Optimization Techniques
Paper Code – BMS-09
Lecture – 05(Unit -1)
Topic-Multiple Variables Optimization – Kuhn Tucker Condition

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Unit-01

Classical Optimization Techniques: Single variable optimization, Multi-variable with no constraints. Non-linear programming: One Dimensional Minimization methods. Elimination methods: Fibonacci method, Golden Section method

Unit-02

Unit-02

Linear Programming: Constrained Optimization Techniques: Simplex method, Solution of System of Linear Simultaneous equations, Revised Simplex method, Transportation problems, Karmarkar’s method, Duality Theorems, Dual Simplex method, Decomposition principle.

Optimization by Numerical Methods.

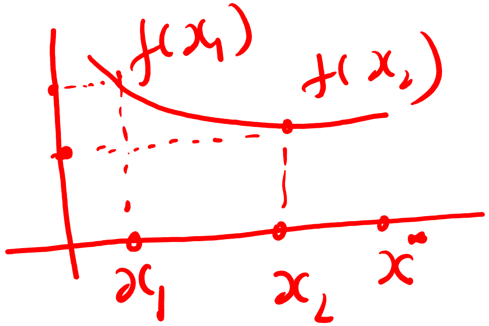
Unimodel function - A unimodel function is one that has only one maximum or minimum in a given interval.

Mathematically, A function $f(x)$ is unimodel if

(i) $x_1 < x_2 < x^*$ $\Rightarrow$ $f(x_2) < f(x_1)$

(ii) $x_2 > x_1 > x^*$ $\Rightarrow$ $f(x_1) < f(x_2)$

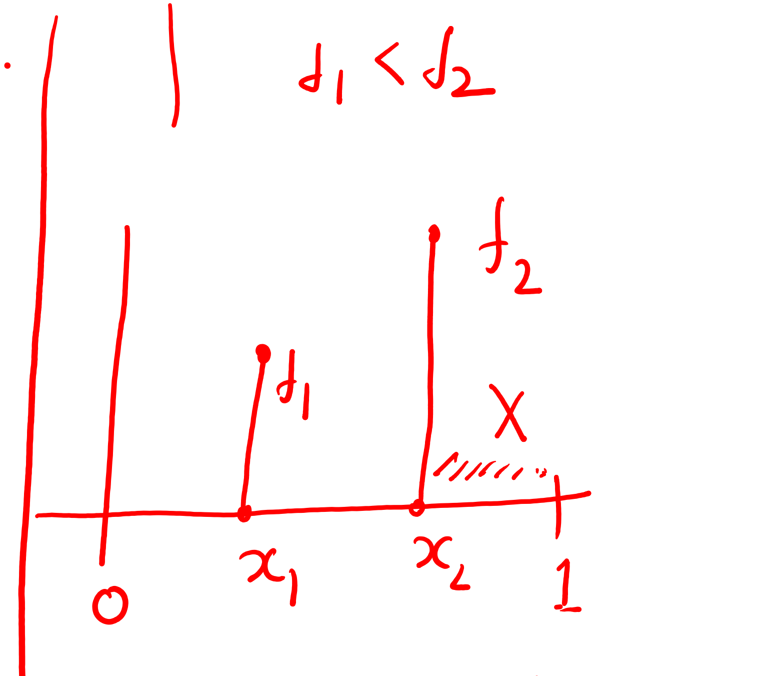
where x^* is the minimum point.



A unimodel function can be non-differential or even a discontinuous function. If a function is known to be unimodel in a given range/interval, the interval in which the minimum lies can be narrowed down provided that the functional values are known at two different point in the range.

For example, consider the normalized interval [0,1] and two function values within the interval are given. there are three possible outcomes, namely, $f_1 < f_2$ or $f_1 > f_2$ or $f_1 = f_2$. If the outcomes is that $f_1 < f_2$, the minimum x can

not lie to the right of x_2.
mean, the part of the interval $[x_2, 1]$ can be neglected and a new small interval of uncertainity $[0, x_2]$ as shown in the figure 1(a)



1(a)

② if $f(x_1) > f(x_2)$, then the interval $[0, x_1]$ can be discarded and new interval of uncertainity $[x_1, 1]$ as shown in the figure 1(b)

1(b)

(3) if $f_1 = f_2$, the interval $[0, x_1]$ and $[x_2, 1]$ can both neglect, the new interval of uncertainity as $[x_1, x_2]$ as shown in the figure 1(c).

