

一. 因每小时清除积雪总量为定值 K . 且清除扫过路上所有积雪. 故车速受雪的厚度 d 影响. 设 $d(x, t)$ 为 x 处, t 时刻雪的厚度. 则

$$d_x(x, t) = 0, \text{ i.e. 降雪是均匀的.}$$

$$\Rightarrow d(x, t) = Kt.$$

$$d_t(x, t) = K, \text{ i.e. 单位时间降雪量为 } K$$

$$\text{由 } K = v t_1 \cdot \Delta s = v t_1 \cdot w \cdot d \text{ 故车速速度与 } d \text{ 成反比. 设 } v(t) = \frac{K}{t} \propto \frac{1}{d}$$

(1): 设 t_0 为出发时间.

$$\text{则 } S_1 = \int_{t_0}^{t_0+t_1} v(t) dt = 2 S_2, \text{ 其中 } S_2 = \int_{t_0+t_1}^{t_0+t_2} v(t) dt.$$

$$\text{解: } t_0^2 + t_0 - 1 = 0$$

$$\text{从而 } t_0 = \frac{\sqrt{5}-1}{2} \text{ 故扫雪车于 } \frac{\sqrt{5}-1}{2} \text{ 时出发.}$$

(2): 设若 $t_1 < t_2$. 则 $t(t_1 < t_2)$ 时刻. 设第 i 辆车在位置 $S_i(t)$ 处. 有:

$$S_1(t) = \int_{t_1}^t v(t) dt, \quad S_2(t) = \int_{t_2}^t v(t-t_1) dt.$$

$$\text{若 } S_1(t) = S_2(t), \text{ 即为:}$$

$$\frac{t^*}{t_1} = \frac{t^* - t_1}{t_2 - t_1}$$

$$\Rightarrow t^* = \frac{t_1^2}{2t_1 - t_2}.$$

故若 $t_2 \geq 2t_1$. 则不可能相遇. 否则在 $t^* = \frac{t_1^2}{2t_1 - t_2}$ 时相遇. 此时.

$$S = S_1(t^*) = K \ln \frac{t_1}{2t_1 - t_2} \quad \text{由 } \int_0^1 w \cdot Kt v(t) dt = K \Rightarrow K = \frac{1}{w}$$

$$\text{故 } S = \frac{1}{w} \ln \frac{t_1}{2t_1 - t_2}, \quad t^* = \frac{t_1^2}{2t_1 - t_2}.$$

$$\text{二. (1): } |a(t)| = \frac{d^2 r(t)}{dt^2} \quad |v(t)| = \frac{dr(t)}{dt}, \quad |v_w(t)| = \frac{2\pi}{Q} \cdot r(t)$$

$$\text{故 } a(t) = \frac{d^2 r(t)}{dt^2} \cdot \vec{u}(t) \quad v(t) = \frac{2\pi}{Q} r(t) \vec{n}(t) + \frac{dr(t)}{dt} \vec{u}(t)$$

$$(2): \frac{d^2 r(t)}{dt^2} = f(r)$$

$$\text{设此时从 } (R, 0) \text{ 处掉入. } Q(t) = \frac{2\pi}{Q} t$$

$$(3): r'(t) = -kr \quad r(0) = R.$$

$$\text{解得 } r(t) = R \cos \sqrt{k} t$$



