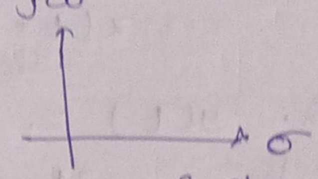
Laplace transform:-

↳ to convert time domain eqⁿ to frequency domain

$$x(t) \underset{}{\overset{L.T}{\rightleftharpoons}} x(s)$$

$$s = \sigma + j\omega$$



S-plane.

$$\boxed{X(s) = \int_{-\infty}^{\infty} x(t)\, e^{-st}\, dt}$$

where,
$\sigma \rightarrow$ damping factor
$\omega \rightarrow$ frequency

Advantages:-

a) Any differential or integral eqⁿ in time domain is transformed into a linear eqⁿ if solved in s-domain and hence calculation becomes easier.

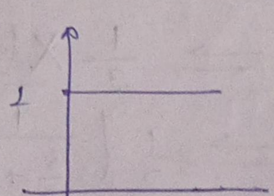
Q. Find Laplace transform of

$$x(t) = e^{-at} u(t)$$

Sol:-

$$X(s) = \int_{-\infty}^{\infty} x(t)\, e^{-st}\, dt$$

$$= \int_{-\infty}^{\infty} e^{-at} \cdot u(t) \cdot e^{-st}\, dt$$



$$u(t) = \begin{cases} 0 & t < 0 \\ 1 & t \geq 0 \end{cases}$$

So

$$X(s) = \int_{0}^{\infty} e^{-(s+a)t}\, dt$$

$$= \frac{\left[e^{-(s+a)t}\right]_0^{\infty}}{-(s+a)} = \frac{-1}{(s+a)}\left[\frac{1}{e^{\infty}} - \frac{1}{e^{0}}\right]$$

$$\boxed{X(s) = \frac{1}{s+a}}$$

Properties of Laplace transform:-

1) Time shifting

$$x(t-t_0) \xrightleftharpoons{L.T.} X(s)\, e^{-st_0}$$

$$x(t+t_0) \xrightleftharpoons{L.T.} X(s)\, e^{st_0}$$

Q. $y(t) = e^{-3t} u(t-4)$

sol:-

$$e^{-3t} u(t) = \frac{1}{s+3}$$

$$\cancel{e^{-3t}}\, u(\cancel{t}-4) \rightleftharpoons \cancel{\left(\frac{1}{s+3}\right)} \left(\cancel{e}\right)$$

$$e^{-3(t-4)} u(t-4) \rightleftharpoons \left(\frac{1}{s+3}\right)\left(e^{-4s}\right)$$

$$e^{-3t} \cdot u(t-4) \rightleftharpoons \frac{e^{-4s} \cdot e^{-12}}{s+3}$$

$$y(t) \rightleftharpoons \frac{e^{-4(s+3)}}{s+3}$$

(2) Time scaling

$$x(at),\ a \neq 0 \rightleftharpoons \frac{1}{|a|} X\left(\frac{s}{a}\right)$$

Q. If $x(t) = e^{-3t} u(t)$ find L-T of $x(2t)$

Sol:- $x(t) = e^{-3t} \cdot u(t) \rightleftharpoons \frac{1}{s+3}$

$$x(2t) = e^{-3(2t)} \cdot u(2t) \rightleftharpoons \frac{1}{2} X\left(\frac{s}{2}\right)$$

$$\rightleftharpoons \frac{1}{2}\left(\frac{1}{\frac{s}{2}+3}\right)$$

$$\rightleftharpoons \frac{1}{\cancel{2}}\left(\frac{\cancel{2}}{s+6}\right)$$

$$e^{-6t} \cdot u(2t) \rightleftharpoons \frac{1}{s+6}$$

$u(2t) = u(t)$ as it always have value 1.

$$e^{-6t} u(t) \rightleftharpoons \frac{1}{s+6}$$

3) Frequency shifting :-

$$e^{-at} x(t) \rightleftharpoons X(s+a)$$

Q. If $x(t) = e^{-2t} u(t)$ find L.T of $e^{-4t} x(t)$

sol:-

$$e^{-2t} u(t) \rightleftharpoons \frac{1}{s+2} = X(s)$$

$$e^{-4t} \cdot x(t) \rightleftharpoons \frac{1}{(s+4)+2} = \frac{1}{s+6}$$

4) Differentiation in frequency.

$$t^n x(t) \rightleftharpoons (-1)^n \frac{d^n X(s)}{ds^n}$$

Q. $y(t) = t e^{-at} u(t)$ find its L.T.

sol.

let $x(t) = e^{-at} \cdot u(t) \rightleftharpoons X(s) = \frac{1}{s+a}$

$$t \cdot e^{-at} \cdot u(t) \rightleftharpoons (-1)^1 \frac{d}{ds}\left(\frac{1}{s+a}\right)$$

$$\rightleftharpoons -\left[-\frac{1}{(s+a)^2}\right]$$

$$\rightleftharpoons \frac{1}{(s+a)^2}$$

5) Integration in frequency.

$$\frac{x(t)}{t} \rightleftharpoons \int_s^\infty X(s)\,ds$$

Q. $f(t) = \frac{(1-e^t)}{t} \cdot u(t)$ find its L.T.

sol:- let $x(t) = (1-e^t) u(t)$

$$X(s) = \frac{1}{s} - \frac{1}{s+1}$$

$$f(t) = \frac{x(t)}{t} \rightleftharpoons F(s) = \int_s^\infty X(s)\,ds$$

$$F(s) = \int_s^\infty \left(\frac{1}{s} - \frac{1}{s-1}\right) ds$$

$$= \left[\log s - \log(s-1)\right]_s^\infty$$

$$F(s) = \left[\log\left(\frac{s}{s-1}\right)\right]_s^\infty$$

$$= \log\left[\lim_{s\to\infty}\left(\frac{s}{s-1}\right)\right] - \log\left(\frac{s}{s-1}\right)$$

$$= \log\left[\lim_{s\to\infty} \frac{1}{1}\right] - \log\left[\frac{s}{s-1}\right]$$

$$= \log(1) - \log\left(\frac{s}{s-1}\right)$$

$$= -\log\left(\frac{s}{s-1}\right)$$

$$= \log\left(\frac{s-1}{s}\right)$$

6) Differentiation in time.

$$\frac{d^n x(t)}{dt^n} \rightleftharpoons s^n X(s) - s^{n-1} x(0^-) - s^{n-2} x'(0^-) - s^{n-3} x''(0^-) - - -$$

Q. where, $x(0^-) = x(t)\big|_{t=0^-}$

$$x'(0^-) = \frac{d}{dt}x(t)\Big|_{t=0^-}$$

$$x''(0^-) = \frac{d^2}{dt^2}x(t)\Big|_{t=0^-}$$

Q. $\frac{d^2x(t)}{dt^2} + 4\frac{d\,x(t)}{dt} - 3x(t) = 0$

where $x(0^-) = -3$, $x'(0^-) = 0$

find its L.T.

sol:

L.T:- $s^2 X(s) - s\,x(0^-) - s^0 x'(0^-) + 4\{sX(s) - s^0 x(0^-)\}$

$- 3X(s) = 0$

$(s^2 X(s) + 3s - 0) + 4sX(s) + 12 - 3X(s) = 0$

$$X(s)\left[s^2+4s-3\right] = -(3s+12)$$

$$X(s) = \frac{-(3s+12)}{s^2+4s-3}$$

Some important Laplace transform pairs:-

	$x(t)$		$X(s)$
1	$\delta(t)$	$\longrightarrow$	1
2	$u(t)$ or $u(-t)$	$\longrightarrow$	$1/s$ or $-1/s$
3	$e^{-at}u(t)$	$\longrightarrow$	$1/s+a$
4)	$\sin\omega_0 t \cdot u(t)$	$\longrightarrow$	$\frac{\omega_0}{s^2+\omega_0^2}$
5)	$\cos\omega_0 t \cdot u(t)$	$\longrightarrow$	$\frac{s}{s^2+\omega_0^2}$
6)	$t^n u(t)$	$\longrightarrow$	$\frac{n!}{s^{n+1}}$
7)	$r(t) = t\,u(t)$	$\longrightarrow$	$\frac{1}{s^2}$
8)	$\sinh\omega_0 t \cdot u(t)$	$\longrightarrow$	$\frac{\omega_0}{s^2-\omega_0^2}$
9)	$\cosh\omega_0 t \cdot u(t)$	$\longrightarrow$	$\frac{s}{s^2-\omega_0^2}$

Inverse Laplace transform:-

$$x(t) \xrightarrow{L.T} X(s)$$

$$X(s) \xrightarrow{I.L.T} x(t)$$

$$X(s) = \int_{-\infty}^{\infty} x(t)\, e^{-st}\, dt$$

$$x(t) = \frac{1}{2\pi j}\int_{\sigma-j\omega}^{\sigma+j\omega} X(s)\, e^{st}\, ds$$

Q. find I.L.T. for $X(s) = \frac{}{(s+1)(s+2)^2}$

Sol:- $\frac{s+3}{(s+1)(s+2)^2} = \frac{A}{s+1} + \frac{B}{s+2} + \frac{C}{(s+2)^2}$

If $s+1=0 \Rightarrow s=-1 \Rightarrow A = \frac{-1+3}{(-1+2)^2} = 2$

If $(s+2)^2=0 \Rightarrow s=-2 \Rightarrow C = \frac{-2+3}{-2+1} = -1$

Let $s=0$

$\frac{3}{(1)(2)^2} = \frac{A}{1} + \frac{B}{2} + \frac{C}{(2)^2}$

$\frac{3}{4} = \frac{2}{1} + \frac{B}{2} + \frac{1}{4} \Rightarrow B = \left(\frac{3}{4} + \frac{1}{4} - 2\right)2$

$= \cancel{\left(\frac{6+1-8}{4}\right)\times 2} = -\cancel{2}$

$= -2$

$X(s) = \frac{s+3}{(s+1)(s+2)^2} = \frac{2}{s+1} + \frac{2}{(s+2)} + \frac{(-1)}{(s+2)^2}$

$x(t) = 2\left[e^{-t}u(t)\right] - 2\left[e^{-2t}.u(t)\right] - t\, e^{-2t}u(t)$

Q. Consider following d.e of a signal

$x(t) \Rightarrow \frac{d^2x(t)}{dt^2} - 3\frac{dx(t)}{dt} + 2x(t) = 0$

with $x(0^-) = -1$ and $x'(0^-) = 0$ find $x(t)$.

Sol:-

L.T. $\Rightarrow s^2X(s) - s\,x(0^-) - s^0x'(0^-) - 3\{sX(s) - s^0x(0^-)\}$

$+ 2X(s) = 0$

$s^2X(s) - s(-1) - 0 - 3\{sX(s) + 1\} + 2X(s) = 0$

$s^2X(s) + s - 3sX(s) - 3 + 2X(s) = 0$

$X(s)\{s^2 - 3s + 2\} = -(s-3)$

$$X(s) = \frac{-(s-3)}{s^2-3s+2} = \frac{-s+3}{(s-1)(s-2)}$$

$$\frac{-s+3}{(s-1)(s-2)} = \frac{A}{(s-1)} + \frac{B}{(s-2)}$$

Let $s-1=0$, then $A = \frac{-1+3}{1-2} = -2$

$s=1$

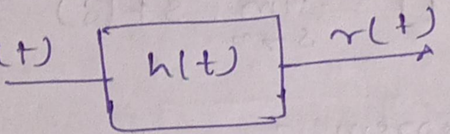
$s-2=0$ then $B = \frac{-2+3}{2-1} = 1$

$s=2$

$$X(s) = \frac{-2}{s-1} + \frac{1}{s-2}$$

I.L.T. $x(t) = -2(e^{t}\cdot u(t)) + e^{2t}u(t)$

⑦ Convolution property :-



$e(t) \otimes h(t)$

$f_1(t) \longrightarrow F_1(s)$

$f_2(t) \longrightarrow F_2(s)$

$f_1(t) \otimes f_2(t) \longrightarrow F_1(s)\cdot F_2(s)$

Short cut for Region of convergence :-

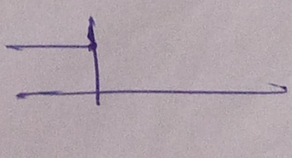
Step-1 :- compare σ with the real part of co-efficient of t in power of e

Step-2 :- check if the signal is left sided or right sided and decide < or >.

For. ex: $e^{-at}u(-t)$ is left sided

since,

$u(-t) =$



Illustrations :-

① $f(t) = e^{(5+3j)t} \underbrace{u(-t-5)}_{\text{left sided}}$

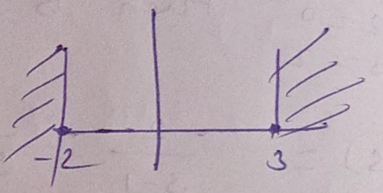
R.O.C :- $\sigma < 5$

② $f(t) = \underset{\text{I}}{e^{3t} u(t)} + \underset{\text{II}}{e^{-2t} u(-t)}$, find its ROC.

sol :-

ROC I $\rightarrow$ $\sigma > 3$

ROC II $\rightarrow$ $\sigma < -2$



No common ROC

so, L.T does not exist.

