1. Optimize the following problems by Langragian method.

(a) $f = 1 - 8x_1 - 6x_2 - 4x_3 + 2x_1^2 + 2x_2^2 + x_3^2 + 2x_1x_2 + 2x_2x_3 + 2x_1x_3$

subject to $x_1 + 2x_2 + 2x_3 = 5$.

Soln

$$L = f + \lambda g$$

$$\frac{\partial L}{\partial x_1} = -8 + 4x_1 + 2x_2 + 2x_3 + \lambda(1) = 0 \quad -(1)$$

$$\frac{\partial L}{\partial x_2} = -6 + 4x_2 + 2x_1 + 2x_3 + \lambda(2) = 0 \quad -(2)$$

$$\frac{\partial L}{\partial x_3} = -4 + 2x_3 + 2x_2 + 2x_1 + \lambda(2) = 0 \quad -(3)$$

$$\frac{\partial L}{\partial \lambda} = x_1 + 2x_2 + 2x_3 - 5 = 0 \quad -(4)$$

Subtract eq (3) from eq (2)

$$-6 + 4x_2 + 2\cancel{x_1} + 2\cancel{x_3} + 2\lambda + 4 - 2\cancel{x_3} - 2x_2 - 2\cancel{x_1} - \cancel{2\lambda} = 0$$

$$-2 + 2x_2 = 0$$

$$2x_2 = 2$$

$$\boxed{x_2 = 1}$$

Subtract eq (2) from eq (1)

$$-\cancel{8} + 4x_1 + \cancel{2} + 2\cancel{x_3} + \lambda + \cancel{6} - 4 - 2x_1 - 2\cancel{x_3} - 2\lambda = 0$$

$$2x_1 - 4 - \lambda = 0$$

$$x_1 = \frac{\lambda + 4}{2}$$

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Put the value of x_1 and x_2 in eq (4)

$$\frac{\lambda}{2} + 2 + 2 + 2x_3 - 5 = 0$$

$$\frac{\lambda}{2} + 2x_3 = 1$$

$$2x_3 = 1 - \frac{\lambda}{2}$$

$$\boxed{x_3 = \frac{1}{2} - \frac{\lambda}{4}}$$

Put x_1, x_2, x_3 in eq — (1)

$$-8 + 4\left(\frac{\lambda}{2} + 2\right) + 2 + 2\left(\frac{1}{2} - \frac{\lambda}{4}\right) + \lambda = 0$$

$$\cancel{-8} + 2\lambda + \cancel{8} + 2 + 1 - \frac{\lambda}{2} + \lambda = 0$$

$$3\lambda - \frac{\lambda}{2} = -3$$

$$\frac{5\lambda}{2} = -3$$

$$\lambda = -\frac{6}{5}$$

$$\boxed{\lambda = -1.2}$$

Put λ to get the value of x_1 and x_3

$$x_1 = \frac{-1.2}{2} + 2 = -0.6 + 2 = \boxed{1.4}$$

$$x_3 = \frac{1}{2} + \frac{1.2}{4} \; 0.3 = \boxed{0.8}$$

$$\boxed{x_2 = 1}$$

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So the extreme points are $(1.4, 1, 0.8, -1.2)$.

for sufficient condition $\rightarrow$

$$\begin{vmatrix} 4-z & 2 & 2 & 1 \\ 2 & 4-z & 2 & 2 \\ 2 & 2 & 2-z & 2 \\ 1 & 2 & 2 & 0 \end{vmatrix} = 0$$

$$4-z\begin{vmatrix} 4-z & 2 & 2 \\ 2 & 2-z & 2 \\ 2 & 2 & 0 \end{vmatrix} - 2\begin{vmatrix} 2 & 2 & 1 \\ 2 & 2-z & 2 \\ 2 & 2 & 0 \end{vmatrix} + 2\begin{vmatrix} 2 & 2 & 1 \\ 4-z & 2 & 2 \\ 2 & 2 & 0 \end{vmatrix}$$

$$-1\begin{vmatrix} 2 & 2 & 1 \\ 4-z & 2 & 2 \\ 2 & 2-z & 2 \end{vmatrix} = 0$$

$$\Rightarrow (4-z)\{4-z(-4) - 2(-4) + 2(\cancel{4}-\cancel{4}+2z)\}$$

$$-2\{2(-4) - 2(-2) + 2(\cancel{2}-\cancel{2}+z)\}$$

$$+2\{2(-2) - (4-z)(-2) + 2(2)\}$$

$$-1\{2(\cancel{4}-\cancel{4}+2z) - (4-z)(4-2+z) + 2(4-2)\} = 0$$

$$\Rightarrow (4-z)\{-16+4z+8+4z\} - 2\{-8+4+2z\} + 2\{\cancel{-4}+8-2z+\cancel{4}\}$$

