Unit-I

Important input signals:-

(i) step signal:-

$$f(t) = \begin{cases} 0; & t<0 \\ K; & t \geq 0 \end{cases}$$

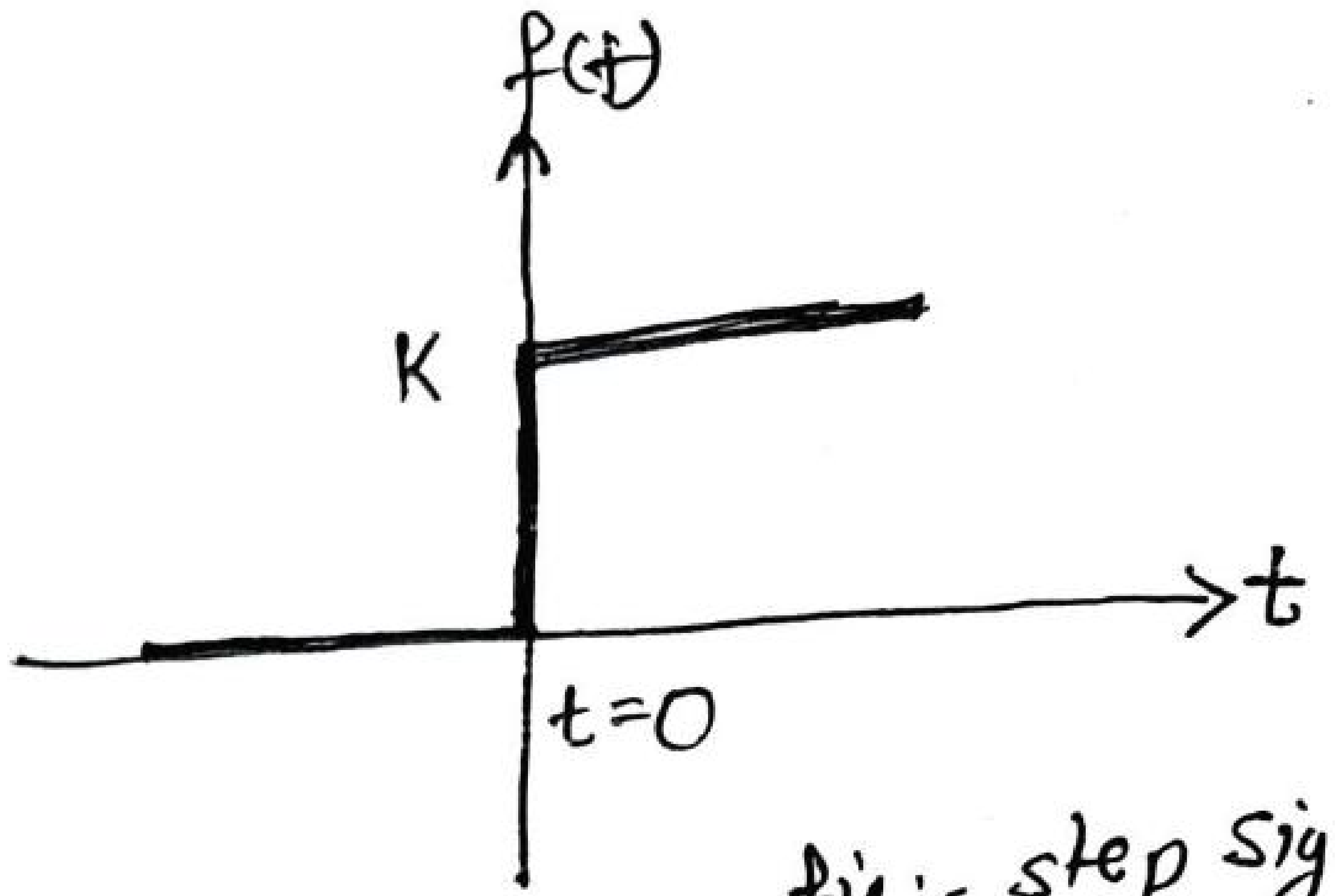

fig:- step signal

unit step signal:-

$$u(t) = \begin{cases} 0; & t<0 \\ 1; & t \geq 0 \end{cases}$$

Laplace transform of step signal—

$$F(S) = \mathcal{L}[F(t)] = \int_0^\infty f(t)\, e^{-st}\, dt$$

$$F(S) = \int_0^\infty K \cdot e^{-st}\, dt$$

$$= K\left[\frac{e^{-st}}{-s}\right]_0^\infty$$

$$= \frac{K}{-s}\left[e^{-\infty} - e^{0}\right]$$

$$= -\frac{K}{s}\left[0-1\right]$$

$$= \frac{K}{S}$$

$$\boxed{\mathcal{L}[F(t)] = F(S) = \frac{K}{S}}$$

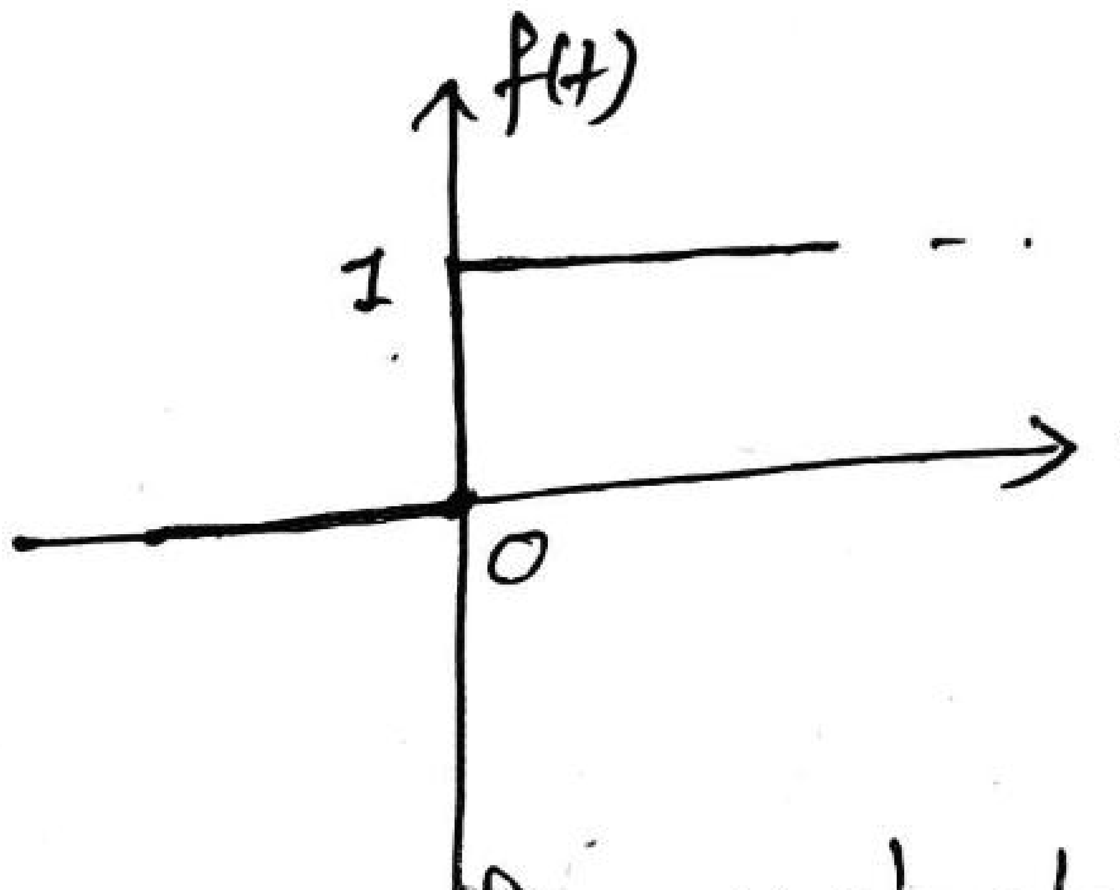

fig:- Unit step signal

(2) Ramp Signal:-

$$f(t) = \begin{cases} 0; & t<0 \\ Kt; & t \geq 0 \end{cases}$$

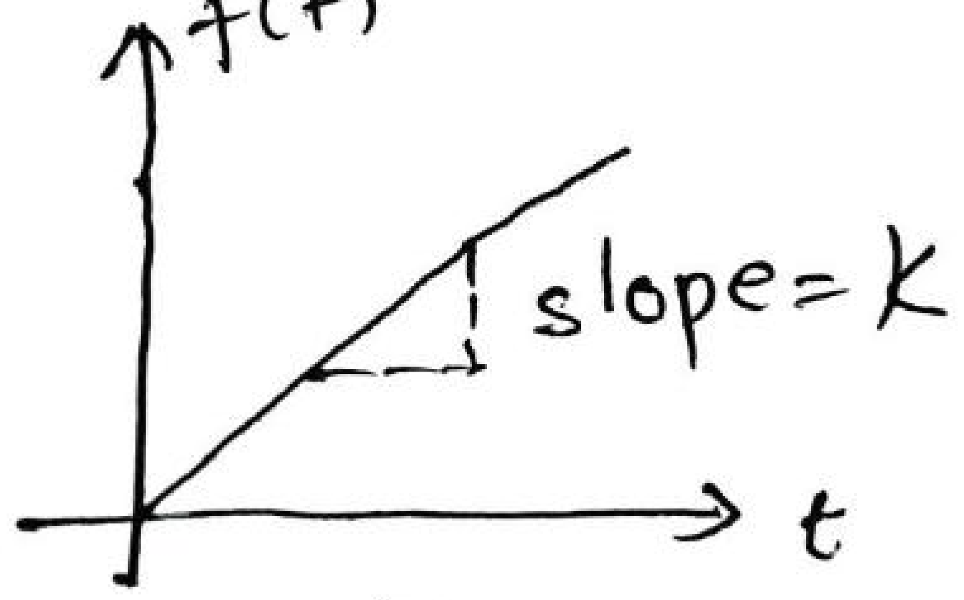

fig:- Ramp signal

Unit Ramp signal:-

$$f(t) = \begin{cases} 0; & t<0 \\ t; & t \geq 0 \end{cases}$$

Laplace transform of ramp signal—

$$F(S) = \mathcal{L}[F(t)] = \int_0^\infty f(t) \cdot e^{-st}\, dt$$

$$F(S) = \int_0^\infty Kt \cdot e^{-st}\, dt$$

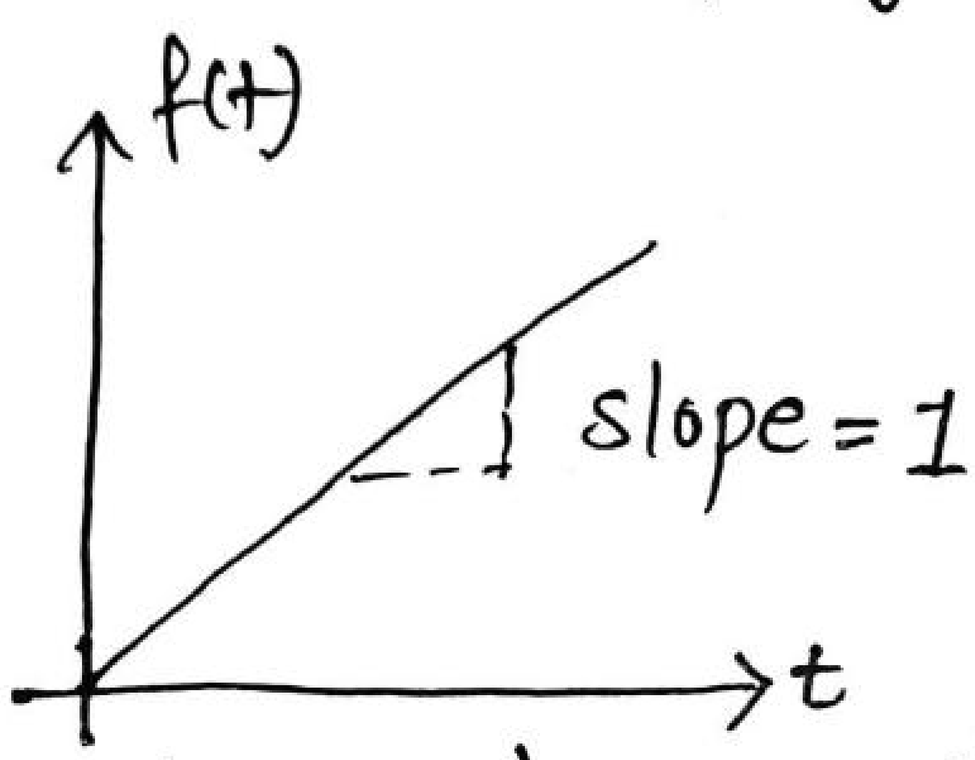

fig - Unit ramp signal

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$$= \frac{K}{s^2}$$

$$\boxed{\mathcal{L}[f(t)] = F(S) = \frac{K}{s^2}}$$

③ Impulse signal :-

$$\delta(t) = \begin{cases} 0; & t \neq 0 \\ 1; & t = 0 \end{cases}$$

and area under the pulse is unity

$$\int_{-\infty}^{\infty} \delta(t)\, dt = 1$$

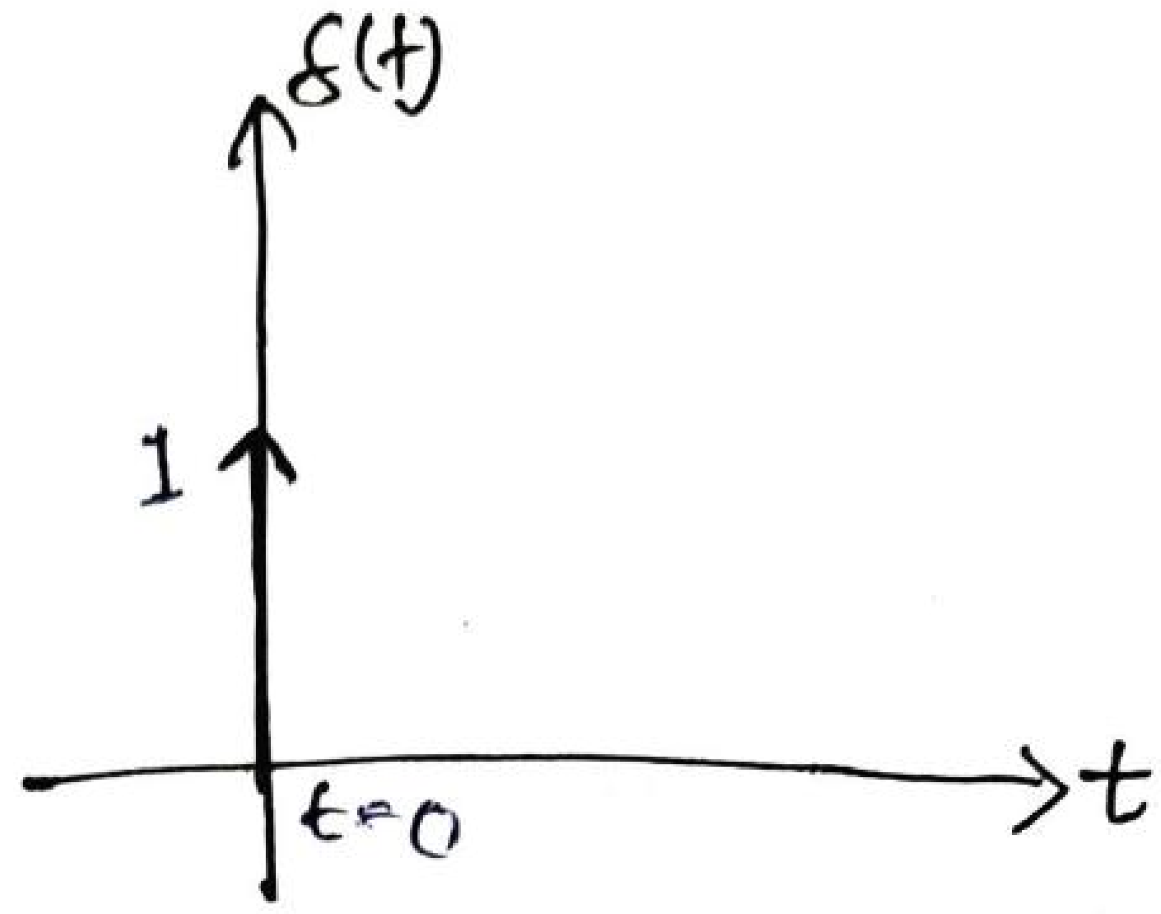


fig - Unit impulse signal

Laplace transform of impulse signal →

→ we can write function in terms of unit step function as

$$\delta(t) = \frac{du(t)}{dt}$$

taking Laplace transform of both sides, we get -

$$\mathcal{L}[\delta(t)] = \mathcal{L}\left[\frac{du(t)}{dt}\right]$$

now, $\mathcal{L}\left[\frac{df(t)}{dt}\right] = SF(S) - f(0^-)$

$$\mathcal{L}[\delta(t)] = s\mathcal{L}[u(t)] - u(t)\Big|_{t=0^-}$$

$$= s \cdot \frac{1}{s}$$

$$= 1$$

$$\mathcal{L}[\delta(t)] = 1$$

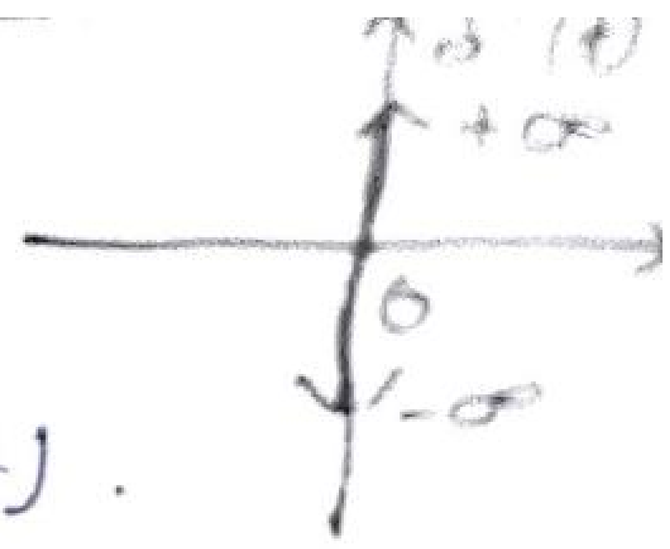
$$\boxed{\mathcal{L}[\delta(t)] = 1}$$

other Basic Signals :-

① Unit doublet signal :-

if a unit impulse signal δ(t) is differentiated with respect to time t, we get -

$$\frac{d\delta(t)}{dt} = \begin{cases} \delta'(t) = +\infty \text{ and } -\infty ; & t=0 \\ 0 ; & t \neq 0 \end{cases}$$



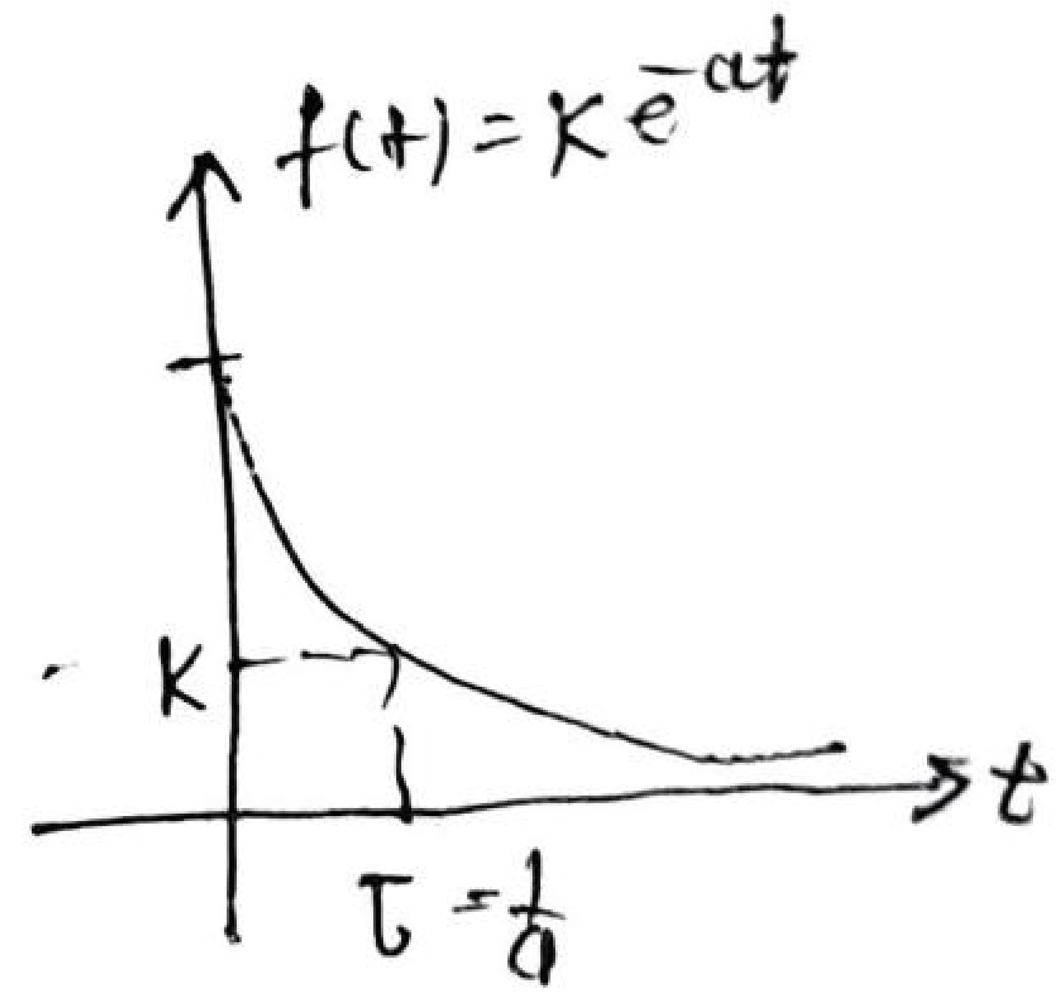
This signal is called unit doublet signal $\delta'(t)$.

→ We can write very important relationship as,
Derivative of unit impulse signal = Unit impulse signal
mathematically -

$$\boxed{\frac{d}{dt}[\delta(t)] = \delta'(t) \text{ or } \int \delta'(t)\,dt = \delta(t)}$$

② **Exponential Signal:-**

$$f(t) = \begin{cases} 0 ; & t<0 \\ K e^{-at} ; & t \geq 0 \end{cases}$$



③ **Sinusoidal signal:-**

$$f(t) = 0 ; \quad t<0$$
$$= A \sin\omega t ; \quad t \geq 0$$

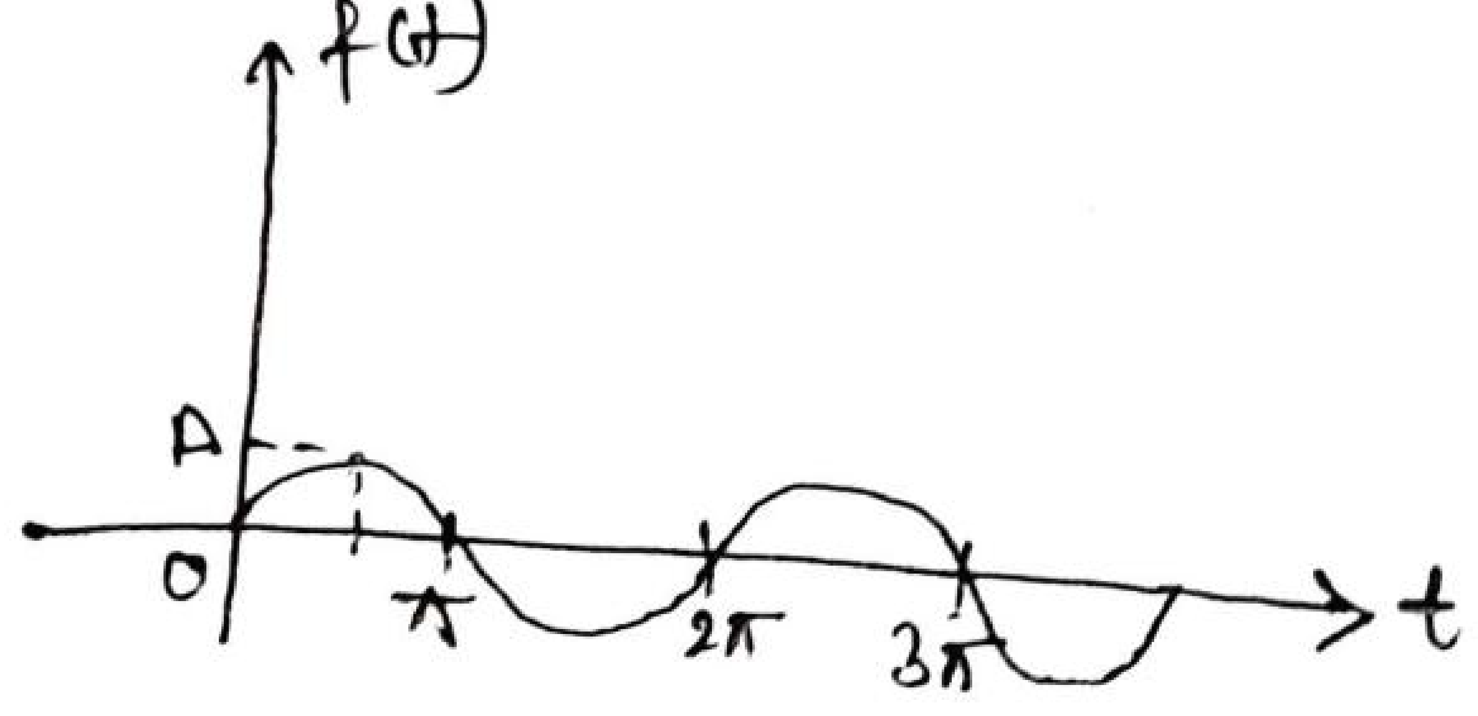


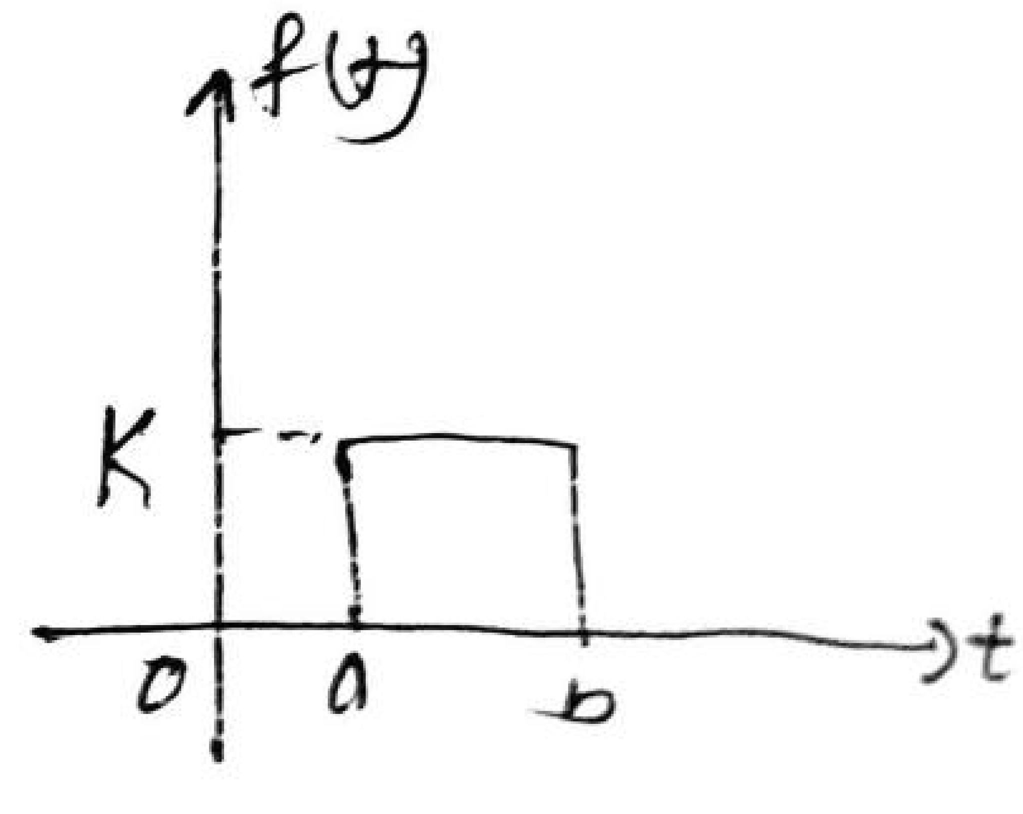
fig:- Sinusoidal signal

④ **Gate signal (or Gate function):-**

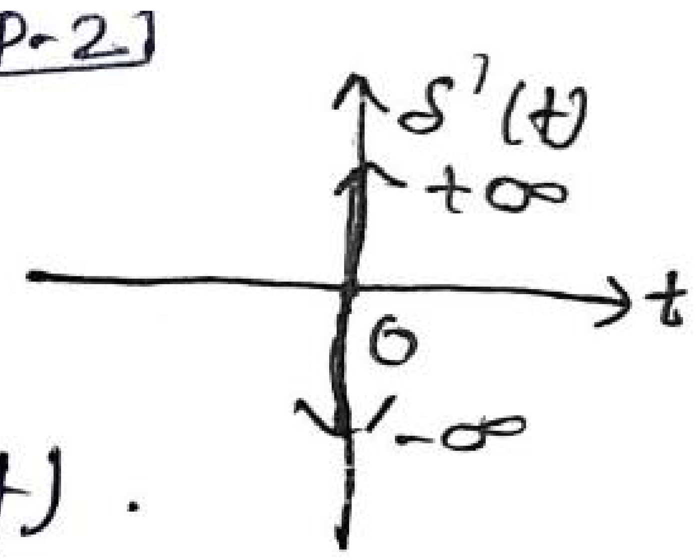
It is a rectangular pulse as shown in figure, starting at $t=a$ and ending at $t=b$ and expressed as -

$$f(t) = G_{a,b}(t) = K[u(t-a) - u(t-b)]$$

where, K is magnitude of the signal.



$$\frac{d\delta(t)}{dt} = \begin{cases} \delta'(t) = +\infty \text{ and } -\infty; & t=0 \\ 0; & t \neq 0 \end{cases}$$



This signal is called unit doublet signal $\delta'(t)$.

∘) We can write very important relationship as,

Derivative of unit impulse signal = Unit impulse signal

mathematically -

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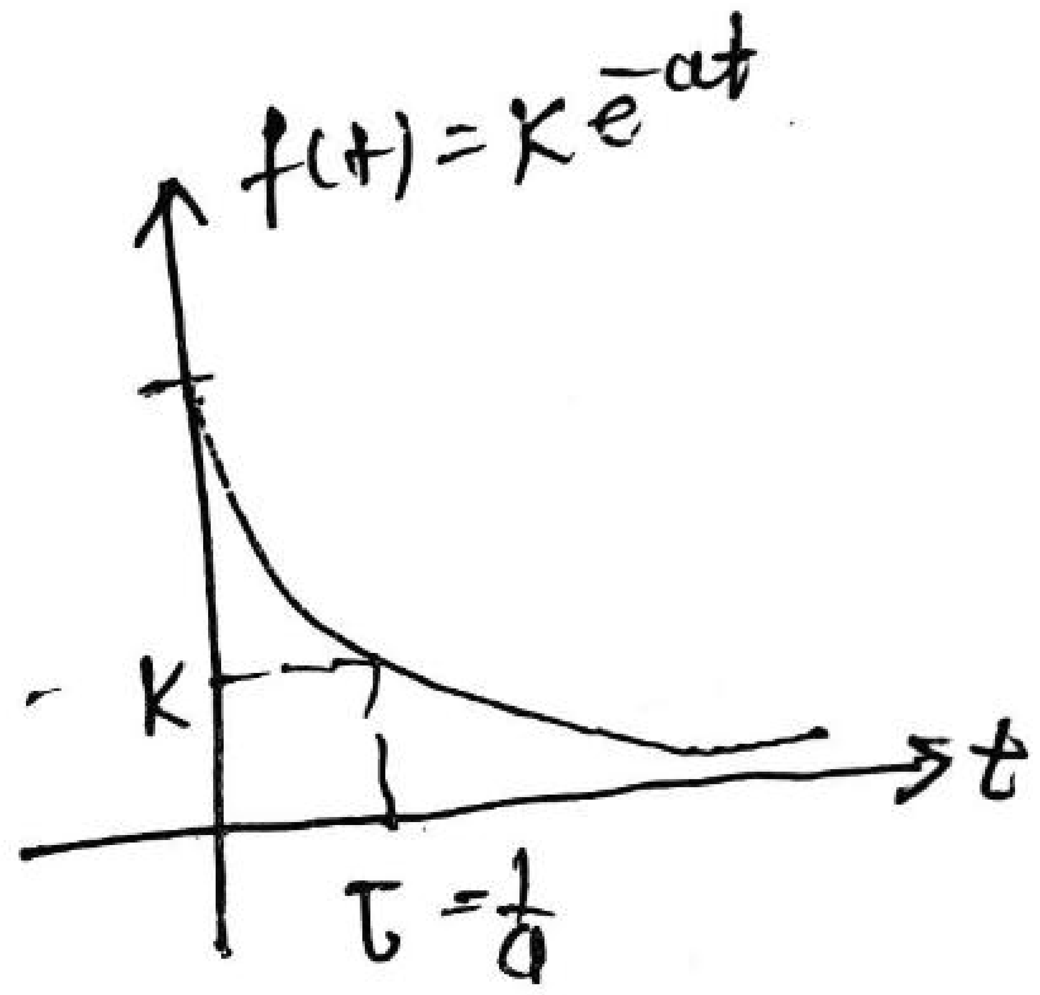

③ **Sinusoidal signal:-**

$f(t) = 0; \quad t<0$

$= A \sin\omega t; \quad t \geq 0$

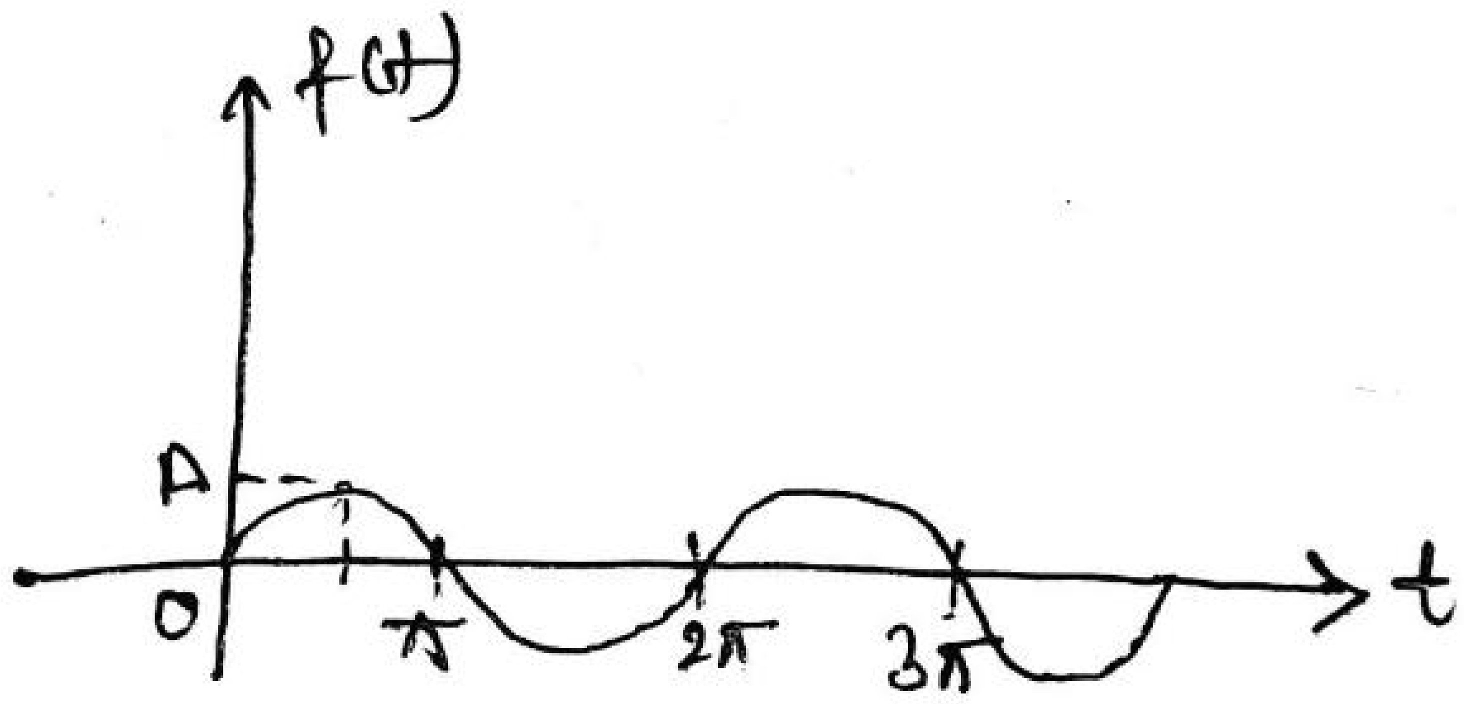

fig:- Sinusoidal signal

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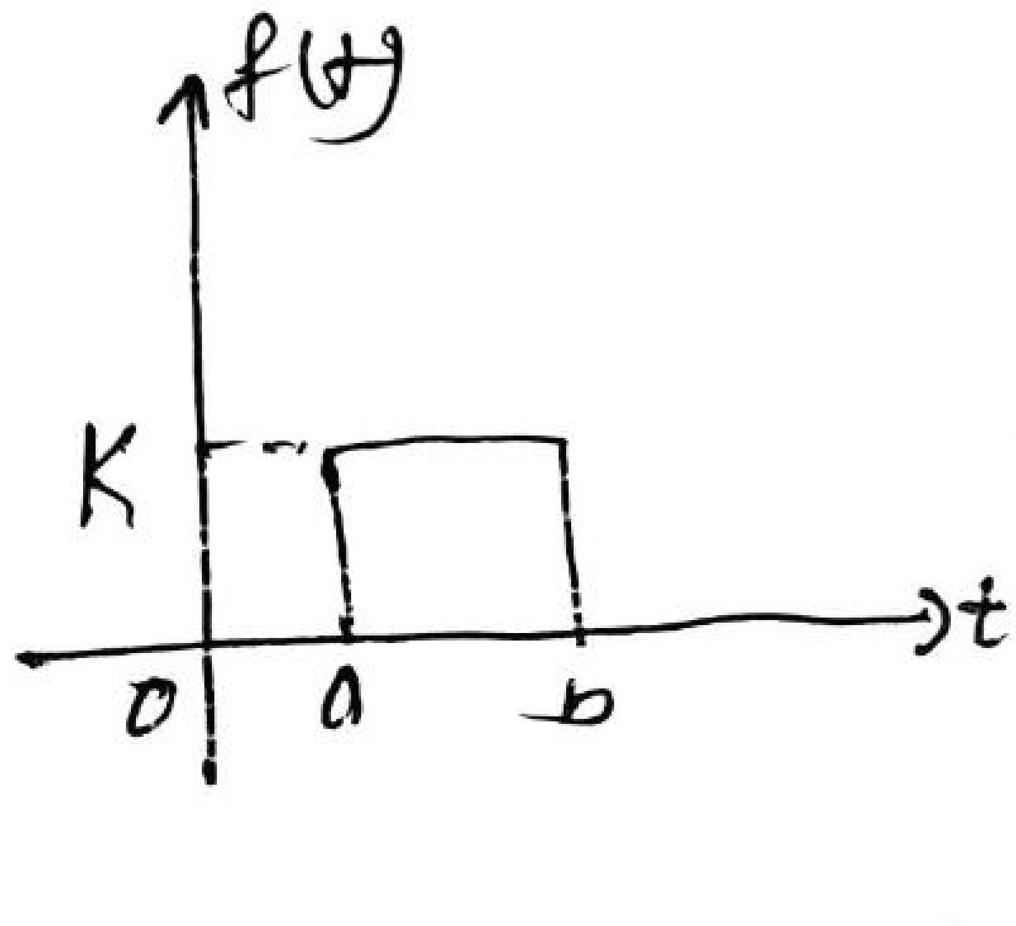

where, K is magnitude of the Signal.

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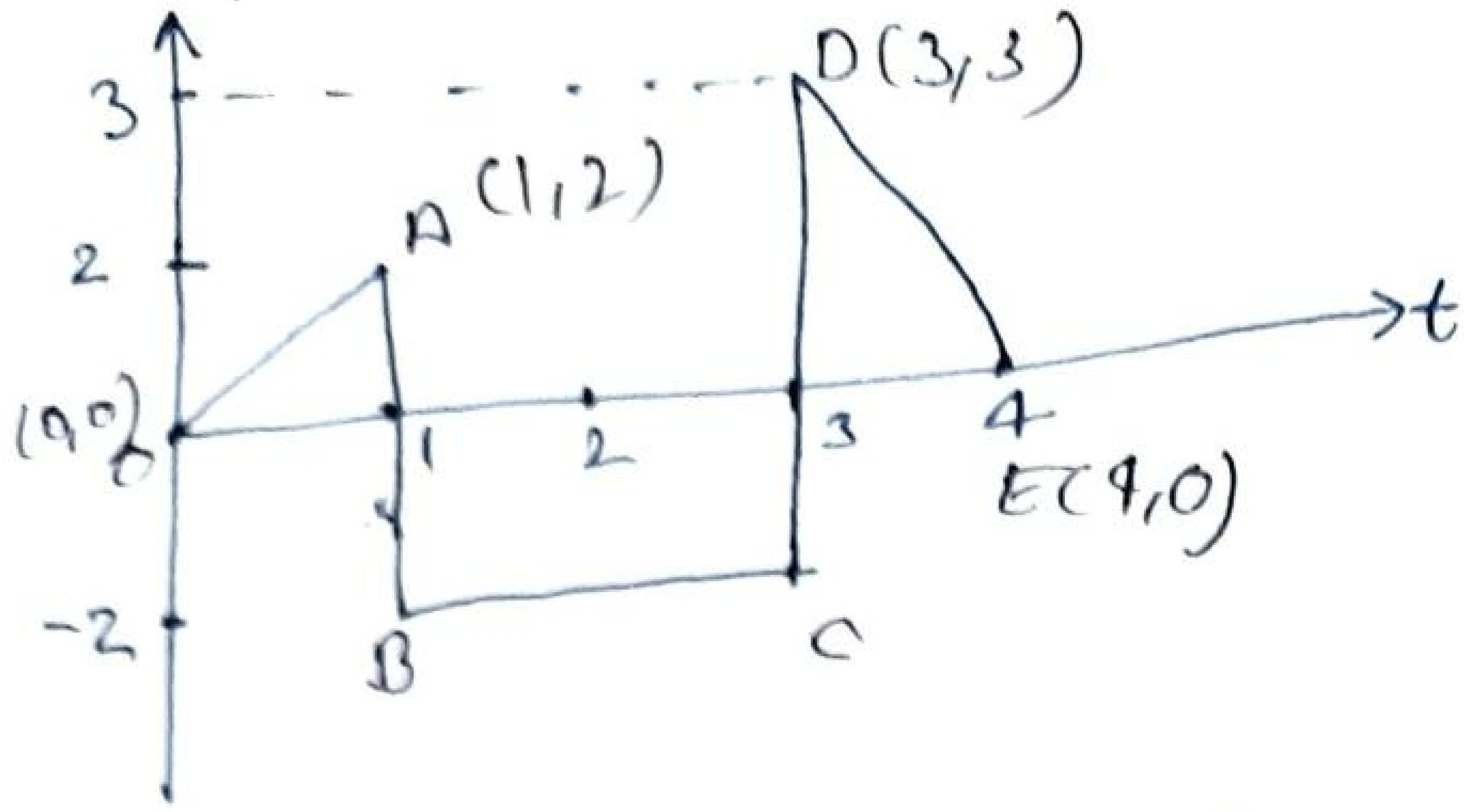
Question:- Express the given waveform shown in figure in step signals.



