P-13

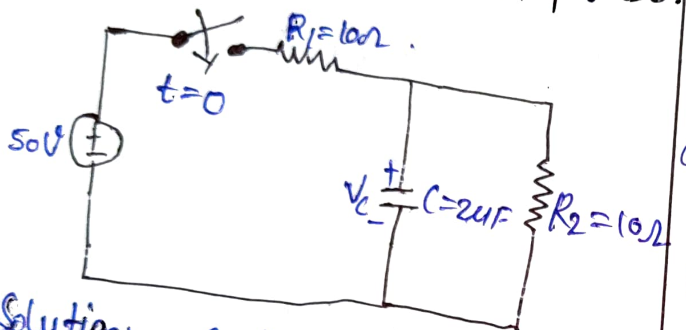
$$q(0^+) = \frac{42}{11} \text{ coulomb}$$

$$V_{C_1}(0^+) = 5 + \frac{42}{11} = \frac{97}{11} \text{ V}$$

$$V_{C_2}(0^+) = 0 + \frac{42}{11} \times \frac{1}{2} = \frac{21}{11} \text{ V}$$

$$V_{C_3}(0^+) = -2 + \frac{42}{11} \times \frac{1}{3} = \frac{-8}{11} \text{ V}.$$

Question:- In figure, the switch is closed at $t=0$. Find the current i and capacitor Voltage V_c, for $t>0$.



Solution:- By KVL, we can write,

$$10(i_1 + i_2) + 0.01\frac{di_1}{dt} + 5i_1 - 100 = 0 \rightarrow (1)$$

and $10(i_1 + i_2) + 5i_2 - 100 = 0 \rightarrow (2)$

putting i_2 from eq^n (2) in eq^n (1), we get –

$$10\left[i_1 + \frac{100 - 10i_1}{15}\right] + 0.01\frac{di_1}{dt} + 5i_1 - 100 = 0$$

or, $\frac{di_1}{dt} + 833 i_1 = 3333$

The solution of this equation is given by –

$i_1 = i_{1h} + i_{1p}$

$i_{1h} = Ke^{-833t}$ A.

&

$i_{1p} = \frac{3333}{833} = 4.0$ A

$i_1 = Ke^{-833t} + 4$

Consider current passes through R_2 is i_1.

By KVL, we can write,

$$R_1 \cdot i + \frac{1}{C}\int_{-\infty}^{t}(i - i_1)\,dt - 50 = 0 \rightarrow (1)$$

and

$$R_2 \cdot i_1 - \frac{1}{C}\int_{-\infty}^{t}(i - i_1)\,dt = 0 \rightarrow (2)$$

On differentiation, we have.

$$R_1\frac{di}{dt} + \frac{1}{C}(i - i_1) = 0 \rightarrow (3)$$

and $R_2\frac{di_1}{dt} - \frac{1}{C}(i - i_1) = 0 \rightarrow (4)$

step 1:- Calculation of i.

putting i_1 from eq^n (3) in eq^n (4), we get,

$$R_2\frac{d}{dt}\left(R_1 C\frac{di}{dt} + i\right) - \frac{1}{C}\left[i - \left(R_1 C\frac{di}{dt} + i\right)\right] = 0$$

or, $R_1 R_2 C\frac{d^2 i}{dt^2} + R_2\frac{di}{dt} - \frac{i}{C} + R_1\frac{di}{dt} + \frac{i}{C} = 0$

or, $R_1 R_2 C\frac{d^2 i}{dt^2} + (R_1 + R_2)\frac{di}{dt} = 0 \rightarrow (5)$

putting given values of R_1, R_2 and C, then we get

$$(10)(10)(2\times10^{-6})\frac{d^2 i}{dt^2} + (10+10)\frac{di}{dt} = 0$$

or, $\frac{d^2 i}{dt^2} + 10^5\frac{di}{dt} = 0 \rightarrow (6)$

The auxiliary eq^n is given by

$$m^2 + 10^5 m = 0$$

$$m = 0, \ -10^5$$

Therefore, the solution of eq^n (6) is given by

$$i = K_1 e^0 + K_2 e^{-10^5 t} \longrightarrow (7)$$

<u>step II</u>:- Calculation of i_1.

From eq^n (4), we have

$$i_1 = R_1 C \frac{di}{dt} + i \longrightarrow (8)$$

putting i from eq^n (7) in eq^n (8), we get -

$$i_1 = 10 \times 2 \times 10^{-6} \frac{d}{dt} (K_1 + K_2 e^{-10^5 t}] + K_1 + K_2 e^{-10^5 t}$$

$$= 2 \times 10^{-5} [-10^5 K_2 \ e^{-10^5 t}] + K_1 + K_2 e^{-10^5 t}$$

$$i_1 = K_1 - K_2 e^{-10^5 t} \longrightarrow (9)$$

<u>step III</u>:- Calculation of V_c,

we have $V_c = \frac{1}{C} \int_{-\infty}^{t} (i - i_1) dt = R_2 i_1$

$$\longrightarrow (10)$$

From eq^n (9) and (10), we have

$$V_C = 10 (K_1 - K_2 e^{-10^5 t})$$

$$\longrightarrow (11)$$

<u>step IV</u>:- Calculation of K_1 and K_2.

we have $v_c(0^-) = 0$

As $t \to \infty$, the capacitor becomes as open circuited, leaving 20Ω in series with the $50V$, i.e,

$$i(\infty) = \frac{50}{20} = 2.5A$$

$V_c(\infty)$ = Voltage across 10Ω

$$= 10 \times 2.5 = 25V$$

Now for,

$V_c(0^-) = 0$, from eq^n (11), we have

$$25 = 10 (K_1 - K_2 e^{-\infty})$$

$$25 = 10 (K_1 - 0)$$

$$\boxed{K_1 = 2.5}$$

Therefore, the current is given by eq^n (7), after putting K_1 and K_2 as -

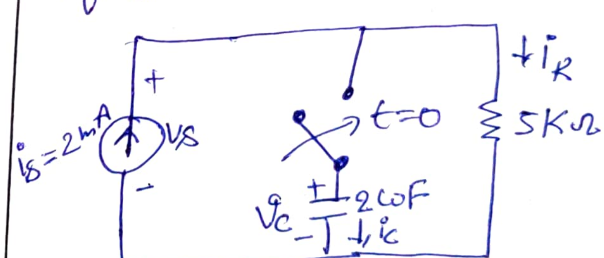
$$i = 2.5 (1 + e^{-10^5 t}) A$$

and V_c is given by eq^n (11), after putting K_1 and K_2 as -

$$V_c = 10 (2.5 - 2.5 \ e^{-10^5 t})$$

$$\boxed{V_c = 25 (1 - e^{-10^5 t}) V}$$

<u>Question</u>:- In figure, the switch is closed at $t=0$. The capacitor has no charge for $t<0$. Find i_R, i_c, V_c and V_s for all instants if $i_s = 2mA$.



<u>Solution</u>:- <u>step I</u>:- To obtain initial condition.

For $t<0$, the circuit becomes as shown in figure, in this case capacitor behaves as open circuited.

For $t<0$, $i_R = 2mA$, $i_c = 0$, $V_c = 0$,

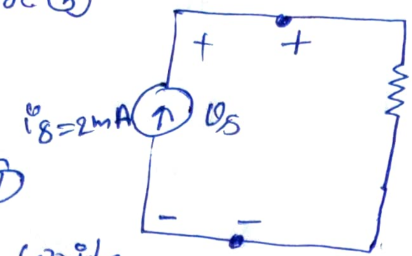
$$V_s = 2 \times 10^{-3} \times 5 \times 10^3 = 10V.$$

<u>step II</u>:- To obtain i_R, i_c, V_c and V_s for $t>0$.

