Delay in Modeling Spiking Neurons

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Delay in Modeling Spiking Neurons

You just do nothing for a very long time…

I was told I could include a joke in my talk
That was the joke

Outline

I. A little about Neuroscience
II. Mathematics and Neuroscience
III. Neurons and Hodgkin-Huxley Equations
IV. Heat Equation
V. Neurons have to work together
VI. A little about Delay in Neuroscience
VII. Numerical Experiments with Delay
VIII. About CSUMS Program

Neuroscience

Neuroscience is the study of the brain.
It is **interdisciplinary**:
medicine, biology, chemistry,
computer science, mathematics, physics,
psychology, philosophy.

10 Unsolved Mysteries Of The Brain.

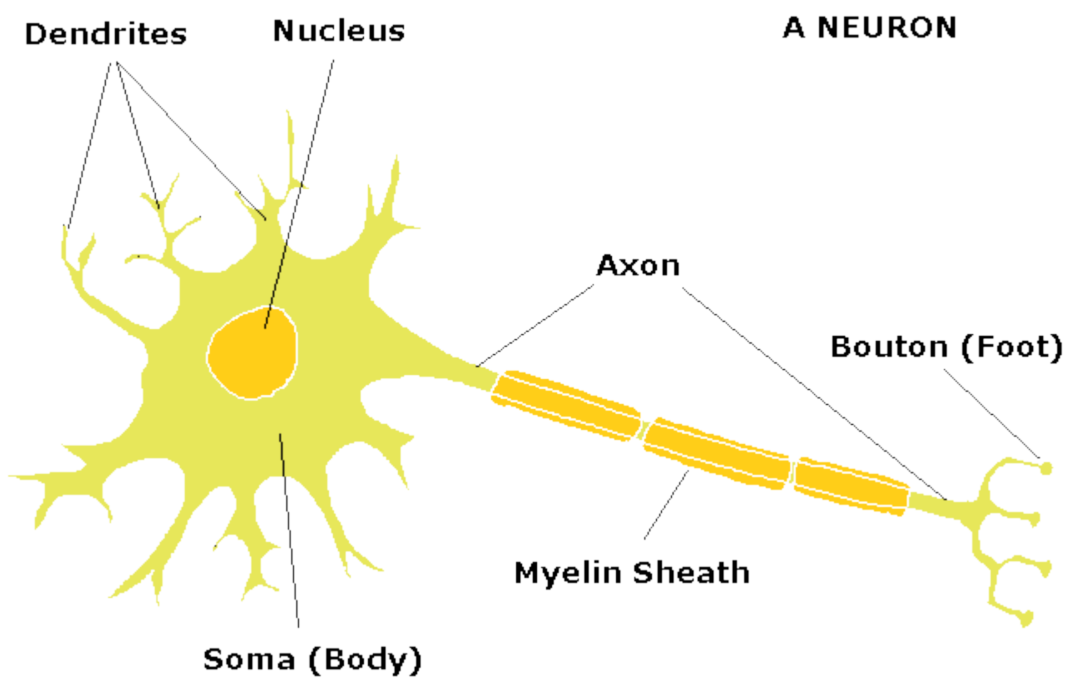
What we know and don't know.

1. How is information coded in neural activity?
2. How are memories stored and retrieved?
3. What does the baseline activity in the brain represent?
4. How do brains simulate the future?
5. What are emotions?
6. What is intelligence?
7. How is time represented in the brain?
8. Why do brains sleep and dream?
9. How do the specialized systems of the brain integrate with one another?
10. What is consciousness?

David Eagleman, Discover Magazine, 08/2007

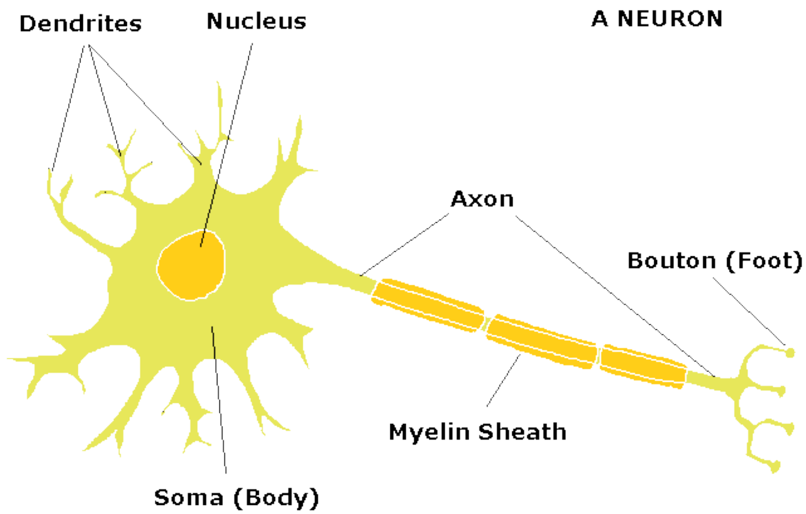
Neurons

Neurons do most of the work related to transmission of signals inside the brain.

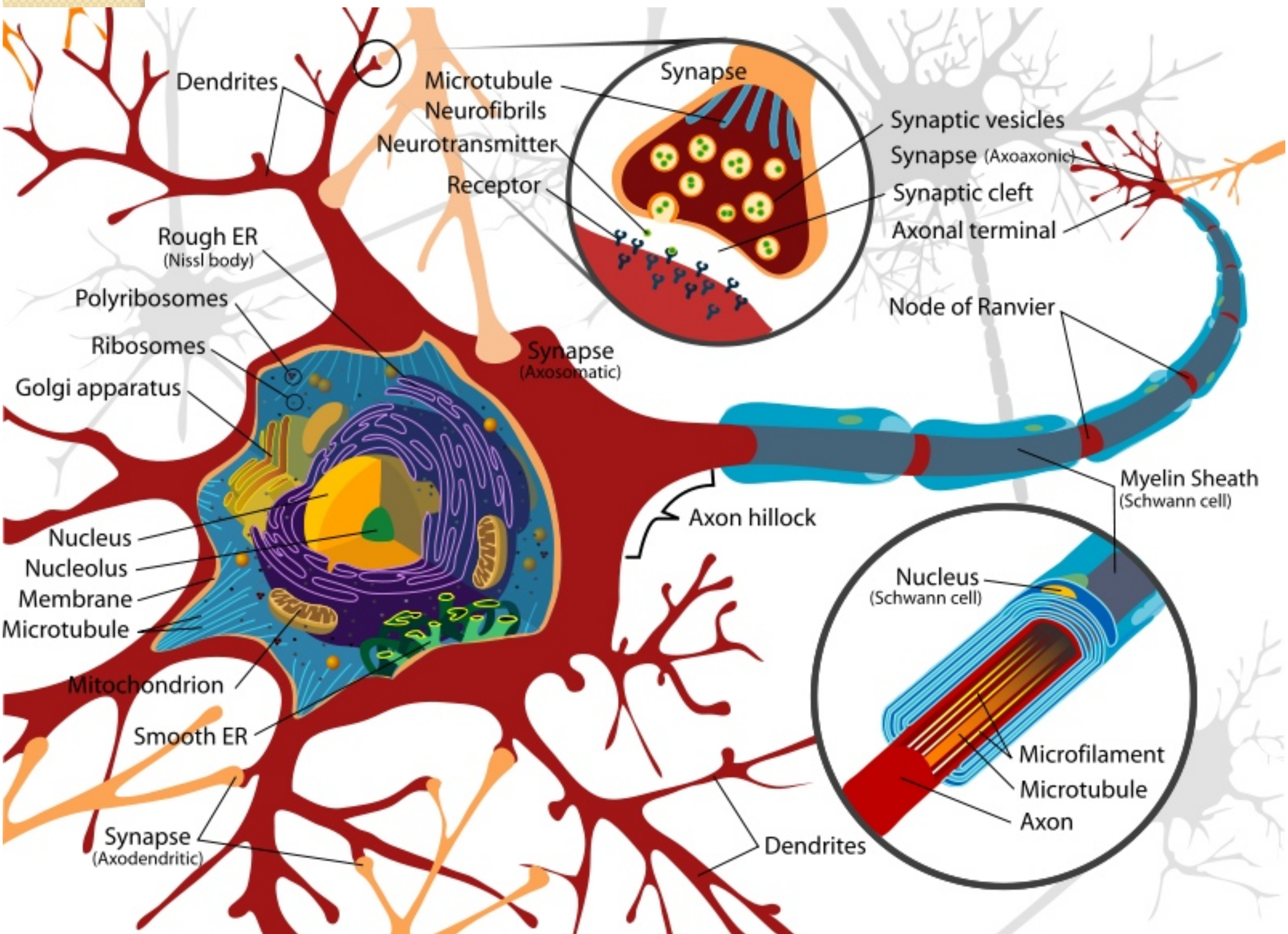


"*I often say when you can measure what you are speaking about and express it in numbers you know something about it; but when you cannot measure it, when you cannot express it in numbers, your knowledge of it is of a meagre and unsatisfactory kind.*"

Lord Kelvin



Neuroscience for Kids
http://faculty.washington.edu/chudler/neurok.html


Dendrites
Microtubule
Neurofibrils
Synapse
Synaptic vesicles
Synapse (Axoaxonic)
Neurotransmitter
Receptor
Synaptic cleft
Axonal terminal
Rough ER
(Nissl body)
Polyribosomes
Node of Ranvier
Ribosomes
Synapse
(Axosomatic)
Golgi apparatus
Myelin Sheath
(Schwann cell)
Axon hillock
Nucleus
Nucleolus
Nucleus
(Schwann cell)
Membrane
Microtubule
Mitochondrion
Smooth ER
Microfilament
Microtubule
Axon
Synapse
(Axodendritic)
Dendrites

Charlie Rose Brain Series

10 Episodes: 1 hour each.

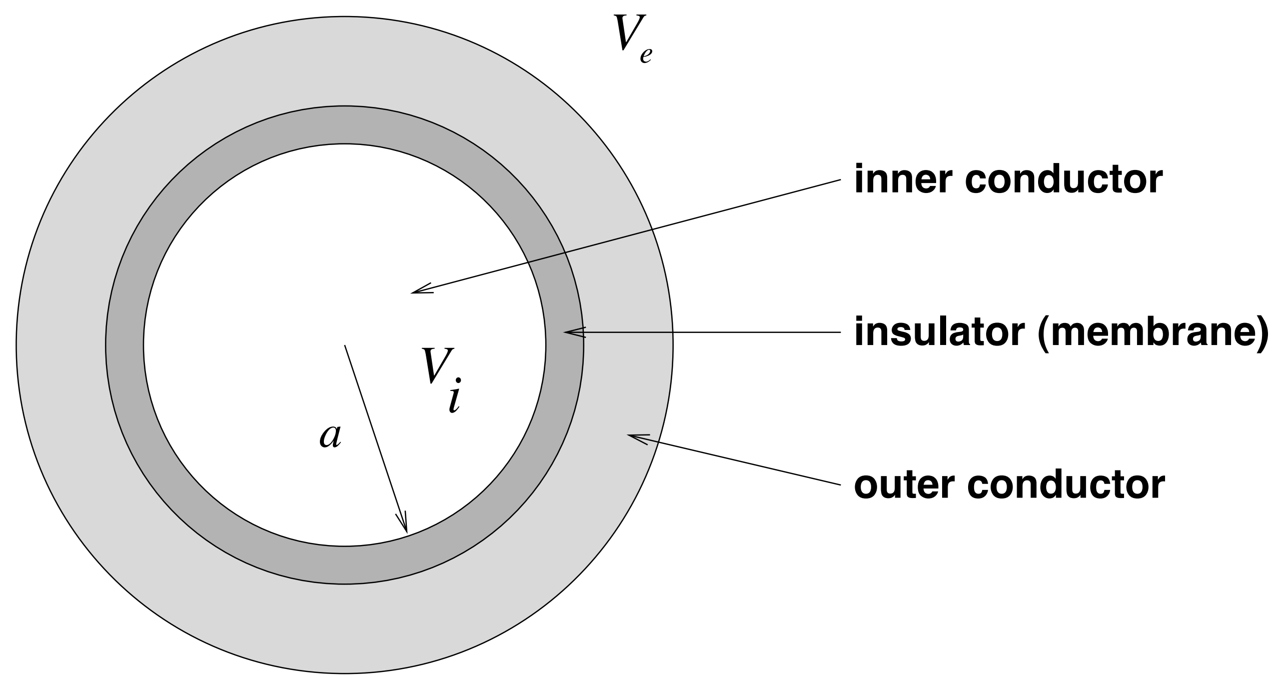
http://www.charlierose.com/view/collection/10702

Gerald D. Fischbach
Simons Foundation
James Harris Simons

http://simonsfoundation.org/

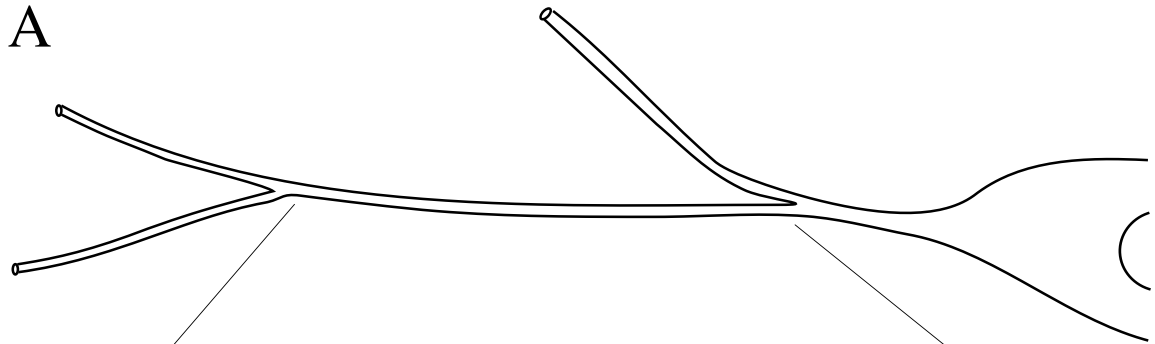
Action Potential

http://www.blackwellpublishing.com/matthews/channel.html

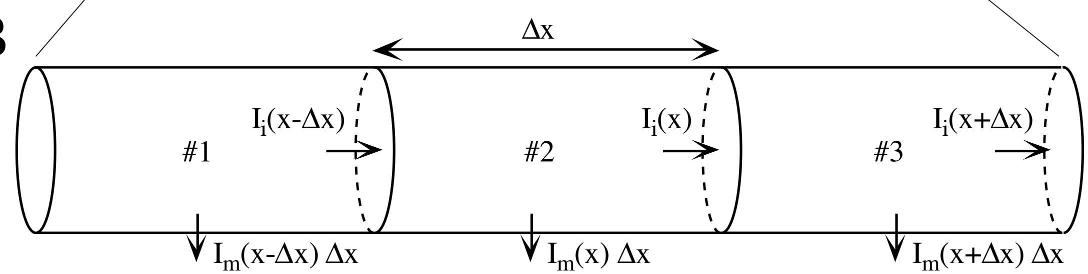


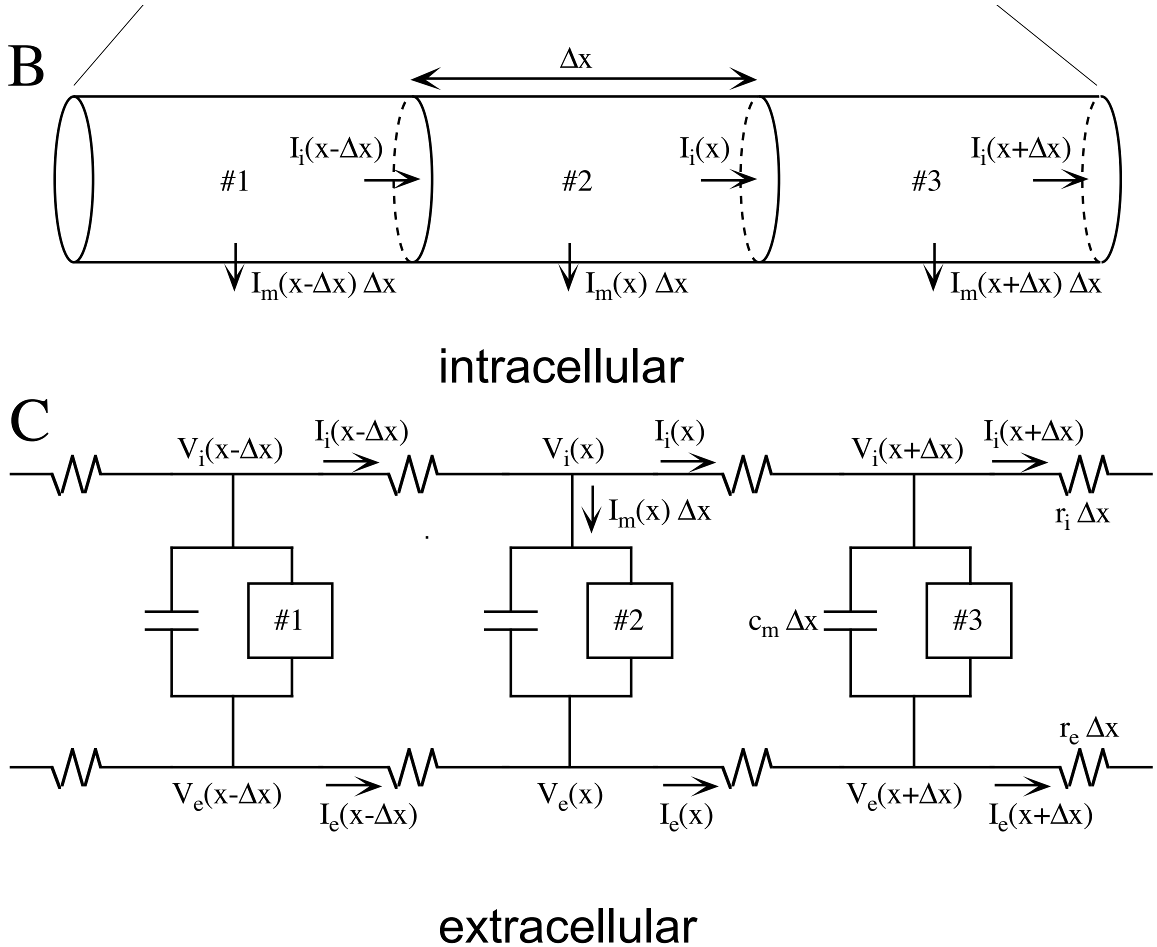
V_e
inner conductor
insulator (membrane)
V_i
a
outer conductor

