dB/dBW/dBm cheat sheet

- dB is a logarithmic **ratio of powers**. So saying $P_1 = 2 \cdot P_2$ is equivalent to saying $P_1/P_2 \triangleq 3$ dB, because

$$10 \cdot \log_{10}\left(\frac{P_1}{P_2}\right) \text{ dB} = 10 \cdot \log_{10}(2) \text{ dB} = 3 \text{ dB}.$$

  The unit dBi can be treated just like dB, because it measures the gain of an antenna relative to an isotropic antenna which has the gain of 1 (0 dB).

- dBm and dBW is a (logarithmic) **unit to measure powers**. Since dB is a ratio of powers, dBm and dBW are defined by forming the ratio of the power you want to express relative to a reference power, which is 1 W for dBW and 1 mW for dBm. Formally speaking,

$$\begin{aligned} P_\mathrm{T}|_{\text{dBm}} &= 10 \cdot \log_{10}\left(\frac{P_\mathrm{T}}{1\ \text{mW}}\right) \\ P_\mathrm{T}|_{\text{dBW}} &= 10 \cdot \log_{10}\left(\frac{P_\mathrm{T}}{1\ \text{W}}\right) = P_\mathrm{T}|_{\text{dBm}} - 30 \text{ dB} \end{aligned}$$

Consequently, **two things are okay**:

- Starting with a **power** (dBW or dBm) you can *add* and *subtract* **ratios** (dB or dBi) as often as you like and you still have a **power** (dBW or dBm):

$$\begin{aligned} P_1 \cdot G_1/L_1 = P_2 \quad &\Leftrightarrow \quad 10 \cdot \log_{10}\left(\frac{P_1 \cdot G_1/L_1}{1\ \text{W}}\right) = 10 \cdot \log_{10}\frac{P_2}{1\ \text{W}} \\ &\Leftrightarrow \quad P_1|_{\text{dBW}} + G_1|_{\text{dB}} - L_1|_{\text{dB}} = P_2|_{\text{dBW}} \end{aligned}$$

- *Subtracting* **two powers** (dBW or dBm) which is equivalent to computing their **ratio** (dB):

$$\begin{aligned} \frac{P_1}{P_2} \triangleq 10 \cdot \log_{10}\left(\frac{P_1}{P_2}\right) \text{ dB} &= 10 \cdot \log_{10}\left(\frac{P_1}{1\ \text{W}}\right) - 10 \cdot \log_{10}\left(\frac{P_2}{1\ \text{W}}\right) = P_1|_{\text{dBW}} - P_2|_{\text{dBW}} \\ &= 10 \cdot \log_{10}\left(\frac{P_1}{1\ \text{mW}}\right) - 10 \cdot \log_{10}\left(\frac{P_2}{1\ \text{mW}}\right) = P_1|_{\text{dBm}} - P_2|_{\text{dBm}} \end{aligned}$$

  Both powers must have the same unit, do not mix dBW and dBm.

Short hand notation for **things that are okay**:

dBW $\pm$ dB	= dBW
dBm $\pm$ dB	= dBm
dBW $-$ dBW	= dB
dBm $-$ dBm	= dB

On the other hand, the following things **are not okay**:

- **Never ever multiply dBW with dB!** I have seen students claiming that a power of P_T =10 W with an antenna gain of G_T =10 gives an effective radiated power of 100 dBW. **No!** 100 dBW is 10 Gigawatts! Multiply in linear scale, that becomes addition in logarithmic scale:

$$\begin{aligned} P_\mathrm{T} \cdot G_\mathrm{T} &= 10\ \mathrm{W} \cdot 10 = 100\ \mathrm{W} \triangleq 20\ \mathrm{dBW} \\ \Leftrightarrow P_\mathrm{T}|_\mathrm{dBW} + G_\mathrm{T}|_\mathrm{dBi} &= 10\ \mathrm{dBW} + 10\ \mathrm{dBi} = 20\ \mathrm{dBW} \end{aligned}$$

- **Never add a bunch of quantities in dBW or dBm**! Adding powers in log scale means multiplying them in linear scale. If you add 10 dBW with 3 dBW and 6 dBW then you have

$$10\ \mathrm{dBW} + 3\ \mathrm{dBW} + 6\ \mathrm{dBW} \triangleq 10\ \mathrm{W} \cdot 2\ \mathrm{W} \cdot 4\ \mathrm{W} = 80\ \mathrm{W}^3$$

  What unit is Watt cubed? I don't know, you tell me...

- A power **cannot** be measured in dB. A gain/loss **cannot** be measured in dBm or dBW.