1. $W=\int_0^x k\frac{q}{r^2}dr=-\frac{k}{r}q\Big|_0^x=-\frac{kq}{x}$

2. $\forall \varepsilon>0\ \exists A_0, B_0>0\ \forall A>A_0,\ B>B_0\ .\ C>\max\{A_0, B_0\}$

$\int_a^{+\infty}(k_1f(x)+k_2g(x))dx=\lim\limits_{A\to+\infty}\int_a^A(k_1f(x)+k_2g(x))dx=\lim\limits_{A\to+\infty}k_1\int_a^A f(x)dx+\lim\limits_{A\to+\infty}k_2\int_a^A g(x)dx$

$=k_1\int_a^{+\infty}f(x)dx+k_2\int_a^{+\infty}g(x)dx$

$\int_a^{+\infty}(k_1f(x)+k_2g(x))dx<k_1\cdot\frac{\varepsilon}{k_1}+k_2\cdot\frac{\varepsilon}{k_2}=2\varepsilon$ 收敛. #

3.

(1) $\int_0^{+\infty}e^{-2x}\sin 5x\,dx=-\frac{1}{5}e^{-2x}\cos 5x\Big|_0^{+\infty}-\int_0^{+\infty}\frac{2}{5}e^{-2x}\cos 5x=\frac{1}{5}-\frac{2}{5}\int_0^{+\infty}e^{-2x}d\frac{1}{5}\sin 5x$

$=\frac{1}{5}-2\cdot\frac{1}{25}\sin 5x\,e^{-2x}\Big|_0^{+\infty}+\frac{2}{5}\cdot\int_0^{+\infty}-2e^{-2x}\frac{1}{5}\sin 5x\,dx$

$=\frac{1}{5}-\frac{4}{25}\int_0^{+\infty}e^{-2x}\sin 5x\,dx$

$\Rightarrow\int_0^{+\infty}e^{-2x}\sin 5x\,dx=\frac{5}{29}$

(2) $\int_0^{+\infty}e^{-3x}\cos 2x\,dx=\int_0^{+\infty}e^{-3x}d\frac{1}{2}\sin 2x=\frac{1}{2}e^{-3x}\sin 2x\Big|_0^{+\infty}+\frac{3}{2}\int_0^{+\infty}e^{-3x}\sin 2x\,dx$

$=\frac{3}{2}\int_0^{+\infty}e^{-3x}d\frac{1}{2}(-\cos 2x)=-\frac{3}{4}e^{-3x}\cos 2x\Big|_0^{+\infty}-\frac{9}{4}\int_0^{+\infty}e^{-3x}\cos 2x\,dx$

$=\frac{3}{4}-\frac{9}{4}\int_0^{+\infty}e^{-3x}\cos 2x\,dx$

$\Rightarrow\int_0^{+\infty}e^{-3x}\cos 2x\,dx=\frac{3}{13}$

(3) $\int_{-\infty}^{+\infty}\frac{1}{x^2+x+1}dx=\int_{-\infty}^{+\infty}\frac{1}{(x+\frac{1}{2})^2+(\frac{\sqrt3}{2})^2}d(x+\frac{1}{2})=\frac{2}{\sqrt3}\arctan\frac{2x+1}{\sqrt3}\Big|_{-\infty}^{+\infty}=\frac{2}{\sqrt3}\pi.$

(4). $\int_0^{+\infty}\frac{dx}{(x^2)^2+(a^2+b^2)x^2+a^2b^2}=\int_0^{+\infty}\frac{dx}{(x^2+\frac{a^2+b^2}{2})^2-(\frac{a^2-b^2}{2})^2}=\frac{1}{a^2-b^2}\left(\int_0^{+\infty}\frac{dx}{x^2+b^2}-\int_0^{+\infty}\frac{dx}{x^2+a^2}\right)$

$=\frac{1}{a^2-b^2}\left(\frac{1}{b}\arctan\frac{x}{b}-\frac{1}{a}\arctan\frac{x}{a}\right)\Big|_0^{+\infty}=\frac{\pi}{2ab(a+b)}$

原式 $=\frac{\pi}{2ab(a+b)}$

(5). $\int_0^{+\infty} x e^{ax^2} dx = \frac{1}{2a} e^{ax^2} \Big|_0^{+\infty}$

当 $a\geq 0$ 时发散. $a<0$ 时收敛到 $\frac{-1}{2a}$

$-p$ $-p+1$

(6) ①当 $p=1$ 时 $\int_2^{+\infty} \frac{1}{x\ln x} dx = \int_2^{+\infty} \frac{d\ln x}{\ln x} = \ln\ln x \Big|_2^{+\infty}$ 发散.

②当 $p\neq 1$ 时 $\int_2^{+\infty} \frac{1}{x\ln^p x} dx = \frac{1}{1-p} \cdot \ln^{1-p} x \Big|_2^{+\infty}$

i. 当 $p>1$. $\int_2^{+\infty} \frac{1}{x\ln^p x} dx = \frac{1}{p-1} \cdot \frac{1}{\ln^{p-1} 2}$

ii. 当 $p<1$ $\int_2^{+\infty} \frac{1}{x\ln^p x} dx$ 发散.

故当 $p\leq 1$ 时 发散. 当 $p>1$ 时, 收敛到 $\frac{1}{(p-1)\ln^{p-1} 2}$

7. $\int_{-\infty}^{+\infty} \frac{dx}{(x^2+1)^{3/2}} \overset{x=\tan\theta}{=\!=} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos\theta \, d\theta = 2$

8. $\int_0^{+\infty} \frac{dx}{(e^x+e^{-x})^2} \overset{t=e^x}{=\!=} \int_1^{+\infty} \frac{dt}{t(t+t^{-1})^2} = \int_1^{+\infty} \frac{t}{(1+t^2)^2} dt = -\frac{1}{2} \frac{1}{1+t^2} \Big|_1^{+\infty} = \frac{1}{4}$

9. $\int_0^{+\infty} \frac{1}{x^4+1} dx$