Thermodynamics and Tornado Prediction

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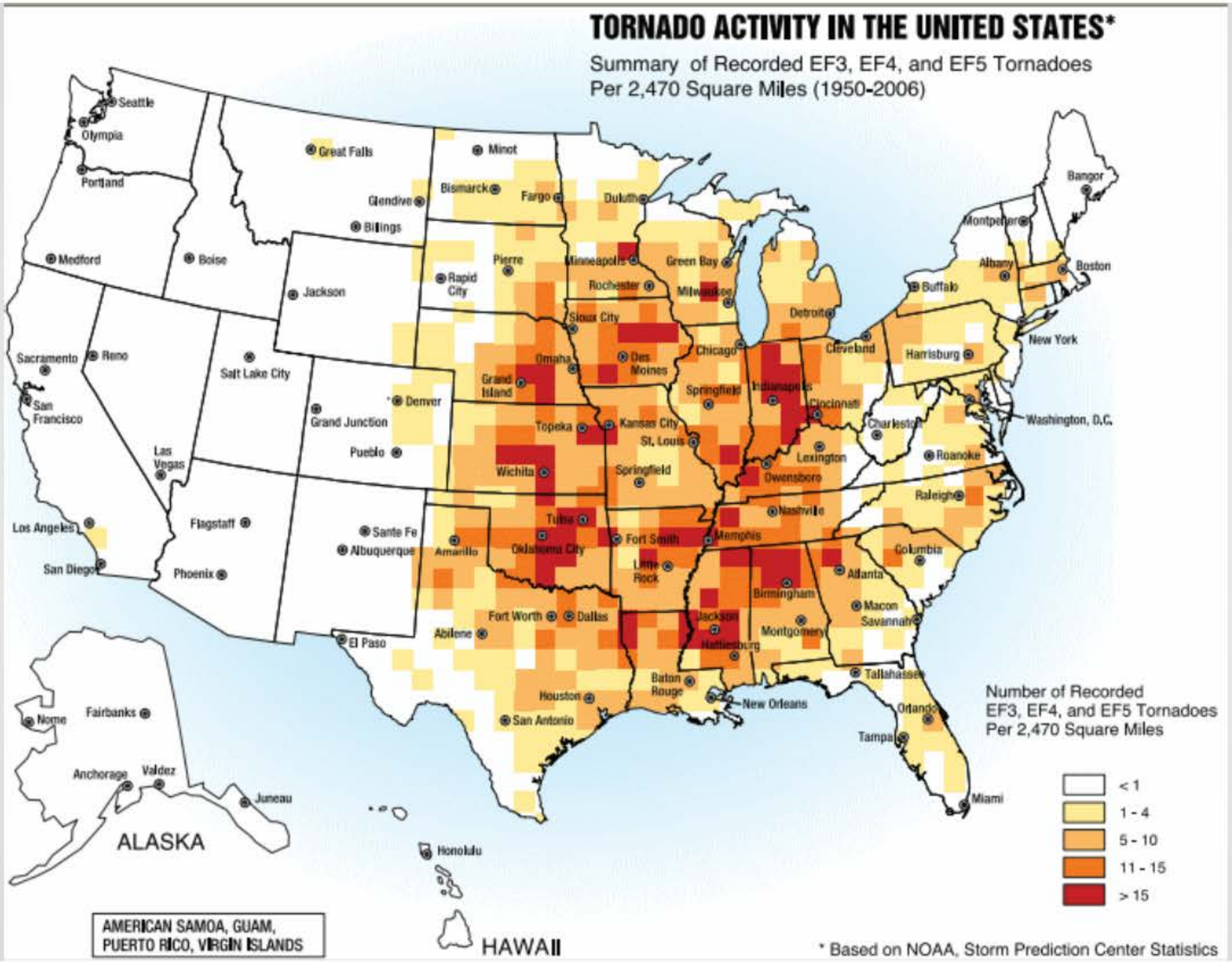
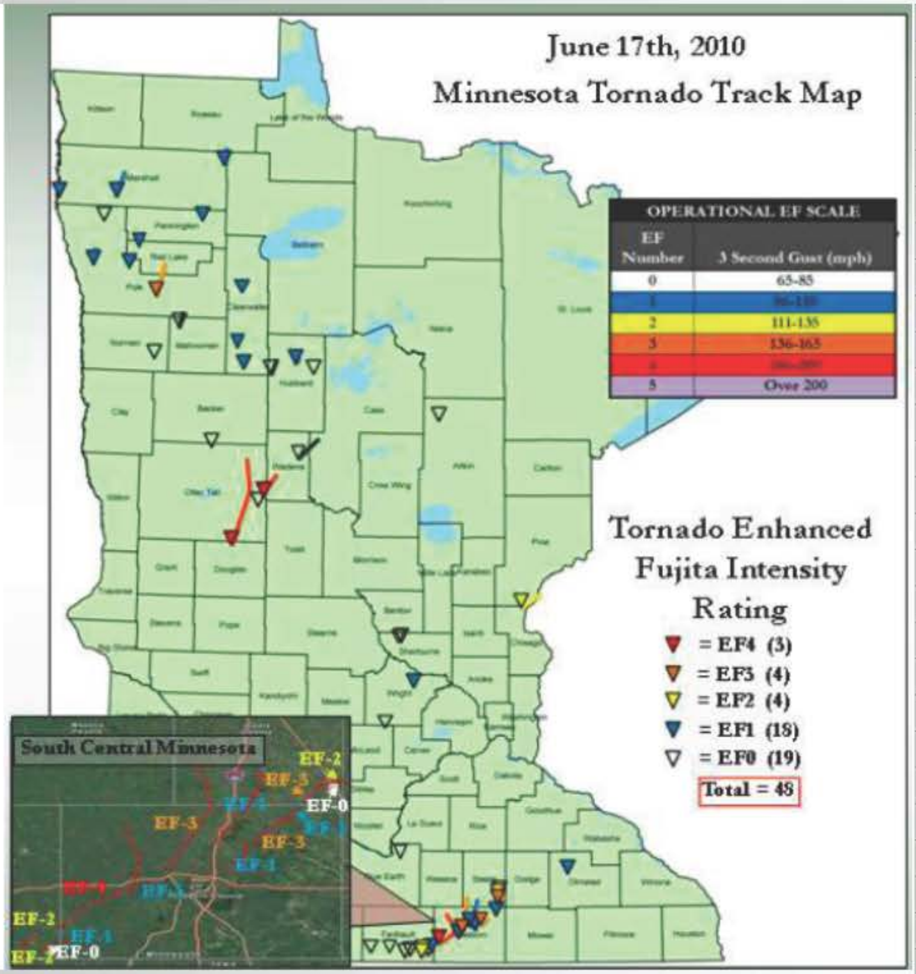
Why this research is important