Solved:- The equation of line passing through two point is given by

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

therefore equation of line OA is given by —

$$f_1(t) - 0 = \frac{2-0}{1-0}(t-0)$$

or, $f_1(t) = 2t$

The second line BC is parallel to t-axis and below -2. so, the equation is given by

$$f_2(t) = -2$$

The third line DE has two points of co-ordinate (3,3) & (4,0). so, the equation is given by —

$$f_3(t) - 3 = \frac{0-3}{4-3}(t-3)$$

$$f_3(t) = -3t + 12$$

Step II:- To express the waveform in step signals.

Using gate function, we have

$$f(t) = f_1(t)\cdot G_{0,1}(t) + f_2(t)\, G_{1,3}(t) + f_3(t)\cdot G_{3,4}(t)$$

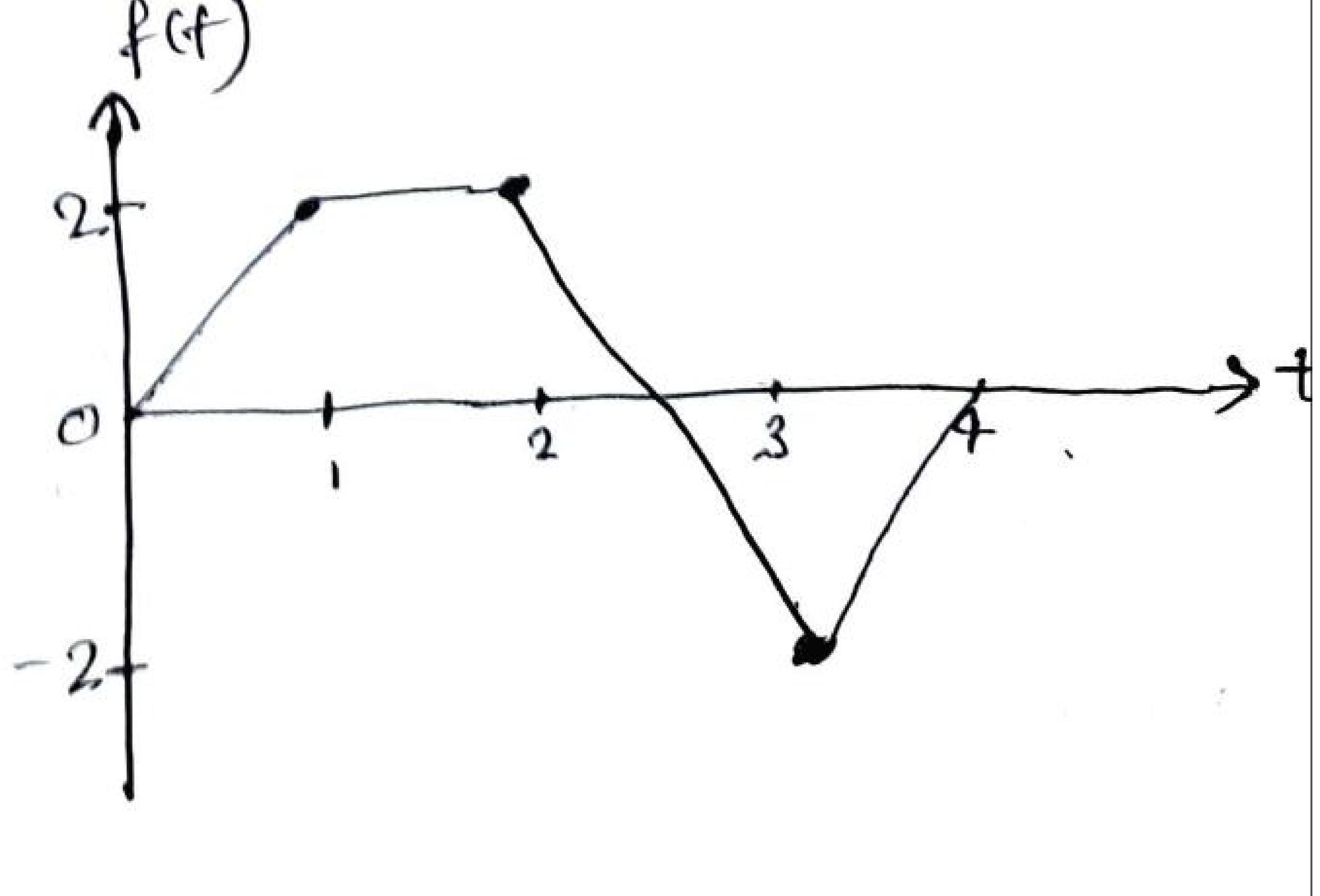
$$= 2t\,[u(t-0) - u(t-1)] + (-2)[u(t-1) - u(t-3)] + (-3t+12)[u(t-3) - u(t-4)]$$

$$= 2t\,u(t) - 2t\,u(t-1) + 2u(t-1) + 2u(t-3) + (-3t+12)\,u(t-3) - (-3t+12)\,u(t-4)$$

$$f(t) = 2t\,u(t) - (2t+2)\,u(t-1) + (-3t+14)\,u(t-3) + 3(t-4)\,u(t-4)$$

Ans

Questions⇒ Write an equation for the f(t) in terms of steps, ramps and other related function as needed as shown in figure.



Classification of signals:-

"A function of one or more independent variables which contains some information is called a signal."

(i) Continuous-time signal:-

In case of continuous-time signals the independent variable is continuous, and thus these signal are defined for continue of values of the independent variable. For continuous-time signals, the independent variable is time t. A continuous time signal is represented by $x(t)$.

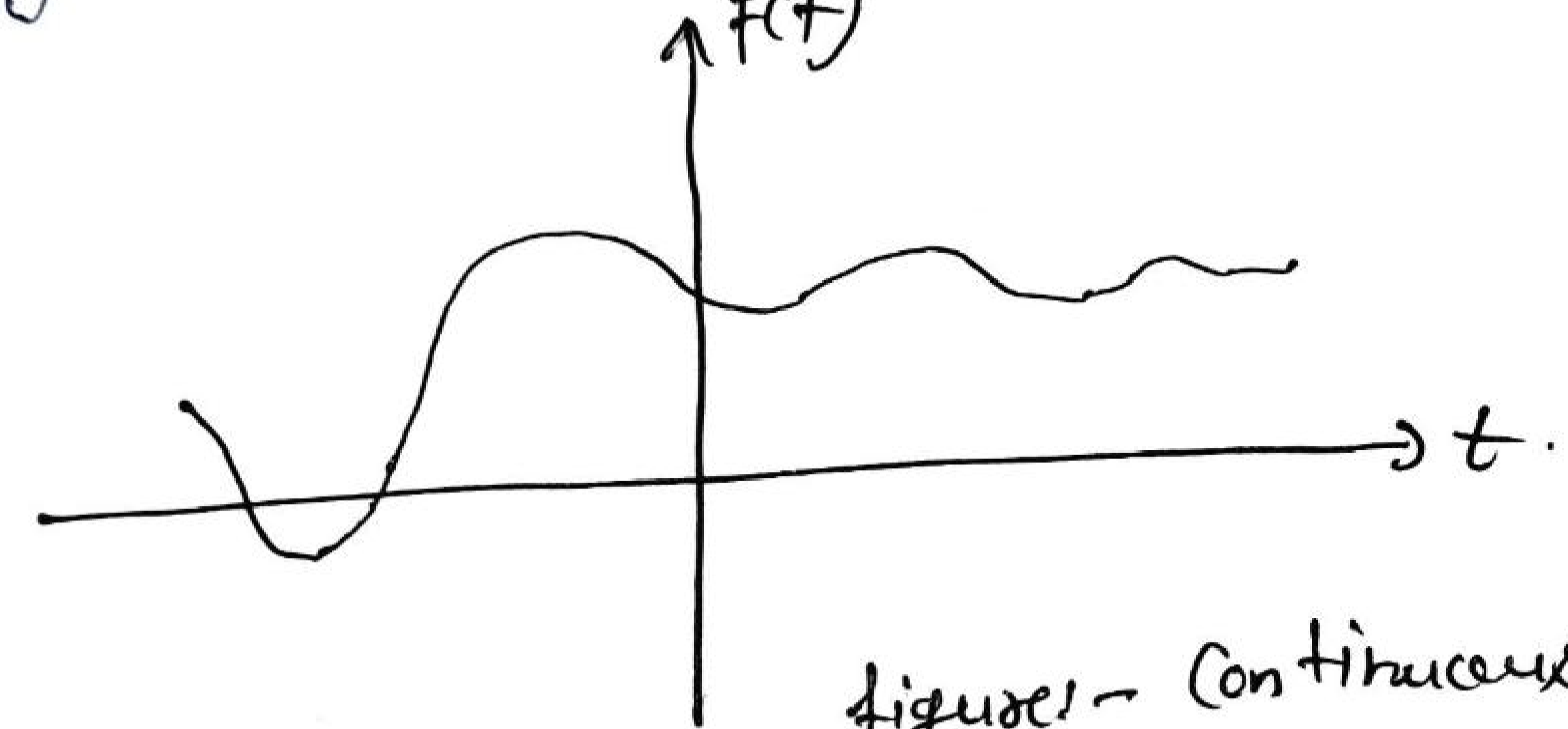
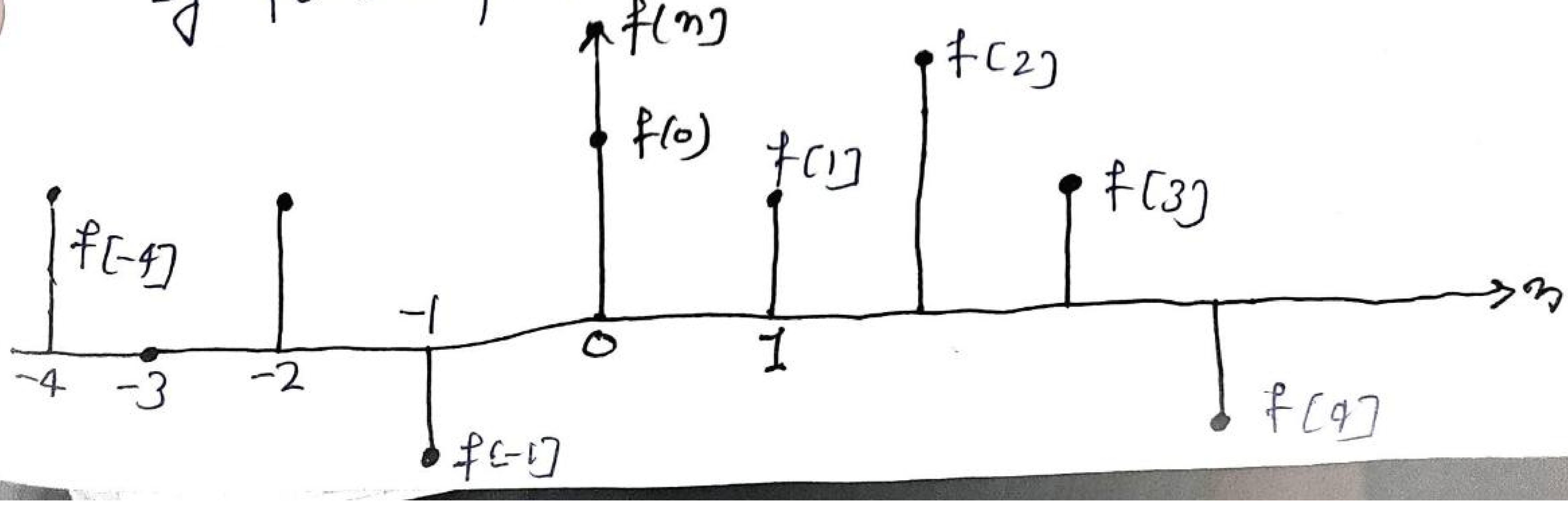


figure:- Continuous time signal

(ii) Discrete-time signal:-

A discrete-time signals are defined only at discrete instants of times, and consequently, for these signals, the independent variable takes on only a discrete set of values. A discrete-time signal is defined only at discrete instant of time, otherwise zero. Thus independent variable has discrete values and let $t = nT$, n is integer $(0, \pm 1, \pm 2, --)$ and T is the sampling time. Thus, a discrete time signal is defined as $f[nT]$, and for the sake of convenience, it is denoted by $f[n]$ or $f(n)$.



we may further classify both continuous-time and discrete-time signals as –

① Even and odd signals,
② Periodic and unperiodic signals,
③ Deterministic & non deterministic signals,
④ Energy and power signals,
⑤ Causal and non-causal signals,
⑥ Analog and digital signals.

① Even and odd signals:-

continuous even and odd signals:-

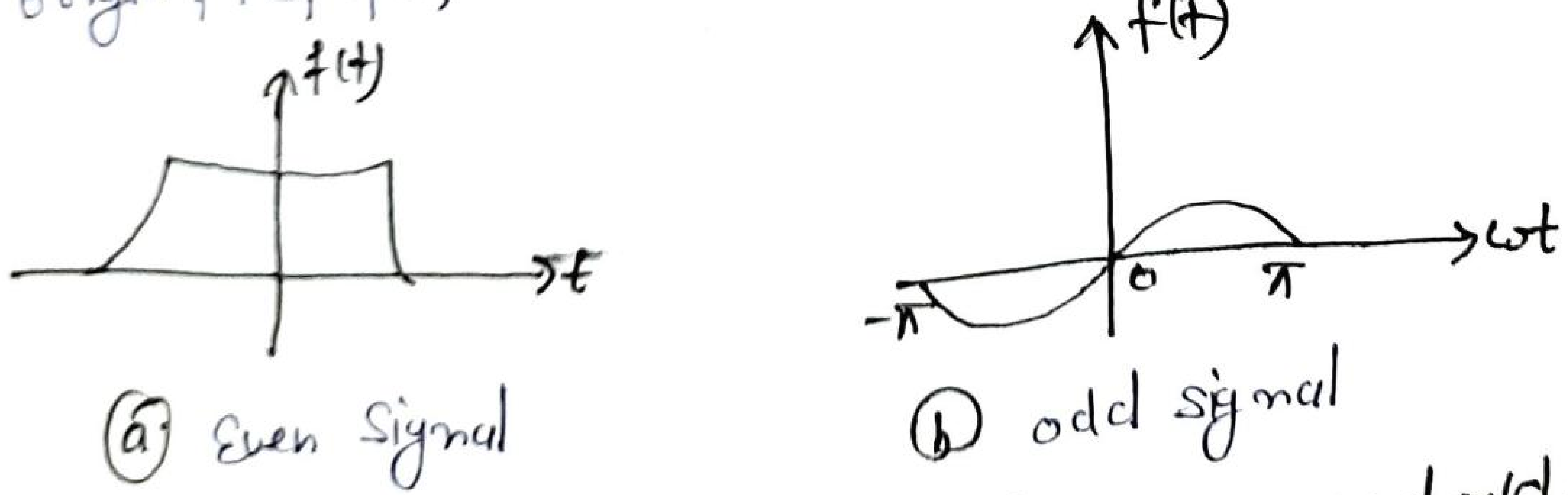
An even signal is that type of signal which exhibits symmetry in the time domaine. this type of signal is identical about the origin. A continuous signal $f(t)$ is said to be even.

if $f(t) = f(-t)$; for all t

and signal $f(t)$ is said to be odd if

$$f(-t) = -f(t) \text{ for all } t.$$

An odd signal is that type of signal which exhibits anti-symmetry. this type of signal is not identical about the origin. This may be noted that an odd continuous time signal will be zero at origin, i.e, $f(0)=0$ at $t=0$.



ⓐ Even Signal ⓑ odd signal

figure:- Continuous-time even and odd signals.

Decomposition of signal:- Any arbitrary signal can be decomposed as the sum of its even and odd parts.

$$f(t) = f_e(t) + f_o(t) \longrightarrow ①$$

We know that for even signal

$$f_e(-t) = f_e(t). \longrightarrow ②$$

and for odd signal –

$$f_o(-t) = -f_o(t) \rightarrow (3)$$

Replacing t by −t in eqn (1), we get

$$f(-t) = f_e(-t) + f_o(-t) = f_e(t) - f_o(t) \rightarrow (4)$$

Solving eqn (1) & (4) we get

$$f_e(t) = \frac{1}{2}[f(t) + f(-t)]$$

$$f_o(t) = \frac{1}{2}[f(t) - f(-t)]$$

Discrete even and odd signal:-

The discrete signal f[n] is said to be even if

$$f[n] = f[-n]; \text{ for all } n$$

and signal f(n) is said to be odd if

$$f(-n) = -f(n); \text{ for all } n$$

that is, an odd discrete time signal will be zero at origin, i.e, $f[0]=0$ at $n=0$. figure shows even and odd discrete-time signals.

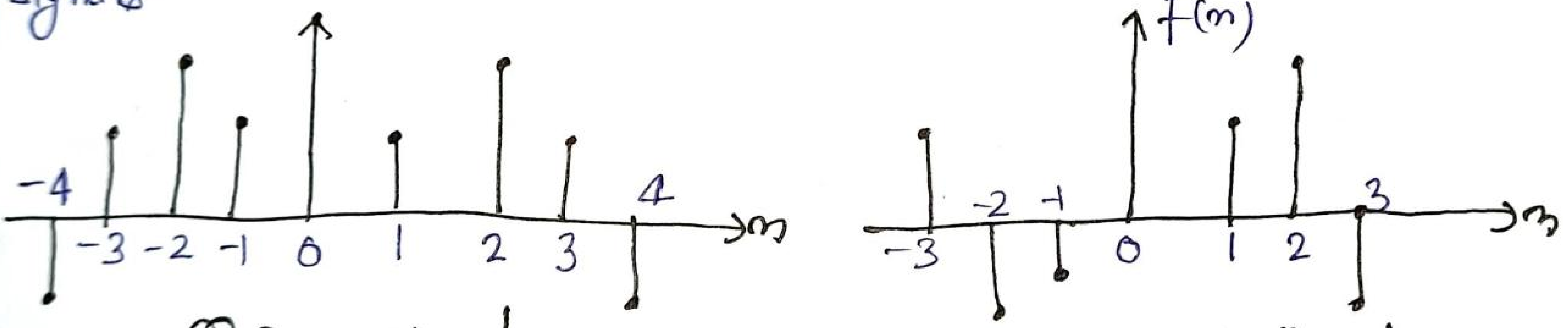


figure - Discrete time even & odd signals.

Decomposition of discrete signal f[n] can be done as in the case of continuous-time case as follows:-

$$f[n] = f_e[n] + f_o[n] \rightarrow (5)$$

we know that for even signal-

$$f_e[-n] = f_e[n] \rightarrow (6)$$

and for odd signal -

$$f[-n] = -f_o[n] \rightarrow (7)$$

Replacing n by −n in eqn (5) we get

$$f[-n] = f_e[-n] + f_o[-n] = f_e[n] - f_o[n] \rightarrow (8)$$

Solving eqn (5) & (8), we get

$$f_e[n] = \frac{1}{2}[f[n] + f[-n]]$$

$$f_o[n] = \frac{1}{2}[f[n] - f[-n]]$$

(2) Periodic & Unperiodic Signal:-

A Continuous signal $f(t)$ is said to be periodic if and only if

$$f(t) = f(t+T) \text{ for } -\infty < t < \infty, \text{ i.e, all } t. \rightarrow (9)$$

where T is the period of the signal.

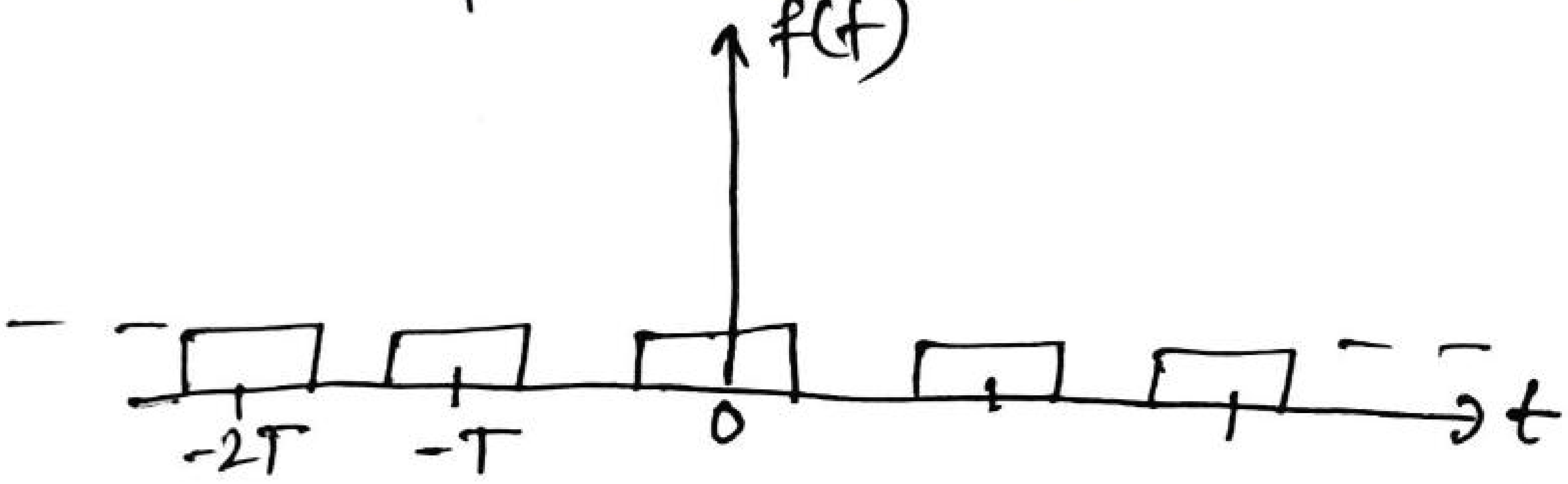


figure:- A continuous time periodic signal.

