4. (1) $\iint_D (x^3+3x^2y+y^3)\,dxdy = \int_0^1 dx\int_0^1 (x^3+3x^2y+y^3)\,dy = \int_0^1 (x^3+\frac{3}{2}x^2+\frac{1}{4})\,dx = 1$

(2) $\iint_D xye^{x^2+y^2}\,dxdy = \int_c^d dy\int_a^b xye^{x^2+y^2}\,dx = (e^{b^2}-e^{a^2})\int_c^d \frac{1}{2}ye^{y^2}\,dy$

$= \frac{1}{4}(e^{b^2}-e^{a^2})(e^{d^2}-e^{c^2})$

(3) $\iiint_D \frac{dx\,dy\,dz}{(x+y+z)^3} = \int_1^2 dx\int_1^2 dy\int_1^2 \frac{1}{(x+y+z)^3}\,dz = \frac{1}{2}\int_1^2 dx\int_1^2 \left(\frac{1}{(x+y+1)^2}-\frac{1}{(x+y+2)^2}\right)dy$

$= \frac{1}{2}\int_1^2 \left(\frac{1}{x+2}+\frac{1}{x+4}-\frac{2}{x+3}\right)dx = \frac{1}{2}\ln\frac{128}{125}$

5. (1) $\int_a^b dx\int_a^x f(x,y)\,dy = \int_a^b dy\int_y^b f(x,y)\,dy$

(2) $\int_0^{2a} dx\int_{\sqrt{2ax-x^2}}^{\sqrt{2ax}} f(x,y)\,dy = \int_0^a dy\int_{\frac{y^2}{2a}}^{a-\sqrt{a^2-y^2}} f(x,y)\,dx + \int_0^a dy\int_{a+\sqrt{a^2-y^2}}^{2a} f(x,y)\,dx$

$+ \int_a^{2a} dy\int_{\frac{y^2}{2a}}^{2a} f(x,y)\,dx$

(3) $\int_0^{2\pi} dx\int_0^{\sin x} f(x,y)\,dy = \int_0^1 dy\int_{\arcsin y}^{\pi-\arcsin y} f(x,y)\,dx + \int_0^{-1} dy\int_{\pi-\arcsin y}^{2\pi+\arcsin y} f(x,y)\,dx.$

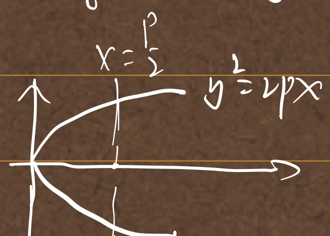
(4) $\int_0^1 dy\int_0^{2y} f(x,y)\,dx + \int_1^3 dy\int_0^{3-y} f(x,y)\,dx = \int_0^2 dx\int_{\frac{1}{2}x}^1 f(x,y)\,dy + \int_0^2 dx\int_1^{3-x} f(x,y)\,dy$

$= \int_0^2 dx\int_{\frac{1}{2}x}^{3-x} f(x,y)\,dy$

(5) $\int_0^1 dx\int_0^{1-x} dy\int_0^{x+y} f(x,y,z)\,dz = \int_0^1 dz\int_0^1 dx\int_0^{1-x} f(x,y,z)\,dy - \int_0^1 dz\int_0^z dx\int_0^{z-x} f(x,y,z)\,dy.$

(6) $\int_{-1}^1 dx\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} dy\int_{\sqrt{x^2+y^2}}^1 f(x,y,z)\,dz = \int_0^1 dz\int_{-z}^{z} dy\int_{-\sqrt{z^2-y^2}}^{\sqrt{z^2-y^2}} f(x,y,z)\,dx.$

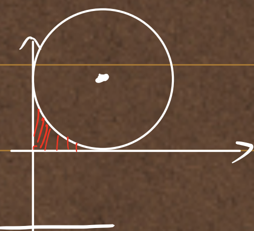
6. (1) $\iint_D xy^2\,dxdy$ 积分区域为




$\iint_D xy^2\,dxdy = \int_0^{\frac{p}{2}} dx\int_{-\sqrt{2px}}^{\sqrt{2px}} xy^2\,dy = \int_0^{\frac{p}{2}} \frac{1}{3}xy^3\Big|_{y=-\sqrt{2px}}^{y=\sqrt{2px}}\,dx = \int_0^{\frac{p}{2}} \frac{4\sqrt{2}}{3}p^{\frac{3}{2}}x^{\frac{5}{2}}\,dx$

$=\frac{4\sqrt{2}}{3}p^{\frac{3}{2}}\cdot\frac{2}{7}x^{\frac{7}{2}}\Big|_0^{\frac{p}{2}}=\frac{1}{21}p^5$

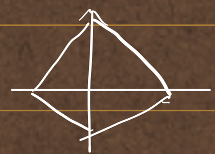
(2). $\iint_D \frac{dxdy}{\sqrt{2a-x}}$ 积分区域



$$\iint_D \frac{dxdy}{\sqrt{2a-x}}=\int_0^a dx\int_0^{a-\sqrt{a^2-(x-a)^2}}\frac{1}{\sqrt{2a-x}}dy=\int_0^a\frac{a}{\sqrt{2a-x}}dx-\int_0^a\frac{\sqrt{2ax-x^2}}{\sqrt{2a-x}}dx$$

$$=-2a\sqrt{2a-x}\Big|_0^a-\frac{2}{3}x^{\frac{3}{2}}\Big|_0^a=(2\sqrt{2}-\frac{8}{3})a^{\frac{3}{2}}$$

(3). $\iint_D e^{x+y}dxdy$ 积分区域

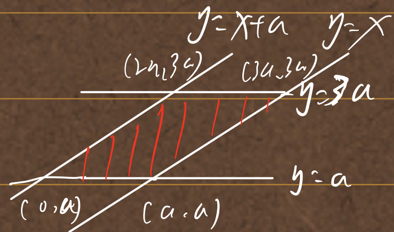


$$\iint_D e^{x+y}dxdy=\int_0^1 dx\int_{x-1}^{1-x}e^{x+y}dy+\int_{-1}^0 dx\int_{-x-1}^{x+1}e^{x+y}dy$$

$$=\int_0^1(e-e^{2x-1})dx+\int_{-1}^0(e^{2x+1}-e^{-1})dx$$

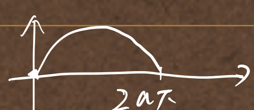
$$=e-\frac{1}{e}$$

(4). $\iint_D(x^2+y^2)dxdy$ 积分区域




$$\iint_D(x^2+y^2)dxdy=\int_a^{3a}dy\int_{y-a}^{y}(x^2+y^2)dx=\int_a^{3a}(2ay^2-a^2y+\frac{1}{3}a^3)dy=14a^4$$

(5) 积分区域.



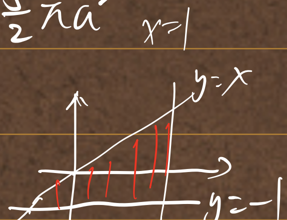

$y=\frac{1-\cos t}{t-\sin t}x$

$$\iint_D y\,dxdy=\int_0^{a\pi}dx\int_0^{y(x)}y\,dy=\frac{1}{2}\int_0^{a\pi}\left(\frac{1-\cos t}{t-\sin t}\right)^2x^2dx=\frac{a^3}{2}\int_0^{\pi}(1-\cos t)^3dt$$

$$=\frac{a^3}{2}\int_0^{\pi}(1-3\cos t+3\cos^2t-\cos^3t)dt=\frac{a^3}{2}\int_0^{2\pi}(\frac{5}{2}+\frac{3}{2}\cos 2t)dt$$

$$=\frac{5}{2}\pi a^3$$

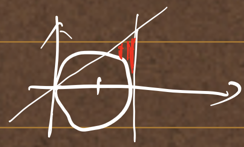
(6). 积分区域



$$\iint_D y[1+xe^{\frac{1}{2}(x^2+y^2)}]\,dxdy = \int_{-1}^{1}dx\int_{-1}^{x} y[1+xe^{\frac{1}{2}(x^2+y^2)}]\,dy$$

$$= \int_{-1}^{1}\left(\frac{1}{2}x^2 + xe^{x^2} - xe^{\frac{1}{2}(x^2+1)} - \frac{1}{2}\right)dx = \left(\frac{1}{6}x^3 + \frac{1}{2}e^{x^2} - e^{\frac{1}{2}(x^2+1)} - \frac{1}{2}x\right)\Big|_{-1}^{1}$$

$$= -\frac{2}{3}$$

(7). 积分区域 

$$\iint_D x^2y\,dxdy = \int_1^2 dx\int_{\sqrt{2x-x^2}}^{x} x^2y\,dy = \int_1^2 (x^4-x^3)\,dx = \left(\frac{1}{5}x^5 - \frac{1}{4}x^4\right)\Big|_1^2 = 2.45$$

