Transformer – I

1.1 PRINCIPLE OF TRANSFORMER OPERATION

A transformer is a static device which consists of two or more stationary electric circuits interlinked by a common magnetic circuit for the purpose of transferring electrical energy between them. The transfer of energy from one circuit to another takes place without a change in frequency.

Consider two coils 1 and 2 wound on a simple magnetic circuit as shown in Fig. 1.1. These two coils are insulated from each other and there is no electrical connection between them. Let T_1 and T_2 be the number of turns in coils 1 and 2 respectively. When a source of alternating voltage V_1 is applied to coil 1, an alternating current I_0 flows in it. This alternating current produces an alternating flux Φ_M in the magnetic circuit. The mean path of this flux is shown in Fig. 1.1 by the dotted line. This alternating flux links the turns T_1 of coil 1 and induces in them an alternating voltage E_1 by **self-induction**.

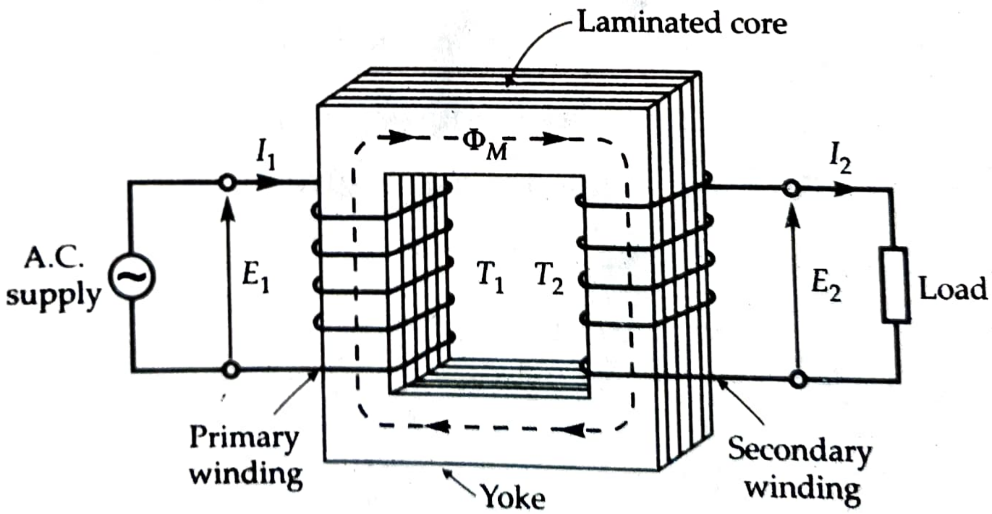

Fig. 1.1 Arrangement of a simple transformer.

Let us make the following simplifying *assumptions* for an *ideal transformer* :

(*a*) There are no losses either in the electric circuits or in the magnetic circuit.

(*b*) The whole of the magnetic flux Φ is confined to the magnetic circuit, so that there is no leakage flux.

(*c*) The permeability of the core is infinite.

Thus, *all* the flux produced by coil 1 also links T_2 turns of coil 2 and induces in them an alternating voltage E_2 by **mutual induction**. If coil 2 is connected to a load then an alternating current will flow through it and energy will be delivered to the load. Thus, electrical energy is transferred from coil 1 to coil 2 by a common magnetic circuit. *Since there is no relative motion between the coils, the frequency of the induced voltage in coil 2 is exactly the same as the frequency of the applied voltage to coil 1.*

Coil 1 which receives energy from the source of a.c. supply is called the *primary coil* or *primary winding* or simply the *primary*. Coil 2, which is connected to load and delivers energy to the load, is called the *secondary coil* or *secondary winding* or simply the *secondary*. The circuit symbol for a two-winding transformer is shown in Fig. 1.2. The two vertical bars are used to signify tight magnetic coupling between the windings.

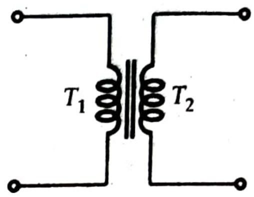

Fig. 1.2 Circuit symbol for a 2-winding transformer.

1.2 E.M.F. EQUATION OF A TRANSFORMER

Let the flux at any instant be given by

$$\Phi = \Phi_m \sin \omega t \qquad (1.2.1)$$

The instantaneous e.m.f. induced in a coil of T turns linked by this flux is given by Faraday's law as

$$e = -\frac{d}{dt}(\Phi T) = -T\frac{d\Phi}{dt} = -T\frac{d}{dt}(\Phi_m \sin \omega t) = -T\omega\Phi_m \cos \omega t$$

$$= T\omega\Phi_m \sin(\omega t - \pi/2) \qquad (1.2.2)$$

Equation (1.2.2) may be written as $e = E_m \sin(\omega t - \pi/2)$ (1.2.3)

where $E_m = T\omega\Phi_m$ = maximum value of induced e.m.f. e.

For a sine wave, the r.m.s. value of e.m.f. is given by

$$E_{rms} = E = E_m/\sqrt{2}$$

$$\therefore \quad E = \frac{T\omega\Phi_m}{\sqrt{2}} = \frac{T(2\pi f)\Phi_m}{\sqrt{2}}$$

or

$$E = 4.44\,\Phi_m f\,T \qquad (1.2.4)$$

Equation (1.2.4) is called the *e.m.f. equation of a transformer*.

The e.m.f. induced in each winding of the transformer can be calculated from its e.m.f. equation. Let subscripts 1 and 2 be used for primary and secondary quantities. The primary r.m.s. voltage is

$$E_1 = 4.44\, \Phi_m f\, T_1 \tag{1.2.5}$$

The secondary r.m.s. voltage is

$$E_2 = 4.44\, \Phi_m f\, T_2 \tag{1.2.6}$$

where Φ_m is the maximum value of flux in webers (Wb), f is the frequency in hertz (Hz) and E_1 and E_2 are in volts.

If B_m = maximum flux density in the magnetic circuit (core) in tesla (T).

A = area of cross-section of the core in square metres (m^2)

then
$$B_m = \frac{\Phi_m}{A} \tag{1.2.7}$$

It should be noted that

1 tesla (T) = 1 Wb/m^2.

The winding with higher number of turns will have a high voltage and is called the high-voltage (*hv*) winding. The winding with the lower number of turns is called the low-voltage (*lv*) winding.

Also, from Eq. (1.2.4),

$$\frac{E}{f} = 4.44\, \Phi_m T = \text{constant}$$

It is to be noted that the induced voltage per unit frequency is constant but not same on both primary and secondary side for a given transformer.

1.3 VOLTAGE RATIO AND TURNS RATIO

The ratio E/T is called **voltage per turn.**

From Eq. (1.2.5), primary volts per turn,
$$\frac{E_1}{T_1} = 4.44\, \Phi_m f \tag{1.3.1}$$

From Eq. (1.2.6), secondary volts per turn,
$$\frac{E_2}{T_2} = 4.44\, \Phi_m f \tag{1.3.2}$$

Equations (1.3.1) and (1.3.2) show that ***the voltage per turn in both the windings is same.*** That is,

$$\frac{E_1}{T_1} = \frac{E_2}{T_2} \tag{1.3.3}$$

Also,
$$\frac{E_1}{E_2} = \frac{T_1}{T_2} \tag{1.3.4}$$

The ratio $\frac{T_1}{T_2}$ is called ***turns ratio.***

The ratio of primary to secondary turns $\left(\frac{T_1}{T_2}\right)$ which equals the ratio of primary to secondary induced voltages $\left(\frac{E_1}{E_2}\right)$, indicates how much the primary voltage is lowered or raised. The turn ratio, or the induced voltage ratio, is called the **transformation ratio** and is denoted by the symbol a. Thus,

$$a = \frac{E_1}{E_2} = \frac{T_1}{T_2} \tag{1.3.5}$$

In a practical voltage transformer, there is a very small difference between the terminal voltage and the induced voltage. Therefore, we can assume that $E_1 = V_1$ and $E_2 = V_2$. Equation (1.3.5) is modified as

$$\frac{V_1}{V_2} = \frac{T_1}{T_2} = a \tag{1.3.6}$$

If a voltage ratio or turns ratio is specified, this is always put in the order input : output, which is primary : secondary. It is to be noted from Eq. (1.3.6) that almost any desired voltage ratio can be obtained by adjusting the number of turns.

