Delay in Neuronal Spiking

Misha Shvartsman
University of St. Thomas, St Paul, MN
Pavel Bělík
Augsburg College, Minneapolis, MN

2012 SIAM Conference on Uncertainty Quantification

April 4, 2012

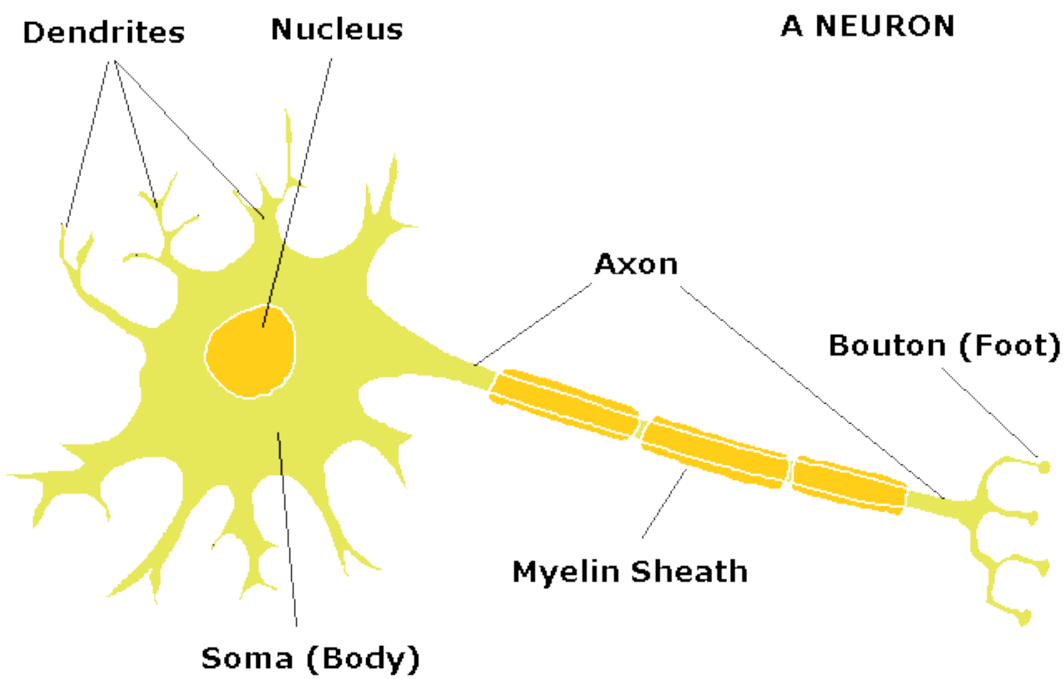
Outline

I. **Neurons and Structural Properties**
II. **Action Potential and Passive State**
III. **Delay and Memory Models**
IV. **Numerical Modeling with Delay**

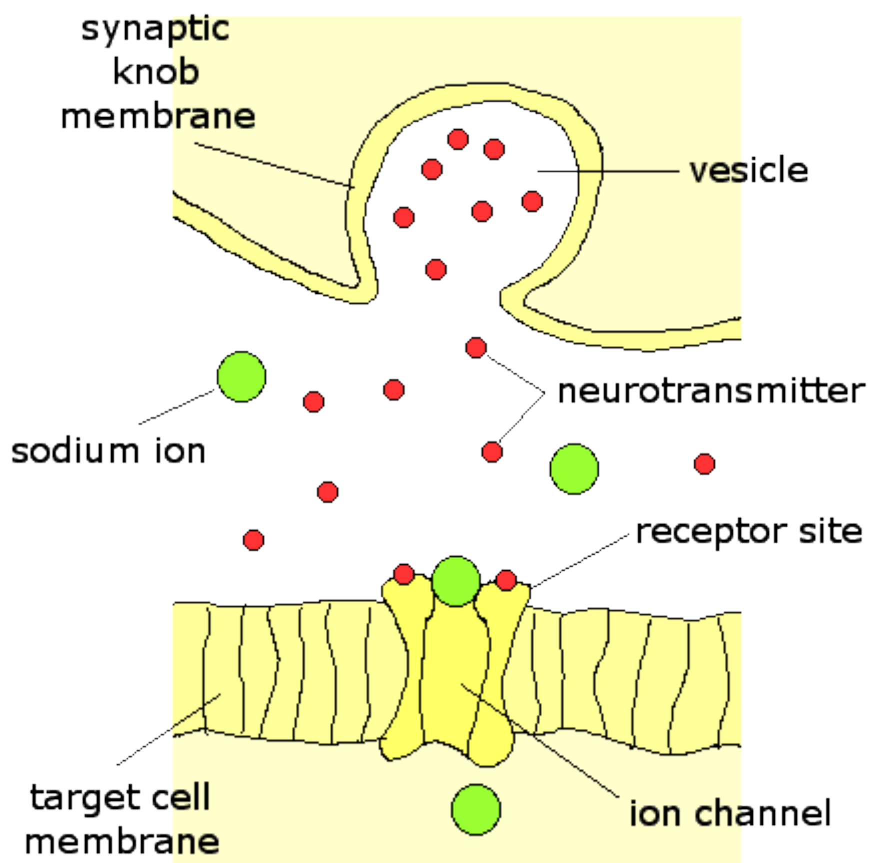
Central Nervous System:

10^{11} Neurons
10^{15} Synapses

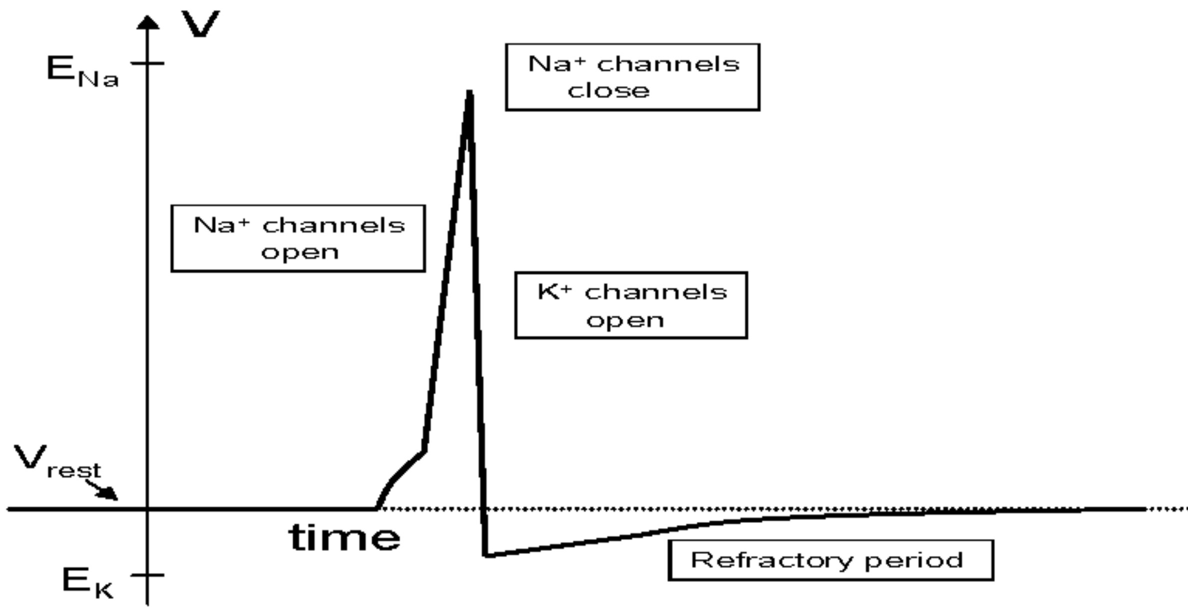
Neuron


A NEURON
Dendrites
Nucleus
Axon
Bouton (Foot)
Myelin Sheath
Soma (Body)

Synapse


synaptic
knob
membrane
vesicle
neurotransmitter
sodium ion
receptor site
target cell
membrane
ion channel

