Probability of 8/99 lottery

1.Introduction

The computations below describe the distribution and prove the fairness of a 8/99 lottery draw containing 3 different colors of winning balls. The game consists of a virtual customer selecting 6 yellow, 1 red and 1 orange number and receiving prizes depending on the amount of yellow balls hit, while getting his prizes multiplied by the red and orange numbers. This paper aims to offer a comprehensive and rigorous explanation of the mathematics behind the probability of hitting respective winning combinations and prove the fairness of the draw.

2.Calculations

We assume that the winning numbers are sampled from a pool of 99 number(1:99) without replacement with each number having equal probabiltiy .

For the calculation of the probability of each winning scenario firstly we will consider only the 6 yellow numbers and their respective probabilites . For this we are going to be using the Hypergeometric distrbution . Also we consider a winning scenario by having 3 correct guessed yellow numbers or more .

$$\begin{gathered}\textit{Let } \boldsymbol{X} \textit{ be the number of succesfully guessed numbers}\\ \textit{Let } \boldsymbol{k} \textit{ be the number of guesses we want to have and } \boldsymbol{K} \textit{ be amount of yellow numbers}\\ \textit{Let } \boldsymbol{n} \textit{ be the number of draws and } \boldsymbol{N} \textit{ the population size}\\ \textit{Therefore the distribution is described by:}\end{gathered}$$

$$\Pr\left(X = k\right) = \frac{\binom{K}{k}\binom{N-K}{n-k}}{\binom{N}{n}}$$

$$\Pr(X = 3) = \frac{\binom{6}{3}\binom{99-6}{6-3}}{\binom{99}{6}} = 0.0023161$$

$$\Pr(X = 4) = \frac{\binom{6}{4}\binom{99-6}{6-4}}{\binom{99}{6}} = 0.0000573$$

$$\Pr\left(X=5\right)=\frac{\binom{6}{5}\binom{99-6}{6-5}}{\binom{99}{6}}=4.97979E-07$$

$$\Pr\left(X=6\right)=\frac{\binom{6}{6}\binom{99-6}{6-6}}{\binom{99}{6}}=8.92435E-10$$

Therefore given the probabilites of winning scenarios the probability we have that:

$$\Pr(X\geq 3)=0.0023739E(X)=n\frac{K}{N}=0.3636$$

This only includes the base scenario of the 6 yellow numbers, now we need to consider the extra 2 numbers For this we can assume the draw of the further 2 balls to be done after the 6 first in the same population without replacement. The probability here is just the succesful cases divided over all possible draws:
Let **Z** be the number of red number drawn and **Y** the number of orange numbers drawn.
Let **z** be the number of all red numbers in the population and **y** the number of all orange numbers
Let N be the population left after the first 6 draws$\Pr(Z=1)=\frac{z}{N}=\frac{1}{93}$

$$\Pr(Y=1)=\frac{y}{N}=\frac{1}{93}$$

$$\Pr\left(Z=1\ and\ Y=0\right)=\frac{z}{N}*\frac{N-1-y}{N-1}=\frac{1}{93}*\frac{91}{92}$$

$$\Pr\left(Z=0\ and\ Y=1\right)=\frac{y}{N}*\frac{N-1-z}{N-1}=\frac{1}{93}*\frac{91}{92}$$

$$\Pr\left(Z=1\ and\ Y=1\right)=\frac{z}{N}*\frac{y}{N-1}=\frac{1}{93}*\frac{1}{92}$$

These scenarios can be seen as subscenarios for each of the scenarios of the Hypergeometric distribution
For example:

$$\Pr\left(X=3\ and\ Z=1\ and\ Y=0\right)=\Pr(X=3)*\frac{1}{93}*\frac{91}{92}=2.46342E-05$$

$$\Pr\left(X=3\ and\ Z=0\ and\ Y=1\right)=\Pr(X=3)*\frac{1}{93}*\frac{91}{92}=2.46342E-05$$

$$\Pr\left(X=3\ and\ Z=1\ and\ Y=1\right)=\Pr(X=3)*\frac{1}{93}*\frac{1}{92}=2.70705E-07$$

