NP prevedbe

- prevedbe NP problemov

Cook-Levine:

katerikoli NP problem lahko rešimo v polinom. času, če lahko SAT v $O(p(n))$.

Zahtevnost problemov

težji

D

H. obhod SAT

H. obhod, SAT

NP : $O(p(n))$

DP

$O(p(n))$

Eulerjev obhod

urejanje $O(n \cdot \log n)$

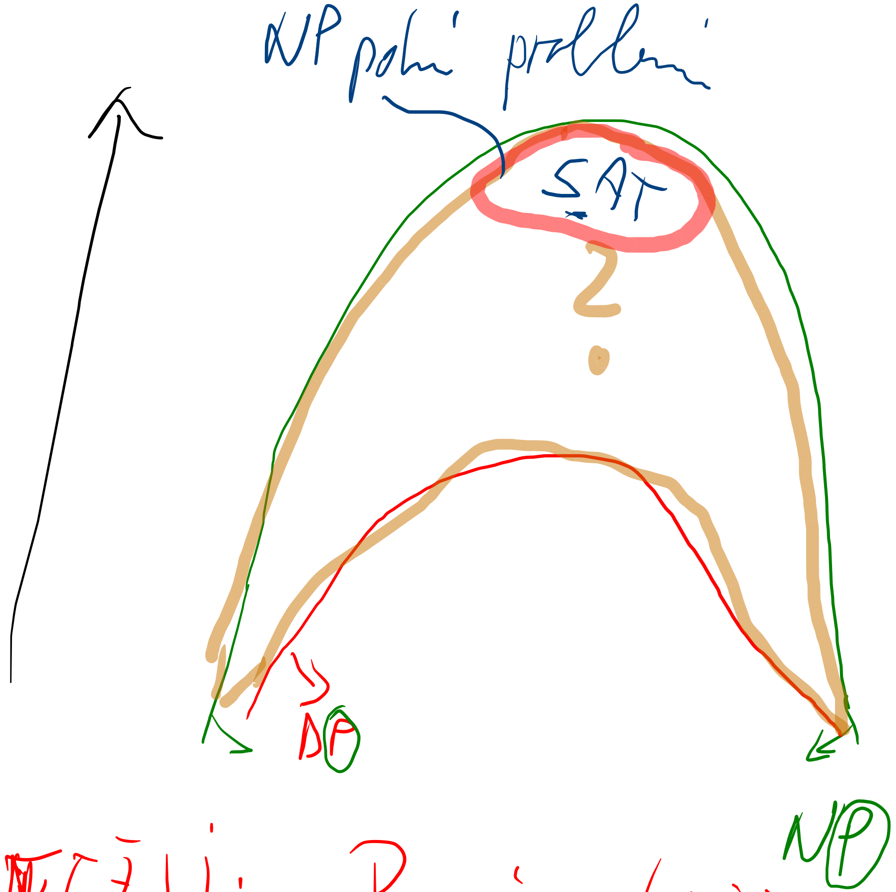
min $O(n)$

najdi $O(\log n)$

manj

učinkovito ≡ v $O(p(n))$ času.

$\boxed{O(\log n)}$



Ali RDEČA in ZELENA SOVP.?

– POIŠČEMO NAJTEŽJI RJAV PROBLEM IN GA POSKUSIMO REŠITI

TEŽJI: P_A je težji od P_B, če rešitev P_A omogoča učinkovito rešitev P_B.

$\boxed{\text{NP}}$ $\boxed{\text{PREVEDBA}}$

P_A – želimo pokazati, da je med najtežjimi.

1) MORAMO pokazati: $P_A \in NP$.

2) Želimo pokazati, da je vsaj tako težek kot katerikoli NP-poln problem.

- NP-poln: SAT

$\boxed{S_A}$

2) Želimo pokazati, da je vsaj tako težek kot katerikoli NP-poln problem.

- Vzamemo $S_{NP} \in$ NP-poln in pokažemo, da je S_A vsaj tako težek kot S_{NP}

• Tehnično:

$S_A \rightarrow S_{NP}$

- Vzamemo problem S_{NP} in naredimo algoritem, ki reši S_{NP} tako, da za reševanje uporabi S_A kot podprogram.

$3SAT \in NP$-počin

SAT:

$$\Delta = \bigwedge_{i=1}^{m} \Delta_i$$

$$\Delta_i = \bigvee_{j=1}^{k_i} x_{ij}^{(p_{ij})}$$

$\boxed{3SAT}$

$$\Delta_i = \bigvee_{j=1}^{3} x_{ij}^{(p_{ij})}$$

$$\Delta = (x_1) \wedge (x_2 \vee \overline{x_1}) \wedge (\overline{x_1} \vee \overline{x_2} \vee x_3) \wedge (\overline{x_2} \vee x_3 \; x_1 \vee x_4)$$

$$\overline{x} = [1, 1, 1, 0]$$

$$|x| = n$$

NP PREVEDBA

$P_{\cancel{A}}^{3SAT}$ želimo pokazati, da je najtežji med njimi.

1) MORAMO pokazati: $P_{\cancel{A}}^{3SAT} \in NP$.

· Ned. program:

Nedeterministično izpiše $\vec{x}$, ki je certifikat.

$O(n)$

Preverjanje certifikata: $O(m \cdot 3)$ velikost problema $= \underline{|S|}$

$\sim O(|\text{velikost problema}|)$

- Vzamemo problem P_{NP}^{SAT} in naredimo algoritem, ki reši P_{NP}^{SAT} tako, da za reševanje uporabi P_{A}^{3SAT} kot podprogram.

RešiSAT(s)

$O(p(б))$ $\quad s' = \text{PretvoriSATv3SAT}(s)$ (1)

$O(p(б))$ $\quad y = \text{Reši3SAT}(s')$

$O(p(б))$ $\quad x = \text{Izluščix}(y)$ (3)

$O(p(б))$ $\quad$ Return x

$|s| = б$

$|s'| = p(б)^2$

$O(p(\sigma))$ $s' = \text{Pretvori SAT v 3SAT}(s)$ (1)

$$s = \bigwedge_{i=m}^{m} s_i = \bigwedge_{i=1}^{m_1} s_{i_1} \cdot \bigwedge_{i=1}^{m_2} s_{i_2} \cdot \bigwedge_{i=1}^{m_3} s_{i_3} \cdot \bigwedge_{i=1}^{m_4} s_{i_4}$$

$$s_{i_3} = \bigvee_{j=1}^{3} x^{(i_3 j)}_{i_3 j} = s'_{i_3}$$

$$m_1 + m_2 + m_3 + m_4 = m$$

$$s' = \bigwedge s'_{i_1} \cdot \bigwedge s'_{i_2} \cdot \bigwedge s'_{i_3} \cdot \bigwedge s'_{i_4}$$

$$|s'_{i_3}| = |s_{i_3}|$$

$O(p(\sigma))$

$s' = \text{Pretvori SAT v 3SAT}(s)$

(1)

$$s = \bigwedge_{i=m}^{m} s_i = \bigwedge_{i=1}^{m_1} s_{i_1} \cdot \bigwedge_{i=1}^{m_2} s_{i_2} \cdot \bigwedge_{i=1}^{m_3} s_{i_3} \cdot \bigwedge_{i=1}^{m_4} s_{i_4}$$

$$s_{i_2} = \left(x_{i_2,1}^{(i_2,1)} \vee x_{i_2,2}^{(i_2,2)}\right) \equiv \left(x_1^{p_1} \vee x_2^{p_2}\right) \quad \boxed{|s'_{i_2}| = 2|s_{i_2}|}$$

$$s'_{i_2} = \left(x_1^{p_1} \vee x_2^{p_2} \vee y\right) \wedge \left(x_1^{p_1} \vee x_2^{p_2} \vee \bar{y}\right)$$

$$s' = \bigwedge s'_{i_1} \cdot \bigwedge s'_{i_2} \cdot \bigwedge s'_{i_3} \cdot \bigwedge s'_{i_4}$$

$O(p(\sigma))$ $\quad \underline{\varphi}' = \text{Pretvori SAT v 3SAT}(\varphi)$

(1)

$$\varphi = \bigwedge_{i=1}^{m} \varphi_i = \bigwedge_{i=1}^{m_1} \varphi_{i_1} \cdot \bigwedge_{i=1}^{m_2} \varphi_{i_2} \cdot \bigwedge_{i=1}^{m_3} \varphi_{i_3} \cdot \bigwedge_{i=1}^{m_4} \varphi_{i_4}$$

$$\varphi_{i_1} = x_{i_1} \equiv X \rightsquigarrow (X \vee \_ \vee \_) \rightsquigarrow$$

$$(x \vee y_1 \vee y_2) \wedge \ldots \wedge (x \vee y_1 \vee y_2) \rightsquigarrow (X \vee \_) \rightsquigarrow (Y \_ \cancel{x})$$

$$\varphi' = \bigwedge \varphi'_{i_1} \cdot \bigwedge \varphi'_{i_2} \cdot \bigwedge \varphi'_{i_3} \cdot \bigwedge \varphi'_{i_4}$$

$$|\varphi'_{i_1}| = 4 \cdot |\varphi_{i_1}|$$

$O(p(\sigma))$ $\quad \underline{s'} = \text{PretvoriSATv3SAT}(s)$ (1)

$$s = \bigwedge_{i=m}^{m} s_i = \bigwedge_{i=1}^{m_1} \underline{s_{i_1}} \cdot \bigwedge_{i=1}^{m_2} \underline{s_{i_2}} \cdot \bigwedge_{i=1}^{m_3} \underline{s_{i_3}} \cdot \bigwedge_{i=1}^{m_4} \underline{s_{i_4}}$$

$$\underline{s_{i_4}} = (x_1 \vee x_2 \cdots x_{k-2} \vee x_{k-1} \vee x_{\underline{k}}) \qquad \underline{k > 3}$$

$$= (x_1 \vee x_2 \cdots \vee x_{k-2} \vee y) \wedge (\bar{y} \vee x_{k-1} \vee x_k)$$

$y \Rightarrow$ TRUE

$$|s_{i_4}| = 3 \cdot (k-3) \cdot |s_{i_4}| = O(p(\sigma))$$

y-vsem

NP-polni: SAT ⇝ 3SAT, MILP, KLIKA, SUBSET SUM, RAZDELITEV, ...

ODLOČITVENI IN OPT. PROBLEMI:

- $3SAT(s)$ ~ odločitveni T/F
- $MAX\text{-}3SAT(s)$: m = število stavkov v s

$MAX\text{-}3SAT(s) = m \iff 3SAT(s) = TRUE$

- $k\text{-}3SAT(s,k)$ – ali obstaja $\vec{x}$, da je k stavkov hkrati T

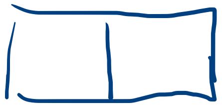
$k\text{-}3SAT(s,m) \equiv 3SAT(s)$

Rodovne funkcije

- Rodovne funkcije

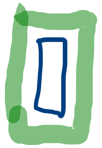
Reference:

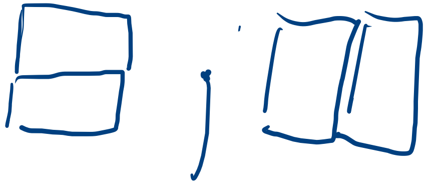
uporabljamo za tvorjenje vseh
možnih primerov problem

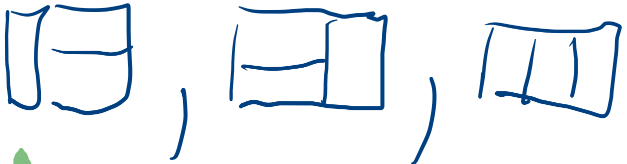
Imamo domine: 

- postavimo 

• Imamo trak višine 2 in dolžine n. Na koliko načinov lahko postavimo domine na trak?

