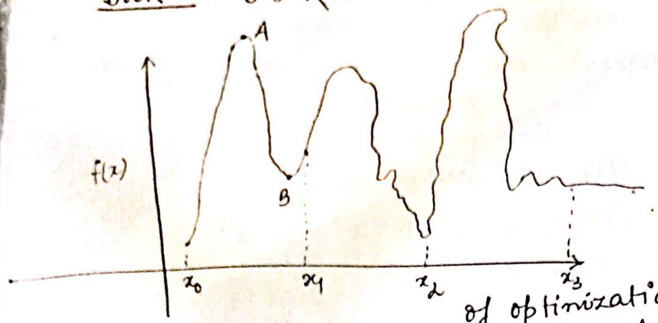
Optimization Techniques (BAS-26)

Book - S.S Rao



$[x_0, x_1] \Rightarrow$ ~~A and B~~ points of optimization known as local maxima or local minima.

$[x_0, \infty) \Rightarrow$ points of optimization known as global maxima or global minima.

A Function of one variable is said to have a relative or local minima at a point $x = x^*$ if $f(x^*) \leq f(x^*+h)$ for all sufficiently positive and negative values of h. Similarly a point $x = x^*$ is a relative or local maxima if $f(x^*) \geq f(x^*+h)$ for all values of h ~~suff~~ sufficiently close to '0'.

Similiarly a function $f(x)$ is said to have a global or absolute minimum at a point x^* if $f(x^*) \leq f(x)$ for all x in the domain over which $f(x)$ is define. Similarly a point x^* will be global maxima of $f(x)$ if $f(x^*) \geq f(x)$ for all values of x in the ~~of~~ domain.

$f(x) = x^3 + 3x + 5$

One Variable Optimization

<u>Qu^n</u>: $f(x) = 12x^5 - 45x^4 + 40x^3 + 5$

Determine the maximum value of the function

<u>Sol^n</u>:

$$f(x) = 12x^5 - 45x^4 + 40x^3 + 5$$
$$f'(x) = 60x^4 - 180x^3 + 120x^2$$

put $f'(x) = 0$

$$x^4 - 3x^3 + 2x^2 = 0$$
$$x^2(x^2 - 3x + 2) = 0$$
$$x = 0, 1, 2$$

$$f''(x) = 240x^3 - 540x^2 + 240x.$$

$f''(0) = 0$

$f''(1) = 240 - 540 + 240 = -60$ <u>(-)ve</u>.

$f''(2) = 240 \times 8 - 540 \times 4 + 240 \times 2.$

$= 1920 - 2160 + 480$

$= 2400 - 2160$

$= 240$ <u>(+)ve</u>

Maximum value of function.

So $f(1) = 12 - 45 + 40 + 5$

$= 12.$ <u>Ans</u>.

$f'''(x) = 720x^2 - 1080x + 240$

$f'''(0) = 240$ (+)ve.

<u>Extreme Points</u>:- If $f(x)$ has an extreme point (maxima/minima) at $x = x^*$ if first partial derivative of $f(x)$ exist at x^* and then

$$\frac{\partial}{\partial x_1}(x^*) = 0$$

$$\frac{\partial (x^*)}{\partial x_1} = 0$$

$$\frac{\partial (x^*)}{\partial x_n} = 0$$

here $x = \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_n \end{Bmatrix}$

$x^* = \begin{Bmatrix} x_1^* \\ x_2^* \\ \vdots \\ x_n^* \end{Bmatrix}$

$f(x) = f(x_1, x_2, \ldots\ldots, x_n) = x_1^2 + x_2^2 + x_3^2 + \ldots + 2x_1x_2 + \ldots + x_n^2$

Sufficient Condition : A sufficient condition ~~is~~ for a stationary point $x = x^*$ to be an extreme point is that the matrix of second order partial derivative (Hessian Matrix) of $f(x)$ evaluated at x^* is that

(i) Positive definite when x^* is a relative minimum point.

(ii) Negative definite when x^* is a relative maximum point.

$f(x) = f(x_1, x_2, x_3)$

$$H(x^*) = \begin{bmatrix} \frac{\partial^2 f}{\partial x_1^2} & \frac{\partial^2 f}{\partial x_1 \partial x_2} & \frac{\partial^2 f}{\partial x_1 x_3} \\ \frac{\partial^2 f}{\partial x_2 \partial x_1} & \frac{\partial^2 f}{\partial x_2^2} & \frac{\partial^2 f}{\partial x_2 \partial x_3} \\ \frac{\partial^2 f}{\partial x_3 \partial x_1} & \frac{\partial^2 f}{\partial x_3 \partial x_2} & \frac{\partial^2 f}{\partial x_3^2} \end{bmatrix}$$

(1) $H(x^*)$ is (+)ve definite means all eigen values should be (+)ve or all minor of $H(x^*)$ should be positive.

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Ques : Find the maxima or minima of the function
$f(x) = f(x_1, x_2, x_3) = x_1^2 + x_2^2 + x_3^2 - 4x_1 - 8x_2 - 12x_3 + 5($

Soln :

$$\frac{f(x)}{\partial x_1} = 2x_1 - 4 = 0 \qquad x_1 = 2$$

$$\frac{f(x)}{\partial x_2} = 2x_2 - 8 = 0 \qquad x_2 = 4$$

$$\frac{f(x)}{\partial x_3} = 2x_3 - 12 = 0 \qquad x_3 = 6$$

extreme points $X^* = (2, 4, 6)$

$$H(X^*) = \begin{vmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{vmatrix}$$

$\Rightarrow$ $D_1 = 2 > 0$ of order 1

$D_2 = 4 > 0$ " " 2

$D_3 = 8 > 0$ " " 3

all (+)ve so Hessian matrix is (+)ve ~~pos~~ definite

then $x^* = (2, 4, 6)$ is minimum points.

$f(x^*,$

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(Constraint Optimizing Problem)
with Equality Constraints

g_{11}

Lagrange Method :

suppose we want to maximize/ g_{12}

minimize the function $f(x)$ subject to the constraint $g_i(x)$

After

then poly

Lagrange Multiplier / Function of

$F(x, \lambda) = f(x) - \lambda g_i(x)$ po

then find $\frac{\partial F}{\partial x} = 0$, $\frac{\partial F}{\partial \lambda} = 0$ poly we get extremepoints (x^*, λ^*)

After that, we have Hessian Matrix

$$H(x^*, \lambda^*) = \begin{bmatrix} \frac{\partial^2 f}{\partial x_1^2} - z & \frac{\partial^2 f}{\partial x_1 \partial x_2} & \frac{\partial^2 f}{\partial x_1 \partial x_3} \\ \frac{\partial^2 f}{\partial x_2 \partial x_1} & \frac{\partial^2 f}{\partial x_2^2} - z & \frac{\partial^2 f}{\partial x_2 \partial x_3} \\ \frac{\partial^2 f}{\partial x_3 \partial x_1} & \frac{\partial^2 f}{\partial x_3 \partial x_2} & \frac{\partial^2 f}{\partial x_3^2} - z \end{bmatrix}$$

$f(x_1, x_2, x_3)$.

$g_1(x) = 0$

$g_i(x) = 0$

$x_1 + x_2 = 10$

$x_1 + x_2 \geq 0$

$x_1 + x_2 \leq 0$

$$H(x^*, \lambda^*) = \begin{bmatrix} \frac{\partial^2 f}{\partial x_1^2} - z & \frac{\partial^2 f}{\partial x_1 \partial x_2} & \frac{\partial^2 f}{\partial x_1 \partial x_3} & \frac{\partial g_1}{\partial x_1} & \frac{\partial g_2}{\partial x_1} & \cancel{\frac{\partial g_1}{\partial x_3}} \\ \frac{\partial^2 f}{\partial x_2 \partial x_1} & \frac{\partial^2 f}{\partial x_2^2} - z & \frac{\partial^2 f}{\partial x_2 \partial x_3} & \frac{\partial g_1}{\partial x_2} & \frac{\partial g_2}{\partial x_2} & \cancel{\frac{\partial g_2}{\partial x_3}} \\ \frac{\partial^2 f}{\partial x_3 \partial x_1} & \frac{\partial^2 f}{\partial x_3 \partial x_2} & \frac{\partial^2 f}{\partial x_3^3} - z & \frac{\partial g_1}{\partial x_3} & \frac{\partial g_2}{\partial x_3} & \phi \\ g_{11} \rightarrow & g_{12} \rightarrow & g_{13} & 0 & 0 & \phi \\ g_{21} & g_{22} & g_{23} & 0 & 0 & \phi \end{bmatrix}$$

$g_{11} = \frac{\partial g_1}{\partial x_1}$

$g_{12} = \frac{\partial g_1}{\partial x_2}$

After solving the Hessian matrix, we find the polynomial expression in term of z. On the basis of roots of polynomial, we can say that extreme points has maxima or minima if all roots of polynomial are (+) ve, we say (x^*, λ^*) is minimum point.

Ques: optimize the problem

$Z = 4x_1^2 + 2x_2^2 + x_3^2 - 4x_1x_2$ subject to the $x_1 + x_2 + x_3 = 15$

Sol^n:

$$f(x_1, x_2, x_3, \lambda) = 4x_1^2 + 2x_2^2 + x_3^2 - 4x_1x_2 - \lambda(x_1 + x_2 + x_3 - 15)$$

$$\frac{\partial f}{\partial x_1} = 8x_1 - 4x_2 - \lambda = 0 \qquad \Rightarrow x_1 = \frac{2\lambda}{4} = \frac{\lambda}{2}$$

$$\frac{\partial f}{\partial x_2} = 4x_2 - 4x_1 - \lambda = 0$$

$$\frac{\partial f}{\partial x_3} = 2x_3 - \lambda = 0 \qquad \Rightarrow x_3 = \frac{\lambda}{2}$$

$$\frac{\partial f}{\partial \lambda} = -(x_1 + x_2 + x_3 - 15) = 0 \qquad \Rightarrow x_2 = \frac{3\lambda}{4}$$

$$x_1 + x_2 + x_3 = 15$$

$$\frac{\lambda}{2} + \frac{3\lambda}{4} + \frac{\lambda}{2} = 15$$

$$2\lambda + 3\lambda + 2\lambda = 60$$

$$\lambda = \frac{60}{7}$$

$$\Rightarrow x_1 = \frac{30}{7}, \quad x_2 = \frac{45}{7}, \quad x_3 = \frac{30}{7}$$

then extreme point $\left(\frac{30}{7}, \frac{45}{7}, \frac{30}{7}, \frac{60}{7}\right)$.

$$\boxed{g = x_1 + x_2 + x_3 - 15}$$

$$H(x^*, \lambda^*) = \begin{bmatrix} 8-z & -4 & 0 & 1 \\ -4 & 4-z & 0 & 1 \\ 0 & 0 & 2-z & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$

$$= (8-z)\begin{vmatrix} \cancel{-4} & \cancel{0} & 1 \\ 4-z & 0 & 1 \end{vmatrix}$$

$$= (8-z)\begin{vmatrix} 4-z & 0 & 1 \\ 0 & 2-z & 1 \\ 1 & 1 & 0 \end{vmatrix} - (-4)\begin{vmatrix} -4 & 0 & 1 \\ 0 & 2-z & 1 \\ 1 & 1 & 0 \end{vmatrix} - 1\begin{vmatrix} -4 & 4-z & 0 \\ 0 & 0 & 2-z \\ 1 & 1 & 1 \end{vmatrix}$$

$$(8-z)\{(z-4)+(z-2)\} + 4\{4+(z-2)\} - \{-4(z-2)-(4-z)(z-2$$

$$= (8-z)(2z-6) + 4(z+2) - \{-4z+8-4z+8+z^2-2z\}$$

$$= 16z - 48 - 2z^2 + 6z + 4z + 8 + 10z - z^2 - 16$$

$$= -3z^2 + 36z - 56$$

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$$z = \frac{-36 \pm \sqrt{36^2 - 4\times 3\times 56}}{-6}$$

$$z = \frac{-36 \pm 24.98}{-6}$$

$$z = 1.83667,\ 10.1633$$

here both root of z are (+)ve

$\Rightarrow$ extreme point of this problem has minimum at these point.

Ques: $f(x_1,x_2,x_3) = 3x_1^2 - 2x_2^2 + 6x_3^2 + 6x_1x_2 - 4x_2x_3 - 4x_1 + 12x$
$-12x_3 + 10$, subject to

$$3x_1 + 2x_2 - 4x_3 = 6$$

Soln: hence Lagrange's function:

$$F(x,\lambda) = 3x_1^2 - 2x_2^2 + 6x_3^2 + 6x_1x_2 - 4x_2x_3 - 4x_1 + 12x_2 - 12x_3 +$$
$$-\lambda(3x_1 + 2x_2 - 4x_3 - 6$$

$$\frac{\partial F}{\partial x_1} = 6x_1 + 6x_2 - 4 - 3\lambda = 0$$

$$\frac{\partial F}{\quad} = -4x_2 + 6x_1 - 4x_3 + 12 \ldots$$

$$\frac{\partial F}{\partial x_3} = 12x_3 - 4x_2 - 12 + 4\lambda = 0$$

$$\frac{\partial F}{\partial \lambda} = -(3x_1 + 2x_2 - 4x_3 - 6) = 0$$

$$3x_1 + 2x_2 - 4x_3 = 6$$

from (I) & (II)

$$10x_2 + 4x_3 - 16 - \lambda = 0$$

$$\lambda = 10x_2 + 4x_3 - 16$$

$$12x_3 - 4x_2 - 12 + 40x_2 + 16x_3 - 64 = 0$$

$$36x_2 + 28x_3 - 76 = 0$$

$$18x_2 + 14x_3 - 38 = 0$$

$$\boxed{9x_2 + 7x_3 - 19 = 0}$$

$$6x_1 + 6x_2 - 3\lambda = 4$$

$$6x_1 + 4x_2 - 8x_3 = 12 \quad \text{III}$$

$$- \quad + \quad -$$

$$2x_2 + 8x_3$$

$$-3(10x_2 + 4x_3 - 16) + 8 = 0$$

$$-28x_2 - 4x_3 + 56 = 0$$

$$\boxed{7x_2 + x_3 - 14 = 0}$$

$$7x_2 + x_3 - 14 = 0$$

$$49x_2 + 7x_3 - 128 = 0$$

$$9x_2 + 7x_3 - 19 = 0$$

$$- \quad - \quad +$$

$$40x_2 - 109 = 0$$

$$x_2 = \frac{109}{4}$$

$$x_3 = \frac{-905}{28}$$

$$7x_3 = 19 - \frac{9 \times 109}{4}$$

$$7x_3 = \frac{76 - 981}{4}$$

$$7x_3 = -\frac{905}{4}$$

$$x_3 = -\frac{905}{28}$$

$$\lambda = \frac{1090}{4} - \frac{905}{7} - 16$$

$$= \frac{1090 \times 7 - 905 \times 4 - 16 \times 28}{28}$$

$$= \frac{3562}{28} = \frac{1781}{14}$$

$6x_1 = 3\lambda + 4 - 6x_2$

$= \frac{3\times1781}{14} + 4 - 6\times\frac{109}{4}$

$6x_1 = \frac{6\times1781 + 4\times28 - 42\times109}{28}$

$x_1 = \frac{6220}{6\times28} = \frac{3110}{3\times28} = \frac{1555}{42}$

$x_1 = \frac{1555}{42}$

$x_2 = \frac{109}{4}$

$x_3 = -\frac{905}{28}$

$\lambda = \frac{1781}{14}$

$\frac{\cancel{3}\times1555}{\cancel{42}\,14} + \frac{\cancel{2}\times109}{\cancel{4}\,\cancel{2}}\times7 + \frac{\cancel{4}\times905}{\cancel{28}\,\cancel{7}}\times2$

$=$

$3\lambda = 4$

$x_3 = 12$

3

$(x_3 - 16) + 8$

$= 0$

$= 0$

0

1

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Kuen ~~Koch~~ Tucker Condition
(optimizing problem with enequality constraints)

suppose we want to minimize $f(x)$ subject to constraints $g_j(x) \le 0$, $j = 1$ to m then K.T. conditions can be states that

$L = L + \lambda_1 g_1 + \lambda_2 g_2 + \cdots + \lambda_j g_j$ and $x = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}$

then

$\frac{\partial L}{\partial x_1} = \frac{\partial f}{\partial x_i} + \lambda_1 \frac{\partial g_1}{\partial x_i} + \lambda_2 \frac{\partial g_2}{\partial x_i} + \cdots + \lambda_i \frac{\partial g_j}{\partial x_i}$

then we can write

$\frac{\partial f}{\partial x_i} + \sum_{j=1}^{m} \lambda_j \frac{\partial g_j}{\partial x_i} = 0$, $i = 1$ to n — (I)

$\lambda_j g_j = 0$ — (II)

$g_i \le 0$ — (III)

$\lambda_j \ge 0$ — (IV)

(i) mini $f(x)$ with $g_j(x) \geq 0$

$\lambda_j g_j = 0$ -

$g_j \geq 0$

$\lambda_j \leq 0$

(ii) max $f(x)$ with $g_j(x) \leq 0$

$\lambda_j g_j = 0$ -

$g_j(x) \leq 0$

$\lambda_j \leq 0$

(iii) max $f(x)$. with $g_j(x) \geq 0$

$\lambda_j g_j = 0$ -

$g_j \geq 0$

$\lambda_j \geq 0$ $\lambda \leq 0$

Ques:

minimize $f(x_1, x_2) = x_1^2 + x_2^2 + 40x_1 + 20x_2$

subject to $x_1 - 50 \geq 0$, $x_1 + x_2 - 100 \geq 0$

Solⁿ. $g_1(x) = x_1 - 50 \geq 0$, $g_2(x) = x_1 + x_2 - 100 \geq 0$

$L = x_1^2 + x_2^2 + 40x_1 + 20x_2 + \lambda_1(x_1 - 50) + \lambda_2(x_1 + x_2 - 100)$

$\frac{\partial L}{\partial x_1} = 2x_1 + 40 + \lambda_1 + \lambda_2 = 0$

$\frac{\partial L}{\partial x_2} = 2x_2 + 20 + \lambda_2 = 0$ > (I)

COR $\lambda_1 = 0$ $\lambda_2 = 0$
$\lambda_1 \neq 0$ λ
$\lambda_2 \neq 0$

$(x_1 - 50)\lambda_1 = 0$

$(x_1 + x_2 - 100)\lambda_2 = 0$ > (II)

$x_1 - 50 \geq 0$

$x_1 + x_2 - 100 \geq 0$ > (III)

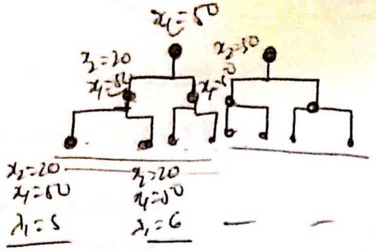
$\lambda_1 \leq 0$

$\lambda_2 \leq 0$ > (IV)

case I from (II)

$x_1 - 50 = 0$ or $\lambda_1 = 0$

$\Rightarrow x_1 = 50$



let $\lambda_1 = 0$

from (I)

$2x_1 + 40 + \lambda_2 = 0$

$2x_2 + 20 + \lambda_2 = 0$

$\Rightarrow x_1 = \frac{-(140 + \lambda_2)}{2}$

$x_2 = \frac{-(20 + \lambda_2)}{2}$

put it (II)

$\left\{ \frac{-(140+\lambda_2)}{2} + \frac{-(20+\lambda_2)}{2} - 100 \right\} \lambda_2 = 0$

$\frac{(-60 - 2\lambda_2 - 200)}{2} \lambda_2 = 0$

$(\lambda_2 + 130)\lambda_2 = 0$

$\lambda_2 = 0$ or $\lambda_2 = -130$

let $\lambda_2 = 0$

$x_1 = -20$

$x_2 = -10$

it voilates the condition (III)

so it is not solution

now let $\lambda_2 = -130$

$x_1 = 45$

$x_2 = 55$

$45 - 50 \geq 0$

$-5 \geq 0$

then $x_1 - 50 = 0$

$x_1 = 50$ ✓

$\lambda_1 + \lambda_2 = -140$

$2x_2 + 20 + \lambda_2 = 0$

$(x_2 - 50)\lambda_2 = 0$

either $\lambda_2 = 0$

$x_2 = 50$

let $\lambda_2 = 0$ ✓

$\lambda_1 = -140$ ✓

$x_2 = -10$ ✓

voilates (III) eqn's 2nd part

let $x_2 = 50$ ✓

$\lambda_1 + \lambda_2 = -140$

$\lambda_2 = -120$ ✓

so $\lambda_1 = -20$ ✓

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Consider the following optimizing problem