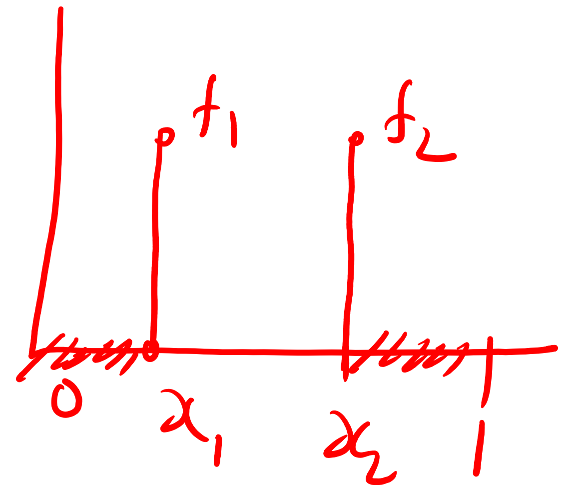


Figure 1(c)

Fibonacci number

here we have a sequence of Fibonacci numbers $\{F_n\}$ as given below

$$F_0 = F_1 = 1 \text{ and}$$

$$F_n = F_{n-1} + F_{n-2}, \quad n = 2, 3, 4, \ldots$$

so, other number of sequence are

$$F_2 = F_1 + F_0 = 1 + 1 = 2$$

$$F_3 = F_2 + F_1 = 2 + 1 = 3$$

$$F_4 = F_3 + F_2 = 3 + 2 = 5$$

$$F_5 = F_4 + F_3 = 5 + 3 = 8$$

$$F_6 = 13$$

$$F_7 = 21$$

$$F_8 = 34$$

$$F_9 = 55$$

$$F_{10} = 89$$

$$\vdots$$

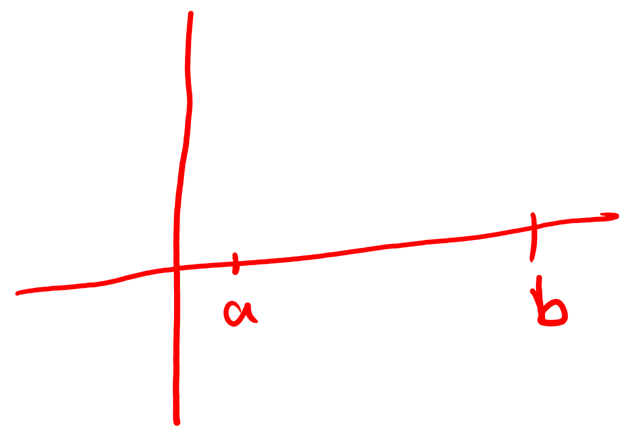
Fibonacci method

Let L_0 be the length of initial interval of uncertainity defined by $a \le x \le b$ and n be the total no. of experiments to be conducted.

So, defined as $L_2^* = \frac{F_{n-2}}{F_n} \times L_0$ and

$$x_1 = a + L_2^*$$

$$x_2 = b - L_2^*$$



Q1 Minimize $f(x) = 0.65 - \left[\frac{0.75}{1+x^2}\right] - 0.65\, x \tan^{-1}\left(\frac{1}{x}\right)$ in the interval $[0, 3]$ by the Fibonacci method with 6 experiment.

Ans given that

$$f(x) = 0.65 - \left[\frac{0.75}{1+x^2}\right] - 0.65\, x \cdot \tan^{-1}\left(\frac{1}{x}\right)$$

