Unit - 03 (Electromagnetic Wave Propagation)

- Wave propogation in lossy dielectrics
- Plane Wave in lossless dielectrics
- Plane Wave in free space
- Plane Wave in good conductors
- Power and the poynting vector
- Reflection and refraction of a plane wave at normal and oblique incidence.

→ Waves are means of transporting energy or information.

→ All forms of EM energy share three fundamental characteristics – They all travel at high velocity.
– They assume the properties of waves while traveling.
– They radiate outwards from a source.

→ Typical example of EM waves includes – Radio waves
– TV signals
– Radar beams
– light rays.

→ Different Medias :

- → Lossy dielectrics ($\sigma \neq 0$, $\varepsilon = \varepsilon_0$, $\mu = \mu_0$)
- → Lossless dielectrics ($\sigma = 0$, $\varepsilon = \varepsilon_r \varepsilon_0$, $\mu = \mu_r \mu_0$ or $\sigma \ll \omega\varepsilon$)
- → Free Space ($\sigma \neq 0$, $\varepsilon = \varepsilon_r \varepsilon_0$, $\mu = \mu_r \mu_0$)
- → Good Conductors ($\sigma = \infty$, $\varepsilon = \varepsilon_0$, $\mu = \mu_r \mu_0$ or $\sigma \gg \omega\varepsilon$)

{ω is angular frequency of the wave}

Wave Propagation in Lossy Dielectrics:

- A lossy medium can be described as a medium where some fraction of power of electromagnetic wave is lost as it propagates.

OR

A lossy dielectric is a medium in which an EM wave, as its propagates, loses power owing to imperfect dielectric.

- This power is due to poor conduction.

- A lossy dielectric is a partially conducting medium (imperfect dielectric or imperfect conductor) with $\sigma \neq 0$.

Parameters:

- For lossy medium → Conductivity $\sigma \neq 0$, $\sigma << \omega\varepsilon$
  → Permeability $(\mu) = \mu_0 \mu_r$
  → Permittivity $(\varepsilon) = \varepsilon_0 \varepsilon_r$

- Consider a linear, isotropic, homogenous, lossy dielectric medium that is charge free (macroscopic $\rho_v = 0$).

$$\nabla \cdot E_s = 0 \quad \text{—(1)}$$

$$\nabla \cdot D_s = 0 \quad \text{—(2)}$$

$$\nabla \times E_s = -j\omega \mu H_s \quad \text{—(3)}$$

$$\nabla \times H_s = (\sigma + j\omega\varepsilon) E_s \quad \text{—(4)}$$

Taking curl of eq^n (3)

$$\nabla \times \nabla \times E_s = -j\omega \mu (\nabla \times H_s)$$

(Applying the Vector identity) ⇒ $\nabla \times (\nabla \times A) = \nabla(\nabla \cdot A) - \nabla^2 A$

$$\nabla(\underbrace{\nabla \cdot E_s}_{0}) - \nabla^2 E_s = -j\omega \mu (\sigma + j\omega\varepsilon) E_s$$

$$\nabla^2 E_s = +j\omega \mu (\sigma + j\omega\varepsilon) E_s$$

$$\nabla^2 E_s = \gamma^2 E_s$$

where, $\gamma^2 = j\omega\mu(\sigma + j\omega\varepsilon)$

$$\boxed{\nabla^2 E_s - \gamma^2 E_s = 0}$$

⇒ γ is known as PROPAGATION CONSTANT.

- Similarly for H field ⇒ $\boxed{\nabla^2 H_s - \gamma^2 H_s = 0}$

- $\left.\begin{array}{l}\nabla^2 E_s - \gamma^2 E_s = 0 \\ \nabla^2 H_s - \gamma^2 H_s = 0\end{array}\right\}$ ⇒ Both the equations are known as Helmholtz's Equations OR Wave equations

- Propagation Constant can be defined as the change in the amplitude or phase of the wave per unit distance of a wave travelling in a specific direction.

