

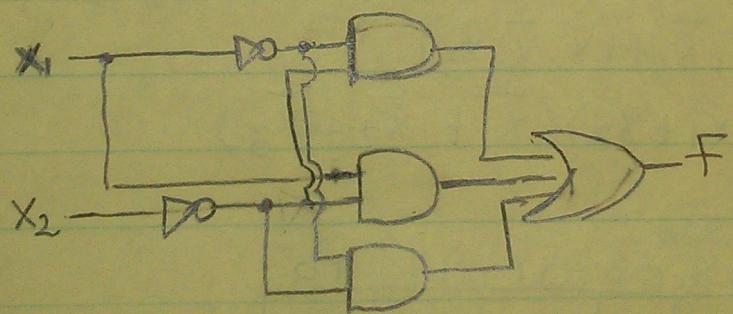
# CS 211 Assignment 4 Solutions

(1)

$$\begin{aligned} \text{LHS} &= x_1 \bar{x}_3 + x_1 x_3 + \bar{x}_2 \bar{x}_3 + \bar{x}_2 x_3 \\ &= x_1 (\bar{x}_3 + x_3) + \bar{x}_2 (\bar{x}_3 + x_3) \\ &= x_1 + \bar{x}_2 \\ \text{RHS} &= \bar{x}_1 \bar{x}_2 + x_1 x_2 + x_1 \bar{x}_2 \\ &= \bar{x}_1 \bar{x}_2 + x_1 (x_2 + \bar{x}_2) \\ &= \bar{x}_1 \bar{x}_2 + x_1 (\bar{1} + \bar{x}_2) = x_1 + \bar{x}_1 \bar{x}_2 + x_1 \bar{x}_2 \\ &= \bar{x}_1 \bar{x}_2 + x_1 (\bar{x}_1 + x_1) \bar{x}_2 = x_1 + \bar{x}_1 \bar{x}_2 . \quad \square \\ &= x_1 + (\bar{x}_1 + x_1) \bar{x}_2 \end{aligned}$$

(2)

②  $F(x_1, x_2) = x_1 \bar{x}_2 + \bar{x}_1 \bar{x}_2 + \bar{x}_1 x_2$



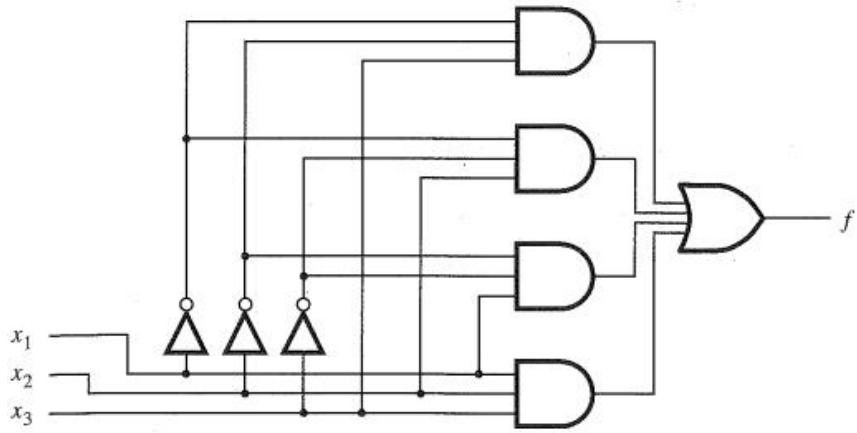
(3)

Minterms:  $\bar{x}_1 \bar{x}_2 \bar{x}_3$ ,  $\bar{x}_1 x_2 \bar{x}_3$ ,  $\bar{x}_1 \bar{x}_2 x_3$ ,  $x_1 x_2 x_3$ ,  
 Maxterms:  $x_1 + x_2 + \bar{x}_3$ ,  $\bar{x}_1 + x_2 + x_3$ ,  $\bar{x}_1 + x_2 + \bar{x}_3$ ,  
 $\bar{x}_1 + \bar{x}_2 + x_3$

