There are simple formulas for the characteristic polynomials of matrices of order 2 and 3.

(a) Suppose $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$. Then

$$Ch(A) = \lambda^2 - (a_{11} + a_{22})\lambda + |A|$$
$$= \lambda^2 - tr(A) + |A|$$

(b) Suppose $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$

$$Ch(A) = \lambda^3 - tr(A)\lambda^2 + (A_{11} + A_{22} + A_{33})\lambda - |A|$$

(Here A_{11}, A_{22} & A_{33} are cofactors of A corresponding to a_{11}, a_{22} & a_{33})

Ques: Find the characteristic equation of

$$A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$$

$$A_{11} = (-1)^{1+1} \begin{vmatrix} 3 & -3 \\ -4 & -4 \end{vmatrix} = -24$$

$$A_{22} = (-1)^{2+2} \begin{vmatrix} 1 & 3 \\ -2 & -4 \end{vmatrix} = 2$$

$$A_{33} = (-1)^{3+3} \begin{vmatrix} 1 & 1 \\ 1 & 3 \end{vmatrix} = 2$$

$tr(A) = 1+3-4 = 0$

$$|A| = 1(-12-12) - 1(-4-6) + 3(-4+6)$$
$$= -24 + 10 + 6$$
$$= -8$$

$\therefore$

$$Ch(A) = \lambda^3 - (A_{11} + A_{22} + A_{33})\lambda^2$$

$$Ch(A) = \lambda^3 - tr(A)\lambda^2 + (A_{11} + A_{22} + A_{33})\lambda - |A|$$
$$= \lambda^3 - 0 \cdot \lambda^2 + (-24+2+2)\lambda - (-8)$$
$$= \lambda^3 - 20\lambda + 8$$

Therefore characteristic eqn is

$$\lambda^3 - 20\lambda + 8 = 0$$

Then by Caley Hamilton theorem

$$A^3 - 20A + 8I = 0$$
$$A^{-1}(A^3 - 20A + 8I) = 0$$

$$A^{-1} = \begin{bmatrix} 3 & 1 & 3/2 \\ -5/4 & -1/4 & -3/4 \\ -1/4 & -1/4 & -1/4 \end{bmatrix}$$

Ques Find the characteristic polynomial of
$A = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 3 & 2 \\ 1 & 3 & 9 \end{bmatrix}$
and hence the inverse.

Sol^n

$tr(A) = 13$

$A_{11} = 21$, $A_{22} = 7$, $A_{33} = 3$

$|A| = 17$

$Ch(A) = \lambda^3 - 13\lambda^2 + 31\lambda - 17$

characteristic eq^n is

$\lambda^3 - 13\lambda^2 + 31\lambda - 17 = 0$

using caley-hamilton theorem

$A^3 - 13A^2 + 31A - 17I = 0 \quad - (1)$

multiplying (1) with A^{-1}, we have

$A^2 - 13A + 31I - 17A^{-1} = 0$

$\Rightarrow A^{-1} = \frac{1}{17}\left(A^2 - 13A + 31I\right)$

$= \frac{1}{17}\left(\begin{bmatrix} 3 & 10 & 22 \\ 2 & 15 & 29 \\ 10 & 37 & 89 \end{bmatrix} - \begin{bmatrix} 13 & 13 & 26 \\ 0 & 39 & 26 \\ 13 & 39 & 117 \end{bmatrix} + \begin{bmatrix} 31 & 0 & 0 \\ 0 & 31 & 0 \\ 0 & 0 & 31 \end{bmatrix}\right)$

$= \frac{1}{17}\left(\begin{bmatrix} 3-13+31 & 10-13+0 & 22-26+0 \\ 2-0+0 & 15-39+31 & 24-26+0 \\ 10-13+0 & 37-39+0 & 89-117+31 \end{bmatrix}\right)$

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$$= \frac{1}{17}\begin{bmatrix} 21 & -3 & -4 \\ 2 & 7 & -2 \\ -3 & -2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 21/17 & -3/17 & -4/17 \\ 2/17 & 7/17 & -2/17 \\ -3/17 & -2/17 & 3/17 \end{bmatrix}.$$

Ques:- Find the characteristic equation of the matrix

$$A = \begin{bmatrix} 5 & 3 \\ 2 & 10 \end{bmatrix}.$$

and hence the inverse.

sol^n

$$tr(A) = 5 + 10 = 15$$

$$|A| = 50 - 6 = 44$$

Characteristic poly

$$Ch(A) = \lambda^2 - tr(A)\lambda + |A|$$

$$= \lambda^2 - 15\lambda + 44$$

Characteristic eq^n is

$$\lambda^2 - 15\lambda + 44 = 0$$

using caley - hamilton theorem

$$A^2 - 15A + 44I = 0 \quad \text{—} (1)$$

multiplying (1) with A^{-1} we have

$$A - 15I + 44A^{-1} = 0$$

$$A^{-1} = \frac{1}{44}\left[-A + 15I\right]$$

$$= \frac{1}{44}\left[\begin{bmatrix} -5 & -3 \\ -2 & -10 \end{bmatrix} + \begin{bmatrix} 15 & 0 \\ 0 & 15 \end{bmatrix}\right]$$

$$= \frac{1}{44}\begin{bmatrix} 10 & -3 \\ -2 & 5 \end{bmatrix} = \begin{bmatrix} 5/22 & -3/44 \\ -1/22 & 5/44 \end{bmatrix}$$