Mathematics and Physics of Global Warming

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August 7, 2013

Outline

I. Goals and Introduction
II. Principles for Radiative Balance
III. Model for Greenhouse Effect
IV. Arrhenius Law, Radiative Forcing and Climate Sensitivity
V. Linear and Quadratic approximations for Climate Sensitivity at various time scales
VI. Conclusions

Global warming is the rise in the average temperature of Earth's atmosphere.

It is also **interdisciplinary** science including physics, mathematics, chemistry, physical chemistry, biology, geography, astronomy, geology, and even political science.

Goals

- Learn basic processes involved in calculation of Earth temperature and assemble information on current models
- Analyze a power law for climate sensitivity based on the Arrhenius equation at different time scales

Temperature T

Concentration of carbon dioxide C

$T = f(C)$

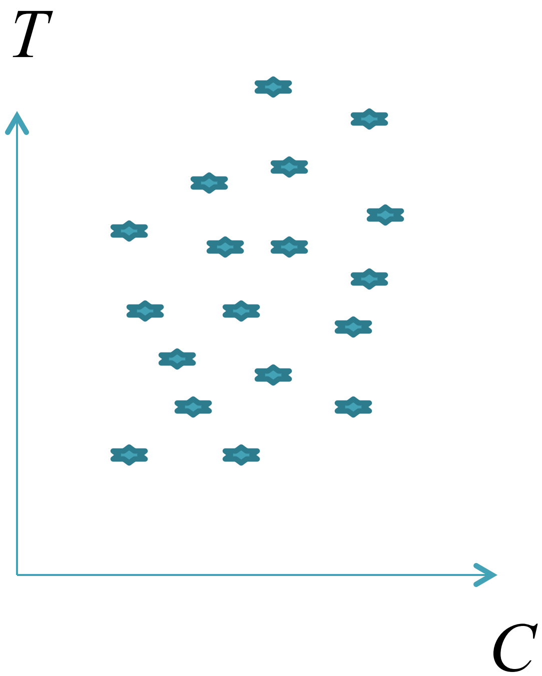
Evolution of T and C in time t

$T(t) = f(C(t))$

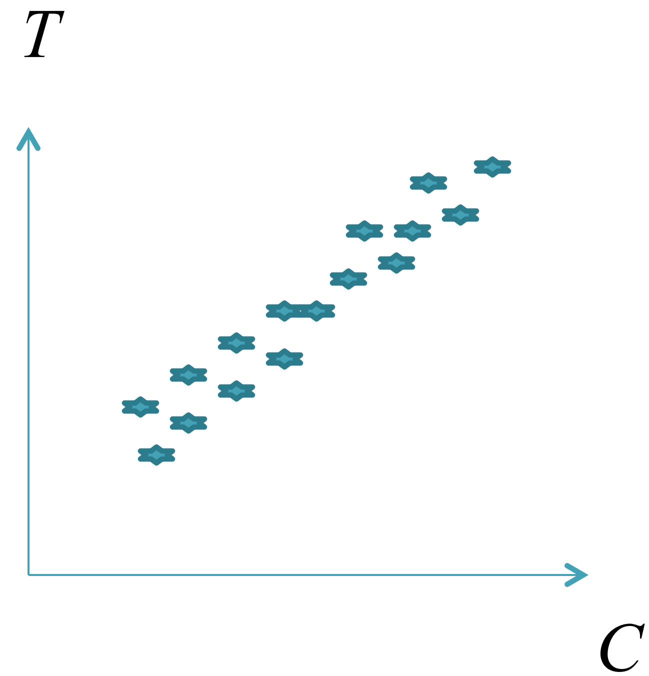
$$\frac{dT}{dt} = \frac{df}{dC}\frac{dC}{dt}$$

