Personnel scheduling: Models and complexity

Кадравае планаванне: мадэлі і складанасць

Peter Brucker, Rong Qu, Edmund Burke

November 2010

Classification of companies according to different personnel scheduling problems:

- **Permanence** centred planning “пастаянная”, “перадвызначаная”
- **Fluctuation** centered planning “хісткая”, “гнуткая”, “зменлівая”
- **Mobility** centered planning “цэнтралізаваная”
- **Project** centered planning “праэкта-арыентаваная”

The problem: no mathematical models in the literature for general personnel scheduling problems

Праблема: адсутнасць агульнай матэматычнай мадэлі задачы кадравага планавання

Proposing a model

planning horizon $[\mathbf{0}, \boldsymbol{T}]$ divided into periods $[t, t+1[$

$\boldsymbol{m}$ tasks, $j = 1, \ldots, m$

$\boldsymbol{D_j(t)}$ - number of employees to perform task j in time period $[t, t+1[$ - **the demand profile** for task j

set $\boldsymbol{E}$ of $\boldsymbol{n}$ employees

subset $\boldsymbol{Q_e}$ of tasks for which $\boldsymbol{e}$ is qualified

working pattern for $\boldsymbol{e}$ is a zero-one vector $\boldsymbol{w_e(t)_{t=0}^{T-1}}$ and an assignment a task from $\boldsymbol{Q_e}$ for each $\boldsymbol{w_e(t) = 1}$ - represented by binary vectors $\boldsymbol{\pi(j, t)}$

Flexible demand modification:

each task j has a duration $\boldsymbol{p_j}$, must be processed within a **time window** $[\boldsymbol{L_j}, \boldsymbol{R_j}] \subseteq [\mathbf{0}, \boldsymbol{T}], R_j - L_j \geq p_j$

hard constraints

(specify feasible working patterns)

(жорсткія абмежаванні абавязкова мусяць быць выкананыя)

soft constraints

(penalties may be applied)

(няжорсткія абмежаванні накладаюць штрафы)

Project centred planning model

the demand $\boldsymbol{D_j(t)}$ for each task is not fixed for the time periods $[\boldsymbol{t}, \boldsymbol{t+1}[$
a schedule for task $\boldsymbol{j}$ is defined by starting time $\boldsymbol{S_j}$ and processing time $\boldsymbol{p_j}$
$\boldsymbol{C_j = S_j + p_j}$
at least $\boldsymbol{D_j(k)}$ employees in period $[\boldsymbol{S_j + k - 1}, \boldsymbol{S_j + k}[$
precedence constraints $(\boldsymbol{i}, \boldsymbol{j}) \in \boldsymbol{A}$: $\boldsymbol{S_i + p_i \leq S_j}$
$\boldsymbol{C_{max} = max_{j=1}^{m} C_j}$

Problems #1 - #4

- #1 A nurse rostering problem
- #2 A problem with restricted task changes
- #3 A problem with flexible demand
- #4 A multi-day personnel scheduling problem

Complexity

Polynomially solvable cases

NP-complete cases

Can be solved efficiently by network flow techniques