Verification in Isabelle/HOL of Hopcroft's algorithm for minimizing DFAs including runtime analysis

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supervised by Prof. Dr. Tobias Nipkow and Dr. Peter Lammich

Technical University of Munich
Chair for Logic and Verification

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Outline

1. Living in Munich
   1.1 The city
   1.2 Technical University of Munich

2. Hopcroft's algorithm
   2.1 DFA minimization by example
   2.2 Towards a formal definition...

Figure: Location of Munich

Figure: Some photos of Munich

Kloster
Scheyern
SEIT 1119
KLOSTER-GOLD
DUNKEL

Kloster
Scheyern
KLOSTER-GOLD
DUNKEL

13 28
J W
Augustiner-
Bräu München
Edelstoff

Kloster
Scheyern
KLOSTER-GOLD
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SPEZIAL
HERZOGLICH BAYERISCHES
BRAUHAUS
TEGERNSEE

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SPEZIAL
BRAUHAUS
TEGERNSEE

WIRTS MADL
Bayrisch Hell
WIRTS MADL
Bayrisch Hell

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DUNKEL

SPEZIAL
BRAUHAUS
TEGERNSEE

WIRTS MADL
Bayrisch Hell
WIRTS MADL
Bayrisch Hell

HB
HOFBRÄUHAUS

Figure: Technical University of Munich (TUM)

