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Local min at x^* if $f(x^*) \le f(x^* \pm h)$
Local max at x^* if $f(x^*) \ge f(x^* + h)$
global min at x^* if $f(x^*) \le f(x)$
global max at x^* if $f(x^*) \ge f(x)$

$\boxed{\text{THEOREM}}$ fn is defined in interval $a \le x \le b$ & has relative min at $x = x^*$, where $a < x^* < b$ & if derivative $\frac{d f(x)}{dx} = f'(x)$ exist as finite no. at $x = x^*$ then $f'(x^*) = 0$

$$\Rightarrow \quad f'(x^*) = \lim_{h \to 0} \frac{f(x^*+h) - f(x^*)}{h} \quad \text{——} (1)$$

Since, x^* is relative min,

$$f(x^*) \le f(x^* + h)$$

$$\frac{f(x^* + h) - f(x^*)}{h} \ge 0 \quad \text{if } h > 0$$

$$\frac{f(x^* + h) - f(x^*)}{h} \le 0 \quad \text{if } h < 0$$

Since, in Eq (1) lt $h \to 0$ through +ve values

$$f'(x^*) \ge 0 \quad \text{——} (2)$$

$h \to 0$ through -ve value

$$f'(x^*) \le 0 \quad \text{——} (3)$$

To satisfy eq 2 & 3.

$$f(x^*) = 0$$

Que. Max. & Min value of fn.

$f(x) = 12x^5 - 45x^4 + 40x^3 + 5$

$f'(x) = 60x^4 - 180x^3 + 120x^2$

$= 60(x^4 - 3x^3 + 2x^2)$

$= 60x^2(x^2 - 3x + 2)$

$= 60x^2(x^2 - 2x - x + 2) \Rightarrow 60x^2(x[x-2) - 1(x-$

$= 60x^2(x-2)(x-1)$

$f'(x) = 0$ at $0, 1, 2$.

$f''(x) = 60(4x^3 - 9x^2 + 4x)$

$x = 1$, $f''(x) = -60$ — relative max $\boxed{f(1) = 12}$

$x = 2$; $f''(x) = 60(32 - 36 + 8) = 240$ — rel min $\boxed{f(2) = -11}$

$x = 0$ $= 0$

Que.

$$W = C_p T_1 \left[\left(\frac{P_2}{P_1}\right)^{(k-1)/k} + \left(\frac{P_3}{P_2}\right)^{(k-1)/k} - 2 \right]$$

$$\frac{dW}{dp_2} = C_p T_1 \cancel{\frac{(k-1)}{k}} \left[\left(\frac{1}{P_1}\right)^{k-1/k} \frac{(k-1)}{k} (P_2)^{-1/k} + (P_3)^{(k-1)/k} \right.$$

$$\left. + \frac{1-k}{k} \left(\frac{1}{P_2}\right)^{\frac{2k-2}{k} - 1} \right.$$

$$= C_p T_1 \left[\left(\frac{1}{P_1}\right)^{\frac{k-1}{k}} \frac{(k-1)}{k} (P_2)^{-1/k} + (P_3)^{k-1/k} \left(\frac{1-k}{k}\right) (P_2)^{\frac{1-2k}{k}} \right] = 0.$$

$$C_p T_1 \frac{(k-1)}{k} \left[\left(\frac{1}{P_1}\right)^{\frac{k-1}{k}} (P_2)^{-1/k} - (P_3)^{k-1/k} (P_2)^{\frac{1-2k}{k}} \right] = 0$$

$$\left(\frac{1}{P_1}\right)^{\frac{k-1}{k}} (P_2)^{-1/k} = (P_3)^{k-1/k} (P_2)^{\frac{1-2k}{k}}.$$

$$(P_2)^{\frac{-1}{k} - \frac{1+2k}{k}} = (P_3)^{k-1/k} \Big/ \left(\frac{1}{P_1}\right)^{\frac{k-1}{k}}$$

$k+1$ ... $\frac{2k-2-k}{k}$ $\frac{k-1}{k}$

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$$(p_2)^{\frac{2(k-1)}{k}} = (p_3 p_1)^{\frac{k-1}{k}}$$

$$p_2 = (p_1 p_3)^{1/2}$$

Multivariable opt. without constraints

Q Find Second order Taylor's series app.

$$f(x_1, x_2, x_3) = x_2^2 x_3 + x_1 e^{x_3}$$

$$X^* = \{1, 0, -2\}^T$$

$$f(x) = f\begin{pmatrix}1\\0\\-2\end{pmatrix} + df\begin{pmatrix}1\\0\\-2\end{pmatrix} + \frac{d^2}{2!} f\begin{pmatrix}1\\0\\-2\end{pmatrix}$$

$$f\begin{pmatrix}1\\0\\-2\end{pmatrix} = e^{-2}$$

$$df\begin{pmatrix}1\\0\\-2\end{pmatrix} = h_1 \frac{\partial f}{\partial x_1}\begin{pmatrix}1\\0\\-2\end{pmatrix} + h_2 \frac{\partial f}{\partial x_2}\begin{pmatrix}1\\0\\-2\end{pmatrix} + h_3 \frac{\partial f}{\partial x_3}\begin{pmatrix}1\\0\\-2\end{pmatrix}$$

$$= h_1 e^{x_3} + h_2(2x_2x_3) + h_3 x_2^2 + h_3 x_1 e^{x_3}\Big]\begin{pmatrix}1\\0\\-2\end{pmatrix}$$

$$= h_1 e^{-2} + h_3 e^{-2}$$

$$d^2 f\begin{pmatrix}1\\0\\-2\end{pmatrix} = \sum_{i=1}^{3}\sum_{j=1}^{3} h_i h_j \frac{\partial^2 f}{\partial x_i \partial x_j}\begin{pmatrix}1\\0\\-2\end{pmatrix}$$

$$= h_1^2 \frac{\partial^2 f}{\partial x_1^2} + h_2^2 \frac{\partial^2 f}{\partial x_2^2} + h_3^2 \frac{\partial^2 f}{\partial x_3^2} + 2h_1h_2 \frac{\partial^2 f}{\partial x_1 \partial x_2} +$$

$$2h_2h_3 \frac{\partial^2 f}{\partial x_2 \partial x_3} + 2h_1h_3 \frac{\partial^2 f}{\partial x_1 \partial x_3}\Bigg)\begin{pmatrix}1\\0\\-2\end{pmatrix}$$

$$= h_1^2(0) + h_2^2(2x_3) + h_3^2(x_1 e^{x_3}) + 2h_1h_2(0) + 2h_2h_3(2x_2)$$
$$+ 2h_1h_3(e^{x_3})$$

$$= -4h_2^2 + e^{-2}h_3^2 + 2h_1 h_3 e^{-2}.$$

Qu4 Minimize $f(x_1, x_2) = x_1 - x_2 + 2x_1^2 + 2x_1x_2 + x_2^2$ by taking starting pt as $x_1 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$$x_{i+1} = x_i - \underbrace{(H)_i^{-1}}_{\text{Hessian}} \underbrace{\nabla f_i}_{\left\{\frac{\delta f}{\delta x_i}\right\}_{x = x_i}}$$

$$H = \begin{bmatrix} \frac{\delta^2 f}{\delta x_1^2} & \frac{\delta^2 f}{\delta x_1 \delta x_2} \\ \frac{\delta^2 f}{\delta x_2 \delta x_1} & \frac{\delta^2 f}{\delta x_2^2} \end{bmatrix} = \begin{bmatrix} 4 & 2 \\ 2 & 2 \end{bmatrix}$$

$$\frac{\delta f}{\delta x_1} = 1 + 4x_1 + 2x_2 \qquad \frac{\delta^2 f}{\delta x_1 \delta x_2} = 2$$

$$\frac{\delta^2 f}{\delta x_1^2} = 4 \qquad \frac{\delta f}{\delta x_2} = -1 + 2x_1 + 2x_2$$

$$\frac{\delta^2 f}{\delta x_2^2} = 2 \qquad \frac{\delta^2 f}{\delta x_2 \delta x_1} = 2.$$

$$H^{-1} = \frac{1}{4}\begin{pmatrix} 2 & -2 \\ -2 & 4 \end{pmatrix} = \begin{pmatrix} 1/2 & -1/2 \\ -1/2 & 1 \end{pmatrix}$$

$$x_2 = x_1 - (H)^{-1} \nabla f_1$$

$$= \begin{pmatrix} 0 \\ 0 \end{pmatrix} - \begin{pmatrix} \frac{1}{2} & -1/2 \\ -1/2 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$$= \begin{bmatrix} 0 \\ 0 \end{bmatrix} - \begin{bmatrix} 1 \\ -3/2 \end{bmatrix}$$

$$\boxed{x_2 = \begin{bmatrix} -1 \\ 3/2 \end{bmatrix}}$$

$$\nabla f_i = \begin{matrix} \frac{\delta f}{\delta x_1} \\ \frac{\delta f}{\delta x_2} \end{matrix}$$

$$\nabla f_2 = \begin{matrix} 1 - 4 + 3 \\ = 0 \\ -1 - 2 + 3 \end{matrix}$$

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Qu. Acc to principle of min. PE

$PE(U)$ = Strain Energy − WD by ext for.