1.4 STEP-UP AND STEP-DOWN TRANSFORMERS

A transformer in which the output (secondary) voltage is greater than its input (primary) voltage is called a **step-up transformer.**

A transformer in which the output (secondary) voltage is less than its input (primary) voltage is called a **step-down transformer.**

The same transformer can be used as a step-up transformer or a step-down transformer depending on the way it is connected in the circuit. When the transformer is used as a step-up transformer, the low voltage winding is the primary. In a step-down transformer, the high-voltage winding is the primary.

A transformer may receive energy at one voltage and deliver it at the same voltage. Such a transformer is called a **one-to-one (1 : 1) transformer.** For a 1 : 1 transformer $T_1 = T_2$ and $|E_1| = |E_2|$. Such a transformer is used to isolate two circuits.

EXAMPLE 1.1 *A 3300/250 V, 50 Hz, single-phase transformer is built on a core having an effective cross-sectional area of 125 cm² and 70 turns on the low-voltage winding. Calculate (a) the value of the maximum flux density, (b) the number of turns on the high voltage winding.*

SOLUTION. $E_1 = 3300\text{ V}, \quad E_2 = 250\text{ V}, \quad f = 50\text{ Hz}$

$$A = 125\text{ cm}^2 = 125 \times 10^{-4}\text{ m}^2$$

$$E_2 = 4.44\,\Phi_m f T_2 = 4.44\, B_m A f T_2$$

$$B_m = \frac{E_2}{4.44\,AfT_2} = \frac{250}{4.44 \times 125 \times 10^{-4} \times 50 \times 70} = 1.287 \text{ teslas (T)}$$

$$\frac{E_1}{E_2} = \frac{T_1}{T_2}$$

$$T_1 = \frac{E_1}{E_2} \times T_2 = \frac{3300}{250} \times 70 = 924$$

EXAMPLE 1.2 *A transformer with 800 primary turns and 200 secondary turns is supplied from a 100 V a.c. supply. Calculate the secondary voltage and the volts per turn.*

SOLUTION. $T_1 = 800,\ T_2 = 200,\ V_1 = 100$ V

$$\frac{V_1}{V_2} = \frac{T_1}{T_2},\quad V_2 = V_1 \times \frac{T_2}{T_1} = 100 \times \frac{200}{800} = 25 \text{ V}$$

$$\text{Volts per turn} = \frac{V_1}{T_1} = \frac{100}{800} = 0.125$$

$$\text{or}\quad \text{Volts per turn} = \frac{V_2}{T_2} = \frac{25}{200} = 0.125$$

EXAMPLE 1.3 *A transformer with an output voltage of 4200 V is supplied at 230 V. If the secondary has 2000 turns, calculate the number of primary turns.*

SOLUTION. $V_2 = 4200 \text{ V},\ V_1 = 230 \text{ V},\ T_2 = 2000$

$$\frac{V_1}{V_2} = \frac{T_1}{T_2}$$

$$T_1 = \frac{T_2 V_1}{V_2} = 2000 \times \frac{230}{4200} = 109.52 \text{ turns}$$

In practice, it is not possible for a winding to have part of a turn (that is, turns cannot be fractional). Therefore, the number of turns should be a whole number. In our case we shall take $T_1 = 110$.

1.5 CONSTRUCTION OF SINGLE-PHASE TRANSFORMERS

A single-phase transformer consists of primary and secondary windings put on a magnetic core. Magnetic core is used to confine flux to a definite path. Transformer cores are made from thin sheets (called *laminations*) of high-grade silicon steel. The laminations reduce eddy-current loss and the silicon steel reduces hysteresis loss. The laminations are insulated from one another by heat resistant enamel insulation coating. *L*-type and *E*-type laminations are used. The laminations are built up into stack and the joints in the laminations are staggered to minimize airgaps (which require large exciting currents). The laminations are tightly clamped.

There are *two* basic types of transformer constructions, the *core type* and the *shell type*.

1.5.1 Core-type Construction

In the core-type transformer, the magnetic circuit consists of two vertical *legs* or *limbs* with two horizontal sections, called *yokes*. To keep the leakage flux to a minimum, half of each winding is placed on each leg of the core as shown in Fig. 1.3. The low-voltage winding is placed next to the core and the high-voltage winding is placed around the low-voltage winding to reduce the insulating material required. Thus, the two windings are arranged as *concentric coils*. Such a winding is, therefore, called *concentric winding* or *cylindrical winding*.

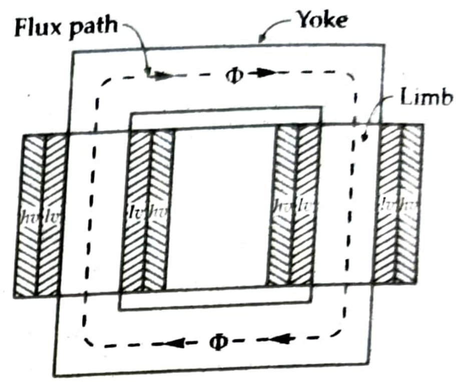

Fig. 1.3 Core-type transformer.

1.5.2 Shell-type Transformer

In the shell-type transformer (Fig. 1.4), both primary and secondary windings are wound on the central limb, and the two outer limbs complete the low-reluctance flux paths. Each winding is subdivided into sections. Low-voltage (*lv*) and high-voltage (*hv*) subsections are alternately put in the form of a sandwich. Such a winding is, therefore, called *sandwich* or *disc winding*.

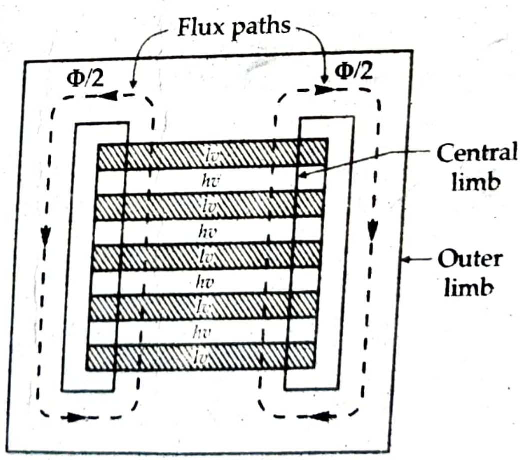

Fig. 1.4 Shell-type transformer.

A core must be made up of at least two types of laminations. The laminations for the core-type transformers are of *U* and *I* shape as shown in Fig. 1.5(*a*). The *U*-shaped laminations are first stacked together for the required length. Half of the prewound low voltage (*lv*) coil is placed around the limbs. The *lv* coil is further

The core-type transformer is easier to dismantle for repair. The shell-type transformer gives better support against electromagnetic forces between the current-carrying conductors. These forces are of considerable magnitude under short-circuit conditions. Shell-type transformers provide a shorter magnetic path, and hence magnetizing current is lesser than that in the core-type transformer. The natural cooling is poor in a shell-type transformer due to the embedding of the coils.

1.6 IDEAL TRANSFORMER

An **ideal transformer** is an imaginary transformer which has the following properties :

(*i*) Its primary and secondary winding resistances are negligible.

(*ii*) The core has infinite permeability (μ) so that negligible mmf is required to establish the flux in the core.

(*iii*) Its leakage flux and leakage inductances are zero. The entire flux is confined to the core and links both windings.

(*iv*) There are no losses due to resistance, hysteresis and eddy currents. Thus, the efficiency is 100 per cent.

It is to be noted that practical (commercial) transformer has none of these properties inspite of the fact that its operation is close to ideal.

An ideal iron-core transformer is shown in Fig. 1.7. It consists of two coils *wound in the same direction* on a common magnetic core. The winding connected to the supply, $\mathbf{V}_1$, is called the *primary*. The winding connected to the load, $\mathbf{Z}_L$, is called the *secondary*.

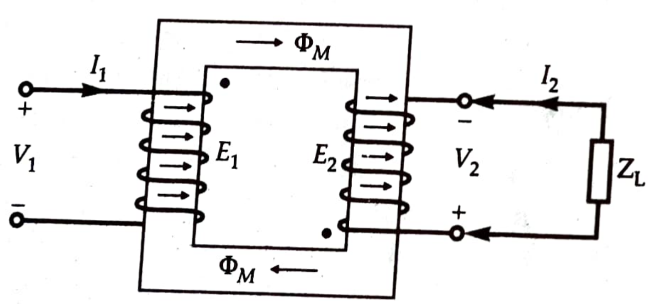

Fig. 1.7 Ideal iron-core transformer.