1. Living in Munich
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2. Hopcroft's algorithm
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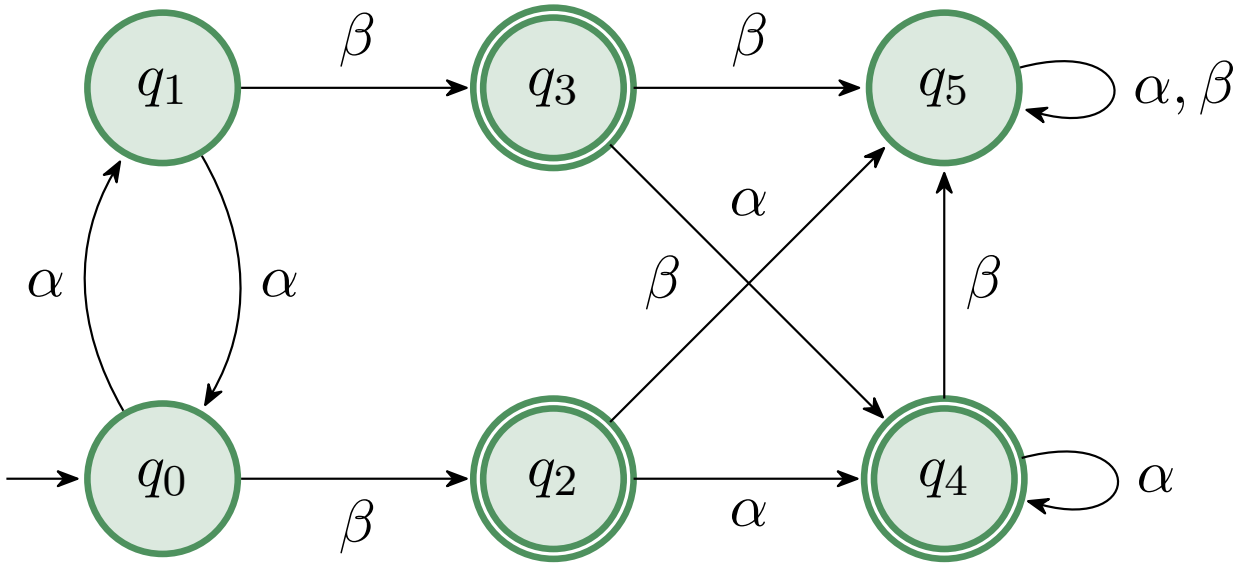
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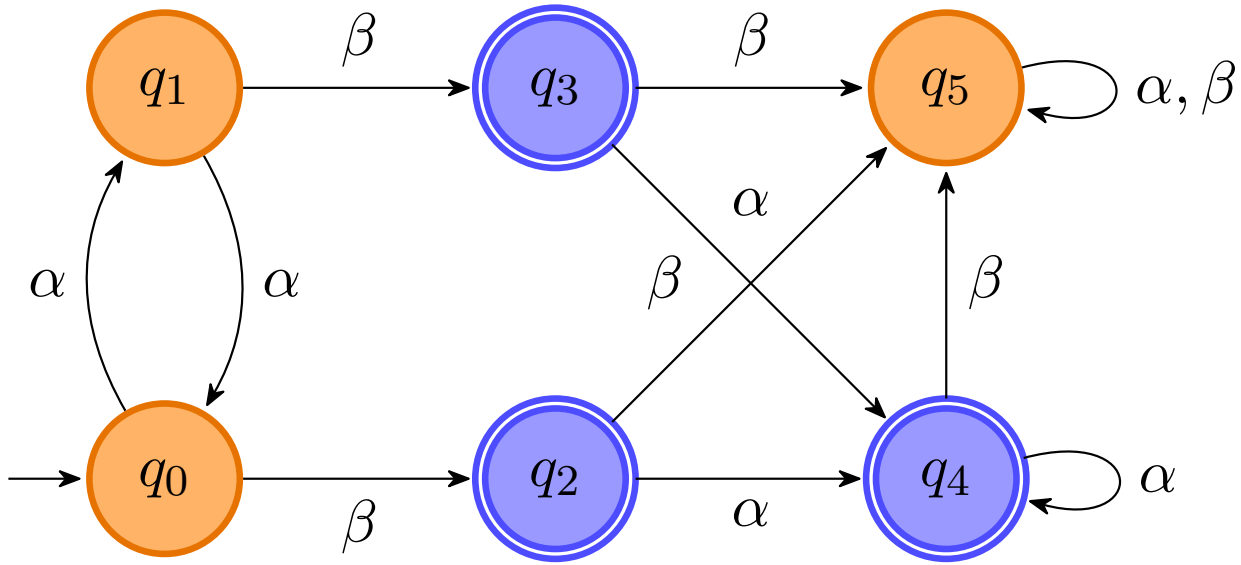
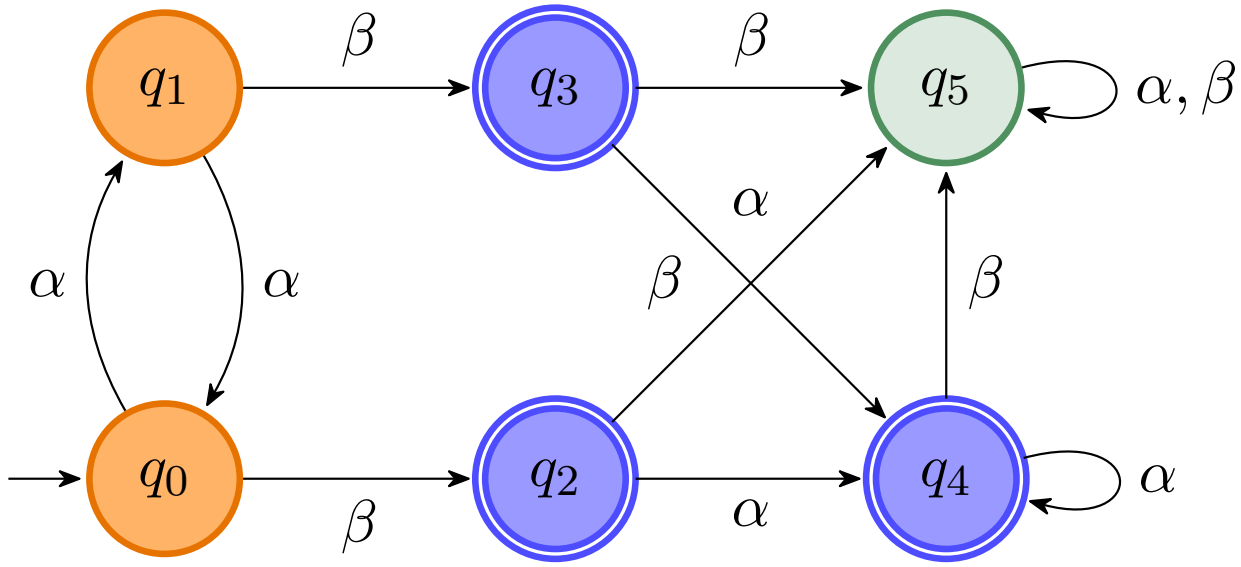
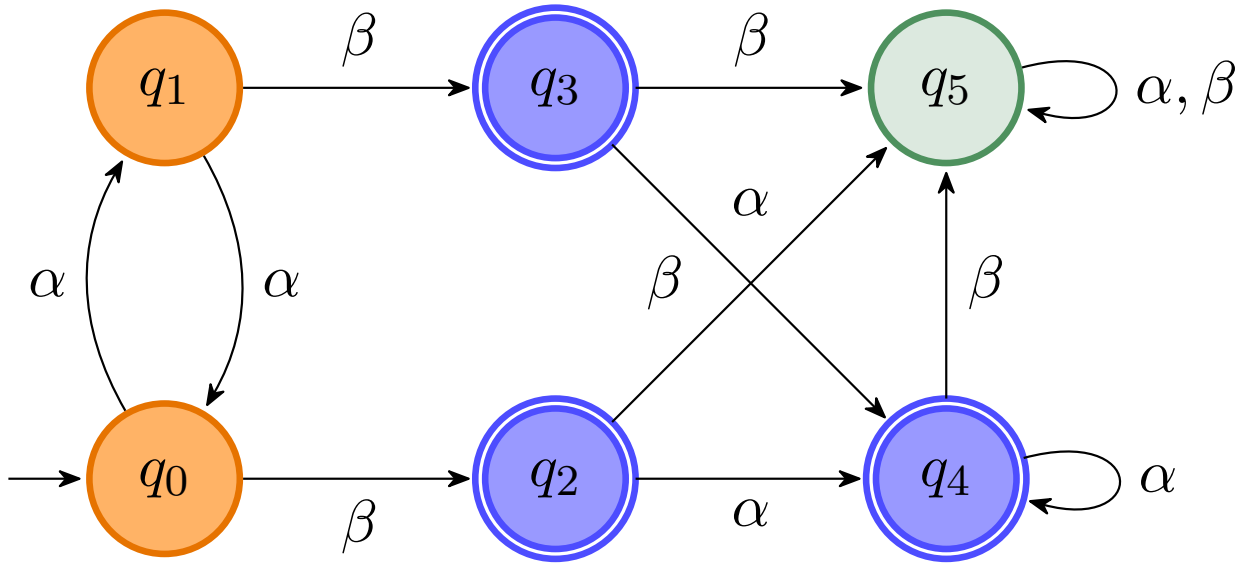


