BAS – 26 (Optimization Techniques) Sheet 1 (UNIT -1 & 2) B. Tech. IV semester

1. Optimize the following problems by Langragian Method:

(a) $f = 1 - 8x_1 - 6x_2 - 4x_3 + 2x_1^2 + 2x_2^2 + x_3^2 + 2x_1x_2 + 2x_2x_3 + 2x_1x_3$ subject to $x_1 + 2x_2 + 2x_3 = 5.$

(b) $f = 10 - 8x_1 - 6x_2 - 4x_3 + 2x_1^2 + 2x_2^2 + x_3^2 + 2x_1x_2 + 2x_2x_3 + 2x_1x_3$ subject to $x_1 + 2x_2 + 2x_3 = 5.$

(c) $f(x_1, x_2, x_3) = 2x_1^2 + x_2^2 + 3x_3^2 + 10x_1 + 8x_2 + 6x_3 - 100$ subject to $x_1 + x_2 + x_3 = 20$

(d) $f = 9 - 8x_1 - 6x_2 - 4x_3 + 2x_1^2 + 2x_2^2 + x_3^2 + 2x_1x_2 + 2x_1x_3$ subject to $x_1 + x_2 + 2x_3 = 3$

(e) $f = 9 - 8x_1 - 6x_2 - 4x_3 + 2x_1^2 + 2x_2^2 + x_3^2 + 2x_1x_2 + 2x_1x_3$ subject to the $x_1 + x_2 + 2x_3 = 4$

(f) $f(x_1, x_2, x_3) = 3x_1^2 - 2x_2^2 + 6x_3^2 + 6x_1x_2 - 4x_2x_3 - 4x_1 + 12x_2 - 12x_3 + 10,\ 3x_1 + 2x_2 - 4x_3 = 6.$

(g) $f = 2x_1 - 3x_2 + 4x_3 + 4x_1^2 + 5x_2^2 - 6x_3^2 + 2x_1x_3 + 4x_2x_3 - 6x_1x_2,\quad 2x_1 - 3x_2 + 4x_3 = 10.$

(2)Minimize $f(X) = (1/2)(x_1^2 + x_2^2 + x_3^2)$ subject to the conditions $x_1 - x_2 = 0$ *and* $x_1 + x_2 + x_3 - 1 = 0.$

(3) Find the all extreme points of the function $f = 5x_1^3 + 2x_2^3 + 3x_3^3 + 4x_1^2 + 8x_2^2 + 12x_3^2 - 15$. Show that the function is maximum or minimum for any two extreme points that at least one variable is non-zero.

(4) Find the all extreme points of the function $(x_1, x_2, x_3) = 3x_1^3 - x_2^3 + x_3^3 - 4x_1 + 12x_2 - 24x_3$. Discuss the nature of extreme point $(\frac{2}{3}, -2, 2\sqrt{2})$.

(5) Find the all extreme points of the Optimizing problem: $f(x_1, x_2) = x_1^3 - 3x_1x_2^2 + x_2^4 + x_2^2$. Also discuss the nature of two non-zero extreme points.

(6) Discuss the nature of two non – zero extreme points of the problem:

$$f = x_1^3 - 3x_1x_2^2 + 6x_2^3 + x_2^2 + \left(\frac{1}{3}\right)x_3^3 - x_2x_3^2 .$$

(7) Find the all extreme points of the function and discuss the nature of two extreme points with $x_3 = \frac{2}{\sqrt{5}}$.

$$f(x_1, x_2, x_3) = \left(\frac{1}{3}\right)x_1^3 - 6x_2^3 + 10x_3^3 - 5x_1 + 12x_2 - 24x_3 + 2x_1^2.$$

(8) Optimization the following problems by method of Kuhn – Tucker conditions. Explain all cases.

