Preliminaries to Complex Analysis

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$Holomopic \iff Analysis : Power - Series$

Theorem 2.5: *Given a power series* $\sum_{n=0}^{\infty} a_n z^n$*, there exists* $0 \leq R \leq \infty$ *s.t.* :

1. *If* $|z| < R$*, the series converges absolutely.*
2. *(ii) If* $|z| > R$ *the series diverges.*

(R):$1/R = \limsup |a_n|^{1/n}$

The number R is called the radius of convergence of the power series

. the region $|z| < R$ the disc of convergence.

2.6:Remark: On the boundary of the disc of convergence, $|z| = R$, the situation is more delicate as one can have either convergence or divergence.

Theorem 2.6: *The power series* $f(z) = \sum_{n=0}^{\infty} a_n z^n$ *defines a holomorphic function in its disc of convergence. The derivative of* f *is also a power series obtained by differentiating term by term the series for* f*, that is*

$$f'(z) = \sum_{n=0}^{\infty} n a_n z^{n-1}$$

Moreover, f' *has the same radius of convergence as* f*.*

*2.7:*A power series is infinitely complex differentiable in its disc of convergence, and the higher derivatives are also power series obtained by termwise differentiation.

if f has a power series expansion at every point of Ω ,we say f is analytic on Ω

Integration along curves

γ:A parametrized curce is a function z(t) which maps a closed inteval $[a, b] \subset R$ to the complex plane.

(orientation of $C_r(z_0)$): The positive orientation (counterclockwise) is the one that is given by the standard parametrization

$$z(t) = z_0 + re^{it}, \qquad \text{where } t \in [0, 2\pi]$$

while the negative orientation (clockwise) is given by

$$z(t) = z_0 + re^{-it}, \qquad \text{where } t \in [0, 2\pi]$$

Theorem 3.1: *Integration of continuous functions over cuvers satisfies the following properties:*

(i) It is linear, that is, if $\alpha, \beta \in \mathbb{C}$, then

$$\int_\gamma (\alpha f(z) + \beta g(z))dz = \alpha \int_\gamma f(z)dz + \beta \int_\gamma g(z)dz$$

(ii) If γ^- is γ with the reverse orientation, then

$$\int_\gamma f(z)dz = -\int_{\gamma^-} f(z)dz$$

(iii) One has the inequality

$$\left| \int_\gamma f(z)dz \right| \leq \sup_{z \in \gamma} |f(z)| \cdot \text{length}(\gamma)$$

and the $\text{length}(\gamma) = \int_a^b |z'(t)|\, dt$

$\int_\gamma f(z)dz$*:* Given a smooth curve γ in $\mathbb{C}$ parametrized by $z : [a, b] \to \mathbb{C}$, and f a continuous function on γ, we define the integral of f along γ by

$$\int_\gamma f(z)dz = \int_a^b f(z(t))z'(t)dt$$

Corollary 3.3: *1.If γ is a closed curve in an open set Ω, and f is continuous and has a primitive in Ω, then*

$$\int_\gamma f(z)dz = 0$$

2.If f is holomorphic in a region Ω and $f' = 0$, then f is constant.