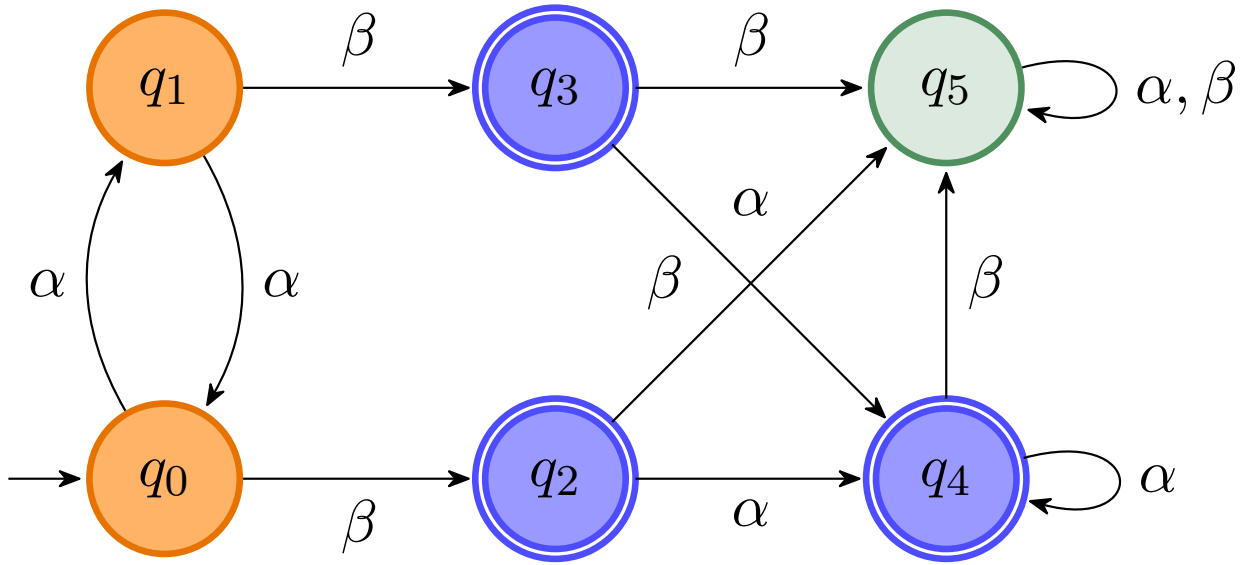
Splitter	Partition	Workset
–	$\{q_0, q_1, q_5\}\{q_2, q_3, q_4\}$	$(\alpha, \{q_0, q_1, q_5\})$ $(\beta, \{q_0, q_1, q_5\})$



