Verification in Isabelle/HOL of Hopcroft's algorithm for minimizing DFAs including runtime analysis

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supervised by Prof. Dr. Tobias Nipkow and Dr. Peter Lammich

Technical University of Munich
Chair for Logic and Verification

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Outline

Figure: Location of Munich

Figure: Some photos of Munich

Figure: Technical University of Munich (TUM), Garching campus

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DUNKEL


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Bayrisch Hell
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HB
HOFBRÄUHAUS

Understanding Refinement

Definition 1

Refinement is a systematic process of refining a high-level abstract specification into a concrete implementation.

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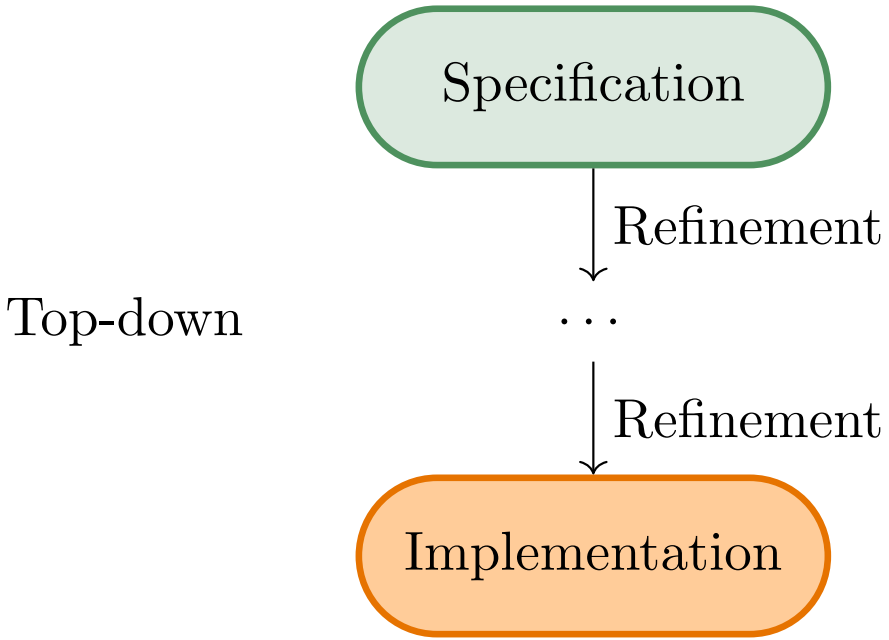

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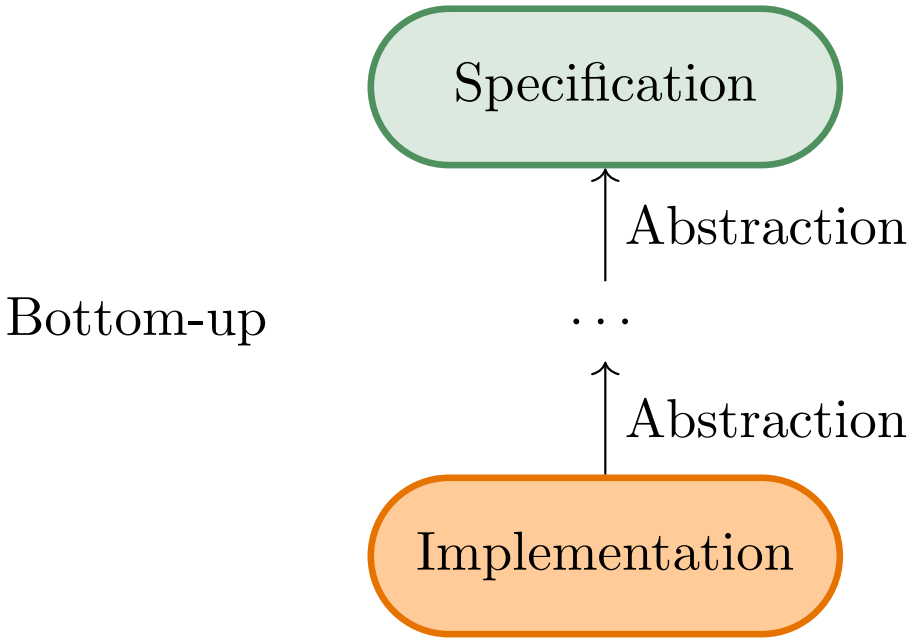
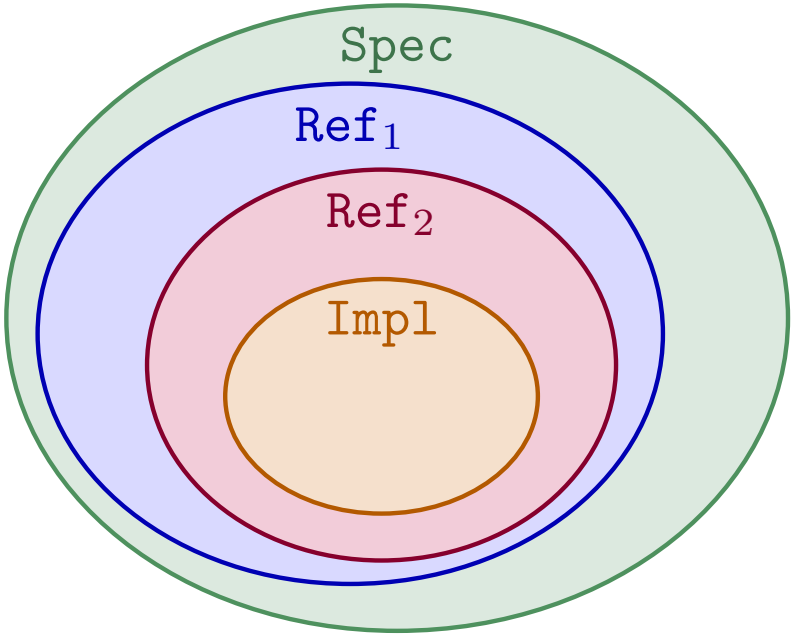
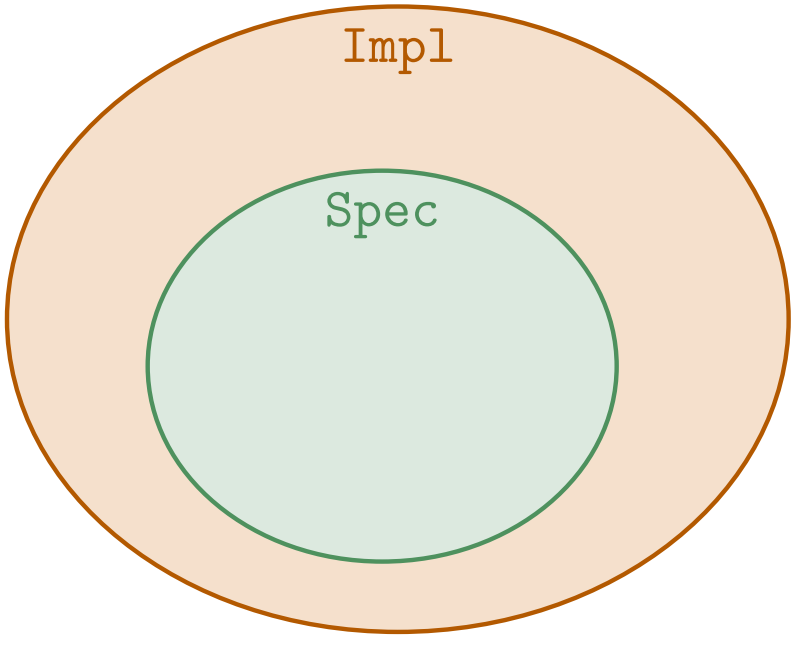


Each refinement step preserves the intended behavior.

$$\texttt{Spec} \rightarrow \texttt{Ref}_1 \rightarrow \texttt{Ref}_2 \rightarrow \texttt{Impl}$$



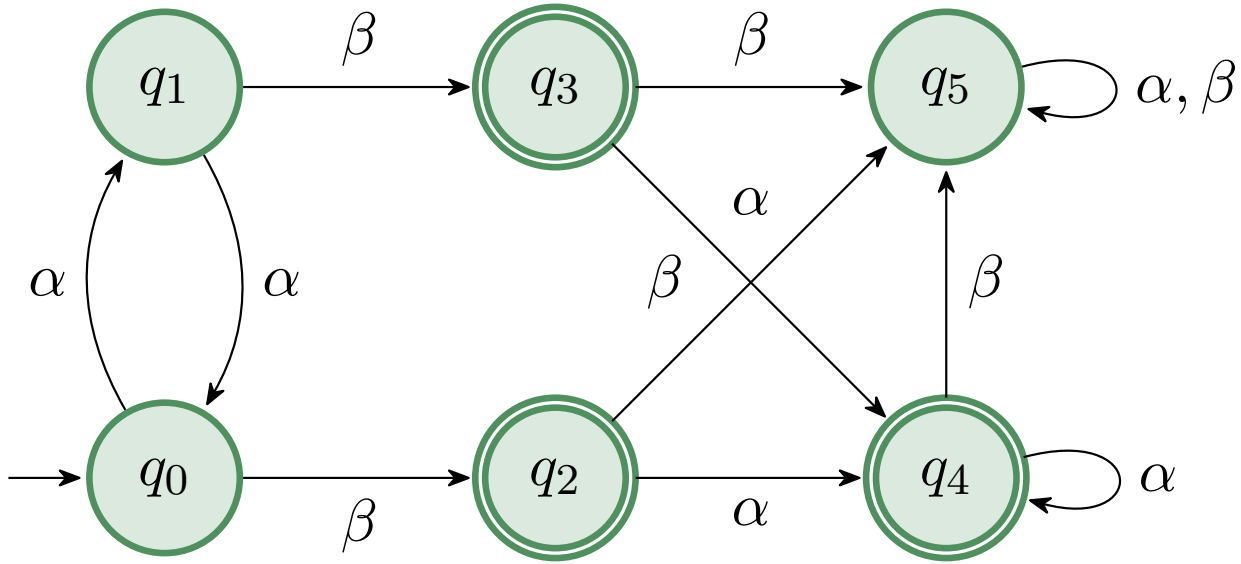


The Isabelle Refinement Framework

Isabelle Refinement Framework

- Stepwise refinement approach to verified program development
- Formal and mathematical
- Ensures correctness at each step

Comes with the Isabelle Collection Framework, which provides an extensive library of reusable verified functional data structures.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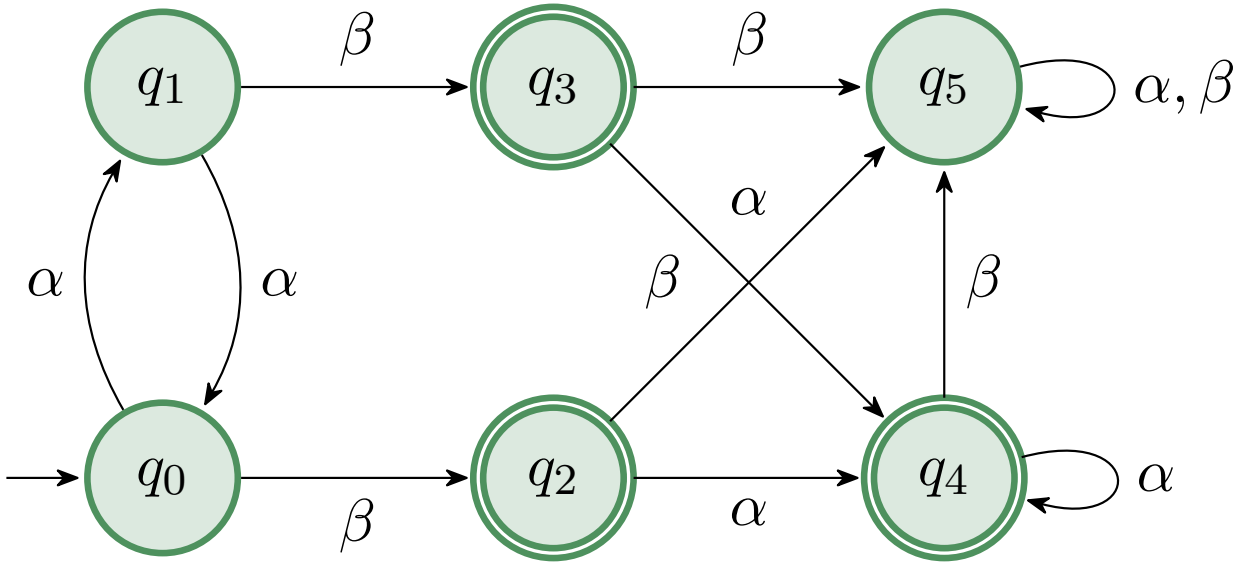
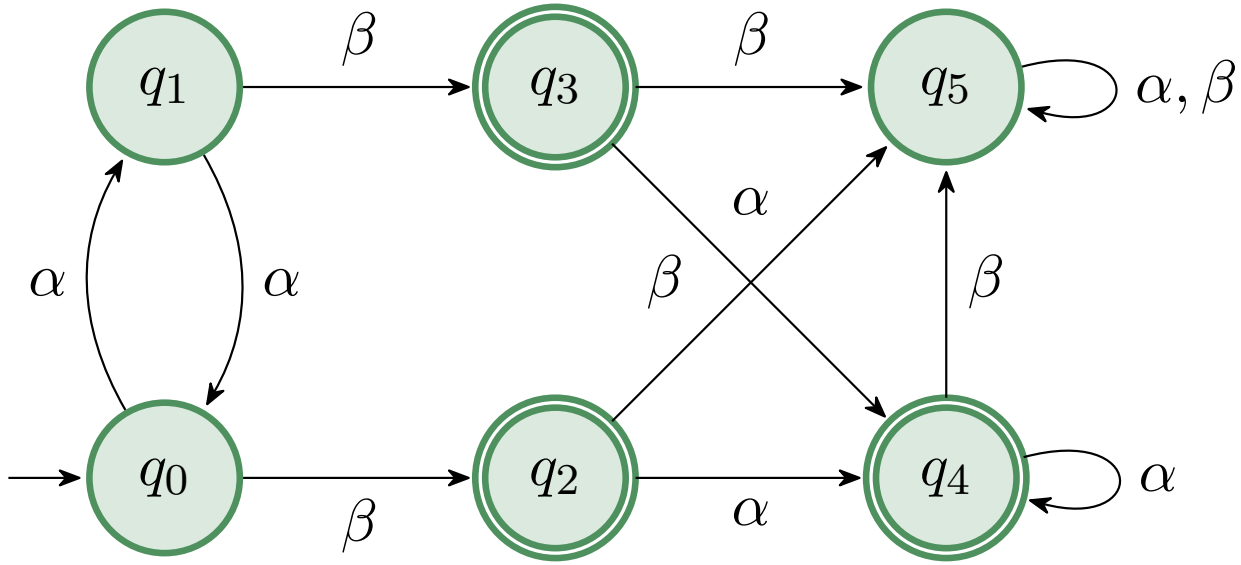
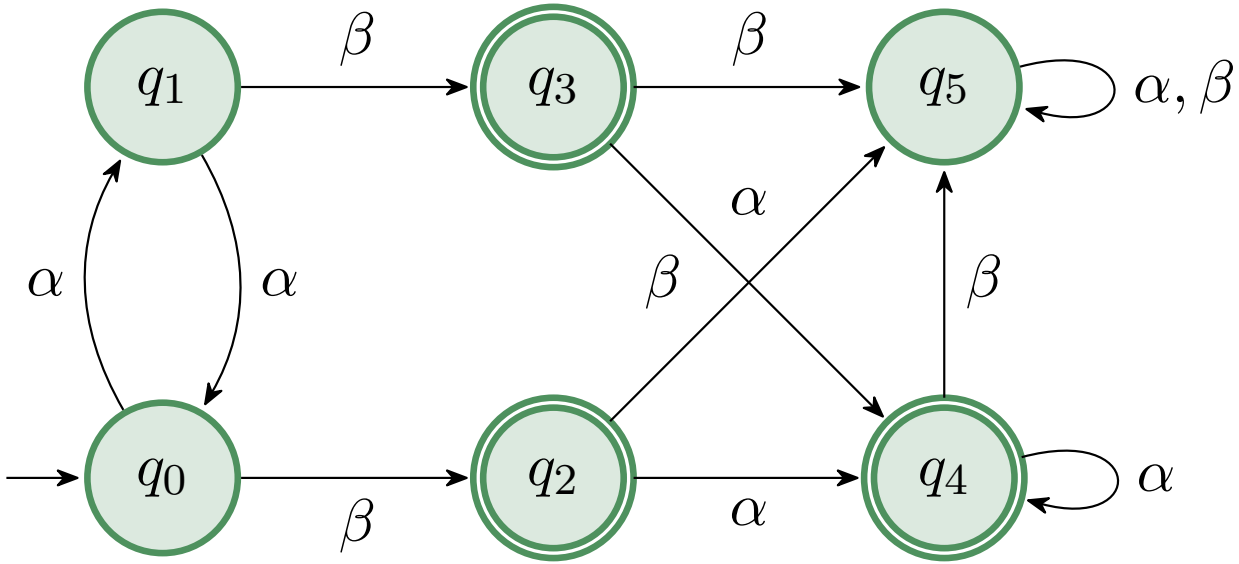


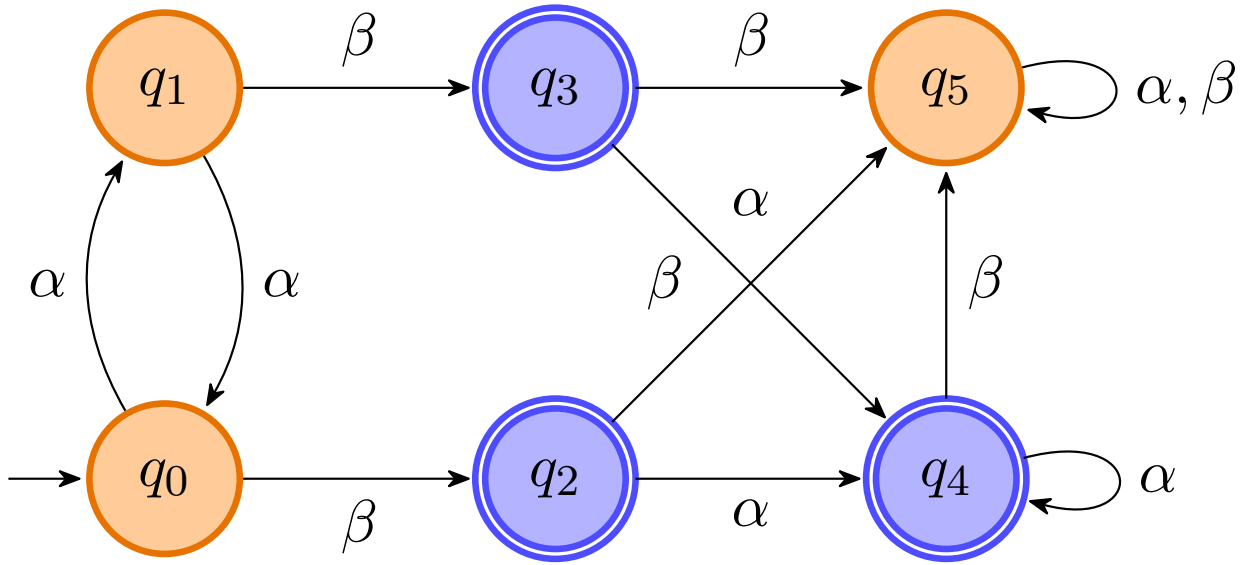
- Successively partitions the set of states into equivalence classes



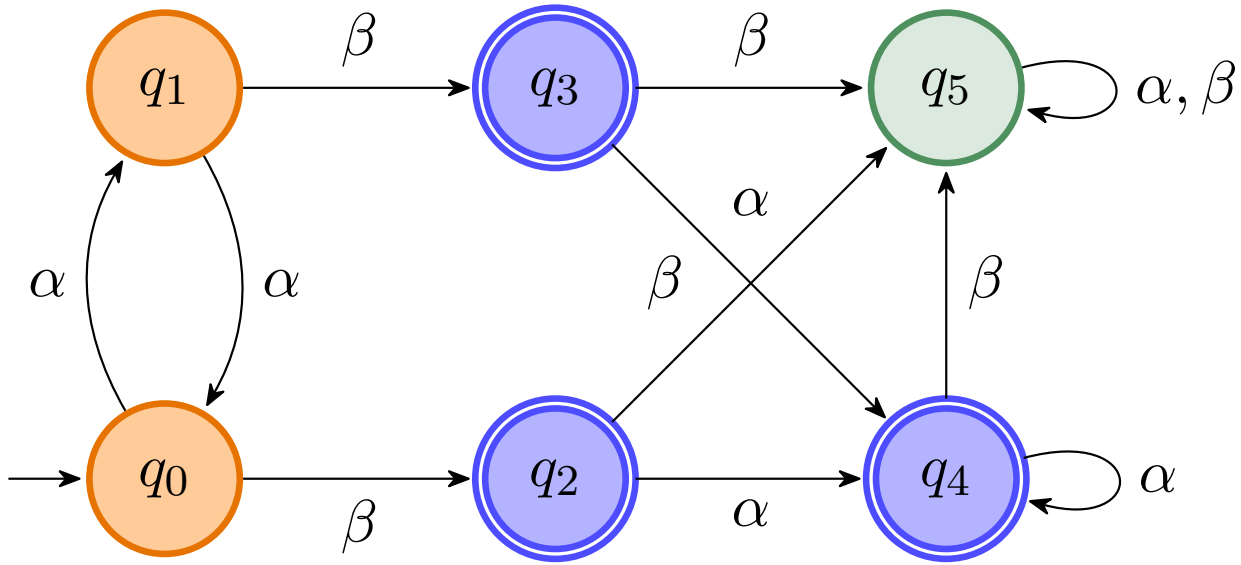
- Successively partitions the set of states into equivalence classes
- Initial partition: accepting and non-accepting states



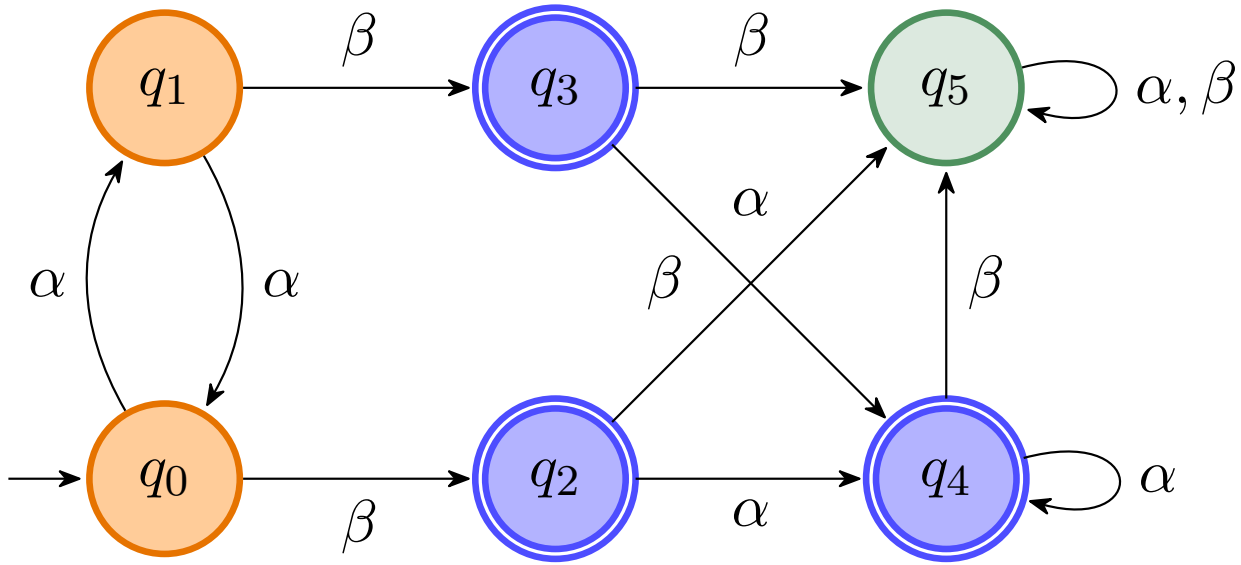
- Successively partitions the set of states into equivalence classes
- Initial partition: accepting and non-accepting states
- Each iteration: pick a splitter and split all blocks of the current partition



Splitter	Partition	Workset
–	$\{q_0, q_1, q_5\}\{q_2, q_3, q_4\}$	$(\alpha, \{q_0, q_1, q_5\})$ $(\beta, \{q_0, q_1, q_5\})$



Splitter	Partition	Workset
–	$\{q_0, q_1, q_5\}\{q_2, q_3, q_4\}$	$(\alpha, \{q_0, q_1, q_5\})$ $(\beta, \{q_0, q_1, q_5\})$
$(\beta, \{q_0, q_1, q_5\})$	$\{q_0, q_1\}\{q_5\}\{q_2, q_3, q_4\}$	$(\alpha, \{q_0, q_1\})$ $(\alpha, \{q_5\})$



Splitter	Partition	Workset
–	$\{q_0, q_1, q_5\}\{q_2, q_3, q_4\}$	$(\alpha, \{q_0, q_1, q_5\})\ (\beta, \{q_0, q_1, q_5\})$
$(\beta, \{q_0, q_1, q_5\})$	$\{q_0, q_1\}\{q_5\}\{q_2, q_3, q_4\}$	$(\alpha, \{q_0, q_1\})\ (\alpha, \{q_5\})$
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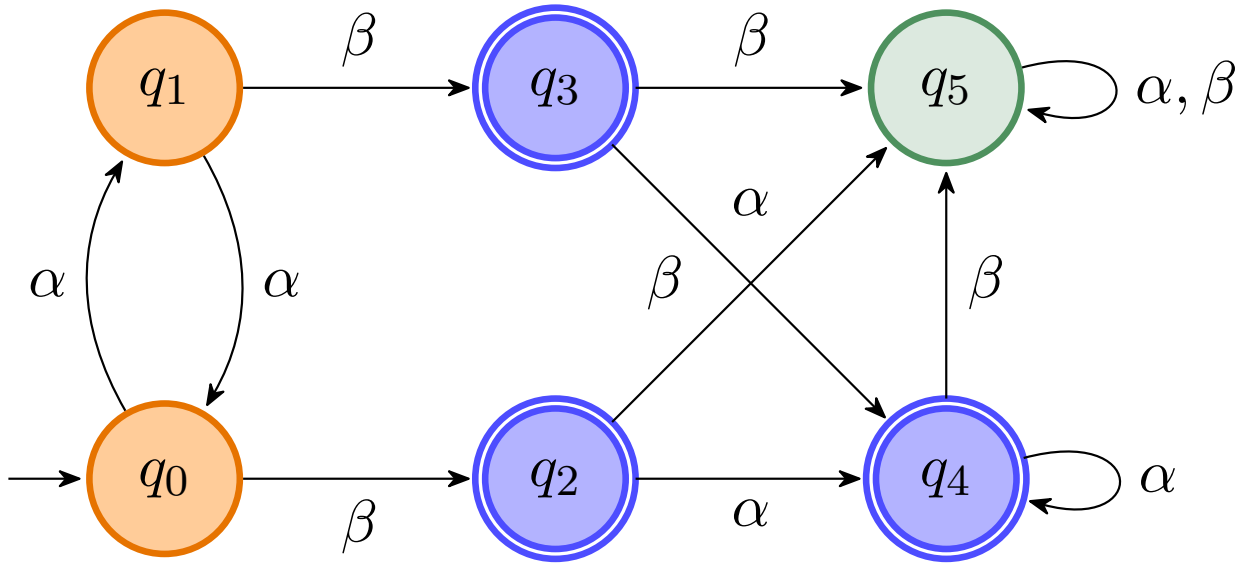
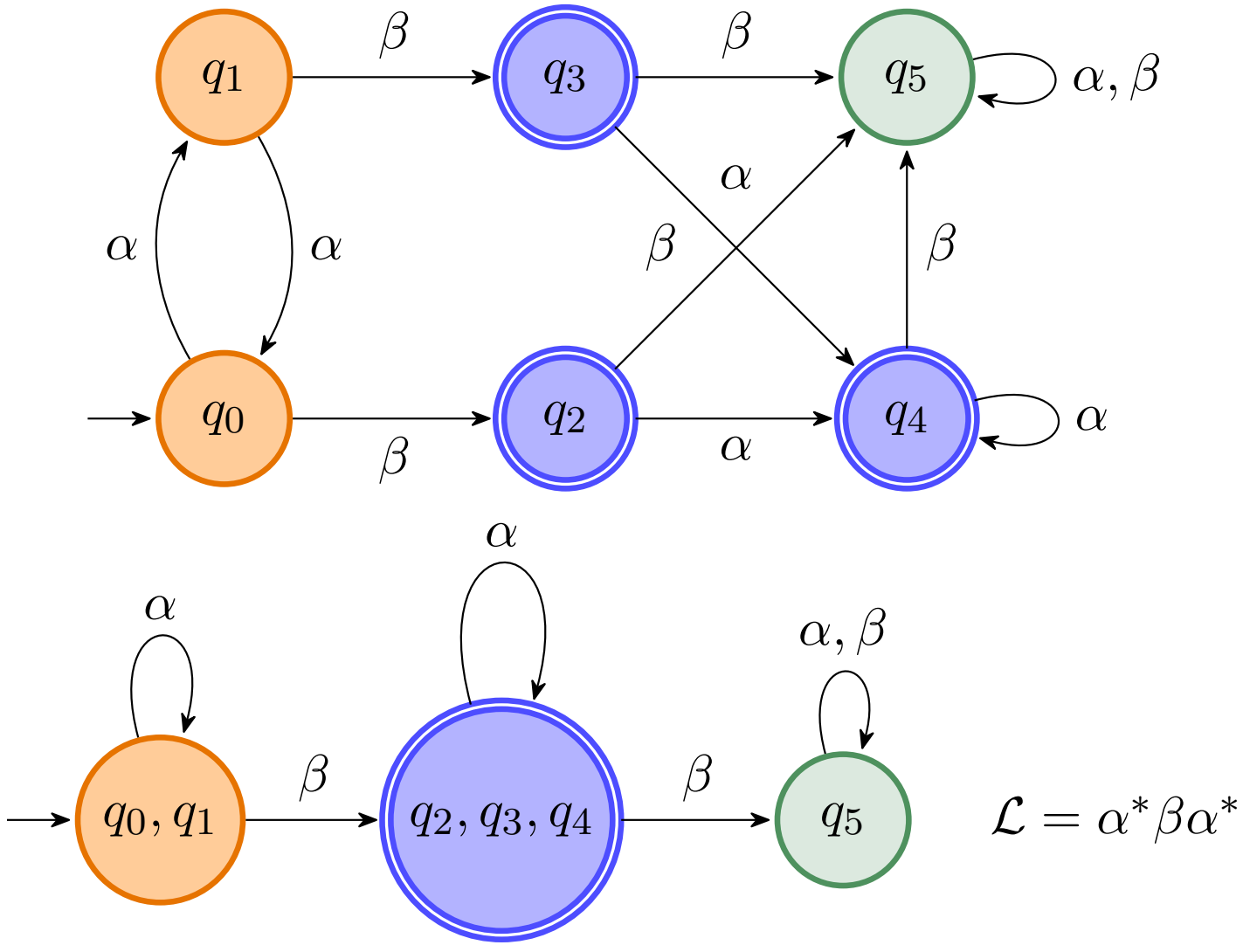


Splitter	Partition	Workset
–	$\{q_0, q_1, q_5\}\{q_2, q_3, q_4\}$	$(\alpha, \{q_0, q_1, q_5\})\ (\beta, \{q_0, q_1, q_5\})$
$(\beta, \{q_0, q_1, q_5\})$	$\{q_0, q_1\}\{q_5\}\{q_2, q_3, q_4\}$	$(\alpha, \{q_0, q_1\})\ (\alpha, \{q_5\})$
$(\alpha, \{q_0, q_1\})$	$\{q_0, q_1\}\{q_5\}\{q_2, q_3, q_4\}$	$(\alpha, \{q_5\})$
$(\alpha, \{q_5\})$	$\{q_0, q_1\}\{q_5\}\{q_2, q_3, q_4\}$	$\varnothing$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q_2, q_3, q_4
q_5
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$\mathcal{L} = \alpha^* \beta \alpha^*$

Formalization

Coming soon