4.	Házi feladat	

1.

a, $\lim_{x \to \frac{\pi}{2}-0} \frac{\ln\left(\frac{\pi}{2}-x\right)}{tg(x)} = A \quad \frac{-\infty}{\infty} \quad (-1) \lim_{x \to \frac{\pi}{2}-0} \frac{-\ln\left(\frac{\pi}{2}-x\right)}{tg(x)} \quad \frac{\infty}{\infty}$

L'Hospital

$f'(x) = -1 \cdot \left(\frac{1}{\pi/2 - x} \cdot -1\right) = \frac{2}{\pi - 2x}$

$g'(x) = \frac{1}{\cos^2 x}$

$A = -1 \cdot \frac{2\cos^2 x}{\pi - 2x} \xrightarrow{x \to \frac{\pi}{2}-0} 0$

b, $\lim_{x \to 0+0} \frac{1 - \sqrt{\cos x}}{1 - \cos\sqrt{x}} = A \quad \left(\frac{0}{0}\right)$ L'Hospital

$f'(x) = -1 \cdot \frac{1}{2\sqrt{\cos x}} \cdot (-\sin x) = \frac{\sin x}{2\sqrt{\cos x}}$

$g'(x) = -1 \cdot -\sin(\sqrt{x}) \cdot \frac{1}{2\sqrt{x}} = \frac{\sin\sqrt{x}}{2\sqrt{x}}$

$A = \lim_{x \to 0+0} \frac{\frac{\sin x}{2\sqrt{\cos x}}}{\frac{\sin\sqrt{x}}{2\sqrt{x}}} = \lim_{x \to 0+0} \frac{2\sqrt{x}\sin x}{2\sqrt{\cos x}\sin\sqrt{x}}$

$= \lim_{x \to 0+0} \frac{\sin x \sqrt{x}}{\sin\sqrt{x}\sqrt{\cos x}} \quad \left(\frac{0}{0}\right)$

$f''(x) = \cos x\sqrt{x} + \frac{1}{2\sqrt{x}}\sin x = \frac{\sqrt{x}\cos x \sin x}{2\sqrt{x}} = \frac{\cos x \sin x}{2}$

$g''(x) = \cos\sqrt{x} \cdot \frac{1}{2\sqrt{x}} \cdot \sqrt{\cos x} + \sin\sqrt{x} \cdot \frac{1}{2\sqrt{x}} \cdot (-\sin x)$

$= \frac{\cos\sqrt{x} \cdot \sqrt{\cos x} - \sin\sqrt{x}\sin x}{2\sqrt{x}}$

$A = \lim_{x \to 0+0} \frac{\cos x \sin x \sqrt{x}}{\cos\sqrt{x}\sqrt{\cos x} - \sin\sqrt{x}\sin x} = \frac{0}{1} = 0$

c, $\lim\limits_{x\to+\infty} x\cdot\left(\operatorname{arc\,tg} x-\frac{\pi}{2}\right)=A \quad (+\infty\cdot 0)$

$$\lim_{x\to+\infty}\frac{\left(\operatorname{arctg} x-\frac{\pi}{2}\right)}{\frac{1}{x}} \quad \left(\frac{0}{0}\right) \text{ L'Hospital}$$

$$f'(x)=\frac{1}{1+x^2} \qquad g'(x)=-\frac{1}{x^2}$$

$$A=\lim_{x\to+\infty}\frac{\frac{1}{1+x^2}}{-\frac{1}{x^2}}=\lim_{x\to+\infty}-\frac{x^2}{1+x^2}$$

$$=-1\cdot\lim_{x\to+\infty}\frac{x^2}{x^2\left(\frac{1}{x^2}+1\right)}$$

$$=-1\cdot\lim_{x\to+\infty}\frac{1}{\frac{1}{x^2}+1}\to\frac{1}{0+1}\cdot-1=-1$$

d, $\lim\limits_{x\to 0}(\operatorname{ch} x)^{\frac{1}{\operatorname{sh} x}}=A \qquad 1^{\frac{1}{0}}$

$$(\operatorname{ch} x)^{\frac{1}{\operatorname{sh} x}}=\left(e^{\ln \operatorname{ch} x}\right)^{\frac{1}{\operatorname{sh} x}}=e^{\frac{\ln(\operatorname{ch} x)}{\operatorname{sh} x}}$$

$$A=e^{\lim\limits_{x\to 0}\frac{\ln(\operatorname{ch} x)}{\operatorname{sh}(x)}=B} \quad \frac{0}{0} \text{ L'Hospital}$$

$$f'(x)=\frac{1}{\operatorname{ch} x}\cdot\operatorname{sh} x=\frac{\operatorname{sh} x}{\operatorname{ch} x}$$

$$g'(x)=\operatorname{ch}(x)$$

$$B=\lim_{x\to 0}\frac{\operatorname{sh}(x)}{\operatorname{ch}(x)}=0$$

$$A=e^0=1$$

2.

a, $f(x) = e^{2x} - (4x+1) = e^{2x} - 4x - 1 \quad (x \in \mathbb{R})$

$f'(x) = e^{2x} \cdot 2 - 4$

$f''(x) = e^{2x} \cdot 4 > 0 \quad (x \in \mathbb{R})$

$f''(0) = 4 \Rightarrow$ f konvex a teljes értelmezési tartományon

b, $f(x) = \frac{4x}{x^2-1} \quad (x \in \mathbb{R} \setminus \{-1, 1\})$

$f'(x) = \frac{4(x^2-1) - 2x(4x)}{(x^2-1)^2} = \frac{-4 \cdot (x^2+1)}{(x^2-1)^2}$

$= -4 \cdot \frac{(x^2+1)}{x^4 - 2x^2 + 1}$

$f''(x) = \frac{2x(x^4-2x^2+1) - ((x^2+1) \cdot (4x^3-4x))}{(x^4-2x^2+1)^2} \cdot (-4)$

$= \frac{2x^5 - 4x^3 + 2x - (4x^5 - 4x^3 + 4x^3 - 4x)}{(x^4-2x^2+1)^2}$

$= \frac{8x^5 + 16x^3 - 24x}{(x^4-2x^2+1)^2} = \frac{8x(x^4+2x^2-3)}{(x^4-2x^2+1)^2} \quad x_1 = 0$

$y = x^2$

$y_{1,2} = \frac{-2 \pm \sqrt{4+12}}{2} = \frac{-2 \pm 4}{2}$ ↗ -6 ↯ ↘ 1

$1 = x^2$

$x_1 = -1$ ↯ $x_2 = 1$ ↯

	$(-\infty; -1)$	$(-1; 0)$	0	$(0; 1)$	$(1; \infty)$
f''	$-$	$+$	0	$-$	$+$
f	⌒	⌣	0	⌒	⌣
			inf.		

3.

$$f(x) := \frac{x^2+4}{x} \quad (x \in \mathbb{R} \setminus \{0\})$$

$$\lim_{x \to \pm\infty} \frac{x^2+4}{x} = \lim_{x \to \pm\infty} x + \frac{4}{x} \to \pm\infty$$

A határértékek léteznek, de nem végesek, ezért nem léteznek $+\infty$-ben és $-\infty$-ben f-nek aszimptotái