9. 解：首先，$P(X=k)=\frac{\lambda^k}{k!}e^{-\lambda},\lambda>0,k=0,1,2,\cdots$

记 $S_1(\lambda)=\sum\limits_{k为奇}\frac{\lambda^k}{k!}$，$S_2(\lambda)=\sum\limits_{k为偶}\frac{\lambda^k}{k!}$，则有

$$\begin{cases} S_1(\lambda)+S_2(\lambda)=e^{\lambda} \\ S_1(-\lambda)+S_2(-\lambda)=e^{-\lambda} \end{cases}，\quad 解得：\begin{cases} S_1(\lambda)=\frac{1}{2}(e^{\lambda}-e^{-\lambda}) \\ S_2(\lambda)=\frac{1}{2}(e^{\lambda}+e^{-\lambda}) \end{cases}.$$

$$E(X\big|X为奇数)=\frac{\sum_{k=0}^{\infty}(2k+1)P(X=2k+1)}{\sum_0^{\infty}\frac{\lambda^{2k+1}}{(2k+1)!}e^{-\lambda}}=\frac{\lambda S_2(\lambda)}{S_1(\lambda)}=\lambda\frac{e^{\lambda}+e^{-\lambda}}{e^{\lambda}-e^{-\lambda}}.$$

10. 解：(1) $P(U=n,Z=N)=P(U=n,V=N-n)=\begin{cases}(1-\beta)^{N-2}\beta^2,1\le n\le N \\ 0,其它\end{cases}$；

(2) $P(U=n\big|Z=N)=\dfrac{P(V=N-n)}{P(Z=N)}=\dfrac{(1-\beta)^{N-2}\beta^2}{\sum_{n=1}^{N-1}(1-\beta)^{N-2}\beta^2}=\dfrac{1}{N-1}$.

22. 解：首先，N 的所有可能取值为 $1,2,3,\cdots$.

记随机变量 X_0 的密度函数为 $f(x)$，分布函数为 $F(x)$，则有：

$$\begin{aligned}\forall k\ge 2，P(N=k)&=P(X_k>X_0,X_1\le X_0,\cdots,X_{k-1}\le X_0)\\ &=\int_{-\infty}^{\infty}P(X_k>x,X_1\le x,\cdots,X_{k-1}\le x)f(x)dx\\ &=\int_{-\infty}^{\infty}(1-F(x))F(x)^{k-1}f(x)dx\\ &=\int_0^1(1-F(x))F(x)^{k-1}dF(x)\\ &=\int_0^1F(x)^{k-1}dF(x)-\int_0^1F(x)^k dF(x)\\ &=\tfrac{1}{k}-\tfrac{1}{k+1}\end{aligned}$$

另外，$P(N=1)=\int_{-\infty}^{\infty}(1-F(x))f(x)dx=\frac{1}{2}$.

从而，$E(N)=\frac{1}{2}+\sum\limits_{k=2}^{\infty}k(\frac{1}{k}-\frac{1}{k+1})=\frac{1}{2}+\sum\limits_{k=2}^{\infty}\frac{1}{k+1}=\sum\limits_{k=2}^{\infty}\frac{1}{k}$.

26. 解：(1) 一方面，$f(x)$ 是凸函数，那么：$\exists a\in R\ s.t.\ f(x)\ge ax+f(0),\forall x\in R$；

另一方面，$f(x)$ 是凹函数，那么：$\exists c\in R\ s.t.\ f(x)\le cx+f(0),\forall x\in R$.

由以上两方面，不难得到：$a=c$，从而 $f(x)=ax+f(0)$.

(2) 由条件有：$P(X>s+t)=P(X>s)P(X>t), \forall s,t>0$

记 $f(s)=P(X>s)$，则有 $f(s+t)=f(s)f(t)$，且 $f(0)=1$

进一步，记 $g(s)=\ln f(s)$，则有 $g(s+t)=g(s)+g(t)$，且 $g(0)=0$

$$g^{'}(s)=\lim_{t\to 0}\frac{g(s+t)-g(s)}{t}=\lim_{t\to 0}\frac{g(t)}{t}=g^{'}(0):=a<0$$

那么，$g(s)=as+g(0)=as$，$P(X>s)=f(s)=e^{as}$，故 X 是指数随机变量.

29. 解：$P(Y=k)=P(\text{前}k-1\text{个取}n\text{个},\text{后}N-k\text{个取}n\text{个})=\dfrac{C_{k-1}^{n}C_{N-k}^{n}}{C_{N}^{2n+1}}$；

由于 $P(Y=k)=P(Y=N+1-k)$，故 $EY=\frac{N+1}{2}$.

$$E(Y^2)=\sum_{k=n+1}^{N-n}\frac{C_{k-1}^{n}C_{N-k}^{n}}{C_{N}^{2n+1}}k^2=\sum_{k=n+1}^{N-n}\frac{C_{k}^{n+1}C_{N-k}^{n}}{C_{N+1}^{2n+2}}\frac{k(n+1)(N+1)}{2n+2}$$

$$=\sum_{k=n+1}^{N-n}\frac{C_{k}^{n+1}C_{N-k}^{n}}{C_{N+1}^{2n+2}}\frac{k(n+1)(N+1)}{2n+2}=\frac{N+1}{2}\cdot\sum_{k=n+1}^{N-n}\frac{C_{k}^{n+1}C_{N-k}^{n}}{C_{N+1}^{2n+2}}k$$

其中，$\displaystyle\sum_{k=n+1}^{N-n}\frac{C_{k}^{n+1}C_{N-k}^{n}}{C_{N+1}^{2n+2}}k=\sum_{k=n+1}^{N-n}\frac{C_{k}^{n+1}C_{N+1-k}^{n+1}}{C_{N+2}^{2n+3}}\frac{(n+1)(N+2)}{(N+1-k)(2n+3)}k$

$$=\frac{(n+1)(N+2)}{2n+3}\sum_{k=n+1}^{N-n}\frac{C_{k}^{n+1}C_{N+1-k}^{n+1}}{C_{N+2}^{2n+3}}\left(\frac{N+1}{N+1-k}-1\right)$$

又 $\displaystyle\sum_{k=n+1}^{N-n}\frac{C_{k}^{n+1}C_{N+1-k}^{n+1}}{C_{N+2}^{2n+3}}=1$,

$$\sum_{k=n+1}^{N-n}\frac{C_{k}^{n+1}C_{N+1-k}^{n+1}}{C_{N+2}^{2n+3}}\cdot\frac{N+1}{N+1-k}=\sum_{k=n+1}^{N-n}\frac{C_{k}^{n+1}C_{N-k}^{n}}{C_{N+1}^{2n+2}}\cdot\frac{(N+1)(2n+3)}{(n+1)(N+2)}$$

$$=\frac{(N+1)(2n+3)}{(n+1)(N+2)}$$

所以，$\displaystyle\sum_{k=n+1}^{N-n}\frac{C_{k}^{n+1}C_{N-k}^{n}}{C_{N+1}^{2n+2}}k=\frac{(n+1)(N+2)}{2n+3}\sum_{k=n+1}^{N-n}\frac{C_{k}^{n+1}C_{N+1-k}^{n+1}}{C_{N+2}^{2n+3}}\left(\frac{N+1}{N+1-k}-1\right)$

$$=\frac{(n+1)(N+2)}{2n+3}\left(\frac{(N+1)(2n+3)}{(n+1)(N+2)}-1\right)=\frac{Nn+2N+1}{2n+3}$$

$$E(Y^2)=\frac{N+1}{2}\cdot\frac{Nn+2N+1}{2n+3}$$

故，$Var(Y)=E(Y^2)-(EY)^2=\frac{N+1}{2}\cdot\frac{Nn+2N+1}{2n+3}-\frac{(N+1)^2}{4}$

$$=\frac{(N-2n-1)(N+1)}{4(2n+3)}.$$