1. A.B独立

(1): $p(A)=0.3$. $p(B)=0.6$. $p(AB|A\cup B)=\frac{p(AB)}{p(A\cup B)}=\frac{p(AB)}{p(A)+p(B)-p(AB)}=\frac{0.3\times0.6}{0.3+0.9-0.3\times0.6}=\frac{9}{51}$

(2): $p(A\cup B\cup C)=p(A)+p(B)+p(C)-p(AB)-p(AC)-p(BC)+p(ABC)$

$=3p(A)-3p^2(A)=\frac{9}{16}$

$\Rightarrow p(A)=\frac{1}{4}$ 或 $\frac{3}{4}$

2. (1): 甲加奶为A. 品正确为B. 则 $p(A)=0.6$. $p(B)=0.7$

则 $p=p(A)p(B)+p(\bar{A})p(\bar{B})=0.6\times0.7+0.4\times0.3=0.54$

(2): $p(C)=0.8$ 为品正确

$p(A|(ABC\cup\bar{A}\bar{B}\bar{C}))=\frac{p(ABC)}{p(ABC)+p(\bar{A}\bar{B}\bar{C})}=\frac{0.6\times0.7\times0.8}{0.6\times0.7\times0.8+0.4\times0.3\times0.2}=\frac{14}{15}$

3.

$\xi \backslash \eta$	0	1	2
0	a	b	b
1	b	a	b
2	b	b	a

$3a+6b=1$

(1) $F(1,1)=2a+2b$

(2): $\zeta=\max\{\xi,\eta\}$, 求 (ξ,ζ) 的联合分布列

$\xi \backslash \zeta$	0	1	2
0	a	b	b
1	0	a+b	b
2	0	0	a+2b

$p(\zeta=\xi=0)=p(\xi=0,\eta=0)=a$ $p(\zeta=2,\xi=0)=p(\xi=0,\eta=2)=b$

$p(\zeta=1,\xi=0)=p(\xi=0,\eta=1)=b$ $p(\zeta=1,\xi=1)=p(\eta\le1,\xi=1)=a+b$

$p(\zeta=2,\xi=1)=p(\xi=1,\eta=2)=b$ $p(\zeta=2,\xi=2)=p(\xi=2,\eta\le2)=a+2b$

其余为0.

(3): $\begin{cases}(a+2b)^2=a\\(a+2b)^2=b\end{cases}\Rightarrow a=b=\frac{1}{9}$

4. $p(x)=\begin{cases}e^{-x} & x>0\\0 & x\le0\end{cases}$

(1): 设 $\begin{cases}u=\frac{x}{x+y}\\v=\frac{x+y}{x+y+z}\\w=x+y+z.\end{cases}\Rightarrow\begin{cases}x=uvw\\y=vw-uvw\\z=w-vw\end{cases}$

则 $f(u,v,w)=\exp(\quad\ \ )$

$p_1(x,y,z) = \exp(-(x+y+z))$

$$\left|\frac{\partial(x,y,z)}{\partial(u,v,w)}\right| = \left|\det\begin{vmatrix} vw & uw & uv \\ -vw & w(1-u) & v(1-u) \\ & -w & 1-v \end{vmatrix}\right| = vw^2$$

故 $p_2(u,v,w) = vw^2 e^{-w}$ $\quad (u,v,w) \in (0,1)\times(0,1)\times(0,+\infty)$ 其余为0.

(2). $p_u(x) = 1$ $\quad p_v(x) = 2x$ $\quad p_w(x) = \frac{1}{2}x^2 e^{-x}$.

而 $p_2(u,v,w) = p_u p_v p_w$ 故 Y_1, Y_2, Y_3 相互独立

5. $\xi \in \{0,1,\dots,n\}$

k封信全放错的个数为$D(k)$. 则 $D(k) = (k-1)D(k-2) + (k-1)D(k-1)$

$D(1) = 0$. $\quad D(2) = 1$

$$p(\xi = k) = \frac{C_n^k D(n-k)}{A_n^n} \qquad E\xi = \sum_{k=0}^{n} \frac{C_n^k D(n-k)}{A_n^n} = \frac{1}{A_n^n}$$