1 Problem 1

1.1 $Y = AC + \overline{A}\,\overline{B}C$

$$\begin{aligned} Y &= AC + \overline{A}\,\overline{B}C \\ Y &= C(A + \overline{A}\,\overline{B}) \\ Y &= C(A + \overline{A}\,\overline{B}) \end{aligned}$$

Truth Table:

A	B	C	AC	$\overline{A}\,\overline{B}C$	Y
0	0	0	0	0	0
0	0	1	0	1	1
0	1	0	0	0	0
0	1	1	0	0	0
1	0	0	0	0	0
1	0	1	1	0	1
1	1	0	0	0	0
1	1	1	1	0	1

K-Map:

<table>
<tr><td></td><td colspan="5">AB</td></tr>
<tr><td rowspan="3">C</td><td></td><td>00</td><td>01</td><td>11</td><td>10</td></tr>
<tr><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td></tr>
<tr><td>1</td><td>1</td><td>0</td><td>1</td><td>1</td></tr>
</table>

1.2 $Y = \overline{A}\,\overline{B} + \overline{A}B\overline{C} + \overline{(A + \overline{C})}$

$$\begin{aligned} Y &= \overline{A}\,\overline{B} + \overline{A}B\overline{C} + \overline{(A + \overline{C})} \\ Y &= \overline{A}\,\overline{B} + \overline{A}B\overline{C} + \overline{A}C \\ Y &= \overline{A}(\overline{B} + B\overline{C} + C) \\ Y &= \overline{A}(\overline{B} + \overline{\overline{B\overline{C}}} + C) \\ Y &= \overline{A}(\overline{B} + \overline{\overline{B} + C} + C) \\ Y &= \overline{A}((\overline{B} + C) + (\overline{\overline{B} + C})) \\ Y &= \overline{A} \end{aligned}$$

Truth Table:

A	B	C	$\overline{A}\,\overline{B}$	$\overline{A}B\overline{C}$	$\overline{(A+\overline{C})}$	Y
0	0	0	1	0	0	1
0	0	1	1	0	1	1
0	1	0	0	1	0	1
0	1	1	0	0	1	1
1	0	0	0	0	0	0
1	0	1	0	0	0	0
1	1	0	0	0	0	0
1	1	1	0	0	0	0

K-Map:

		AB			
		00	01	11	10
C	0	1	1	0	0
	1	1	1	0	0

2 Problem 2

2.1 Part 1

A	B	C	Y	Minterm
0	0	0	1	$\overline{A}\,\overline{B}\,\overline{C}$
0	0	1	0	-
0	1	0	0	-
0	1	1	0	-
1	0	0	0	-
1	0	1	0	-
1	1	0	0	-
1	1	1	1	ABC

$Y = \overline{A}\,\overline{B}\,\overline{C} + ABC$

2.2 Part 2

A	B	C	Y	Minterm
0	0	0	1	$\overline{A}\,\overline{B}\,\overline{C}$
0	0	1	0	-
0	1	0	1	$\overline{A}B\overline{C}$
0	1	1	0	-
1	0	0	1	$A\overline{B}\,\overline{C}$
1	0	1	0	-
1	1	0	1	$AB\overline{C}$
1	1	1	0	-

$Y = \overline{A}\,\overline{B}\,\overline{C} + \overline{A}B\overline{C} + A\overline{B}\,\overline{C} + AB\overline{C}$

3 Problem 3

3.1 Part 1

Already minimal

3.1.1 Part 2

K-Map:

		AB			
		00	01	11	10
C	0	1	1	1	1
	1	0	0	0	0

$Y = \overline{C}$

4 Problem 4

f is true except when x_1, $\overline{x_2}$ and x_3 are all true (term 5), i.e. it is the negation of $x_1\overline{x_2}x_3$:

$$\begin{aligned} f(x_1, x_2, x_3) &= \overline{x_1\overline{x_2}\,x_3} \\ f(x_1, x_2, x_3) &= \overline{x_1} + x_2 + \overline{x_3} \end{aligned}$$

K-Map:

		x_1x_2			
		00	01	11	10
x_3	0	1	1	1	1
	1	1	1	1	0

$f(x_1, x_2, x_3) = \overline{x_1} + x_2 + \overline{x_3}$

5 Problem 5

5.1 Part 1

Sum of Products:

x_1	x_2	x_3	$f(x_1, x_2, x_3)$	Minterm
0	0	0	0	-
0	0	1	1	$\overline{x_1}\,\overline{x_2}x_3$
0	1	0	0	-
0	1	1	0	-
1	0	0	1	$x_1\overline{x_2}\,\overline{x_3}$
1	0	1	1	$x_1\overline{x_2}x_3$
1	1	0	1	$x_1x_2\overline{x_3}$
1	1	1	0	-

$Y = \overline{x_1}\,\overline{x_2}x_3 + x_1\overline{x_2}\,\overline{x_3} + x_1\overline{x_2}x_3 + x_1x_2\overline{x_3}$

K-Map:

	x_1x_2				
		00	01	11	10
x_3	0	0	0	1	1
	1	1	0	0	1

$Y = x_1\overline{x_2} + x_1\overline{x_3} + \overline{x_2}x_3$

Minimize:

$$
\begin{aligned}
Y &= \overline{x_1}\,\overline{x_2}x_3 + x_1\overline{x_2}\,\overline{x_3} + x_1\overline{x_2}x_3 + x_1x_2\overline{x_3} \\
Y &= \overline{x_1}\,\overline{x_2}x_3 + x_1\overline{x_2}(\overline{x_3} + x_3) + x_1x_2\overline{x_3} \\
Y &= \overline{x_1}\,\overline{x_2}x_3 + x_1\overline{x_2} + x_1x_2\overline{x_3} \\
Y &= x_1\overline{x_2} + \overline{x_1}\,\overline{x_2}x_3 + x_1x_2\overline{x_3} \\
Y &= x_1\overline{x_2} + \overline{x_1}\,\overline{x_2}x_3 + x_1x_2\overline{x_3}
\end{aligned}
$$

Product of sums:

x_1	x_2	x_3	$f(x_1,x_2,x_3)$	Maxterms
0	0	0	0	$x_1 + x_2 + x_3$
0	0	1	1	-
0	1	0	0	$x_1 + \overline{x_2} + x_3$
0	1	1	0	$x_1 + \overline{x_2} + \overline{x_3}$
1	0	0	1	-
1	0	1	1	-
1	1	0	1	-
1	1	1	0	$\overline{x_1} + \overline{x_2} + \overline{x_3}$

