Optimization Techniques

Paper Code – BMS-09

Lecture – 06(Unit -1)

Topic-Multiple Variables Optimization – Kuhn Tucker Condition

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Unit-01

Classical Optimization Techniques: Single variable optimization, Multi-variable with no constraints. Non-linear programming: One Dimensional Minimization methods. Elimination methods: Fibonacci method, Golden Section method

Unit-02

Unit-02

Linear Programming: Constrained Optimization Techniques: Simplex method, Solution of System of Linear Simultaneous equations, Revised Simplex method, Transportation problems, Karmarkar's method, Duality Theorems, Dual Simplex method, Decomposition principle.

Golden Section Method

The G.S.M. is same as the Fibonacci method except that in the Fibonacci method the total number of experiments to be specified before beginning the calculation, where as this is not required in the Golden Section Method.

So, we start with the assumption that are going to conduct a large no. of experiments. Of course, the total number of experiments can be divided during the computation.

the intervals of uncertainity remaining at the end of different number of experiments can be conducted as follows:—

$$L_2 = \lim_{N \to \infty} \frac{F_{N-1}}{F_N} L_0 \quad — (I)$$

$$L_3 = \lim_{N \to \infty} \frac{F_{N-2}}{F_N} L_0 = \lim_{N \to \infty} \frac{F_{N-2}}{F_{N-1}} \times \frac{F_{N-1}}{F_N} \times L_0$$

$$= \lim_{N \to \infty} \left(\frac{F_{N-1}}{F_N} \right)^2 L_0$$

as $N \to \infty$

$$\frac{F_{N-1}}{F_N} \simeq \frac{F_{N-2}}{F_{N-1}} \quad — (II)$$

similarly,

$$L_4 = \lim_{N \to \infty} \left(\frac{F_{N-1}}{F_N} \right)^3 L_0 .$$

So, this result can be generalized as

$$L_K = \lim_{N \to \infty} \left(\frac{F_{N+1}}{F_N} \right)^{K-1} L_0 \quad \text{— (III)}$$

and using the relation, $F_N = F_{N-1} + F_{N-2}$

$$\Rightarrow \frac{F_N}{F_{N-1}} = 1 + \frac{F_{N-2}}{F_N} \quad \text{— (IV)}$$

if we defind $\gamma = \lim_{N \to \infty} \frac{F_N}{F_{N-1}}$

so, (IV) can be written as $N \to \infty$

$$\gamma \cong 1 + \frac{1}{\gamma}$$

that is

$$r^2 - r - 1 = 0$$

this gives $r = 1.618$

so, $L_K = \left(\frac{1}{r}\right)^{K-1} L_0 = \left(\frac{1}{1.618}\right)^{K-1} L_0$

$L_K = (0.618)^{K-1} L_0$, this is the stopping criteria.

Q. Minimize the function

$$f(x) = 0.65 - \left[\frac{0.75}{1+x^2}\right] - 0.65\,x\,\tan^{-1}(1/x)$$

in the interval $[0, 3]$, by Golden Section Method

by taking $n = 6$.

Ans here $n = 6$,

$$x_1 = a + L_2^*$$

$$x_2 = b - L_2^*$$

$$L_2^* = \frac{F_{N-2}}{F_N} L_0$$

$$= \frac{F_{N-2}}{F_{N-1}} \times \frac{F_{N-1}}{F_N} L_0$$

$$= \frac{L_0}{r^2} = \frac{L_0}{(1.618)^2}$$

$$\Rightarrow L_2^* = 0.382\, L_0$$

$$L_0 = 3 - 0 = 3$$

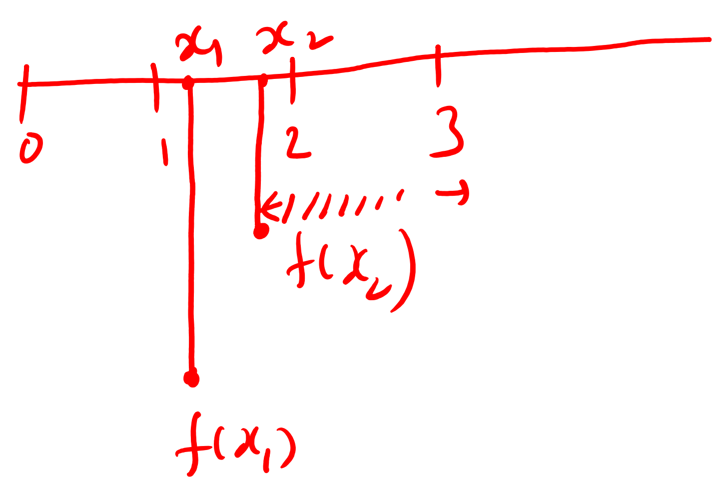
$$x_1 = a + L_2^* = 0 + 0.382 \times 3$$