Tornado 06/17/2010

Courtesy Minnesota Dept. of Natural Resources


TORNADO ACTIVITY IN THE UNITED STATES*
Summary of Recorded EF3, EF4, and EF5 Tornadoes
Per 2,470 Square Miles (1950-2006)
Seattle
Olympia
Portland
Medford
Boise
Great Falls
Glendive
Billings
Minot
Bismarck
Fargo
Duluth
Pierre
Rapid City
Jackson
Minneapolis
Green Bay
Rochester
Milwaukee
Sioux City
Omaha
Des Moines
Chicago
Detroit
Cleveland
Grand Island
Springfield
Indianapolis
Cincinnati
Denver
Topeka
Kansas City
St. Louis
Sacramento
Reno
Salt Lake City
San Francisco
Grand Junction
Pueblo
Las Vegas
Wichita
Springfield
Owensboro
Lexington
Charleston
Roanoke
Raleigh
Nashville
Tulsa
Los Angeles
Flagstaff
Santa Fe
Albuquerque
Amarillo
Oklahoma City
Fort Smith
Memphis
Columbia
San Diego
Phoenix
Little Rock
Atlanta
Birmingham
Fort Worth
Dallas
Macon
Savannah
Abilene
Jackson
Montgomery
Hattiesburg
El Paso
Tallahassee
Baton Rouge
Houston
New Orleans
San Antonio
Orlando
Tampa
Miami
Bangor
Montpelier
Albany
Boston
Buffalo
New York
Harrisburg
Washington, D.C.
Nome
Fairbanks
Anchorage
Valdez
Juneau
ALASKA
Honolulu
HAWAII
AMERICAN SAMOA, GUAM, PUERTO RICO, VIRGIN ISLANDS
Number of Recorded EF3, EF4, and EF5 Tornadoes Per 2,470 Square Miles
< 1
1 - 4
5 - 10
11 - 15
> 15
* Based on NOAA, Storm Prediction Center Statistics



DI No.	Damage Indicator (DI)	Degrees of Damage (DOD)
1	Small Barns or Farm Outbuildings (SBO)	8
2	One- or Two-Family Residences (FR12)	10
3	Manufactured Home – Single Wide (MHSW)	9
4	Manufactured Home – Double Wide (MHDW)	12
5	Apartments, Condos, Townhouses [3 stories or less] (ACT)	6
6	Motel (M)	10
7	Masonry Apartment or Motel Building (MAM)	7
8	Small Retail Building [Fast Food Restaurants] (SRB)	8
9	Small Professional Building [Doctor's Office, Branch Banks] (SPB)	9
10	Strip Mall (SM)	9
11	Large Shopping Mall (LSM)	9
12	Large, Isolated Retail Building [K-Mart, Wal-Mart] (LIRB)	7
13	Automobile Showroom (ASR)	8
14	Automobile Service Building (ASB)	8
15	Elementary School [Single Story; Interior or Exterior Hallways] (ES)	10
16	Junior or Senior High School (JHSH)	11
17	Low-Rise Building [1–4 Stories] (LRB)	7
18	Mid-Rise Building [5–20 Stories] (MRB)	10
19	High-Rise Building [More than 20 Stories] (HRB)	10
20	Institutional Building [Hospital, Government or University Building] (IB)	11
21	Metal Building System (MBS)	8
22	Service Station Canopy (SSC)	6
23	Warehouse Building [Tilt-up Walls or Heavy-Timber Construction] (WHB)	7
24	Electrical Transmission Lines (ETL)	6
25	Free-Standing Towers (FST)	3
26	Free-Standing Light Poles, Luminary Poles, Flag Poles (FSP)	3
27	Trees: Hardwood (TH)	5
28	Trees: Softwood (TS)	5

June 16-18th 2010 Storm

EF0	EF1	EF2	EF3	EF4	EF5	Total
48	28	9	4	4	0	93

Original Fujita Scale : Wind

Speed = $6.3(F+2)^{1.5}$, F0: 18 m/s

F4: 92 m/s, F5: 117 m/s, F12: 330 m/s

- 1st tornado in South Dakota, Last touched town in Iowa
- 4 fatalities in Minnesota alone, highest in MN since July 5th 1978
- $117.7 Million in damage

Why is it important to know energy distribution in a tornado vortex?

Energy of a Thunderstorm : $10\ \text{km} \times 10\ \text{km} \times 15\ \text{km}$

Average Density: $0.75\ \text{kg/m}^3$, Mass $\approx 1.125\times10^{12}\ \text{kg}$

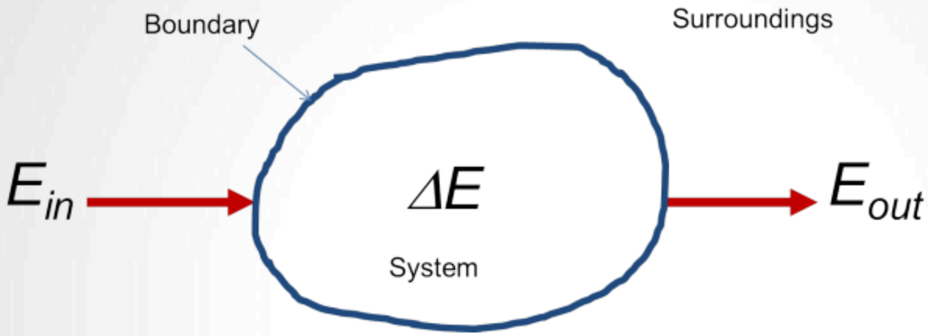
CAPE $\approx 1.5\times10^{3}\ \text{J/kg}$, Energy $\approx 1.125\times1.5\times10^{15}\ \text{J} = 1.687\ \text{TJ}$

CAPE vs Latent Heat Release :

CAPE $\approx 1.5\ \text{kJ/kg}$,

Latent Heat $\approx 2260\ \text{kJ/kg}$

Thermodynamic System



Equilibrium Systems vs Non-Equilibrium Systems

Air Parcel in a Tornado Vortex can be thought of as a Thermodynamic System

Equilibrium and 4 Laws of Thermodynamics

0th Law: Absolute Temperature T exists

1st Law: Internal Energy U exists

$\delta Q = dU + \delta W, \quad \delta W = PdV$

2nd Law: Entropy S exists

$$dS = \frac{\delta Q}{T}$$

3rd Law: $S \to 0$ as $T \to 0$

$$dU = TdS - PdV = \frac{\partial U}{\partial S} dS + \frac{\partial U}{\partial V} dV$$

$$= \langle T, -P \rangle \cdot \langle dS, dV \rangle = \text{forces} \cdot \text{fluxes}$$

$U = U(S,V)$

S,V: **extensive**

T,P: **intensive**

Helmholtz Free Energy:

$F = U - TS$

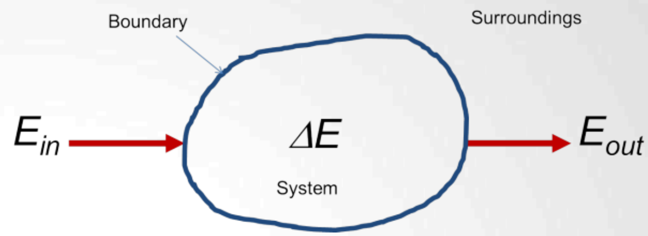
Units for these functions: J

$$dF = dU - TdS - SdT = TdS - PdV$$

$$-TdS - SdT = -PdV - SdT$$

$F = F(T,V)$!!!!

Non - Equilibrium Thermodynamics



Local Hypothesis : Small air parcel is in state of equilibrium for a short period of time. So thermodynamic variables will depend on position $\boldsymbol{x}$ and time t

Helmholtz Free Energy :

$$F(\boldsymbol{x}, t) = U(\boldsymbol{x}, t) - T(\boldsymbol{x}, t)S(\boldsymbol{x}, t)$$

Note the units for these functions are J/kg

so F is a local quantity (free energy **density**)

We wish to use Local Equilibrium Idea to test the influence of tornado parameters on Helmholtz free energy

1. C. A. Doswell, D. M. Schultz, On the Use of Indices and Parameters in Forecasting Severe Storms, E-Journal of Severe Storms Meteorology, **Vol. 3, No. 3, 2006**

2. G. P. Bystrai, I. A. Lykov, S. A. Okhotnikov, Thermodynamics of nonequilibrium processes in a tornado: synergistic approach, **http : / / arxiv.org / pdf / 1109.5019v1.pdf, 09 / 23 / 2011**

Rate Equation for the Free Energy Density $F = U - TS = F(T,V)$

The vortex layer at the height h is a non-equilibrium air parcel with

$$\hat{F}(t) = F(\theta(t, \boldsymbol{\xi}(t)), V(t), t)$$

$$\frac{d\hat{F}}{dt} = \frac{\partial F}{\partial \theta}\frac{\partial \theta}{\partial t} + \frac{\partial F}{\partial \theta}\frac{\partial \theta}{\partial \boldsymbol{\xi}} \cdot \frac{d\boldsymbol{\xi}}{dt} + \frac{\partial F}{\partial V}\frac{dV}{dt} + \frac{\partial F}{\partial t}$$

$$d\hat{F} = \ldots + \frac{\partial F}{\partial \theta}\frac{\partial \theta}{\partial \boldsymbol{\xi}} \cdot d\boldsymbol{\xi} + \frac{\partial F}{\partial V} dV + \ldots$$

Intensive ("Forces"): $\frac{\partial F}{\partial \theta}\frac{\partial \theta}{\partial \boldsymbol{\xi}}$, Extensive ("Fluxes"): $d\boldsymbol{\xi}$

Thermodynamic Fluxes and Thermodynamic Forces

$$\frac{d\hat{F}}{dt} = \frac{\partial F}{\partial \theta}\frac{\partial \theta}{\partial t} + \frac{\partial F}{\partial \theta}\frac{\partial \theta}{\partial \boldsymbol{\xi}} \cdot \frac{d\boldsymbol{\xi}}{dt} + \frac{\partial F}{\partial V}\frac{dV}{dt} + \frac{\partial F}{\partial t}$$

$$\frac{d\hat{F}}{dt} = -S\cancel{\frac{\partial \theta}{\partial t}} - P\cancel{\frac{dV}{dt}} - \theta \boldsymbol{X} \cdot \boldsymbol{J} - \cancel{\theta\sigma}$$

$\boldsymbol{J} = -\dfrac{d\boldsymbol{\xi}}{dt}$ is a vector of thermodynamic **fluxes**

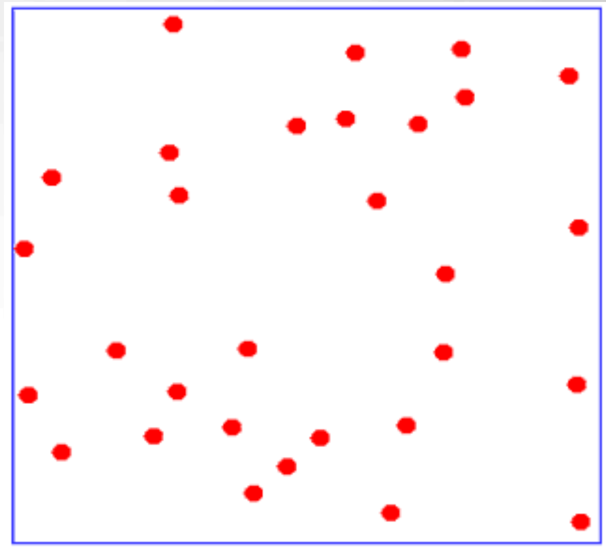
$\boldsymbol{X}$ is a vector of thermodynamic **forces**

Onsager Relations:

$\boldsymbol{J} = L\boldsymbol{X}$, L is a matrix

$$\frac{d\hat{F}}{dt} = -\theta \boldsymbol{X} \cdot \boldsymbol{J}, \qquad \boldsymbol{X} = (\text{CAPE, SRH}), \quad \boldsymbol{J} = L\boldsymbol{X}$$

Moist Air is a homogeneous mixture of **Dry air** and **Water Vapor,** we assume that both are **Ideal Gases**



$$\rho_m = \rho_d + \rho_v$$

$$P_m = P_d + P_v = \rho_d R_d T + \rho_v R_v T = \rho_d R_d T \left(1 + \frac{R_v}{R_d}\frac{\rho_v}{\rho_d}\right)$$

$$= \rho_m R_d T \left(1 + \left(\frac{R_v}{R_d} - 1\right)\frac{\rho_v}{\rho_m}\right) = \rho_m R_d \theta$$

θ is **Virtual Temperature :** Shows how a 2 - gas system (dry air and water vapor) would behave if it was a 1 - gas system

