Seismology and Seismic Imaging

5. Ray tracing in practice

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Introduction

- Although 1-D whole Earth models are an acceptable approximation in some applications, lateral heterogeneity is significant in many regions of the Earth (e.g. subduction zones) and therefore needs to be accounted for.
- Ray tracing in laterally heterogeneous media is non-trivial, and many different schemes have been devised in the last few decades.
- I will briefly discuss the following schemes:
  - Ray tracing
  - Finite difference solution of the eikonal equation
  - Shortest Path Ray tracing (SPR)

Initial value ray tracing

- From before, the ray equation is given by:

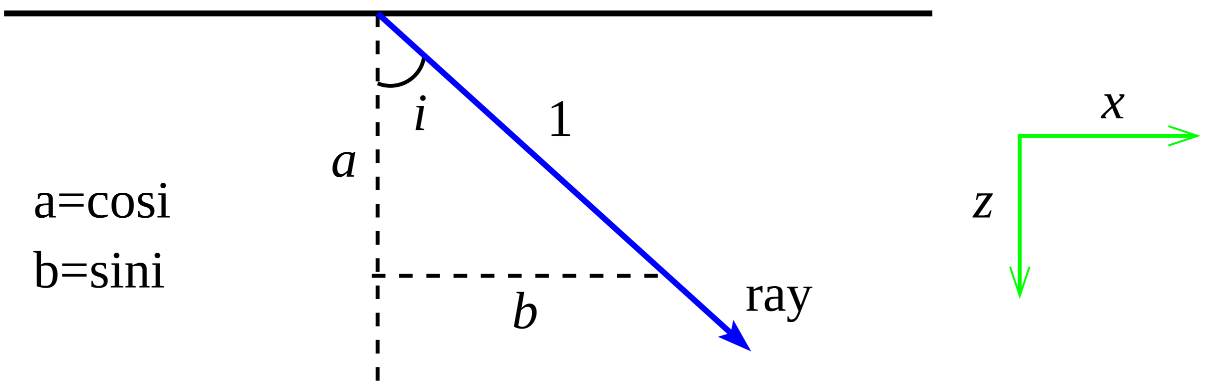
$$\frac{\mathrm{d}}{\mathrm{d}s}\left[U\frac{\mathrm{d}\mathbf{r}}{\mathrm{d}s}\right] = \nabla U$$

where U is slowness, $\mathbf{r}$ is the position vector and s is path length.

- The quantity $\mathrm{d}\mathbf{r}/\mathrm{d}s$ is a unit vector in the direction of the ray, so in 2-D Cartesian coordinates:

$$\frac{\mathrm{d}\mathbf{r}}{\mathrm{d}s} = [\sin i, \cos i]$$

where i is the ray inclination angle.



- Substitution of this expression into the ray equation yields:

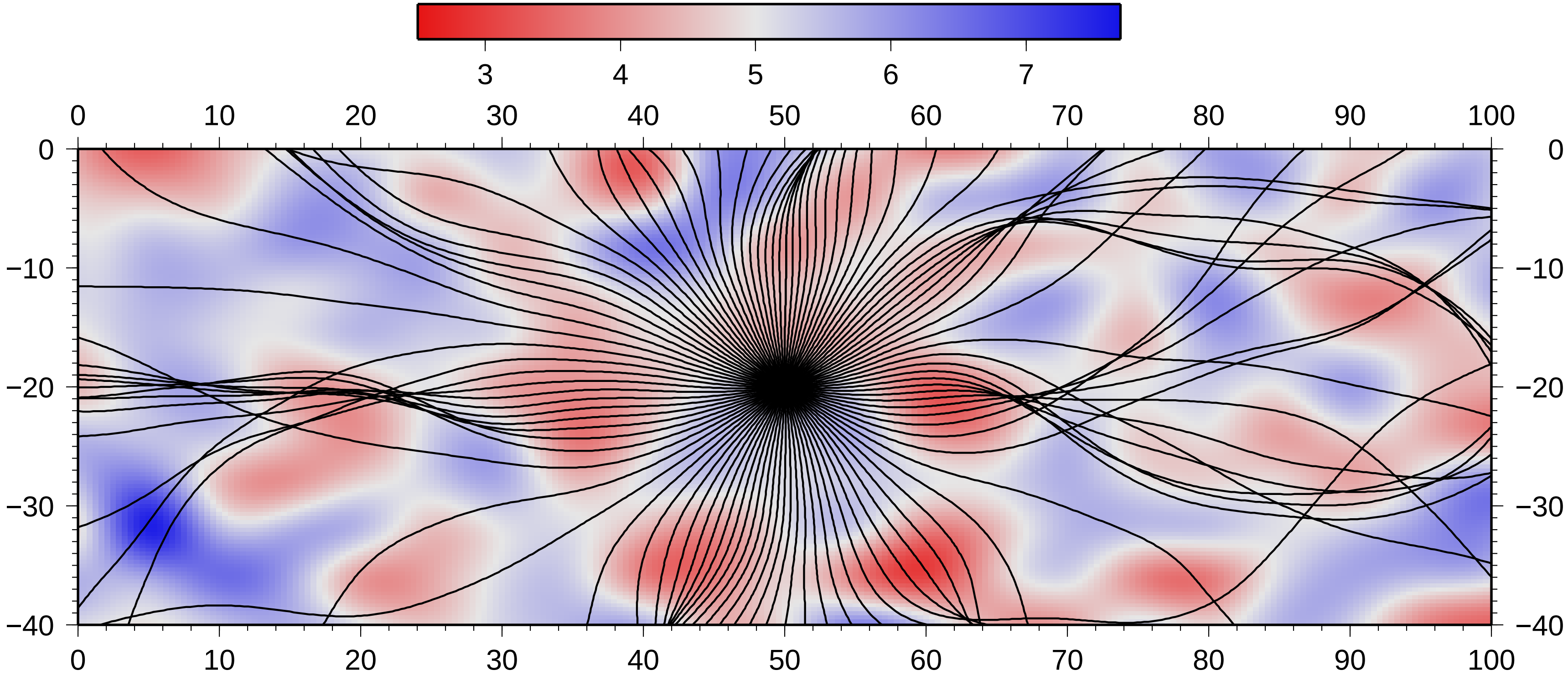
$$\frac{\mathrm{d}i}{\mathrm{d}s} = \frac{1}{U}\left[\cos i\frac{\partial U}{\partial x} - \sin i\frac{\partial U}{\partial z}\right]$$

- Since $\mathrm{d}\mathbf{r}/\mathrm{d}s = [\mathrm{d}x/\mathrm{d}s, \mathrm{d}z/\mathrm{d}s] = [\sin i, \cos i]$,

