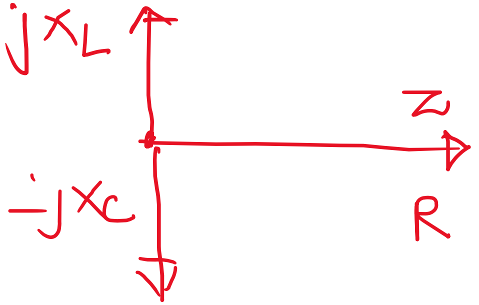
$I_0 \longrightarrow I_{0max}$

$$\bar{v} = \bar{v}_R + \underbrace{\bar{v}_L + \bar{v}_c}$$

$$\bar{v} = \bar{v}_R + \bar{0}$$

$$\boxed{\bar{v} = \bar{v}_R}$$

$X = X_L - X_c$
$= 0$



$\underbrace{Z = R}$

$Z = R + jX_L - jX_c$
$= R + j(X_L - X_c)$
$= R + j0$
$= R$

series ckt "Resonance"

Condition : $X_L = X_c$

General cond^n

$X_{net} = 0$

Reactive component $= 0$

$Z = R + jX$

$X = 0$

$$\boxed{X_L = X_C}$$

"f_r" Resonance freq.

$\omega_r = 2\pi f_r$

$$\Rightarrow \quad \omega_r L = \frac{1}{\omega_r C}$$

$$\Rightarrow \quad 2\pi f_r L = \frac{1}{2\pi f_r C}$$

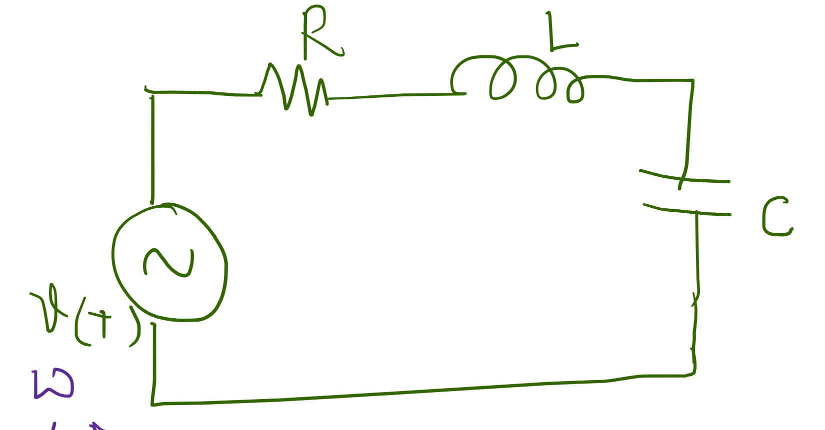
$$\Rightarrow \quad f_r^2 = \frac{1}{(2\pi)^2 LC}$$

$$\Rightarrow \quad \boxed{f_r = \frac{1}{2\pi\sqrt{LC}}}$$

$Z = R \pm j X_{net}$

$X_{net} = 0$

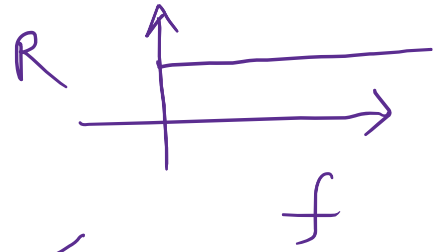
$Z = R$



$$v(t) = V_0 \sin \omega t$$

$$Z = R + j(X_L \sim X_C)$$

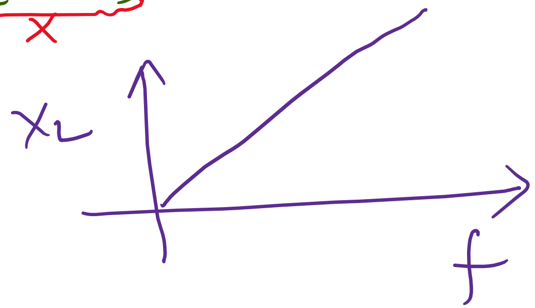
$$|Z| = \sqrt{R^2 + \underbrace{(X_L - X_C)^2}_{x}}$$



