Ans2 (c) $\begin{pmatrix}0\\ .8\end{pmatrix}$ $\begin{pmatrix}-1\\ -1.92\end{pmatrix}$ $\begin{pmatrix}1.92\\ 3.76\end{pmatrix}$ $\begin{pmatrix}-0.2\\ -0.32\end{pmatrix}$ $\begin{pmatrix}-1.064\\ -2.12\end{pmatrix}$ $\begin{pmatrix}.52\\ 1.12\end{pmatrix}$ $\begin{pmatrix}.172\\ .232\end{pmatrix}$ $\begin{pmatrix}.884\\ 1.704\end{pmatrix}$
−5.216 −2.2415 −95.69 2.0827 −4.3108 −14.18 −3.322 −27.05

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$\begin{pmatrix}-.264\\ -0.52\end{pmatrix}$ $\begin{pmatrix}1.128\\ 2.264\end{pmatrix}$
2.8166 −41.53

Roll No. | | | | | | | | | | | |

B. TECH.
EVEN SEMESTER / SUMMER TERM
MAJOR EXAMINATION 2017 - 2018

Optimization Technique

Time: 3 Hrs.
Note: Attempt all questions. Each questions carry equal marks. **Max. Marks: 50**

1. Attempt any five parts of the following: **(5× 2 = 10)**

$x_1 = 2, z = 10$, $x_2 = 0$

(a) Solve the following Linear Programming problem by Simplex method:
$Maximize\ z = 5x + 3y$ subject to $\boxed{x} + y \le 2$, $\boxed{5}x + 2y \le 10$, $3x + 8y \le 12$, $x \ge 0,\ y \ge 0$.
1 or 5 is entering variable

(b) Consider the problem: $Minimize\ z = x + y$ subject to $x + 2y \ge 7$, $4x + y \ge 6$, $x \ge 0$, $y \ge 0$. Solve by dual simplex method. $x_1 = 7/2,\ x_2 = 22/7$ (−2) is 1st entering (−7/2) is 2nd entering variable.

(c) Give the dual of LP problem with proper justifications: $Min\ z = 2x_1 - 3x_2 + 7x_3$ subject to the constraints: $3x_1 + 4x_2 - 5x_3 \ge 5$, $4x_1 - 4x_2 - 8x_3 = 7$, $3x_1 + 7x_2 - 15x_3 \le 5$
$x_1, x_2 \ge 0$ and x_3 is unrestricted. $Max\ z' = -5y_1 + 7(y_2 - y_3) + 5y_4$
$-3y_1 + 4y_2 - 4y_3 - 3y_4 \ge -2$
$-4y_1 - 4y_2 + 4y_3 + 7y_4 \ge 7$

(d) Find the all extreme points of the Optimizing problem:
$$f(x_1, x_2) = x_1^3 - 3x_1x_2^2 + x_2^4 + x_2^2.$$
Also discuss the nature of two non-zero extreme points.
(0,0), (1,1), (1,−1), (−1,−1), (−1, 1)

(e) Find the minimum of the function $f(x) = x^2 - \frac{x^3}{5} - Sin^{-1}(Sin(x))$ in the range $(-1, 3)$ by Fibonacci method with taking n = 4. Also discuss the validity of results.
$x_1 = -1 + 1.6666 = 0.6666$ $f(x_1) = -0.2854$
$x_2 = 1.3333$ ~~= 2.6666 f(x_2) = .2296~~, ~~$x_3 = 4.3333$~~ $f(x_2) = -0.0296$

(f) Discuss the nature of two non – zero extreme points of the problem:
$$f = x_1^3 - 3x_1x_2^2 + 6x_2^3 + x_2^2 + \left(\tfrac{1}{3}\right)x_3^3 - x_2x_3^2 .$$

(g) Find the initial basic feasible solution to the following transportation problem using Vogel's approximation method

							Available
	9	12	9	8	4	3	5
	7	3	6	8	9	4	8
	4	5	6	8	10	14	6
	7	3	5	7	10	9	7
	2	3	8	10	2	4	3
Require	3	4	5	7	6	4	

2. **Attempt any two parts of the following:** (2× 5 = 10)

(a) Minimize $f(x_1, x_2, x_3) = x_1^2 + 3x_2^2 + 5x_3^2 - 2x_1x_2 + 3x_2x_3 - 7x_1 - 8x_2$ with starting point $\begin{Bmatrix}2\\3\\4\end{Bmatrix}$ by Univariate method up to three iterations given that $\varepsilon = 0.01$.

