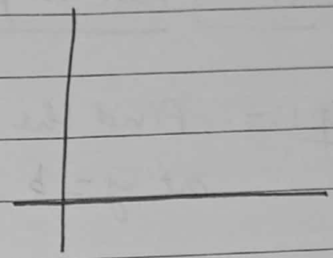
Unit-3:-



A function $f(x,y)$ is said to tends to a limit l as $x \to a$ and $y \to b$ if and only if the limit l is independent of path followed by the point x, y then,

$$\lim_{x \to (} \lim_{(x,y) \to (a,b)} f(x,y) = l.$$

or

The function $f(x,y)$ in region R is said to tends to a limit l as $x \to a$ and $y \to b$ if and only if corresponding to a positive no. ε, there exists another (+) no. δ such that

$$|f(x,y) - l| < \varepsilon \text{ whenever,}$$

$$0 < \sqrt{(x-a)^2 + (y-b)^2} < \delta$$

Working rule to find the limit :- (limit does not exist)

Step 1:- find the limit of $f(x, y)$ along $x \to a$ at $y = b$

Step 2: find the limit of $f(x,y)$ along $y \to b$ at $x = a$.

if $a = 0$, $b = 0$, in that case

Step 3:- we put $y = mx$ and take limit along $x \to 0$

Step 4: we put $y = mx^n$ and take the limit along $x \to 0$.

Conclusion:-

(i) if $f_1 \neq f_2$, then limit does not exist.

if $f_1 = f_2$

(ii) find f_3

if $f_1 = f_2 \neq f_3$

limit does not exist.

(iii) if $f_1 = f_2 = f_3$

find f_4, if $f_1 = f_2 = f_3 \neq f_4$ (limit does not exist)

Que: Evaluate:-

$$\lim_{(x,y)\to(0,0)} \frac{x^2y}{x^4+y^2}$$

Sol^n:

Step 1:- put $y=0$,

$$\lim_{x\to 0} \frac{0}{x^4+0} = 0 = f_1.$$

Step 2:- put $x=0$

$$\lim_{y\to 0} \frac{0}{0+y^2} = 0 = f_2.$$

Step 3:- put $y = mx$

$$\lim_{x\to 0} \frac{x^2\times mx}{x^4+m^2x^2} = \frac{mx^3}{x^2(x^2+m^2)}$$

$$= \frac{mx}{x^2+m^2} = \frac{0}{m^2} = 0$$

Step 4:- put $y = mx^2$

$$\lim_{x\to 0} \frac{m}{1+m^2} \quad \frac{x^2\times mx^2}{x^4+m^2x^4} = \frac{\cancel{x^4}(m)}{\cancel{x^4}(1+m^2)} = \frac{m}{1+m^2}$$

$\Rightarrow$ limit depends on path
limit does not exist.

Evaluate:-

$$\lim_{(x,y)\to(0,0)} \frac{y^2-x^2}{y^2+x^2}$$

$x=0$

$$\lim_{y\to 0} \frac{y^2}{y^2} = 1.$$

$y=0$

$$\lim_{x\to 0} \frac{-x^2}{x^2} = -1.$$

does not exist.

Evaluate:

$$\lim_{(x,y)\to(0,0)} \frac{x+y}{2x-y}$$

$x=0$

$$\lim_{y\to 0} \frac{y}{-y} = -1$$

$y=0$

$$\lim_{x\to 0} \frac{x}{2x} = \frac{1}{2}$$

limit does not exist A.

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Limit:-

$(x,y) \to (a,b)$

$f(x,y) \to l$

$\forall \ \varepsilon > 0 \quad \exists \quad \delta >$

$|f(x,y) - l| < \varepsilon$ where $0 < \sqrt{(x-a)^2 + (y-b)^2} < \delta$

$(x,y) \to (a,b)$.

Ques. Show that

$$\lim_{(x,y)\to(a,b)} (x^2+y^2) \sin \frac{1}{x^2+y^2} = 0.$$

$x \to r\cos\theta$
$y \to r\sin\theta.$

$|f(x,y) - l| = \left| (x^2+y^2) \sin\left(\frac{1}{x^2+y^2}\right) \right|$

lim

$|f(x,y) - l| = x^2+y^2 \left| \sin \frac{1}{x^2+y^2} \right|$

$\leq x^2 + y^2$ —— (i)

Now, choose a nbd of $(0,0)$.

$\sqrt{(x-0)^2 + (y-0)^2} < \delta$

$\Rightarrow x^2+y^2 < \delta^2$

from (i),

$|f(x,y) - l| < \varepsilon = \delta^2$

$\Rightarrow \forall \ \varepsilon > 0, \ \exists \ \delta = \sqrt{\varepsilon}$ such that,

$|f(x,y) - l| < \varepsilon$ whenever

$$0 < \sqrt{(x-0)^2 + (y-0)^2} < \delta$$

$\Rightarrow$ limit exist and is equal to 0.

Qn Show that :-

$$\lim_{(x,y)\to(0,0)} \frac{xy}{\sqrt{x^2+y^2}} = 0.$$

$$|f(x,y) - l| = \left| \frac{xy}{\sqrt{x^2+y^2}} \right|$$

$$= \frac{|xy|}{\sqrt{x^2+y^2}} < \frac{x^2+y^2}{\sqrt{x^2+y^2}} = \sqrt{x^2+y^2}$$

$$= \varepsilon = \delta.$$

$\because$ $(x-y)^2 \geq 0$

$= x^2+y^2-2xy \geq 0$

$x^2+y^2 \geq 2|xy| > |xy|$

Now changing the nbd of $(0,0)$

$$\sqrt{x^2+y^2} < \delta$$

$\Rightarrow$ $\forall \varepsilon > 0 \ \exists \ \delta = \varepsilon > 0$ s.t.

$|f(x,y) - l| < \varepsilon$ whenever

$$0 < \sqrt{(x-0)^2 + (y-0)^2} < \delta$$

$\Rightarrow$ limit exists and is equal to 0.

Qu Evaluate :-

$$\lim_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2}$$

~~Along~~ Along $x = 0$, $\lim_{y\to 0} \frac{0}{\,} = 0$

$y = 0$, $\lim_{x\to 0} = 0$.

Along $y = mx$

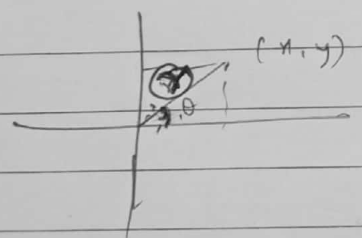
$$\lim_{x\to 0} \frac{x \times mx}{x^2+m^2x^2} = \frac{m\cancel{x^2}}{\cancel{x^2}(1+m^2)} = \frac{m}{1+m^2};$$ → depends on path

Along $y = mx^2$

$$\lim_{x\to 0} \frac{x \times mx^2}{x^2+m^2x^4} = \frac{m\cancel{x^2}\,x}{\cancel{x^2}(1+m^2x^2)} = 0.$$

So limit does not exist.

Polar form:- $x = r\cos\theta$
$y = r\sin\theta$



$$\lim_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2} = \lim_{r\to 0} \frac{\cancel{r^2}\cos\theta\sin\theta}{\cancel{r^2}(\sin^2\theta+\cos^2\theta)}$$

$= \cos\theta\sin\theta$

limit depends on θ and limit does not exist.

Que: Evaluate $\lim_{(x,y)\to(0,0)} \frac{x^2 y}{x^2+y^2}$

Along $y = mx$

$= \lim_{x\to 0} \frac{x^2 \times mx}{x^2+m^2x^2} = \frac{mx^3 x}{x^2(m^2+1)} = 0.$

Along $y = mx^2$

$\lim_{x\to 0} \frac{x^2 \times mx^2}{x^2+m^2x^4} = \frac{mx^4}{x^2(1+m^2x^2)} = 0$ ✓

polar

$\lim_{r\to 0} \frac{r^2 \cos^2\theta \sin\theta}{}$

if ~~$\cos^2\theta$ an~~ $\cos^2\theta \sin\theta$ is bounded

put limit on r else limit doe not exist.

~~lim~~ $= 0.$ ✓

Que: $\lim_{(x,y)\to(0,0)} \frac{x^2y^2}{x^3+y^3}$

$x = r\cos\theta$
$y = r\sin\theta$

$\lim_{r\to 0} \frac{r^2\cos^2\theta \times r^2\sin^2\theta}{r^3(\cos^3 + \sin^3\theta)}$

$\lim_{r\to 0} \frac{r\cos^2\theta \times \sin^2\theta}{\cancel{\theta}(\cos\theta+\sin\theta)(1-\sin\theta\cos\theta)}$

At $\theta = \frac{3\pi}{4}$

$\lim_{r\to 0} \left(\frac{\frac{1}{4}}{0}\right) \to 0 f(0)$

$f(0)$ is unbounded

$\Rightarrow$ Limit does not exist.

Que: $\lim\limits_{(x,y)\to(0,0)} \dfrac{\sin^{-1}(x+2y)}{\tan^{-1}(3x+6y)}$

$\lim\limits_{(x,y)\to(0,0)} \dfrac{\sin^{-1}(x+2y)}{\tan^{-1}3(x+2y)}$

$x+2y = t$

as $(x,y)\to(0,0) \Rightarrow t\to 0$

$\lim\limits_{t\to 0} \dfrac{\sin^{-1} t}{\tan^{-1} 3t} = \dfrac{1}{3}$ ✓

Partial Derivatives:-

$f(x), \quad x \in D$

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

$\delta x = h$

$x \to x+h$

$f(x) \to f(x+h)$

$$\lim_{\delta x \to 0} \frac{\delta f}{\partial x} = f(a+h) - f(a)$$

Let $z = f(x,y)$ be a function of two variables. If we keep y constant and x varies, then z becomes a function of x only. The derivability of z w.r.t x keeping y as constant is called partial derivative w.r.t x keeping y as constant. Symbolically denoted by

$$\frac{\partial z}{\partial x} \text{ or } \frac{\partial f}{\partial x}, z_x, f_x$$

here, $\boxed{\dfrac{\partial f}{\partial x} = \lim_{h \to 0} \dfrac{f(x+h, y) - f(x,y)}{h}}$

Similarly, keeping x as constant, y varies then derivative of z w.r.t y is symbolically denoted by

$$\frac{\partial f}{\partial y}, \frac{\partial z}{\partial y}, z_y \text{ or } f_y.$$

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$$\frac{\partial f}{\partial y} = \lim_{k \to 0} \frac{\cancel{f(y,}\, f(x, y+k) - f(x,y)}{k}$$

Ans: $z = x^2 + 3xy + y^2$

$$\frac{\partial z}{\partial x} = \lim_{h \to 0} \frac{f(x+h, y) - f(x,y)}{h}$$

$$\frac{\partial z}{\partial x} = \underline{2x + 3y}\ \text{Ans}$$

$$\frac{\partial z}{\partial y} = \underline{3x + 2y}\ \text{Ans}$$

- **Higher order derivative :-**

$z = f(x,y)$

$$\frac{\partial^2 z}{\partial x^2}, \quad \boxed{\frac{\partial^2 z}{\partial x \partial y}, \quad \frac{\partial^2 z}{\partial y \partial x}}, \quad \frac{\partial^2 z}{\partial y^2}$$

→ where z_x and z_y exists and cont.

Some notation :-

$$\frac{\partial z}{\partial x} = p, \quad \frac{\partial z}{\partial y} = q, \quad \frac{\partial^2 z}{\partial x^2} = r, \quad \frac{\partial^2 z}{\partial x \partial y} = s,$$

$$\frac{\partial^2 z}{\partial y^2} = t$$

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eg:- $u = e^{xyz}$, find $\frac{\partial^3 u}{\partial x \partial y \partial z}$ $\frac{\partial^3 u}{\partial z \partial y \partial x}$

Solⁿ:

$\frac{\partial u}{\partial z} = xy \times e^{xyz}$

Now w.r.t y,

$= e^{xy} \times x + xy \times e^{xy} \times x$

$= e^{xy} x + x^2 y \times e^{xy}$

$\Rightarrow \frac{\partial u}{\partial x} = e^{xyz} \times yz$

w.r.t y

$\Rightarrow yz \times e^{xyz} \times xz + e^{xyz} \times z$

$xyz^2 \times e^{xyz} + e^{xyz} \times z$

w.r.t z,

$\Rightarrow xyz^2 \times e^{xyz} \times xy + e^{xyz} \times xy \times 2z + e^{xyz} + z \times e^{xyz} \times xy$

Ques: 1 $z(x+y) = x^2 + y^2$

show:. $\left(\frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right)^2 = 4\left(1 - \frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right)$

Sol:- $(2x - 2y)^2 = 4(1 - 2x - 2y)$

$\Rightarrow 4x^2 + 4y^2 - 8xy = 4 - 8x - 8y$

Ques 2 · $u = \sin^{-1}\left(\frac{x}{y}\right) + \tan^{-1}\left(\frac{y}{x}\right)$, value of $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$

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Sol^n. $z = \dfrac{x^2+y^2}{(x+y)}$

$\dfrac{\partial z}{\partial x} = \dfrac{(x+y)2x - (x^2+y^2)}{(x+y)^2} = \dfrac{x^2+2xy-y^2}{(x+y)^2}$

$\dfrac{\partial z}{\partial y} = \dfrac{(x+y)2y - (x^2+y^2)}{(x+y)^2} = \dfrac{2xy+y^2-x^2}{(x+y)^2}$

Now evaluate-

Sol^n 2: $x\left(\dfrac{\partial u}{\partial x}\right) = \left[\dfrac{1}{\sqrt{1-\frac{x^2}{y^2}}} \times \dfrac{1}{y} - \dfrac{1}{1+\frac{y^2}{x^2}} \times y \times \dfrac{1}{x^2}\right] \times x$

$y\left(\dfrac{\partial u}{\partial y}\right) = \left[-\dfrac{1}{\sqrt{1-\frac{x^2}{y^2}}} \times x \times \dfrac{1}{y^2} + \dfrac{1}{1+\frac{y^2}{x^2}} \times \dfrac{1}{x}\right] \times y.$

$= \cancel{\dfrac{1}{\sqrt{1-\frac{x^2}{y^2}}}} \times \dfrac{x}{y} - \cancel{\dfrac{1}{1+\frac{y^2}{x^2}}} \times \dfrac{y}{x} - \cancel{\dfrac{1}{\sqrt{1-\frac{x^2}{y^2}}}} \times \dfrac{x}{y} + \cancel{\dfrac{1}{1+\frac{y^2}{x^2}}} \times \dfrac{y}{x}$

$= 0$

Ques: $y = f(x+at) + g(x-at)$

Find, $\frac{\partial^2 y}{\partial t^2} / \frac{\partial^2 y}{\partial x^2}$.

Sol^n. $\frac{\partial y}{\partial t} = f'(x+at) \times a - g'(x-at) \times a$

$a[f'(x+at) - g'(x-at)]$

$\frac{\partial^2 y}{\partial t^2} =$

$\hookrightarrow a[f''(x+at) \times a + g''(x-at) a]$

$= a^2 [f''(x+at) + g''(x-at)]$ — (1)

$\cancel{\frac{\partial^2 y}{\partial x^2}}$. $\frac{\partial y}{\partial x} = f'(x+at) + g'(x-at)$

$\frac{\partial^2 y}{\partial x^2} = f''(x+at) + g''(x-at)$ — (2)

(1) / (2)

$= \underline{a^2}$ Ans

$x^2 = r^2 - y^2$

Ques: ~~z = x~~ $z = x \log(x+r) - r$ where $r^2 = x^2 + y^2$

$x^2 = r^2 - y^2$

Prove that:-

$2 \times x \times \frac{\partial x}{\partial y} =$

(i) $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = \frac{1}{x+r}$

(ii) $\frac{\partial^3 z}{\partial x^3} = -\left(\frac{x}{r^3}\right)$

$\frac{\partial r}{\partial x} = \frac{1}{2}(x^2+y^2)^{-\frac{1}{2}} \times 2x = \frac{1}{(x^2+y^2)}$

$r = \sqrt{x^2+y^2}$ $\quad \frac{1}{2}(x^2+y^2)^{-1/2}$ $\quad \frac{\partial r}{\partial x} =$

Sol^n: (i) $\frac{\partial z}{\partial x} = x \times \frac{1}{x+r} \times \left(1 + \frac{1}{2\sqrt{x^2+y^2}} \times 2x\right) + \log(x+r) - \frac{1}{2\sqrt{x^2+y^2}} \times 2x$

$\frac{\partial z}{\partial y} = x\left[\frac{1}{x+r} \times \left(1 + \frac{1}{2\sqrt{x^2+y^2}} \times 2y\right)\right] + \log(x+r)\frac{\partial x}{\partial y} - \frac{1}{2\sqrt{x^2+y^2}} \times 2y \Big]$.

$r^2 = x^2+y^2$

diff w.r.t 'x'

$2r\frac{\partial r}{\partial x} = 2x$

$\frac{\partial r}{\partial x} = \frac{x}{r}$ — (i)

$\frac{\partial r}{\partial y} = \frac{y}{r}$ — (ii)

$\frac{\partial z}{\partial x} = \frac{x}{(x+r)}\left(1 + \frac{\partial r}{\partial x}\right) + \log(x+r) - \frac{\partial r}{\partial x}$

$= \frac{x}{x+r}\left(1 + \frac{x}{r}\right) + \log(x+r) - \frac{x}{r}$

$\frac{x}{\cancel{x+r}}\left(\frac{\cancel{x+r}}{r}\right) + \log(x+r) - \cancel{\frac{x}{r}} = \log(x+r)$

Differentiability :-

$f(x)$, $x = a$

$$\lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

let (x,y), $(x+h, y+k)$ be two neighbouring points in the domain of function f, the change δf in the func. f as the point changes from $(x,y) \to (x+h, y+k)$ is denoted by

$$\delta f = \frac{f(x+h, y+k) - f(x,y)}{(x,y)}, \quad \text{then}$$

the func. f is said to be differiantiable at (x,y) if the change δf can be expressed in the form

$\boxed{\delta f = Ah + Bk + h\phi + k\varphi}$, where A, B - constant and independent of h and k. Also,

φ and ϕ tends to zero and $h, k \to 0$

Remark :- Assuming $f(x,y)$ is differentiable

(i) $\delta f = f(x+h, y+k) - f(x,y)$

as $h, k \to 0$, $\delta f \to 0$

$$\lim_{\substack{h \to 0 \\ k \to 0}} f(x+h, y+k) = f(x,y)$$

$\Rightarrow$ Every differentiable funct. is continuous.

(ii) $\delta f = \underline{f(x+h, y+k) - f(x,y)}$

$= Ah + Bk + h\phi + k\psi$

$h \to \delta x \quad , \quad k \to \delta y$

Assuming y is const. $\delta y = k = 0$

$\delta f = A\delta x + \delta x \phi$

$\frac{\delta f}{\delta x} = A + \phi$

as $\delta x \to 0$, $\phi \to 0$ ($\because f$ is diff.)

$\lim_{\delta x \to 0} \frac{\delta f}{\delta x} = A$

$$\boxed{\frac{\partial f}{\partial x} = A}$$

(iii) $\delta f = Ah + Bk + h\phi + k\psi$

$h \to \delta x, \ k \to \delta y$

assume ~~x~~ as const.

Similarly,

$\lim_{\delta y \to 0} \frac{\delta k}{\delta y} = B$

$$\boxed{\frac{\partial f}{\partial y} = B}$$

Que: $f(x,y) = \begin{cases} \dfrac{x^3-y^3}{x^2+y^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0). \end{cases}$

Sol^n: ~~lim~~

$$\left(\frac{\partial f}{\partial x}\right)_{(0,0)} = \lim_{h \to 0} \frac{f(0+h,0) - f(0,0)}{h} = 1$$

$$= .$$

$$\left(\frac{\partial f}{\partial y}\right)_{(0,0)} = \lim_{k \to 0} \frac{f(0,k) - f(0,0)}{k} = -1$$