Action Potential



Extreme States are Unstable

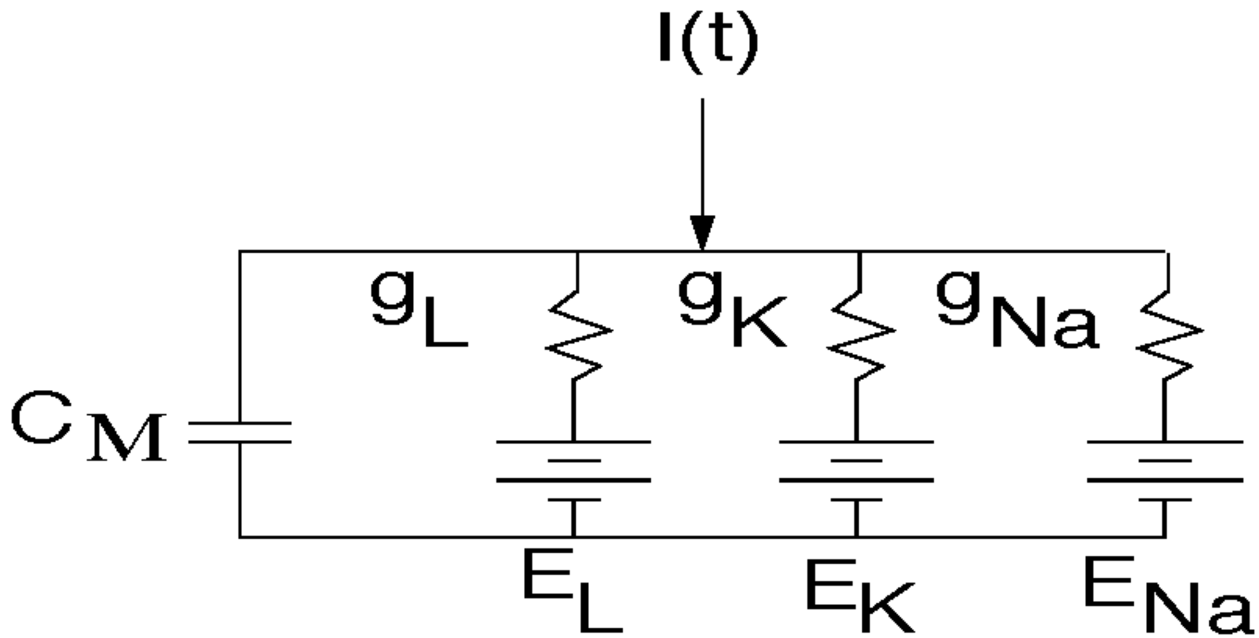
Important States are between extreme states and *they* "ignite" the spikes

(Still) Open Problems

David Eagleman (2007), Eric R. Kandel (2009)

- How do the specialized systems of the brain integrate with one another?
- What can computational models contribute to understanding synaptic plasticity?
- What firing patterns do neurons actually use to initiate long-term plasticity at various synapses?

"Mechanics" of Axon Potential



$I_m \sim V_{xx}$ $\quad$ $I_m \sim V_t$ $\quad$ and $\quad$ $I_m \sim V/r$

Governing Equations of Neuronal Process: Hodgkin-Huxley (applied current is zero)

$$c_M \frac{dV}{dt} = -\bar{g}_{Na} m^3 h (V - E_{Na}) - \bar{g}_K n^4 (V - E_K) - \bar{g}_L (V - E_L)$$

$$\frac{dn}{dt} = \phi[\alpha_n(V)(1-n) - \beta_n(V)n]$$

$$\frac{dm}{dt} = \phi[\alpha_m(V)(1-m) - \beta_m(V)m]$$

$$\frac{dh}{dt} = \phi[\alpha_h(V)(1-h) - \beta_h(V)h]$$

Empirical Properties of Rate Constants (for Gating Variables)

$$\alpha_n(V) = 0.01(V+55)/(1-\exp(-(V+55)/10))$$
$$\beta_n(V) = 0.125\exp(-(V+65)/80)$$
$$\alpha_m(V) = 0.1(V+40)/(1-\exp(-(V+40)/10))$$
$$\beta_m(V) = 4\exp(-(V+65)/18)$$
$$\alpha_h(V) = 0.07\exp(-(V+65)/20)$$
$$\beta_h(V) = 1/(1+\exp(-(V+35)/10)).$$