- Propagation constant is a complex quantity, so it can be represented as; $\gamma = \alpha + j\beta$

$\because \gamma^2 = j\omega\mu(\sigma + j\omega\varepsilon)$

$$\alpha^2 - \beta^2 + 2\alpha j\beta = j\omega\mu(\sigma + j\omega\varepsilon)$$

$$-Re(\gamma^2) = \beta^2 - \alpha^2 = \omega^2\mu\epsilon \quad \text{— (A)}$$

$$|\gamma^2| = \beta^2 + \alpha^2 = \omega\mu\sqrt{\sigma^2 + \omega^2\varepsilon^2} \quad \text{— (B)}$$

From eqn (A) and eqn (B) we get

Attenuation constant ⇒ $\alpha = \omega\sqrt{\frac{\mu\epsilon}{2}\left[\sqrt{1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2} - 1\right]}$

Phase constant ⇒ $\beta = \omega\sqrt{\frac{\mu\epsilon}{2}\left[\sqrt{1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2} + 1\right]}$

→ Attenuation Constant – It is a measure of the spatial rate of decay of the wave in the medium; measured in nepers per meter (Np/m).

→ Phase Constant – It is a measure of the phase shift per unit length in radians / meter.

Derivation of α and β:

$$\nabla^2 E - \gamma^2 E = 0 \quad \text{and} \quad \gamma = \alpha + j\beta = \sqrt{j\omega\mu(\sigma + j\omega\epsilon)}$$

$$\Rightarrow \alpha^2 + j2\alpha\beta - \beta^2 = j\omega\mu\sigma - \omega^2\mu\epsilon$$

$$\Rightarrow \alpha^2 - \beta^2 = -\omega^2\mu\epsilon \quad \text{--- (1)}$$

$$\Rightarrow 2\alpha\beta = \omega\mu\sigma$$

$$\sqrt{\alpha^2 + \beta^2} = \sqrt{\sqrt{(\sigma\omega\mu)^2 + \omega^2\mu^2\epsilon^2}}$$

$$\Rightarrow \alpha^2 + \beta^2 = \sqrt{(\sigma\omega\mu)^2 + \omega^2\mu^2\epsilon^2}$$

$$\Rightarrow \alpha^2 + \beta^2 = \sqrt{\omega^4\mu^2\epsilon^2\left(1 + \frac{\sigma^2\omega^2\mu^2}{\omega^4\mu^2\epsilon^2}\right)}$$

$$\Rightarrow \alpha^2 + \beta^2 = \omega^2\mu\epsilon\sqrt{1 + (\sigma/\omega\epsilon)^2} \quad \text{--- (2)}$$

eqn (1) + eqn (2), we get

$$\Rightarrow 2\alpha^2 = \omega^2\mu\epsilon\sqrt{1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2} - \omega^2\mu\epsilon$$

$$\Rightarrow \alpha^2 = \frac{\omega^2\mu\epsilon}{2}\left(\sqrt{1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2} - 1\right)$$

$$\Rightarrow \boxed{\alpha = \omega\sqrt{\frac{\mu\epsilon}{2}\left[\sqrt{1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2} - 1\right]}}$$

eqn (2) − eqn (1), we get

$$\Rightarrow 2\beta^2 = \omega^2\mu\epsilon\sqrt{1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2} + \omega^2\mu\epsilon$$

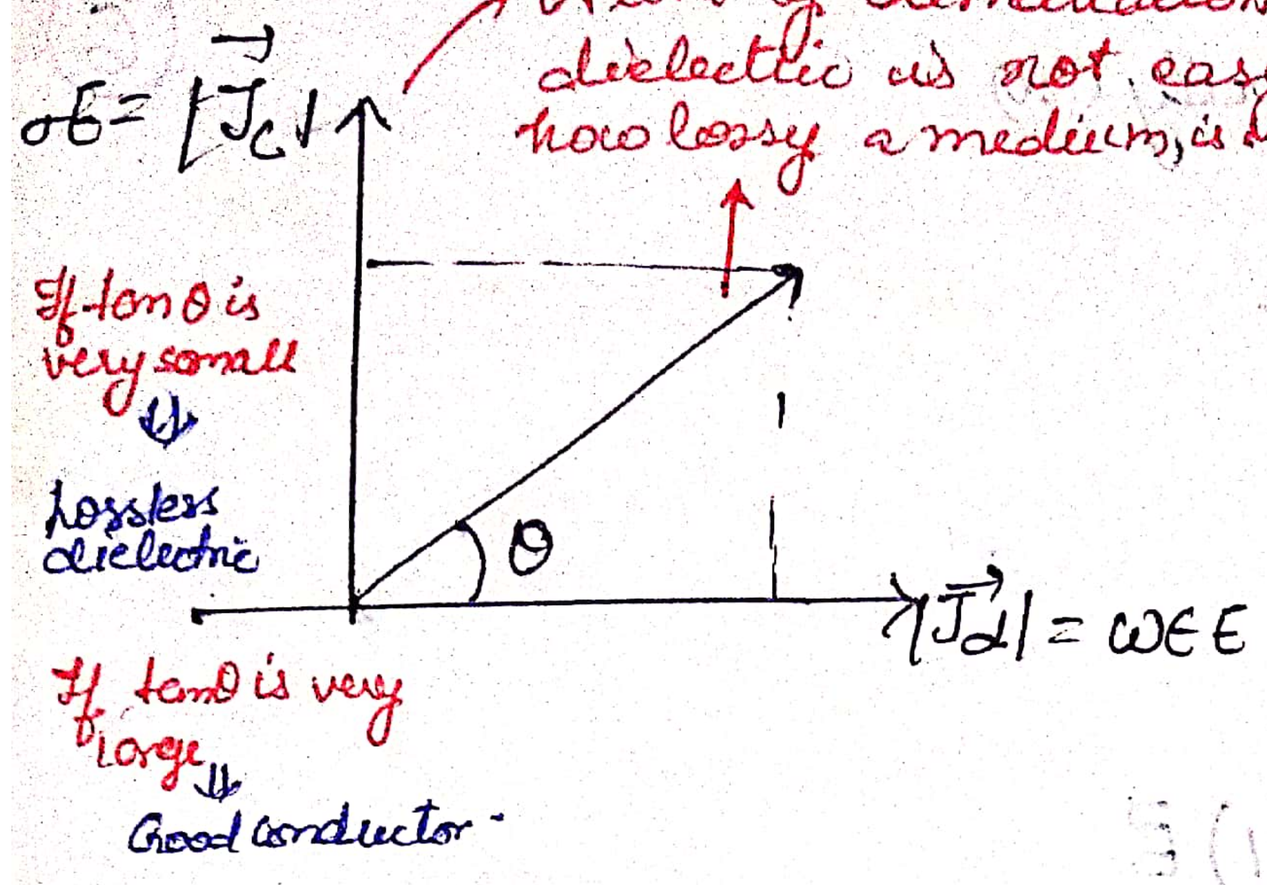
$$\Rightarrow \beta^2 = \frac{\omega^2\mu\epsilon}{2}\left[\sqrt{1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2} + 1\right]$$

$$\Rightarrow \boxed{\beta = \omega\sqrt{\frac{\mu\epsilon}{2}\left[1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2 + 1\right]}}$$

where, $\boxed{\frac{\sigma}{\omega\epsilon} = \tan\theta}$

$\Rightarrow$ $\tan\theta$ is known as the loss tangent and θ is the loss angle of the medium illustrated in figure on next page.

A line of demarcation between good conductor and lossy dielectric is not easy to make, $\tan\theta$ may be used to determine how lossy a medium is.



$$\vec{J_d} = \frac{\partial \vec{D}}{\partial t} = j\omega\epsilon\vec{E} \quad -(1)$$

$$J_c = \sigma\vec{E} \quad -(2)$$

$$\boxed{\frac{|J_c|}{|J_D|} = \frac{\sigma E}{\omega\epsilon E} = \frac{\sigma}{\omega\epsilon}}$$

- Frequency at which magnitude of conduction current density and displacement current are equal. ⇒ $J_c = J_d$
  ⇒ $\frac{\sigma}{\omega\epsilon} = 1$

$$\boxed{f = \frac{\sigma}{2\pi\epsilon}} \quad \{\omega = 2\pi f\}$$

⇒ $\epsilon = \epsilon_0\epsilon_r$

→ <u>INTRINSIC IMPEDANCE</u> :- It describes the magnitude of the magnetic and electric fields present in the free space.

$$\eta = \frac{E_0}{H_0} = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}}$$

$$\eta = \sqrt{\frac{j\omega\mu}{\left(\frac{\sigma}{j\omega\epsilon} + 1\right) j\omega\epsilon}}$$

$$\eta = \sqrt{\frac{\mu/\epsilon}{1 - j\frac{\sigma}{\omega\epsilon}}}$$

$$|\eta| = \frac{\sqrt{\mu/\epsilon}}{\left[\left(1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2\right)^{1/2}\right]^{1/2}} = \frac{\sqrt{\mu/\epsilon}}{\left[1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2\right]^{1/4}}$$