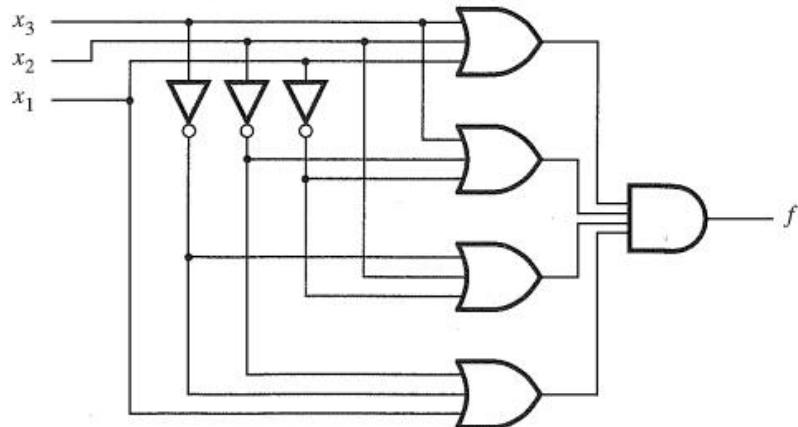
(4)

| 4) | $x_1$ | $x_2$ | $x_3$ | $f(x_1, x_2, x_3)$ |
|----|-------|-------|-------|--------------------|
|    | 0     | 0     | 0     | 0                  |
|    | 0     | 0     | 1     | 1                  |
|    | 0     | 1     | 0     | 1                  |
|    | 0     | 1     | 1     | 0                  |
|    | 1     | 0     | 0     | 1                  |
|    | 1     | 0     | 1     | 0                  |
|    | 1     | 1     | 0     | 0                  |
|    | 1     | 1     | 1     | 1                  |

SOP =  $\bar{x}_1 \bar{x}_2 x_3 + \bar{x}_1 x_2 \bar{x}_3 + x_1 \bar{x}_2 \bar{x}_3 + x_1 x_2 x_3$   
 POS =  $(x_1 + x_2 + x_3)(x_1 + \bar{x}_2 + \bar{x}_3)(\bar{x}_1 + x_2 + \bar{x}_3)(\bar{x}_1 + \bar{x}_2 + x_3)$



(a) Sum-of-products realization



(b) Product-of-sums realization

Figure 2.21 Implementation of the function in Figure 2.20.

(5)

$$f(x_1, x_2, x_3) = \text{Im}(1, 2, 4, 7) =$$

$$\bar{x}_1 \bar{x}_2 x_3 + \bar{x}_1 x_2 \bar{x}_3 + x_1 \bar{x}_2 \bar{x}_3 + x_1 x_2 x_3$$

| $x_3$ | $\bar{x}_1 \bar{x}_2$ | 00 | 01 | 11 | 10 |
|-------|-----------------------|----|----|----|----|
| 0     | 0                     | 1  |    |    |    |
| 1     | 1                     |    | 1  |    |    |

| $x_1 x_2 x_3$ | F |
|---------------|---|
| 000           | 0 |
| 001           | 1 |
| 010           | 1 |
| 011           | 0 |
| 100           | 1 |
| 101           | 0 |
| 110           | 0 |
| 111           | 1 |

this is the simplest form possible,  
no simplifications available

(6)

$$6) F(x_1, x_2, x_3) = \overline{M}(0, 2, 5) = (x_1 + x_2 + x_3)(\bar{x}_1 + \bar{x}_2 + x_3)(\bar{x}_1 + x_2 + \bar{x}_3)$$

$$\Rightarrow F(x_1, x_2, x_3) = (\bar{x}_1 \bar{x}_2 \bar{x}_3) + (\bar{x}_1 x_2 \bar{x}_3) + (x_1 \bar{x}_2 x_3)$$

$$\Rightarrow F(x_1, x_2, x_3) = (\bar{x}_1 \bar{x}_3) + (x_1 \bar{x}_2 x_3)$$

$$= (x_1 + x_3)(\bar{x}_1 + x_2 + \bar{x}_3)$$

K-map for F

|  |  | $x_3 \backslash x_1 x_2$ | 00 | 01 | 11 | 10 |
|--|--|--------------------------|----|----|----|----|
|  |  | 0                        | 0  | 1  | 1  | 0  |
|  |  | 1                        | 1  | 0  | 1  | 1  |

thus K-map for  $\bar{F}$

|  |  | $x_3 \backslash x_1 x_2$ | 00 | 01 | 11 | 10 |
|--|--|--------------------------|----|----|----|----|
|  |  | 0                        | 1  | 0  | 0  | 1  |
|  |  | 1                        | 0  | 1  | 0  | 0  |

thus  $\bar{F}$

$$\bar{F} = \bar{x}_1 \bar{x}_2 + x_1 \bar{x}_2 x_3 = (x_1 + x_3)(\bar{x}_1 + x_2 + \bar{x}_3)$$

