一. (1): $p(E|E\cup F)=\dfrac{p(E\cap(E\cup F))}{p(E\cup F)}=\dfrac{p(E)}{p(E\cup F)}=\dfrac{p(E\cap F)+p(E\cap F^c)}{p(F)+p(E\cap F^c)}\geqslant\dfrac{p(EF)}{p(F)}$

因为 $\frac{a+c}{b+c}>\frac{a}{b}$ $b>a$ 时. 当且仅当 $E\cap F^c=\phi$ 时取"="

(2): $p(AB|\bar{C})=\dfrac{p(AB\bar{C})}{p(\bar{C})}=\dfrac{p(AB)-p(ABC)}{p(\bar{C})}=\dfrac{0.5}{0.7}=\dfrac{5}{7}$

二. $p(\xi=k)=\dfrac{\lambda^k}{k!}e^{-\lambda}$ $\eta_N\sim B(N,p)$

(1): $p=\sum_{l=k}^{\infty}p(\xi=l)p(\eta_l=k).=\sum_{l=k}^{\infty}\dfrac{\lambda^l}{l!}e^{-\lambda}\cdot C_l^k p^k q^{l-k}$

$=\sum_{l=k}^{\infty}\dfrac{1}{k!}\left(\dfrac{p}{q}\right)^k(\lambda q)^k\dfrac{(\lambda q)^{l-k}}{(l-k)!}e^{-\lambda}=e^{-\lambda}\dfrac{(\lambda p)^k}{k!}\sum_{l=0}^{\infty}\dfrac{(\lambda q)^l}{l!}$

$=e^{-\lambda p}\dfrac{(\lambda p)^k}{k!}$

故k件中的概率 $p=e^{-\lambda p}\dfrac{(\lambda p)^k}{k!}$

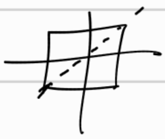
(2): $p(\text{卖}m|\text{中}k)\,p(\text{中}k)=p(\text{中}k|\text{卖}m)\,p(\text{卖}m)$

$\Rightarrow p=\dfrac{p(\text{中}k|\text{卖}m)\,p(\text{卖}m)}{p(\text{中}k)}=\dfrac{C_m^k p^k q^{m-k}\cdot e^{-\lambda}\frac{\lambda^m}{m!}}{e^{-\lambda p}\frac{(\lambda p)^k}{k!}}=e^{-\lambda q}\dfrac{(\lambda q)^{m-k}}{(m-k)!}$

$q=1-p.$ $m\geqslant k.$

$m<k$ 时 $p=0.$

三. $f(x,y)=c(1+xy)$ $(x,y)\in(-1,1)^2$



(1): $1=\int_{-1}^{1}\int_{-1}^{1}c(1+xy)dydx=4c\Rightarrow c=\frac{1}{4}$

(2): $p(x|y=\frac{1}{2})=\frac{1}{4}(1+\frac{1}{2}x)=\frac{1}{8}x+\frac{1}{4}$

(3): $p(X>Y)=\int_{-1}^{1}\int_{-1}^{x}\frac{1}{4}(1+xy)dydx=\int_{-1}^{1}(\frac{1}{8}x+\frac{1}{8}x^3+\frac{1}{4})dx=\frac{1}{2}$

(4): $p_X(x)=\int_{-1}^{1}\frac{1}{4}(1+xy)dy=\frac{1}{2}=p_Y(y).$ $\frac{d}{dx}\sqrt{x}$

$p_{X^2}(x)=\frac{1}{2}\cdot\frac{d}{dx}(\sqrt{x})\times 2=\frac{1}{2\sqrt{x}}$ $p_{Y^2}(y)=\frac{1}{2\sqrt{y}}$

$\begin{cases}u=x^2\\v=y^2\end{cases}\Rightarrow\begin{cases}x=\sqrt{u}\\y=\sqrt{v}\end{cases}$ $\dfrac{\partial(x,y)}{\partial(u,v)}=\begin{vmatrix}\frac{1}{2\sqrt{u}} & \\ & \frac{1}{2\sqrt{v}}\end{vmatrix}=\dfrac{1}{4\sqrt{uv}}$

4. $Z \sim N(0,1)$

(1): $p(y)=\int_{-\infty}^{+\infty}\frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}}\cdot\frac{1}{\sqrt{2\pi}}e^{-\frac{(y-x)^2}{2}}dx=\frac{1}{2\pi}\int_R \exp\{-x^2+xy-\frac{1}{2}y^2\}dx$

$=\frac{1}{2\pi}e^{-\frac{1}{4}y^2}\int_R \exp\{-(x-\frac{1}{2}y)^2\}d(x-\frac{1}{2}y)$

$=\frac{1}{2\sqrt{\pi}}e^{-\frac{1}{4}y^2}$

$P_Y(x)=\frac{1}{2\sqrt{\pi}}e^{-\frac{1}{4}x^2}$

(2): $E(ZY|Z=x)=\int_R xy\frac{\frac{1}{\sqrt{2\pi}}e^{-\frac{(y-x)^2}{2}}}{\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}x^2}}dy=xe^{\frac{1}{2}x^2}\int_R ye^{-\frac{(y-x)^2}{2}}dy=xe^{\frac{1}{2}x^2}\int_R(x+t)e^{-\frac{t^2}{2}}dt$

$=xe^{\frac{1}{2}x^2}(\sqrt{2\pi}x+1)=(\sqrt{2\pi}x+1)xe^{\frac{1}{2}x^2}$

(3): $r_{Z,Y}=\frac{Cov(Z,Y)}{\sqrt{VarZ}\sqrt{VarY}}$

$EZ=0$, $VarZ=1$. $EY=\frac{1}{2\sqrt{\pi}}\int_R ye^{-\frac{1}{4}y^2}dy=0$. $VarY=\frac{1}{2\sqrt{\pi}}\int_R y^2e^{-\frac{1}{4}y^2}dy=2$

$EZY=0$. 对称性. 故 $Cov(Z,Y)=0$

$r_{Z,Y}=0$

(4): $\begin{cases} u=ax+y \\ v=ax-y \end{cases}$

五: