Arguments

Class code:b7xpd4s

In mathematics and logic an argument is not a dispute. It is a sequence of statements ending in a conclusion.

For example, the following is an argument:

If Socrates is a man, then Socrates is mortal.

Socrates is a man.

$\therefore$ Socrates is mortal.

- An *argument* is a sequence of statements, and
- an *argument form* is a sequence of statement forms.
- All statements in an argument and all statement forms in an argument form, except for the final one, are called ***premises*** (or *assumptions* or *hypotheses*). The final statement or statement form is called the ***conclusion***.
- The symbol ∴, which is read “therefore,” is normally placed just before the conclusion.

To say that an **argument form is *valid*** means that no matter what particular statements are substituted for the statement variables in its premises, if the resulting premises are all true, then the conclusion is also true.

To say that an **argument is valid** means that its form is valid. Otherwise it is ***invalid***.

Testing an Argument Form for Validity

- Identify the premises and conclusion of the argument form.
- Construct a truth table showing the truth values of all the premises.
- A row of the truth table in which all the premises are true is called a ***critical row***. If there is a critical row in which the conclusion is false, then it is possible for an argument of the given form to have true premises and a false conclusion, and so the argument form is invalid. If the conclusion in every critical row is true, then the argument form is valid.

Example:

$$p \rightarrow q$$

$$\sim q \rightarrow p$$

$$\sim r$$

$$\therefore \quad q$$

p	q	r	$\sim q$	$\sim r$	$p \rightarrow q$	$\sim q \rightarrow p$	$\sim r$	q
T	T	T	F	F	T	T	F	
T	T	F	F	T	T	T	T	
T	F	T	T	F	F	T	F	
T	F	F	T	T	F	T	T	
F	T	T	F	F	T	T	F	
F	T	F	F	T	T	T	T	
F	F	T	T	F	T	F	F	
F	F	F	T	T	T	F	T	

Example:

$$p \to q$$
$$\sim q \to p$$
$$\sim r$$
$$\therefore \; q$$

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T	T	F	F	T	T	T	T	
T	F	T	T	F	F	T	F	
T	F	F	T	T	F	T	T	
F	T	T	F	F	T	T	F	
F	T	F	F	T	T	T	T	
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T	T	F	F	T	T	T	T	T
T	F	T	T	F	F	T	F	
T	F	F	T	T	F	T	T	
F	T	T	F	F	T	T	F	
F	T	F	F	T	T	T	T	T
F	F	T	T	F	T	F	F	
F	F	F	T	T	T	F	T	

Example:

Let p:Roger studies.

q:Roger plays racket ball. r:Roger passes discrete mathematics.

Write the argument from the following argument form,

$$p \to r$$
$$\sim q \to p$$
$$\sim r$$
$$\therefore\ q$$

If Roger studies, then he will pass discrete mathematics.

If Roger doesn't play racket ball, then he will study.

Roger failed discrete mathematics.

Therefore, Roger plays racket ball.

Example:

Check the validity of the following argument form

$$\begin{array}{ll} & p \to q \\ & p \\ \therefore & q \end{array}$$

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$$\begin{array}{ll} & p \to q \\ & p \\ \therefore & q \end{array}$$

p	q	$p \to q$	p	q
T	T	T	T	
T	F	F	T	
F	T	T	F	
F	F	T	F	

Example:

Check the validity of the following argument form

$$p \to q$$
$$p$$
$$\therefore \quad q$$

p	q	$p \to q$	p	q
T	T	T	T	T
T	F	F	T	
F	T	T	F	
F	F	T	F	

Example:

Check the validity of the following argument form

$$\begin{array}{ll} & p \to q \\ & p \\ \therefore & q \end{array}$$

Modus ponens

Example:

Check the validity of the following argument form

$$\begin{array}{ll} & p \rightarrow q \\ & \sim q \\ \therefore & \sim p \end{array}$$

Example:

Check the validity of the following argument form

$$p \to q$$
$$\sim q$$
$$\therefore \quad \sim p$$

Modus Tollens

Exercise:

Check the validity of the following argument form

$$p \rightarrow q \vee \sim r$$

$$q \rightarrow p \wedge r$$

$$\therefore p \rightarrow r$$

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Check the validity of the following argument form

$$p \rightarrow q \vee \sim r$$

$$q \rightarrow p \wedge r$$

$$\therefore p \rightarrow r$$

p	q	r	$q \vee \sim r$	$p \wedge r$	$p \rightarrow q \vee \sim r$	$q \rightarrow p \wedge r$	$p \rightarrow r$
T	T	T	T	T	T	T	
T	T	F	T	F	T	F	
T	F	T	F	T	F	T	
T	F	F	T	F	T	T	F
F	T	T	T	F	T	F	
F	T	F	T	F	T	F	
F	F	T	F	F	T	T	
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Rule of Inference

An argument form consisting of two premises and a conclusion is called a **syllogism**. The first and second premises are called the **major premise** and **minor premise**, respectively. The most famous form of syllogism in logic is called **modus ponens**.