A periodic discrete-time signal $f[n]$ is said to be periodic if and only if

$$f[n] = f[n+N] \text{ for } -\infty < n < \infty, \text{ i.e, all } n.$$

where N is the period of the signal.

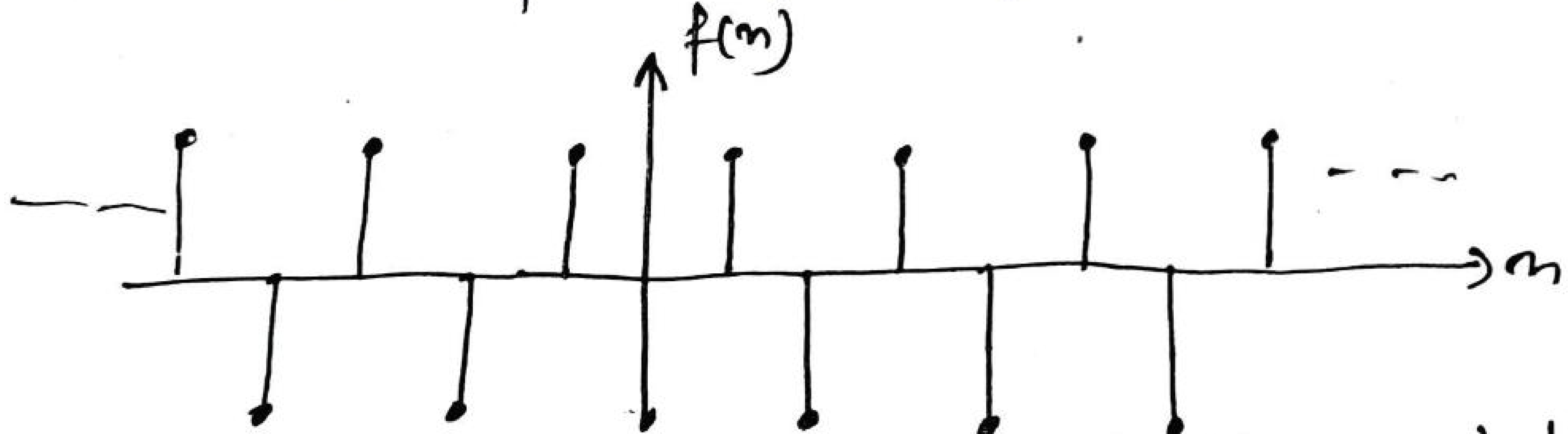


figure:- A discrete time periodic signal

(3) Deterministic and Non deterministic signals:-

A signal whose value is specified for all time either in mathematical form or a graphical form is referred to deterministic signal. The nature and amplitude of such a signal at any time can be predicted.

o) A signal, whose value can not be predicted precisely but are known only in terms of probabilistic description such as mean squared value, mean value and so, on is referred to random or non deterministic signal. In otherwords, a non-deterministic signal is one whose occurrence is always random in $\frac{(p_s)}{}$ nature. The pattern of such a signal is quite irregular.

④ <u>Energy and Power signals</u>:—

a) The energy signal is one which has finite energy and zero average power. Hence, $f(t)$ is an energy signal. if: $0<E<\infty$ and $P=0$

b) The power signal, is one which has finite average power and infinite energy. Hence, $f(t)$ is a power signal if: $0<P<\infty$ and $E=\infty$.

Energy Signal	Power signal
① Total energy of the signal is finite and non-zero, i.e, $0<E_\infty<\infty$.	① The average power of the signal is finite and non zero, i.e $0<P_\infty<\infty$.
② These signal are time limited.	② The signals can exist over infinite time
③ power of energy signal is zero. i.e, $P_\infty=0$	③ Energy of the power signal is infinite, i.e, $E_\infty=\infty$.
④ Unperiodic signals and deterministic signals are energy signals.	④ Periodic signals and random signals are power signals.
⑤ The total energy of the signal is given by $E_\infty = \lim_{T\to\infty}\int_{-T}^{T}\|f(t)\|^2 dt = \int_{-\infty}^{\infty}\|f(t)\|^2 dt$	⑤ Average power of the signal is given by $P_\infty = \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}\|f(t)\|^2 dt = \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2}\|f(t)\|^2 dt$.

⑤ <u>Causal and Non-causal signals</u>! —

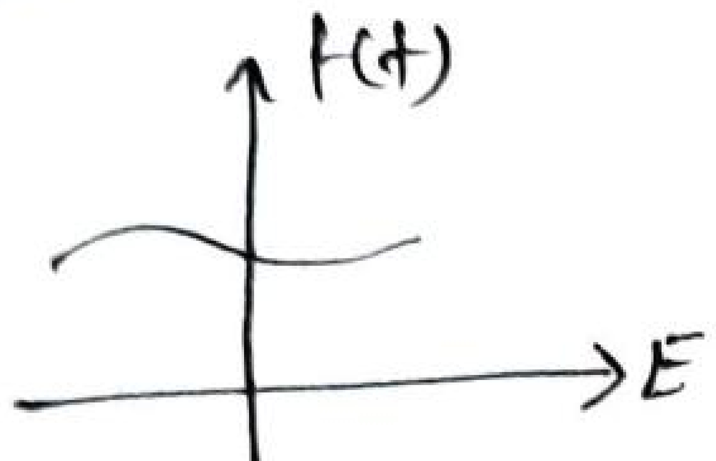
A Continuous-time signal $f(t)$ is said to causal if $f(t)=0$ for $t<0$ for, i.e, $f(t)$ does not start before $t=0$. On the other hand, a signal, which is zero for all $t\geq 0$ is called an anticausal signals.

For example:— $u(t)$, $e^{at}u(t)$, etc are causal signals.

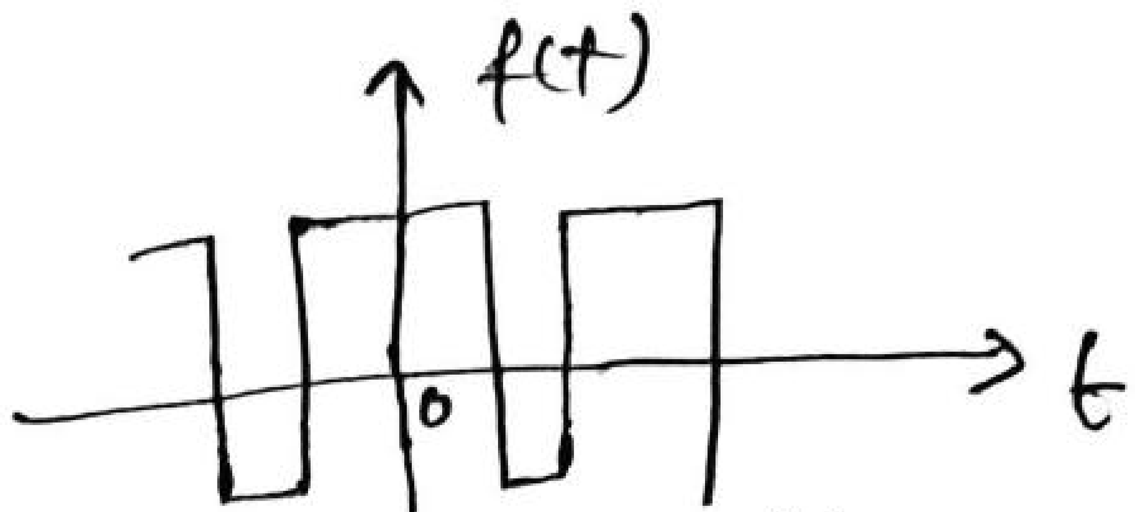
$u(-t)$, $e^{a|t|}$ etc. are non causal signals.

⑥ <u>Analog and Digital signal</u>:— A signal whose amplitude can take on any value in a continuous range is an analog signal. It means that an analog signal amplitude can take on an infinite number of values.

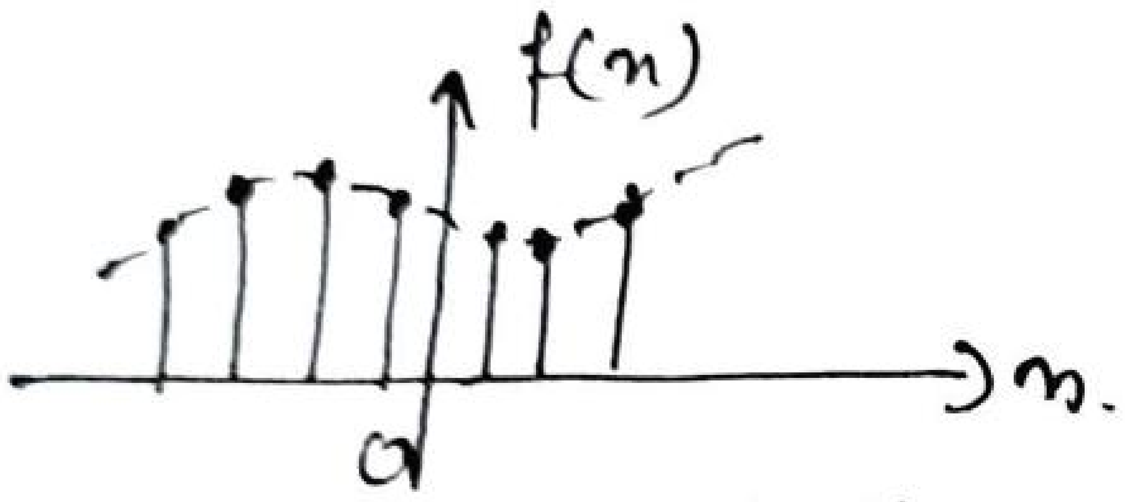
A digital signal is one whose amplitude can take on only a finite number of values. It is clear that analog is not necessarily continuous-time and digital need not be discrete-time.



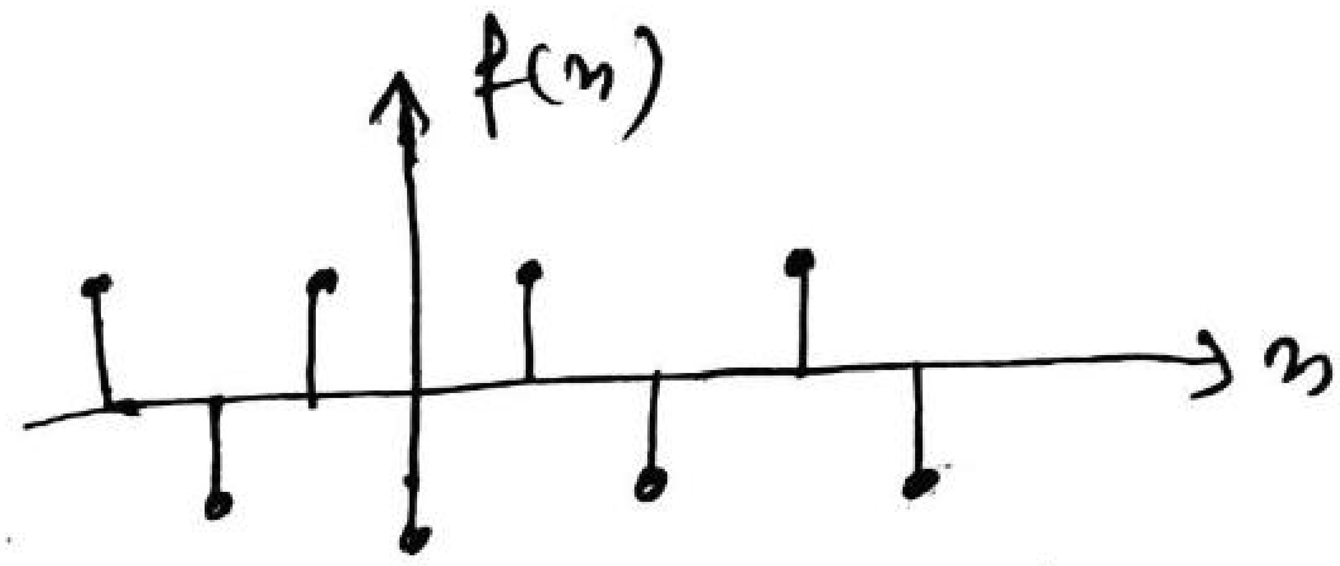
(a) Analog continuous time signal



(b) Digital, continuous time signal



(c) Analog discrete time signal



(d) Digital, discrete-time signal

Transformation of the independent variable:—

(i) Time shifting.
(ii) Time scaling.
(iii) Time inversion or folding or reflection or reversal.

(i) Time shifting:- A signal may be shifted in time. It means that a may be either advanced in the time axis or delayed in the time axis.

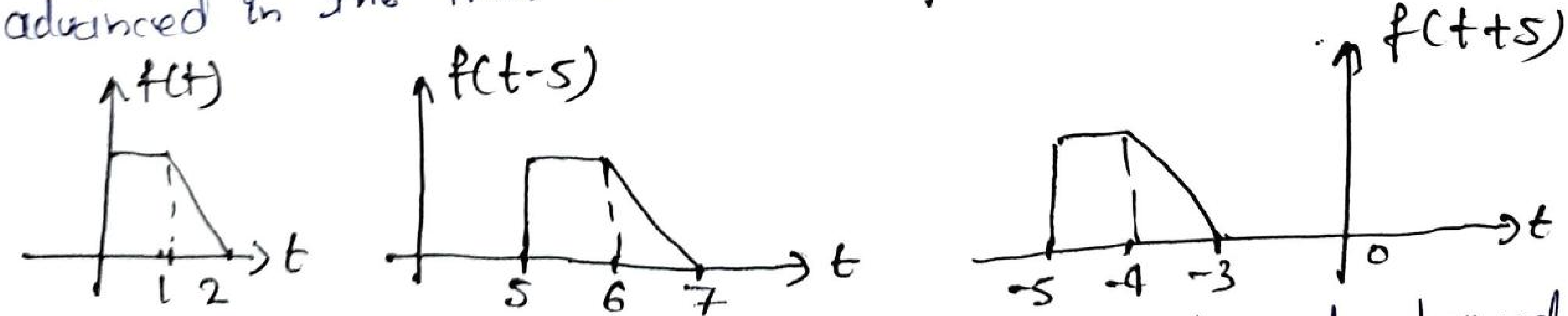
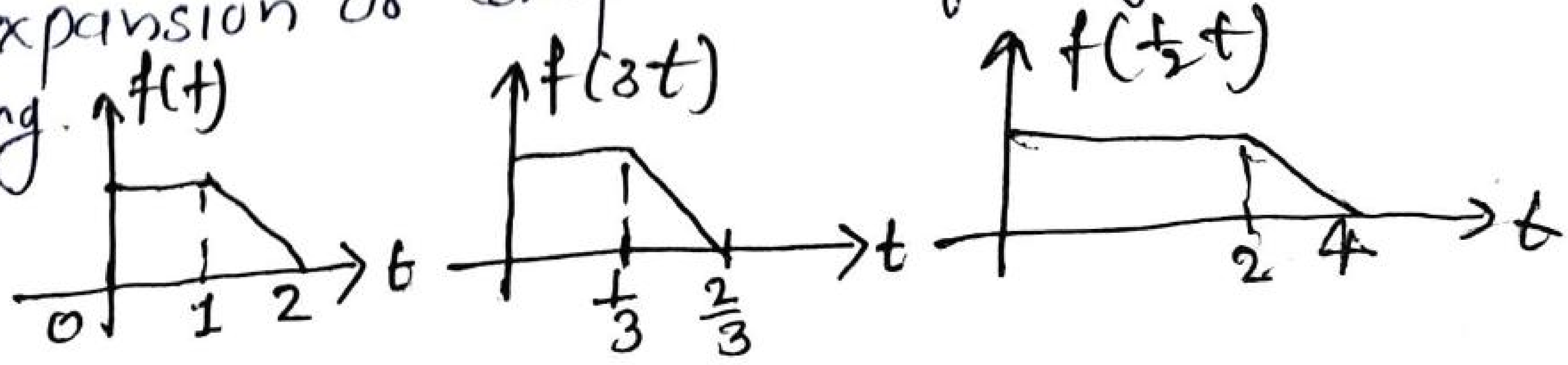


figure:- signal and their respective delayed and advanced signals.

(ii) Time-scaling:- The expansion or compression of a signal in time is called as time-scaling.



(iii) Time Inversion or Folding:-

Time inversion is obtained by replacing the independent variable t by -t in case of continuous time signal. Consequently, the mirror image of the signal about the vertical axis is time inversion of the signal. It means that to time invert a signal, we must replace t with -t in case of continuous time signal and n with -n in case of discrete-time signal.

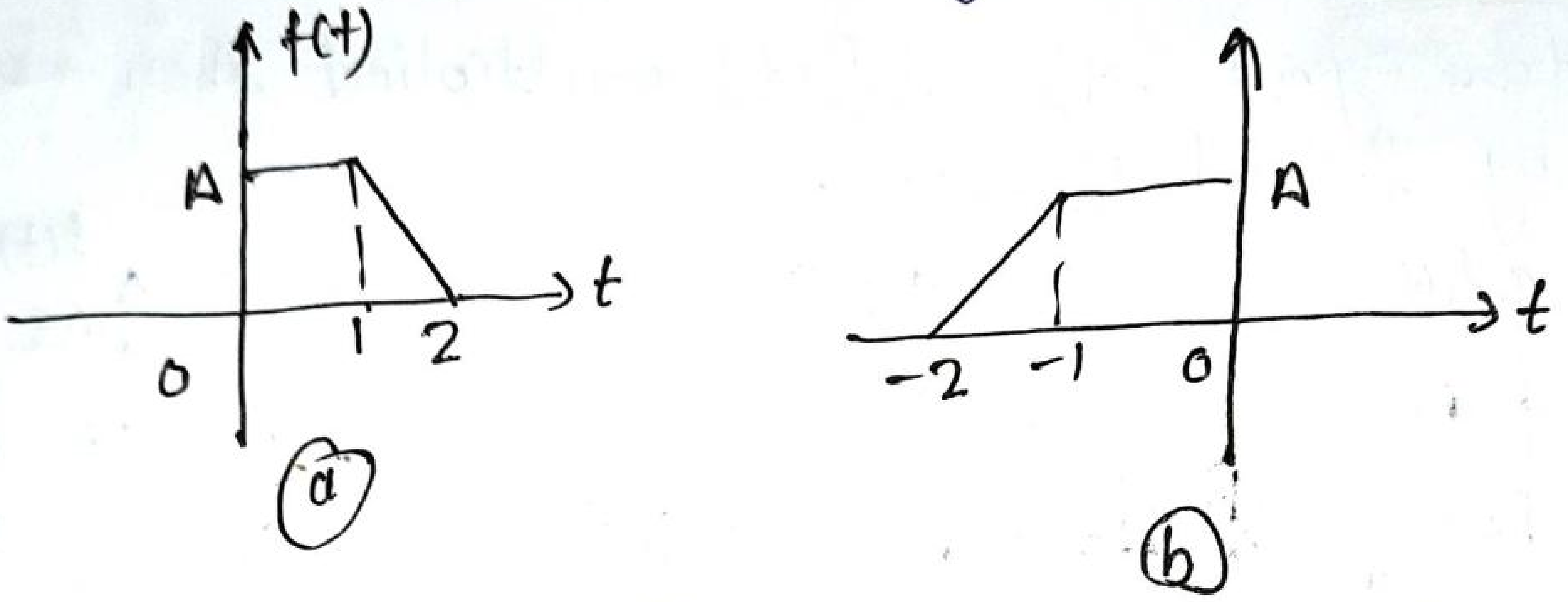


figure:- Time-inversion or Folding.

Transformation of the dependent variable:-

(i) Amplitude scaling.
(ii) addition.
(iii) Multiplication.
(iv) Differentiation and first difference.
(v) Integration and running sum.
(vi) Convolution integral and convolution sum.

(1) Amplitude scaling:- In the amplitude scaling, a signal F(t) results from a signal f(t) on amplitude scaling by a real value K as $F(t) = Kf(t)$.

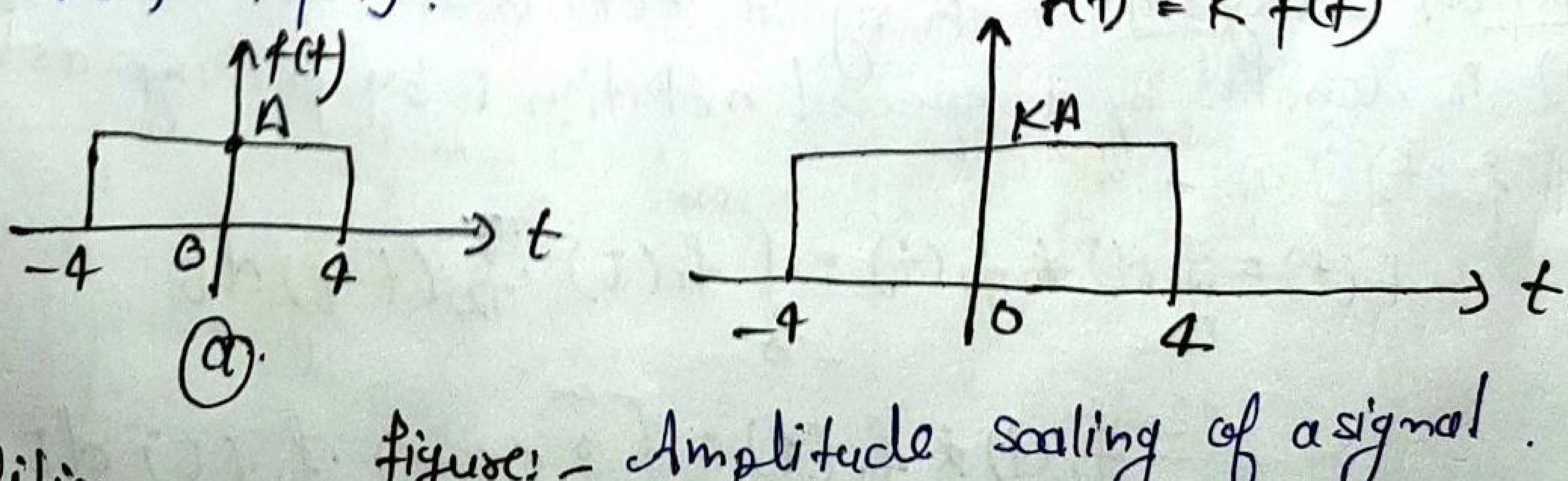


figure:- Amplitude scaling of a signal.

(2) addition:- Consider, two signals $f_1(t)$ and $f_2(t)$ added then result f(t) is given by - $f(t) = f_1(t) + f_2(t)$

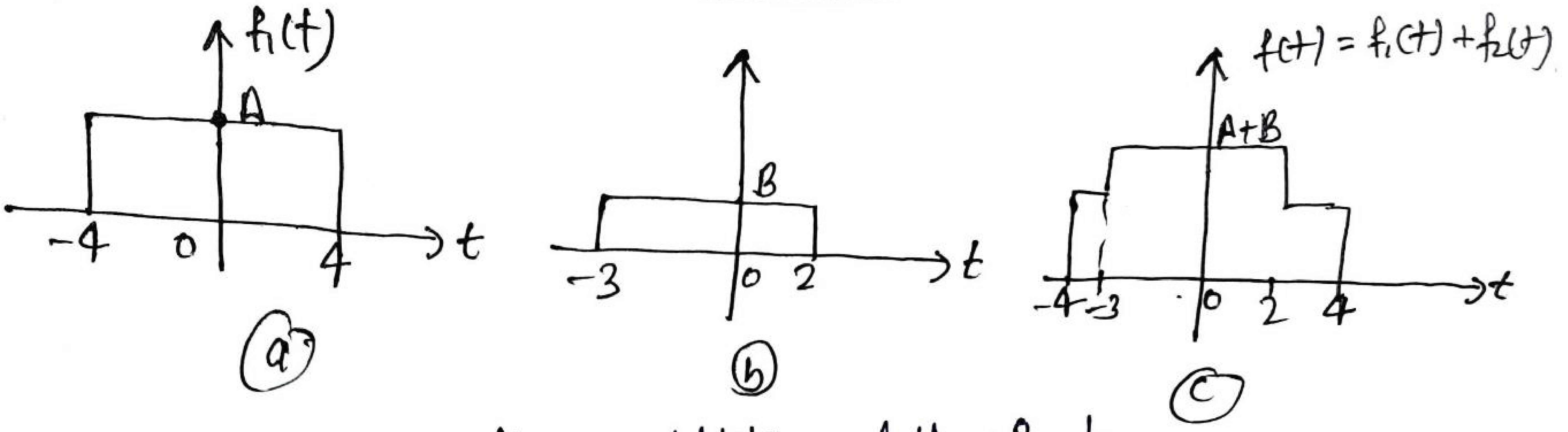


figure- Addition of the signals

③ **Multiplication:-**

Consider two signal $f_1(t)$ and $f_2(t)$ multiplied then results $f(t)$ is given by $f(t) = f_1(t) \cdot f_2(t)$.

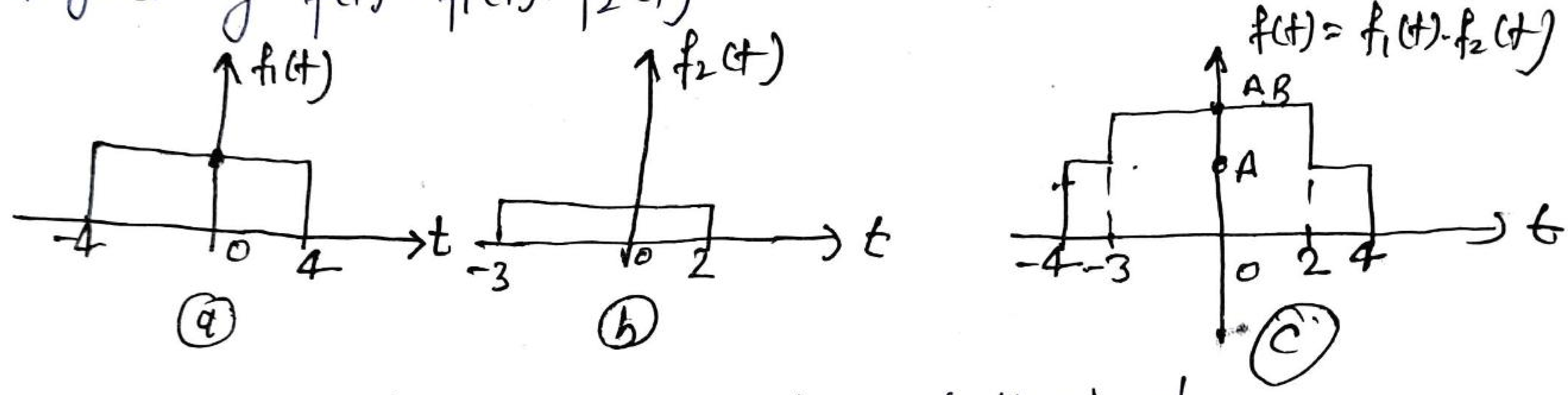


figure- Multiplication of the signal

④ **First differentiation:-**

The first differential of $f(t)$ is given by -

$$f(t) = f'(t) = \frac{d}{dt}[f(t)]$$

⑤ **Running Integration:-**

The running integration of a signal $f(t)$ is given by -

$$F(t) = \int_{-\infty}^{t} f(\tau)\, d\tau.$$

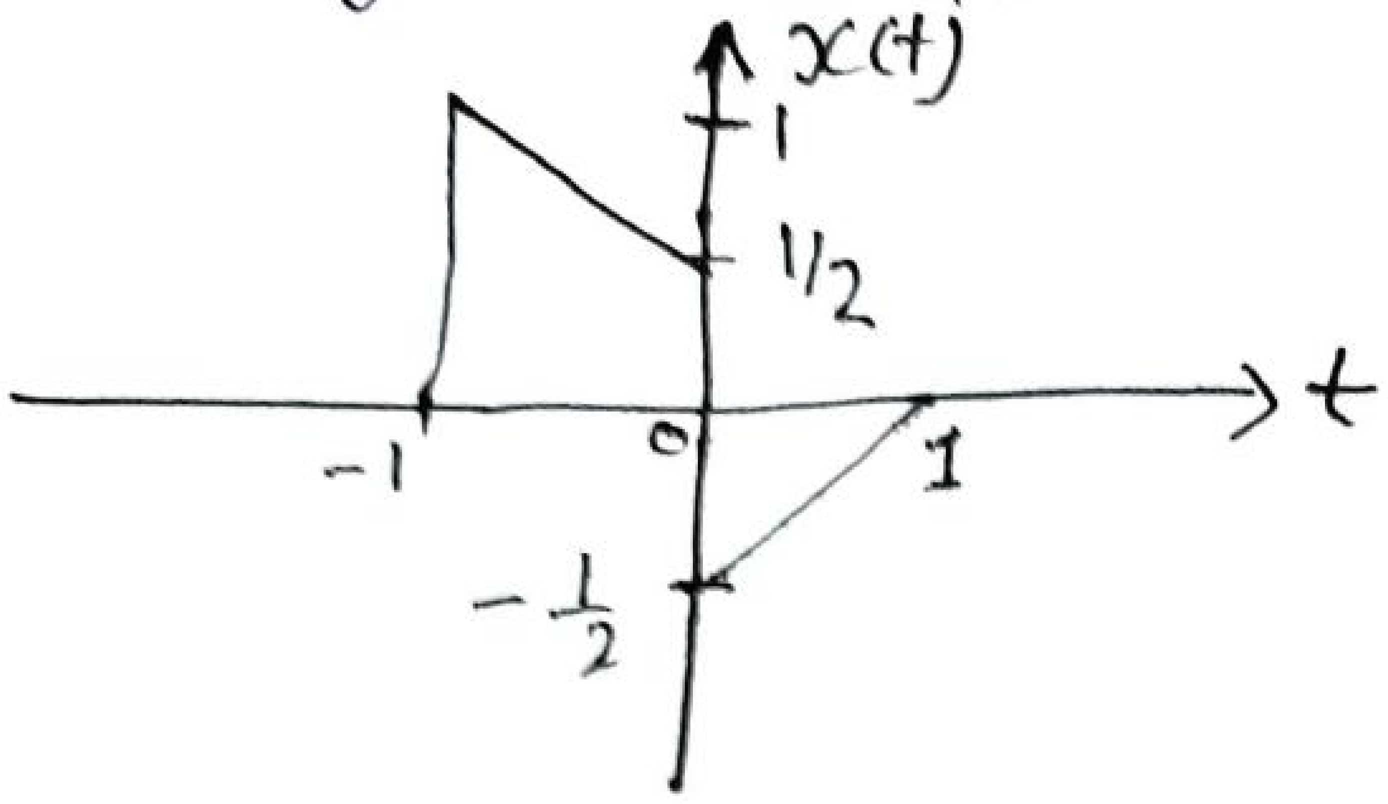
where τ is variable for integration.

⑥ **Convolution Integral:-** A signal $F(t)$ is the convolution of $f_1(t)$ and $f_2(t)$ is denoted by a special notation [by putting a star b/w $f_1(t)$ & $f_2(t)$] as -

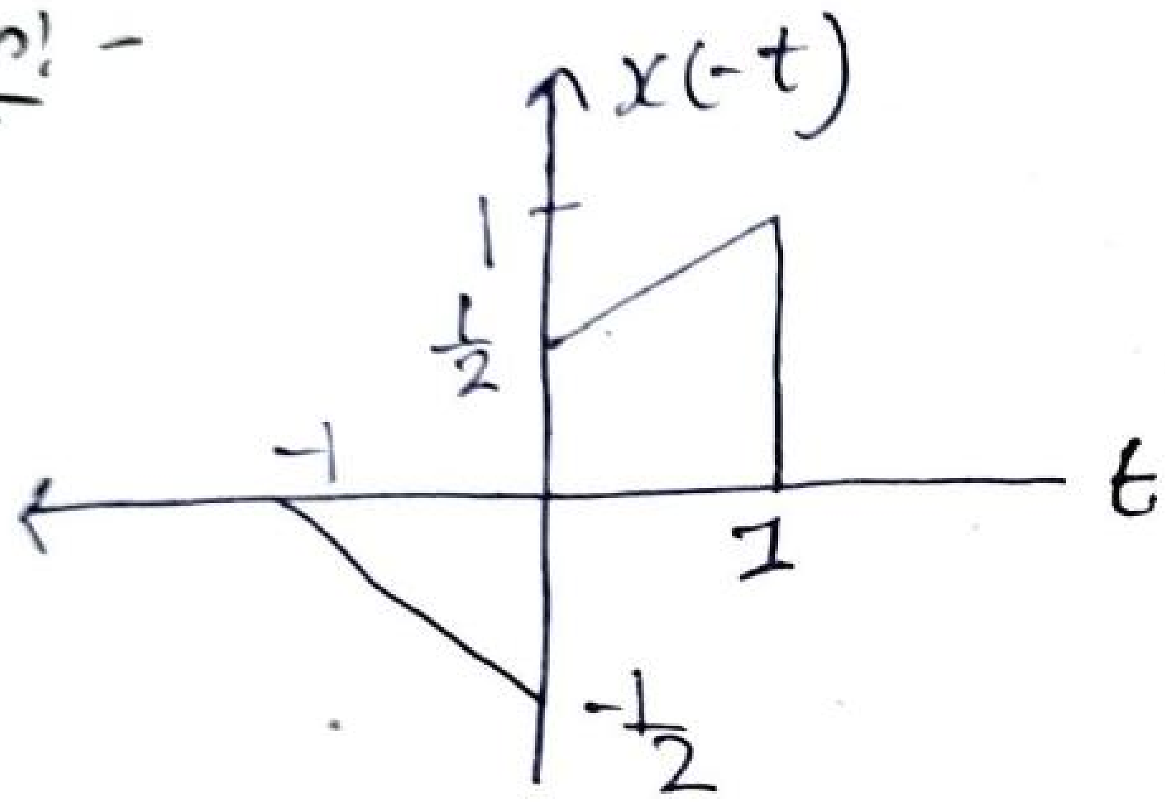
$$F(t) = f_1(t) * f_2(t) = \int_{-\infty}^{\infty} f_1(\tau) \cdot f_2(t-\tau)\, d\tau$$

$$= f_2(t) * f_1(t) = \int_{-\infty}^{\infty} f_1(t-\tau) \cdot f_2(\tau)\, d\tau.$$

Q:- Decompose the signal $x(t)$ as shown in figure, into even and odd parts through a systematic sequence of diagrammatic approach.



Solution:-

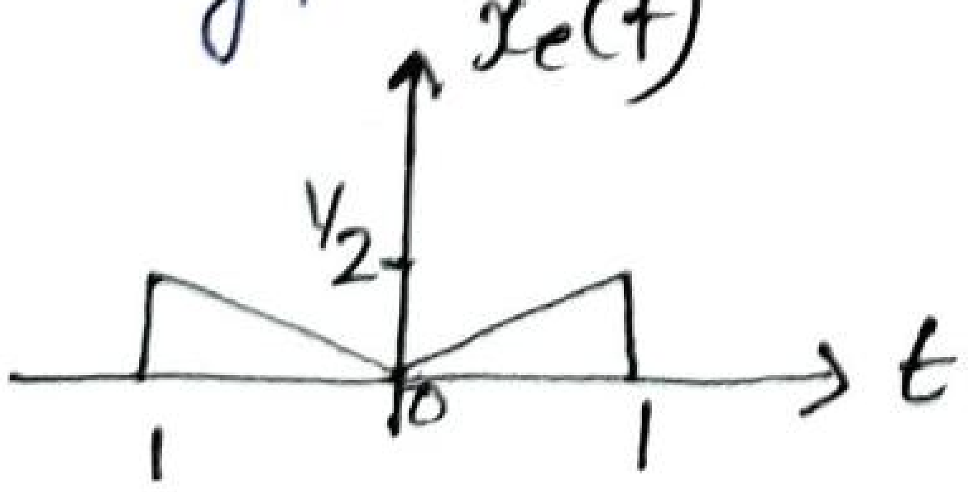


We know that the even and odd parts of the signal is given by

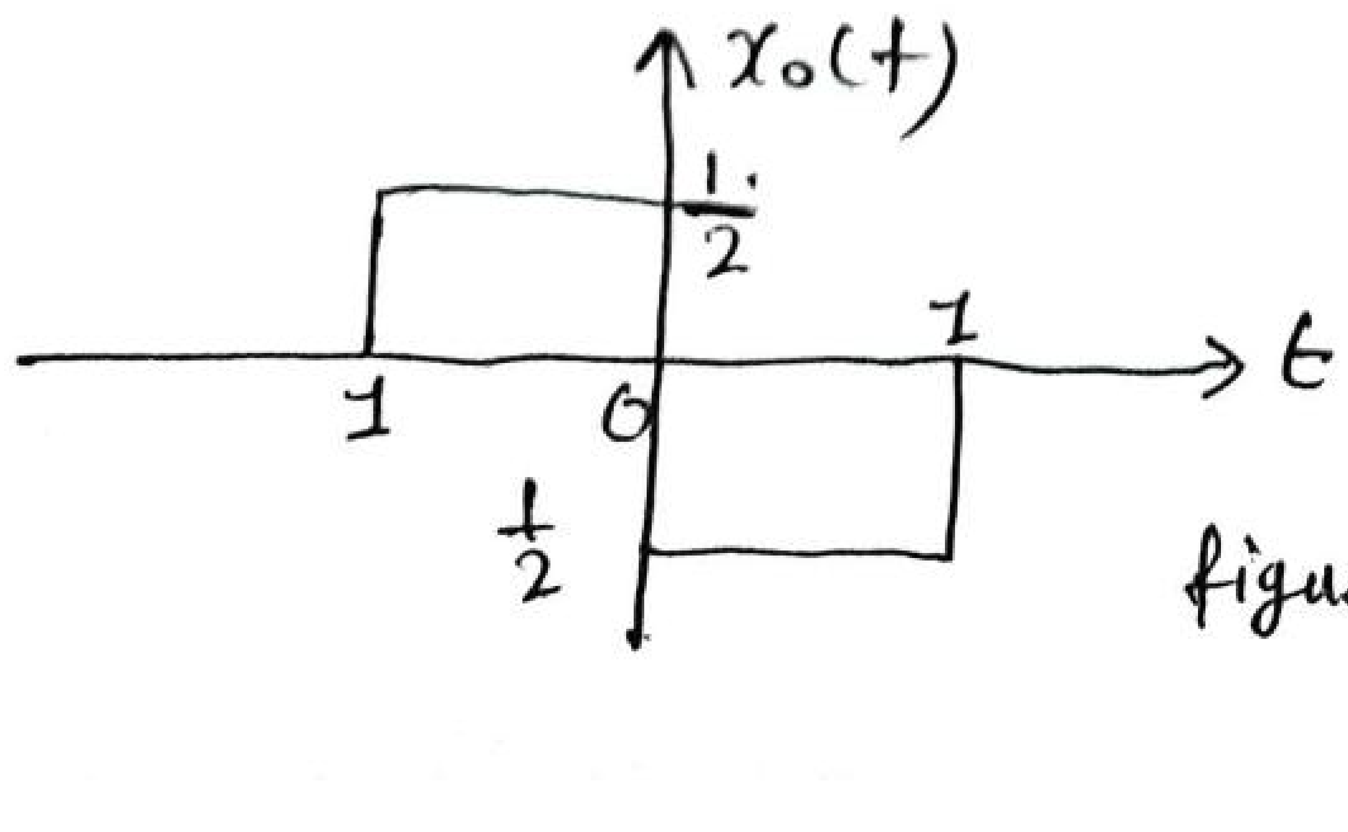
$$x_e(t) = \frac{1}{2}[x(t) + x(-t)]$$

& , $x_o(t) = \frac{1}{2}[x(t) - x(-t)]$

Now, the even and odd parts of the signal are as shown in figure (b) & (c) respectively,

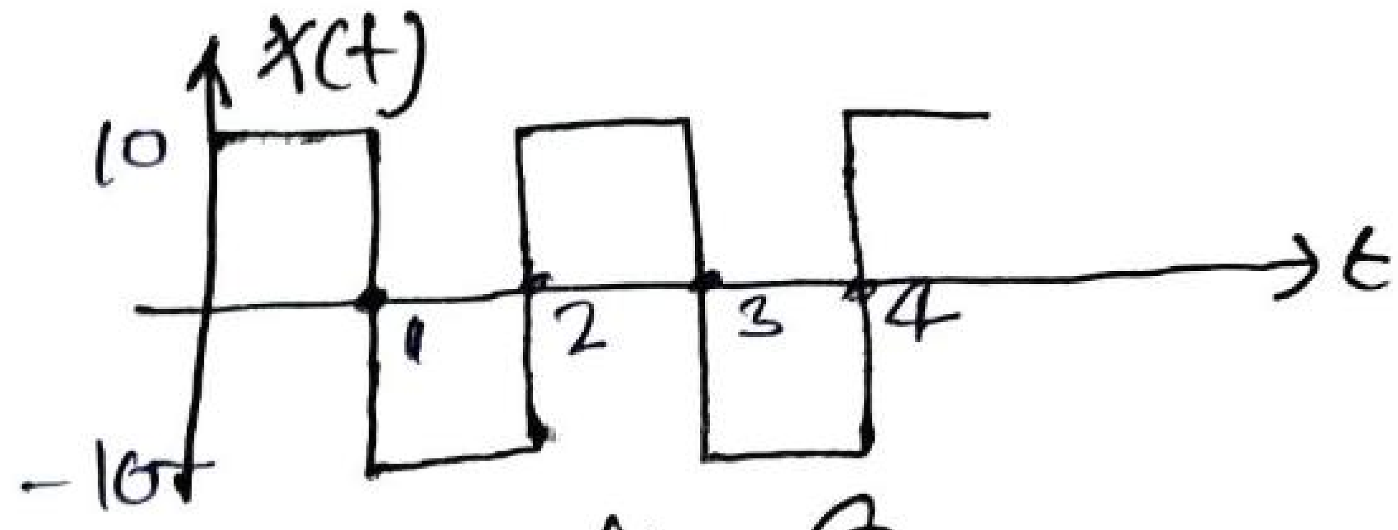


figure(d)

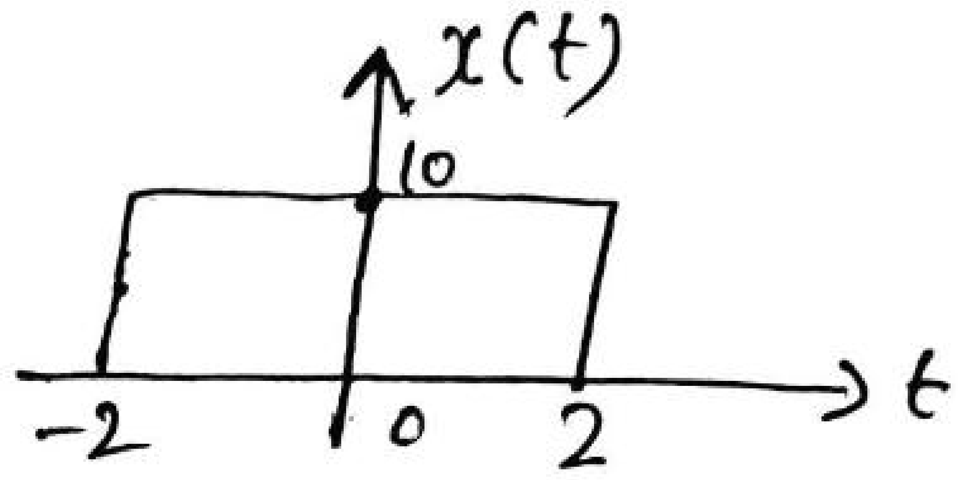


figure(b)

Q2 check whether the [P-7] signal shown in figure (a) & (b) are energy signals or power signals. Also determine energy & power for each signal.



figure(a)



figure(b).

Solution:- (a) $P = \frac{1}{T}\int_0^T x^2(t)\,dt$

$$= \frac{1}{2}\int_0^2 (10)^2 dt$$

$$= \frac{1}{2}\cdot 100\,[t]_0^2 = 50[2-0] = 100$$

$$E = \lim_{T\to\infty}\int_{-T/2}^{T/2}[x(t)]^2 dt$$

$$= \lim_{T\to\infty}\left[\int_{-T/2}^{0} 0.dt + \int_0^{T/2}(10)^2 dt\right]$$

$$= \lim_{T\to\infty} 100\,[t]_0^{T/2}$$

$$= \lim_{T\to\infty} 100\left[\frac{T}{2}\right] = \infty$$

(b) $P_\infty = \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2}[x(t)]^2 dt$

$$= \lim_{T\to\infty}\frac{1}{T}\int_{-2}^{2} 100\,dt = 0$$

$$E = \int_{-\infty}^{\infty} x^2(t)\,dt = \int_{-\infty}^{-2} 0 dt + \int_{-2}^{2}(10)^2 dt + \int_{2}^{\infty} 0 dt$$

$$= 100[t]_{-2}^{2} = 100[2-(-2)] = 400$$

General descriptions of signals:—

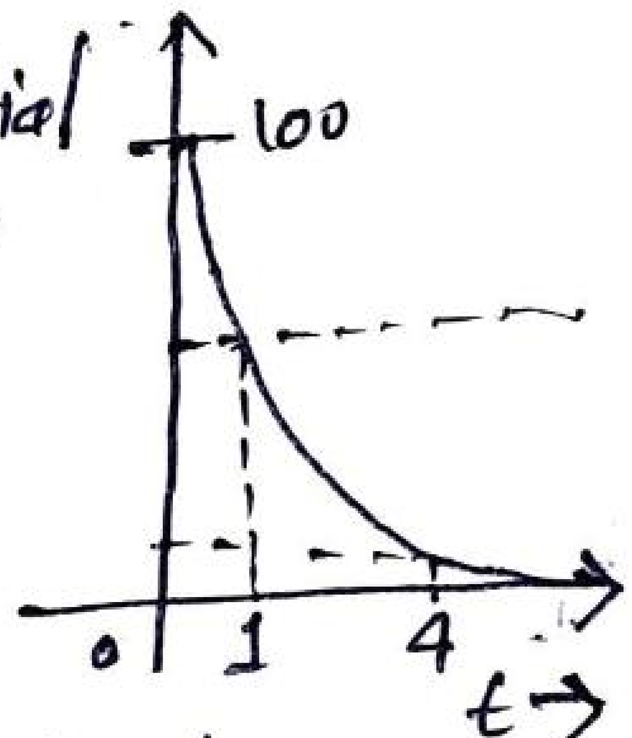
(i) Time Constant,
(ii) r.m.s Value,
(iii) D.C Value,
(iv) Duty cycle,
(v) Creast factor,
(vi) Form factor.

(i) Time Constant:—

∘> In many physical problems, it is important to know how quikly a waveform decays. A useful measure of the decay of ~~an exponential waveform is given by~~ the decay of an exponential is the time constant T.

Consider the exponential waveform is given by

$$r(t) = K e^{-t/T} u(t)$$



From a plot of r(t) in figure, we see that when t=T,

$$r(T) = 0.37\, r(0).$$

Also, $r(4T) = 0.02\, r(0)$

observe that the larger the time constant, the longer it requires for the waveform to reach 37% of its peak value. In circuit analysis, common time constants for R-L and R-C circuits are the $\frac{R}{L}$ and RC respectively.

(ii) r.m.s Value:—

The r.m.s or root mean square value of a periodic waveform e(t) is defined as—

$$e_{r.m.s} = \sqrt{\left[\frac{1}{T}\int_0^T e^2(t)\,dt\right]}$$

(iii) D.C Value:—

The d.c value of a waveform has meaning only when the waveform is periodic. It is the average value of the waveform over one period.

$$e_{dc} = e_{ave} = \frac{1}{T}\int_0^T e(t)\,dt.$$

(iv) duty cyle:—

∘> The duty cycle, D is defined as the ratio of the time duration of the positive cycle to of a periodic waveform to the period, T, i.e.

$$D = \frac{t_0}{T}.$$

(v) Creast Factor:—

∘> The creast factor is defined as the ratio of maximum value of the period waveform and its r.m.s value, i.e,

$$\text{creast factor} = \frac{\text{maximum value of the waveform}}{\text{r.m.s value of the waveform}}$$

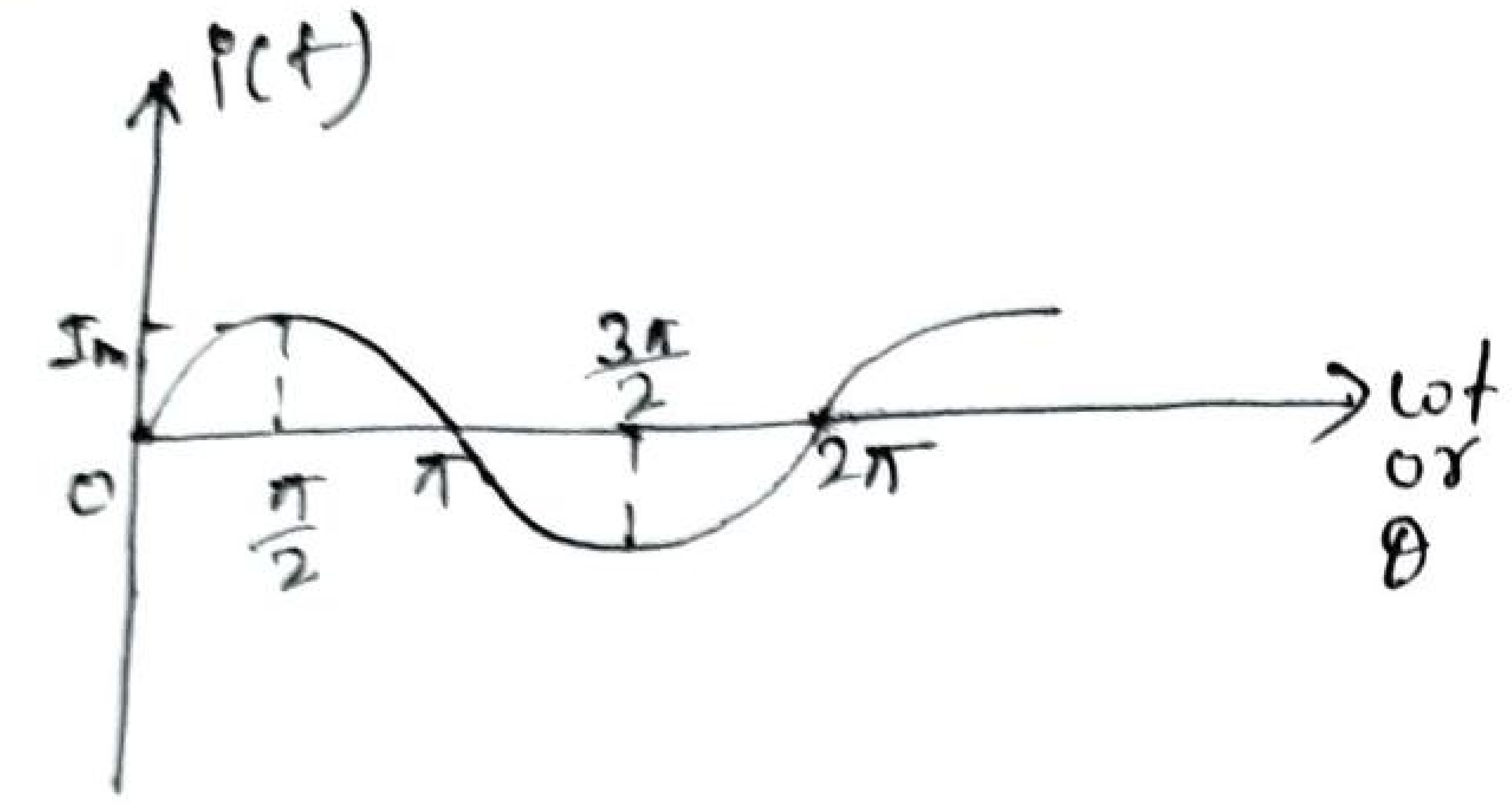
(vi) form factor:—

It is the ratio of r.m.s value and the average value of the periodic waveform. It may be expressed as

$$\text{Form factor} = \frac{\text{r.m.s value of the waveform}}{\text{average value of the waveform}}$$

Question:- Find the r.m.s value of given waveform shown in figure, and also find the form factor.

Solved:-



Step I :- To obtain r.m.s Value-

$$I = I_m \sin\omega t = I_m \sin\theta$$

$$i_{r.ms} = \sqrt{\left[\frac{1}{T}\int_0^T i^2(t)\,d\theta\right]}$$

$$T = 2\pi$$

$$i_{r.ms} = \sqrt{\left[\frac{1}{2\pi}\int_0^{2\pi} I_m^2 \sin^2\theta\, d\theta\right]}$$

$$= \sqrt{\frac{I_m^2}{4\pi}\int_0^{2\pi}(1-\cos 2\theta)\,d\theta}$$

(since, $\cos 2\theta = 1 - 2\sin^2\theta$)

$$= \sqrt{\frac{I_m^2}{4\pi}\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{2\pi}}$$

$$= \sqrt{\frac{I_m^2}{4\pi}\left[\left(2\pi - \sin\frac{4\pi}{2}\right) - 0\right]}$$

$$= \frac{I_m}{\sqrt{2}} = 0.707 \times \text{maximum Value}$$

Step II :- To obtain average Value i_{av}.

The average Value is given by -

$$i_{av} = \frac{1}{T}\int_0^T i(t)\,dt.$$

$$i_{av} = \frac{1}{\pi}\int_0^{\pi} I_m \sin\theta\, d\theta.$$

$$i_{av} = \frac{I_m}{\pi}\int_0^{\pi}\sin\theta\, d\theta = \frac{I_m}{\pi}[-\cos\theta]_0^{\pi}$$

$$= \frac{I_m}{\pi}[(-\cos\pi) - (-\cos 0^\circ)]$$

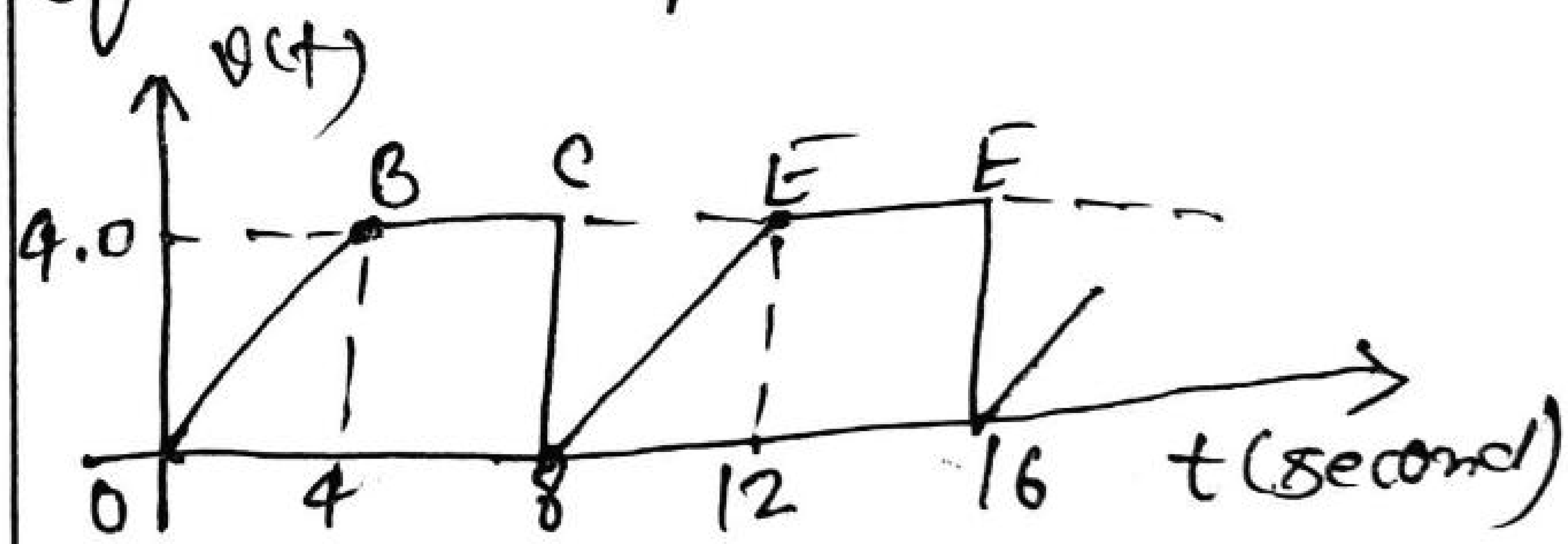
$$= \frac{2I_m}{\pi} = 0.637 \times \text{maximum Value } I_m.$$

Step III :- To obtain form factor.

$$\text{Form factor} = \frac{\text{r.m.s Value}}{\text{average Value}}$$

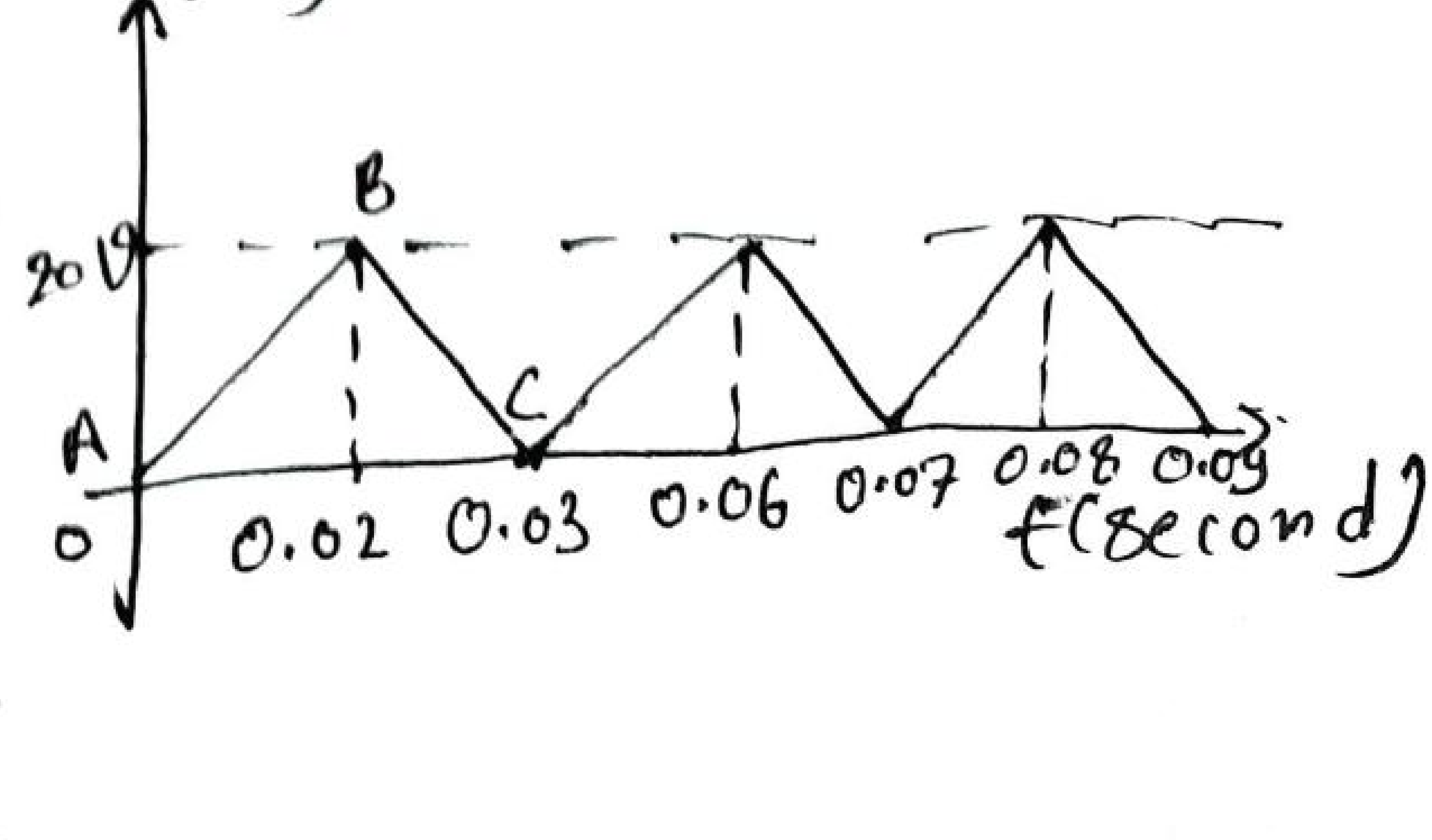
$$= \frac{\frac{I_m}{\sqrt{2}}}{\frac{2I_m}{\pi}} = 1.11$$

Question:- obtain the form factor of the waveform shown in figure



Ans form factor = 1.12

Question:- Find the form factor of the waveform shown in figure.



Step and impulse Response:-

- The problem of obtaining the step and impulse is stated as follows: Given a network with zero initial energy, we are required to solve for a specified response (current or voltage) due to a given excitation function u(t) or δ(t), which either can be a current or a voltage source.

- If the excitation is a step of voltage, the physical analogy is that of a switch closing at time t=0 which connects of a 1V battery to a circuit. The physical analogy of an impulse excitation is that of a very short (compared to the time constants of the circuit) pulse with large amplitude.

Solution of Network equations:-

We apply our knowledge of differential equations to the analysis of linear networks. There are two important points in network analysis -

(i) The writing of network equations and
(ii) the solution of these equations.