Since the ideal transformer has zero primary and zero secondary impedance, the voltage induced in the primary $\mathbf{E}_1$ is equal to the applied voltage $\mathbf{V}_1$. Similarly, the secondary voltage $\mathbf{V}_2$ is equal to the secondary induced voltage $\mathbf{E}_2$. The current $\mathbf{I}_1$ drawn from the supply is just sufficient to produce mutual flux Φ_M and the required magnetomotive force (mmf) $\mathbf{I}_1 T_1$ to overcome the demagnetizing effect of the secondary mmf $\mathbf{I}_2 T_2$ as a result of connected load.

By Lenz's law $\mathbf{E}_1$ is equal and opposite to $\mathbf{V}_1$. Since $\mathbf{E}_2$ and $\mathbf{E}_1$ are both induced by the same mutual flux, $\mathbf{E}_2$ is in the same direction as $\mathbf{E}_1{}'$ but opposite to $\mathbf{V}_1$. The magnetizing current $\mathbf{I}_\mu$ lags $\mathbf{V}_1$ by 90° and produces Φ_M in phase with $\mathbf{I}_\mu$. $\mathbf{E}_1$ and $\mathbf{E}_2$ lag Φ_M by 90° and are produced by Φ_M. $\mathbf{V}_2$ is equal in magnitude to $\mathbf{E}_2$ and is opposite to $\mathbf{V}_1$. Figure 1.8 shows the no-load phasor diagram of the ideal transformer.

For an ideal transformer, if

$$a = \text{transformation ratio} = \text{turn ratio}$$

then,
$$a = \frac{T_1}{T_2} = \frac{\mathbf{E}_1}{\mathbf{E}_2} = \frac{\mathbf{V}_1}{\mathbf{V}_2} = \frac{\mathbf{I}_2}{\mathbf{I}_1} \tag{1.6.1}$$

$$\therefore \quad \mathbf{I}_1 T_1 = \mathbf{I}_2 T_2 \tag{1.6.2}$$

$$\mathbf{E}_1 \mathbf{I}_1 = \mathbf{E}_2 \mathbf{I}_2 = \mathbf{S}_2 = \mathbf{S}_1 \tag{1.6.3}$$

$$\mathbf{V}_1 \mathbf{I}_1 = \mathbf{V}_2 \mathbf{I}_2 = \mathbf{S}_2 = \mathbf{S}_1 \tag{1.6.4}$$

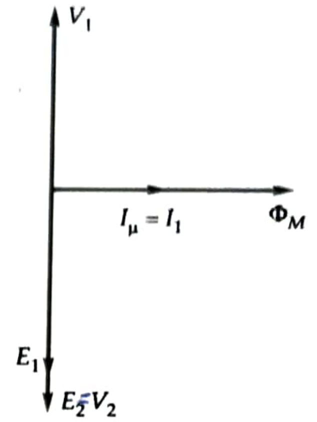

Fig. 1.8 No-load phasor diagram of an ideal transformer.

Equation (1.6.2) states that the demagnetizing ampere-turns of the secondary are equal and opposite to the magnetizing mmf of the primary of an *ideal* transformer.

Equation (1.6.3) shows that the voltamperes (apparent power) drawn from the primary supply is equal to the voltamperes (apparent power) transferred to the secondary without any loss in an *ideal* transformer. In other words,

$$\text{input voltamperes} = \text{output voltamperes}$$

Also,
$$\frac{V_1 I_1}{1000} = \frac{V_2 I_2}{1000}$$

$$(kVA)_1 = (kVA)_2 \tag{1.6.5}$$

or $\quad$ **input kilovoltamperes = output kilovoltamperes**

Thus, *the kVA input of an ideal transformer is equal to the kVA output. That is, kVA is same on both the sides of the transformer.*

EXAMPLE 1.4 *A 25 kVA transformer has a voltage ratio of 3300 / 400 V. Calculate the primary and secondary currents.*

SOLUTION.
$$\text{kVA} = \frac{V_1 I_1}{1000}$$

$$I_1 = \frac{1000 \times \text{kVA}}{V_1} = \frac{1000 \times 25}{3300} = 7.58 \text{ A}$$

Also,
$$\text{kVA} = \frac{V_2 I_2}{1000}$$

$$I_2 = \frac{1000 \times \text{kVA}}{V_2} = \frac{1000 \times 25}{400} = 62.5 \text{ A}$$

EXAMPLE 1.5 *A 125 kVA transformer having primary voltage of 2000 V at 50 Hz has 182 primary and 40 secondary turns. Neglecting losses, calculate (a) the full-load primary and secondary currents, (b) the no-load secondary induced e.m.f. and (c) the maximum flux in the core.*

SOLUTION. (*a*) $$\text{kVA} = \frac{V_1 I_1}{1000} = \frac{V_2 I_2}{1000}$$

$$I_1 = \frac{1000 \times \text{kVA}}{V_1} = \frac{1000 \times 125}{2000} = 62.5 \text{ A}$$

$$I_2 T_2 = I_1 T_1$$

$$I_2 = \frac{I_1 T_1}{T_2} = \frac{62.5 \times 182}{40} = 284.4 \text{ A}$$

(*b*) $$\frac{E_2}{T_2} = \frac{E_1}{T_1}$$

$$E_2 = \frac{E_1 T_2}{T_1} = \frac{2000 \times 40}{182} = 439.6 \text{ V}$$

(c) $$E_1 = 4.44\, \Phi_m f T_1$$

$$\Phi_m = \frac{E_1}{4.44\, f T_1} = \frac{2000}{4.44 \times 50 \times 182} = 0.0495 \text{ Wb}$$

1.7 TRANSFORMER ON NO-LOAD

A transformer is said to be on *no-load* when the secondary winding is open-circuited. The secondary current is thus zero. When an alternating voltage is applied to the primary, a small current I_0 flows in the *primary*. The current I_0 is called the *no-load current* of the transformer. It is made up of two components I_μ and I_W. The component I_μ is called the *magnetizing component*. It magnetizes the core. In other words, it sets up a flux in the core and therefore I_μ is in phase with Φ_M. The current I_μ is also called *reactive*, or *wattless component of no-load current*.

The component I_W supplies the hysteresis and eddy-current losses in the core and the negligible I^2R loss in the primary winding. The current I_W is called the *active component* or *wattful component of no-load current*. It is in phase with the applied voltage V_1. The no-load current I_0 is small of the order of 3 to 5 percent of the rated current of the primary.

1.8 PHASOR DIAGRAM OF TRANSFORMER ON NO-LOAD

An approximate phasor diagram for a transformer under no-load conditions is shown in Fig. 1.9. The flux Φ_M is taken as the reference phasor.

For a transformer on no-load, we have

$$\Phi = \Phi_M \sin \omega t$$
$$e_1 = E_{1m} \sin(\omega t - \pi/2)$$
$$e_2 = E_{2m} \sin(\omega t - \pi/2)$$

Since $\mathbf{E}_1$ and $\mathbf{E}_2$ are induced by the same flux Φ, they will be in phase with each other. E_2 differs in magnitude from E_1 because $E_2 = E_1 \frac{T_2}{T_1} = \frac{E_1}{a}$.

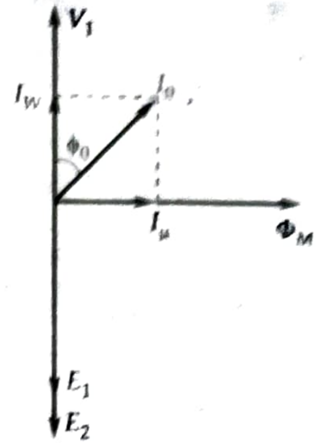

Fig. 1.9 Phasor diagram at no-load.

The above equations show that $\mathbf{E}_1$ and $\mathbf{E}_2$ lag behind Φ by 90°. If the voltage drops in the primary winding are neglected $\mathbf{E}_1$ will be equal and opposite to the applied voltage $\mathbf{V}_1$. $\mathbf{I}_\mu$ is in phase with Φ and $\mathbf{I}_W$ is in phase with $\mathbf{V}_1$. The phasor sum of $\mathbf{I}_\mu$ and $\mathbf{I}_W$ is $\mathbf{I}_0$. Angle ϕ_0 is called the *no-load power factor angle,* hence the power factor on no-load is $\cos \phi_0$.