(b) Minimize $f(x_1, x_2) = x_1 - x_2 + 5x_1^2 + 10x_1x_2 + x_2^2$ with starting point $\begin{Bmatrix}-1\\1\end{Bmatrix}$ by Steepest Descent method up to two iterations.

(c) Minimize $f(x_1, x_2) = 15x_1^2 - 18x_1x_2 + \left(\frac{3}{5}\right)x_2^2 - \left(\frac{5}{3}\right)x_1 - 7x_2$ in the range $-2 \le x_1 \le 2$ and $-4 \le x_2 \le 4$ by using random search method up to 10 iterations given the set of values as $\{(r_1, r_2) = (0.50, 0.60), (.25, .26), (.98, .97), (.45, .46), (.234, .235), (.63, .64), (.543, .544), (.712, 0.713), (.434, .435), (.782, .783)\}$.

3. **Attempt any two parts of the following:** (2× 5 = 10)

(a) Minimize $f(x_1, x_2) = x_1^4 - 2x_1^2x_2 + x_1^2 + x_2^2 - 2x_1 + x_2 + 1$ with stating point $\begin{Bmatrix}1.5\\-1.0\end{Bmatrix}$ by Newton's method up to two iterations.

(b) Minimize $f(x_1, x_2) = 4x_1^2 + 3x_2^2 - 5x_1x_2 - 8x_1$ with starting point $\begin{Bmatrix}2\\-2\end{Bmatrix}$ by Univariate method up to two iterations given that $\varepsilon = 0.01$.

(c) Minimize $f(x_1, x_2) = (10x_1 + 6x_2 - 9)^2 + (6x_1 + 10x_2 - 11)^2$ with starting point $\begin{Bmatrix}-1.0\\1.0\end{Bmatrix}$ by Newton's method up to two iterations.

4. **Attempt any two parts of the following:** (2 × 5 = 10)

(a) Minimize $f(X) = x_1x_2x_3^{-3} + 17x_1^2x_2^{-3}x_3 + 34x_1^{-3}x_3 + 51x_1x_2$, $x_i \ge 0$, $i = 1,2,3$ by geometric programming method.

(b) Derive the geometric dual of the problem: Minimize $f(X) = 5x_1x_2x_3 + 2x_1^2x_2^{-2}x_3^{-2} + 5x_1^{-2}x_2^{-3}x_3^{-5} + 7x_1^2x_3^{-4} + 8x_1x_2^{-1/2}$, $x_i \ge 0$, $i = 1,2,3$.

(c) Derive the geometric dual of the problem: Minimize $f(X) = x_1x_2^{-2}x_3^{-1} + 2x_1^{-1}x_2^{-3}x_4 + 10x_1x_3x_4$ subject to $3x_1^{-1}x_3x_4^{-2} + 4x_3x_4 \le 1$,

$$5x_1^{-1}x_2^{-2}x_3 \le 1, \quad x_i \ge 0, \quad i = 1,2.$$

5. **Attempt any two parts of the following:** (2 × 5 = 10)

(a) Minimize $f(X) = x_1^{-2}x_2^{-1} + \frac{1}{4}x_1^2x_2^{-1}x_3^{-1} + x_1^{-1}x_3^2x_4$ subject to $\frac{3}{4}x_1x_2 + \frac{3}{8}x_2x_3x_4^{-3} \le 1$ $x_i \ge 0$, $i = 1,2,3$ by geometric programming method.

(b) Minimize $f(X) = 10x_2x_3x_4^4 + 40x_1^2x_3^{-1} + 5x_2x_3^2$ subject to

$$5x_2^{-5}x_3^{-1} \le 1,$$

$$10x_1^{-1}x_2^3x_4^{-1} \le 1, \quad x_i > 0, \quad i = 1 \text{ to } 4$$

by geometric programming method.

(c) Minimize x_1 subject to $-4x_1^2 + 7x_2 \le 1$ $\quad x_1 + x_2 \ge 1$ and $x_1 > 0, x_2 > 0$

by procedure of complementary geometric programming method.

