1. 解：首先，$\mu = EX(t) = EX_0 \cdot E(-1)^{N(t)} = 0$. 不妨设 $t \geq s$，则有

$$E(X(s)X(t)) = E({X_0}^2(-1)^{N(s)+N(t)}) = E((-1)^{2N(t)+N(t-s)})$$
$$= E(-1)^{N(t-s)} = \sum_{k\text{为偶}} \tfrac{(\lambda(t-s))^k}{k!} e^{-\lambda(t-s)} - \sum_{k\text{为奇}} \tfrac{(\lambda(t-s))^k}{k!} e^{-\lambda(t-s)}$$
$$= \sum_{k \geq 0} \tfrac{(-\lambda(t-s))^k}{k!} e^{-\lambda(t-s)} = e^{-\lambda(t-s)} \cdot e^{-\lambda(t-s)} = e^{-2\lambda(t-s)}$$

故 $X = (X(t), t \geq 0)$ 是平稳过程. 从而,

$$r_X(t) = E(X(0)X(t)) = e^{-2\lambda t}.$$

$$\frac{1}{T}\int_0^T \left(r_X(t) - \mu^2\right)dt = \frac{1}{T}\int_0^T e^{-2\lambda t} dt = \frac{1}{T}\left(\frac{1-e^{-2\lambda T}}{2\lambda}\right) \to 0, T \to \infty.$$

由推论 8.3 可知，$X = (X(t), t \geq 0)$ 满足均值遍历性.

2. 解： 首先，$\mu = EX(t) = 0$. 不妨设 $t \geq s$，则有

$$E(X(s)X(t)) = e^{-\frac{\alpha(t+s)}{2}} \cdot E(B(e^{\alpha t})B(e^{\alpha s})) = e^{-\frac{\alpha(t+s)}{2}} \cdot e^{\alpha s} = e^{-\frac{\alpha(t-s)}{2}},$$

故 $X = (X(t), t \geq 0)$ 是平稳过程. 从而,

$$r_X(t) = E(X(0)X(t)) = e^{-\frac{\alpha t}{2}}.$$

$$\frac{1}{T}\int_0^T \left(r_X(t) - \mu^2\right)dt = \frac{1}{T}\int_0^T e^{-\frac{\alpha t}{2}} dt = \frac{1}{T}\left(-\frac{2}{\alpha}\left(1-e^{-\frac{\alpha T}{2}}\right)\right) \to 0, T \to \infty.$$

由推论 8.3 可知，$X = (X(t), t \geq 0)$ 满足均值遍历性.

4. 解：首先，$EX_n = \mu$. 由于 $(\xi_n, n \geq 0)$ 是不相关的随机变量序列，则有

$$r_X(m) = E(X_k X_{k+m}) = E\left(\frac{\xi_k + \cdots + \xi_0}{k+1} \cdot \frac{\xi_{k+m} + \cdots + \xi_m}{k+1}\right)$$

$$= \frac{(E\xi_k + \cdots + E\xi_0)(E\xi_{k+m} + \cdots + E\xi_m)}{(k+1)^2} = \mu^2,$$

进一步，有

$$\frac{1}{n}\sum_{m=1}^{n}\left(r_X\left(n\right)-\mu^2\right)=0,$$

由推论 8.1 可知，$X=\left(X_n,n\geq 0\right)$满足均值遍历性.