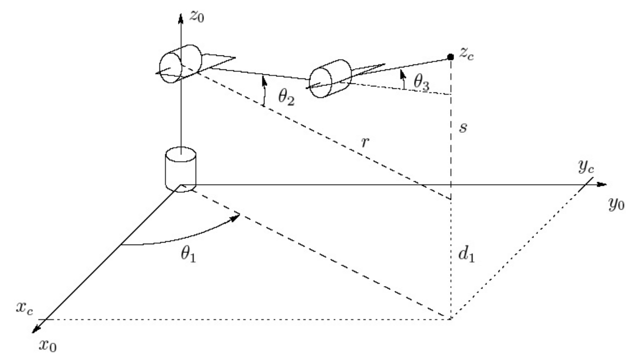
UNIVERSITY of HOUSTON | ECE

ECE 5397/6397: Intro to Robotics

HW 4, Due March 3 The manipulator Jacobian

1. Calculate the manipulator Jacobian of the anthropomorphic manipulator at the position z_c.
   a. Write out the J matrix in terms of z_i and o_i.
   b. Write out the z_i and o_i values.
   c. Write out the J values. Calculate the cross products. You may use your previous calculations for the A and T matrices.



$$T_1^0 = \begin{bmatrix} c_1 & 0 & s_1 & 0 \\ s_1 & 0 & -c_1 & 0 \\ 0 & 1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_2^0 = \begin{bmatrix} c_1c_2 & -c_1s_2 & s_1 & r_2c_1c_2 \\ c_2s_1 & -s_1s_2 & -c_1 & r_2c_2s_1 \\ s_2 & c_2 & 0 & d_1 + r_2s_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_3^0 = \begin{bmatrix} c_1c_{23} & -c_1s_{23} & s_1 & c_1(r_2c_2 + r_3c_{23}) \\ c_{23}s_1 & -s_1s_{23} & -c_1 & (r_2c_2 + r_3c_{23})s_1 \\ s_{23} & c_{23} & 0 & d_1 + r_2s_2 + r_3s_{23} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$J = \begin{bmatrix} z_0 \times (o_c - o_0) & z_1 \times (o_c - o_1) & z_2 \times (o_c - o_2) \\ z_0 & z_1 & z_2 \end{bmatrix}$$

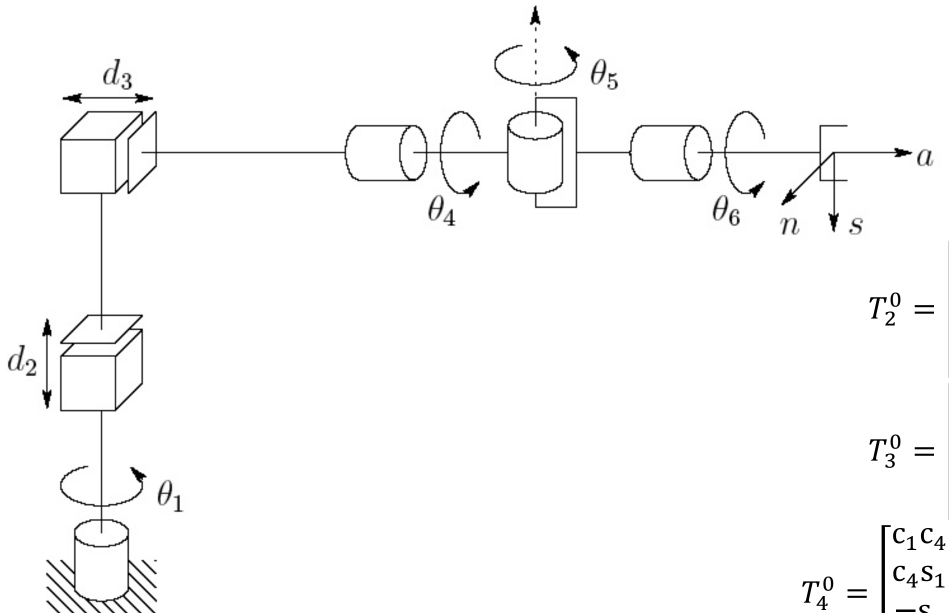
$$z_0 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, z_1 = \begin{bmatrix} s_1 \\ -c_1 \\ 0 \end{bmatrix}, z_2 = \begin{bmatrix} s_1 \\ -c_1 \\ 0 \end{bmatrix}, o_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, o_1 = \begin{bmatrix} 0 \\ 0 \\ d_1 \end{bmatrix}, o_2 = \begin{bmatrix} r_2c_1c_2 \\ r_2c_2s_1 \\ d_1 + r_2s_2 \end{bmatrix},$$

$$o_c = \begin{bmatrix} c_1(r_2c_2 + r_3c_{23}) \\ (r_2c_2 + r_3c_{23})s_1 \\ d_1 + r_2s_2 + r_3s_{23} \end{bmatrix}$$

$$J = \begin{bmatrix} -(c_2r_2 + c_{23}r_3)s_1 & -r_2s_{12} - r_3s_{123} & -r_3s_{123} \\ c_1(c_2r_2 + c_{23}r_3) & r_2c_{12} + r_3c_{123} & r_3c_{123} \\ 0 & r_2c_2 + r_3c_{23} & r_3c_{23} \\ 0 & s_1 & s_1 \\ 0 & -c_1 & -c_1 \\ 1 & 0 & 0 \end{bmatrix}$$

2. Calculate the manipulator Jacobian of the cylindrical robot with spherical wrist manipulator at the position z_6.

   a. Write out the J matrix in terms of z_i and o_i.
   b. Write out the z_i and o_i values.
   c. Calculate the cross products. You may use your previous calculations for the A and T matrices.



$$T_1^0 = \begin{bmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_2^0 = \begin{bmatrix} c_1 & 0 & -s_1 & 0 \\ s_1 & 0 & c_1 & 0 \\ 0 & -1 & 0 & d_1 + q_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_3^0 = \begin{bmatrix} c_1 & 0 & -s_1 & -q_3 s_1 \\ s_1 & 0 & c_1 & q_3 c_1 \\ 0 & -1 & 0 & d_1 + q_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_4^0 = \begin{bmatrix} c_1 c_4 & s_1 & -c_1 s_4 & -q_3 s_1 \\ c_4 s_1 & -c_1 & -s_1 s_4 & q_3 c_1 \\ -s_4 & 0 & -c_4 & d_1 + q_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_5^0 = \begin{bmatrix} c_1 c_4 c_5 + s_1 s_5 & -c_1 s_4 & -c_5 s_1 + c_1 c_4 s_5 & -q_3 s_1 \\ c_4 c_5 s_1 - c_1 s_5 & -s_1 s_4 & c_1 c_5 + c_4 s_1 s_5 & q_3 c_1 \\ -c_5 s_4 & -c_4 & -s_4 s_5 & d_1 + q_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_6^0 = \begin{bmatrix} c_6 s_1 s_5 + c_1(c_4 c_5 c_6 - s_4 s_6) & -s_1 s_5 s_6 - c_1(c_6 s_4 + c_4 c_5 s_6) & -c_5 s_1 + c_1 c_4 s_5 & -q_3 s_1 \\ c_4 c_5 c_6 s_1 - c_1 c_6 s_5 - s_1 s_4 s_6 & -c_6 s_1 s_4 + (-c_4 c_5 s_1 + c_1 s_5) s_6 & c_1 c_5 + c_4 s_1 s_5 & q_3 c_1 \\ -c_5 c_6 s_4 - c_4 s_6 & -c_4 c_6 + c_5 s_4 s_6 & -s_4 s_5 & d_1 + q_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$J = \begin{bmatrix} z_0 \times (o_c - o_0) & z_1 & z_2 & z_3 \times (o_c - o_3) & z_4 \times (o_c - o_4) & z_5 \times (o_c - o_5) \\ z_0 & 0 & 0 & z_3 & z_4 & z_5 \end{bmatrix}$$

$$z_0 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, z_1 = z_2 = z_3 = \begin{bmatrix} -s_1 \\ c_1 \\ 0 \end{bmatrix}, z_4 = \begin{bmatrix} -c_1 s_4 \\ -s_1 s_4 \\ -c_4 \end{bmatrix}, z_5 = \begin{bmatrix} -c_5 s_1 + c_1 c_4 s_5 \\ c_1 c_5 + c_4 s_1 s_5 \\ -s_4 s_5 \end{bmatrix},$$

$$o_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, o_1 = \begin{bmatrix} 0 \\ 0 \\ d_1 \end{bmatrix}, o_2 = \begin{bmatrix} 0 \\ 0 \\ d_1 + q_2 \end{bmatrix}, o_3 = o_4 = o_5 = o_c = \begin{bmatrix} -q_3 s_1 \\ q_3 c_1 \\ d_1 + q_2 \end{bmatrix}$$

$$J = \begin{bmatrix} -c_1 q_3 & -s_1 & -s_1 & 0 & 0 & 0 \\ -s_1 q_3 & c_1 & c_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -s_1 & -c_1 s_4 & -c_5 s_1 + c_1 c_4 s_5 \\ 0 & 0 & 0 & c_1 & -s_1 s_4 & c_1 c_5 + c_4 s_1 s_5 \\ 1 & 0 & 0 & 0 & -c_4 & -s_4 s_5 \end{bmatrix}$$

With non zero d_6

$$T_1^0 = \begin{bmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_2^0 = \begin{bmatrix} c_1 & 0 & -s_1 & 0 \\ s_1 & 0 & c_1 & 0 \\ 0 & -1 & 0 & d_1 + q_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_3^0 = \begin{bmatrix} c_1 & 0 & -s_1 & -q_3 s_1 \\ s_1 & 0 & c_1 & q_3 c_1 \\ 0 & -1 & 0 & d_1 + q_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_4^0 = \begin{bmatrix} c_1 c_4 & s_1 & -c_1 s_4 & -q_3 s_1 \\ c_4 s_1 & -c_1 & -s_1 s_4 & q_3 c_1 \\ -s_4 & 0 & -c_4 & d_1 + q_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_5^0 = \begin{bmatrix} c_1 c_4 c_5 + s_1 s_5 & -c_1 s_4 & -c_5 s_1 + c_1 c_4 s_5 & -q_3 s_1 \\ c_4 c_5 s_1 - c_1 s_5 & -s_1 s_4 & c_1 c_5 + c_4 s_1 s_5 & q_3 c_1 \\ -c_5 s_4 & -c_4 & -s_4 s_5 & d_1 + q_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_6^0$$

$$= \begin{bmatrix} c_6 s_1 s_5 + c_1(c_4 c_5 c_6 - s_4 s_6) & -s_1 s_5 s_6 - c_1(c_6 s_4 + c_4 c_5 s_6) & -c_5 s_1 + c_1 c_4 s_5 & -(q_3 + d_6 c_5) s_1 + d_6 c_1 c_4 s_5 \\ c_4 c_5 c_6 s_1 - c_1 c_6 s_5 - s_1 s_4 s_6 & -c_6 s_1 s_4 + (-c_4 c_5 s_1 + c_1 s_5) s_6 & c_1 c_5 + c_4 s_1 s_5 & c_1(q3 + d6 c_5) + d_6 c_4 s_1 s_5 \\ -c_5 c_6 s_4 - c_4 s_6 & -c_4 c_6 + c_5 s_4 s_6 & -s_4 s_5 & d_1 + q_2 - d_6 s_4 s_5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$J = \begin{bmatrix} z_0 \times (o_c - o_0) & z_1 & z_2 & z_3 \times (o_c - o_3) & z_4 \times (o_c - o_4) & z_5 \times (o_c - o_5) \\ z_0 & 0 & 0 & z_3 & z_4 & z_5 \end{bmatrix}$$

$$z_0 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, z_1 = z_2 = z_3 = \begin{bmatrix} -s_1 \\ c_1 \\ 0 \end{bmatrix}, z_4 = \begin{bmatrix} -c_1 s_4 \\ -s_1 s_4 \\ -c_4 \end{bmatrix}, z_5 = \begin{bmatrix} -c_5 s_1 + c_1 c_4 s_5 \\ c_1 c_5 + c_4 s_1 s_5 \\ -s_4 s_5 \end{bmatrix},$$

$$o_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, o_1 = \begin{bmatrix} 0 \\ 0 \\ d_1 \end{bmatrix}, o_2 = \begin{bmatrix} 0 \\ 0 \\ d_1 + q_2 \end{bmatrix}, o_3 = o_4 = o_5 = \begin{bmatrix} -q_3 s_1 \\ q_3 c_1 \\ d_1 + q_2 \end{bmatrix},$$

$$o_c = \begin{bmatrix} -(q_3 + d_6 c_5) s_1 + d_6 c_1 c_4 s_5 \\ c_1(q_3 + d_6 c_5) + d_6 c_4 s_1 s_5 \\ d_1 + q_2 - d_6 s_4 s_5 \end{bmatrix}$$

$$z_0 \times (o_c - o_0) = \begin{bmatrix} -c_1(q_3 + d_6 c_5) - d_6 c_4 s_1 s_5 \\ -(q_3 + d_6 c_5) s_1 + d_6 c_1 c_4 s_5 \\ 0 \end{bmatrix}$$

$$z_3 \times (o_c - o_3) = \begin{bmatrix} -d_6 c_1 s_4 s_5 \\ -d_6 s_1 s_4 s_5 \\ -d_6 c_4 s_5 \end{bmatrix}, z_4 \times (o_c - o_4) = \begin{bmatrix} d_6(c_1 c_4 c_5 + s_1 s_5) \\ d_6(c_4 c_5 s_1 - c_1 s_5) \\ -d_6 c_5 s_4 \end{bmatrix}, z_5 \times (o_c - o_5) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$J_{11} \begin{bmatrix} -q_3 c_1 - d_6 c_1 c_5 - d_6 c_4 s_1 s_5 & -s_1 & -s_1 & -d_6 c_1 s_4 s_5 & d_6 c_1 c_4 c_5 + d_6 c_4^2 s_1 s_5 + d_6 s_1 s_4^2 s_5 & 0 \\ -q_3 s_1 - d_6 c_5 s_1 + d_6 c_1 c_4 s_5 & c_1 & c_1 & -d_6 s_1 s_4 s_5 & d_6 c_4 c_5 s_1 - d_6 c_1 c_4^2 s_5 - d_6 c_1 s_4^2 s_5 & 0 \\ 0 & 0 & 0 & -d_6 c_1^2 c_4 s_5 - d_6 c_4 s_1^2 s_5 & -d_6 c_1^2 c_5 s_4 - d_6 c_5 s_1^2 s_4 & 0 \end{bmatrix}$$