Slope of Vorticity Lines Derived from Numerical Models as a Tornado Predictor

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1 Introduction

One can estimate the relationship between vorticity and the length scale of tornadic thunderstorms using Doppler radar data. Case studies have indicated that there may be values of exponents that are strongly correlated with thresholds of tornado activity. In a recent paper [8], Cai defines the pseudovorticity by $\zeta_{pv} = \frac{\Delta V}{L}$ where $\Delta V = |(V_r)_{max} - (V_r)_{min}|$ is the difference between the maximum and minimum radial velocity of the mesocyclone (rotating updraft) and L is the distance between them. Using mobile Doppler radar data from past tornadic and non-tornadic storms, for fixed times Cai filtered the data from the highest resolution to the smallest resolution determined by the diameter of the mesocyclone (ϵ is the finest resolvable scale of the filtered radar data) to obtain data points $(\ln(\epsilon), \ln(\zeta_{pv}))$. He then calculated the regression line for each storm and found that the steeper slopes (smaller negative values) are indicative of tornadic storms and the threshold for the slope of strong tornados was approximately m = -1.6. Cai′s study comparing mobile Doppler radar data from tornadic and non-tornadic storms indicates that the steeper slopes (smaller negative values) are indicative of tornadic storms. As those mesocyclones that produced tornados become stronger approaching tornadogenesis the slope of the line decreased. Cai found the threshold for strong tornados was slope $m = -1.6$. For tornadic mesocyclones, this suggests a power law of the form, $\zeta \propto r^b$, where r is the radius of the vortex. Cai observed that the exponent can be thought of as a fractal dimension associated with the vortex. For high-resolution mobile Doppler radar data, there has been some attempt to interpret this as a giving a power law for the drop-off of the velocity as a function of the radius of the vortex. In several papers devoted to analyzing mobile radar data associated to strong or violent tornados Josh Wurman computed $v \propto r^b$. Wurman(2000) [36] has calculated the exponent b in $v \propto r^b$ from high resolution mobile Doppler radar data obtained from a tornado, rated F2-F4, and found b varying from -0.5 to -0.7. From data he obtained in the intercept of the Spencer, South Dakota tornado (May 31, 1998), Wurman (2005) calculated [35] the value of b=-0.67. The tornado was rated EF4. These values were calculated from the data taken at one instant during the tornados existence. There may have been considerable variation during the tornados lifespan. It is interesting to note that the (threshold) power law for vorticity in strong tornados given by Cai differs from the power law Wurman calculated for the velocity of the two strong/violent tornados by 1. This is consistent with the vorticity being the derivative of the velocity. Hence this suggests that the results are consistent. Power laws suggest self similarity. We agree that to truly understand strong atmospheric vortices their self similarity is a fact that needs to be exploited. Using *Mathematica* we revisited Serrin′s

"Swirling Vortex" model (Serrin 1972) [31] and investigated solutions to the Navier Stokes and Euler equations in spherical coordinates, with $v \propto r^b$ where b is not necessarily -1 (Belik, et. al.) [4]. Recent studies of radar data (Markowski 2008) [?] and numerical simulations (Straka 2007) [32]reveal arching vortex lines (or vortex tubes) in the rear flank of supercell storms. These appear to be correlated with tornadogenesis. As more and more vortex lines enter this region, viscous interactions between neighboring vortex lines are believed to lead to mergers. This should also lead to a strengthening of the vortex and increase in the vorticity as well. Several theories have been given for the production of the arching vortex lines (Markowski 2008; Straka 2007). One theory involves the production of vorticity along the edge of the rear flank gust front. These vortex lines are captured by the updraft and tilted vertically. As this tilting occurs, stretching creates a pressure drop in the near ground vortex that draws air down and pins the vortex to the ground. In a second theory the origin of the vorticity is in the shear between different layers in the rear-flank downdraft. The vortex line is pulled in opposite directions, up by the updraft into the mesocyclone, and down to the ground on adjacent sides by the downdraft creating an arching vortex. It seems reasonable to assume a combination of these two processes should be present, with a reconnection of vortex lines produced by the two different processes. It also apears possible that surface winds interacting (transversly across) with a gust front (rear flank) could also result in vertical shear as in Lee (1997a, 1997b, 2000) [24], [25], [26]. This would create a vortex sheet that could be stretched and roll-up into a tornado vortex. However there may be other sources as well. We believe that the fractal dimension of 1.6 that Cai found in his study and have also been observed in several other studies comes from the interactions of vortices prroduced in these shear regions due to Kelvin-Helmholtz instability. The observation of Trapp(1999) [33] that "parcels that nearly conserve angular momentum penetrate closer to the central axis of the tornadic mesocyclones, resulting in large tangential velocities" supports this observation.

The paper is organized as follows. In §2 we describe the tornadogenesis problem, and results on Kelvin Helmholtz instability due to Baker and Shelley. In §4 we address geometric self-similarity of tornados in radar data. We also describe Cai's power law for strong tornados and give a heuristic argument supporting it.

In §3 we discuss the vortex gas theory in two and three dimensions of Onsager and Chorin, and give arguments for its role in modeling tornadogenesis and tornado maintenance. In §5 we discuss suction vortices. In section §6 we give conclusions. In §7 we give describe future work.

2 Tornadogenesis

The search to understand tornadogenesis invariably leads to the question "Where does the vorticity in the tornado originate?" The most penetrating studies of this question have lead to the study of two types of vorticity: barotropic vorticity and baroclinic vorticity. Barotropic vorticity is vorticity that exists in the ambient enviornment and is frozen in the fluid and stretched and advected by the fluid. Baroclinic vorticity is vorticity is that is generated by density currents in the fluid and is stretched and advected by the fluid. Definitive discussions of the role of barotropic and baroclinic vorticity in tornadogenesis and the mathematical decomposition of vorticity into barotropic and baroclinic parts and its consequences are given in a series of papers by Davies-Jones(1982, 2000, 1996, 2006a, 2006b, 2008) [12], [14], [13], [15], [16], [17]. Both types of vorticity or combinations thereof have been suggested as the origins of the vorticity in tornadogenesis (cite references). Based on film footage of tornados showing sheets of precipitation spiraling into tornados, Fujita suggested a "barotropic" method called "Fujita's recycling hypothesis" in (Fujita 1973, 1975). In this process the precipitation falling near the interface between the updraft and downdraft dragged vorticity to the surface into the tornado and the precipitation rich air was recycled into the thunderstorm updraft by the tornado. This hypothesis was explored in a paper by Davies-Jones (2008), and papers by Marcowski, et.al., (2003) [?] . The numerical model and experiment of Davies-Jones (2008) shows that tornadogenesis can take place by a purely barotropic process. Numerical studies have also found baroclinic vorticity can be important if not dominant in tornadogenesis in a recycling type process as well. Additional evidence supporting the recycling hypothesis is the observation that downdrafts associated to tornadic storms are generally warmer than downdarfts associated to storms that were nontornadic this would make the updraft more buoyant in the tornado producing storms and less buoyant in storms that do not produce tornados (Markowski, 2002a). Numerical and radar studies of supercell thunderstorms storms have shown the existence of arching vortex lines in the vicinity of the rearflank downdraft and mesocyclone (Markowski 2008; Straka 2007) [**?**], [32]. However the process by which the arching vortex lines form, whether barotropic, baroclinic, or some combination of the two is a matter of investigation and may vary as a storm evolves.

