Ques:- find Eigen values and Eigen Vector of matrix $A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$.

Solⁿ The characteristic equation of matrix A

$$|A - \lambda I| = 0$$

$$\Rightarrow \begin{vmatrix} 2-\lambda & 2 & 1 \\ 1 & 3-\lambda & 1 \\ 1 & 2 & 2-\lambda \end{vmatrix} = 0$$

$$\Rightarrow -(\lambda-1)(\lambda^2 - 6\lambda + 5) = 0$$

$$\Rightarrow \lambda = 1, 1, 5 \quad \text{(Eigen values of A).}$$

Case I:- Let $X_1 = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ be eigen vector

for $\lambda = 5$ Then

$$|A - \lambda I| X_1 = 0$$

$$\Rightarrow \begin{bmatrix} 2-5 & 2 & 1 \\ 1 & 3-5 & 1 \\ 1 & 2 & 2-5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -3 & 2 & 1 \\ 1 & -2 & 1 \\ 1 & 2 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$A = \begin{bmatrix} -3 & 2 & 1 \\ 1 & -2 & 1 \\ 1 & 2 & -3 \end{bmatrix}$$

$$\sim \begin{bmatrix} -3 & 2 & 1 \\ 3 & -6 & 3 \\ 3 & 6 & -9 \end{bmatrix} \quad \begin{matrix} R_2 \to 3R_2 \\ R_3 \to 3R_3 \end{matrix}$$

$$\sim \begin{bmatrix} -3 & 2 & 1 \\ 0 & -4 & 4 \\ 0 & 8 & -8 \end{bmatrix} \quad \begin{matrix} R_2 \to R_2 + R_1 \\ R_3 \to R_3 + R_1 \end{matrix}$$

$$\sim \begin{bmatrix} -3 & 2 & 1 \\ 0 & -4 & 4 \\ 0 & 0 & 0 \end{bmatrix} \quad R_3 \to R_3 + 2R_2$$

$$\Rightarrow \begin{bmatrix} -3 & 2 & 1 \\ 0 & -4 & 4 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{matrix} -3x + 2y + z = 0 \\ -4y + 4z = 0 \end{matrix}$$

$$\frac{x}{12} = \frac{y}{0-(-12)} = \frac{z}{12-0}$$

$$\Rightarrow \frac{x}{12} = \frac{y}{12} = \frac{z}{12}$$

$$\Rightarrow \frac{x}{1} = \frac{y}{1} = \frac{z}{1} = k$$

$$\Rightarrow x = y = z = k$$

for $k = 1$

$$X_1 = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}^T$$

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Case II :- for $\lambda = 1$, Let $X_2 = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ is

eigen vector

i.e. $[A - \lambda I] X_2 = 0$

$$\begin{bmatrix} 2-1 & 2 & 1 \\ 1 & 3-1 & 1 \\ 1 & 2 & 2-1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$\begin{bmatrix} 1 & 2 & 1 \\ 1 & 2 & 1 \\ 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$

$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$x + 2y + z = 0$$

Let $y = k_1$ & $z = k_2$

$$x = -2k_1 - k_2$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2k_1 - k_2 \\ k_1 \\ k_2 \end{bmatrix}$$

$$= \begin{bmatrix} -2k_1 \\ 1k \\ 0 \end{bmatrix} + \begin{bmatrix} -k_2 \\ 0 \\ k_2 \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} k_1 + \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} k_2$$

$\Rightarrow$ $[-2, 1, 0]^T$ & $[-1, 0, 1]^T$ are two eigen vector corresponding to the eigen values $\lambda = 1$

<u>Ques</u> Find the eigen values and eigenvectors of the matrix $A = \begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$

<u>Sol^n</u> The characteristic eq^n

$$|A - \lambda I| = 0$$

$$\Rightarrow \begin{vmatrix} 3-\lambda & 1 & 4 \\ 0 & 2-\lambda & 6 \\ 0 & 0 & 5-\lambda \end{vmatrix} = 0.$$

$$\Rightarrow (3-\lambda)(2-\lambda)(5-\lambda) = 0$$

$$\Rightarrow \lambda = 2, 3 \,\&\, 5$$

<u>Case I :-</u> Let $X_1 = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ be eigen vector corres to eigen value $\lambda = 2$.

$$\Rightarrow [A - \lambda I] X_1 = 0$$

$$\begin{bmatrix} 1 & 1 & 4 \\ 0 & 0 & 6 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$R_3 \rightarrow R_3 - \frac{1}{2} R_2$$

$$\begin{bmatrix} 1 & 1 & 4 \\ 0 & 0 & 6 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \quad 0x + y + 4z = 0$$
$$0\cdot x + 0\cdot y + 1\cdot z = 0$$

$$\Rightarrow \frac{x}{1-0} = \frac{y}{0-1} = \frac{z}{0-0}$$

$$\Rightarrow \frac{x}{1} = \frac{y}{-1} = \frac{z}{0} = k$$

$$\Rightarrow x = 1\cdot k, \quad y = -1\cdot k, \quad z = 0\cdot k$$

for $k = 1$

$x = 1$, $y = -1$, $z = 0$

$\Rightarrow X_1 = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$ is an eigen vector corresponding to eigen value $\lambda = 2$

Case 2 :- Let $X_2 = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ be eigen vector for $\lambda = 3$

i.e.

$$\begin{bmatrix} 3-3 & 1 & 4 \\ 0 & 2-3 & 6 \\ 0 & 0 & 5-3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 & 4 \\ 0 & -1 & 6 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$R_2 \rightarrow R_1 + R_2$$

$$\begin{bmatrix} 0 & 1 & 4 \\ 0 & 0 & 10 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_3 \to R_3 - \frac{1}{5} R_2$

$$\begin{bmatrix} 0 & 1 & 4 \\ 0 & 0 & 10 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$0.x + y + 4z = 0$

$0.x + 0.y + 10.z = 0$

$$\frac{x}{10 - 0} = \frac{y}{4.0 - 0.10} = \frac{z}{0.0 - 1.0}$$

$$\frac{x}{10} = \frac{y}{0} = \frac{z}{0}$$

$$\frac{x}{1} = \frac{y}{0} = \frac{z}{0} = k$$

$x = 1.k, \quad y = 0.k, \quad z = 0.k$

for $k = 1$

$x = 1, \quad y = 0, \quad z = 0$

$X_2 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ is eigen vector

case 3:- Let $X_3 = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$. be eigen vector for $\lambda = 5$

$$\therefore \begin{bmatrix} 3-5 & 1 & 4 \\ 0 & 2-5 & 6 \\ 0 & 0 & 5-5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -2 & 1 & 4 \\ 0 & -3 & 6 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

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$-2x + y + 4z = 0$

$0x + 3y + 6z = 0$

$$\frac{x}{6+12} = \frac{y}{0+12} = \frac{z}{6-0}$$

$$\frac{x}{18} = \frac{y}{12} = \frac{z}{6}$$

$$\Rightarrow \frac{x}{3} = \frac{y}{2} = \frac{z}{1} = k$$

$x = 3k, \quad y = 2k, \quad z = k$

for $k = 1$

$x = 3, \quad y = 2, \quad z = 1$

$$X_3 = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$$