Seismic waves and Snell's law
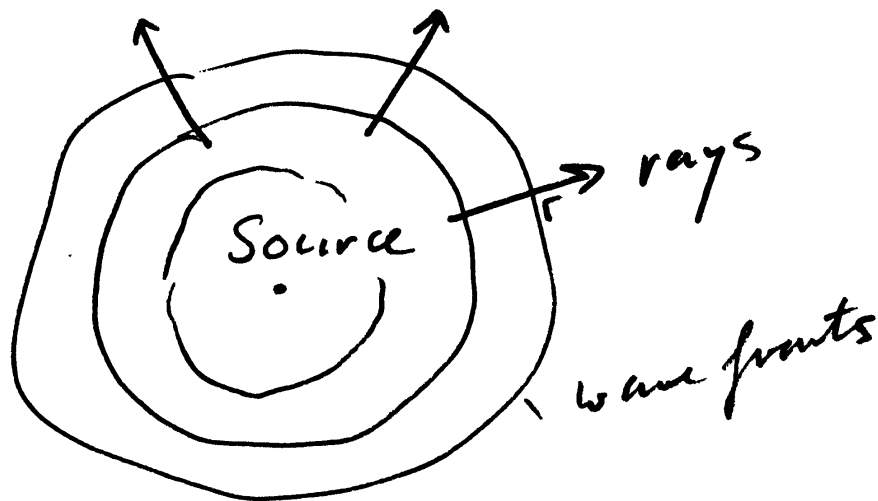

A wave front is a surface connecting all points of equal travel time from the source.

Rays are the normals to the wavefronts, and they point in the direction of the wave propagation.

While the mathematical description of the wavefronts is rather complex, that of the rays is simple. For many applications is it convenient to consider rays rather than wavefronts.

But before proceeding it is important
to understand that the two approaches
are not exactly equivalent.

Consider a planar wavefront passing
through a slow anomaly. Can this
anomaly be detected by a seismic
network located on the opposite side?

wavefront

negative anomaly   seismic network

negative anomaly

negative anomaly
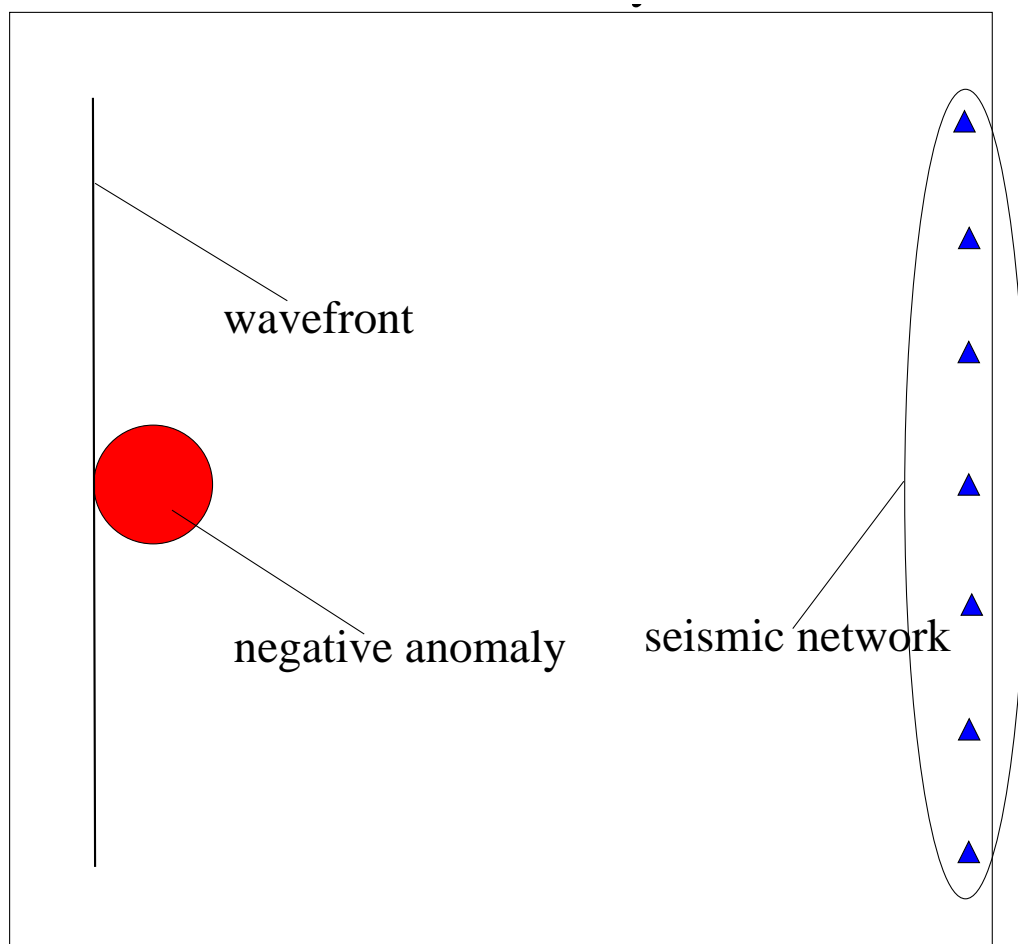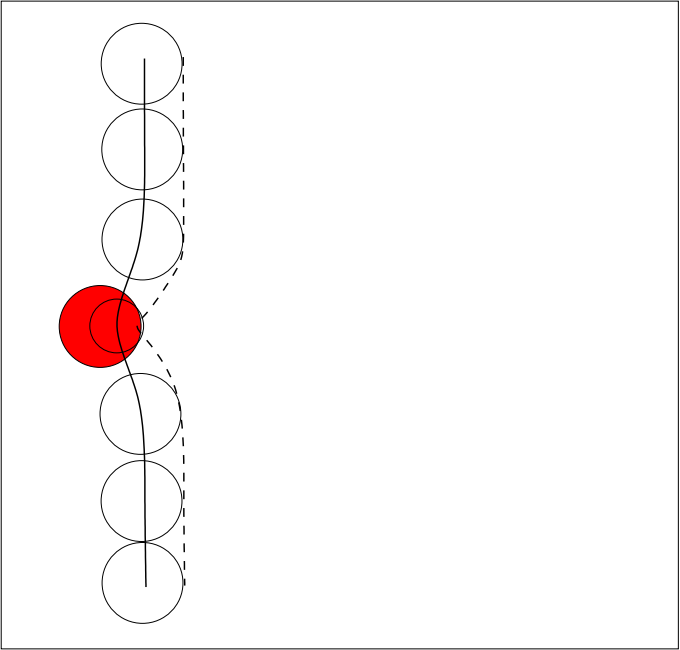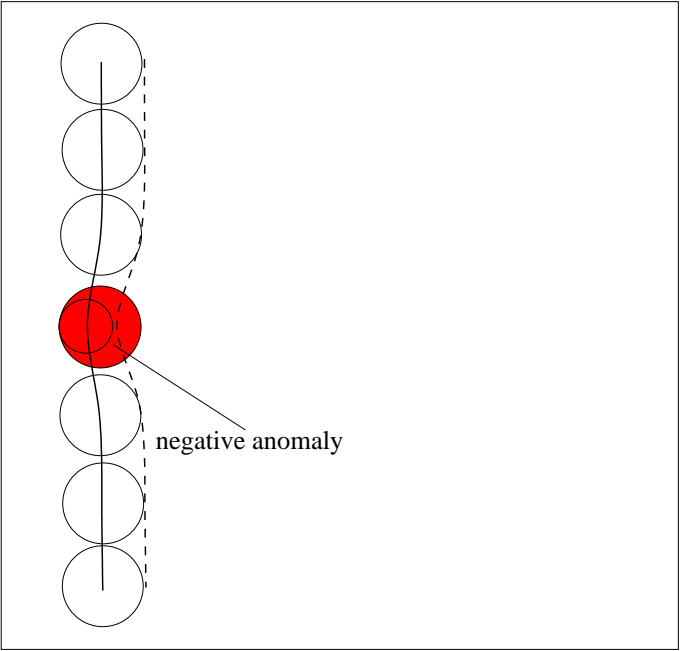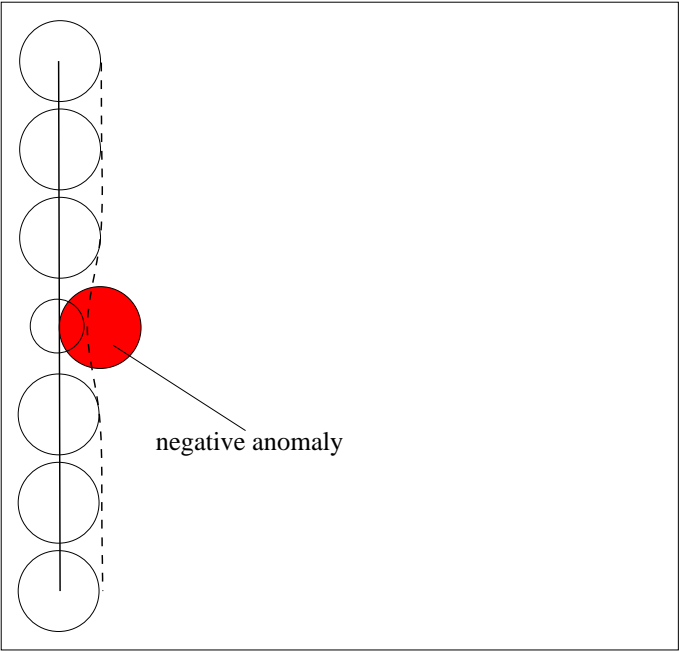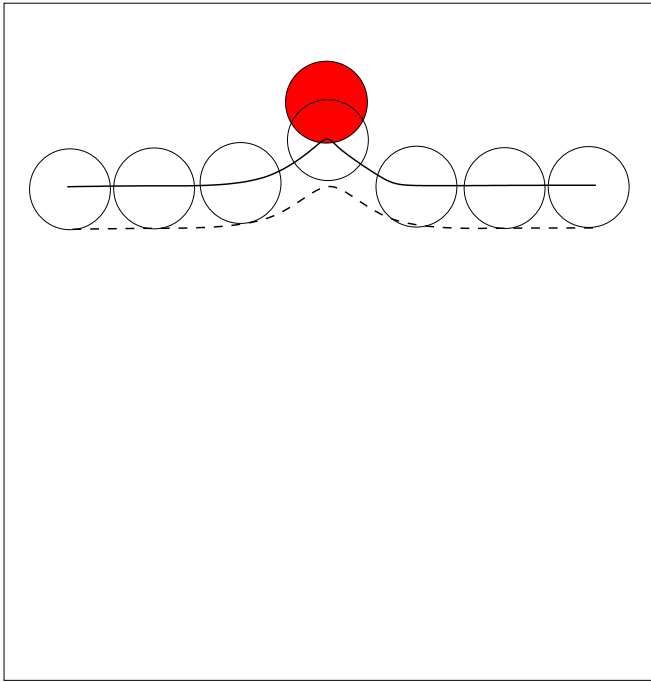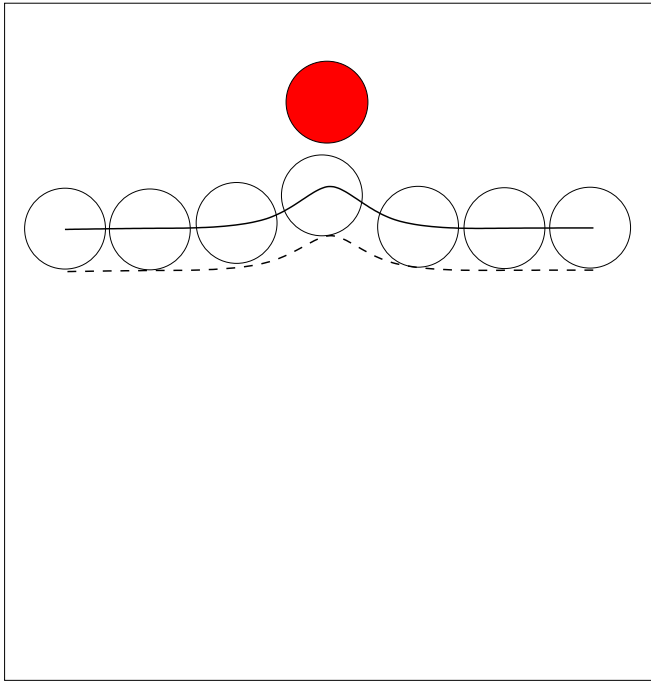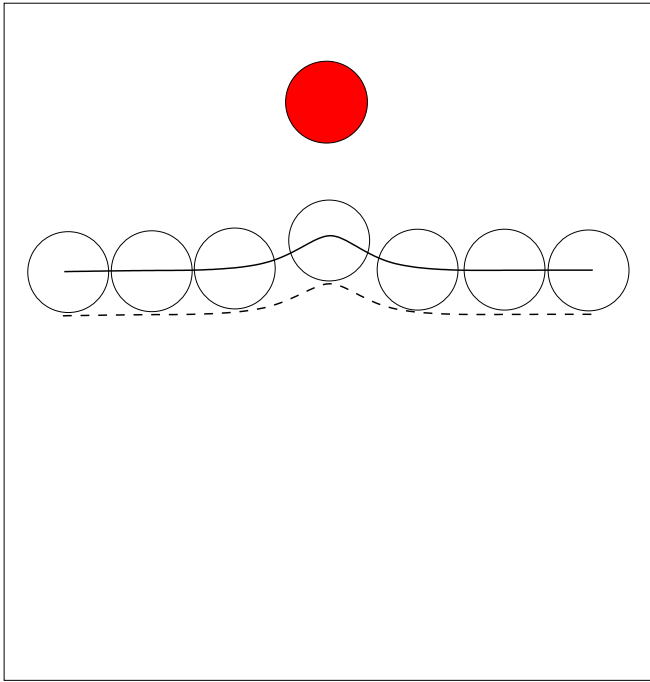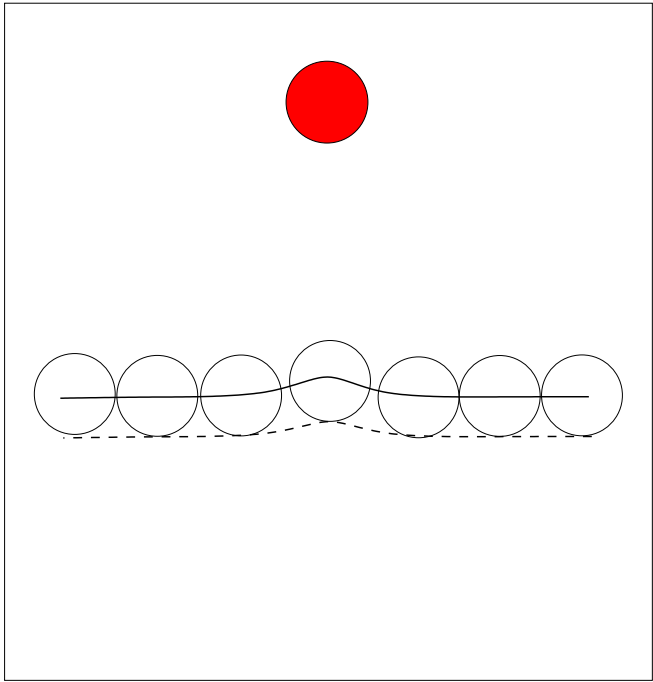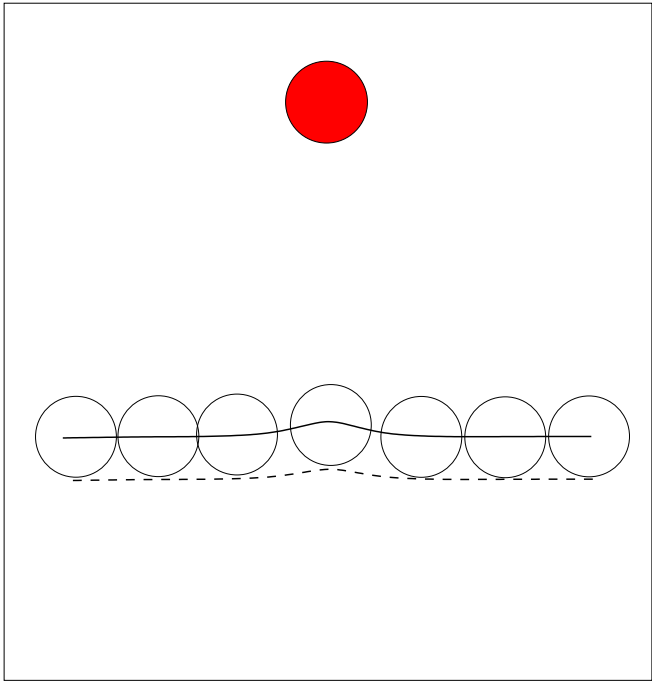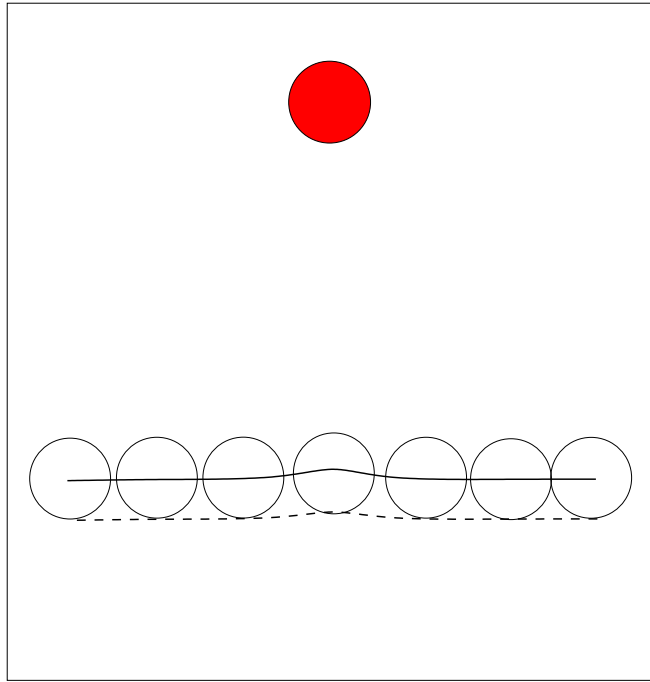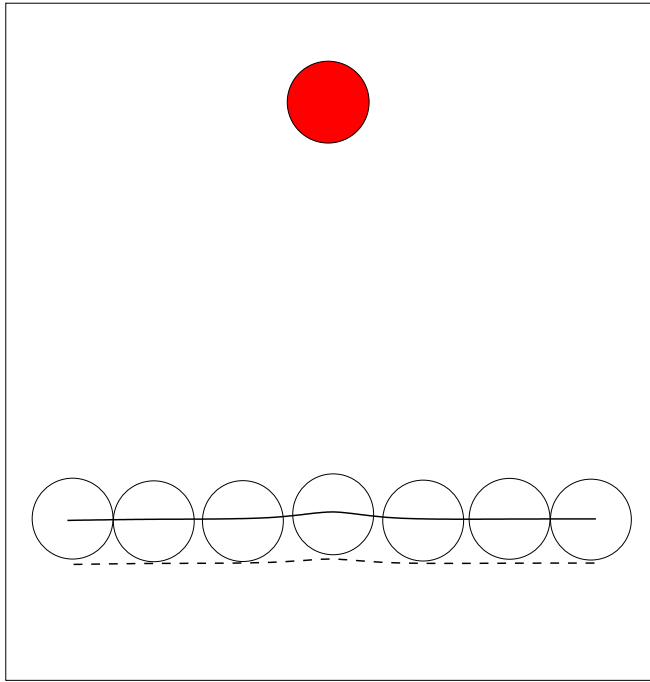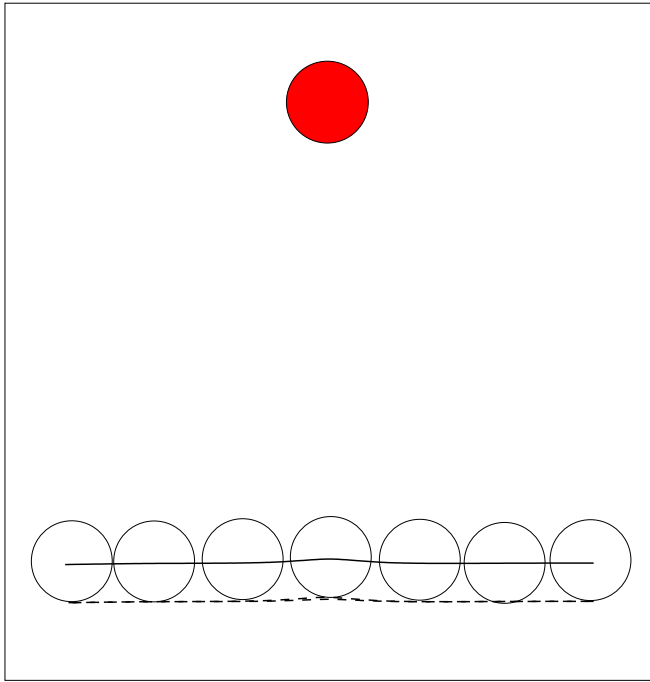
With increasing distance from the anomaly, the wavefronts undergo healing (show animation). This effect is often referred to as the Wavefront Healing.

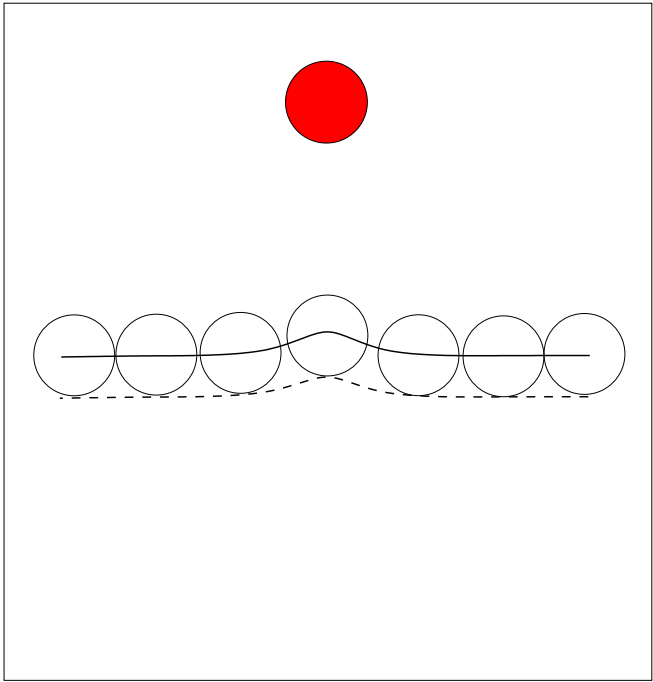On the other hand, according to the ray theory the travel time from point A to B is given by:

$$T_A^B = \int_A^B \frac{dS}{C(s)},$$

where dS is the distance measured along the ray, and C is the seismic velocity.

Thus, a ray traveling through a slow anomaly will arrive after a ray traveling through the rest of the medium.

Just like in optics

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The angle of reflection equals the angle of incidence, and the angle of refraction is related through the velocity ratio:

$$\frac{\sin i_{\text{air}}^{\text{incoming}}}{V_{\text{air}}} = \frac{\sin i_{\text{air}}^{\text{reflected}}}{V_{\text{air}}} = \frac{\sin i_{\text{glass}}^{\text{refracted}}}{V_{\text{glass}}}$$

Seismic rays too obey Snell's law. But conversions from P to S and vice versa can also occur.

Phase conversions

Consider a down-going P-wave arriving to an interface, part of its energy is reflected, part of it is transmitted to the other side, and part of the reflected and transmitted energies are converted into S_v-wave.
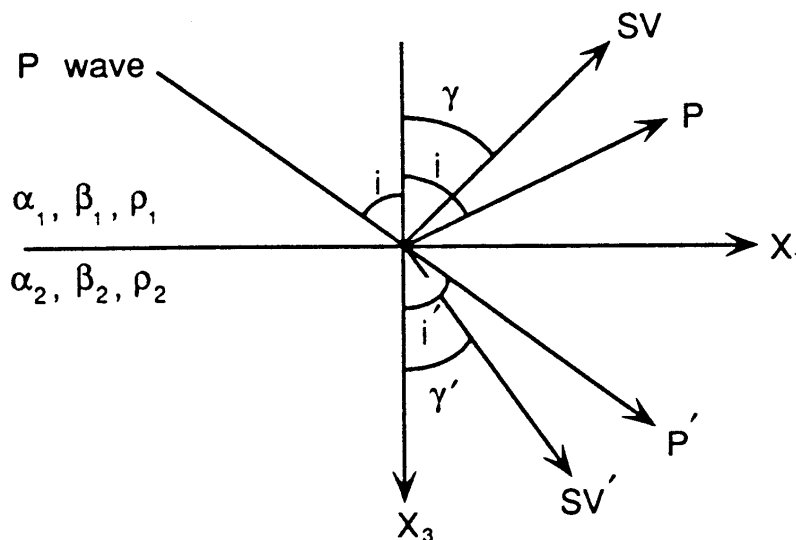
The incidence angle of the reflected and transmitted waves are controlled by an extended form of the Snell's law:

$$\frac{\sin i}{\alpha_1} = \frac{\sin \gamma}{\beta_1} = \frac{\sin i^{'}}{\alpha_2} = \frac{\sin \gamma^{'}}{\beta_2} \equiv P$$

The ray parameter and the horizontal slowness

The ray parameter, P, is constant along the ray, and is the same for all rays (reflected, refracted and converted) originated from the same incoming ray.

Consider a plane wave that propagates in the k direction. The apparent velocity c_1, measured at the surface is larger than the actual velocity, c.
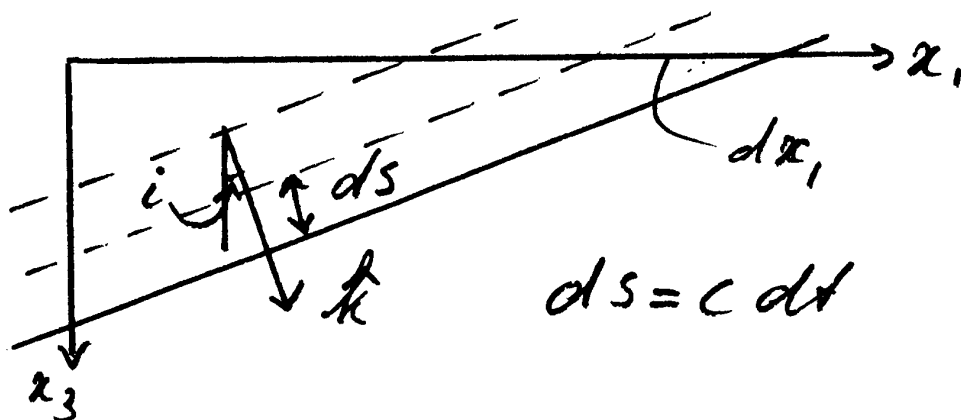
$$c_1 = \frac{c}{\sin i} > c$$

$$\sin i = \frac{ds}{dx_1} = \frac{cdt}{dx_1} = \frac{c}{c_1} \Rightarrow$$

$$P \equiv \frac{\sin i}{c} = \frac{1}{c_1}$$

Thus, the ray parameter may be thought as the horizontal slowness.

Snell's law for radial earth

The radial earth ray parameter is given by:

$$P \equiv \frac{R \sin i}{V}$$

Next we present a geometrical proof showing that P is constant along the ray.

A geometrical construction showing that $R\sin i/V$ is constant along the ray.
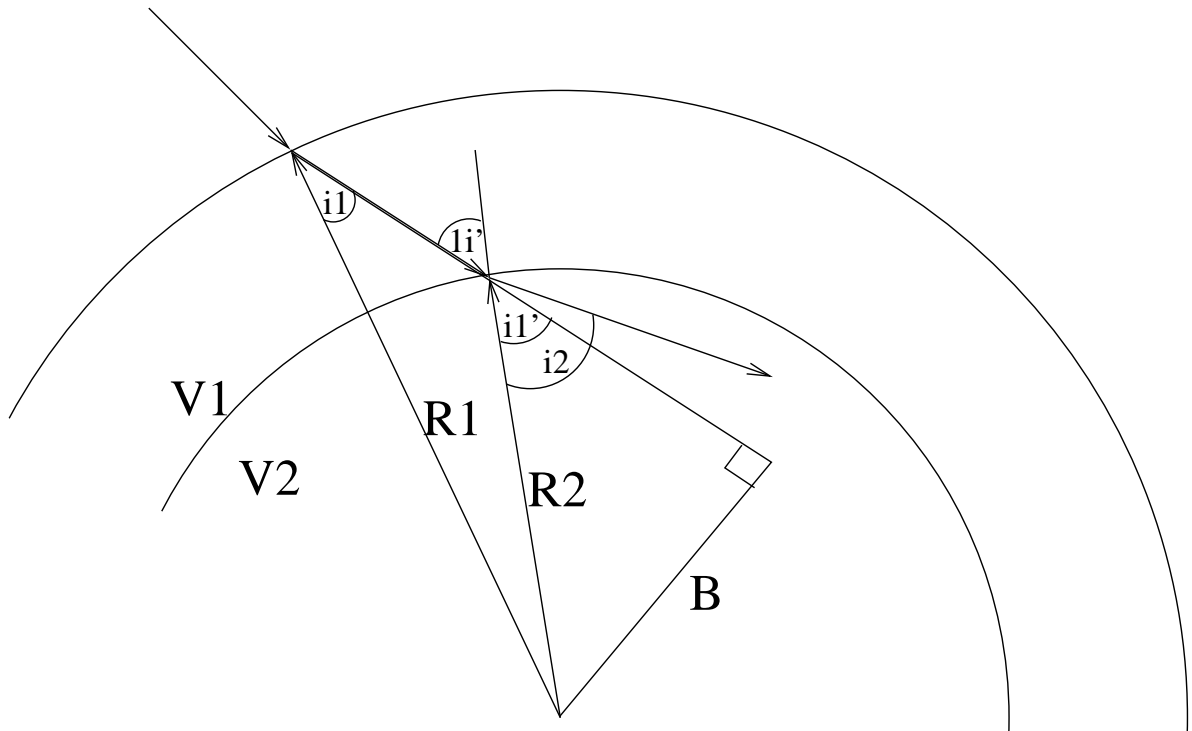
From the two triangles:

$$B = R_2 \sin i_1' = R_1 \sin i_1$$

From Snell's law across a plane boundary:

$$\frac{\sin i_1'}{\sin i_2} = \frac{V_1}{V_2}$$

$$\Rightarrow \frac{R\sin i}{V} = \text{constant} = \text{ray parameter}$$

How can P be measured?
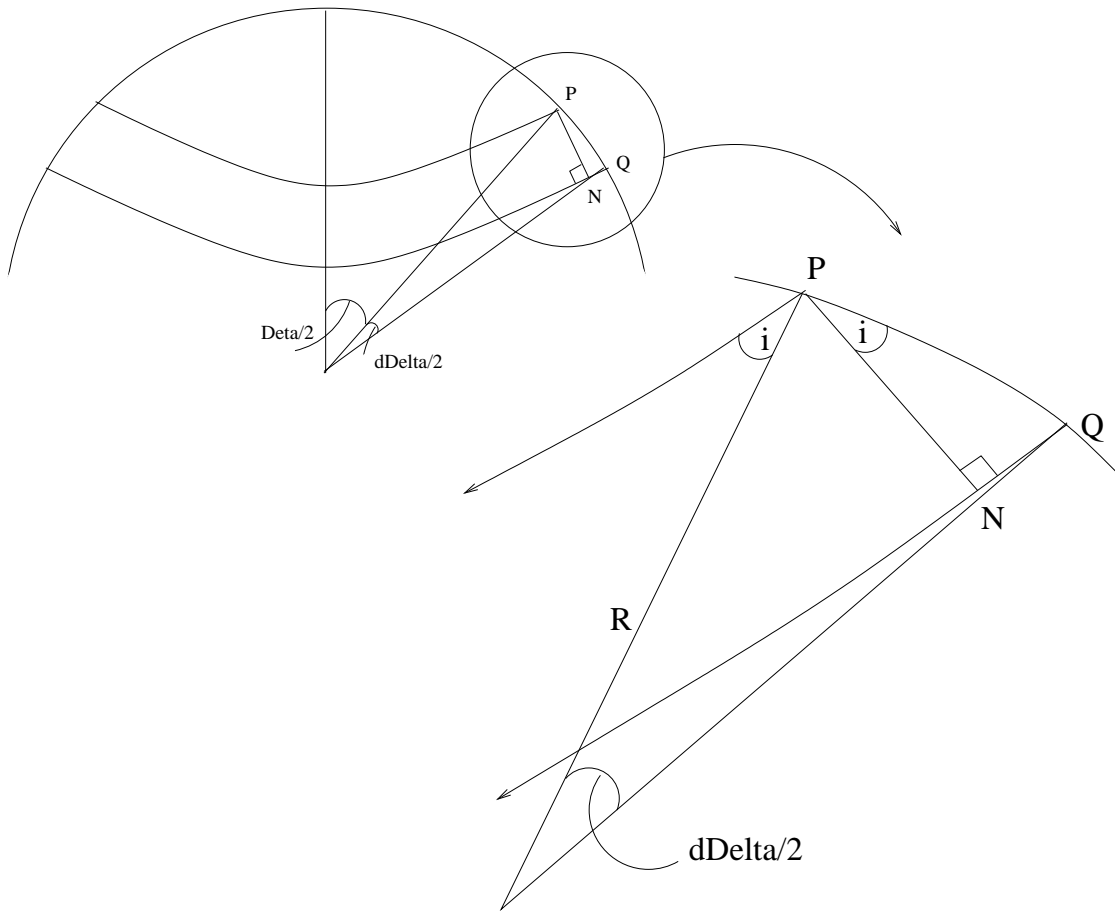
$$\sin i = \frac{QN}{QP} = \frac{V\,dT/2}{R\,d\Delta/2} \Rightarrow$$

$$\frac{dT}{d\Delta} = \frac{R\sin i}{V} = P$$

So P is the slope of the travel time curve (T-versus-Δ). While the units of the flat earth ray parameter is s/m, that of the radial earth is s/rad.

The bottoming point

With this definition for the ray parameter in a spherical earth we can get a simple expression that relates P to the minimum radius along the ray path. This point is known as the turning point or the bottoming point.
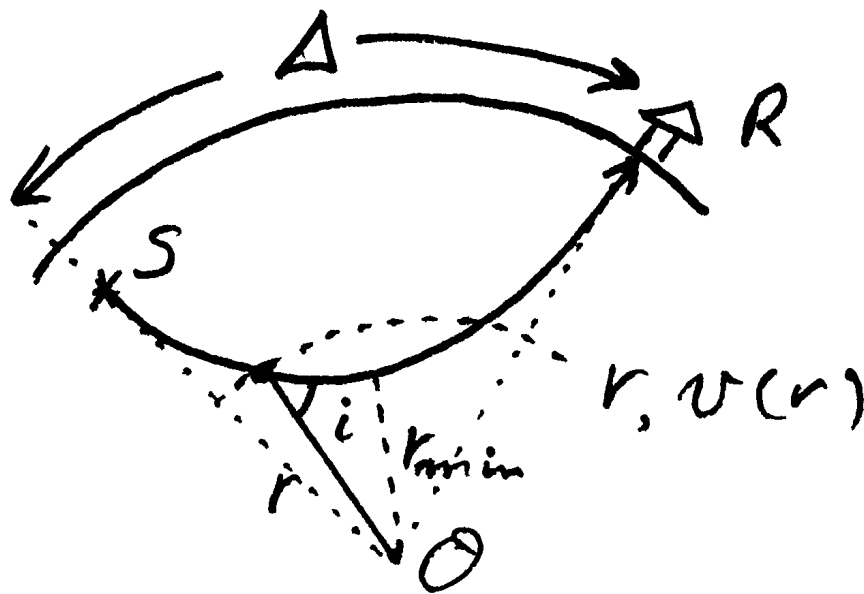
$$\frac{R_{min} \sin 90}{V(R_{min})} = \frac{R_{min}}{V(R_{min})} = P$$

Travel time curves

The ray parameter of a seismic wave arriving at a certain distance can thus be determined from the slope of the travel time curve.
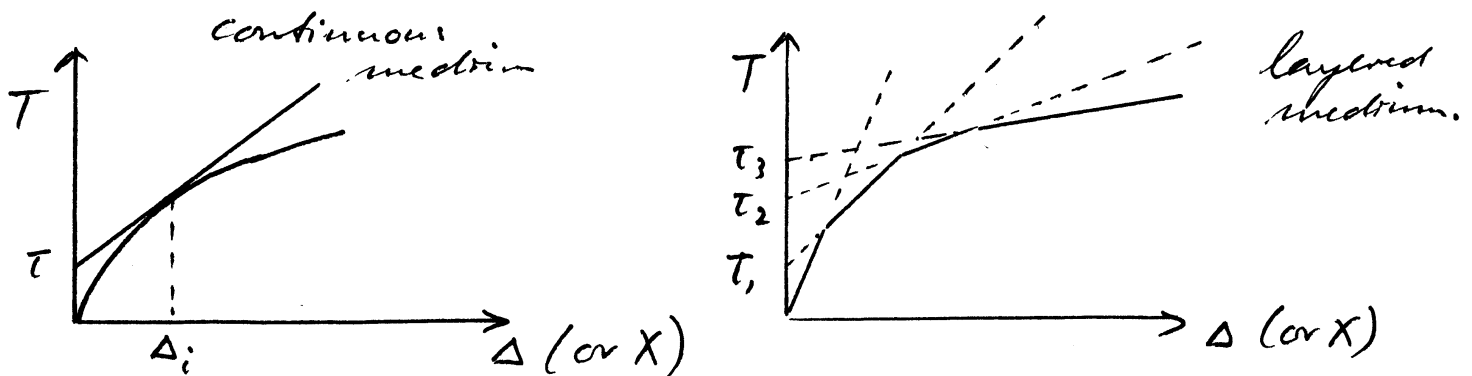

The straight line tangent to the travel time curve at Δ can be written as a function of the intercept time τ and the slope P.

$$P = \frac{dT}{d\Delta} \Rightarrow T(\Delta) = \tau + \frac{dT}{d\Delta}\Delta = \tau + P\Delta.$$

This equation forms the basis of what is known as the τ - P method.

P, the local slope of the travel time curve, contains information about the horizontal slowness, and the intercept time τ, contains information about the layer thickness.

Additional important information comes from the amplitude of the reflected and refracted waves. This and additional aspects of travel time curves will be discussed next week.

Reflection and refraction from a horizontal velocity contrast

Consider a seismic wave generated near the surface and recorded by a seismic station at some distance.
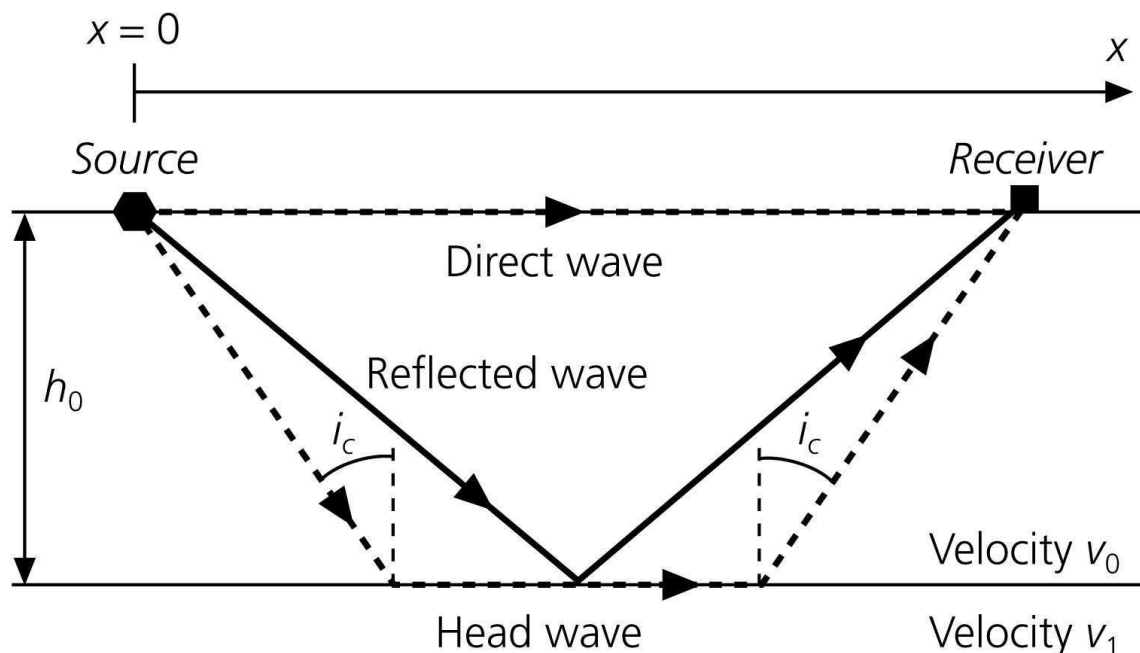
In the simple case of a 2 layer medium, the following arrivals are expected:

the arrival of the direct wave

the arrival of the reflected wave

the arrival of the refracted wave

Next we develop the equations describing the travel time of each ray.

○ The travel time of the direct wave is simply the horizontal distance divided by the seismic velocity of the top layer.

$$t = \frac{X}{V_0}$$

This is a surface wave!!!
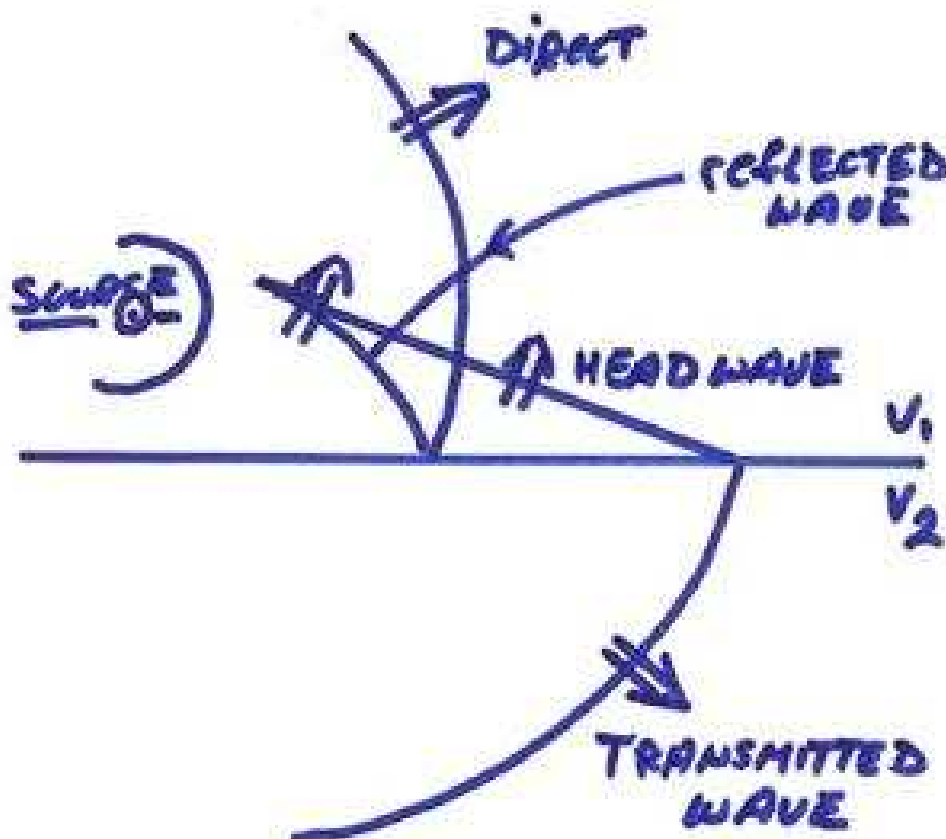
○ The travel time of the reflected wave is given by:

$$t = \frac{2}{V_0}\sqrt{h_0^2 + \left(\frac{X}{2}\right)^2} \Rightarrow$$

$$t^2 = \left(\frac{2h_0}{V_0}\right)^2 + \left(\frac{X}{V_0}\right)^2$$

So the travel time curve of this ray is a hyperbola!!!

○ The refracted wave traveling along the interface between the upper and the lower layer is a special case of Snell's law, for which the refraction angle equals 90 deg. We can write:

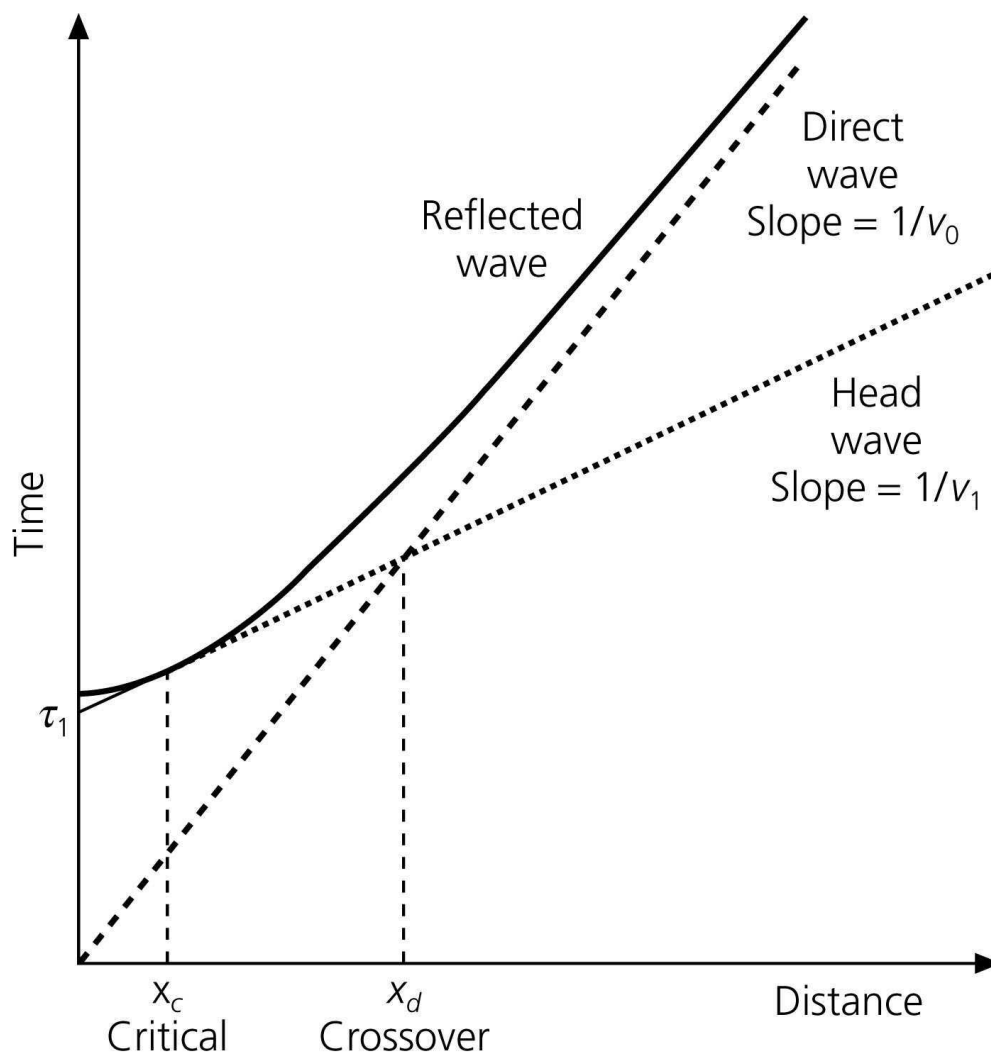$$\frac{\sin i_c}{V_0} = \frac{\sin 90}{V_1} \Rightarrow \sin i_c = \frac{V_0}{V_1}, \qquad (1)$$

where i_c is the critical angle. The refracted ray that is returned to the surface is a head wave.

$$t = \frac{2h_0}{V_0 \cos i_c} + \frac{X - 2h_0 \tan i_c}{V_1} =$$

$$\frac{2h_0 \sqrt{V_1^2 - V_0^2}}{V_0 V_1} + \frac{X}{V_1}$$

So this is an equation of a straight line with a slope of $1/V_1$, and the intercept is a function of the layer thickness and the velocities above and below the interface.

Refracted waves start arriving after a critical distance X_{crit}, but they overtake the direct waves at a crossover distance X_{co}.

The critical distance is:

$$X_{\text{crit}} = 2h_0 \tan i_c$$

The crossover distance is:

$$\frac{X_{\text{co}}}{V_0} = \frac{X_{\text{co}}}{V_1} + \frac{2h_0\sqrt{V_1^2 - V_0^2}}{V_1 V_0} \Rightarrow$$

$$X_{\text{co}} = 2h_0 \frac{\sqrt{V_1 + V_0}}{\sqrt{V_1 - V_0}}$$

Note that at distances greater than X_{co} the refracted waves arrive before the direct waves even though they travel a greater distance. Why?

Reflection in a multilayerd medium

For a single layer we found:

$$T^2 = T_0^2 + \left(\frac{X}{V_0}\right)^2,$$

where: $T_0 = 2h_0/V_0$.

Similarly, for a multilayerd medium:

$$T_n^2 = T_{0,n}^2 + \left(\frac{X}{V_{rms,n}}\right)^2,$$
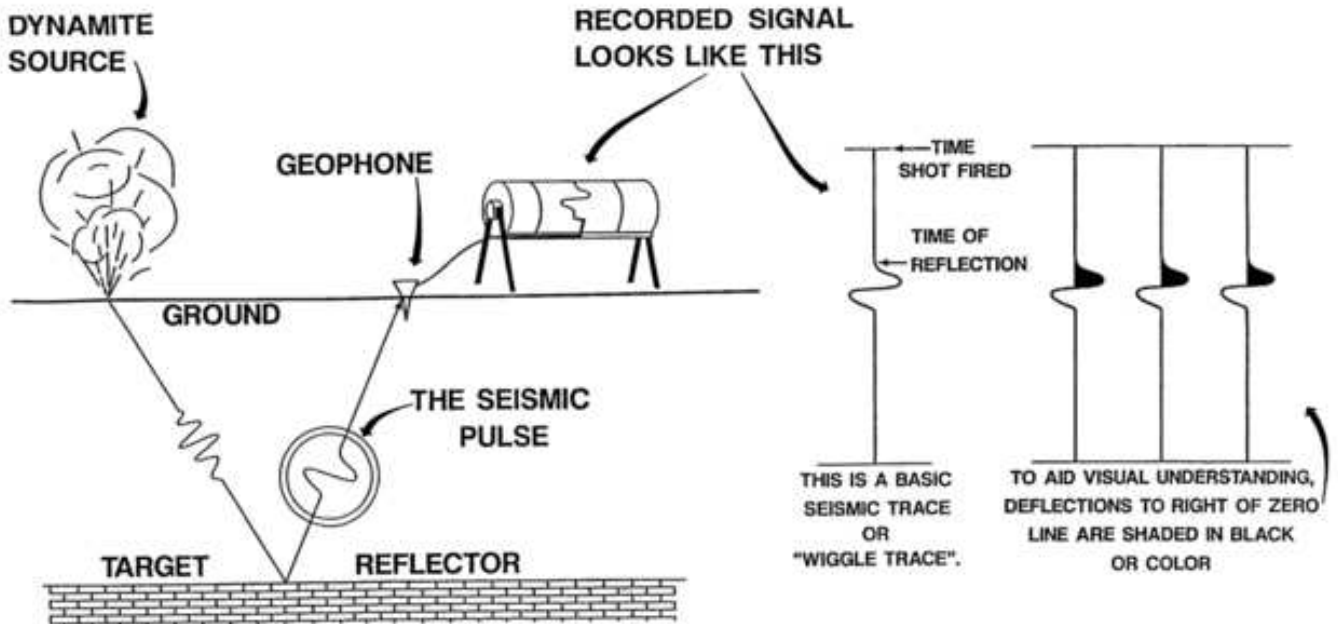
where:

$$T_{0,n} = \sum_n \frac{2h_n}{V_n},$$

and:

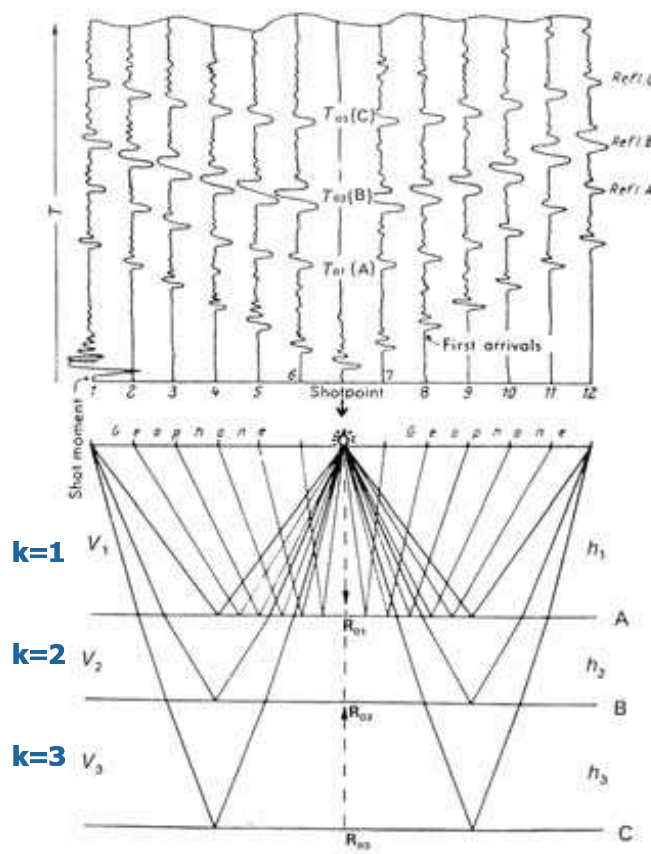$$V_{rms,n}^2 = \frac{\sum_n V_n^2 \frac{2h_n}{V_n}}{\sum_n \frac{2h_n}{V_n}}$$

On a T^2-versus-X^2 plot, the reflectors appear as straight lines with slopes that are inversely proportional to $V_{rms,n}^2$.

So how do we do it?

Data acquisition:

Next, the traces from several geophones are gathered:

And here is a piece of a real record: