

$$5. \text{ 两点: } p(x) = \frac{x-0.9}{1.1-0.9} 0.8912 + \frac{x-1.1}{0.9-1.1} 0.7833$$

$$p(1) = \frac{0.1}{0.2} \times 0.8912 + \frac{0.1}{0.2} \times 0.7833 = 0.83725.$$

$$\text{三点: } p(x) = \frac{(x-0.9)(x-1.1)}{(1.3-0.9)(1.3-1.1)} 0.9636 + \frac{(x-0.9)(x-1.3)}{(1.1-0.9)(1.1-1.3)} 0.8912 + \frac{(x-1.1)(x-1.3)}{(0.9-1.1)(0.9-1.3)} 0.7833$$

$$p(1) = \frac{-0.01}{0.08} \times 0.9636 + \frac{0.03}{0.04} 0.8912 + \frac{0.03}{0.08} \times 0.7833 = 0.846875$$

差商表

x	$f(x)$	一阶	二阶	三阶	四阶	五阶
0.7	0.6442					
0.9	0.7833	0.6955				
1.1	0.8912	0.5395	-0.78			
1.3	0.9636	0.362	-0.8875	-0.5375		
1.5	0.9975	0.1695	-0.9625	-0.375	0.8125	
1.7	0.9817	-0.029	-0.9925	-0.15	1.125	1.5625

$$p(x) = 0.6442 + 0.6955(x-0.7) - 0.78(x-0.7)(x-0.9) - 0.5375(x-0.7)(x-0.9)(x-1.1) \\ + 0.8125(x-0.7)(x-0.9)(x-1.1)(x-1.3) + 1.5625(x-0.7)(x-0.9)(x-1.1)(x-1.3)(x-1.5)$$

$$p(1) = 0.887453125.$$

8. x	0	1	2	3
f	0	0	0	0

$$1) : S'(0) = 1 \quad S'(3) = 0.$$

$$\text{不妨设 } S_{01}(x) = (a_1x + b_1)x(x-1) \quad S_{12}(x) = (a_2x + b_2)(x-1)(x-2).$$

$$S_{23}(x) = (a_3x + b_3)(x-2)(x-3)$$

$$\begin{array}{l} \text{从而: } \left\{ \begin{array}{l} S'_{01}(0) = -b_1 = 1 \\ S'_{01}(1) = S'_{12}(1) \\ S'_{12}(2) = S'_{23}(2) \\ S'_{23}(3) = 3a_3 + b_3 = 0 \\ S''_{01}(1) = S''_{12}(1) \\ S''_{12}(2) = S''_{23}(2) \end{array} \right. \quad \text{即} \quad \left\{ \begin{array}{l} b_1 = -1 \\ a_1 + b_1 = -a_2 - b_2 \\ 2a_2 + b_2 = -2a_3 - b_3 \\ 3a_3 + b_3 = 0 \\ 6a_1 + 2(b_1 - a_1) = 6a_2 + 2(b_2 - 3a_2) \\ 12a_2 + 2(b_2 - 3a_2) = 12a_3 + 2(b_3 - 5a_3) \end{array} \right. \Rightarrow \left\{ \begin{array}{l} a_1 = 0.8 \\ b_1 = -1 \\ a_2 = -0.4 \\ b_2 = 0.6 \\ a_3 = -0.2 \\ b_3 = 0.6 \end{array} \right.$$

$$1) (0.8x - 1)x(x-1)$$

$$x(1.5x - 1)$$

$$\text{故 } S(x) = \begin{cases} (0.8x - 1.1)x(x-1) & x \in [0, 1) \\ (-0.4x + 0.6)(x-1)(x-2) & x \in [1, 2) \\ (-0.2x + 0.6)(x-2)(x-3) & x \in [2, 3] \end{cases}$$

(2) $S''(0)=1$, $S''(3)=0$, 如(1)中设法有:

$$\begin{cases} a_1 + b_1 = -a_2 - b_2 \\ 2a_2 + b_2 = -2a_3 - b_3 \\ 2a_1 + b_1 = b_2 \\ 3a_2 + b_2 = a_3 + b_3 \\ 2(b_1 - a_1) = 1 \\ 4a_3 + b_3 = 0 \end{cases} \Rightarrow \begin{cases} a_1 = -\frac{19}{90} \\ b_1 = \frac{13}{45} \\ a_2 = \frac{5}{90} \\ b_2 = \frac{-6}{45} \\ a_3 = -\frac{1}{90} \\ b_3 = \frac{2}{45} \end{cases}$$

$$\text{故 } S(x) = \begin{cases} \frac{1}{90}(-19x + 26)x(x-1) & x \in [0, 1) \\ \frac{1}{90}(5x - 12)(x-1)(x-2) & x \in [1, 2) \\ \frac{1}{90}(-x + 4)(x-2)(x-3) & x \in [2, 3] \end{cases}$$

1b. $y = ax + b$. $a = \frac{\sum x f(x) - n \bar{x} \bar{f}(x)}{\sum x^2 - n \bar{x}^2}$

$$\bar{x} = \frac{1}{6} \sum x_i = 66.1 \quad \bar{y} = \frac{1}{6} \sum y_i = 243.$$

$$\sum x_i y_i = 101176.3 \quad \sum x_i^2 = 28365.28$$

$$\Rightarrow a = 2.03 \quad b = 108.817$$

$$\text{故 } y = 2.03x + 108.817.$$