Indeed,

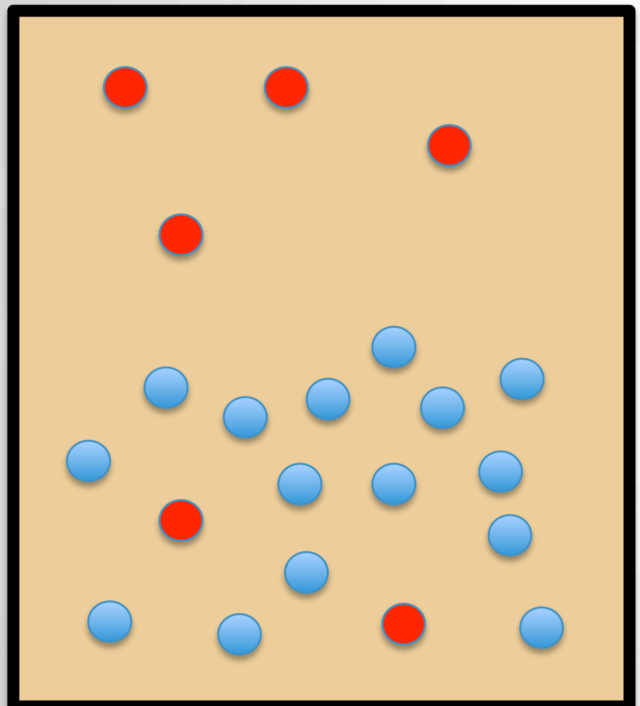
$$\rho_m R_d \theta = \rho_m R_d T\left(1+\left(\frac{R_v}{R_d}-1\right)\frac{\rho_v}{\rho_m}\right)$$

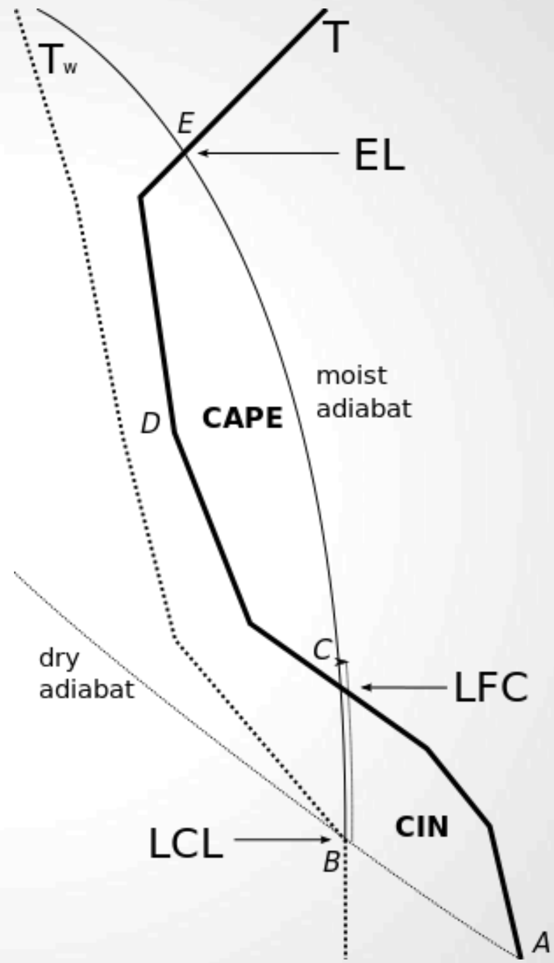
$$= \rho_m R_d T + R_d T\,\frac{R_v - R_d}{R_d}\rho_v = \rho_m R_d T + T\left(R_v - R_d\right)\rho_v$$

$$= \left(\rho_d + \rho_v\right)R_d T + T\left(R_v - R_d\right)\rho_v = \rho_d R_d T + \rho_v R_v T$$

$$= P_d + P_v = P_m$$

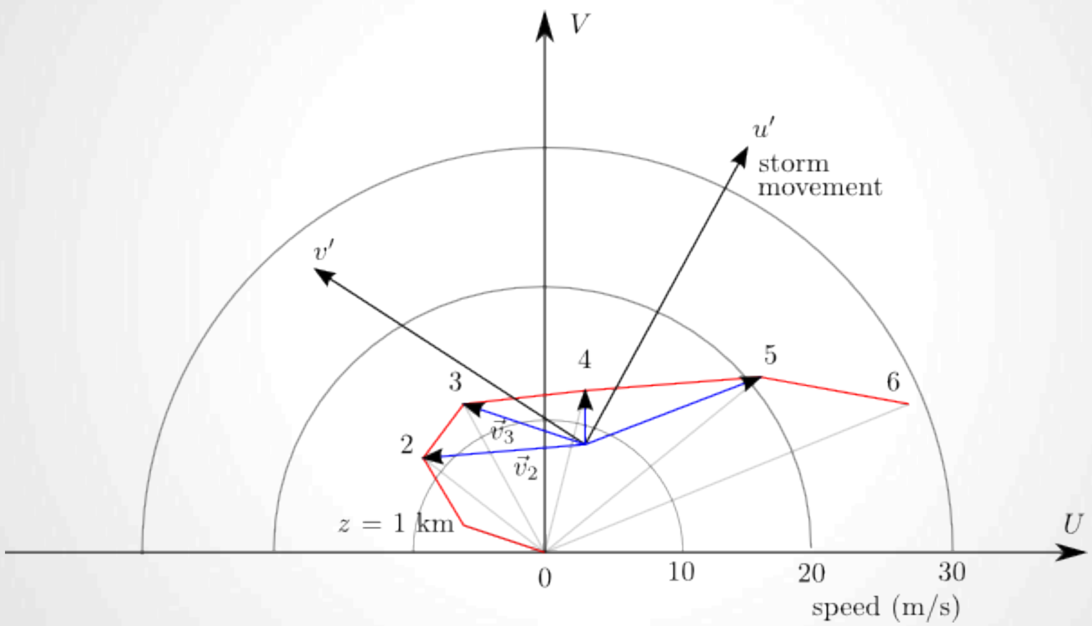
$$\mathbf{CAPE} = \int_{z_1}^{z_2} \frac{\theta - \theta_{env}}{\theta_{env}} g \; dz$$





SRH (Storm Relative Helicity)

$H = \boldsymbol{u} \cdot \boldsymbol{\omega}, \quad \boldsymbol{\omega} = \nabla \times \boldsymbol{u}, \quad$ so H is a local quantity

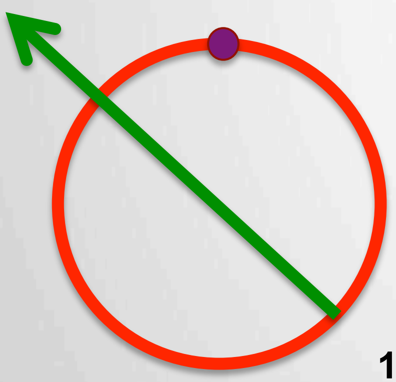


Integrated Version: $HI \approx v\Delta u$ in $\mathrm{m^2/s^2 = J/kg}$

[Soundings]