$$\boxed{|\eta| = \frac{\sqrt{\mu/\epsilon}}{\left[1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2\right]^{1/4}}}$$

and $\theta_\eta = \tan^{-1}\left(\frac{0}{\sqrt{\mu/\epsilon}}\right) - \frac{1}{2}\tan^{-1}(-\sigma/\omega\epsilon)$

$$\theta_\eta = -\frac{1}{2}\tan^{-1}(-\sigma/\omega\epsilon)$$

$$\boxed{\theta_\eta = \frac{1}{2}\tan^{-1}(\sigma/\omega\epsilon)}$$ ⇒ Angle of impedance

$$\boxed{\theta_\eta = \frac{\theta}{2}}$$

P1

⇒ Phase Velocity or the Wave Velocity (u)

$$u = \frac{\omega}{\beta}, \quad \lambda = \frac{2\pi}{\beta}$$

⇒ Complex permittivity

$$\nabla \times \vec{H} = (\sigma + j\omega\epsilon)\vec{E}$$

$$= j\omega\varepsilon\left(\frac{\sigma}{j\omega\varepsilon} + 1\right)\vec{E}$$

$$= j\omega\boxed{\varepsilon\left(1 - j\frac{\sigma}{\omega\varepsilon}\right)}\vec{E}$$

⇓

Complex permittivity

$$\boxed{\varepsilon_c = \varepsilon\left(1 - j\,\sigma/\omega\varepsilon\right)}$$

$$\boxed{\varepsilon_c = \varepsilon - \frac{j\sigma}{\omega}}$$

$$\boxed{\varepsilon_c = \varepsilon' - j\varepsilon''}$$

where $\varepsilon' = \varepsilon$

$\varepsilon'' = \frac{\sigma}{\omega}$

$$\boxed{\frac{\varepsilon''}{\varepsilon'} = \frac{\sigma}{\omega\varepsilon} = \tan\theta}$$

$$\frac{\alpha}{\beta} = \frac{\omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2} - 1\right]}}{\omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left(\sigma/\omega\varepsilon\right)^2} + 1\right]}}$$

$$\frac{\alpha}{\beta} = \sqrt{\frac{1+\tan^2\theta - 1}{1+\tan^2\theta + 1}} = \sqrt{\frac{\sec^2\theta - 1}{\sec^2\theta + 1}}$$

$$\frac{\alpha}{\beta} = \sqrt{\frac{1-\cos^2\theta}{1+\cos^2\theta}} = \sqrt{\frac{2\sin^2\frac{\theta}{2}}{2\cos^2\frac{\theta}{2}}} = \tan\frac{\theta}{2}$$

$$\boxed{\frac{\alpha}{\beta} = \tan\frac{\theta}{2}} \Rightarrow \boxed{\theta_n = \frac{\theta}{2}} \Rightarrow \boxed{\frac{\alpha}{\beta} = \tan\theta_n}$$

Plane Waves in Free Space:-

→ $\sigma = 0$

→ $\varepsilon = \varepsilon_0$

→ $\mu = \mu_0$

$\alpha = 0$

$\beta = \omega\sqrt{\mu_0\varepsilon_0} = \frac{\omega}{c}$

$\boxed{u = \frac{1}{\sqrt{\mu_0\varepsilon_0}} = c}$, $\lambda = \frac{2\pi}{\beta}$

$\Downarrow$

$$\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}\left(\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^{2}} - 1\right)}$$

$\Rightarrow \boxed{\alpha = 0}$

$$\beta = \omega\sqrt{\frac{\mu\varepsilon}{2}\left(\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^{2}} + 1\right)}$$

$$\boxed{\beta = \omega\sqrt{\mu_0\varepsilon_0} = \frac{\omega}{c}}$$

$$u = \frac{\omega}{\beta} = \frac{\omega}{\omega/c} = c$$

$\boxed{u = c}$ OR $\Rightarrow \boxed{u = \frac{1}{\sqrt{\mu_0\varepsilon_0}}}$

$\Downarrow$

→ The fact that EM wave travel in free space at the speed of light is significant.

→ It provides some evidence that light is the manifestation of an EM wave.

⇒ Light is characteristically electromagnetic.

$$\therefore \eta = \frac{\sqrt{\mu/\varepsilon}}{\left[1+\left(\sigma/\omega\varepsilon\right)^{2}\right]^{1/4}} \Rightarrow \eta_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 120\pi = 377\,\Omega$$

$\boxed{\eta_0 = 377\,\Omega}$ ⇒ η_0 is intrinsic impedance of free space.