$$-(4z+4+z^2-2z-8) = 0$$

$$(4-z)\{8z-8\} - 2\{2z-4\} + 16 - 4z - z^2 - 2z + 4 = 0$$

$$32z - 32 - 8z^2 + \cancel{8z} - \cancel{4z} + 8 + 16 - \cancel{4z} - z^2 - 2z + 4 = 0$$

$$30z - 9z^2 - 4 = 0$$

$$9z^2 + 4 - 30z = 0$$

$$9z^2 - 30z + 4 = 0$$

$$z = \frac{30 \pm \sqrt{(30)^2 - 4 \times 9 \times 4}}{2 \times 9}$$

$$z = \frac{30 \pm \sqrt{756}}{18} = \frac{30 \pm 27.5}{18}$$

$$\boxed{z = 3.195,\ 0.1389}$$

$$z > 0$$

we get relative minima.

Q2 $f = 10 - 8x_1 - 6x_2 - 4x_3 + 2x_1^2 + 2x_2^2 + x_3^2 + 2x_1 + 2x_2x_3 + 2x_1x_3$

subject to $x_1 + 2x_2 + 2x_3 = 5$

Sol^n

$L = f + \lambda g$

$L = f + \lambda(x_1 + 2x_2 + 2x_3 - 5)$

$\frac{\partial L}{\partial x_1} = -8 + 4x_1 + 2x_2 + 2x_3 + \lambda(1) = 0$ —(1)

$\frac{\partial L}{\partial x_2} = -6 + 4x_2 + 2x_1 + 2x_3 + \lambda(2) = 0$ —(2)

$\frac{\partial L}{\partial x_3} = -4 + 2x_3 + 2x_2 + 2x_1 + \lambda(2) = 0$ —(3)

$\frac{\partial L}{\partial \lambda} = x_1 + 2x_2 + 2x_3 - 5 = 0$ —(4)

Subtract eq (3) from eq (2)

$-6 + 4x_2 + \cancel{2x_1} + \cancel{2x_3} + \cancel{2\lambda} + 4 - \cancel{2x_3} - 2x_2 - \cancel{2x_1} - \cancel{2\lambda} = 0$

$-2 + 2x_2 = 0$

$2x_2 = 2$

$\boxed{x_2 = 1}$

Subtract eq (2) from eq (1)

$-\cancel{8} + 4x_1 + \cancel{2} + \cancel{2x_3} + \lambda + \cancel{6} - 4 - 2x_1 - \cancel{2x_3} - 2\lambda = 0$

$2x_1 - 4 - \lambda = 0$

$x_1 = \frac{\lambda + 4}{2}$

$\boxed{x_1 = \frac{\lambda}{2} + 2}$

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Put value of x_1 and x_2 in eq (4)

$$\frac{-1}{2}+2+2+2x_3-5=0$$

$$\frac{-\lambda}{2}+2x_3=1$$

$$2x_3=1-\lambda/2$$

$$\boxed{x_3=\frac{1}{2}-\frac{\lambda}{4}}$$

Put the value of x_1, x_2, x_3 in equation (1)

$$-8+4\left(\frac{\lambda}{2}+2\right)+2+2\left(\frac{1}{2}-\frac{\lambda}{4}\right)+\lambda=0$$

$$-8+2\lambda+8+2+1-\frac{\lambda}{2}+\lambda=0$$

$$3\lambda-\frac{\lambda}{2}=-3$$

$$\frac{5\lambda}{2}=-3$$

$$\boxed{\lambda=-\frac{6}{5}=-1.2}$$

Put the value of x_1, x_2 and x_3.

$$\boxed{\begin{aligned} x_1 &= 1.4 \\ x_2 &= 1 \\ x_3 &= 0.8 \end{aligned}}$$

So the extreme points are $(1.4, 1, 0.8, -1.2)$.

for sufficient condition

$$\begin{vmatrix} 4-z & 2 & 2 & 1 \\ 2 & 4-z & 2 & 2 \\ 2 & 2 & 2-z & 2 \\ 1 & 2 & 2 & 0 \end{vmatrix} = 0$$

$$4-z\begin{vmatrix} 4-z & 2 & 2 \\ 2 & 2-z & 2 \\ 2 & 2 & 0 \end{vmatrix} - 2\begin{vmatrix} 2 & 2 & 1 \\ 2 & 2-z & 2 \\ 2 & 2 & 0 \end{vmatrix} + 2\begin{vmatrix} 2 & 2 & 1 \\ 4-z & 2 & 2 \\ 2 & 2 & 0 \end{vmatrix}$$

$$-1\begin{vmatrix} 2 & 2 & 1 \\ 4-z & 2 & 2 \\ 2 & 2-z & 2 \end{vmatrix} = 0$$

$$\Rightarrow (4-z)\{(4-z)(-4)-2(-4)+2(2z)\} - 2\{2(-4)-2(-2)+2(2-2+z)\}$$

$$+2\{2(-2)-(4-z)(-2)+2(2)\}.$$

$$-1\{2(4-4+2z)-(4-z)(4-2+z)+2(4-z)\}=0$$

$$\Rightarrow (4-z)\{-16+4z+8+4z\} - 2\{-4+2z\} + 2\{8-2z\}$$

$$-(4z+4+z^2-2z-8)=0$$

$$(4-z)\{8z-8\} - 2\{2z-4\} + 16-4z-z^2-2z+4=0$$

$$32z-32-8z^2+8z-4z+8+16-4z-z^2-2z+4=0$$

$$30z-9z^2-4=0$$

$$9z^2+4-30z=0$$

$$Z = \frac{30 \pm \sqrt{756}}{18}$$

$$Z = \frac{30 \pm 27.5}{18}$$

$$\boxed{Z = 3.195,\ 0.1389}$$

$Z > 0$

we get absolute minima

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Q3 $f(x_1, x_2, x_3) = 2x_1^2 + x_2^2 + 3x_3^2 + 10x_1 + 8x_2 + 6x_3 - 100$

subject to $x_1 + x_2 + x_3 = 20.$

Solution

$$L = f + \lambda g.$$

$$L = f + \lambda(x_1 + x_2 + x_3 - 20).$$

$$\frac{\partial L}{\partial x_1} = 4x_1 + 10 + \lambda(1) = 0 \quad -(1)$$

$$\frac{\partial L}{\partial x_2} = 2x_2 + 8 + \lambda(1) = 0 \quad -(2)$$

$$\frac{\partial L}{\partial x_3} = 6x_3 + 6 + \lambda(1) = 0 \quad -(3)$$

$$\frac{\partial L}{\partial \lambda} = x_1 + x_2 + x_3 - 20 = 0 \quad -(4)$$

$$x_1 = \frac{-10-\lambda}{4}$$

$$x_2 = \frac{-8-\lambda}{2}$$

$$x_3 = \frac{-6-\lambda}{6}$$

Put x_1, x_2, x_3 in eq 4.

$$\frac{-10-\lambda}{4} + \frac{-8-\lambda}{2} + \frac{-6-\lambda}{6} - 20 = 0$$

$$30 + 3\lambda + 48 + 6\lambda + 12 + 2\lambda + 240 = 0$$

$$11\lambda + 330 = 0$$

$$\lambda = \frac{-330}{11}$$

$$\boxed{\lambda = -30}$$

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Put the value of λ to find x_1, x_2, x_3.

$$x_1 = \frac{-10+30}{4} = \frac{20}{4} = 5.$$

$$x_2 = \frac{-8+30}{2} = \frac{22}{2} = 11.$$

$$x_3 = -\frac{-6+30}{6} = \frac{24}{6} = 4.$$

for sufficient condition

$$\begin{vmatrix} 4-z & 0 & 0 & 1 \\ 0 & 2-z & 0 & 1 \\ 0 & 0 & 6-z & 1 \\ 1 & 1 & 1 & 0 \end{vmatrix} = 0$$

$$4-z \begin{vmatrix} 2-z & 0 & 1 \\ 0 & 6-z & 1 \\ 1 & 1 & 0 \end{vmatrix} - 0 + 0 - 1 \begin{vmatrix} 0 & 0 & 1 \\ 2-z & 0 & 1 \\ 0 & 6-z & 1 \end{vmatrix} = 0.$$

$$(4-z)\{2-z\{0-1\} - 0 + 1\{0+z-6\} - 1\{0-(2-z)(z-6)\} = 0.$$

$$(4-z)\{z-2+z-6\} + \{(2-z)(z-6)\} = 0$$

$$(4-z)\{2z-12\} + 2z - 12 - z^2 + 6z = 0$$

$$8z - 48 - 2z^2 + 12z + 2z - 12 - z^2 + 6z = 0$$

$$-3z^2 + 28z - 60 = 0$$

$3z^2 - 28z + 60 = 0$

$$z = \frac{28 \pm \sqrt{(28)^2 - 4 \times 3 \times 60}}{6}$$

$$z = \frac{28 \pm 8}{6}$$

$$z = \frac{36}{6}, \frac{20}{6}$$

$$\boxed{z = 6, 3.33}$$

$$\boxed{z > 0}$$

So we get relative minima -

Q4 $f = 9 - 8x_1 - 6x_2 - 4x_3 + 2x_1^2 + 2x_2^2 + x_3^2 + 2x_1x_2 + 2x_1x_3$

subject to $x_1 + x_2 + 2x_3 = 3$.

Sol^n $L = f + \lambda g$

$L = f + \lambda(x_1 + x_2 + 2x_3 - 3)$

$$\frac{\partial L}{\partial x_1} = -8 + 4x_1 + 2x_2 + 2x_3 + \lambda = 0$$

$$\frac{\partial L}{\partial x_2} = -6 + 4x_2 + 2x_1 + \lambda = 0$$

$$\frac{\partial L}{\partial x_3} = -4 + 2x_3 + 2x_1 + 2\lambda = 0$$

$$\frac{\partial L}{\partial \lambda} = x_1 + x_2 + 2x_3 - 3 = 0$$

$\lambda + 4x_1 + 2x_2 + 2x_3 = 8$ — (1)

$\lambda + 2x_1 + 4x_2 = 6$ — (2)

$2\lambda + 2x_1 + \quad + 2x_3 = 4$ — (3)

$x_1 + x_2 + 2x_3 = 3$ — (4)

Subtract (2) from (1)

$\cancel{\lambda} + 4x_1 + 2x_2 + 2x_3 - \cancel{\lambda} - 2x_1 - 4x_2 = 8 - 6$

$2x_1 - 2x_2 + 2x_3 = 2$ — (4)

Multiply equation (2) with 2 and subtract eq (3)

$2\lambda + 4x_1 + 8x_2 - 2\lambda - 2x_1 - 2x_3 = 12 - 4$

$2x_1 + 8x_2 - 2x_3 = 8$ — (5)

Multiplying equation 1 with 2 and substract equation 3.

$$\cancel{2x} + 8x_1 + 4x_2 + 4x_3 - \cancel{2x} - 2x_1 - 2x_3 = 16 - 4$$

$$6x_1 + 4x_2 + 2x_3 = 12 \quad -(6)$$

Substract equation 5 from eq 4

$$2x_1 - 2x_2 + 2x_3 - 2x_1 - 8x_2 + 2x_3 = 2 - 8$$

$$-10x_2 + 4x_3 = -6$$

$$2x_3 - 5x_2 = -3 \quad -(7)$$

from eq 4

$$\boxed{x_1 = 1 - x_3 + x_2} \quad -(8)$$

Put eq 8 in eq α.

$$1 - x_3 + x_2 + x_2 + 2x_3 = 3$$

$$2x_2 + x_3 = 2 \quad -(9)$$

Solving 7 and 9

$$\cancel{2x_3} - 5x_2 = -3$$

$$\cancel{2x_3} + 4x_2 = 4$$

$$-9x_2 = -7$$

$$\boxed{x_2 = 7/9}$$

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$$2x_3 - \frac{5\times 7}{9} = -3$$

$$2x_3 - \frac{35}{9} = -3$$

$$2x_3 = -3 + \frac{35}{9} = \frac{-27+35}{9}$$

$$2x_3 = \frac{8}{9}$$

$$\boxed{x_3 = \frac{4}{9}}$$

$$x_1 = 1 - \frac{4}{9} + \frac{7}{9}$$

$$x_1 = \frac{9-4+7}{9}$$

$$\boxed{x_1 = \cancel{\frac{12}{9}}\ \frac{4}{3}}$$

Put the value of x_1, x_2, x_3 in equation (4)

$$\lambda + 4\times\frac{4}{3} + 2\times\frac{7}{9} + 2\times\frac{4}{9} = 8$$

$$\lambda + \frac{48+14+8}{9} = 8$$

$$\lambda + \frac{70}{9} = 8$$

$$\lambda = 8 - \frac{70}{9} = \frac{72-70}{9}$$

$$\boxed{\lambda = \frac{2}{9}} > 0$$

extreme points are $\left(\frac{12}{9}, \frac{7}{9}, \frac{4}{9}\right)$.

sufficient condition is

$$\begin{vmatrix} 4-z & 2 & 2 & 1 \\ 2 & 4-z & 0 & 1 \\ 2 & 0 & 2-z & 2 \\ 1 & 1 & 2 & 0 \end{vmatrix} = 0$$

$$(4-z)\begin{vmatrix} 4-z & 0 & 1 \\ 0 & 2-z & 2 \\ 1 & 2 & 0 \end{vmatrix} - 2\begin{vmatrix} 2 & 2 & 1 \\ 0 & 2-z & 2 \\ 1 & 2 & 0 \end{vmatrix} + 2\begin{vmatrix} 2 & 2 & 1 \\ 4-z & 0 & 1 \\ 1 & 2 & 0 \end{vmatrix}$$

$$-1\begin{vmatrix} 2 & 2 & 1 \\ 4-z & 0 & 1 \\ 0 & 2-z & 2 \end{vmatrix} = 0$$

$$(4-z)\{(4-z)\times -4 + 1(z-2)\} - 2\{2\times -2 + (4-2+z)\}$$

$$+2\{2(-2) - (4-z)(-2) + 1(2)\}$$

$$-1\{2(z-2) - 4(4-2+z)\}$$

$$(4-z)\{4z-16+z-2\} - 2\{-4+4-2+z\} + 2\{-4+8-2z+2\}$$

$$-1\{2z-4-8-4z\} = 0$$

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$(4-z)\{5z-10\}+2-2z+2\{6-2z\}-1\{-2z-12\}=0$

$20z-7z-5z^2+18z+2-\cancel{2z}+12-4z+\cancel{2z}+12=0$

$34z-5z^2-46=0$

$5z^2-34z+46=0$

$$z=\frac{34\pm\sqrt{(34)^2-4\times46\times5}}{2\times5}$$

$$z=\frac{34\pm\sqrt{15.36}}{10}$$

$$\boxed{z=4.936,\ 1.864}$$

$$\boxed{z>0}$$

so we get the relative minima at point $\left(\frac{4}{3},\frac{2}{9},\frac{14}{9}\right)$

(2) $f = 9 - 8x_1 - 6x_2 - 4x_3 + 2x_1^2 + 2x_2^2 + x_3^2 + 2x_1x_2 + 2x_1x_3$ subject to $x_1 + x_2 + 2x_3 = 4$

Sol^n

$$L = f + \lambda g$$

$$L = f + \lambda(x_1 + x_2 + 2x_3 - 4)$$

$$\frac{\partial L}{\partial x_1} = -8 + 4x_1 + 2x_2 + 2x_3 + \lambda = 0$$

$$\frac{\partial L}{\partial x_2} = -6 + 4x_2 + 2x_1 + \lambda = 0$$

$$\frac{\partial L}{\partial x_3} = -4 + 2x_3 + 2x_1 + 2\lambda = 0$$

$$\frac{\partial L}{\partial \lambda} = x_1 + x_2 + 2x_3 - 4 = 0$$

$$\lambda + 4x_1 + 2x_2 + 2x_3 = 8 \quad \text{—— (1)}$$

$$\lambda + 2x_1 + 4x_2 = 6 \quad \text{—— (2)}$$

$$2\lambda + 2x_1 + 2x_3 = 4 \quad \text{—— (3)}$$

$$x_1 + x_2 + 2x_3 = 4 \quad \text{—— (4)}$$

Subtract (2) from (1)

$$\lambda + 4x_1 + 2x_2 + 2x_3 - \lambda - 2x_1 - 4x_2 = 8 - 6$$

$$2x_1 - 2x_2 + 2x_3 = 2 \quad \text{—— (4)}$$

multiply eq-(2) with 2 and subtract equation (3)

$$2\lambda + 4x_1 + 8x_2 - 2\lambda - 2x_1 - 2x_3 = 12 - 4$$

$$2x_1 + 8x_2 - 2x_3 = 8 \quad \text{—— (5)}$$

from eq-(4) $\quad x_1 = 1 - x_3 + x_2$ —— (6)

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equation (1) $\times 2$ − equation (3)

$$2\lambda + 8x_1 + 4u_2 + 4u_3 - 2\lambda - 2x_1 - 2u_3 = 16 - 4$$

$$6x_1 + 4u_2 + 2u_3 = 12 \quad \text{— (6)}$$

Substract equation (5) from equation (4)

$$2x_1 - 2u_2 + x_3 - 2u_1 - 8u_2 + 2u_3 = 2 - 8$$

$$-10u_2 + 4u_3 = -6$$

$$2u_3 - 5u_2 = -3 \quad \text{— (7)}$$

$$x_1 = 1 - u_3 + x_2 \rightarrow (8)$$

on solving 7 and 8 put in eq (2)

$$1 - x_3 + x_4 + x_2 + 2u_3 = 4$$

$$2u_2 + u_3 = 3 \quad \text{— (9)}$$

multiply eq (9) with 2 and subtract eq (7)

$$4u_2 + \cancel{2u_3} - 6 - \cancel{2u_3} + 5u_2 - 3 = 0$$

$$9u_2 = 9$$

$$\boxed{u_2 = 1}$$

$$\boxed{x_1 = 1}$$

$$\boxed{u_3 = 1}$$

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Put in eq (1) x_1, x_2, x_3.

$\lambda + 4 + 2 + 2 = 0$

$\boxed{\lambda = 0}$ extreme points are $(1,1,1)$

Sufficient condition is

$$\begin{vmatrix} 1-z & 2 & 2 & 1 \\ 2 & 4-z & 0 & 1 \\ 2 & 0 & 2-z & 2 \\ 1 & 1 & 2 & 0 \end{vmatrix} = 0$$

$$1-z \begin{vmatrix} 4-z & 0 & 1 \\ 0 & 2-z & 2 \\ 1 & 2 & 0 \end{vmatrix} - 2 \begin{vmatrix} 2 & 2 & 1 \\ 0 & 2-z & 2 \\ 1 & 2 & 0 \end{vmatrix} + 2 \begin{vmatrix} 2 & 2 & 1 \\ 4-z & 0 & 1 \\ 1 & 2 & 0 \end{vmatrix} - 1 \begin{vmatrix} 2 & 2 & 1 \\ 4-z & 0 & 1 \\ 0 & 2-z & 2 \end{vmatrix} = 0$$

$(1-z)\{(4-z)\times -4 + 1(2-z)\} - 2\{2\times -2 + 1(4-2+2)\} + 2\{2(-2) - \cancel{(4-z)}(-2) + 1(2)\} = 0$

$-1| 2(2-z) - 4(4-z+2)\} = 0$

$(1-z)\{4z - 16 + 2 - z\} - 2\{-4 + 4 - 2 + 2\} + 2\{-4 + 8 - 2z + 2\}$

$-1\{2z - 4 - 8 - 4z\} = 0$

$(1-z)\{5z - 18\} + 2 - 2z + 2\{6 - 2z\} - 1\{-2z - 12\} = 0$

$20z - 72 - 5z^2 + 18z + 2 - \cancel{2z} + 12 - 4z + \cancel{2z} + 12 = 0$

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$$34z - 5z^2 - 46 = 0$$

$$5z^2 - 34z + 46 = 0$$

$$z = \frac{34 \pm \sqrt{(34)^2 - 4 \times 46 \times 5}}{2 \times 5}$$

$$z = \frac{34 \pm \sqrt{236}}{10}$$

$$z = \frac{34 \pm 15.36}{10}$$

$$z = 4.936,\ 1.864$$

$$\boxed{z > 0}$$

So we find relative minima at point $(1, 1, 1)$.

(8) $f = 2x_1 - 3x_2 + 4x_3 + 4x_1^2 + 5x_2^2 - 6x_3^2 + 2x_1x_2 + 4x_2x_3 - 6x_1x_3$

$g = 2x_1 - 3x_2 + 4x_3 = 10.$

Sol^n $L = f + \lambda g$

$$\frac{\partial L}{\partial x_1} = 2 + 8x_1 + 2x_3 - 6x_2 + 2\lambda = 0 \quad \text{—(1)}$$

$$\frac{\partial L}{\partial x_2} = -3 + 10x_2 + 4x_3 - 6x_1 - 3\lambda = 0 \quad \text{—(2)}$$

$$\frac{\partial L}{\partial x_3} = 4 - 12x_3 + 2x_1 + 4x_2 + 4\lambda = 0 \quad \text{—(3)}$$

$$\frac{\partial L}{\partial \lambda} = 2x_1 - 3x_2 + 4x_3 - 10 = 0 \quad \text{—}(\alpha)$$

$$8x_1 - 6x_2 + 2x_3 + 2\lambda = -2 \quad \text{—(4)}$$

$$-6x_1 + 10x_2 + 4x_3 - 3\lambda = 3 \quad \text{—(5)}$$

$$2x_1 + 4x_2 - 12x_3 + 4\lambda = -4 \quad \text{—(6)}$$

$$2x_1 + 4x_2 - 12x_3 + 4\lambda = -4$$

$$16x_1 - 12x_2 + 4x_3 + 4\lambda = -4 \quad \text{(multiply eq(4) with (2)}$$

$$-14x_1 + 16x_2 - 16x_3 = 0$$

$$14x_1 - 16x_2 + 16x_3 = 0 \quad \text{—(7)}$$

eq (4) × 3

$$24x_1 - 12x_2 + 6x_3 + 6\lambda = -\cancel{6}$$
$$-12x_1 + 20x_2 + 8x_3 - 6\lambda = \cancel{6}$$ (multiply eq (5) × 2)

$$12x_1 + 8x_2 + 14x_3 = 0 \quad \text{--- (8)}$$

multiply eq 8 with (2)

$$24x_1 + \cancel{16x_2} + 28x_3 = 0$$
$$14x_1 - \cancel{16x_2} + 16x_3 = 0$$

$$38x_1 + 44x_3 = 0 \quad \text{--- (9)}$$

from eq (7)

$$14x_1 - 16x_2 + 16x_3 = 0$$

$$\boxed{x_2 = \frac{14x_1 + 16x_3}{16}} \quad \text{--- (10)}$$

Put eq (10) in eq (2)

$$2x_1 - 3\left\{\frac{14x_1 + 16x_3}{16}\right\} + 4x_3 = 10$$

$$2x_1 - \frac{21}{8}x_1 - 3x_3 + 4x_3 = 10$$

$$\left(\frac{16-21}{8}\right)x_1 + x_3 = 10$$

$$\boxed{x_3 = 10 + \frac{5x_1}{8}} \quad \text{--- (11)}$$

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Put eq (11) in eq (9)

$$38x_1 + 44(10 + \frac{5x_1}{8}) = 0$$

$$38x_1 + 440 + \frac{5\times 44}{8} x_1 = 0$$

$$38x_1 + \frac{220x_1}{8} = -440$$

$$65.5x_1 = -440$$

$$\boxed{x_1 = -6.7176}$$

$$x_3 = 10 + \frac{5\times -6.7176}{8}$$

$$\boxed{x_3 = 5.8016}$$

$$\boxed{x_2 = -0.0764}$$

$$\boxed{\lambda = 19.8396}$$

Sufficient condition is

$$\begin{vmatrix} 8-z & -6 & 2 & 2 \\ -6 & 10-z & 4 & -3 \\ 2 & 4 & -12-z & +4 \\ 2 & -3 & +4 & 0 \end{vmatrix} = 0$$