A **rule of inference** is a form of argument that is valid. Thus, **modus ponens** and **modus tollens** are both rules of inference. The following are additional examples of rules of inference that are frequently used in deductive reasoning.

modus ponens

The most famous form of syllogism in logic is called **modus ponens.** It has the following form:

If p then q.

p

$\therefore q$

Here is an argument of this form:

If the sum of the digits of 371,487 is divisible by 3, then 371,487 is divisible by 3.

The sum of the digits of 371,487 is divisible by 3.

$\therefore$ 371,487 is divisible by 3.

modus tollens

Another form of syllogism in logic is called **modus tollens.** It has the following form:

If p then q.

$\sim q$

$\therefore \sim p$

Here is an argument of this form:

If the sum of the digits of 371,48 is divisible by 3, then 371,48 is divisible by 3.

371,48 is not divisible by 3.

$\therefore$ the sum of the digits of 371,48 is not divisible by 3.

Generalization

The following argument forms are valid:

a. p

$\therefore p \vee q$

b. q

$\therefore p \vee q$

You reason as follows:

Anton is a junior.

∴ Anton is a junior or Anton is a senior

Specialization

The following argument forms are valid:

a. $p \wedge q$

$\therefore p$

b. $p \wedge q$

$\therefore q$

You reason as follows:

Ana knows numerical analysis and Ana knows graph algorithms.
∴ Ana knows graph algorithms.

Elimination

The following argument forms are valid:

a. $p \vee q$
$\sim q$
$\therefore p$

b. $p \vee q$
$\sim p$
$\therefore q$

For instance, suppose you know that for a particular number x,
$x - 3 = 0$ or $x + 2 = 0$.
If you also know that x is not negative, then $x \neq -2$, so
$x + 2 \neq 0$.
By elimination, you can then conclude that
$\therefore\ x - 3 = 0$.

Transitivity

The following argument form is valid:

$$p \to q$$
$$q \to r$$
$$\therefore p \to r$$

If 18,486 is divisible by 18, then 18,486 is divisible by 9.

If 18,486 is divisible by 9, then the sum of the digits of 18,486 is divisible by 9.

$\therefore$ If 18,486 is divisible by 18, then the sum of the digits of 18,486 is divisible by 9.

Proof by Division into Cases

The following argument form is valid:

$p \vee q$

$p \to r$

$q \to r$

$\therefore r$

x is positive or x is negative.

If x is positive, then $x^2 > 0$.

If x is negative, then $x^2 > 0$.

$\therefore\ x^2 > 0$.

Rules of Inference

A ***rule of inference*** is a form of argument that is valid.

Modus Ponens	$p \to q$ p $\therefore q$	**Elimination**	$p \vee q$ $\sim q$ $\therefore\ p$	$p \vee q$ $\sim p$ $\therefore\ q$
Modus Tollens	$p \to q$ $\sim q$ $\therefore \sim p$	**Transitivity**	$p \to q$ $q \to r$ $\therefore p \to r$	
Generalization	$a)\ p$ $\therefore p \vee q$ $b)\ q$ $\therefore p \vee q$	**Proof by Division into Cases**	$p \vee q$ $p \to r$ $q \to r$ $\therefore r$	
Specialization	$a)\ p \wedge q$ $\therefore p$ $a)\ p \wedge q$ $\therefore q$			
Conjunction	p q $\therefore\ p \wedge q$	**Contradiction Rule**	$\sim p \to c$ $\therefore\ p$	$p \to c$ $\therefore\ \sim p$

Examples:

Which rule of inference is used in the following argument?

If Margaret Thatcher is the president of the United States, then she is at least 35 years old.

Margaret Thatcher is the president of the United States.

∴ She is at least 35 years old.

Example:

Which rule of inference is used in the following argument?

If you have access to the network, then you can change your grade.

You cannot change your grade.

∴ you do not have access to the network.

Use the valid argument forms listed in Table 2.3.1 to deduce the conclusion from the premises, giving a reason for each step

a) $p \vee q$

b) $q \to r$

c) $p \wedge s \to t$

d) $\sim r$

e) $\sim q \to u \wedge s$

f) $\therefore\ t$

Solution

	$q \to r$	From (b)
	$\sim r$	From (d)
g)	$\therefore \sim q$	By Modus tollens
	$p \vee q$	From (a)
	$\sim q$	From (g)
h)	$\therefore p$	By elimination
	$\sim q \to u \wedge s$	From (e)
	$\sim q$	From (g)
i)	$\therefore u \wedge s$	By Modus ponens
j)	$\therefore s$	By specialization
	p	From (h)
	s	From (j)
k)	$\therefore p \wedge s$	By conjunction law
	$p \wedge s \to t$	From (c)
	$p \wedge s$	From (k)
	$\therefore t$	By Modus ponens

Fallacies

A *fallacy* is an error in reasoning that results in an invalid argument.