⇒ Both E and H fields are everywhere normal to the direction of wave propagation. That means that the fields lie in a plane that is transverse or orthogonal to the direction of wave propagation.

→ They form a EM wave that has no electric or magnetic field components along the direction of propagation, such waves are called Transverse Electromagnetic Waves (TEM)

→ A combination of E and H is called a uniform plane wave because E (or H) has the same magnitude throughout any transverse plane defined by z = constant

→ The direction in which the electric field points is the polarisation of a TEM wave.

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Plane Waves in Lossless Dielectrics: (Perfect dielectric)

→ $\sigma << \omega\epsilon$

→ $\sigma = 0$

→ $\epsilon = \epsilon_0 \epsilon_r$

→ $\mu = \mu_0 \mu_r$

⇒ substituting above values in following eq'ns we get;

$$\alpha = \omega \sqrt{\frac{\mu\epsilon}{2}\left(\sqrt{1+\cancel{\left(\frac{\sigma}{\omega\epsilon}\right)^2}^{\,0}} - 1\right)}$$

$$\Rightarrow \boxed{\alpha = 0}$$

and $$\beta = \omega \sqrt{\frac{\mu\epsilon}{2}\left(\sqrt{1+\cancel{\left(\frac{\sigma}{\omega\epsilon}\right)^2}^{\,0}} + 1\right)}$$

$$\Rightarrow \boxed{\beta = \omega\sqrt{\mu\epsilon}}$$

$$u = \frac{\omega}{\beta} = \frac{\omega}{\omega\sqrt{\mu\epsilon}} = \frac{1}{\sqrt{\mu\epsilon}}$$

$$\Rightarrow \boxed{u = \frac{1}{\sqrt{\mu\epsilon}}} \quad \Rightarrow \boxed{\lambda = \frac{2\pi}{\beta}}$$

$$\eta = \frac{\sqrt{\mu/\epsilon}}{\left[1+\cancel{\left(\frac{\sigma}{\omega\epsilon}\right)^2}^{\,0}\right]^{1/4}} \Rightarrow \sqrt{\mu/\epsilon}\,\angle 0^\circ$$

$$\boxed{\eta = \sqrt{\mu/\epsilon}\,\angle 0^\circ}$$

and thus E and H are in time phase with each other

Summary:

$$\boxed{\begin{aligned} \alpha &= 0 \\ \beta &= \omega\sqrt{\mu\epsilon} \\ u &= \frac{1}{\sqrt{\mu\epsilon}} \\ \eta &= \sqrt{\frac{\mu}{\epsilon}}\,\angle 0^\circ \end{aligned}}$$

Plane Waves in Good Conductor :- (10)

$\sigma \approx \infty$, $\mu = \mu_0 \mu_r$

$\varepsilon = \varepsilon_0 \varepsilon_r$, $\sigma \gg \omega\varepsilon$

Attenuation Constant $\Rightarrow$ $\alpha = \omega \sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2} - 1\right]}$

$\because \sigma/\omega\varepsilon \gg 1$

$$\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}\left(\frac{\sigma}{\omega\varepsilon} - 1\right)}$$ $\{\sigma/\omega\varepsilon$ is so large that 1 is ignored$\}$

$$\boxed{\alpha = \omega\sqrt{\frac{\mu\sigma}{2\omega}}}$$

$$\Rightarrow \alpha = \sqrt{\frac{\omega^2\mu\sigma}{2\omega}} = \sqrt{\frac{\omega\mu\sigma}{2}}$$

Similarly β can also be Obtained

$$\boxed{\alpha = \beta = \sqrt{\frac{\omega\mu\sigma}{2}} = \sqrt{\pi f \mu \sigma}}$$

Intrinsic Impedance, $\Rightarrow \eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}} = \sqrt{\frac{j\omega\mu}{\sigma\left(1 + \frac{j\omega\varepsilon}{\sigma}\right)}}$

$$\Rightarrow \eta = \sqrt{\frac{j\omega\mu}{\sigma}} \Rightarrow |\eta| = \sqrt{\frac{\omega\mu}{\sigma}}$$

$$\Rightarrow |\eta| = \sqrt{\frac{\omega\mu}{\sigma}}, \quad \theta_\eta = \frac{1}{2}\tan^{-1}\left(\frac{\frac{\omega\mu}{\sigma}}{0}\right)$$

