Addition of Matrices :-

If A and B be two matrices of the same order, then their sum, A+B is defined as the matrix, each element of which is the sum of the corresponding elements of A and B.

e.g.

$$A = \begin{bmatrix} 4 & 2 & 5 \\ 1 & 3 & -6 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 & 2 \\ 3 & 1 & 4 \end{bmatrix}$$

$$A+B = \begin{bmatrix} 4+1 & 2+0 & 5+2 \\ 1+3 & 3+1 & -6+4 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 2 & 7 \\ 4 & 4 & -2 \end{bmatrix}$$

Ex Show that any square matrix can be expressed as the sum of two matrices, one symmetric and the other ~~skew~~ skew symmetric.

Solution:- Let A be a given square matrix.

Then, $A = \frac{1}{2}(A+A') + \frac{1}{2}(A-A')$

Now, $(A+A')' = A' + A = A + A'$

$\therefore$ $(A+A')$ is a symmetric matrix.

Also, $(A-A')' = A' - A = -(A-A')$

$\therefore$ $A-A'$ is a skew symmetric matrix.

$\Rightarrow$ Square Matrix = Symmetric matrix + Skew symmetric

Example:- Write matrix A given below as the sum of a symmetric and a skew matrix

$$A = \begin{bmatrix} 1 & 2 & 4 \\ -2 & 5 & 3 \\ -1 & 6 & 3 \end{bmatrix}$$

Sol^n $A = \begin{bmatrix} 1 & 2 & 4 \\ -2 & 5 & 3 \\ -1 & 6 & 3 \end{bmatrix}$, $A' = \begin{bmatrix} 1 & -2 & -1 \\ 2 & 5 & 6 \\ 4 & 3 & 3 \end{bmatrix}$

$$A + A' = \begin{bmatrix} 1 & 2 & 4 \\ -2 & 5 & 3 \\ -1 & 6 & 3 \end{bmatrix} + \begin{bmatrix} 1 & -2 & -1 \\ 2 & 5 & 6 \\ 4 & 3 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 0 & 3 \\ 0 & 10 & 9 \\ 3 & 9 & 6 \end{bmatrix}$$

$$\Rightarrow \frac{1}{2}(A + A') = \begin{bmatrix} 1 & 0 & 3/2 \\ 0 & 5 & 9/2 \\ 3/2 & 9/2 & 3 \end{bmatrix}$$

$$A - A' = \begin{bmatrix} 1 & 2 & 4 \\ -2 & 5 & 3 \\ -1 & 6 & 3 \end{bmatrix} - \begin{bmatrix} 1 & -2 & -1 \\ 2 & 5 & 6 \\ 4 & 3 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 4 & 5 \\ -4 & 0 & -3 \\ -5 & 3 & 0 \end{bmatrix}$$

$$\frac{1}{2}(A - A') = \begin{bmatrix} 0 & 2 & 5/2 \\ -2 & 0 & -3/2 \\ -5/2 & 3/2 & 0 \end{bmatrix}$$

$$\because A = \frac{1}{2}(A + A') + \frac{1}{2}(A - A') = \begin{bmatrix} 1 & 0 & 3/2 \\ 0 & 5 & 9/2 \\ 3/2 & \frac{9}{2} & 3 \end{bmatrix} + \begin{bmatrix} 0 & 2 & 5/2 \\ -2 & 0 & -3/2 \\ -5/2 & 3/2 & 0 \end{bmatrix}$$

Properties of Matrix Addition:-

(i) Only the matrices of same order can be added or subtracted.

(ii) Commutative law:

$$A+B = B+A$$

(iii) Associative law:

$$A+(B+C) = (A+B)+C$$

Scalar Multiplication of a Matrix:-

$$A = \begin{bmatrix} 2 & 3 & 4 \\ 4 & 5 & 6 \\ 6 & 7 & 9 \end{bmatrix}$$

$$3A = 3\begin{bmatrix} 2 & 3 & 4 \\ 4 & 5 & 6 \\ 6 & 7 & 9 \end{bmatrix} = \begin{bmatrix} 6 & 9 & 12 \\ 12 & 15 & 18 \\ 18 & 21 & 27 \end{bmatrix}$$

Multiplication of matrices:-

The product of two matrices A and B is only possible if the number of column in A is equal to the number of rows in B.

Let $A = [a_{ij}]$ be an $m \times n$ matrix and $B = [b_{ij}]$ be an $n \times p$ matrix. Then the product AB of these matrices is an $m \times p$ matric $C = [c_{ij}]$ where

$$c_{ij} = a_{i1} b_{1j} + a_{i2} b_{2j} + \dots + a_{in} b_{nj}.$$

→ $(AB)' = B'A'$

If A and B are two matrices conformal for product, then show that $(AB)' = B'A'$, where dash represents transpose of a matrix.

Proof:- Let $A = a_{ij}$ be an $m \times n$ matrix and $B = b_{ij}$ be $n \times p$ matrix

Since AB is $m \times p$ matrix, $(AB)'$ is a $p \times m$ matrix. further B' is $p \times n$ matrix and A' is $n \times m$ matrix and therefore $B'A'$ is ~~matrix of~~ $p \times m$ matrix. Thus $(AB)'$ and $B'A'$ are of same order.

$\because$ (j,i)th element of $(AB)'$ = (i,j)th element of AB

$$= \sum_{k=1}^{n} a_{ik} b_{kj} \quad — (1)$$

$\because$ B' & A' are confor for then $B'A'$ can be defin

$$B'A'[j,i] = \sum_{k=1}^{k} b_{jk} \cdot a_{ki} \quad — (ii)$$

thus from (1) & (ii) we have

$$(AB)' = B'A'$$

Properties of Matrix Multiplication :-

1. Multiplication of matrices is not commutative

$$AB \neq BA$$

eg: $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$

$$A \cdot B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$B \cdot A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$

Here, $A \cdot B \neq B \cdot A$

2. Matrix multiplication is associative, if conformability is assured.

$$A(BC) = (AB)C$$

3. Matrix multiplication is distributive w.r.t. addition

$$A.(B+C) = AB + AC$$

4. Multiplication of square matrix A by unit matrix

$$AI = IA = A.$$