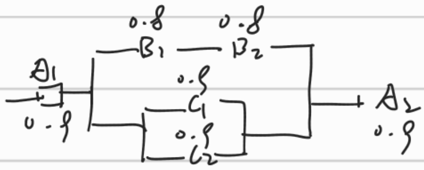
一.



D表示中间部分连通.

则 $P(\bar{D}) = [1-P(B_1B_2)]\times[P(\bar{C_1})P(\bar{C_2})] = 0.0036$

故正常的概率: $P = P(D)P(A_1)P(A_2) = 0.807084$.

二.

Y	0	1	2
P	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

$Z_0=1$

(1): Z_1可能为 0, 1, 2.

$$P(Z_2=0) = P(Z_1=0) + P(Z_1=1, Z_2=0) + P(Z_1=2, Z_2=0)$$
$$= \frac{1}{4} + \frac{1}{2}\times\frac{1}{4} + \frac{1}{4}\times\frac{1}{4}\times\frac{1}{4}$$
$$= \frac{25}{64}$$

(2): $P(Z_2=4) = P(Z_1=2, Z_2=4) = \frac{1}{4}\times\frac{1}{4} = \frac{1}{16}$

三. $P(Z_i=k) = (1-p_i)p_i^{k-1}$ $k=1,2\cdots$ $i=1,2\cdots$

$$P(Z_1<Z_2<Z_3) = \sum_{k=2}^{\infty} P(Z_1<k, Z_2=k, Z_3>k)$$
$$= \sum_{k=2}^{\infty} P(Z_1<k)P(Z_2=k)P(Z_3>k)$$
$$= \sum_{k=2}^{\infty}\Big[\sum_{i=1}^{k-1}(1-p_1)p_1^{i-1}\Big]\times\Big[(1-p_2)p_2^{k-1}\Big]\times\Big[\sum_{i=k+1}^{\infty}(1-p_3)p_3^{i-1}\Big]$$
$$= \sum_{k=2}^{\infty} (1-p_1)\frac{1-p_1^{k-1}}{1-p_1}\times(1-p_2)p_2^{k-1}\times(1-p_3)\frac{p_3^k}{1-p_3}$$
$$= \sum_{k=2}^{\infty}(1-p_2)(1-p_1^{k-1})p_2^{k-1}p_3^k = (1-p_2)p_3\sum_{k=1}^{\infty}\big[(p_2p_3)^k-(p_1p_2p_3)^k\big]$$
$$= (1-p_2)p_3\left(\frac{p_2p_3}{1-p_2p_3} - \frac{p_1p_2p_3}{1-p_1p_2p_3}\right) = \frac{(1-p_1)(1-p_2)p_2p_3^2}{(1-p_2p_3)(1-p_1p_2p_3)}$$

四. $p(Y=x) = e^{-x}$. $x>0$ $x+y+z$ $-x$

(1)
(Y_1,Y_2,Y_3)的联合分布 $p_1(x,y,z) = e^{-(x+y+z)}$.

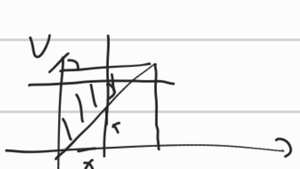
考虑变换 $\begin{cases} u = \frac{x}{x+y+z} \\ v = \frac{x+y}{x+y+z} \\ w = x+y+z \end{cases}$ 则 $\left|\frac{\partial(u,v,w)}{\partial(x,y,z)}\right| = \begin{vmatrix} \frac{y+z}{w^2} & -\frac{x}{w^2} & -\frac{x}{w^2} \\ \frac{z}{w^2} & \frac{z}{w^2} & -\frac{x+y}{w^2} \\ 1 & 1 & 1 \end{vmatrix} = \frac{1}{w^2}$

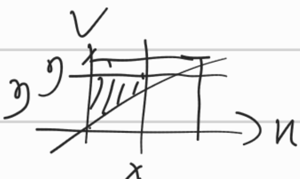
故 $\left|\frac{\partial(x,y,z)}{\partial(u,v,w)}\right| = w^2$