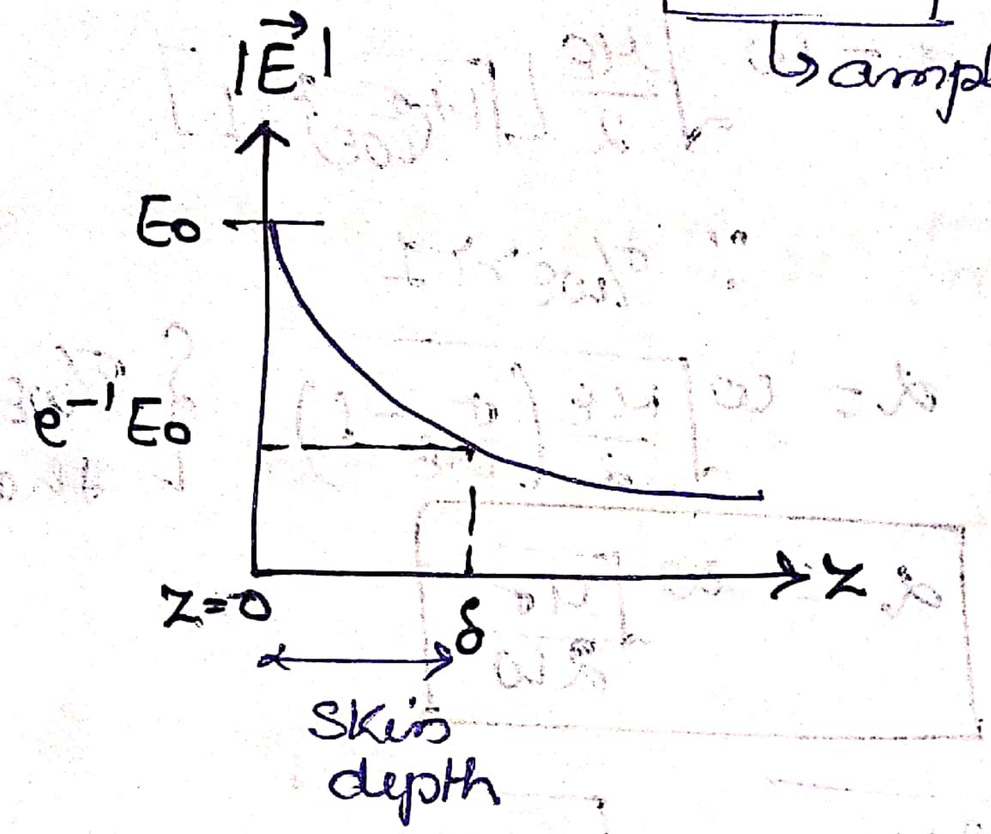
$$\Rightarrow \theta_\eta = \frac{\tan^{-1}(\infty)}{2} = \frac{\pi}{4}$$

$$\Rightarrow \boxed{\eta = \sqrt{\frac{\omega\mu}{\sigma}} \angle 45^\circ}$$

Skin depth OR Depth of Penetration (δ)

It is distance at which peak amplitude of electric field becomes $\frac{1}{e}$ times or it reduces by 37%.

$$\vec{E}(z,t) = \underbrace{E_0 e^{-\alpha z}}_{\text{amplitude}} \cos(\omega t - \beta z)\, a_E$$



$$\Rightarrow E_0 \cancel{e^{-1}} = E_0 \cancel{e^{-\alpha(0)}}$$

$$\Rightarrow E_0 e^{-\alpha z} = E_0 e^{-1}$$ → cond^n of skin depth

$$\Rightarrow \alpha z = 1$$

$$\Rightarrow \delta \alpha = 1 \quad (\because z = \delta)$$

$$\boxed{\delta = \frac{1}{\alpha}}$$ → Lossy and Good Conductor

↳ when α is present then only amplitude will reduced.

Expression of Electric field and Magnetic field of Plane wave:

From the Wave Equation:

$$\nabla^2 E_s - \gamma^2 E_s = 0 \quad \left[\text{Seems like } \{D^2 - \gamma^2 = 0\} \Rightarrow D = \pm\gamma\right]$$

$\Downarrow$

Solution

$\Downarrow$

$$E(z) = E_0^{+} e^{-\gamma z} + E_0^{-} e^{+\gamma z}$$

$\Downarrow$ Forward Travelling Wave ($+z$ dirn) $\quad$ $\Downarrow$ Backward Travelling Wave ($-z$ dirn)

→ If wave is travelling in (Forward) $+z$ direction, then,

$$E(z) = E_0 e^{-\gamma z}$$

$$\Rightarrow E(z) = E_0 e^{-(\alpha + j\beta) z}$$

$$\Rightarrow E(z) = \underbrace{E_0 e^{-\alpha z}}_{\text{amplitude part}} \underbrace{e^{-j\beta z}}_{\text{phase part}}$$

Including time factor in above equation -

$$E_s(z,t) = E_0 e^{-\alpha z} \cdot e^{-j\beta z} \cdot e^{j\omega t}$$

$$E_s(z,t) = Re\left[E_0 e^{-\alpha z} e^{-j\beta z} \cdot e^{+j\omega t}\right]$$

$$E_s(z,t) = E_0 e^{-\alpha z} Re\left[e^{-j\beta z} \cdot e^{j\omega t}\right]$$

$$E_s(z,t) = E_0 e^{-\alpha z} Re\left[e^{j(\omega t - \beta z)}\right]$$

$$E_s(z,t) = E_0 e^{-\alpha z} \cos(\omega t - \beta z)$$

$$\Rightarrow \boxed{\vec{E}_s(z,t) = E_0 e^{-\alpha z} \cos(\omega t - \beta z)\hat{a}_E} \rightarrow \text{dirn of } \vec{E}$$

E_0 → Peak value; ω → frequency; β → dirn of wave propagation

~~cos(t)~~ $\because \omega t - \beta z = \text{constant}$

$$\frac{d}{dt}(\omega t - \beta z) = 0 \Rightarrow \omega - \beta \frac{dz}{dt} = 0$$

$$\Rightarrow \omega = \beta V_p \Rightarrow \boxed{V_p = \frac{\omega}{\beta}}$$

Similarly from $\nabla^2 H_s - \gamma^2 H_s = 0 \Rightarrow$ Expression for $\vec{H}$ can be obtained.

POWER AND POYNTING THEOREM:

→ Energy can be transported from one point (transmitter) to another (receiver) point by means of EM waves.

→ Poynting Theorem states that - The net power flowing out of a given volume is equal to the time rate of decrease in the energy stored within v minus the Ohmic losses.

→ It can be obtained from Maxwell's Equations:

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad \{\vec{B} = \mu \vec{H}\}$$

$$\Rightarrow \nabla \times \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t} \quad \text{--- (1)}$$

$$\Rightarrow \nabla \times \vec{H} = \vec{J_c} + \vec{J_d}$$

$$\nabla \times \vec{H} = \sigma \vec{E} + \varepsilon \frac{\partial \vec{E}}{\partial t} \quad \text{--- (2)}$$

Taking dot product of $\vec{H}$ and eqn (1), we get

$$\vec{H} \cdot (\nabla \times \vec{E}) = -\mu \vec{H} \frac{\partial \vec{H}}{\partial t} = -\mu \frac{1}{2} \frac{\partial H^2}{\partial t} \quad \text{--- (3)}$$

Taking dot product of $\vec{E}$ and eq (2), we get

$$\vec{E} \cdot (\nabla \times \vec{H}) = \sigma E^2 + \frac{\varepsilon}{2} \frac{\partial E^2}{\partial t} \quad \text{--- (4)}$$

(Vale Know $\Rightarrow \nabla \cdot (\vec{A} \times \vec{B}) = \vec{B} \cdot (\nabla \times \vec{A}) - \vec{A} (\nabla \times \vec{B})$

$$\Rightarrow \nabla (\vec{E} \times \vec{H}) = \vec{H} (\nabla \times \vec{E}) - \vec{E} (\nabla \times \vec{H}) \quad \text{--- (5)}$$

putting ~~eqn~~ values from eqn (4) and (3) in eqn (5), we get

$$\Rightarrow \nabla (\vec{E} \times \vec{H}) = -\frac{\mu}{2} \frac{\partial H^2}{\partial t} - \sigma E^2 - \frac{\varepsilon}{2} \frac{\partial E^2}{\partial t}$$

$$\Rightarrow \nabla (\vec{E} \times \vec{H}) = -\sigma E^2 - \frac{\partial}{\partial t} \frac{1}{2} \mu H^2 - \frac{\partial}{\partial t} \frac{1}{2} \varepsilon E^2$$

$$\nabla\cdot(\vec{E}\times\vec{H}) = -\sigma E^2 - \frac{\partial}{\partial t}\frac{1}{2}\varepsilon E^2 - \frac{\partial}{\partial t}\frac{1}{2}\mu H^2$$

