11. 解：由于变量ξ的生成函数为

$$\phi(s)=\frac{1}{2-s}=\frac{1}{2}\times\left[1+\frac{s}{2}+\left(\frac{s}{2}\right)^2+\left(\frac{s}{2}\right)^3+\cdots\right],$$

所以，ξ的概率分布为：

$$p_0=\frac{1}{2},\ \ p_1=\frac{1}{2^2},\dots,p_k=\frac{1}{2^{k+1}},\dots,$$

$$\mu=E\xi=\sum_{k=1}^{\infty}k\cdot\frac{1}{2^{k+1}}=1.$$

从而，由定理 5.2 可知，$EZ_n=\mu^n=1$．进而，$EW_1=E(Z_0+Z_1)=2$，$E(2W_2-W_3)=2$．

12. 解：(1) 首先，变量ξ的生成函数为

$$\phi(s)=(1-p)\sum_{k=0}^{\infty}p_k s^k=\frac{1-p}{1-ps}.$$

记 $\alpha_n=P(Z_n=0)=\phi_n(0)$，则有 $\alpha_n=\dfrac{1-p}{1-p\alpha_{n-1}}$．当 $p=\frac{1}{2}$ 时，$\alpha_n=\dfrac{n}{n+1}$．当 $p\neq\frac{1}{2}$ 时，

$$\frac{\alpha_n-1}{\alpha_n-\frac{1-p}{p}}=\frac{\dfrac{1-p}{1-p\alpha_{n-1}}-1}{\dfrac{1-p}{1-p\alpha_{n-1}}-\frac{1-p}{p}}=\frac{p}{1-p}\frac{\alpha_{n-1}-1}{\alpha_{n-1}-\frac{1-p}{p}}.$$

记 $x_n=\dfrac{\alpha_n-1}{\alpha_n-\frac{1-p}{p}}$，从而有 $x_n=\dfrac{p}{1-p}\cdot x_{n-1}$，其中

$$x_1=\frac{\alpha_1-1}{\alpha_1-\frac{1-p}{p}}=\frac{1-p-1}{1-p-\frac{1-p}{p}}=\frac{-p^2}{-p^2+2p-1}=\frac{p^2}{(p-1)^2}.$$

求解可得，$x_n=\left(\dfrac{p}{1-p}\right)^{n+1}$，从而可解得：

$$\alpha_n=(1-p)\frac{(1-p)^n-p^n}{(1-p)^{n+1}-p^{n+1}}.$$

综上可得，

$$\alpha_n=\begin{cases}\dfrac{n}{n+1}, p=\frac{1}{2}\\ (1-p)\dfrac{(1-p)^n-p^n}{(1-p)^{n+1}-p^{n+1}}, p\neq\frac{1}{2}\end{cases}.$$

从而,

$$P(T=n)=\alpha_n-\alpha_{n-1}=\begin{cases}\dfrac{1}{(n+1)n}, p=\frac{1}{2}\\ (1-p)\left[\dfrac{(1-p)^n-p^n}{(1-p)^{n+1}-p^{n+1}}-\dfrac{(1-p)^{n-1}-p^{n-1}}{(1-p)^n-p^n}\right], p\neq\frac{1}{2}\end{cases}$$

$$=\begin{cases}\dfrac{1}{(n+1)n}, p=\frac{1}{2}\\ \dfrac{p^{n-1}(1-p)^n(2p-1)^2}{\left((1-p)^{n+1}-p^{n+1}\right)\left((1-p)^n-p^n\right)}, p\neq\frac{1}{2}\end{cases}$$

$$=\begin{cases}\dfrac{1}{(n+1)n}, p=\frac{1}{2}\\ \dfrac{(2p-1)^2}{p^2}\dfrac{\left(\frac{1-p}{p}\right)^n}{\left(\left(\frac{1-p}{p}\right)^{n+1}-1\right)\left(\left(\frac{1-p}{p}\right)^n-1\right)}, p\neq\frac{1}{2}\end{cases}.$$

当 $p=\frac{1}{2}$ 时,

$$ET=\sum_{n=1}^{\infty}nP(T=n)=\sum_{n=1}^{\infty}\frac{1}{n+1}=\infty.$$

当 $p\neq\frac{1}{2}$ 时,

$$P(T=n)=\frac{(2p-1)^2}{p^2}\frac{\left(\frac{1-p}{p}\right)^n}{\left(\left(\frac{1-p}{p}\right)^{n+1}-1\right)\left(\left(\frac{1-p}{p}\right)^n-1\right)}$$

(此处记 $\left(a=\frac{1-p}{p}\right)$) $$:=\frac{(2p-1)^2}{p^2}\frac{a^n}{(a^{n+1}-1)(a^n-1)}$$

$$=\frac{(2p-1)^2}{p^2}\frac{1}{(a^{n+1}-1)\left(1-\frac{1}{a^n}\right)}.$$

不难判断级数 $ET=\sum_{n=1}^{\infty} nP(T=n)<\infty$，即收敛.

13. 解：(1) $\mu=E\xi=\frac{7}{6}$，$EZ_{30}=E\left(Z_{30}^{(1)}+Z_{30}^{(2)}+Z_{30}^{(3)}\right)=3EZ_{30}^{(1)}=3\times\left(\frac{7}{6}\right)^{30}$.

(2) $\lim_{n\to\infty}P(Z_n=0)=\lim_{n\to\infty}P\left(Z_n^{(1)}=0\right)\lim_{n\to\infty}P\left(Z_n^{(2)}=0\right)\lim_{n\to\infty}P\left(Z_n^{(3)}=0\right)=\tau^3$，其中 τ 为方程 $\phi(s)=\frac{1}{3}+\frac{1}{6}s+\frac{1}{2}s^2=s$ 的最小正解, 可解得 $\tau=\frac{2}{3}$. 从而 $\lim_{n\to\infty}P(Z_n=0)=\frac{8}{27}$.

(3) $P(Z_6=2|Z_5=2)=C_2^3\times\frac{1}{3}\times\frac{1}{2}+\frac{1}{6}\times\frac{1}{6}=\frac{13}{36}$.

14. 解：(1) $P\left(\text{直到}n+\tfrac{1}{2}\text{分钟都没有白细胞产生}\right)=\prod_{k=0}^{n}\left(\tfrac{1}{4}\right)^{2^k}=\left(\tfrac{1}{4}\right)^{\sum_{k=0}^{n-1}2^k}=2^{2-2^{n+1}}$.

(2) 灭绝概率 τ 为方程 $\phi(s)=\frac{1}{12}+\frac{2}{3}s+\frac{1}{4}s^2=s$ 的最小正解, 解之可得 $\tau=\frac{1}{3}$.