A

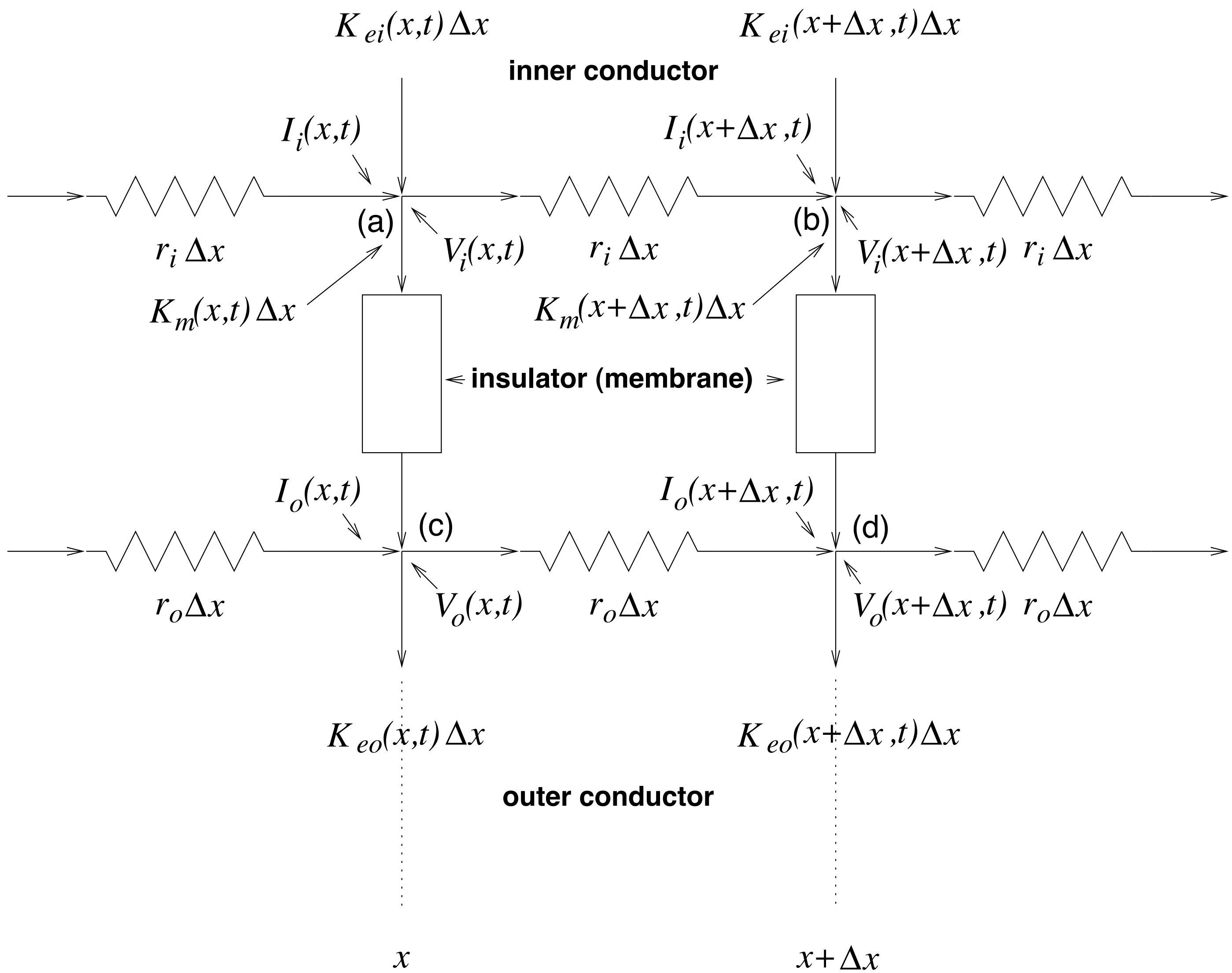


B

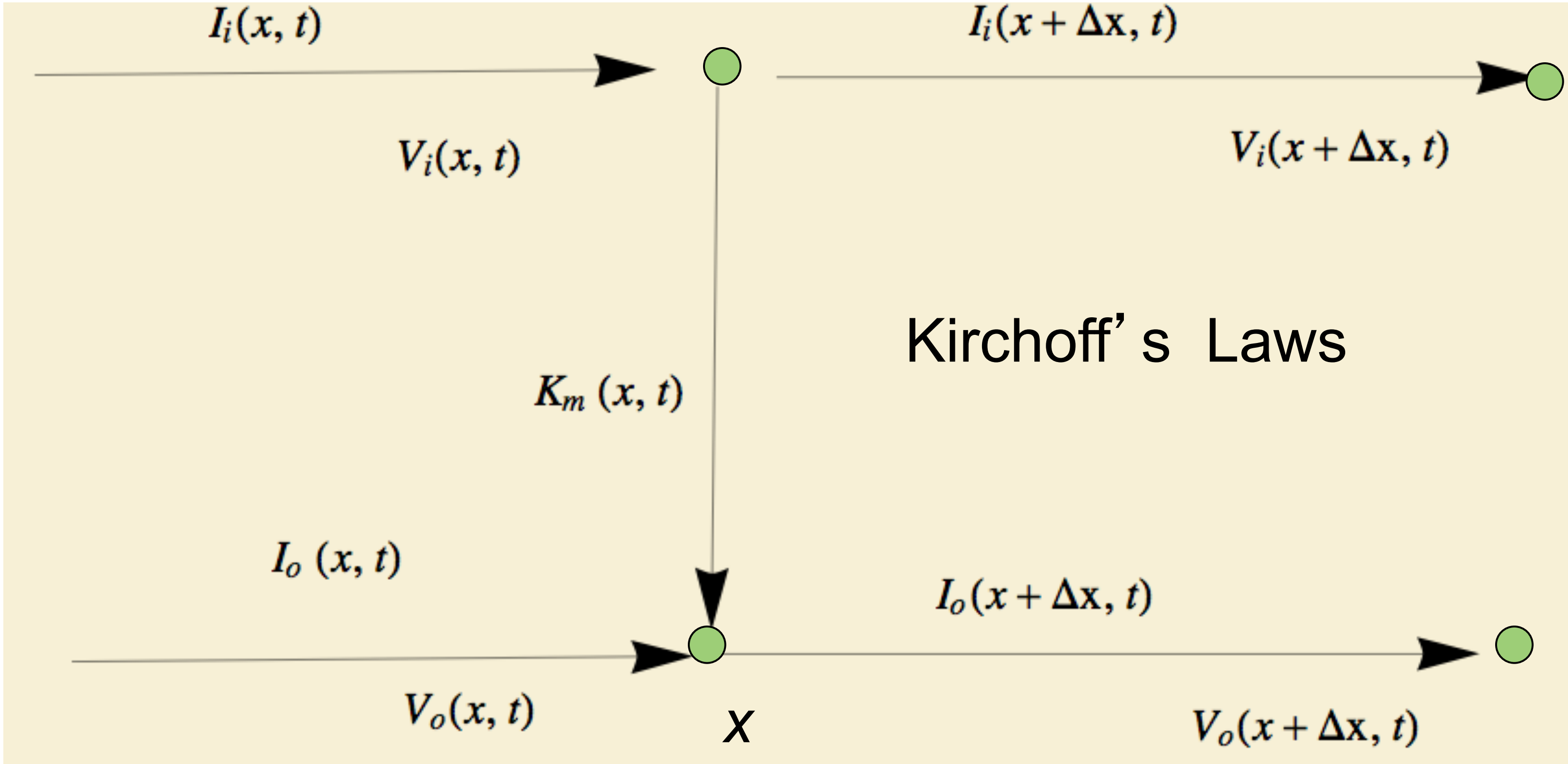




B
Δx
#1
#2
#3
I_i(x-Δx)
I_i(x)
I_i(x+Δx)
I_m(x-Δx) Δx
I_m(x) Δx
I_m(x+Δx) Δx
intracellular
C
V_i(x-Δx)
V_i(x)
V_i(x+Δx)
I_i(x-Δx)
I_i(x)
I_i(x+Δx)
I_m(x) Δx
r_i Δx
c_m Δx
#1
#2
#3
r_e Δx
V_e(x-Δx)
V_e(x)
V_e(x+Δx)
I_e(x-Δx)
I_e(x)
I_e(x+Δx)
extracellular



$K_{ei}(x,t)\,\Delta x$
$K_{ei}(x+\Delta x,t)\Delta x$
inner conductor
$I_i(x,t)$
$I_i(x+\Delta x,t)$
(a)
(b)
$V_i(x,t)$
$V_i(x+\Delta x,t)$
$r_i\,\Delta x$
$r_i\,\Delta x$
$r_i\,\Delta x$
$K_m(x,t)\,\Delta x$
$K_m(x+\Delta x,t)\Delta x$
insulator (membrane)
$I_o(x,t)$
$I_o(x+\Delta x,t)$
(c)
(d)
$V_o(x,t)$
$V_o(x+\Delta x,t)$
$r_o\Delta x$
$r_o\Delta x$
$r_o\Delta x$
$K_{eo}(x,t)\,\Delta x$
$K_{eo}(x+\Delta x,t)\Delta x$
outer conductor
x
$x+\Delta x$



$$I_i (x, t) = I_i (x + \Delta x, t) + K_m (x, t) \Delta x$$

$$I_o (x, t) + K_m (x, t) \Delta x = I_o (x + \Delta x, t)$$

$$V_i (x, t) - V_i (x + \Delta x, t) = r_i \Delta x \, I_i (x + \Delta x, t)$$

$$V_o (x, t) - V_o (x + \Delta x, t) = r_o \Delta x \, I_o (x + \Delta x, t)$$

$$V_m (x, t) = V_i (x, t) - V_o (x, t)$$

$$I_i(x,t) = I_i(x+\Delta x,t) + K_m(x,t)\,\Delta x$$

$$I_o(x,t) + K_m(x,t)\,\Delta x = I_o(x+\Delta x,t)$$

$$V_i(x,t) - V_i(x+\Delta x,t) = r_i\,\Delta x\,I_i(x+\Delta x,t)$$

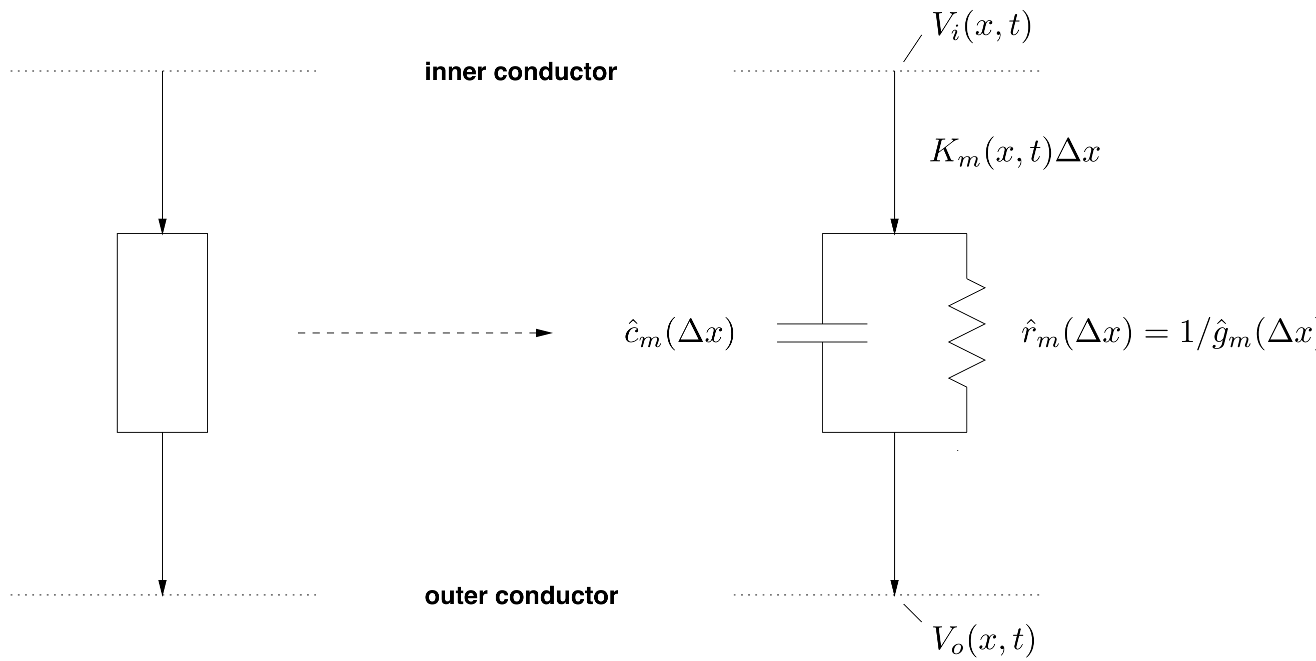
$$V_o(x,t) - V_o(x+\Delta x,t) = r_o\,\Delta x\,I_o(x+\Delta x,t)$$

