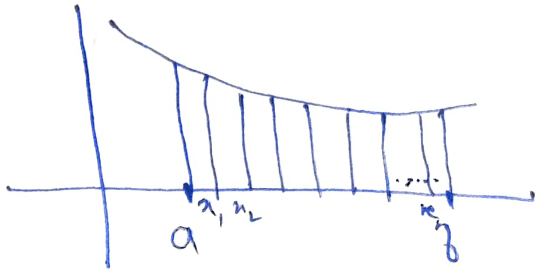
multiple integral $\begin{cases} \to \text{Double integral} \\ \to \text{Tripal integral.} \end{cases}$

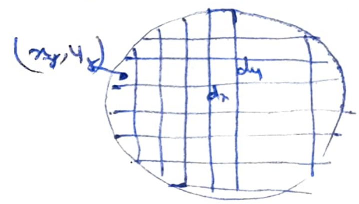


Finite integral :- (one variable

$$I = \int_a^b f(x)\,dx$$

$$= \lim_{\substack{n \to \infty \\ \delta x_i \to 0}} \sum_{i=1}^{n} f(x_i)\,\delta x_i$$

double integral :- Let $f(x,y)$ be defined over region



$$\lim_{\substack{n \to \infty \\ \delta A \to 0}} \sum_{r=1}^{n} f(x_r, y_r) \cdot dA_r = \iint_R f(x,y)\,dA \quad -(1)$$

$$\lim_{\substack{n \to \infty \\ \delta A \to 0}} \sum_{r=1}^{n} f(x_r, y_r)\,dA_r = \iint_R f(x,y)\,dA$$

$$\boxed{\iint_R f(x,y)\,dA = \int_{x_1}^{x_2}\left(\int_{y_1}^{y_2} f(x,y)\,dy\right)dx}$$

Q Evaluate $\displaystyle\int_0^5 \int_0^{x^2} x(x^2+y^2)\,dx\,dy$

$$= \int_0^5 \left(\int_0^{x^2} x(x^2+y^2)\,dy\right)dx$$

$$= \int_0^5 \left[x^3 y + x\frac{y^3}{3}\right]_0^{x^2} dx$$

$$= \int_0^5 \left(x^5 + \frac{x^7}{3}\right)dx = \left[\frac{x^6}{6} + \frac{x^8}{24}\right]_0^5$$

$$= \left(\frac{5^6}{6} + \frac{5^8}{24}\right) \text{ Ans}$$

Change of order of integration:—

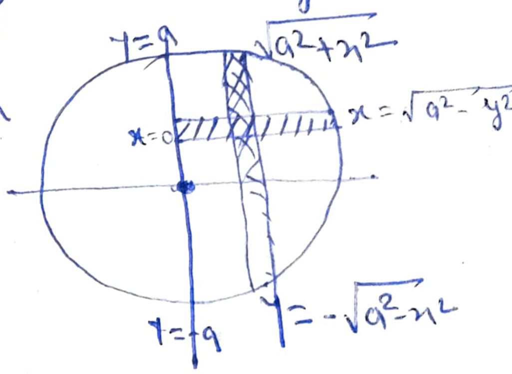
~~Some time it is not~~

Q Change the order of integration in the integral

$$I = \int_{-a}^{a} \int_{0}^{\sqrt{a^2-y^2}} f(x,y)\, dx\, dy$$

Sol^n Here the elementary strip is $\parallel^r$ to x-axis as $x=0$ to $x = \sqrt{a^2-y^2}$ (i.e. $x^2+y^2=a^2$) and this strip slides from $y=-a$ to $y=a$