Solution of first order Homogenous Differential equations:-

Let, $\frac{dx(t)}{dt} + Qx(t) = 0$

$$\frac{dx(t)}{x(t)} = -Q\,dt$$

on integration, we get -

$$\ln x(t) = -Qt + K_1$$

$$K_1 = \ln K$$

$$\ln x(t) = \ln e^{-at} + \ln K$$

$$\ln x(t) = \ln (K \cdot e^{-at}).$$

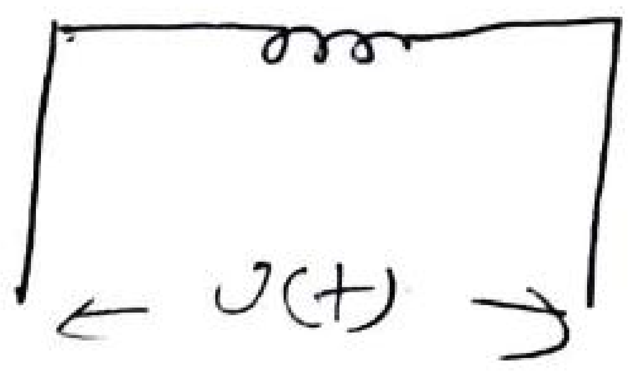
$$x(t) = Ke^{-Qt}$$

Solution of first order Non Homogeneous differential equation:-

Initial Condition for an Inductor :⇒

o) The Voltage - current relationship for an

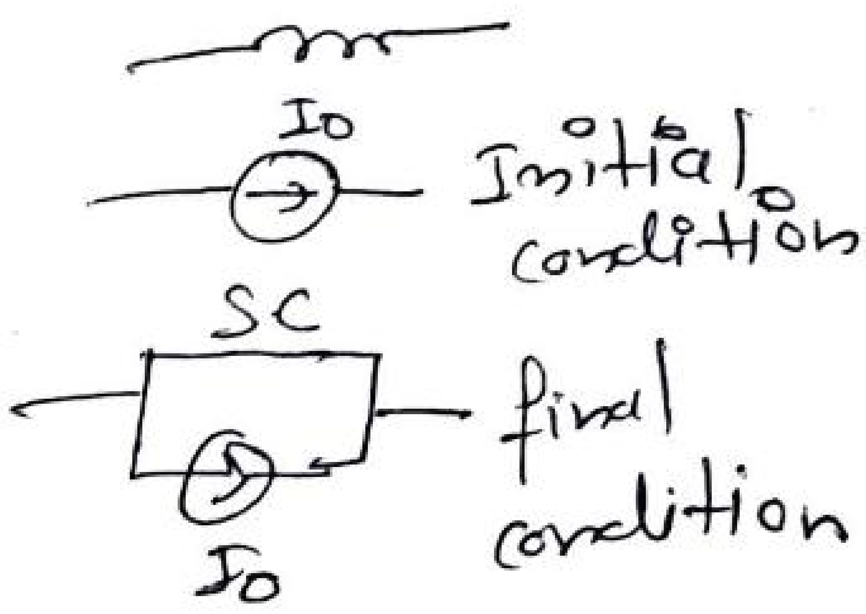
$$\left[V_L = L\frac{di}{dt} \right]$$



$$V_L dt = L di$$

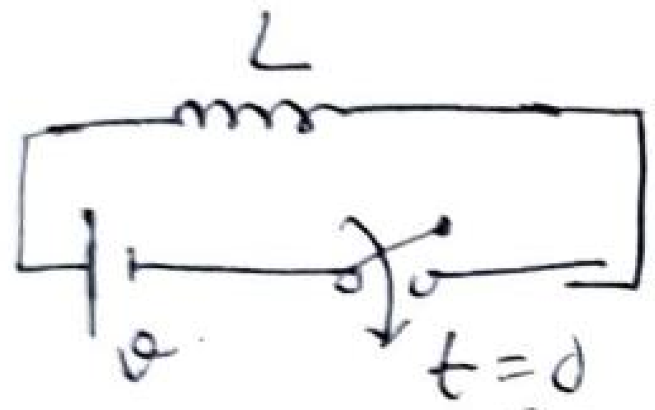
$$\frac{V}{L} dt = di$$

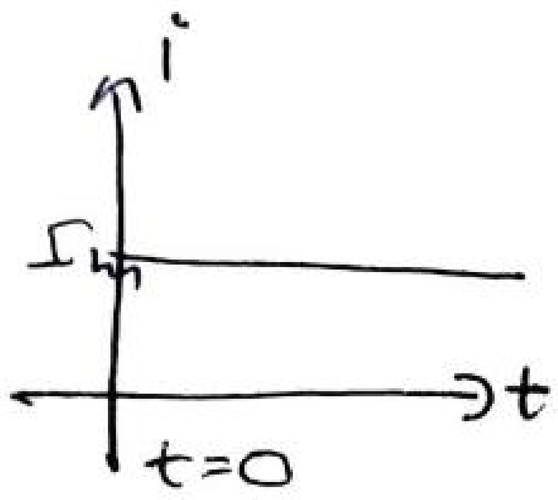
$$\boxed{\frac{1}{L}\int v dt = i(t)}$$



$$\frac{1}{L}\int v dt + i(0) = i(t)$$

$$i_L(0^+) = \frac{1}{L}\int_{0^-}^{0^+} V(\tau)\, d\tau = i_L(0^-)$$





$$v = L\frac{di}{dt} \quad \frac{-I_n - 0}{0 - 0} = \infty$$

$t = 0^-$, ⟶ O·C

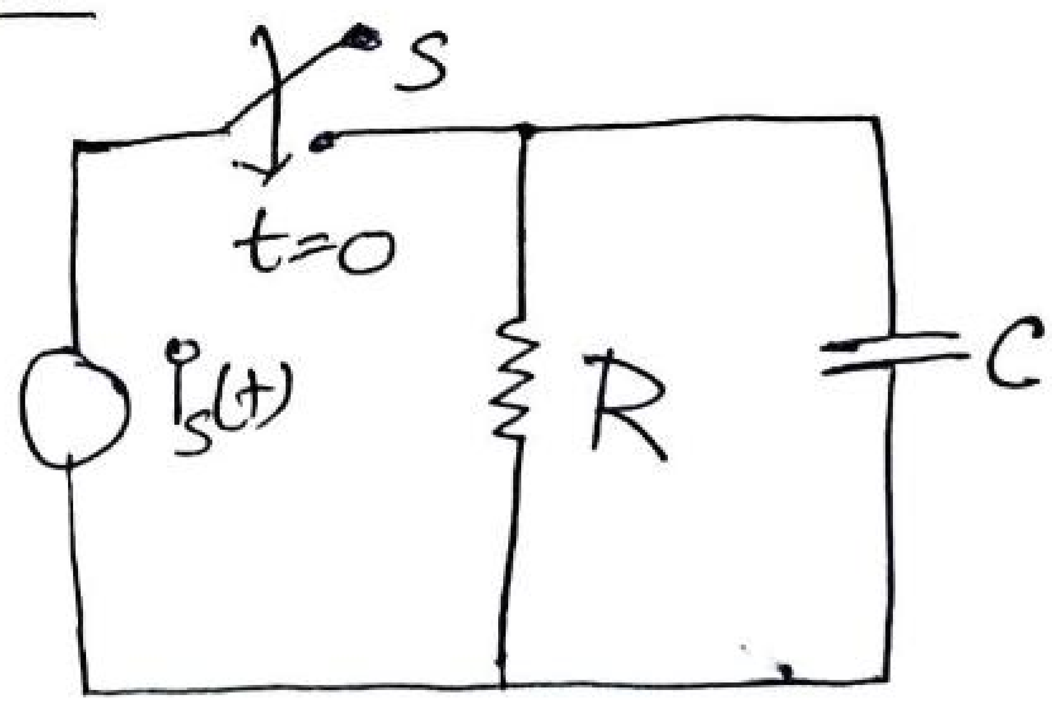
$t = 0^+$, ⟶ O·C

$t = \infty$

$v = L\frac{di}{dt}$

⟶ short circuits

Final Conditions for sinusoidal Excitations :-



The source current is -

$$i_s(t) = (I_0 \sin\omega_0 t)\, u(t)$$

the steady state solution would take the form →

$$V_{ip}(t) = |V(j\omega_0)| \sin[\omega_0 t - \phi(\omega_0)]$$

$$V(j\omega_0) = \frac{I_0}{|Y(j\omega_0)|} = \frac{I_0}{(G^2 + \omega_0^2 C^2)^{1/2}}$$

$$\& \ \phi(\omega_0) = \tan^{-1}\left(\frac{\omega_0 C}{G}\right)$$

so that $$v_p(t) = \frac{I_0}{(G^2 + \omega_0^2 C^2)^{1/2}} \sin\left[\omega_0 t - \tan^{-1}\left(\frac{\omega_0 C}{G}\right)\right]$$

$$\frac{dx(t)}{dt} + Px(t) = Q$$

where P is constant and Q may be a function of independent variable or a constant. The equation is not altered if every term is multiplied by e^{Pt}, i.e,

$$e^{Pt}\frac{dx(t)}{dt} + e^{Pt}Px(t) = Qe^{Pt}.$$

since, $d(x, y) = xdy + ydx$.

$$\frac{d}{dt}[x(t)e^{Pt}] = Qe^{Pt}$$

$$x(t)\,e^{Pt} = \int Qe^{Pt}dt + K$$

$$x(t) = e^{-Pt}\int Qe^{Pt}dt + Ke^{-Pt}$$

If Q is a constant, then –

$$x(t) = e^{-Pt}Q\cdot\frac{e^{Pt}}{P} + Ke^{-Pt}$$

$$\boxed{x(t) = \frac{Q}{P} + Ke^{-Pt}}$$

Solution of second order differential equation:–

A second order homogeneous differential equation with constant coefficient may be written in the general form–

$$a_0\frac{d^2x(t)}{dt^2} + a_1\frac{dx(t)}{dt} + a_2x(t) = 0 \quad \rightarrow ①$$

$$x(t) = Ke^{mt} \quad \rightarrow ⑪$$

Putting $x(t)$ from eq^n ② in eq^n ①, we get–

$$a_0m^2Ke^{mt} + a_1mKe^{mt} + a_2Ke^{mt} = 0$$

$$a_0m^2 + a_1m + a_2 = 0$$

$$m_1, m_2 = \frac{-a_1}{2a_0} \pm \frac{1}{2a_0}\sqrt{a_1^2 - 4a_0a_2}$$

$$x(t) = K_1e^{m_1t} + K_2e^{m_2t}$$

$$\boxed{x(t) = K_1e^{m_1t} + K_2e^{m_2t}}$$

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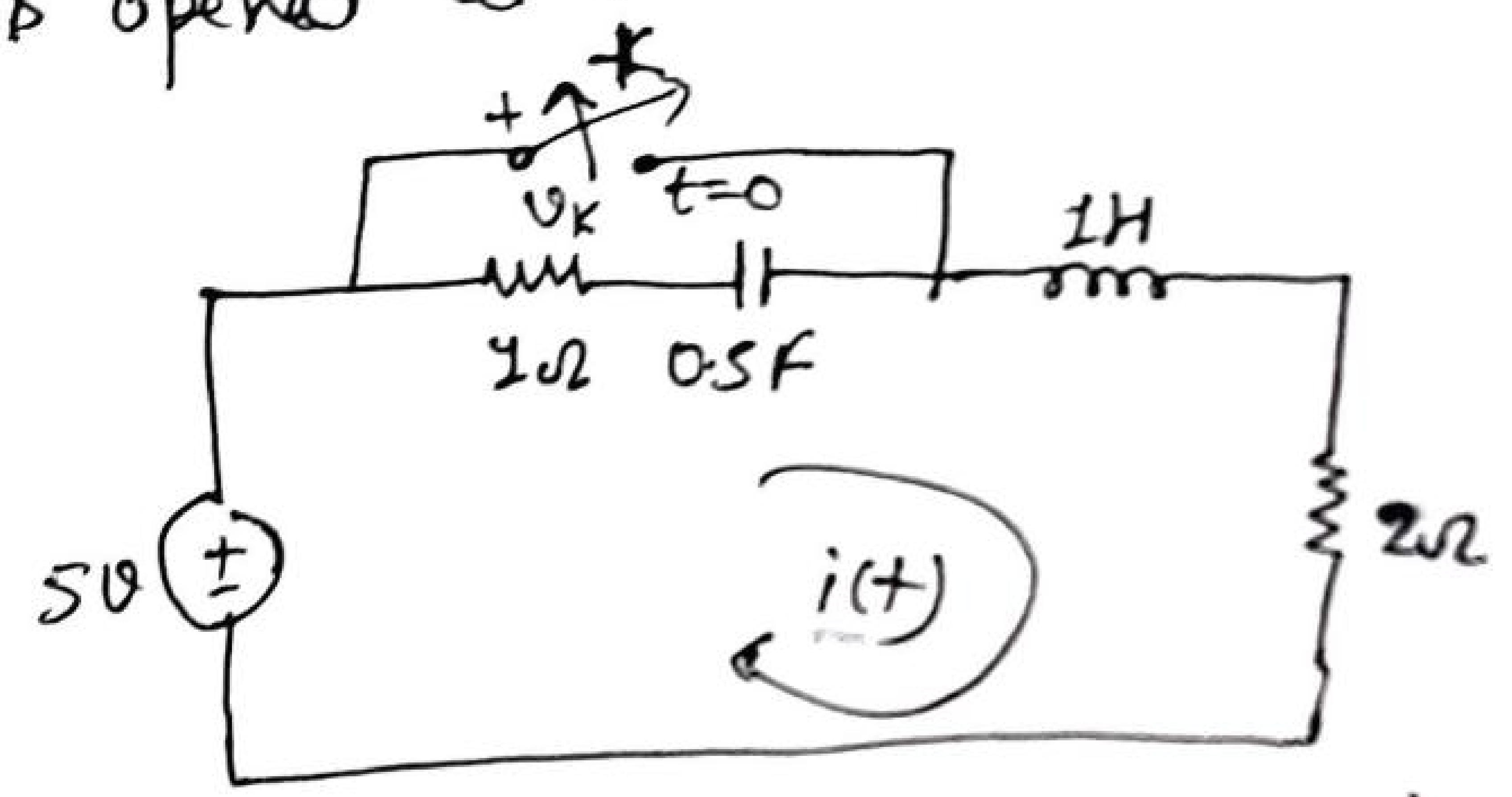
If $m_1 = m_2$, the solution is given by –

$$x(t) = (K_1 + K_2 t)e^{mt}$$

where, $m_1 = m_2 = m$

Solved numerical problems:–

Question:– For the circuit shown in figure. obtain $V_K(0^+)$, $\frac{dV_K(0^+)}{dt}$ and $\frac{d^2V_K(0^+)}{dt}$. The circuit was in steady state when the switch K is opened at $t=0$.



Solution:– **Step I:–** To obtain initial current through inductor.

$$i(0^-) = \frac{5}{2} = 2.5\,A$$

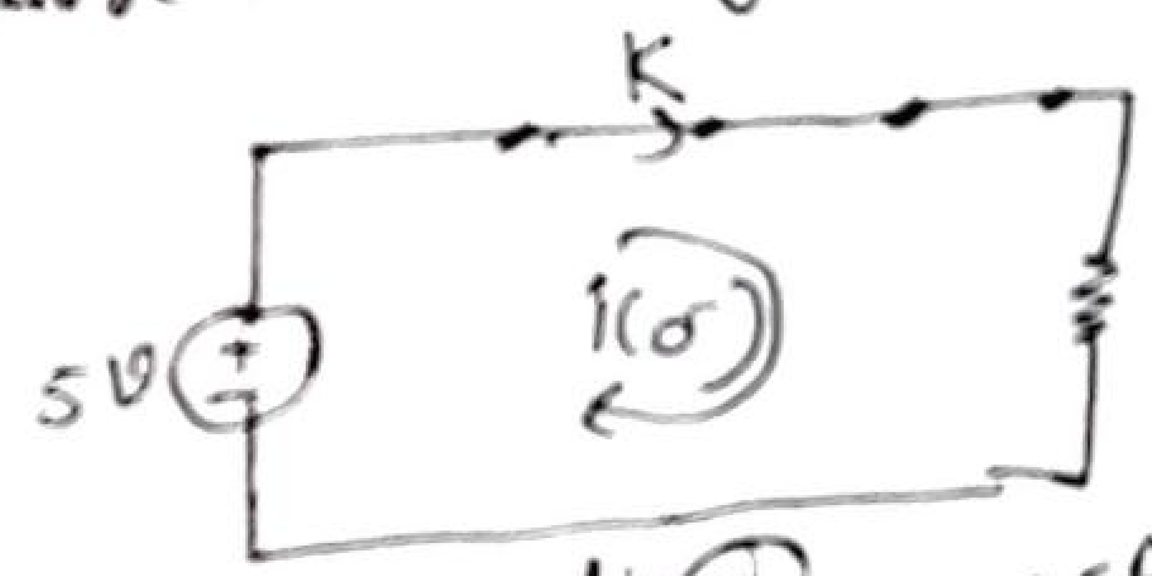


fig (a)

Step II:– To obtain $i(t)$ at $t=0^+$ and $V_K(0^+)$.

To find the initial condition at $t=0^+$, the circuit becomes as shown in fig (b).

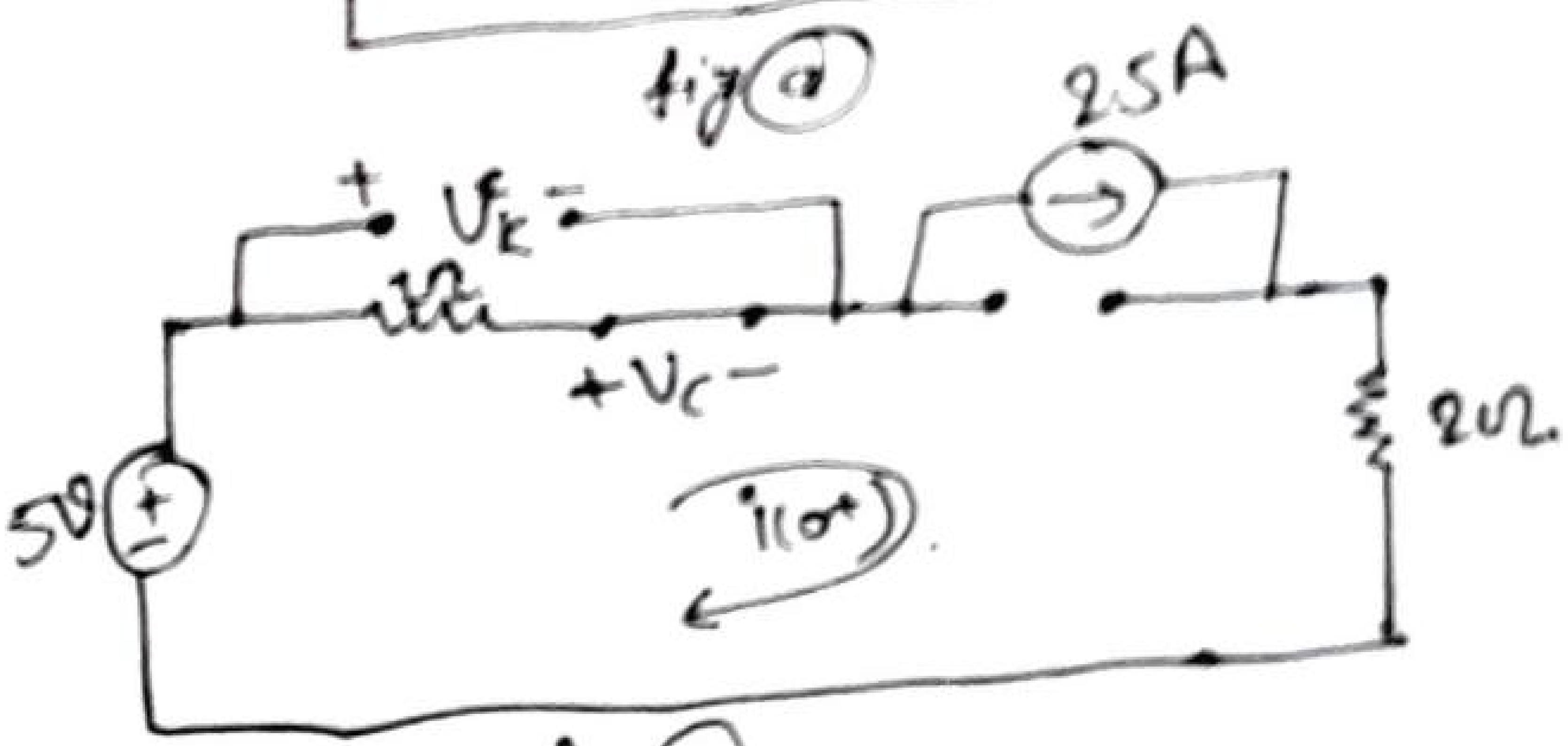


fig (b)

$i(0^+) = 2.5A$ & $V_C(0^+) = 0$

$V_K(0^+) = 2.5 \times 1 = 2.5$ volt

Step III:– To obtain $\frac{dV_K(0^+)}{dt}$.

In this case circuit becomes as shown in figure (c).

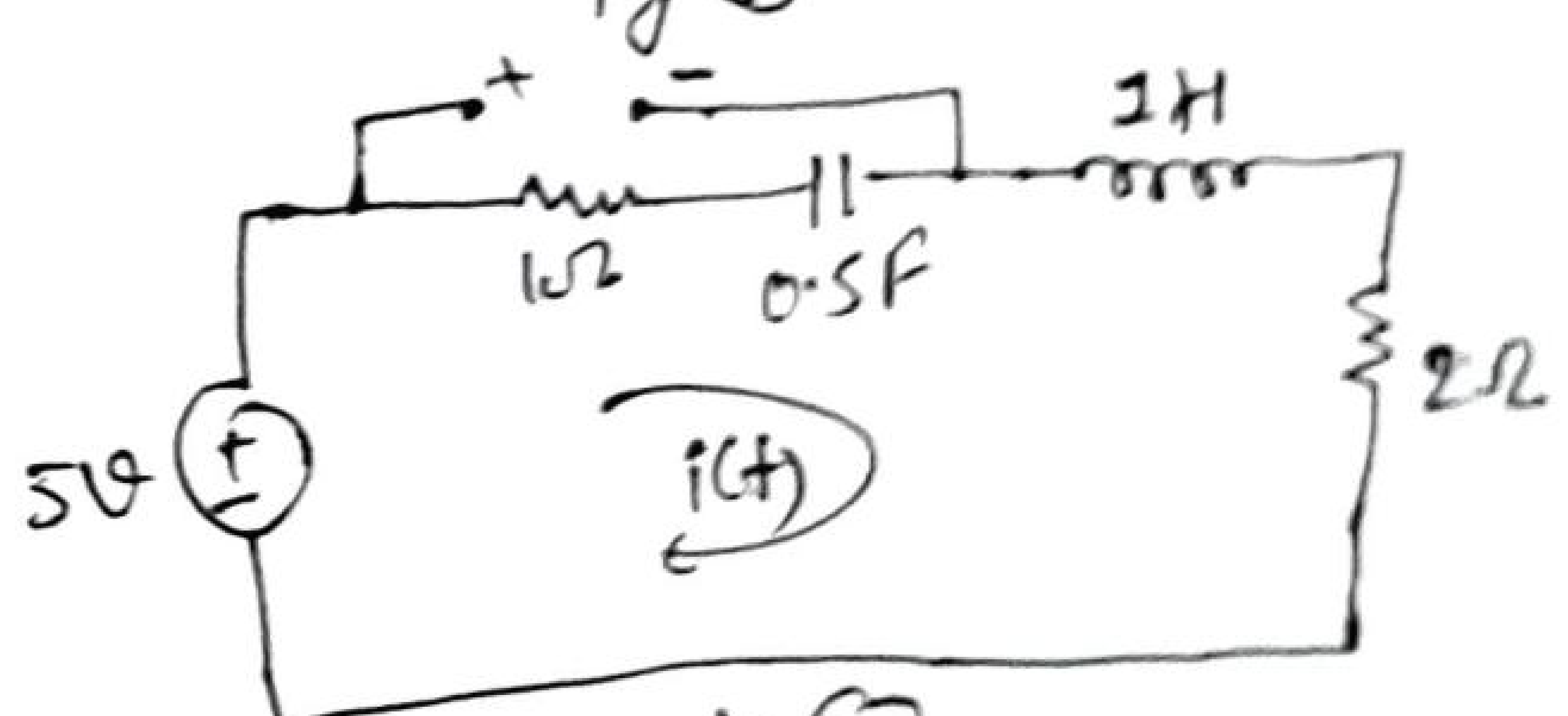


fig (c)

From this circuit, we have –

$$V_K = \frac{1}{0.5}\int_{-0}^{t} i(t)\,dt + 1 \cdot i(t) \longrightarrow (1)$$

differentiating eqⁿ (1), we get

$$\frac{dv_K}{dt} = 2i(t) + \frac{di(t)}{dt} \rightarrow (2)$$

At $t=0^+$ the eqⁿ (2) becomes as -

$$\frac{dv_K(0^+)}{dt} = 2\,i(0^+) + \frac{di(0^+)}{dt} \rightarrow (3)$$

By KVL, we can write -

$$3i(t) + \frac{1}{0.5}\int_{-\infty}^{t} i(t)\,dt + 1\cdot\frac{di(t)}{dt} = 5 \rightarrow (4)$$

At $t=0^+$, the eqⁿ (4) becomes as -

$$3i(0^+) + 2\int_{-\infty}^{0^+} i(t)\,dt + \frac{di(0^+)}{dt} = 5 \rightarrow (5)$$

Since $2\int_{-\infty}^{0^+} i(t)\,dt$ = Voltage across the capacitor at $t=0^+$ is equal to zero, putting the value $i(0^+) = 2.5A$ in eqⁿ (5) we get -

$$3(2.5) + 0 + \frac{di(0^+)}{dt} = 5$$

$$\frac{di(0^+)}{dt} = 5 - 7.5 = -2.5\ A/sec \rightarrow (6)$$

Step IV:- To obtain $\frac{d^2v_K(0^+)}{dt^2}$.