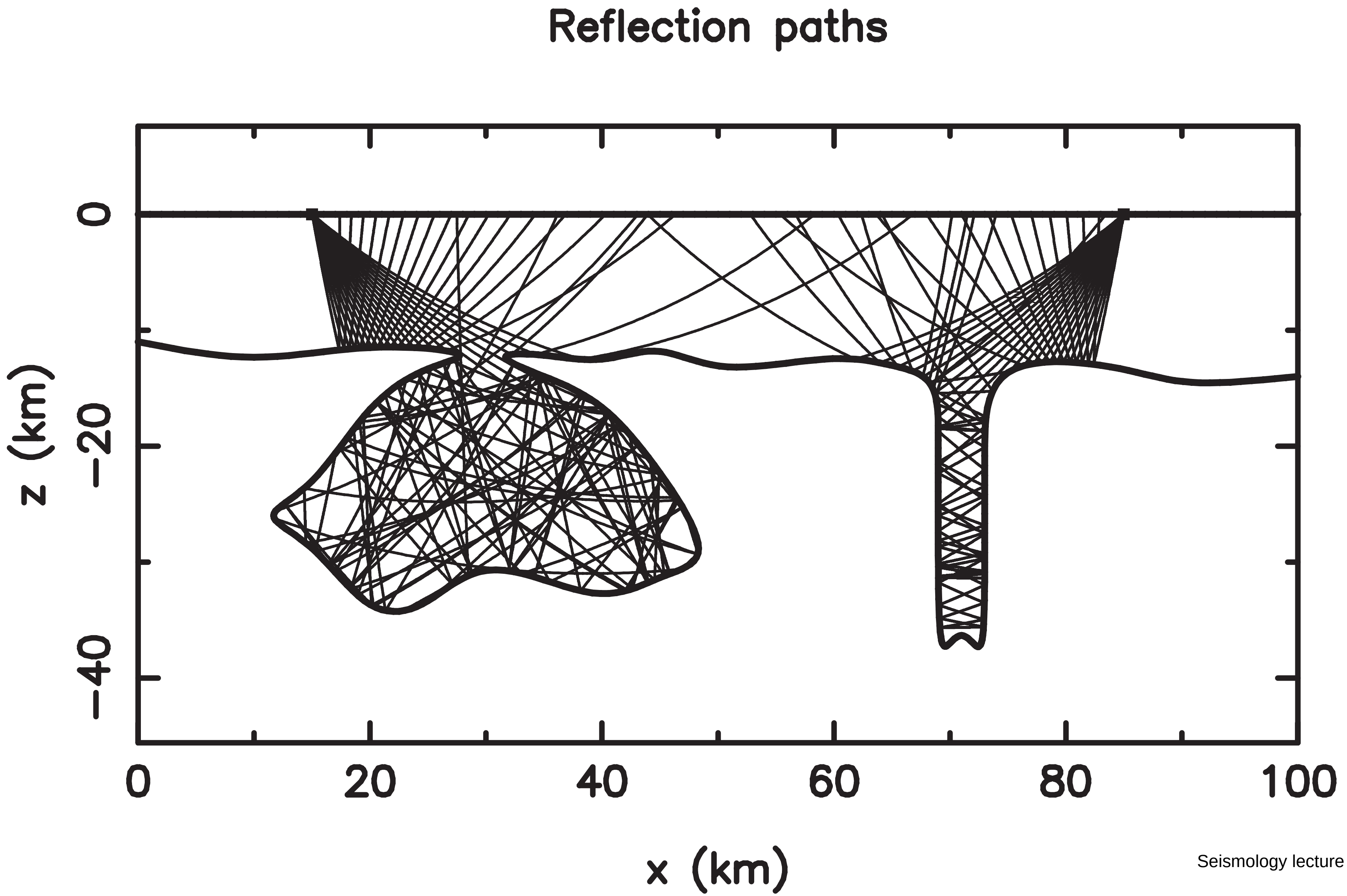
$$\begin{aligned}\frac{\mathrm{d}x}{\mathrm{d}t} &= v \sin i \\ \frac{\mathrm{d}z}{\mathrm{d}t} &= v \cos i \\ \frac{\mathrm{d}i}{\mathrm{d}t} &= -\cos i \frac{\partial v}{\partial x} + \sin i \frac{\partial v}{\partial z}\end{aligned}$$

  where $v = v(x, z)$ is wavespeed.
- The above coupled system of ordinary differential equations represents an initial value form of the ray equation.

- The example below shows a fan of 100 rays traced by solving the initial value ray equations using a 4th order Runge Kutta scheme.