after switching, the circuit becomes

as shown in figure (b)

we have



$i_R + i_C = 2\times10^{-3}$ $\longrightarrow (1)$

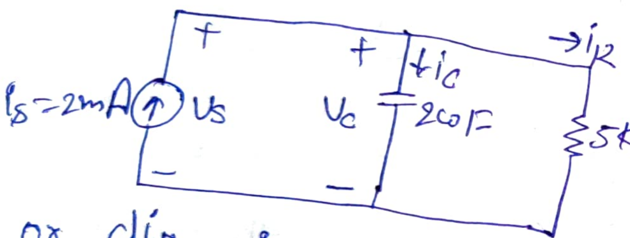
By KVL, we can write

$$Ri_R - \frac{1}{C}\int_{-\infty}^{t} i_C\,dt = 0$$

$$Ri_R - \frac{1}{C}\int_{-\infty}^{t} (2\times10^{-3} - i_R)\,dt = 0$$

On differentiation, we get

$$R\frac{di_R}{dt} - \frac{1}{C}(2\times10^{-3} - i_R) = 0$$



$$\text{or, } \frac{di_R}{dt} + \frac{i_R}{RC} = \frac{2\times10^{-3}}{RC}$$

The solution of this eq^n is given by -

$$i_R = i_{hR} + i_{pR}$$

where, $i_{hR} = K e^{-t/RC}$

&

$$i_{pR} = \frac{\frac{2\times10^{-3}}{RC}}{\frac{1}{RC}} = 2\times10^{-3}$$

$$i_R = K e^{-t/RC} + 2\times10^{-3} \longrightarrow (2)$$

For calculation of K using initial condition.

we have, $V_C(0^-) = 0$ (initial condition)

We know that,

$$v_C = \frac{1}{C}\int_{-\infty}^{t} i_C\,dt$$

$$= V_C(0^-) + \frac{1}{C}\int_{0}^{t} i_C\,dt$$

$$V_C = \frac{1}{C}\int_{0}^{t} i_C\,dt \longrightarrow (3)$$

From eq^n (1), (2) and (3) we have

$$V_C = \frac{1}{C}\int_0^t (-K e^{-t/RC}\,dt)$$

$$= RK\left[e^{-t/RC}\right]_0^t = RK(e^{-t/RC} - 1)$$

$$\text{or, } V_C = RK(e^{-t/RC} - 1) \longrightarrow (4)$$

As $t\to\infty$, $V_C(\infty) = R\,i_R(\infty)$ (since, at steady state condition capacitor behaves as open circuited).

$$\therefore 5\times10^3 \times i_R(\infty) = 5\times10^3 \times K(e^{-\infty} - 1)$$

From eq^n (2), we have -

$$i_R(\infty) = K e^{-\infty} + 2\times10^{-3} = 2\times10^{-3}A.$$

$$5\times10^3 \times 2\times10^{-3} = 5\times10^3 \times K(-1)$$

$$\text{or, } \boxed{K = -2\times10^{-3}}$$

putting the value of K in eq^n (2), we get -

$$i_R = 2\times10^{-3}\left(1 - e^{-t/5\times10^3\times2\times10^{-6}}\right)$$

$$i_R = 2\times10^{-3}(1 - e^{-100t})\,A \longrightarrow (5)$$

now, $i_R(0^+) = 0A$, $i_R(\infty) = 2mA$.

For i_C; putting the value of i_R from eq^n (5) in eq^n (1), we get -

$$i_C = 2\times10^{-3} - 2\times10^{-3}(1 - e^{-100t})$$

$$\text{or, } i_C = 2\times10^{-3} e^{-100t}\,A$$

Now, $i(0^+) = 2mA$, $i(\infty) = 0A$.

For V_C; Putting $K = -2\times10^{-3}$ in eq^n (4), we get -

$$V_C = -2\times10^{-3}\times5\times10^3(e^{-100t} - 1)$$

$$\text{or, } V_C = 10(1 - e^{-100t})\,V$$

now, $V_C(0^+) = 0V$, $V_C(\infty) = 10V$

For V_s; From circuit diagrams, we have -

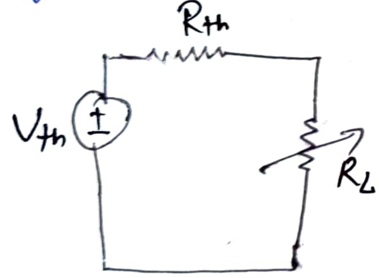
$$V_s = V_c = \text{Voltage across resistor of } 5k\Omega$$

$$V_s = 10(1-e^{-100t})\,V.$$

now, $V_s(0^+) = 0V$, $V_s(\infty) = 10V$.