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Roll No. | | | | | | | | | | | |

Name of the Course: B. Tech-II year
Odd Semester/Summer Term
Minor Examination: 2017-18
Subject Name: Optimization Techniques

Time: 2 hrs. **Max. Marks: 30**
Note: Answer all questions.

Q.1 Attempt any three parts of the following. Q. 1(a) is compulsory.

(a) Solve the following Linear Programming Problem by decomposition principle 4

Maximize $f = 7x - 9y + 5z + 8w$ subject to

$$5x + 2y + 5z + 11w \le 20,$$
$$9y + 13z + 15w \le 14,$$
$$5z + 2w \le 10,$$
$$w \ge 1,$$
$$3z + 5w \le 15,$$
$$6x + 5y \le 30,$$
$$y \ge 5, \quad and \quad x, y, w, z \ge 0.$$

(b) Determine the basic feasible solution by Vogel's method of the transportation problem: 3

	A	B	C	D	Available
I	6	1	9	3	70
II	11	5	2	8	55
III	10	12	4	7	90
Required	85	35	50	45	

Also find the optimum basic feasible solution of the above transportation problem.

(c) Find the all extreme points of the Optimizing problem: 3

$$f(x_1, x_2) = x_1^3 - 3x_1x_2^2 + x_2^4 + x_2^2.$$

Also discuss the nature of two non-zero extreme points.

(d) Discuss the nature of two non – zero extreme points of the problem: 3

$$f = x_1^3 - 3x_1x_2^2 + 6x_2^3 + x_2^2 + \left(\frac{1}{3}\right)x_3^3 - x_2x_3^2.$$

Q.2 Attempt any three parts of the following. Q. 2(a) is compulsory.

(a) Minimize $z = 4x_1 + 2x_2$ by dual Simplex method 4

subject to constraints

$$x_1 + x_2 = 1,$$
$$3x_1 - x_2 \ge 2,$$
$$x_1, x_2 \ge 0.$$

(b) $Maximize\ z = x + 2y + 2z$
subject to constraints

$$5x + 2y + 3z \leq 15,$$
$$x + 4y + 2z \leq 12,$$
$$2x + z \leq 8,$$
$$x \geq 0, y \geq 0, z \geq 0.$$

3

(c) Obtain the dual of the problem

$$\text{Min } z = x_1 - 3x_2 + 7x_3$$

subject to the constraints

$$x_1 - 3x_2 + 4x_3 = 5,$$
$$x_1 - 2x_2 \leq 3,$$
$$x_1 - 2x_2 - x_3 \geq 4$$

$x_1, x_3 \geq 0$ and x_2 is unrestricted.

3

(d) Solve by Karmarkar's method to the following L. P. problem:

$$Minimize\ f = 3x_1 + 11x_2 - 13x_3$$

Subject to

$$3x_1 - 7x_3 = 0,$$
$$x_1 + x_2 + x_3 = 1$$

$x_i \geq 0,\ i = 1,2,3.$

Use the value of $\varepsilon = 0.05$ for testing the convergence of the procedure.

Q.3 Attempt any three parts of the following. Q. 3(a) is compulsory.

(a) Maximize $f(x_1, x_2, x_3) = 32x_1 + 50x_2 - 10x_2^2 + x_2^3 - x_1^4 - x_2^4$ 4
subject to,

$$3x_1 + x_2 \leq 11,$$
$$2x_1 + 5x_2 \leq 16,$$
$$x_1 \geq 0, x_2 \geq 0.$$

by applying Kuhn –Tucker conditions. Also discuss all cases.

(b) Find the minimum of the function $f(x) = x^2 - \frac{x^3}{5} - Sin^{-1}(Sin(x))$ in the range $(-1,3)$ 3
by Fibonacci method with taking n = 6. Also discuss the validity of results.

(c) Find the minimum of the function $0.5 - \frac{0.75}{1+x^2} - 0.65x\ \tan^{-1}(\frac{1}{x^2})$ in the range (-1, 4) by 3
Golden section method (n = 7).

(d) Minimize the objective function 3
$f(x_1, x_2, x_3) = 2x_1 - 3x_2 + 4x_3 + 4x_1^2 + 5x_2^2 - 6x_3^2 + 2x_1x_3 + 4x_2x_3 - 6x_1x_2$
subject to condition

$$2x_1 - 3x_2 + 4x_3 = 10$$

by Lagrange method. Also discuss the sufficient conditions.

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Roll No. | | | | | | | | | | | |

B. Tech.

EVEN SEMESTER

MAJOR EXAMINATION 2017 - 2018

Optimization Technique

Time: 3 Hrs.

Max. Marks: 50

Note: Attempt all questions. Each questions carry equal marks.

1. **Attempt any five parts of the following:** (5× 2 = 10)

$z=10$, $x_1=2$, $x_2=0$

(a) Solve the following Linear Programming problem by Simplex method: *Maximize* $z = 5x + 3y$ subject to $x + y \le 2,\ 5x + 2y \le 10,\ 3x + 8y \le 12,\ x \ge 0,\ y \ge 0.$

1 or 5 is entering variable

(b) Consider the problem: *Minimize* $z = x + y$ subject to $x + 2y \ge 7,\ 4x + y \ge 6,\ x \ge 0,\ y \ge 0.$ Solve by dual simplex method. $x_1 = 7/2$, $x_2 = 22/7$; 2 in 1st it., 7/2 in second it.

y_2' is unrestricted; $5y_1 - 8y_2' - 6y_4 = 5$

(c) *Give the dual of LP problem with proper justifications: Min* $z = 3x_1 - 4x_2 + 5x_3$ *subject to the constraints:*

$$3x_1 + 4x_2 + 5x_3 \ge 3, \quad 4x_1 + 2x_2 + 8x_3 = 3, \quad x_1 + 4x_2 + 6x_3 \le 5.$$

$x_1, x_2 \ge 0$ *and* x_3 *is unrestricted.*

Min $z' = -3y_1 + 3y_2' + 5y_4$; $-3y_1 + 4y_2' + y_4 \ge -3$; $-4y_1 + 2y_2' + 4y_4 \ge 4$

$(\frac{2}{3}, 2, 2\sqrt{2}), (\frac{2}{3}, -2, 2\sqrt{2}), (-\frac{2}{3}, 2, 2\sqrt{2}), (-\frac{2}{3}, -2, 2\sqrt{2})$, $+4 = 8$

(d) Find the all extreme points of the function $(x_1, x_2, x_3) = 3x_1^3 - x_2^3 + x_3^3 - 4x_1 + 12x_2 - 24x_3$. Let the point $(\frac{2}{3}, -2, 2\sqrt{2})$ is the extreme point of the function, show the nature of this extreme point $(\frac{2}{3}, -2, 2\sqrt{2})$.

$$\begin{bmatrix} 18x_1 & 0 & 0 \\ 0 & -6x_2 & 0 \\ 0 & 0 & 6x_3 \end{bmatrix} = \begin{bmatrix} 12 & 0 & 0 \\ 0 & 12 & 0 \\ 0 & 0 & 12\sqrt{2} \end{bmatrix}, \quad D_1 = 12,\ D_2 = 144,\ D_3 = 9$$ Point is minimum.

$x_2 = 2.5625$, $f(x_2) = -110.024$, $x_3 = x_1 + 3.5 - x_2 = 2.875$, $f(x_3) = -50.696$

(e) Find the minimum of the function $f(x) = 10x^5 - 40x^4 + 30x^3 + 5$ by Fibonacci method in the given interval (1, 3.5) up to fifth iteration. Also test the accuracy. $n = 5$, $L_2^* = .9375$, $x_1 = 1.9375$, $f(x_1) = -67.4478$

(f) Discuss the nature of all extreme points of the function $f(x_1, x_2) = \left(\frac{1}{3}\right)x_1^3 + \left(\frac{5}{2}\right)x_1^2 - 6x_1 + x_2^2 - 2x_2$.

$$\begin{bmatrix} 2x_1 + 5 & 0 \\ 0 & 2 \end{bmatrix}$$ $f_{x_1} = x_1^2 + 5x_1 - 6$, $f_{x_2} = 2x_2 - 2$, $(1,1), (-6,1)$ | $(1,1)$, 7, 14, minima. $(-6,1)$, -7, -14, maxima

(g) Find the initial basic feasible solution to the following transportation problem using Vogel's approximation method 141,

$x_4 = x_1 + x_3 - x_2 = 2.75$, $f(x_4) = \cancel{-101.78}\ -85.9863$

						Available	Row min
9	12	9	8	4	3	5	1
7	3	6	8	9	4	8	1
4	5	6	8	10	14	6	1
7	3	5	7	10	9	7	2
2	3	8	10	2	4	3	1
Require 3	4	5	7	6	4		
2	2	1	1	2	1		

$f(X_2 - d_2 S_2) = 28.75 - 2d_2 + 3d_2^2$

$d_2 = 1/3$, $X_3 = \begin{bmatrix} 11/2 \\ 5/3 \\ 3 \end{bmatrix}$, $f(X_3) = 28.4167$, $f_3^- = 28.0672$, $f_3^+ = 28.7672$

$-S_3$ is the correct direction

$f(X_3 - d_3 S_3) = 28.41 - 3.5 d_3 + 5 d_3^2$

2. **Attempt any two parts of the following:** (2× 5 = 10)

$d_3 = 7/2$, $X_4 = (11/2, 5/3, -1/2)$

(a) Minimize $f(x_1, x_2, x_3) = x_1^2 + 3x_2^2 + 5x_3^2 - 2x_1x_2 + 3x_2x_3 - 7x_1 - 8x_2$ with starting point $\begin{Bmatrix} 1 \\ 2 \\ 3 \end{Bmatrix}$ by

$2 + 1.5 + 1.5$

$X_1 = \{1, 2, 3\}$, $f(X_1) = 49$, $f^+ = 48.9101$, $f^- = 49.0901$ | $f^+ < f$ → S_1 is the correct direction; $f_2^+ = 28.7703$, $f_2 = 28.75$, $f_2^- = 28.7303$, $-S_2$ is direction

$f(X_1 + d_1 S_1) = d_1^2 - 9d_1 + 49 \Rightarrow d_1 = +9/2 \Rightarrow X_2 = \left(\frac{11}{2}, 2, 3\right)$

3(a) To Check X_2 is optimum, $g_2 = \{-6.0596, -0.02\}$

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Univariate method up to three iterations given that $\varepsilon = 0.01$.

$\nabla f = \begin{Bmatrix} 1+4x_1+2x_2 \\ -1+2x_1+2x_2 \end{Bmatrix}$, $\nabla f_{(0,1)} = \begin{Bmatrix} 3 \\ 1 \end{Bmatrix}$, $S_1 = -\nabla f_1 = \begin{Bmatrix} -3 \\ -1 \end{Bmatrix}$, $f(X_1+\lambda_1 S_1) = -10\lambda_1 + 25\lambda_1^2$, $\lambda_1 =$

$X_2 = \begin{Bmatrix} -3/5 \\ 4/5 \end{Bmatrix} = \begin{Bmatrix} -0.6 \\ 0.8 \end{Bmatrix}$

(b) Minimize $f(x_1, x_2) = x_1 - x_2 + 2x_1^2 + 2x_1x_2 + x_2^2$ with starting point $\begin{Bmatrix} 0 \\ 1 \end{Bmatrix}$ by Steepest Descent method up to two iterations.

$f(X_2 + \lambda_2 S_2) = f\left(\frac{-3-\lambda_1}{5}, \frac{4+3\lambda_1}{5}\right) = \frac{1}{5}(-5 - 2\lambda_1 + \lambda_1^2)$, $\lambda_1 = 1$

$S_2 = -\nabla f_2 = \begin{bmatrix} -1/5 \\ 3/5 \end{bmatrix}$

$\lambda_1 + r_1(\lambda_1 - \lambda_1) = -5 + r_1(10) =$ $f(-4/5, 7/5)$

$-10 + r_2(20) =$

Please give the value of r_1, r_2

(c) Minimize $f(x_1, x_2) = 12x_1^2 - 8x_1x_2 + \left(\frac{1}{5}\right)x_2^2 - \left(\frac{1}{2}\right)x_1 - 2x_2$ in the range $-5 \le x_1 \le 5$ and $-10 \le x_2 \le 10$ by using random search method up to 10 iteration.