$$= 1.1460$$

$$x_2 = b - L_2^* = 3 - 0.382 \times 3 = 1.8540$$

$$f(x_1) = -0.208654$$

$$f(x_2) = -0.115124$$



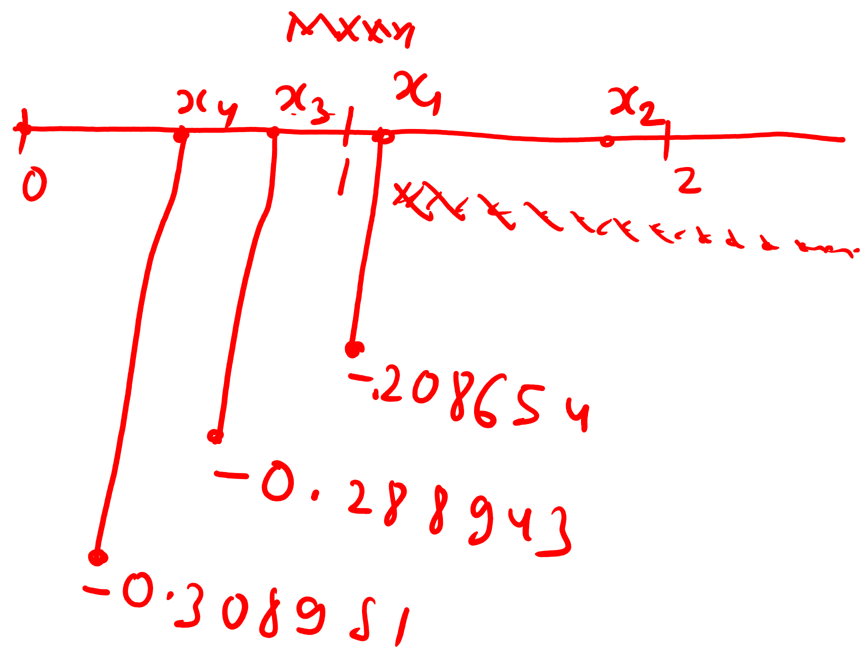
So, we can see that

$f(x_1) < f(x_2)$, So, we ignore the interval $[x_2, 3]$ and new interval is $[0, x_2]$

$x_3 = a + (x_2 - x_1) = 0 + 1.8540 - 1.1460 = 0.7080$

$f(x_3) = -0.288943$

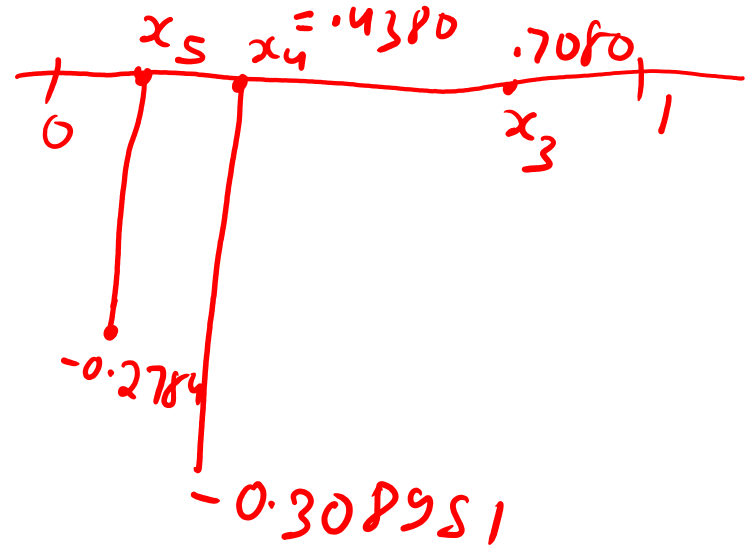
So, we can say here



$f(x_3) < f(x_1)$, so, delete the interval $[x_1, x_2]$ and new interval is $[0, x_1]$

$x_4 = a + x_1 - x_3 = 0 + 1.1460 - 0.7080 = 0.4380$

$f(x_4) = -0.308951$, so, we can say here $f(x_4) < f(x_3)$, so delete the proportion $[x_3, x_1]$ and new interval is $[0, x_3]$.



$$x_5 = 0 + x_3 - x_4$$

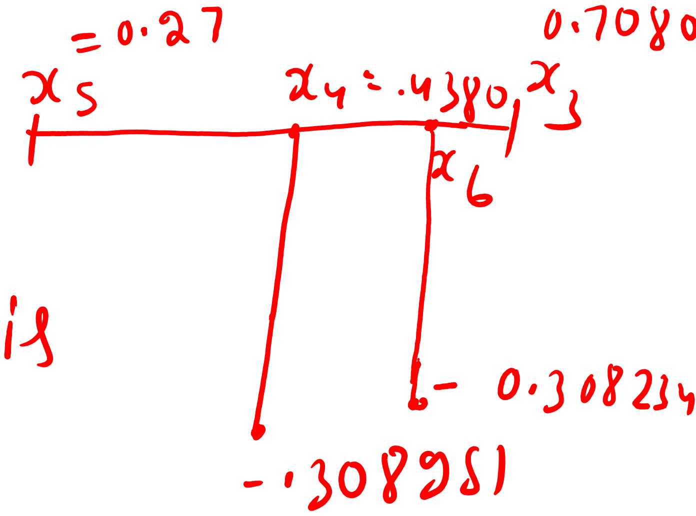
$$= 0.2700$$

$$f(x_5) = -0.278434$$

So, we can see here, $f(x_5) > f(x_4)$, so, delete the portion $[0, x_5]$ and new interval is $[x_5, x_3]$

$$x_6 = x_5 + x_3 - x_4 = 0.5400$$

$f(x_6) = -0.308234$



as $f(x_6) > f(x_4)$, ignore the portion $[x_6, x_3)$ and new interval is

$[x_5, x_6] = [0.2700, .5400]$

Ans $x_6 = -0.308234$.

stoping critera $L_k = (0.618)^{k-1} L_0$ (theory [illegible])

$k=6$, $\dfrac{L_6}{L_0} = (0.618)^5 = 0.09145 \rightarrow (1)$

by Experiment

$$\frac{L_b}{L_0} = \frac{(1.54 - .27)}{3} = \frac{0.27}{3} = 0.09 \quad \text{— (ii)}$$

(i) and (ii) approximate equal.