Climate Sensitivity: $\frac{df}{dC}$

Statistical Test for $T = f(C)$



No correlation

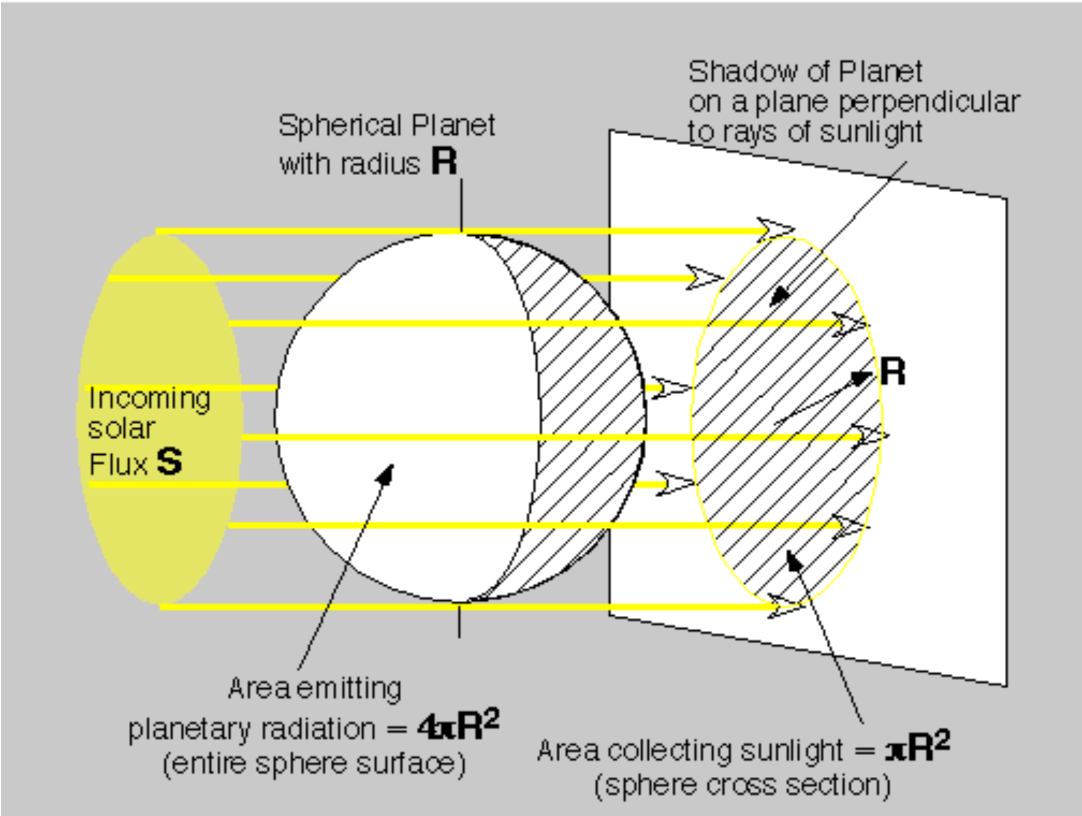


There exists correlation

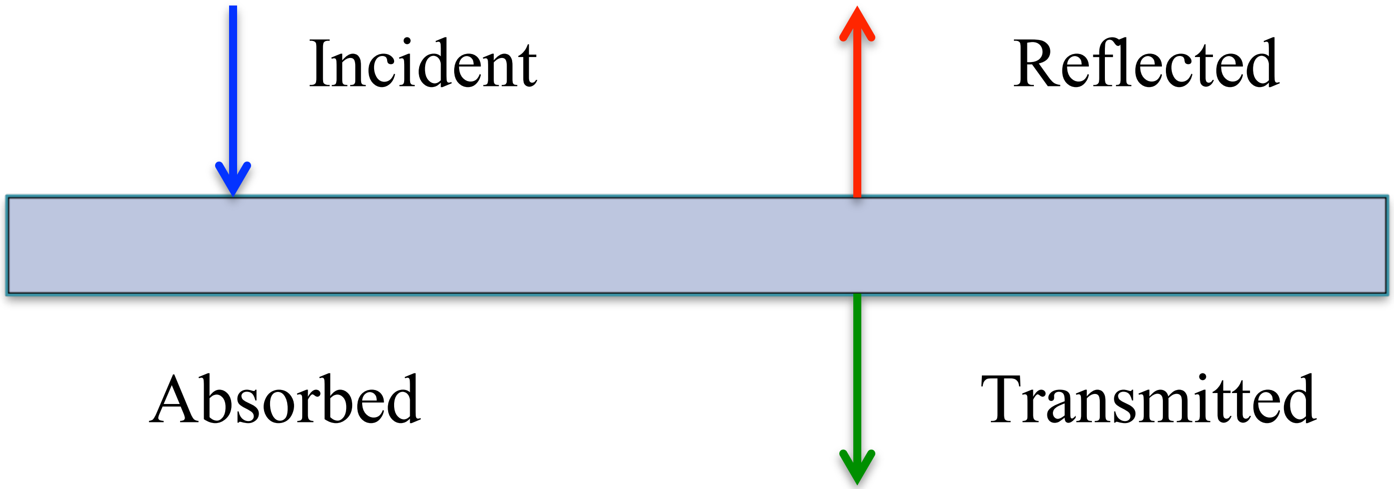
Radiative Flux. S: incoming flux density. R: Earth's radius. The flux of the field $\mathbf{F}=\langle 0,0,S\rangle$ across an Earth hemisphere:

$$\iint_{\text{Earth Hemisphere}} \mathbf{F}\cdot\boldsymbol{n}\,ds = \iint_{\text{Equatorial Plane}} S\,dA = \pi R^2 S$$

A Spherical Planet Receiving the Sun's Radiation



Radiation Balance



Incident = Absorbed + Transmitted + Reflected

Black Body:

Incident = Absorbed

Not all of the incoming flux $\pi R^2 S$ reaches the Earth, part of it is lost to atmosphere (albedo)

Incoming (and Absorbed) radiation: $(1-a)\pi R^2 S$

Emitted radiation: $4\pi R^2 G$

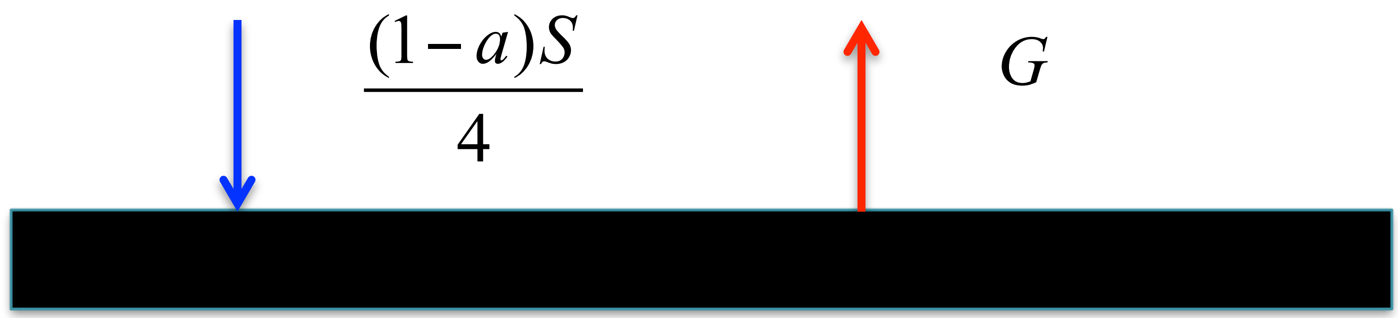
Equilibrium

$$(1-a)\pi R^2 S = 4\pi R^2 G$$

$$(1-a)S = 4G$$

$$\boxed{\frac{(1-a)S}{4} = G}$$

No Greenhouse Effect



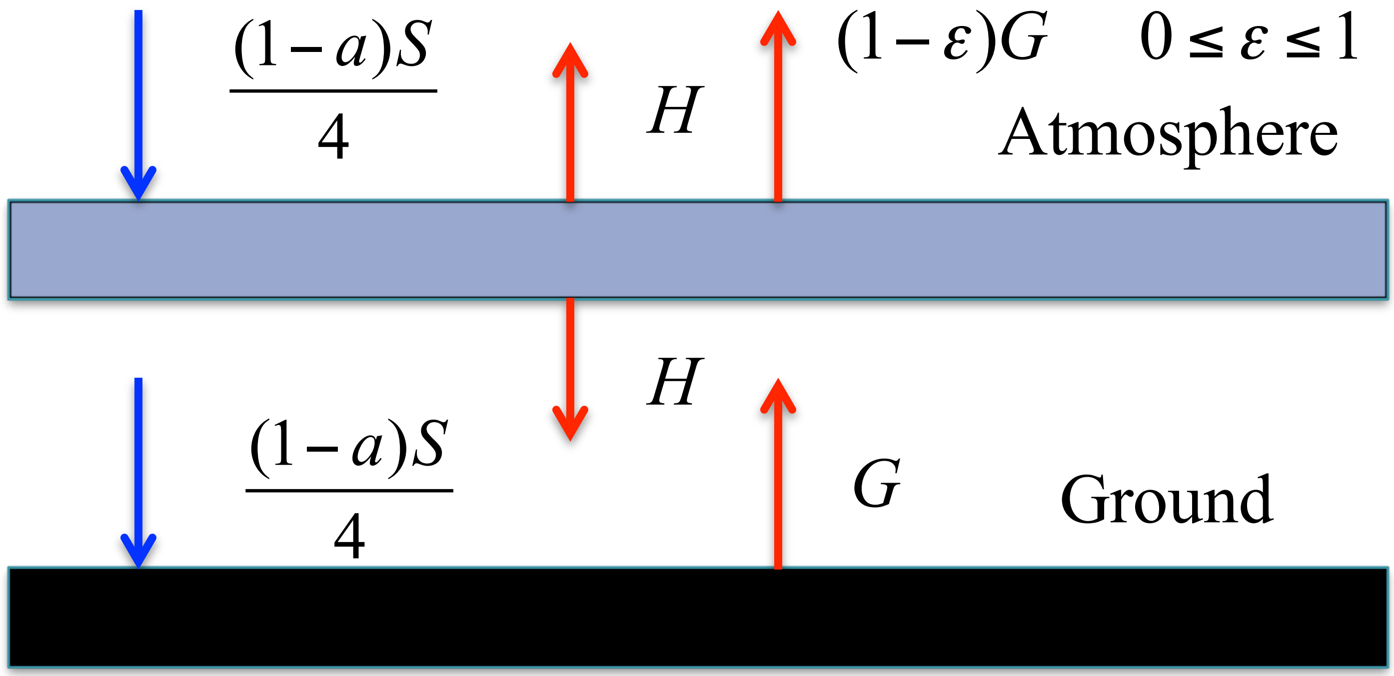
Ground Level

$$\frac{(1-a)S}{4} = G = \sigma T^4$$

$$S = 1366, \quad \sigma = 5.67 \cdot 10^{-8}, \quad a = 0.3$$

$$T \approx 255 \text{ K}$$

Greenhouse Effect



$$\frac{(1-a)S}{4} = H + (1-\varepsilon)G \qquad G = \frac{2}{2-\varepsilon}\frac{(1-a)S}{4}$$

$$G = \frac{(1-a)S}{4} + H$$

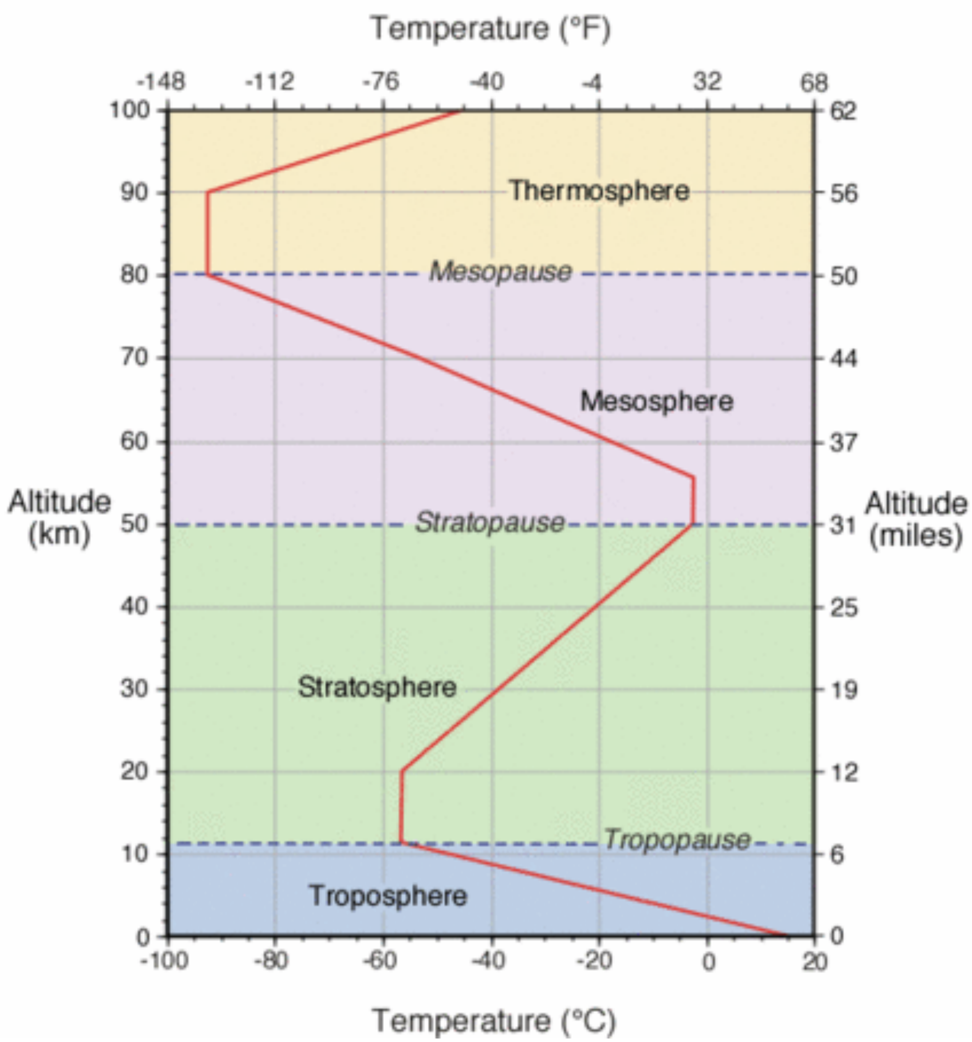
$\varepsilon = 0$
$T \approx 255$ K

$\varepsilon = 1$
$T \approx 303$ K

$\varepsilon = 0.77$
$T \approx 288$ K

$$G = \frac{2}{2-\varepsilon}\frac{(1-a)S}{4}$$

ε and C connection??