2x1:  - 1 način

2x2:  - 2 načina

2x3:  - 3 načini

2x0: - 1 način

$$T = | + \square + \boxed{\,|\,} + \boxminus + \dots \qquad \to \infty$$

$\oplus \sim$ unija $\quad +$

stik $\sim \quad \times$

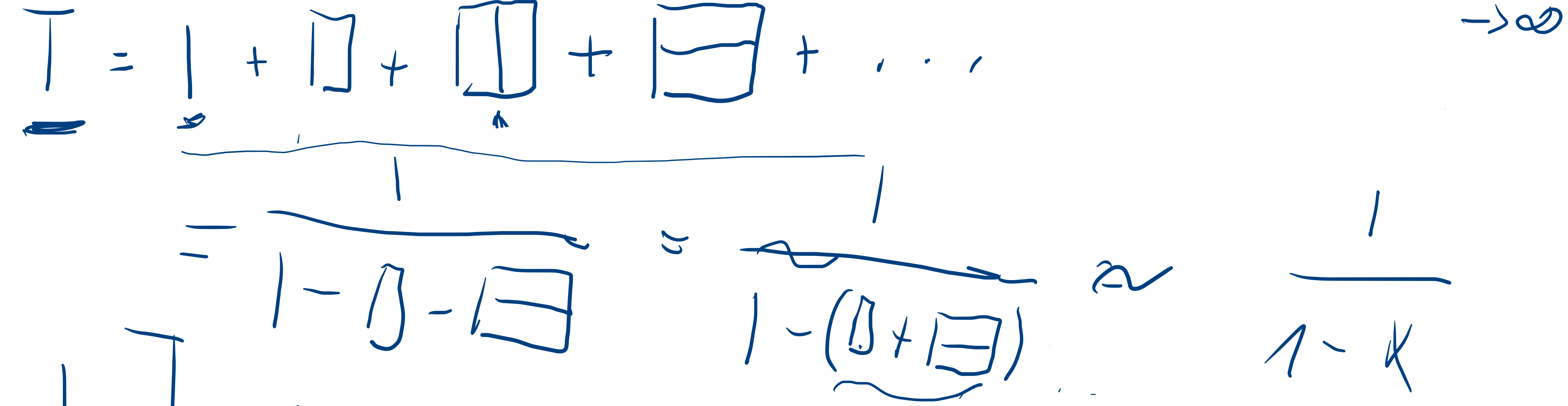
$$= | + \square\left(| + \square + \dots\right) + \boxminus\left(| + \square + \dots\right)$$

$$= | + \square \cdot T + \boxminus \cdot T$$

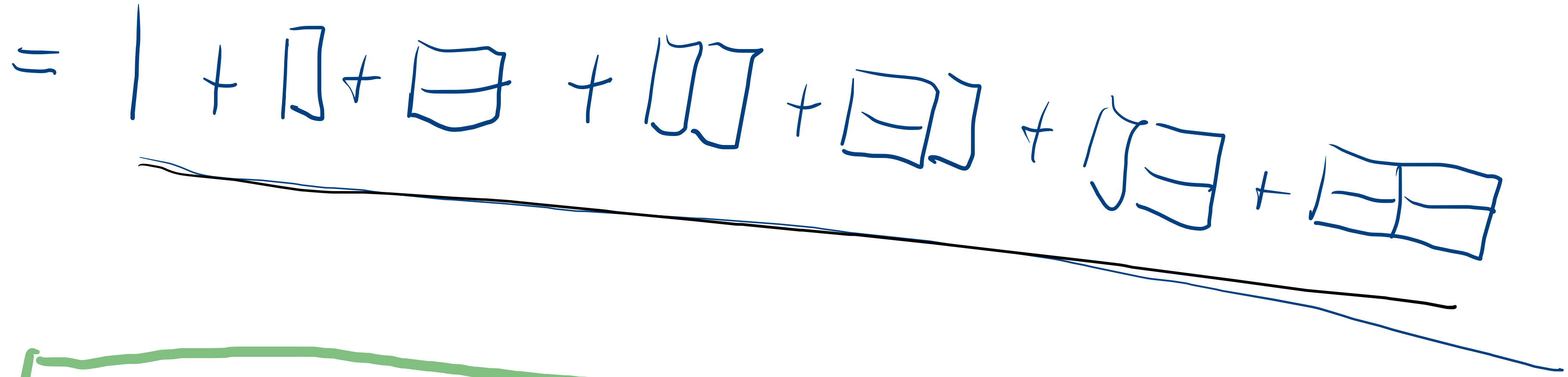
$$1 = T \cdot \left(1 - \square - \boxminus\right)$$

$$T = \frac{1}{1 - \square - \boxminus} = \frac{1}{1 - (\square + \boxminus)} \approx \frac{1}{1 - \cancel{x}}$$

$$\frac{1}{1 - \cancel{x}} = 1 + x + x^2 + x^3 + \dots$$



$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots$$



VPR 1: Koliko konfiguracij je na traku $2 \times n$, kjer imamo j vodoravnih domin in m ▭^k.

n-domin

$\text{▯} = \text{▯}^1$; $\text{⊟} = \text{▭}^2$

$$T = \frac{1}{1-(\text{▯}+\text{⊟})} = \frac{1}{1-(\text{▯}^1+\text{▭}^2)}$$

$$= \sum_{i=0}^{\infty} \underbrace{(\text{▯}^1+\text{▭}^2)}^{i} = \sum_{i=0}^{\infty} \left(\sum_{k=0}^{i} \binom{i}{k} \cdot \text{▯}^k \cdot \text{▭}^{2(i-k)} \right)$$

$$= \sum_{i,k \geq 0} \binom{i}{k} \cdot \text{▯}^k \, \text{▭}^{2(i-k)} \quad \Big/ \; m = i-k$$

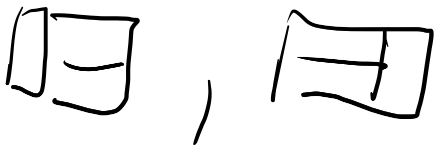
VPR1: Koliko konfiguracij je na traku $2 \times n$, kjer imamo j vodoravnih domin in ▯^{k} ▭.

$$= \sum_{i,k \geq 0} \binom{i}{k} \cdot ▯^{k} \, ▭^{2(i-k)} \quad \Big| \; m = i-k$$

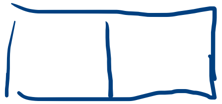
$$= \sum_{k,m \geq 0} \binom{m+k}{k} ▯^{k} \, ▭^{2m}$$

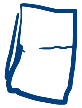
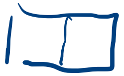
To so tiste položitve, ko $\boxed{k + 2m = n}$

2×3: 1▯, 2▭ :



2×10: 4▯ 6▭ : 35

Imamo domine: 

- postavimo  

- Imamo trak višine 2 in dolžine n. Na koliko načinov lahko postavimo domine na trak?

$$T = \frac{1}{1 - (\square + \square)}$$

$\square = z$

$\square = z$

$$= \frac{1}{1 - z - z^2} \Rightarrow$$

Rodovna f. za Fib. števila

$F_0 = 0 \quad F_1 = 1$

$F_{i+2} = F_{i+1} + F_i$

$F = $ ~~~~

$$F(z) = \sum_{i=0}^{\infty} F_i z^i = 0 + 1z + 1z^2 + 2z^3 + \dots + F_n z^n + \dots$$

$$\begin{aligned}
&\cdot & F(z) &= F_0 + F_1 z + F_2 z^2 + F_3 z^3 + F_4 z^4 + \dots \\
&- & z \cdot F(z) &= F_0 \cdot z + F_1 z^2 + F_2 z^3 + F_3 \cdot z^4 + \dots \\
&- & z^2 F(z) &= F_0 z^2 + F_1 z^3 + F_2 z^4 + \dots
\end{aligned}$$

$$F(z)(1 - z - z^2) = F_0 + (F_1 - F_0) z$$

$$F(z) = \frac{z}{1 - z - z^2}$$

$$F(z) = \frac{z}{1 - z - z^2} = \frac{A}{1 - \alpha z} + \frac{B}{1 - \beta z}$$

$$A - A\beta z + B - B\alpha z = z \Rightarrow$$

$$\Rightarrow \bullet \; A + B = 0$$

$$\bullet \; A\beta + B\alpha = 1$$

$$1 + \alpha\beta z^2 - (\alpha + \beta) z = 1 - z - z^2 \Rightarrow$$

$$\bullet \; \alpha\beta = \pm 1$$

$$\bullet \; \alpha + \beta = \pm 1$$

$$\boxed{F(z) = \frac{z}{1-z-z^2}} = \frac{A}{1-\alpha z} + \frac{B}{1-\beta z}$$

$$= \frac{1}{\sqrt{5}} \cdot \frac{1}{1-\phi z} - \frac{1}{\sqrt{5}} \cdot \frac{1}{1-\hat{\phi} z}$$

$$\phi = \frac{1+\sqrt{5}}{2} \qquad \hat{\phi} = \frac{1-\sqrt{5}}{2}$$

$$= \frac{1}{\sqrt{5}} \left(\sum_{i=0} \left(\phi^i z^i - \hat{\phi}^i z^i \right) \right) = \boxed{\frac{1}{\sqrt{5}} \cdot \sum_i \left(\phi^i - \hat{\phi}^i \right) z^i}$$