(7)

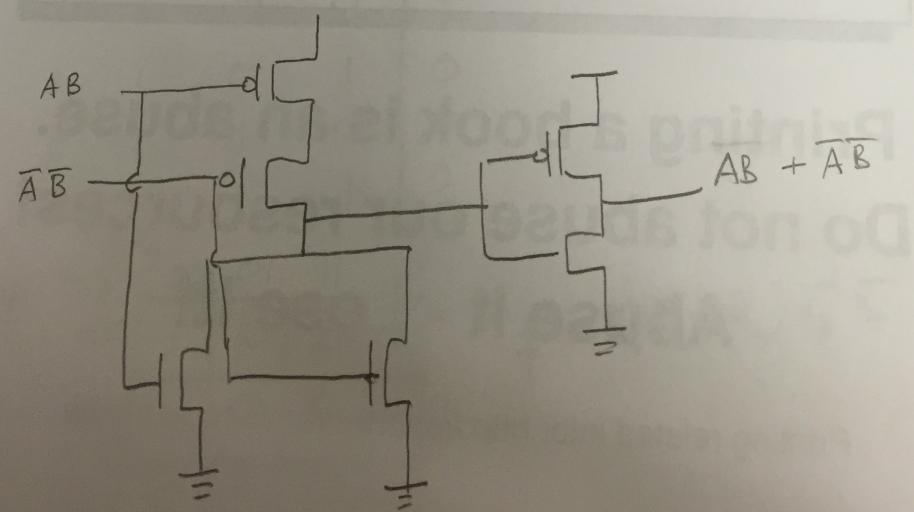
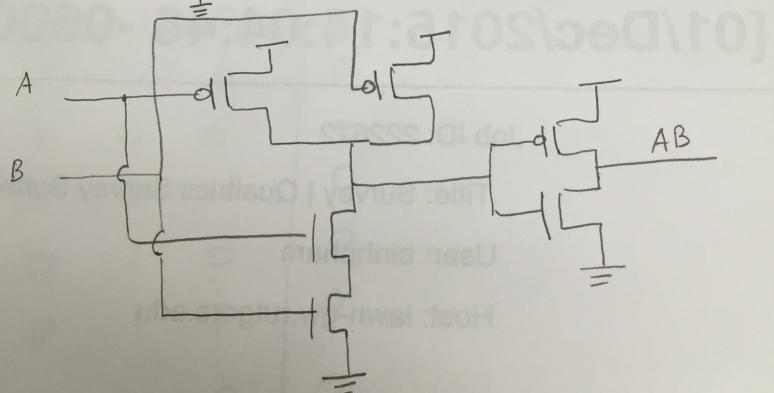
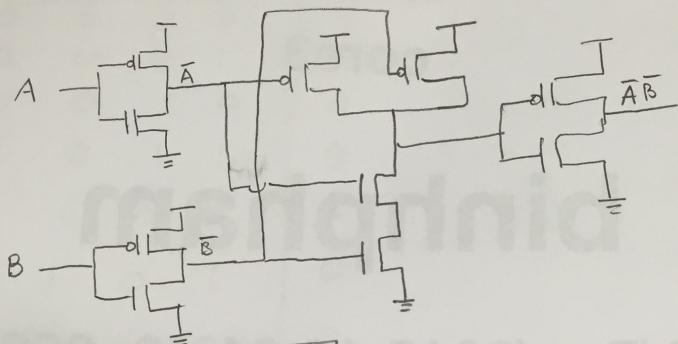
| $x_1$ | $x_2$ | $x_3$ | F |
|-------|-------|-------|---|
| 0     | 0     | 0     | 1 |
| 0     | 0     | 1     | 1 |
| 0     | 1     | 0     | 1 |
| 0     | 1     | 1     | 1 |
| 1     | 0     | 0     | 1 |
| 1     | 0     | 1     | 0 |
| 1     | 1     | 0     | 0 |
| 1     | 1     | 1     | 0 |

|  |  | $x_1 \backslash x_2 x_3$ | 00 | 01 | 11 | 10 |
|--|--|--------------------------|----|----|----|----|
|  |  | 0                        | 1  | 1  | 1  | 0  |
|  |  | 1                        | 1  | 0  | 0  | 0  |

$$f = \bar{x}_1 + \bar{x}_2 \bar{x}_3$$

(8)

$$A \text{ XNOR } B = \overline{\overline{A} \oplus B} = \overline{\overline{A} \overline{B}} + AB$$



(9)

| A | B | F |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |

Thus, it's a XOR gate.

(10)

| $x_1$ | $x_2$ | $x_3$ | $x_4$ | F | $\bar{x}_3 \bar{x}_4$ | $\bar{x}_3 x_4$ | $x_3 \bar{x}_4$ | $x_3 x_4$ |
|-------|-------|-------|-------|---|-----------------------|-----------------|-----------------|-----------|
| 0     | 0     | 0     | 0     | 1 | 1                     | 0               | 0               | 0         |
| 0     | 0     | 0     | 1     | 0 | 00                    | 1               | 0               | 0         |
| 0     | 0     | 1     | 0     | 0 | 01                    | 1               | 0               | 0         |
| 0     | 0     | 1     | 1     | 0 | 01                    | 1               | 0               | 0         |
| 0     | 1     | 0     | 0     | 1 | 0                     | 0               | 0               | 0         |
| 0     | 1     | 0     | 1     | 0 | 11                    | 0               | 0               | 0         |
| 0     | 1     | 1     | 0     | 0 | 11                    | 0               | 0               | 0         |
| 0     | 1     | 1     | 1     | 0 | 10                    | 1               | 0               | 0         |
| 1     | 0     | 0     | 0     | 1 | 0                     | 0               | 0               | 0         |
| 1     | 0     | 0     | 1     | 0 | 0                     | 0               | 0               | 0         |
| 1     | 0     | 1     | 0     | 0 | 0                     | 0               | 0               | 0         |
| 1     | 0     | 1     | 1     | 0 | 0                     | 0               | 0               | 0         |
| 1     | 1     | 0     | 0     | 0 | 0                     | 0               | 0               | 0         |
| 1     | 1     | 0     | 1     | 0 | 0                     | 0               | 0               | 0         |
| 1     | 1     | 1     | 0     | 0 | 0                     | 0               | 0               | 0         |
| 1     | 1     | 1     | 1     | 0 | 0                     | 0               | 0               | 0         |

$$F = \bar{x}_3 \bar{x}_4 (\bar{x}_1 + \bar{x}_2)$$

(11)

| $x_1$ | $x_2$ | $x_3$ | $h$ | $g$ |
|-------|-------|-------|-----|-----|
| 0     | 0     | 0     | 0   | 0   |
| 0     | 0     | 1     | 1   | 1   |
| 0     | 1     | 0     | 1   | 1   |
| 0     | 1     | 1     | 0   | 0   |
| 1     | 0     | 0     | 1   | 1   |
| 1     | 0     | 1     | 0   | 0   |
| 1     | 1     | 0     | 0   | 0   |
| 1     | 1     | 1     | 1   | 1   |

Thus, the two circuits are equivalent.

(12)

$S_0 \sim S_{n-1}$ : n XORs  $G_0 \sim G_{n-1}$ : n ANDs

$P_0 \sim P_{n-1}$ : n ORs  $C_n$ : 1 OR, n ANDs

(13)

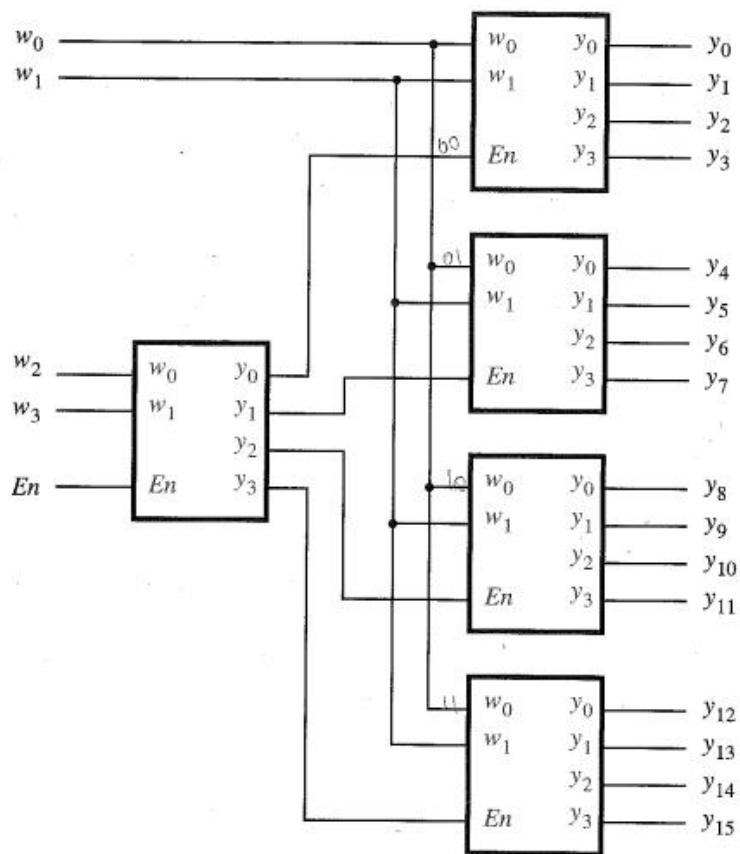


Figure 6.18 A 4-to-16 decoder built using a decoder tree.

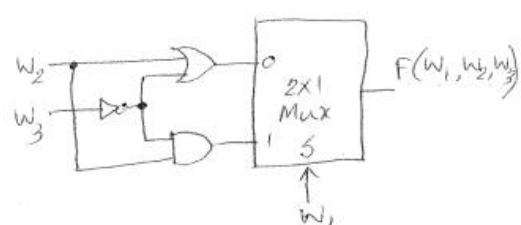
(14)

$$F(w_1, w_2, w_3) = \sum m(0, 2, 3, 6)$$

$$= \bar{w}_1 \bar{w}_2 \bar{w}_3 + \bar{w}_1 w_2 \bar{w}_3 + \bar{w}_1 w_2 w_3 + w_1 w_2 \bar{w}_3$$

$$= \bar{w}_1 (w_2 w_3 + \bar{w}_3) + w_1 (w_2 \bar{w}_3)$$

$$= \bar{w}_1 (w_2 + \bar{w}_3) + w_1 (w_2 \bar{w}_3)$$



(15)

Excitation Table

Truth Table

| M | N | $Q_{t+1}$   |
|---|---|-------------|
| 0 | 0 | 0           |
| 0 | 1 | 1           |
| 1 | 0 | $\bar{Q}_t$ |
| 1 | 1 | $\bar{Q}_t$ |

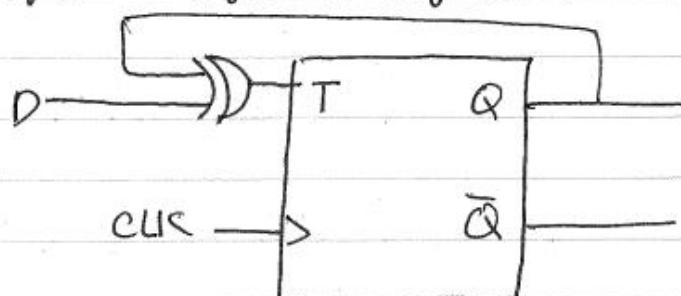
| $Q_t$ | $Q_{t+1}$ | M | N |
|-------|-----------|---|---|
| 0     | 0         | 0 | 0 |
| 0     | 1         | 0 | 1 |
| 0     | 1         | 1 | X |
| 1     | 0         | 0 | 0 |
| 1     | 0         | 1 | X |
| 1     | 1         | 0 | 1 |

(16)

Excitation table:

| $Q_t$ | $Q_{t+1}$ | D | T |
|-------|-----------|---|---|
| 0     | 0         | 0 | 0 |
| 0     | 1         | 1 | 1 |
| 1     | 0         | 0 | 1 |
| 1     | 1         | 1 | 0 |

$$T = Q'_t D + Q_t D' = Q_t \oplus D$$



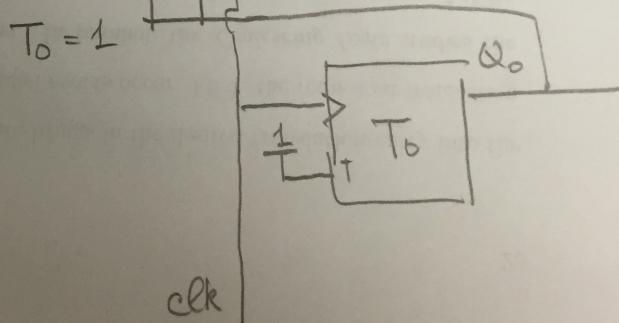
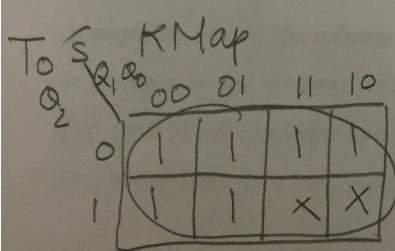
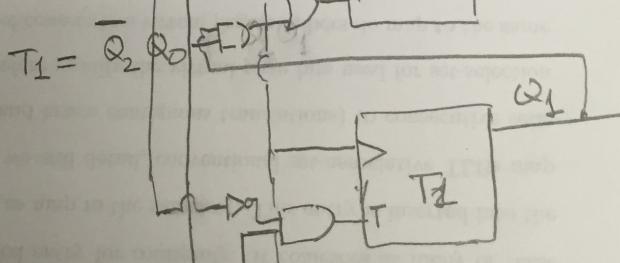
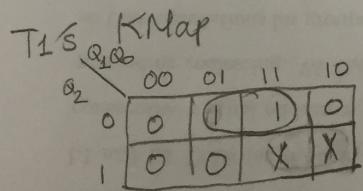
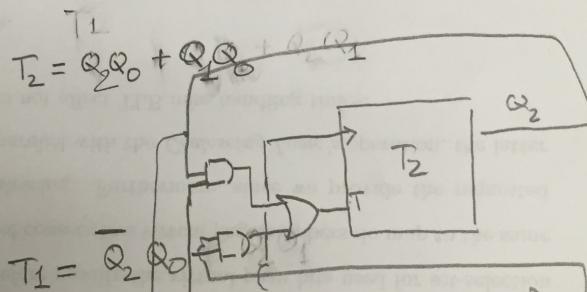
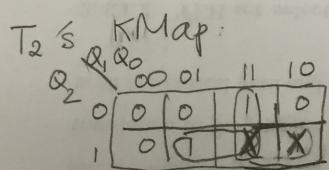
(17)

| A | B | Q | Comment |
|---|---|---|---------|
| 0 | 0 | 1 | set     |
| 0 | 1 | Q | hold    |
| 1 | 0 | 0 | reset   |
| 1 | 1 | Q | hold    |

(18)

State table:

| Prev State |       |       | Next State |        |        | T's Excitation |       |       |
|------------|-------|-------|------------|--------|--------|----------------|-------|-------|
| $Q_2$      | $Q_1$ | $Q_0$ | $Q'_2$     | $Q'_1$ | $Q'_0$ | $T_2$          | $T_1$ | $T_0$ |
| 0          | 0     | 0     | 0          | 0      | 1      | 0              | 0     | 1     |
| 0          | 0     | 1     | 0          | 1      | 0      | 0              | 1     | 1     |
| 0          | 1     | 0     | 0          | 1      | 1      | 0              | 1     | 1     |
| 0          | 1     | 1     | 1          | 0      | 0      | 1              | 0     | 1     |
| 1          | 0     | 0     | 1          | 0      | 1      | 1              | 0     | 1     |
| 1          | 0     | 1     | 0          | 0      | 0      | X              | X     | X     |
| 1          | 1     | 0     | X          | X      | X      | X              | X     | X     |
| 1          | 1     | 1     | X          | X      | X      |                |       |       |



(19)

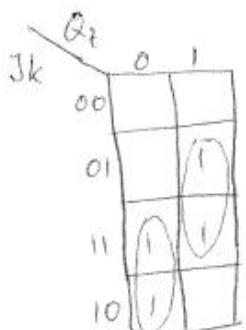
Output of flip-flop charges only on clock. On the other hand, latch reacts immediately with change in input.

(20)

Excitation Table

| $Q_t$ | $Q_{t+1}$ | $J$ | $k$ | $T$ |
|-------|-----------|-----|-----|-----|
| 0     | 0         | 0   | x   | 0   |
| 0     | 1         | 1   | x   | 1   |
| 1     | 0         | x   | 1   | 1   |
| 1     | 1         | x   | 0   | 0   |

$$\text{So, } f_T(J, k, Q_t) = \Sigma M(3, 5, 6, 7)$$



$$f_T = J\bar{Q}_t + kQ_t$$