Averaging

We wish to thank Prof. J. Abraham for wealth of information on models of global warming

Arrhenius Law

$$\Delta F = \alpha \ \ln \frac{C}{C_0}$$

ΔF : Radiative Forcing, $\mathrm{W/m^2}$

$\alpha = 5.35$

C : Current level of carbon dioxide

C_0 : Base level of carbon dioxide

Climate Sensitivity

$$\Delta T = \lambda \ \Delta F, \quad 0.3 \le \lambda \le 0.8$$

$$\Delta T = \lambda\alpha \ \ln\frac{C}{C_0} = b \ \ln\frac{C}{C_0}$$

Power Law:

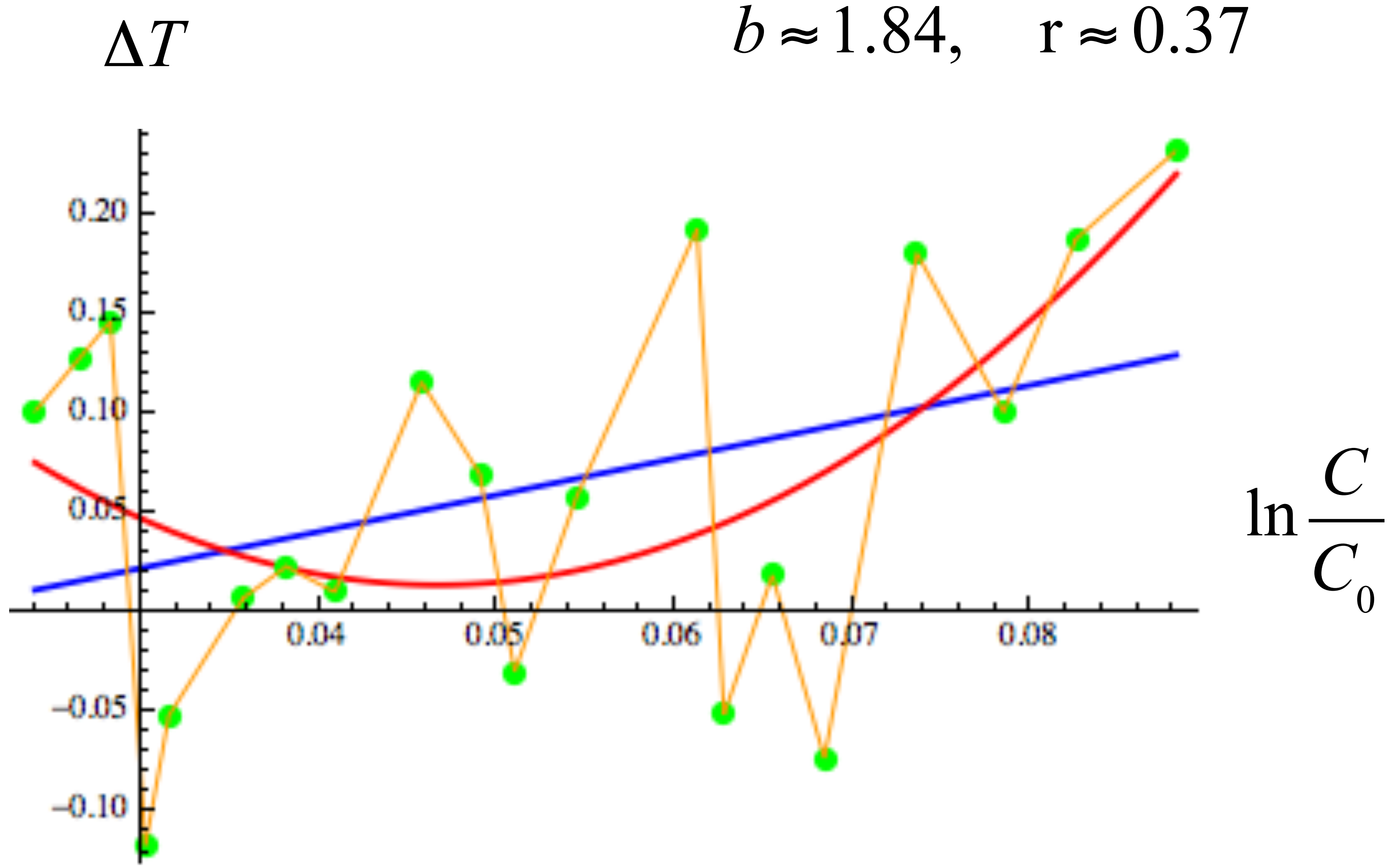
$$e^{\Delta T} = \left(\frac{C}{C_0}\right)^b \qquad\qquad e^{\Delta T(t)} = \left(\frac{C(t)}{C_0}\right)^b$$