1) R const

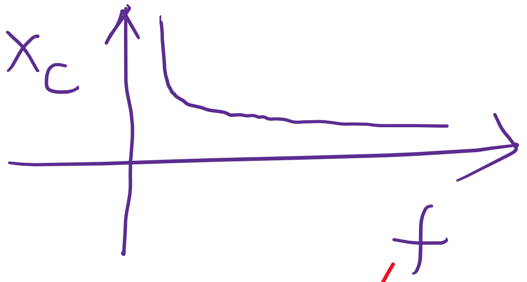
2) $X_L = 2\pi f L$

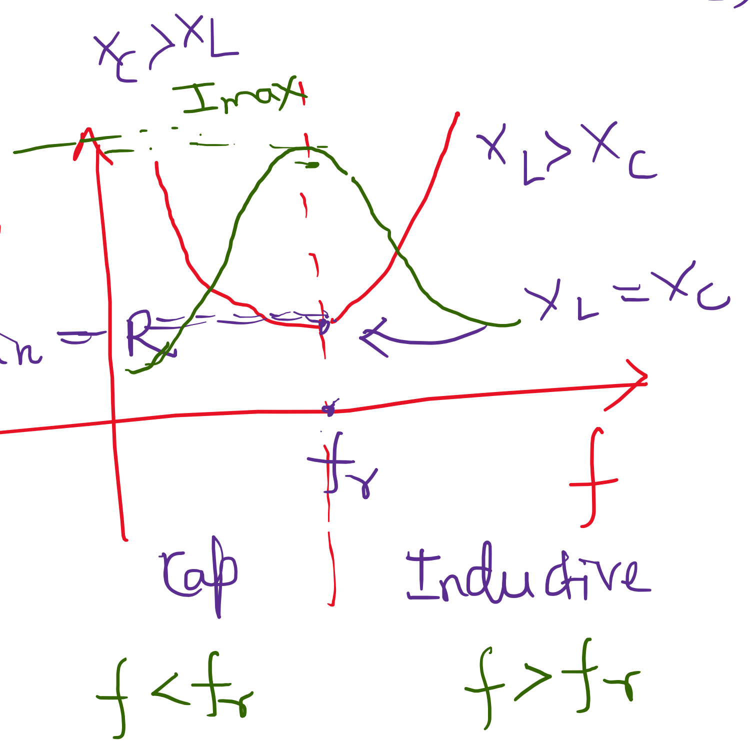
$X_L \propto f$



3) $X_C = \dfrac{1}{2\pi f C}$

$X_C \propto \dfrac{1}{f}$

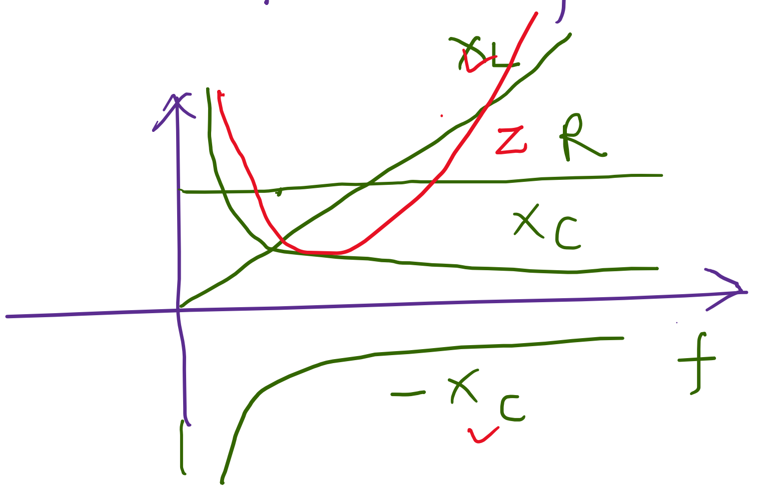




$$I = \frac{V_0}{|Z|}$$

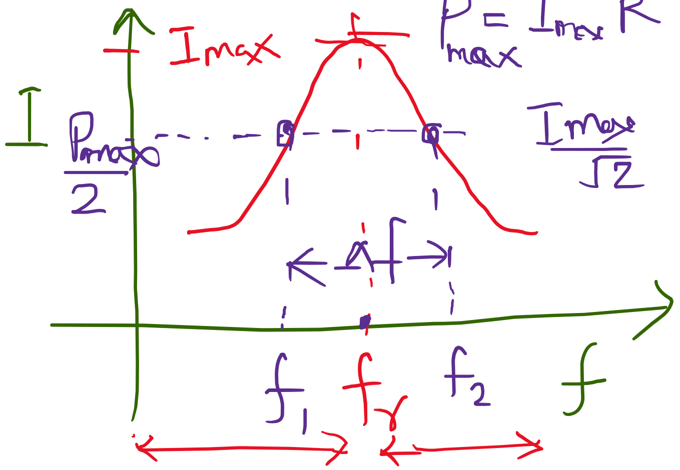
Resonance Curve

I v/s f



$X_L + (-X_C)$

Resonance Curve



$P_{max} = I_{max}^2 R$, (R)