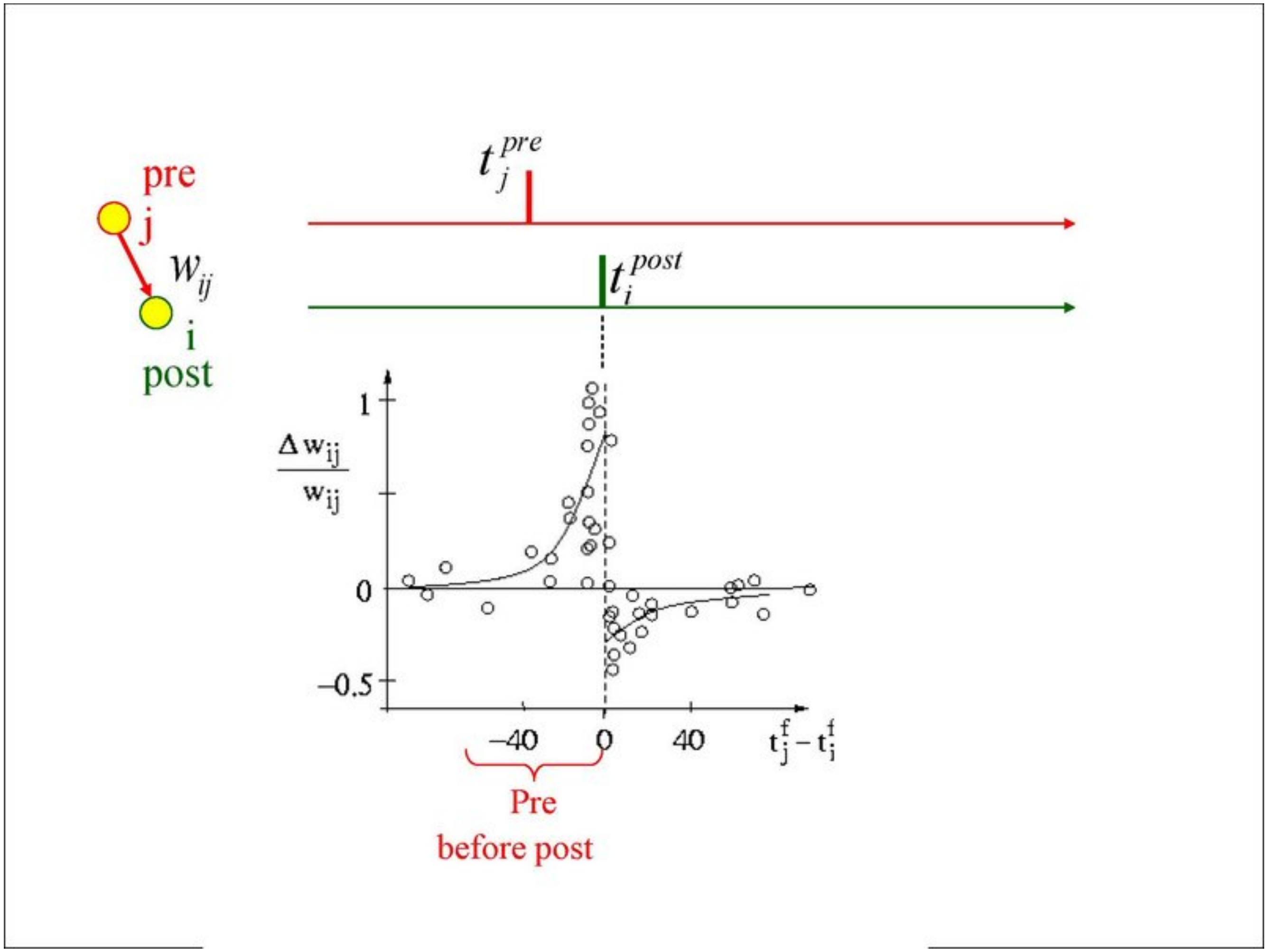
Questions:

- How specific neuron is affected by delay (memory)
- How its communication with the network is affected by delay (memory)