Maximize

$f(x_1, x_2) = 4x_1 + 7x_2 - x_1^2 + 2x_2^2$ subject to

$2x_1 + 5x_2 \le 6$

$2x_1 - 15x_2 \le 12$ by method of Kuhn-Tucker conditions. Explain all cases

Soln

$g_1(x) = 2x_1 + 5x_2 - 6 \le 0$

$g_2(x) = 2x_1 - 15x_2 - 12 \le 0$

$L = 4x_1 + 7x_2 - x_1^2 + 2x_2^2 + \lambda_1(2x_1 + 5x_2 - 6) + \lambda_2(2x_1 - 15x_2 - 12)$

$$\left.\begin{aligned} \frac{\partial L}{\partial x_1} &= 4 - 2x_1 + 2\lambda_1 + 2\lambda_2 = 0 \\ \frac{\partial L}{\partial x_2} &= 7 - 4x_2 + 5\lambda_1 - 15\lambda_2 = 0 \end{aligned}\right\} \text{(I)}$$

$$\left.\begin{aligned} \lambda_1(2x_1 + 5x_2 - 6) &= 0 \\ \lambda_2(2x_1 - 15x_2 - 12) &= 0 \end{aligned}\right\} \text{(II)}$$

$$\left.\begin{aligned} 2x_1 + 5x_2 - 6 &\le 0 \\ 2x_1 - 15x_2 - 12 &\le 0 \end{aligned}\right\} \text{(III)}$$

$$\left.\begin{aligned} \lambda_1 &\le 0 \\ \lambda_2 &\le 0 \end{aligned}\right\} \text{(IV)}$$

from (II)

$\lambda_1 = 0$ or $2x_1 + 5x_2 - 6 = 0$

$2x_1 + 5x_2 = 6$

from (I)

$4 - 2x_1 + 2\lambda_2 = 0$

$7 - 4x_2 - 15\lambda_2 = 0$

$8 - 4x$

$11 - 2x_1 - 4x_2 - 13\lambda_2$

$60 - 30x_1 + 30\lambda_2 = 0$

$14 - 8x_2 - 30\lambda_2 = 0$

$74 - 30x_1 - 8x_2 = 0$

$37 - 15x_1 - 4x_2 = 0$

$\lambda_2($

$2x_1 = 2\lambda_1 + 2\lambda_2 + 4$

$x_1 = \lambda_1 + \lambda_2 + 2$

$4x_2 = 5\lambda_1 - 15\lambda_2 + 7$

$\lambda_1 \left(2\lambda_1 + 2\lambda_2 + 4 + \frac{25\lambda_1 - 75\lambda_2 + 35}{4} - 6 \right) = 0$

$\lambda_2 \left(2\lambda_1 + 2\lambda_2 + 4 - \frac{75\lambda_1 - 225\lambda_2 + 105}{4} - 6 \right) = 0$

$\lambda_1 (8\lambda_1 + 8\lambda_2 - 8 + 25\lambda_1 - 75\lambda_2 + 35) = 0$

$\lambda_2 (8\lambda_1 + 8\lambda_2 - 8 - 75\lambda_1 + 225\lambda_2 - 105) = 0$

$\lambda_1 (33\lambda_1 - 67\lambda_2 - 27) = 0$

$\lambda_2 (-67\lambda_1 + 233\lambda_2 - 113) = 0$

$\lambda_1 = 0 \quad , \quad \lambda_2 = 0$ ✓

$\lambda_1 = 0 \quad , \quad \lambda_2 = \frac{113}{233}$ ×

$\lambda_1 = \frac{27}{33} \quad , \quad \lambda_2 = 0$ ×

$\lambda_1 = \qquad \lambda_2 = \frac{5538}{3260}$ ×

$\cancel{2211\lambda_1} - 4489\lambda_2 - 1809 = 0$

$\cancel{-2211\lambda_1} + 7689\lambda_2 - 3729 = 0$

$3200\lambda_2 = 5538$

$\lambda_2 = \frac{5538}{3200}$ ×

$x_1 = 2$

$x_2 = \frac{7}{4}$ ×

$4 + \frac{35}{4} - 6 \le 0$ ×

$x_1 = \frac{33}{8}$

$x_2 = -\frac{9}{20}$

$\frac{33}{4} - \frac{9}{4} - 6$

$\frac{32}{4} - 6$

$\frac{8-6}{2 \ 70}$

—×—×—

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03/01/2018

Optimization by Numerical Methods

Unimodel Function

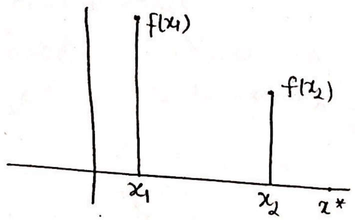
— A Unimodel function is one that has only one maxima or minima in a given interval.

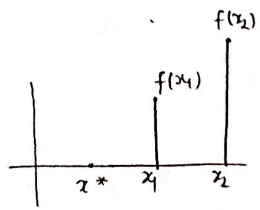
Mathematically, A function $f(x)$ is unimodel if

(i) $x_1 < x_2 < x^*$ implies that $f(x_2) < f(x_1)$ and

(ii) $x_2 > x_1 > x^*$ implies that $f(x_1) < f(x_2)$, x^* is the minimum point.

A unimodel function can be non-differentiable or even a discontinuous function ~~It is a~~ If a function is known to be unimodel in a given range / interval, the interval in which the minimum lies can be narrowed down provided that the functional values are known at two different point in the range.





For example,

Consider the interval [0,1] and two concerned value within the interval are even, there are 3 possibility —

means

$f(x_1) < f(x_2)$ or $\quad f_1 < f_2$

$f(x_1) > f(x_2)$ or $\quad f_1 > f_2$

$f(x_1) = f(x_2)$ $\quad f_1 = f_2$

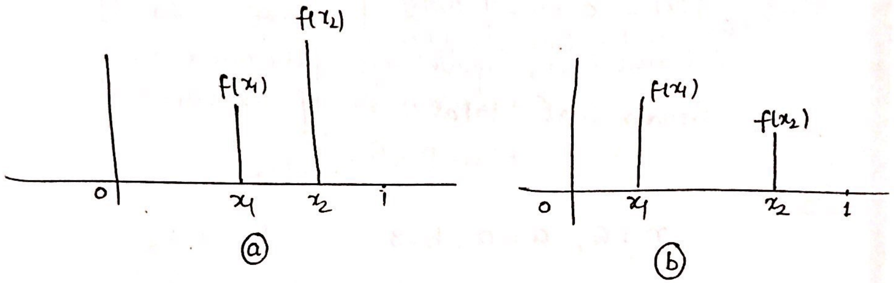
the outcomes, is that

$$f(x_1) < f(x_2)$$

the minimize x not lie to the right of x_2 means the part of the interval $[x_2, 1]$ can be neglected and a new interval of uncertainity $[0, x_2]$ as shown in fig. A.

- If $f(x_1) > f(x_2)$, the interval $[0, x_1]$ can be neglect so new interval is $[x_1, 1]$ as shown in fig. B

- If $f(x_1) = f(x_2)$, the intervals $[0, x_1]$ and $[x_2, 1]$ both can be neglect and new interval is $[x_1, x_2]$.



Fibonacci Numbers :—

Here we have a sequence of fibonacci numbers $\{F_n\}$ as given below :—

$F_0 = 1$, $F_1 = 1$ and

$F_n = F_{n-1} + F_{n-2}$, $n = 2, 3, 4, 5, \ldots$

For $n = 2$

$F_2 = F_1 + F_0 = 1 + 1 = 2$

$n = 3$

$F_3 = F_2 + F_1 = 2 + 1 = 3$

$n = 4$

$F_4 = F_3 + F_2 = 3 + 2 = 5$

$\vdots$

Fibonacci Method :— let L_0 define interval of uncertaintly given by $a \le x \le b$, n be the total no. of experiment to be conducted define

$$L_2^* = \frac{F_{n-2}}{F_n} L_0 \quad \text{—— (1)}$$

and place the first two experiments x_1 and x_2 which is located at a distance of L_2^* from each end of L_0. This gives

$$x_1 = a + L_2^* = a + \frac{F_{n-2}}{F_n} L_0$$

$$x_2 = b - L_2^* = b - \frac{F_{n-2}}{F_n} L_0$$

Ques. $f(x) = 0.65 - \left[\frac{0.75}{1+x^2}\right] - 0.65\ x \tan^{-1}\left(\frac{1}{x}\right)$ in interval $[0,3]$ given by fibonacci method. Given that total no of experiments $n = 6$.

Solⁿ:

$n = 6$, $a = 0$, $b = 3$

$L_0 = b - a = 3 - 0 = 3$

$n = 6$

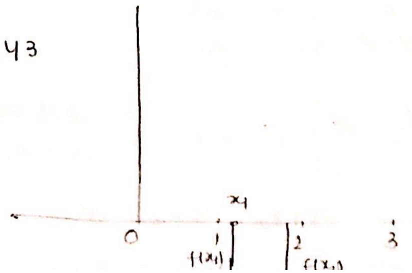
$$L_2^* = \frac{F_{n-2}}{F_n} L_0 = \frac{F_4}{F_6} \times 3 = \frac{5}{13} \times 3 = 1.153846$$