$$\Pr\left(X=3\ and\ Z=0\ and\ Y=0\right)=\Pr(X=3)-\Pr\left(X=3\ and\ Z+\ Y>0\right)=0.002266616$$

$$\Pr\left(X=4\ and\ Z=1\ and\ Y=0\right)=\Pr(X=4)*\frac{1}{93}*\frac{91}{92}=6.15633E-07$$

$$\Pr\left(X=4\ and\ Z=0\ and\ Y=1\right)=\Pr(X=4)*\frac{1}{93}*\frac{91}{92}=6.15633E-07$$

$$\Pr\left(X=4\ and\ Z=1\ and\ Y=1\right)=\Pr(X=4)*\frac{1}{93}*\frac{1}{92}=6.84037E-09$$

$$\Pr\left(X=4\ and\ Z=0\ and\ Y=0\right)=\Pr(X=4)-\Pr\left(X=4\ and\ Z+\ Y>0\right)=0.0000560$$

$$\Pr\left(X=5\ and\ Z=1\ and\ Y=0\right)=\Pr\left(X=5\right)*\frac{1}{93}*\frac{91}{92}=5.35333E-09$$

$$\Pr\left(X=5\ and\ Z=0\ and\ Y=1\right)=\Pr\left(X=5\right)*\frac{1}{93}*\frac{91}{92}=5.35333E-09$$

$$\Pr\left(X=5\ and\ Z=1\ and\ Y=1\right)=\Pr\left(X=5\right)*\frac{1}{93}*\frac{1}{92}=5.94815E-11$$

$$\Pr\left(X=5\ and\ Z=0\ and\ Y=0\right)=\Pr\left(X=5\right)-\Pr\left(X=5\ and\ Z+\ Y>0\right)=0.000000487$$

$$\Pr\left(X=6\ and\ Z=1\ and\ Y=0\right)=\Pr\left(X=6\right)*\frac{1}{93}*\frac{91}{92}=9.59379E-12$$

$$\Pr\left(X=6\ and\ Z=0\ and\ Y=1\right)=\Pr\left(X=6\right)*\frac{1}{93}*\frac{91}{92}=9.59379E-12$$

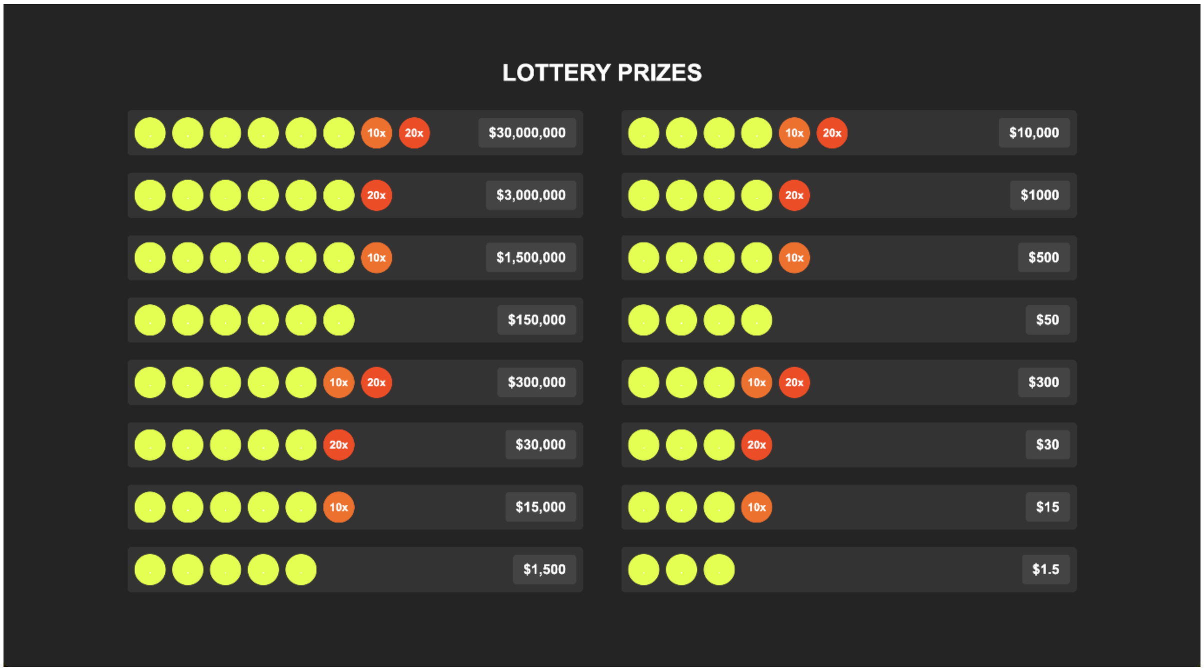
$$\Pr\left(X=6\ and\ Z=1\ and\ Y=1\right)=\Pr\left(X=6\right)*\frac{1}{93}*\frac{1}{92}=1.06598E-13$$

