4. (1): $\lim\limits_{x\to 0}\lim\limits_{y\to 0} f(x,y) = \lim\limits_{x\to 0}\frac{x}{x} = 1 \neq \lim\limits_{y\to 0}\lim\limits_{x\to 0} f(x,y) = \frac{-y}{y} = -1$ 不$\exists$

(2) $\lim\limits_{\substack{x\to 0\\ y\to 0}} \frac{xy}{x^2+y^2} = \lim\limits_{t\to 1}\frac{1}{1+t^2} = \frac{1}{2}$

(3) $\lim\limits_{y\to 0}\lim\limits_{x\to 0} f(x,y) = 0 \neq \lim\limits_{x\to 0}\lim\limits_{y\to 0} = 1$ ∴不存在

(4) $\lim\limits_{\substack{x\to 0\\ y\to 0}} f(x,y) = \lim\limits_{t\to 0}\frac{t^6}{t^4+t^8} = \lim\limits_{t\to 0}\frac{t^2}{1+t^4} \to 0$

7. (1) $\lim\limits_{(x,y)\to(0,1)} \frac{1-xy}{x^2+y^2} = 1$

(2) $\lim\limits_{(x,y)\to(0,0)} \frac{1+x^2+y^2}{x^2+y^2} = \lim\limits_{t\to 0}\frac{1+t}{t}$ 不存在.

(3) $\lim\limits_{(x,y)\to(0,0)} \frac{\sqrt{1+xy}-1}{xy} = \lim\limits_{(x,y)\to(0,0)} \frac{xy}{xy(\sqrt{1+xy}+1)} = \frac{1}{2}$

(4) $\lim\limits_{(x,y)\to(0,0)} \frac{x^2+y^2}{\sqrt{1+x^2+y^2}-1} = \lim\limits_{(x,y)\to(0,0)} \frac{(x^2+y^2)(\sqrt{1+x^2+y^2}+1)}{x^2+y^2} = 2$

(5) $\lim\limits_{(x,y)\to(0,0)} \frac{\ln(x^2+e^{y^2})}{x^2+y^2} = \lim\limits_{(x,y)\to(0,0)} \frac{(x^2+y^2+1)}{x^2+y^2} = 1$

(6) $\lim\limits_{(x,y)\to(0,0)} \frac{\sin(x^3+y^3)}{x^2+y^2} = \lim\limits_{(x,y)\to(0,0)} \frac{x^3+y^3}{x^2+y^2} = 0$

(7) $\lim\limits_{(x,y)\to(0,0)} \frac{1-\cos(x^2+y^2)}{(x^2+y^2)x^2y^2} = 2\lim\limits_{(x,y)\to(0,0)} \frac{\left[\frac{x^2+y^2}{2}\right]^2}{(x^2+y^2)x^2y^2} = \frac{1}{2}\lim\limits_{(x,y)\to(0,0)} \frac{(x^2+y^2)}{x^2y^2} = 0$

(8) $\lim\limits_{\substack{x\to+\infty\\ y\to+\infty}} \frac{x^2+y^2}{e^{x+y}} = 0.$

8. (1) $\lim\limits_{x\to 0}\lim\limits_{y\to 0} f(x,y) = \lim\limits_{x\to 0}\frac{2x^2}{2x^2+2} = 0 = \lim\limits_{y\to 0}\lim\limits_{x\to 0} f(x,y)$

令 $x=r\cos\theta$, $y=r\sin\theta$, $r\to 0^+$ $\theta\in[0,2\pi)$

$$\lim_{(x,y)\to(0,0)} \frac{x^2y^2}{x^2y^2+(x-y)^2} = \lim_{r\to 0} \frac{r^4\sin^2\theta\cos^2\theta}{r^4\sin^2\theta\cos^2\theta + r^2(\sin\theta-\cos\theta)^2} = \lim_{r\to 0} \frac{r^2\sin 2\theta}{r^2\sin^2 2\theta + 4 - 4\sin 2\theta}$$

$$= \lim_{\varepsilon\to 0} \frac{\varepsilon}{4-4\sin 2\theta}$$ 不存在。 二重不存在。二次相同为 0

(2). $\lim\limits_{x\to 0}\lim\limits_{y\to 0} f(x,y) = \lim\limits_{x\to 0}(1+x^2) = 1 \neq \lim\limits_{y\to 0}\lim\limits_{x\to 0} -(1+y^2) = -1$

令 $x=r\cos\theta$, $y=r\sin\theta$, $r\to 0$

$\lim\limits_{(x,y)\to(0,0)} f(x,y) = \lim\limits_{r\to 0}(r^2+1)\cos 2\theta = \cos 2\theta$ 不存在。

二重极限不存在。二次极限为 1 与 -1.

12. 对给定的 x_0, $\forall \varepsilon>0$ $\exists \delta = \frac{\varepsilon}{L} > 0$

s.t. $\forall y_1, y_2$ $|y_1-y_2|<\delta$. 有 $|f(x_0,y_1)-f(x_0,y_2)| \le L\cdot\frac{\varepsilon}{L} = \varepsilon$.

故 f 对 y 一致连续. 故 f 对 y 连续. 又 f 对 x 连续. 由 x_0 的任意性.

f 在 D 上连续.