Differentiating eqⁿ (2), we get

$$\frac{d^2v_K}{dt^2} = 2\frac{di(t)}{dt} + \frac{d^2i(t)}{dt^2} \rightarrow (7)$$

At $t=0^+$, the eqⁿ (7) becomes as —

$$\frac{d^2v_K(0^+)}{dt^2} = 2\frac{di(0^+)}{dt} + \frac{d^2i(0^+)}{dt^2} \rightarrow (8)$$

Now differentiating eqⁿ (4), we get

$$3\frac{di(t)}{dt} + 2i(t) + \frac{d^2i(t)}{dt^2} = 0$$

At $t=0^+$, the eqⁿ (9) becomes as

$$3\frac{di(0^+)}{dt} + 2\,i(0^+) + \frac{d^2 i(0^+)}{dt^2} = 0$$

putting $\frac{di(0^+)}{dt}$ from eqn (6) & $i(0^+) = 2.5A$ in eqn (10), we get –

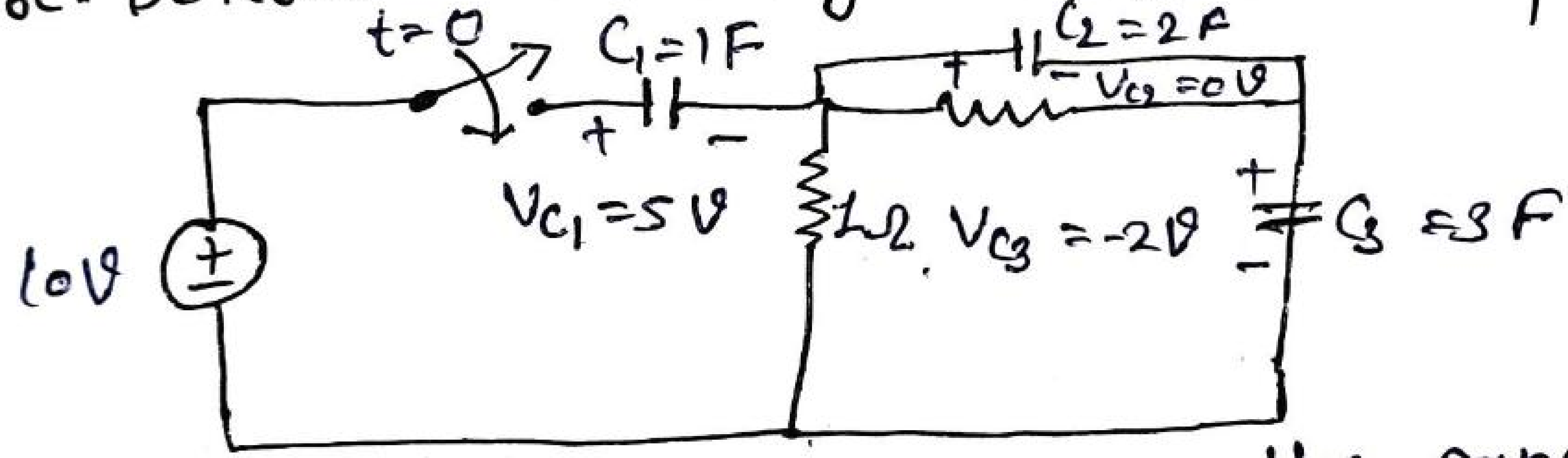
$$3(-2.5) + 2(2.5) + \frac{d^2 i(0^+)}{dt^2} = 0$$

$$\frac{d^2 i(0^+)}{dt^2} = 7.5 - 5.0 = 2.5\ A/sec^2$$

Therefore, from eqn (8), we get –

$$\frac{d^2 v_K(0^+)}{dt^2} = 2(-2.5) + 2.5 = -2.5\ V/sec^2.$$

Question:- The initial voltage across the capacitors are indicated in figure. Determine the voltage across the capacitors at $t = 0^+$.



Solution:- Consider the voltage across the capacitor as V_{C_1}, V_{C_2} and V_{C_3} with the polarities as at $t = 0^-$.

By KVL, we can write

$$V_{C_1} + V_{C_2} + V_{C_3} = 10 \rightarrow (i)$$

At $t = 0^+$,

$$\frac{1}{C_1}\int_{-\infty}^{0^+} i(t)\,dt + \frac{1}{C_2}\int_{-\infty}^{0^+} i(t)\,dt + \frac{1}{C_3}\int_{-\infty}^{0^-} i(t)\,dt = 10.$$

$$\frac{1}{1}\int_{-\infty}^{0} i(t)\,dt + \frac{1}{1}\int_{0^-}^{0^+} i(t)\,dt + \frac{1}{2}\int_{-\infty}^{0^-} i(t)\,dt + \frac{1}{2}\int_{0^-}^{0^+} i(t)\,dt + \frac{1}{3}\int_{-\infty}^{0^-} i(t)\,dt + \frac{1}{3}\int_{0^-}^{0^+} i(t)\,dt = 10$$

$$5 + \frac{q(0^+)}{1} + 0 + \frac{q(0^+)}{2} + (-2) + \frac{q(0^+)}{3} = 10$$

or,

$$q(0^+) = \frac{42}{11}\ \text{Coulomb}.$$

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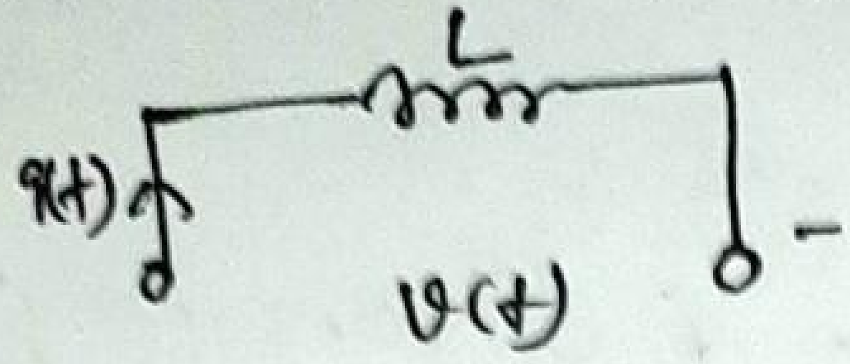
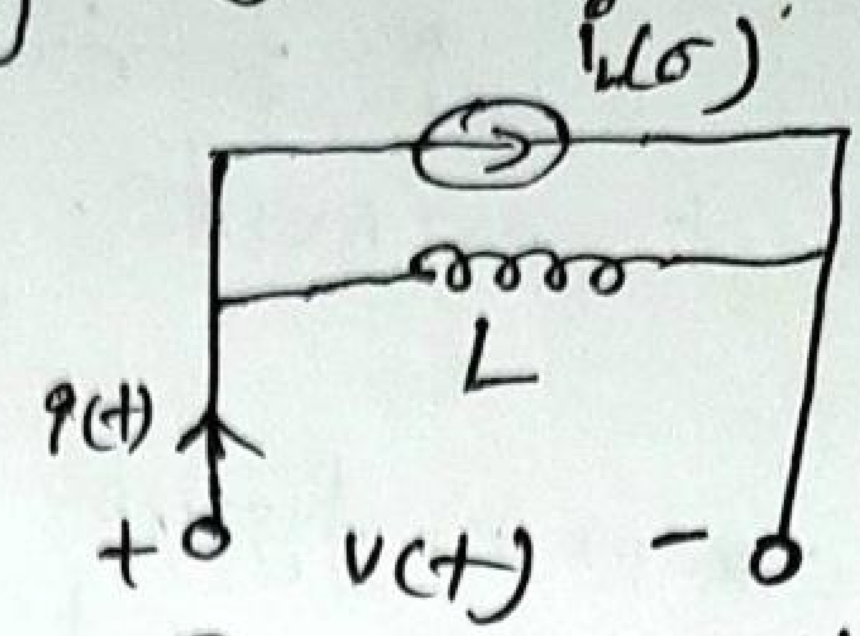
The v-i relationship for an inductor is a dual relationship b/w voltage and current when compared to a capacitor. The unit of L is Henrys. The initial current $i_L(0^-)$ can be regarded as an independent current source, as shown in figure (b)



(a) Inductor

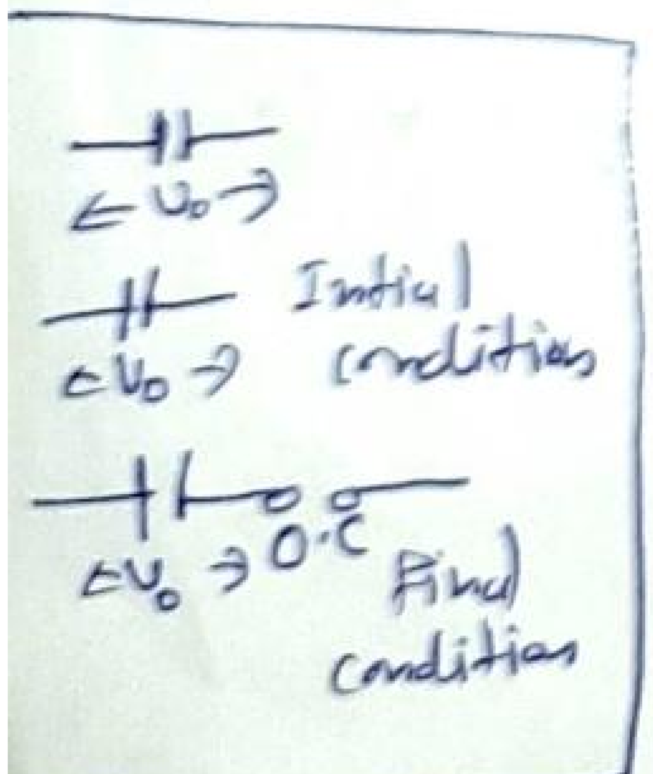


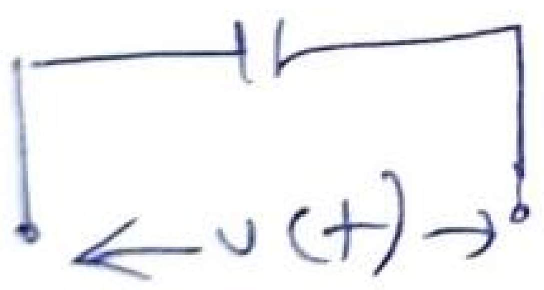
(b) Inductor with initial current.

Initial and Final Conditions:-

- In the solution of network differential equations, the complementary function is called the transient solution or free response. The particular integral is known as the force response. In the case of constant or periodic excitations, the forced response at $t=\infty$ is the steady-state or final solution.
- There are two ways to obtain initial conditions at $t=0^+$ for a network: (i) Through the differential equations describing the network, (ii) through knowledge of the physical behaviour of the R L and C elements in the network.

(i) Initial condition for a capacitor:-





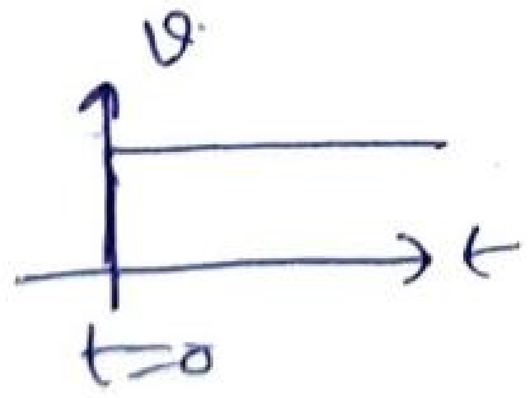
$$i = C\frac{dv}{dt}$$

$$i\,dt = C\,dv$$

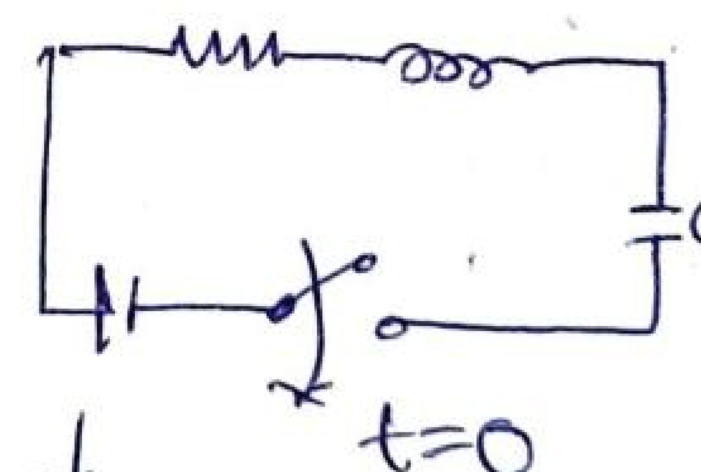
$$dv = \frac{i}{C}\,dt$$

$$\int dv = \frac{1}{C}\int i\,dt$$

$$\left[v = \frac{1}{C}\int i\,dt + i(0) \right]$$



important point:-



t=0

switch is closed

$t=0^-$ → Before switch is closed

$t=0^+$ → After switch is closed

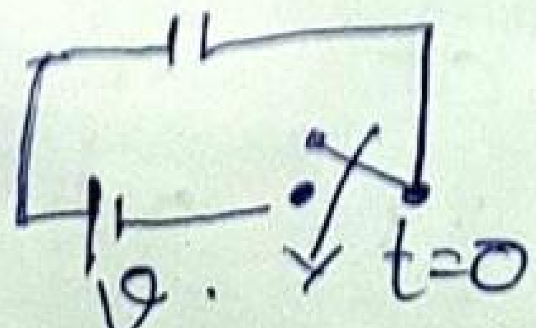
initial

$t=\infty$ → final condition

$t=0^- \rightarrow$ S.C

$t=0^+ \rightarrow$ S.C

$t=\infty$ → O.C



$$i = C\frac{dv}{dt} = \frac{v_0 - 0}{0-0} = \infty$$

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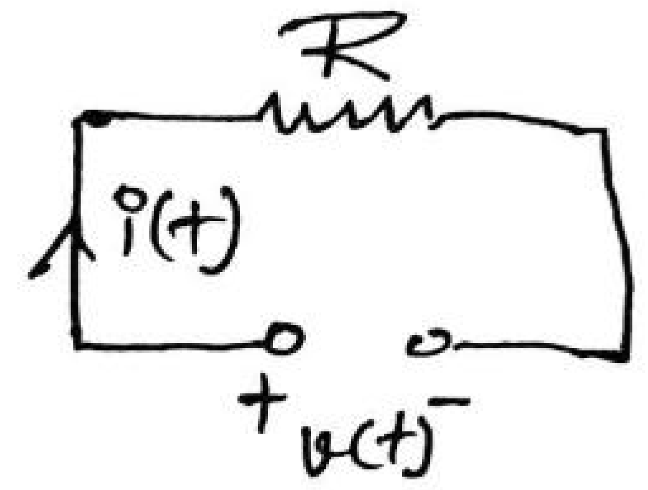
Network Analysis

(I) Network elements :-

In this section, we will study the Voltage current (V-i) relationships that exist for the basic network elements.

(i) Resistor :-

The resistor shown in figure defines a linear proportionality relationship b/w voltage and currents, i.e.



$$v(t) = R\,i(t)$$

or, $$i(t) = G\,v(t)$$

where $G = \frac{1}{R}$ = conductance, mho ($\mho$)

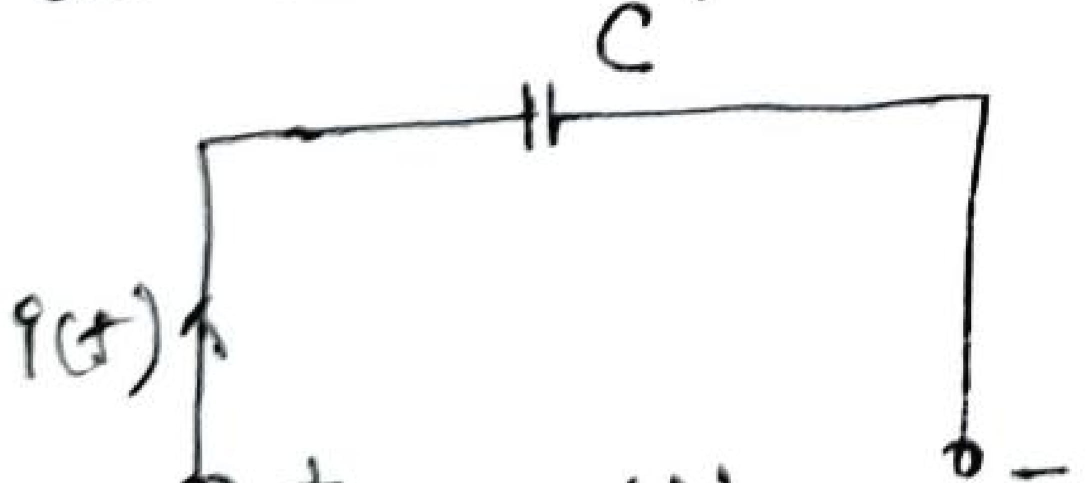
(ii) Capacitor :-

For the capacitor shown in figure (a) the v-i relationships are

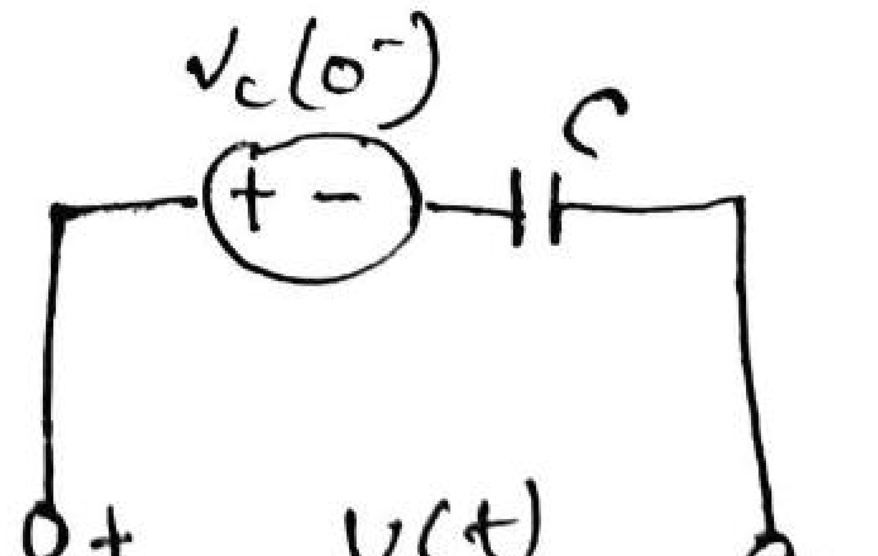
$$i(t) = C\frac{dv(t)}{dt}$$

$$v(t) = \frac{1}{C}\int_{-\infty}^{t} i(\tau)\,d\tau = V_C(0^-) + \int_{0^-}^{t} i(\tau)\,d\tau .$$

Where C is in farads. The initial value $V_C(0^-)$ is the voltage across the capacitor just before the switching action. It can be regarded as an independent voltage source, as shown in figure b. We should point out also that $V_C(0^-) = V_C(0^+)$ for all excitations except impulse and derivatives of impulse.



(a) capacitor



(b) Capacitor with initial voltage.

(iii) Inductor :-

The v-i relationships for an inductor are given as-

$$v(t) = L\frac{di(t)}{dt}$$

$$i(t) = \frac{1}{L}\int_{-\infty}^{t} v(\tau)\,d\tau = i_L(0^-) + \frac{1}{L}\int_{0^-}^{t} v(\tau)\,d\tau$$