Observational analysis of videos of the tornadogenesis phase in large diameter tornados forming under low cloud bases, suggests vortices (vortex lines) which enter the developing tornado make a partial revolution about the ambient tornado vortex before *kinking*. This *kinking* is necessary to conserve energy as the vortex stretches and/or interacts with other vortices (Chorin 1994) [9]. In this process some energy is transmitted too much smaller scales, the so-called inertial range, beginning the Kolmogorov cascade to the viscous range and then dissipating as heat. However, before kinking up, as the vortex stretches much energy is transmitted too the ambient vortex as kinetic energy of the flow and this increases the vorticity of the tornado. As more and more vortices successively enter the developing tornado this process repeats itself many times gradually increasing the vorticity of the tornado vortex. This raises the vorticity of the ambient vortex (assuming the vortex lines are produced uniformly) eventually achieving quasi-equilibrium with its environment. This process transfers energy from the smaller scales to the larger scales (inverse cascade). As more vortex lines enter the ambient tornado vortex, large vortices tend to form; the stronger vortices at the core and slightly weaker vortices wrapping around them. This could manifest itself as multiple vortices or as a large single vortex. The stretching of the vortices that enter the tornado eventually leads to the dissipation of their vorticity in a Kolmogorov cascade.

In a study of Kelvin-Helmholtz instability, Baker and Shelly (1990) [3], considered a thin vortex layer of thickness h in a two dimensional model. Above the layer and below the layer the fluid flowed in opposite directions. As the flow proceeded the region between the two layers rolled up into a double-branched spiral shaped vortex, with an approximately elliptical cross-section (Kirchhoff ellipse) at its core. They identified a relationship between the thickness of the vortex sheet, h, and the cross sectional area of the vortices, A. They found A scales like $O(h^{1.55})$ as h goes to 0, specifically, $A = 8.58h^{1.55}$ and that the vorticity scales like $O(h^{-1})$ as h goes to 0. They observed the roll-up eventually changes the structure of the flow so that no new vorticity gets added to the core of the roll-up. They also comment that once the cores have formed they will interact with one another and in some cases form larger structures. From our perspective, the two dimensional model gives the cross sections of the vortex lines (tubes) that eventually arch and amalgamate together to form the tornado. For persistent shear layers, after a sequence of roll-ups, a sequence of new vortices would form. The new vortices would lie in a linear sequence in a thin vortex layer that would itself have local roll-ups just as the earlier layer did. Locally, the vortices from the first roll-up would wrap-around each other and roll-up into a new vortex. Thus forming a sequence of new-generation roll-up vortices made up of roll-ups of earlier generation roll-up vortices. By the Baker and Shelley result, the new roll-up vortices would satisfy the condition, A scales like $O(h^{1.55})$ were h is the thickness of the new layer. If the above process occurs repeatedly the result would be a self-similar fractal vortex. The vortex thus produced would be geometrically self-similar and would have the property that A scales like $O(h^{1.55})$. If the thin vortex layer bends, the layer locally looks like a plane and the above roll-up process will continue. This process creates structure in the cross-sections of the vortices that lie within the vortex layer. We assume that as the vortex layer arches and is lifted by the updraft and pinned or pulled down by the downdraft, these vortex lines within the vortex sheet become the arching vortex lines. After the tilting by the updraft, the vertical vortex layer may wrap up, it begins to form the tornado vortex which gradually strengthens as the roll-ups continue. One can regard this process as an energy cascade from smaller to larger scales. If this tilting into the vertical occurs where the storm relative velocity of the mean winds and the vorticity are nearly collinear the vortex lines will twist about one another creating a helical flow. Such vortices would be more resistant to dissipation due to stretching. Assuming self-similarity, it is tempting to think that the vorticity inside the layer scales like as $O(r^{-1})$, where r is the radius of the vortex. In fact this assumption is consistent with the weak tornados forming from some modes of non-supercell tornado genesis, assuming the vortex sheet does not roll-up and that it forms vertically due to shear instability.

One can distinguish two cases: one where the vortex layer forms in the vertical and does not roll up but the vortices in the layer do roll up (in this case the vortices acquire little helicity), and the other case where the vortex layer forms horizontally tilts into the vertical as this tilting occurs the vortices within it acquire helicity and the vortex sheet then rolls up and the vortices within the layer roll up as well.

Moffat (1969) showed that the twisting of the vortices is measured by the helicity of the vortex. Helicity is measured by the integral of the dot product of the velocity and the vorticity. It is thought that helicity of a flow inhibits the dissipation of energy and helps maintain the intensity of the flow (Levich,1985), (Lilly, 1983, 1986). The use of the helicity as a tool in studying supercell thunderstorms was in initiated by D. Lilly (1986) and R. Davies Jones (1984, 1990). It has been used in a slightly modified form as a parameter to study its effect on supercell storms (R. Davies Jones,1984, 1990), (Droegemeier, 1993). Lilly thought of a vortex as a coiled spring that unwinds as it stretches. If we think of the helicity as measuring how much the vortex is wound up, the stretching unwinds the spring and releases energy to the surrounding flow. This unwinding could manifest itself as vortex breakdown and/or the fractalization and kinking up that is predicted in the vortex gas theory (see below).

In an series of papers Lee and Wilhelmson (Lee 1997a, 1997b, 2000) studied non-supercell tornado genesis due to vertical shear in the boundary layer. They considered a weak cold pool (outflow boundary)advancing from the west into an ambient flow from the south to the north. This lead to a south to north oriented vortex sheet forming at the interface of the two flows. They noted first generation vortices rolling up into stronger second generation vortices. It seems plausible that if one did finer grid simulations of the situation considered by Lee and Wilhelmson, the first generation vortices would have formed from roll-ups at smaller scales, etc., resulting in a self-similar structure. The structure of the resulting vortex sheet resembled that of Baker and Shelley. For example, consider a thin vertical layer (south to north oriented) where winds are nonexistent or weakly from the north to the west of the layer and from the south to the east of the layer. Assume the height of the layer extends through the boundary layer. Under these circumstances a vortex sheet is produced that is vertical, if convection moves over the vertical vortex sheet tornadogenesis can occur due to stretching of vortices within the sheet. The first of three non-supercell tornados studied by Roberts (1995) was of this form. The tornados that form under these circumstances generally are weak. Looking down on the vortex sheet from above one sees a cross-section that resembles the set-up of Baker and Shelley (Baker 1990). In that paper (Baker 1990) the vorticity scales like $1/r$ where r is the thickness of the vertical layer. If the vortices in the layer have formed from a succession of roll-ups of smaller vortices in thinner layers, they will be self similar and the vorticity would scale like $1/r$ where r is the radius of the vortex. If the vortex is stretched, the vortex cross section will decrease and the vorticity will increase, hence the slope of the psuedo-vorticity (vorticity) line will decrease. Hence -1 provides an upper bound for the slope of the psuedo-vorticity line of the vortex sheet (as above) if it is stretched. While the cross-sectional area of the roll-up vortices scales like $r^{1.55}$. In Cai's paper the pseudo-vorticity for weak tornadoes scales like $r^{-1.02}$. Since non-supercell tornados are generally weak, the results of (Baker 1990) and Cai (2005) are consistent for weak non-supercell tornados forming as above. (However with sufficient stretching the non-supercell tornado vortices have achieved EF3 strength. (Roberts, 1995)) This suggests that the roll-up process that produces supercell tornados is different than the processes that produce the type of non-supercell tornados described above. It is natuaral to ask how the results of Lee and Wilhelmson might apply to tornado genesis in supercell storms? Radar (Bluestein, 2000, 2003a, 2003b) [6], [5], [7], (Dowell, 1997, 2002a, 2002b), [18], [19], [20] and observational analysis (Brandes, 1986), (Wakimoto, 1996), (Wilson, 1986) of supercell thunderstorms have shown vortices along the rear flank gust front of both tornadic and non-tornadic supercell storms. The role of these vortices in tornado formation is a matter of current research. Numerical simulations of supercell tornado genesis have shown vortices that appear to form along the edge of the rear flank gust front or a secondary gust front roll up into a tornado vortex. The positioning of the vortices with respect to the updraft appears to be crucial, in that stretching of vorticity appears to be a trigger for the rollup. This will occur if the vortices are under the updraft. Numerical experiments performed by A. Chorin (Chorin 1973)and Krasny (Krasny 1995), suggest that a vortex sheet containing a finite linear sequence of equally spaced planar cyclonically rotating mutually interacting vortices can roll up into a cyclonic spiral vortex sheet. In the models of Chorin [10] and of Krasny, the vortices move under the cumulative influence of their neighbors. The trigger for tornado formation observed in (Wakimoto, 1996) was stretching of a vortex along the rear flank gust front that moved under a strong updraft in the flanking line. The tornado formation appeared to be delayed from the stretching event and the tornado appeared to have originated in the boundary layer. Observations of tornado forming near Bassett, Neb., (Bluestein, 2000) suggested that interaction of a larger vortex(500m scale) and a vortex of smaller scale(100m-200m scale), located along the edge of a bulge in the rear flank gust front, interacted and the smaller vortex was absorbed by the larger vortex, may have been the trigger for the tornado formation. They suggested that the time scales for the interaction of the vortices was on the order of ten's of seconds, much faster than the doppler radars could scan. They also suggest that the numerical simulations that can detect vortices on the scale of 10 meters may be necessary to resolve the vortices in these interactions. In addition they suggest that the likely mechanism for the tornado genesis process was the barotropic instablity due to roll up of vortex sheets. They suggested the origin of the vorticity was tilting of streamwise vorticity along the rear flank gust front (Dowell 2002b). The tilting of streamwise vorticity would result in vortices that have large helicity and would be more resistant to dissipation due to stretching. From this one could conclude subsequent contraction of the large vortex due to stretching and associated convergence appeared to draw other vortices through their mutual interaction and due to the successive stretching of vortices as they intern move under the updraft into the larger vortex, leading to intensification of the ambient vortex and tornado genesis. In a numerical simulation of Adlerman and Droegemeier ([2]), the resulting vorticity distribution, in one of the tornado genesis phases, resembled the two dimensional vortex sheet roll-up in Chorin (Chorin 1973) and Krasny (Krasny 1995). A remarkable photo taken by Gene Moore shows this kind of roll-up (Grazulis 1997, p. 1349) [23]. In the photo a sequence of vortices appear to be spiraling into a tornado as it crosses a lake. These feeder vortices have cross-sections too small to be resolved in all but the highest resolution runs and appear to be very intense. Tracks left in corn fields appear to show vortices spiraling into the tornados and then dissipating as they stretch.

Numerical simulations of near ground rotation and the impact of surface flow on tornado genesis has been done by D. C. Lewellen and W. S. Lewellen (2007). They perform several experiments impeding flow into a low level non tornadic vortex and cause it to intensify to tornadic levels. In one of their experiments they simulated the effects of the rear flank downdraft spiraling into the low level flow resulting in the a near surface vortex breakdown and intensification. They refer to the process they modeled as corner flow collapse. Given the spiraling rear flank downdraft creates a vortex sheet, that could also act as a source of vorticity for the tornado, there may be several purposes that vortex sheet roll up that could

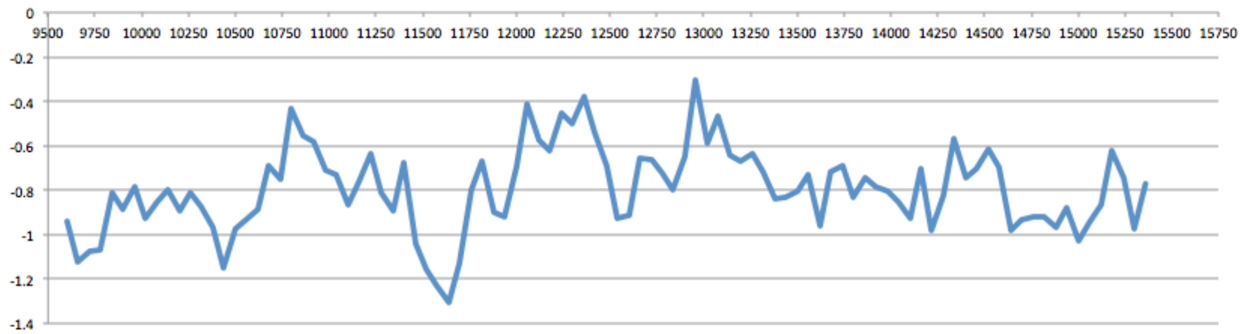

Figure 1: Time series for slopes of vorticity lines derived from nested grid May20, Del City, ARPS output.

play in tornado genesis.

Another possible source for the fractal dimension of the tornadic vortex is intense stretching by the updraft. Turbulence theory suggests as the vortex lines are stretched they kink, becoming fractalized (Chorin 1994). This process acts along the axis of the vortex line.

If the above two processes occur simultaneously we may approximate this with a compound process by breaking the processes up into an alternating sequence, first the roll-up process then the stretching process, etc. We hypothesis that the above roll-up processes take place at many different scales, including within the tornado vortex itself. The roll-up process has been studied in papers by Rotunno, (Rotunno 1984), John Snow (Snow 1978). Snow concludes that, ”the subsidiary vortices are integral parts of the overall flow pattern and should not be viewed as interacting independent vortices”.

