

# Damage detection using transmissibility compressed by principal component analysis enhanced with distance measure

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## Abstract

Detecting structural damage in operational conditions still encounters some difficulties, especially in early-stage, as environmental varieties impose challenges in real engineering applications and may require large computational efforts in the structural health monitoring and potential maintenance. Unlike conventional strategies employing frequency response function or response data, a damage detection methodology is addressed in this study by employing transmissibility functions that retains a strong interrelation with structural damage or deterioration, in order to avoid the measurement of excitation, together with principal component analysis that leads to reduction in computational costs. In this procedure, transmissibility is extracted from the structural responses and main features are selected by principal component analysis for less computational costs. Then, via distance measures damage indicators are constructed for both intact and damaged states, and finally a numerical simulation with a clamped-clamped beam and a four-story benchmark are adopted to verify the applicability of the proposed procedure. The results demonstrate a good performance in structural damage detection.

## Keywords

Damage detection, transmissibility, principal component analysis, distance measure, structural health monitoring

## 1. Background and motivation

Intensive investigations have been carried out during the last decades for structural health monitoring (SHM). SHM is a process of implementing a damage detection strategy for anticipated deterioration/defects in mechanical and civil engineering structures, often manufactured with highly integrated advanced technologies and generally subjected to higher and complex loadings on a daily basis, for which longer lifecycle periods are always pursued. Previous studies started with visual inspection, penetrating liquids, etc., followed by static and dynamic (vibration-based) methods, after the advent of digital computers in the early 70's, with which signal processing could be analyzed conveniently and in a fast way. From there on, a large quantity of SHM research output has been extensively published. SHM approaches have been progressed not only with time-series data, but also with frequency-domain data. Vibration-based methods are commonly applied in mechanical engineering, e.g. chain-like systems, and civil engineering, e.g. bridges (Sohn et al., 2004; Maeck et al., 2000; Montalvão et al., 2006).

SHM methodologies can be divided into two main categories: physical model and data model/statistical model. For a physical model, normally a finite element analysis (FEA) is undertaken and different levels and patterns, such as fatigue in adhesively bonded joints (Wahab et al., 2001), crack initiation (Hojjati-Talemi et al., 2014), fretting wear (Ferjaoui et al., 2015), are

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numerically analyzed in order to provide a pre-design assessment as a reference for further analysis, especially in fatigue life-cycle prediction (Hojjati-Talemi et al., 2014). Generally, FEA analysis is validated with experimental results and model updating intends to minimize the differences between FEA and experimental responses (Wahab et al., 1999, 2001; Wahab, 2001) by optimizing the FEA model. In this direction, low cost but relatively fine performance in analyzing actual engineering problems fostered other numerical techniques development, e.g. boundary element methods (BEM), mesh free approaches, extended finite element methods (XFEM), isogeometric analysis (IGA) (Cottrell et al., 2009). Drawbacks to that direction, however, are clear. Firstly, the precise comprehension of the investigated object should be made a priori, while in realistic structures this is arduous or impractical, particularly in some engineering applications, such as dams and wind turbines, which require more collaborations for acquiring the physical model. Secondly, the physical model methodology requires the excessive computational expenses in actual engineering applications, as the elements of the studied objects are commonly excessively extensive, for instance, aerospace crafts finite element model may require millions of elements. These disadvantages restrict the wide and profound applicability of the physical model.

On the other hand, data/statistical models have been raised decades ago for overcoming the disadvantages of physical models-based methodologies, to simplifying the analysis. For a data model/statistical model, the SHM problem is described as a pattern recognition paradigm consisting of a four-part process: (1) Operational evaluation; (2) Data acquisition, fusion, and cleansing; (3) Feature extraction and information condensation and (4) Statistical mode development for feature discrimination (Sohn et al., 2004) and the principal issues are feature extraction and feature discrimination. From this starting point, statistical techniques such as Artificial Neural Networks (ANNs), PCA and so on have been introduced into SHM (Figueiredo et al., 2009; Farrar and Worden, 2013).

For feature extraction, obtaining modal parameters from experimental modal analysis (EMA) still occupies a significant role in view of their low cost in experimental testing. Nevertheless, damage identification under real operating conditions of the structure during its daily utilization is not an easy task, due to the difficulties and troubles of drawing out controlled forced excitation tests on the structures. In this case, it would be preferable to have output-only response measurements and therefore an output-only damage identification procedure should be implemented. Within output-only SHM approaches, such as methodologies based on Operational Modal Analysis (OMA), the transmissibility

concept has been raised some years ago (by the end of the 20th century), as it depends only on the output signals. Booming research on transmissibility has been intensively developed in both mechanical and civil engineering applications for damage identification.