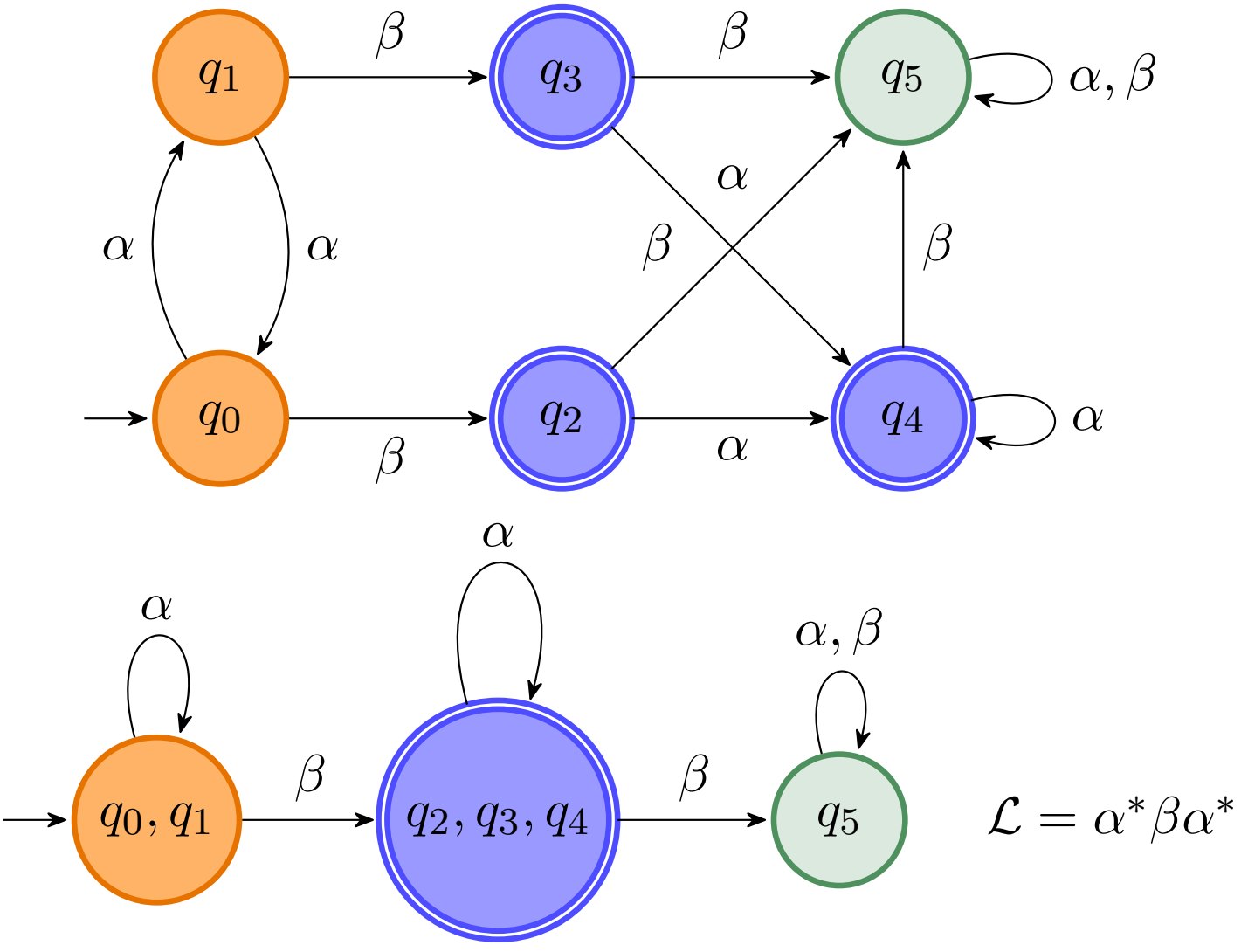
Splitter	Partition	Workset
–	$\{q_0, q_1, q_5\}\{q_2, q_3, q_4\}$	$(\alpha, \{q_0, q_1, q_5\})$ $(\beta, \{q_0, q_1, q_5\})$
$(\beta, \{q_0, q_1, q_5\})$	$\{q_0, q_1\}\{q_5\}\{q_2, q_3, q_4\}$	$(\alpha, \{q_0, q_1\})$ $(\alpha, \{q_5\})$



