Simplex Method

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The Simplex method provides a systematic algorithm by moving from one basic feasible solution to another (say movement from one vertex to another vertex) in a prescribed manner so that the value of the objective function in every step is improved.

* If the objective function is improved at each step/jump, then no basis can ever repeated and there is no need to go back to vertex that already covered.
* Since the no. of vertex is finite, the process must lead to the optimal vertex in a finite number of steps.
* In standard form, the L.P.P. can be stated as

$$\text{Max } Z = CX^T$$

subject to

$$AX^T = B^T, \quad B^T \geq 0$$

with

$$X^T \geq 0$$

where

$$C = (c_1, c_2, \dots c_n, 0, 0, \dots 0)$$

$$X = (x_1, x_2, \dots x_n, x_{n+1}, x_{n+2}, \dots, x_{n+m})$$

$$B = (b_1, b_2, \dots, b_m)$$

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} & 1 & 0 & 0 & \cdots & 0 \\ a_{21} & a_{22} & \cdots & a_{2n} & 0 & 1 & & \cdots & 0 \\ \vdots & & & \vdots & & & & & \\ a_{m1} & a_{m2} & \cdots & a_{mn} & 0 & 0 & & \cdots & 1 \end{bmatrix}$$

This is the L.P.P. with $m+n$ variables and m constraints

For starting simplex method, write n variables equal to zero, say,

$x_1 = x_2 = \cdots = x_n = 0$, and solve for remaing m variables as the no. of equation/ no. of constraints are m. So, get

$x_{n+1} = b_1, x_{n+2} = b_2, \ldots, x_{n+m} = b_m$ and

the value of objective function is zero.

here, $x_{n+1}, x_{n+2}, \ldots, x_{n+m}$ are basic variables and $x_1, x_2, \ldots, x_n$ are non-basic variables.

here, define net evaluation

$$z_j = c_B X_j - c_j = Z_j - c_j \quad , \quad (Z_j = c_B X_j)$$

For optimality Test

$\Delta_j \geq 0$, the sol^n will be optimal

(i) Alternative sol^n $\rightarrow$ Alternative sol^n will exist if any non-basic Δ_j is always zero.

(ii) if at least one Δ_j is negative, the solution is not optimal and go further step.

(iii) If corresponding to any negative Δ_j, all elements of the coloumn x_j are negative, then the sol^n under test will be unbounded.

Incoming vector $\rightarrow$ most negative value of Δ_j

out going vector $\rightarrow$ corresponding to minimum ratio ($x_i > 0$),