3. Attempt any two parts of the following: (2× 5 = 10)

$X_2 = X_1 - J_1^{-1} g_1 = \begin{Bmatrix} -\frac{599}{300} = -1.99003, 3.96013 \end{Bmatrix}$

$g_1 = \nabla f_1 = \begin{Bmatrix} 2(-1+x_1+200x_1^3 - 200x_1x_2) \\ 200(x_1^2 - x_2) \end{Bmatrix}_{(-2,0)} = \{-1206, -300\}$

(a) Minimize $f(x_1, x_2) = 100(x_1^2 - x_2)^2 + (1 - x_1)^2$ with starting point $\begin{Bmatrix} -2.0 \\ 2.5 \end{Bmatrix}$ by Newton's method up to two iterations.

$J_1^{-1} = \begin{bmatrix} 1.66\times10^{-3} & -6.64\times10^{-3} \\ -6.644\times10^{-3} & 3.15\times10^{-2} \end{bmatrix}$, $J_1 = \begin{bmatrix} 2+1200x_1^2 - 400x_2 & -400x_1 \\ -400x_1 & 200 \end{bmatrix} = \begin{bmatrix} 3802 & 800 \\ 800 & 200 \end{bmatrix}$, $J_1^{-1} = \begin{bmatrix} \frac{1}{602} & -\frac{2}{301} \\ -2/301 & 1901/60200 \end{bmatrix}$

$X_3 = (-1/4, -5/24)$, $\lambda_2 = 43/24 = 1.7917$

$f(x_1) = 32$

(b) Minimize $f(x_1, x_2) = 4x_1^2 + 3x_2^2 - 5x_1x_2 - 8x_1$ with starting point $\begin{Bmatrix} 2 \\ -2 \end{Bmatrix}$ by Univariate method up to two iterations given that $\varepsilon = 0.01$.

$f^+ = \begin{pmatrix} 2.01 \\ -2.0 \end{pmatrix} = 32.1804 \Rightarrow -S_1$ is the direction

$f_2^+ = 11.0428$, $f_2^- = 11.8578$, $f_2^+ < f_2$, S_2 is the direct

$f^- = 31.8284$, $f^- < f_1$, $f(X_1 - \lambda_1 S_1) = 4\lambda_1^2 - 18\lambda_1 + 32 = (9/4 = \lambda_1)$ $X_2 = \begin{pmatrix} -1/4 \\ -2 \end{pmatrix}$, $f(X_2) = \frac{47}{4} = 11.75$

(c) Minimize $f(x_1, x_2) = (x_1 + 2x_2 - 7)^2 + (2x_1 + x_2 - 5)^2$ with starting point $\begin{Bmatrix} -1.5 \\ 1.5 \end{Bmatrix}$ by Newton's method up to two iterations.

One iteration Problem

$J_1 = \begin{bmatrix} 10 & 8 \\ 8 & 10 \end{bmatrix}$, $g_1 = \nabla f_1 = \begin{Bmatrix} -34 + 10x_1 + 8x_2 \\ -38 + 8x_1 + 10x_2 \end{Bmatrix}_{(-1,5)} = \begin{Bmatrix} -37 \\ -35 \end{Bmatrix}$

$J_1^{-1} = \begin{bmatrix} 10 & -8 \\ -8 & 10 \end{bmatrix} \frac{1}{36}$, $X_2 = \{1, 3\}$

$f = 10.1941$

4. Attempt any two parts of the following: (2 ×5 = 10)

$x_1 = 1.4563$

(a) Minimize $f(X) = x_1x_2x_3^{-2} + 2x_1^{-1}x_2^{-1}x_3 + 5x_2 + 3x_1x_2^{-2}$, $x_i \ge 0, i = 1,2,3$ by geometric programming method.

$\Delta_1 - \Delta_2 + \Delta_4 = 0$, $-2\Delta_1 + \Delta_2 = 0$, $\left(\frac{1}{7}, \frac{2}{7}, \frac{3}{7}, \frac{1}{7}\right)$

2nd $= 1.4563$, $\Delta_1 - \Delta_2 + \Delta_3 - 2\Delta_4 = 0$,

$x_2 = 0.8738$

(b) Derive the geometric dual of the problem: $f(X) = 10x_1x_2x_3 + 20x_1^5x_2^2x_3^{-1} + 5x_1^{-1}x_2^{-3}x_3^{-5} + 7x_1^2x_3^{-4} + 8x_1x_2^{-2}$, $x_i \ge 0, i = 1,2,3$.

