AER 1318: Assignment 3 Multistep and Multigrid code

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1 Subsonic problem

1.1 Replicate figure 5.18 (textbook)

The following figure replicates Figure 5.18 in the textbook. The figure shows the convergence history for three cases: Blue line) Multistep solution with $\beta = 0.6$ and Courant number of $C = 3$, Red line) Multistep solution with $\beta = 0.6$ and Courant number of $C = 7$, and green line) Four grid W-cycle multigrid solution with $\beta = 0.6$ and Courant number of $C = 7$. As observed, the multigrid solution reduces dramatically the number of iterations required for convergence compared to the multistep solutions.

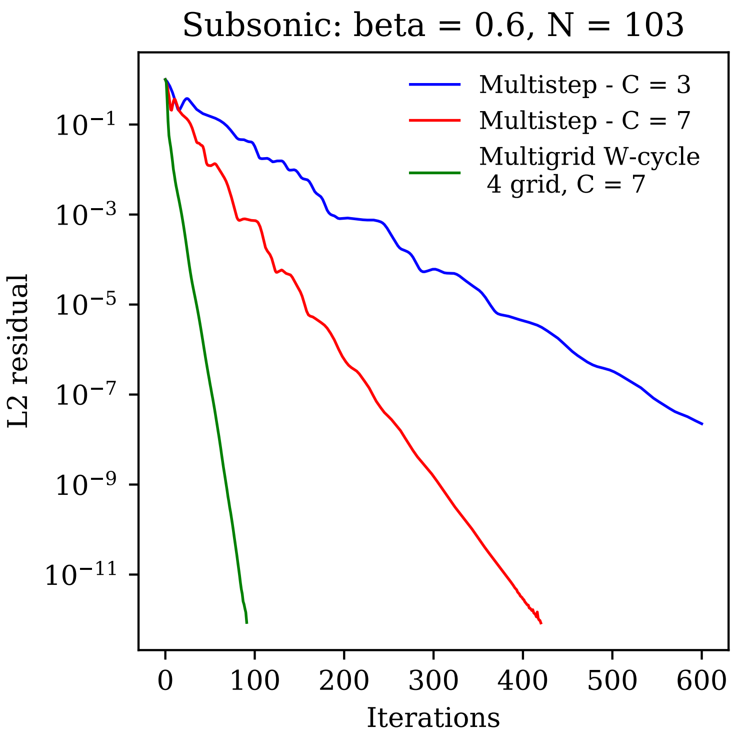


Figure 1: Replicate figure 5.18 from the textbook

1.2 Comparison W-cycle VS V-cycle

The figure below illustrates the performance of W-cycle and V-cycle multigrids. As observed, W-cycle multigrids require fewer iterations compared with the V-cycle multigrids. Different grid levels have been tested. The W-cycle that requires fewer iterations is the cycle that uses four grids in the coarsest mesh, five in the intermediate mesh, and six grids in the finest mesh.

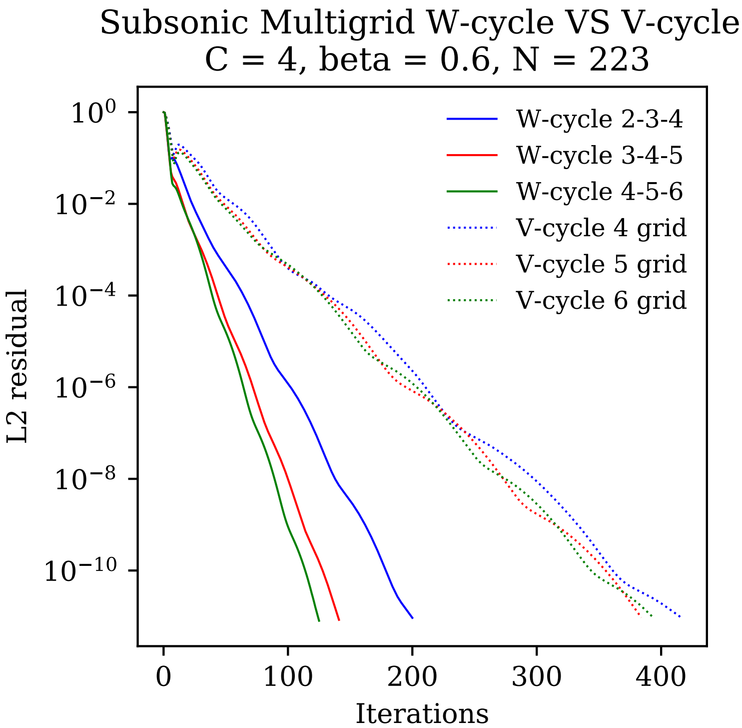


Figure 2: Performance of W-cycle compared with V-cycle multigrid

1.3 Effect of the Courant number

The following figure shows the effect of the Courant number on the solution of the subsonic problem. The effect of the Courant number has been evaluated for two types of W-cycle multigrids. The first cycle (solid line) consisted of a W-cycle multigrid that uses 2 grids in the coarsest mesh, 3 grids in the intermediate mesh, and 4 grids in the finest mesh. The second cycle (dots) consisted of a W-cycle multigrid that uses 3 grids in the coarsest mesh, 4 grids in the intermediate mesh, and 5 grids in the finest mesh. For both types of W-cycle, it can be observed that a larger Courant number accelerates the solution and reduces the number of iterations or multigrid cycles. A Courant number of $C = 7$ could be considered as optimal since it requires the fewest number of iterations compared with the other two values of Courant number ($C = 1$ and $C = 4$)

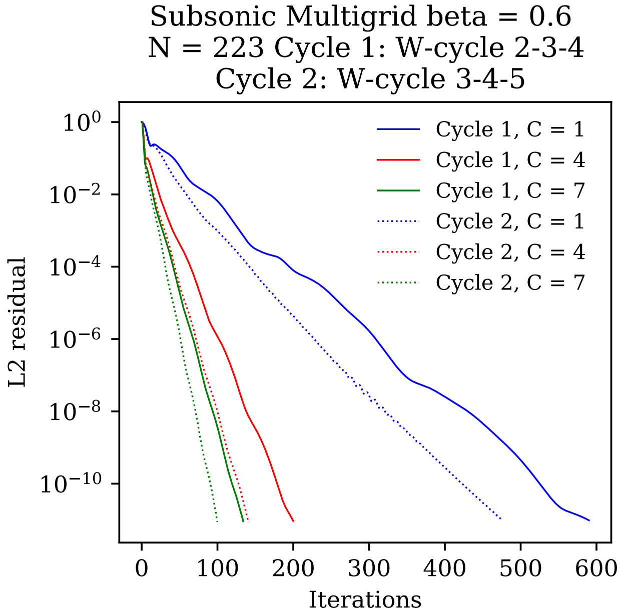


Figure 3: Effect of the Courant number on the solution of the subsonic channel

1.4 Effect of the smoothing factor

The figure below presents the effect of the smoothing factor β on the solution of the subsonic problem. Three values of smoothing factors were evaluated: $\beta = 0.5$, $\beta = 0.6$, and $\beta = 0.7$. As observed, the smoothing factor of $\beta = 0.5$ increases notably the number of multigrid cycles (iterations). While the smoothing factors of $\beta = 0.6$ and $\beta = 0.7$ provides almost the same performance in terms of the number of convergence and required multigrid cycles. There is no effect of increasing the smoothing factor respect to the recommended value of $\beta = 0.6$.

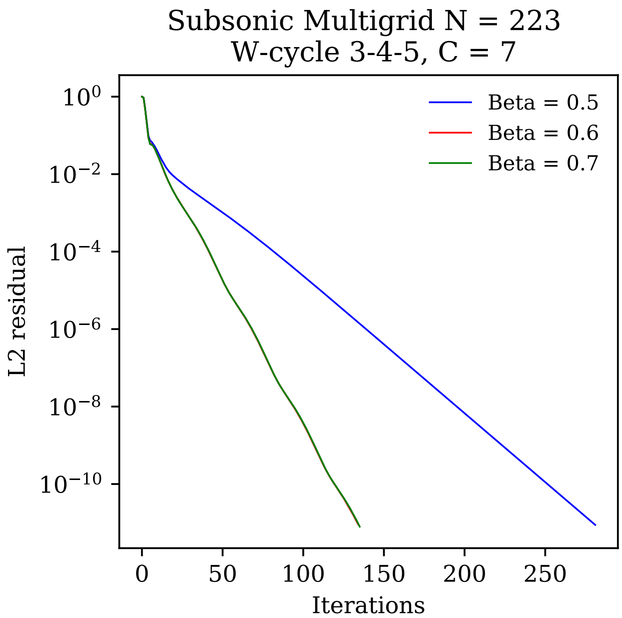
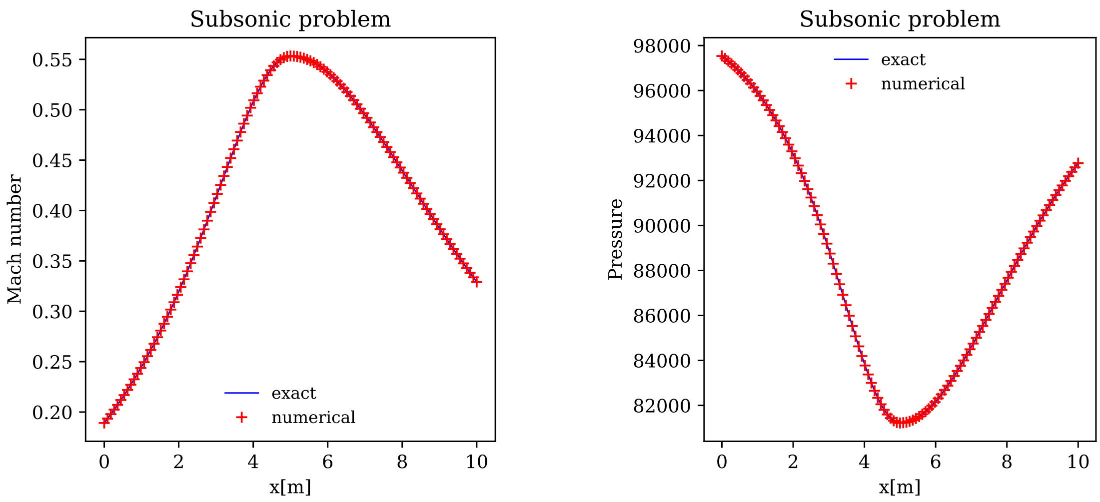


Figure 4: Effect of β on the solution of the subsonic channel



(a) Mach number

(b) Pressure

Figure 5: Mach number and pressure for the subsonic channel

2 Transonic problem

2.1 Replicate figure 5.21 from the textbook

The figure below replicates the results presented in the figure 5.21 from the textbook. The figure shows the convergence history for three cases: Blue line) Multistep solution with $\beta = 0.6$ and Courant number of $C = 3$, Red line) Multistep solution with $\beta = 0.6$ and Courant number of $C = 7$, and green line) Four grid W-cycle multigrid solution with $\beta = 0.6$ and Courant number of $C = 7$. As observed, the multigrid solution reduces dramatically the number of iterations required for convergence compared to the multistep solutions.

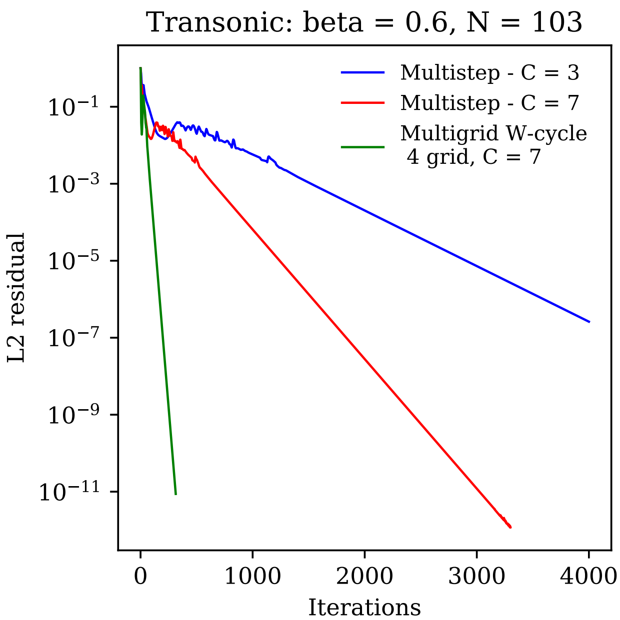


Figure 6: Replicate figure 5.18 from the textbook

2.2 Comparison W-cycle VS V-cycle

For the transonic problem, a similar trend was found respect to the difference between W-cycle and V-cycle multigrids. As observed, W-cycles reduces dramatically the number of iterations. For the transonic case is more evident the difference since the W-cycle multigrid reduces the number of iterations by a factor of about two. In addition, the convergence trend for the V-cycle multigrid shows an oscillatory behaviour, especially in the first iterations 500 iterations, which was not observed for the subsonic problem.

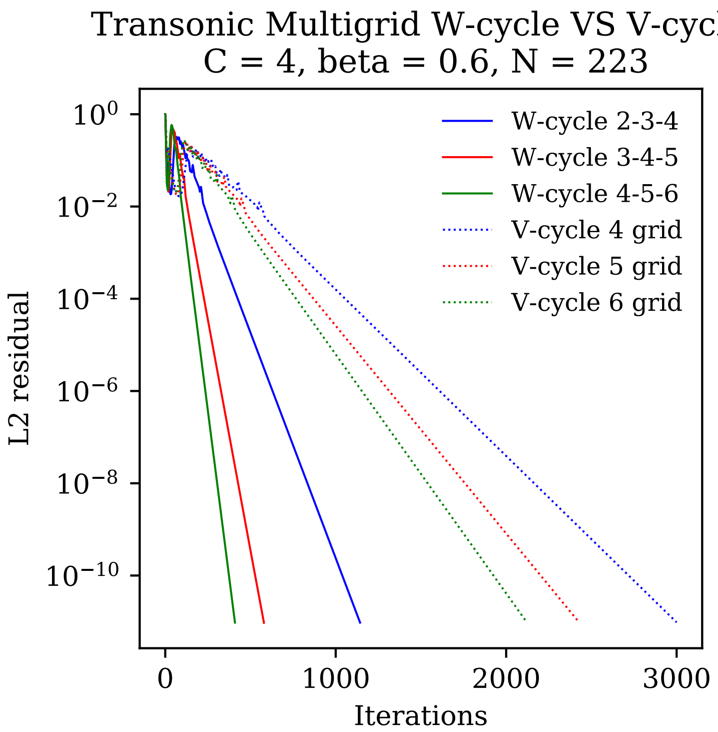


Figure 7: Performance of W-cycle compared with V-cycle multigrid

2.3 Effect of the Courant number

Respect to the Courant number, it can appreciated in Figure 8, a trend similar to the one found in the analysis of the convergence dependency on the Courant number, performed for the subsonic problem. From the performed analysis, it can be concluded that a Courant number of $C = 7$ is optimal, since it requires the fewest number of iterations compared with the other two Courant number values.