$Y = (x_1 + x_2 + x_3)(x_1 + \overline{x_2} + x_3)(x_1 + \overline{x_2} + \overline{x_3})(\overline{x_1} + \overline{x_2} + \overline{x_3})$

5.2 Part 2

Sum of Products:

x_1	x_2	x_3	$f(x_1,x_2,x_3)$	Minterm
0	0	0	1	$\overline{x_1}\,\overline{x_2}\,\overline{x_3}$
0	0	1	x	-
0	1	0	1	$\overline{x_1}x_2x_3$
0	1	1	1	$\overline{x_1}x_2x_3$
1	0	0	1	$x_1\overline{x_2}\,\overline{x_3}$
1	0	1	x	-
1	1	0	0	-
1	1	1	0	-

$Y = \overline{x_1}\,\overline{x_2}\,\overline{x_3} + \overline{x_1}x_2x_3 + \overline{x_1}x_2x_3 + x_1\overline{x_2}\,\overline{x_3}$

K-Map:

	x_1x_2				
		00	01	11	10
x_3	0	1	1	0	1
	1	x	1	0	x

$Y = \overline{x_1} + \overline{x_2}$

6 Problem 6

Truth Table:

A	B	C	D	Y
0	0	0	0	x
0	0	0	1	x
0	0	1	0	x
0	0	1	1	0
0	1	0	0	0
0	1	0	1	x
0	1	1	0	0
0	1	1	1	x
1	0	0	0	1
1	0	0	1	0
1	0	1	0	x
1	0	1	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	x
1	1	1	1	1

K-Map:

	AB				
		00	01	11	10
CD	00	x	0	1	1
	01	x	x	1	0
	11	0	x	1	1
	10	x	0	x	x

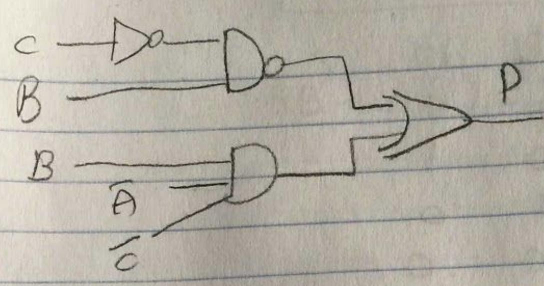
Minimal:

$Y = AB + AC + BD$

Problem 7

Problem 7

Circuit



Minimal-cost expression

$$P = \bar{A}B\bar{C} + \overline{(B\bar{C})} = \bar{A}B\bar{C} + \bar{B} + C$$

$$= \bar{A}\cancel{B}\cancel{\bar{C}} + C + \bar{B} = \bar{A} + \bar{B} + C$$

Truth table

A	B	C	P
0	0	0	1
0	0	1	1
0	1	0	1 ✓
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	1

K-map

A \ BC	00	01	11	10
0	1	1	1	1
1	1	1	1	0

$\Rightarrow C$

$\Rightarrow \bar{A}$

$\downarrow \bar{B}$

Problem 8

problem 8

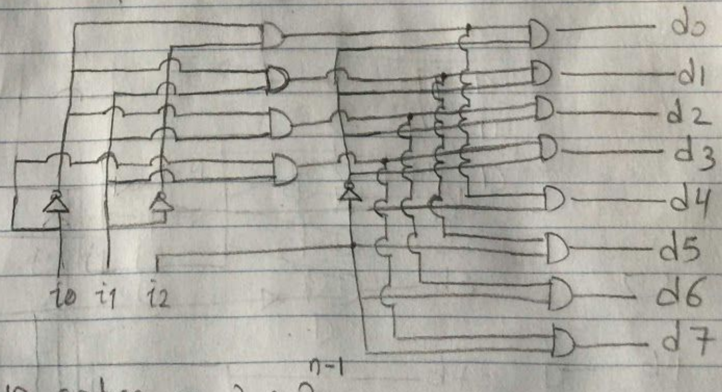
2-to-4 decoder truth table

i1	i0	d3	d2	d1	d0
0	0	0	0	0	1
0	1	0	0	1	0
1	0	0	1	0	0
1	1	1	0	0	0

3-to-8 decoder truth table

i2	i1	i0	d7	d6	d5	d4	d3	d2	d1	d0
0	0	0					0	0	0	1
0	0	1		0			0	0	1	0
0	1	0					0	1	0	0
0	1	1					1	0	0	0
1	0	0	0	0	0	1				
1	0	1	0	0	1	0				
1	1	0	0	1	0	0		0		
1	1	1	1	0	0	0				

Circuit of 3-to-8 decoder



Cost of n-to-2^n decoder

\# 2-i/p AND gates = $3 * 2^{n-1}$

\# NOT gates = n

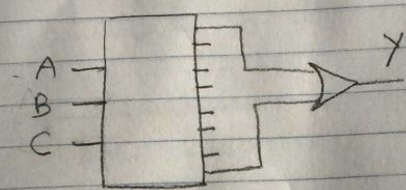
Problems 9 and 10

Problem 9

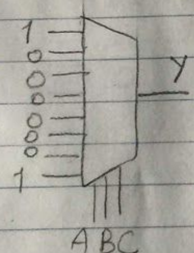
$$Y = \bar{A}\bar{B} + (\bar{A}B + A\bar{B})\bar{C}\bar{D}$$

