Meromorphic Functions and the Ldgarithm

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(Singularitiy): z_0 is a ingularitiy of f while f has no defination at z_0

(Types of Singularities): 1.Removable Singularitiy:0 for $f(z) = \frac{\sin z}{z}$ 2.Pole:0 for $f(z) = \frac{1}{z}$

3.Essential Singularity:0 for $e^{1/z}$

Theorem Laurent-serie: $f(z) = f(z_0) + \sum\limits_{n=-\infty}^{-1} a_n(z-z_0)^n + \sum\limits_{n=0}^{\infty}(z-z_0)^n$ *the first part is called the Principal-Part, the second part is called the Analytic-Part.espacially* a_{-1} *is called the residue of* f *at* z_0

Theorem 1.1: f *hol on* Ω, $f(z_0) = 0 \Rightarrow \exists u = O(z_0, r)$ $\exists g(z)$ *defines on* u, $\exists n > 0$ *s.t.* $f(z) = (z-z_0)^n g(z)$ *the* n *called the order of* z_0

Theorem 1.2: f *hol on* Ω, z_0 *is a pole of* $f \Rightarrow \exists h(z)$ *hol on* $\Omega, \exists n > 0$ *s.t.* $f(z) = (z-z_0)^{-n}h(z)$ n *called the order of pole* z_0

Theorem 1.3: *if* f *has a pole of order* $n \Rightarrow$ $f(z) = \frac{a_{-n}}{(z-z_0)^n} + \cdots + \frac{a_{-1}}{z-z_0} + g(z)$ g *is hol on a neiborhood of* z_0

Theorem 1.4: $res_{z_0} f = \lim\limits_{z \to z_0}(z-z_0)f(z) = \lim\limits_{z \to z_0} \frac{1}{(n-1)!}\frac{d^n}{dz^n}(z-z_0)^n f(z)$

The residue formula

Theorem 2.1: z_0 *is a pole of* f *and* f *hol on* $\Omega - \{z_0\}, z_0 \in Int(C),\ C \cup Int(C) \subset \Omega \quad \Rightarrow$

$res_{z_0} f = \frac{1}{2\pi i} \int_C f(z)dz$

Corollary 2.2: $\int_C f(z)dz = 2\pi i \sum\limits_{k=1}^{N} res_{z_k} f$

Corollary 2.3: $\int_\gamma f(z)dz = 2\pi i \sum\limits_{k=1}^{N} res_{z_k} f$