

Verification in Isabelle/HOL of Hopcroft's algorithm for minimizing DFAs including runtime analysis

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Outline

1. Living in Munich

1.1 The city

1.2 Technical University of Munich

2. Hopcroft's algorithm

2.1 DFA minimization by example

2.2 Towards a formal definition...



Figure: Location of Munich

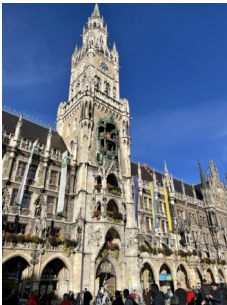


Figure: Some photos of Munich



Figure: Technical University of Munich (TUM), Garching campus



Figure: Technical University of Munich (TUM), Garching campus

















1. Living in Munich

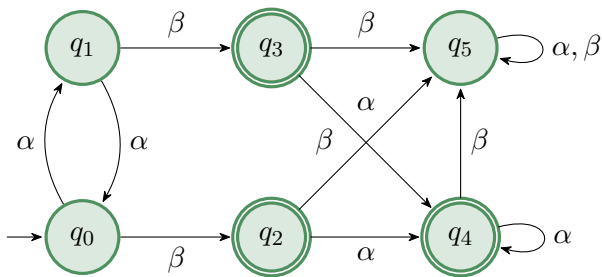
1.1 The city

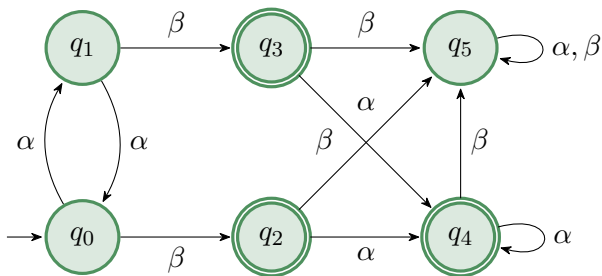
1.2 Technical University of Munich

2. Hopcroft's algorithm

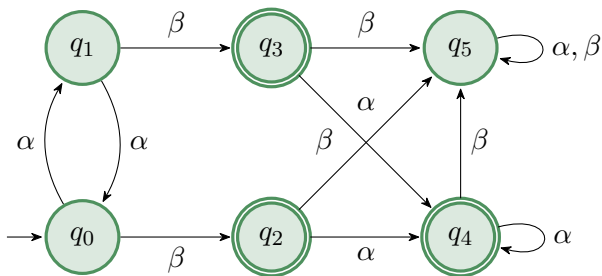
2.1 DFA minimization by example

2.2 Towards a formal definition...

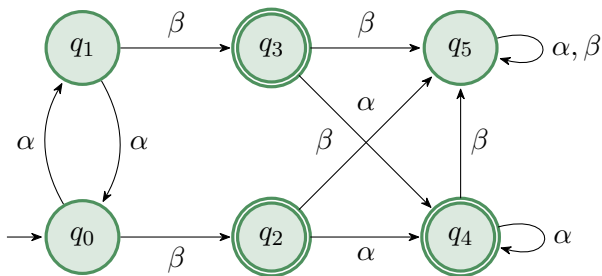




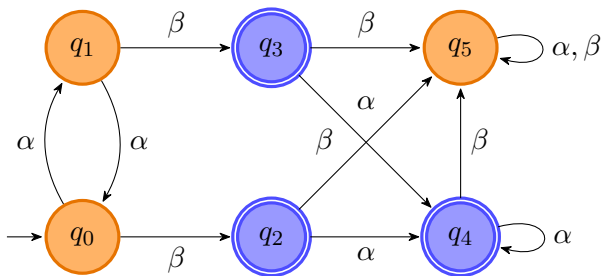
- Successively partitions the set of states into equivalence classes



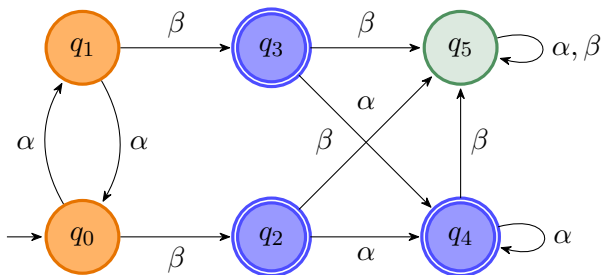
- Successively partitions the set of states into equivalence classes
- Initial partition: accepting and non-accepting states