Maximum power transfer Theorem:-

"According to the maximum power transfer theorem, the condition for maximum power flow through load resistor R_L can be achieved when the load resistor equals the thevenin's equivalent resistance of the circuit."



(i) power through R_L -

$$P = I_L^2 R_L \longrightarrow (i)$$

(ii)

$$I_L = \frac{V_{th}}{R_{th}+R_L} \quad \text{(thevenin's theorem)}$$

$$P = \left(\frac{V_{th}}{R_{th}+R_L}\right)^2 \cdot R_L \longrightarrow (ii)$$

differentiate the above eq^n w.r.t R_L and equating it to zero.

$$\frac{dP}{dR_L} = V_{th}^2 \left[\frac{(R_{th}+R_L)^2 - 2R_L(R_{th}+R_L)}{(R_{th}+R_L)^4}\right]$$

$$R_{th}^2 - R_L^2 = 0$$

$$\therefore \boxed{R_L = R_{th}} \longrightarrow (iii).$$

this is the required condition for max power flow.

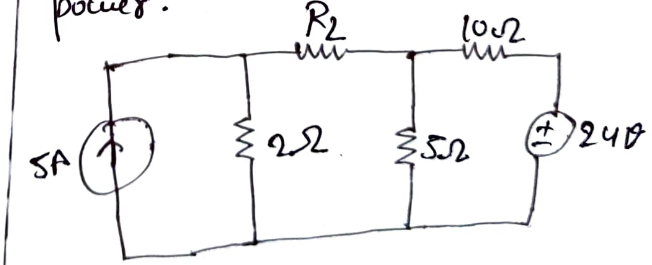
putting the value of $R_L = R_{th}$ in eq^n (2)

$$P_{max} = \frac{V_{th}^2}{(R_{th}+R_{th})^2} \cdot R_{th}$$

$$P_{max} = \frac{V_{th}^2}{4R_{th}^2} \cdot R_{th}$$

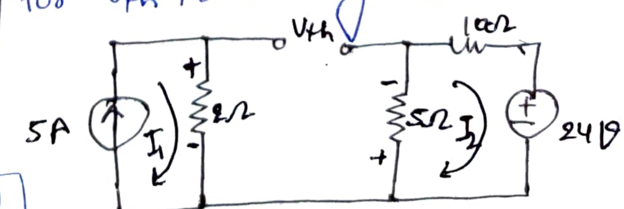
$$\boxed{P_{max} = \frac{V_{th}^2}{4R_{th}}}$$

Question:- In the given network find the value of R_L which will absorb the maximum power from the source. Also find the maximum power.



Solved:- $R_L = ?$

For V_{th}, removing R_L -



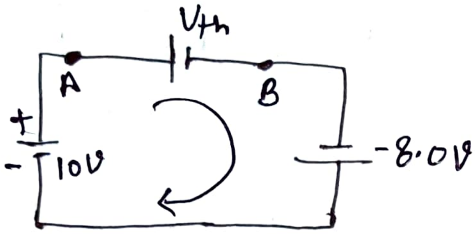
$$\boxed{I_1 = 5A}$$

applying KVL in mesh (2) -

$-15I_2 = 24$

$I_2 = \frac{-24}{15}$

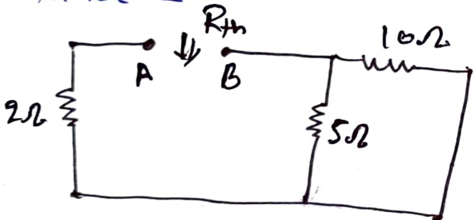
$\boxed{I_2 = -1.6A}$



$-V_{th} - 8 + 10 = 0$

$\boxed{V_{th} = 2V}$

Remove R_L & replace all the active sources by their internal resistance –



$\boxed{R_{th} = 5.33\Omega}$

(i) $\boxed{R_L = R_{th} = 5.33\Omega}$ for maximum power flow.

