

例 10.7.1.

$$1) \quad 3x^2 + 2y^2 + z^2 - 4xy - 4yz + 6x + 4y - 4z + 12 = 0.$$

$$\text{令 } A = \begin{bmatrix} 3 & -2 & 0 \\ -2 & 2 & -2 \\ 0 & -2 & 1 \end{bmatrix} \quad |\lambda E - A| = \begin{vmatrix} \lambda-3 & 2 & 0 \\ 2 & \lambda-2 & 2 \\ 0 & 2 & \lambda-1 \end{vmatrix} = (\lambda+1)(\lambda-2)(\lambda-5)$$

$$\lambda_1 = -1 \quad \lambda_2 = 2 \quad \lambda_3 = 5$$

$$\text{当 } \lambda = -1, \text{ 特征向量为 } x_1 = [1, 2, 2]^T$$

$$\text{当 } \lambda = 2, \text{ 特征向量为 } x_2 = [2, 1, 2]^T$$

$$\text{当 } \lambda = 5, \text{ 特征向量为 } x_3 = [-2, 2, 1]^T$$

$$\text{令 } P = \begin{bmatrix} 1/3 & 2/3 & 2/3 \\ 2/3 & 1/3 & -2/3 \\ 2/3 & 2/3 & 1/3 \end{bmatrix}$$

$$P^T A P = \begin{bmatrix} -1 & & \\ & 2 & \\ & & 5 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = P \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} \Rightarrow \begin{cases} x = \frac{1}{3}x' + \frac{2}{3}y' + \frac{2}{3}z' \\ y = \frac{2}{3}x' + \frac{1}{3}y' + \frac{2}{3}z' \\ z = \frac{2}{3}x' + \frac{2}{3}y' + \frac{1}{3}z' \end{cases}$$

代换得:

