Short questions

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1 Question 1

Consider the following matrices coming from a linear system model.

$$\begin{aligned} A_1 &= \begin{bmatrix} 3 & 0 \\ 0 & -6 \end{bmatrix}, & A_2 &= \begin{bmatrix} 3 & 0.1 \\ 0 & -6 \end{bmatrix}, \\ B_1 &= \begin{bmatrix} 1 \\ 0 \end{bmatrix}, & B_2 &= \begin{bmatrix} 0 \\ 1 \end{bmatrix}. \end{aligned} \tag{1}$$

1. Is the pair (A_1, B_1) controllable?
2. Is the pair (A_1, B_1) stabilizable?
3. Is the pair (A_1, B_2) controllable?
4. Is the pair (A_1, B_2) stabilizable?
5. Is the pair (A_2, B_1) controllable?
6. Is the pair (A_2, B_1) stabilizable?
7. Design B such that (A_1, B) is controllable.

2 Question 2

Given a second-order system $\ddot{x}(t) = u(t)$, where $x \in \mathbb{R}$, design a controller $u(t)$ such that $x(t) \to cos(t)$ as $t \to \infty$.

3 Question 3

Given a second-order system $\ddot{x}(t) = u(t)$, where $x \in \mathbb{R}^2$ design a controller $u(t)$ such that $x(t) \to \mathcal{C}$, where $C := \{x_1^2 + x_2^2 = 1\}$ as $t \to \infty$.

4 Question 4

Given the following linear sytem

$$\Sigma_{\text{linear}} := \begin{cases} \dot{x}(t) &= -\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} x(t) \\ y(t) &= \begin{bmatrix} 1 & 0 \end{bmatrix} x(t) \end{cases}, \tag{2}$$

- Is it observable?

- Is it stable in the sense of Lyapunov?
- Find the analytic solution of $x(t)$, and check that your prediction checks out with the numerical simulation.
- What can you say about the behavior of $z(t) := x_1(t) - x_2(t)$? Find the analytic solution of $z(t)$, and check that your prediction checks out with its numerical solution. Is the behavior of $z(t)$ coherent with $x(t)$?

5 Question 5

Given a second-order system $\ddot{x}(t) = u(t) = \cos(t)$, where $x \in \mathbb{R}$, and $y(t) = x_1(t)$, please, design an observer to estimate $x_2(t)$.

6 Question 6

Given the following system

$$\Sigma_{\text{linear}} := \begin{cases} \dot{x}(t) &= \begin{bmatrix} 0 & 1 \\ -2 & 0 \end{bmatrix} x(t) \\ y(t) &= \begin{bmatrix} 4 & -7 \end{bmatrix} x(t) \end{cases}, \tag{3}$$

please, design an observer to estimate $x(t)$.

7 Question 7

Given the following system

$$\Sigma_{\text{linear}} := \begin{cases} \dot{x}(t) &= \begin{bmatrix} 1 & 1 \\ -2 & 1 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t) \\ y(t) &= \begin{bmatrix} 1 & 0 \end{bmatrix} x(t) \end{cases}, \tag{4}$$

please, design a controller such that $x(t) \to 0$, as $t \to \infty$.