**HOMEWORK**

(1) Translate the statement and proof of problem1 in Hw10.lean to a pen-and-paper statement and proof.
(2) Translate the statement and proof of problem2 in Hw10.lean to a pen-and-paper statement and proof.
(3) Translate the statement and proof of problem3 in Hw10.lean to a pen-and-paper statement and proof.
(4) Below is a theorem and proof. Translate the statement as a theorem problem4 and give a proof of it in Lean in Hw10.lean.

    **Theorem.** Let $f : A \to B$ and $g : B \to C$ be functions. If $d : B \to A$ is a left inverse to $f$ and $e : C \to B$ is a left inverse to $g$, then $d \circ e$ is a left inverse to $g \circ f$.

    *Proof.* To be a left inverse we need to show that
    $$(d \circ e) \circ (g \circ f) = \mathrm{id}_A$$
    for all $a$. Using the fact that composition is associative, we can rewrite the left-hand side as
    $$(d \circ e) \circ (g \circ f) = d \circ (e \circ g) \circ f$$
    By definition of a left inverse, we know that $e \circ g = \mathrm{id}_B$ so we can rewrite again
    $$d \circ (e \circ g) \circ f = d \circ \mathrm{id}_B \circ f$$
    Since composition with the identity function is the identity we have
    $$d \circ \mathrm{id}_B \circ f = d \circ f$$
    Finally, as $d$ is a left inverse to $f$, we have
    $$d \circ f = \mathrm{id}_A$$
    □

(5) Below is a theorem and proof. Translate the statement as a theorem labeled problem5 give a proof of it in Lean in Hw10.lean.

    **Theorem.** Being in bijection is symmetric. More precisely, if there exists a bijection $A \cong B$, then there also exists a bijection $B \cong A$.

    *Proof.* Let $f : A \to B$ be a bijection. Since $f$ is a bijection, it has an inverse $f^{-1} : B \to A$ which satisfies $f \circ f^{-1} = \mathrm{id}_B$ and $f^{-1} \circ f = \mathrm{id}_A$. We claim that $f^{-1}$ is also bijection. But the conditions we just listed imply that $f$ is the inverse of $f^{-1}$. Since $f^{-1}$ has an inverse, it is a bijection also. □