EFFECT OF ADDING OMITTED VARIABLES IN MEDIATION AND MODERATION ANALYSES: A CASE STUDY

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Abstract

Mediation and moderation analyses have been widely implemented in social sciences. Both techniques use regression analysis for modeling. Consequently, these techniques are subjected to assumptions of linearity, normality, constant variance, and independence of errors. When these assumptions are violated, models generate distorted results on the magnitude of effects and the causal relationships. To remediate the appearing inconsistencies, a common practice is the inclusion of variables that exert specific effects given their nature. In this project, a case study is analyzed based on the work developed by Mukuka et al. (2021), where we explore the results of adding moderators. In this work, we compare the robustness and assumptions fulfillment from the original model with our proposed model. Results show that the developed model increases the coefficient of determination (R^2) by approximately 43.96%, and decreases Root Mean Square Error (RMSE) and Mean Absolute Error (MAE) by 7.9 and 6.12 units, respectively.

Keywords: Mediator; Moderator; Regression assumptions; Multiple linear regression.

1. Introduction

In social sciences, researchers build models to study the relationships between independent and dependent variables. Commonly used models are mediation and moderation analyses. Mediation analysis investigates intermediate variables in the relationship between independent and dependent variables (Miočević et al., 2018). Moderation deals with the intensity that the independent variable holds on the dependent variable (MacKinnon, 2017).

1.1. Mediation Analysis

Mediation analysis is used to evaluate intermediate variables called mediators (*M*) that transmit the effect of an independent variable (*X*) on a dependent variable (*Y*). From the top and middle panels of Figure 1, the effects of interest in the single mediator model can be computed using these equations:

$$Y = i_1 + cX + e_1 \tag{1}$$

$$M = i_2 + aX + e_2 \tag{2}$$

$$Y = i_3 + c'X + bM + e_3 \tag{3}$$

Where the intercepts are i_1, i_2 and i_3, *c* is the total effect of *X* on *Y*, *a* is the coefficient relating the independent variable to the mediator *M*, *b* is the coefficient relating the mediator *M* to *X*, *c'* is the direct effect of *X* on *Y* and e_1, e_2, and e_3 are error terms assumed to follow a normal distribution. Equation 1 serves as a point of comparison in the magnitude of the total effect by mediator *M* in Equations 2 and 3. The mediated effect (also known as indirect effect) is often computed as the product of *ab*, and when added to *c'* conveys the total effect when the mediator is present.

1.2. Moderation Analysis

Moderation analysis is used the evaluate the impact on the effect that *X* has on *Y* when in presence variables called moderators *W*. Sharma, Durand, and Gur-Arie (1981) defined three types of moderator effects. The first one is the *homologizer* which does not change the true relation that the *X* has on *Y* but does change the error variance. The second kind is called a *quasi-moderator* because moderator W changes the form of the relationship between X and Y, and is also a significant predictor of Y. Finally, the third type of moderator is called a *pure moderator* because the variable is not a significant predictor of Y, but it causes the formation of the relationship between X and Y to change as a function of W.

This moderation effect is usually expressed as:

$$Y = i_4 + c_1X + c_2W + c_3XW + e_4 \tag{4}$$

where Y is the dependent variable, X is the independent variable, W is the moderator, and XW is the interaction of the moderator and the independent variable; e_1 is the residual, and c_1, c_2, and c_3 represent the relationships that the dependent variable has with the independent variable, the moderator, and the interaction of moderator and independent variable respectively.

1.3. Linear Regression Assumptions

Mediation and moderation analyses rely on regression models to make causal inferences in the relationship between X and Y. Hence, the validity of such models lies in meeting the traditional multiple linear regression model assumptions: linearity, normality, homoscedasticity, and independence of errors. Falling short in certain assumptions would trigger questions regarding the validity of the inferences and conclusions made about the model. In other words, these assumptions guarantee the robustness of predictions and conclusions made about the considered variables (Navidi, 2010).

The linearity assumption states that the relationship between X and Y is linear, or at least approximately linear. This condition is essential because, if violated, it jeopardizes the interpretation of the regression coefficients (Darlington & Hayes, 2017). On the other hand, normality implies that errors in the estimation of the fitted model are normally distributed.

However, this assumption is usually the most neglected in linear regression analysis and is sometimes considered the least critical.

The last two assumptions relate to constant variance and independence of errors. The first one is usually referred to as homoscedasticity and states that errors in estimating Y are equally variable. When this assumption is not met, it is said that errors are heteroscedastic, which is a condition that can affect the validity of inference, reduce the statistical power of hypothesis testing, and diminish the accuracy of confidence intervals for regression coefficients (Breusch & Pagan, 1979). Finally, errors are considered independent when information about one of the variables does not provide information about the other. In other words, for all pairs of observations, there is no indication of how the error in the first case could be used to estimate the next one (Montgomery et al., 2012). Like heteroscedasticity, a lack of independence affects the accuracy of the standard error in the regression coefficients. However, usually, independence problems could be easily solved when omitted variables are identified.

1.4. Omitted variables

Much of statistics focuses on the association between two variables, one independent *X* and the other dependent *Y*. However, research is often interested in the existing relationship when more variables *Z* are added to the analysis. Omitted variables *Z* often refer to confounding, covariate, mediating, and moderating variables.

We provide a brief definition of each.

- A confounder is a variable related to the dependent and independent variable; however, their identification is not always possible, resulting in false accentuations or dampening specific effects relevant to the study (MacKinnon, 2017).
- A covariate is a dependent variable that usually has low or null relation to the independent variable. Adding a covariate into a regression model would usually

improve the accuracy of the predicted variable because it explains its variability better (MacKinnon, 2017).

- A mediator is a variable that transmits the effect of an independent variable to a dependent variable (Baron and Kenny, 1986).
- A moderator is a variable that affects the direction or magnitude of the relationship that the two variables hold (Baron and Kenny, 1986). Figure 1 shows the relationship between mediators and moderators.

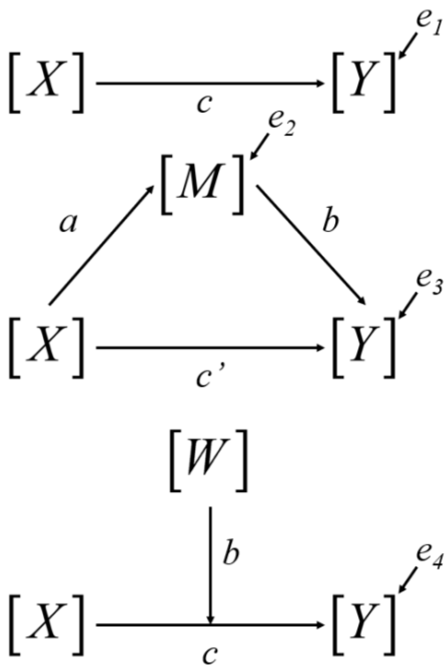


Figure 1. Top panel: Total effect of the independent variable on the outcome. Middle panel: Single mediator model. Bottom panel: Single moderator model. The intercepts are included in the two models, but not in the figure.

The following paper addresses the importance of adding omitted variables in a case study on mathematics education (Mukuka et al., 2021). Where incorporating moderation variables into the mediation model not only aids fulfillment of linear regression assumptions but also provides robustness to the regression model.

The rest of the paper is structured as follows. Section 2 establishes the problem definition for the case study. Section 3 provides the analytical solution, including the construction of the proposed mediation and moderation model. Section 4 discusses the new model's results in the problem context. Finally, Section 5 presents the conclusions of this work.

2. Problem definition

This article is based on a case study from a quasi-experiment developed by Mukuka et al. (2021), where six public secondary schools from Zambia were analyzed; three were randomly allocated to the experimental group while the other three were allocated to the control group. Students assigned to the control group were taught using an expository instructional approach based on daily lectures and question-and-answer techniques.

Meanwhile, those assigned to the experimental group were taught using the Student Teams-Achievement Division (STAD) model of cooperative learning, based on students working cooperatively in heterogeneous groups. They are faced with contextualized problems and hands-on activities, and at the end of each topic, a quiz is given to them in which they were not allowed to help one another. In the end, the group with the highest average or that attained the desired performance level earned an award or recognition.

The students' scores from this study were collected after six weeks of exposure to the different learning methods. Three main results confirm these: Mathematical Reasoning Test (MRT), mathematical reasoning dimensions adequacy (conjecturing, justifying, and mathematizing), and self-efficacy beliefs.

Firstly, the MRT was a test given to the students prior and posterior to the experiment based on two class topics: quadratic equations and quadratic functions. The MRT posterior to the experiment consisted of 16 test items organized under seven questions. Three mathematical reasoning dimensions conformed these 16 items: conjecturing, justifying, and mathematizing. Table 1 shows the specific test items allocated to the three dimensions. If students scored less than 50% for each of the three dimensions, they were classified under the 'inadequate reasoning' category; otherwise, they were classified under the 'adequate reasoning' category.

This categorization is based on the standard pass rate criteria set by the Zambian tertiary institutions (Mukuka et al., 2020b).

Table 1 Finalized Mathematical Reasoning dimensions and allocated MRT items

MR Dimension	Question number															
	1		2	3		4		5		6		7				
	(a)	(b)	(a)	(a)	(b)	(a)	(b)	(a)	(b)	(a)	(b)	(a)	(b)	(c)	(d)	(e)
Conjecturing			x	x		x				x				x		
Justifying	x		x	x	x		x			x			x	x		
Mathematising		x						x	x	x	x	x			x	x

Finally, the students were given a questionnaire prior and posterior to the experiment to measure their mathematics self-efficacy and task-specific self-efficacy beliefs. These scores were based on how often students felt confident to succeed in various aspects of mathematical learning and how confident they were in answering each of the mathematical reasoning questions on quadratic equations and functions, respectively. The student's results can be found on the MR Dataset.sav (available on http://dx.doi.org/10.17632/3472zggczv.1). Demographic variables such as a respondent’s identity, gender, school type, and age have equally been specified in the dataset.

By making use of this dataset, Mukuka et al. (2021) developed an analysis based on a “Parallel Multiple Mediation Model” where an independent variable, (*group*), was modeled as influencing the dependent variable (*posttest*), both directly and indirectly through two mediators, (*post_efficacy*) and (*post_specific*). Parameters were obtained using the PROCESS custom dialogue version 3.4 embedded in SPPS, resulting in the regression coefficients shown in Figure 2. Further results in their mediation analysis are summarized in Table 2.

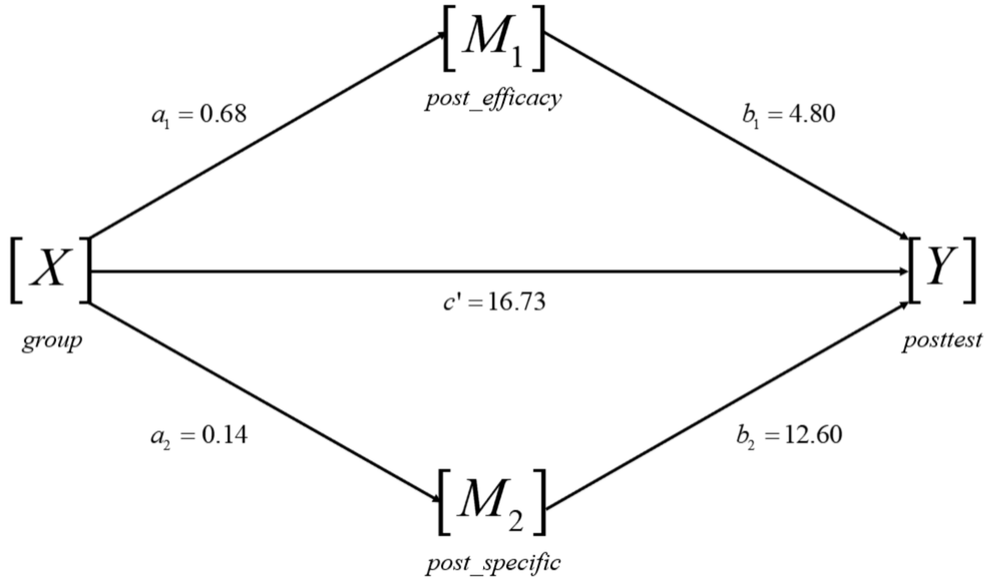


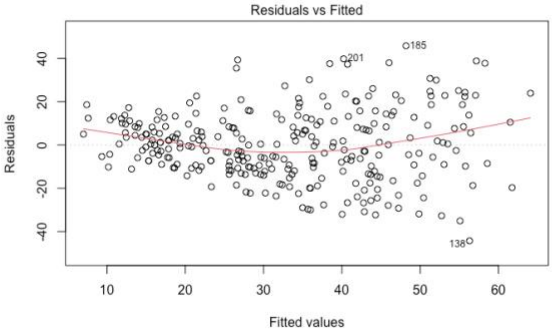
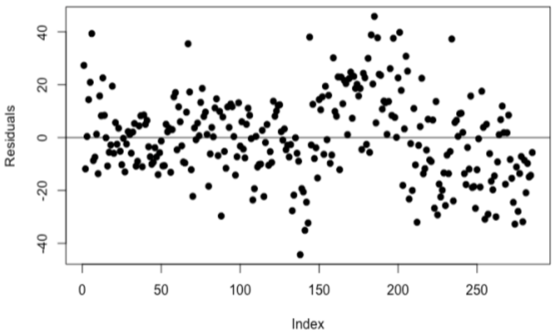
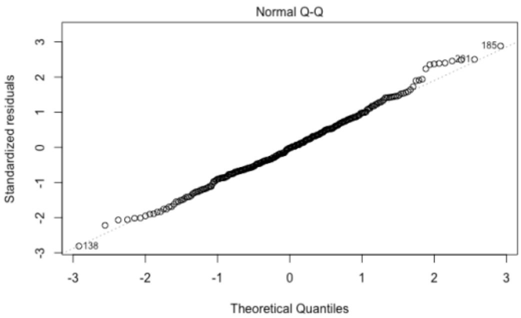
Figure 2. Mediation Model for the indirect effect of Teaching Method (*group*) on Mathematical Reasoning (*posttest*) given Self-Efficacy beliefs (*post_efficacy* & *post_specific*) as proposed by Mukuka et al. (2021).

Table 2. Summary of the mediation and regression parameters from the PROCESS output as proposed by Mukuka et al. (2021).

		Consequent										
		M_1 (post_efficacy)				*M_2 (post_specific)*				*Y (posttest)*		
Antecedent		*Coeff.*	*SE*	*p*		*Coeff.*	*SE*	*p*		*Coeff.*	*SE*	*p*
X (group)	a	0.68	0.05	0.00	a	0.14	0.06	0.03	c'	16.73	2.54	0.00
M_1 (post_efficacy)	---	---	---	---	---	---	---	---	b_1	4.80	2.64	0.07
M_2 (post_specific)	---	---	---	---	---	---	---	---	b_2	12.60	1.85	0.00
Intercept	a_0	2.40	0.03	0.00	a_0	2.37	0.05	0.00	b_0	-19.7	6.45	0.00
		$R^2 = 0.44$ $F(1,284) = 223.67$ $p < 0.001$				$R^2 = 0.02$ $F(1,284) = 5.03$ $p = 0.03$				$R^2 = 0.42$ $F(1,282) = 66.80$ $p < 0.001$		

Focusing on the results of the original model ($Y = X + M_1 + M_2$), which explains the direct effects of the variables X (*group*), M_1(*post_efficacy*) and M_2 (*post_specific*) have on the response variable Y(*posttest*), it is noticeable that its coefficient of determination (R^2) is barely over 40% which means that the predictor variables do not reduce the uncertainty on predicting the posttest scores.

Table 3. Graphical testing of multiple regression model assumptions ($Y \sim X + M_1 + M_2$)

Assumptions	Original Model
Linearity Homoscedasticity	
Independence	
Normality	

Further analyses were made to verify the adequacy of this complete model ($Y = X + M_1 + M_2$) and a series of inconsistencies were encountered. First, K-fold cross-validation was performed, where the results confirm that this model has low coefficient stability and a low ability to generalize inferences. Additionally, its Root Mean Square Error (RMSE) conveys that, in general, the model's prediction of the posttest score is 16.09 points off. Similarly, its Mean Absolute Error showed that, on average, the forecast of this model has a 12.84 point difference from its actual value.

Also, we analyzed the satisfiability of the Multiple Linear Regression assumptions: linearity, errors' independence, homoscedasticity, and normality. These were tested graphically on the model (see Table 3), and it is visible that the homoscedasticity assumption is not met, which leads to the conclusion that this model has a high risk of generating distortion in the interpretation of results and weakening the overall statistical power of the analysis. In other words, the results of this model have an increased possibility of Type I error and inconsistent F-test results, leading to erroneous conclusions (Aguinis et al., 1999; Osborne & Waters, 2002).

Table 4. Correlations between all post-treatment scores.

	posttest	***conjecturing***	***justifying***	***mathematizing***	***post_efficacy***	***post_specific***
posttest	1.00	**0.67**	**0.76**	**0.79**	0.50	0.43
conjecturing		1.00	0.58	0.48	0.40	0.29
justifying			1.00	0.48	0.36	0.35
mathematizing				1.00	0.41	0.27
post_efficacy					1.00	0.35
post_specific						1.00

Furthermore, there is a lack of error independence, leading to an underestimation of standard errors and the labeling variables as statistically significant when not (Keith, 2006). To correct these unsatisfiability of assumptions and attain higher predictability on the results of students' posttest scores, a new model was developed.

3. Methodology

For the new model development, we considered that Mukuka et al. (2021) excluded some important post-experiment scores. These correspond to the adequacy of the student's mathematical reasoning dimensions (conjecturing, justifying, and mathematizing). These variables are considered post-treatment moderating variables because they are consequences of the treatment and directly affect the dependent variable (Koschate-Fischer & Schwille, 2018).

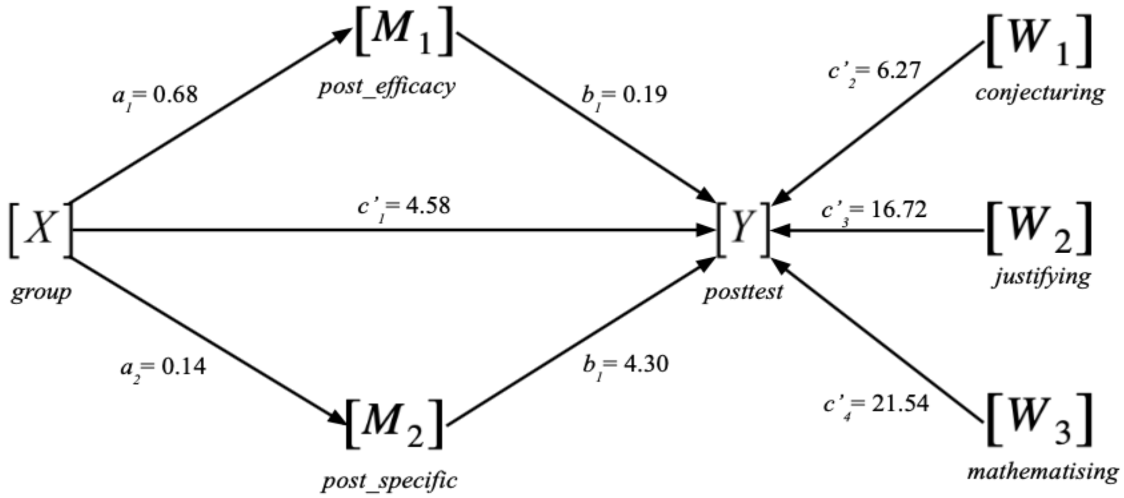


Figure 3. Mediation Model for the indirect effect of Teaching Method (*group*) given Self-Efficacy beliefs (*post_efficacy* & *post_specific*) and the direct moderation effect of the Mathematical Reasoning Dimensions (*conjecturing, justifying & mathematising*) on Mathematical Reasoning Test scores.

In experimental studies, the omission of a mediator-response confounding factor variable likely leads to an inadequate estimation of the indirect effect (Fritz et al., 2016). Therefore, adding these omitted variables into the model is necessary to avoid this issue. Moreover, Table 4 shows a high linear association between these moderating variables and the dependent variable, indicating that they are necessary for the model.

Table 5. Summary of the regression parameters from the original complete model developed by Mukuka et al. (2021) and the modified version including the moderating variables at the last level of the mediation analysis.

		Consequent						
		Original model			*Modified model*			
		Y (posttest)				*Y (posttest)*		
Antecedent		*Coeff.*	*SE*	*p*		*Coeff.*	*SE*	*p*
X (group)	c'	16.73	2.54	0.00	c_1'	4.58	1.39	0.00
M_1 (post_efficacy)	b_1	4.80	2.64	0.07	b_1	0.19	1.35	0.88
M_2 (post_specific)	b_2	12.60	1.85	0.00	b_2	4.30	0.98	0.00
W_1 (conjecturing)	---	---	---	---	c_2'	6.27	1.29	0.00
W_2 (justifying)	---	---	---	---	c_3'	16.72	1.37	0.00
W_3 (mathematizing)	---	---	---	---	c_4'	21.54	1.34	0.00
Intercept	a_0	-19.7	6.45	0.00	b_0	8.32	3.68	0.02
		$R^2 = 0.42$ $F(1{,}282) = 66.80$ $p < 0.001$				$R^2 = 0.85$ $F(1{,}282) = 265.90$ $p < 0.001$		

4. Results

Parameters for the modified model ($Y = X + M_1 + M_2 + W_1 + W_2 + W_3$) were obtained, resulting in the regression coefficients shown in Figure 3. Comparison of the original model and the one developed in this work appears in Table 5. In this work, it is visible that the coefficient of determination R^2 increased significantly in the modified model. This situation means that by adding the moderating variables, the total variability of the posttest scores that is explained by the regression model corresponds to 84.8%.

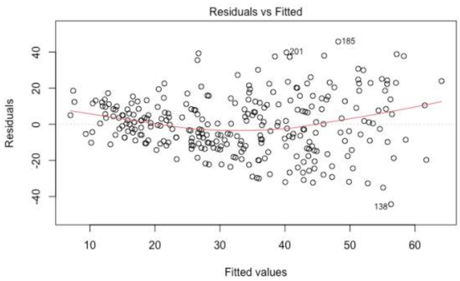
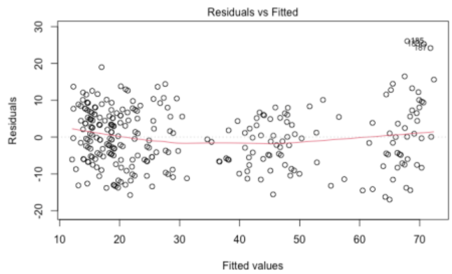
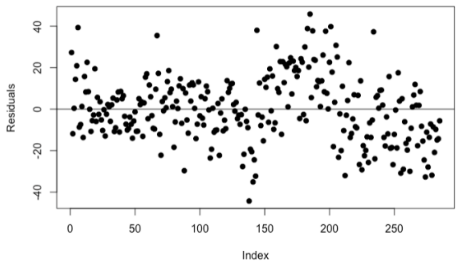
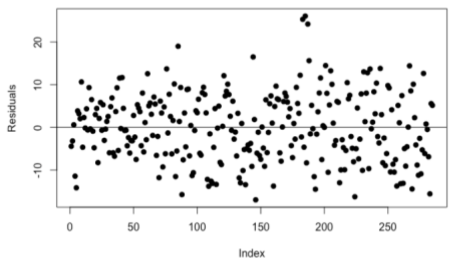
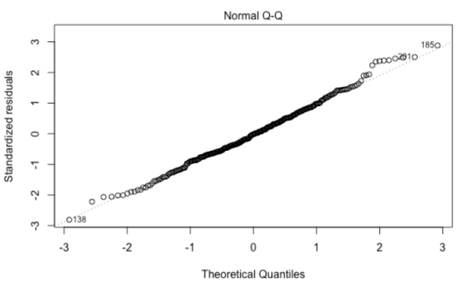
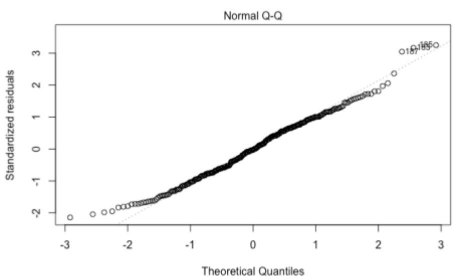
Additionally, the robustness of the regression is tested by performing K-fold cross-validation. The results conveyed a higher coefficient of determination R^2 of 84.80% compared to the original model of 40.84%. Also, the RMSE and MAE resulted in lower values from the original model (see Table 6).

