Birla Insititute of Technology & Science – Pilani, K.K. Birla Goa Campus
First Semester 2019-2020

PHY F313 | **Lab test-1 (Open Book)** | **Time: 90 mins**

Computational Physics 29 th August 2019

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1. Here is a root finding problem in a classic *surface area minimization* problem. The minimization problem and its solution are given below. The solution reaches completion only after finding roots of a specific function described in the solution below.

   The surface minimization problem : Consider a surface connecting two rings of equal radii, R, separated a distance L as shown in figure below. We want to find the surface with minimal area. The immediate goal is to find a surface $s(z)$ that gives the cross-sectional radius of the surface as a function if height z (measured vertically from the lower ring). Take $z = 0$ on the lower ring and $z = L$ on the upper ring.

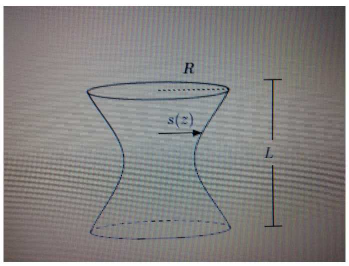

Figure 1:

Solution to the above problem : Area element of the above surface will be $dA = sd\phi\sqrt{dz^2 + s'(z)^2dz^2}$. If we integrate the expression for the area in ϕ and z, the expression for the surface area will be

$$A = 2\pi \int_0^L \sqrt{(s(z) + s'(z)^2)\, dz} \tag{1}$$

The above expression gives the surface area for a specific surface $s(z)$. A is a function of the function $s(z)$. A has to be minimized with respect to the function s(z) for obtaining the surace with least area. This can be done through methods that some of you are familiar with and others may not be. It does not matter. However minimizing A with respect to $s(z)$ yields a differential equation

$$1 + (\frac{ds}{dz})^2 - s\frac{d^2s}{dz^2} = 0 \tag{2}$$

which has to be solved subject to the boundary conditions $s(0) = R$ and $s(L) = R$.

The general solution of the above differential equation will yield the surface that will minimize the surface area. The solution to the above differential equation is of the form :

$$s(z) = \alpha cosh(\frac{z - \beta}{\alpha}) \tag{3}$$

where α and β are constants. These constants have to be determined from the above boundary conditions. From $z = 0$ boundary we get

$$s(0) = \alpha cosh(\frac{\beta}{\alpha}) = R \tag{4}$$

and from $z = L$ boundary we get

$$s(L) = \alpha cosh(\frac{L - \beta}{\alpha}) = R \tag{5}$$

The $z = 0$ boundary condition gives the following

$$\beta = \alpha cosh^{-1}\left(\frac{s(0)}{\alpha}\right) \tag{6}$$

Substituting for β in the $z = L$ boundary condition, we get

$$\alpha cosh\left(\frac{L}{\alpha} - cosh^{-1}\frac{R}{\alpha}\right) = R \tag{7}$$

Root finding problem :

Roots of the above equation (7) will give the α (and hence β also) for the surface with the minimum area and need to be found to complete the solution.

Use *Secant method* to find α and β . Take $R = 5cm$ and $L = 5cm$

(a) Write the root finding code. *Before you go ahead and find roots of equation (7) test your code against some $f(x) = 0$ whose roots are known to you.*

(b) Find the value(s) of α from Eq. (7) till 4 significant figures.

(c) Find the value(s) of β and thus write down the equation of the surface with the least area in the above problem.

All your results should be entered in the doc format kept on photon. Submit (a) this result file , and (b) a file containing the code on photon.

You can use the net for syntax, etc.