$$V_m(x,t) = V_i(x,t) - V_o(x,t)$$

$$\frac{\partial}{\partial x} V_m(x,t) = r_o\,I_o(x,t) - r_i\,I_i(x,t)$$

$$\frac{\partial^2}{\partial x^2} V_m(x,t) = r_o\,\frac{\partial}{\partial x} I_o(x,t) - r_i\,\frac{\partial}{\partial x} I_i(x,t)$$

$$\frac{\partial^2}{\partial x^2} V_m(x,t) = (r_o + r_i)\,K_m(x,t)$$



$$K_m(x,t) = \hat{c}_m(\Delta x)\frac{\partial}{\partial t}[V_i(x,t) - V_o(x,t)]$$

$$+[V_i(x,t) - V_o(x,t)]/\hat{r}_m(\Delta x)$$

$$= \quad \hat{c}_m(\Delta x)\frac{\partial}{\partial t}V_m(x,t) + V_m(x,t)\,\hat{g}_m(\Delta x)$$

$$\frac{1}{r_o + r_i} \frac{\partial^2}{\partial x^2} V_m(x, t) = K_m(x, t)$$

$$K_m(x, t) = \hat{c}_m(\Delta x) \frac{\partial}{\partial t} V_m(x, t) + V_m(x, t)\, \hat{g}_m(\Delta x)$$

$$\hat{c}_m(\Delta x) \frac{\partial}{\partial t} V_m(x, t) = \frac{1}{r_o + r_i} \frac{\partial^2}{\partial x^2} V_m(x, t) - V_m(x, t)\, \hat{g}_m(\Delta x)$$

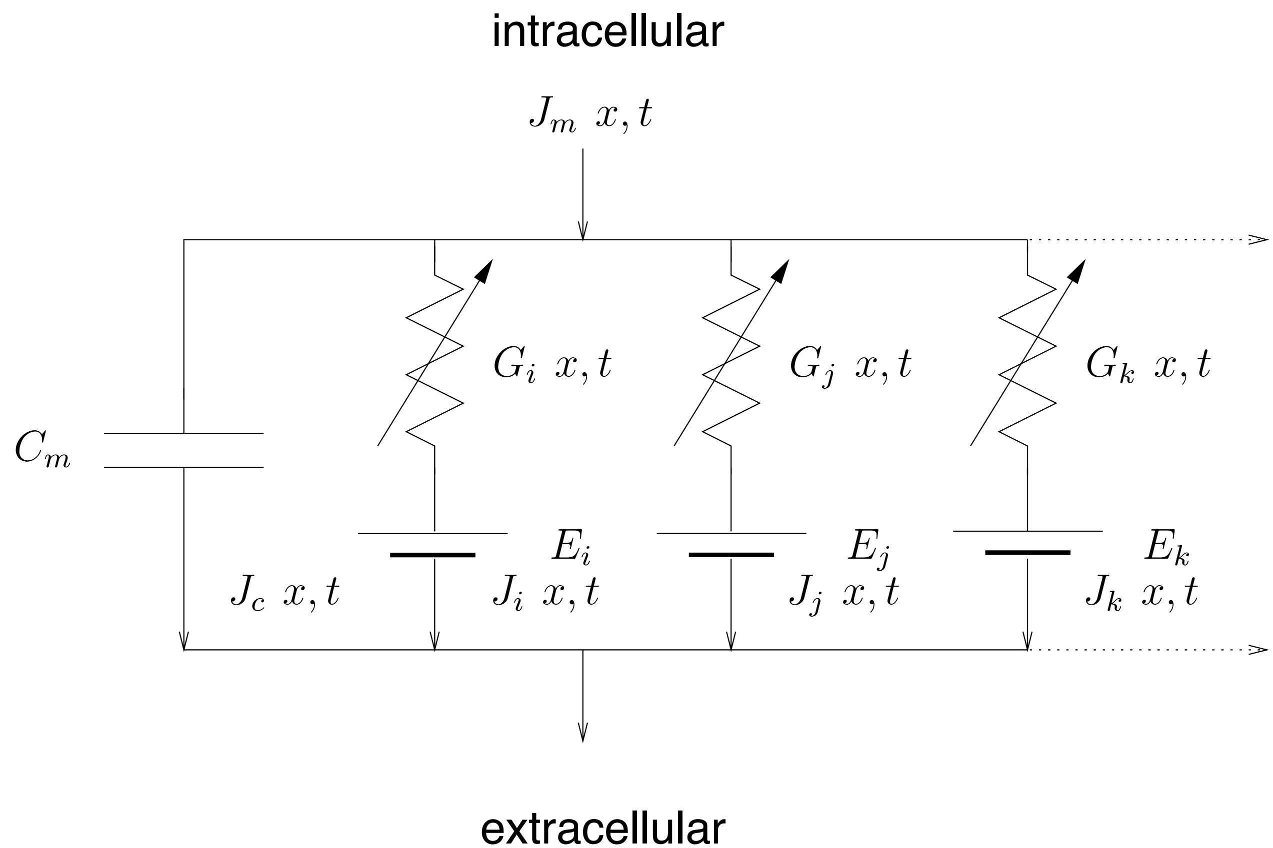
$$\frac{\partial}{\partial t} V_m(x, t) = M \frac{\partial^2}{\partial x^2} V_m(x, t)$$

Heat Equation

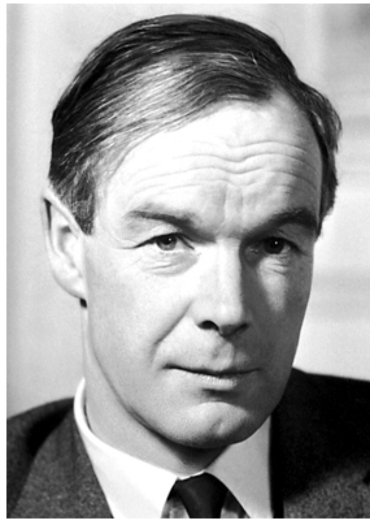
$$V_m(x, t) = e^{-Mt} \sin x \qquad V_m(x, t) \rightarrow 0$$

How can a heat equation make "waves" ??

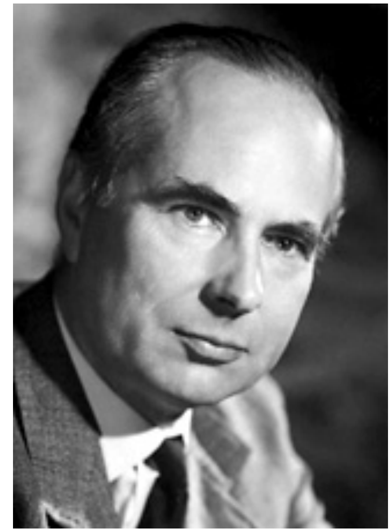
$$\frac{1}{2\pi a r_i}\frac{\partial^2 V_m}{\partial x^2} = C_m \frac{\partial V_m}{\partial t} + \sum_k (V_m - E_k)\ G_k(x,\ t,\ V_m)$$



Hodgkin



Huxley



$$C \rightleftarrows O$$

$$\frac{dn}{dt} = \alpha(V_m,\ t)(1-n) - \beta(V_m,\ t)\,n$$

$$G_k(x,\ t,\ V_m) = C\,n^{\text{integer exponent}}(t)$$

The Nobel Prize in Physiology or Medicine 1963 was awarded jointly to Sir John Carew Eccles, Alan Lloyd Hodgkin and Andrew Fielding Huxley "*for their **discoveries concerning the ionic mechanisms** involved in excitation and inhibition in the peripheral and central portions of the nerve cell membrane*"

