一.(1)

$E Y_n = P(\text{方向阳}) E Y_{[\frac{n}{2}]} + P(\text{方向为阴})(E Y_{[\frac{n}{2}]} + E X_{[\frac{n}{2}]})$. $P(\text{方向为阳}) = 1-(1-p)^{[\frac{n}{2}]}$

故 $E Y_n = E Y_{[\frac{n}{2}]} + (1-(1-p)^{[\frac{n}{2}]}) E X_{[\frac{n}{2}]}$

(2): $E X_n = (1-p)^n \times 1 + [1-(1-p)^n] E Y_n$.

故 $E Y_n = E Y_{[\frac{n}{2}]} + (1-(1-p)^{[\frac{n}{2}]})[(1-p)^{[\frac{n}{2}]} + (1-(1-p)^{[\frac{n}{2}]}) E Y_{[\frac{n}{2}]}]$

$\Rightarrow E Y_n = [1 + (1-(1-p)^{[\frac{n}{2}]})^2] E Y_{[\frac{n}{2}]} + (1-p)^{[\frac{n}{2}]}[1-(1-p)^{[\frac{n}{2}]}]$

二.(1): $m_r \le 1 \Rightarrow m_{r-1} \le 2 \cdots m_i \le 2^{r-i}$. 不然若 $m_r > 1$. 则无法决出冠军.

(2): $m_1 \le 2^{r-1}$ 此时至多有 $2^r$ 个队. 令 $2^r \ge 4 \Rightarrow r \ge 2$. 设 $p(r)$ 为 $r$ 时, 1 最终获冠的概率.

① $r=2$ 时. $p(2) = \left(\frac{v_1}{v_1+v}\right)^2$

② $r=3$ 时: $p(3) = \frac{C_3^2}{C_4^2} \cdot \frac{C_2^2}{C_3^2} \cdot \frac{v_1}{v_1+v} + \frac{C_3^1}{C_4^2}\left(\frac{v_1}{v_1+v}\right) \cdot \frac{C_2^2}{C_3^2}\left(\frac{v_1}{v_1+v}\right) + \frac{C_3^2}{C_4^2} \cdot \frac{C_2^1}{C_3^2}\left(\frac{v_1}{v_1+v}\right)\left(\frac{v_1}{v_1+v}\right)$

$+ \frac{C_3^1}{C_4^2}\left(\frac{v_1}{v_1+v}\right) \cdot \frac{C_2^1}{C_3^2}\left(\frac{v_1}{v_1+v}\right)\left(\frac{v_1}{v_1+v}\right)$

$= \frac{1}{6} \cdot \frac{v_1}{v_1+v} + \frac{1}{2}\left(\frac{v_1}{v_1+v}\right)^2 + \frac{1}{6}\left(\frac{v_1}{v_1+v}\right)^3$

令 $t = \frac{v_1}{v_1+v} > \frac{1}{2}$. 则 $(\frac{1}{6}t^3 + \frac{1}{2}t^2 + \frac{1}{6}t) - t^2 = \frac{1}{6}(t^3 - 3t^2 + t) < -\frac{3}{4} < 0$. i.e. $p(3) < p(2)$.

故 bb 试 3 场时, 1 获冠军概率小; bb 试 2 场时. 获冠军概率大.

(3): $f_1 = \frac{C_{n-1}^{2k}}{C_n^{2k}} + \left(1 - \frac{C_{n-1}^{2k}}{C_n^{2k}}\right)\frac{v_1}{v_1+v}$

$f_2 = \left[\frac{C_{n-1}^{2j}}{C_n^{2j}} + \left(1 - \frac{C_{n-1}^{2j}}{C_n^{2j}}\right)\frac{v_1}{v_1+v}\right] \cdot \left[\frac{C_{n-j-1}^{2k-2j}}{C_{n-j}^{2k-2j}} + \left(1 - \frac{C_{n-j-1}^{2k-2j}}{C_{n-j}^{2k-2j}}\right)\frac{v_1}{v_1+v}\right]$ 令

$f_1 - f_2 = \frac{v_1}{v_1+v} + \frac{C_{n-1}^{2k}}{C_n^{2k}} \frac{v}{v_1+v} - \left[\frac{C_{n-1}^{2j}}{C_n^{2j}} \frac{v}{v_1+v} + \frac{v_1}{v_1+v}\right]\left[\frac{C_{n-j-1}^{2k-2j}}{C_{n-j}^{2k-2j}} \frac{v}{v_1+v} + \frac{v_1}{v_1+v}\right]$

$= t + (1 - \frac{2k}{n})(1-t) - [(1 - \frac{2j}{n})(1-t) + t][(1 - \frac{2k-2j}{n-j})(1-t) + t]$

$= -\frac{4j(k-j)}{n(n-j)}(t-1)^2 + \left(\frac{2k-2j}{n} - \frac{2k-2j}{n-j}\right)(t-1)$

$= -\frac{2j(k-j)}{n(n-j)}[2(t-1)^2 + (t-1)] > 0$. $t-1 \in (-\frac{1}{2}, 0)$

(4): 用 $f_s$ 表示 s 轮结束后队一未被淘汰的概率. 在总局数中. 1 与其它队 bb 赛次数越少, 越有可能获胜.

因为 1 队胜的概率为 $f_r = \left(\frac{v_1}{v_1+v}\right)^t$. 其中 $t$ 是 1 队的 bb 赛次数. 故应选 bb 赛次数尽可能少的安排.

$t' = t^{\frac{2m_1}{n}} \le t = t^{\frac{2k}{n}}$

$m_i = k$. $m_l = 1$ 已经最大. 且当 $m_i > k$. $f_i$ … $f_i$ … 时. 且由(3)知 $m_i \geq k$.

故 $m_i = k$. 此时此方案最优.