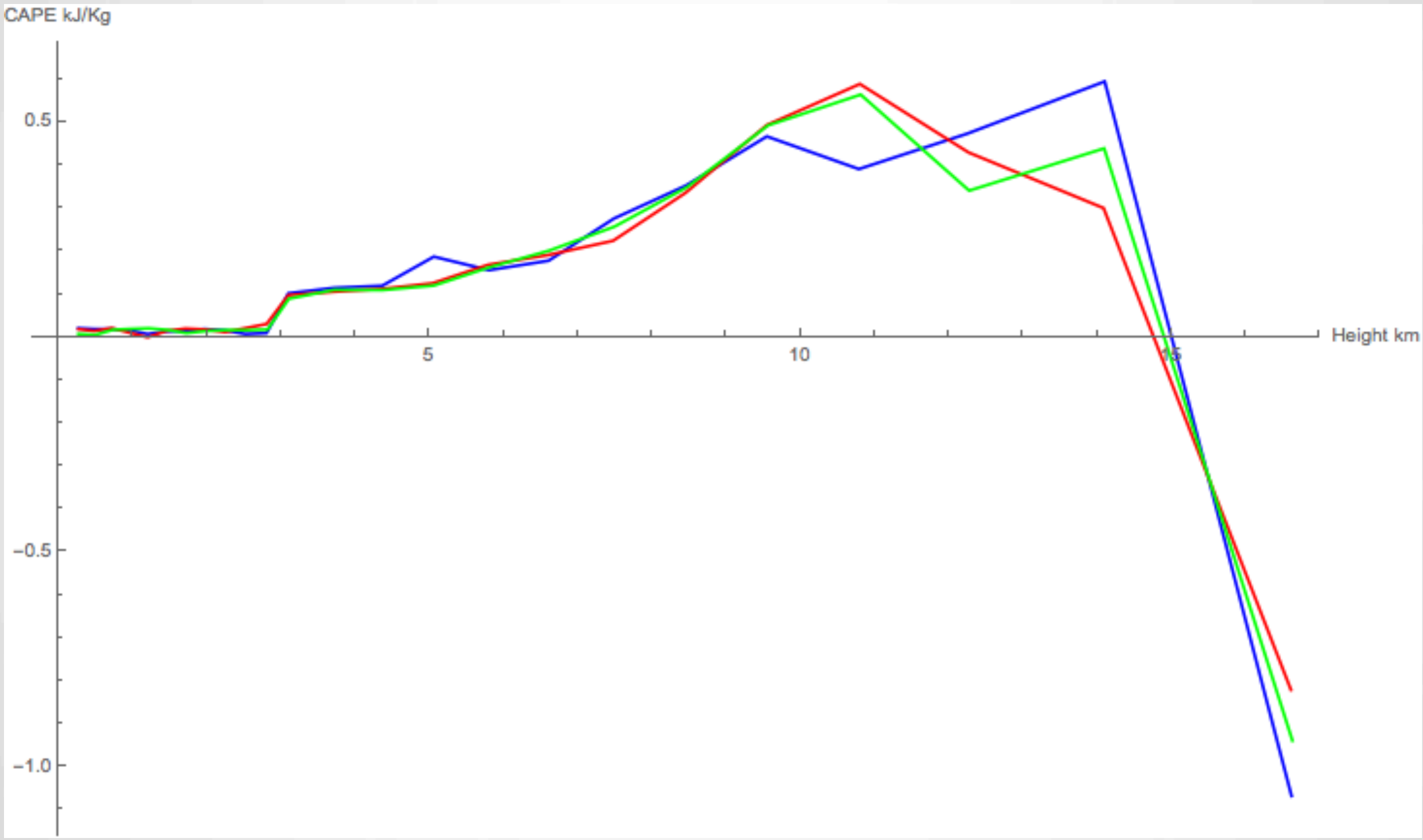
06/17/2010
Minneapolis
Station
7:00 PM

[Indices]

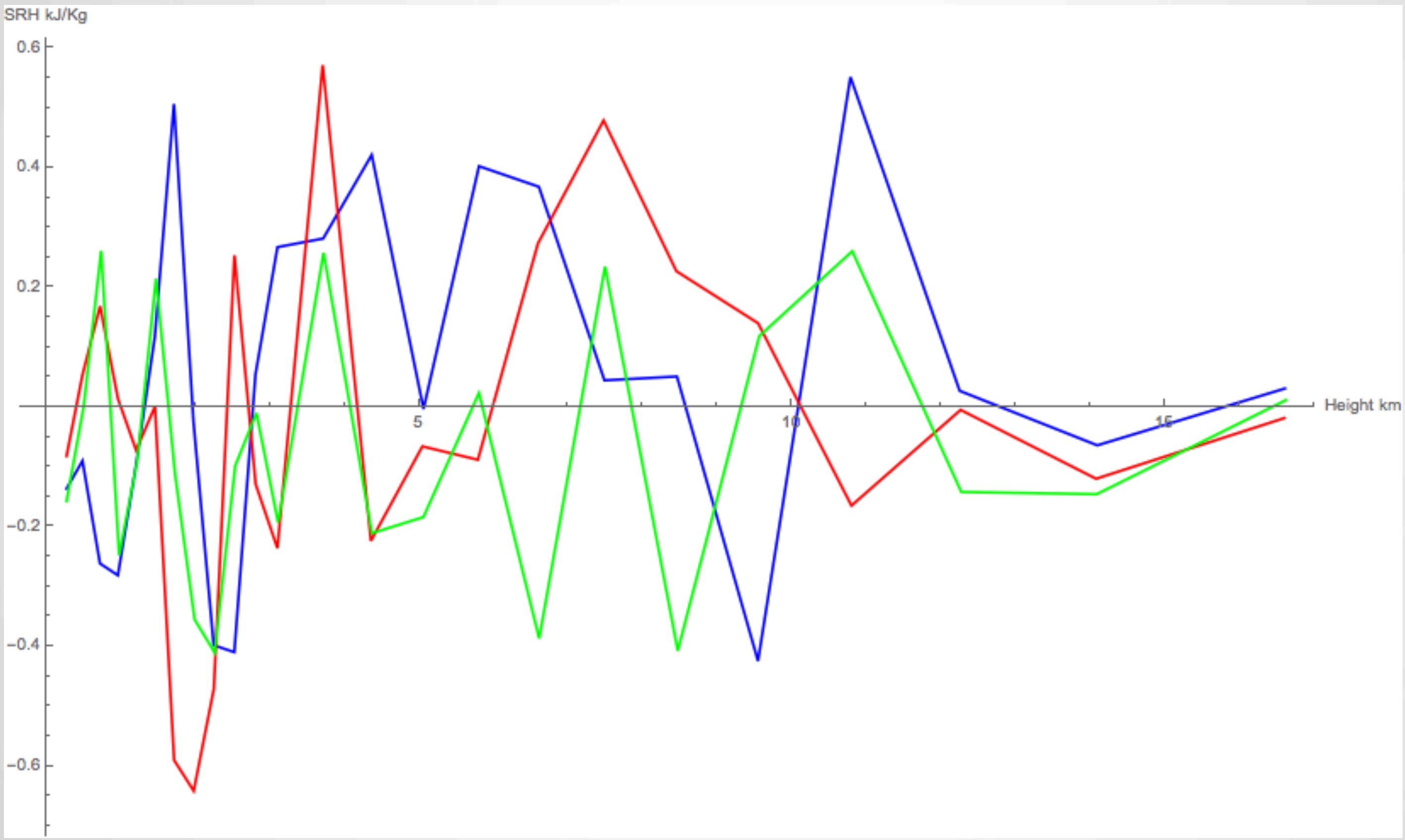


PR HPA	HGT M	TE C	DEW C	W D DEG	W S M/S
973.	270.	29.1	20.8	176.3	8.7
950.	484.	26.5	19.6	164.8	13.4
925.	720.	24.7	20.8	167.8	17.4
900.	960.	22.1	19.6	171.6	20.4
875.	1204.	20.9	13.3	177.9	22.5
850.	1455.	21.0	8.8	184.8	23.7
825.	1713.	19.4	7.0	191.1	24.4
800.	1976.	17.2	-0.7	200.4	25.5
775.	2246.	15.3	-8.5	209.8	25.6
750.	2522.	14.1	-8.4	213.3	23.5
725.	2807.	12.0	-5.7	212.7	23.7
700.	3099.	9.0	-3.6	219.5	24.7
650.	3706.	4.4	-2.2	232.5	24.8
600.	4353.	-0.2	-4.9	237.4	23.0
550.	5044.	-4.7	-8.2	240.4	24.3

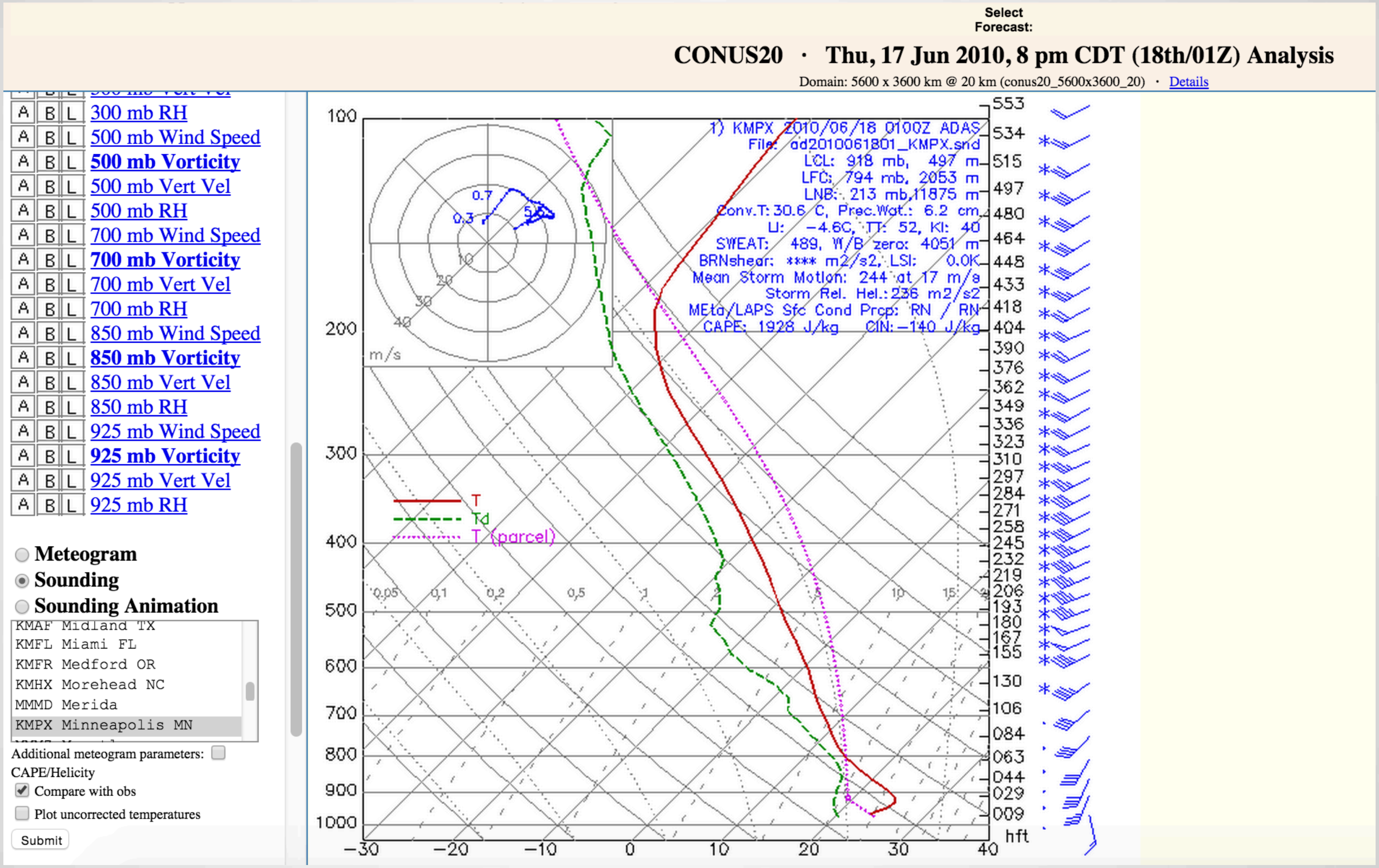
CAPE. Blue: 4 PM, Red: 7 PM, Green: 10 PM



SRH. Blue: 4 PM, Red: 7 PM, Green: 10 PM



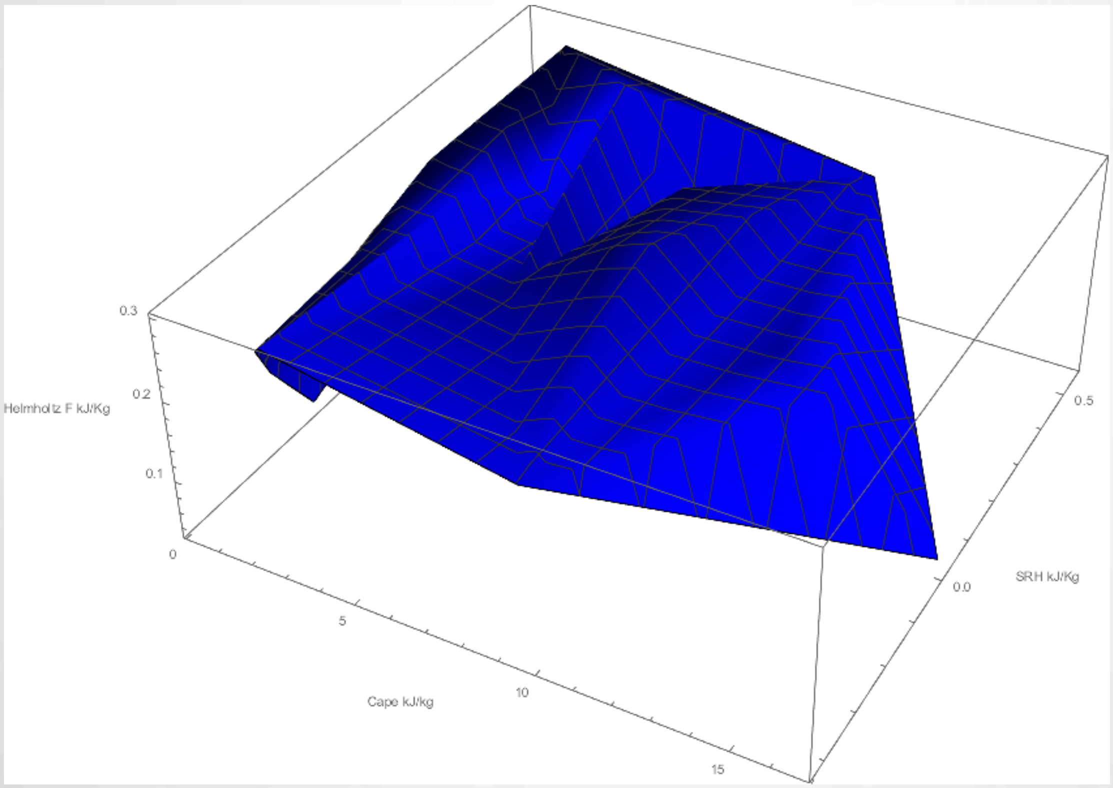
CAPS



Select Forecast:
CONUS20 · Thu, 17 Jun 2010, 8 pm CDT (18th/01Z) Analysis
Domain: 5600 x 3600 km @ 20 km (conus20_5600x3600_20) · Details
300 mb RH
500 mb Wind Speed
500 mb Vorticity
500 mb Vert Vel
500 mb RH
700 mb Wind Speed
700 mb Vorticity
700 mb Vert Vel
700 mb RH
850 mb Wind Speed
850 mb Vorticity
850 mb Vert Vel
850 mb RH
925 mb Wind Speed
925 mb Vorticity
925 mb Vert Vel
925 mb RH
Meteogram
Sounding
Sounding Animation
KMAF Midland TX
KMFL Miami FL
KMFR Medford OR
KMHX Morehead NC
MMMD Merida
KMPX Minneapolis MN
Additional meteogram parameters:
CAPE/Helicity
Compare with obs
Plot uncorrected temperatures
Submit
1) KMPX 2010/06/18 0100Z ADAS
File: ad2010061801_KMPX.snd
LCL: 918 mb, 497 m
LFC: 794 mb, 2053 m
LNB: 213 mb,11875 m
Conv.T: 30.6 C, Prec.Wat.: 6.2 cm
LI: -4.6C, TT: 52, KI: 40
SWEAT: 489, W/B zero: 4051 m
BRNshear: **** m2/s2, LSI: 0.0K
Mean Storm Motion: 244 at 17 m/s
Storm Rel. Hel.: 236 m2/s2
MEta/LAPS Sfc Cond Prcp: RN / RN
CAPE: 1928 J/kg CIN: -140 J/kg
T
Td
T (parcel)
m/s
hft

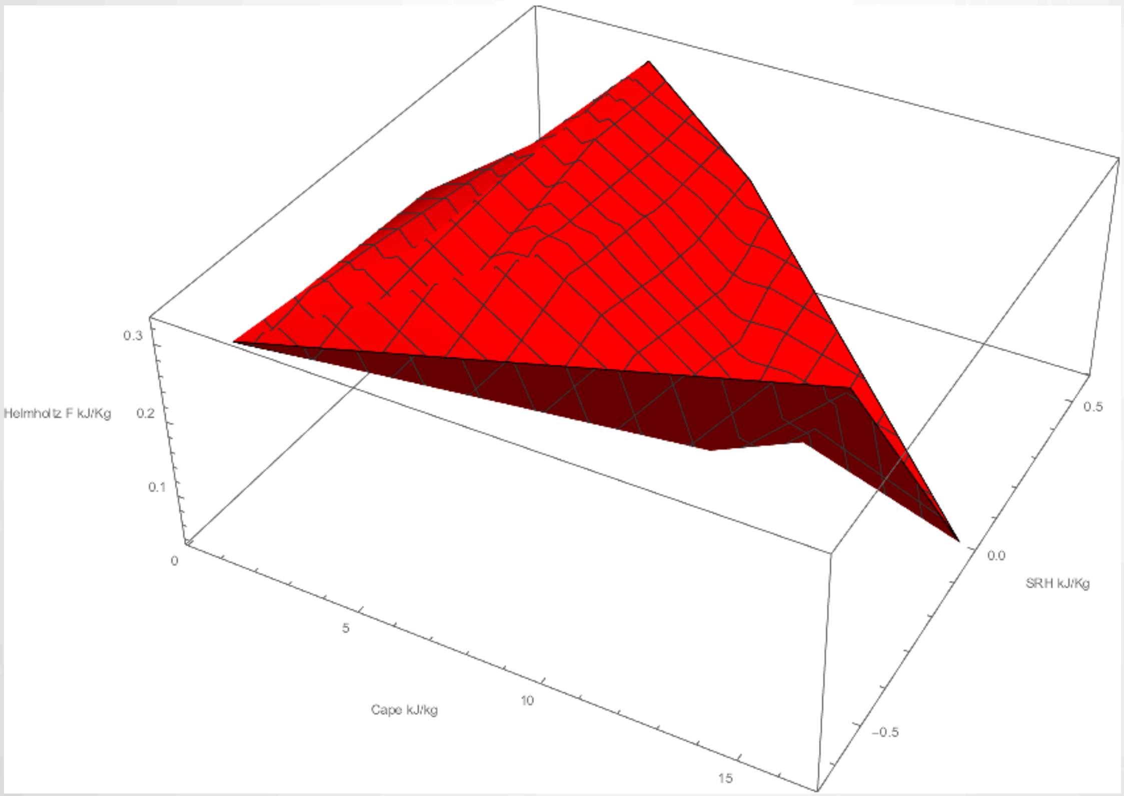
Helmholtz free energy as a function of **CAPE** and **SRH**

