

Note: $z_j - c_j \geq 0$, only possible when $c_j \geq 0$ for every j in the maximization objective function.

(1) To find the leaving vector or outgoing vector from the basis-

Minimum of $x_B = \min\{-3, -5\} = -5$.

means, outgoing vector is s_2

i.e., the outgoing vector is selected corresponding to the basic variable having the most -ve value.

If the values of all basic variables are +ve, the process ends and get the optimum solution.

(2) To find the entering in the basis-

$$\max_j \left\{ \frac{\Delta_j}{a_{rj}}, \quad a_{rj} < 0 \right\}, \quad a_{rj} \text{ is coefficients of variables.}$$

Suppose, get s_2 is a outgoing variables, means, we have fixed a second equation.

$$\Rightarrow r=2,$$

$$\Rightarrow \max_j \left\{ \frac{\Delta_j}{a_{2j}}, \quad a_{2j} \leq 0 \right\}$$

$$\Rightarrow \text{Max} \left\{ \frac{\Delta_1}{a_{21}}, \frac{\Delta_2}{a_{22}}, \dots, \frac{\Delta_n}{a_{2n}}, \quad a_{21} \leq 0, a_{22} \leq 0, \dots, a_{2n} \leq 0 \right\}$$

Here, a_{21}, a_{22}, \dots are the coefficient of x_1, x_2, \dots in the second equation.

For example in above problem-

$$\text{Max} \left\{ \frac{4}{-}, \frac{6}{-1}, \frac{18}{-2} \right\} = \text{Max} \{ -6, -9 \} = -6 = \frac{\Delta_2}{a_{22}}.$$

$\Rightarrow x_2$ is an entering variable in the basis