$$\Pr\left(X=6\ and\ Z=0\ and\ Y=0\right)=\Pr\left(X=6\right)-\Pr\left(X=5\ and\ Z+\ Y>0\right)=0.000000001$$

3. Fair game

The idea of this game is to have no house edge, meaning that whatever the price is to participate should be the expected outcome. For every 25 euro in an account 1 ticket of entry is granted. With an estimated annual interest rate of 2% the weekly interest rate should be 0.02/365. Therefore the expected winnings every ticket should have weekly is:

$$E(T)=25*\frac{0.02}{365}=0.0096$$



Expectation is calculated by multiplying the probabilities of every scenario by the payout of the respective winning set of numbers.

$$Let\ X\ be\ the\ winnings\ from\ every\ ticket.$$
$$Let\ x_i\ be\ the\ \ win\ from\ the\ event\ with\ probability\ p_{xi}$$

$$E(X) = \sum_{i=1}^{16} x_i * p_{xi}$$

$$E(X) = 0.002266616*1.5 + 2.46342E - 05*15 + 2.46342E - 05*30 + 2.70705E - 07*300 + 0.0000560*50 + 6.15633E - 07*500 + 6.15633E - 07*1000 + 6.84037E - 09*10000 + 0.000000487*1500 + 5.35333E - 09*15000 + 5.35333E - 09*30000 + 5.94815E - 11*300000 + 0.000000001*150000 + 9.59379E - 12*1500000 + 9.59379E - 12*300000 + 1.06598E - 13*30000000 = 0.00954990$$

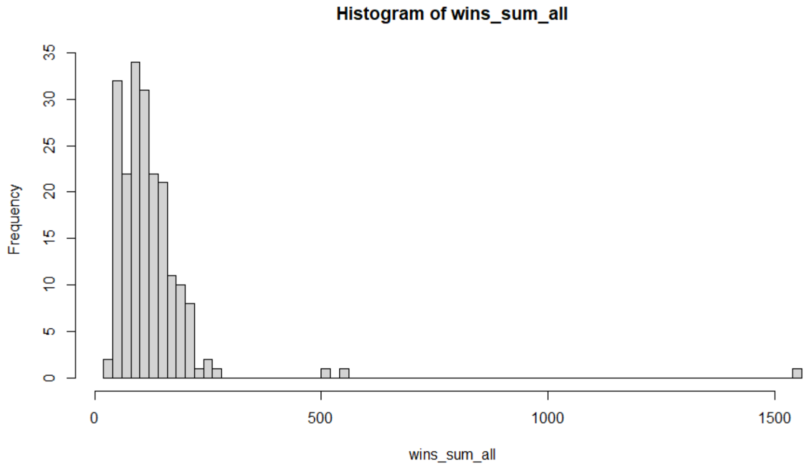
The difference of 0.00005 between the expectations is small enough for it to be disregarded as a edge.

4.Simulated output

The R code used to simulate these results is in the repository.

By simulating 10000000 runs the real return of a ticket equals to 0.0094052 which is within the expected scope given the low probability of the big prizes.

Here is a histogram showing the distribution of the annual payments expected to be received by a user with 300 tickets.



The mean of the simulation hovers around 140-150 which is aligned with what we should expect