As to transmissibility, the team of Maia (Ribeiro et al., 1999, 2000b, 2005; Maia et al., 2001, 2006, 2011c, 2012; Fontul et al., 2004a, 2004b; Lage et al., 2014b; Zhou et al., 2015b) gave a systematic theory in the last years. Transmissibility has been firstly raised for avoiding the measurement of excitation. Ribeiro et al. (1999) addressed a methodology of extracting transmissibility from the measured responses only. The idea was based on the fact that transmissibilities related different groups of responses for a given number of applied forces. For multiple-degree-of-freedom systems the number of known responses needed to be at least the same as the number of input forces and/or moments (generalized forces) applied to the structure (Ribeiro et al., 2000b, 2005; Maia et al., 2001, 2006). Then, transmissibility in structural coupling and harmonic and random process were studied (Fontul et al., 2004a, 2004b). After few years, a thorough survey of whys and wherefores of transmissibility was given (Maia et al., 2011c), where the concept of transmissibility and its potentialities, virtues, limitations and possible applications were generally overviewed. Later on, transmissibility coherence was also introduced into the transmissibility theory (Zhou et al., 2015b). Apart from the theory development, the team of Maia also has extended the investigation of transmissibility to other applications, such as response prediction (Ribeiro et al., 2000a), force identification (Lage et al., 2010), FRF estimation (Urgueira et al., 2011), force reconstruction (Lage et al., 2014a), and damage identification (Sampaio et al., 1999, 2000, 2001; Maia et al., 2007, 2011a; Zhou et al., 2015a).

For transmissibility applications, the major part was concerned with damage identification. Three reviews can be taken as a starting point (Maia et al., 2011b,c; Chesne and Deraemaeker, 2013; Zhou, 2015). Maia et al. (2011b, 2011c) reviewed the transmissibility concept and application from its theory development aspect, while Chesne and Deraemaeker (2013) gave a critical review about the transmissibility application, especially a damage identification historical survey that recalled those investigations made by researchers from the starting point of Worden's work (Worden et al., 2000, 2003; Chen et al., 1994, 2003; Manson et al., 2003a, b; Worden, 1997). In those studies, auto-associative neural networks, novelties, and multi-layer perception (MLP) were used to detect and localize the damages. However, specific frequency band used in those methodologies restricted their general application, and a pseudo-faults via adding mass to simulate

damages in the actual structures was studied (Papatheou et al., 2010). Note that even this method proved to be efficient for some structures, while there was no solid proof for a general case of its application. Unlike using algorithms or novelties, the team of Maia tried to directly extract widely applicable damage indicator from the transmissibility (Sampaio et al., 1999, 2000, 2001; Maia et al., 2007, 2011a; Zhou et al., 2015a). Since transmissibility was sensitive to structural damage, the variation of the transmissibility between the intact and damaged states has been used as an indicator for detecting damage (Sampaio et al., 1999); such as the curvature of mode shapes and the second derivative of the transmissibility integrated over a specific frequency band, which was constructed as an indicator for localizing damages (Sampaio et al., 2000). A few years later, response vector assurance criterion (RVAC) and damage quantification indicator (DRQ) were put forward together with the transmissibility, in a similar way to the use of FRFs (Maia et al., 2007, 2011a). Later, the peak of Mahalanobis squared distance of transmissibility under damaged states compared to the intact one was taken as a damage feature (Zhou et al., 2015a, 2015b).

During the development of these cited outputs above, other researchers also did studies on transmissibility (in some papers it was described as transmittance, power spectral density transmissibility) for damage identification (Schulz et al., 1997; Brown and Adams, 2003; Kess and Adams, 2007; Zhou et al., 2012, 2014; Zhao et al., 2015). Schulz et al. (1997) employed the feature of the transmittance difference in amplitude between the healthy and in-service complex normalized form for determining structural damages. Brown and Adams (2003) addressed the transmissibility that relied on the damage center manifold for nonlinearity diagnosis in an analytical manner. Continuously, Kess and Adams (2007) estimated the operational and environmental variability effects using transmissibility and detected damages on a woven composite plate analytically. Zhou et al. (2012) and Zhou and Perera (2014) used power spectral density transmissibility to detect damages by using modal assurance criterion and artificial neural networks, respectively; Zhao et al. (2015) discussed damage detection and localization by using transmissibility-based nonlinear feature analytically. In addition to damage identification and the applications described above, transmissibility has also been extended to operational modal analysis (Steenackers et al., 2007; Yan and Ren, 2012) and model updating (Devriendt and Guillaume, 2008).

