Ques:- test the consistency and solve
$5x+3y+7z=4,\ 3x+26y+2z=9,\ 7x+2y+10z=5.$

sol^n

we have
$$\begin{bmatrix} 5 & 3 & 7 \\ 3 & 26 & 2 \\ 7 & 2 & 10 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 9 \\ 5 \end{bmatrix}$$

~~operate~~
$$[A:b] = \left[\begin{array}{ccc|c} 5 & 3 & 7 & 4 \\ 3 & 26 & 2 & 9 \\ 7 & 2 & 10 & 5 \end{array}\right]$$

operate $5R_2, 3R_1$
$$[A:b] \approx \left[\begin{array}{ccc|c} 15 & 9 & 21 & 12 \\ 15 & 130 & 10 & 45 \\ 7 & 2 & 10 & 5 \end{array}\right]$$

$R_2 \to R_2 - R_1$
$$[A:b] \approx \left[\begin{array}{ccc|c} 15 & 9 & 21 & 12 \\ 0 & 121 & -11 & 33 \\ 7 & 2 & 10 & 5 \end{array}\right]$$

operate $\frac{7}{3}R_1, 5R_3, \frac{1}{11}R_2$
$$[A:b] \approx \left[\begin{array}{ccc|c} 35 & 21 & 49 & 28 \\ 0 & 11 & -1 & 3 \\ 35 & 10 & 50 & 25 \end{array}\right]$$

$R_3 \to R_3 - R_1 + R_2$
$$\sim \left[\begin{array}{ccc|c} 35 & 21 & 49 & 28 \\ 0 & 11 & -1 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$\Rightarrow \rho(A) = \rho(A:b) = 2$

$\Rightarrow$ System is consistent.

$$\Rightarrow \begin{bmatrix} 35 & 21 & 49 \\ 0 & 11 & -1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 28 \\ 3 \\ 0 \end{bmatrix}$$

$$\Rightarrow 35x + 21y + 49z = 28$$

$$11y - z = 3$$

$$\Rightarrow z = 11y - 3$$

Let $z = t$

$$y = \frac{t+3}{11}$$

& $x = \frac{7}{11} - \frac{16}{11}t$

t is arbitrary $\Rightarrow$ it has infinite no. of solⁿ.

Ques: ~~We have~~ Investigate the value of λ and μ so that the equation

$2x + 3y + 5z = 9,\ 7x + 3y - 2z = 8,\ 2x + 3y + \lambda z = \mu$

~~we have~~ (i) no solⁿ (ii) unique solⁿ (iii) Infinite solⁿ

$$\begin{bmatrix} 2 & 3 & 5 \\ 7 & 3 & -2 \\ 2 & 3 & \lambda \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ 8 \\ \mu \end{bmatrix}$$

solⁿ for unique solⁿ

$\rho(A) = \rho(A:b) =$ no of variable $= 3$

$\Rightarrow \rho(A) = 3$

$\Rightarrow |A| \neq 0$

$$\begin{vmatrix} 2 & 3 & 5 \\ 7 & 3 & -2 \\ 2 & 3 & \lambda \end{vmatrix} \neq 0 \Rightarrow \lambda \neq 5$$

(i) system have unique solⁿ when $\lambda \neq 5$, & we can choose any value of μ.

(ii) If $\lambda = 5$, then we have

$$A:b = \left[\begin{array}{ccc|c} 2 & 3 & 5 & 9 \\ 7 & 3 & -2 & 8 \\ 2 & 3 & 5 & \mu \end{array}\right]$$

for $\lambda = 5$

$$\rho(A) = 2$$

If $\mu \neq 9$

$\Rightarrow \rho(A:b) = 3$

$\Rightarrow \rho(A) \neq \rho(A:b)$

for $\lambda = 5$ & $\mu \neq 9$

(iii) If $\lambda = 5$ & $\mu = 9$

$\Rightarrow \rho(A) = \rho(A:b) = 2$

$\Rightarrow$ system has ~~unique sol~~ infinite solⁿ.

Eigen values, Eigen vectors and characteristic equation.

__Eigen values__:- An scalar λ is said to be eigen value of an square matrix A of order n if

$$|A - \lambda I| = 0$$

or $\det(A - \lambda I) = 0$

eg $A = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$

find the eigen values of A

$$A - \lambda I = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 5-\lambda & 4 \\ 1 & 2-\lambda \end{bmatrix}$$

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 5-\lambda & 4 \\ 1 & 2-\lambda \end{vmatrix} = 0$$

$$(5-\lambda)(2-\lambda) - 4 = 0$$

$$10 - 7\lambda + \lambda^2 - 4 = 0$$

$$\lambda^2 - 7\lambda + 6 = 0$$

$$\lambda^2 - 6\lambda - \lambda + 6 = 0$$

$$\Rightarrow \lambda = 1, 6$$

$\Rightarrow$ A has two eigen value 1 & 6.

<u>eigen vectors</u> :- Let λ be an eigen value of A, then a non-zero vector X is said to be eigen vector of A corresponding to the eigen value λ if

$$\boxed{AX = \lambda X}$$

$$\Rightarrow \boxed{[A - \lambda I]X = 0}$$

eg:

Find eigen value and respective eig. vectors of the matrix

$$A = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$$

eigen values are 1 & 6

For $\lambda = 1$, Let $X = [x\ y]^T$ be a vector s.t.

$$[A - \lambda I] X = 0$$

$$\begin{bmatrix} 5-1 & 4 \\ 1 & 2-1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 4 & 4 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$4(x+y) = 0$, $x + y = 0$

Let $x = 1$, then $y = -1$

$\Rightarrow X = [1\ -1]^T$ is an eigen vector corresponding to eigen value '1'

For $\lambda = 6$, Let $X = [u\ v]^T$ be eigen vectors

s.t.

$$\begin{bmatrix} 5-6 & 4 \\ 1 & 2-6 \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 4 \\ 1 & -4 \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$-u + 4v = 0$

$4v = u$

$\cancel{-2u + 4v = 0}$

$\cancel{u = 4v}$

Let $v = 1$, $u = 4$ $\Rightarrow X = [4, 1]^T$ be the eigen vector corr. to $\lambda = 6$