

## Short questions

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## 1 Question 1

Consider the following matrices coming from a linear system model.

$$\begin{aligned} A_1 &= \begin{bmatrix} 3 & 0 \\ 0 & -6 \end{bmatrix}, & A_2 &= \begin{bmatrix} 3 & 0.1 \\ 0 & -6 \end{bmatrix}, \\ B_1 &= \begin{bmatrix} 1 \\ 0 \end{bmatrix}, & B_2 &= \begin{bmatrix} 0 \\ 1 \end{bmatrix}. \end{aligned} \tag{1}$$

1. Is the pair  $(A_1, B_1)$  controllable?
2. Is the pair  $(A_1, B_1)$  stabilizable?
3. Is the pair  $(A_1, B_2)$  controllable?
4. Is the pair  $(A_1, B_2)$  stabilizable?
5. Is the pair  $(A_2, B_1)$  controllable?
6. Is the pair  $(A_2, B_1)$  stabilizable?
7. Design  $B$  such that  $(A_1, B)$  is controllable.

## 2 Question 2

Given a second-order system  $\ddot{x}(t) = u(t)$ , where  $x \in \mathbb{R}$ , design a controller  $u(t)$  such that  $x(t) \rightarrow \cos(t)$  as  $t \rightarrow \infty$ .

## 3 Question 3

Given a second-order system  $\ddot{x}(t) = u(t)$ , where  $x \in \mathbb{R}^2$  design a controller  $u(t)$  such that  $x(t) \rightarrow \mathcal{C}$ , where  $\mathcal{C} := \{x_1^2 + x_2^2 = 1\}$  as  $t \rightarrow \infty$ .

## 4 Question 4

Given the following linear system

$$\Sigma_{\text{linear}} := \begin{cases} \dot{x}(t) &= - \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} x(t), \\ y(t) &= \begin{bmatrix} 1 & 0 \end{bmatrix} x(t) \end{cases}, \tag{2}$$

- Is it observable?

- Is it stable in the sense of Lyapunov?
- Find the analytic solution of  $x(t)$ , and verify that your prediction checks out with the numerical simulation.
- What can you say about the behavior of  $z(t) := x_1(t) - x_2(t)$ ? Find the analytic solution of  $z(t)$ , and verify that your prediction checks out with its numerical solution. Is the behavior of  $z(t)$  coherent with  $x(t)$ ?

## 5 Question 5

Given a second-order system  $\ddot{x}(t) = u(t) = \cos(t)$ , where  $x \in \mathbb{R}$ , and  $y(t) = x_1(t)$ , please, design an observer to estimate  $x_2(t)$ .

## 6 Question 6

Given the following system

$$\Sigma_{\text{linear}} := \begin{cases} \dot{x}(t) &= \begin{bmatrix} 0 & 1 \\ -2 & 0 \end{bmatrix} x(t) \\ y(t) &= \begin{bmatrix} 4 & -7 \end{bmatrix} x(t) \end{cases}, \quad (3)$$

please, design an observer to estimate  $x(t)$ .

## 7 Question 7

Given the following system

$$\Sigma_{\text{linear}} := \begin{cases} \dot{x}(t) &= \begin{bmatrix} 1 & 1 \\ -2 & 1 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t) \\ y(t) &= \begin{bmatrix} 1 & 0 \end{bmatrix} x(t) \end{cases}, \quad (4)$$

please, design a controller such that  $x(t) \rightarrow 0$ , as  $t \rightarrow \infty$ .