As cited above, although lots of transmissibility-based methodologies have been developed for damage identification, accuracy in detecting, localizing and quantifying structural damages is still an open problem to the scientific community, especially in the earliest

stages of damage identification. Several problems still arise in damage identification upon transmissibility. Firstly, how to select the effective transmissibility, as in a complex structure, the total transmissibility will occupy a big storage and not all of the transmissibility terms have the same sensitivity towards potential damage or deterioration detection? Secondly, how to reduce the environmental uncertainties influences, as the operational condition varies according to the environment, namely due to wind loading, changes in temperature, traffic loading, and so on, which impose effects on the final dynamic responses? Similar questions can be raised more for improving the transmissibility theory, while further investigation is necessary to pave the way of transmissibility-based damage identification methodology in real engineering applications.

As transmissibility selection still does not have a clear framework the computational time will demand a big cost if all transmissibilities are taken into account. Therefore, investigation should be conducted to reduce the computational costs and to maintain the quality of the transmissibility in use. A promising direction is to reduce the dimensions, which can greatly decrease the computational costs in data mining, and Principal Component Analysis (PCA) is a well-known alternative commonly used for main feature selection in pattern recognition, data compression and so on. And it is based on the spectral analysis of the second-order moment matrix that statistically characterizes a random vector (Du and Swamy, 2006). In the past decades, PCA has been widely used in SHM (Boe and Golinval, 2003; Yan et al., 2005a, 2005b; Yan and Golinval, 2006; VietHa and Golinval, 2010a, 2010b, 2010c; VietHa et al., 2014; Ni et al., 2006; Trendafilova et al., 2008; Borguet and Leonard, 2009; Bellino et al., 2010; Kesavan et al., 2012; Yang and Nagarajaiah, 2012; Bandara et al., 2014; Tibaduiza et al., 2016).

The team of Golinval did a continuous study of applying PCA in damage diagnosis during the last years. Boe and Golinval (2003) took the Euclidean measure between response and the response on the PCA projected hyperplane as a novelty index for damage detection, and the angle between subspaces for detection of the damaged sensor. Since local damages do not affect much the global modal responses through damaged sensor detection, it is feasible to localize the damage. However, no clear evidence led to the applicability of that procedure for large-scale structures. Later, Yan et al. (2005a, 2005b) tried to use the residual error to account for the difference between the response and the projected data that has been mapped by PCA to the projected space and re-mapped them back to the original space for damage detection. Similar to their previous work, both Euclidean and Mahalanobis norms were considered as novelty indices, using upper and lower

control limits for setting thresholds. They showed the well performance of that methodology in both linear and nonlinear structural damage diagnosis. One year later, Yan and Golinval (2006) extended the previous angle change index to characteristic subspace damage detection via Hankel matrix factorized Singular Value Decomposition (SVD); a residual error index was also considered together with residual covariance. A few years later, VietHa and Golinval (2010a) did a study of sensitivity analysis of FRF together with PCA for damage localization, and quantification with average distance and second derivative of sensitivity change from the reference state. Kernel PCA was also discussed for fault detection in the presence of nonlinearity (VietHa and Golinval, 2010b). After several years, VietHa et al. (2014) summarized their previous work and applied it in the Champangshiehl Bridge.

In parallel to the papers discussed above, similar works were also presented by other researchers. Ni et al. (2006) employed artificial neural networks to associate the damage severities with the structural feature of FRF compressed by PCA and applied it to a tall building model under seismic condition. Trendafilova et al. (2008) employed the FRF mapping vector by PCA as a damage sensitive feature and took its difference between damaged state and intact state as a measure to detect damages and applied this to an aircraft. Bellino et al. (2010) also employed residual changes to construct a novelty index to detect damage, but applied it in time-varying systems. Bandara et al. (2014) constructed a damage index with the ratio between PCA compressed FRF and its reference under intact state and incorporated it with artificial neural networks in detecting damages in a frame. Tibaduiza et al. (2016) considered four damage indices for the PCA projected data originally measured by an active piezoelectric system and applied them to an aircraft skin panel and an aircraft turbine blade.

As cited above, the dynamic response in time domain or FRF is the most commonly used feature in PCA-based methods, although disadvantages are clear. As it is well known, dynamic response in time does not hold a strong interrelation with the structural damage or deterioration, as it also includes environmental uncertainties, loading conditions and so on. FRFs retain a strong relation with damage, but they require the measurement of the excitation, which is challenging in real complex structures under operational conditions.

The main contribution of the present study is to introduce for the first time PCA into transmissibility-based damage detection procedure that uses distance measures for the PCA compressed transmissibility to detect damages. The merits of this study are as follows. Firstly, to employ transmissibility as damage sensitive feature, so as to avoid the measurement of excitation

and potentially pave the way for real engineering application. Secondly, to adopt PCA in transmissibility, similarly to the conventional work of using PCA to compress FRF. This gives an enrichment in data reduction/computation that has a profound influence in further engineering applications.

In this paper, an early stage damage detection procedure using transmissibility compressed by PCA along with a distance measure and correlation analysis is developed. Firstly, transmissibility definition, estimation and selection are systematically illustrated. Then, PCA is used to compress the transmissibility of the whole structural system into a low dimensional representation. Afterwards, the use of a distance measure, giving a better discrimination of the compressed transmissibility, is described. Finally, damage sensitive indicators are constructed and used to separate the intact patterns apart from the damaged patterns; comparisons with a conventional index are provided. For verification of the proposed methodology, numerical analysis of clamped-clamped beam and an experiment on a laboratory structure are employed. Finally conclusions are presented.

## 2. Theoretical background

### 2.1. Transmissibility derivation

Considering an elastic system, for instance, a loaded linear multiple-degree-of-freedom (MDOF) system, the dynamic equilibrium equation of damped vibration can be written by the well-known second order differential equation:

$$\mathbf{M}\ddot{\mathbf{x}}(t) + \mathbf{C}\dot{\mathbf{x}}(t) + \mathbf{K}\mathbf{x}(t) = \mathbf{f}(t) \quad (1)$$

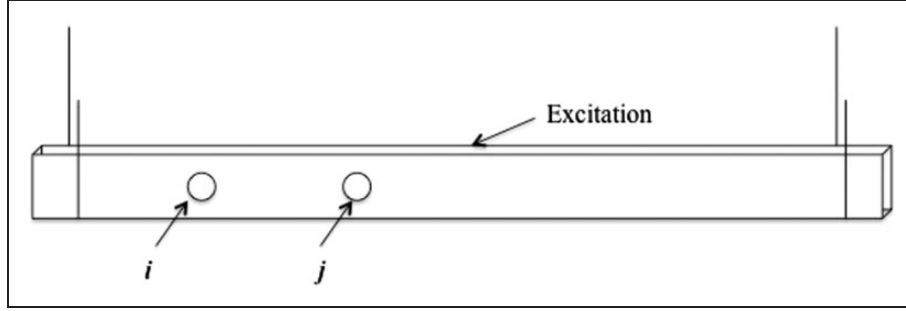
where  $\mathbf{M}$ ,  $\mathbf{C}$  and  $\mathbf{K}$  are the mass, damping, and stiffness matrices of the system, respectively,  $\mathbf{f}(t)$  include all possible types of time dependent loading, i.e. the input force vector; and  $\mathbf{x}(t)$  contains the responses of each degree-of-freedom of the system. Fourier transform or Laplace transform can be used to solve the above differential equation in order to calculate the displacement and velocity as well as the acceleration.

**2.1.1. Transmissibility definition.** For a harmonic force at a given coordinate, for instance a free-free beam tested with a single applied force as shown in Figure 1, the direct transmissibility between one point  $i$  and a reference point  $j$  can be defined as:

$$T_{(ij)}(\omega) = \frac{X_i(\omega)}{X_j(\omega)} \quad (2)$$

where  $X_i$  and  $X_j$  are the complex amplitudes of the system responses,  $x_i(t)$  and  $x_j(t)$ , respectively, and  $\omega$  is the frequency.





**Figure 1.** A simple schematic diagram of a free-free beam experiment.

**2.1.2. Transmissibility estimation.** For estimating transmissibility, one can utilize several approaches, for instance frequency response functions (FRFs):

$$T_{(i,j)}^b(\omega) = \frac{X_i(\omega)}{X_j(\omega)} = \frac{X_i(\omega)/F_b(\omega)}{X_j(\omega)/F_b(\omega)} = \frac{H_{(i,b)}(\omega)}{H_{(j,b)}(\omega)} \quad (3)$$

where  $H$  refers to the FRFs and  $b$  is the excitation node. Note that the use of FRFs will restrict further application of this transmissibility estimation approach in real engineering structures due to the unavailability of information about the excitation. In order to resolve this drawback, the use of cross- and auto-spectra is another alternative (taking averages):