3 Slopes of Vorticity Lines as a Tornado Predictor

Preliminary results from numerical simulations using ARPS have produced results similar to those of Cai. We did several nested grid runs consisting of only two grids, the coarse grid 100km by 100km of $\Delta x = \Delta y = 1000m$, $\Delta z = 500m$, and a second grid (80km by 80 km)with $\Delta x = \Delta y = 333m$, $\Delta z = 500m$ centered in the coarse grid. Using the May20th 1977, Del City sounding, we did a run four hours, and compared the results of the runs with the results of Adlerman's thesis [1]. Let ζ_ϵ, be the low level vertical vorticity maximum corresponding to the grid scale ϵ at a given time in the run. We considered the points $(\ln(\epsilon), \ln(\zeta_\epsilon)$ corresponding to the different scales. We computed the slope of the vorticity lines as follows: $m = -\ln(\zeta_{333}/\zeta_{1000})/\ln(3)$. We formed a time series for the 4-hour runs (Fig. 2)with $\Delta t = 60sec$. The slopes with the most negative values corresponded roughly with the tornado cyclones in Adlerman's thesis. The values of the slopes varied from -1.0 to roughly -1.3 during the short tornado phases. The tornados in Adlerman's thesis had windspeeds in the EF1-2 range. Spikes in the vorticity were noted at times delayed from the times in the simulation in chapter 4 of Adlerman's thesis by about 1000 s. We think this delay is due to the coarser grid used in our simulations. Also, the strongest tornado in the runs of Adlerman occurred during a a period where the fractal dimension of the vorticity was roughly -1.3. Slopes greater than 1 correspond to velocity increasing with radius suggesting that at least the fine grid is inside the radius of maximum winds. Therefore, using several more large scale grids (as Cai did in his paper) would be help identify the radius of maximun winds. Choosing the location of the initial bubble at (60,60) kept the main storm well away from the edge of the fine grid. ”Noise” from other storms crossing from the fine grid to the coarse grid caused spikes in the vorticity during the first two and a half hours of the run. The time series for the slopes of the vorticity line during the last 6000 seconds of the run are given in Figure 1. The coordinates of the vorticity maxima on both grids were calculated and converted to the coarse grid scale. Both the fine grid vorticity maximums and the coarse grid vorticity maximums were at very near the same point and associated with the same storm during this time. Slopes less than minus one occur at times 9660, 10440, 11640 and 15000 s. The slope the vorticity lines at these times were respectively: -1.123, -1.148, -1.306, -1.02. Adlerman comments that there is a delay of about 1000 seconds from fine scale run of $\Delta x = \Delta y = 105m$ to the more coarse scale run of $\Delta x = \Delta y = 500m$. The tornado times in Adlerman's thesis are at 7380, 8400, 10620, and 13600 s. Using the strongest vortex at 11640 in our run as corresponding to the strongest vortex in Adlerman's run, we see the delay is approximately 1000 s. With the delay of 1200s from 13600 s to 15000 s. The "tornadic" vortex in Adlerman's thesis at time 8400 s should correspond to the vortex at 9660 s. This leaves the vortex at 10440 s with no corresponding vortex in Adlerman's thesis.

This suggests that one might use a "Cai type" scaling criterion from the vorticity data generated by nested grids in a mesoscale numerical weather forecast to predict tornados.

Tornados appear to have not only self-similarity of their vorticity, but they also appear to have geometric self-similarity. This appears in Doppler radar and reflectivity data (Bluestein 2000; Nova 2004, Wurman) and in some high-resolution numerical simulations of tornadic storms (Wicker, Nova, 2004; Adlerman 2002, [2]). The power law suggests a geometric self-similarity in the vortex structure as well. Recent results of Wurman (2000, 2005) suggest that, for strong tornados, v scales like $r^{0.6}$, where r is the radius of the vortex. It is not clear what the fractal dimension of the boundary of the vortex is. In the discussion above, the doubly branched spiral that results from the roll up in the thin vortex layer, that gives rise to the vortex lines that become the arching vortex lines, is similar in shape to the hook echo that is associated with the mesocyclone or tornado vortex. In two dimensions energy can cascade from smaller scales to larger scales, as tornados and the roll-up vortex tubes have a nearly two-dimensional structure one might expect the small scales to have a strong influence on the formation of the larger scales. The dimension of the cross-section of the geometric vortex can be thought of as 1.6, by the Baker and Shelley result. The initially horizontal vortex sheet containing the vortex lines tilts into the vertical. As the vortex lines arch into the vertical, the sheet then rolls up into the hook echo shaped object associated with the mesocyclone. As the vortex lines role along it they group together into the tornado vortex or pre-tornado vortex. There is also evidence of this in photos (Grazulis 1997, p. 1349). In a one-dimensional study of the roll up of vortex sheets, Chorin (1973) showed that vortex sheets consisting of cyclonically rotating vortices rolled up into a cyclonic vortex. (The roll up in Chorin (1973) should be interpreted as the roll up in Adlerman (2002).) Vortices were spaced along a segment representing the vortex sheet. The rolled up vortex sheet resembled the hook echo region of a supercell thunderstorm. With vortices of opposite sign grouped and placed in the different halves of the segment the vortex sheet rolled up into a cyclonic, anti-cyclonic couplet. This resembled the radar reflectivity couplets that suggest arching vortex lines.

The connection between tornadoes and nearly continuously (periodically) produced arching vortex lines is that the vortex lines stir or pump the tornado and increase the vorticity (See section 4.2). How frequently vortex lines are produced, their strength, and the stretching of the vortices determine the eventual strength of the tornado. This can be seen from the point of view of the vortex gas theory below. We assume the vortices are all of the same sign (rotation), as the arching vortex lines tend to segregate themselves with the positive and negative collections grouping together, the positive parts forming the cyclonic tornado.

We now give a heuristic argument to support Cai$'$s power law for strong tornados, ζ scales like $O(h^{-1.6})$. From Kelvin$'$s Circulation Theorem vorticity times the cross-sectional area of a vortex tube is constant for Eulerian barotropic flows. Hence, $\zeta = C/A$ were A the cross sectional area of the vortex. Recent numerical and radar studies of tornadic storms suggest that vortex lines produced on the rear flank gust front of a supercell thunderstorm (captured by the updraft) form arches to produce counter-rotating vortices. The vortex lines could also be produced by the shear between different layers in rear flank downdraft, and then pulled in opposite directions by the updraft (into the mesocyclone and up) and downdrafts (to the surface). A result of a numerical study of Kelvin-Helmholtz instability by Baker and Shelley (1990), identfies a relationship between the thickness of the vortex sheet h and the cross sectional area of the vortices A. They found that A scales like $O(h^{1.55})$ as h goes to 0. Cai$'$s paper suggests that the tornados are fractal. If the arching vortex lines combine to form the self-similar (fractal) vortex, the self-similarity of the vortex suggests the largest scales are similar to the smallest scales. Therefore, ζ scales like $O(h^{-1.55})$. If the vortex is stretched, the vortex cross section will decrease and the the vorticity will increase, hence the slope of the psuedo-vorticity (vorticity) line will decrease. Hence -1.55 provides a upper bound for the slope of the psuedo-vorticity line of the vortex sheet if it is stretched as it roll's up. In Cai's paper the threshhold for pseudo-vorticity for strong tornados scales like $r^{-1.6}$. This argument could be reversed, assuming self similarity and Cai's power law for strong tornados. For this argument to apply to tornados, the vortices that flow into the tornado must be resistant to the effects of stretching. This is thought to be the case if the flow is helical.

4 Vortex gases

In this section we address the question of how high energy negative temperature vortices increase the energy of a developing tornado. We use an argument from the Chorin's book "Vorticity and Turbulence". We give an overview of the vortex gas theory. We have followed the development in the book's of Chorin [9], Chorin and Marsden, and Newton [30]. In the back of our minds is the work of Doug Lilly and his energy balance analysis (Lilly, 1986a) and finding some analogue in turbulence theory that one could apply to the development of rotation in tornados. The interaction of large numbers of vortices in two and three-dimensional space has been studied by modeling the vortices as part of a vortex gas. This theory has its origins in the works of Helmholtz (1858) and Kelvin (Thompson 1869) in the 1800$'$s. The theory is the analogue of the classical statistical mechanics of gases, which attempts to explain the macroscopic behavior of gases by using the statistics of microscopic modeled behavior of molecules. In the vortex gas case the molecules are replaced by vortices. These could be arching vortex lines (tubes). Just as in the case of gases there is a notion of entropy and a notion of temperature. Onsager (1949) first suggested the notion of temperature for vortex gases. In this theory, negative temperatures are hotter than positive temperatures. Vortices with negative temperatures are smooth. "Those with positive temperature form tightly folded structures, and if reconnection is allowed, break down into small loops "(Chorin(1993) [11]). Those with infinite temperature are fractal. The infinite temperature is between positive and negative temperature. The closer a negative temperature is to zero the warmer it is. From this point of view, a tornadic vortex that begins with kinked up or fractal vortices in it and gradually over time forms into a cylindrical vortex with smooth vortices, would be heating up. Initially smooth vortices that kink up are cooling down. This is what happens to vortices that are stretched. The stronger (hotter) a vortex is, the more resistant it is to kinking up when it is stretched. For a discussion of these ideas see Chorin (1994, 1993). As smooth slender vortices enter the developing tornado vortex, they are being stretched and are cooling down. They kink up, and in doing so they lose energy in the form of kinetic energy to the mean flow of the developing tornado. This adds to the internal energy of the tornado. While the smooth slender vortex "cools" down the ambient tornado "heats" up. As this process repeats itself many times the tornado eventually achieves quasi-equilibrium with its environment. A critique of the notion of negative temperature in two dimensions has been given by Frohlich (1982). This result was critiqued by Miller, et.al. (1992).