$= \left[\frac{1}{2}k_2 x_1^2 + \frac{1}{2}k_3(x_2 - x_1)^2 + \frac{1}{2}k_1 x_2^2\right] - Px_2$

Necessary cond. for min of U.

$\frac{\delta U}{\delta x_1} = k_2 x_1 - k_3(x_2 - x_1) = 0$ —— ①

$\frac{\delta U}{\delta x_2} = k_3(x_2 - x_1) + k_1 x_2 - P = 0$ —— ②

$k_2 x_1 - \cancel{k_3 x_2} + \cancel{k_3 x_1} = 0$

$\cancel{k_3 x_2} - \cancel{k_3 x_1} + k_1 x_2 - P = 0$

Saddle point

$r = \frac{\delta^2 z}{\delta x^2}$ $s = \frac{\delta^2 z}{\delta x \delta y}$

$t = \frac{\delta^2 z}{\delta y^2}$

1) $rt - s^2 > 0$, $r > 0$ → fn min
2) $rt - s^2 > 0$, $r < 0$ → fn max
3) $rt - s^2 < 0$ → saddle pt

fn neither max nor min

Qu. $f(x,y) = x^3 + 2xy + y^3$

$\frac{\delta f}{\delta x} = 3x^2 + 2y$ $\frac{\delta^2 f}{\delta x^2} = 6x = r$

$\frac{\delta f}{\delta y} = 2x + 3y^2$ $\frac{\delta^2 f}{\delta y^2} = 6y = t$

$\frac{\delta^2 f}{\delta x \delta y} = 2 = s$

$rt - s^2 = 36xy - 4$

a $(0,0) = -4 \leq 0$.

Multivariable opt. with eq. constraints

Max/ Min $f(x)$

Subject to

$g_j(X) = b_j$

$j = 1, 2, \dots n$

$X = (x_1, \dots x_n)^T$

Step

① Convert this constraint to no constraints

$f(x), g(x) \quad\quad f(x)$.

→ Direct Sub. Method

→ Lagrange Multiplier Method.

⇒ Direct Sub. Method

$f(x), \ g_j(x) = b_j$

$g_1(x) = b_1$

$\vdots$

$g_m(x) = b_m$

$m < n \rightarrow$ Soln.

if $m > n \rightarrow$ No soln

Q Find min value of $x^2+y^2+z^2$ subject $x+y+2z=12$

Min $f(x) = x^2+y^2+z^2$

subject to $x+y+2z = 12$

$X = (x, y, z)^T$

Direct Sub. method.

$$z = \frac{12-x-y}{2}.$$

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$$F(x) = x^2 + y^2 + \left(\frac{12-x-y}{2}\right)^2$$

$$= \frac{4x^2+4y^2 + 144 + x^2+y^2 - 24x + 2xy - 24y}{4}$$

$$\frac{\delta F}{\delta x} = \frac{\delta F}{\delta y} = 0.$$

$$\frac{\delta F}{\delta x} = \frac{1}{4}\left[8x + 2x - 24 + 2y\right] = \frac{1}{\cancel{4}\,2}\left[\cancel{10x}^{5} + \cancel{2}y - \cancel{24}^{12}\right]$$

$$\frac{\delta F}{\delta y} = \frac{1}{4}\left[8y + 2y + 2x - 24\right] = \frac{1}{\cancel{4}}\left[\cancel{10y}^{5} + \cancel{2}x - \cancel{24}^{12}\right]$$

$$5x + y - 12 = 0 \times 5$$

$$5y + x - 12 = 0$$

$$\cancel{25x + 5y = 0}$$

$$\cancel{x + 5y = 0}$$

$$\boxed{x = y}$$

$$\frac{1}{2}\left[5x + x - 12\right] = 0.$$

$$\cancel{5x}\cdot \cancel{6}x = 12^{2}$$

$$\boxed{x = 2 = y}$$

$$z = \frac{12-4}{2} \Rightarrow \boxed{z = 4}$$

Extreme pt $\rightarrow (2,2,4)$

$$\frac{\delta^2 F}{\delta x^2} = \frac{1}{4}[10] = \frac{5}{2} \qquad \frac{\delta^2 F}{\delta x \delta y} = \frac{1}{4}[2] = \frac{1}{2}$$

$$\frac{\delta^2 F}{\delta y^2} = \frac{1}{4}[10] = \frac{5}{2} \qquad \frac{\delta^2 F}{\delta y \delta x} = \frac{1}{2}$$

$$H = \begin{bmatrix} \frac{\delta^2 F}{\delta x^2} & \frac{\delta^2 F}{\delta x \delta y} \\ \frac{\delta^2 F}{\delta y \delta x} & \frac{\delta^2 F}{\delta y^2} \end{bmatrix} = \begin{bmatrix} \frac{5}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{5}{2} \end{bmatrix}$$

$H_1 = \frac{5}{2}$ $\quad H_2 = \frac{28}{4} - \frac{1}{4} = 6 > 0$

H is pos. definite
(2,2,4) will be min pt

f(x) at (2,2,4)
= 20

Q $X = (x, y, z)$

Max $f(x,y,z) = 8xyz$

Sub. to $x^2 + y^2 + z^2 = 1$

$z^2 = 1 - x^2 - y^2$

$z = \sqrt{1 - x^2 - y^2}$

$F(x) = 8xy\sqrt{1-x^2-y^2}$

$\frac{\delta F}{\delta x} = 8y\left[\sqrt{1-x^2-y^2} + x.\frac{1}{\cancel{2}}\frac{(-\cancel{2}x)}{\sqrt{1-x^2-y^2}}\right] = 0$

$\frac{\delta F}{\delta y} = 8x\left[\sqrt{1-x^2-y^2} + y.\frac{1}{\cancel{2}}\frac{(-\cancel{2}y)}{\sqrt{1-x^2-y^2}}\right] = 0$

$1 - 2x^2 - y^2 = 0 \quad \times 2$

$1 - x^2 - 2y^2 = 0$

$\cancel{2} - \cancel{4x^2} - \cancel{2y^2} = 0$

$1 - x^2 - \cancel{2y^2} = 0$

$- \quad + \quad +$

$1 - 3x^2 = 0$

$\boxed{x = 1/\sqrt{3}}$

Lagrange Multiplier Method

Statement Min/Max $f(x)$
Subject to $g_j(x) = b_j$
$X = (x_1, x_2 \ldots x_n)$
$j = 1$ to m.

Step ① Convert problem into no constraint

$$L(x, \lambda_1, \lambda_2 \ldots \lambda_m)$$
$$= f(x) + \lambda_1 (g_1(x) - b_1) + \lambda_2 (g_2(x) - b_2) \ldots + \lambda_m (g_m(x) - b_m)$$

Necessary cond

$$\frac{\delta L}{\delta x_i} = 0 = \frac{\delta L}{\delta \lambda_j} \qquad i = 1, 2, \ldots n \qquad j = 1 \text{ to } m$$

Sufficient cond

$H|_{x^*} \rightarrow$ +ve definite $\rightarrow$ relative min
-ve definite $\rightarrow$ relative max

$$H = \begin{matrix} \frac{\delta^2 L}{\delta x_1^2} & \frac{\delta^2 L}{\delta x_1 \delta x_2} & \ldots & \frac{\delta^2 L}{\delta x_1 \delta \lambda_1} & \ldots & \frac{\delta^2 L}{\delta x_1 \delta \lambda_m} \\ \vdots & & & & & \\ \frac{\delta^2 L}{\delta \lambda_1 \delta x_1} & \frac{\delta^2 L}{\delta \lambda_1 \delta x_2} & \ldots & \frac{\delta^2 L}{\delta \lambda_1^2} & \ldots & \frac{\delta^2 L}{\delta \lambda_1 \delta \lambda_m} \\ \vdots & & & & & \\ \frac{\delta^2 L}{\delta \lambda_m \delta x_1} & \ldots & & & & \frac{\delta^2 L}{\delta \lambda_m^2} \end{matrix}$$

Q Let x be radius & y be length of the cup.

