BAS – 26 (Optimization Techniques) Sheet 1 (UNIT -3 & 4) B. Tech. IV semester

1. **Solve by Newton Methods**

Minimize $f(x_1, x_2) = 100(x_2 - x_1^2)^2 + (1 - x_1)^2$ from the starting point {-1.2, 1.0}

Minimize $f(x_1, x_2) = 4x_1^2 + 3x_2^2 - 5x_1x_2 - 8x_1$ from the starting point $\begin{Bmatrix} -1.2 \\ 1.0 \end{Bmatrix}$.

Minimize $f(x_1, x_2) = x_1^4 - 2\,x_1^2x_2 + x_1^2 + x_2^2 - 2\,x_1 + x_2 + 1$ with stating point $\begin{Bmatrix} 1.5 \\ -1.0 \end{Bmatrix}$ up to two iterations.

Minimize $f(x_1, x_2) = (\,x_1 + 2x_2 - 7)^2 + (2\,x_1 + x_2 - 5)^2$ with starting point $\begin{Bmatrix} -1.5 \\ 1.5 \end{Bmatrix}$ up to two iteration

Minimize $f(x_1, x_2) = (\,10\,x_1 + 6\,x_2 - 9)^2 + (6\,x_1 + 10\,x_2 - 11)^2$ with starting point $\begin{Bmatrix} -1.0 \\ 1.0 \end{Bmatrix}$

2. **Solve by univariate method**

Minimize $f(x_1, x_2) = 2x_1^2 + x_2^2$ from the starting point {1, 2} .

Minimize $f(x_1, x_2) = x_1 - x_2 + 2x_1^2 + x_2^2 + 2x_1x_2$ from the starting point {0, 0} using that $\varepsilon = 0.01$.

Minimize $f(x_1, x_2, x_3) = x_1^2 + 3\,x_2^2 + 5\,x_3^2 - 2\,x_1x_2 + 3\,\,x_2x_3 - 7\,x_1 - 8\,x_2$ with point $\begin{Bmatrix} 1 \\ 2 \\ 3 \end{Bmatrix}$ given $\varepsilon = 0.01$.

Minimize$f(x_1, x_2) = 4x_1^2 + 3\,x_2^2 - 5\,x_1x_2 - 8\,x_1$ with starting point $\begin{Bmatrix} 2 \\ -2 \end{Bmatrix}$ given $\varepsilon = 0.01$

3. **Solve by steepest desecent method**

Minimize $f(x_1, x_2) = 2x_1^2 + x_2^2$ by using the steepest desecent method with starting point {1, 2}.

Minimize $f(x_1, x_2) = 6x_1^2 + 2x_2^2 - 6x_1x_2 - x_1 - x_2$ by using the with starting point {1, 2}.

$f(x_1, x_2) = x_1 - x_2 + 2x_1^2 + 2x_1x_2 + x_2^2$ with starting point $\begin{Bmatrix} 0 \\ 1 \end{Bmatrix}$

Minimize $f(x_1, x_2) = -3x_1 - 2x_2 + 2x_1^2 + 2x_1x_2 + \left(\frac{3}{2}\right)x_2^2$ with starting point $\begin{Bmatrix} 1 \\ -1 \end{Bmatrix}$

4. **Solve by random search method**

Minimize $f(x_1, x_2) = 12x_1^2 - 8x_1x_2 + \left(\frac{1}{5}\right)x_2^2 - \left(\frac{1}{2}\right)x_1 - 2x_2$ in the range $-5 \le x_1 \le 5$ and $-10 \le x_2 \le 10$ up to 10 iteration.

Minimize $f(x_1, x_2) = 15x_1^2 - 18x_1x_2 + \left(\frac{3}{5}\right)x_2^2 - \left(\frac{5}{3}\right)x_1 - 7x_2$ in the range $-2 \le x_1 \le 2$ and $-4 \le x_2 \le 4$ by using random search method up to 10 iterations given the set of values as $\{(r_1, r_2) = (0.50, 0.60), (.25, .26), (.98,\ .97), (.45, .46),\ (.234, .235),\ (.63, .64),\ (.543, .544), (.712,\ 0.713), (.434, .435), (.782,\ .783)\}$.

Minimize $f(x_1, x_2) = 2x_1^3 - 8x_1^2\,x_2 + \left(\frac{1}{5}\right)x_2^2 - 5\,x_1 - 7\,Sin^{-1}\left(\frac{x_1}{x_2}\right)$ in the range $-5 \le x_1 \le 5$ and $-10 \le x_2 \le 10$ by using random search method up to 6 iterations given that set of values as $\{(r_1, r_2) = (.50, 0.60), (.25, .26), (.98,\ .97), (.45, .46),\ (.234, .235),\ (.63, .64)\}$.

5. Derive the geometric dual of

problem: $f(X) = 20\, x_2 x_3 x_4^4 + 20 x_1^2 x_3^{-1} + 5 x_2 x_3^2$ subject to $5\, x_2^{-5} x_3^{-1} \le 1$, $10 x_1^{-1} x_2^3\, x_4^{-1} \le 1, x_i > 0,\ \ i = 1\ to\ 4.$

Minimize $f(X) = 2x_1x_2 + 2x_1x_2^{-1}x_3 + 4x_1^{-1}x_2^{2}x_3^{-1/2}$ subject to $\sqrt{3}\ x_2^{-1} + 3x_1^{-1}x_3^{-1/2} \le 1$ and $x_i \ge 0,\ \ i = 1,2,3$

Minimize$f(X) = x_1x_2 + 2x_1^{-1}x_3 + 5x_3 + 10x_2^{-1}$, $x_i \ge 0, i = 1,2,3$ by geometric programming method.