Initial value theorem :-

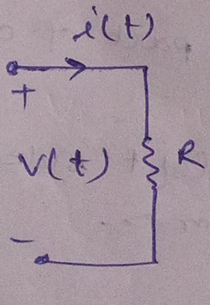
$$\lim_{t \to 0} f(t) = \lim_{s \to \infty} s F(s)$$

Final value theorem :-

$$\lim_{t \to \infty} f(t) = \lim_{s \to 0} s F(s)$$

Representation of circuit elements in s-domain :-

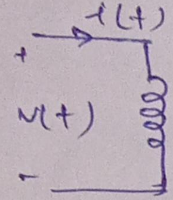
① Resistor



$v(t) = i(t) R$

$V(s) = i(s) R$

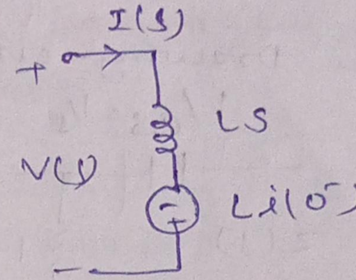
2) Inductor :-



$$v(t) = L\frac{di(t)}{dt}$$

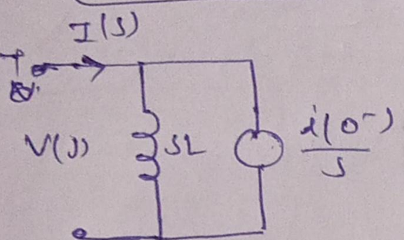
$$V(s) = L[sI(s) - s^0 i(0^+)]$$

$$= LsI(s) - L\,i(0^-)$$

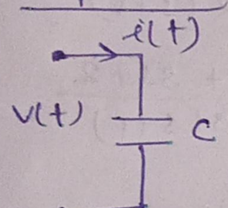


$$I(s) = \frac{V(s)}{sL} + \frac{\cancel{L}\,i(0^-)}{s\cancel{L}}$$

$$\boxed{I(s) = \frac{V(s)}{sL} + \frac{i(0^-)}{s}}$$



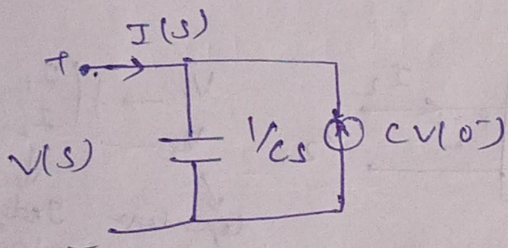
3) Capacitor :-



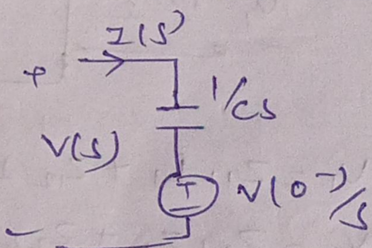
$$i(t) = C\frac{dv}{dt}$$

$$I(s) = C[sV(s) - s^0 v(0^-)]$$

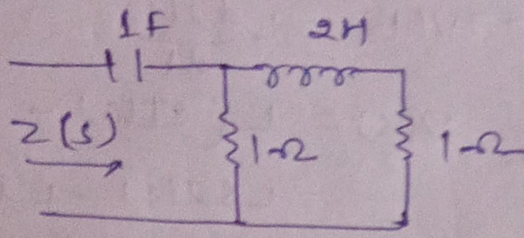
$$= \frac{V(s)}{1/sC} - CV(0^-)$$



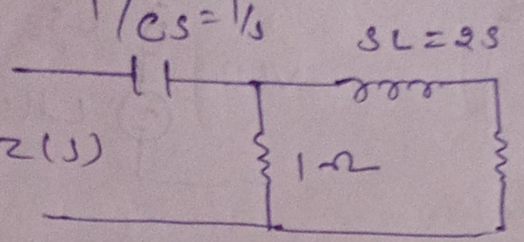
$$V(s) = \frac{I(s)}{Cs} + \frac{V(0^-)}{s}$$



Q. In the following circuit find $Z(s)$



Sol:- Drawing circuit in s-domain



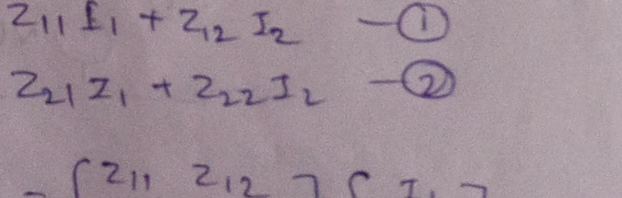
$$Z(s) = \frac{1}{s} + \{1 \parallel (2s+1)\}$$

$$= \frac{1}{s} + \frac{1\times(2s+1)}{2s+1+1} = \frac{1}{s} + \frac{2s+1}{2(s+1)}$$

$$= \frac{2s+2+s(2s+1)}{2s(s+1)} = \frac{2s^2+3s+2}{2s(s+1)}$$

Two Port Network :-

1) Z-Parameter



$V_1, V_2 \rightarrow f(I_1, I_2)$

dependent — independent variable

$$V_1 = Z_{11} I_1 + Z_{12} I_2 \quad (1)$$
$$V_2 = Z_{21} I_1 + Z_{22} I_2 \quad (2)$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

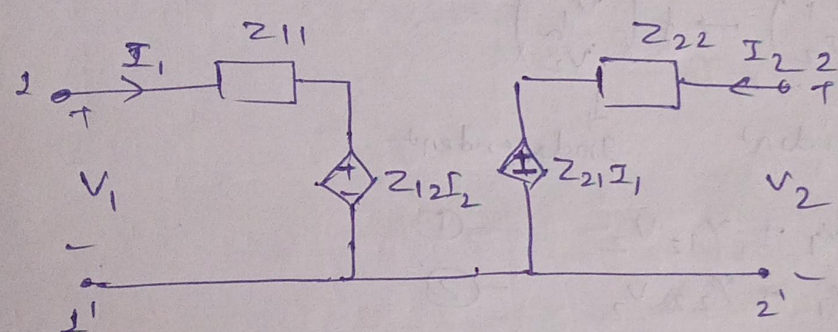
$$Z_{11} = \frac{V_1}{I_1}\Big|_{I_2=0}$$ open ckt. driving point input impedance

$$Z_{12} = \frac{V_1}{I_2}\Big|_{I_1=0}$$ Open ckt. reverse transfer impedance

$$Z_{21} = \frac{V_2}{V_1}\Big|_{I_2=0}$$ Open ckt. forward transfer impedance.

$$Z_{22} = \frac{V_{22}}{I_2}\Big|_{I_1=0}$$ open ckt. driving pt. o/p impedance.

Electrical Equivalent circuit of Z-Parameter :-



Q. A two port is described by $V_1 = I_1 + 2V_2$
$I_2 = -2I_1 + 0.4V_2$. find impedance parameters.

Sol:-

$$V_1 = I_1 + 2V_2 \quad -(1)$$

$$I_2 = -2I_1 + 0.4V_2 \quad -(2)$$

$$V_2 = \frac{2I_1}{0.4} + \frac{1}{0.4}I_2 \quad -(2)$$

$$= 5I_1 + 2.5I_2 \quad -(2)$$

put it in eqn (1)

$$V_1 = I_1 + 2(5I_1 + 2.5I_2)$$

$$= I_1 + 10I_1 + 5I_2$$

$$V_1 = 11I_1 + 5I_2 \quad -(3)$$

NOW,

$V_1 = 11I_1 + 5I_2$

$V_2 = 5I_1 + 2.5I_2$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 11 & 5 \\ 5 & 2.5 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$[Z] = \begin{bmatrix} 11 & 5 \\ 5 & 2.5 \end{bmatrix}$$

2.) Y- Parameter or Admittance Parameter

$$\underbrace{\begin{pmatrix} I_1 \\ I_2 \end{pmatrix}}_{\text{dependent}} \rightarrow f \underbrace{\begin{pmatrix} V_1 \\ V_2 \end{pmatrix}}_{\text{independent}}$$

