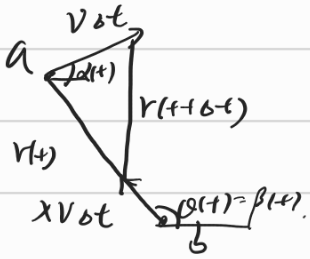
一、(1): $\begin{cases} y_a'(t) = v_a(t)\sin\alpha(t) \\ x_a'(t) = v_a(t)\cos\alpha(t) \end{cases}$ $\begin{cases} y_b'(t) = v_b(t)\sin\beta(t) \\ x_b'(t) = v_b(t)\cos\beta(t) \end{cases}$



(2): $\beta(t) = \theta(t)$.

(3): 令 $v = v_a$. 则 $v_a(t) = v$ $v_b(t) = \lambda v$. 如图,有:

$$r(t+\Delta t) = \sqrt{(r(t)-\lambda v\Delta t)^2 + (v\Delta t)^2 - 2(r(t)-\lambda v\Delta t)v\Delta t\cos(\pi-\theta(t)+\alpha(t))}$$

$$= \sqrt{r^2(t) + 2r(t)\Delta t\left(-\lambda v + v\cos(\theta(t)-\alpha(t))\right) - (\Delta t)^2\left(v^2+2\lambda v\cos(\theta(t)-\alpha(t))\right)}$$

$$= r(t) + (-\lambda+\cos(\theta-\alpha))v\Delta t + o(\Delta t)$$

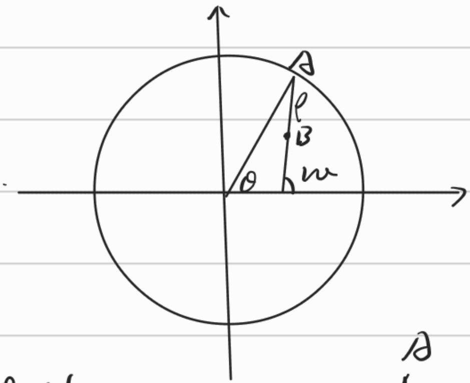
故 $\frac{dr(t)}{dt} = (-\lambda+\cos(\theta(t)-\alpha(t)))v$

即 $r'(t) = (-\lambda+\cos(\theta(t)-\alpha(t)))v_a$.

(4): 则 $r'(t) = (-1+\cos\theta(t))v$

$x_a(t) = vt$ $y_a(t) = 0$. $x_b(t) = x_0 + \int_0^t \lambda v\sin\theta(s)\,ds$

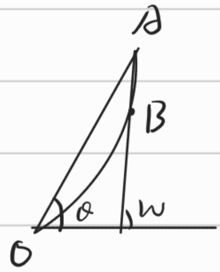
由 $\tan\theta(t) = \frac{y_b(t)}{x_b(t)-vt}$ 可解出 $\theta(t)$. 再代入 $r(t) = (-1+\cos\theta(t))v$ 得到 $r(t)$.



二、(1): 设A以w_0为角速度转,则 $v_a = aw_0$, $v_b = naw_0$.

$\frac{dy(t)}{dx(t)} = \tan w \Rightarrow \frac{dy}{dt} = \tan w\frac{dx}{dt}$.

$A(a\cos\theta(t), a\sin\theta(t))$ 其中 $\theta(t) = w_0 t$. 则 $\theta \propto t$.



从而 $\frac{dy}{d\theta} = \tan w\frac{dx}{d\theta}$ 由A、B间速度关系有:

$$n(a\theta) = \int_0^\theta \sqrt{(x'(s))^2+(y'(s))^2}\,ds$$

$$= \int_0^\theta \frac{1}{\cos w}x'(s)\,ds = \frac{1}{\cos w}x'(s)\Big|_{s=0}^{s=\theta} = \frac{x(\theta)}{\cos w}$$

故 $x(\theta) = na\theta\cos w$, $y(\theta) = na\theta\sin w$

从而 $\frac{dx}{d\theta} = na\cos w$, $\frac{dy}{d\theta} = na\sin w$

(2): $y' = \frac{dy/d\theta}{dx/d\theta} = \tan w$. 故. 切线方程: $v - y = \tan w(u-x)$.

法线方程: $v - y = -\cot w(u-x)$.

(3): $(x-a\cos\theta)^2 + (y-a\sin\theta)^2 = \rho^2 \Rightarrow \frac{\rho^2(\theta)}{a^2} = (n\cos w - \cos\theta)^2 + (n\sin w - \sin\theta)^2$