P_{165}:

1. (1): y' ln $e^x - y^2 - 2xy\cdot y' = 0 \Rightarrow \frac{dy}{dx} = \frac{y^2 - e^x}{\ln y - xy}$

(2). $y\ln x =$ $\Rightarrow y'\ln x + \frac{y}{x} = \ln y + \frac{x}{y}\cdot y' \Rightarrow \frac{dy}{dx} = \frac{\frac{y}{x} - \ln y}{\frac{x}{y} - \ln x}$

(3): $\frac{x + y\cdot y}{x^2+y^2}$ $-\frac{y}{x^2} + \frac{y'}{x}) / (1 + \frac{y^2}{x^2}) = (-y + x\cdot y') / (x^2+y^2)$

$\Rightarrow \frac{dy}{dx} = \frac{x}{x-}$

(4): $1 + ($ $\frac{1}{a} +$ $2 = \frac{y'}{a} \Rightarrow \frac{dy}{dx} = \frac{a^2}{(x+y)^2}$ $\frac{d^2y}{dx^2} = \frac{-2a^2(1 + \frac{a^2}{(x+y)^2})}{(x+y)^3} = \frac{-2a^2((x+y)^2 + a^2)}{(x+y)^5}$

(5). $\frac{1}{z} - \frac{x}{z^2}\cdot\frac{d}{d}$ $\frac{\frac{1}{z}}{1+\frac{x}{z}} \Rightarrow \frac{dz}{dx} = \frac{z}{x} - \frac{z^2}{x(x+y)}$

$-\frac{x}{z^2}\cdot\frac{dz}{dy} =$ $= \frac{-x}{y^2+xy} \Rightarrow \frac{dz}{dy} = \frac{z^2}{y(x+y)}$

(6). $\frac{\partial z}{\partial x}\cdot e^z = y$ $xy\frac{\partial z}{\partial x} \Rightarrow \frac{\partial z}{\partial x} = \frac{yz}{e^z - xy} \Rightarrow \frac{\partial^2 z}{\partial x^2} = \frac{-e^z y^2 z^2 + y^2 z}{(e^z - xy)^3}$

$\frac{\partial z}{\partial y}\cdot e^z = x$ $xy\cdot\frac{\partial z}{\partial y} \Rightarrow \frac{\partial z}{\partial y} = \frac{xz}{e^z - xz} \Rightarrow \frac{\partial^2 z}{\partial x\partial y} = \frac{(1-z)xyze^z + ze^z(e^z - xy)}{(e^z - xz)^3}$

(10): $f_1 + f_2 + $ $\frac{z}{x}) f_3 = 0 \Rightarrow \frac{\partial z}{\partial x} = -\frac{f_1 + f_2 + f_3}{f_3}$

$f_2 + (1 + \frac{\partial}{\partial}$ $= 0 \Rightarrow \frac{\partial z}{\partial y} = -\frac{f_2 + f_3}{f_3}$

$f_{11} + f_{22} +$ $f_3 + (1 + \frac{\partial z}{\partial x})^2 f_3 = 0 \Rightarrow \frac{\partial^2 z}{\partial x^2} = -\frac{f_2^2}{f_3^2} - \frac{f_{11} + f_{22}}{f_3}$

$f_{22} + \frac{\partial^2 z}{\partial x\partial y}\cdot f_3$ $\frac{\partial z}{\partial y})(1 + \frac{\partial z}{\partial x}) f_3 = 0 \Rightarrow \frac{\partial^2 z}{\partial x\partial y} = -\frac{f_{22}}{f_3} - \frac{f_2 f_{33}}{f_3^3}(f_1 + f_2)$ x

4. $\frac{\partial\varphi}{\partial x} = (1 + \frac{1}{y}\cdot\frac{\partial}{\partial}$ $1 + (-\frac{1}{x^2}\cdot z + \frac{1}{x}\cdot\frac{\partial z}{\partial x})\varphi_2 = 0$

$\frac{\partial\varphi}{\partial y} = (-\frac{1}{y^2}z +$ $)\varphi_1 + (1 + \frac{1}{x}\cdot\frac{\partial z}{\partial y})\varphi_2 = 0$

$\Rightarrow \frac{\partial z}{\partial x} = \frac{\frac{z}{x^2}\varphi_2 - \varphi}{\frac{1}{y}\varphi_1 + \frac{1}{x}}$ $\frac{\partial z}{\partial y} = \frac{\frac{z}{y^2}\cdot\varphi_1 - \varphi_2}{\frac{\varphi_1}{y} + \frac{1}{x}\varphi_2}$