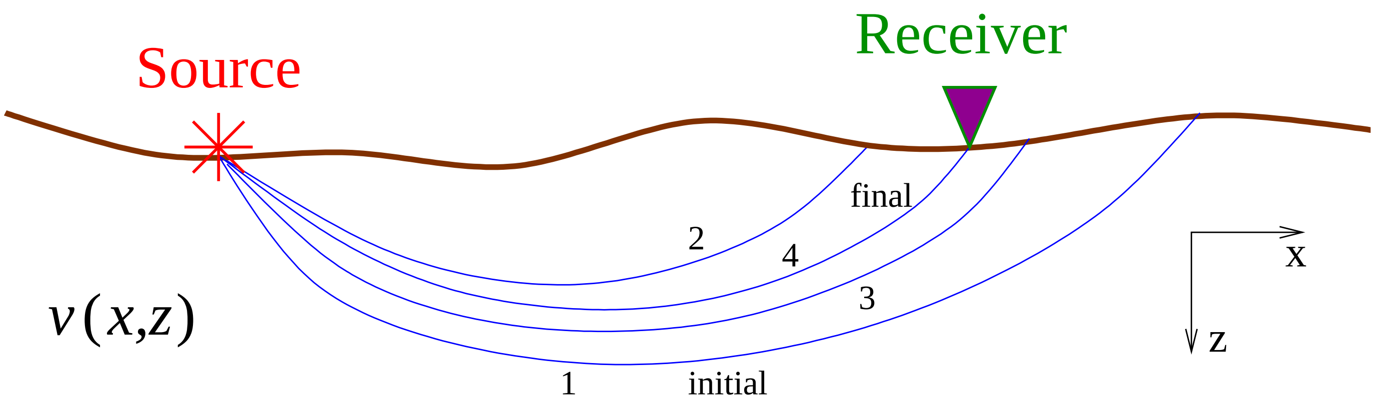


Initial value ray tracing is powerful

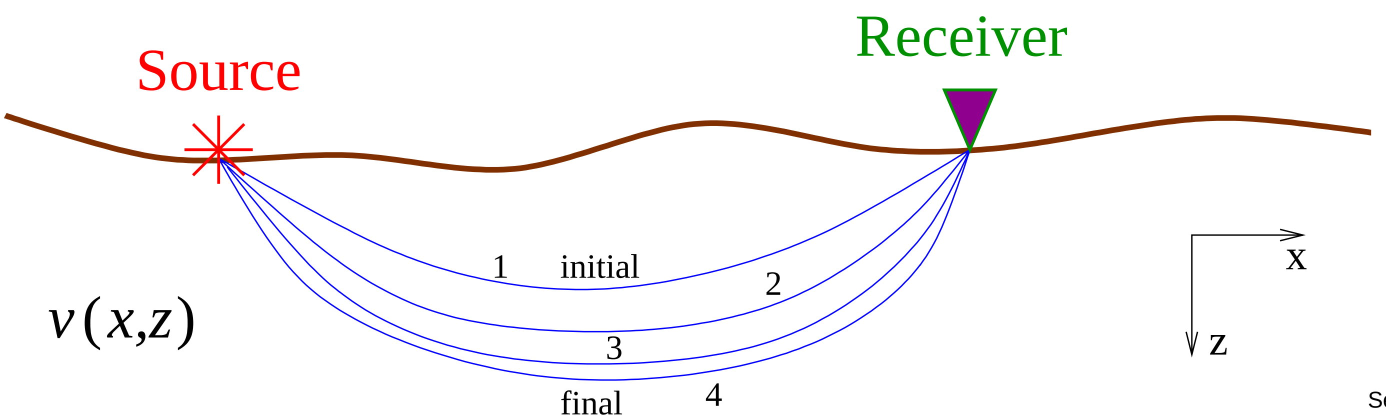


Shooting and bending methods

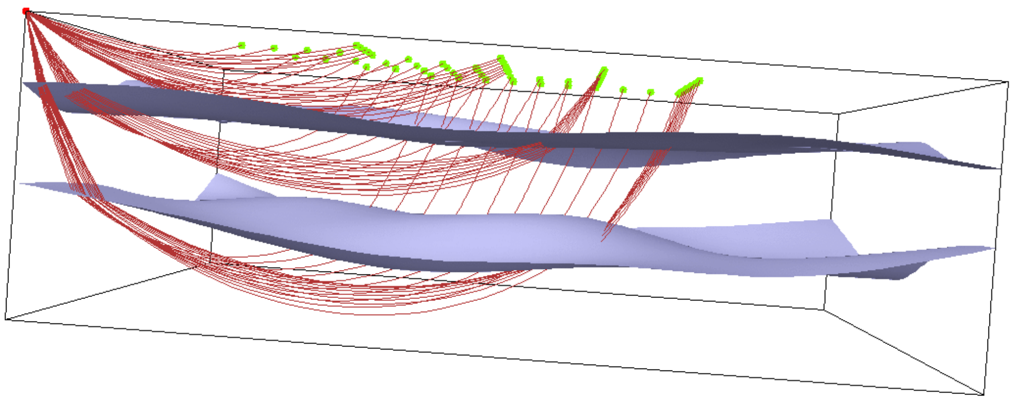
- Shooting

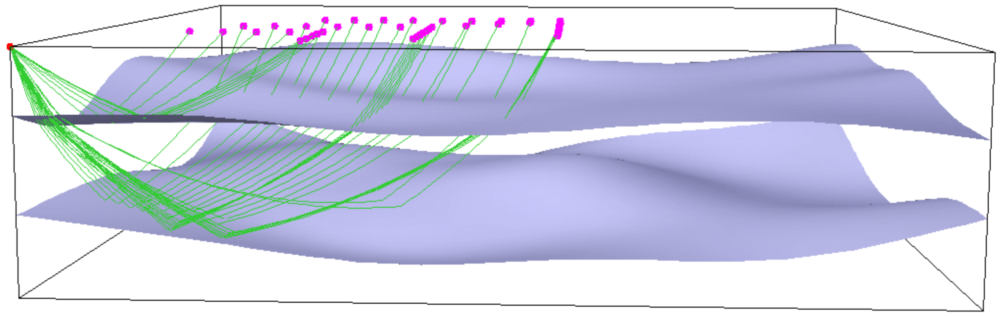


- Bending

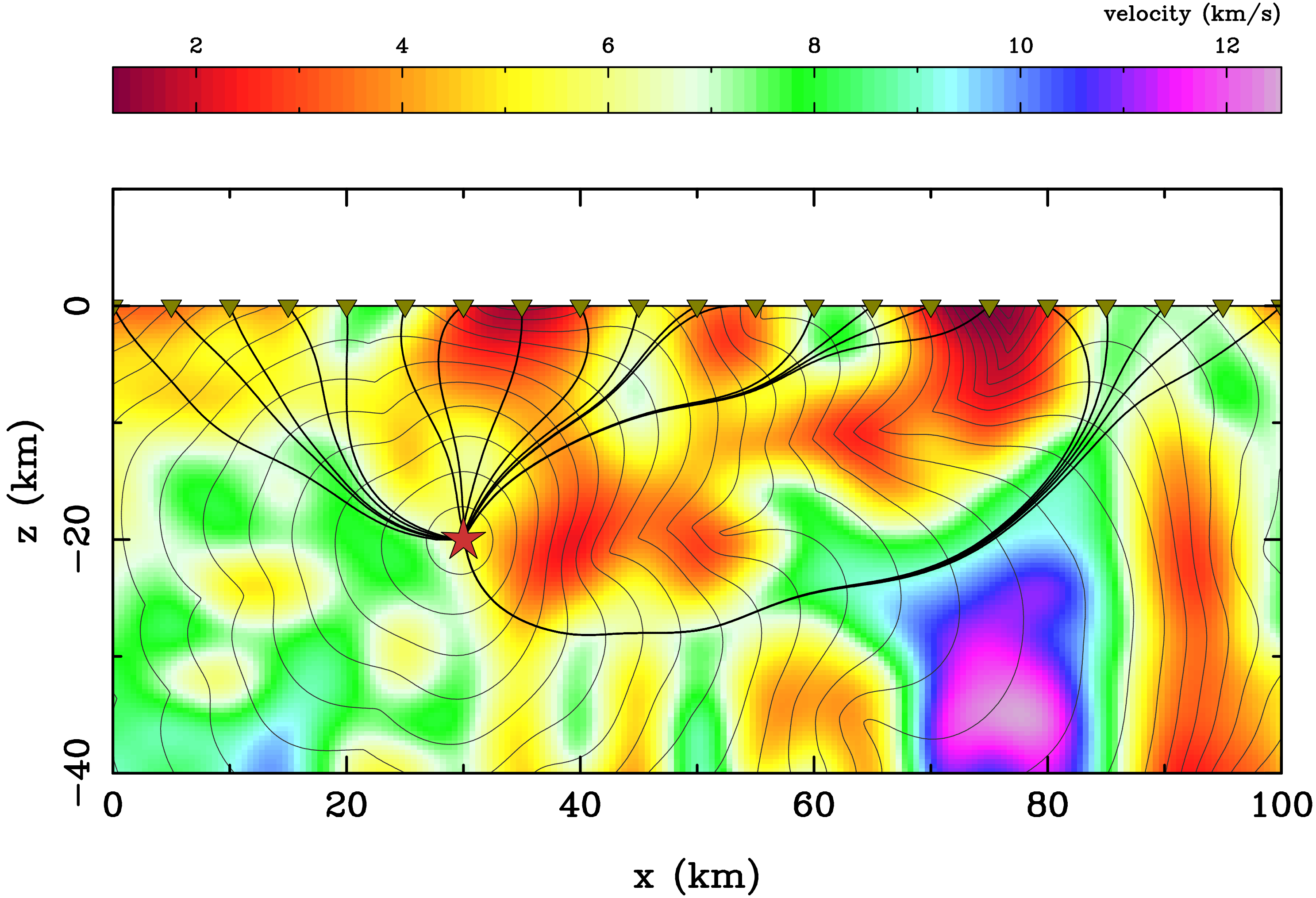


- Ray tracing becomes less robust as the complexity of the medium increases.
- Can find a limited class of later arrivals.



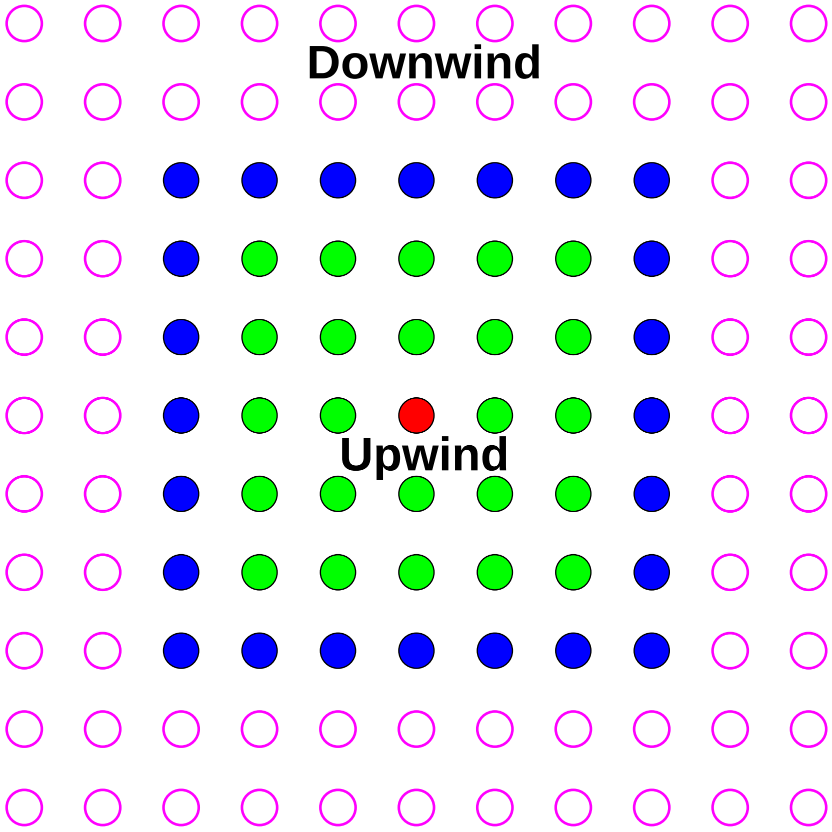


The failure of ray tracing



Eikonal solvers

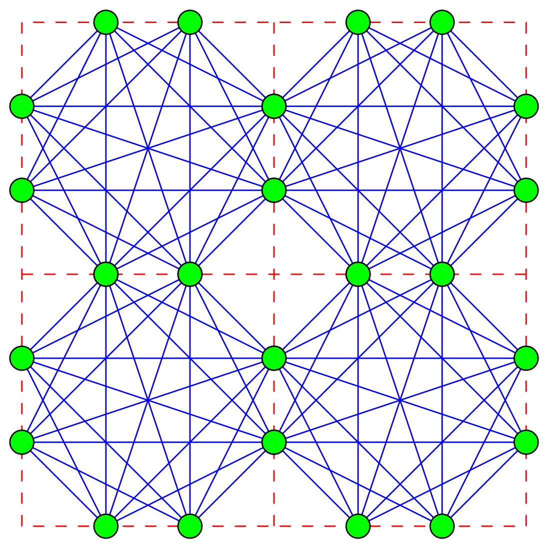
- Seek finite difference solution of eikonal equation throughout a gridded velocity field (Vidale, 1988,1990).
- Very fast but first arrival only.
- Stability is an issue.