Taking Volume integral both side

$$\boxed{\int_V \nabla\cdot(\vec{E}\times\vec{H}) = -\int_V \sigma E^2 dv - \frac{\partial}{\partial t}\int_V \frac{1}{2}\varepsilon E^2 dv - \frac{\partial}{\partial t}\int_V \frac{1}{2}\mu H^2 dv} \quad (A)$$

$\Downarrow$

Expression for Poynting theorem

Where $\Rightarrow$ $\int_V \nabla\cdot(\vec{E}\times\vec{H})$ = Total Power leaving the Volume

$-\frac{\partial}{\partial t}\int_V \frac{1}{2}\varepsilon E^2 dv$ = Rate of decrease in energy stored in $\vec{E}$

$-\frac{\partial}{\partial t}\int_V \frac{1}{2}\mu H^2 dv$ = Rate of decrease in energy stored in $\vec{H}$

$-\int \sigma E^2 dv$ = Ohmic power dissipated

Applying Divergence theorem in eqn (A)

$$\boxed{\oint_S \underbrace{(\vec{E}\times\vec{H})}_{\vec{P}}\cdot ds = -\int_V \sigma E^2 \cdot dv - \frac{\partial}{\partial t}\int_V \frac{1}{2}\varepsilon E^2 dv - \frac{\partial}{\partial t}\int_V \frac{1}{2}\mu H^2 \cdot dv}$$

POYNTING VECTOR ($\vec{P}$)

The term $\vec{E}\times\vec{H}$ is known as poynting Vector $\vec{P}$, measured in Watts per square meter (W/m^2).

$$\vec{P} = \vec{E}\times\vec{H}$$

Plane Waves in Good Conductors:

⇒ Assume wave is travelling in (+z) direction

$$\vec{E} = E_0 e^{-\alpha z} \cos(\omega t - \beta z)\hat{a}_x$$

$$\vec{J_d} = J_{d0} e^{-\alpha z} \cos(\omega t - \beta z)\hat{a}_y$$

$$\Rightarrow \vec{J_d} = \frac{E_0}{|\eta| \angle \theta_n} e^{-\alpha z} \cos(\omega t - \beta z)\hat{a}_y$$

$$\Rightarrow \vec{J_d} = \frac{E_0}{|\eta|} e^{-\alpha z} \cos(\omega t - \beta z - \theta_n)\hat{a}_y$$

$$\Rightarrow \because \vec{P} = \vec{E} \times \vec{H}$$

$$\therefore \vec{P} = \frac{E_0^2}{|\eta|} e^{-2\alpha z} \cos(\omega t - \beta z)\cos(\omega t - \beta z - \theta_n) \cdot \hat{a}_z$$

⇒ Time average Poynting Vector = $\vec{P}_{avg}$

$$\Rightarrow \vec{P}_{avg} = \frac{1}{T}\int \vec{P} \cdot dt$$

$$\Rightarrow \vec{P}_{avg} = \frac{1}{T}\int_T^T \frac{E_0^2}{2|\eta|} e^{-2\alpha z} \cos\theta_n \hat{a}_z + \underbrace{\frac{1}{T}\int \frac{E_0^2}{2|\eta|} \cos(2\omega t - 2\beta z - \theta_n)\hat{a}_z}_{\text{Ave. Value of cosine} = 0} \to 0$$

$$\Rightarrow \vec{P}_{avg} = \frac{E_0^2}{2|\eta|} e^{-2\alpha z} \cos\theta_n \hat{a}_z \frac{1}{T}\int_0^T dt$$

$$\Rightarrow \boxed{\vec{P}_{avg} = \frac{E_0^2}{2|\eta|} e^{-2\alpha z} \cos\theta_n \cdot \hat{a}_z}$$

NOTE: For lossless medium, $\alpha = 0$, $\theta_n = 0$

$$\boxed{\vec{P}_{avg} = \frac{E_0^2}{2|\eta|} a_k \quad \frac{W}{m^2}}$$

a_k → dirn of wave propagation

⇒ **Total POWER (P)**

$$\boxed{P = \int_S \vec{P}_{avg} \cdot \vec{ds}}$$