The modeling of vortices in three dimensions has been done using the Ising model, by A. Chorin [11]. Chorin uses this simplified approach to the vortex gas to study the relationship of stretching and temperature of the vortex and other quantities associated to a vortex gas. In this approach the vortices appear as either horizontal or vertical segments joining adjacent points in a three dimensional lattice. The lattice is formed from the points in three dimensional space with integer coordinates. As time advances the vortex configuration is allowed to change subject to certain restrictions. In studying vortices with no self intersections, the vortices are not allowed to intersect themselves at future times. The future configurations of the vortices can be studied using a Monte Carlo Markov chain algorithm, the so called Metropolis flow algorithm.

This discussion and notation follows that of Paul K. Newton in chapter 4 section 1 of his book, **The N-Vortex Problem**, A. Chorin in chapter four of his book, **Vorticity and Turbulence**, and Chorin and Marsden, Chapter 2 section 1.

We want to develop the vortex gas theory. Naively, the Euler equation for incompressible fluid flow is a limit of the vortex gas equation.

The Euler equation is

$$\frac{DV}{Dt} = \frac{\partial V}{\partial t} + (V\nabla)V = -p + f.$$

to obtain the vorticity equation we take the curl of the above equation to obtain

$$\frac{D\omega}{Dt} = (\omega\nabla)V.$$

It is possible to extract the vertical component of vorticity(ζ) from the above equation to obtain (Klemp,1987),

$$\frac{\partial\zeta}{\partial t} = -V * \nabla\zeta + \zeta\frac{\partial w}{\partial z} + \omega_H * \frac{\nabla_H w}{\partial t}$$

where w is the vertical component of the velocity, ω_H is the horizontal component of the velocity, and ∇_H is the horizontal gradient.

The three terms on the right side of the above equation are called the advection term, the stretching term, and the tilting term respectively. These terms give the contribution to the increase in vertical vorticity due to the advection, stretching and tilting of vorticity.

If we want to model the possible flow behavior of an intense vortex at the surface, we assume the flow is basically horizontal-two dimensional. Regarding the tornado vortex as two dimensional may be regarded as extremely crude and ignoring the tornado itself. However comparing the tracks left by suction spots in tornados and plots of interacting two dimensional vortices suggest that there is a connection between the two (see figure 5 and figure 6). We proceed to develop the two dimensional theory to model this behavior, then develop the three dimensional theory. The two dimensional vorticity equation for incompressible fluid flow is

$$\frac{D\omega}{Dt} = 0.$$

The incompressibility condition $div(V) = 0$ implies that, $\frac{\partial u}{\partial x} = -\frac{\partial v}{\partial y}$.

If the domain we are working in is simply connected (has no holes), then there exists a function, ψ, such that, $u = \frac{\partial \psi}{\partial y}$ and $v = -\frac{\partial \psi}{\partial x}$.

Assume the vorticity is concentrated at the points,

$$\mathbf{x_1}, \mathbf{x_2}, ..., \mathbf{x_n},$$

and

$$\mathbf{x} = (x, y), \mathbf{x_i} = (x_i, y_i)$$

with circulation,

$$\Gamma_1, \Gamma_2, ..., \Gamma_n.$$

then

$$\zeta = \Sigma_i \Gamma_i \delta(\mathbf{x} - \mathbf{x_i}).$$

The solution to the equation,

$$\zeta = -\Delta\psi$$

is given by,

$$\psi = -\Sigma_i \frac{\Gamma_i}{2\pi} \log ||\mathbf{x} - \mathbf{x_i}||.$$

The velocity field induced by the jth vortex is,

$$V_j = (\frac{\Gamma_j}{2\pi}(\frac{y_j - y}{r^2}), -\frac{\Gamma_j}{2\pi}(\frac{x_j - x}{r^2}))$$

where

$$r = ||\mathbf{x} - \mathbf{x_j}||.$$

If one assumes that each of the vortices moves under the influence of the combined velocity field of the remaining vortices, then

$$\frac{dx_i}{dt} = \frac{1}{2\pi}\Sigma_{i\neq j}\frac{\Gamma_j(y_j - y_i)}{r_{ij}^2}$$

$$\frac{dy_i}{dt} = -\frac{1}{2\pi}\Sigma_{i\neq j}\frac{\Gamma_j(x_j - x_i)}{r_{ij}^2}$$

$$r_{ij} = ||\mathbf{x_j} - \mathbf{x_i}||.$$

These equations form a Hamiltonian system that has rigorous connections with the Euler equations. The Hamiltonian is

$$H = -\frac{1}{4\pi}\Sigma_{i\neq j}\Gamma_i\Gamma_j \log||\mathbf{x_j} - \mathbf{x_i}||.$$

The Hamiltonian is conserved,

$$\frac{dH}{dt} = 0.$$

This implies if all the vortices are of the same sign, they cannot merge together.

Using this representation we can model the behavior of vortex configurations in the plane (Lim), (Newton), (Machioro). For a pair of vortices of equal circulations the vortices move about the midpoint of the segment joining them (Lim, page 92). For a line of vortices, of equal circulations, the vortices stay in a line (Lim, page 99) . If one has a half line of vortices located at integer points on the x-axis and terminating at a $x = 0$, the vortex line rolls up into a spiral (Chorin, 1973).

It is possible to develop a two dimensional theory of vortex gases, by proceeding in analogy with development of the Boltzman distribution in the theory of statistical mechanics in three dimensions. The particles are replaced by vortices and the assumptions on the distribution of vortices in a region in 2-space is used to define a distribution in the corresponding phase space. The entropy of the distribution is defined and under the assumption of energy conservation the entropy is maximized using a Lagrange multiplier argument. This leads the most probable distribution of the vortices under the constraints applied in the Lagrange multiplier argument. Consider a 2-dimensional region occupied by a vortex gas. Let us assume that there are N vortices. Divide the region up into smaller square regions, of area A. Let us assume that there are m sub-regions. In each of these sub-regions there will be a number of vortices, say n_i vortices in the i-th sub-region. Then

$$\Sigma_i n_i = N.$$

In phase space, the dimension is 2N . The probability of a given distribution of vortices within the region occupied by the gas is

$$W = (\frac{N!}{n_1!n_2!...n_m!})A^N.$$

The entropy S is defined by,

$$S = \log(W).$$

Assume that each box has reached a statistical equilibrium, with energy

$$n_iE_i.$$

If we assume that energy, E, and the number of vortices are conserved, then

$$E = \Sigma_i n_i E_i$$

$$N = \Sigma_i n_i.$$

Using Lagrange multipliers we can find the most probable distribution by maximizing the entropy. Assuming N is very large we use Stirling's formula to approximate the factorials,

$$n! = (\frac{n}{e})^n.$$

We obtain

$$n_i = e^{-\alpha}e^{-\beta E_i},$$

where α and β are the Lagrange multipliers. Define $Z = \Sigma_i e^{-\beta E_i}$. Then P_i, the probability that the vortex is in the i-th box, is $\frac{n_i}{N} = P_i$, and $P_i = \frac{e^{-\beta E_i}}{Z}$. Note that,

$$\langle E\rangle = \Sigma_i E_i P_i = \Sigma_i E_i\frac{e^{-\beta E_i}}{Z}.$$

Let i-th box of the partition. Then the entropy of the equilibrium partition is

$$S = -\Sigma P_i \log P_i.$$

The theory of three dimensional vortex gases is much more difficult and has been developed only in special cases. Chorin developed a theory of vortex gases in three dimensions assuming the vortices were made up of vertical and horizontal line segments connecting points on an integer lattice in three dimensional space. He thought of the vortex as supported on an oriented self-avoiding random walk on an integer lattice.