$I_1 = Y_{11}V_1 + Y_{12}V_2$ — ①

$I_2 = Y_{21}V_1 + Y_{22}V_2$ — ②

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$[Y] = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}$$

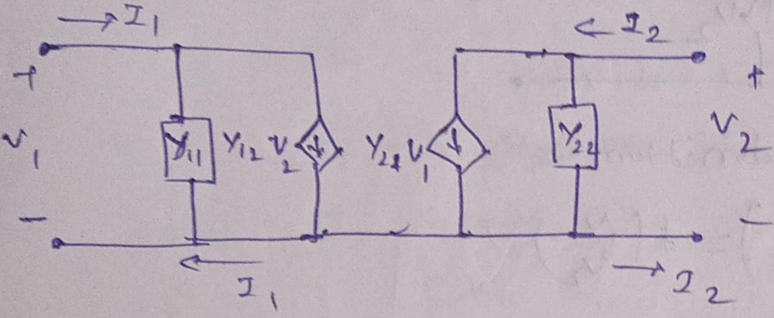
$Y_{11} = \frac{I_1}{V_1} \Big|_{V_2 = 0}$ Short ckt driving point input admittance ~~impedance~~

$Y_{12} = \frac{I_2}{V_1} \Big|_{V_2 = 0}$ short ckt ~~forward~~ transfer admittance

$Y_{21} = \frac{I_1}{V_2} \Big|_{V_1 = 0}$ → short ckt reverse transfer admittance

$$Y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0}$$ short ckt. driving point o/p admittance

Electrical Equivalent Circuit of Y-parameter.



Relation b/w Z and Y parameter :-

from Y. parameter.

$$[I] = [Y][V] \quad -①$$

from Z - parameter

$$[V] = [Z][I]$$

$$[I] = [Z^{-1}][V] \quad -②$$

from eq^n ① and ②

$$[Y] = [Z^{-1}] \text{ or } [Z] = [Y^{-1}]$$

$$[Z^{-1}] = \frac{\text{Adj } Z}{|Z|}$$

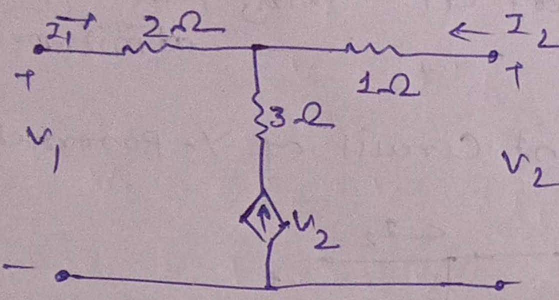
$$[Z] = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}$$

$$\text{adj } Z = \begin{bmatrix} Z_{22} & -Z_{12} \\ -Z_{21} & Z_{11} \end{bmatrix}$$

$$Y_{11} = \frac{Z_{22}}{|Z|} \neq \frac{1}{Z_{11}} \qquad Y_{12} = \frac{-Z_{12}}{|Z|}$$

$$Y_{21} = \frac{-Z_{21}}{|Z|} \qquad Y_{22} = \frac{Z_{11}}{|Z|}$$

Q.2 For the ckt. below find admittance matrix



Sol:- For admittance,

$$\begin{pmatrix} I_1 \\ I_2 \end{pmatrix} = f\begin{pmatrix} V_1 \\ V_2 \end{pmatrix}$$

$$V_2 = -(I_1 + I_2) \quad — ①$$

Applying KVL in outer loop.

$$V_1 - 2I_1 + I_2 - V_2 = 0$$

$$V_1 = 2I_1 - I_2 + V_2$$

$$= 2I_1 - I_2 - I_1 - I_2$$

$$= I_1 - 2I_2 \quad — ②$$

$$V_1 = I_1 - 2I_2$$

$$V_2 = -I_1 - I_2$$

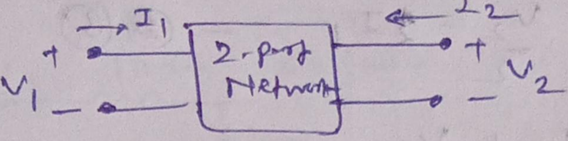
$$[Z] = \begin{bmatrix} 1 & -2 \\ -1 & -1 \end{bmatrix} \qquad |Z| = -1 - 2 = -3$$

$$[Y] = [Z^{-1}] = \frac{\text{adj } Z}{|Z|}$$

$$= -\frac{1}{3}\begin{bmatrix} -1 & 2 \\ 1 & 1 \end{bmatrix}$$

$$[Y] = \begin{bmatrix} 1/3 & -2/3 \\ -1/3 & -1/3 \end{bmatrix}$$

3) h - parameter :-



$$\begin{pmatrix} V_1 \\ I_2 \end{pmatrix} = f\begin{pmatrix} I_1 \\ V_2 \end{pmatrix}$$

$$V_1 = h_{11} I_1 + h_{12} V_2$$

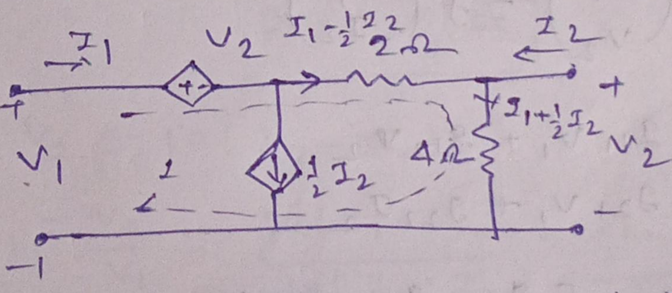
$$I_2 = h_{21} I_1 + h_{22} V_2$$

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

$$\downarrow$$

$$[H] = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \rightarrow \text{h. parameter matrix.}$$

Q. For the circuit show find the hybrid parameter matrix ([h]).



Sol:- using supermesh concept applying KVL

$$V_1 - V_2 - 2(I_1 - \tfrac{1}{2} I_2) - 4(I_1 + \tfrac{1}{2} I_2) = 0$$

$$-V_2 - 2I_1 - 4I_1 + I_2 - 2I_2 = 0$$

$$V_1 - V_2 - 6I_1 - I_2 = 0$$

$$V_1 = 6I_1 + I_2 + V_2 \quad \text{—} \ (1)$$

$$V_2 = 4(I_1 + \tfrac{1}{2} I_2)$$

$$= 4I_1 + 2I_2 \quad \text{—} \ (2)$$

putting it in eq^n (1)

$$V_1 = 6I_1 + I_2 + 4I_1 + 2I_2$$

$$= 10I_1 + 3I_2$$

$V_2 = 4I_1 + 2I_2$

$I_2 = \frac{V_2}{2} - 2I_1$ — (3)

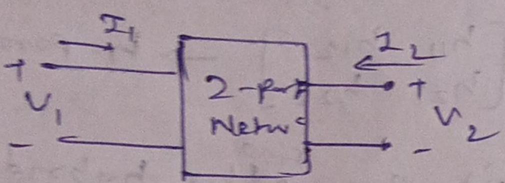
$V_1 = 6I_1 + \frac{V_2}{2} - 2I_1 + V_2$