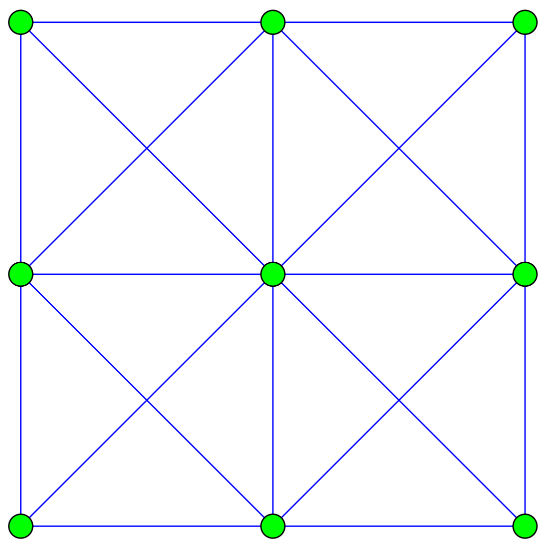


Shortest Path Ray tracing (SPR)

- A network or graph is formed by connecting neighbouring nodes with traveltime path segments (Moser, 1991).
- Find path of minimum traveltime between source and receiver through network using Dijkstra-like algorithms.

Not as fast as eikonal solvers, but tends to be more stable.



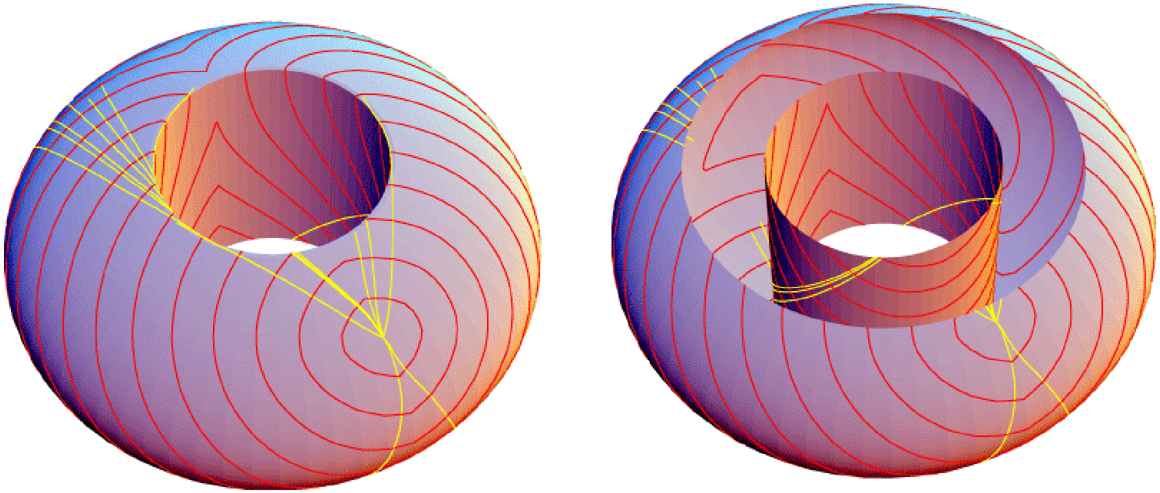


The Fast Marching Method (FMM)

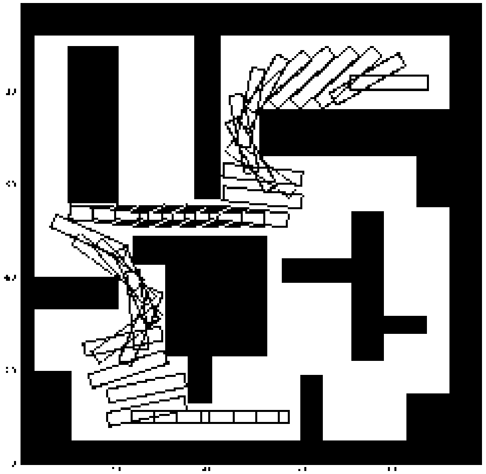
- FMM = grid based numerical scheme for tracking the evolution of monotonically advancing interfaces via FD solution of the eikonal equation.
- Only computes the first arrival in continuous media, but combines unconditional stability and rapid computation.
- $\Rightarrow$ It will always work regardless of the complexity of the medium. This is a very desirable feature.
- First introduced by James Sethian (1996), who subsequently applied it to a range of problems in the physical sciences.

Medical imaging

Geodesics

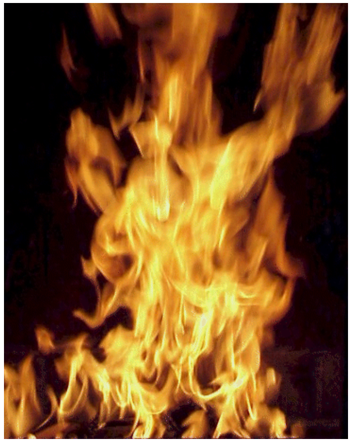


Robotic navigation

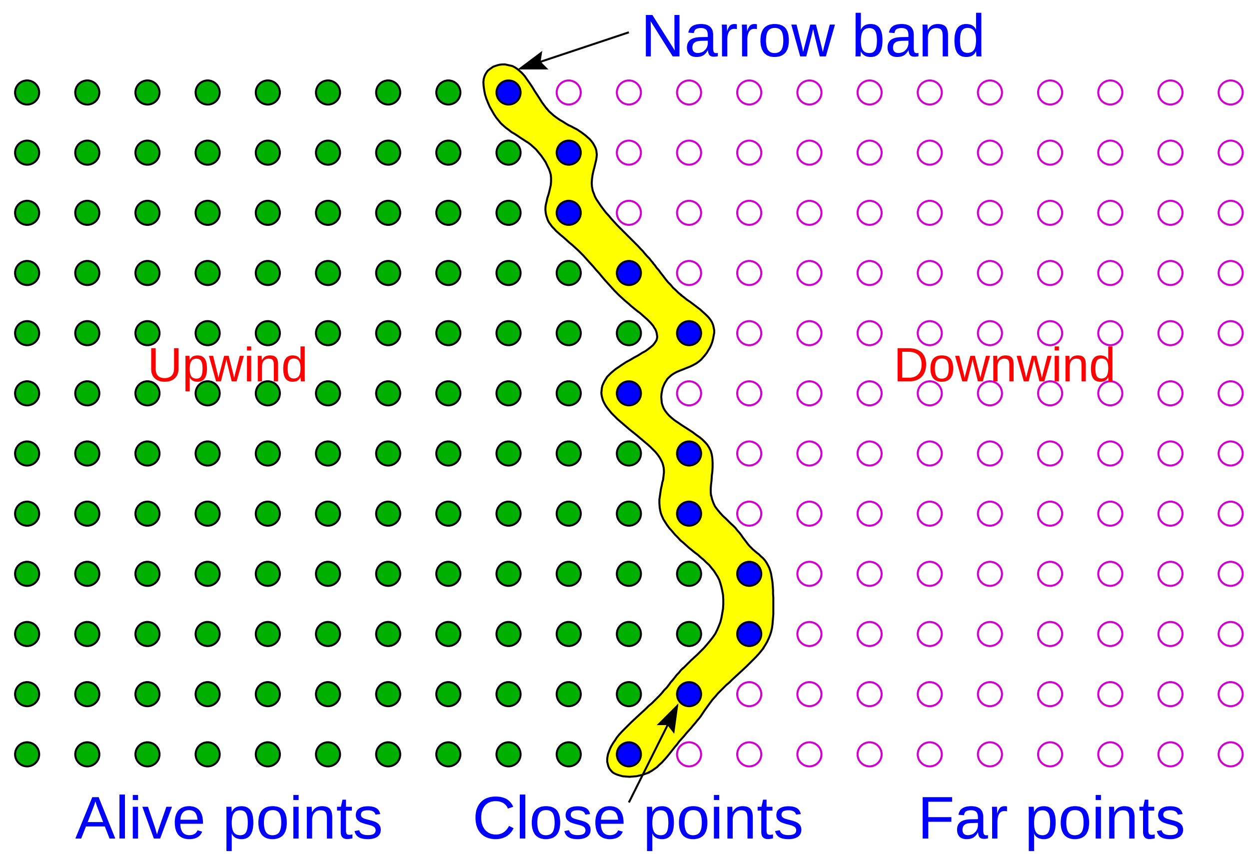


FMM in continuous media

Narrow band sweeps through grid like a forest fire



Entropy condition:
Once a point burns, it stays burnt



- Heap sort algorithm used to locate grid points in narrow band with minimum traveltime $\Rightarrow O(M \log M)$ operation count for FMM.