From the phasor diagram of Fig. 1.9.

$$I_W = I_0 \cos \phi_0 \qquad (1.8.1)$$

$$I_\mu = I_0 \sin \phi_0 \qquad (1.8.2)$$

$$I_0 = \sqrt{I_W^2 + I_\mu^2} \qquad (1.8.3)$$

$$\cos \phi_0 = \frac{I_W}{I_0} \qquad (1.8.4)$$

Also, core loss $= V_1 I_0 \cos \phi_0 = V_1 I_W$ W (1.8.5)

Magnetizing (reactive) voltamperes

$$= V_1 I_0 \sin \phi_0 = V_1 I_\mu \text{ VAr} \qquad (1.8.6)$$

EXAMPLE 1.6 *Find the active and reactive components of no-load current, and the no-load current of a 440/220 V single-phase transformer if the power input to the hv winding is 80 W. The low-voltage winding is kept open. The power factor of the no-load current is 0.3 lagging.*

SOLUTION. $P_o = V_1 I_o \cos \phi_o$

Active component of no-load current

$$I_W = I_o \cos \phi_o = \frac{P_o}{V_1} = \frac{80}{440} = 0.182 \text{ A}$$

No-load current $$I_o = \frac{P_o}{V_1 \cos \phi_o} = \frac{80}{440 \times 0.3} = 0.606 \text{ A}$$

current I_2 produces a flux Φ_2 which opposes the main flux Φ_M. A portion of this flux is also diverted to the surrounding medium. This leakage flux is called the **secondary leakage flux Φ_{L_2}**. It only links the secondary turns and induces an emf E_{L_2} in the secondary. Thus, each leakage flux links one winding only and it is caused by the current in that winding alone. The flux which passes completely through the core and links *both* windings is called mutual flux and is shown as Φ_M in Fig. 1.12.

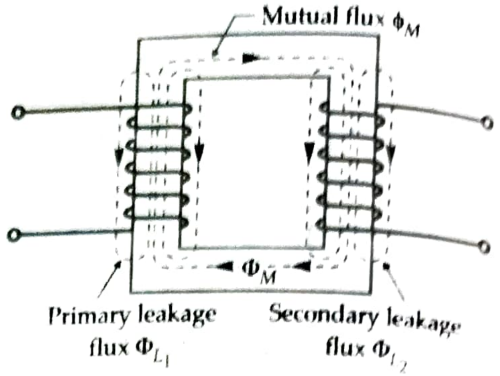

Fig. 1.12

It should be noted that the induced voltages E_{L_1} and E_{L_2} due to leakage fluxes Φ_{L_1} and Φ_{L_2} are different from induced voltages E_1 and E_2 caused by the mutual flux Φ_M. As the leakage flux linking with each winding produces a self-induced emf in that winding, hence the effect of leakage flux is equivalent to an inductance in series with each winding such that the voltage drop in each series inductance is equal to that produced by the leakage flux. In other words, a transformer with magnetic flux leakage is equivalent to an ideal transformer with inductive reactances X_1 and X_2 connected in series with the primary and secondary windings respectively as shown in Fig. 1.13. The quantities X_1 and X_2 are known as **primary and secondary leakage reactances respectively.**

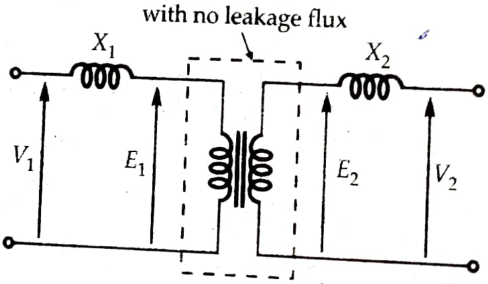

Fig. 1.13 Representation of leakage fluxes with reactances X_1 and X_2.

It should be noted that X_1 and X_2 are fictitious quantities introduced as a convenience in representing the effects of primary and secondary leakage fluxes.

1.13 REFERRED VALUES

In order to simplify calculations, it is *theoretically* possible to transfer voltage, current, and impedance of one winding to the other and combine them into single values for each quantity. Thus, we have to work in one winding only which is more convenient.

Let us transfer the resistance of the secondary winding R_2 to the primary side. Suppose that R_2' is the resistance of the secondary winding referred or reflected to the primary winding. This reflected resistance R_2' should produce the same effect in primary as R_2 produces in secondary. Therefore the power consumed by R_2' when carrying the primary current is equal to the power consumed by R_2 due to the secondary current.

That is, $$I_2'^2 R_2' = I_2^2 R_2$$

$$R_2' = \left(\frac{I_2}{I_2'}\right)^2 R_2$$

But $$I_2 T_2 = I_2' T_1$$

$$\therefore \quad \frac{I_2}{I_2'} = \frac{T_1}{T_2} = a$$

and $$R_2' = \left(\frac{T_1}{T_2}\right)^2 R_2 = a^2 R_2 \tag{1.13.1}$$

Let X_2' be the reactance of the secondary winding reflected or referred to the primary side. For X_2' to produce the same effect in the primary side as X_2 in the secondary side, each must absorb the same reactive voltamperes (VAr).

$$\text{VAr} = VI \sin\phi = IZ.\,I.\frac{X}{Z} = I^2 X$$

Equating the reactive voltamperes consumed by X_2' and X_2 gives

$$(I_2')^2 X_2' = I_2^2 X_2$$

$$X_2' = \left(\frac{I_2}{I_2'}\right)^2 X_2$$

$$\therefore \quad X_2' = \left(\frac{T_1}{T_2}\right)^2 X_2' = a^2 X_2 \tag{1.13.2}$$

Let R_{e_1}, X_{e_1}, and Z_{e_1} represent the effective resistance, effective reactance, and effective impedance respectively of the whole transformer referred to the primary, then

$$R_{e_1} = \text{primary resistance} + \text{secondary resistance referred to primary}$$

$$\therefore \quad R_{e_1} = R_1 + R_2' = R_1 + R_2\left(\frac{T_1}{T_2}\right)^2 = R_1 + a^2 R_2 \tag{1.13.3}$$

$$X_{e_1} = \text{primary reactance} + \text{secondary reactance referred to primary}$$

$$X_{e_1} = X_1 + X_2' = X_1 + X_2\left(\frac{T_1}{T_2}\right)^2 = X_1 + a^2 X_2 \tag{1.13.4}$$

Z_{e_1} = primary impedance + secondary impedance referred to primary

$$Z_{e_1} = Z_1 + Z_2' = Z_1 + Z_2\left(\frac{T_1}{T_2}\right)^2 = Z_1 + a^2 Z_2 \tag{1.13.5}$$

Also, $$Z_{e_1} = \sqrt{R_{e_1}^2 + X_{e_1}^2}, \quad \mathbf{Z}_{e_1} = R_{e_1} + jX_{e_1} \tag{1.13.6}$$

The load impedance Z_L referred to primary is $a^2 Z_L$. It may also be taken into account by adding its resistive and reactive components to R_{e_1} and X_{e_1} respectively.

Equivalent values referred to secondary

The equivalent values referred to secondary can also be found in the same manner. If R_{e_2}, X_{e_2} and Z_{e_2} denote the equivalent resistance, equivalent reactance, and equivalent impedance respectively of the *whole* transformer referred to secondary, then

R_{e_2} = secondary resistance + primary resistance referred to secondary

$$R_{e_2} = R_2 + R_1' = R_2 + R_1\left(\frac{T_2}{T_1}\right)^2 = R_2 + \frac{R_1}{a^2} \tag{1.13.7}$$

X_{e_2} = secondary resistance + primary resistance referred to secondary

$$X_{e_2} = X_2 + X_1' = X_2 + X_1\left(\frac{T_2}{T_1}\right)^2 = X_2 + \frac{X_1}{a^2} \tag{1.13.8}$$

Z_{e_2} = secondary impedance + primary impedance referred to primary

$$Z_{e_2} = Z_2 + Z_1' = Z_2 + Z_1\left(\frac{T_2}{T_1}\right)^2 = Z_2 + \frac{Z_1}{a^2} \tag{1.13.9}$$

Also, $$\mathbf{Z}_{e_2} = R_{e_2} + jX_{e_2}$$

$$Z_{e_2} = \sqrt{R_{e_2}^2 + X_{e_2}^2}$$

$$R_{e_1} = R_1 + R_2\left(\frac{T_1}{T_2}\right)^2$$

1.14 DERIVATION OF EQUIVALENT CIRCUIT OF A TRANSFORMER

Figure 1.15 shows the complete equivalent circuit of a transformer. An exact equivalent circuit referred to the primary can be deduced as follows :

(i) All secondary resistances and reactances are reflected to the primary as a multiple of the square of the transformation ratio. That is, the quantities R_2, X_2 and $\mathbf{Z}_L$ in the secondary become a^2R_2, a^2X_2 and $a^2\mathbf{Z}_L$ respectively, when referred to the primary.