This power law says that warming is self-smilar, or b is independent of time scale (Notices of AMS, 2010, **57**, # 10, p. 1278). This motivated us to study b for different time scales

$$\Delta T = b_0 + b\ \ln\frac{C}{C_0}$$

$1961-1980$

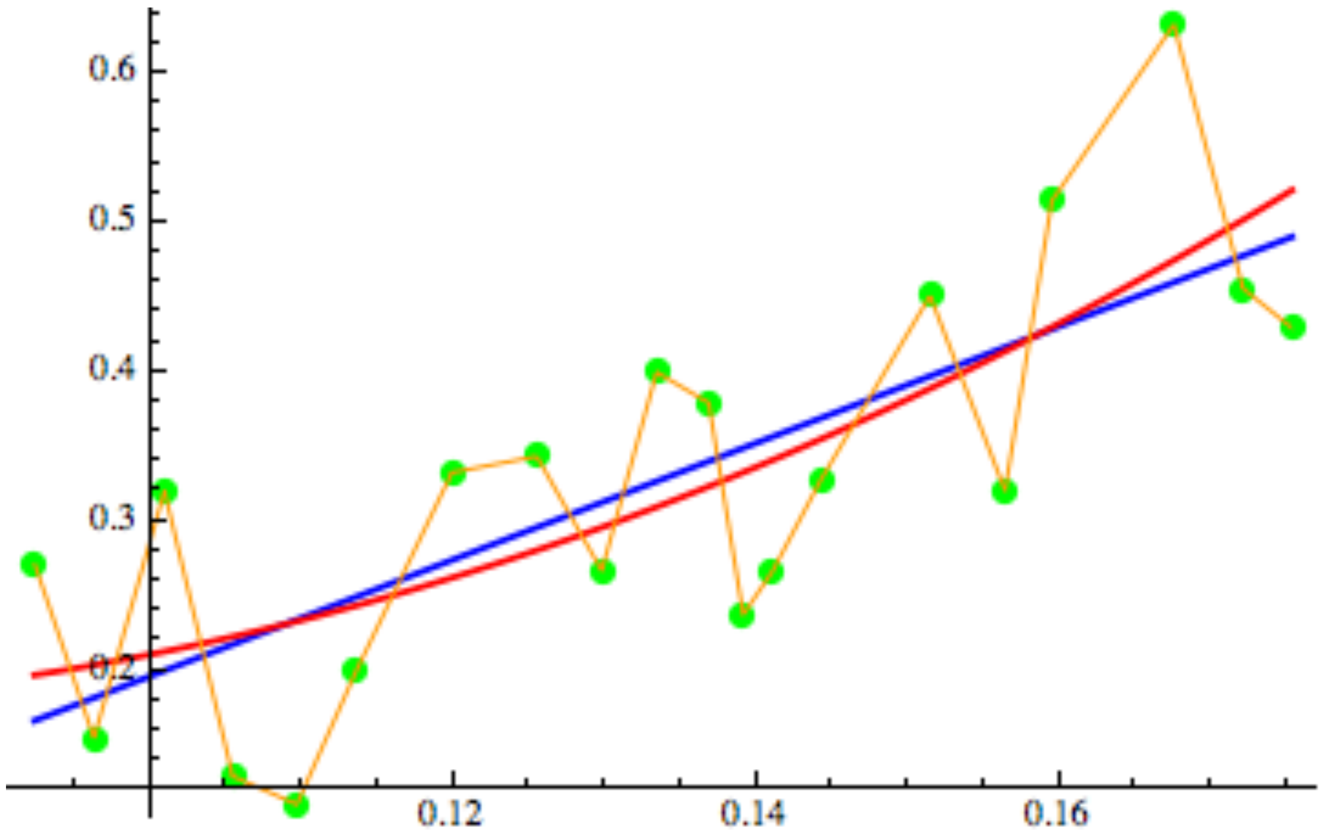
$b \approx 1.84, \quad r \approx 0.37$



$$\Delta T = b_0 + b \ \ln\frac{C}{C_0} \qquad 1981-2000$$

$$b \approx 3.9, \qquad r \approx 0.75$$

ΔT