Updating grid points

- The eikonal equation $|\nabla_{\mathbf{x}} T| = s(\mathbf{x})$ is solved using an entropy satisfying upwind scheme.

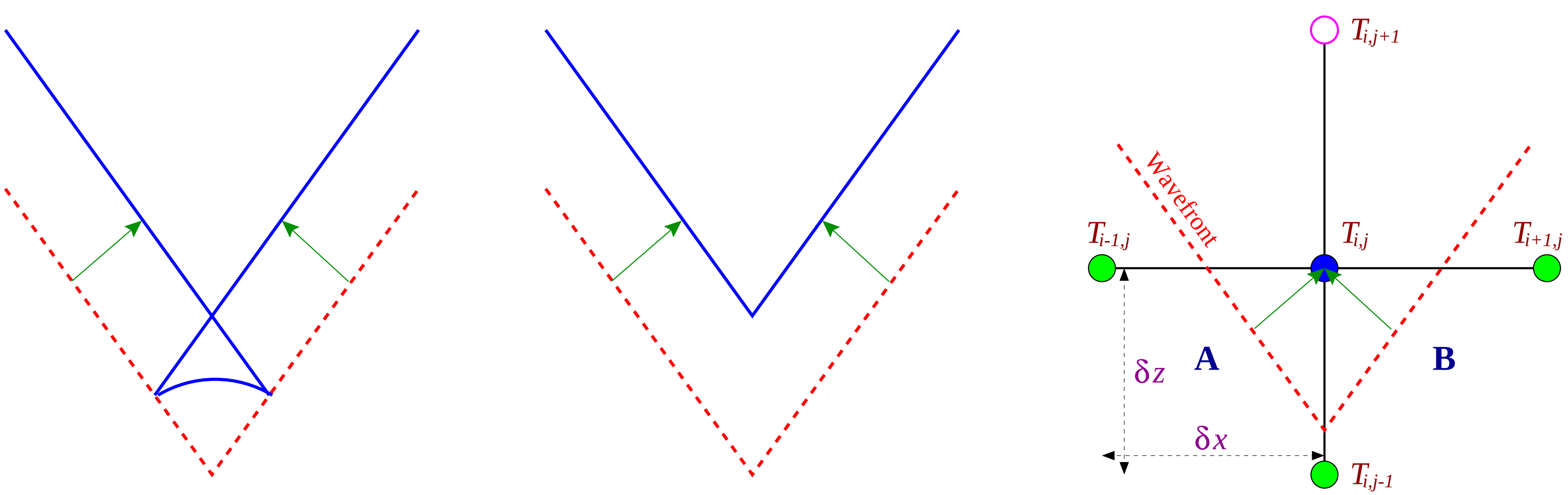
$$\begin{bmatrix} \max(D_a^{-x}T, -D_b^{+x}T, 0)^2 + \\ \max(D_c^{-y}T, -D_d^{+y}T, 0)^2 + \\ \max(D_e^{-z}T, -D_f^{+z}T, 0)^2 \end{bmatrix}_{ijk}^{\frac{1}{2}} = s_{i,j,k}$$

$$D_1^{-x}T_i = \frac{T_i - T_{i-1}}{\delta x} \qquad D_2^{-x}T_i = \frac{3T_i - 4T_{i-1} + T_{i-2}}{2\delta x}$$

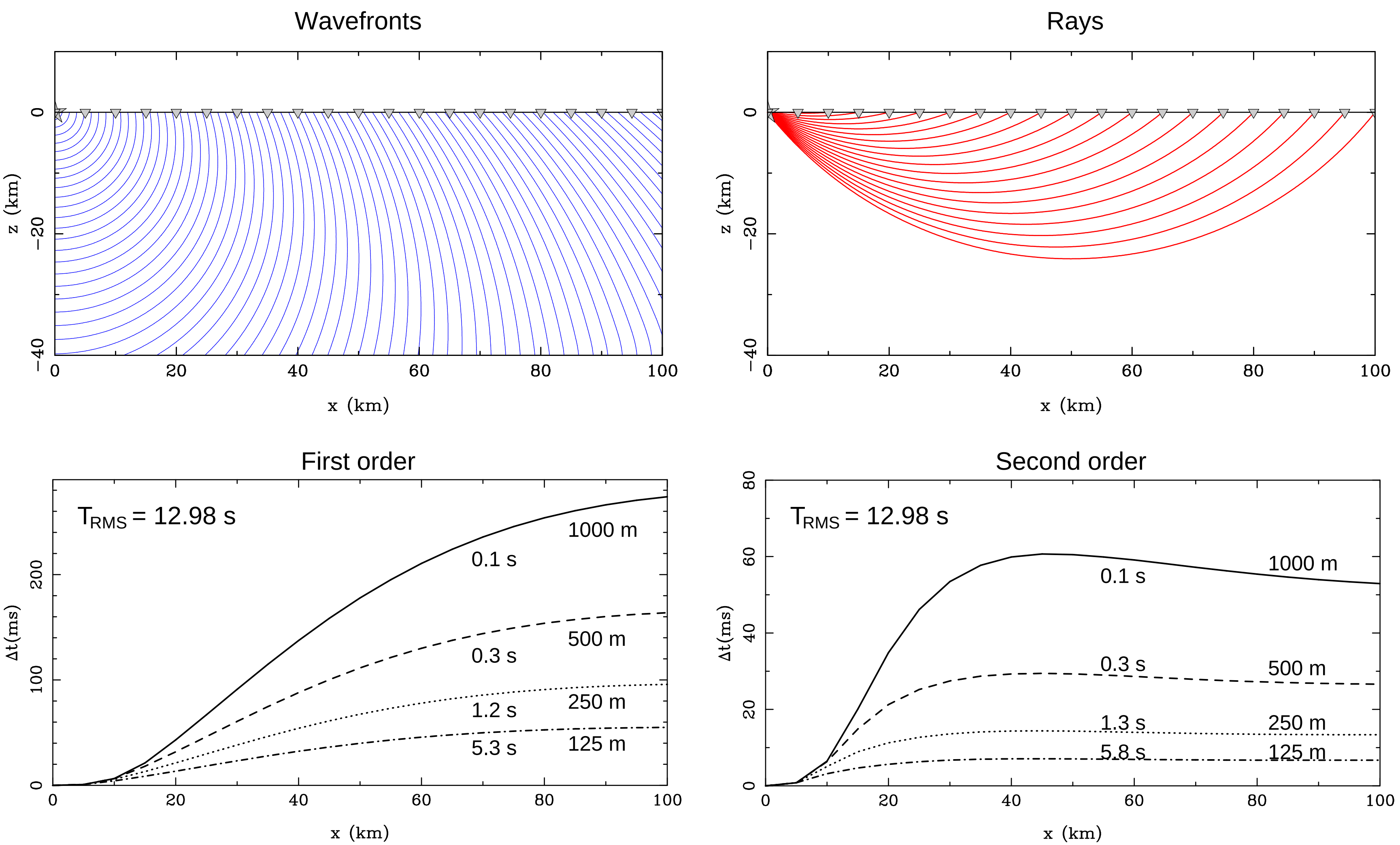
- D_1 or D_2 are used depending on availability of upwind traveltimes.

Stability

- The unconditional stability of FMM is due in part to its ability to handle propagating wavefront discontinuities.

