Verification in Isabelle/HOL of Hopcroft's algorithm for minimizing DFAs including runtime analysis

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supervised by Prof. Dr. Tobias Nipkow and Dr. Peter Lammich

Technical University of Munich
Chair for Logic and Verification

September 2023

Outline

Figure: Location of Munich

Figure: Some photos of Munich

Figure: Technical University of Munich (TUM), Garching campus

Figure: Technical University of Munich (TUM), Garching campus


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DUNKEL


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Understanding Refinement

Definition 1

Refinement is a systematic process of refining a high-level abstract specification into a concrete implementation.

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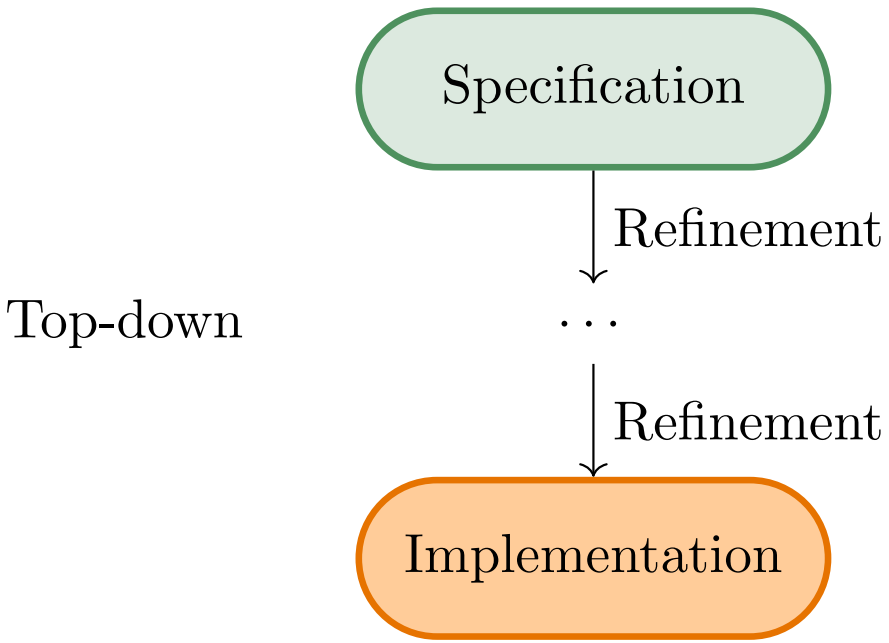

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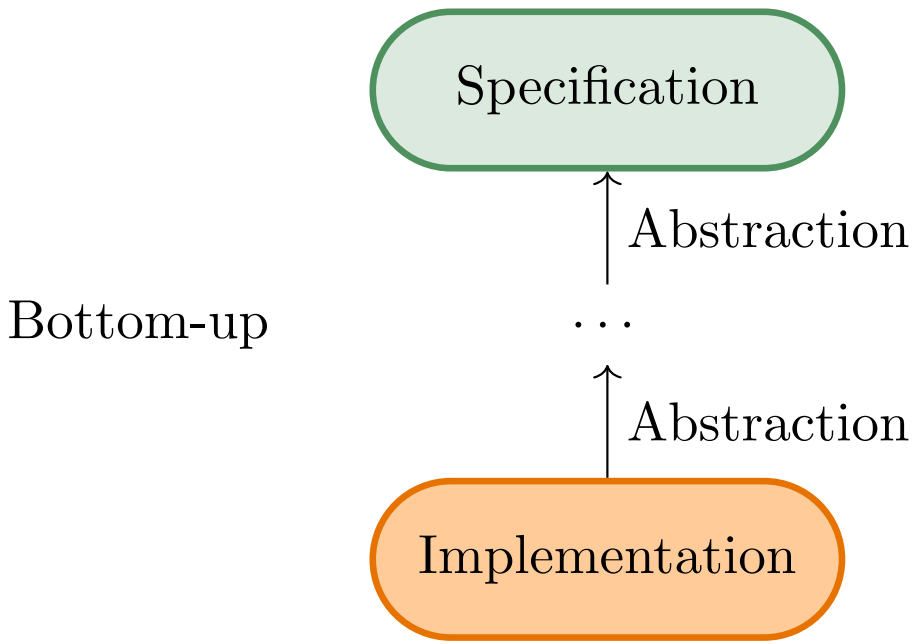
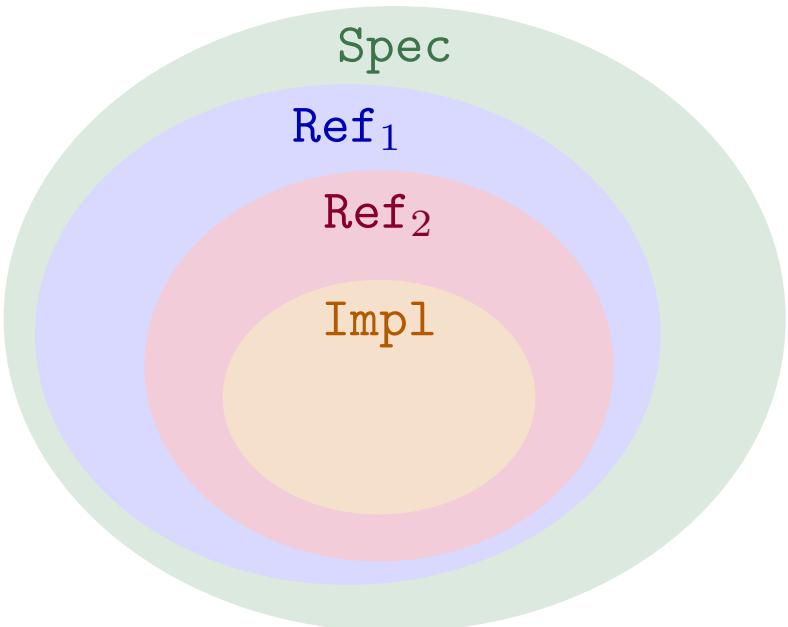
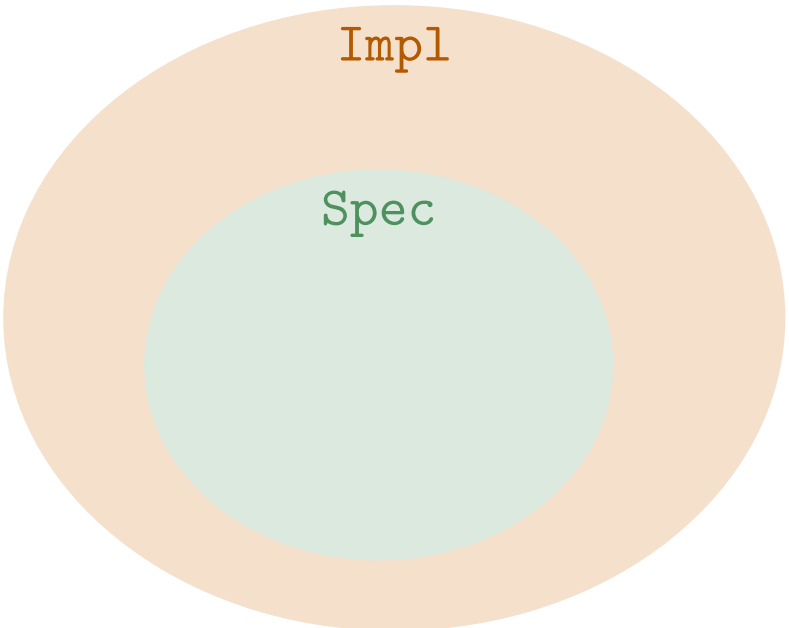
Refinement is a systematic process of refining a high-level abstract specification into a concrete implementation.