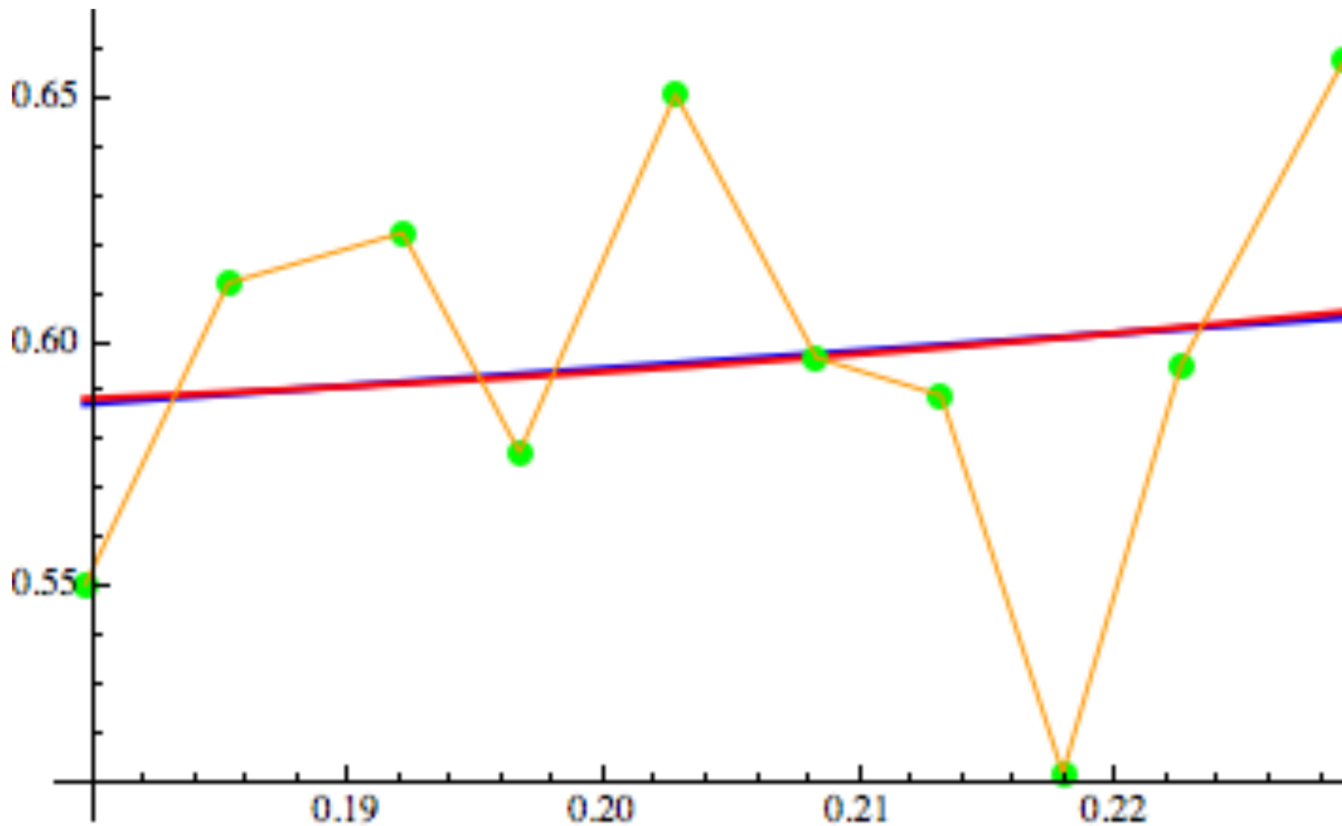
$\ln\frac{C}{C_0}$

$$\Delta T = b_0 + b \ \ln \frac{C}{C_0}$$

$$2001 - 2010$$

$$b \approx 0.36, \quad r \approx 0.13$$

ΔT

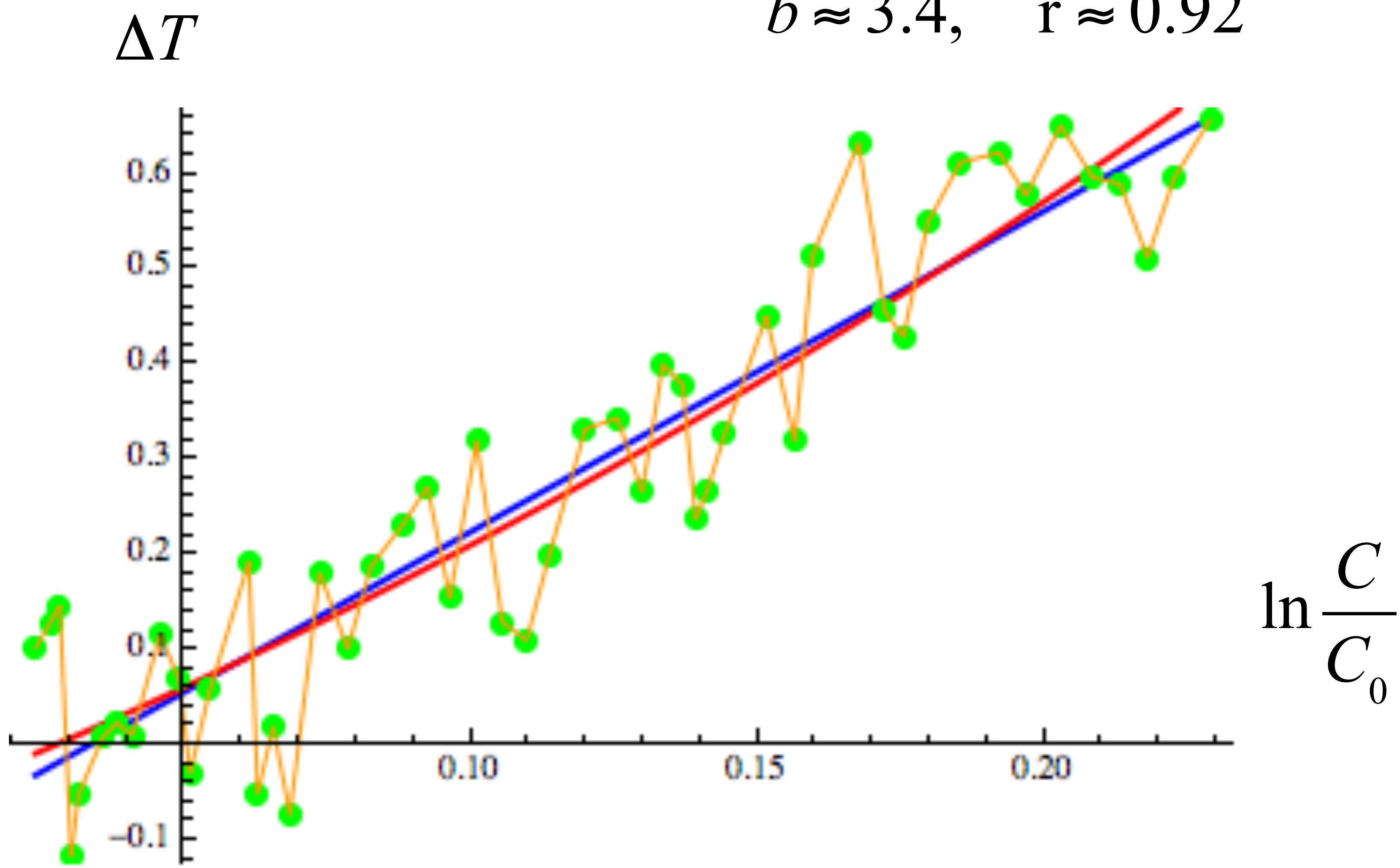


$\ln \frac{C}{C_0}$

$$\Delta T = b_0 + b\ \ln\frac{C}{C_0}$$

1961 – 2010

$b \approx 3.4, \quad \mathrm{r} \approx 0.92$



Summary

$$\Delta T = b_0 + b \ln \frac{C}{C_0}$$

$b \approx 1.84$, r ≈ 0.37, 1961-1980

$b \approx 3.90$, r ≈ 0.75, 1981-2000

$b \approx 0.36$, r ≈ 0.13, 2001-2010

$b \approx 3.4$, r ≈ 0.92, 1961-2010

Conclusions :

1. Longer (50 years) time scale has strong correlation beween warming and CO_2
2. Shorter (10 and 20 years) time scale has weak correlation beween warming and CO_2
3. Variability in b indicates that the power law is not (time) scale invariant.
4. Climate sensitivity depends on time scale
5. Climate sensitivity depends on CO_2 concentration for small time scales