16. (1) $\frac{\partial^2 z}{\partial x^2}=\frac{\partial}{\partial x}\left(\frac{-\frac{y}{x^2}}{1+\frac{y^2}{x^2}}\right)=\frac{\partial}{\partial x}\left(\frac{-1}{x^2+y^2}\right)=\frac{2x}{(x^2+y^2)^2}$

$\frac{\partial^2 z}{\partial x\partial y}=\frac{\partial}{\partial x}\left(\frac{x}{x^2+y^2}\right)=\frac{y^2-x^2}{(x^2+y^2)^2}$

$\frac{\partial^2 z}{\partial y^2}=\frac{\partial}{\partial y}\left(\frac{x}{x^2+y^2}\right)=\frac{-2yx}{(x^2+y^2)^2}$

(6) $\frac{\partial u}{\partial x}=yze^{x+y+z}+xyze^{x+y+z}$ $\quad \frac{\partial^2 u}{\partial x^2}=2yze^{x+y+z}+xyze^{x+y+z}$.

归纳得，有 $\frac{\partial^k u}{\partial x^k}=kyze^{x+y+z}+xyze^{x+y+z}$

$$\frac{\partial^{p+q+r}u}{\partial x^p\partial y^q\partial z^r}=\frac{\partial^{p+q}}{\partial x^p\partial y^q}\left(rxye^{x+y+z}+xyze^{x+y+z}\right)$$

$$=\frac{\partial^p}{\partial x^p}\left(rqxe^{x+y+z}+rxye^{x+y+z}+qxze^{x+y+z}+xyze^{x+y+z}\right)$$

$$=pqre^{x+y+z}+qrxe^{x+y+z}+prye^{x+y+z}+rxye^{x+y+z}+$$
$$pqze^{x+y+z}+qxze^{x+y+z}+pyze^{x+y+z}+xyze^{x+y+z}$$

$$=e^{x+y+z}(xyz+rxy+qxz+pyz+qrx+pry+pqz+pqr)$$

18. (1). $\frac{\partial z}{\partial x}=-kn^2e^{-kn^2x}\sin ny$

$\frac{\partial^2 z}{\partial y^2}=\frac{\partial}{\partial y}\left(ne^{-kn^2x}\cos ny\right)=-n^2e^{-kn^2x}\sin ny$

故 $k\frac{\partial^2 z}{\partial y^2}=\frac{\partial z}{\partial x}$.

(2). $\frac{\partial^2 u}{\partial x^2}=\frac{\partial}{\partial x}\left(\frac{\frac{z}{y}}{1+\frac{x^2}{y^2}}\right)=-\frac{2xyz}{(x^2+y^2)}$

$\frac{\partial^2 u}{\partial y^2}=\frac{\partial}{\partial y}\left(\frac{-zx}{x^2+y^2}\right)=\frac{2xyz}{(x^2+y^2)}$

$\frac{\partial^2 u}{\partial z^2}=\frac{\partial}{\partial z}\left(\arctan\frac{x}{y}\right)=0$ $\qquad$ 故 $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}=0$