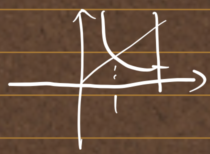
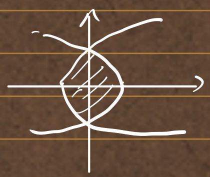
1. (1) $y=\frac{1}{x}$ $y=x$ $x=2$



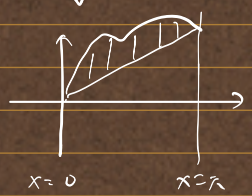
$$S=\int_1^2 x\,dx-\int_1^2\frac{1}{x}dx=\left(\frac{1}{2}x^2-\ln x\right)\Big|_1^2=\frac{3}{2}-\ln 2$$

(2). $y^2=4(x+1)$ $y^2=4(1-x)$



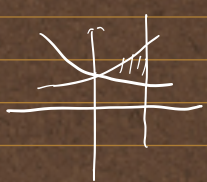
$$S=4\int_{-1}^0 2\sqrt{x+1}=\frac{16}{3}(x+1)^{\frac{3}{2}}\Big|_{-1}^0=\frac{16}{3}$$

(3) $y=x$, $y=x+\sin^2 x$ $x=0$ $x=\pi$



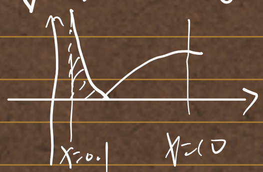
$$S=\int_0^\pi \sin^2 x\,dx=\int_0^\pi\frac{1-\cos 2x}{2}dx=\left(\frac{1}{2}x-\frac{1}{4}\sin 2x\right)\Big|_0^\pi=\frac{\pi}{2}$$

(4). $y=e^x$ $y=e^{-x}$ $x=1$



$$S=\int_0^1(e^x-e^{-x})dx=(e^x+e^{-x})\Big|_0^1=e+\frac{1}{e}-2$$

(5). $y=|\ln x|$ $y=0$ $x=0.1$ $x=10$



$$S=\int_{0.1}^1(-\ln x)dx+\int_1^{10}\ln x\,dx=-x(\ln x-1)\Big|_{0.1}^1+x(\ln x-1)\Big|_1^{10}$$
$$=-8.1+9.9\ln 10$$

(6). $\begin{cases}x=2t-t^2\\ y=2t^2-t^3\end{cases}$ $t\in[0,2]$.

$$S=\int_0^2|y(t)\cdot x'(t)|dt=2\left|\int_0^2(t^4-3t^3+2t^2)dt\right|=2\left|\left(\frac{1}{5}t^5-\frac{3}{4}t^4+\frac{2}{3}t^3\right)\Big|_0^2\right|=\frac{8}{15}$$

(7) $\begin{cases}x=a\cos^3 t\\ y=a\sin^3 t\end{cases}$ $0\le t\le 2\pi$.

$$S=\int_0^{2\pi}|a\sin^3 t\cdot 3a\cos^2 t\sin t|dt=3a^2\left|\int_0^{2\pi}\sin^4 t\cos^2 t\,dt\right|=3a^2\int_0^{2\pi}$$
$$=3a^2\int_0^{2\pi}\left|\frac{1}{16}-\frac{1}{16}\cos 4t-\frac{1}{8}\cos 2t\right|dt+\frac{3}{16}a^2\int_0^{2\pi}(1-\sin^2 2t)\,d\sin 2t$$
$$=3a^2\left(\frac{1}{16}t-\frac{1}{64}\sin 4t-\frac{1}{16}\sin 2t\right)\Big|_0^{2\pi}+\frac{3}{16}a^2\left(t-\frac{1}{3}\sin^3 2t\right)\Big|_0^{2\pi}$$

$= \frac{3}{8}a^2\pi$

(8). $r = a\theta \quad \theta = 0 \quad \theta = 2\pi$

$$S = \frac{1}{2}\int_0^{2\pi} r^2(\theta)\, d\theta = \frac{a^2}{2}\int_0^{2\pi} \theta^2 d\theta = \frac{a^2}{6}\theta^3\Big|_0^{2\pi} = \frac{4}{3}a^2\pi^3$$

(9) $r = ae^{\theta} \quad \theta = 0 \quad \theta = 2\pi$

$$S = \frac{1}{2}\int_0^{2\pi} r^2(\theta)\, d\theta = \frac{a^2}{2}\int_0^{2\pi} e^{2\theta} d\theta = \frac{a^2}{4}e^{2\theta}\Big|_0^{2\pi} = \frac{a^2}{4}(e^{4\pi}-1)$$