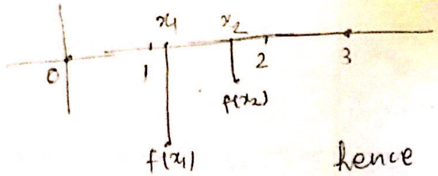
$$x_1 = a + L_2^* = 0 + 1.153846 = 1.153846$$

$$x_2 = b - L_2^* = 3 - 1.153846 = 1.846154$$

$f(x_1) = -0.207270$

$f(x_2) = -0.115843$





hence

$f(x_2) > f(x_1)$

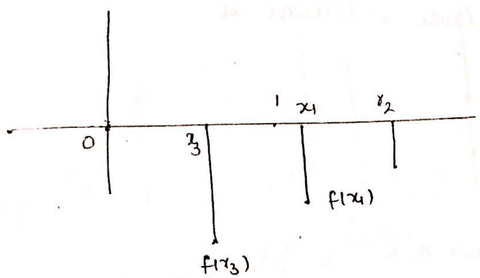
hence $f(x_2) > f(x_1)$

ignore the interval $[x_2, 3]$

and now interval is $[0, x_2]$

$$x_3 = a + (x_2 - x_1)$$
$$= 0 + (1.846154 - 1.153846)$$
$$= 0.692308$$

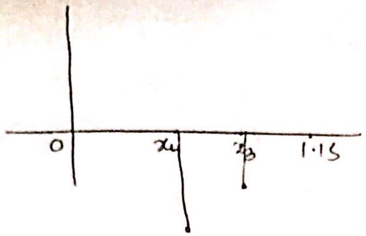
$$f(x_3) = -0.291364$$



hence $f(x_1) > f(x_3)$

ignore $[x_1, x_2]$ and new interval is $[0, x_1]$

$$x_4 = a + (x_1 - x_3)$$
$$= 0 + (1.153846 - 0.692308)$$
$$= 0.461538$$
$$f(x_4) = -0.309811$$



$f(x_3) > f(x_4)$ — new interval [0,1

ignore $[x_3, x_4]$

now $x_5 = 0 + x_3 - x_4$

$= 0.230720$

$f(x_5) = -0.263678$

$L_6 = 0.461540 - 0.230770$

the ratio of final interval is

$$\frac{L_6}{L_0} =$$

for $j = n$

$$\frac{L_n}{L_0} = \frac{\cancel{F_0}}{F_n} \Rightarrow \text{have } n=6, \Rightarrow \frac{L_6}{L_0} = \frac{1}{F_6}$$

$$\cancel{\frac{L_n}{L_0}} = \cancel{\left(\frac{F_{n+1}}{F_n}\right)} \qquad \frac{L_6}{L_0} = (0.618)^{6-1}$$

gnot

Solve himself

$n = 6$
$a = 0$
$b = 3$

$$L_2^* = \frac{F_{n-2}}{F_n} \times L_0$$

$L_0 = b - a$
$= 3 - 0$
$= 3$

$$= \frac{F_4}{F_2} \times 3 = \frac{5}{13} \times 8$$

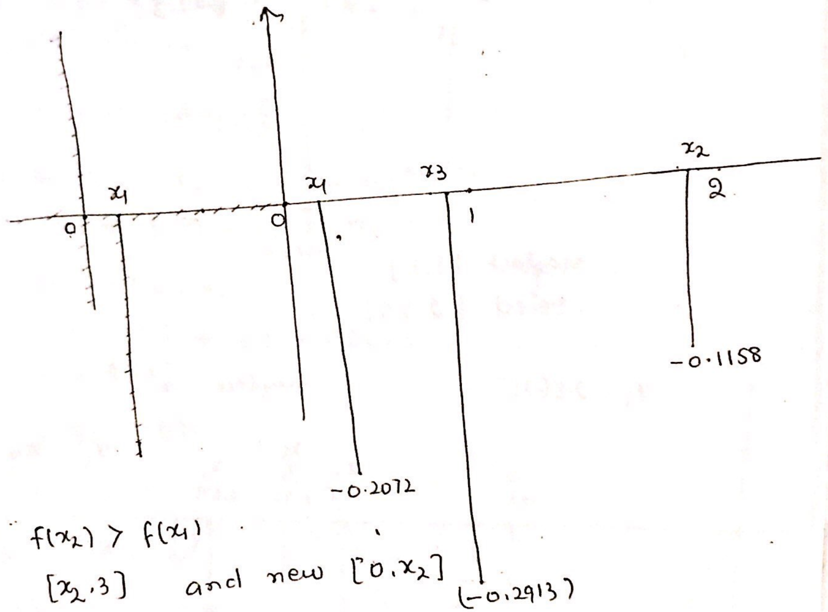
$$= 1.153846$$

$x_1 = a + L_2^* = 0 + 1.153846 = 0.153846$

$x_2 = b - L_2^* = 3 - 1.153846 = 1.846154$

$f(x_1) = -0.207270$

$f(x_2) = -0.115843$



$\because f(x_2) > f(x_1)$

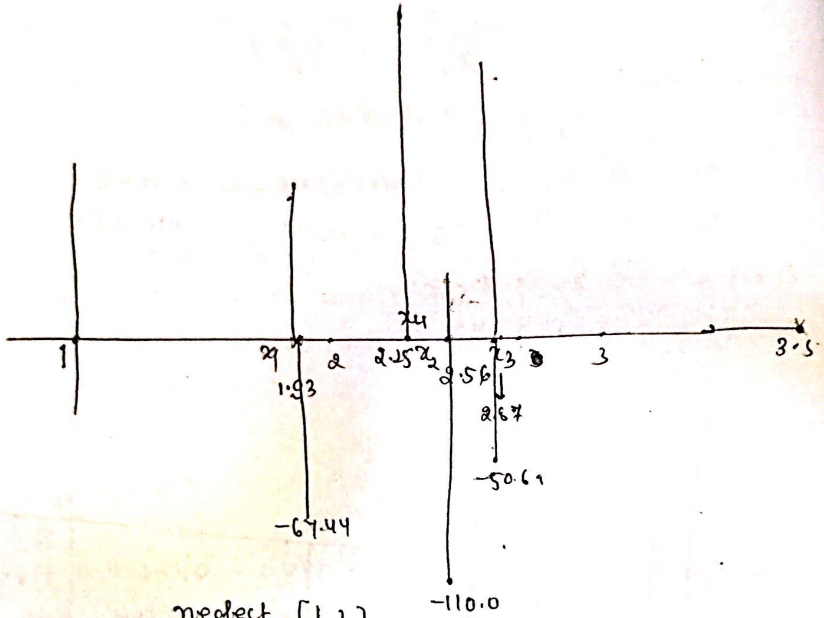
more $[x_2, 3]$ and new $[0, x_2]$

$x_3 = a + (x_2 - x_1)$
$= 0 + (1.846154 - 1.153846)$
$= 0.692308$

$f(x_3) = -0.291364$

since $f(x_1) > f(x_3)$

Date 09/01/2019

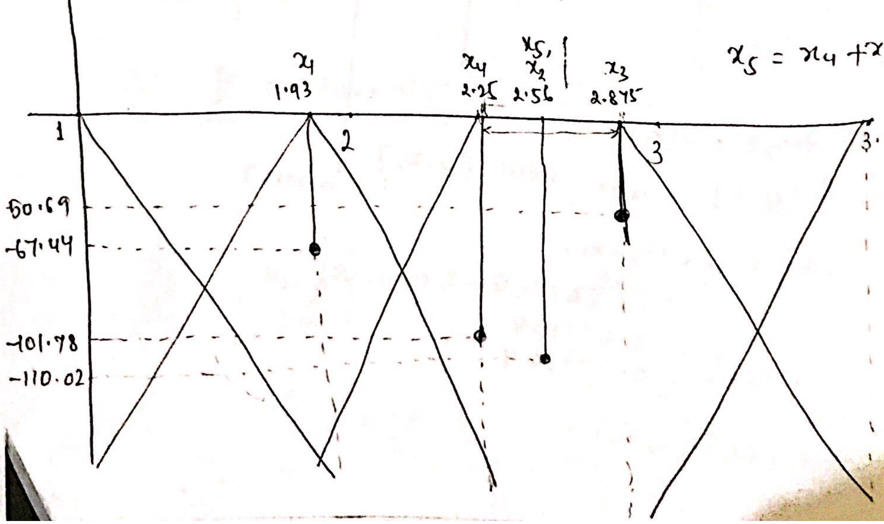


neglect [1,2]
select [2,3.5]

$x_5 = 2.5625$

neglect x

$x_5 = x_4 + x_3 - x_2$ f1



Minimize the function $f(x) = 10x^5 - 40x^4 + 30x^3 + 5$ in $(1, 3.5)$ upto 5^{th} iteration.

1 1 2 3 5 8

Sol^n.

$a = 1$ $n = 5$
$b = 3.5$

$L_0 = b - a$
$= 3.5 - 1 = 2.5$

$$L_2^* = \frac{F_{n-2}}{F_n} L_0 = \frac{F_3}{F_5} \times 2.5 = \frac{3}{8} \times 2.5 = 0.9375$$

$x_1 = a + L_2^* = 1 + 0.9375 = 1.9375$
$x_2 = b - L_2^* = 3.5 - 0.9375 = 2.5625$

$f(x_1) = -67.44783$
$f(x_2) = -110.02433$

new interval ~~[2, 3.5]~~ $[x_1, 3.5]$

$x_3 \cancel{= a + (x_2 - x_1)}$ $x_3 \cancel{= a + (x_2 - x_1)}$
$\cancel{= 1 + 2.5625 - 1.9375 = 1.625}$

$x_3 = x_1 + (3.5 - x_2)$
$= 1.9375 + 3.5 - 2.5625$
$= 2.875$

$f(x_3) = -50.69611$

neglect $[x_3, 3.5]$
new is $[x_1, x_3]$

$x_4 = x_1 + x_3 - x_2$
$= 1.9375 + 2.875 - 2.5625$
$= 2.25$

$f(x_4) = -101.78711$

neglect $[x_1, \cancel{x_4}]$
new $[\cancel{x_4}, x_3]$

$x_5 = x_4 + \cancel{x_3} - x_2$
$= 2.25 \cancel{+} 2.875 - 2.5625$
$= \cancel{2}.5625$

neglect $[1, x_1]$

select $[x_1, 3.5]$

$x_3 = x_1 + 3.5 - x_2$
$= 2.875$

neglect $[x_3, 3.5]$

select $[x_1, x_3]$

$x_4 = x_1 + x_3 - x_2$
$= 2.25$

neglect $[x_1, x_4]$

select $[x_4, x_3]$

$x_5 = x_4 + x_3 - x_2$
$= 2.5625$

Date
11/01/2019

Golden Section Method

The GSM is same as the Fibonacci method except that in the fibonacci method the total number of experiments to be conducted has ~~too~~ & to specified before beginning the calculation.

Whereas this is not required in the golden section method. In GSM we start with the assumption that we are going to conduct a large number of experiments, ofcourse the total no. of experiment can be decided during the calculation ~~in~~.

The interval of uncertainity remaining at the end of different number of experiments can be conducted as follow :—

Fibonacci decided
Golden — not decided

$$L_2 = \lim_{N\to\infty} \frac{L_{N-1}}{L_N} \times L_0 \quad \text{—— (1)}$$

$$L_3 = \lim_{n\to\infty} \frac{F_{n-2}}{F_n} \times L_0 = \lim_{n\to\infty} \frac{F_{n-2}}{F_{n-1}} \times \frac{F_{n-1}}{F_n} \times L_0$$

$$= \lim_{N\to\infty} \left(\frac{F_{N-1}}{F_N}\right)^2 L_0 \text{ , [the ratio } \frac{F_{n-2}}{F_{n-1}} \text{ and } \frac{F_{n-1}}{F_n} \text{ ~~have taken~~]}$$

have taken to be same. for large number of N.

$$L_4 = \lim_{N\to\infty} \frac{L_{N-3}}{F_N} \times L_0 = \lim_{N\to\infty} \frac{F_{N-3}}{F_{N-2}} \times \frac{F_{n-2}}{F_{n-1}} \times \frac{F_{n-1}}{F_n} \times L_0$$

$$= \left(\frac{F_{n-1}}{F_n}\right)^3 L_0$$

this result can be generalized as follow :—

$$L_K = \lim_{N\to\infty} \left(\frac{F_{n-1}}{F_n}\right)^{K-1} L_0$$

$$F_N = F_{N-1} + F_{N-2}$$

$$\Rightarrow \frac{F_N}{F_{N-2}} = 1 + \frac{F_{n-2}}{F_{n-1}}$$

by defining the ratio

$$\gamma = \lim_{N\to\infty} \frac{F_N}{F_{N-1}}$$

$$\gamma = 1 + \frac{1}{\gamma}$$

$$\gamma^2 - \gamma - 1 = 0$$

$\gamma = \frac{1 \pm \sqrt{1+4}}{2}$

this gives

$$\gamma = \cancel{1.1618} \quad 1.6180$$

~~Prod~~ Procedure:—

The location of first 2 experiments are defin[ed]
as $L_2^* = \frac{F_{N-2}}{F_N} \times L_0 = \frac{F_{N-2}}{F_{N-1}} \times \frac{F_{N-1}}{F_N} \times L_0$

$= \frac{1}{\gamma^2} L_0$

$= 0.382 L_0$

Ques: Minimize the function $f(x) =$

$0.65 - \left[\frac{0.35}{1+x^2}\right] - 0.65\, x \times \tan^{-1}\left(\frac{1}{x}\right)$ in the interval $[0, 3]$.

$x_1 = a + L_2^*$
$x_2 = b - L_2^*$

Soln:

$a = 0$
$b = 3$

$L_0 = b - a$
$= 3 - 0 = 3$

$L_2^* = \frac{F_{N-2}}{F_N} \times L_0$

$= \frac{F_{N-2}}{F_{N-1}} \times \frac{F_{N-1}}{F_N} \times L_0$

$= \left(\frac{1}{\gamma}\right)^2 L_0$

$= 0.382 \times 3 = 1.146$

$x_1 = a + L_2^* = 0 + 1.146 = 1.146$
$x_2 = b - L_2^* = 3 - 1.146 = 1.854$

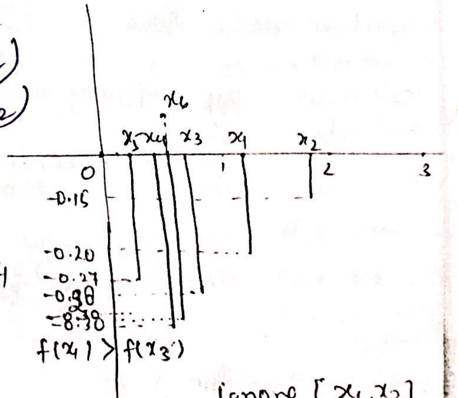
$f(x_1) = -0.208654$
$f(x_2) = -0.115124$

$f(x_2) > f(x_1)$
$f(x_1) < f(x_2)$

ignore $[x_2, 3]$
select $[0, x_2]$

$x_3 = \cancel{(x_2 - 0) + x_1}$
$= \cancel{(x_2 - 0) + x_1} = (x_2 + 0) - x_1$
$= 1.854 - 1.146$
$= 0.708$

$f(x_3) = -0.288943$



$f(x_1) > f(x_3)$

ignore $[x_1, x_2]$
new $[0, x_1]$

$x_4 = x_1 + 0 - x_3$
$= 1.146 - 0.708$
$= 0.438$

ignore $[x_3, x_4]$
select $[0, x_3]$

$f(x_4) = -0.308951$

$x_5 = (x_3 + 0 - x_4)$
$= 0.708 - 0.438$
$= 0.27$

$f(x_5) = -0.278434$

ignore $[0, x_5]$
new $[x_5, x_3]$

$x_6 = x_3 + x_5 - x_4$
$= 0.708 + 0.27 - 0.438$
$= 0.54$

$f(x_6) = -0.308234$ Ans

Date
| 01 | 2019 |

Linear Programming Problems :- An optimizing problem of the form.

Maximize / Minimize

$f(z) = CX$ subject to

$A_1 X \leq b_1$
$A_2 X \geq b_2$
$A_3 X = b_3$ where $X = \begin{Bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{Bmatrix}$

means $x_1, x_2, \ldots\ldots, x_n$ variables and optimizing function and constraints are linear.

Standard Form of LPP

Maximize $z = f(x)$ subject to

$$Ax \le b$$
$$x \ge 0$$

* some times, we can say primal problems.

Date
17/01/2019

Rules for converting primal problem to Dual problem

Primal	Dual
$\text{Max } z = fx$	$\text{Min } z = bX$
$AX \le b$	$AX \ge f$
$X \ge 0$	$X \ge 0$

Rule 1 : (i) First converting the objective function to maximize function if not.

(ii) If a constants of the ~~found~~ form $Ax \ge c_1$. convert it in the form $-Ax \le -c_1$ by multiplying (−) sign.

(iii) If a constants of the form $Ax = c_1$, convert it in the form $Ax \le c_1$

$$Ax \le c_1 \longrightarrow Ax \le c_1$$
$$Ax \ge c_1 \longrightarrow -Ax \le -c_1$$

(iv) every unrestricted variable convert into restricted variable

x_1 is unrestricted

let x_1' and x_1'' such that

$x_1 = x_1' - x_1''$ given that

$x_1' \ge 0,\ x_1'' \ge 0$

Then dual of the problem ~~it~~ can be obtained

(i) Transposing the coefficient matrix

(ii) Interchanging the role of constant terms and the coefficient of the objective concerned.

(iii) Reverting the enequality.

(iv) Minimizing the objective concerned in place of Maximizing concerned.

Qun: Find the dual of the following function.

$$\text{Min } Z = 2x_2 + 5x_3 \quad \text{s.t.}$$

$$x_1 + x_2 \geq 2$$

$$2x_1 + x_2 + 6x_3 \leq 6$$

$$+x_1 - x_2 + 3x_3 = 4$$

and $x_1, x_2, x_3 \geq 0$

Let $z = -z'$

$\Rightarrow$ Max $z' = -2x_2 - 5x_3$ s.to

Revision

$-x_1 - x_2 \le -2$ y_1

$2x_1 + x_2 + 6x_3 \le 6$, y_2

$x_1 - x_2 + 3x_3 \le 4$. y_3

$\boxed{x_1 - x_2 + 3x_3 \ge 4}$

$-x_1 + x_2 - 3x_3 \le -4$ y_4,

$x_1 \ge 0$, $x_2 \ge 0$, $x_3 \ge 0$ min

then dual of the problem is

Min $z' = -2y_1 + 6y_2 + 4y_3 - 4y_4$ Min z' Ma

$-y_1 + 2y_2 + y_3 - y_4 \ge 0$ (comparing to $-2x_2 - 5x_3$) M.Z

$-y_1 + y_2 + y_3 + y_4 \ge -2$

$6y_2 + 3y_3 - 3y_4 \ge -5$ $y_3 - y_4 = \alpha$ $-\alpha$

and $y_1 \ge 0$, $y_2 \ge 0$, $y_3 \ge 0$, $y_4 \ge 0$ y_4 -

Min $z' = -2y_1 + 6y_2 + 4y_3'$ $y_3' = y_3 - y_4$

such that

$-y_1 + 2y_3 + y_3' \ge 0$ Revision

$-y_1 + y_2 - y_3' \ge -2$ y_3'

$6y_2 + 3y_3' \ge -5$ Max

and $y_1 \ge 0$, $y_2 \ge 0$, $\cancel{y_3 \ge 0, y_4 \ge 0}$ Min

y_3' is unrestricted.

Ques: Min $Z = 2x_1 + 3x_2$ subject to

$2x_1 + 3x_2 \ge 2$

$3x_1 + x_2 = 3$

(i) $x_1 \ge 0$, $x_2 \ge 0$

(ii) $x_1 \ge 0$, x_2 is unrestricted

Scanned with CamScanner

Sol^n. Let $z = -z'$

$\Rightarrow$ Max $z' = -2x_1 - 3x_2$ s.t.

$-2x_1 - 3x_2 \le -2$ $\quad y_1$

$3x_1 + x_2 \le 3$ $\quad y_2$

$\boxed{3x_1 + x_2 \ge 3}$

$-3x_1 - x_2 \le -3$ $\quad y_3$

$x_1 \ge 0,\ x_2 \ge 0$

then dual of the problem is

Min $z' = -2y_1 + 3y_2 - 3y_3$

$-2y_1 + 3y_2 - 3y_3 \ge -2$

$-3y_1 + y_2 - y_3 \ge 3$

and $y_1 \ge 0,\ y_2 \ge 0,\ y_3 \ge 0$

Min $z' = -2y_1 + 3y_2'$

such that $y_2' = y_2 - y_3$

$-2y_1 + 3y_2' \ge -2$

$-3y_1 + y_2' \ge 3$

and $y_1 \ge 0$, y_2' is unrestricted.

1/2019 Solve by S.M (Simplex Method)

4 $\text{Min } Z = x_1 - 3x_2 + 2x_3$ subject to

$$3x_1 - x_2 + 3x_3 \le 7$$
$$-2x_1 + 4x_2 \le 12$$
$$-4x_1 + 3x_2 + 8x_3 \le 10$$
$$x_1, x_2, x_3 \ge 0$$

$\text{Max } Z' = -x_1 + 3x_2 - 2x_3$ $\quad Z' = -Z$

$$3x_1 - x_2 + 3x_3 + s_1 = 7$$
$$-2x_1 + 4x_2 + s_2 = 12$$
$$-4x_1 + 3x_2 + 8x_3 + s_3 = 10$$
$$x_1, x_2, x_3 \ge 0$$

pivot

Basic/Identity Matrix

$\rightarrow \text{Max } z' = -x_1 + 3x_2 - 2x_3 + 0s_1 + 0s_2 + 0s_3$

Basic variables	C_B	X_B/b	x_1 (−1)	x_2 (3)	x_3 (−2)	s_1 (0)	s_2 (0)	s_3 (0)	Minimum Ratio
s_1	0	7	3	−1	3	1	0	0	—
$s_2 \to x_2$	0	12	−2	4	0	0	1	0	$12/4 = 3$ min
s_3	0	10	−4	3	8	0	0	1	$10/3 = 3.3$
			1	−3	2	0	0	0	

Net Evaluation

$\Delta_j = Z - C_j$

$\Delta_j = C_B X_B - C_j$

divide 2nd row by 4
add ① and new 2nd row

Mean =

$\Delta_1 = C_B X_1 - C_1$

[Δ_1 = net evaluation for x_1

$$D_1 = (0,0,0)\begin{pmatrix}3\\-2\\4\end{pmatrix} - (-1)$$

$$= \begin{pmatrix}1\\1\\1\end{pmatrix}$$

(III) row + (−3) × new 2nd row

$\Delta_2 = C_B X_2 - C_2$

$$\Delta_2 = (0,0,0)\begin{pmatrix}-1\\4\\3\end{pmatrix} - (3)$$

$$= \begin{pmatrix}-3\\-3\end{pmatrix}$$

10 compute for Incoming variables

Min. of net evaluation

$\min\{\Delta j\}$

$\min\{1, -3, 2, 0, 0, 0\} \Rightarrow x_2$

Calculation of outgoing variables

$\min\left\{\frac{b}{x_i}, x_i > 0\right\}$

x_i means coefficient of outgoing vectors

Min (3, 3.33)

$\Rightarrow s_2$ is a outgoing variable

Date 25/01/2019

pivot को 1
rest को 0

pivot-2

			-1	3	-2	0	0	0	
s_1	0	10	$3-2/4 = 5/2$	0	3	1	1/4	0	$\frac{10\times 2}{5}$ 4
x_2	3	3	$-2/4$	1	0	0	1/4	0	
s_3	0	1	$-5/2$	0	8	0	$-3/4$	0	
	$z' = 0\times10+3\times3+0\times1 = 9$		$-\frac{1}{2}$	0	2	0	$\frac{3}{4}$	0	
x_1	-1	4	1	0	$\frac{6}{5}$	$\frac{2}{5}$	$\frac{1}{10}$	0	
x_2	3	5	0	1	3/5	1/5	3/10	0	
s_3	0	11	0	0	11	1	$-1/2$	0	
	$z' = 11$		0	0	13/5	1/5	3/10	0	

$x_1 = 4, x_2 = 5, x_3 = 0,$
$s_1 = 0, s_2 = 0, s_3 = 11$

An. $\cancel{x_1 = 0}$ $\cancel{x_2 = 2}$ $\cancel{x_3 = 5}$ $\cancel{s_1 = 0}$ $\cancel{s}$

$$\text{Max } z = 3x_1 + 2x_2 + 5x_3$$
$$x_1 + 2x_2 + x_3 \le 430$$
$$3x_1 + 2x_3 \le 460$$
$$x_1 + 4x_2 \le 420$$

$x_1 = 0$, $x_2 = 100$, $x_3 = 230$, $s_1 = 0$, $s_2 = 0$, $s_3 = 20$

$$\text{Max } z = 3x_1 + 2x_2 + 5x_3 + 0s_1 + 0s_2 + 0s_3$$

$$x_1 + 2x_2 + x_3 + s_1 = 430$$
$$3x_1 + 2x_3 + s_2 = 460$$
$$x_1 + 4x_2 + s_3 = 420$$

pivot 1

			3	2	5	0	0	0	
Basic Variables	C_B	x_B/b	x_1	x_2	x_3	s_1	s_2	s_3	Minimum Ratio
s_1	0	430	1	2	1	1	0	0	430/1 = 430
s_2	0	460	3	0	$\boxed{2}$	0	1	0	460/2 = 230
s_3	0	420	1	4	0	0	0	1	
			−3	−2	−5 ↑	0	0	0	
s_1	0	200	−1/2	$\boxed{2}$ new pivot	0	1	−1/2	0	200/2 = 100
x_3	5	230	3/2	0	1	0	1/2	0	420/4 = 105
s_3	0	420	1	4	0	0	0	1	
			9/2	−2 ↑	0	0	5/2	0	
x_2	2	100	−1/4	1	0	1/2	−1/4	0	
x_3	5	230	3/2	0	1	0	1/2	0	
s_3	0	20	2	0	0	−2	1	1	
		$z = 200 + 1150 + 0 = 1350$	4	0	0	1	2	0	

28|01|2019

Dual Simplex Method :

Find the solⁿ of the problem

Max $Z = -4x_1 - 6x_2 - 18x_3$ s.t.

$x_1 + 3x_3 \geq 3$

$x_2 + 2x_3 \geq 5$

$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0$

-6 -9

$-x_1 - 3x_3 \leq -3$; $-x_1 - 3x_3 + s_1 = -3$

$-x_2 - 2x_3 \leq -5$; $-x_2 - 2x_3 + s_2 = -5$

pivot

			-4	-6	-18	0	0	
Basic variable	C_B	X_B	x_1	x_2	x_3	s_1	s_2	~~Minimum Ratio~~
s_1	0	-3	-1	0	-3	1	0	
s_2	0	-5	0	$\boxed{-1}$	-2	0	1	pivot
	z = 0		4	6	18	0	0	
s_1	0	-3	-1	0	$\boxed{-3}$	1	0	
x_2	-6	5	0	1	2	0	-1	(multiply by (-1))
			4	0	6	0	6	
x_3	-18	1	1/3	0	1	-1/3	0	÷ by (-3)
x_2	-6	3	-2/3	1	0	2/3	-1	

ote : 1: To find ~~limt~~ ~~livi~~ leaving vector / outgoing vector from the bases .

$min[x_B] = \{-3, -5\} = -5 \Rightarrow s_2$ is a leaving vector

means the outgoing vector is selected corresp- -onding to the basic variables having the most negative value. If the values of all basic variables are positive then the process ends and step and find the optimal solution

→ To find the entering vector in the basis.

$$\max_j \left\{ \frac{\Delta_j}{a_{rj}}, a_{rj} < 0 \right\}$$

, a_{rj} is the coefficients — ① of the variables in the equation.

Suppose we get S_2 is a outgoing variable means we have fired a second equation

means $r = 2$

from ①

$$= \max_j \left\{ \frac{\Delta_1}{a_{r1}}, a_{2j} < 0 \right\} = \max \left\{ \frac{4}{\dots}, \frac{6}{-1}, \frac{18}{-2} \right\}$$

$$\max \{-6, -9\} = -6$$

$\Rightarrow x_2$ is a entering vector

s_1 is leaving vector :

$$\max \left\{ \frac{4}{-1}, \frac{6}{-3}, \frac{6}{0} \right\} = \max \{ \cancel{3, -6} \} \{-4, -2\}$$

$= -2$.

$\Rightarrow$

ques: Min $z = 2x_1 + x_2$ s.t.

$3x_1 + x_2 \geq 3$

$4x_1 + 3x_2 \geq 6$ $\quad x_1 \geq 0,\ x_2 \geq 0$

$x_1 + 2x_2 \geq 3$

Soln:

Max $z' = -2x_1 - x_2$

$-3x_1 - x_2 \leq -3$ $\quad -3x_1 - x_2 + s_1 = -3$

$-4x_1 - 3x_2 \leq -6$ $\quad -4x_1 - 3x_2 + s_2 = -6$

$-x_1 - 2x_2 \leq -3$ $\quad -x_1 - 2x_2 + s_3 = -3$

			-2	-1	0	0	0
Basic variable	C_B	x_B	x_1	x_2	s_1	s_2	s_3
s_1	0	-3	-3	-1	1	0	0
s_2	0	-6	-4	$\boxed{-3}$	0	1	0
s_3	0	-3	-1	-2	0	0	1
			2	1	0	0	0

min $\{-3, -6, -3\} = -6$

max $\left\{\frac{2}{-4}, \frac{1}{-3}\right\} = \{-0.5, -0.33\}$

$= -\frac{1}{3}$

						pivot	
s_1	0	-1	-5/3	0	1	-1/3	0
x_2	-1	2	4/3	1	0	-1/3	0
s_3	0	1	5/3	0	0	-2/3	1
			+2/3	0	0	+1/3	0

$\min\{-1, 2, 1\} = -1$

$\max\left\{\frac{-2/3}{-5/3}, \frac{-1/3}{-1/3}\right\}$

$\max\{-\frac{2}{5}, 1\} = -\frac{2}{5}$

s_2	0		
x_2	-1		
s_3	0		

Q: Use dual Simplex Method to solve L.P.P.