$$(8-z)\begin{vmatrix} 10-z & 4 & -3 \\ 4 & -12-z & 4 \\ -3 & 4 & 0 \end{vmatrix} + 6\begin{vmatrix} -6 & 2 & 2 \\ 4 & -12-z & 4 \\ -3 & 4 & 0 \end{vmatrix} + 2\begin{vmatrix} -6 & 2 & 2 \\ 10-z & 4 & -3 \\ -3 & +4 & 0 \end{vmatrix}$$

$$-2\begin{vmatrix} -6 & 2 & 2 \\ 10-z & 4 & -3 \\ 4 & -12-z & +4 \end{vmatrix} = 0$$

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$$(8-z)\{(10-z)(-16)-4(12)-3(16-(-3x-(12+z)))\}$$
$$+6\{-6(-16)-4(-8)-3(8+2(12+z))\}$$
$$+2\{-6(12)-(10-z)(-8)-3(-6-8)\}$$
$$-2\{-6(16-(-3x-(12+z)))-(10-z)\{8+2(12+z)\}+4(-6-8)\}$$

$$\Rightarrow (8-z)\{-160+16z-48-3(16-36-3z)\}$$
$$+6\{96+32-3(8+24+2z)\}$$
$$+2\{-72+80-8z+42\}$$
$$-2\{-6(16-36-3z)-(10-z)(32+2z)-56\}$$

$$\Rightarrow (8-z)\{-160+16z-48-48+108+9z\}$$
$$+6\{96+32-24-72-6z\}$$
$$+2\{50-8z\}-2\{+6(-20+3z)-(10-z)(32+2z)$$
$$-56\}$$

$$=(8-z)\{-148+25z\}+6\{32-6z\}+(100-16z)$$
$$-2\{-120+18z-320-20z+32z+2z^2-56\}$$

$$=-1184+200z+148z-25z^2+192-36z+100-16z-2\{-498+30z+2z^2\}$$
$$=100+236z-29z^2=0$$

$$29z^2 - 236z - 100 = 0$$

$$z = \frac{236 \pm \sqrt{(236)^2 - 4 \times 29 \times -100}}{58}$$

$$z = \frac{236 \pm 259.4}{58}$$

$$\boxed{z = 8.541, \; -0.4034}$$

- $z > 0$ and $z < 0$

So it is neither +ve definite nor negative definite so at point $(-6.7176, -0.0764, 5.0046)$ or at any other point there is no maxima or minima.

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(2) Minimize $f(x) = \frac{x_1^2+x_2^2+x_3^2}{2}$ subject to the conditions $x_1 - x_2 = 0$ and $x_1 + x_2 + x_3 - 1 = 0$.

Solⁿ

$L = f + \lambda_1 g_1 + \lambda_2 g_2$

$\frac{\partial L}{\partial x_1} = x_1 + 1(\lambda_1) + 1(\lambda_2) = 0$ — (1)

$\frac{\partial L}{\partial x_2} = x_2 + (-1)\lambda_1 + \lambda_2 = 0$ — (2)

$\frac{\partial L}{\partial x_3} = x_3 + 0(\lambda_1) + \lambda_2 = 0$ — (3)

$\frac{\partial L}{\partial \lambda_1} = x_1 - x_2 = 0 \Rightarrow \boxed{x_1 = x_2}$ — (4)

$\frac{\partial L}{\partial \lambda_2} = x_1 + x_2 + x_3 - 1 = 0$ — ~~(5)~~

$\Rightarrow x_1 + x_2 + x_3 = 1$ — (5)

Put eq (4) in eq (5)

$2x_2 + x_3 = 1$

from eq (3)

$\boxed{x_3 = -\lambda_2}$

$\boxed{x_2 = \frac{1+\lambda_2}{2}}$

$\boxed{x_1 = \frac{1+\lambda_2}{2}}$

Put x_1 in eq (1)

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$$\frac{1+\lambda_2}{2} + \lambda_1 + \lambda_2 = 0 \quad \text{--- (6)}$$

Put x_2 in eq (2).

$$\frac{1+\lambda_2}{2} - \lambda_1 + \lambda_2 = 0 \quad \text{--- (7)}$$

Adding 6 and 7.

$$1+\lambda_2 + 2\lambda_2 = 0$$

$$\boxed{\lambda_2 = -\tfrac{1}{3}}$$

$$\boxed{\lambda_1 = 0}$$

$$\boxed{x_1 = x_2 = \frac{\cancel{2}}{3\times\cancel{2}} = \tfrac{1}{3}}$$

$$\boxed{x_3 = \tfrac{1}{3}}$$

Sufficient condition is

$$\begin{vmatrix} 1-z & 0 & 0 & 1 & 1 \\ 0 & 1-z & 0 & -1 & 1 \\ 0 & 0 & 1-z & 0 & 1 \\ 1 & -1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \end{vmatrix} = 0$$