Problem 10

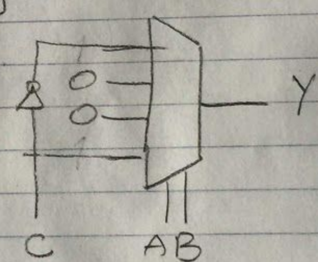
1



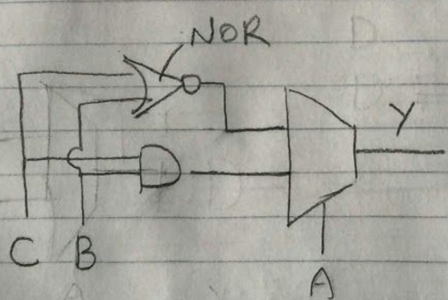
2



3



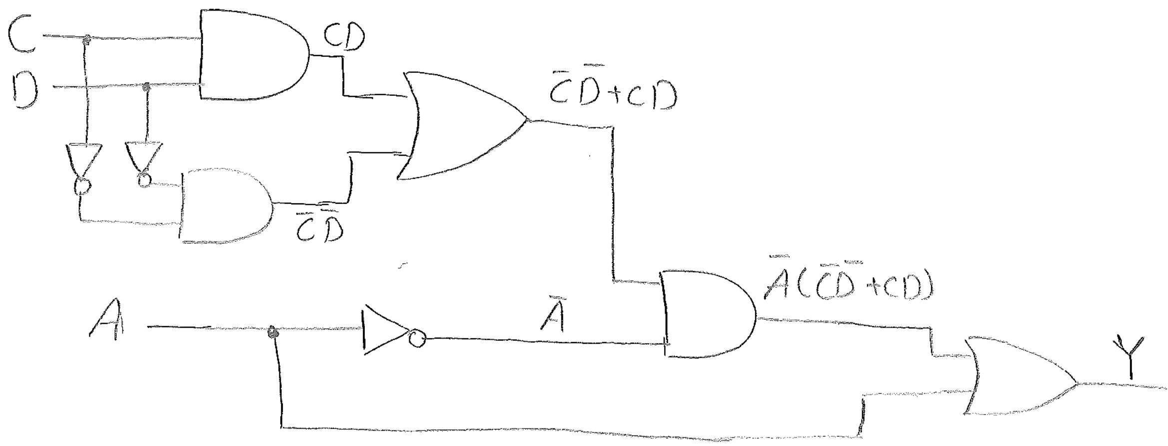
4



(Assuming NOR is okay)

Problem 11

$$Y = A + \bar{A}(\bar{C}\bar{D} + CD)$$



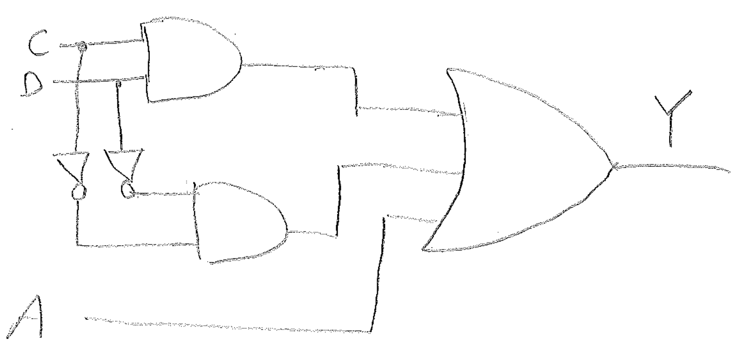
simplification: $a + \bar{a}B = a + B \Rightarrow Y = A + \bar{C}\bar{D} + CD$

or using K-map

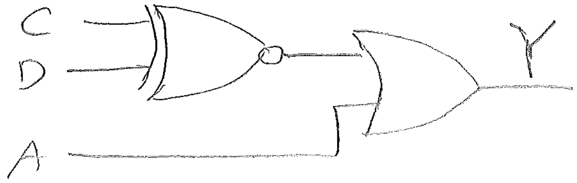
A \ CD	00	01	11	10
0	1		1	
1	1	1	1	1

$\Rightarrow A + \bar{C}\bar{D} + CD$

Circuit:

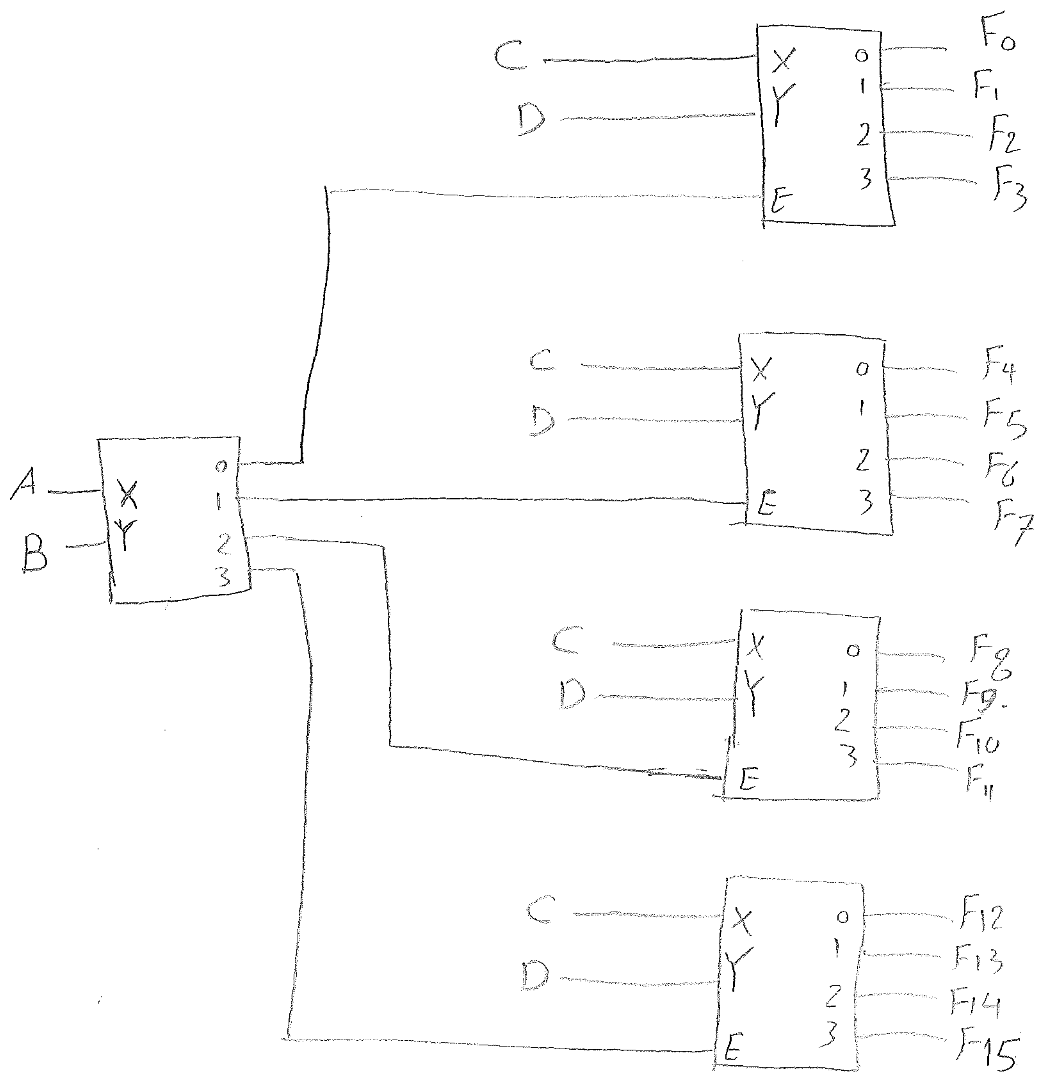


more simplification: $\bar{C}\bar{D} + CD = \overline{C \oplus D} \equiv XNOR$



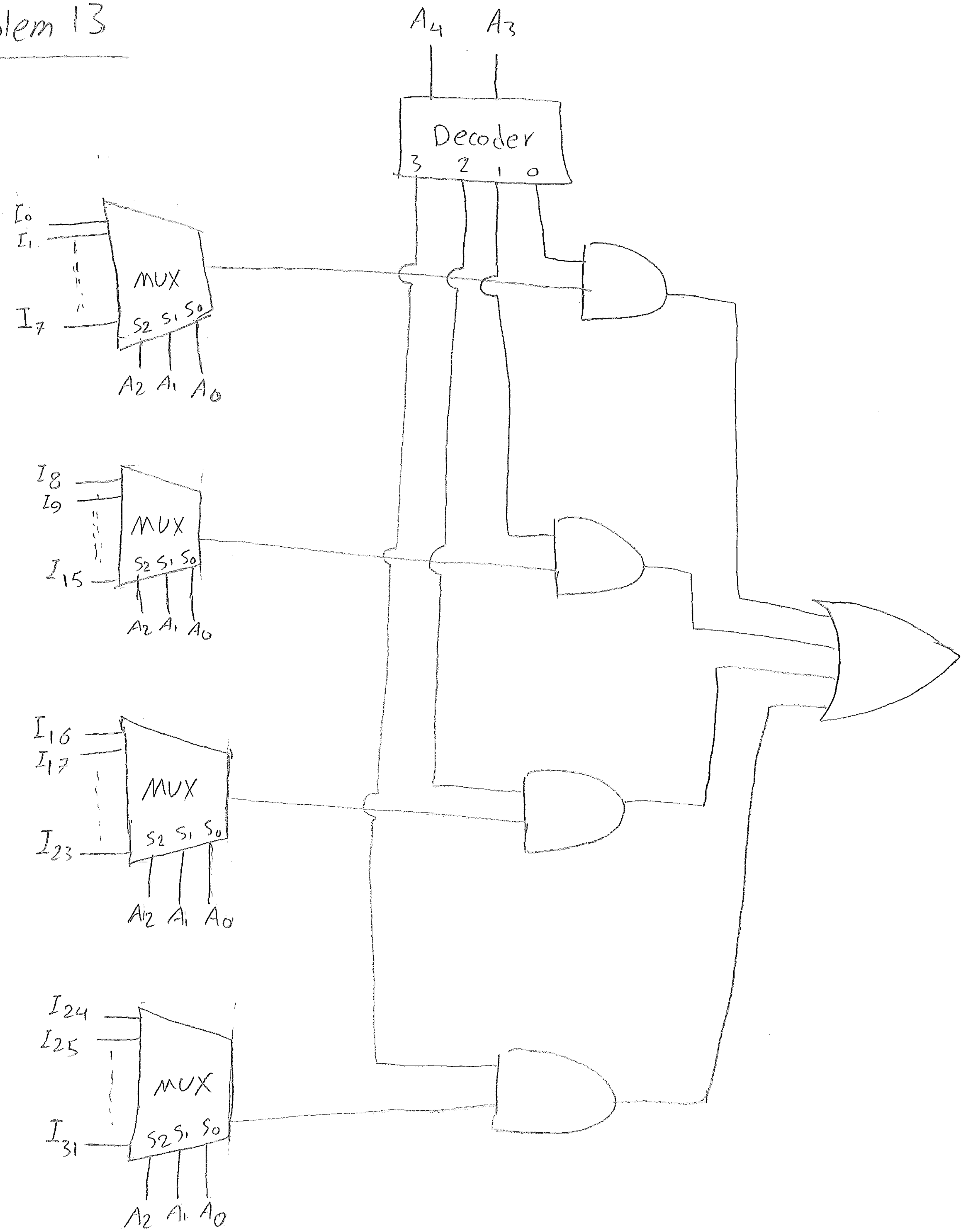
Problem 12 :

E = Enable of the circuit



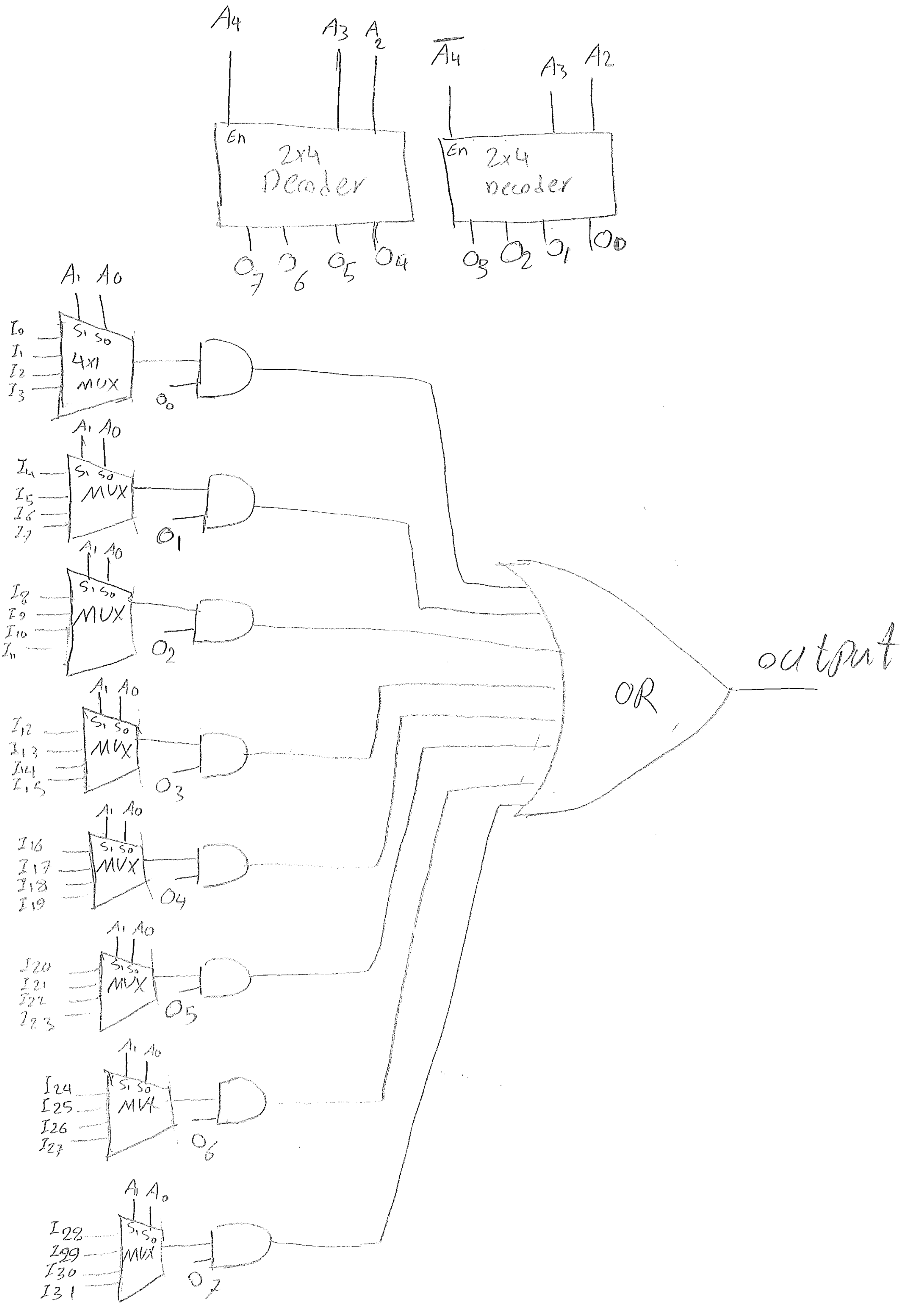
Problem 13

①



A_4
A_3
Decoder
3 2 1 0
I_0
I_1
I_7
MUX
S_2 S_1 S_0
A_2 A_1 A_0
I_8
I_9
I_{15}
MUX
S_2 S_1 S_0
A_2 A_1 A_0
I_{16}
I_{17}
I_{23}
MUX
S_2 S_1 S_0
A_2 A_1 A_0
I_{24}
I_{25}
I_{31}
MUX
S_2 S_1 S_0
A_2 A_1 A_0

2



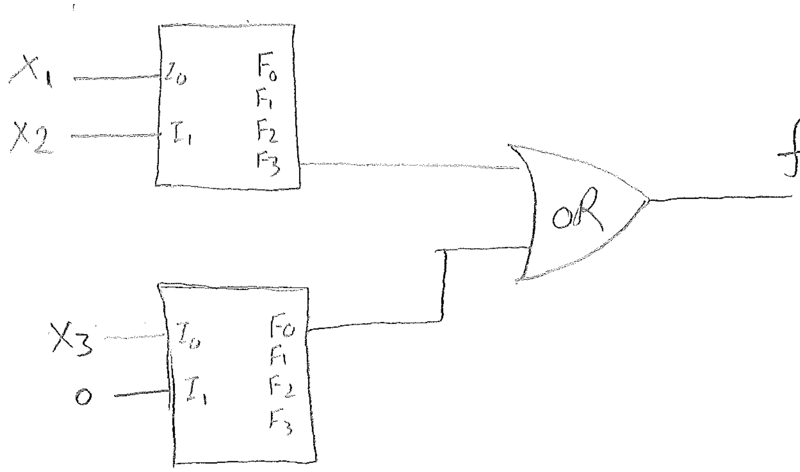
A4
A3
A2
En
2x4 Decoder
O7
O6
O5
O4
Ā4
A3
A2
En
2x4 Decoder
O3
O2
O1
O0
A1
A0
S1
S0
4x1 MUX
I0
I1
I2
I3
MUX
I4
I5
I6
I7
I8
I9
I10
I11
I12
I13
I14
I15
I16
I17
I18
I19
I20
I21
I22
I23
I24
I25
I26
I27
I28
I29
I30
I31
OR
output

Problem 14

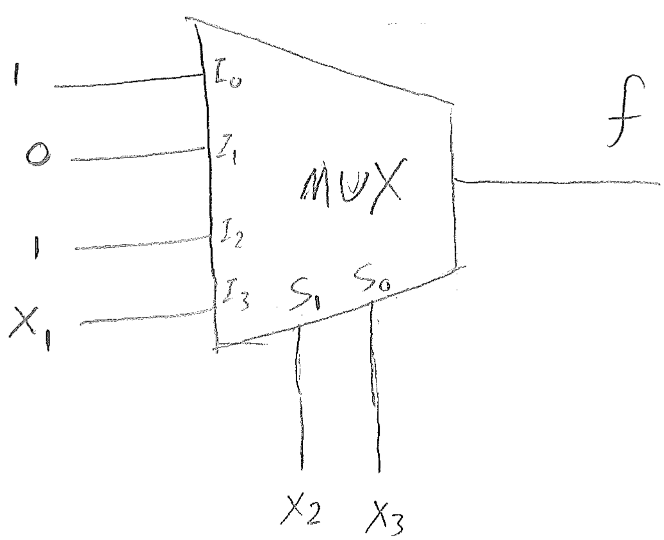
(1)

X_1 \ X_2X_3	00	01	11	10
0	1			1
1	1		1	1

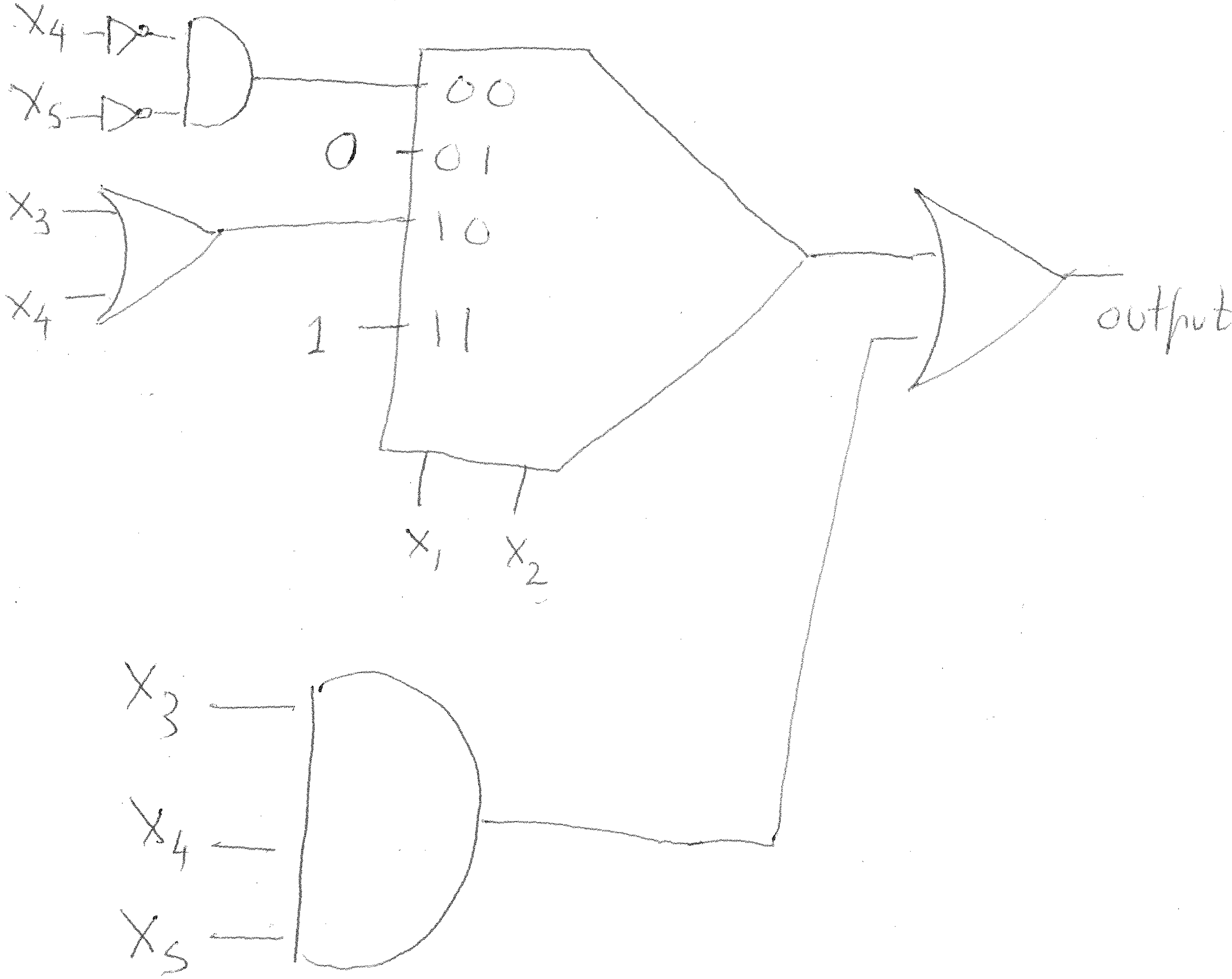
$$= \overline{X_3} + X_1 X_2$$



(2)



Problem 15

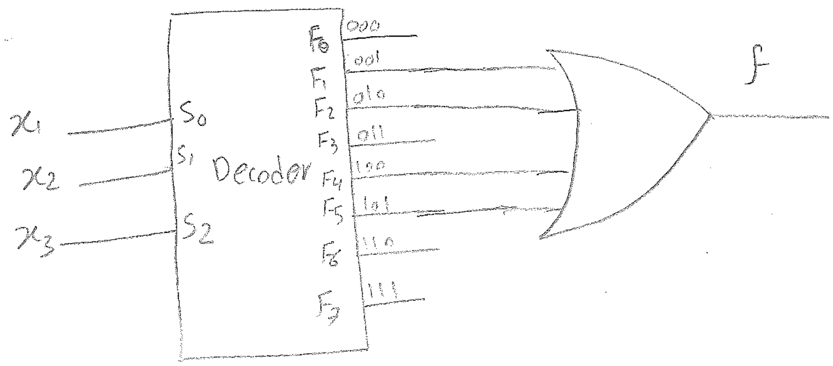


Problem 16:

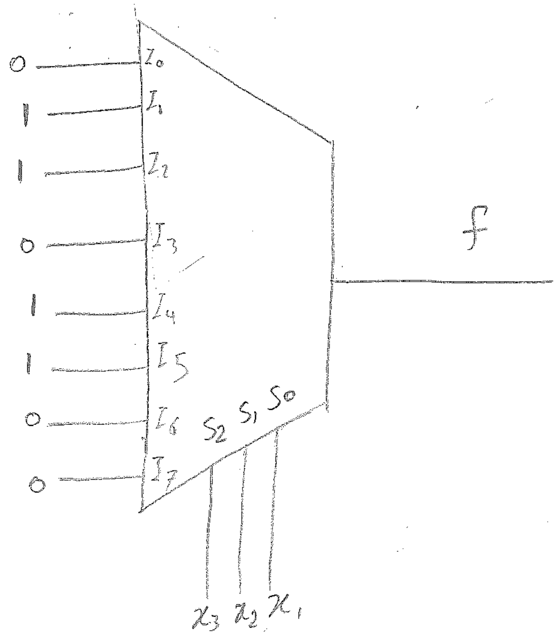
$$f = x_1\overline{x_2} + \overline{x_2}x_3 + \overline{x_1}\,x_2\,\overline{x_3}$$

(1)

$$f = x_1\overline{x_2}x_3 + x_1\overline{x_2}\,\overline{x_3} + x_1\overline{x_2}x_3 + \overline{x_1}\,\overline{x_2}x_3 + \overline{x_1}\,x_2\,\overline{x_3}$$

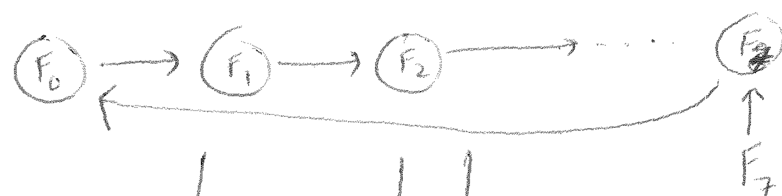


(2)



Problem 17

Need to maintain 3 bits. Say A B C. 8 states. $F_0, \ldots, F_7$ where F_0 is 000 and F_8 is 100 and rest are in gray sequence.



Use ABC to maintain state.

Table:

A	B	C	A_N	B_N	C_N
0	0	0	0	0	1
0	0	1	0	1	1
0	1	1	0	1	0
0	1	0	1	1	0
1	1	0	1	1	1
1	1	1	1	0	1
1	0	1	1	0	0
1	0	0	0	0	0

Expression for A_N

C \ AB	00	01	11	10
0		1	1	
1			1	1

$$A_N = B\bar{C} + AC$$

Exp for B_N

C \ AB	00	01	11	10
0		1	1	
1	1	1		

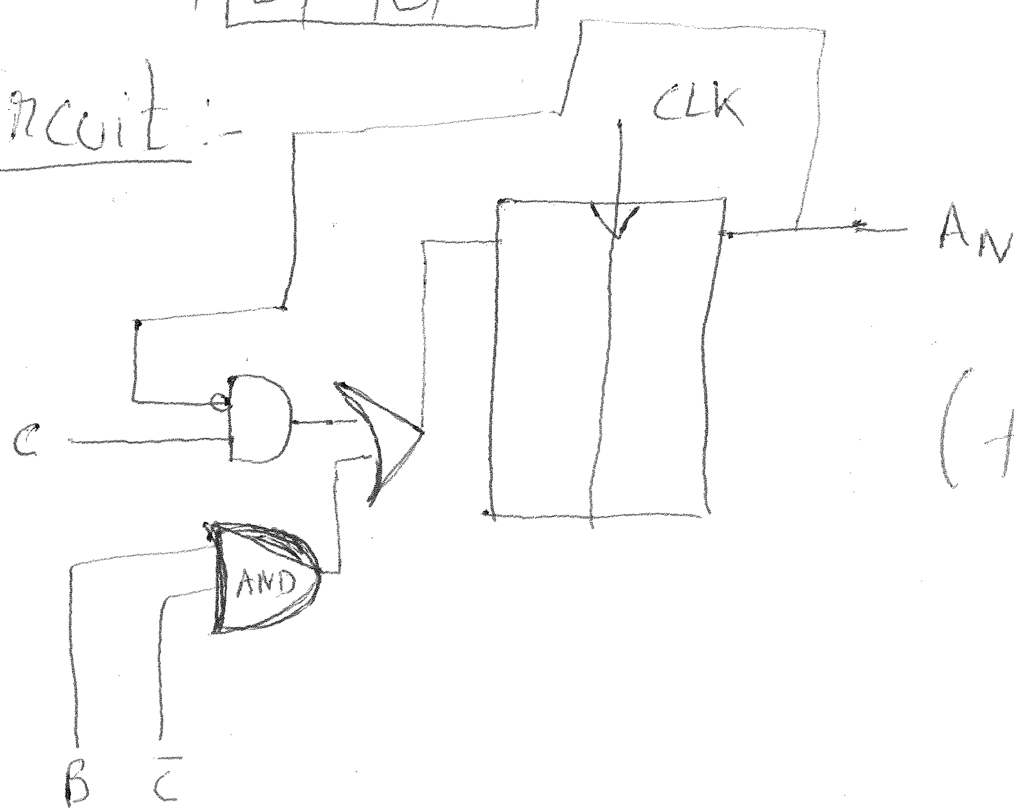
$$B_N = B\bar{C} + \bar{A}C$$

C_N :

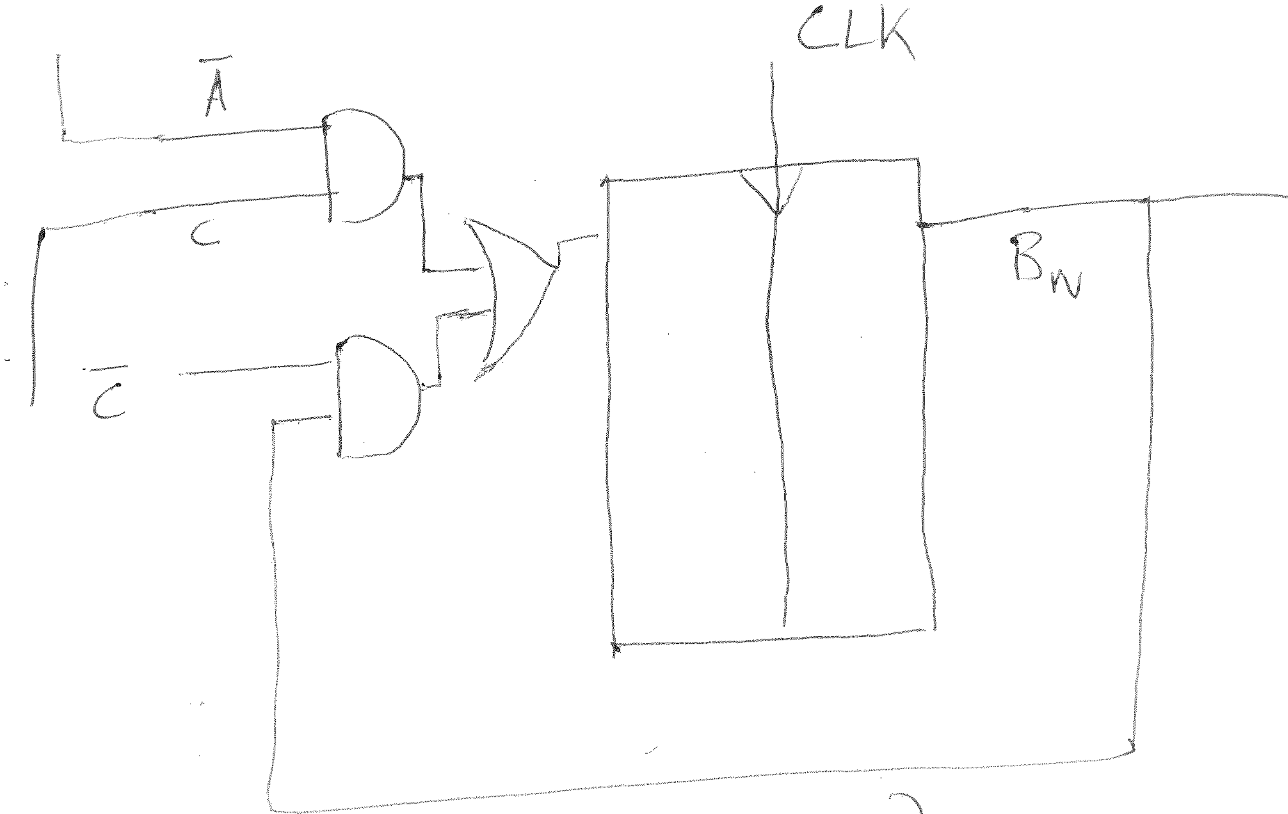
C \ AB	00	01	11	10
0	1		1	
1	1		1	

$$C_N = \bar{A}\bar{B} + AB$$

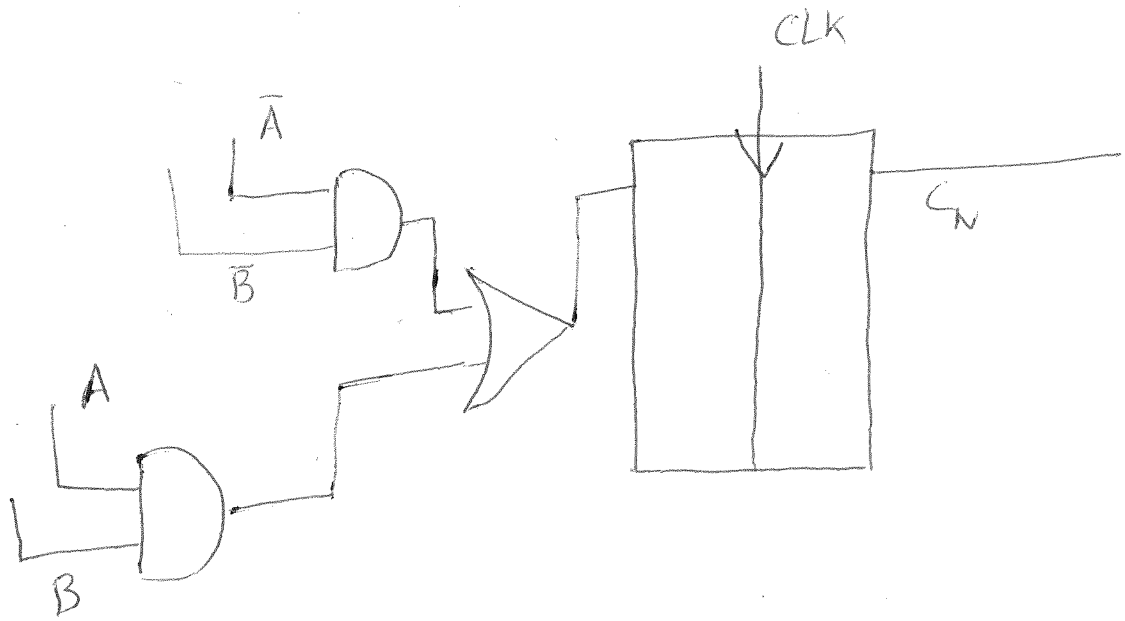
Circuit :-



(for A_N)

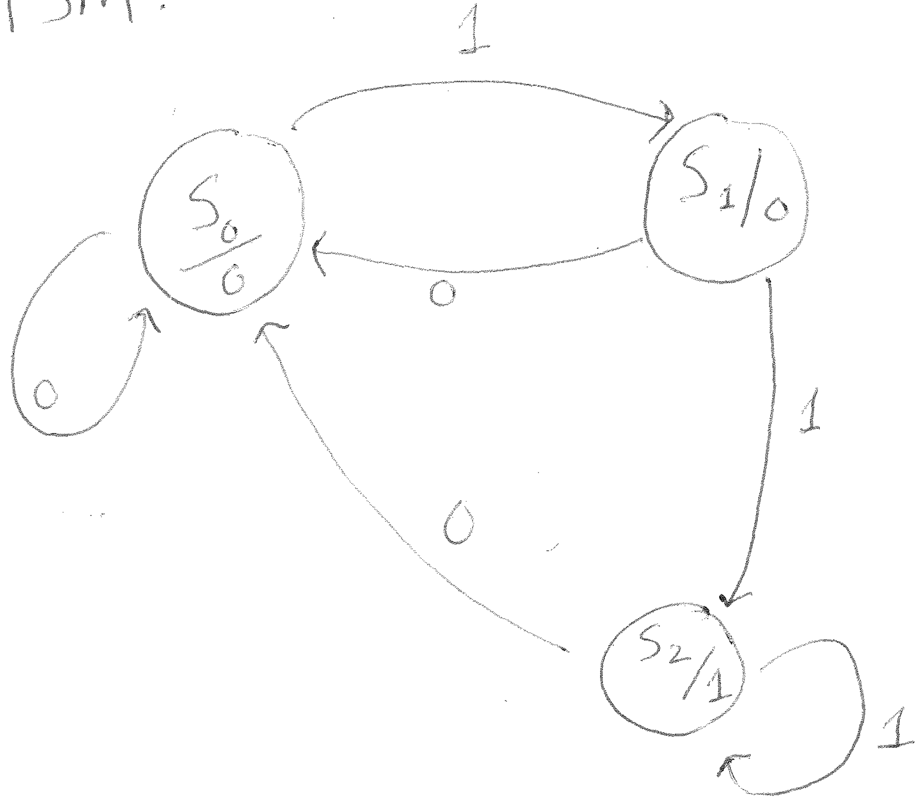


(for B_N)



(for C_N)

Problem 18: Let's first draw the FSM:



	A	B
S_0 =	0	0
S_1 =	0	1
S_2 =	1	0

D ≡ input

Y ≡ output

Table

A	B	D	A_N	B_N	Y
0	0	0	0	0	0
0	0	1	0	1	0
0	1	0	0	0	0
0	1	1	1	0	1
1	0	0	0	0	0
1	0	1	1	0	1

Karnaugh maps to obtain expressions for $A_N = f(A, B, D)$ and $B_N = g(A, B, D)$

D \ AB	00	01	11	10
0				
1		1	X	1

$$A_N = D(B + A)$$

D \ AB	00	01	11	10
0			X	
1	1		X	

$$B_N = D\bar{A}\bar{B}$$

$$Y = A_N \overline{B_N}$$

(is 1 only when $A_N B_N = 10$)

Circuit :-