Each refinement step preserves the intended behavior.

$$\texttt{Spec} \rightarrow \texttt{Ref}_1 \rightarrow \texttt{Ref}_2 \rightarrow \texttt{Impl}$$



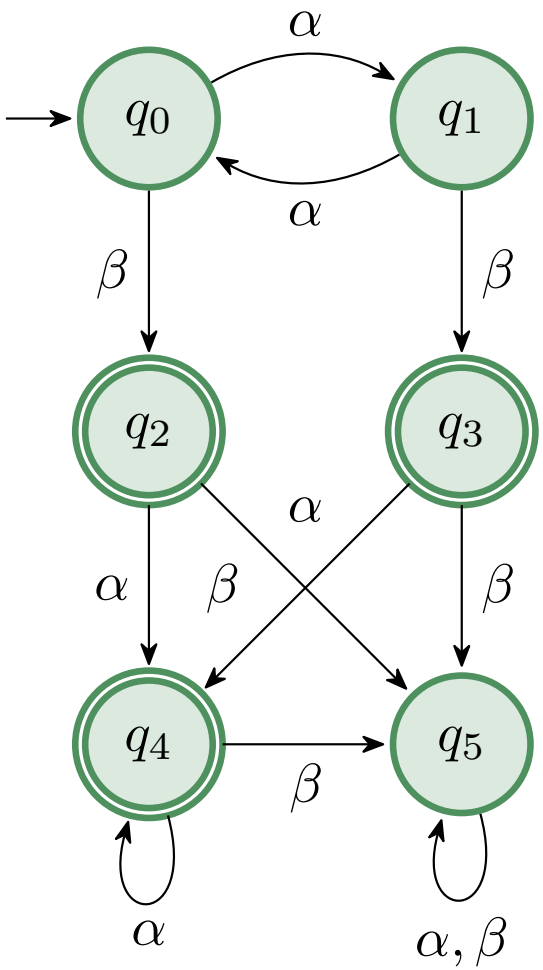


The Isabelle Refinement Framework

Isabelle Refinement Framework

- Stepwise refinement approach to verified program development
- Formal and mathematical
- Ensures correctness at each step

Comes with the Isabelle Collection Framework, which provides an extensive library of reusable verified functional data structures (for data refinement).


α
q_0
q_1
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β
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q_2
q_3
α
α
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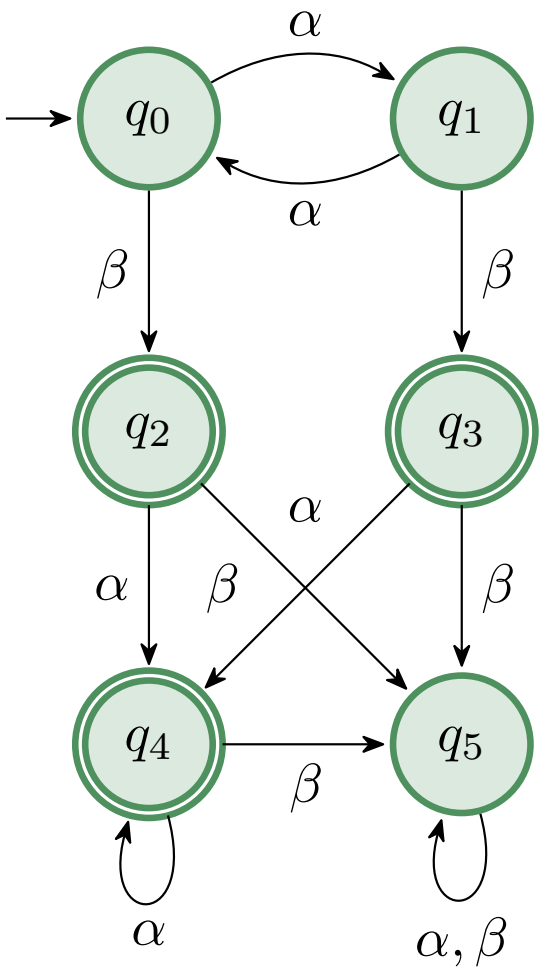
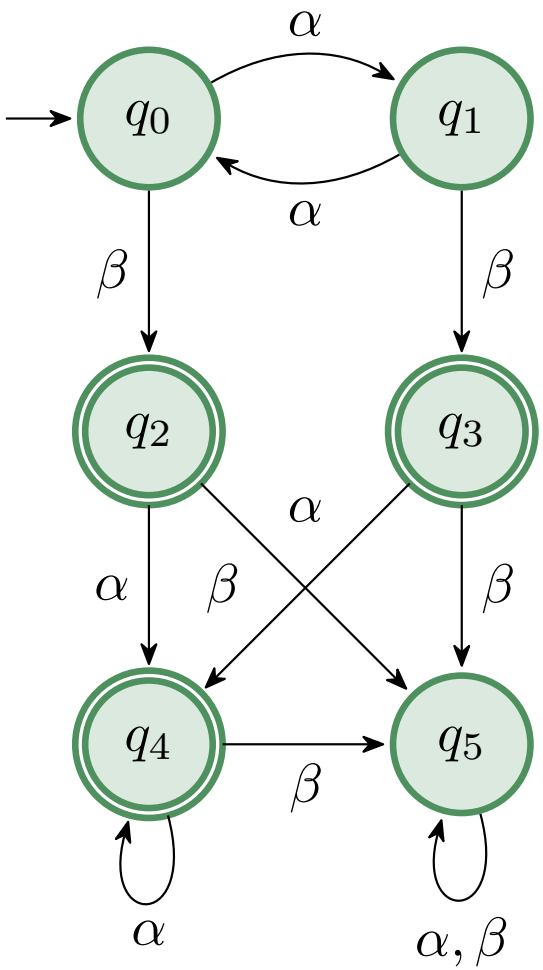
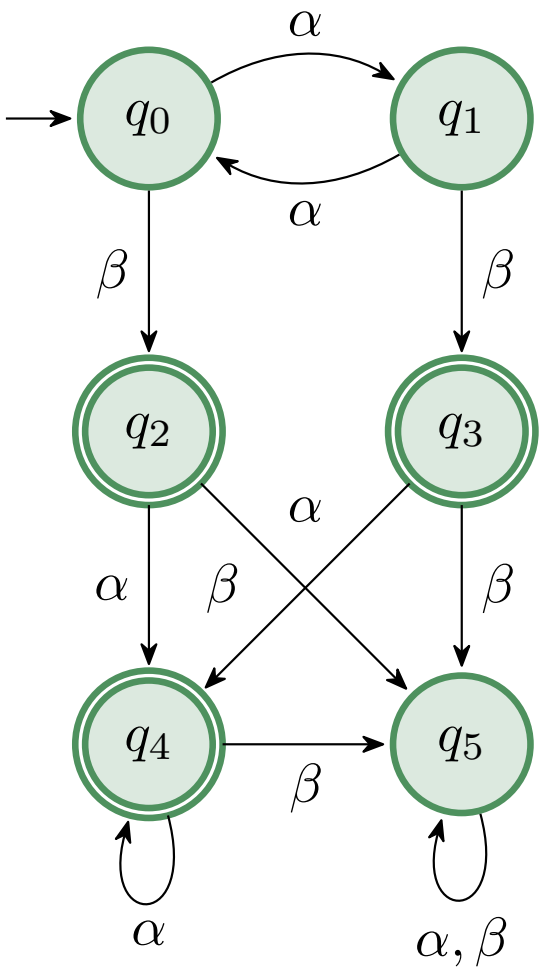
α
α, β



- Successively partitions the set of states into equivalence classes



- Successively partitions the set of states into equivalence classes
- Initial partition: accepting and non-accepting states



- Successively partitions the set of states into equivalence classes
- Initial partition: accepting and non-accepting states
- Each iteration: pick a splitter and split all blocks of the current partition