$(\sqrt{2}R) = |Z|$

half Power freq $= f_1 \ f_2$

$\Delta f = f_2 - f_1$ 'BW'

$$\underbrace{\frac{I_{max}^2 R}{2} = \left(\frac{I_{max}}{\sqrt{2}}\right)^2 \cdot R}$$

$X_L = X_C$

$Z_{min} = R$

$X_{net} = 0$

I_{max}

f_r

$P = I_{max}^2 \cdot R$

$\cos\phi = 1$

$v\,\hat{}\,i = \phi = 0$

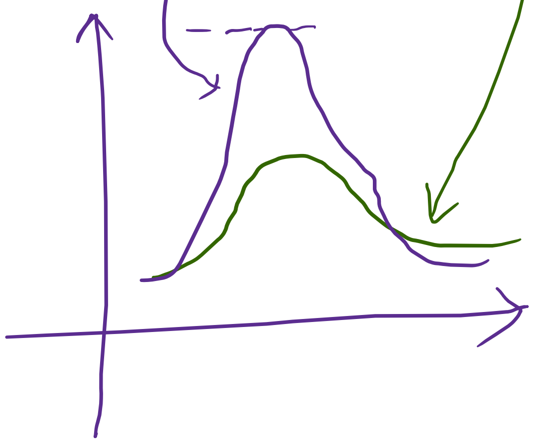
upf → Resonance

At f_1, f_2

1. $P_{max}/2$
2. $I_{max}/\sqrt{2} = 0.707 I_{max}$
3. $|Z| = \sqrt{2}R$

$I_{max} = \frac{V_0}{R}$ ✓

R → low   R → high



selective R ↓

Bandwidth ↓

At f_1, f_2

$$|Z| = \sqrt{2} R$$

$$\Rightarrow \sqrt{R^2 + X^2} = \sqrt{2} R$$

$$\Rightarrow R^2 + X^2 = 2R^2$$

$$\Rightarrow X^2 = R^2$$

$$\Rightarrow \boxed{X = \pm R}$$

$$\boxed{\omega_2 - \omega_1 = \frac{R}{L}}$$

$$2\pi(f_2 - f_1) = R/L$$

$$\boxed{BW = \Delta f = \frac{R}{2\pi L}}$$

$$f_1 = f_r - \frac{\Delta f}{2} =$$

$$f_2 = f_r + \Delta f/2$$

f_1: $X_L - X_C = -R$ f_2: $X_L - X_C = +R$

$\omega_1 = 2\pi f_1$ $f_1 < f_r$ cap $X_C > X_L$ $\omega_2 = 2\pi f_2$

$$\omega_1 L - \frac{1}{\omega_1 C} = -R \qquad \omega_2 L - \frac{1}{\omega_2 C} = R$$

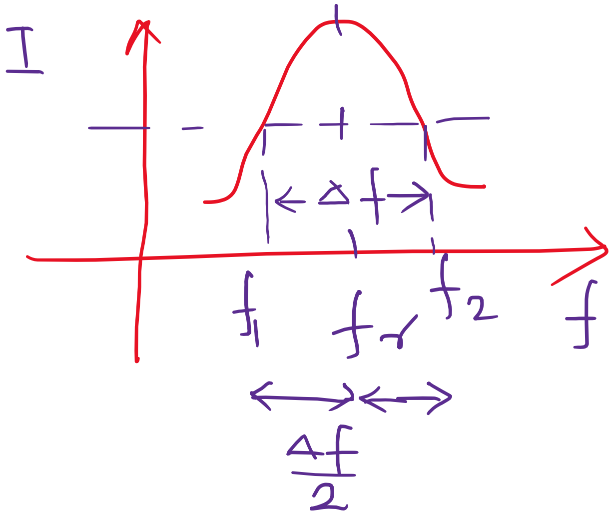
$$\omega_1^2 LC - 1 + R\omega_1 C = 0$$

$$\omega_1^2 + \frac{R}{L}\omega_1 - \frac{1}{LC} = 0 \qquad \omega_2^2 - \frac{R}{L}\omega_2 - \frac{1}{LC} = 0$$