Max. $f(x,y) = \pi x^2 y$

subject to $2\pi x^2 + 2\pi xy = A_0 = 24\pi$

$2\pi x^2 + 2\pi xy - 24\pi = 0$

$L(x,y,\lambda) = \pi x^2 y + \lambda(2\pi x^2 + 2\pi xy - 24\pi)$

$\frac{\delta L}{\delta x} = 2\pi xy + \lambda(4\pi x + 2\pi y) = 0$

$\frac{\delta L}{\delta y} = \pi x^2 + \lambda(2\pi x) = 0$

$\frac{\delta L}{\delta \lambda} = 2\pi x^2 + 2\pi xy - 24\pi = 0$

$2\pi xy + 4\pi x\lambda + 2\pi\lambda y = 0$

$\pi x^2 + 2\pi x\lambda = 0 \quad \times 2$

$2\pi x^2 + 2\pi xy - 24\pi = 0$

$\cancel{2\pi x^2} + 4\pi x\lambda = 0$

$2\pi xy - 4\pi x\lambda - 24\pi = 0$

$2\pi xy + 4\pi x\lambda + 2\pi\lambda y = 0$

$\cancel{6\pi xy = 24\pi}$ 4

$2\cancel{4}\cancel{\pi}xy + \cancel{2\pi}\lambda y = \cancel{24\pi}$ 12

$2xy + \lambda y = 12$

$\cancel{\pi}x^2 = -2\cancel{\pi}x\lambda$

$\lambda = \frac{-1}{2}x$

$\cancel{2}\cancel{\pi}x^2 + \cancel{2\pi}xy = 24\pi$ 12

$x^* = 2, \quad y^* = 4, \quad \lambda^* = -1$

$f^* = 16\pi$

272
64
208

2nd derivative $z^* < 0$, Maxima
$z^* > 0$, minima

$$\frac{\delta^2 L}{\delta x^2} = 2\pi y + \lambda(4\pi) = 4\pi$$

$$\frac{\delta^2 L}{\delta x \delta y} = 2\pi x + \lambda(2\pi) = 2\pi$$

$$\frac{\delta^2 L}{\delta y \delta x} = 2\pi x + 2\pi\lambda = 2\pi$$

$$\frac{\delta^2 L}{\delta y^2} = 0$$

$$\frac{\delta g_1}{\delta x} = 4\pi x + 2\pi y = 16\pi$$

$$\frac{\delta g_1}{\delta y} = 4\pi$$

$$H = \begin{vmatrix} 4\pi - z & 2\pi & 16\pi \\ 2\pi & 0-z & 4\pi \\ 16\pi & 4\pi & 0 \end{vmatrix}$$

$$= 4\pi - z[-16\pi^2] - 2\pi[-64\pi^2] + 16\pi[8\pi^2 + 16\pi z]$$

$$= -64\pi^3 + 16\pi^2 z + 128\pi^3 + 144\pi^3 + 256\pi^2 z$$

272
64
208

$$= 272\pi^2 z + 192\pi^3 = 0$$

$$z = \frac{-12}{17}\pi$$

Multivariable opt. with inequality constraint

Min $f(x)$

subject to

$g_j(x) \le 0 \quad j = 1, 2, \ldots n$

$x = (x_1, x_2, \ldots x_n) \in R^n$

$\downarrow$ Lagrange method

$$L(x, y, \lambda) = f(x) + \sum_{j=1} \lambda_j \left(g_j(x) + s_j^2\right)$$

$$g_j(x) + s_j^2 = 0.$$

$$g_j(x) + s_j^2 = 0$$

$$\downarrow$$

optimality cond $\leftarrow \frac{\delta L}{\delta x_i} = 0 \qquad i = 1, 2, \ldots n$

feasible cond $\leftarrow \frac{\delta L}{\delta \lambda_j} = 0 \Rightarrow g_i(x) + s_j^2 = 0.$

$\rightarrow g_i(x) < 0$

complementary slack var cond $\leftarrow \frac{\delta L}{\delta s_j} = 0 \Rightarrow 2\lambda_j s_j = 0$

$$\lambda_j s_j = 0$$

Case I

$\lambda_j = 0, \quad s_j \neq 0.$

$g_i(x) + s_j^2 = 0.$

$\Rightarrow$ Inactive constraint

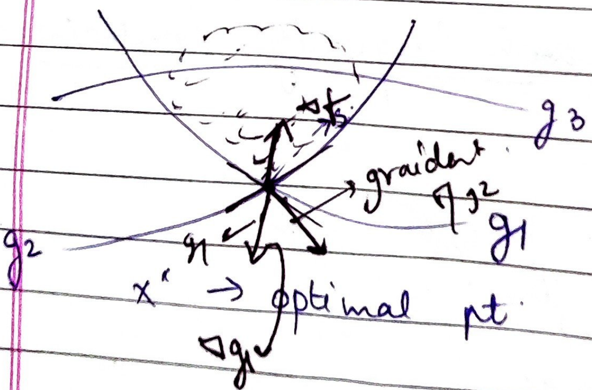
Case II

$\lambda_j \neq 0, \quad s_j = 0.$

$g_i(x) = 0$

Active constraint

$n = 2, \quad m = 3$



x^* → optimal pt

$g_3 \rightarrow$ inactive con

$g_1, g_2 \rightarrow$ active

$$\frac{\delta L}{\delta x_i} = 0$$

$$\frac{\delta f}{\delta x_i} + \lambda_1 \frac{\delta g_1}{\delta x_i} + \lambda_2 \frac{\delta g_2}{\delta x_i} + \cancel{\lambda_3 \delta g_3}$$

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$\Rightarrow \nabla f + \lambda_1 \nabla g_1 + \lambda_2 \nabla g_2$

$$\boxed{\nabla f = -\lambda_1 \nabla g_1 - \lambda_2 \nabla g_2}$$

Q $f = 2x_1 x_2$ at $(1,3)^T$

$S \rightarrow$ feasible vetor $S = (1,1)^T$

$$\nabla f \Big|_{1,3} = \begin{pmatrix} 2x_2 \\ 2x_1 \end{pmatrix}_{1,3} = \begin{pmatrix} 2 \\ 6 \end{pmatrix}$$

$$S^T \nabla f = 2 \cdot 1 + 6 \cdot 1 > 0$$

$$8 > 0$$

Q Find max. value

Optimize problem $F = \pi x_1^2 x_2$ subject to

constraints ~~to~~ $2\pi x_1^2 + 2\pi x_1 x_2 = A_0$

$$g_1 = 2\pi x_1^2 + 2\pi x_1 x_2 - A_0$$

Let Lagrange fn.

$$L = f + \lambda g_1$$

$$L = \pi x_1^2 x_2 + \lambda (2\pi x_1^2 + 2\pi x_1 x_2 - A_0)$$

For maxima/ minin

$$\frac{\delta L}{\delta x_1} = \frac{\delta L}{\delta x_2} = 0 = \frac{\delta L}{\delta \lambda}$$

$$\frac{\delta L}{\delta x_1} = 2\pi x_1 x_2 + \lambda(4\pi x_1 - 2\pi x_2) = 0$$

$$\frac{\delta L}{\delta x_2} = \pi x_1^2 + \lambda(2\pi x_1) = 0 \qquad \times 2$$

$$\frac{\delta L}{\delta \lambda} = 2\pi x_1^2 + 2\pi x_1 x_2 - A_0 = 0$$

$$2\pi x_1 x_2 + \cancel{4\pi\lambda x_1} - 2\pi\lambda x_2 = 0$$

$$2\pi x_1^2 + \cancel{4\pi\lambda x_1} = 0$$

$$2\pi x_1 x_2 - 2\pi\lambda x_2 - 2\pi x_1^2 = 0,$$

$$2\pi x_1 x_2 - 2\pi\lambda x_2 - 2\pi x_1^2 = 0$$

$$+ 2\pi x_1 x_2 - A_0 + \cancel{2\pi x_1^2} = 0.$$

$$4\pi x_1 x_2 - 2\pi\cancel{\lambda x_2} - A_0 = 0.$$

$$-2\pi x_1 x_2 \mp \cancel{2\pi\lambda x_2} + 4\pi\lambda x_1 = 0$$

$$2\pi x_1 x_2 - A_0 - 4\pi\lambda x_1 = 0.$$

$$2\pi x_1 x_2 - 4\pi\lambda x_1 = A_0$$

$$2\pi x_1 (x_2 - 2\lambda) = A_0.$$

$$2\pi x_1 = A_0. \qquad \pi x_1 (x_1 + 2\lambda) = 0$$

$$x_2 - 2\lambda = A_0. \qquad x_1 = 0$$

$$\boxed{x_1 = -2\lambda}$$

$$2\pi\cdot(-2\lambda) x_2 + \lambda(4\pi(-2\lambda) - 2\pi x_2) = 0$$

$$-4\pi\lambda x_2 - 8\pi\lambda^2 - 2\pi\lambda x_2 = 0$$

$$-6\cancel{\pi}\cancel{\lambda}x_2 = 8\cancel{\pi}\lambda^{\cancel{2}}$$

$$-6\pi\lambda x_2 - 8\pi\lambda^2 = 0.$$

$$\cancel{2(}\quad 2\pi\lambda(-3x_2 - 4\lambda) = 0.$$

$$\boxed{x_2 = \frac{4}{3}\lambda}$$

$$2\pi\cdot 4\lambda^2 + 2\pi(-2\lambda)\left(\frac{4}{3}\lambda\right) - A_0 = 0.$$

$$8\pi\lambda^2 - \frac{32}{3}\pi\lambda^2 - A_0 = 0.$$

$$24\pi\lambda^2 - 32\pi\lambda^2 - 3A_0 = 0.$$

$$\boxed{A_0 = 24\lambda}$$

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$2\cancel{x}(4\lambda)^{2} + \cancel{2x}(-2\lambda)(-4\lambda) - 24\cancel{x} = 0,$