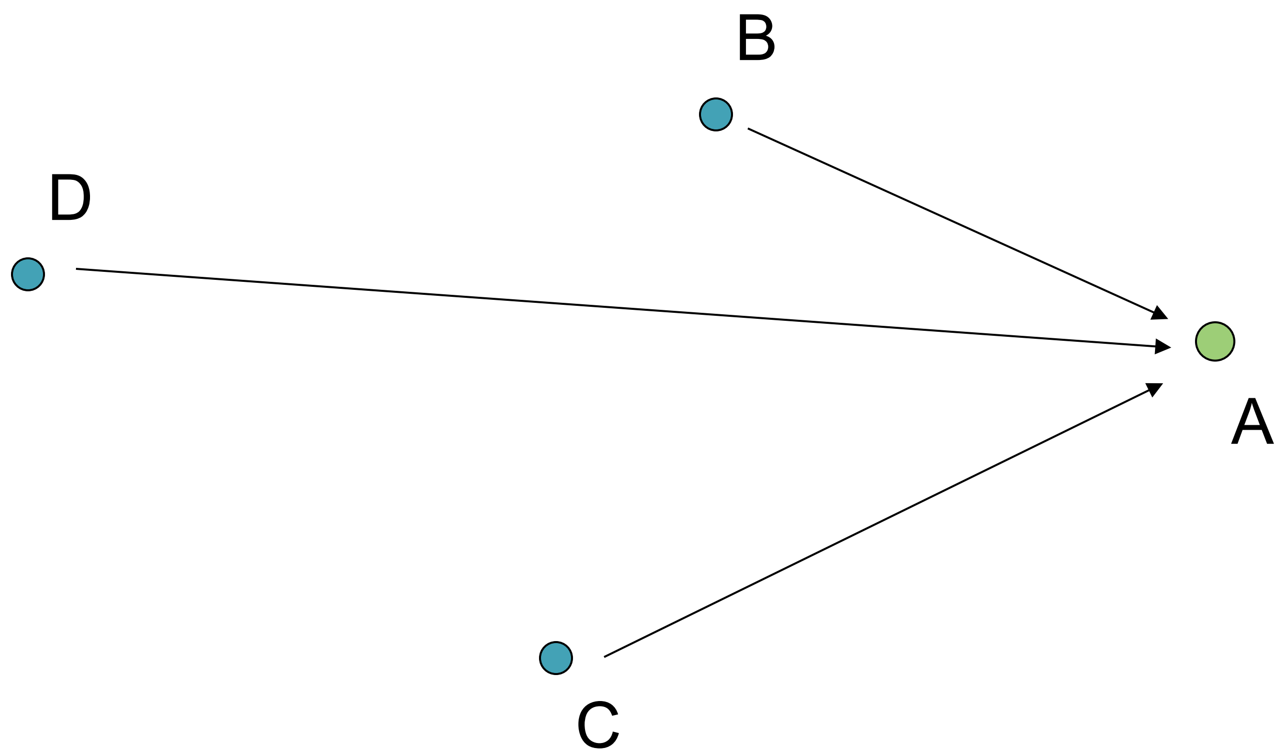
Delay

- STDP: Spike -Timing Dependent Plasticity
- Conduction Delay

STDP: Spike-Time Dependent plasticity



Conduction Delay



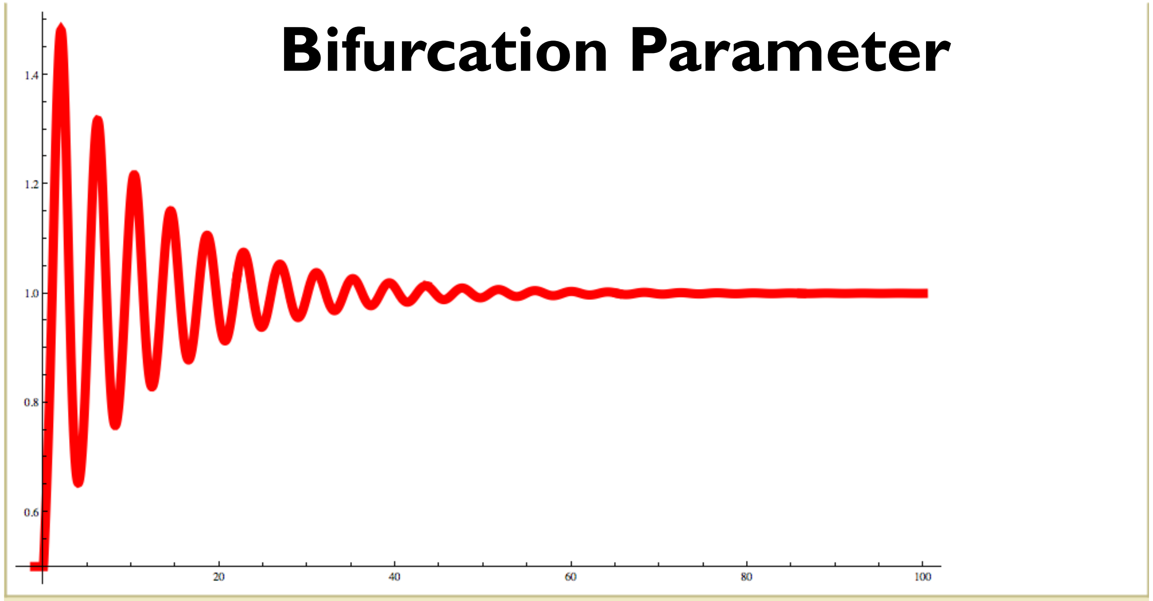
Analogy between 2 modes of Circuit Equation and Delayed Logistic Model