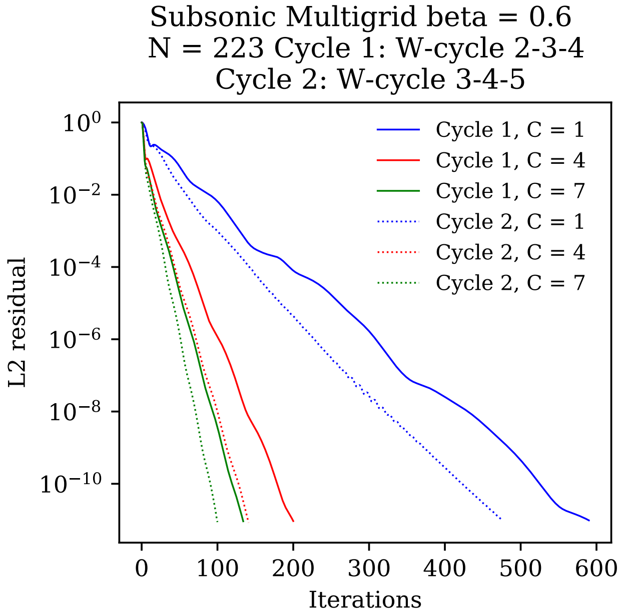


Figure 8: Effect of the Courant number on the solution of the transonic problem

2.4 Effect of the smoothing factor

Respect to the smoothing factor, it was found a different trend compared with the one found for the subsonic problem. In Figure 9, it can be appreciated that the difference in the convergence trends when using different smoothing factors is minimal, in contrast with the subsonic problem when it was observed that a smoothing factor of $\beta = 0.6$ was the optimal since it required a lower number of iterations.

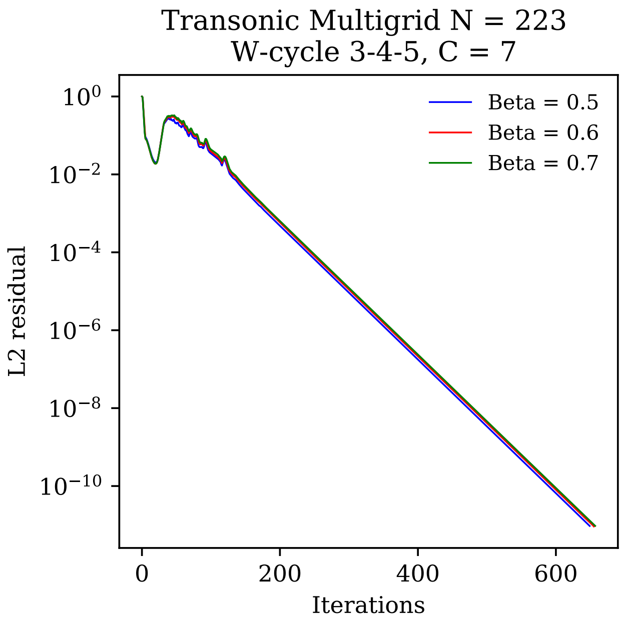
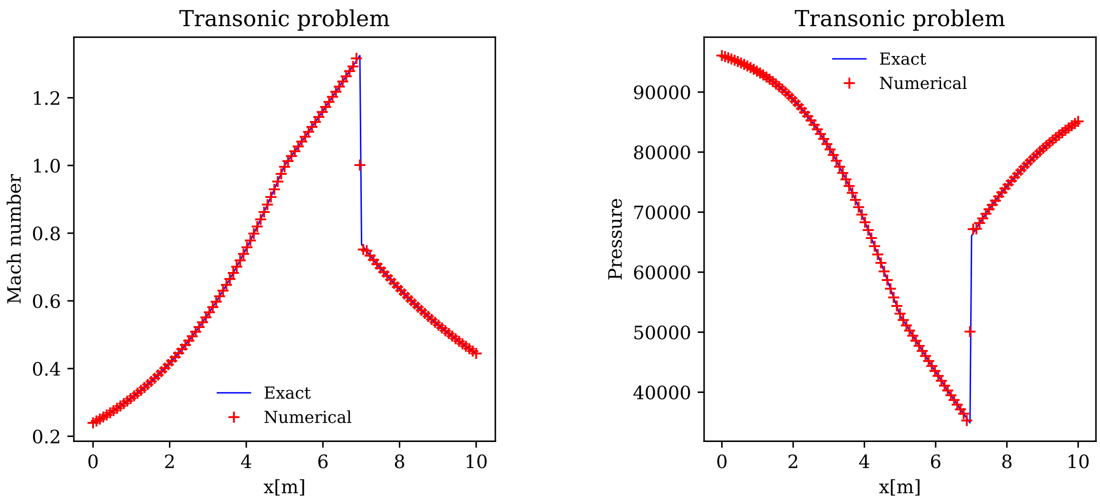


Figure 9: Effect of β on the solution of the transonic problem



(a) Mach number

(b) Pressure

Figure 10: Mach number and pressure for the transonic problem