$x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} =$ $\frac{x\varphi_1}{\frac{1}{x}\varphi_2} + \frac{\frac{z}{y}\varphi_1 - y\varphi_2}{\frac{\varphi_1}{y} + \frac{1}{x}\varphi_2} = z - xy$

故 $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = z$

5 (3) $\frac{\partial u}{\partial x} = (u + \frac{\partial u}{\partial x}\cdot$ $f_1 + \frac{\partial v}{\partial x} f_2$

$\frac{\partial v}{\partial x} = (\frac{\partial u}{\partial x} - 1)$ $+ 2vy\cdot\frac{\partial v}{\partial x} g_2$

$$\Rightarrow \frac{\partial u}{\partial x} = \frac{u f_1 - g_2 f}{(1-2vyg_2)(1-x)} \quad f_2$$

$$\frac{\partial v}{\partial x} = \frac{u f g_1 - (1-x)}{(1-2vyg_2)(1-xf_1)} \quad f_2$$

7. $\frac{\partial F}{\partial y} = (1-\frac{\partial x}{\partial y})F_1 + \quad \frac{\partial z}{\partial y})F_2$

$\frac{\partial G}{\partial y} = (\frac{\partial x}{\partial y} \cdot y + x) G_1 + \quad \cdot\frac{1}{y} - \frac{1}{y^2}\cdot z) G_2$

$\Rightarrow \frac{\partial x}{\partial y} = \frac{y^3 G_1(F_1+F_2) - F_1 \quad y + z G_2) - y^3 F_y G_1}{y^2 F_2 G_1 - \quad G_2} \quad (\frac{-F_x}{F_1}) + 1 + \frac{F_z}{F_y} - \frac{F_y}{F_1}$

$\frac{\partial z}{\partial y} = \frac{y^3 G_1(F_1+F_2) - F_1 \quad y + z G_2) - y^3 F_y G_1}{y^2 F_2 G_1 - \quad 2}$

8. $\frac{\partial f}{\partial r} = \frac{\partial f}{\partial x}\cdot\frac{\partial x}{\partial r} + \frac{\partial f}{\partial y}\frac{\partial}{\partial r} \quad \frac{\partial f}{x}\cdot\cos\theta + \frac{\partial f}{\partial y}\cdot\sin\theta$

$\frac{\partial f}{\partial \theta} = \frac{\partial f}{\partial x}\cdot\frac{\partial x}{\partial \theta} + \frac{\partial f}{\partial y}\cdot\frac{\partial y}{\partial \theta} - \quad \cdot(-r\sin\theta) + \frac{\partial f}{\partial y}\cdot r\cos\theta$

$\frac{\partial^2 f}{\partial r^2} = \cos\theta(\frac{\partial^2 f}{\partial x^2}\cdot\frac{\partial x}{\partial r} \quad \frac{\partial^2 f}{\partial y}\cdot\frac{\partial y}{\partial r}) + \sin\theta(\frac{\partial^2 f}{\partial y^2}\frac{\partial y}{\partial r} + \frac{\partial^2 f}{\partial y\partial x}\frac{\partial x}{\partial r})$

$= \cos^2\theta\frac{\partial^2 f}{\partial x^2} + 2\sin\theta \quad \frac{\partial^2 f}{\partial x\partial y} + \sin^2\theta\frac{\partial^2 f}{\partial y^2}$

$\frac{\partial^2 f}{\partial \theta^2} = -r\sin\theta(\frac{\partial^2 f}{\partial x^2}\cdot\frac{\partial}{\partial} \quad \frac{\partial^2 f}{\partial x\partial y}\cdot\frac{\partial y}{\partial \theta}) + r\cos\theta(\frac{\partial^2 f}{\partial y^2}\cdot\frac{\partial y}{\partial \theta} + \frac{\partial^2 f}{\partial y\partial x}\frac{\partial x}{\partial \theta}) - r\cos\theta\frac{\partial f}{\partial x} + r\sin\theta\frac{\partial f}{\partial y}$

$= r^2\sin^2\theta\frac{\partial^2 f}{\partial x^2} - 2r \quad \cos\theta\frac{\partial^2 f}{\partial x\partial y} + r^2\cos^2\theta\frac{\partial^2 f}{\partial y^2}$.