$8\lambda^2 - 16\lambda^2 - 24 = 0.$

$8(\lambda^2 - 2\lambda^2 - 3) = 0$

$-\lambda^2 = 3.$

$$H = \begin{bmatrix} 4\pi-2 & 2\pi & 16\pi \\ 2\pi & 0-2 & 4\pi \\ 16\pi & 4\pi & 0 \end{bmatrix} \qquad \begin{bmatrix} L_{11}-2 & L_{12} & g_{11} \\ L_{21} & L_{22}-2 & g_{12} \\ g_{11} & g_{12} & 0 \end{bmatrix}$$

$= 4\pi-2[-16\pi^2] - 2\pi[-64\pi^2] + 16\pi[8\pi^2 + \cancel{16}\pi^2 z]$

$= -64\pi^3 + 16\pi^2 z + 128\pi^3 + 1\cancel{28}\pi^3 + 256\pi^2 z.$

$= 272\pi^2 z + 192\pi^3 = 0,$

$272\cancel{\pi^2}z = -192\pi^{\cancel{3}}.$

$z = \dfrac{\cancel{-192\pi}\ \cancel{96}\ \cancel{48}\ \cancel{24}\ 12}{\cancel{272}\ \cancel{+36}\ \cancel{68}\ \cancel{34}\ 17.} < 0.$

$\begin{array}{r} 2\,\overset{1}{7}\,2 \\ 2\cancel{6}4 \\ \hline 208 \end{array}$

$\begin{array}{r} 128 \\ 128 \\ \cancel{256} \\ \cancel{264}/2 \end{array}$

Maxima.

So, the given pt $(2, 4, -1)$ is maximum pt & max. value 16π

~~Qu~~ optimisation with unequality constraints if Minimize $f(X)$ subject to $g_i(X) \le 0$, $j = 1$ to m

The Kuhn Tucker conditions can be stated as $i = 1$ to n ; $j = 1$ to m

① $\frac{\delta f}{\delta x_i} + \sum_{j=1}^{m} \frac{\delta g_j}{\delta x_i}$

② $\lambda_j g_j = 0$

③ $g_j \le 0$

④ $\lambda_j \ge 0$.

Qu

$\frac{5x}{5}$.

$$L = x_1^2 + x_2^2 + \lambda_1 (4x_2 + 4)$$

$$\frac{\delta L}{\delta x_1} = 0 = \frac{\delta L}{\partial x_2} = 0$$

$$\frac{\delta L}{\partial \lambda_1} = g_1 = 0 \qquad \frac{\delta L}{\delta \lambda_2} = g_2 = 0.$$

Suppose we have two constraints. (g_1, g_2)

$$\frac{\delta f}{\delta x_1} + \lambda_1 \frac{\delta g_1}{\delta x_1} + \lambda_2 \frac{\delta g_2}{\delta x_1} = 0$$

$$\frac{\delta f}{\delta x_2} + \lambda_1 \frac{\delta g_1}{\delta x_2} + \lambda_2 \frac{\delta g_2}{\delta x_2} = 0$$

Min $f(x)$ sub. to $g_j \geq 0$ ① ② ③ $g_j \geq 0$ ④ $\lambda_j \leq 0$.
Max $f(x)$ sub. to $g_j \leq 0$ ① ② ③ $g_j \leq 0$ ④ $\lambda_j \leq 0$.
Max $f(x)$ sub. to $g_j \geq 0$. ① ② ③ $g_j \geq 0$ ④ $\lambda_j \geq 0$

Qu Find soln of problem Min $f(x_1, x_2) = x_1^2 + x_2^2 + 40x_1 + 20x_2$ s/to

$\cancel{g_1} = x_1 - 50 \geq 0$

$x_1 + x_2 - 100 \geq 0$

$L = f(x) + \lambda_1 g_1 + \lambda_2 g_2$
$= x_1^2 + x_2^2 + 40x_1 + 20x_2 + \lambda_1 (x_1 - 50)$
$+ \lambda_2 (x_1 + x_2 - 100)$

$\frac{5x}{5}$ $\cancel{\delta L}$. KT conditions are.

$\frac{\delta L}{\delta x_1} = 2x_1 + 40 + \lambda_1 + \lambda_2 = 0$. —— ①

$\frac{\delta L}{\delta x_2} = 2x_2 + 20 + \lambda_2 = 0$ —— ②

$\frac{\delta L}{\delta \lambda_1} = x_1 - 50 \geq 0$ —— ③

$\frac{\delta L}{\delta \lambda_2} = x_1 + x_2 - 100 \geq 0$. —— ④

$\lambda_1 \leq 0$, $\lambda_2 \leq 0$

$\lambda_1 (x_1 - 50) = 0$ —— ⑤

$\lambda_2 (x_1 + x_2 - 100) = 0$ —— ⑥

From ⑤

$\lambda_1 = 0$; $x_1 = 50$

Case I $\lambda_1 = 0$.

Put in ①.

$$\cancel{2}x_1 + \overset{20}{\cancel{40}} + \cancel{\lambda_2} = 0.$$
$$\cancel{2}x_2 + \overset{10}{\cancel{20}} + \cancel{\lambda_2} = 0$$

$x_1 - x_2 = 0.$

$x_1 = x_2;$

$\boxed{\lambda_2 = -130}$

From (6)

$\lambda_2 (2x_2 - 100) = 0.$

$\lambda_2 = 0, \quad x_2 = 50.$

Subcase I $\quad \lambda_1 = 0, \ \lambda_2 = 0$

Subcase II $\quad \lambda_1 = 0, \ \lambda_2 = -130$

I $\quad x_1 = -20 - \frac{\lambda_1}{2} \qquad x_2 = -10 - \frac{\lambda_2}{2}.$

$x_1 = -20, \quad x_2 = -10$

from (3) $\quad -20 - 50 \geq 0$

violates the condition.

II $\quad x_1 = -20 - \frac{\overset{65}{\cancel{130}}}{\cancel{2}1} < 0. \qquad x_2 = -10 - \frac{130}{2} \leq 0$

violates the condition

Case II, $\quad x_1 = 50.$

$\lambda_1 + \lambda_2 = -140. \quad$ — From (1).

From (2).

$\lambda_2 = -2x_2 - 20.$

From (6)

$-2x_2 - 20 \cancel{\lambda_2} (50 + x_2 - 100) = 0.$

$-2x_2 = 20 = 0 \qquad \boxed{x_2 = 50}$

$\boxed{x_2 = -10}$

Subcase I

$x_1 = 50, x_2 = -10.$

$50 - 50 \geq 0$ satisfies.

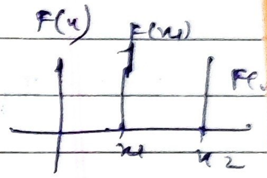
$50 - 10 - 100 \geq 0 \rightarrow$ violates.

Subcase II $x_1 = 50$, $x_2 = 50$.

$50 - 50 \geq 0$

$100 - 100 \geq 0$ } satisfies.

Unimodal fn: f(x) is unimodal



1) $x_1 < x_2 < x^* \Rightarrow F(x_2) < F(x_1)$

2) $x_2 > x_1 > x^* \Rightarrow F(x_1) < F(x_2)$

where x^* is min. pt.

A unimodal fn can be non differentiable or even a discontinuous fn if fn is known to be unimodel in given range & interval the interval in which min. lies can be narrowed down provided that functional values are known the functional values are known at to different pt. in the given range

e.g. consider the normalized interval [0,1] and two functional values within the given interval are known

There are 3 possible outcomes namely $f(x_1) < f(x_2)$ or $f(x_1) > f(x_2)$ or $f(x_1) = f(x_2)$

If the outcome is that $\Rightarrow f_1 < f_2$, the minimum x cannot lie to the interval right of x_2 (bcoz fn is unimodal fn)

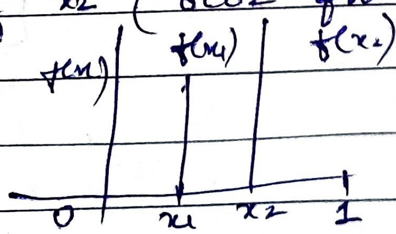


fig 1(a)

means part of the interval $[x_2, 1]$ can be neglected & a new small interval of uncertainity $[0, x_2]$

as shown in fig 1(a)

⇒ If $f(x_1) > f(x_2)$ in fig 1(b) then interval $[0, x_1]$ can be ignored and new interval of uncertainity $[x_1, x_2]$ as shown in fig.

