Q $J_{101\times101} = [a_{ij}]$, $a_{ij} = 1$ then

$|I - J| = ?$

Soln (a) 101

(b) 1

(c) 0

(d) 100

(e) NOTA

Soln ① consider λ is an eigen value of A

$\Rightarrow$ there exist a non-zero vector X such that

$$AX = \lambda X$$

also $IX = X$

for any scalar a_0, a_1 such that

$$(a_1 A + a_0 I)X = (a_1\lambda + a_0)X$$

$\Rightarrow$ $a_1\lambda + a_0$ is an eigen value of $a_1 A + a_0 I$.

② nulity of a square matrix = number of eigen value which is zero.

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$\because \quad \rho(J) = 1$

$\Rightarrow \eta(J) = n - \rho(J)$

$= 101 - 1$

$= 100$

$\Rightarrow$ J has eigen value zero with multiplicity 100

$\because$ Sum of eigen values = trace(J)

$\Rightarrow 0 + 0 + \cdots + 0 (100 \text{ times}) + \lambda = 101$

$\Rightarrow \lambda = 101$

$J \xrightarrow{e.v.} 0, 0, \ldots, 0, 101$

$I \xrightarrow{e.v} 1, 1, \ldots, 1, 1$

$I - J \xrightarrow{e.v} 1, 1, \ldots, 1, -100$

$|I - J| = (1)^{100} \cdot (-100) = -100$

Ques:- $J_{n \times n} = [a_{ij}]$, $a_{ij} = 1$ then

$\left| I - \frac{1}{n} J \right| = ?$

(a) -101
(b) 1
(c) 0
(d) -1

$$J \xrightarrow{e.v.} 0, 0, \ldots, 0, (n-1 \text{ time}), n$$

$$\frac{1}{n} J \longrightarrow 0, 0, \ldots, 0, (n \text{ times}), 1$$

$$I \longrightarrow 1, 1, \ldots, 1, \quad , 1$$

$$I - \frac{1}{n} J \longrightarrow 1, 1, \ldots, 1, 0$$

$$\left| I - \frac{1}{n} J \right| = (1)^{n-1} \cdot 0 = 0$$

Inverse of a matrix using elementary transformation (Gauss - Jordan method) :-

$$\because \quad \underline{A = A \cdot I}$$

Apply same elementary transformation on A of left side and I of right side. Transform the matrix A on left side, to identity matrix. The similar transformation, transfor the identity matrix on right side to A^{-1}.

ex Inverse of $A = \begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & -5 \\ 2 & 5 & 0 \end{bmatrix}$, using Gauss - Jordan method.

sol

$$A = AI$$

$$\begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & -5 \\ 2 & 5 & 0 \end{bmatrix} = A \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_2 \to R_2 + 3R_1, \quad R_3 \to R_3 - 2R_1$$

$$\begin{bmatrix} 1 & 3 & -2 \\ 0 & 9 & -11 \\ 0 & -1 & 4 \end{bmatrix} = A \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$$

$$R_3 \to 9R_3 + R_2$$

$$\begin{bmatrix} 1 & 3 & -2 \\ 0 & 9 & -11 \\ 0 & 0 & 25 \end{bmatrix} = A \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ -15 & 1 & 9 \end{bmatrix}$$

$$R_1 \to R_1 - \frac{1}{3}R_2$$

$$\begin{bmatrix} 1 & 0 & 5/3 \\ 0 & 9 & -11 \\ 0 & 0 & 25 \end{bmatrix} = A \begin{bmatrix} 0 & -1/3 & 0 \\ 3 & 1 & 0 \\ -15 & 1 & 9 \end{bmatrix}$$

$$R_1 \to R_1 - \frac{1}{15}R_3, \quad R_2 \to R_2 + \frac{11}{25}R_3$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 25 \end{bmatrix} = A \begin{bmatrix} 1 & -2/5 & -3/5 \\ -\frac{18}{5} & \frac{36}{25} & \frac{99}{25} \\ -15 & 1 & 9 \end{bmatrix}$$

$$R_2 \to \frac{1}{9}R_2, \quad R_3 \to \frac{1}{25}R_3$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = A \begin{bmatrix} 1 & -\frac{2}{5} & -\frac{3}{5} \\ -\frac{2}{5} & \frac{4}{25} & \frac{11}{25} \\ -\frac{3}{5} & \frac{1}{25} & \frac{9}{25} \end{bmatrix}$$

Q. $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$, Find A^{-1}

using elementary transformation.

$$A^{-1} = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ -4 & 3 & -1 \\ \frac{5}{2} & -\frac{3}{2} & \frac{1}{2} \end{bmatrix}$$