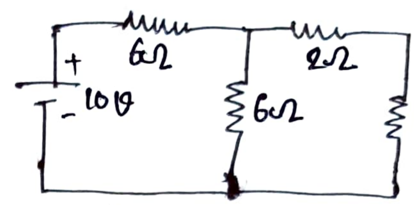
(ii) $P_{max} = \frac{V_{th}^2}{4R_{th}}$

$= \frac{2V}{4 \times 5.33}$

$= \frac{1}{10.66}$

$\boxed{P_{max} = 0.187\,W}$

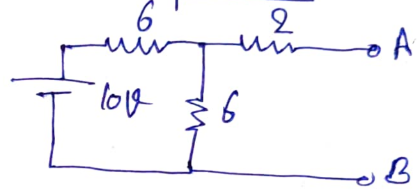
Question:- Find the value of R_L in given networks. Also find the maximum power in R_L.



$R_L = R_{th}$ for max power transfer

figure – DC network ckt.

Solved:- for V_{th}



$6I + 6I - 10 = 0$

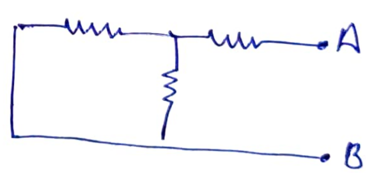
$12I = 10$

$I = \frac{10}{12} = \frac{5}{6}A$

$V_{th} = 10 - 6 \times \frac{5}{6} = 5V$

$\boxed{V_{th} = 5V}$

for R_{th}



$R_{th} = 6 \| 6 + 2 = 5\Omega$

thevenin's ckt diagram →

for maximum power –

$R_L = R_{th}$

$R_L = 5\Omega$

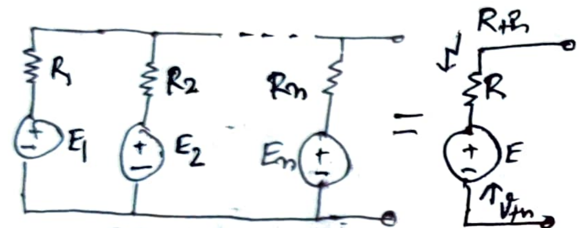
$P = I_L^2 \cdot R_L$

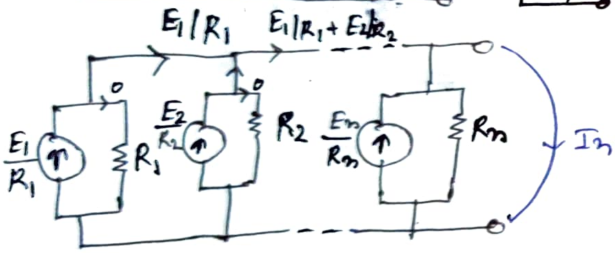
$I_L = \frac{V_{th}}{R_{th} + R_L}$

$P_{max} = \left(\frac{5}{5+5}\right)^2 \times 5 = \frac{25}{100} \times 5 = 1.25W$

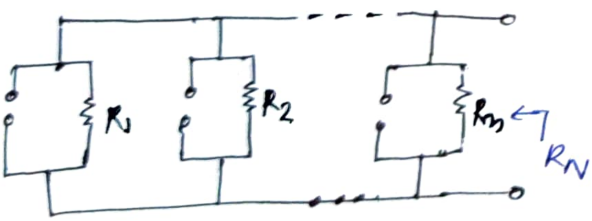
Millman's theorem :-

"If n voltage sources with voltages $E_1, E_2, E_3, \ldots, E_n$ and internal resistance $R_1, R_2, R_3, \ldots, R_n$ are connected in parallel, then these voltage source can be ~~replaced~~ replaced by a single voltage source E in series with resistance R."