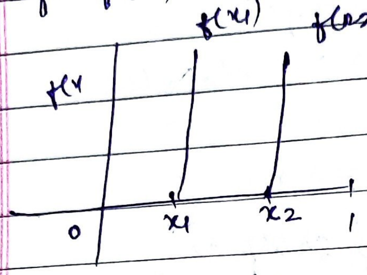
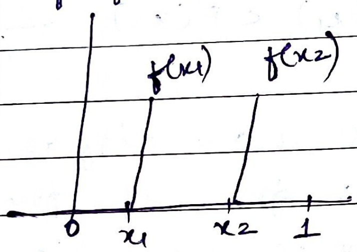


fig 1(b)

⇒ If $f(x_1) = f(x_2)$, the interval $[0, x_1]$ & $[x_2, 1]$ can both be ignored to find the new interval of uncertainity as $[x_1, x_2]$



⇒ <u>Fibonacci Series.</u>

Here we have a sequence of fibonacci numbers $\{f_n\}$ as given below

$F_0 = F_1 = 1$ & $F_n = F_{n-1} + F_{n-2}$; $n = 2, 3, 4, \ldots$

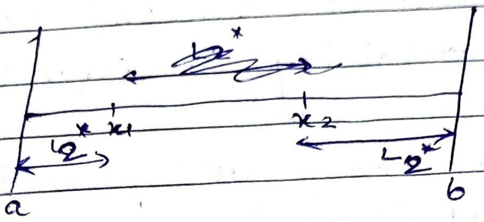
⇒ <u>Fibonacci methods</u>

In this method let L_0 be initial interval of uncertainity defined by $a \le x \le b$ & n be the total number of experiment to be conducted. Now define $\boxed{L_2^* = \frac{F_{n-2}}{F_n} \times L_0}$

and $x_1 = a + L_2^*$

$x_2 = b - L_2^*$

Here x_1 & x_2 which are located at a distance of L_2^* from each end of L_0



$L_0 = b - a$

Que Minimize func $f(x) = 0.65 - \left[\frac{0.75}{1+x^2}\right] - 0.65x \tan^{-1}(1/x)$

in the interval [0,3] by Fibonacci method with taking total nos of exper. $n=6$

$n = 6$

$[0, 3]$ $\quad L_0 = 3 - 0 = 3.$

$[a, b]$

$L_2^* = \frac{F_4}{F_6} \times 3 = \frac{5}{13} \times 3 = 1.153846$

then poss. two experiments are given by

$x_1 = a + L_2^* = 1.153846$

$x_2 = 1.846154$

40.914°

$F(x_1) = 0.65 - \left[\frac{0.75}{2.33136}\right] - \cancel{0.6}\ 0.750\,(40.914)$ $\quad 0.0124$

$= 0.65 - 0.3217 - 0.0093$

~~= 0.31~~

$f(x_1) = -0.207270$

$f(x_2) = -.115843$

Since, $f(x_1) < f(x_2)$

So we can delete the interval $[x_2, 3]$

& new interval of uncertainity $[0, x_2]$

So, 3rd exper. can be placed as

$x_3 = a + (x_2 - x_1)$

$= 0 + 0.692308 \quad = 0.692308$

$f(x_3) = -0.291364$

In 3rd expe: $f(x_3) < f(x_1)$

$\Rightarrow$ Delete interval $[x_1, x_2]$ and new interval of uncertainity is $[0, x_1]$

Now, 4th exp. can be placed as

$x_4 = a + (x_1 - x_3)$

$= 0 + 0.461538$

$f(x_4) = -0.309811$

$\Rightarrow f(x_4) < f(x_3)$

So delete interval $[x_3, x_1]$ & now interval of uncertainity is $[0, x_3]$

$x_5 = 0.230767$

$f(x_5) = -0.263678$

$\Rightarrow f(x_5) > f(x_4)$

So delete interval $[0, x_5]$ & now interval of uncertainity is $[x_5, x_3]$

$x_6 = x_5 + (x_3 - x_4)$

$= 0.461540$

$f(x_6) = -0.309810$

$f(x_6) > f(x_4)$

~~Delete~~ so delete interval $[x_6, x_3]$

so new interval of uncertainity $[x_5, x_6]$

Verification

$L_6 = x_6 - x_5$

$= \cancel{0.2307}$ 0.23082

Ratio of final interval to initial interval is

$$\frac{L_6}{L_0} = 0.076923 \quad \text{—①}$$

$$\frac{L_n}{L_0} = \frac{F_1}{F_n} = 0.076923 \quad \text{—②}$$

From ① & ②

$$\boxed{\frac{L_6}{L_0} = \frac{F_1}{F_6}}$$ verified.

$$\boxed{x = 0.461540}$$

Golden Section Method

It is same as the Fibonacci method, except that in the Fibonacci method, the total no of experiments to be conducted has to be specified before beginning the calculation whereas this is not required in the golden section method.

In this method we start with assumptions that we are going to conduct a large no. of experiments of course the total no of experiments can be decided during computation. The intervals of uncertainty remaining at the end of diff. no of exper. can be conducted as follows

$$L_2 = \lim_{N \to \infty} \frac{F_{N-1}}{F_N} L_0$$

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$$L_3 = \lim_{N\to\infty} \frac{F_{N-2}}{F_{N-1}} L_2$$

$$= \lim_{N\to\infty} \frac{F_{N-2}}{F_{N-1}} \times \frac{F_{N-1}}{F_N} \times L_0$$

$$= \lim_{N\to\infty} \left(\frac{F_{N-1}}{F_N}\right)^2 L_0 \quad \text{--- (1)}$$ The ration

$\frac{F_{N-2}}{F_{N-1}}$ & $\frac{F_{N-1}}{F_N}$ has been taken to be

same ~~as~~ for large values of N

So, this result can be taken as

$$L_k = \lim_{N\to\infty} \left(\frac{F_{N-1}}{F_N}\right)^{k-1} L_0 \quad \text{--- (2)}$$

$$F_N = F_{N-1} + F_{N-2}$$

$$\Rightarrow \frac{F_N}{F_{N-1}} = 1 + \frac{F_{N-2}}{F_{N-1}}$$

$$= 1 + \frac{1}{\frac{F_{N-1}}{F_{N-2}}}$$

$$r = \lim_{N\to\infty} \frac{F_N}{F_{N-1}}$$

$$r = 1 + \frac{1}{r}$$

$$r^2 - r - 1 = 0$$

$$r = 1.618$$

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Stopping criteria for this method

$$L_K = \left(\frac{1}{r}\right)^{k-1} L_0$$

$$= (0.618)^{k-1} L_0$$

~~For~~ To start this method, let

Define $L_2^* = \frac{F_{N-2}}{F_N} L_0$

$$= \frac{F_{N-2}}{F_{N-1}} \times \frac{F_{N-1}}{F_N} L_0$$

$$= \frac{1}{r} \cdot \frac{1}{r} L_0$$

$$= \frac{1}{r^2} L_0$$

$$L_2^* = \frac{1}{(1.618)^2} L_0 = 0.382 L_0.$$

So,

$$x_1 = a + L_2^*$$

$$x_2 = b - L_2^*$$

Min fn. $F(x) = 0.65 - \left[\frac{0.75}{1+x^2}\right] - 0.65x \tan^{-1}\frac{1}{x}$ in interval $[0,3]$ by Golden section method

$$L_0 = 3$$

$$L_2^* = 1.146$$

$$x_1 = 1.146$$

$$x_2 = 1.854$$

Solution of Linear Programming Problems By Graphical Method

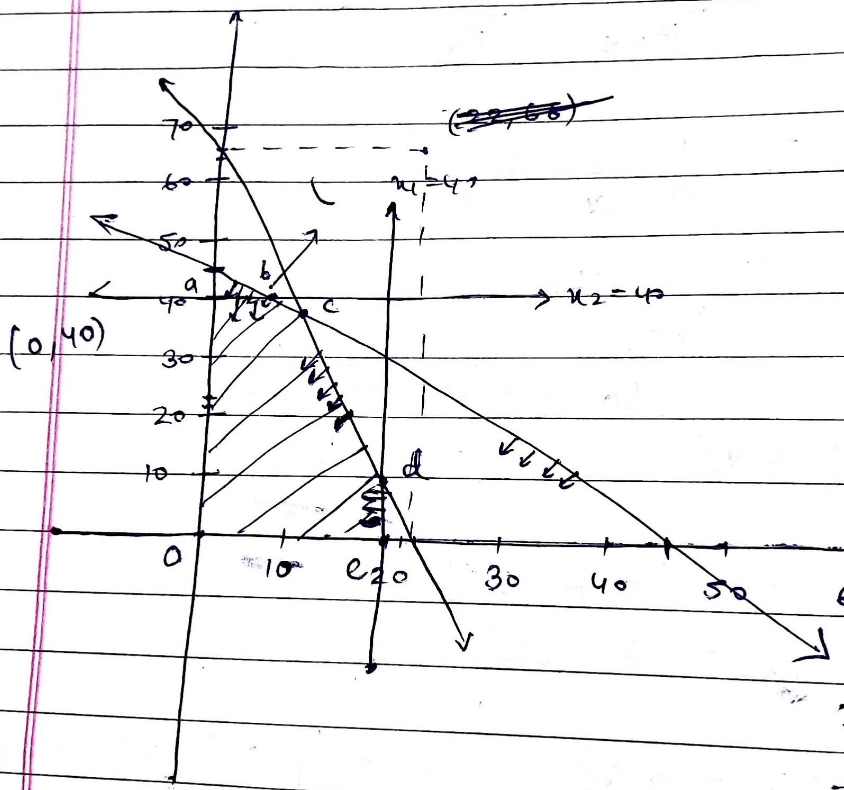
Max. $z = 8000x_1 + 7000x_2$ subject to

$3x_1 + x_2 \leq 66$

$x_1 + x_2 \leq 45$

$x_1 \leq 20$

$x_2 \leq 40$ & $x_1 \geq 0, x_2 \geq 0$



$x_1 + x_2 = 45$

$x_1 = 5$

$x_1 + x_2 = 45.$
$3x_1 + x_2 = 66$
$-2x_1 \ / \ 21$

$x_1 = 0, \quad x_2 = 66$

$x_2 = 0 \quad ; x_1 = 22.$

Feasible region → abcdeo

A(0,40), Max z = 0

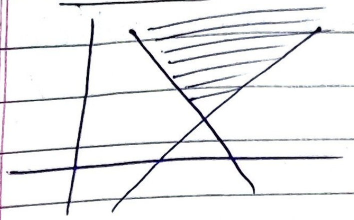
B(5,40)

C,

D

E

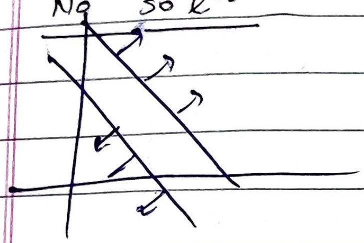
Unbounded sol^n



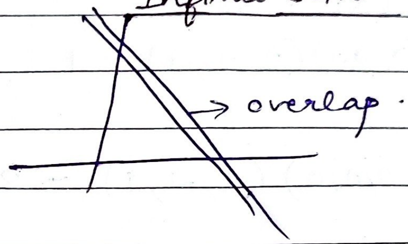
Q: $x_1 - x_2 \leq 1$

$x_1 + x_2 \geq 3$

No sol^n



Infinite soln



Find soln of problem by simplex method

Max $z = x_1 + 2x_2 + x_3$ subject to

$2x_1 + x_2 - x_3 \leq 2$

$2x_1 - x_2 + 5x_3 \leq 6$ $\qquad$ s = slack variable

$4x_1 + x_2 + x_3 \leq 6$

$x_1, x_2, x_3 \geq 0$

Work in term of $AX = B$

$2x_1 + x_2 - x_3 + s_1 = 2$

$2x_1 - x_2 + 5x_3 + s_2 = 6$

$4x_1 + x_2 + x_3 + s_3 = 6$

~~s_1~~, ~~s_2~~, ~~s_3~~ cost coefficient

		z	1	2	1	0	0	0	
Basic Variable	C_B	X_B	x_1	x_2	x_3	s_1	s_2	s_3	Min Ratio
s_1	0	2	2	$\boxed{1}$	-1	1	0	0	2/1 = 2
s_2	0	6	2	-1	5	0	1	0	6/-1
s_3	0	6	4	1	1	0	0	1	6/1 = 6
	$Z = C_B X_B = 0$		-1	-2	-1	0	0	0	

$x_1 = 0,\ x_2 = 0,\ x_3 = 0.$

$s_1, s_2, s_3 = 2, 6, 6 \rightarrow$ Basic solution

Δj = Net evalation

$= z_j - c_j$

$= c_B X_j - c_j$

$\Delta_1 = c_B X_1 - c_1$

$= (0,0,0)(2,2,4) - 1$

$\Delta_2 = (0,0,0)(1,-1,1) - 2$

Test for optimality

If all $\Delta j \geq 0$, solution is optimal

Alternative soln

Qu Max $z = 3x_1 + 2x_2 + x_3$ s/to

$x_1 + 2x_2 + x_3 \leq 430$

$3x_1 + 2x_3 \leq 460$

$x_1 + 4x_2 \leq 420$, $x_1, x_2, x_3 \geq 0$

$x_1 + 2x_2 + x_3 + s_1 = 430$

$3x_1 + 2x_3 + s_2 = 460$

$x_1 + 4x_2 + s_3 = 420$

det $x_1 = x_2 = x_3 = 0$

$s_1 = 430$, $s_2 = 460$, $s_3 = 420$

$s_1, s_2, s_3 \geq 0$

Basic var.	c_B	x_B	x_1	x_2	x_3	s_1	s_2	s_3	Min ratio
		$z \rightarrow$	3	2	1	0	0	0	
s_1	0	430	1	2	1	1	0	0	430
s_2	0	460	$\boxed{3}$	0	2	0	1	0	153.3 $\rightarrow$
s_3	0	420	1	4	0	0	0	1	420
		$z = 0$	$\uparrow -3$	-2	-1	0	0	0	
s_1	0	$\frac{830}{3}$	0	2	1/3	1	$-1/3$	0	$\frac{830}{6} = 138$
x_1	3	$\frac{460}{3}$	1	0	2/3	0	1/3	0	
s_3	0	$\frac{800}{3}$	0	$\boxed{4}$	$-2/3$	0	$-1/3$	1	$\frac{800}{3\times4} = 66 \leftarrow$
		$z = 460$	0	$\uparrow -2$	1	0	1	0	
s_1	0	$\frac{430}{3}$	0	0	2/3	1	1/6	$-1/2$	
x_1	3	$\frac{460}{3}$	1	0	2/3	0	1/3	0	
x_2	2	$\frac{800}{12}$	0	1	$-1/6$	0	$-1/12$	1/4	
			0	0	2/3	0	5/6	1/2	

$\frac{430}{3}$ 1290 460 83

200 $\frac{\cancel{800}}{\cancel{12}} \times 2$ 3 $\frac{400}{3}$

830

$\frac{1}{3} + \frac{1}{3}$

all net evaluation (Δ_i) are 0 or pos.

So we stop here

$x_1 = \frac{460}{3}$, $x_2 = \frac{800}{12}$, x_3

$$Z = \frac{\cancel{3} \times 460}{\cancel{3}} + \frac{2 \times 200}{3}$$

Q. $\text{Max } z = -2x_1 - x_2 + 5x_3$

$x_1 - 2x_2 + x_3 + s_1 = 8$

$-3x_1 + 2x_2 + s_2 = 18$

$2x_1 + x_2 - 2x_3 + s_3 = 4$

Let $x_1 = x_2 = x_3 = 0$

$s_1 = 8$, $s_2 = 18$, $s_3 = 4$

B.V	CB	XB	x_1	x_2	x_3	s_1	s_2	s_3	Min Ratio
		$z \rightarrow$	-2	-1	5	0	0	0	
s_1	0	8	1	-2	$\boxed{1}$	1	0	0	$8/1 = 8 \leftarrow$
s_2	0	18	-3	2	0	0	1	0	—
s_3	0	4	2	1	-2	0	0	1	—
	$z = 0$		2	1	$-5\uparrow$	0	0	0	—
x_3	5	8	1	-2	1	1	0	0	$9 \rightarrow$
s_2	0	18	-3	$\boxed{2}$	0	0	1	0	—
s_3	0	20	4	-3	0	2	0	0	
			7	$\uparrow -9$	0	5	0	0	
x_3	5	26	-2	0	1	1	1	0	
x_2	-1	9	-3/2	1	0	0	1/2	0	
s_3	0	47	-1/2	0	0	2	3/2	1	
			$\uparrow -13/2$	0	0	5	9/2	0	

$\Delta_1 \rightarrow$ -ve & corresponding colm all are neg.

