$\mathrm{BC}_1\!\left(X_P^\bullet\right)$