$\text{Min } z = 3x_1 + 2x_2$ subject to

$x_1 + x_2 \geq 1$

$x_1 + x_2 \leq 7$

$x_1 + 2x_2 \geq 10$

$x_2 \leq 3$

$x_1, x_2 \geq 0$

Standard form to use Dual Simplex Method

$\text{Max } z' = -3x_1 - 2x_2$

$-x_1 - x_2 \leq -1$

$x_1 + x_2 \leq 7$

$-x_1 - 2x_2 \leq -10$

$x_2 \leq +3$

$\text{Max } z' = -3x_1 - 2x_2$ subject to

$-x_1 - x_2 + s_1 = -1$
$x_1 + x_2 + s_2 = 7$
$-x_1 - 2x_2 + s_3 = -10$
$x_2 + s_4 = 3$

Basic Variable	C_B	X_B	x_1 (−3)	x_2 (−2)	s_1 (0)	s_2 (0)	s_3 (0)	s_4 (0)
s_1	0	−1	−1	−1	1	0	0	0
s_2	0	7	1	1	0	1	0	0
~~s_3~~	0	−10	−1	$\boxed{-2}$	0	0	1	0
s_4	0	3	0	1	0	0	0	1
	$z' = 0$		3	2	0	0	0	0
s_1	0	(4)	$\boxed{-1/2}$	0	1	0	−1/2	0
s_2	0	(2)	+1/2	0	0	+1	+1/2	0
x_2	−2	(5)	1/2	1	0	0	−1/2	0
s_4	0	(−2)	−1/2	0	0	1	1/2	0
	$z' = 10$		2	0	0	0	1/2	1
x_1	−3	+12	1	0	−2	0	1	0
s_2	0	6	0	0	0	1	0	0
x_2	−2	+17	0	1	1	1	+1	0
s_4	0	14	0	0	0	1	1	1
	$z' = -36 - 34$ = −70		0	0	0	−1	−1	1

$x_4 = 12$, $x_2 = 17$, $s_1 = 0$, $s_2 = 6$, $s_3 = 0$, $s_4 = 14$

~~$s_1 = 4$~~ ~~$x_1 = 0$~~
~~$s_2 = 2$~~
~~$x_2 = 5$~~
~~$s_4 = 2$~~

$s_3 = 0$

$x_1 = 4$
$x_2 = 3$
$z = 18$
$s_2 = 0$
$s_1 = 6$

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$\max\left\{\frac{3}{-1}, \frac{2}{-2}\right\} = \max\{-3, -1\}$

$\max\left[\frac{2}{-1/2}\right] = \max\{-4\}$

s_1	0	6	0	0	1	-1	-1	0
s_2	0	0	0	0	0	2	1	0
x_2	-2	3	0	1	0	1	0	0
x_1	-3	4	1	0	0	-2	-1	0

$s_1 = 6$
$s_2 = 0$
$x_1 = 4$
$x_2 = 3$

Ques: Use dual simplex Method to solve L.P.P.

Max $Z = -2x_1 - 2x_2 - 4x_3$ subject to

$2x_1 + 3x_2 + 5x_3 \geq 2$

$3x_1 + x_2 + 7x_3 \leq 3$

$x_1 + 4x_2 + 6x_3 \leq 5$

$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0$.

Solution of L.P.P. by Karmarkar's Method

Minimize $f = C^T X$

subject to $[a] X = 0$

$x_1 + x_2 + \dots + x_n = 1$, $x \geq 0$, where

$X = \{x_1, x_2, \dots, x_n\}^T$, $C = \{c_1, c_2, \dots, c_n\}^T$

$[a]$ is an $m \times n$ matrix

Karmarkar's Method requires L.P.P. in the following form –

Step 1 : starting initial basic ~~sequence~~ feasible solution

$$x^1 = \left\{ \frac{1}{n}, \frac{1}{n}, \frac{1}{n}, \cdots, \frac{1}{n} \right\}^T$$

① Test for optimality since $f=0$ at the starting point, we stop the procedure if the following condition is satisfy

Norm $\|C^T X^{(K)}\| \leq \varepsilon$, $K=1$

$\varepsilon = 0.001$ or 0.005

$$\|C^T X^{(1)}\| \leq \varepsilon$$

Compute the next point $X^{(K+1)}$ for calculating

First calculate $y^{(K+1)}$.

Thus

$$y^{(K+1)} = \left[\frac{1}{n}, \frac{1}{n}, \frac{1}{n}, \cdots, \frac{1}{n} \right] - \frac{\alpha((I) - [P]^T([P][P]^T)^{-1}[P](x}{\|C\| \sqrt{n(n-1)}}$$

Here $\|C\|$ is the length of vector 'C'.

I is the identity matrix of order n and

$D(X^{(K)})$ is an $m \times n$ matrix with off diagonal entry $= 0$ and diagonal component is equal to the component of the vector $(X^{(K)})$. Here $[P]$ is an $(m+1) \times n$ matrix with first 'm' rows are given by matrix $[a][D(X^{(K)})]$ — and last row is composed with entry 1 means

$$[P] = \begin{bmatrix} [a][D(X^{(K)})] \\ 1 \quad 1 \quad \cdots \quad 1 \end{bmatrix}$$

and value of the parameter α is usually chos $\frac{1}{4}$.

After calculating y^{K+1}, we can calculate $(X)^{K+1}$ as

$$X^{(K+1)} = \frac{x_i^{(K)} y_i^{(K+1)}}{\sum_{r=1}^{n} x_r^{(K)} y_r^{(K+1)}} \quad , i = 1, 2, \ldots, n.$$

Ques: Find the solⁿ of following by Karmarkar's method

Minimize $f = 2x_1 + x_2 - x_3$ subject to

$$x_2 - x_3 = 0 \qquad x_i \geq 0,$$
$$x_1 + x_2 + x_3 = 1 \qquad i = 1, 2, 3,.$$
$$\varepsilon = 0.05, \quad \alpha = \frac{1}{4}$$

$x^{(K)})] C$

Step 1

Choose $X^{(1)} = \begin{Bmatrix} \frac{1}{3} \\ \frac{1}{3} \\ \frac{1}{3} \end{Bmatrix}$ $\qquad C = \{2, 1, -1\}$

$$||C^T X^{(K)}|| \leq \varepsilon$$

$$||C^T X^{(1)}|| \leq 0.05$$

$$[2, 1, -1] \begin{bmatrix} \frac{1}{3} \\ \frac{1}{3} \\ \frac{1}{3} \end{bmatrix} \leq 0.05$$

$$\frac{2}{3} + \frac{1}{3} - \frac{1}{3} \leq 0.05$$

$$0.666.. \leq 0.05 \checkmark$$

means we can say that convergance criteria is not satisfied. so we go to next step.

$$D(X^{(1)}) = \begin{bmatrix} \frac{1}{3} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{3} \end{bmatrix}$$

$$[a] = [0, 1, -1]$$

$$[a]D[X^{(1)}] = [0 \ 1 \ -1] \begin{bmatrix} \frac{1}{3} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{3} \end{bmatrix}$$

$$= [0 \ \ \tfrac{1}{3} \ \ -\tfrac{1}{3}]$$

$$P = \begin{bmatrix} 0 & \frac{1}{3} & -\frac{1}{3} \\ 1 & 1 & 1 \end{bmatrix}, \quad P^T = \begin{bmatrix} 0 & 1 \\ \frac{1}{3} & 1 \\ -\frac{1}{3} & 1 \end{bmatrix}$$

$$P \cdot P^T = \underset{2\times 3}{\begin{bmatrix} 0 & \frac{1}{3} & -\frac{1}{3} \\ 1 & 1 & 1 \end{bmatrix}} \underset{3\times 2}{\begin{bmatrix} 0 & 1 \\ \frac{1}{3} & 1 \\ -\frac{1}{3} & 1 \end{bmatrix}}$$

$$= \begin{bmatrix} \frac{2}{9} & 0 \\ 0 & 3 \end{bmatrix}$$

$$([P][P^T])^{-1} = \begin{bmatrix} 9/2 & 0 \\ 0 & 1/3 \end{bmatrix}$$

$$[P]^T([P][P^T])^{-1}[P] = A = \begin{bmatrix} 0 & 1 \\ \frac{1}{3} & 1 \\ -\frac{1}{3} & 1 \end{bmatrix} \begin{bmatrix} 9/2 & 0 \\ 0 & 1/3 \end{bmatrix} \begin{bmatrix} 0 & \frac{1}{3} & -\frac{1}{3} \\ 1 & 1 & 1 \end{bmatrix}$$

~~3×2×2×2×2×3~~

$$= \begin{bmatrix} 0 & \frac{1}{3} \\ \frac{3}{2} & \frac{1}{3} \\ -\frac{3}{2} & \frac{1}{3} \end{bmatrix} \begin{bmatrix} 0 & \frac{1}{3} & -\frac{1}{3} \\ 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{5}{6} & -\frac{1}{6} \\ \frac{1}{3} & -\frac{1}{6} & \frac{5}{6} \end{bmatrix}$$

$$[P]^T \left([P][P^T]\right)^{-1}[P] = \cdots$$

$$I - A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{5}{6} & -\frac{1}{6} \\ \frac{1}{3} & -\frac{1}{6} & \frac{5}{6} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{2}{3} & -\frac{1}{3} & -\frac{1}{3} \\ -\frac{1}{3} & \frac{1}{6} & \frac{1}{6} \\ -\frac{1}{3} & \frac{1}{6} & \frac{1}{6} \end{bmatrix}$$

$$D(x')C = \begin{bmatrix} \frac{1}{3} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{3} \end{bmatrix}_{3\times 3} \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}_{3\times 1}$$

$$= \begin{bmatrix} \frac{2}{3} \\ \frac{1}{3} \\ -\frac{1}{3} \end{bmatrix}$$

$$[I - A]\, D(x')C = \begin{bmatrix} \frac{2}{3} & -\frac{1}{3} & -\frac{1}{3} \\ -\frac{1}{3} & \frac{1}{6} & \frac{1}{6} \\ -\frac{1}{3} & \frac{1}{6} & \frac{1}{6} \end{bmatrix} \begin{bmatrix} \frac{2}{3} \\ \frac{1}{3} \\ -\frac{1}{3} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{4}{9} \\ -\frac{2}{9} \\ -\frac{2}{6} \end{bmatrix}$$

$-\frac{1}{3}$

use $\alpha = \frac{1}{4}$

$$y^{(2)} = \begin{bmatrix} 1/3 \\ 1/3 \\ 1/3 \end{bmatrix} - 4 \begin{bmatrix} \frac{4}{9} \\ -\frac{2}{9} \\ -\frac{2}{9} \end{bmatrix} \cdot \frac{1}{\sqrt{3(2)}\sqrt{6}}$$

x_i

if $C = \{x_1, x_2, x_3, \dots, x_n\}$

$$\|C\| = \sqrt{x_1^2 + x_2^2 + x_3^2 + \dots + x_n^2}$$

$C = \{2, 1, -1\}$

$$\|C\| = \sqrt{4+1+1} = \sqrt{6}$$

$$y^{(2)} = \begin{Bmatrix} \frac{34}{108} \\ \frac{37}{108} \\ \frac{37}{108} \end{Bmatrix}$$

Now

$$x_i^{(k+1)} = \frac{x_i^k \, y_i^{k+1}}{\sum_{r=1}^{n} x_r^{(k)} \, y_r^{(k+1)}} \quad , i = 1, 2, \dots, n$$