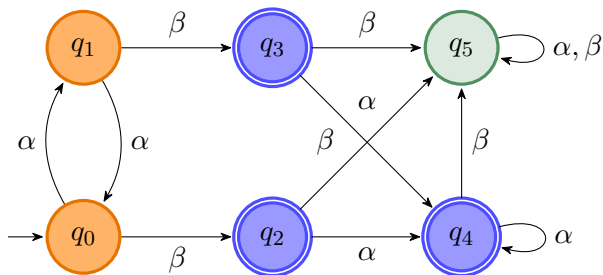
- Successively partitions the set of states into equivalence classes
- Initial partition: accepting and non-accepting states
- Each iteration: pick a splitter and split all blocks of the current partition



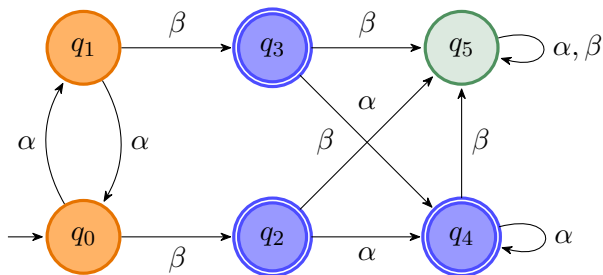
Splitter	Partition	Workset
—	$\{q_0, q_1, q_5\} \{q_2, q_3, q_4\}$	$(\alpha, \{q_0, q_1, q_5\}) (\beta, \{q_0, q_1, q_5\})$



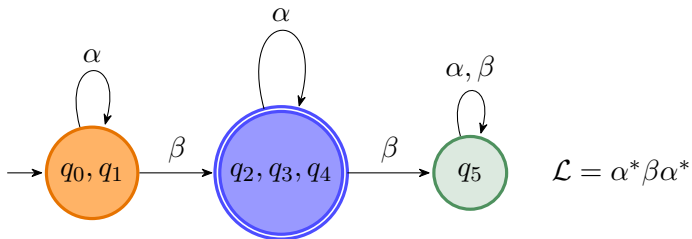
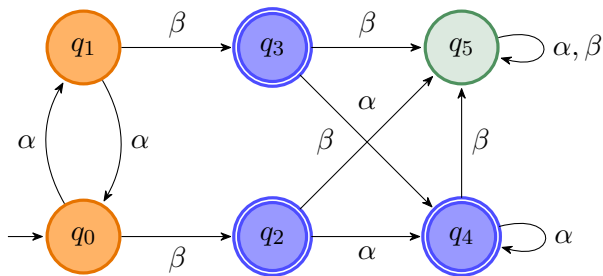
Splitter	Partition	Workset
—	$\{q_0, q_1, q_5\} \{q_2, q_3, q_4\}$	$(\alpha, \{q_0, q_1, q_5\}) (\beta, \{q_0, q_1, q_5\})$
$(\beta, \{q_0, q_1, q_5\})$	$\{q_0, q_1\} \{q_5\} \{q_2, q_3, q_4\}$	$(\alpha, \{q_0, q_1\}) (\alpha, \{q_5\})$



Splitter	Partition	Workset
—	$\{q_0, q_1, q_5\} \{q_2, q_3, q_4\}$	$(\alpha, \{q_0, q_1, q_5\}) (\beta, \{q_0, q_1, q_5\})$
$(\beta, \{q_0, q_1, q_5\})$	$\{q_0, q_1\} \{q_5\} \{q_2, q_3, q_4\}$	$(\alpha, \{q_0, q_1\}) (\alpha, \{q_5\})$
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Splitter	Partition	Workset
—	$\{q_0, q_1, q_5\} \{q_2, q_3, q_4\}$	$(\alpha, \{q_0, q_1, q_5\}) (\beta, \{q_0, q_1, q_5\})$
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$(\alpha, \{q_5\})$	$\{q_0, q_1\} \{q_5\} \{q_2, q_3, q_4\}$	\emptyset



Formalization