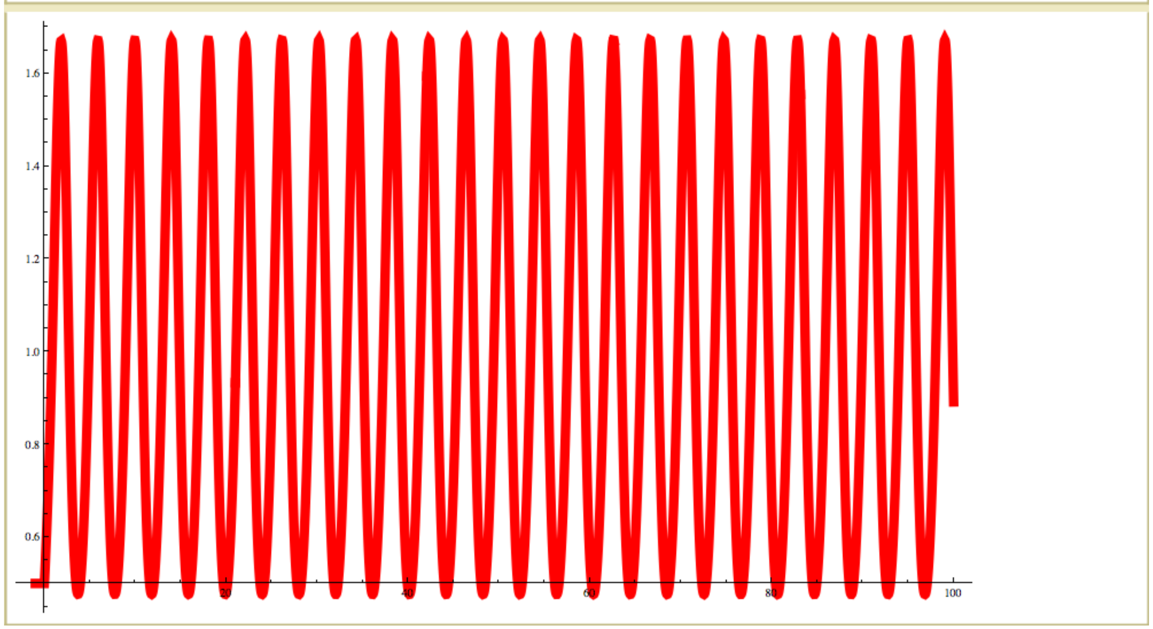
$$C\frac{\partial V}{\partial t}+\frac{V}{r}=0$$

$$C\frac{\partial V}{\partial t}+\frac{V-E}{r}=0$$

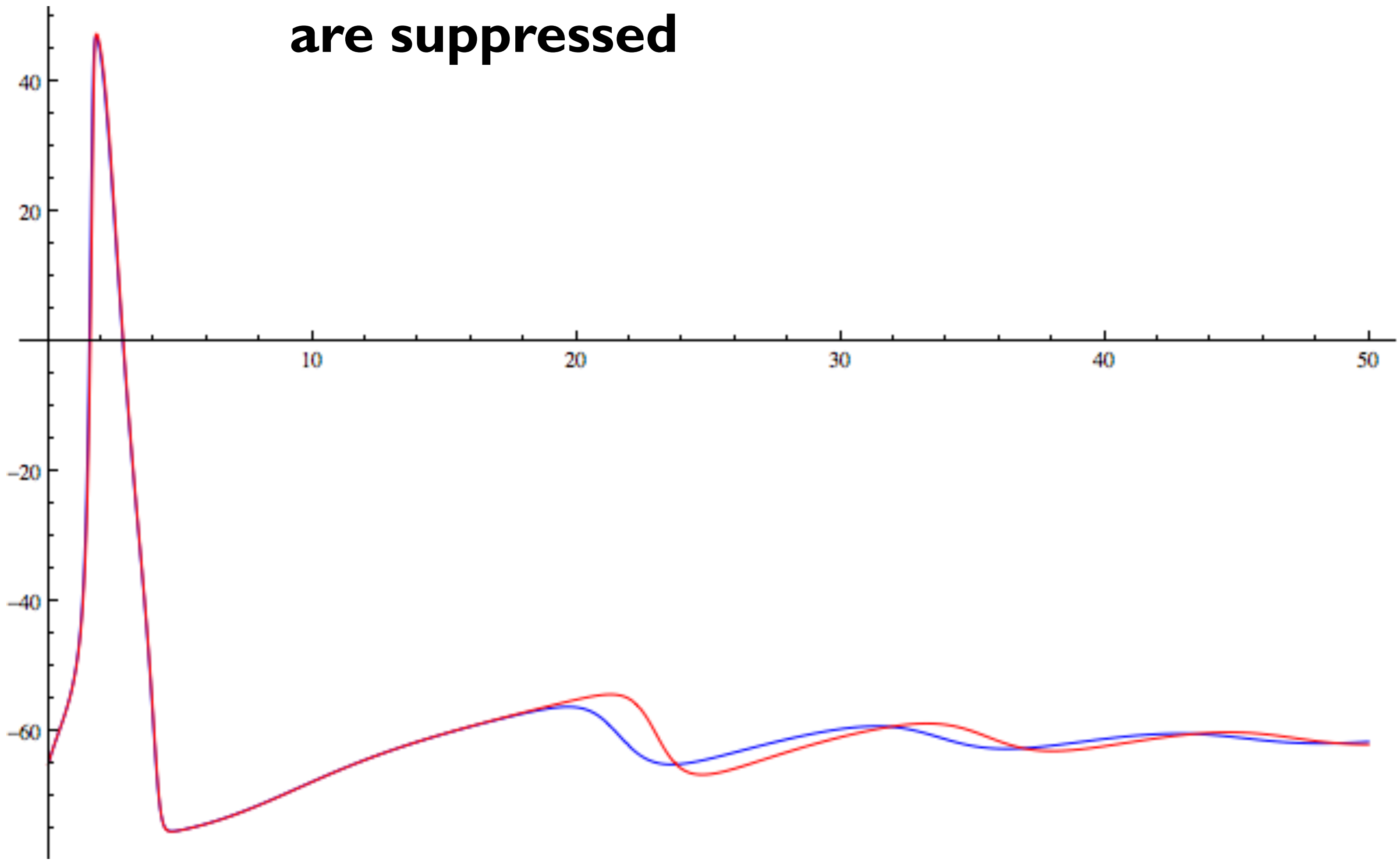
$$\frac{\partial V}{\partial t}=kV\left(1-\frac{V(t-\tau)}{E}\right)$$