从而 $p_2(u,v,w) = p_1(uw, vw-uw, w-vw)|J| = w^2e^{-w}$ $0<u\le v<1$, $w>0$

$P_2(u,v) = \int_0^{+\infty} p_2(u,v,w)\,dw = 2$

故 $p_2(u,v) = \begin{cases} 2 & 0<u\le v<1 \text{ 时} \\ 0 & \text{其它} \end{cases}$





$$F(x,y)=\begin{cases} 0 & y\le x \text{ 或 } x\le 0. \\ 1 & y>x\ge 1 \\ 2x-x^2 & y\ge 1>x>0 \\ 2xy-x^2 & 1>y>x>0 \end{cases}$$

(2) $P(W_1=\frac{1}{2})\neq 0$. $P(W_2=\frac{1}{3})\neq 0$. 但 $P(W_1=\frac{1}{2}, W_2=\frac{1}{3})=0$. 故不独立.

五: (1) $\xi,\eta\sim U(-1,1)$

$E\xi=E\eta=0$. $Var\xi=Var\eta=E\xi^2=\int_0^1 x^2dx=\frac{1}{3}$

$D(U+V)=D(3\xi-3\eta)=Var(3\xi)+Var(3\eta)=9Var\xi+9Var\eta=6$

$D(U^2+V^2)=D(5\xi^2-8\xi\eta+5\eta^2)=25Var\xi^2+64Var\xi\eta+25Var\eta^2$

$=50\times((E\xi^2)^2-(E\xi)^4)+64(E\xi^2E\eta^2-(E\xi E\eta)^2)$

$=50\times(\frac{1}{3}-0)+64\times(\frac{1}{3})=\frac{114}{9}$

(2) $\because\xi,\eta\sim N(0,1)$. $U=3\xi+2\eta$ $V=2\xi+3\eta$

$r_{(U+V),(U^2+V^2)}=\frac{Cov(U+V,U^2+V^2)}{\sqrt{Var(U+V)}\sqrt{Var(U^2+V^2)}}$

$Cov(U+V,U^2+V^2)=E(U+V)(U^2+V^2)-E(U+V)E(U^2+V^2)$

$=E5(\xi+\eta)(13\xi^2+13\eta^2+24\xi\eta)-E5(\xi+\eta)E(13\xi^2+13\eta^2+24\xi\eta)$

$=E(65(\xi^3+\eta^3)+185\xi\eta^2+185\xi^2\eta)$

$=130E\xi^3+185E\xi E\eta^2+185E\xi^2E\eta$

$=130E\xi^3$

$E\xi^3=\int_R x^3p(x)dx=\int_R x^3\frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}}dx=\sqrt{\frac{2}{\pi}}\int_0^{+\infty}x^3e^{-\frac{x^2}{2}}dx=2\sqrt{\frac{2}{\pi}}$

故 $Cov(U+V,U^2+V^2)=\sqrt{\frac{8}{\pi}}$

$Var(U+V)=Var(6(\xi+\eta))=36Var(\xi+\eta)=72Var\xi=72$

$Var(U^2+V^2)=Var(13\xi^2+13\eta^2+24\xi\eta)=338Var\xi^2+576Var\xi\eta$

$=338E\xi^2+576[(E\xi^2)^2-(E\xi)^4]$

$E\xi^2=\frac{1}{\sqrt{2\pi}}\int_R x^2e^{-\frac{x^2}{2}}dx=\frac{2}{\sqrt{2\pi}}\int_0^{+\infty}e^{-\frac{x^2}{2}}dx=4$

故 $Var(U^2+V^2)=10568$

故 $r=0.00184$

六. $\xi_i\sim B(1,0.9)$ $S_n=\sum_{i=1}^{n}\xi_i$, X

$\text{七}.\ \xi_i \sim P(\lambda_i).\quad \lambda = \sum_{i=1}^{n} \lambda_i.\quad E\xi_i = \lambda_i < \infty$

$E\xi_n = \lambda_n.\quad Var\xi_n = \lambda_n \quad \sum_{n=1}^{n} \frac{Var\xi_n}{n^2} = \frac{\sum_{n=1}^{n} \lambda_k}{n^2}$

$\xi \sim P(\lambda) \qquad \forall \varepsilon > 0,\ \exists n > 0. \qquad P(|\xi - \mu| > \delta) < \varepsilon$