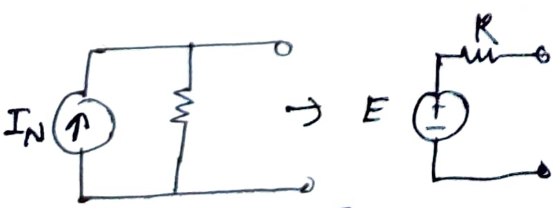
$$I_N = \frac{E_1}{R_1} + \frac{E_2}{R_2} + \cdots + \frac{E_n}{R_n}.$$



$\therefore$

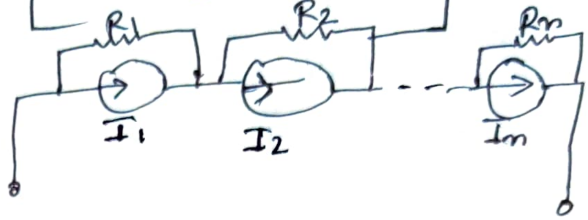
$$R_N = R_1 \| R_2 \| \cdots \| R_N$$

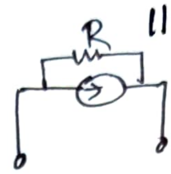
$$\frac{1}{R_N} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N}$$



$$E = I_N R_N = \frac{\frac{E_1}{R_1} + \frac{E_2}{R_2} + \cdots + \frac{E_n}{R_n}}{\frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n}}$$

$$\boxed{E = \frac{\sum_{i=1}^{n} E_i G_i}{\sum_{i=1}^{n} G_i}}, \quad R = R_n$$





$\boxed{\cancel{I = I_N R_1 + I_2 R_2 + \cdots + I}}$

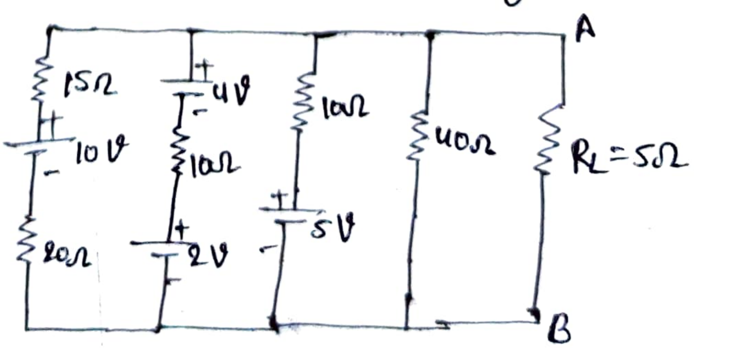
$$I = \frac{I_1 R_1 + I_2 R_2 + \cdots + I_n R_n}{R_1 + R_2 + \cdots + R_n}$$

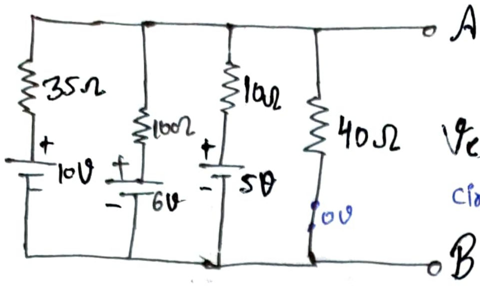
$$I = \frac{\sum_{i=1}^{n} I_i R_i}{\sum_{i=1}^{n} R_i}$$

$$R = R_1 + R_2 + \cdots + R_n$$

$$R = \sum_{i=1}^{n} R_i$$

Question:- Using millman's theorem, calculate current through 5Ω resistor.





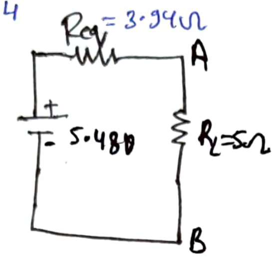
V_{eq} = open circuits

$$V_{eq} = \frac{\frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3} + \frac{V_4}{R_4}}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}}$$

$$= \frac{\frac{10}{35} + \frac{6}{10} + \frac{5}{10} + \frac{0}{40}}{\frac{1}{35} + \frac{1}{10} + \frac{1}{10} + \frac{1}{40}}$$

$$= \frac{1.3857}{0.253}$$

$$= 5.48V$$



$$\frac{1}{R_{eq}} = \frac{1}{35} + \frac{1}{10} + \frac{1}{10} + \frac{1}{40}$$

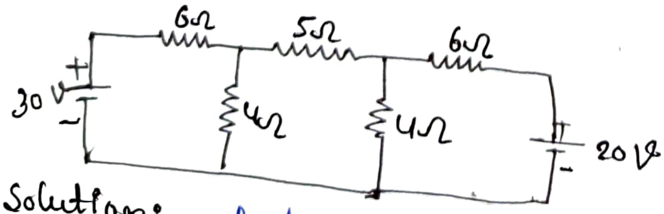
$$R_{eq} = \frac{1}{\frac{1}{35} + \frac{1}{10} + \frac{1}{10} + \frac{1}{40}} = \frac{1}{0.253}$$

$$\boxed{R_{eq} = 3.94\Omega}$$

$$I_L = \frac{5.48}{3.94+5} = 0.61A$$

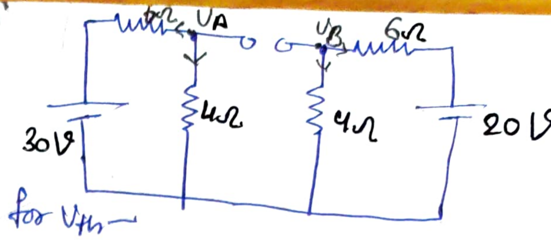
$$\boxed{I_L = 610mA}$$

Question:- Find I_L in 5Ω resistance or R_L.



Solution:- find $I_L = ?$, $R_L = 5\Omega$.

$$\therefore I_L = \frac{V_{th}}{R_{th} + R_L}$$



for V_{th} –

$$\frac{V_A - 30}{6} + \frac{V_A - 0}{4} = 0$$

$$\frac{2V_A - 60 + 3V_A}{12} = 0$$

$$5V_A = 60$$

$$\boxed{V_A = 12V}$$

$$\Rightarrow \frac{V_B - 20}{6} + \frac{V_B - 0}{4} = 0$$

$$\Rightarrow \frac{2V_B - 40}{12} + 3A = 0$$

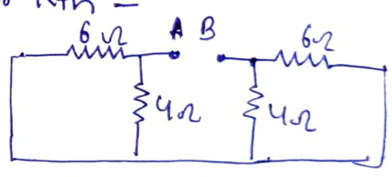
$$5V_B = 40$$

$$\boxed{V_B = 8V}$$

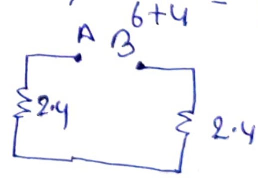
$$V_{th} = V_A - V_B$$

$$V_{th} = 12 - 8 = 4V$$

for R_{th} –

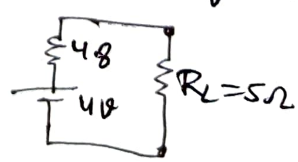


$$6 \| 4 \Rightarrow \frac{6 \times 4}{6+4} = \frac{24}{10} = 2.4$$



$$\boxed{R_{th} = 4.8\Omega}$$

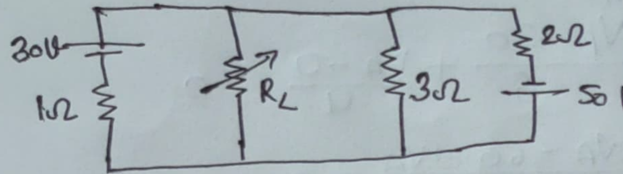
Thevenin's equivalent ckt.

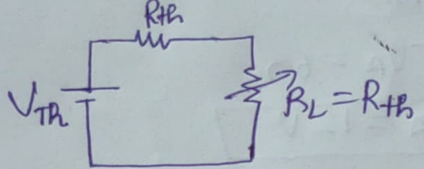


$$I_L = \frac{4}{(4.8+5)} = \frac{4}{9.8}$$

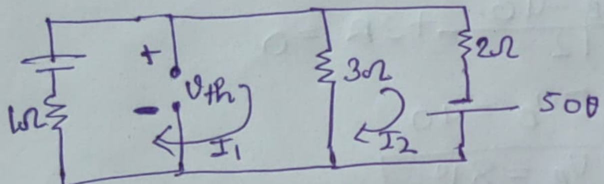
$$I_L = 0.004A$$

Qu find the value of resistance R_L for maximum power transfer calculate maximum power.