Waves are just a Figment of our Imagination

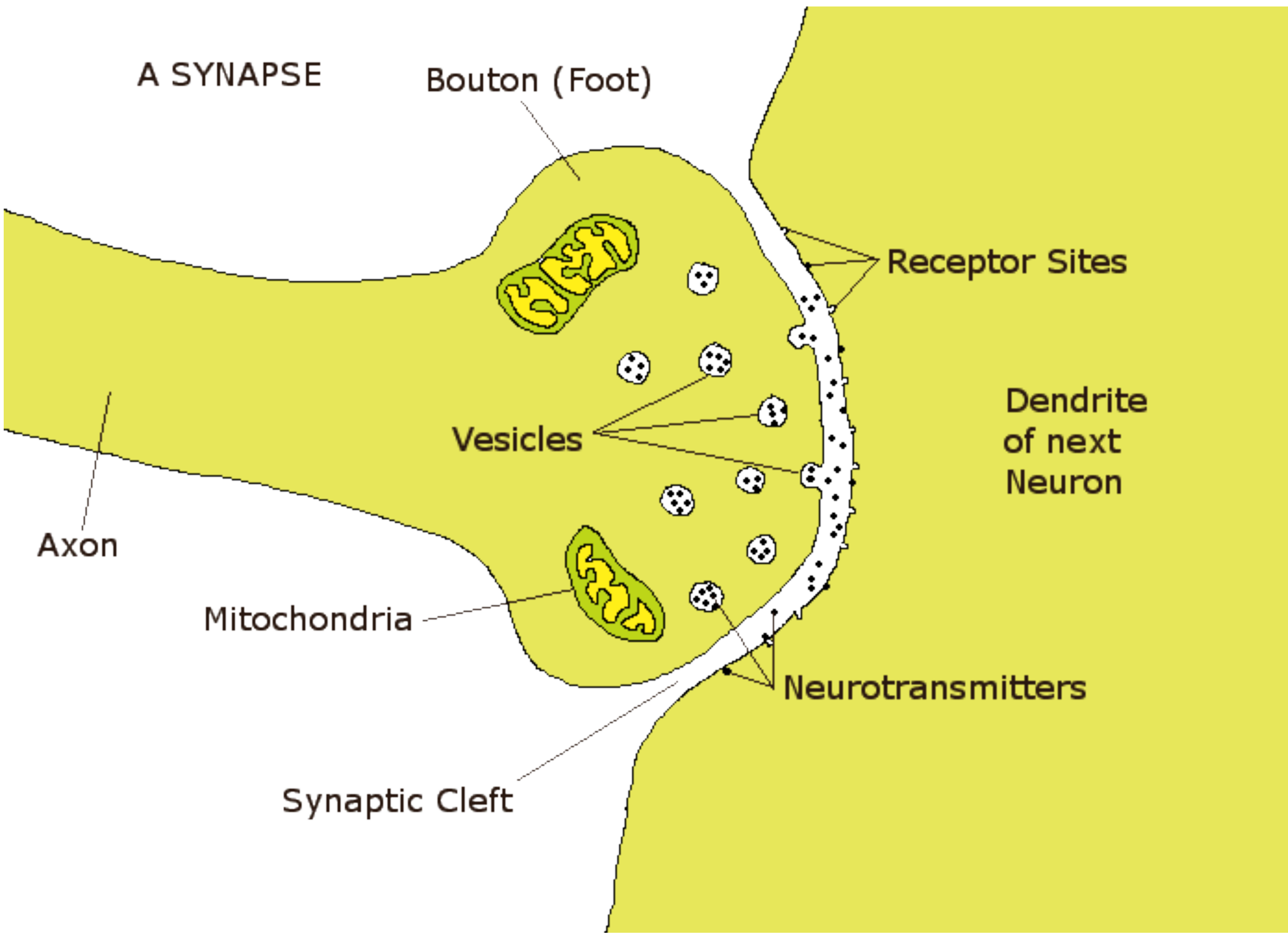
http://www.blackwellpublishing.com/matthews/actionp.html

$$V_m(x, t) = U(x - ct)$$

$$\frac{1}{2\pi a r_i}\frac{\partial^2 V_m}{\partial x^2} = C_m \frac{\partial V_m}{\partial t} + (V_m - E_k)\, G_k(x, t, V_m)$$

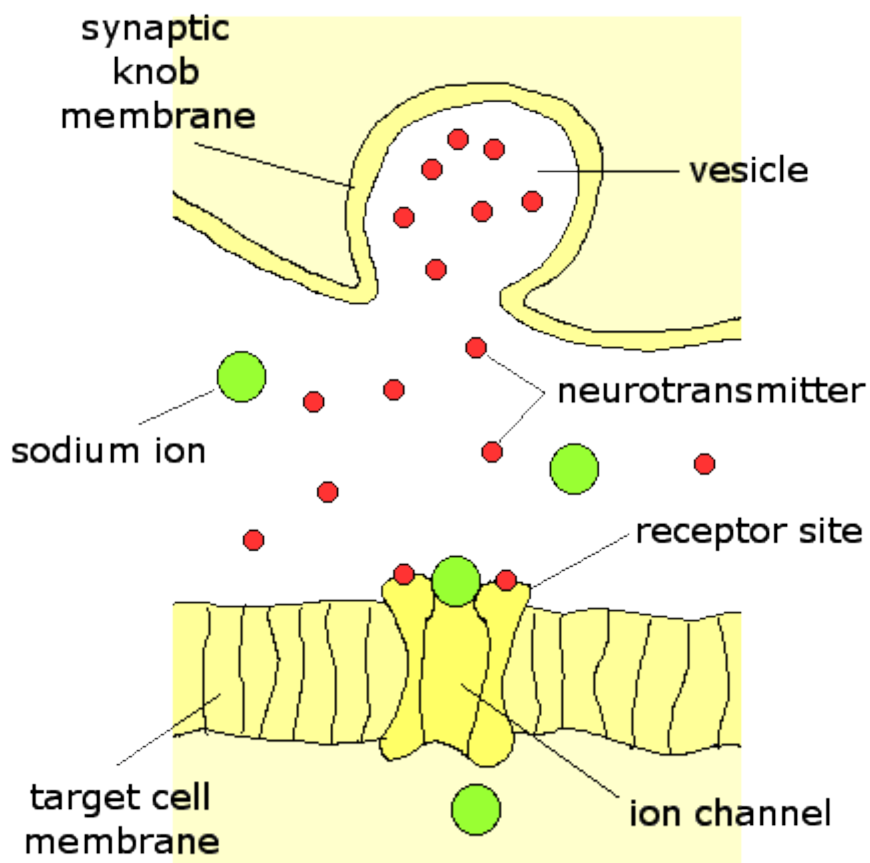
$$G_k(x, t) = \frac{\dfrac{1}{2\pi a\, r_i}\dfrac{\partial^2 V_m}{\partial x^2} - C_m \dfrac{\partial V_m}{\partial t}}{V_m - E_k}$$

Synapse



A SYNAPSE
Bouton (Foot)
Receptor Sites
Vesicles
Dendrite of next Neuron
Axon
Mitochondria
Neurotransmitters
Synaptic Cleft

Synapse



synaptic
knob
membrane
vesicle
neurotransmitter
sodium ion
receptor site
target cell
membrane
ion channel

Eric R. Kandel
The Nobel Prize in Physiology or Medicine 2000

11 open problems in the biology of memory, J. of Neuroscience **29**(41):12748-12756 (2009)

3. What can computational models contribute to understanding synaptic plasticity?
5. What firing patterns do neurons actually use to initiate long-term plasticity at various synapses?

Eduardo Izquierdo: Unlike the 23 problems proposed by David Hilbert, the vagueness of Kandel's open questions suggest an even grander question: What is needed in the study of biological memory to allow us to propose more concrete and formal questions?

Neurons talk to each other

- 10^{11} Neurons and 10^{15} Synapses

Need to Know:

- How a specific neuron works
- How neurons communicate with each other (networks)

Our Question:

- **How a specific neuron interacts with its network when delay is present**

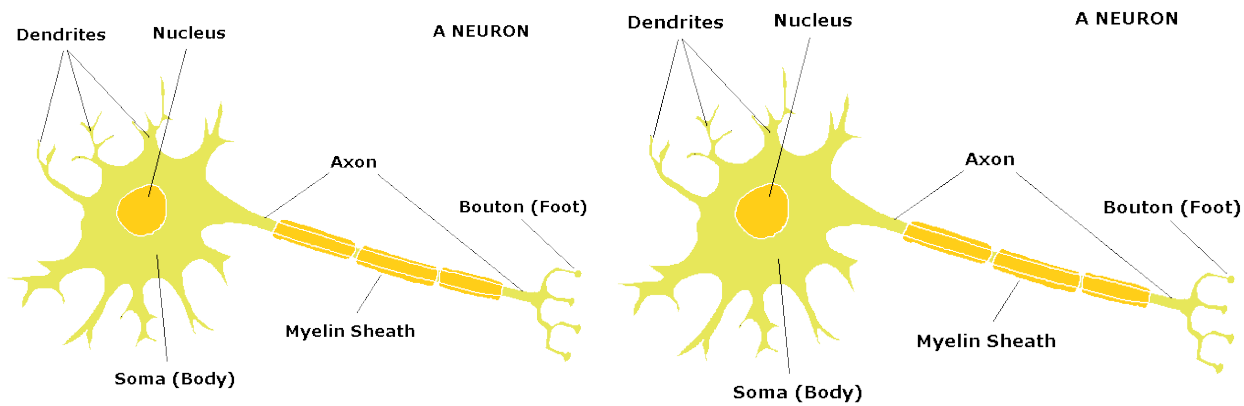
Types of Delay

- **Modeled within the Network:**
  - STDP: Spike-Timing Dependent Plasticity
  - Conduction Delay: Delay in the signal itself
- **Modeled within the Cable Equation:**
  - Boundary Conditions
  - Within the PDE model itself (discrete)

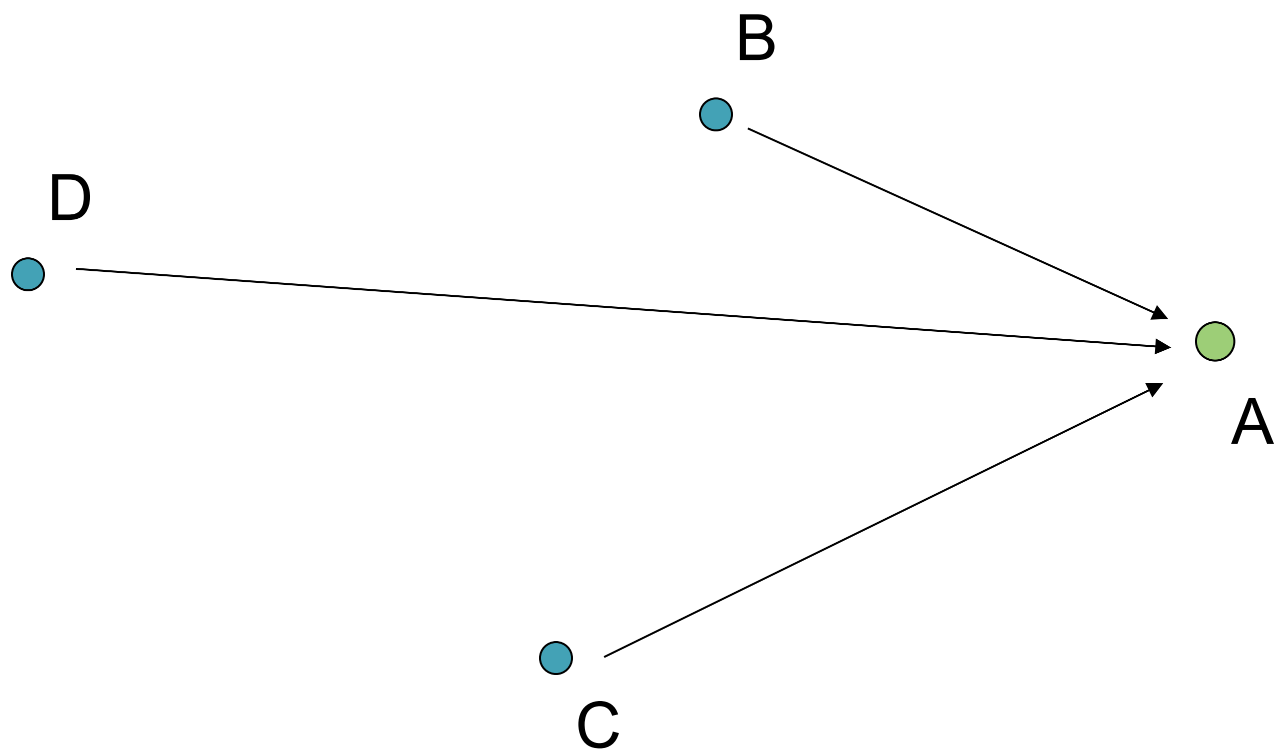
STDP: Spike-Time Dependent plasticity

A

B



Conduction Delay



PDE Delay

$$\frac{1}{2\pi a r_i}\frac{\partial^2 V_m}{\partial x^2} = C_m \frac{\partial V_m}{\partial t}$$

$$+ \sum_k (V_m - E_k)\ G_k(x,\ t - \tau,\ V_m)$$

τ is the delay

Emulating the Network

A.2 Spiking Neurons. Each neuron in the network is described by the simple spiking model (Izhikevich, 2003)

$$v' = 0.04v^2 + 5v + 140 - u + I \tag{A.1}$$

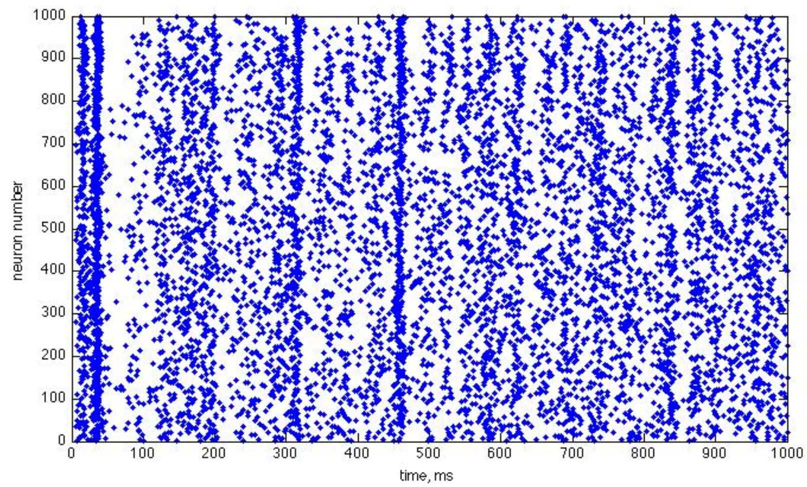
$$u' = a(bv - u) \tag{A.2}$$

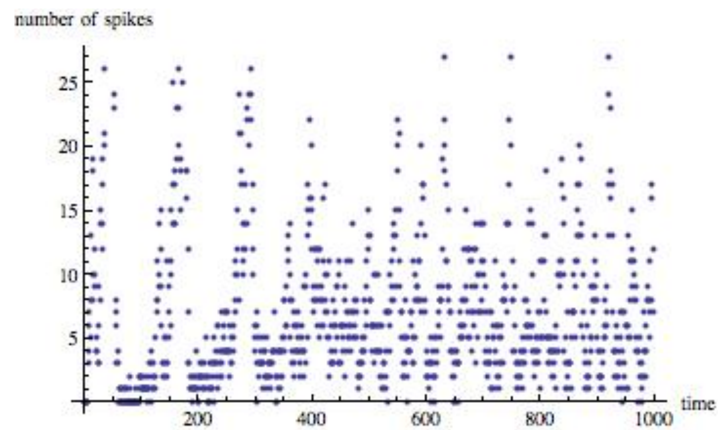
with the auxiliary after-spike resetting

$$\text{if } v \geq +30 \text{ mV, then } \begin{cases} v \leftarrow c \\ u \leftarrow u + d. \end{cases} \tag{A.3}$$

Simple Network Simulation

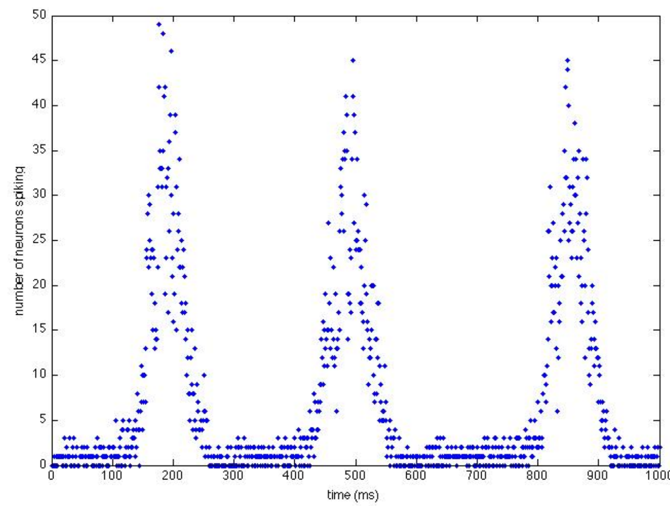
STDP





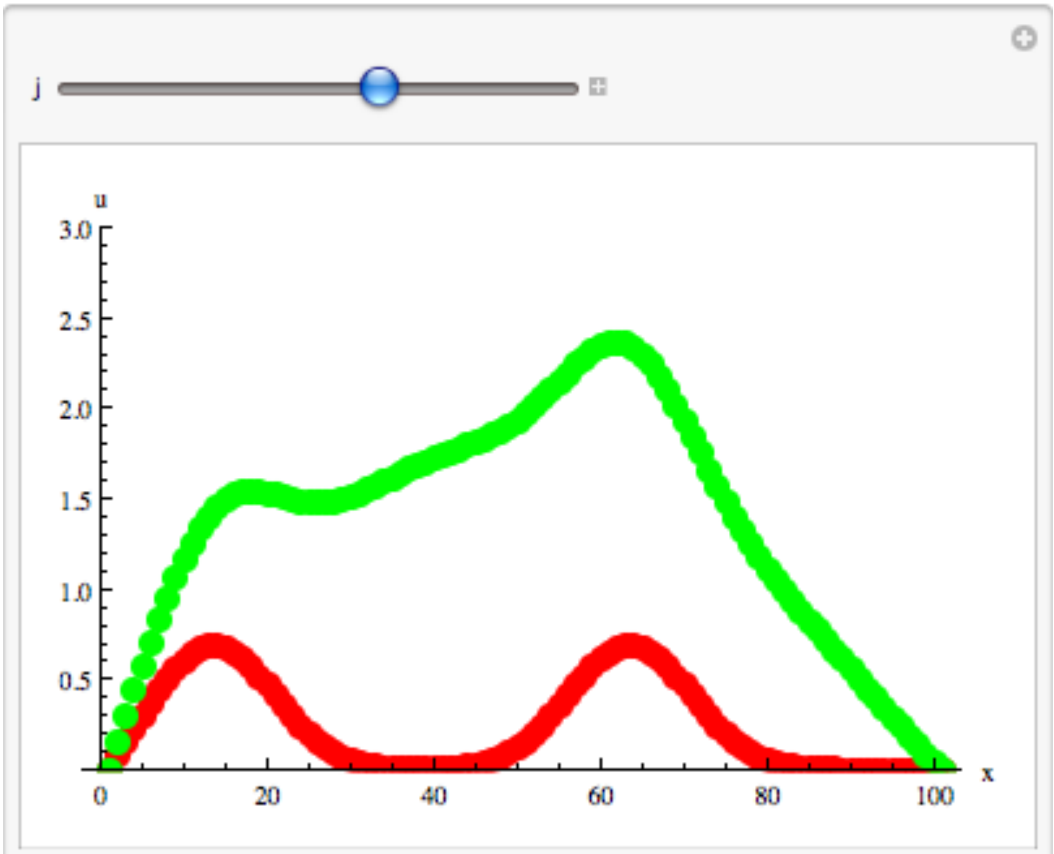
Simulation of randomly connected 1000 coupled spiking neurons