(a) Maximize $f(x_1, x_2) = 4x_1 + 7x_2 - x_1^3 - 2x_2^2$ subject to $2x_1 + 5x_2 \le 6,\ 2x_1 - 15x_2 \le 12.$

(b) Maximize $f(x_1, x_2) = -x_1^2 - x_2^2 + x_1x_2 + 7x_1 + 4x_2$ with $x_1 - 5 \le 0, 2x_1 + 3x_2 - 70 \le 0.$

(c) Maximize $f(x_1, x_2) = 32x_1 + 50x_2 - 10x_2^2 + x_2^3$ subject to $3x_1 + x_2 \le 11,\ 2x_1 + 5x_2 \le 16,$ $x_1 \ge 0, x_2 \ge 0.$

(d) Maximize $8x_1 + 4x_2 + x_1x_2 - x_1^2 - x_2^2$ subject to $2x_1 + 3x_2 \le 24$, $-5x_1 + 12x_2 \le 24$, $x_2 \le 5$

(e) Minimize $f(X) = x_1^2 + x_2^2 + x_3^2$ subject to $x_1 + x_2 + x_3 \ge 5$ and $x_3 \ge 2$.

(f) Minimize $z = (x_1 - 1)^2 + (x_2 - 2)^2$ subject to $-x_1 + x_2 = 1$ $\quad x_1 + x_2 \le 2$ $\quad x_1 \ge 0,\ x_2 \ge 0$.

(g) Maximize $z = -x_1^2 - x_2^2 + 4x_1 + 6x_2$ subject to $x_1 + x_2 \le 6$ $\quad x_1 \le 3$, $x_2 \le 4$ $\quad x_1 \ge 0,\ x_2 \ge 0$.

(h) Minimize $z = (x_1 - 3)^2 + (x_2 - 5)^2$ subject to $x_1 + x_2 \le 7$ $\quad x_1 \ge 0,\ x_2 \ge 0$.

(i) Minimize $z = x_1^2 + 2x_2^2 + 3x_3^2$ subject to $x_1 - x_2 - 2x_3 \le 12$, $\quad x_1 + 2x_2 - 3x_3 \le 8$ $\quad x_1 \ge 0,\ x_2 \ge 0$.

(9) Find the minimum of the function $f(x) = x^5 - 5x^3 - 20x + 5$ by the Golden section method in the interval (0, 5).

(10) Find the minimum of the function $f(x) = 12x^5 - 45x^4 + 40x^3 + 5$ by Fibonacci method in the interval (0, 4).

(11) Find the minimum of the function $f(x) = x^2 - \frac{x^3}{5} - Sin^{-1}(Sin(x))$ in the range $(-1, 3)$ by Fibonacci method with taking n = 6. Also discuss the validity of results.

(12) Find the minimum of the function $x^3/16 - 27x/4 + (Sin x)^2$ in the range (0, 10) by Fibonacci method with n = 7

(13) Find the minimum of $0.5 - \dfrac{0.75}{1+x^2} - 0.65x\ \tan^{-1}(\dfrac{1}{x^2})$ in the range (-1, 4) by Golden section method with n = 7.

(14) Find the minimum of the function $f(x) = 10x^5 - 40x^4 + 30x^3 + 5$ by Fibonacci method in the given interval (1, 3.5) up to 6 iterations. Also test the accuracy.

(15) Find the minimum of the function $f(x) = x^3 + x^2 - x - 2$ in the interval -4 to 4 by Fibonacci method.

(16) Find the minimum of the function $f(x) = -\frac{1.5}{x} + \frac{6\times10^{-6}}{x^9}$ in the interval -4 to 4 by Golden section method.

(17) Find the maximum of the function $f(x) = x^2 - \frac{x^3}{5} - Sin^{-1}(Sin(x))$ in the range $(2, 6)$ by golden section method.

(18) Obtain the dual of the following problems. Explain properly.

(a) Min $z = 2x_1 + 3x_2 + 4x_3$ subject to the condition $2x_1 + 3x_2 + 5x_3 \ge 2$, $3x_1 + x_2 + 7x_3 = 3$, $\quad x_1 + 4x_2 + 6x_3 \le 5$.
$x_1, x_2 \ge 0$ and x_3 is unrestricted.

(b) Min $z = 3x_1 - 4x_2 + 5x_3$ subject to $3x_1 + 4x_2 + 5x_3 \ge 3$, $4x_1 + 2x_2 + 8x_3 = 3$, $x_1 + 4x_2 + 6x_3 \le 5$. $x_1, x_2, x_3 \ge 0$.

(c) Min $z = x_1 - 3x_2 + 7x_3$ subject to the constraints $x_1 - 3x_2 + 4x_3 = 5$, $\quad x_1 - 2x_2 \le 3$,
$x_1 - 2x_2 - x_3 \ge 4$, $x_1, x_3 \ge 0$ and x_2 is unrestricted.

(d) Min $z = 3x_1 + x_2 + 2x_3 - x_4$ subject to the conditions: $\quad 2x_1 - x_2 + 3x_3 + x_4 = 1$, $x_1 + x_2 - x_3 + 4x_4 = 3$

$x_1, x_2, x_3 \ge 0$ and x_4 is unrestricted.

BAS – 26 (Optimization Techniques) Sheet 2 (UNIT - 2) B. Tech. IV semester

(1) Solve the following problems by using simplex method:

(a) Maximize $f = 3x + 2y + 5z$ subject to subject to

$$x + 2y + z \le 430, \quad 3x + 2z \le 460, \quad x + 4y \le 420, \quad x \ge 0, \ y \ge 0, \ z \ge 0.$$

(b) $Maximize\ z = 5x + 3y$ subject to

$$x + y \le 2, \ 5x + 2y \le 10, \ 3x + 8y \le 12, \ x \ge 0, \ y \ge 0.$$

(c) Maximize $z = 19x + 7y$ subject

$$7x + 6y \le 42, \ 5x + 9y \le 45, \ x - y \le 4, \ x \ge 0, \ y \ge 0.$$

(d) $Maximize\ z = x + 2y + 2z$ subject to constraints

$$5x + 2y + 3z \le 15, \quad x + 4y + 2z \le 12, \quad 2x + z \le 8, \quad x \ge 0, y \ge 0, z \ge 0.$$

(e) $Minimize\ z = -10x - 15y - 8z$ subject to

$$x + 2y + 2z \le 200, \quad 2x + y + z \le 220, \quad 3x + y + 2z \le 180,$$

$$x \ge 0, y \ge 0, z \ge 0.$$

(f) **Maximize** $f = 2x_1 + 4x_2 + 3x_3$ subject to conditions

$$3x_1 + 4x_2 + 3x_3 \le 3600, \ 2x_1 + x_2 + 3x_3 \le 2400, \quad x_1 + 3x_2 + 3x_3 \le 4800, \quad x_1 \ge 0, \ x_2 \ge 0, x_3 \ge 0$$

(g) Min $\boldsymbol{z} = x_1 - 3x_2 + 2x_3$*, subject to*

$$3x_1 - x_2 + 3x_3 \le 7, \quad -2x_1 + 4x_2 \le 12, \quad -4x_1 + 3x_2 + 8x_3 \le 10$$

$$x_1, x_2, x_3 \ge 0$$

(h) Max $\boldsymbol{z} = -x_1 + 2x_2$*, subject to,* $-x_1 + x_2 \le 1, \ -x_1 + 2x_2 \le 4,$ $x_1, x_2 \ge 0.$

(2) Solve the following Linear Programming Problem by decomposition principle

(a) Maximize $f = 8x_1 + 3x_2 + 8x_3 + 6x_4$ subject to

$$4x_1 + 3x_2 + x_3 + 3x_4 \le 16, \quad 4x_1 - x_2 + x_3 \le 12,$$

$$x_1 + 2x_2 \le 8, \quad 3x_1 + x_2 \le 10, \quad 2x_3 + 3x_4 \le 9,$$

$$4x_3 + x_4 \le 12. \qquad x_i \ge 0, \quad i = 1, 2, 3, 4.$$

(b) Maximize $f = 7x - 9y + 5z + 8w$ subject to

$$5x + 2y + 5z + 11w \le 20,$$

$$9y + 13z + 15w \le 14,$$

$$5z + 2w \le 10, \quad w \ge 1,$$

$$3z + 5w \le 15, \quad 6x + 5y \le 30, \quad y \ge 5,$$

$$and \quad x, y, w, z \ge 0.$$

(c) Maximize $f = 10x - 20y + 5z + 30w$ subject to

$$4x - 3y + 5z + 10w \le 20,$$

$$7y + 10z + 5w \le 6, 3z + w \le 70,$$

$$z + w \le 45,$$

$$z \le 20, w \le 40, z \ge 0, w \ge 0,$$

$$x + y \le 300, \quad x - 2y \le 200,$$

$$2x + y \ge 100, \quad y \le 200, \quad x, y \ge 0.$$

(d) Max $\boldsymbol{z} = x_1 + x_2 + 8000x_3 + 7000x_4$ *subject to,*

$$8x_1 + 3x_2 + 500x_3 + 100x_4,$$

$$8x_1 + 10x_2 - 200x_4$$

$$x_1 + 2x_2 \le 2000, \quad x_1 + x_2 \le 1500, \quad x_2 \le 600$$

$$3x_3 + x_4 \le 66, \quad x_3 + x_4 \le 45, \quad x_3 \le 20, \quad x_4 \le 40$$

(3) Solve the following L. P. problem by Karmarkar's method. Use the value of $\varepsilon = 0.05$ for

(1) testing the convergence of the procedure and $\propto = 1/4$.