$\Rightarrow$ This prob has unbounded soln

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Primal Dual Problem

Every lpp has associated with it another linear lpp. So, the original prob. is called Primal & associated prob. is known as dual Prob. If the optimal soln to one is known then optimal soln of the other is also available. The dual of the dual is known as primal problem.

Symmetric Primal Dual Problem

Let Primal Prob. is

$$\text{Max } Z_x = c_1x_1 + c_2x_2 + \dots c_nx_n$$

s/to

$$a_{11}x_1 + a_{12}x_2 + \dots a_{1n}x_n \leq b_1$$

$$a_{21}x_2 + a_{22}x_2 + \dots a_{2n}x_n \leq b_2$$

$$\vdots$$

$$a_{m1}x_1 + a_{m2}x_2 + \dots a_{mn}x_n \leq b_m$$

And.

$$x_1, x_2, \dots x_n \geq 0$$

OR $\quad \text{Max } Z_x = CX$

$$AX \leq B \;;\; X \geq 0$$

So, the dual of the above prob. is obtained by

1- Transposing the coeff. matrix
2- Interchanging the role of constant terms and the coeff. of the objective fns.
3- Reverting the inequalities
4- Minimizing the obj. fn in place of maximization

So, the dual of the above problem is

find $w_1, w_2 \dots w_m$ such that $\text{Min } Z_w = b_1w_1 + \dots b_mw_m$

$$a_{11}w_1 + a_{21}w_2 + \dots a_{m1}w_m \geq c_1$$

$$a_{12}w_1 + a_{22}w_2 + \dots a_{m2}w_m \geq c_2$$

$$\vdots$$

$$a_{1n}w_1 + a_{2n}w_2 + \dots a_{mn}w_m \geq c_n$$

$w_1, w_2 \dots w_m \geq 0$

Que Find the dual of the given primal prob.

$$\text{Min } z_x = 2x_2 + 5x_3 \quad s/to$$

$$x_1 + x_2 \geq 2$$

$$2x_1 + x_2 + 6x_3 \leq 6$$

$$x_1 - x_2 + 3x_3 = 4 \rightarrow \boxed{y_3 = y_3' - y_3''}$$

$$x_1, x_2, x_3 \geq 0$$

Standard Primal Prob:

$$\text{Max } z_x' = -2x_2 - 5x_3 \quad , \quad z_x = -z_x'$$

s/to

$$-x_1 - x_2 \leq -2 \quad y_1$$

$$2x_1 + x_2 + 6x_3 \leq 6 \quad y_2$$

$$x_1 - x_2 + 3x_3 \leq 4 \quad y_3'$$

$$-x_1 + x_2 - 3x_3 \leq -4 \quad y_3''$$

$$x_1, x_2, x_3 \geq 0$$

$x_1 - x_2 + 3x_3 \geq$

convert

$x_1 - x_2 + 3x_3 \leq 4$

$x_1 - x_2 + 3x_3 \geq 4$

So, dual of this problem is :- $4y_3$.

$$\text{Min } z_y' = -2y_1 + 6y_2 + \overbrace{4(y_3' - y_3'')}$$

s/to

$$-y_1 + 2y_2 + (y_3' - y_3'') \geq 0$$

$$-y_1 + y_2 - y_3' + y_3'' \geq -2$$

$$6y_2 + 3(y_3' - y_3'') \geq -5$$

$$y_1, y_2 \geq 0, \; y_3 = y_3' - y_3''$$

y_3 is unrestricted

Or x_1 is unrestricted

$$x_1 = x' - x_2''$$

Que $\text{Min } z_x = 2x_1 + 3x_2 + 4x_3 \quad s/to$

$$2x_1 + 3x_2 + 5x_3 \geq 2$$

$$3x_1 + x_2 + 7x_3 = 3.$$

$$x_1 + 4x_2 + 6x_3 \leq 5$$

$$x_1, x_2, x_3 \geq 0 \qquad x_3 \text{ is unrestricted.}$$

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Dual Simplex method.

Find soln of prob.

$$Max\ z = -4x_1 - 6x_2 - 18x_3 \quad s/to$$

$$x_1 + 3x_3 \geq 3$$

$$x_2 + 2x_3 \geq 5$$

$$x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0$$

$$Max\ z = -4x_1 - 6x_2 - 18x_3 \quad s/to$$

$$-x_1 - 3x_3 \leq -3$$

$$-x_2 - 2x_3 \leq -5$$

$$-x_1 - 3x_3 + s_1 = -3$$

$$-x_2 - 2x_3 + s_2 = -5.$$

To find leaving / outgoing vector min of x_B

$= \{x_{B1}, x_{B2} \ldots x_{Bn}\}$

Min of $x_B = \{-5, -3\} = -5$

$\Rightarrow$ s_2 is outgoing vector from basic variables

To find entering vector in the basis

$$\underset{j}{Max} \left\{ \frac{\Delta j}{a_{rj}},\ a_{rj} < 0 \right\}$$

Here, a_{rj} is the coefficient of variables.

Here $r = 2,\ a_{2j} < 0$

① $Max \{ \frac{4}{\ } -$

② $Max \{ \frac{6}{\ } -$

③ $\{ -$

④ $\{ -$

⑤ $\{ -$

$$\left\{ \frac{4}{\ },\ \frac{6}{-1},\ \frac{18}{-2} \right\}$$

$$\{-6, -9\}$$

			c_j	-4	-6	-18	0	0
B.V	C_B	x_B		x_1	x_2	x_3	S_1	S_2
S_1	0	-3		-1	0	-3	1	0
→ S_2	0	-5		0	$\boxed{-1}$	-2	0	1
	$z = 0$			4_{Δ_1}	$6 \uparrow_{\Delta_2}$	18_{Δ_3}	0_{Δ_4}	0_{Δ_5}
→ S_1	0	-3		$-1_{\Delta_{11}}$	$0_{\Delta_{12}}$	$\boxed{-3}_{\Delta_{13}}$	$1_{\Delta_{14}}$	$0_{\Delta_{15}}$
x_2	-6	5		0	1	2	0	-1
				4_{Δ_1}	0_{Δ_2}	6_{Δ_3}	0_{Δ_4}	6_{Δ_5}
x_3	-18	1		1/3	0	1	-1/3	0
x_2	-6	3		-2/3	1	0	+2/3	-1
	$z = -18-18 = -36$			+2	0	0	2	6

$\boxed{x_2 = 3}$ $\boxed{x_3 = 1}$

Q Find soln of prob.

$$\text{Min } z = 2x_1 + x_2$$

s/to $3x_1 + x_2 \geq 3$

$4x_1 + 3x_2 \geq 6$

$x_1 + 2x_2 \geq 3$

$x_1, x_2, x_3 \geq 0$

Max $z^* = -2x_1 - x_2$.

$-3x_1 - x_2 \leq -3$

$-4x_1 - 3x_2 \leq -6$

$-x_1 - 2x_2 \leq -3,$

Max $= \{-\frac{1}{2},$

$-3x_1 - x_2 + S_1 = -3$

$-4x_1 - 3x_2 + S_2 = -6$

$-x_1 - 2x_2 + S_3 = -3.$

		y^0	-2	-1	0	0	0
B.V	C_B	X_B	x_1	x_2	S_1	S_2	S_3
S_1	0	-3	-3	-1	1	0	0
→ S_2	0	-6	$\boxed{-4}$	$\boxed{-3}$	0	1	0
S_3	0	-3	-1	-2	0	0	1
	$z=0$		2	1	0	0	0
→ S_1	0	-1	$\boxed{-5/3}$	0	1	-1/3	0
x_2	-1	2	4/3	1	0	-1/3	0
S_3	0	1	5/3	0	0	-2/3	1
			2/3	0	0	1/3	0
x_1	-2	3/5	1	0	-3/5	1/5	0
x_2	-1	-2/3	0	0	-4/5	-1/15	0
S_3	0						

$-3+\frac{4}{3}$

$-3+2$

$\frac{8}{3}-1$

$2-\frac{4}{3}$

$x_1 = 3/5$ $\qquad z = \frac{12}{5}$

$x_2 = 6/5$

$3+\frac{4}{3}$

$-5/3$

$-1, -\frac{5}{3}\frac{3}{2}$

$-1, -2$

$\frac{-1}{3} \times \frac{3}{-5}$

$\frac{4}{3} \times \frac{-3}{5}$

$-\frac{1}{3} + \frac{4}{15}$

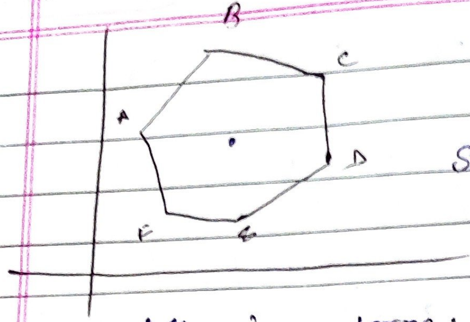
$\frac{-5+4}{15}$

$2-\frac{8}{3}$

$6-$

Karmarkar's Method

(Interior Method)