STDP with Conduction Delay

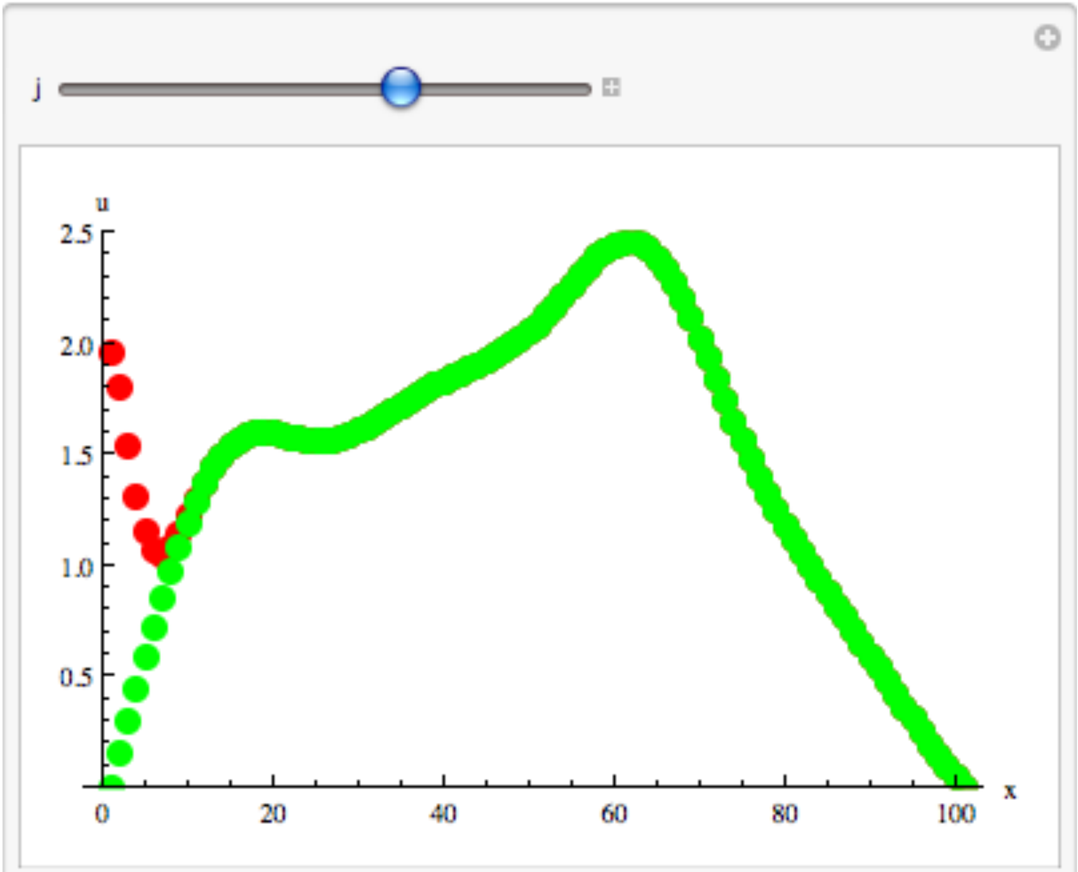


Simulation of 1000 polychronized neurons with maximal delay = 20ms

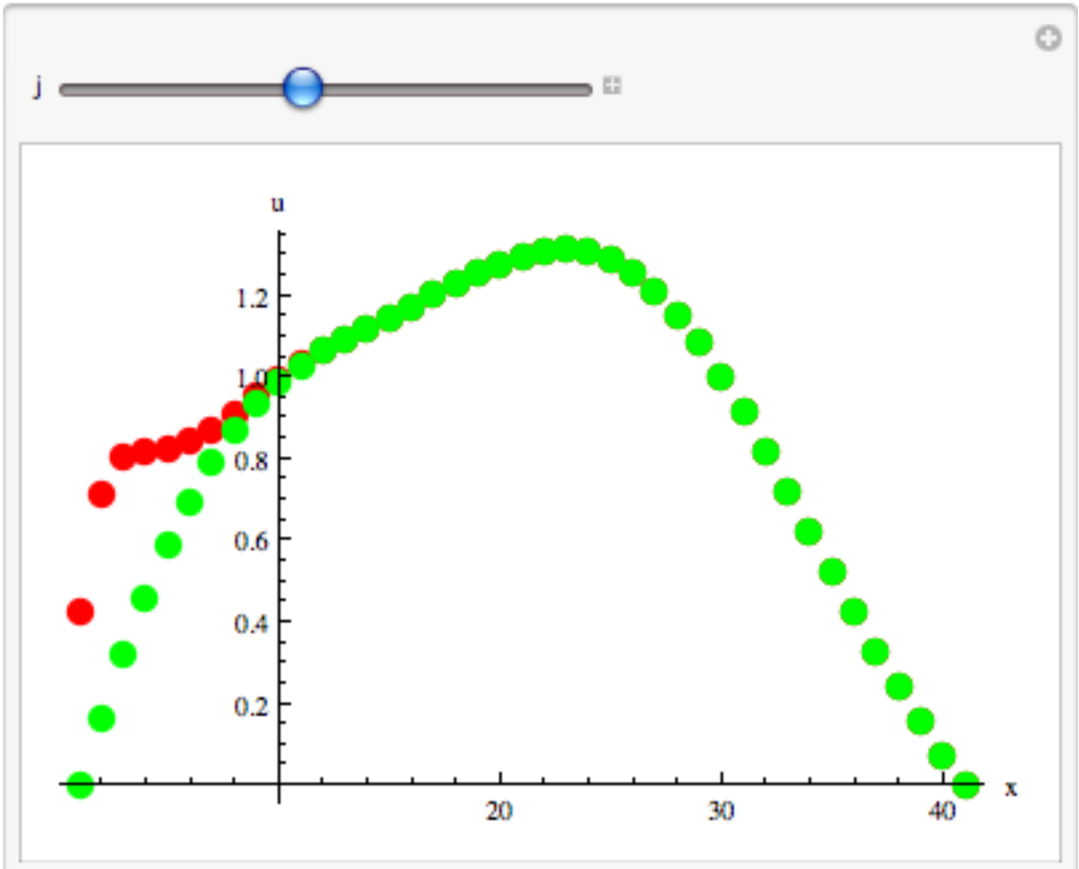
Passive without PDE delay
G = 0 (Red) vs G < 0



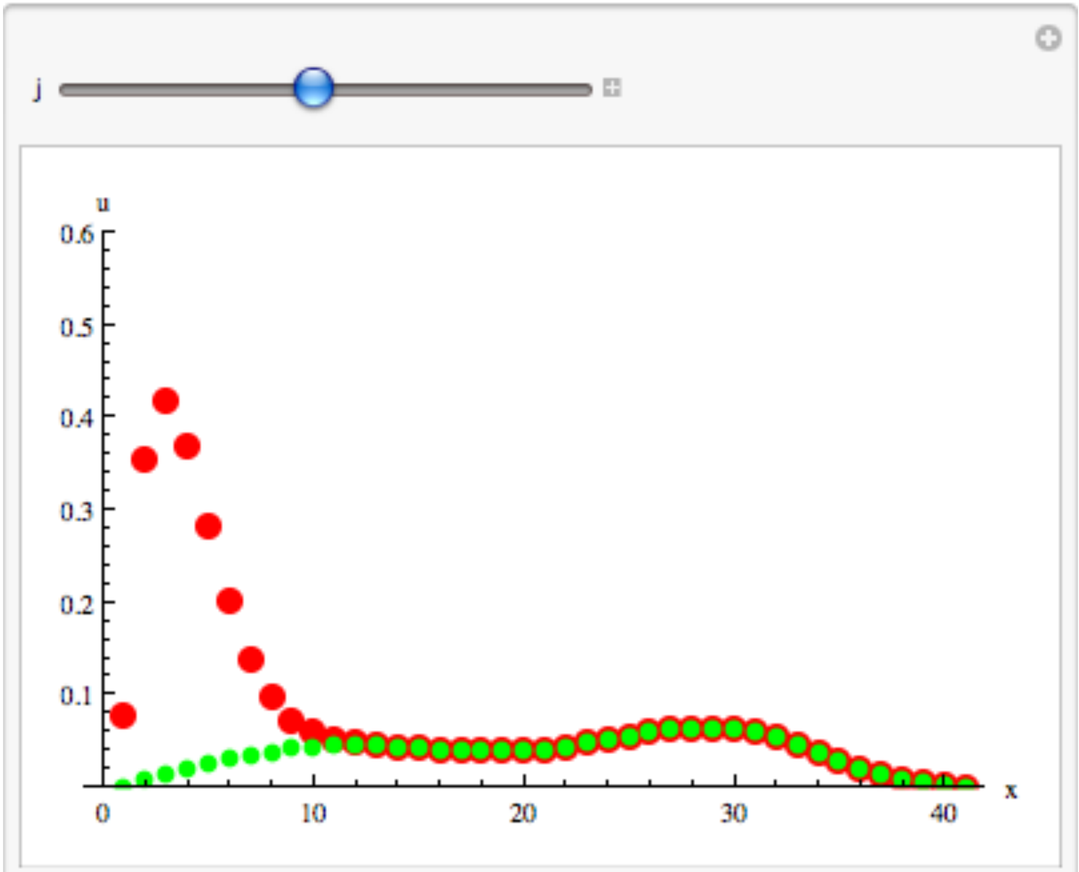
Passive without PDE delay
STDP (Red) vs No STDP



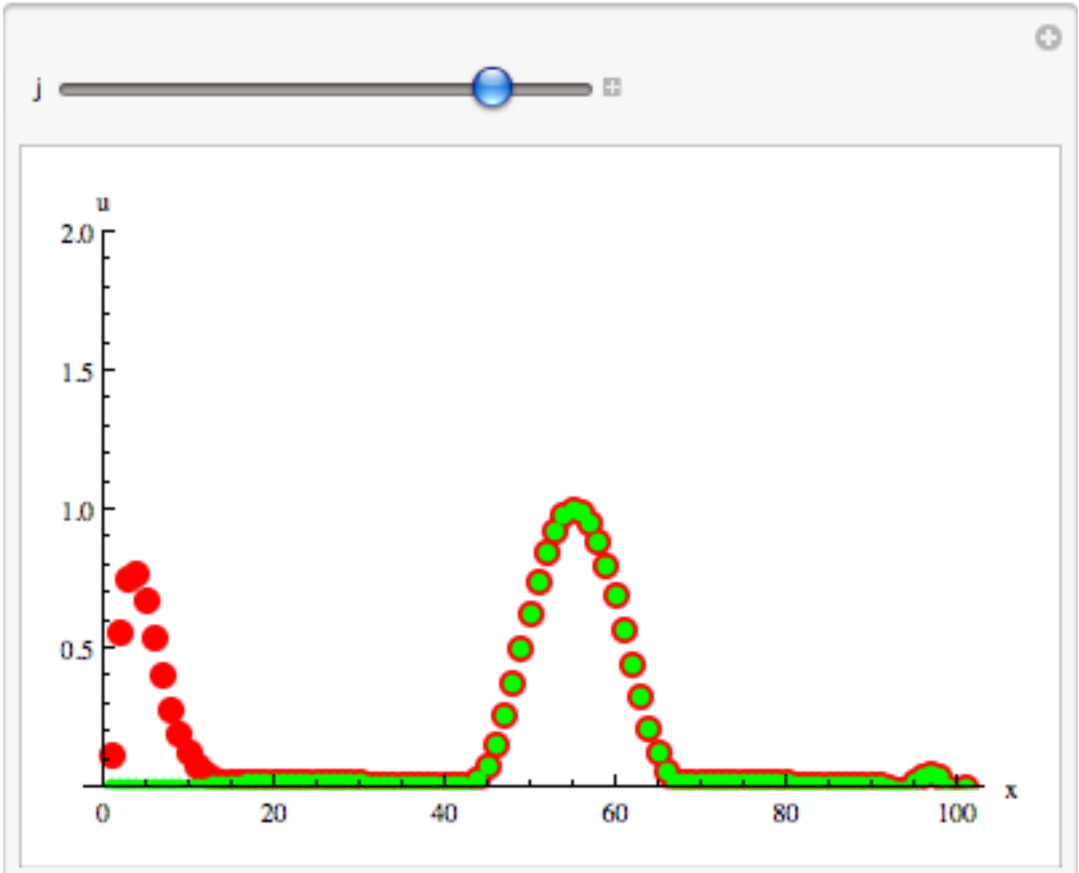
Passive without PDE delay
Polychronization (Red) vs No STDP