$\hookrightarrow x_1^k y_1^{k+1} + x_2^k y_2^{k+1} + x_3^k y_3^{k+1}$

put $k=1$

$$= x_1^{(1)} y_1^{(2)} + x_2^{(1)} y_2^{(2)} + x_3^{(1)} y_3^{(2)}$$

$$= \frac{1}{3}\left(\frac{34}{108}\right) + \frac{1}{3}\left(\frac{37}{108}\right) + \frac{1}{3}\left(\frac{37}{108}\right) = \frac{1}{3}$$

$$x_2 = \frac{x_1^{(1)} y_1^{(2)}}{1/3} + \frac{x_2^{(1)} y_2^{(2)}}{1/3} + \frac{x_3^{(1)} y_3^{(2)}}{1/3}$$

$$= \frac{\frac{1}{3}\left(\frac{34}{108}\right)}{1/3} + \frac{\frac{1}{3}\left(\frac{37}{108}\right)}{1/3} + \frac{\frac{1}{3}\left(\frac{37}{108}\right)}{1/3} = 1$$

Ques.

$$\text{Min } f = 2x_1 + 11x_2 - 9x_3 \quad \text{subject to}$$
$$3x_1 - 4x_3 = 0$$
$$x_1 + x_2 + x_3 = 1$$

Sol^n:

choose

$$X^{(1)} = \begin{Bmatrix} \frac{1}{3} \\ \frac{1}{3} \\ \frac{1}{3} \end{Bmatrix}, \quad C = [2 \;\; 11 \;\; -$$

$$\varepsilon = 0.05, \quad \alpha = \frac{1}{4}$$

$$\| C^T X^{(K)} \| \le \varepsilon$$
$$\| C^T X^{(1)} \| \le 0.05$$
$$[2, 11 \; -9] \begin{bmatrix} \frac{1}{3} \\ \frac{1}{3} \\ \frac{1}{3} \end{bmatrix} \le 0.05$$
$$\frac{2}{3} + \frac{11}{3} - 3 \le 0.05$$
$$\frac{13}{3} - 3 \le 0.05 \Rightarrow \frac{4}{3} \le 0.05$$
$$1.33 \le 0.05$$

means we can say that convergance criteria is not satisfied. so we go to next step.

$$D(x^{(1)}) = \begin{bmatrix} \frac{1}{3} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{3} \end{bmatrix}$$

$$[a] = [3 \;\; 0 \;\; -4]$$

$$[a]\, D[x^{(1)}] = [3 \; 0 \; -4]_{1\times 3} \begin{bmatrix} \frac{1}{3} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{3} \end{bmatrix}_{3\times 3}$$

$$= \left[1 \;\; 0 \;\; -\frac{4}{3}\right]$$

$$P = \begin{bmatrix} 1 & 0 & -\frac{4}{3} \\ 1 & 1 & 1 \end{bmatrix}_{2\times 3}, \quad P^T = \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ -\frac{4}{3} & 1 \end{bmatrix}_{3\times 2}$$

$$PP^T = \begin{bmatrix} \frac{25}{9} & -\frac{1}{3} \\ -\frac{1}{3} & 3 \end{bmatrix}$$

$$(PP^T)^{-1} = \begin{bmatrix} \frac{27}{74} & \frac{3}{74} \\ \frac{3}{74} & \frac{25}{74} \end{bmatrix}$$

Now

$$A = P^T(PP^T)^{-1}P = \begin{bmatrix} 0.7888 & 0.3784 & -0.6122 \\ 0.3784 & 0.3378 & 0.2838 \\ -0.6182 & 0.2838 & 0.8784 \end{bmatrix}$$

so

$$I - A = \begin{bmatrix} 0.2162 & -0.3784 & 0.6122 \\ -0.3784 & 0.6622 & -0.2838 \\ 0.6182 & -0.2838 & 0.1216 \end{bmatrix}$$

$$D(x')c \quad \begin{bmatrix} \frac{1}{3} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{3} \end{bmatrix} \begin{bmatrix} 2 \\ 11 \\ -9 \end{bmatrix} = \begin{bmatrix} \frac{2}{3} \\ \frac{11}{3} \\ -3 \end{bmatrix}$$

$$[I-A] \cdot D(x')c$$

$$= \begin{bmatrix} 0.2162 & -0.3784 & 0.6122 \\ -0.3784 & 0.6622 & -0.2838 \\ 0.6182 & -0.2838 & 0.1216 \end{bmatrix} \begin{bmatrix} \frac{2}{3} \\ \frac{11}{3} \\ -3 \end{bmatrix}$$

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$$= \begin{bmatrix} -3.0799 \\ 3.0272 \\ -0.9933 \end{bmatrix}$$

$\|c\| = \sqrt{4+121+81}$

$c = [2, 11, -9]^T$

use $\alpha = \frac{1}{4}$

$$y^{(2)} = \begin{bmatrix} 1/3 \\ 1/3 \\ 1/3 \end{bmatrix} - 4 \begin{bmatrix} \frac{4}{9} \\ \frac{-2}{9} \\ \frac{-2}{9} \end{bmatrix} \cdot \frac{1}{14.352}$$

Date 11/02/2019

Solution by Decomposition Principle

Suppose, we have the
minimum $f(x_1, x_2, y_1, y_2) = c_1 x_1 + c_2 x_2 + c_3 y_1 + c_4 y_2$
subject to

$$a_{11} x_1 + a_{12} x_2 + a_{13} y_1 + a_{14} y_2 \le b_1$$

$$a_{21} x_1 + a_{22} x_2 + a_{23} y_1 + a_{24} y_2 \le b_2$$

$$\boxed{\begin{array}{l} a_{31} x_1 + a_{32} x_2 \le b_3 \\ a_{41} x_1 + a_{42} x_2 \le b_4 \end{array}} \qquad \boxed{\begin{array}{l} a_{51} y_1 + a_{52} y_2 \le b_5 \\ a_{61} y_1 + a_{62} y_2 \le b_6 \end{array}}$$

$$x_1, x_2 \ge 0$$
$$y_1, y_2 \ge 0$$

Graphically solve

Qun: Minimize $f = x_1 + 2x_2 + 2y_1 + 3y_2$
subject to

$$x_1 + x_2 + y_1 + y_2 \le 1000$$
$$x_1 + y_1 \le 500$$
$$\begin{cases} x_1 + x_2 \le 600 \\ x_1 - 2x_2 \le 0 \end{cases}$$
$$-2y_1 + y_2 \le 0$$
$$x_1 \ge 0,\ x_2 \ge 0,\ y_1 \ge 0,\ y_2 \ge 0$$

Ans: The problem can be state as
Maximize $f(x) = c_1^T x_1 + c_2^T x_2$ subject to

$A_1 x_1 + A_2 x_2 \le b_0$

(i) – $B_1 x_1 \le b_1$

(ii) – $B_2 x_2 \le b_2$

$x_1 \ge 0, x_2 \ge 0$

Subsidiary constant

where $X_1 = \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix}$, $X_2 = \begin{Bmatrix} y_1 \\ y_2 \end{Bmatrix}$

$c_1 = \begin{Bmatrix} 1 \\ 2 \end{Bmatrix}$, $c_2 = \begin{Bmatrix} 2 \\ 3 \end{Bmatrix}$

$A_1 = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$, $A_2 = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$

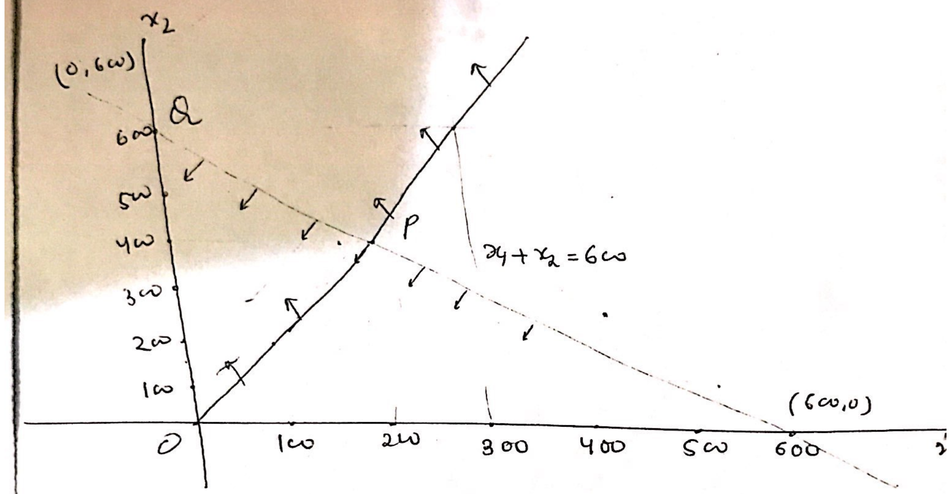
$b_0 = \begin{bmatrix} 1000 \\ 500 \end{bmatrix}$, $b_1 = \begin{bmatrix} 600 \\ 0 \end{bmatrix}$ $B_2 = [-2, 1]$

$b_2 = \{0\}$

$$\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} + \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1000 \\ 500 \end{bmatrix}$$

$$A_1 X_1 + A_2 X_2 \le b_0$$

$$B_1 = \begin{bmatrix} 1 & 1 \\ 1 & -2 \end{bmatrix}$$



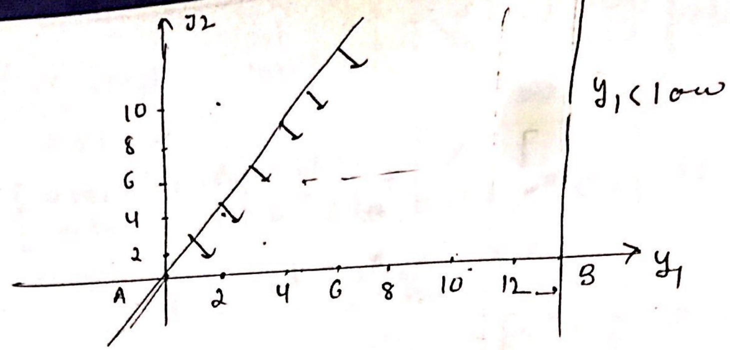
~~Q1~~ ~~AOPQ~~ Area OPQ
~~O(0,0)~~

Q(0, 0)

~~A(0, 1000)~~ P ~~(0, 600)~~

~~B(500, 1000)~~ P(400, 200)

Q(0, 600)



A (0,0)
B (1000,0)
C = (1000,2000)