(ii) All voltages are reflected from secondary to primary directly as the product of the transformation ratio. That is, $\mathbf{V}_2$ and $\mathbf{E}_2$ in the secondary become $a\mathbf{V}_2$ and $a\mathbf{E}_2$ respectively, when referred to the primary.

(iii) All secondary currents are reflected to the primary inversely as the transformation ratio. Thus, $\mathbf{I}_2$ in the secondary becomes $\frac{\mathbf{I}_2}{a}$ when referred to the primary.

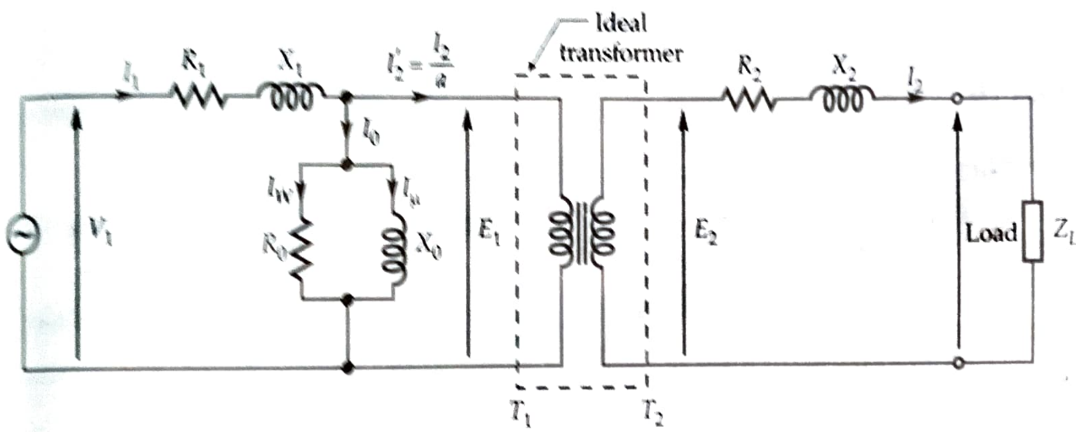

Fig. 1.15 Complete circuit model (equivalent circuit) of a real transformer.

Figure 1.16 shows the equivalent circuit with all *secondary* values referred (reflected) to the primary. This circuit is called the **exact equivalent circuit of the transformer referred to the primary.** With this model, it is not possible to add directly the primary impedance ($R_1 + jX_1$) to the secondary impedance reflected to the primary side.

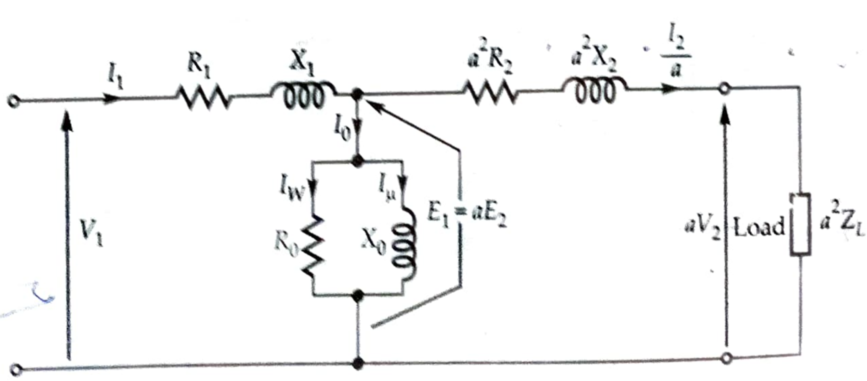

Fig. 1.16 Exact equivalent circuit of a transformer referred to the primary.

Figure 1.17 shows the approximate equivalent circuit referred to the primary. The justification for approximation is as follows :

The no-load current I_0 is usually less than 5 per cent of the full-load primary current. The voltage drop produced by I_0 in $(R_1 + jX_1)$ is negligible for practical purposes. Therefore, it is immaterial that the shunt branch $R_0 \parallel X_0$ is connected before the primary series impedance $(R_1 + jX_1)$ or after it. The currents I_W and I_μ are not much affected. Therefore, the parallel branch $R_0 \parallel X_0$ is connected to the input terminals as shown in Fig. 1.17.

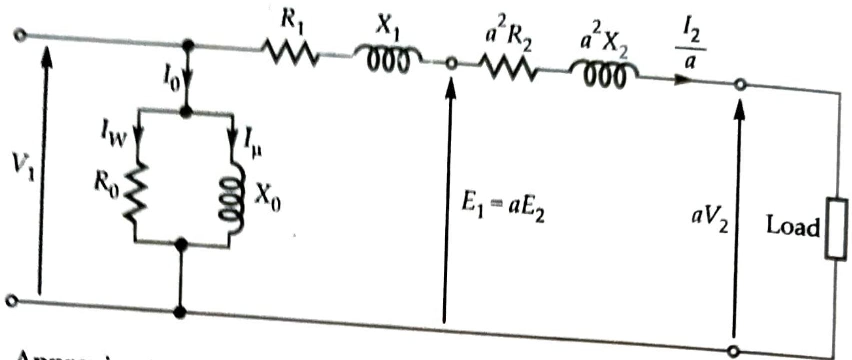

Fig. 1.17 Approximate equivalent circuit of the transformer referred to the primary.

This approximation gives the following advantages :

(*a*) The primary and secondary impedance reflected to the primary can be added conveniently.

(*b*) The calculations for voltage regulation of the transformer become easier.

The following results are obtained from the approximate equivalent circuit of Fig. 1.17.

$$R_{e_1} = R_1 + a^2R_2 \; ; \quad X_{e_1} = X_1 + a^2X_2 \; ; \quad \mathbf{Z}_{e_1} = R_{e_1} + jX_{e_1} \; ; \quad \mathbf{I}_1 = \frac{\mathbf{V}_1}{\mathbf{Z}_{e_1} + a^2\mathbf{Z}_L}$$

where $\mathbf{Z}_L$ is the load impedance.

1.15 APPROXIMATE EQUIVALENT CIRCUIT REFERRED TO THE SECONDARY

The approximate equivalent circuit of the transformer referred to the secondary can also be found in the same manner. Here we transfer all primary resistances and reactances to the secondary. The approximate equivalent circuit referred to the secondary is shown in Fig. 1.18.

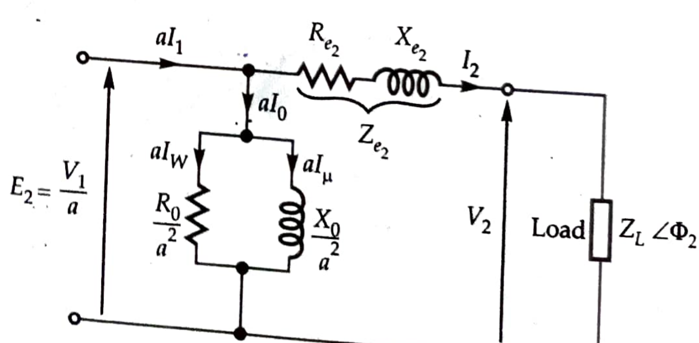

Fig. 1.18 Approximate equivalent circuit of the transformer referred to the secondary.

The phasor sum of $\mathbf{V}_2$ and $\mathbf{I}_2\mathbf{Z}_{e_2}$ is $\dfrac{\mathbf{V}_1}{a}$ (or $\mathbf{E}_2$) and $\dfrac{\mathbf{V}_1}{a}$ represents the secondary no-load voltage.

Also, $$\mathbf{E}_2 = \frac{\mathbf{V}_1}{a} = \mathbf{V}_2 + \mathbf{I}_2\mathbf{Z}_{e_2} \qquad (1.15.2)$$

The significance of Eq. (1.15.2) is that it permits the calculation of the voltage regulation of the transformer in terms of secondary (load) voltages.

1.16 FURTHER SIMPLIFICATION TO APPROXIMATE EQUIVALENT CIRCUIT

Since the no-load current of a transformer is about 3 to 5 per cent of the full-load primary current, its effect can be neglected. Hence for all practical purposes, the parallel circuit containing R_0 and X_0 can be omitted on full load without loss of accuracy. This is the further simplification to the approximate equivalent circuit of the transformer. The simplified circuit is shown in Fig. 1.20.