Converse Error

The general form of the previous argument is as follows:

$$p \to q$$

$$q$$

$$\therefore p$$

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The general form of the previous argument is as follows:

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$$q$$
$$\therefore p$$

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

Fallacies

Example: Show that the following argument is invalid:

If Zeke is a cheater, then Zeke sits in the back row.

Zeke sits in the back row.

$\therefore$ Zeke is a cheater

If x is divisible by 9 then x is divisible by 3

x is divisible by 3

$\therefore$ x is divisible by 9

Fallacies

Inverse Error

Note that this argument has the following form:

$$p \to q$$

$$\sim p$$

$$\therefore \sim q$$

Fallacies

Example: Consider the following argument:

If interest rates are going up, stock market prices will go down.

Interest rates are not going up.

∴ Stock market prices will not go down.

If x is divisible by 9 then x is divisible by 3

x is not divisible by 9

∴ x is not divisible by 3

Example: A More Complex Deduction

You are about to leave for school in the morning and discover that you don't have your glasses. You know the following statements are true:

a) If I was reading the newspaper in the kitchen, then my glasses are on the kitchen table.

b) If my glasses are on the kitchen table, then I saw them at breakfast.

c) I did not see my glasses at breakfast.

d) I was reading the newspaper in the living room or I was reading the newspaper in the kitchen.

e) If I was reading the newspaper in the living room then my glasses are on the coffee table.

Where are the glasses?

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d) I was reading the newspaper in the living room or I was reading the newspaper in the kitchen.

e) If I was reading the newspaper in the living room then my glasses are on the coffee table.

Where are the glasses?

RK=I was reading in the kitchen.

GK = My glasses are on the kitchen table.

SB =I saw my glasses at breakfast.

RL =I was reading the newspaper in the living room.

GC = My glasses are on the coffee table.

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a) If I was reading the newspaper in the kitchen, then my glasses are on the kitchen table.
$RK \rightarrow GK$

b) If my glasses are on the kitchen table, then I saw them at breakfast.
$GK \rightarrow SB$

c) I did not see my glasses at breakfast.
$\sim SB$

d) I was reading the newspaper in the living room or I was reading the newspaper in the kitchen.
$RL \vee RK$

e) If I was reading the newspaper in the living room then my glasses are on the coffee table.
$RL \rightarrow GC$

Where are the glasses?

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GC = My glasses are on the coffee table.

Example: A More Complex Deduction

a) $RK \rightarrow GK$

b) $GK \rightarrow SB$

c) $\sim SB$

d) $RL \vee RK$

e) $RL \rightarrow GC$

	$RK \rightarrow GK$	*from a)*
	$GK \rightarrow SB$	*from b)*
f)	$\therefore RK \rightarrow SB$	*By transitivity*
	$\sim SB$	*from c)*
g)	$\therefore \sim RK$	*by modus tollens*
	$RL \vee RK$	*from d)*
h)	$\therefore RL$	by elimination
	$RL \rightarrow GC$	*from e)*
i)	$\therefore GC$	

Contradiction Rule:

If a statement p implies a contradiction,

Then the statement is false

That it:

$$p \to c$$
$$\therefore \sim p$$

Example: Knights and Knaves

The logician Raymond Smullyan describes an island containing two types of people: knights who always tell the truth and knaves who always lie. You visit the island and are approached by two natives who speak to you as follows:

A says: B is a knight.

B says: A and I are of opposite type.

What are A and B?

A says: B is a knight.
B says: A and I are of opposite type.

Suppose *A* is a knight.

∴ What *A* says is true.

∴ *B* is also a knight.

∴ What *B* says is true.

∴ *A* and *B* are of opposite types.

∴ We have arrived at the following contradiction:

A and B are both knights and A and B are of opposite type.

∴ The supposition is false. by the contradiction rule

∴ A is not a knight.

∴ A is a knave.

∴ What A says is false.

∴ B is not a knight.

∴ B is also a knave.

A and *B* are both knaves.

NOTE:

- If a supposition **leads to a contradiction**, then the negation of the supposition is true.
- If a supposition **does not** lead to a contradiction, then the supposition **maybe true and it may not be true**; so either we exhaust all the possibilities or we cannot reach to any conclusion.