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$$(1-z)\begin{vmatrix} 1-z & 0 & -1 & 1 \\ 0 & 1-z & 0 & 1 \\ -1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{vmatrix} - \begin{vmatrix} 0 & 0 & 1 & 1 \\ 1-z & 0 & -1 & 1 \\ 0 & 1-z & 0 & 1 \\ 1 & 1 & 0 & 0 \end{vmatrix} + \begin{vmatrix} 0 & 0 & 1 & 1 \\ 1-z & 0 & -1 & 1 \\ 0 & 1-z & 0 & 1 \\ -1 & 0 & 0 & 0 \end{vmatrix} = 0$$

A B C

Simplify (A)

$$= (1-z)\begin{vmatrix} 1-z & 0 & -1 & 1 \\ 0 & 1-z & 0 & 1 \\ -1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{vmatrix}$$

$$= (1-z)\left\{ (1-z)\begin{vmatrix} 1-z & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{vmatrix} - \begin{vmatrix} 0 & -1 & 1 \\ 1-z & 0 & 1 \\ 1 & 0 & 0 \end{vmatrix} + \begin{vmatrix} 0 & -1 & 1 \\ 1-z & 0 & 1 \\ 0 & 0 & 0 \end{vmatrix} \right\}$$

$$= (1-z)\left\{ (1-z)\times 0 - \begin{vmatrix} 0 & -1 & 1 \\ 1-z & 0 & 1 \\ 1 & 0 & 0 \end{vmatrix} + 0 \right\}$$

$$= -(1-z)\left\{ \begin{vmatrix} 0 & -1 & 1 \\ 1-z & 0 & 1 \\ 1 & 0 & 0 \end{vmatrix} \right\}$$

$$= (z-1)\left\{ 0 - (1-z)\times 0 + 1\times -1 \right\}$$

$$= 1-z$$

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Solving (B)

$$B = \begin{vmatrix} 0 & 0 & 1 & 1 \\ 1-z & 0 & -1 & 1 \\ 0 & 1-z & 0 & 1 \\ 1 & 1 & 1+z & \end{vmatrix}$$

$$B = (z-1)\begin{vmatrix} 0 & 1 & 1 \\ 1-z & 0 & 1 \\ 1 & 0 & 0 \end{vmatrix} - \begin{vmatrix} 0 & 1 & 1 \\ 0 & -1 & 1 \\ 1-z & 0 & 1 \end{vmatrix}$$

$$B = (z-1)\{0-0+1\} - \{(1-z)(1+1)\}$$

$$B = (z-1) - 2+2z$$

$$B = z-1-2+2z$$

$$\boxed{B = 3z-3}$$

Solving (C)

$$C = \begin{vmatrix} 0 & 0 & 1 & 1 \\ 1-z & 0 & -1 & 1 \\ 0 & 1-z & 0 & 1 \\ -1 & 0 & 0 & 0 \end{vmatrix}$$

$$C = (z-1)\begin{vmatrix} 0 & 1 & 1 \\ 1-z & 0 & 1 \\ 0 & 0 & 0 \end{vmatrix} + \begin{vmatrix} 0 & 1 & 1 \\ 0 & -1 & 1 \\ 1-z & 0 & 1 \end{vmatrix}$$

$$C = z-1 \times 0 + (1-z)(2) = 2-2z$$

$$1-z-3z+3+2-2z=0$$

$$6-6z=0$$

$$\boxed{z=1} > 0$$

so it is positive definite at the point $\left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$.

$$f_{min} = \frac{1}{2}\left(\frac{1}{9}+\frac{1}{9}+\frac{1}{9}\right)$$

$$= \frac{1}{2}\left(\frac{\cancel{3}}{\cancel{9}3}\right) = \frac{1}{6} \text{ Ans}$$

[3]

Find ... function ... point

Soln

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(3) $f = 5x_1^3 + 2x_2^3 + 3x_3^3 + 4x_1^2 + 8x_2^2 + 12x_3^2 - 15$.

find all the extreme points of f(x). show that the function is maximum or minimum for any two extreme point that at least one variable is non-zero.

3).

Sol^n for extreme point

$$\frac{\partial f}{\partial x_1} = 15x_1^2 + 8x_1 = 0 \qquad x_1 = 0,\ x_1 = -\frac{8}{15}$$

$$\frac{\partial f}{\partial x_2} = 6x_2^2 + 16x_2 = 0 \qquad x_2 = 0,\ x_2 = -\frac{8}{3}.$$

$$\frac{\partial f}{\partial x_3} = 9x_3^2 + 24x_3 = 0 \qquad x_3 = 0,\ x_3 = -\frac{8}{3}.$$

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