(10) $r = a\cos\theta + b$

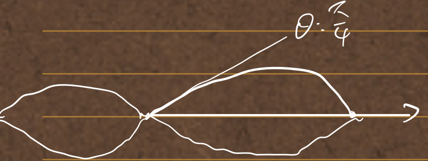
$$S = \frac{1}{2}\int_0^{2\pi}(a\cos\theta+b)^2 d\theta = \frac{a^2}{4}\int_0^{2\pi}(1+\cos 2\theta)\,d\theta + ab\int_0^{2\pi}\cos\theta\, d\theta + \frac{b^2}{2}\int_0^{2\pi} d\theta$$

$$= \frac{1}{2}\pi a^2 + \pi b^2$$

(11) $r = 3\cos\theta,\ r = 1+\cos\theta \quad \theta \in [-\frac{\pi}{3}, \frac{\pi}{3}]$

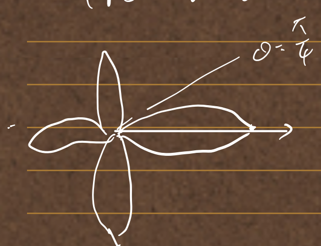
$$S = \left|\frac{1}{2}\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}}(3+4\cos 2\theta - 2\cos\theta)\,d\theta\right| = \frac{1}{2}\left|3\theta + 2\sin 2\theta - 2\sin\theta\right|_{-\frac{\pi}{3}}^{\frac{\pi}{3}} = \pi$$

(12). $r^2 = a^2\cos 2\theta$



$$S = \frac{4}{2}\int_0^{\frac{\pi}{4}} a^2\cos 2\theta\, d\theta = a^2\sin 2\theta\Big|_0^{\frac{\pi}{4}} = a^2$$

(13) $r = a\cos 2\theta$



$$S = 8\times\frac{1}{2}\int_0^{\frac{\pi}{4}} a^2\cos^2 2\theta\, d\theta = 2a^2\int_0^{\frac{\pi}{4}}(\cos 4\theta + 1)\,d\theta = \frac{a^2}{2}\pi$$

(14) $x^3+y^3 = 3axy$. 令 $x = r\cos\theta,\ y = r\sin\theta$

联立 $y = x \Rightarrow x_0 = \frac{3}{2}a \Rightarrow r = \frac{3a\sin 2\theta}{2(\sin^3\theta+\cos^3\theta)} = \frac{3a\sin\theta\cos\theta}{\sin^3\theta+\cos^3\theta} \quad \theta\in[0,\frac{\pi}{2}]$

$$S = \frac{9a^2}{2}\int_0^{\frac{\pi}{2}}\frac{\sin^2\theta\cos^2\theta}{(\sin^3\theta+\cos^3\theta)^2}d\theta = \frac{9}{2}a^2\int_0^{\frac{\pi}{2}}\frac{\tan^2\theta}{(\tan^3\theta+1)^2}d\tan\theta = \frac{3}{2}a^2\left(-\frac{1}{(t^3+1)}\right)\Big|_0^{+\infty}$$

$$= \frac{3}{2}a^2$$

故 $S = \frac{3}{2}a^2$

(14). $x^4+y^4=a^2(x^2+y^2)$ $\Rightarrow r^2=\frac{a^2}{\sin^4\theta+\cos^4\theta}=\frac{a^2(\tan^2\theta+1)}{(\tan^4\theta+1)}\cdot\sec^2\theta$

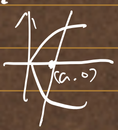
令 $x=r\cos\theta$ $y=r\sin\theta$

$$S=\frac{a^2}{2}\int_0^{2\pi}\frac{\tan^2\theta+1}{\tan^4\theta+1}d\tan\theta=2a^2\int_0^{+\infty}\frac{t^2+1}{t^4+1}dt=2a^2\int_0^{+\infty}\frac{1}{(t-\frac{1}{t})^2+2}d(t-\frac{1}{t})$$

$$=2a^2\cdot\frac{1}{\sqrt{2}}\arctan(t-\frac{1}{t})\Big|_0^{+\infty}=\sqrt{2}a^2\pi$$

故 $S=\sqrt{2}a^2\pi$

2. $S=2\int_0^a\sqrt{4ax}\,dx=4\sqrt{a}\int_0^a\sqrt{x}\,dx=\frac{8}{3}\sqrt{a}\,x^{\frac{3}{2}}\Big|_0^a=\frac{8}{3}a^2$



3. (1) $l=\int_0^4\sqrt{1+\frac{9}{4}x}\,dx=\frac{8}{27}(1+\frac{9}{4}x)^{\frac{3}{2}}\Big|_0^4=\frac{8}{27}(10^{\frac{3}{2}}-1)$

