

$$\text{一. (1): } P(\text{环规, 先罚胜}) = p^2 \cdot (C_2^0 (1-q)^2 + C_2^1 (1-q)q) + C_2^1 p(1-p) C_2^0 (1-q)^2 \\ = p^2 (1-q^2) + 2p(1-p)(1-q)^2$$

$$P(\text{环规, 后罚胜}) = (1-p)^2 (q^2 + C_2^1 q(1-q)) + C_2^1 p(1-p) \cdot q^2 \\ = (2q-q^2)(1-p)^2 + 2p(1-p)q^2$$

$$P(\text{环规, 打平}) = C_2^0 (1-p)^2 C_2^0 (1-q)^2 + C_2^1 p(1-p) C_2^1 q(1-q) + p^2 q^2 \\ = (1-p)^2 (1-q)^2 + 4pq(1-p)(1-q) + p^2 q^2$$

$$P(\text{折板, 先罚胜}) = pq((1-q)(1-p) + q(1-p) + (1-q)p) + [p(1-q) + q(1-p)](1-q)(1-p) \\ = pq(1-pq) + (p+q-2pq)(1-p)(1-q)$$

$$P(\text{折板, 后罚胜}) = qp[p(1-q) + (1-p)q + (1-p)(1-q)] + [q(1-p) + p(1-q)](1-p)(1-q) \\ = pq(1-pq) + (p+q-2pq)(1-p)(1-q)$$

$$P(\text{折板, 打平}) = (1-p)(1-q) \cdot (1-q)(1-p) + [p(1-q) + q(1-p)] \cdot [q(1-p) + p(1-q)] + pq - qp \\ = (p+q-2pq)^2 + p^2 q^2 + (1-p)^2 (1-q)^2$$

$$\text{代入 } p=\frac{3}{4}, q=\frac{1}{2}, \text{ 依次有 } p_1=0.3542 \quad p_2=0.2222 \quad p_3=0.4236$$

$$p_4=0.2847 \quad p_5=0.2847 \quad p_6=0.4306$$

(2): 设  $P_k$  为第一罚点球在加赛  $k$  轮后获胜(环规规则)的概率: 则  $P_1 = p(1-q)$

$$P_{k+1} = \sum_{i=1}^k (pq)^i [(1-p)(1-q)]^{k-i} \cdot p(1-q) = p(1-q)(1-p-q+2pq)^k$$

$$\text{故 } P(\text{先罚胜}) = \sum_{k=1}^{\infty} P_k = \frac{p-pq}{p+q-2pq} \quad \text{同理: } P(\text{后罚胜}) = \frac{q-pq}{p+q-2pq}$$

同上, 设  $P_k$  为折板下, 先罚队在第  $k$  轮加赛后获胜的概率:

$$P_1 = q(1-p), \quad P_{k+1} = \sum_{i=1}^{2k} (pq)^i [(1-p)(1-q)]^{2k-i} \cdot q(1-p), \quad P_{k+2} = \sum_{i=1}^{2k+1} (pq)^i [(1-p)(1-q)]^{2k+1-i} p(1-q).$$

$$\text{故 } P(\text{先罚胜}) = \sum_{k=1}^{\infty} (P_{2k-1} + P_{2k}) = 1 - p(1-q)$$

$$\text{从而 } P(\text{后罚胜}) = p(1-q)$$

二. (1): 设  $P_k(A)$  表示  $A$  在  $A$  的第  $k$  次进攻后胜利的概率.

$$P_1(A) = \alpha + \beta \quad P_k(A) = r^{2k-2}(\alpha + \beta). \quad \text{故 } P(A \text{ 胜利}) = \sum_{k=1}^{\infty} P_k(A) = \frac{\alpha + \beta}{1-r^2} = \frac{1}{1+r}$$

$$(2): p = \alpha + \beta a + r\beta$$

(2): 由 (1) 知, 从某时开始实行突然袭击战术, 该队胜利的概率为  $\frac{\alpha + \beta}{1-r^2} = \frac{1}{1+r}$

$$\text{则 } p(A) = \alpha + \beta [r + \beta \cdot \frac{1}{1+r}] + r \cdot (1 - \frac{1}{1+r}) = \frac{1 + \beta r^2}{1+r}$$

因有  $\frac{1 + \beta r^2}{1+r} > \frac{1}{1+r} > \frac{1}{2}$ , 且  $\beta$  也互相同, 且  $\beta$  不能取  $p(A) > \frac{1}{2}$  的原案判罚分