In three dimensions the equation $\omega = curl\ \mathbf{u}$, can be solved for $\mathbf{u}$ in terms of ω. Assuming the velocity field is divergence free, and the domain is simply connected, there exists a vector potential function, $\mathbf{A}$ such that $\mathbf{u} = curl\ \mathbf{A}$, such that $div\ \mathbf{A} = 0$. Using the vector calculus we have, $\Delta\mathbf{A}= \omega$, therefore

$$\mathbf{A} = (1/4\pi) \int (1/||x - x'||)\omega(x')dx'.$$

There for taking the curl of both sides of the equation gives

$$\mathbf{u} = (-1/4\pi) \int ((\mathbf{x} - \mathbf{x}') \times \omega(\mathbf{x}'))/||\mathbf{x} - \mathbf{x}'||^3 dx'.$$

From this formula it can be shown that the energy

$$E = (1/(8\pi) \int \int (\omega(\mathbf{x}) * \omega(\mathbf{x}')/||\mathbf{x} - \mathbf{x}'||)dx'dx.$$

In the case Chorin considered the formula becomes

$$E = (1/(8\pi)\Sigma_I \Sigma_{J\neq I}(\omega_{\mathbf{I}} * \omega_{\mathbf{J}}/||I - J||) + (1/(8\pi)\Sigma_I E_{II}$$

where I, J are the three dimensional coordinates of the location of the vortex segments making up the vortex, and $|I - J|$ is the distance between them. E_{II} is the "self energy" term. Chorin (Chorin 1993, 1994) carefully defines the velocity field using a cut-off function to prevent singularities and keep the terms in the sum defined. Similar ideas can be used in the two dimensional case.

4.1 Monte Carlo Setup

[9] Consider a vortex in a 3D lattice, call it the old vortex. Label the endpoints B for the beginning and C for the end. Chose at random a lattice point O in the vortex. and rotate the portion of the vortex from O to C by a randomly chosen orthogonal transformation of the lattice. Check that the vortex is not self-intersecting. If it is self-intersecting call it discard it and call the old vortex the new vortex. If it is not self-intersecting call it the new vortex. This step is called the pivot step. Calculate the energies, E_{old} of the and E_{new} of the two vortex configurations. Calculate the probability of accepting the new configuration $p = \min(1, \exp[-(E_{old}$ - $E_{new})/T])$ (Metropolis rejection). If the new vortex is accepted then the new vortex becomes the old one. Repeat the two steps the pivot and the energy computation step, ... Using this (pivot-Metropolis rejection) algorithm he gives an argument showing that a vortex that stretches must fold. He also uses the Monte Carlo algorithm (Pivot-Metropolis rejection) to simulate the properties of vortices of various temperatures (positive and negative) undergoing stretching. Among the results he showed were those consistent with stretching argument above. That as a high temperature vortex is stretched it cools down. Using Chorin's ideas it is possible to model a fluid made up of many vortices, but not without a significant increase in the computational complexity.

4.2 Entropy and Temperature

The entropy of the system is defined to be,

$$S(E) = \log(\Lambda(E)),$$

where $\Lambda(E)$ is the total volume of the energy surface

$$H = E$$

on which systems motion is constrained to move on. The temperature T is defined to be

$$\frac{dS}{dE} = \beta = \frac{1}{T.}$$

From this point on the two and three dimensional theory developments coincide. Define $Z = \Sigma_i e^{-\beta E_i}$. Then P_i, the probability that the vortex is in the i-th box, is $P_i = \frac{e^{-\beta E_i}}{Z}$. Note that,

$$\langle E \rangle = \Sigma_i E_i P_i = \Sigma_i E_i \frac{e^{-\beta E_i}}{Z}.$$

Let i-th box of the partition. Then the entropy of the equilibrium partition is

$$S = -\Sigma P_i \log P_i.$$

Hence,

$$S = -\Sigma P_i \log P_i = -\Sigma P_i \log \frac{e^{-\beta E_i}}{Z} =$$
$$= -\Sigma P_i(-\beta E_i - \log Z) = \beta\langle E \rangle + \log Z.$$

Therefore, Note,

$$-\frac{\partial \log Z}{\partial \beta} = -\frac{\partial \log(\Sigma_i e^{-\beta E_i})}{\partial \beta} = \frac{\Sigma_i E_i e^{-\beta E_i}}{Z} = \langle E \rangle.$$

Hence,

$$\frac{dS}{d\langle E \rangle} = \beta = \frac{1}{T}.$$

We now show vortices with negative temperature are straight and those with infinite temperature are fractal. Let t be a negative temperature near zero. Then $\beta = t^{-1}$ is negative and near $-\infty$. The terms in the sum $Z = \Sigma_i e^{-\beta E_i}$ with negative β that are large are the ones with large energies E_i. From, $P_i = \frac{e^{-\beta E_i}}{Z}$, the most probable energy, E_i, is very large. This implies that the energy

$$E_i = (1/(8\pi)) \int \int (\omega(\mathbf{x}) * \omega(\mathbf{x}')/||\mathbf{x} - \mathbf{x}'||) dx' dx.$$

is large. This will be as large as possible if the dot products are as large as possible, hence the vortex should be straight. Therefore the high temperature vortices (negative temperatures nearest 0) are the straightest. On the other hand as $\beta = t^{-1}$ goes to 0 (i.e. t goes to $+\infty = -\infty$), the the probability distribution P goes to a uniform distribution. Therefore the distribution is fractal. Hence, as negative temperature vortices are stretched, they cool down and kink up as they dissipate.

Next we show, following Chorin (Chorin, 1994) [9], that if T_1 is the temperature of a vortex with energy E_1, and T_2 is the temperature of a vortex with energy E_2, and the vortex systems are combined then, assuming conservation of energy,

a. if $T_2 > T_1 > 0$, then $\frac{dE_2}{dt} < 0$ and $\frac{dE_1}{dt} > 0$.
b. if $T_1 > 0 > T_2$, then $\frac{dE_2}{dt} < 0$ and $\frac{dE_1}{dt} > 0$.
c. if $0 > T_2 > T_1$, then $\frac{dE_2}{dt} < 0$ and $\frac{dE_1}{dt} > 0$.

Hence, for a vortex with a negative temperature, the closer to 0 (the larger negative temperature) the temperature is, the hotter the vortex is. If two vortices interact as a combined system, the hotter vortex (larger negative temperature) will loose energy to the lower energy vortex as the system moves to an equilibrium state.