06 / 17 / 2010   4 PM   Minneapolis Storm



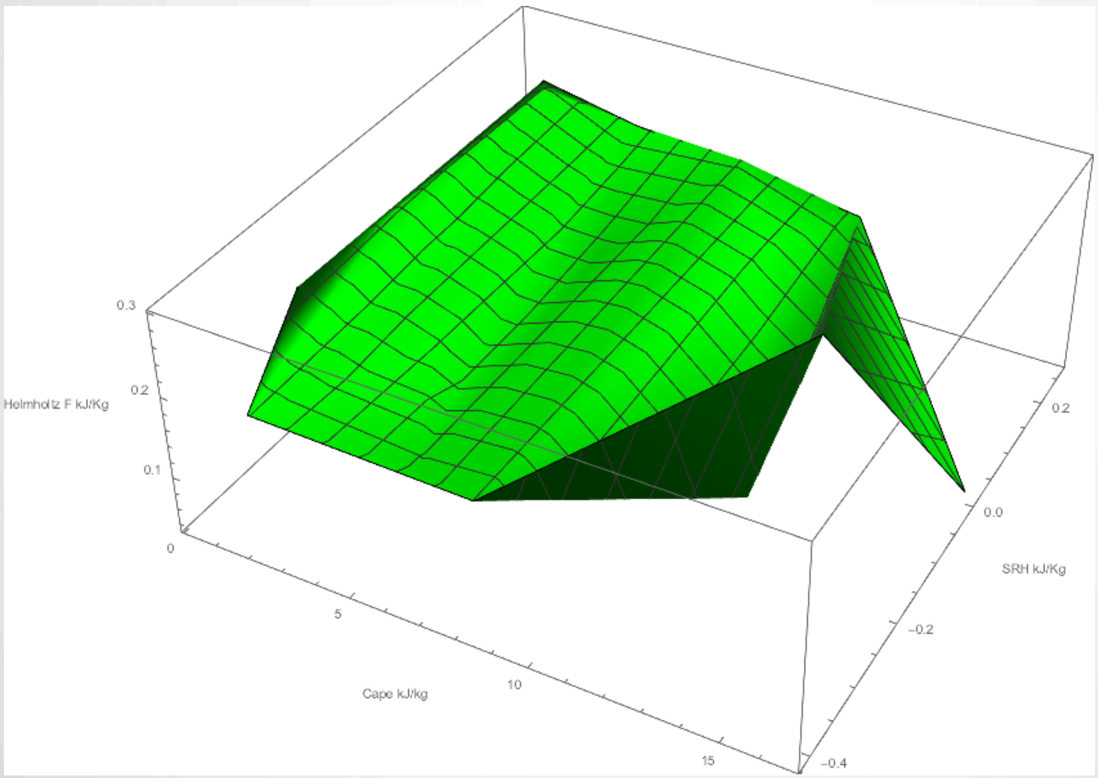
Helmholtz free energy as a function of **CAPE** and **SRH**

06 / 17 / 2010 7 PM Minneapolis Storm



Helmholtz free energy as a function of **CAPE** and **SRH**

06 / 17 / 2010 10 PM Minneapolis Storm



In Bystrai's et al. paper the estimate of

$$\frac{d\hat{F}}{dt} = -S\frac{\partial\theta}{\partial t} - P\frac{dV}{dt} - \theta \boldsymbol{X} \cdot \boldsymbol{J} - \theta\sigma$$

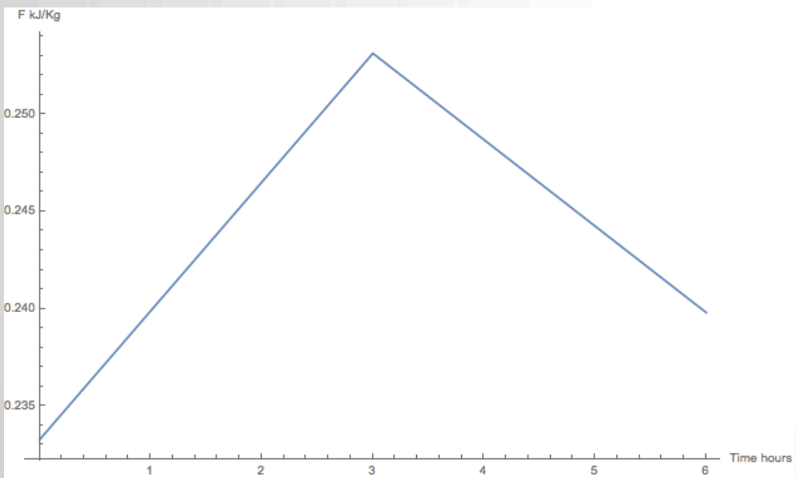
for a typical tornado is given by

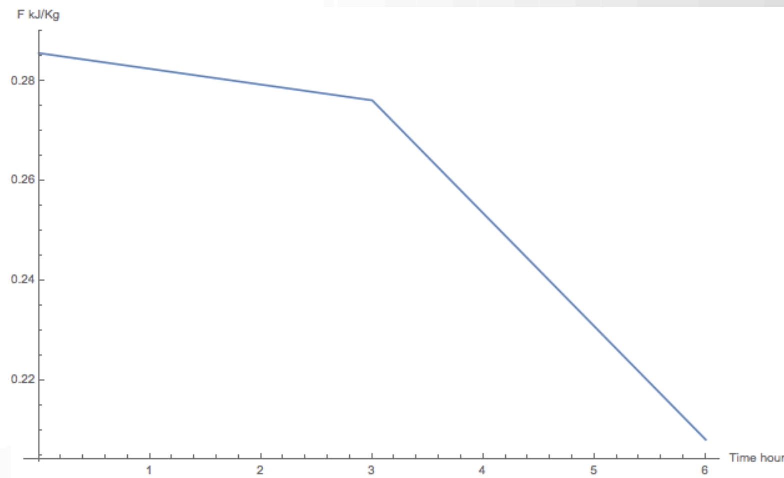
$$0.01 \le \left|\frac{d\hat{F}}{dt}\right| \le 2.0 \quad \frac{J}{kg\ s}$$

Our data is spaced by 3 hours so the time derivative estimate for $\frac{d\hat{F}}{dt}$ is unlikely to be accurate

Helmholtz Free Energy Density F as a function of time t

$h = 1200\ m$ $h = 2500\ m$





Future Plans

1. To calculate the CAPE using precise virtual temperature to avoid distorted values at the low CAPE values
2. To refine the calculation for $\hat{F}(t)$ taking into consideration the updraft component of the flow
3. Evaluate additional parameters such as EHI, STP, LI and their ability to influence $\hat{F}(t)$

Fin