$V_1 = 4I_1 + \frac{3}{2}V_2$ — (4)

from eqn (3) and (4)

$$[h] = \begin{bmatrix} 4 & 3/2 \\ -2 & 1/2 \end{bmatrix}$$

4) g - parameter (Inverse h - parameter)



- $\begin{pmatrix} I_1 \\ V_2 \end{pmatrix} = f\begin{pmatrix} V_1 \\ I_2 \end{pmatrix}$

$I_1 = g_{11}V_1 + g_{12}I_2$

$V_2 = g_{21}V_1 + g_{22}I_2$

$$\begin{bmatrix} I_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ I_2 \end{bmatrix}$$

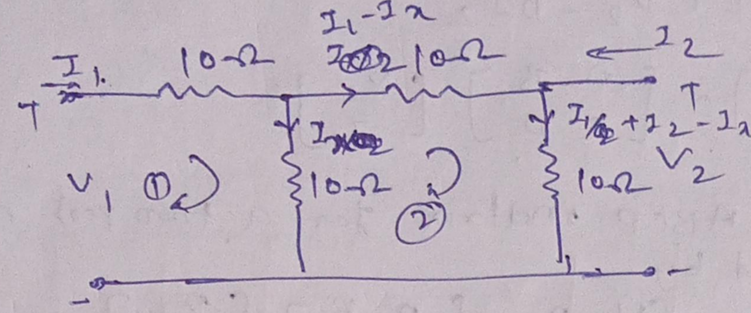
$$[g] = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix}$$

Relation b/w h- and g parameters:-

$[H] = [G]^{-1}$ or $[G] = [H]^{-1}$

Q. for the circuit given below ~~the~~ find the inverse hybrid parameter g_{21}.

sol.



$$g_{21} = \frac{V_2}{V_1}\Big|_{I_2=0}$$

sol:-

$$V_1 - 10I_1 - 10I_x = 0$$

$$V_1 = 10(I_1 + I_x) \quad -①$$

$$-10I_x - 10(I_1 - I_x) - 10(I_1 + I_2 - I_x) = 0$$

$$10I_x + 10I_x + 10I_x - 10I_1 - 10I_1 - 10I_2 = 0$$

But $I_2 = 0$

$$30I_x - 20I_1 = 0$$

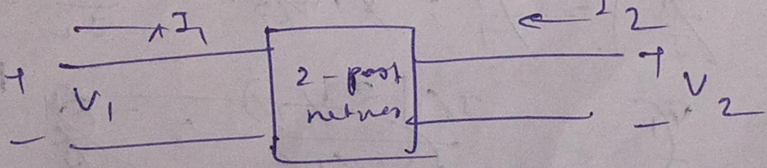
$$\boxed{I_1 = \frac{3}{2} I_x}$$

$$V_2 = 10(I_1 - I_x) \qquad [\because I_2 = 0]$$

$$g_{21} = \frac{V_2}{V_1} = \frac{\cancel{10}(I_1 - I_x)}{\cancel{10}(I_1 + I_x)} = \frac{\frac{3}{2}I_x - I_x}{\frac{3}{2}I_x + I_x}$$

$$= \frac{I_x/2}{5I_x/2} = 0.2$$

5) ABCD Parameter :- or ~~...~~



$$\begin{pmatrix} V_1 \\ I_1 \end{pmatrix} = f\begin{pmatrix} V_2 \\ -I_2 \end{pmatrix}$$

$V_1 = AV_2 - BI_2$

$I_1 = CV_2 - DI_2$

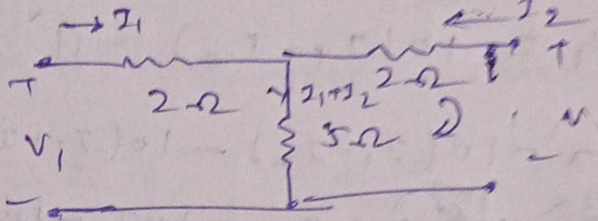
$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

Q. The ABCD matrix for a two port network is defined by

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

then find the parameter B for given network.

2



Sol:-

$$B = \frac{-V_1}{I_2}\bigg|_{V_2=0}$$

$V_1 - 2I_1 - 5I_1 - 5I_2 = 0$

$V_1 = 7I_1 + 5I_2$

Eq $5(I_1+I_2) + 2I_2 = 0$ $[\because V_2 = 0]$

$7I_2 = -5I_1$

$I_2 = -\frac{5}{7}I_1$, $I_1 = -\frac{7}{5}I_2$

$V_1 = 7I_1 - \frac{5\times5}{7}I_1$, $V_1 = 7\left(-\frac{7}{5}I_2\right) + 5I_2$

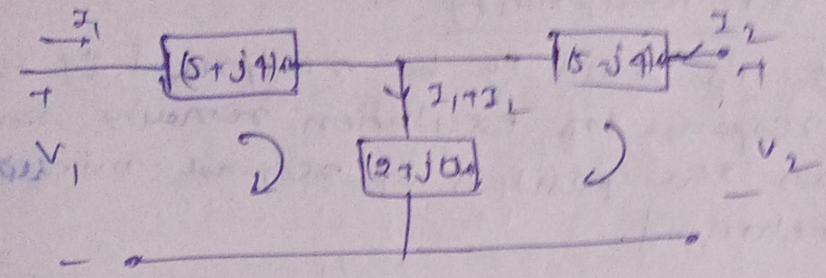
$= I_1\left(\frac{24}{7}\right)$ $= -\frac{24}{5}I_2$

$B = \cancel{\frac{-V_1}{I_1}}\Big| = \cancel{\frac{24}{7}} = \cancel{3.43}$ $-\frac{V_1}{I_2} = \frac{24}{5}$

$\frac{V_1}{I_2} = \frac{I_1(24/7)}{-5/7\, I_1} = -\frac{24}{5}$ $B = 4.8$

$= -4.8$

Q.



Find ABCD parameter.

Sol:-

$V_1 = AV_2 - BI_2$

$I_1 = CV_2 - DI_2$

$V_1 - (5+j4)I_1 - (I_1+I_2)(2+j0) = 0$

$V_1 - (7+j4)I_1 - 2I_2 = 0$

$V_1 = (7+j4)I_1 + 2I_2$ — (1)

$(I_1+I_2)(2+j0) + (I_2)(5-j4) - V_2 = 0$

$2I_1 + I_2(7-j4) - V_2 = 0$

$I_1 = \frac{1}{2}V_2 - I_2(3.5-j2)$

$I_1 = \frac{1}{2}V_2 - (3.5-j2)I_2$ — (2)

put it in eq^n (1)

$V_1 = (7+j4)\left[\frac{1}{2}V_2 - (3.5-j2)I_2\right] + 2I_2$

$= \left(\frac{7}{2}+j2\right)V_2 - \{(7+j4)(3.5-j2) + 2\}I_2$

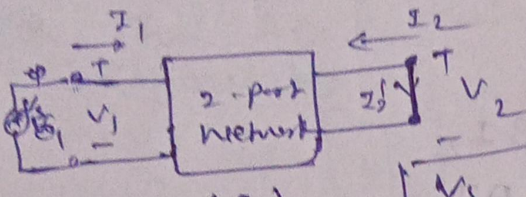
$V_1 = (3.5+j2)V_2 - [30.5 I_2]$ — (A)

$I_1 = 3.5V_2 - (3.5-j2)I_2$ — B

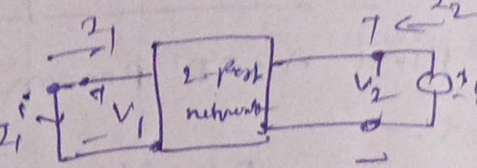
$$[ABCD] = \begin{bmatrix} (3.5+j2) & -30.5 \\ 3.5 & (3.5-j2) \end{bmatrix}$$

Condition for Reciprocity :-

A network is said to be reciprocal if the ratio of response to excitation remains same even when the position of response and excitation are interchanged.