Data: a DFA $\mathcal{A} = (\mathcal{Q}, \Sigma, \delta, q_0, \mathcal{F})$
if $\mathcal{F} = \varnothing \vee \mathcal{Q} \setminus \mathcal{F} = \varnothing$
  return $\mathcal{Q}$
else
$\mathcal{P} := \{\mathcal{F}, \mathcal{Q} \setminus \mathcal{F}\}$
$\mathcal{W} := \{(a, \min\{\mathcal{F}, \mathcal{Q} \setminus \mathcal{F}\}), a \in \Sigma\}$
while $\mathcal{W} \neq \varnothing$ **do**
  Pick (a, C) from $\mathcal{W}$ and remove it
  forall $B \in \mathcal{P}$ **do**
    Split B with (a, C) into B_0 and B_1
    $\mathcal{P} := (\mathcal{P} \setminus \{B\}) \cup \{B_0, B_1\}$
    forall $b \in \Sigma$ **do**
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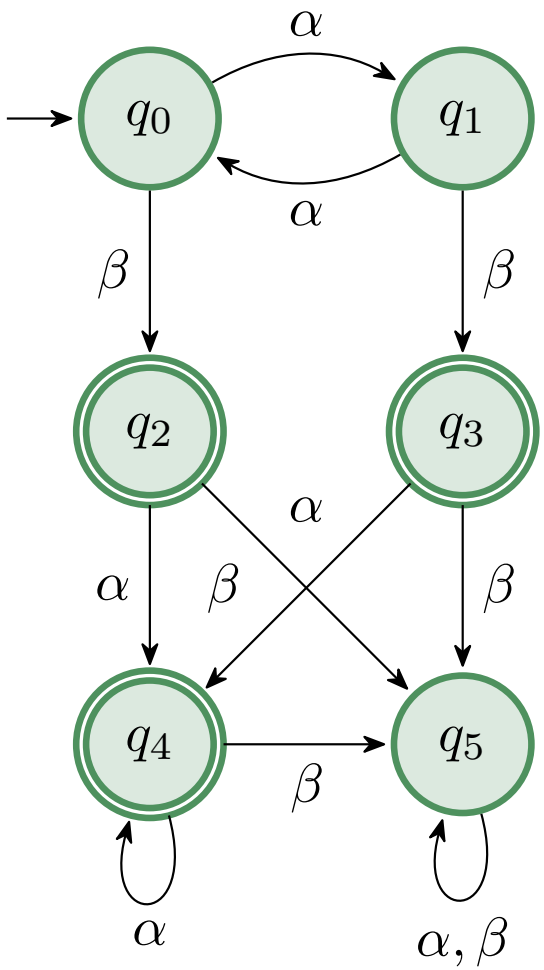

Data: a DFA $\mathcal{A} = (Q, \Sigma, \delta, q_0, \mathcal{F})$
if $\mathcal{F} = \varnothing \vee Q \setminus \mathcal{F} = \varnothing$
  return Q
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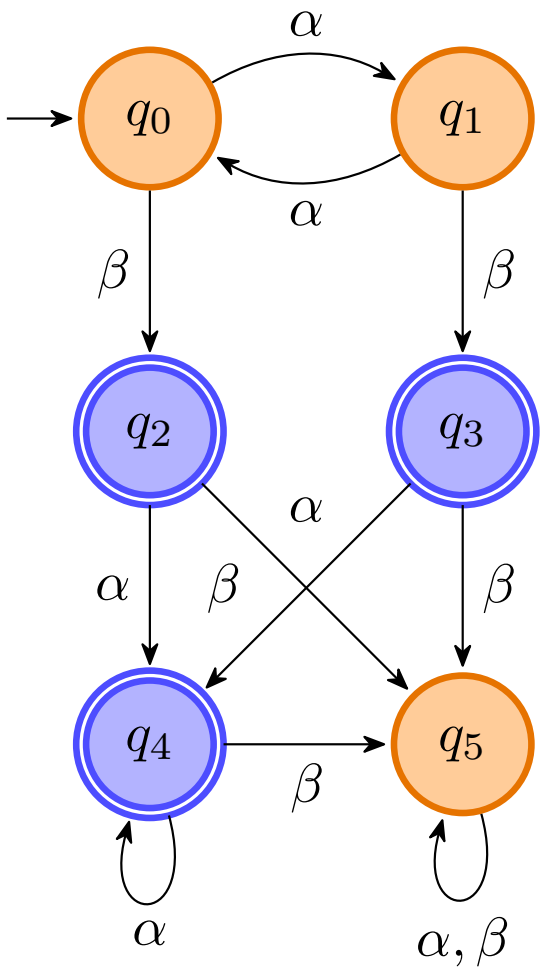

Splitter	Partition	Workset
–	$\{q_0, q_1, q_5\}\{q_2, q_3, q_4\}$	$(\alpha, \{q_0, q_1, q_5\})$ $(\beta, \{q_0, q_1, q_5\})$

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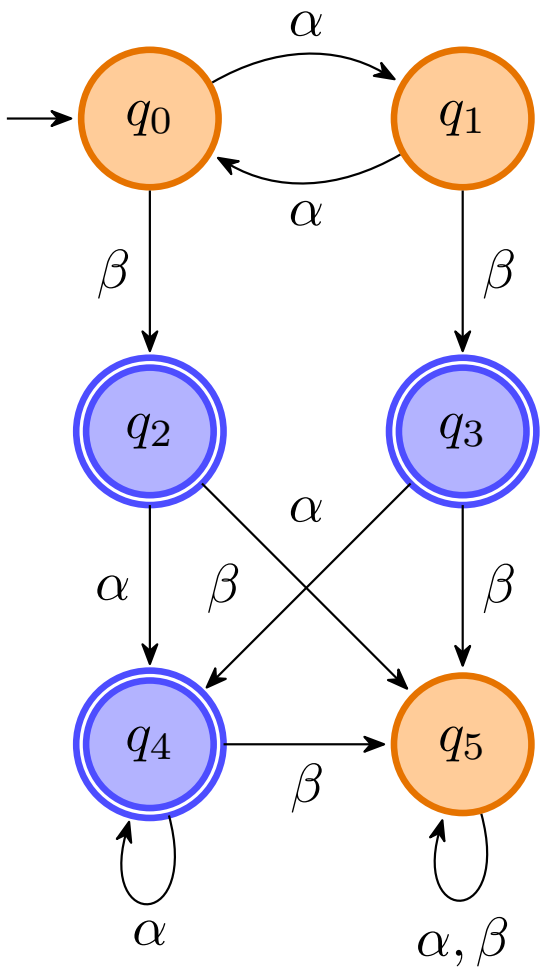

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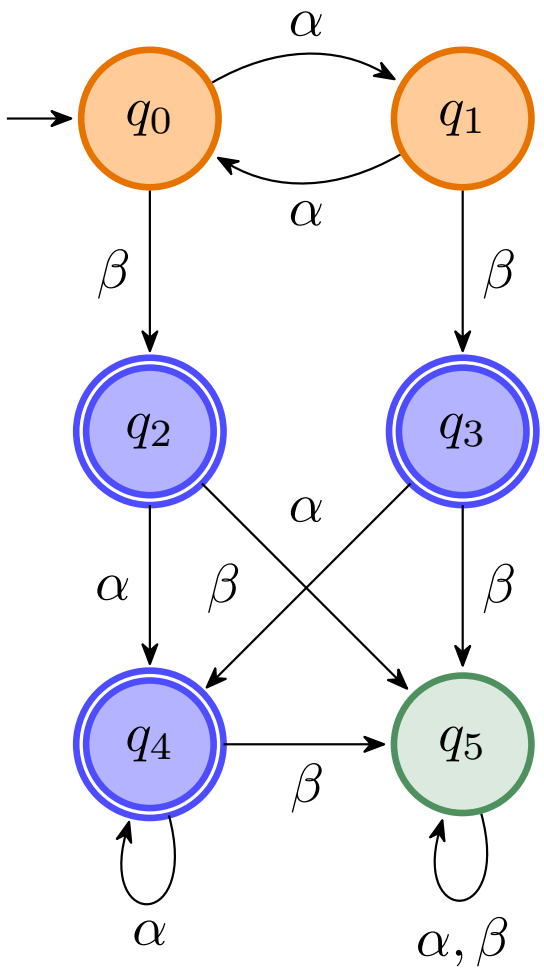

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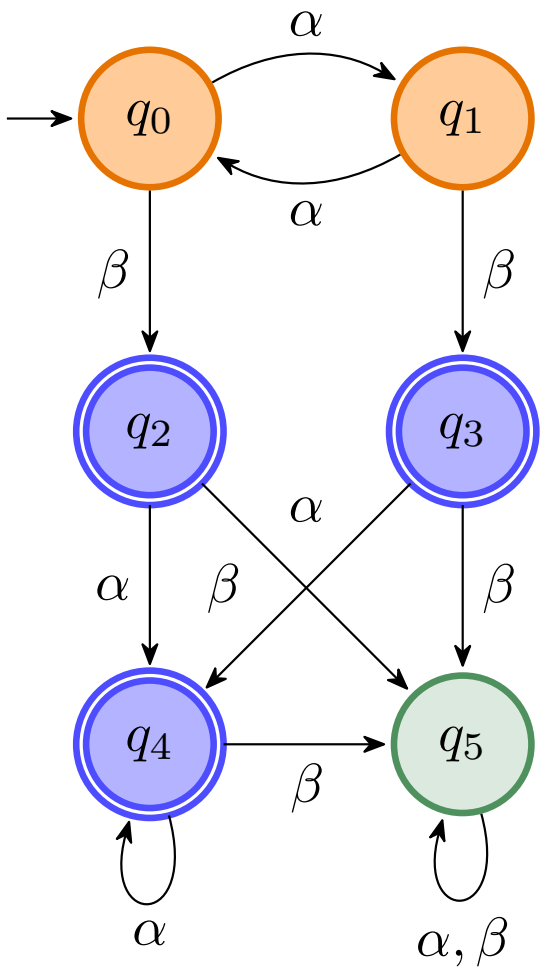

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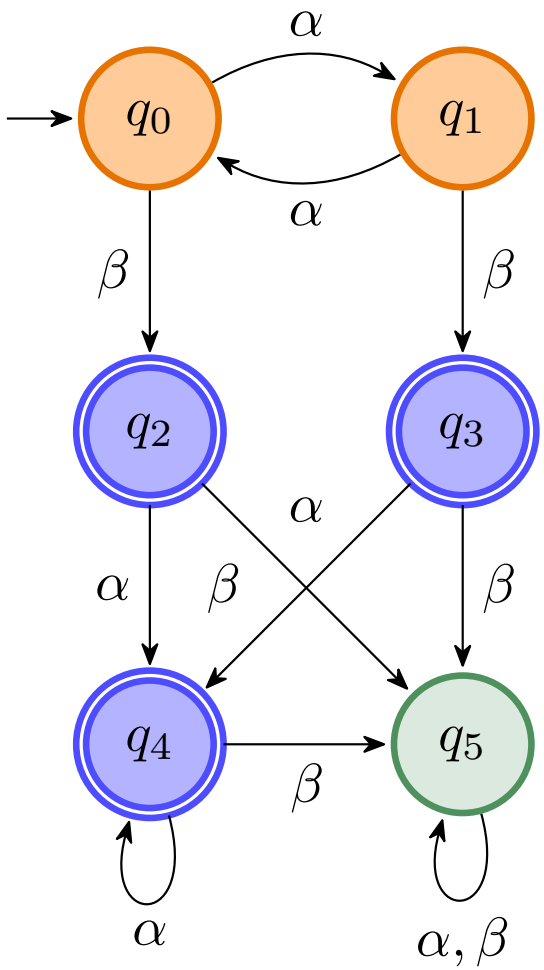

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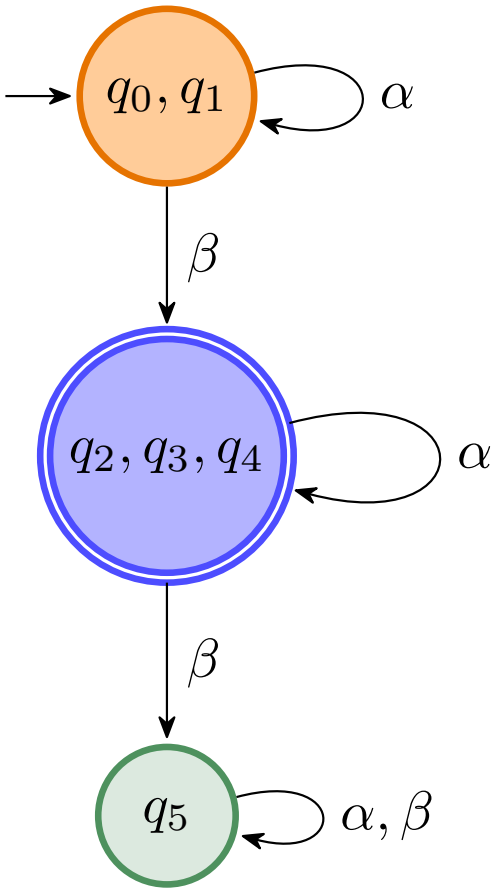
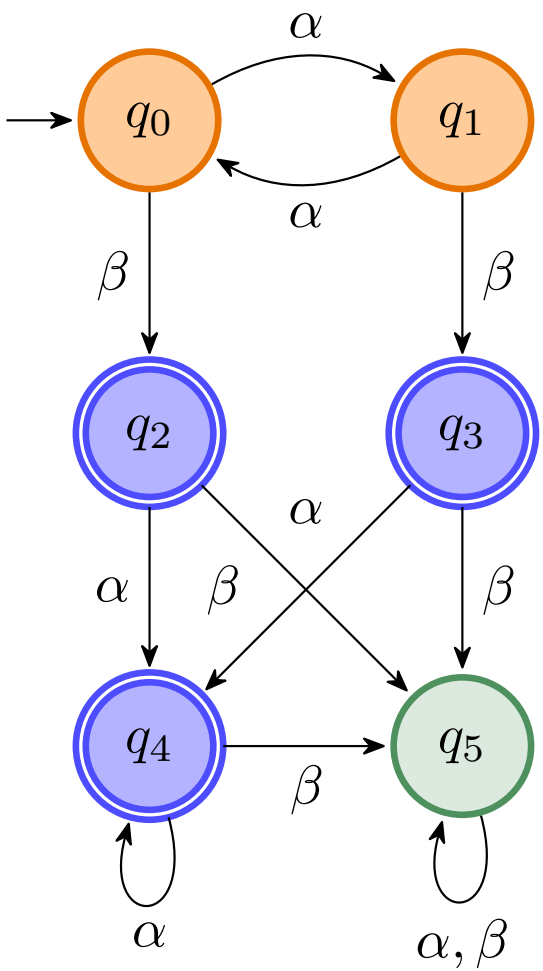
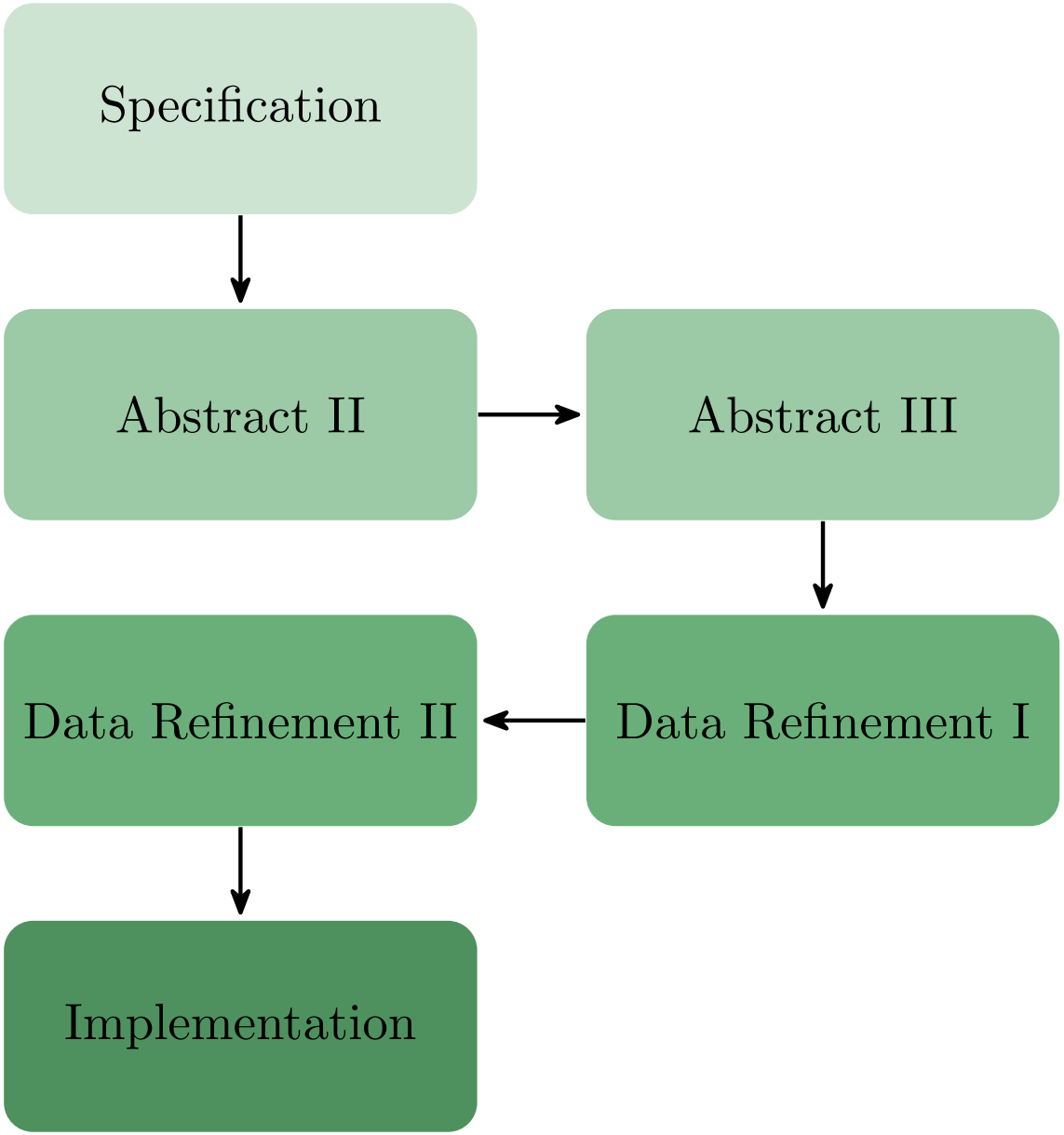
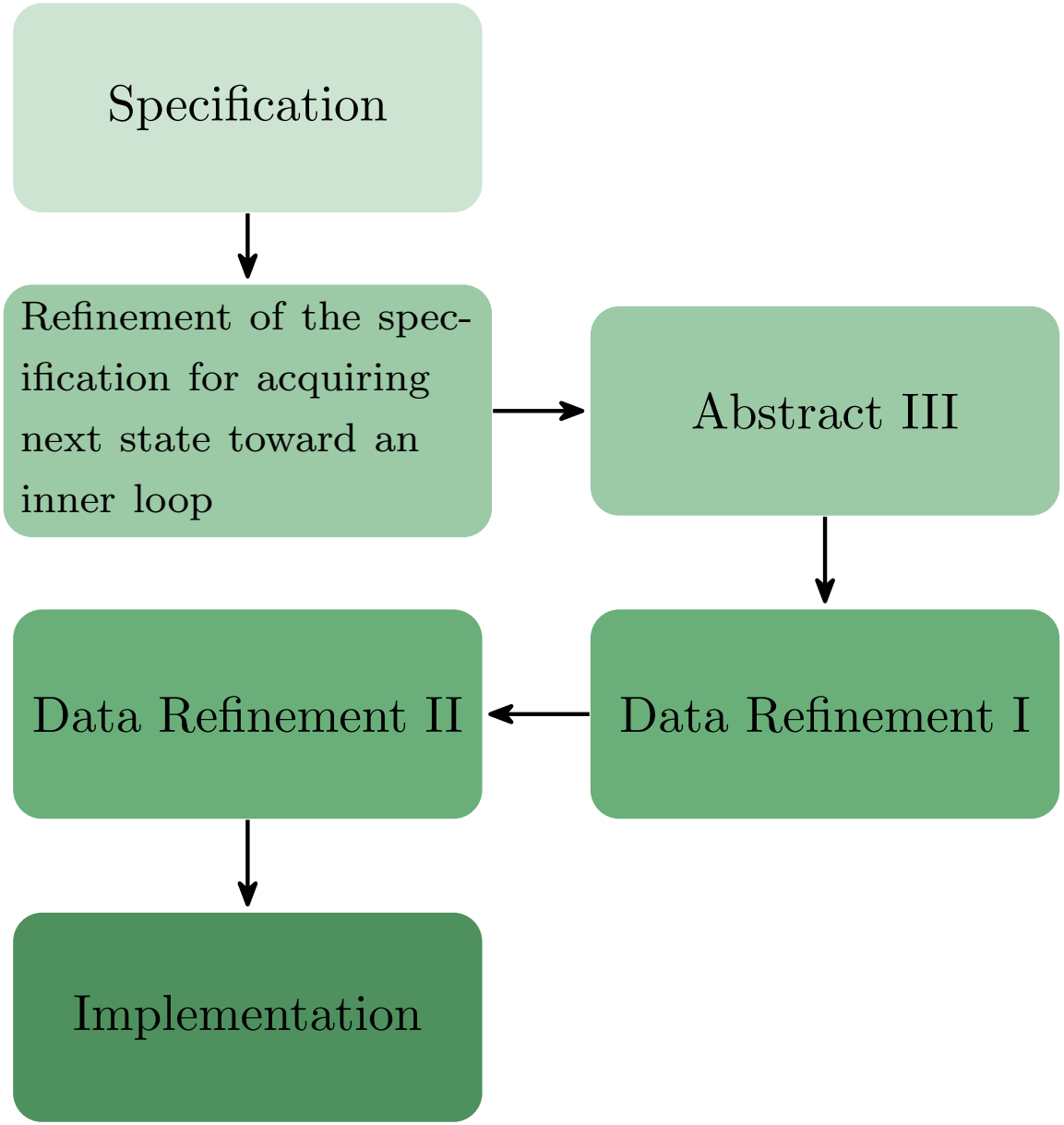
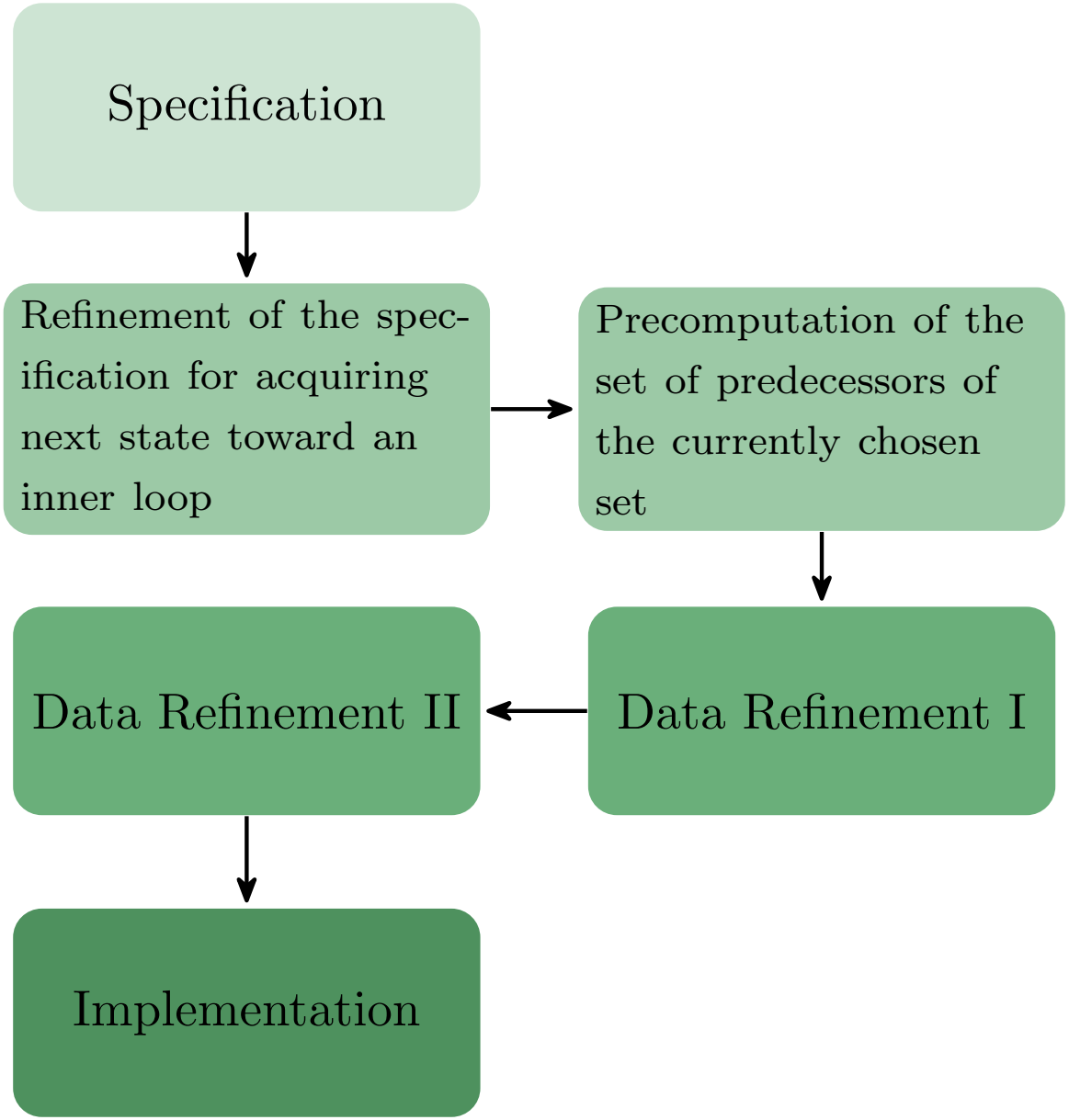


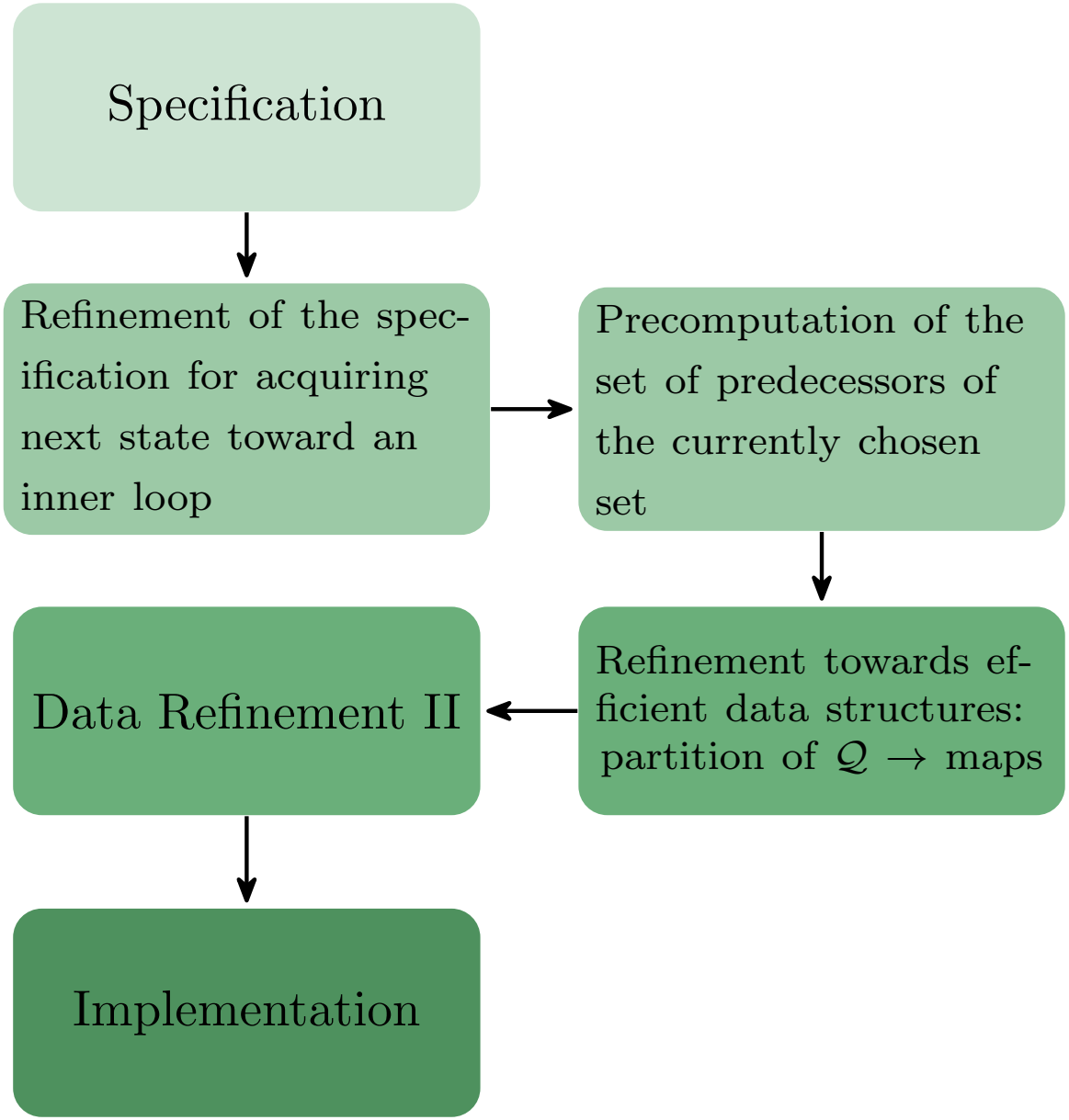


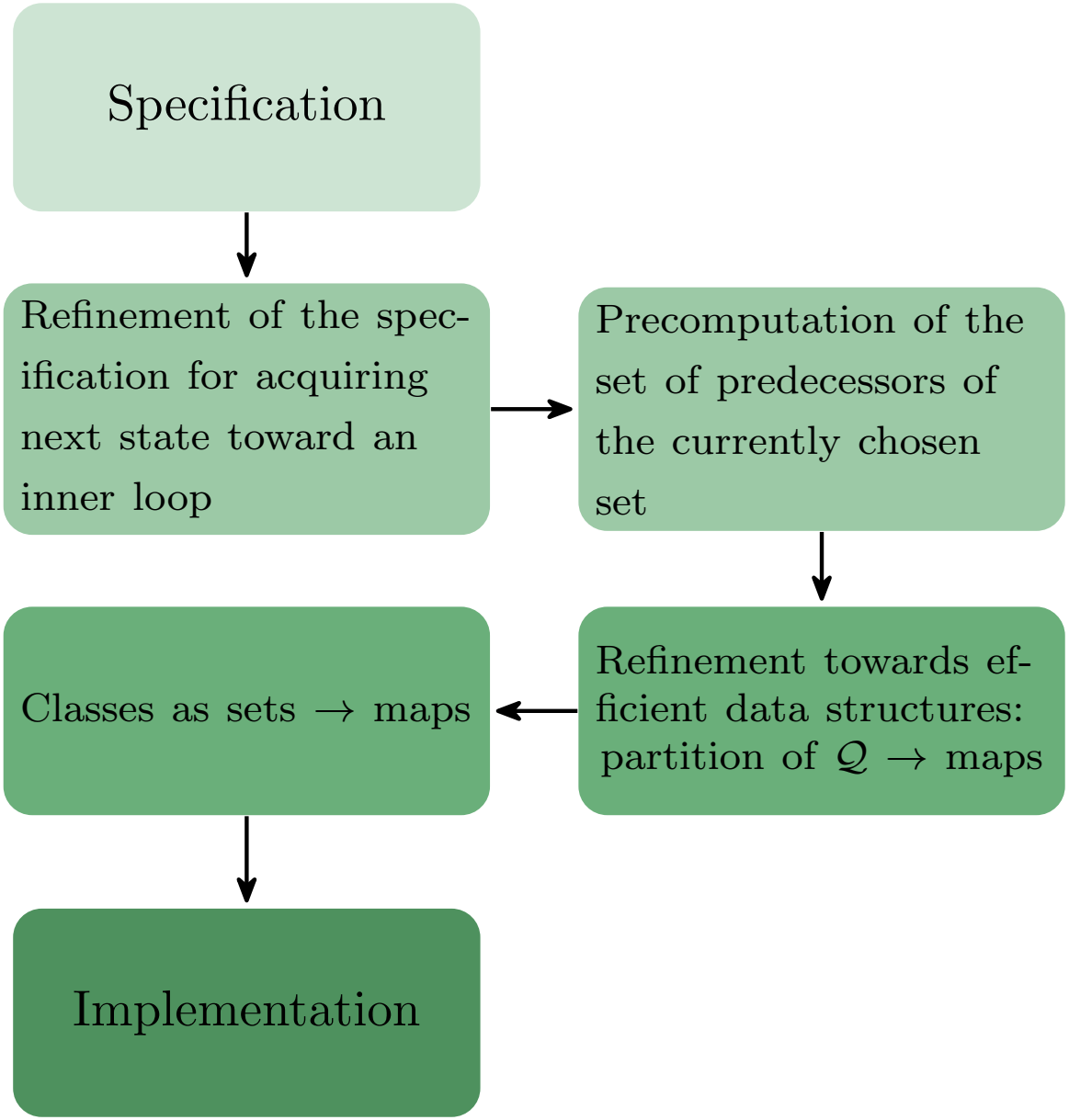
$$\mathcal{L} = \alpha^* \beta \alpha^*$$


Specification
Abstract II
Abstract III
Data Refinement II
Data Refinement I
Implementation


Specification
Refinement of the specification for acquiring next state toward an inner loop
Abstract III
Data Refinement I
Data Refinement II
Implementation


Specification
Refinement of the specification for acquiring next state toward an inner loop
Precomputation of the set of predecessors of the currently chosen set
Data Refinement II
Data Refinement I
Implementation


Specification
Refinement of the specification for acquiring next state toward an inner loop
Precomputation of the set of predecessors of the currently chosen set
Refinement towards efficient data structures: partition of $\mathcal{Q} \to$ maps
Data Refinement II
Implementation


Specification
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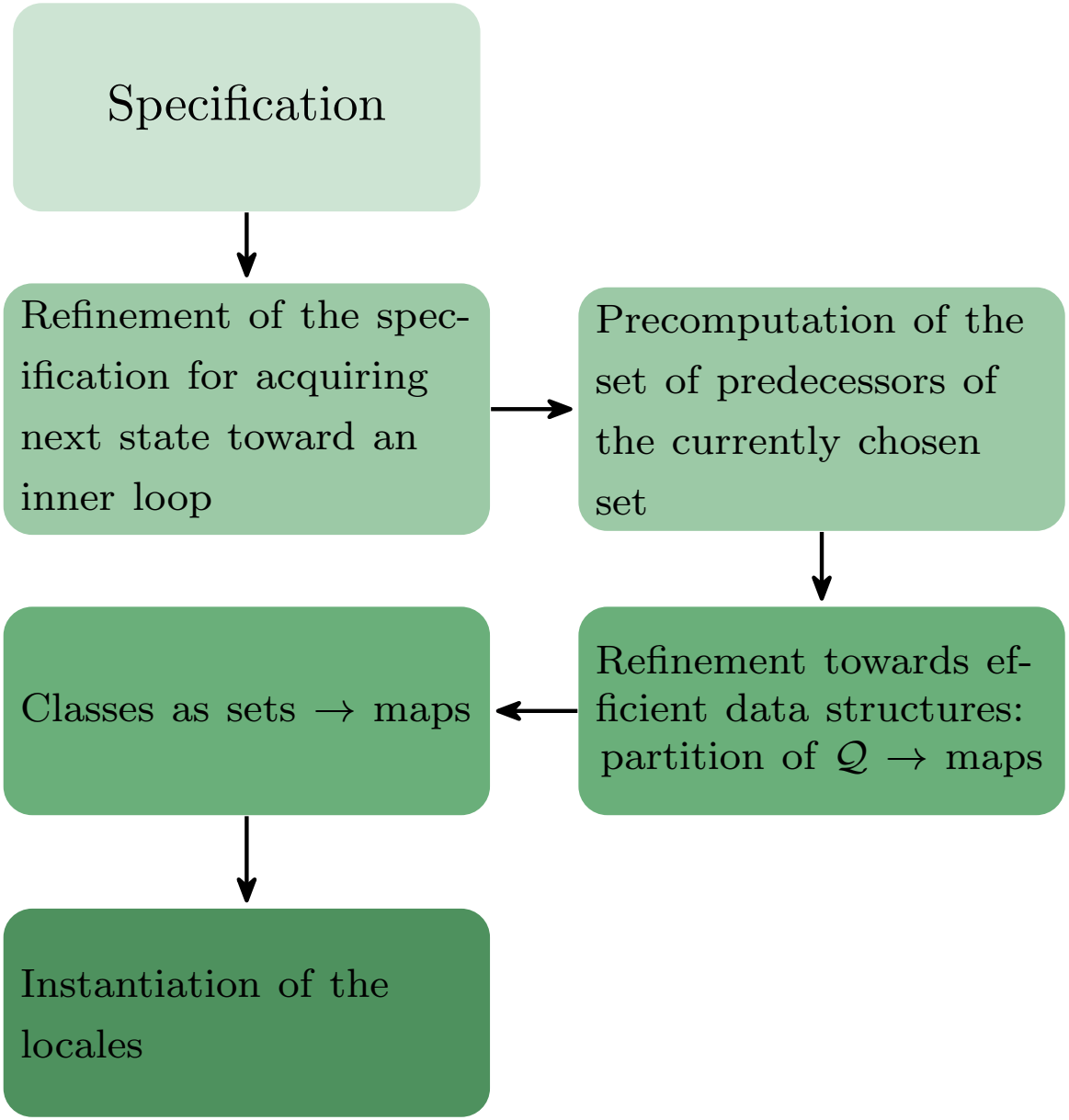
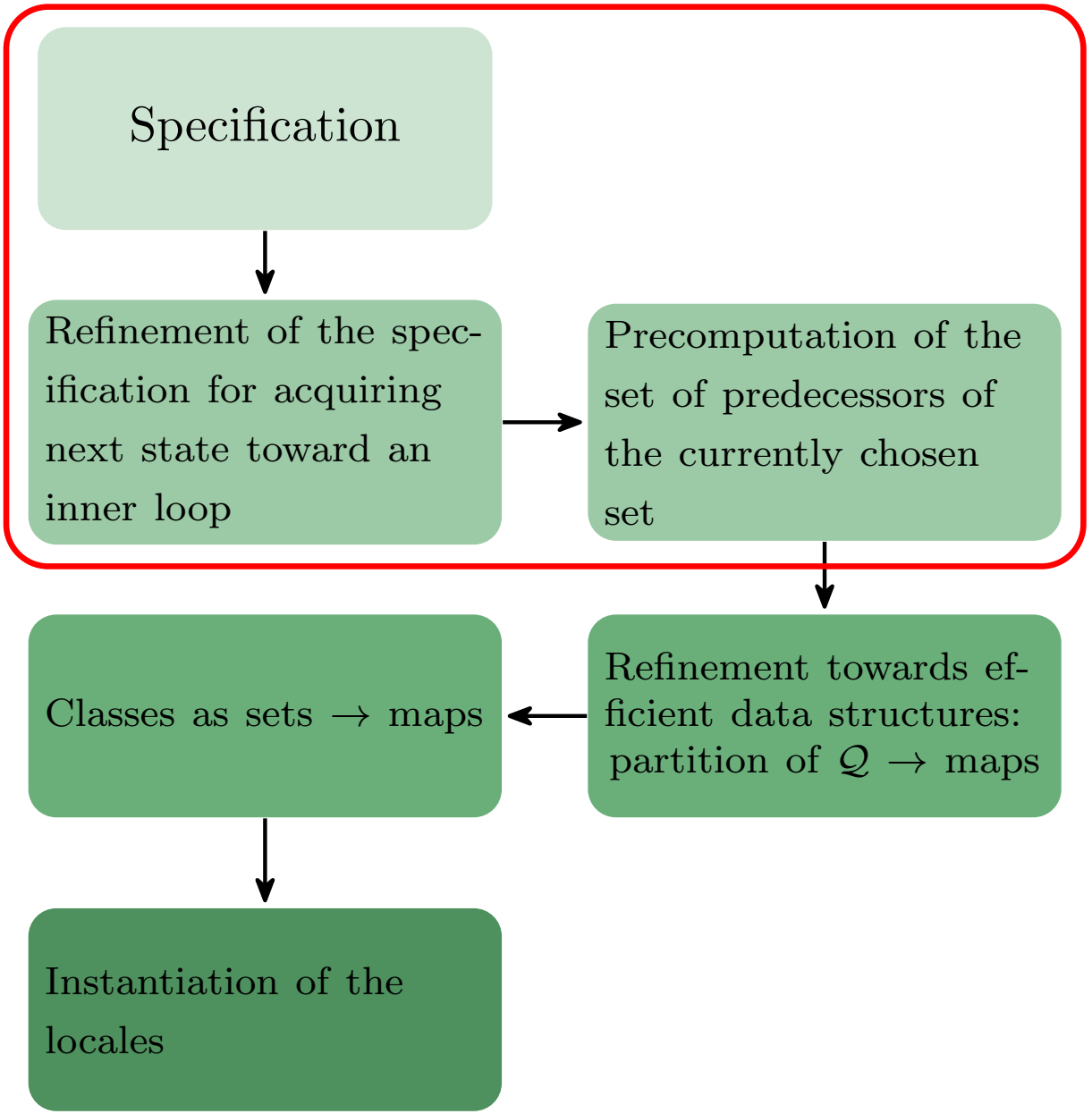
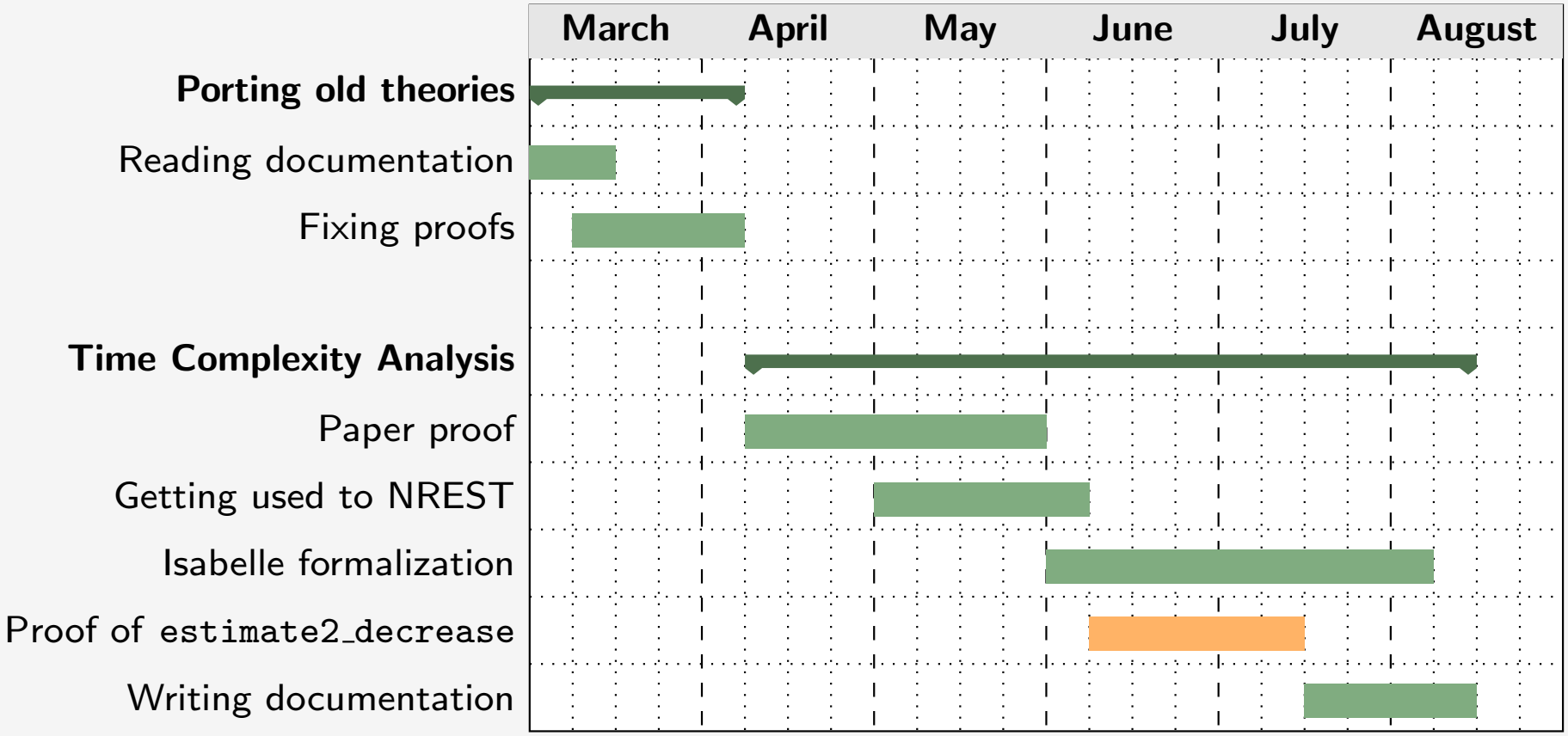
Precomputation of the set of predecessors of the currently chosen set
Refinement towards efficient data structures: partition of $\mathcal{Q}$ → maps
Classes as sets → maps
Implementation


Specification
Refinement of the specification for acquiring next state toward an inner loop
Precomputation of the set of predecessors of the currently chosen set
Refinement towards efficient data structures: partition of $\mathcal{Q}$ → maps
Classes as sets → maps
Instantiation of the locales



What I had time to do

vtrelat@home:~$ echo gantt chart



```
vtrelat@home:~$ show stats
```

Beginning of the project (in SLoC):

Current state (in SLoC) after approx. 900h:

vtrelat@home:~$ show stats

Beginning of the project (in SLoC):

```
$ find . -type f -name '*' | xargs wc -l | grep 'total'
  477020 total
```

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```

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$ find . -type f -name '*.thy' | xargs wc -l | grep 'total'
  139052 total
```

Current state (in SLoC) after approx. 900h:

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$ find . -type f -name '*' | xargs wc -l | grep 'total'
  691703 total
```

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