$$\omega_1 = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 + 1/LC} \qquad \omega_2 = \frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

$$\omega_1 = -\frac{R}{2L} - \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}} \; X \qquad \omega_2 = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

$$\omega_1 = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}} \; ✓ \qquad \omega_2$$



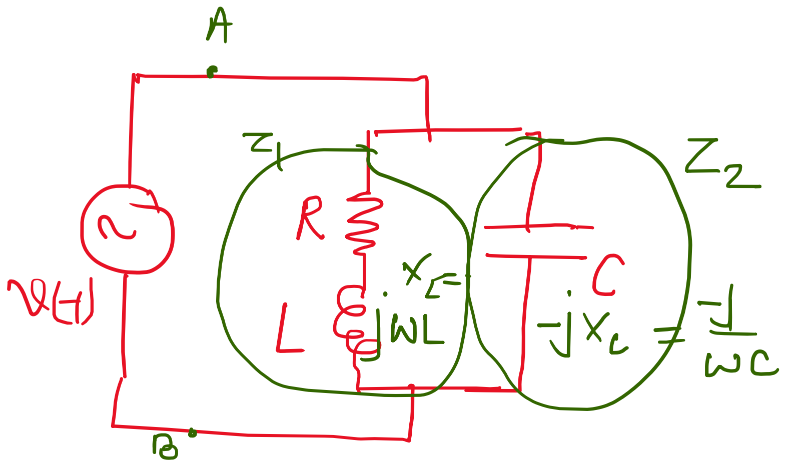
$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

$$\Delta f = \frac{R}{2\pi L}$$

$$f_1 = f_r - \frac{\Delta f}{2} = f_r - \frac{R}{4\pi L}$$

$$f_2 = f_r + \frac{\Delta f}{2} = f_r + \frac{R}{4\pi L}$$

Resonance in || al ckts (or in general any ckt)

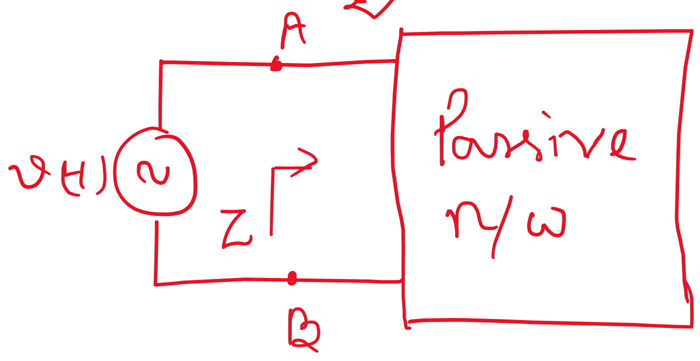


$v(t) = V_0 \sin \omega t$

$Z_1 = R + j\omega L$

$Z_2 = -j/\omega C$

$$Z_{AB} = \frac{Z_1 Z_2}{Z_1 + Z_2} = \frac{(R + j\omega L)(-j/\omega C)}{R + j\omega L - j\frac{1}{\omega C}}$$



✓1) find Z

2) $Z = R + jX$

3) $X = 0$

$Z_{AB} = R + jX$ form

$X = 0 \rightarrow$ Resonance condn

$\Downarrow$

$\underline{Z_{AB} = R}$

$\downarrow$

$\underline{\underline{f_r}}$

$$Z_{AB} = \frac{(R+j\omega L)\left(-j\frac{1}{\omega C}\right)}{R + j\left(\omega L - \frac{1}{\omega C}\right)}$$

$$Z_{AB} = \frac{L}{C R}$$

$$= \frac{(R+j\omega L)\left(-j\frac{1}{\omega C}\right)\left[R - j\left(\omega L - \frac{1}{\omega C}\right)\right]}{\left[R + j\left(\omega L - \frac{1}{\omega C}\right)\right]\left[R - j\left(\omega L - \frac{1}{\omega C}\right)\right]}$$