Splitter	Partition	Workset
–	$\{q_0, q_1, q_5\}\{q_2, q_3, q_4\}$	$(\alpha, \{q_0, q_1, q_5\})\ (\beta, \{q_0, q_1, q_5\})$
$(\beta, \{q_0, q_1, q_5\})$	$\{q_0, q_1\}\{q_5\}\{q_2, q_3, q_4\}$	$(\alpha, \{q_0, q_1\})\ (\alpha, \{q_5\})$
$(\alpha, \{q_0, q_1\})$	$\{q_0, q_1\}\{q_5\}\{q_2, q_3, q_4\}$	$(\alpha, \{q_5\})$



Splitter	Partition	Workset
–	$\{q_0, q_1, q_5\}\{q_2, q_3, q_4\}$	$(\alpha, \{q_0, q_1, q_5\})\ (\beta, \{q_0, q_1, q_5\})$
$(\beta, \{q_0, q_1, q_5\})$	$\{q_0, q_1\}\{q_5\}\{q_2, q_3, q_4\}$	$(\alpha, \{q_0, q_1\})\ (\alpha, \{q_5\})$
$(\alpha, \{q_0, q_1\})$	$\{q_0, q_1\}\{q_5\}\{q_2, q_3, q_4\}$	$(\alpha, \{q_5\})$
$(\alpha, \{q_5\})$	$\{q_0, q_1\}\{q_5\}\{q_2, q_3, q_4\}$	$\varnothing$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q_0, q_1
q_2, q_3, q_4
q_5
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$\mathcal{L} = \alpha^* \beta \alpha^*$

Formalization