(a)



(b)

$$\boxed{\frac{V_s}{I_2'} = \frac{V_s}{I_1'}}$$

In case - (a)

2 $V_1 = V_s \quad I_1 = I_1 \quad I_2 = -I_2' \quad V_2 = 0$

In case - (b)

$V_1 = 0 \quad I_1 = I_1' \quad V_2 = V_s \quad I_2 = I_2$

Now,

i) Z - parameter.

$$V_1 = Z_{11} I_1 + Z_{12} I_2 \quad \text{— (1)}$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2 \quad \text{— (2)}$$

For case - (a)

from eq^n (1)

$$V_s = Z_{11} I_1 + Z_{12} I_2' \quad \text{— (A)}$$

from eq^n (2)

$$0 = Z_{21} I_1 - Z_{22} I_2' \quad \text{— (B)}$$

$$I_1 = \frac{Z_{22} I_2'}{Z_{21}}$$

put it in eq^n (A)

$$V_s = \frac{Z_{11} Z_{22} I_2'}{Z_{21}} - Z_{12} I_2'$$

$$\frac{V_s}{I_2'} = \frac{Z_{11} Z_{22} - Z_{12} Z_{21}}{Z_{21}}$$

For case -(b)

$V_1 = 0, \; I_1 = -I_1', \quad V_2 = V_s \quad I_2 = I_2$

put these values in eqⁿ (1)

$$0 = Z_{11}(-I_1') + Z_{12} I_2$$

$$I_2 = \frac{Z_{11}}{Z_{12}} I_1 \quad \text{— (B)}$$

put the values in eqⁿ (2)

$$V_s = Z_{21}(-I_1') + Z_{22} I_2$$

$$= Z_{21}(-I_1') + Z_{22}\left(\frac{Z_{11}}{Z_{12}}\right) I_1'$$

$$\frac{V_s}{I_1'} = \frac{Z_{11}Z_{22} - Z_{12}Z_{21}}{Z_{12}}$$

Now we know for reciprocity

$$\frac{V_s}{I_2'} = \frac{V_s}{I_1'}$$

so,
$$\frac{Z_{11}\cancel{Z_{22}} - Z_{12}Z_{21}}{Z_{21}} = \frac{Z_{11}Z_{22} - \cancel{Z_{12}Z_{21}}}{Z_{12}}$$

$$\boxed{Z_{21} = Z_{12}} \rightarrow \text{Z - parameter}$$

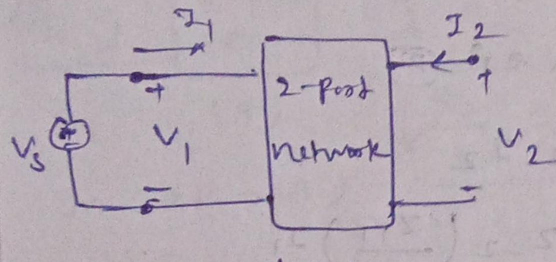
Similarly, $\boxed{Y_{21} = Y_{12}}$ → Y - parameter

$$\boxed{AD - BC = 1} \rightarrow \text{ABCD parameter}$$

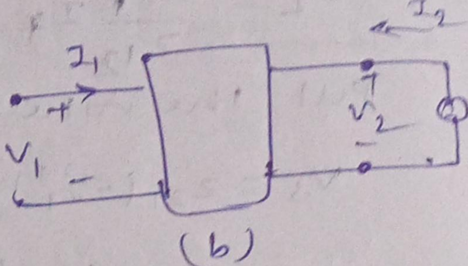
$$\boxed{g_{12} = -g_{21}} \rightarrow \text{g - parameter.}$$

Condition for symmetry :-

A two port network is said to be symmetrical if the ports can be interchanged without changing the port voltage & current.



(a)



(b)

$V_1 = V_1$ $I_1 = 0$ $V_2 = V_s$ $I_2 = I_1$

$$\frac{V_s}{I_1}\Big|_{I_2=0} = \frac{V_s}{I_2}\Big|_{I_1=0}$$

Z-Parameters

for case (a)

$V_1 = V_s$ $I_1 = I_1$ $V_2 = V_2$ $I_2 = 0$

$V_1 = Z_{11}I_1 + Z_{12}I_2$ — (1)

$V_2 = Z_{21}I_1 + Z_{22}I_2$ — (2)

put case-(a) in eq^n (1) & (2)

$V_s = Z_{11}I_1 \rightarrow \frac{V_s}{I_1} = Z_{11}$ — (A)

put case-b in eq^n (1) & (2)

(1) $V_1 = Z_{11}(0) + Z_{12}I_2$

$V_1 = Z_{12}I_2$ — (B)

(2) $V_s = Z_{21}(0) + Z_{22}I_2$

$\frac{V_s}{I_2} = Z_{22}$ — B)

Applying symmetry condⁿ.

$$\boxed{Z_{11} = Z_{22}}$$

Y - Parameter :-

$$I_1 = Y_{11} V_1 + Y_{12} V_2 \quad \text{— (i)}$$
$$I_2 = Y_{21} V_1 + Y_{22} V_2 \quad \text{— (ii)}$$

put case (a) in eqⁿ (i) & (ii)

$$I_1 = Y_{11} V_s + Y_{12} V_2 \quad \& \quad 0 = Y_{21} V_1 + Y_{22} V_2$$

$$\boxed{V_2 = -\frac{Y_{21}}{Y_{22}} V_s}$$

$$\frac{I_1}{V_s} = \frac{Y_{22} Y_{11} - Y_{12} Y_{21}}{Y_{22}} \quad \text{— (a)}$$

put case b in eqⁿ (i) & (ii)

$$I_2 = Y_{21} V_1 + Y_{22} V_s$$

$$0 = Y_{11} V_1 + Y_{12} V_s$$

$$V_1 = -\frac{Y_{12} V_s}{Y_{11}}$$

$$I_2 = Y_{21}\left(-\frac{Y_{12}}{Y_{11}}\right) V_s + Y_{22} V_s$$

$$\frac{V_s}{I_2} = \frac{Y_{11}}{Y_{11} Y_{22} - Y_{21} Y_{12}}$$

Equating both

$$\frac{Y_{22}}{\cancel{Y_{22} Y_{11} - Y_{12} Y_{21}}} = \frac{Y_{11}}{\cancel{Y_{11} Y_{22} - Y_{21} Y_{12}}}$$

$$\boxed{Y_{11} = Y_{22}}$$

Parameter	Cond^n of Reciprocity	Cond^n of Symmetry
Z	$Z_{12} = Z_{21}$	$Z_{11} = Z_{22}$
Y	$Y_{12} = Y_{21}$	$Y_{11} = Y_{22}$
ABCD	$AD - BC = 1$	$A = D$
h	$h_{12} = -h_{21}$	$h_{11}h_{22} - h_{12}h_{22}$

Relation b/w Z and ABCD parameter :-

2 Taking ABCD parameter eq^n.

$$V_1 = AV_2 - BI_2$$

$$I_1 = CV_2 - DI_2$$

$$V_2 = \frac{I_1}{C} + \frac{D}{C}I_2 \quad — (B)$$

$$V_1 = A\left[\frac{I_1}{C} + \frac{D}{C}I_2\right] - BI_2$$

$$V_1 = \frac{A}{C}I_1 + \frac{AD - BC}{C}I_2 \quad — (A)$$

on comparing with Z-parameters.

$$Z_{11} = \frac{A}{C} \qquad Z_{12} = \frac{AD-BC}{C}$$

$$Z_{21} = \frac{1}{C} \qquad Z_{22} = D/C$$

Determine Z parameter from h-parameter

$$V_1 = h_{11}I_1 + h_{12}V_2$$

$$I_2 = h_{21}I_1 + h_{22}V_2$$

$$V_2 = \frac{I_2}{h_{22}} - \frac{h_{21}}{h_{22}}I_1$$

$$V_2 = -I_1\left(\frac{h_{21}}{h_{22}}\right) + \frac{1}{h_{22}}(I_2) \quad — (B)$$

$$V_1 = h_{11} I_1 + h_{12}\left(-\frac{h_{21}}{h_{22}} I_1 + \frac{1}{h_{22}} I_2\right)$$

$$= h_{11}I_1 - \frac{h_{12}h_{21}}{h_{22}} I_1 + \frac{h_{12}}{h_{22}} I_2$$

$$V_1 = \frac{(h_{11}h_{22} - h_{12}h_{21})}{h_{22}} + \frac{h_{12}}{h_{22}} I_2 \quad \text{— (A)}$$

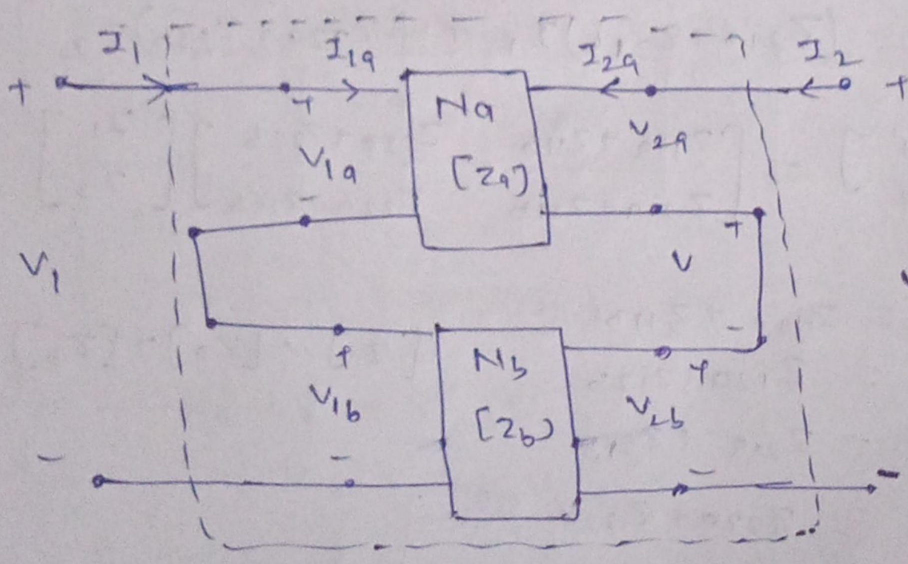
On comparing with z-parameters.

$$z_{11} = \frac{h_{11}h_{22} - h_{12}h_{21}}{h_{22}} \qquad z_{12} = \frac{h_{12}}{h_{22}}$$

$$z_{21} = \frac{-h_{21}}{h_{22}} \qquad z_{22} = \frac{1}{h_{22}}$$

Interconnection of two ports:-

(a) series connection



$$I_1 = I_{1a} = I_{1b} \quad \text{— (i)}$$
$$V_1 = V_{1a} + V_{1b} \quad \text{— (ii)}$$
$$I_2 = I_{2a} = I_{2b} \quad \text{— (iii)}$$
$$V_2 = V_{2a} + V_{2b} \quad \text{— (iv)}$$

for N_a.

$$\begin{bmatrix} V_{1a} \\ V_{2a} \end{bmatrix} = \begin{bmatrix} Z_{11a} & Z_{12a} \\ Z_{21a} & Z_{22a} \end{bmatrix} \begin{bmatrix} I_{1a} \\ I_{2a} \end{bmatrix}$$

for N_b.

$$\begin{bmatrix} V_{1b} \\ V_{2b} \end{bmatrix} = \begin{bmatrix} Z_{11b} & Z_{12b} \\ Z_{21b} & Z_{22b} \end{bmatrix} \begin{bmatrix} I_{1b} \\ I_{2b} \end{bmatrix}$$

In eq^n (ii),

$$V_1 = V_{1a} + V_{1b}$$

2 $$= (Z_{11a} I_{1a} + Z_{12a} I_{2a}) + (Z_{11b} I_{11b} + Z_{12b} I_{2b})$$

$$V_1 = (Z_{11a} + Z_{11b}) I_1 + (Z_{12a} + Z_{12b}) I_2$$

similarly

$$V_2 = (Z_{21a} + Z_{21b}) I_1 + (Z_{22a} + Z_{22b}) I_2$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11a} + Z_{11b} & Z_{12a} + Z_{12b} \\ Z_{21a} + Z_{21b} & Z_{22a} + Z_{22b} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$Z_{11} = Z_{11a} + Z_{11b}$

