SHORT NOTE

A NOTE ON THE USE OF MICROSEISMS IN DETERMINING THE SHALLOW STRUCTURES OF THE EARTH'S CRUST†

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In the VELA UNIFORM Special Issue I of GEOPHYSICS, M. N. Toksöz presented the result of his attempt at determining the shallow structures of the earth's crust from the phase velocity of microseisms. He stated that his attempt resulted only in partial success on account of the fact that the microseisms arrived from more than one direction at the same time with comparable strength. He also concluded that there was no way of improving the results by the use of special arrays because, according to him, there were two unknown parameters, direction and phase velocity, and without the knowledge of one the other cannot be found. I thought this problem was already solved in my paper (Aki, 1957), in which a statistical theory of determining the phase velocity of random waves was given with a successful application to microseisms in Tokyo in the frequency range of 5 to 15 cps. Since Toksöz's conclusion might have given a pessimistic view on the use of microseisms, I feel it is necessary to report a brief summary of my old paper published in a Japanese journal which might not be well circulated in the United States.

My model of random waves is the sum of waves propagating in a horizontal plane in different directions with different powers but with the same phase velocity for a given frequency. It is assumed that the waves with different propagation directions and with different frequencies are statistically independent. If we define a space-correlation function as

$$\phi(r, \lambda) = \overline{u(x, y, t) \cdot u(x + r \cos \lambda, y + r \sin \lambda, t)},$$

and make an azimuthal average of this function as

$$\phi(r) = \frac{1}{\pi} \int_0^{\pi} \phi(r, \lambda) d\lambda,$$

then the power spectrum $\Phi(\omega)$ of the waves in the time domain may be expressed as the Hankel transform of $\phi(r)$ and vice versa;

$$\Phi(\omega) = \frac{\pi\omega}{c(\omega) U(\omega)} \int_0^{\infty} \phi(r) J_o\left(\frac{\omega}{c(\omega)} r\right) r dr,$$

$$\phi(r) = \frac{1}{\pi} \int_0^{\infty} \Phi(\omega) J_o\left(\frac{\omega}{c(\omega)} r\right) d\omega,$$

where J_o is the zero order Bessel function. These relations show that the phase velocity $c(\omega)$ may be obtained by measuring $\phi(r)$ and $\Phi(\omega)$ without the knowledge of the directionality of the waves. A convenient method of determining the value of $c(\omega)$ is to prefilter the waves by a narrow band-pass filter with the center frequency ω_o before computing the space-correlation function. Then the space-correlation function $\phi(r)$ will take the form of

$$J_o\left(\frac{\omega_o r}{c(\omega_o)}\right),$$

if normalized. By varying ω_o for a fixed r, we can determine the phase velocity as a function of frequency. Thus, this method requires only a set of seismographs placed on a semicircle and one at the center of the circle. It is only necessary to form the correlation function between the seismogram at the center and the sum of signals from the seismographs equally spaced on the semicircle.

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The above form of

$$J_o\left(\frac{\omega_o r}{c(\omega_o)}\right)$$

applies only to the vertical motion of the waves. In the study of horizontal motion, we may set the direction of seismograph displacement parallel to the direction connecting the seismograph pair or perpendicular to it. In the former case, the space-correlation function between the pair will take the form

$$J_o\left(\frac{\omega}{c(\omega)}r\right) - J_2\left(\frac{\omega}{c(\omega)}r\right)$$

for the waves polarized in the direction of propagation like Rayleigh waves, and the form

$$J_o\left(\frac{\omega}{c(\omega)}r\right) + J_2\left(\frac{\omega}{c(\omega)}r\right)$$

for the waves polarized perpendicularly to the direction of propagation like Love waves. In the latter case, the same functional forms apply if the wave types are interchanged. Thus, we can determine the type of waves by this method.

This method was applied to the microseisms in Tokyo in the frequency range of 5 to 15 cps. A specifically designed electronic computer was used for the computation of correlation function based on the "plus-minus" method of Y. Tomoda. All the computations were done automatically and the results were displayed on the decatron tubes. It was found that the microseisms in the above frequency range in the central part of Tokyo are coming from all directions with nearly uniform azimuthal distribution, and that the waves prevailing in the horizontal-component records are Love waves. The phase velocities were determined with an accuracy better than 5 percent, and interpreted satisfactorily in terms of the structure at the site.

REFERENCES

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Toksöz, M. Nafi, 1964, Microseisms and an attempted application to exploration: Geophysics, v. 29, p. 154–177.