Consider two vortices, one with energy E_1 and a developing tornado with energy E_2 we may regard this as two disjoint vortex systems, each separately in equilibrium. If a vortex with energy E_1 moves into a developing tornado with energy E_2. Assume their probability densities are in are independent, then the combined energy and entropy of the combined vortex system is,

$$E = E_1 + E_2$$

and

$$S = S_1 + S_2.$$

As the combined vortex system adjusts to equilibrium

$$\frac{dS}{dt} = \frac{dS_1}{dt} + \frac{dS_2}{dt} = \frac{dS_1}{d\langle E_1 \rangle}\frac{d\langle E_1 \rangle}{dt} + \frac{dS_2}{d\langle E_2 \rangle}\frac{d\langle E_2 \rangle}{dt} \geq 0.$$

Conservation of energy implies that,

$$\frac{dE}{dt} = \frac{dE_1}{dt} + \frac{dE_2}{dt} = 0.$$

Hence,

$$\frac{dS}{dt} = \left(\frac{dS_1}{d\langle E_1 \rangle} - \frac{dS_2}{d\langle E_2 \rangle}\right)\frac{d\langle E_1 \rangle}{dt} = \left(\frac{1}{T_1} - \frac{1}{T_2}\right)\frac{d\langle E_1 \rangle}{dt} \geq 0.$$

The three cases a, b, c, above now follow.

Hence, negative temperatures are warmer than positive temperatures, and negative temperatures that are closer to zero are warmer than negative temperatures that are farther from zero. Hence as hot negative temperature vortices move into the developing tornado they stretch and cool down and the ambient vortex heats up. As this process repeats itself many times the tornado vortex achieves a quasi-equilibrium with the enviornment.

Consider [9] a system in equilibrium, and partition the system into boxes, and let $E_{tot} = \frac{1}{2}\Sigma_i m_i U_i^2 + \Sigma_i E_i$, where m_i is the mass of the i-th box in the partition. If $T < 0$, then $\frac{dS_i}{d\langle E_i \rangle} < 0$, hence as the entropy increases the energy moves away from E_i to U_i. This explains the increase in the vorticity of the tornado as hot (negative temperature) vortices enter the tornado. The vortices entering the tornado are stretched and begin to cool down, they kink up, as this happens the entropy increases and energy is transfered to the larger scale flow, increasing the kinetic energy.

An important equation in the study of the two dimensional vortex gas theory is the sinh-poisson equation, also known as the Joyce-Montgomery equation,

$$-\Delta\psi = (N/Z)e^{(-\beta\psi(x))}.$$

Where $\beta < 0$ is the inverse temperature. This is a nonlinear elliptic equation. It is obtained as a limit of the distribution of the vorticity in the vortex gas theory. It can be solved for in polar coordinates (Chorin (1994) pg 82), and in certain cases in euclidean coordinates (Majda (2001),). This equation allows one to recover the two dimensional stream function and then obtain the two dimensional vorticity. For the euclidean case with $\beta = -2 = 1/T$, the solution corresponds to the Kelvin-Stuart cats eye flow, which can be thought of as modeling two dimensional Kelvin-Helmholtz instablity. Stretching a three dimensional vortex with this two dim cross section would lead to an increase in the vorticity. We use this as a justification for assuming the temperature of the vortices produced by the Kelvin-Helmhotz instability is negative.

4.3 Energy Spectrum and Power Laws of Cai and Wurman

Chorin [9] gives two different possible power laws in his book for the dissipation of energy with scale: $E(k) \sim k^{-5/3}$ and $E(k) \sim k^{-2}$. The first is derived by a scaling argument due to Kolmogorov (page 52). The other is derived as an alternative $E(k) = k^{-2}$ and is based on a possible form of the energy cascade (page 56). He later says this is a "better candidate for the "mean field result" (page 64).

Later in his book he incorporates cross-sections into his discussion of vortices (section 7.3, page 142). He does this on page 142 and derives the power law for Kolmogorov $E(k) \sim k^{-5/3}$ using fractal dimension arguments associated with the 3D metropolis rejection algorithm discussed in the previous section. The details are in the book p. 142. His argument (he calls it the filament model) is general enough that he tries to give results for $T = + - \infty, T < 0$, and $T > 0$.

He considers a vortex tube in a homogeneous sparse suspension of tubes. He evaluates the integral $S_r =< \int_{|\mathbf{r}|\leq r} \omega(\mathbf{x}) * \omega_(\mathbf{x}+\mathbf{r}) > d\mathbf{r}$.

Let C_s be the centerline of the within r of $\mathbf{x}$, and $\Sigma(s)$ is a cross- section of the tube. Then

$$S_r =< \omega(\mathbf{x}) * [\int_{C_s} ds \int_{\Sigma(s)} \omega(\mathbf{x}+\mathbf{r}) d\Sigma(s)] > .$$

If $|\Sigma|$ is the mean size of Σ then

$$S_r =< \omega(\mathbf{x}) * [\int_{C_s} |\Sigma| \omega(\mathbf{x}+\mathbf{r}) d\Sigma(s)] >$$

Hence,

$$S_N =< \Sigma_{|I-J|\leq r_N} \omega_I * \omega_J > \approx S_r/|\Sigma|.$$

To obtain the vorticity spectrum $Z(k)$ we integrate the Fourier transform of S_r over a sphere of radius $k = |\mathbf{k}|$. This gives $Z(k) = O(k^{-D-dim\Sigma+2})$. The energy spectrum is $E(k) = Z(k)/k^2 = O(k^{-D-dim\Sigma})$.

We believe we can give "mean" results using the filament model for the cases $E(k) \sim k^{-2}$, the $T = + - \infty$, and the $T < 0$ and $T > 0$. These would be the "mean cases". These come out of the results of Cai (and supported by Wurman). We claim Cai's estimates of the vorticity power laws give "mean" results for the area power laws, in fact Cai's power laws are in some sense "mean" power laws for vorticity. Using the formal steps in the derivation on page 142 one can then derive the formula 7.1 on page 142. Chorin shows the dimension of the cross section $\Sigma = D - D_c$, where D_c is the dimension of the center line (axis of the vortex) and D is is the dimension of the support of the vorticity in the vortex filament. For $T < 0$ Chorin assumes that the axis of the vortex is $D_c = 1$ as do we. For $T > 0$ he assumes the vortex axis is 3 dim as do we. Use D from page 141.

Wurman's [34] results suggest that vortices moving in tornados have negative temperature and that their the horizontal cross sections of are fractal. So we need to replace $D - D_c$ by the negative power in Cai's power law for vorticity. This then gives powers for $T < 0$ of the form $E(k) \sim k^{-\gamma}$, where $2 \leq \gamma$. In the case $T > 0$ we have $D_c = 3$ according to Chorin. This gives $\gamma = 0$. The jump from $T < 0$ to $T > 0$ (vortex cooling down) causes problems for Chorin (page 143 and see page145(ii) top of page). The comment page145(ii) top of page suggests that we use the entropy argument at the bottom of page 76 to get around the energy conservation problems. That is the energy lost in the vortex filament as the temp of the filament goes from $T < 0$ to $T > 0$ goes to the kinetic energy in the surrounding flow. These ideas I believe give us "mean" results using the filament model for the cases $E(k) \sim k^{-2}$, the $T = -\infty$, and the $T < 0$ and $T > 0$. These come out of the results of Cai and the results of Wurman). We believe we can also give interpretations of these situations that support other parts of the paper.