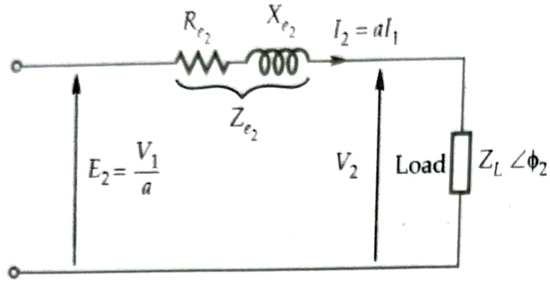

Fig. 1.20 Simplified equivalent circuit of the transformer referred to secondary.

1.17 FULL-LOAD PHASOR DIAGRAM

Figure 1.21 shows the phasor diagram for the exact circuit model of transformer of Fig. 1.15. It is assumed that the load is inductive, which is generally

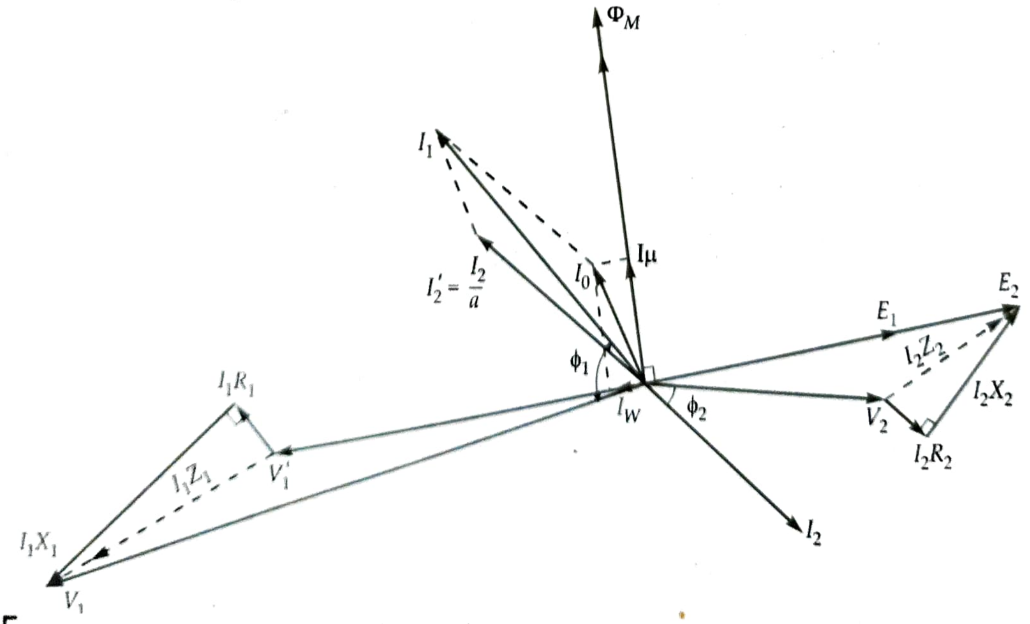

Fig. 1.21 Phasor diagram for the exact equivalent circuit of real transformer of Fig. 1.15.

the case. Let $\cos\phi_2$ be the power factor of the load (lagging). The phasor $\mathbf{V}_2$ is taken as reference. Since the load power factor is lagging, the secondary current $\mathbf{I}_2$ lags behind $\mathbf{V}_2$ by the power factor angle ϕ_2. The secondary current $\mathbf{I}_2$ flows through R_2 and X_2 and produces voltage drops across them equal to I_2R_2 and I_2X_2. The resistive voltage drop I_2R_2 is in phase with $\mathbf{I}_2$ and the inductive voltage drop I_2X_2 leads the current $\mathbf{I}_2$ by 90°. The secondary induced voltage $\mathbf{E}_2$ is the phasor sum of V_2, I_2R_2 and I_2X_2. That is,

$$\mathbf{E}_2 = \mathbf{V}_2 + \mathbf{I}_2\mathbf{Z}_2 \qquad (1.17.1)$$

$$\mathbf{E}_2 = \mathbf{V}_2 + I_2(R_2 + jX_2) \qquad (1.17.2)$$

The primary induced voltage $\mathbf{E}_1$ $(= a\mathbf{E}_2)$ is in time phase with $\mathbf{E}_2$ because both these voltages are induced by the same flux Φ_M. The flux Φ_M leads $\mathbf{E}_1$ by 90°.

The ampere turns of the secondary I_2T_2 must be balanced by a load component of current I_2 in the primary winding such that $I_2'T_1 = I_2T_2$. The current I_2' is called the *secondary current referred (reflected) to the primary*. Thus, the current I_2' represents the component of the primary current to neutralize the demagnetizing effect of the secondary current. The current $\mathbf{I}_2'\left(=\frac{\mathbf{I}_2}{a}\right)$ is therefore 180° out of phase with $\mathbf{I}_2$.

The current $\mathbf{I}_W$ is in phase opposition with $\mathbf{E}_1$ and $\mathbf{I}_\mu$ leads $\mathbf{E}_1$ by 90°. The no-load current $\mathbf{I}_0$ is the phasor sum of $\mathbf{I}_W$ and $\mathbf{I}_\mu$.

$$\mathbf{I}_0 = \mathbf{I}_W + \mathbf{I}_\mu$$

The total primary current $\mathbf{I}_1$ taken from the supply is the phasor sum of $\mathbf{I}_2'$ and $\mathbf{I}_0$.

$$\mathbf{I}_1 = \mathbf{I}_2' + \mathbf{I}_0$$

$$\mathbf{I}_1 = \frac{\mathbf{I}_2}{a} + \mathbf{I}_0$$

Since $\mathbf{E}_1$ is the voltage induced in the primary winding, it is equal and opposite to the component of the applied voltage at the ideal transformer winding. Let $\mathbf{V}_1'$ be the voltage applied to the primary of the ideal transformer to neutralize the effect of induced voltage $\mathbf{E}_1$. Thus $\mathbf{V}_1'$ is equal and opposite to $\mathbf{E}_1$. The phasor sum of I_1R_1, I_1X_1 and $\mathbf{V}_1'$ is equal to the supply voltage $\mathbf{V}_1$. That is,

$$\mathbf{V}_1 = \mathbf{V}_1' + I_1R_1 + jI_1X_1$$

$$\mathbf{V}_1' = -\mathbf{E}_1$$

The resistive voltage drop I_1R_1 is in phase with $\mathbf{I}_1$ and the inductive voltage drop I_1X_1 leads $\mathbf{I}_1$ by 90°. The angle between $\mathbf{V}_1$ and $\mathbf{I}_1$ is ϕ_1. Thus $\cos\phi_1$ is the power factor on the primary side. Power input to the transformer is given by $V_1I_1\cos\phi_1$.

(d) Equivalent leakage impedance referred to the high voltage side

$$Z_{e_1} = R_{e_1} + jX_{e_1} = 0.2 + j0.6 = 0.634 \angle 71.6^\circ \Omega$$

Equivalent leakage impedance referred to the low-voltage side

$$Z_{e_2} = R_{e_2} + jX_{e_2} = 0.008 + j0.024 = 0.0253 \angle 71.6^\circ \Omega$$

Alternatively

$$Z_{e_2} = Z_{e_1}\left(\frac{T_2}{T_1}\right)^2 = (0.634 \angle 71.6^\circ)\left(\frac{220}{1100}\right)^2 = 0.0253 \angle 71.6^\circ \Omega$$

1.30 LOSSES IN TRANSFORMER

The losses which occur in a transformer are :

(a) iron loss or core loss P_i

(b) copper loss or $I^2 R$ loss P_c

Iron loss or core loss P_i

Iron loss occurs in the magnetic core of the transformer. This loss is the sum of hysteresis loss (P_h) and eddy current loss (P_e).

$$P_i = P_h + P_e$$

The hysteresis and eddy current losses are given by

$$P_h = k_h f B_m^x$$

$$P_e = k_e f^2 B_m^2$$

where k_h = proportionality constant which depends upon the volume and quality of the core material and the units used

k_e = proportionality constant whose value depends upon the volume and resistivity of the core material, thickness of laminations and units used.