$\Delta_1 + 5\Delta_2 - \Delta_3 + 2\Delta_4 + \Delta_5 = 0$, $\Delta_1 + 2\Delta_2 - 3\Delta_3$

4rd $= .4854$, $x_1 = 0.3705$, $x_3 = .6477$

$\Delta_1 - \Delta_2 - 5\Delta_3 - 4\Delta_4 = 0$, $\Delta_1 + \Delta_2 + \Delta_3 + \Delta_4 + \Delta_5 = 1$

(c) ~~Derive the geometric dual of the problem:~~ $f(X) = 20x_2x_3x_4^4 + 20x_1^2x_3^{-1} + 5x_2x_3^2$ subject to

$x_3 = .4752$ $\lambda_{01} = \frac{1}{5}$, $\lambda_{02} = \frac{2}{5}$, $\lambda_{03} = \frac{2}{5}$

$$5x_2^{-5}x_3^{-1} \le 1$$

$$10x_1^{-1}x_2^3x_4^{-1} \le 1$$

$$x_i > 0, \quad i = 1 \text{ to } 4.$$

$2\lambda_{02} - \lambda_{22} = 0$; $\lambda_{01} + \lambda_{02} + \lambda_{03} = 1$

$\lambda_{11} = \frac{3}{5}$, $\lambda_{22} = \frac{4}{5}$

$\lambda_{01} + \lambda_{03} - 5\lambda_{11} + 3\lambda_{22} = 0$

$= (100)^{1/5}(50)^{2/5}(12.5)^{2/5}(5)^{3/5}(10)^{4/5}$

$\lambda_{01} - \lambda_{02} + 2\lambda_{03} - \lambda_{11} = 0$

$4\lambda_{01} - \lambda_{22} = 0$

5. Attempt any two parts of the following: (2 ×5 = 10)

(a) Minimize $f(X) = 2x_1x_2 + 2x_1x_2^{-1}x_3 + 4x_1^{-1}x_2^2x_3^{-1/2}$ subject to $\sqrt{3}\, x_2^{-1} + 3x_1^{-1}x_3^{-1/2} \le 1$ and $x_i \ge 0$, $i = 1,2,3$ by geometric programmin g method.

$f^* = (2)^{1/2}(4)^{1/2}(-10)^{-1/2}(20)^{1/2} = (160)^{1/2}/(-10)^{1/2} \Rightarrow \sqrt{(-10)}$ cannot calculate.

(b) Minimize $f(X) = x_1x_2 + 2x_1^{-1}x_3 + 5x_3 + 10x_2^{-1}$, $x_i \ge 0, i = 1,2,3$ by geometric programming method.

$\Delta_1 - \Delta_2 = 0$, $\Delta_1 - \Delta_4 = 0$, $\Delta_3 + \Delta_4 = 0$, $\Delta_1 + \Delta_2 + \Delta_3 + \Delta_4 = 1$ $x_1^{(1)} = \{1, 1\}$

$\Delta_1 = \Delta_2 = \Delta_4 = 1/2$, $\Delta_3 = -1/2$

(c) Minimize x_1 subject to

$Q_1(X) = 1 + x_1^2$, $Q_1(X, \underset{\sim}{X}) = (1 + x_1^2)\left(\frac{x_1}{x_1}\right)^{2x_1^2/(1+x_1^2)}$ $Q_1(X, X^{(1)}) = 2x_1$

$$-x_1^2 + 4x_2 \ge 1$$

$Q_2(X) = 1 + x_1^{-1}x_2$

$$x_1 + x_2 \ge 1 \quad \textit{and} \quad x_1 > 0, x_2 > 0.$$

$Q_2(X, X^{(1)}) = 2x_1^{-1/2}x_2^{1/2}$

Now Problem as minimize x_1 subject to

by procedure of complementary geometric programming method.

$Q_2(X, \underset{\sim}{X}) = \left(1 + \frac{x_2}{x_1}\right)\left(\frac{x_1}{x_1}\right)^{-x_2/(x_1+x_2)}\left(\frac{x_2}{x_1}\right)^{x_2/(x_1+x_2)}$ $2x_1^{-2}x_2 \le 1$

$\frac{1}{2}x_1^{-1/2}x_2^{-1/2} \le 1$