Find V_{th} →



Apply KVL to mesh I_1,

$$4I_1 - 3I_2 = 30$$

Apply KVL to mesh 2,

$$-3I_1 + 5I_2 = 50$$

$I_1 = 27.27A$; $I_2 = 2636\ A.$

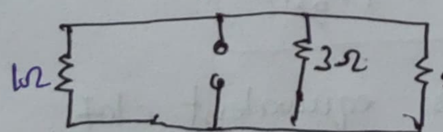
$$-I_1 + 30 - V_{th} = 0$$

$$-27.27 + 30 = V_{th}$$

$$2.73V = V_{Th}$$

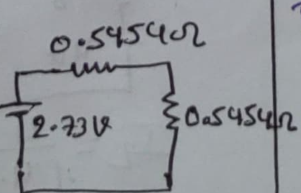
$$\boxed{V_{Th} = 2.73V}$$

Find R_{th} →



$$R_{th} = \left(\frac{1}{1} + \frac{1}{3} + \frac{1}{2}\right)^{-1}$$

$$R_{th} = 0.545454\ \Omega$$

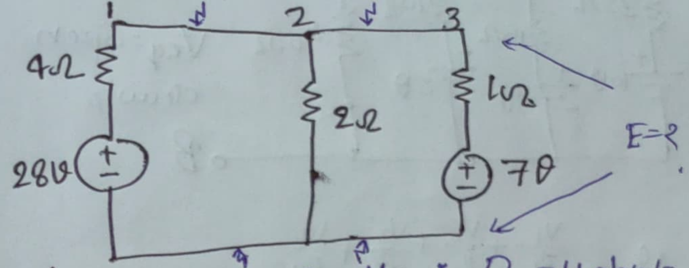


$$I_L = \frac{2.73}{2(0.5454)}$$

$$= 2.502A$$

$$P_{max} = \frac{V_{Th}^2}{4R_{Th}} = \frac{(2.73)^2}{4 \times 0.5454} = 3.4162\ W$$

Question: ⇒ Find the voltage across all the branches in the below given ckt.



Solution: – $E_1 = 28$ volts ; $R_1 = 4$ ohms

$E_2 = 0$ volts ; $R_2 = 20$ ohms

$E_3 = 7$ volts ; $R_3 = 1$ ohm

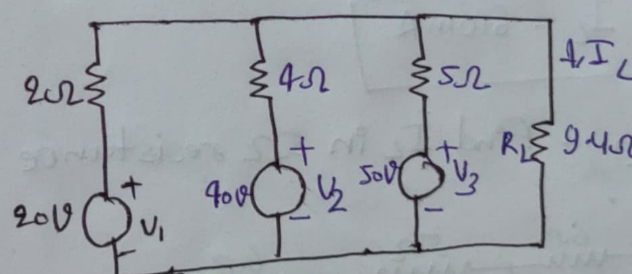
$$E = \frac{\sum_{i=1}^{n} E_i G_i}{\sum_{i=1}^{n} G_i} \Rightarrow E = \frac{\frac{E_1}{R_1} + \frac{E_2}{R_2} + \frac{E_3}{R_3}}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

$G_i \leftarrow \frac{1}{R_i}$

$$E = \frac{\frac{28}{4} + \frac{0}{20} + \frac{7}{10}}{\frac{1}{4} + \frac{1}{20} + \frac{1}{10}}$$

$$\boxed{E = 8\text{ Volts}}$$

Question: – Use millman's theorem to find current I_L through resistor R_L for the network shown below.



$$V_{eq} = \frac{\frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3}}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}} = \frac{\frac{20}{2} + \frac{40}{4} + \frac{50V}{5}}{\frac{1}{2} + \frac{1}{4} + \frac{1}{5}}$$

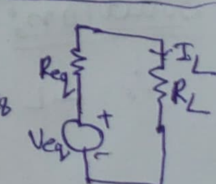
$$\boxed{V_{eq} = 31.57}$$

$$R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

$$= \frac{1}{\frac{1}{2} + \frac{1}{4} + \frac{1}{5}} = \frac{1}{0.95}$$

$$\boxed{R_{eq} = 1.0526\Omega}$$

$$I_L = \frac{V_{eq}}{R_L + R_{eq}}$$



$$I_L = 3.021$$

$$\boxed{I_L = 3.021A}$$