B_m = maximum flux density in the core

and f = frequency of the alternating flux

The exponent x is called **Steinmetz constant**. Its value varies from 1.5 to 2.5 depending upon the magnetic properties of the core material. The total core loss can be written as

$$P_i = P_h + P_e$$

$$P_i = k_h f B_m^x + k_e f^2 B_m^2 \tag{1.30.1}$$

Since the applied voltage is approximately equal to the induced voltage

$$V_1 = E_1 = 4.44\, \phi_m f T_1 = 4.44\, B_m A_i f T_1$$

$$B_m = \frac{V_1}{4.44\, A_i\, f\, T_1}$$

and
$$P_h = k_h f B_m^x = k_h f\left(\frac{V_1}{4.44\, A_i\, f\, T_1}\right)^x$$

$$= k_h\left(\frac{1}{4.44\, A_i T_1}\right)^x f\left(\frac{V_1}{f}\right)^x = K_h V_1^x f^{1-x}$$

where $K_h = k_h\left(\frac{1}{4.44\, A_i T_1}\right)^x$

This relation shows that *the hysteresis loss depends upon both the applied voltage and frequency.*

Also,
$$P_e = k_e f^2 B_m^2 = k_e f^2\left(\frac{V_1}{4.44\, A_i\, f\, T_1}\right)^2 = K_e V_1^2$$

where $K_e = k_e\left(\frac{1}{4.44 A_i T_1}\right)^2$.

This relation shows that *the eddy current loss is proportional to the square of the applied voltage* **and** *is independent of frequency.*

Since, $V_1 = 4.44 B_m A_i f T_1$,

$$V_1 \propto B_m f$$

Therefore for any given voltage if f decreases, B_m increases. Similarly, if f increases, B_m decreases.

The total core loss can be written as

$$P_i = K_h V^x f^{1-x} + K_e V^2 \tag{1.30.2}$$

Copper loss or I^2R loss (P_C)

Copper loss is the I^2R loss which takes place in the primary and secondary windings because of the winding resistances.

Total copper loss in a transformer = Primary winding copper loss + Secondary winding copper loss

$$P_C = I_1^2 R_1 + I_2^2 R_2$$

Since
$$I_1 T_1 = I_2 T_2$$

$$I_1 = I_2 \frac{T_2}{T_1}$$

$$\therefore \quad P_C = I_2^2\left(\frac{T_2}{T_1}\right)^2 R_1 + I_2^2 R_2 = I_2^2\left[R_2 + \left(\frac{T_2}{T_1}\right)^2 R_1\right] = I_2^2 R_{e_2}$$

Thus, copper loss varies as the square of the load current.

Again,

$$P_c = I_1^2 R_1 + I_2^2 R_2 = I_1^2 R_1 + \left(I_1 \frac{T_1}{T_2} \right)^2 R_2 = I_1^2 \left[R_1 + \left(\frac{T_1}{T_2} \right)^2 R_2 \right] = I_1^2 R$$

$$\therefore \quad P_c = I_1^2 R_{e_1} = I_2^2 R_{e_2} \quad (1.$$

Stray loss

Leakage flux in a transformer produces eddy currents in the conduc tanks etc. These eddy currents produce losses.

Dielectric loss

Dielectric loss occurs in insulating materials, that is, in the transformer and the solid insulation of transformers. This loss is significant only in h voltage transformers.

The stray loss and dielectric loss are usually small and negligible.

1.31 SEPARATION OF HYSTERESIS AND EDDY-CURRENT LOSSES

The transformer core loss P_i has two components namely hysteresis loss and eddy-current loss P_e.

$$P_i = P_h + P_e$$

$$P_i = K_h f B_m^x + K_e f^2 B_m^2 \quad (1.31.$$

where K_h = a constant whose value depends upon the ferromagnetic materia

B_m = maximum value of the flux density

f = supply frequency

The exponent x varies in the range 1.5 to 2.5 depending upon the material. F a given B_m the hysteresis loss varies directly as the frequency and the eddy-curren loss varies as the square of the frequency. That is,

$$P_h \propto f \quad \text{or} \quad P_h = af$$

and

$$P_e \propto f^2 \quad \text{or} \quad P_e = bf^2$$

where a and b are constants.

$$\therefore \quad P_i = af + bf^2 \quad (1.31.2$$

For separation of these two losses the no-load test is performed on th transformer. However, the primary of the transformer is connected to a variabl frequency and variable sinusoidal supply and the secondary is open circuited.

Now $\quad V \simeq E = 4.44\, f\, \Phi_m T$

or

$$\frac{V}{f} = 4.44\, B_m A_i T$$

For any transformer T and A_i are constants. Therefore B_m will remain constant if the test is conducted so that the ratio (V/f) is kept constant.

Dividing Eq. (1.31.2) by f, we get

$$P_i/f = bf + a \tag{1.31.3}$$

During this test, the applied voltage V and frequency f are varied together so that (V/f) is kept constant. The core loss is obtained at different frequencies. A graph of (P_i/f) versus frequency f is plotted. This graph is a straight line AB of the form $y = mx + c$, as shown in Fig. 1.25.

The intercept of the straight line on the vertical axis gives a and the slope of the line AB gives b. Thus, knowing the constants a and b, hysteresis and eddy-current losses can be separated.

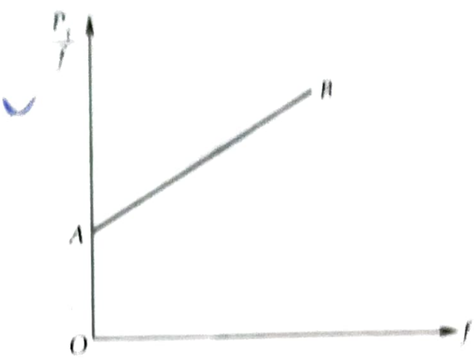

Fig. 1.25 Variation of (P_i/f) with f.

EXAMPLE 1.16 *In a transformer, the core loss is 100 W at 40 Hz and 72 W at 30 Hz. Find the hysteresis and eddy current losses at 50 Hz.*

SOLUTION. $$\frac{P_i}{f} = a + bf$$

$$\frac{100}{40} = a + 40\,b \quad \text{and} \quad \frac{72}{30} = a + 30\,b$$

Solution of these equation gives : $a = 2.1, \quad b = 0.01$

Therefore,

Hysteresis loss at 50 Hz $= af = 2.1 \times 50 = 105$ W

Eddy-current loss at 50 Hz $= bf^2 = 0.01 \times (50)^2 = 25$ W

EXAMPLE 1.17 *At 400 V and 50 Hz the total core loss of a transformer was found to be 2400 W. When the transformer is supplied at 200 V, and 25 Hz, the core loss is 800 W. Calculate the hysteresis and eddy current loss at 400 V and 50 Hz.*

SOLUTION. $$\frac{V_1}{f_1} = \frac{400}{50} = 8$$

$$\frac{V_2}{f_2} = \frac{200}{25} = 8$$

Since $$\frac{V_1}{f_1} = \frac{V_2}{f_2} = 8$$

the flux density B_m remains constant.

Hence $$\frac{P_i}{f} = a + bf$$

$$\frac{2400}{50} = a + 50\,b \quad \text{and} \quad \frac{800}{25} = a + 25\,b$$

Solving these equations, we get : $a = 16$, $b = 0.64$

Therefore, at 50 Hz

$$P_h = af = 16 \times 50 = 800 \text{ W}$$

$$P_e = bf^2 = 0.64 \times (50)^2 = 1600 \text{ W}$$

1.32 OPEN-CIRCUIT AND SHORT-CIRCUIT TESTS

Open-circuit and short-circuit tests are performed to determine the circuit constants, efficiency and regulation without actually loading the transformer. These tests give more accurate results than those obtained by tak ng measurements on fully loaded transformers. Also, the power consumption in these tests is very small compared with the full-load output of the transformer.

1.33 OPEN-CIRCUIT TEST

Figure 1.26 shows the connection diagram for the open circuit test. The high voltage (*hv*) side is left open.

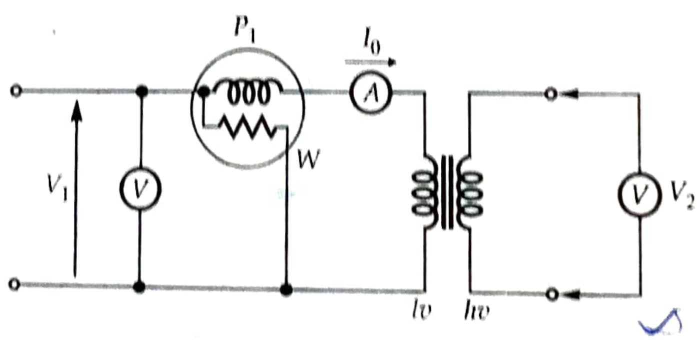

Fig. 1.26 Open-circuit test on a transformer.