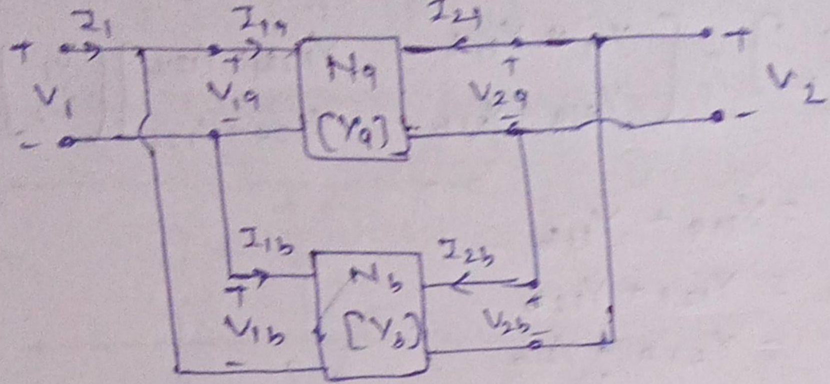
$Z_{12} = Z_{12a} + Z_{12b}$

$Z_{21} = Z_{21a} + Z_{21b}$

$Z_{22} = Z_{22a} + Z_{22b}$

$[Z] = [Z_a] + [Z_b]$

② Parallel connection :-



$$V_1 = V_{1a} = V_{1b} \quad -(i)$$
$$V_2 = V_{2a} = V_{2b} \quad -(ii)$$
$$I_1 = I_{1a} + I_{1b} \quad -(iii)$$
$$I_2 = I_{2a} + I_{2b} \quad -(iv)$$

for N_a

$$\begin{bmatrix} I_{1a} \\ I_{2a} \end{bmatrix} = \begin{bmatrix} Y_{11a} & Y_{12a} \\ Y_{21a} & Y_{22a} \end{bmatrix} \begin{bmatrix} V_{1a} \\ V_{2a} \end{bmatrix}$$

for N_b

$$\begin{bmatrix} I_{1b} \\ I_{2b} \end{bmatrix} = \begin{bmatrix} Y_{11b} & Y_{12b} \\ Y_{21b} & Y_{22b} \end{bmatrix} \begin{bmatrix} V_{1b} \\ V_{2b} \end{bmatrix}$$

$$I_{1a} = Y_{11a} V_{1a} + Y_{12a} V_{2a}$$
$$I_{1b} = Y_{11b} V_{1b} + Y_{12b} \cdot V_{2b}$$

$$\begin{aligned} I_1 &= I_{1a} + I_{1b} \\ &= Y_{11a} V_{1a} + Y_{12a} V_{2a} + Y_{11b} V_{1b} + Y_{12b} V_{2b} \\ &= (Y_{11a} + Y_{11b}) V_1 + (Y_{12a} + Y_{12b}) V_2 \quad -③ \end{aligned}$$

Similarly,

$$I_2 = (Y_{21a} + Y_{21b})V_1 + (Y_{22a} + Y_{22b})V_2$$

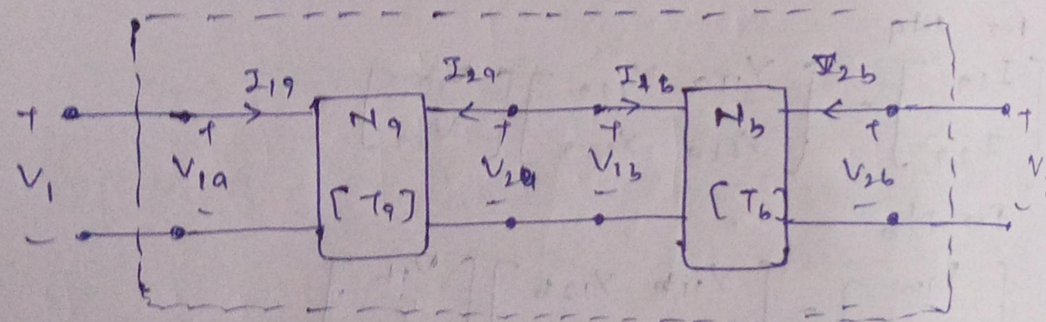
$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11a} + Y_{11b} & Y_{12a} + Y_{12b} \\ Y_{21a} + Y_{21b} & Y_{22a} + Y_{22b} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$Y_{11} = Y_{11a} + Y_{11b}$$
$$Y_{12} = Y_{12a} + Y_{12b}$$
$$Y_{21} = Y_{21a} + Y_{21b}$$
$$Y_{22} = Y_{22a} + Y_{22b}$$

2.)

$$[Y] = [Y_a] + [Y_b]$$

Cascade connection :-



$V_1 = V_{1a}$ $\quad I_{1b} = -I_{2a}$ $\quad I_2 = I_{2b}$

$I_1 = I_{1a}$ $\quad V_{1b} = V_{2a}$ $\quad V_2 = V_{2b}$

for N_a

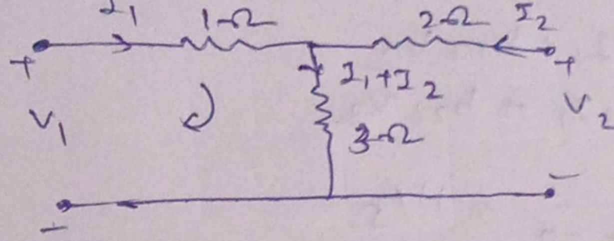
$$\begin{bmatrix} V_{1a} \\ I_{1a} \end{bmatrix} = \begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix} \begin{bmatrix} V_{2a} \\ -I_{2a} \end{bmatrix}$$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix} \begin{bmatrix} V_{1b} \\ I_{1b} \end{bmatrix}$$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix} \begin{bmatrix} A_b & B_b \\ C_b & D_b \end{bmatrix} \begin{bmatrix} V_{2b} \\ -I_{2b} \end{bmatrix}$$

$$[T] = [T_a][T_b]$$

Q. For the network shown below.



Find
a) Z parameter b) Y Parameter c) T-parameter
d) h-parameter e) g-parameter.

Sol:- (a) Z-parameter.

$$V_1 = 4I_1 + 3I_2 \quad -① $$
$$V_2 = 3I_1 + 5I_2.$$

$$[Z] = \begin{bmatrix} 4 & 3 \\ 3 & 5 \end{bmatrix}$$

(b) $$[Y] = [Z]^{-1} = \frac{1}{|Z|} \, adj[Z]$$

$$= \frac{1}{11} \begin{bmatrix} 5 & -3 \\ -3 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 5/11 & -3/11 \\ -3/11 & 4/11 \end{bmatrix}$$

(c) T - parameter

$$V_1 = AV_2 - BI_2$$
$$I_1 = CV_2 - DI_2$$

$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0} = 4/3$$

$$B = -\left.\frac{V_1}{I_2}\right|_{V_2=0} = 11/3$$

$C = \left.\frac{I_1}{V_2}\right|_{I_2=0} = 1/3$

$D = \left.\frac{-I_1}{I_2}\right|_{V_2=0} = 5/3$

e) h- parameter

$V_1 = h_{11} I_1 + h_{12} V_2$

$I_2 = h_{21} I_1 + h_{22} V_2$

$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0} = 11/5$

2 $h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0} = 3/5$

$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0} = -3/5$

$h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0} = 1/5$

$[h] = \begin{bmatrix} 11/5 & 3/5 \\ -3/5 & 1/5 \end{bmatrix}$ $\boxed{|h| = 4/5}$

f) g - parameter

$[g] = [h]^{-1} = \frac{1}{|h|} \{ adj(h)$

$= \frac{5}{4} \begin{bmatrix} 1/5 & -3/5 \\ 3/5 & 11/5 \end{bmatrix}$

$= \begin{bmatrix} 1/4 & -3/4 \\ 3/4 & 11/4 \end{bmatrix}$