(8). $$\iiint_\Omega xy^2z^3\,dxdydz = \iint_{\Omega'} dxdy\int_0^{xy} xy^2z^3\,dz = \iint_{\Omega'} \frac{1}{4}x^5y^6\,dxdy$$

$$= \frac{1}{4}\int_0^1 dx\int_0^x x^5y^6\,dy = \frac{1}{28}\int_0^1 x^{12}\,dx = \frac{1}{364}$$

(9) $$\iiint_\Omega \frac{dxdydz}{(1+x+y+z)^3} = \int_0^1 dz\cdot S_{(z)} = \int_0^1 \frac{1}{2}(1-z)^2\,dz = \frac{1}{6}(z-1)^3\Big|_0^1 = \frac{1}{8}$$

(10) $$\iiint_\Omega z\,dxdydz = \int_0^h z\cdot\pi z\,dz = \frac{1}{3}\pi h^3$$

(11). $$\iiint_\Omega z^2\,dxdydz = \int_0^{\frac{R}{2}} z^2dz\cdot\iint_{\Omega_z} dxdy + \int_{\frac{R}{2}}^{R} z^2dz\iint_{\Omega_z} dxdy$$

$$= \int_0^{\frac{R}{2}} \pi z^2(2Rz-z^2)\,dz + \int_{\frac{R}{2}}^{R} \pi z^2(R^2-z^2)\,dz$$

$$= \frac{1}{2}\pi R\cdot z^4\Big|_0^{\frac{R}{2}} - \frac{1}{5}\pi z^5\Big|_0^{\frac{R}{2}} + \frac{1}{3}\pi R^2\cdot z^3\Big|_{\frac{R}{2}}^{R} - \frac{1}{5}\pi z^5\Big|_{\frac{R}{2}}^{R}$$

$$= \frac{13}{96}\pi R^5$$

(12) $$\iiint_\Omega x^2\,dxdydz = \int_{-a}^{a} x^2dx\cdot\pi bc\left(1-\frac{x^2}{a^2}\right) = \frac{4}{15}\pi a^3bc$$

11. $\iiint_{\Omega} dxdydz = \iint_{\Omega_z} dxdy \int_0^{x^2+y^2} dz = \int_0^1 dx \int_0^{1-x} (x^2+y^2) dy = \int_0^1 (-\frac{4}{3}x^3+2x^2-x+\frac{1}{3}) dx$
$= 1/6$

14. $\iint_D (\sin x^2+\cos y^2) dxdy = \iint_D \sin x^2 dxdy + \iint_D \cos y^2 dxdy$
$= \int_0^1 dx \int_0^1 \sin x^2 dy + \int_0^1 dy \int_0^1 \cos y^2 dx = \int_0^1 \sin x^2 dx + \int_0^1 \cos y^2 dy$
$f(x) = \int_0^1 \sin x^2 dx + \int_0^1 \cos x^2 dx = \frac{\sqrt{2}}{2}\int_0^1 \sin(x^2+\frac{\pi}{4}) dx < \frac{\sqrt{2}}{2}\int_0^1 (x^2+\frac{\pi}{4}) dx = \frac{\sqrt{2}}{2}(\frac{1}{3}+\frac{\pi}{4}) < \sqrt{2}$
$f(x) = \int_0^1 \sin x^2 dx + \int_0^1 \cos x^2 dx \geq \int_0^1 \sin^2 x^2 dx + \int_0^1 \cos^2 x^2 dx = \int_0^1 dx = 1.$
故 $1 \leq f(x) \leq \sqrt{2}$. #

17：证明
$[\int_a^b f(x) dx]^2 = \int_a^b f(x) dx \int_a^b f(y) dy = \iint_{[a,b]\times[a,b]} f(x)f(y) dxdy \leq \frac{1}{2}\iint_{\Omega} [f^2(x)+f^2(y)] dxdy$
$= \frac{1}{2}\int_a^b f^2(x) dx \int_a^b dy + \frac{1}{2}\int_a^b f(y)^2 dy \int_a^b dx$
$= (b-a)\int_a^b f^2(x)$
即 $[\int_a^b f(x) dx]^2 \leq (b-a)\int_a^b f^2(x)$.