# Futher Application

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*(Princiole of Analytic Continuation):*every polynomial of order n have n roots in C

**Theorem 4.8:***f is defined on a region* $\Omega$*,if there is a sequence named* $\{x_n\} \subset \Omega : x_n \to x \in \Omega$ *and* $f(x_n) = 0$ *for all n;then* $f \equiv 0$

**Corollary 4.9:***then if we need to judge function f=g in* $\Omega$*,just to find a sequence* $\{x_n\} \subset \Omega : x_n \to x \in \Omega$ *s.t.* $f(x_n) = g(x_n)$ *for all n*

**Theorem Morera's theorem:***Supposed f is a continuous function in the open disc D s.t. for any triangle T containded in D:* $\int_T f(z)dz = 0$ *then f is Hol on D*

## Application

Sequences of holomorphic functions

**Theorem 5.2:***if* $\{f_n\}_{n=1}^{\infty}$ *is a sequence of holomorphic functions that converges uniformly to a function f in every compact set of* $\Omega$ *then f is holomorphic in* $\Omega$

**Corollary 5.3:***then the sequence of dirivatives* $\{f_n'\}$ *converges uniformly to f' on every compact set of* $\Omega$

Holomorphic functions defined in terms of integrals

**Theorem 5.4:***F(z,s) defined in* $\Omega \times [0, 1]$*,*$\Omega$ *is an open set ,if:1.*$F(z, s)$ *is hol in z for each s.*

*2.F is continuous on* $\Omega \times [0,1]$*:then* $f(z) = \int_o^1 F(z,s)ds$ *is holomorphic*

Schwarz reflection principle

**Theorem Symmetry principle:***if* $f^+$ *and* $f^-$ *are hol on* $\Omega^+$ *and* $\Omega^-$ *respectively, and* $f^+(x) = f^-(x), \forall x \in I = \Omega^+ \cap \Omega^-$ *,then:* $F = \begin{cases} f^+ & x \in \Omega^+ \\ f^+ = f^- & x \in I \\ f^- & x \in \Omega^- \end{cases}$ *is hol on* $\Omega$

**Theorem Schwarz reflection principle:***f hol on* $\Omega^+$ *and extends continuously to I and s.t. f(x) is a real function while x∈I,then there exists a function F s.t. F hol on* $\Omega$

*Proof:*[Srp]define $F(z) = \overline{f(\overline{z})}$ *while* $z \in \Omega^-$

Runge's approximation Theorem