Table 6. Comparison of *K*-fold cross-validation results.

	Original model	**Modified model**
Root Mean Square Error (RMSE)	16.09	8.19
R^2	40.84%	84.80%
Mean Absolute Error (MAE)	12.85	6.73

Finally, the multiple linear regression model assumptions were tested and compared to the original model. Table 7 shows that by adding the Mathematical Reasoning Dimensions into

Table 7. Graphical testing of multiple regression model assumptions for the original ($Y \sim X + M_1 + M_2$) and modified model ($Y \sim X + M_1 + M_2 + W_1 + W_2 + W_3$)

Assumptions	**Original Model**	**Modified Model**
Linearity Homoscedasticity		
Independence		
Normality		

the model, the assumptions of homoscedasticity and independence of error are met. This result leads us to conclude that the modified model has significantly lower distortions on the findings, higher stability in the results, and a higher ability to predict posttest scores.

4. Discussion

The analysis undertaken through the assumptions' testing showed that the original model developed by Mukuka et al. (2021) has some critical inconsistencies since the model does not meet the homoscedasticity and independent error assumptions nor has a trustworthy ability to predict posttest scores.

Through these findings, the initial mediation analysis was modified so that the Mathematical Reasoning Dimensions are included in the model, which implied new findings regarding the study:

1. Enhancing conjecturing, justifying, and mathematizing skills in students is the most influential factor that affects their mathematical reasoning abilities.
2. When the assumptions are met, there is higher stability in the predicted results.
3. The modified model has a higher coefficient of determination and prediction while decreasing RMSE and MAE. Hence, it has a higher ability to predict posttest scores.

It is essential to mention that even though posttest scores seem highly affected by the Mathematical Reasoning Dimensions, they are not directly related. According to a study developed by Mukuka et al. (2020a), even if the students had correctly answered a question, most of them were not able to justify their answers neither by deductive nor inductive reasoning. Inductive reasoning was based on citing numerical values to expressions or giving examples of numbers that can satisfy a given expression, and deductive reasoning was based on logical deductions to arrive at a valid generalization of a given algebraic statement or argument (Mukuka et al., 2020a). For example, in question number 4, 79% of the students answered correctly, yet only 14% of those properly justified their answer. The other 86% had misconceptions about the topic or did not correctly reference the statement (Mukuka et al., 2020a). These results mean that although a student may have a higher test score, it does not necessarily mean that they have correctly developed the conjecturing, justifying, and mathematizing skills.

5. Conclusions

This research established a high focus on the mediation analysis adequacy through meeting the multiple linear assumptions. The initial mediation analysis was modified so that the students' Mathematical Reasoning Dimensions adequacy (conjecturing, justifying, and mathematizing) were included as variables that directly influence the post-test score. The findings of this study revealed that the inclusion of such factors increases R^2 while decreasing MAE and MSE. These results mean that teachers must focus in improving students' conjecturing, justifying, and mathematizing skills while enhancing the application of the STAD model of cooperative learning, so that the students have a complete mathematical reasoning formation. In addition, students will have a better probability of attaining a higher score in the final test.

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