一、(1): $p(A)=p(B)=0.2$. $p(C)=0.4$ A.B.C互斥

$p(A\bar{C}\cup B)=p(A\bar{C})+p(B)-p(A\bar{C}B)=p(A)+p(B)=0.4$ $(A\subset\bar{C})$

(2): ___

二、

三. 独立 ⇒ 不相关 不相关 ⇏ 独立.

(1)

$\xi \backslash \eta$	0	1	2	
0	0.28	0.16	0.16	0.6
1	0.08	0.32	0	0.4
	0.36	0.48	0.16	

$$\begin{cases} c+d=0.4 \\ a+b=0.44 \\ a+c=0.36 \\ b+d=0.48 \\ \underbrace{E\xi E\eta=0.4\times0.8=E\xi\eta} \end{cases} \Rightarrow \begin{cases} a=0.28 \\ b=0.16 \\ c=0.08 \\ d=0.32 \end{cases}$$

(2) 不独立. 因为 $p(\xi=1,\eta=2)\neq p(\xi=1)p(\eta=2)$

4. $p_{Z_1}(x)=\lambda e^{-\lambda x}$ $p_{Z_2}(x)=\mu e^{-\mu x}$. $p_Z(x,y)=\mu\lambda e^{-\lambda x-\mu y}$

(1) $\begin{cases} u=x+y \\ v=\frac{x}{x+y} \end{cases} \Rightarrow \begin{cases} x=uv \\ y=u(1-v) \end{cases}$ $\left|\frac{\partial(x,y)}{\partial(u,v)}\right|=\left|\det\begin{vmatrix} v & u \\ 1-v & -u \end{vmatrix}\right|=u$

$p_Y(u,v)=p_Z(uv,u(1-v))u=\mu\lambda e^{-\lambda uv-\mu u(1-v)}u$

$\Rightarrow p_Y(u,v)=\lambda\mu\cdot u\exp\{-\lambda uv-\mu u+\mu uv\}$

(2). $p_{Y_1}(u)=\int_0^1 p_Y(u,v)dv=\frac{\lambda\mu}{\mu-\lambda}(e^{-\lambda u}-e^{-\mu u})$

$p_{Y_2}(v)=\int_0^{+\infty}p_Y(u,v)du=\lambda\mu\int_0^{+\infty}u\exp\{(\mu v-\lambda v-\mu)u\}du$
$=\lambda\mu\cdot\frac{1}{(\mu v-\lambda v-\mu)^2}$

5. 若要 $5m+7n+3$ h回到出发点的概率为 $P(m,n)=(\frac{1}{3})^{m+n+1}$

故 $E\xi=\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}(\frac{1}{3})^{m+n+1}\cdot(5m+7n+3)$

$$= \sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\left[(5m+3)\left(\frac{1}{3}\right)^{m+1}\cdot\left(\frac{1}{3}\right)^{n} + \left(\frac{1}{3}\right)^{m+1}\cdot 7\cdot n\left(\frac{1}{3}\right)^{n}\right]$$

$$= \sum_{m=0}^{\infty} (5m+3)\frac{3}{2}\cdot\left(\frac{1}{3}\right)^{m+1} + \frac{21}{4}\left(\frac{1}{3}\right)^{m+1}$$

$$= 6.75.$$

6. $Cov(X_k, S_n) = EX_kS_n - EX_kES_n$

$EX_k = \frac{1}{k}$ $ES_n = EX_1X_2 + \cdots + EX_nX_{n+1} = EX_1EX_2 + \cdots + EX_nEX_{n+1}$

$= \frac{1}{1}\times\frac{1}{2} + \frac{1}{2}\times\frac{1}{3} + \cdots + \frac{1}{n}\times\frac{1}{n+1}$

$= 1 - \frac{1}{2} + \frac{1}{2} - \frac{1}{3} + \cdots + \frac{1}{n} - \frac{1}{n+1} = \frac{n}{n+1}$

$EX_kS_n = P(X_k=1)ES_n = \frac{1}{k}\cdot ES_n = \frac{1}{k}\frac{n}{n+1}. = EX_kES_n.$

故 $Cov(X_k, S_n) = 0.$

7. $\xi \sim B(600, \frac{1}{6}).$

$P\left(\frac{\xi - 100}{\sqrt{\frac{500}{6}}} < x\right) = \Phi(x) \Rightarrow \frac{\xi - 100}{\sqrt{\frac{500}{6}}} < 2.625 \Rightarrow$ 找出ξ即可

8. $\bar{\xi} \xrightarrow{P} \mu$ 故 $\frac{S^2}{n} \to \sigma^2 \Rightarrow \frac{S^2}{n-1} \xrightarrow{P} \sigma^2$ 故 $c = \sigma^2$