$n = 6, \quad a = 0, \quad b = 3$

$$L_0 = b - a = 3 - 0 = 3$$

$$L_2^* = \frac{F_{n-2}}{F_n} \times L_0 = \frac{F_4}{F_6} \times 3 = \frac{5}{13} \times 3$$

$$= 1.153846$$

then the possible 1st two experiment are given by

$$x_1 = a + L_2^* = 0 + 1.153846 = 1.153846$$

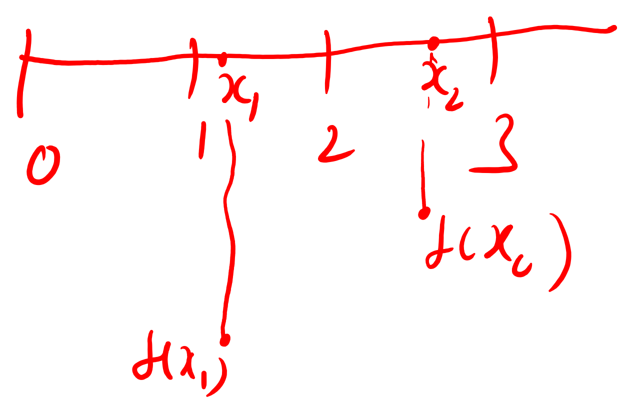
$$x_2 = b - L_2^* = 3 - 1.153846 = 1.846154$$

and

$$f(x_1) = -0.207270$$

$$f(x_2) = -0.115843$$

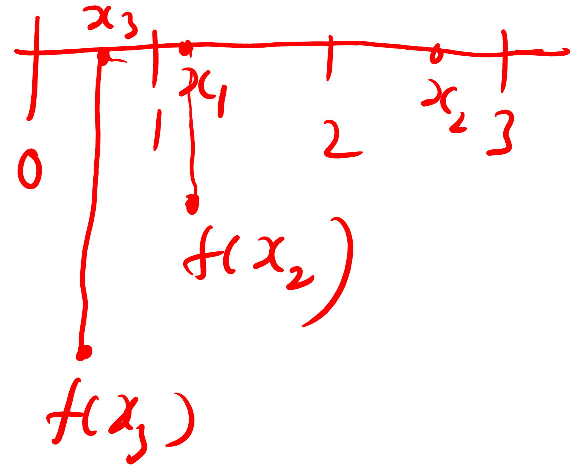
since $f(x_1) < f(x_2)$, so we can delete the interval $[x_2, 3]$ and the new interval is $[0, x_2]$



so, third experiment

$$x_3 = a + (x_2 - x_1)$$

$$= 0.692308$$



$$f(x_3) = -0.291364$$

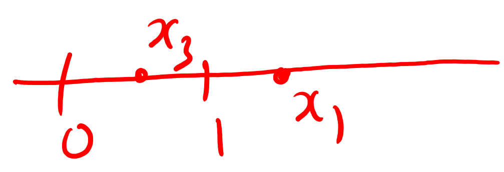
see here $f(x_3) < f(x_2)$,

So, we ignore the interval $[x_1, x_2]$ and new interval is

$[0, x_1]$

$$x_4 = a + x_1 - x_3 = 0.461538$$

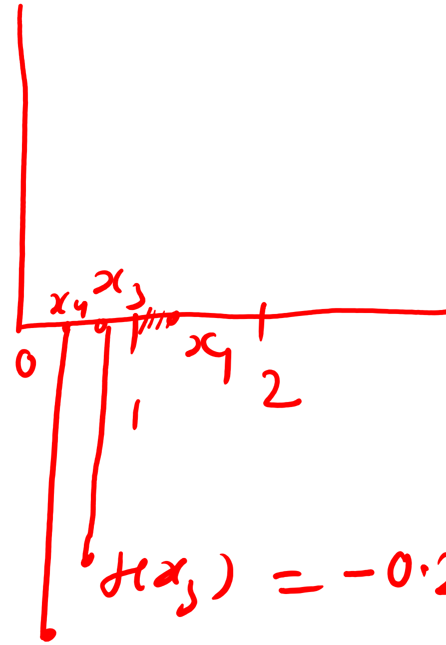
$$f(x_4) = -0.309811$$



we can see here

~~$f(x_4) > f(x_3)$, we delete the interval $[x_3, x_1]$ and new interval $[0, x_3]$.~~

$f(x_4) < f(x_3)$



$f(x_3) = -0.291364$

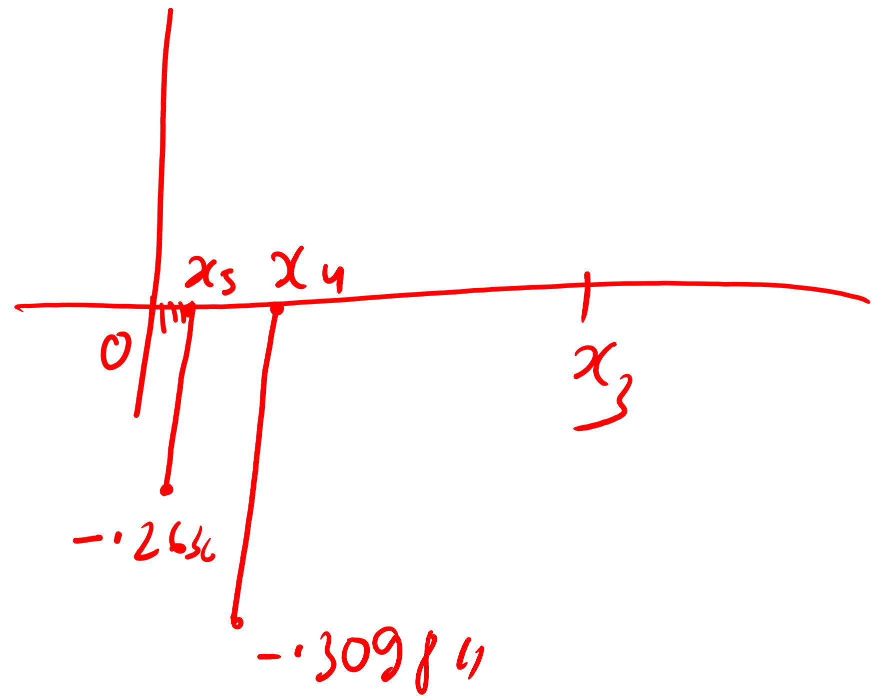
$f(x_4) = -0.309811$

so, we can see here

$$f_3 > f(x_4) \Rightarrow -0.291364 > -0.309811$$

so, we neglect the portion $[x_3, x_1]$ and new interval is

$[0, x_3]$



$$x_5 = x_0 + x_3 - x_4$$

$$= 0.230720$$

and $f(x_5) = -0.263678$

here, we can see that

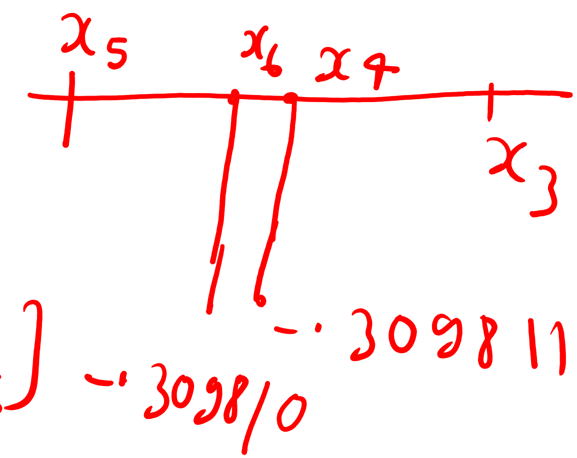
$f(x_5) > f(x_4)$, so.

we can neglect the portion $[0, x_5]$, so new interval is $[x_5, x_3]$,

now, $x_6 = x_5 + x_3 - x_4 = 0.461\ 40$

$f(x_6) = -0.309810$

we can see here,



$f(x_6) > f(x_4)$

so, we delete the portion $[x_5, x_6]$

so, new interval is $[x_6, x_3]$.

here $L_6 = x_3 - x_6 = .692308 - .461540$

$= 0.230768$

so $\frac{L_6}{L_0} = \frac{.230768}{3} = 0.076922.$

Also, we know that

$$\frac{L_n}{L_0} = \frac{F_1}{F_n}$$

here $n = 6$

$$\frac{L_6}{L_0} = \frac{F_1}{F_6} \quad \text{— (1)}$$

if (1) is approximately equal, then we stop the process.

$$L.H.S. = \frac{L_6}{L_0} = 0.076922$$

$$R.H.S. = \frac{1}{F_6} = \frac{1}{13} = 0.076923$$

So, we have $L.H.S. \cong R.H.S.$

So, we stop the process

and $x = x_6 = 0.461540$

minimum value is $f(x_6) = -0.309810$ Ans