Writing the Navier Stokes equation in energy form and taking the Fourier trans form Chorin obtains

$$\partial_t E(k) + 2k^2 R^{-1} E(k) = Q(k),$$

$Q(k)$ where is cubic in $\hat{\mathbf{u}}(k)$ and comes from the nonlinear term in the Navier Stokes Equation. This term represents the transfer of energy between wave numbers and has been studied extensively for the case of homogeneous turbulence [Waleffe,1992] [21] . Assuming a form of the energy $E(k) \sim k^{-\gamma}$, we see that for large scales (small k) an increase of the $E(k)$ corresponds to an increase in γ, which corresponds to an decrease in the power in Cai's power law. Certain terms have been singled out and studied in relation to inverse cascades of energy. These interactions involve three wave numbers. It was found that the net effect of the nonlocal interactions is to transfer energy to larger scales. These interactions occur between modes with helicity of the same sign. Studies have shown that the presence of helicity and low energy dissipation are unlinked unless the helicity is continuously supplied and or generated at the energy containing scales, this is associated with inhomogeneity in the mean field [yokoi, et.al., 1993] [38]. Such an inhomogeneity would be supplied by surface friction and the rear flank and forward flank downdrafts and/or their gust fronts. The increase in the exponents for the power laws for the vorticity(Cai) as the tornado genesis approaches and or the tornado strengthens, suggests the dimension of the cross sections of the tornado cyclone increase as well. From this we infer the high fractal dimension of the cross-sections of the vortices associated to strong tornados is associated with an inverse cascade of energy from smaller energy producing scales. We also conclude the fractal dimension of the cross-sections is a measure of the helicity production of the flow at the energy containing scales.

5 Suction Spots

The vortex line theory we have used here has been used to study the interaction of pairs of cyclonically rotating vortices in the half-plane. The paths of the pairs of interacting vortices (Marchioro 1994, p. 53) form the same type of pattern as the tracks of overlapping suction spots moving through fields as observed by Fujita (Fujita, 1981) [22] and others from the air (Grazulis 1997, p. 1379). The paths in Figure 2 can be modeled using the the two dimensional vortex gas theory with translating and interacting point vortices, see Figure 3. One can identify the two counter-rotating (anti-cyclonic) mirror vortices in the other half plane as the other ends of the arching vortex lines from the original pair. These would be rotat-

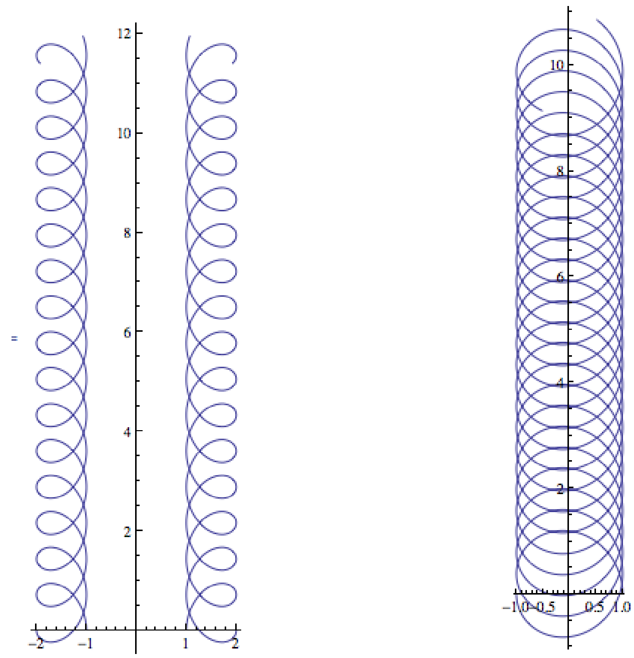

Figure 2: Graphs of numerical solutions of suction vortex tracks

ing in the opposite orientation. The tracks left by the vortices, suggest the vortices either originate in the larger tornado vortex or move into the tornado vortex, intensify due to stretching, make a partial revolution, then dissipate. The periodic entering of the vortices into the tornado suggests that they appear as a result of the rollup of a vortex sheet (ref.). These vortices may be extremely intense. Fujita (Fujita, 1981) documents that these intense vortex paths are from one to two yards in diameter (Check and ref.)(Some as narrow as 30cm in diameter). And that in some cases these intense vortices pulled cornstalks out of clay soil by their roots. We suggest that these vortices have negative temperature and that as they dissipate they transfer energy to the larger vortex. This could manifest itself as a vortex breakdown. Numerical simulations of intense vortices by Fiedler and Rotunno (Fiedler, 1986), Fiedler (Fiedler 1994, 1997) Lewellen, et. al. [27], [28], (Lewellen 2000) [29], and Xia, (Xia, 2003) [37] suggest that the maximum wind speeds in intense narrow vortices undergoing vortex breakdown may exceed the speed of sound in the vertical direction. In some cases the tracks of the vortices appear to originate in the vortex core make a partial revolution about the ambient tornado vortex and dissipate. This might indicate a Hopf bifurcation of the horizontal component of the flow field, creating a two cell flow structure: downdraft core and surrounding updraft. Josh Wurman has studied the flow structure of a number of tornados using mobile Doppler radar. He has found evidence supporting both the creation of vortices inside the tornado and the creation of vortices outside the tornado that then flow into the tornado potentially enhancing the tornados strength. He has also noted that these secondary vortices have a different velocity and shear profile than the parent tornados. The parent strong tornados appear to have a two cell structure and a modified Rankine combined profile, with mean velocity depending linearly on radius inside the tornado core and outside the core a power-law drop-off $v = Cr^{-.5}$ to $v = Cr^{-.6}$. In an extreme case Wurman found a power law drop of of the velocity approaching $v = Cr^{-1}$. Where as the secondary vortices are single cell with extreme values of shear and extreme transient updrafts. This is consistent with these vortices having negative temperatures in the vortex gas sense. [9] [36]

6 Future Work

The power laws Cai proposed in his paper should be explored and refined for both psuedo-vorticity and vorticity involving radar data and mesoscale numerical weather models for both theoretical and prediction purposes. Continue to explore fully the relationship between helicity, temperature (in the vortex gas sense) and the slope of psuedo-vorticity (vorticity) line. Cai's concept of scaling in detecting intense concentrations low-level vorticity should be exploited to the maximum effect possible.

The initial arrangement of the vortices is linear along the vortex sheet, as the roll-up takes place the vortices fill out in a circle-bounded region. The circle-bounded region would be the region where it would be appropriate to use the vortex gas theory. In the linear region one might use a shift on a finite alphabet to study the vortex sheet. The coding of a fractal into the shift on a finite alphabet is used in the study of fractal dimension as presented by Edgar (1992). This is a common way to study dynamical systems. In Edgar′s book he studies the fractal dimension of the boundary of the Heighway dragon fractal, the dimension is 1.52. This fractal has a crude resemblance to radar reflectivity image of the hook echo region of a supercell, the hooks on it representing the successive vortices in the vortex sheet, as in (Nova 2004; Chorin 1994; Adlerman 2002). On can think of a 2 dimensional radar image as a Poincare section of the supercell dynamical system. One often studies the dynamical systems associated with Poincare sections using discrete dynamical systems. These systems often take the form of shifts on finite alphabets, like the system studied in Edgars book. There are several other "dragon" fractals that have a resemblance to the radar reflectivity image of the hook echo region of a supercell thunderstorm.

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