Rate Constant as a Bifurcation Parameter



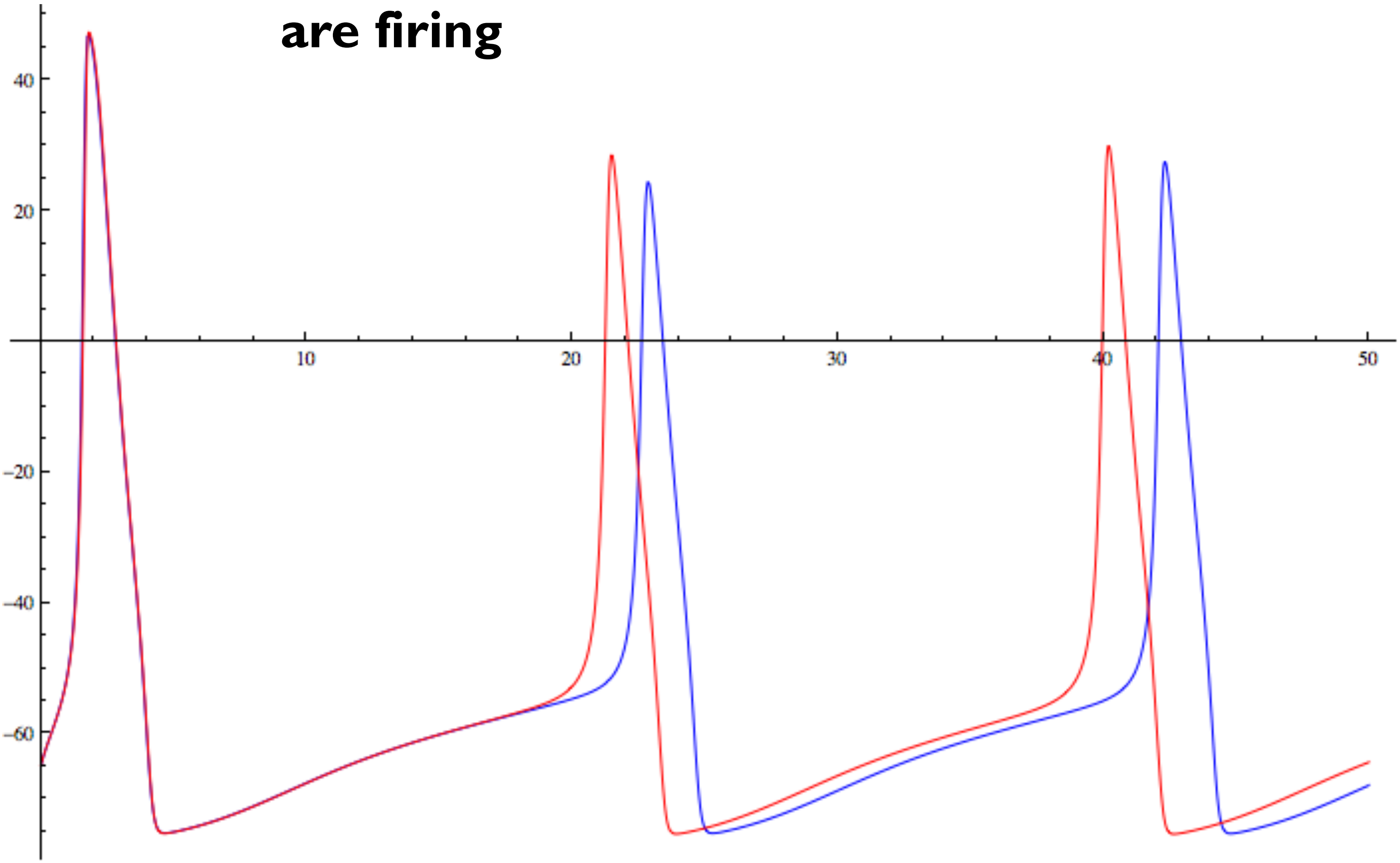


Standard and Delayed potentials are suppressed



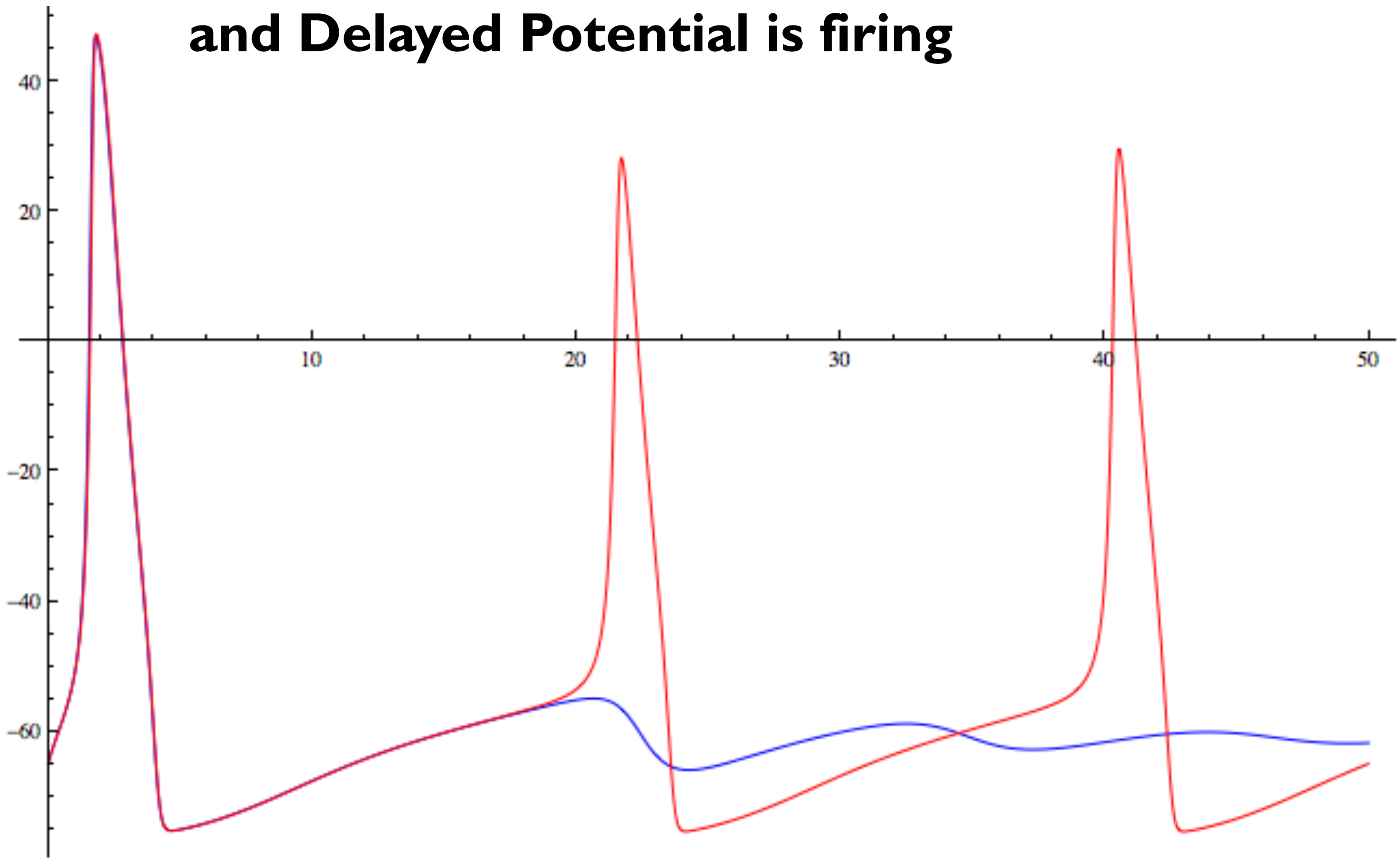
$I_{app} \sim 6.22$

Standard and Delayed potentials are firing



$I_{app} \sim 6.30$

Standard Potential is suppressed and Delayed Potential is firing



$I_{app} \sim 6.28$

Conclusions and Future Work

- Numerical Delay can imitate certain aspects of STDP (constant applied current)
- Conduction Delay may be modeled using a variable (in time) applied current (future work)
- Consider full PDE response to various modes of Delay (present and future work)

References

- Ermentrout-Terman (2010)
- Izhikevich (2007)
- Scholarpedia (2011)