(a) Minimize $f = 2x_1 + 11x_2 - 9x_3$ subject to

$$3x_1 - 4x_3 = 0, \qquad x_1 + x_2 + x_3 = 1, \qquad x_i \geq 0, \quad i = 1, 2, 3.$$

(b) $Minimize\ \ f = 3\,x_1 + 11\,x_2 - 13\,x_3$ subject to

$$3x_1 - 7\,x_3 = 0, \quad x_1 + x_2 + x_3 = 1 \qquad x_i \geq 0, \quad i = 1, 2, 3.$$

(c) $Minimize\ \ f = 2\,x_1 + 13\,x_2 - 11\,x_3$ with

$$4\,x_1 - 3\,x_3 = 0, \qquad x_1 + x_2 + x_3 = 1, \quad x_i \geq 0, \qquad i = 1, 2, 3.$$

(d) $Minimize\ \ f = 4\,x_1 + 15\,x_2 - 13\,x_3$ Subject to

$$3\,x_1 - 4\,x_3 = 0,\ x_1 + x_2 + x_3 = 1,\ x_1 \geq 0,\ \ i = 1, 2, 3\,.$$

(e) Minimize $f = 3x_1 + 5x_2 - 3x_3$ Subject to

$$x_1 - x_3 = 0, \qquad x_1 + x_2 + x_3 = 1, \qquad x_i \geq 0, \quad i = 1, 2, 3.$$

(4)Use dual simplex method to solve following L.P.P.

(a) Maximize $z = -2x_1 - x_2$ subject to

$$3x_1 + x_2 \geq 3, \quad x_1 + 3x_2 \geq 6, \quad x_1 + 2x_2 \geq 3 \qquad and \qquad x_1, x_2 \geq 0\,.$$

(b) $Minimize\ z = x + y$ subject to

$$x + 2\,y \geq 7,\ 4\,x + y \geq 6,\ x \ \geq 0,\ y \ \geq 0.$$

(c) Minimize $z = 4\,x_1 + 2x_2$ subject to

$$x_1 + x_2 = 1, \qquad 3x_1 - x_2 \geq 2, \qquad x_1, x_2 \geq 0.$$

(d) Minimize $Z = x_1 + 2x_2 + 3x_3$ subject to constraints

$$2x_1 - x_2 + x_3 \geq 4, \quad x_1 + x_2 + 2x_3 \geq 8, \quad x_2 - x_3 \geq 2, \qquad x_i \geq 0, \quad i = 1, 2, 3.$$

(e) Minimize $Z = 4x_1 + 2x_2 + 3\,x_3$ subject to

$$2x_1 + \ 4\,x_3 \geq 5, \qquad 2x_1 + 3x_2 + \ x_3 \geq 4, \qquad x_1 \geq 0, x_2 \geq 0, x_3 \geq 0.$$

(f) Min $z = 3\,x_1 + x_2$ subject to

$$x_1 + x_2 \geq 1 \quad 2x_1 + 3x_2 \geq 2, \quad 3x_1 + 4x_2 \geq 6 \quad x_1, x_2 \geq 0$$

(g) Maximum $z = 3\,x_1 - 2\,x_2$ subject to the constraints:

$x_1 + x_2 \leq 1,\ 2x_2 + 2x_2 \geq 4$ *And $x_1, x_2 \geq 0$.*

(h) Min $z = 5\,x_1 + 7\,x_2$ subject to

$$x_1 + x_2 \geq 1, \quad 2x_1 + 3x_2 \geq 2, \quad 4x_1 + x_2 = 4, \quad x_1, x_2 \geq 0\,.$$

(5) Solving the following transportation problems and find optimum solution.

	A	B	C	Available
I	50	30	220	1
II	90	45	170	
III	250	200	50	3
Required	4	2	2	4