$- r\cos\theta\frac{\partial f}{\partial x} - \quad n\theta\frac{\partial f}{\partial y}$

$\frac{1}{r^2}\frac{\partial^2 f}{\partial \theta^2} + \frac{\partial^2 f}{\partial r^2} + \frac{1}{r}\cos\theta \quad + \frac{1}{r}\sin\theta\frac{\partial f}{\partial y} = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$

即 $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = \frac{1}{r^2}\frac{\partial^2 f}{\partial \theta^2} + \frac{\partial^2 f}{\partial r^2} \quad \frac{1}{r}\frac{\partial f}{\partial r}$.

11. $\frac{\partial z}{\partial x} = \frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial x} + \frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial x} \Rightarrow \quad = \frac{\partial z}{\partial u} + \frac{\partial z}{\partial v}$

$\frac{\partial^2 z}{\partial x^2} = \frac{\partial^2 z}{\partial u\partial v}\cdot\frac{\partial v}{\partial x} + \frac{\partial^2 z}{\partial u^2}\cdot\frac{\partial u}{\partial x} + \quad 2\cdot\frac{\partial v}{\partial x} + \frac{\partial^2 z}{\partial v\partial u}\cdot\frac{\partial u}{\partial x} = \frac{\partial^2 z}{\partial u^2} + 2\frac{\partial^2 z}{\partial u\partial v} + \frac{\partial^2 z}{\partial v^2}$

$\frac{\partial z}{\partial y} = \frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial y} + \frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial y} = \frac{1}{\sqrt{y}}(\quad \frac{z}{u} + \frac{\partial z}{\partial v})$

$\frac{\partial^2 z}{\partial y^2} = \frac{1}{\sqrt{y}}(-\frac{1}{2\sqrt{y}}(-\frac{\partial z}{\partial u} + \frac{\partial z}{\partial v}) + \quad \frac{\partial^2 z}{2}\frac{1}{\sqrt{y}} - \frac{\partial^2 z}{\partial u\partial v}\cdot\frac{1}{\sqrt{y}} + \frac{\partial^2 z}{\partial^2 v}\frac{1}{\sqrt{y}} - \frac{\partial^2 z}{\partial v\partial u}\frac{1}{\sqrt{y}})$

$$= \frac{1}{2}\cdot y^{-\frac{3}{2}}\left(\frac{\partial z}{\partial u}-\frac{\partial z}{\partial v}\right)+\quad\left(\frac{\partial^2 z}{\partial u^2}-2\frac{\partial^2 z}{\partial u\partial v}+\frac{\partial^2 z}{\partial v^2}\right)$$

$$\frac{\partial^2 z}{\partial u^2}+2\frac{\partial^2 z}{\partial u\partial v}+\frac{\partial^2 z}{\partial v^2}-\frac{1}{2}\frac{1}{y\sqrt{y}}\cdot\left(\quad-\frac{\partial z}{\partial v}\right)-\frac{\partial^2 z}{\partial u^2}-\frac{\partial^2 z}{\partial v^2}+2\frac{\partial^2 z}{\partial u\partial v}=\frac{1}{2}\frac{1}{y\sqrt{y}}\left(-\frac{\partial z}{\partial u}+\frac{\partial z}{\partial v}\right)$$

$$\Rightarrow \quad \frac{{}^2 z}{v}=0.$$

12.(1)

$$\frac{\partial w}{\partial x}=\frac{1}{z}\cdot\frac{\partial z}{\partial x}-1 \Rightarrow \frac{\partial z}{\partial x}=z\left(\frac{\partial w}{\partial x}+1\right) \text{ 有 } \frac{\partial z}{\partial y}=z\left(\frac{\partial w}{\partial y}+1\right)$$

$$\Rightarrow \frac{\partial z}{\partial x}=z\left(2x\frac{\partial w}{\partial u}-\frac{1}{x^2}\frac{\partial w}{\partial v}+1\right) \quad \frac{\partial z}{\partial y}=z\left(2y\frac{\partial w}{\partial u}-\frac{1}{y^2}\frac{\partial w}{\partial v}+1\right)$$

$$\Rightarrow z\left(2xy\frac{\partial w}{\partial u}-\frac{y}{x^2}\frac{\partial w}{\partial v}+y-2xy\frac{\partial w}{\partial u}+\frac{x}{y^2}\frac{\partial w}{\partial v}-x\right)=(y-x)z$$

$$\Rightarrow \left(\frac{x}{y^2}-\frac{y}{x^2}\right)\frac{\partial w}{\partial v}=0$$

即 $\frac{\partial w}{\partial v}=0$