$\longrightarrow$ ~~discontinuous~~

Let f is diff at (0,0),

$$\delta f = f(0+h, 0+k) - f(0,0) = Ah + Bk + h\phi, k\psi$$

$$\delta f = f(h,k) = Ah + Bk + h\phi + k\psi$$

$$h = \rho\cos\theta, \quad k = \rho\sin\theta$$

as $(h,k) \to (0,0)$, $\delta \to 0$

$$\delta(\cos^3\theta - \sin^3\theta) = \delta(\cos\theta - \sin\theta)$$

$$(\cos\theta - \sin\theta)(\cos^2\theta + \sin^2\theta + \cos\theta\sin\theta) = 0$$

$$(\cos\theta\sin\theta)(\cos\theta - \sin\theta) = 0$$

$\longrightarrow$ dependent on θ

$\Rightarrow$ our supposition is wrong
Hence not diff at (0,0).

Qu show that the func. f where

$$f(x,y) = \begin{cases} \dfrac{xy}{\sqrt{x^2+y^2}}, & \text{if } x^2+y^2 \neq 0 \\ 0, & \text{if } x=y=0 \end{cases}$$

is cont. possess partial derivative but is not diff. at the origin.

Soln:- ~~lim~~ Along ~~...~~ $y=mx$

$$\lim_{x\to 0} \frac{mx^2}{\sqrt{x^2+m^2x^2}} = \frac{mx^2}{x\sqrt{1+m^2}} = \frac{mx}{\sqrt{1+m^2}} = 0$$

Along $y = mx^2$

$$\lim_{x\to 0} \frac{mx^3}{\sqrt{x^2+m^2x^4}} = \frac{m\cancel{x^3}x^2}{\cancel{x}\sqrt{1+m^2x^2}} = 0.$$

$\therefore$ cont. ✓ $\quad$ $f(x,y) = 0$

$\Rightarrow$ ∂ $f_x(0,0) = \lim_{h\to 0} \frac{f(0+h,0) - f(0,0)}{h} = 0$

$f_y(0,0) = \lim_{k\to 0} \frac{f(0,k+0) - f(0,0)}{k} = 0$

Let $f(x,y)$ is diff at $(0,0)$

$$\delta f = f(0+h, 0+k) - f(0,0) = Ah + Bk + h\phi + k\psi$$
$$= f(h,k) - f(0,0) = Ah + Bk + h\phi + k\psi$$

$\Rightarrow$

$$\Rightarrow \quad \frac{hk}{\sqrt{h^2+k^2}} = Ah + Bk + h\phi + k\psi$$

$f(x,y)$ is diff as $h, k \to 0$; $\phi \to, \psi \to 0$

Now, let us change a path

$h = l\cos\theta, \quad k = k\sin\theta$

$$f(l\cos\theta, l\sin\theta) = l(A\cos\theta + B\sin\theta + \phi\cos\theta + \psi\sin\theta)$$

$\lim h,k \to 0$; $\phi \to 0, \psi \to 0$

$\boxed{\cos\theta\sin\theta = 0}$ — Limit depends on path.

Let (a,b) be a point of domain of definition of func. f such that

(i) ~~$f(x)$~~ f_x exist and contn. at (a,b)

(ii) f_y exists at (a,b)

then func. is diff. at (a,b).

Ques: $f(x,y) = \begin{cases} xy \dfrac{x^2-y^2}{x^2+y^2} & , x^2+y^2 \neq 0 \\ 0 & , x=y=0 \end{cases}$

check the ~~cha~~ diff at $(0,0)$

Solⁿ. $\lim_{(h,k)\to(0,0)} \dfrac{f(h,k) - f(0,0) - h f_x(a,b) - k f_y(a,b)}{\sqrt{h^2+k^2}}$

$f_x = \lim_{h\to 0} \dfrac{f(h,0) - f(0,0)}{h}$

$= 0$

$f_y = 0$

$\lim_{(h,k)\to(0,0)} \dfrac{hk \cdot \frac{h^2-k^2}{h^2+k^2}}{\sqrt{h^2+k^2}} = \dfrac{\cancel{hk(h^2+k^2)}}{\cancel{h^2+k^2}} \dfrac{hk \cdot (h^2-k^2)}{\sqrt{h^2+k^2}}$