The right semi-circular region is ~~area of~~ region of integration



Let us choose a strip $\parallel^r$ to y-axis in the region of integration

$$\because x^2+y^2 = a^2$$

$$y^2 = a^2-x^2$$

$$y = \pm\sqrt{a^2-x^2}$$

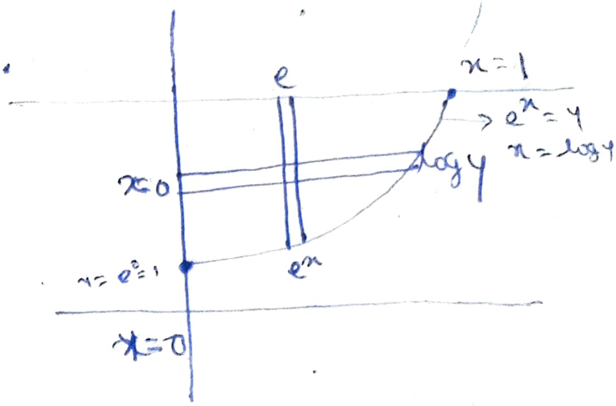
i.e. y varies from $-\sqrt{a^2-x^2}$ to $\sqrt{a^2-x^2}$

& the strip varies from 0 to a

$$\therefore I = \int_{-a}^{a} \int_{0}^{\sqrt{a^2-y^2}} f(x,y)\, dx\, dy = \int_{0}^{a} \int_{-\sqrt{a^2-x^2}}^{\sqrt{a^2-x^2}} f(x,y)\, dy\, dx$$

... blane curve

Q Evaluate $\int_0^1 \int_{e^x}^{e} \frac{1}{\log y}\, dy\, dx$ by changing the order of integration.



In the given region elementary strip is $\parallel^r$ to y-axis as y varies from e^x to e & x from 0 to 1

While changing of order we have to choose elementary strip $\parallel^r$ to x-axis which varies from $x=0$ to $x = \log y$ correspondingly y varies from 1 to e

$$\therefore I = \int_0^1 \int_{e^x}^{e} \frac{1}{\log y}\, dy\, dx = \int_1^e \int_0^{\log y} \left(\frac{1}{\log y}\right) dx\, dy$$

$$= \int^e \frac{1}{\log y} [x]_0^{\log y}\, dy$$

$$= \int_1^e 1\, dy = [y]_1^e = e-1 \quad \text{m}$$

Ques:- Change of order of integration & hence evaluate

$$I = \int_0^a \int_0^{y^2/a} \cancel{\frac{y^2 dx\, dy}{\sqrt{y^4 - a^2x^2}}} \quad \int_0^a \int_{\sqrt{ax}}^{a} \frac{y^2 dx\, dy}{\sqrt{y^4 - a^2 x^2}}$$

~~x varies from x=0 to~~ $x = \frac{y^2}{a} \Rightarrow y^2 = ax$

& ~~y varies from 0 to a~~

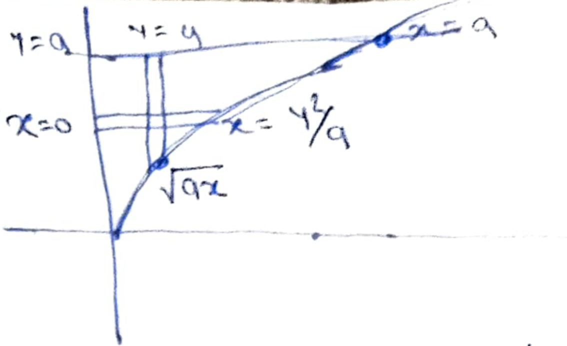
y- varies from $\sqrt{ax}$ to a & x from 0 to a

~~while changing of order of integration we have to check strip $\parallel^{r}$ to y axis~~

~~& y varies from $y=\sqrt{ax}$ to $y=a$~~

~~& x varies from $x=0$ to $x=a$~~

$$\cancel{\therefore I \int_0^a \int_{\sqrt{ax}}^a \frac{y^2}{\sqrt{y^4-a^2x^2}}\,dy\,dx}$$



Q

while changing of order of integration, we have to draw another strip $\perp^{r}$ to x axis then x varies from 0 to y^2/a & y from 0 to a

$$I = \int_0^a \int_0^{y^2/a} \frac{y^2\,dx\,dy}{\sqrt{y^4-a^2x^2}}$$

$$= \int_0^a y^2 \left(\int_0^{y^2/a} \frac{1}{\sqrt{y^4-a^2x^2}}\,dx \right) dy$$

$$= \frac{1}{a}\int_0^a y^2 \left(\int_0^{y^2/a} \frac{1}{\sqrt{\left(\frac{y^2}{a}\right)^2 - x^2}}\,dx \right) dy = \frac{1}{a}\int_0^a y^2 \left[\sin^{-1}\left(\frac{xa}{y^2}\right) \right]_0^{y^2/a} dy$$

$$= \frac{1}{a}\int_0^a y^2 \left[\sin^{-1}(1) - \sin^{-1}(0) \right] dy$$

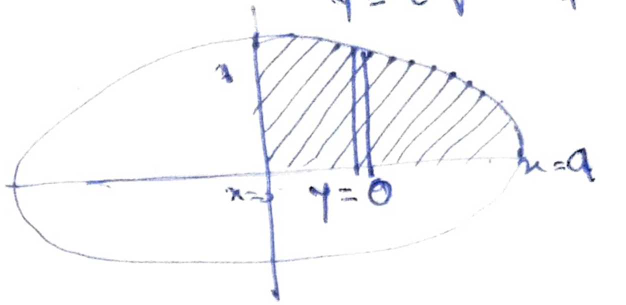
$$= \frac{\pi}{2a}\int_0^a y^2\,dy = \frac{\pi}{2a} \times \left[\frac{y^3}{3}\right]_0^a$$

$$= \frac{\pi}{6} a^2 \text{ Ans}$$

Area enclosed by plane curve

Q. Find the area of a plate in the form of a quadrant of ellipse

$E: \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$



$$\iint_E dx\,dy = \int_0^a \left(\int_0^{b\sqrt{1-x^2/a^2}} dy \right) dx$$

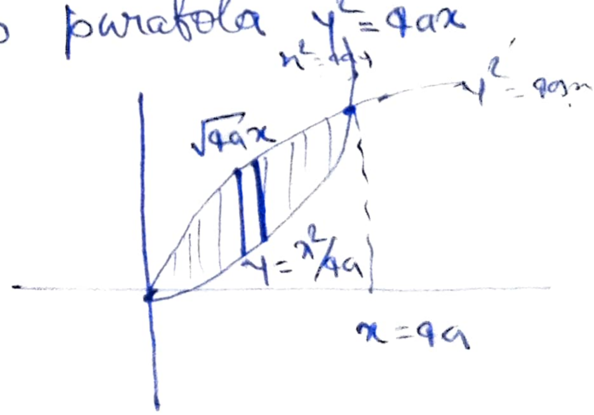
$$= b\int_0^a \left(\sqrt{1-x^2/a^2}\right) dx$$

$$= \frac{b}{a}\int_0^a \sqrt{a^2-x^2}\,dx = \frac{b}{a}\left[\frac{1}{2}x\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}\right]_0^a$$

$$= \frac{b}{a}\left[\frac{1}{2}a\sqrt{a^2-a^2} + \frac{a^2}{2}\sin^{-1}(1) - 0 - 0\right]$$

$$= \frac{ab}{2} \times \frac{\pi}{2} = \frac{1}{4}\pi ab \text{ m}$$

Q. Show that the area b/w parabola $y^2 = 4ax$ & $x^2 = 4ay$ is $\frac{16}{3}a^2$.



$$\iint_R dx\,dy = \int_{x=0}^{4a} \left(\int_{x^2/4a}^{\sqrt{4ax}} dy \right) dx$$

$$= \int_0^{4a} \left(\sqrt{4ax} - x^2/4a\right) dx$$

$$= \frac{16}{3}a^2.$$