Statement of the problem
Karmarkar's method requires lp problem in the following form:

$$\text{Min } f = c^T x \quad s/b \text{ to matrix } ax = 0$$

$$\text{and } x_1 + x_2 + \cdots x_n = 1, \quad x \geq 0$$

$$\text{where } x = [x_1, x_2, \ldots, x_n]^T$$

$$c = [c_1, c_2, \ldots, c_n]^T$$

a is $m \times n$ matrix

In this method, generally starting interior initial pt. is

$$x = \left\{\frac{1}{n}, \frac{1}{n} \cdots \frac{1}{n}\right\}^T$$

<u>Test for optimality</u>
Since $f = 0$ at optimum pt., we stop the procedure if the following converse criteria is satisfied

$$\| c^T x^{(k)} \| \leq E$$

For ~~Now~~ computing the pt., $x^{(k+1)}$, first complete

$$y^{(k+1)} = \left\{\frac{1}{n}, \frac{1}{n}, \frac{1}{n} \cdots \frac{1}{n}\right\}^T - \frac{\alpha\left[(I) - [P]^T ([P][P]^T)^{-1} [P] \right] [D(x^k)] c^T}{\|c\| \sqrt{n(n-1)}} \qquad (I)$$

where, norm of c is length of vector c

$I \rightarrow$ Identity matrix of order n

$[D[x^k]]$ is $n \times n$ matrix with all non diagonal entries $= 0$. & diagonal entries $=$ components of vector $[x^k]$

as the $[D(x^k)]_{ii} = x_i^{(k)}$; $i = 1, 2, \dots n$

Matrix $[P]$ is an $(m+1) \times n$ matrix whose first m rows are given by matrix $[A][D(x^k)]$ and the last row is composed of 1 and value of the parameter α is usually chosen as $\frac{1}{4}$

$$\alpha = 1/4$$

to ensure the convergence. Once y^{k+1} is computed the components of the new pt. x^{k+1} are calculated $x_i^{(k+1)} = \frac{x_i^k y_i^{(k+1)}}{\sum_{r=1}^{n} x_r^k y_r^{(k+1)}}$

Set the new iteration no. as $k = k+1$ and go to step 2

Q. Find soln of following problem by using Karmarkar's method.

$$\text{Min } f = 2x_1 + x_2 - x_3 \text{ s/to}$$

$$x_2 - x_3 = 0$$

$$x_1 + x_2 + x_3 = 1 \quad ; \quad x_i \geq 0$$

use the value of $\varepsilon = 0.05$ for testing convergence of procedure

(I) $x^{(1)} = \begin{Bmatrix} \frac{1}{3} \\ \frac{1}{3} \\ \frac{1}{3} \end{Bmatrix}$ and set $k = 1$

$\frac{2}{3} + \frac{1}{3}$

(II) Since $|f(x^1)| = |c^T x^{(1)}|$

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$$|c^T x^{(1)}| = \left| [2, 1, -1] \begin{bmatrix} \frac{1}{3} \\ \frac{1}{3} \\ \frac{1}{3} \end{bmatrix} \right|$$

$$= \left| \frac{2}{3} + \frac{1}{3} \cdot -\frac{1}{3} \right| = \frac{2}{3} = 0.667 \le \varepsilon$$

So we go to step 3.

Since $[a] = [0, 1, -1]$

$$\|c\| = \sqrt{2^2 + 1^2 + (-1)^2} = \sqrt{6}$$

$$D\, x^{(1)} = \begin{bmatrix} 1/3 & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & 1/3 \end{bmatrix}$$

$$[a][D x^{(1)}] = [0 \quad 1 \quad -1] \begin{bmatrix} 1/3 & 0 & 0 \\ 0 & 1/3 & 0 \\ 0 & 0 & 1/3 \end{bmatrix}$$

$$= \cancel{\begin{bmatrix} 0 \\ 1/3 \\ -1/3 \end{bmatrix}} = [0 \quad 1/3 \quad -1/3]$$

$$P = \begin{bmatrix} 0 & 1/3 & -1/3 \\ 1 & 1 & 1 \end{bmatrix}$$

$$[P][P^T] = \underset{2\times3}{\begin{bmatrix} 0 & 1/3 & -1/3 \\ 1 & 1 & 1 \end{bmatrix}} \underset{3\times2}{\begin{bmatrix} 0 & 1 \\ \frac{1}{3} & 1 \\ -1/3 & 1 \end{bmatrix}}$$

$$= \begin{bmatrix} \frac{2}{9} & 0 \\ 0 & 3 \end{bmatrix}.$$

$[[P][$

$[P^T]$

$\frac{1}{2} + \frac{1}{3}$

$\frac{3+2}{6}$

$-\frac{1}{2} + \frac{1}{3}$

$\frac{-3+2}{}$

$-\frac{1}{2} + \frac{1}{3}$

$\frac{1}{2}$

$$\left[[P][P]^T\right]^{-1} = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & 1/3 \end{bmatrix}$$

$$\underbrace{[P^T]\left[[P][P]^T\right]^{-1}[P]}_{A} = \underset{3\times2}{\begin{bmatrix} 0 & 1 \\ 1/3 & 1 \\ -1/3 & 1 \end{bmatrix}} \underset{2\times2}{\begin{bmatrix} \frac{1}{2} & 0 \\ 0 & 1/3 \end{bmatrix}} \begin{bmatrix} 0 & 1/3 & -1/3 \\ 1 & 1 & 1 \end{bmatrix}$$

$$= \underset{3\times2}{\begin{bmatrix} 0 & 1/3 \\ \frac{3}{2} & 1/3 \\ -3/2 & 1/3 \end{bmatrix}} \underset{2\times3}{\begin{bmatrix} 0 & 1/3 & -1/3 \\ 1 & 1 & 1 \end{bmatrix}}$$

$$= \begin{bmatrix} 1/3 & 1/3 & 1/3 \\ 1/3 & 5/6 & -1/6 \\ 1/3 & -1/6 & 5/6 \end{bmatrix}$$

$\frac{1}{2} + \frac{1}{3}$

$\frac{3+2}{6}$

$$I - A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} \frac{1}{3} & 1/3 & 1/3 \\ 1/3 & 5/6 & -1/6 \\ 1/3 & -1/6 & 5/6 \end{bmatrix}$$

$-\frac{1}{2} + \frac{1}{3}$

$-3+2$

$-\frac{1}{2} + \frac{1}{3}$

$$= \begin{bmatrix} 2/3 & -1/3 & -1/3 \\ -1/3 & 1/6 & 1/6 \\ -1/3 & 1/6 & 1/6 \end{bmatrix}$$

$\frac{1}{2}$

$$\underbrace{(I-A)\,(D(x^k)c}_{B} = \underset{3\times3}{\begin{bmatrix} \frac{2}{3} & -1/3 & -1/3 \\ -1/3 & 1/6 & 1/6 \\ -1/3 & 1/6 & 1/6 \end{bmatrix}} \underset{3\times1}{\begin{bmatrix} \frac{2}{3} \\ 1/3 \\ 1/6 \end{bmatrix}}$$

$$= \begin{bmatrix} 4/9 \\ -2/9 \\ -2/9 \end{bmatrix}$$

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$c^T = (2, 1, -1)$

$$\alpha B = \frac{1}{4}\begin{bmatrix} 4/9 \\ -2/9 \\ -2/9 \end{bmatrix}$$

$\frac{1}{4} \times \frac{2}{9}$

$$= \begin{bmatrix} 1/9 \\ -1/18 \\ -1/18 \end{bmatrix}$$

$$y^2 = \begin{bmatrix} \frac{1}{3} \\ \frac{1}{3} \\ \frac{1}{3} \end{bmatrix} - \begin{bmatrix} \frac{1}{9} \\ -1/18 \\ -1/18 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{2}{9} \\ 7/18 \\ \frac{7}{18} \end{bmatrix}$$

$\frac{1}{3} + \frac{1}{18}$

$\frac{6+1}{18}$

$(2, 1, -1)\ \sqrt{6}$

$\frac{34+7}{\cancel{108}}\ 54$

$$= \begin{bmatrix} 34/108 \\ 37/108 \\ 37/108 \end{bmatrix}$$

$$\sum_{r=1}^{3} (x_4^k) = x_1^k y_1^{k+1} + x_2^k y_2^{k+1} + x_3^k y_3^{k+1}$$

$\cancel{x_i^{(2)}} = \cancel{\frac{1}{3} \times 4/9}$

$$x_i^{k+1} = \frac{x_i^k\, y_i^{k+1}}{\sum_{r=1}^{3} x_r^k\, y_r^{k+1}}$$