(2): $l=\int_1^e\sqrt{1+(\frac{y}{2}-\frac{1}{2y})^2}\,dy=\frac{1}{2}\int_1^e(y+\frac{1}{y})dy=\frac{1}{2}(\frac{1}{2}y^2+\ln y)\Big|_1^e=\frac{1}{4}(e^2+1)$

(3): $l=\int_0^a\sqrt{1+\tan^2x}\,dx=\int_0^a\sec x\,dx=\ln|\sec x+\tan x|\Big|_0^a=\ln|\sec a+\tan a|$

(4) $l=\int_0^{2\pi}3a|\sin t\cos t|dt=\frac{3a}{4}\int_0^{2\pi}|\sin 2t|d2t=6a$

(5) $l=\int_0^{2\pi}a\sqrt{t^2(\cos^2t+\sin^2t)}\,dt=\int_0^{2\pi}at\,dt=2\pi a$

(6) $l=\frac{a^2}{2}\int_0^{2\pi}(1-\cos\theta)^2d\theta=\frac{a^2}{2}\int_0^{2\pi}(1-2\cos\theta+\frac{1+\cos2\theta}{2})d\theta=\frac{3}{2}\pi a^2$

(7). $l=\int_0^{2\pi}a^2\sqrt{\theta^2+1}\,d\theta=a^2(\frac{1}{2}\theta\sqrt{\theta^2+1}+\frac{1}{2}\ln|\theta+\sqrt{\theta^2+1}|)\Big|_0^{2\pi}$

$=\pi\sqrt{4\pi^2+1}\,a^2+\frac{1}{2}a^2\ln|2\pi+\sqrt{4\pi^2+1}|$

8. $l=a\int_0^{3\pi}\sqrt{\sin^6\frac{\theta}{3}+\sin^4\frac{\theta}{3}\cos^2\frac{\theta}{3}}\,d\theta=\frac{a^2}{2}\int_0^{3\pi}(1-\cos\frac{2}{3}\theta)d\theta=\frac{3}{2}\pi a^2$

4. $\begin{cases}x=t-\sin t\\ y=1-\cos t\end{cases}$ $l=\int_0^x\sqrt{2-2\cos t}\,dt=\int_0^x2\sin\frac{t}{2}dt=4-4\cos\frac{x}{2}$

$l(2\pi)=8$. 令 $l(t)=2\Rightarrow t=\frac{2}{3}\pi$. $x(t)=\frac{2}{3}\pi-\frac{\sqrt{3}}{2}$ $y(t)=\frac{3}{2}$

坐标为 $(\frac{2}{3}\pi-\frac{\sqrt{3}}{2}, \frac{3}{2})$

$S_{(1)}$ $S_{上}=\pi ab$ $S_{下}=\pi AB$.

$S(x)=\pi AB+\frac{\pi}{h}(ab-AB)\cdot x$

$V=\int_0^h S(x)dx=\int_0^h[\pi AB+\frac{\pi}{h}(ab-AB)x]dx=\frac{1}{2}\pi h(ab+AB)$

有 $V=\frac{1}{2}\pi h(ab+AB)$

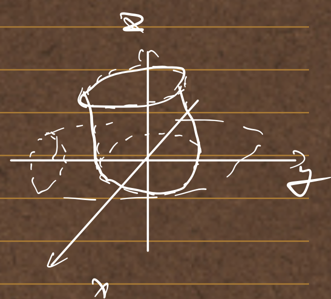
(2).

$S(z)=\frac{ab}{c^2}(c^2-z^2)\pi$

$V=2\int_0^c S(z)dz=2\int_0^c \frac{ab}{c^2}\pi(c^2-z^2)dz=\frac{2ab\pi}{c^2}(c^3-\frac{1}{3}c^3)=\frac{4}{3}\pi abc$

$V=\frac{4}{3}\pi abc$

(3) $y^2=a^2-t^2$ $z^2=a^2-t^2$



$$\int\frac{dx}{x^4+1}\int\frac{dx}{x^4+1}=\int\left(\frac{\frac{\sqrt{2}}{4}x+\frac{1}{2}}{x^2+\sqrt{2}x+1}-\frac{\frac{\sqrt{2}}{4}x-\frac{1}{2}}{x^2-\sqrt{2}x+1}\right)dx$$

$$=\frac{\sqrt{2}}{8}\ln\frac{x^2+\sqrt{2}x+1}{x^2-\sqrt{2}x+1}+\frac{1}{4}\int\left(\frac{1}{x^2+\sqrt{2}x+1}+\frac{1}{x^2-\sqrt{2}x+1}\right)dx$$

$$=\frac{\sqrt{2}}{8}\ln\frac{x^2+\sqrt{2}x+1}{x^2-\sqrt{2}x+1}+\frac{\sqrt{2}}{4}(\arctan(\sqrt{2}x+1)+\arctan(\sqrt{2}x-1))+C$$ 。