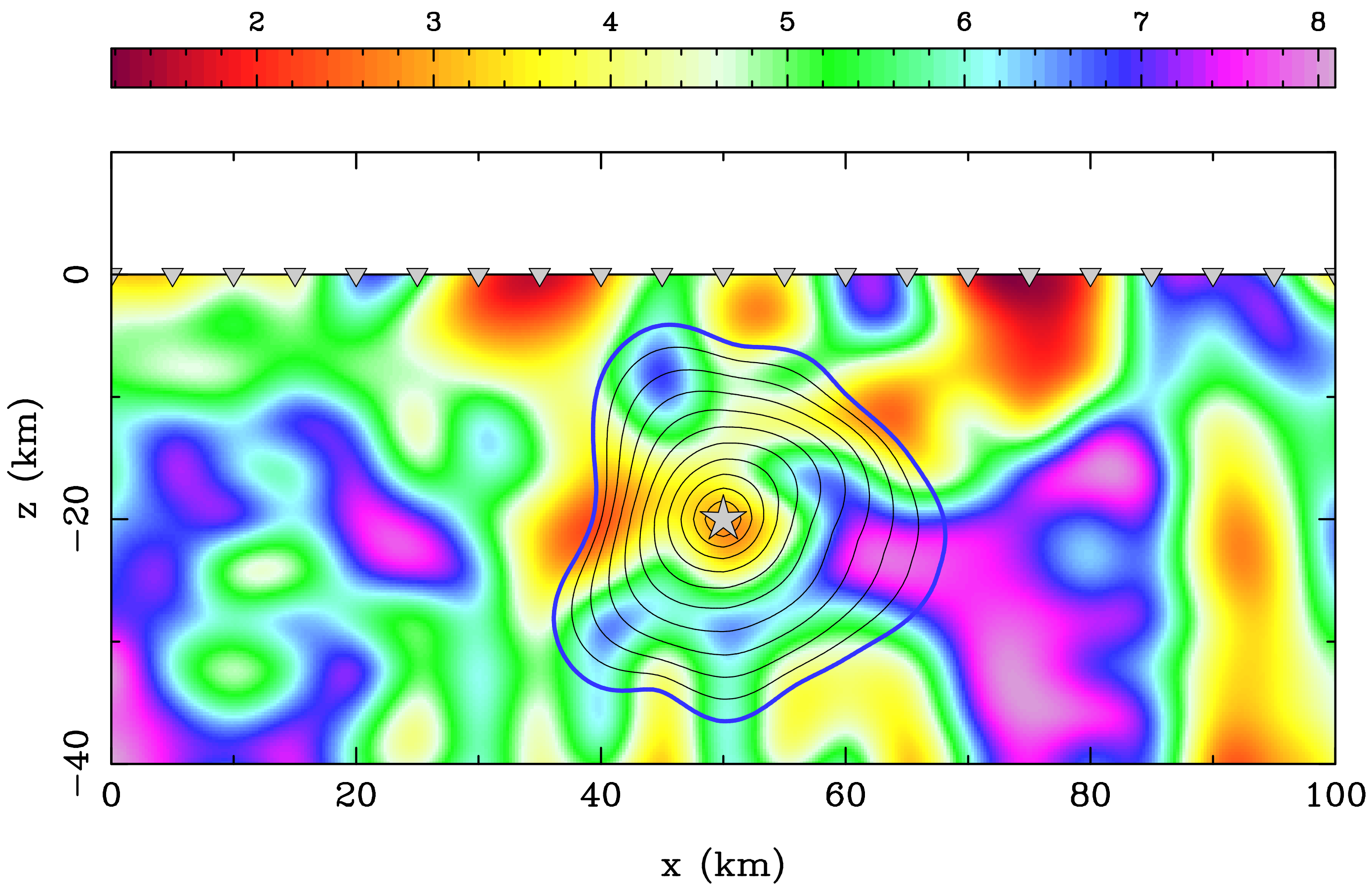


Example



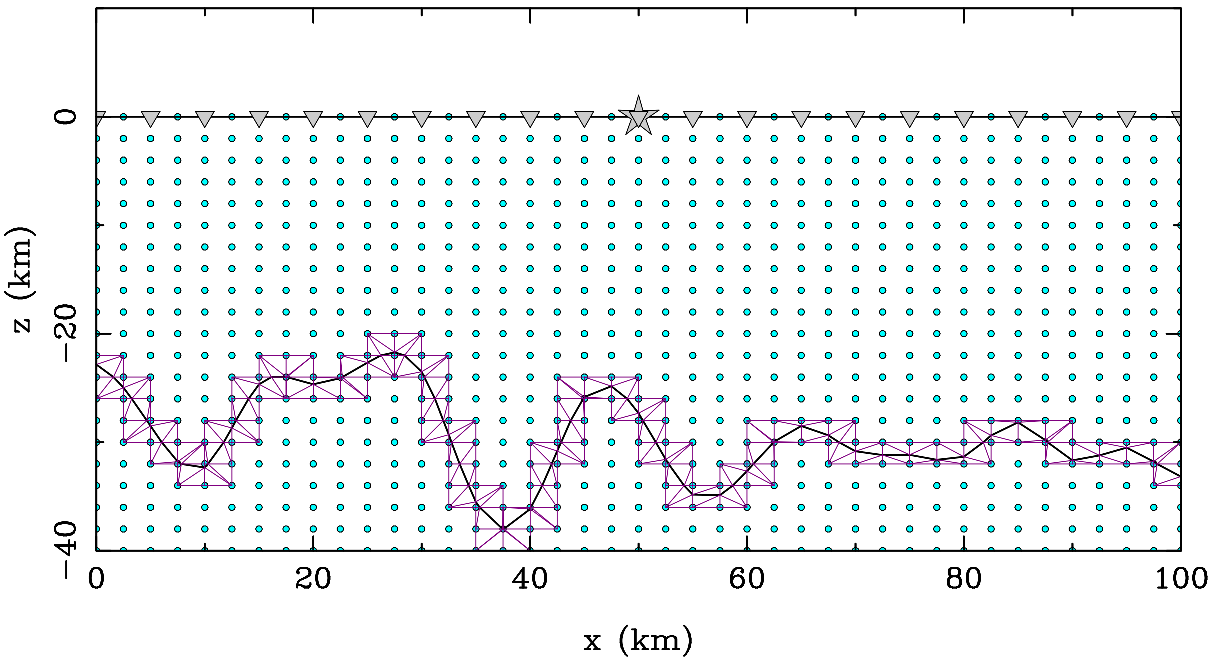
Wavefronts
Rays
First order
Second order
x (km)
z (km)
Δt(ms)
T_RMS = 12.98 s
1000 m
500 m
250 m
125 m
0.1 s
0.3 s
1.2 s
5.3 s
1.3 s
5.8 s

Movie



FMM in layered media

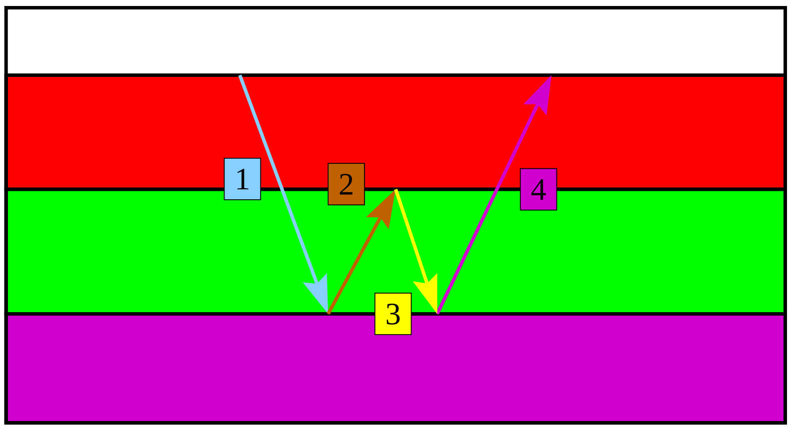
- A locally irregular mesh of triangles is used to suture the velocity nodes to the interface nodes.