$= \dfrac{hk \cdot (h^2-k^2)\left(\sqrt{h^2-k^2}\right)}{h^2+k^2}$

$= \dfrac{hk\,(h^2-k^2)^{3/2}}{h^2+k^2}$

~~$\lim_{h}$~~ let $k = mh$

$\lim_{h\to 0} \dfrac{m\cancel{h^2}\,(h^2-m^2h^2)^{3/2}}{\cancel{h^2+k^2}\,\cancel{h^2}(1+m^2)} = = 0$

let $\underline{h^2 = k^2}$, $\underline{h = k}$

$$\lim_{h \to 0} \frac{h^2 \times 0}{2h^2} = 0 \checkmark$$

Limit exists, hence diff.

Sir's method:

$$f_x(0,0) = \lim_{h \to 0} \frac{f(0+h,0) - f(0,0)}{h}$$

$= 0$

$f_y = 0$

$$f_x = \frac{x^4 y + 4x^2 y^3 - y^5}{(x^2+y^2)^2}$$

$x = r\cos\theta \quad y = r\sin\theta$

$f_x = r(\cos^4\theta \sin\theta + 4\cos^2\theta \sin^3\theta - \sin^5\theta$

$= 0 = f_x(0,0)$

f_x is cont. at $(0,0)$

Hence f is diff. at $(0,0)$.

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Ques: Prove that the point

$f(x,y) = \sqrt{xy}$ is not diff at $(0,0)$.

Sol^n $f_x = \lim_{h \to 0} \frac{f(h,0) - f(0,0)}{h}$

$\Rightarrow 0$

$f_y = \cancel{0} \; 0$

$$\lim_{(h,k)\to(0,0)} \frac{f(h,k) - f(0,0) - hf_x(0,0) - kf_y(0,0)}{\sqrt{h^2+k^2}}$$

$$\lim_{(h,k)\to(0,0)} \frac{\sqrt{hk}}{\sqrt{h^2+k^2}}$$

$$\Rightarrow \sqrt{\frac{hk}{h^2+k^2}}$$

$k = mh$

$$= \lim_{h\to 0} \sqrt{\frac{mh^2}{h^2+m^2h^2}} = \sqrt{\frac{m}{1+m^2}}$$ ~~plan~~

limit value depends on m, so non differentiable.

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By Sir

~~$f_x = \cdot t$~~

~~$f(x,y)$~~ $\cdot f_x(x,y) = \lim_{h \to 0} \dfrac{f(x+h,y) - f(x,y)}{h}$

$f_x = \lim_{h \to 0} \dfrac{\sqrt{|(x+h) y|} - \sqrt{|xy|}}{h}$.

$\sqrt{|y|} \lim_{h \to 0} \dfrac{\sqrt{|x+h|} - \sqrt{|x|}}{h} \times \dfrac{\sqrt{|x+h|} + \sqrt{|x|}}{\sqrt{|x+h|} + \sqrt{|x|}}$

$\sqrt{|y|} \cdot \lim_{h \to 0} \dfrac{|x+h| - |x|}{h(\sqrt{|x+y|} + \sqrt{|x|})}$

as $h \to 0$, if $x+h > 0$, $|x+h| = x+h$, $x > 0$

also if $x+h < 0$, $|x+h| = -(x+h)$, $x < 0$

$$\begin{cases} \dfrac{1}{2} \dfrac{\sqrt{|y|}}{\sqrt{|x|}}, & x > 0 \\ -\dfrac{1}{2} \dfrac{\sqrt{|y|}}{\sqrt{|x|}}, & x < 0 \end{cases}$$

Similarly,

$$f_y = \begin{cases} \dfrac{1}{2} \dfrac{\sqrt{|x|}}{\sqrt{|y|}}, & y > 0 \\ -\dfrac{1}{2} \dfrac{\sqrt{|x|}}{\sqrt{|y|}}, & y < 0 \end{cases}$$

f_x, f_y both are discont. at $(0,0)$ Hence $f(x,y)$ is not diff. at $(0,0)$.

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