$$= \frac{-j\left(\frac{1}{\omega C}\right)\left[R^2 + \omega L\left(\omega L - \frac{1}{\omega C}\right) + j\left\{R\omega L - R\left(\omega L - \frac{1}{\omega C}\right)\right\}\right]}{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

$$= \overbrace{\frac{\frac{1}{\omega C}\left\{R\omega L - R\left(\omega L - \frac{1}{\omega C}\right)\right\}}{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} - j\,\underbrace{\frac{\frac{1}{\omega C}\left[R^2 + \omega L\left(\omega L - \frac{1}{\omega C}\right)\right]}{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

$$Z_{AB} = \underbrace{\overset{?}{R}} + j\,\underset{\overset{\shortparallel}{0}}{X}$$

$$\frac{\frac{1}{\omega C}\left[R^2+\omega L\left(\omega L-\frac{1}{\omega C}\right)\right]}{R^2+\left(\omega L-\frac{1}{\omega C}\right)^2}=0$$

$$\Rightarrow \boxed{R^2+\omega^2L^2-\frac{L}{C}=0}$$

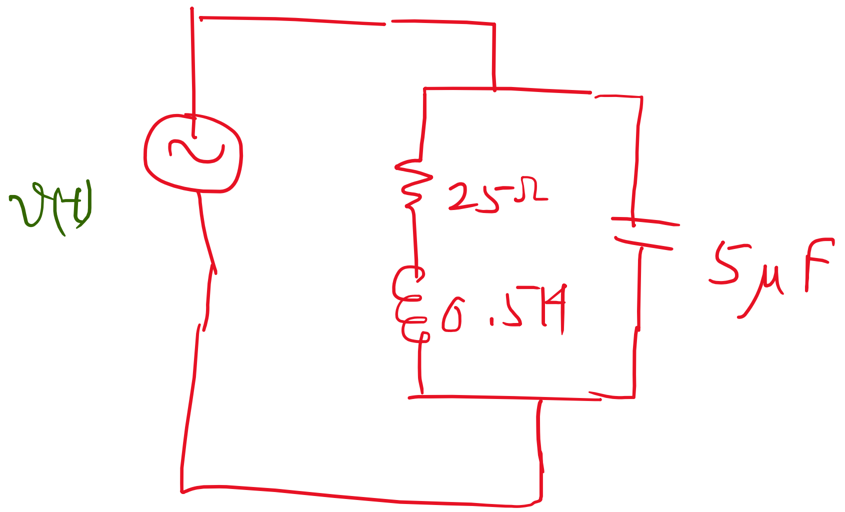
$$\Rightarrow \omega^2L^2=\frac{L}{C}-R^2$$

$$\Rightarrow \omega^2=\frac{1}{LC}-\left(\frac{R}{L}\right)^2$$

$$\boxed{\omega_r=\sqrt{\frac{1}{LC}-\left(\frac{R}{L}\right)^2}}$$

$$\boxed{f_r=\frac{1}{2\pi}\sqrt{\frac{1}{LC}-\left(\frac{R}{L}\right)^2}}$$

Prob: (i) f_r (ii) Z_{tot} at resonance (iii) BW (iv) Q



$$f_r = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \left(\frac{R}{L}\right)^2}$$

$$= \frac{1}{2\pi}\sqrt{\frac{1}{0.5 \times 5 \times 10^{-6}} - \left(\frac{25}{0.5}\right)^2}$$

$$= 100.34 \text{ Hz}$$

$$Z = \frac{L}{CR} = \frac{0.5}{5 \times 10^{-6} \times 25} = 4000\ \Omega$$

$$BW = \frac{R}{2\pi L} = \frac{25}{2\pi \times 0.5} = 7.9 \text{ Hz}$$

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{25}\sqrt{\frac{0.5}{5 \times 10^{-6}}} = 12.65$$

Major Exam

Q.1. (10) Unit 1 + 2 7 part (a), (b), (c), (d), (e) (f) (g) $5 \times 2 = \underline{10}$ ~~4 + 3~~

Q.2 (10)
Q.3 (10)] Unit 3 [(a) (b) (c)] 2 out of 3 (5×2)

Q.4 (10)
Q.5 (10)] Unit 4

Unit - 1 N ✓
2 N + T
3 N + T.
4 T.