A voltmeter *V*, an ammeter *A*, and a wattmeter *W* are connected in the low-voltage (*lv*) side (primary in our case) which is supplied at rated voltage and frequency. Thus, the voltmeter *V* reads the rated voltage V_1 of the primary. Since the secondary is open-circuited, a very small current I_0, called the no-load current, flows in the primary. The ammeter *A*, therefore, reads the no-load current I_0. The power loss in the transformer is due to core loss and a very small I^2R loss in the primary. There is no I^2R loss in the secondary since it is open and $I_2 = 0$. Since the no-load current I_0 is very small (usually 2 to 5 percent of the full-load primary current), the I^2R loss in the primary winding can be neglected. The core loss depends upon the flux. Since the rated voltage V_1 is applied, the flux set up by it will have a normal value so that normal core losses will occur. This core loss is the same at all loads. Therefore the wattmeter which is connected to measure input power reads the core loss (iron loss) P_i only. The readings of the instruments in an open-circuit test are as follows :

Ammeter reading = no-load current I_0

Voltmeter reading = primary rated voltage V_1

Wattmeter reading = iron or core loss P_i

From these measured values the components of the no-load equivalent circuit can be determined.

$$P_i = V_1 I_0 \cos\phi_0$$

The no-load power factor, $\cos\phi_0 = \dfrac{P_i}{V_1 I_0}$

$$I_W = I_0 \cos\phi_0 , \; I_\mu = I_0 \sin\phi_0$$

$$R_0 = \frac{V_1}{I_W}, \quad X_0 = \frac{V_1}{I_\mu}$$

1.34 SHORT-CIRCUIT TEST

In the short-circuit (SC) test (Fig. 1.27), usually the low-voltage side is short-circuited by a thick conductor (or through an ammeter which may serve an additional purpose of indicating rated load current). An ammeter, a voltmeter and a wattmeter are connected on the high-voltage side. The reasons for short-circuiting the *lv* side and taking measurements on the *hv* side are as follows :

(i) The rated current on *hv* side is lower than that on lv side This current can be safely measured with the available laboratory ammeters.

(ii) Since the applied voltage is less than 5 percent of the rated voltage of the winding, greater accuracy in the reading of the voltmeter is possible when the *hv* side is used as the primary.

The high voltage winding is supplied at the reduced voltage from a variable voltage supply. The supply voltage is gradually increased until full-load primary current flows. When the rated full-load current flows in the primary winding rated full-load current will flow in the secondary winding by transformer action.

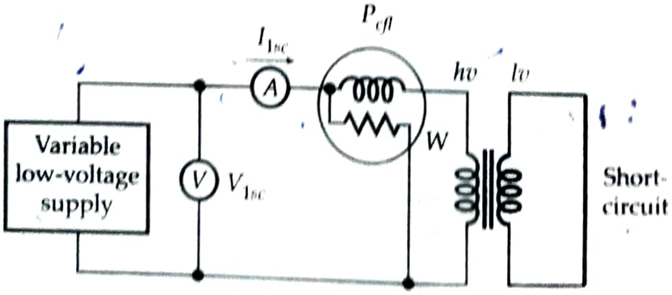

Fig. 1.27 Short-circuit test on a transformer.

Readings of the ammeter, voltmeter and wattmeter are noted. The ammeter reading I_{1SC} gives the full-load primary current. The voltmeter reading V_{1SC} gives the value of the primary applied voltage when full-load currents are flowing in the primary and secondary. Since the applied voltage is low (usually about 5 to 10 percent of the normal rated supply voltage), the flux Φ produced is low. Also, since the core loss is *nearly* proportional to the square of the flux, the core loss is so small that it can be neglected. However, the windings are carrying normal full-load currents and therefore the input is supplying the normal full-load copper losses. Thus the wattmeter gives the full-load copper losses P_{cfl}. The output voltage V_2 is zero because of the short-circuit. Consequently, whole of the primary voltage is used in supplying the voltage drop in the total impedance Z_{e_1} referred to the primary

$$V_{1SC} = I_{1SC} Z_{e_1}$$

If $\cos\phi_{SC}$ = power factor at short-circuit then $P_{cfl} = V_{1SC} I_{1SC} \cos\phi_{SC}$.

The readings of the instruments in a short-circuit test are as follows :

Ammeter reading = full-load primary current, I_{1SC}

Voltmeter reading = short-circuit voltage V_{1SC}

Wattmeter reading = full-load copper loss of the transformer P_{cfl}

From the readings of the instruments on short-circuit test, the following calculations can be made :

Equivalent resistance of the transformer referred to primary

$$R_{e_1} = \frac{P_{cfl}}{I_{1SC}^2}$$

Equivalent impedance referred to primary

$$Z_{e_1} = \frac{V_{1SC}}{I_{1SC}}$$

Equivalent reactance referred to primary

$$X_{e_1} = \sqrt{Z_{e_1}^2 - R_{e_1}^2} \quad \Rightarrow \quad \cos\phi_{sc} = \frac{R_{e_1}}{Z_{e_1}}$$

With short-circuit test performed only on one side the equivalent circuit constants referred to other side can also be calculated as follows :

$$Z_{e_2} = Z_{e_1}\left(\frac{T_2}{T_1}\right)^2 = \frac{Z_{e_1}}{a^2}$$

$$R_{e_2} = R_{e_1}\left(\frac{T_2}{T_1}\right)^2 = \frac{R_{e_1}}{a^2}$$

$$X_{e_2} = X_{e_1}\left(\frac{T_2}{T_1}\right)^2 = \frac{X_{e_1}}{a^2}$$

The equivalent circuit is shown in Fig. 1.29.

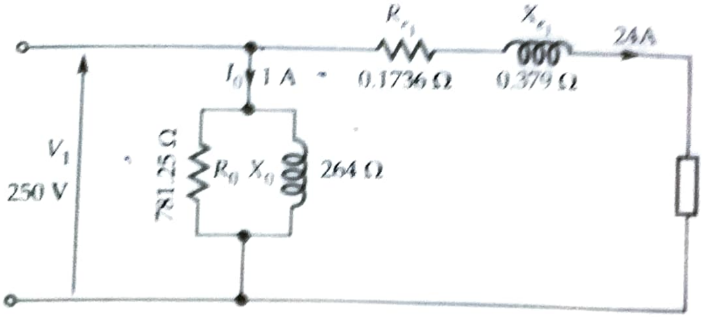

Fig. 1.29 Equivalent circuit of Example 1.19.

EXAMPLE 1.20 *A 50-Hz, 1-phase transformer has a turn-ratio of 6. The resistances are 0.90 Ω and 0.03 Ω, and the reactances 5 Ω and 0.13 Ω for high-voltage and low-voltage windings respectively. Find (a) the voltage to be applied to the high-voltage side to obtain full-load current of 200 A in the low-voltage winding on short-circuit, (b) the power factor on short circuit.*

SOLUTION. $$\frac{T_1}{T_2} = 6, \quad R_1 = 0.9\,\Omega, \quad R_2 = 0.03\,\Omega$$

$$X_1 = 5\,\Omega, \quad X_2 = 0.13\,\Omega$$

$$R_{e_1} = R_1 + R_2\left(\frac{T_1}{T_2}\right)^2 = 0.9 + 0.03(6)^2 = 1.98\,\Omega$$

$$X_{e_1} = X_1 + X_2\left(\frac{T_1}{T_2}\right)^2 = 5 + 0.13(6)^2 = 9.68\,\Omega$$

$$Z_{e_1} = \sqrt{R_{e_1}^2 + X_{e_1}^2} = \sqrt{(1.98)^2 + (9.68)^2} = 9.88\,\Omega$$

$$\cos\phi_{sc} = \frac{R_{e_1}}{Z_{e_1}} = \frac{1.98}{9.88} = 0.2$$

$$I_{sc_2} T_2 = I_{sc_1} T_1$$

$$I_{sc_1} = I_{sc_2}\frac{T_2}{T_1} = 200 \times \frac{1}{6} = 33.33\text{ A}$$

$$V_{sc} = I_{sc_1} Z_{e_1} = \frac{200}{6} \times 9.88 = 329.3\text{ V}$$

1.36 TRANSFORMER EFFICIENCY

The ratio of the output power to the input power in a transformer is known as transformer efficiency (η).

$$\eta \triangleq \frac{\text{output power}}{\text{input power}} = \frac{\text{output power}}{\text{output power + copper loss + iron loss}}\ \text{pu}$$