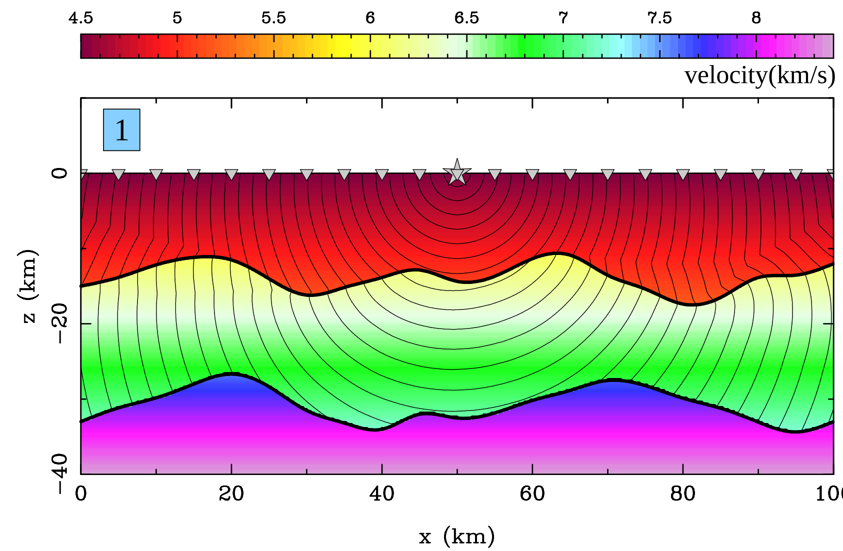


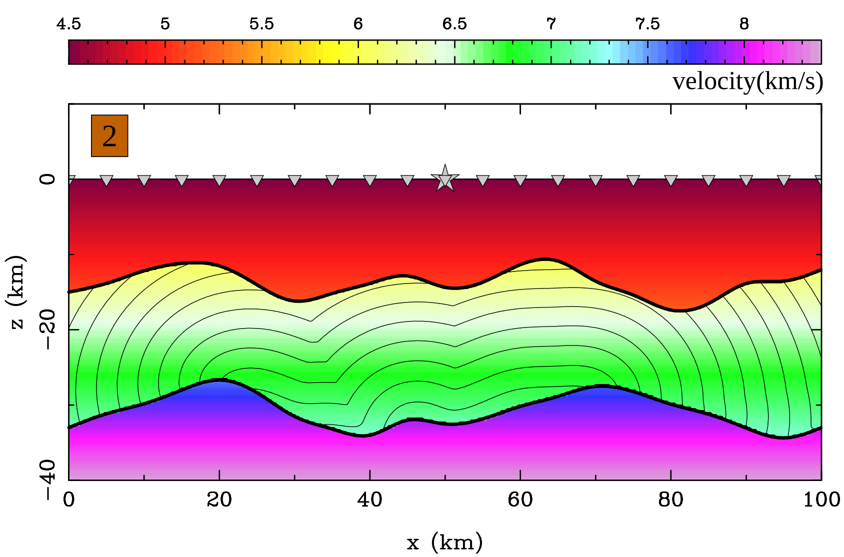
- A first-order entropy satisfying upwind scheme is used to solve the eikonal equation within the irregular mesh.

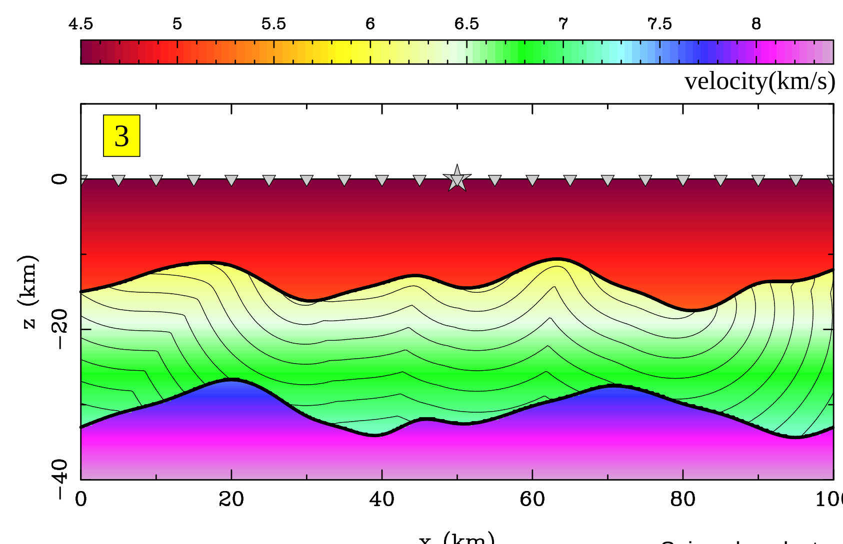
Example

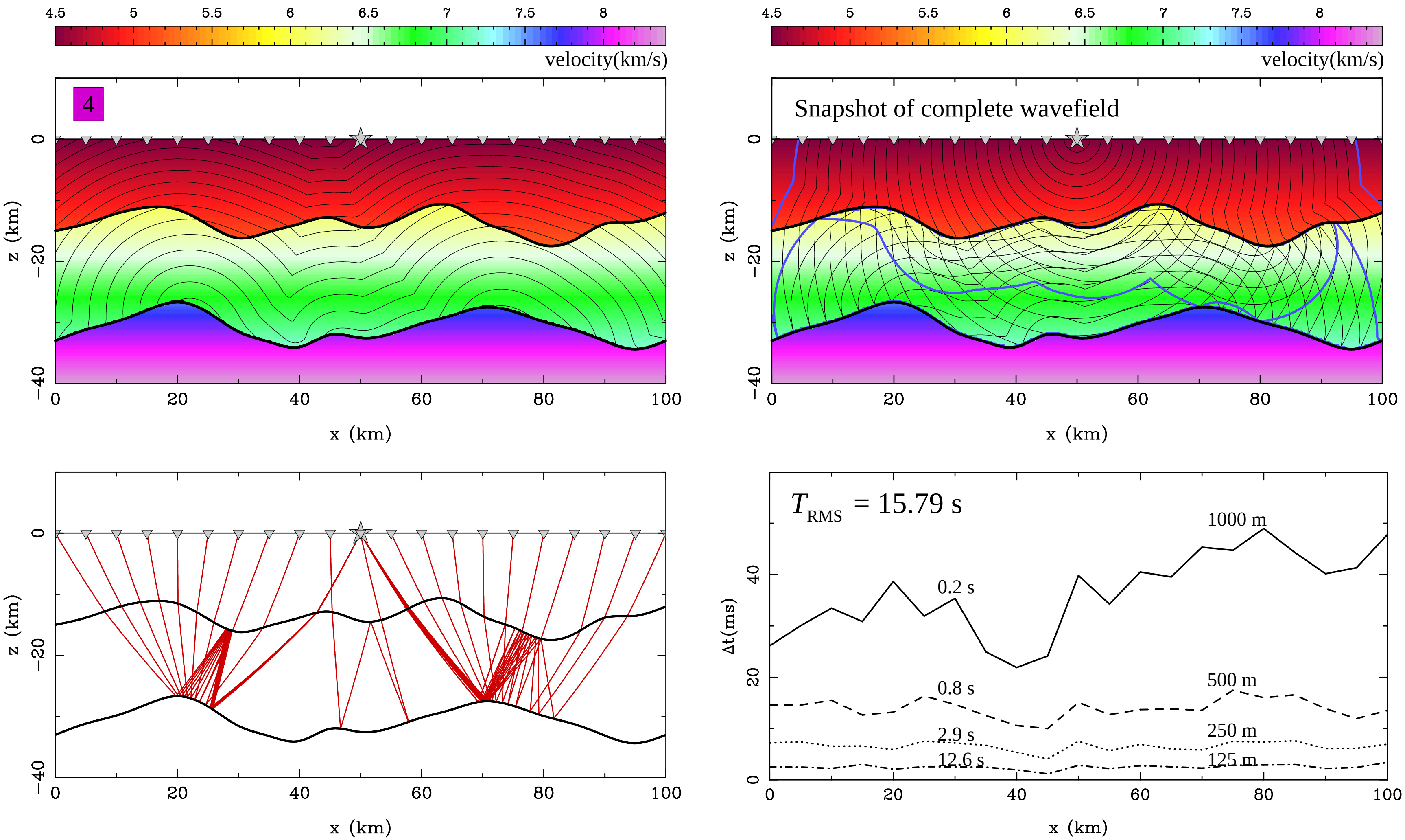
Four branch multiple



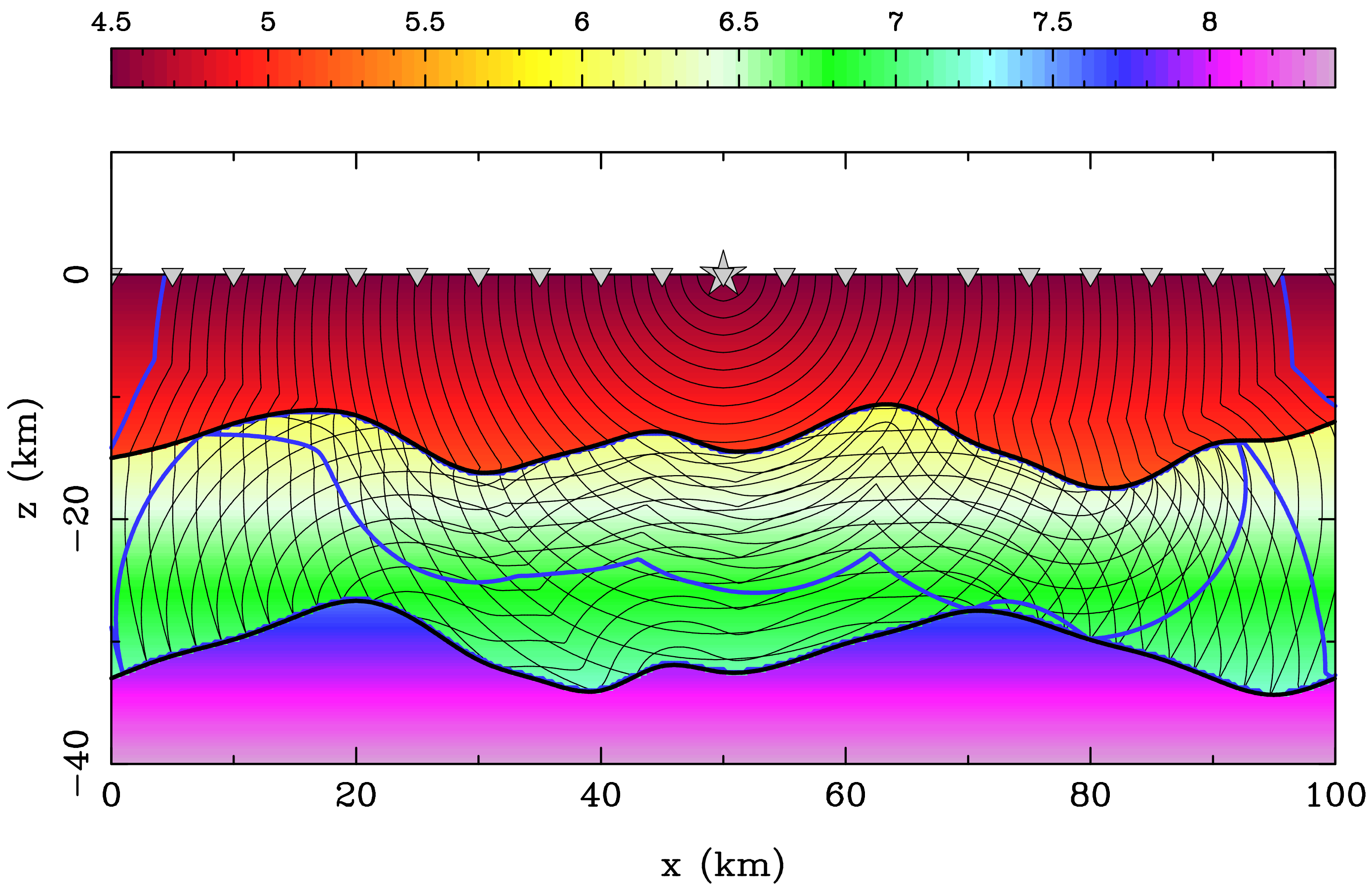


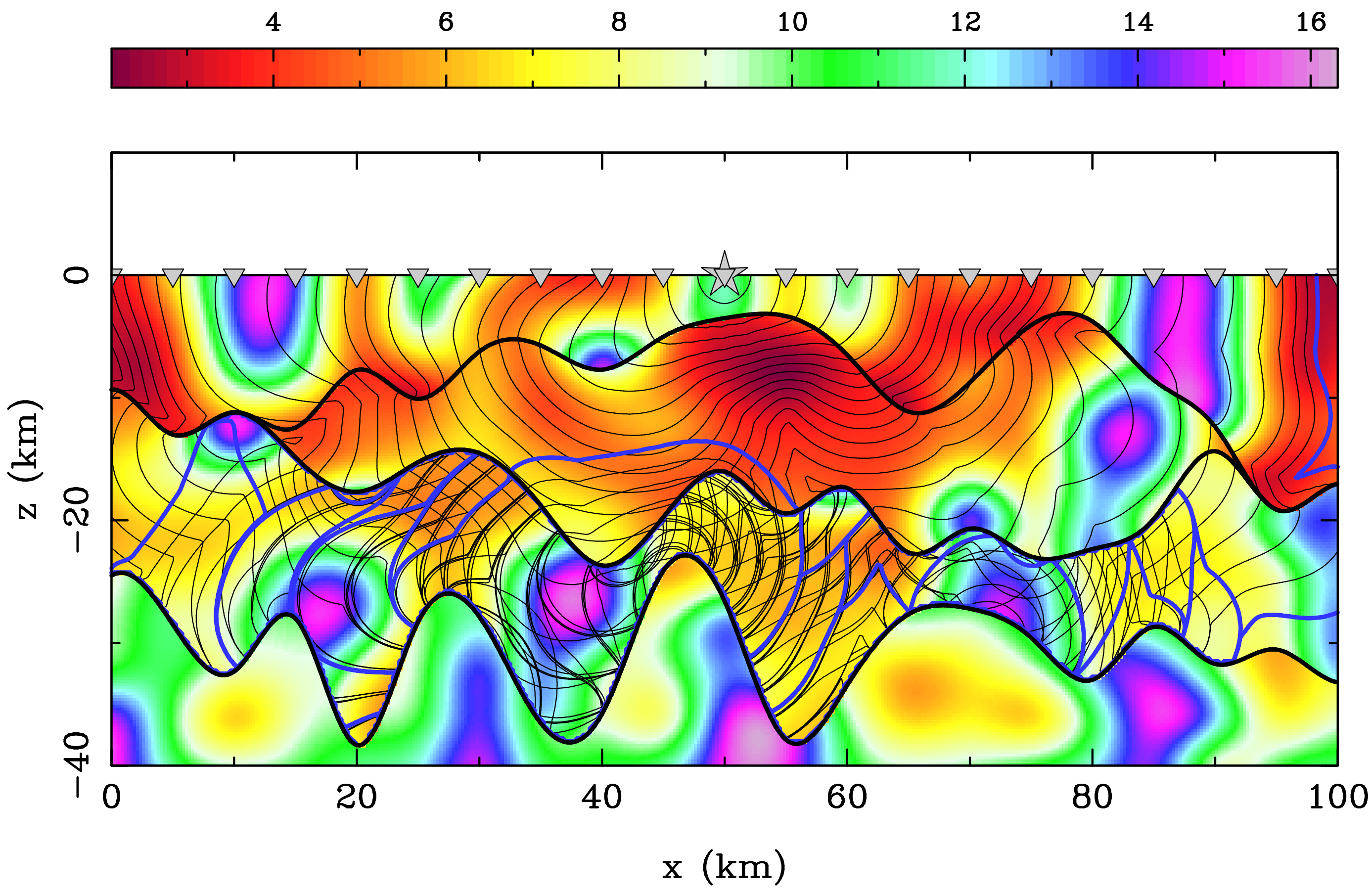





4.5
5
5.5
6
6.5
7
7.5
8
velocity(km/s)
4
Snapshot of complete wavefield
z (km)
x (km)
0
−20
−40
20
40
60
80
100
T_{RMS} = 15.79 s
Δt(ms)
1000 m
0.2 s
500 m
0.8 s
250 m
2.9 s
125 m
12.6 s

Movies




4
6
8
10
12
14
16
0
−20
−40
z (km)
0
20
40
60
80
100
x (km)