Derive the geometric dual of the problem: Minimize $f(X) = x_1 x_2^{-2} x_3^{-1} + 2\, x_1^{-1} x_2^{-3}\, x_4 + 10\, x_1\, x_3\, x_4$ subject to $3x_1^{-1}\, x_3\, x_4^{-2} + 4\, x_3\, x_4 \le 1$, $5\, x_1^{-1} x_2^{-2}\ x_3 \le 1$, $x_i \ge 0$, $i = 1,2.$

Minimize$f(X) = x_1x_2x_3^{-2} + 2x_1^{-1}x_2^{-1}x_3 + 5x_2 + 3x_1x_2^{-2}$, $x_i \ge 0, i = 1,2,3$ by geometric programming method.

Minimize$f(X) = x_1x_2x_3^{-3} + 17x_1^2x_2^{-3}x_3 + 34\, x_1^{-3}x_3 + 51\, x_1x_2$, $x_i \ge 0$, $i = 1,2,3$ by geometric programming method.

Minimize $f(X) =$

$x_1x_2^{-3}x_3^{-1} + 5\, x_1^{-1}x_2^{-2}\, x_3 + 2\, x_1\, x_3\, x_2 + 8x_1x_2^{-1/2} + x_1^{3/2}\, x_3$, $x_i \ge 0$, $i = 1,2,3.$

Derive the geometric dual of the problem: Minimize $f(X) = 5\, x_1x_2x_3 + 2\, x_1^2x_2^{-2}x_3^{-2} + 5x_1^{-2}x_2^{-3}x_3^{-5} + 7x_1^2x_3^{-4} + 8x_1x_2^{-1/2}$, $x_i \ge 0$, $i = 1,2,3.$

Minimize$f(X) = x_1^{-2}\, x_2^{-1} + \frac{1}{4}\, x_1^2\, x_2^{-1}\, x_3^{-1} + x_1^{-1}x_3^2\, x_4$ subject to

$\frac{3}{4}\ x_1\, x_2 + \frac{3}{8}\, x_2\, x_3\, x_4^{-3} \le 1\ \ x_i \ge 0,\ i = 1,2,3$ by geometric programming method.

Derive the geometric dual of the problem: $f(X) = 10\, x_1x_2x_3 + 20\, x_1^5x_2^2x_3^{-1} + 5x_1^{-1}x_2^{-3}x_3^{-5} + 7x_1^2x_3^{-4} + 8x_1x_2^{-2}$, $x_i \ge 0, i = 1,2,3.$

Minimize $z = 7x_1x_2^{-1} + 3x_2x_3^{-2} + 5x_1^{-3}x_2x_3 + x_1x_2x_3$ *and* $x_i \ge 0,\ \ i = 1,2,3$ *by* geometric programming method.

Derive the Geometric dual of the problem: Minimize $f(x_1, x_2) = x_1^{-3}x_2 + x_1^{3/2}x_3^{-1}$ subject to $x_1^{2}x_2^{-1} + \frac{1}{2}x_1^{-2}x_3^{3} \le 1$ and $x_1 > 0,\ x_2 > 0, x_3 > 0.$

find the solution of given Geometric minimization problem

$$\text{Minimize} f(X) = x_1^{-2} + \frac{1}{4}\, x_2^2 x_3 \quad \text{subject to}$$

$$\frac{3}{4}x_1^2x_2^{-2} + \frac{3}{8}x_2x_3^{-2} \le 1,\ \ x_i \ge 0, i = 1,2,3.$$

Derive the geometric dual of the problem : $f(X) = 10\, x_1x_2 + 2\, x_1\, x_2^{-2}x_3^{-1} + 5\, x_1^{-2}\, x_2^2 x_3^{-1/2}$ subject to

$\frac{7}{5}x_1^3x_2^{-1} + 6\, x_1^{-1}\, x_3^{-1/2} \le 1.$

Derive the geometric dual of the problem : $f(X) = x_1^{-\frac{3}{4}} x_2 + x_1^{\frac{3}{2}} x_2^{-2} x_3^{-\frac{1}{3}} + x_1\ x_2^{-3} x_3^{-1}$ subject to $\frac{7}{5} x_1^3 x_2^{-1} + 6\ x_1^{-1}\ x_3^{-1/2} \leq 1.$

Derive the geometric dual of given problem: Min $f(X) = x_1^{-\frac{1}{4}} x_2 + x_1^{\frac{1}{2}} x_2^{-2} x_3^{-\frac{1}{2}} + x_1\ x_2^{-4} x_3^{-1}$ subject to $\frac{3}{5} x_1^3 x_2^{-1} + 3\ x_1^{-2}\ x_3^{-1/2} \leq 1.$

Minimize ***x₁*** subject to $-x_1^{\ 2} + 4x_2 \leq 1$ $\quad x_1 + x_2 \geq 1$ $\quad$ *and* $\quad x_1 > 0, x_2 > 0$.

Minimize x_1 subject to $-4\ x_1^2 + 7\ x_2\ \leq 1$ $\quad x_1 + x_2\ \geq 1$ $\quad$ and $x_1 > 0, x_2 > 0$ by procedure of complementary geometric programming method.

$-4\ x_1^3 + 6\ x_2^2\ \leq 1,\quad x_1 + x_2\ \geq 1$ and $x_1 > 0, x_2 > 0$

Minimize ***x₁*** subject to

Minimize ***x₁*** subject to

$-3\ x_1^2 + 7\ x_2\ \leq 1 \quad x_1 + x_2\ \geq 1$ $\quad$ and $x_1 > 0, x_2 > 0$ by procedure of complementary geometric programming method.