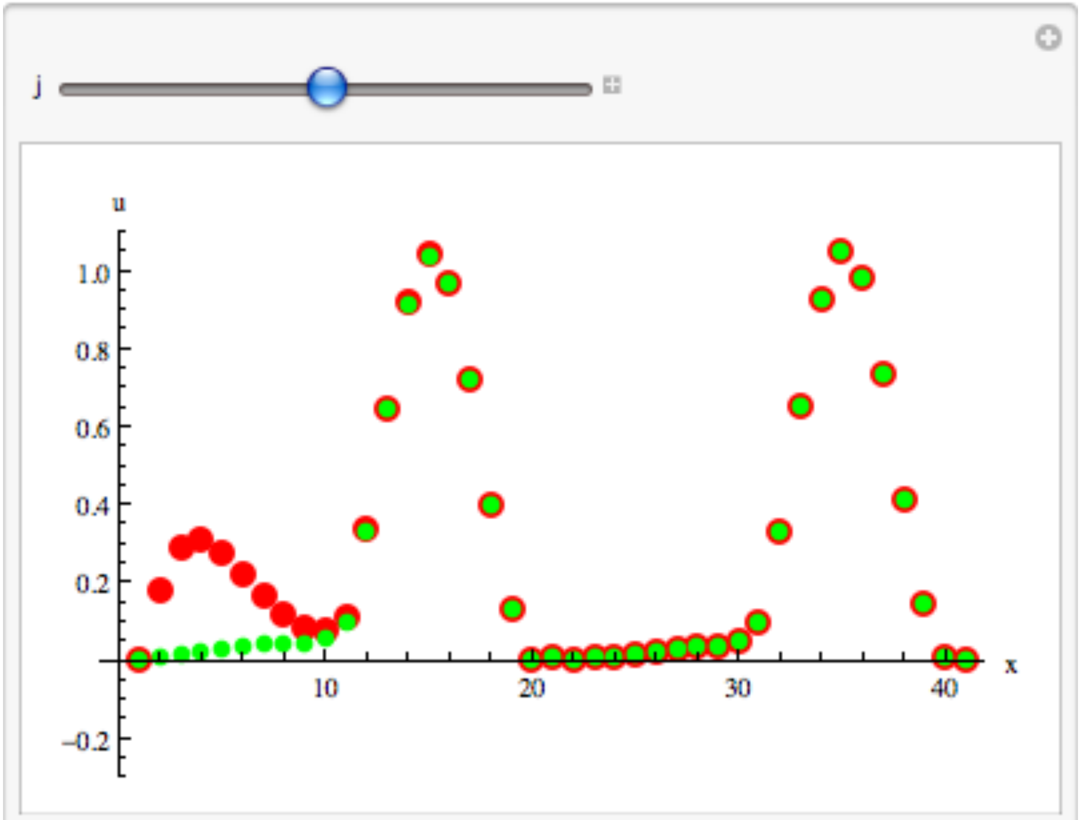
Propagating without PDE delay
Polychronization (Red) vs No STDP



Propagating with PDE delay
STDP (Red) vs No STDP

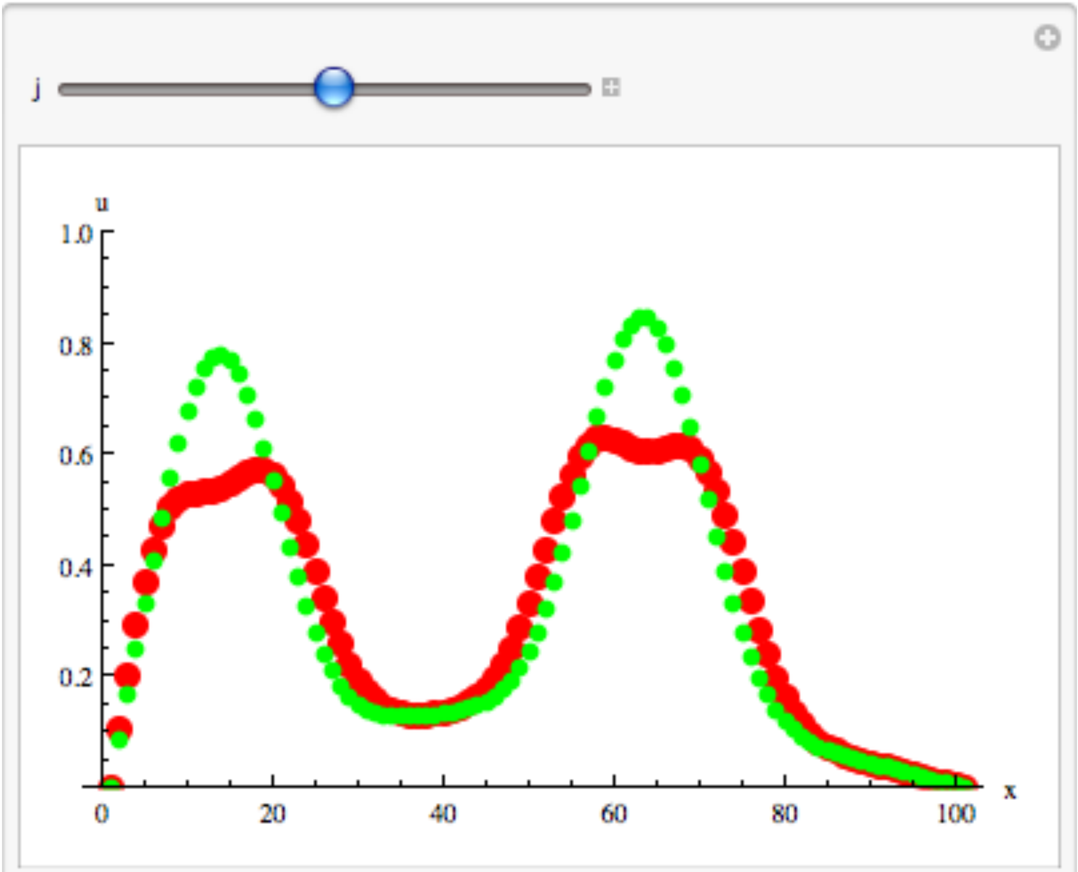


Propagating with PDE delay
Polychronization (Red) vs No STDP

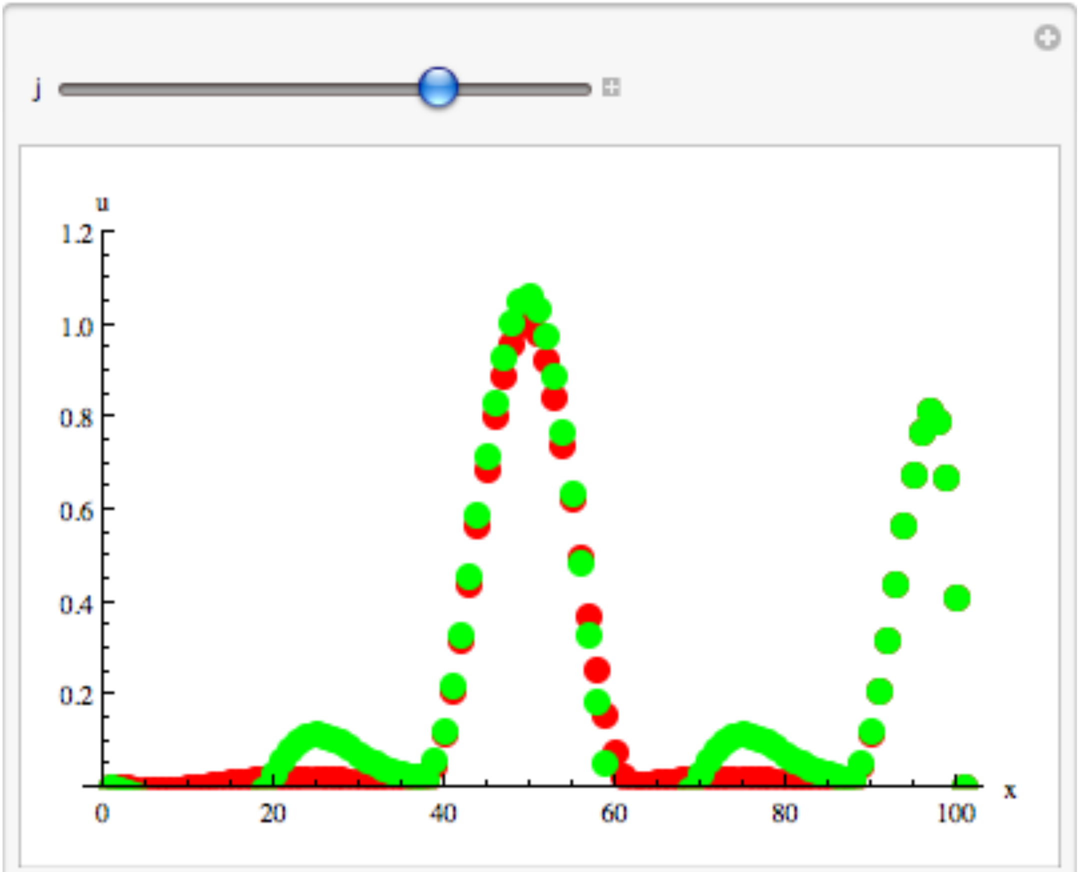


Passive Cable

PDE delay (Red) No PDE Delay



Propagating Potential
PDE delay (Red) vs No PDE Delay



Computational Science Training for Undergraduates in the Mathematical Sciences (CSUMS)

National Science Foundation (Grant DMS-0802959)

Students:
Ann Motl,
Dee Prioleau,
Natasha Wright
(St. Thomas)

Faculty Mentors:
Misha Shvartsman
(St. Thomas)
Pavel Bělík
(Augsburg)

Ben Stottrup (Augsburg College):
Tom Lopez, Justin Gyllen,
Casey Ernst, ???
Identifying boundary in biological systems, Viscosity Modeling and Analysis

Arkady Shemyakin (University of St. Thomas):
Mark Jadin, Pat Persons, Jenny Woychik
Adam Lanser ???
Modeling Multivariate Financial Data Using Copula Functions

Pat Van Fleet (University of St. Thomas):
Susan Ray, Cristen Bonz, Liz Motz
Mutliwavelets and Image Compression