Takže je

$$F_n = \left(\underline{\phi^n} - \hat{\phi}^n\right) \cdot \frac{1}{\sqrt{5}}$$

$$\boxed{\phi \doteq 1{,}6} \Rightarrow F_n \text{ (v čase } \boxed{O(\log n)}\text{)}$$

$$\hat{\phi}^n = 0$$

$n \to \infty$

$$F(z) = \overset{F_c}{\frac{z}{1-z-z^2}} = \overset{F}{\sum_{i=0}^{\infty} F_i z^i}$$

$$F(z) \Rightarrow [z^{20}]F(z) \to F_{20}$$

Fib.: $F(z) = \dfrac{z}{1-z-z^2} = \sum_{i=0}^{\infty} F_i z^i$

Dom.: $T(z) = \dfrac{1}{1-z-z^2} = \sum_{i=0}^{\infty} T_i z^i$

imamo trake $2 \times n$ $\Rightarrow$ imamo n-domin: $\underline{\underline{[z^n]T(z)}}$

$F(z) = T(z) \cdot z$

Fib.: $F(z) = \frac{z}{1-z-z^2} = \sum_{i=0}^{\infty} F_i z^i$

Dom.: $T(z) = \frac{1}{1-z-z^2} = \sum_{i=0}^{\infty} T_i z^i$

$$F(z) = T(z) \cdot z$$

$$\sum_{i=0}^{\infty} F_i z^i = z \cdot \sum_{i=0}^{\infty} T_i z^i = \sum_{i=0}^{\infty} T_i \cdot z^{i+1} = \sum_{i=1}^{\infty} T_{i-1} z^i$$

$$\Rightarrow \quad F_i = T_{i-1} \Rightarrow \boxed{T_i = F_{i+1}}$$

$T_i = F_{i+1}$;

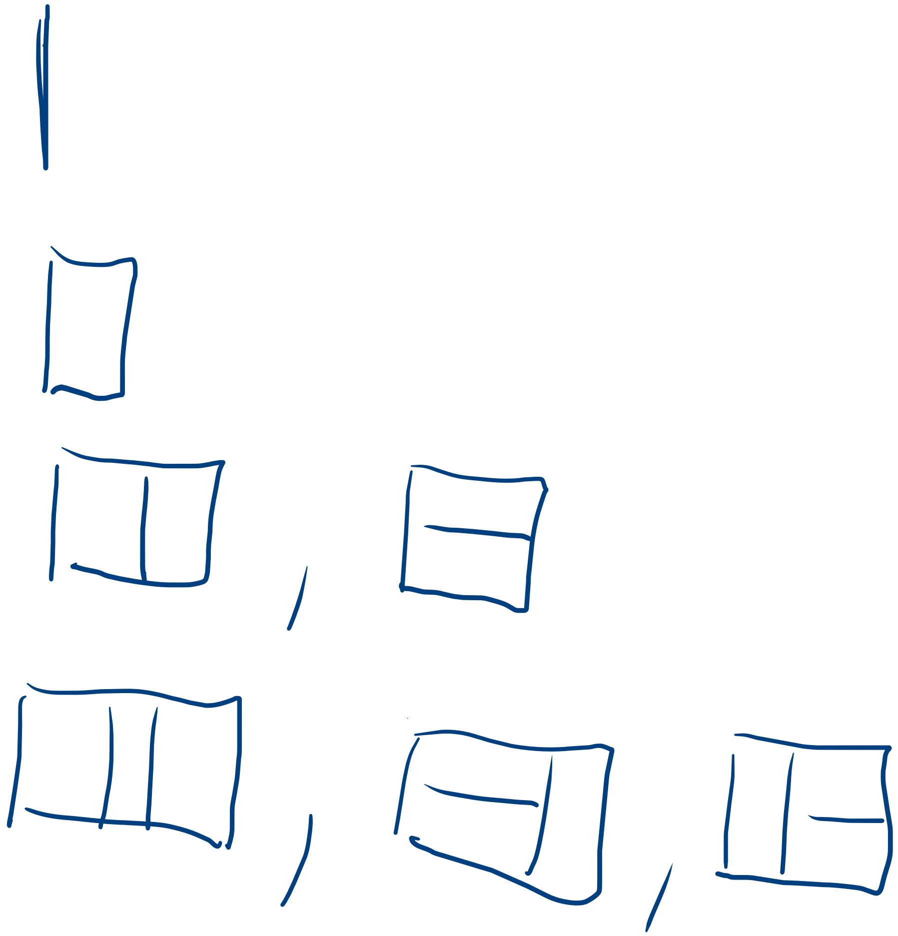
0 - 1

1 - 1

2 - 2

3 - 3

⋮



$$\left[R^{n} \right] T(R) \longrightarrow F_{n+1}$$