**2.1.3. Transmissibility selection.** Transmissibility selection is very important, but until now there is no detailed research about this in previous investigations. The idea of transmissibility selection is to reduce the size of input data. If all  $N \times N$  transmissibility elements for the system are considered, a high CPU time will be required, especially when an industrial application is considered. However, if we can reduce the input data without affecting the capacity of detecting damage, then this will give a potential feasibility for industrial applications. Herein, this section intends to unveil the skills of selecting transmissibility in OMA. Considering the linear MODF system discussed above, the total transmissibility matrix of the structural system corresponding to all damaged and intact scenarios can be expressed as:

$$T^P(\omega)_D = \begin{bmatrix} T_{(1,1)}^P(\omega) & T_{(1,2)}^P(\omega) & \cdots & T_{(1,N-1)}^P(\omega) & T_{(1,N)}^P(\omega) \\ T_{(2,1)}^P(\omega) & T_{(2,2)}^P(\omega) & \cdots & T_{(2,N-1)}^P(\omega) & T_{(2,N)}^P(\omega) \\ \vdots & \vdots & & \vdots & \vdots \\ T_{(N-1,1)}^P(\omega) & T_{(N-1,2)}^P(\omega) & \cdots & T_{(N-1,N-1)}^P(\omega) & T_{(N-1,N)}^P(\omega) \\ T_{(N,1)}^P(\omega) & T_{(N,2)}^P(\omega) & \cdots & T_{(N,N-1)}^P(\omega) & T_{(N,N)}^P(\omega) \end{bmatrix} \quad (5)$$

$$T_{(i,j)}^P(\omega) = \frac{X_i(\omega)}{X_j(\omega)} = \frac{X_i(\omega) \times X_P(\omega)}{X_j(\omega) \times X_P(\omega)} = \frac{G_{(i,P)}(\omega)}{G_{(j,P)}(\omega)} \quad (4)$$

where  $P$  is another output reference node, and  $G$  means auto- and cross- spectrum. Note that  $T(\omega)$  is also called power spectral density transmissibility (PSDT) (Yan and Ren, 2012). Herein, note that the reference  $P$  can be the same as  $i$  and  $j$ . If one chooses the reference to be the same as  $i$  and  $j$ , then one might develop the transmissibility coherence (TC) by analogy of the coherence in FRFs analysis. Details analysis about this can be found in Zhou et al. (2015b).

where  $N$  is the number of degrees of freedom of the structural system, and  $T^P(\omega)_D$  is the transmissibility matrix of the structural system referred to the scenario D (intact/damaged structure).

Note that for an easy transmissibility selection methodology in data analysis, this study intends to employ PCA to extract the principal components of the transmissibility in order to provide a better estimation for further damage detection procedures. The transmissibility selection in this study can be illustrated in the following multiple-stage selection procedure:

*Stage 1: General transmissibility selection.* To neglect the obvious useless transmissibilities, this step depends

$$T^P(\omega)_D = \begin{bmatrix} T^P_{(1,1)}(\omega) & T^P_{(1,2)}(\omega) & \dots & T^P_{(1,N-1)}(\omega) & T^P_{(1,N)}(\omega) \\ T^P_{(2,1)}(\omega) & T^P_{(2,2)}(\omega) & \dots & T^P_{(2,N-1)}(\omega) & T^P_{(2,N)}(\omega) \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ T^P_{(N-1,1)}(\omega) & T^P_{(N-1,2)}(\omega) & \dots & T^P_{(N-1,N-1)}(\omega) & T^P_{(N-1,N)}(\omega) \\ T^P_{(N,1)}(\omega) & T^P_{(N,2)}(\omega) & \dots & T^P_{(N,N-1)}(\omega) & T^P_{(N,N)}(\omega) \end{bmatrix}$$

**Figure 2.** Transmissibility selection.

on the engineer's experience (Zhou et al., 2015b) and one might recall that experience summarized herein-after for Figure 2. Note that the obvious use-less transmissibilities mean those measured data that are not well captured, in real engineering testing, which frequently occur as measurement is not always perfect.

*Stage 2: PCA-based transmissibility compression.* PCA is used to extract the principal components of the whole useful transmissibilities set from stage 1. Details of this stage are discussed in later sections.

If we numerically divide the target structure into several parts, and for each node, transmissibility can be extracted, and the whole transmissibility for all nodes of the structure can be shown in Figure 2. From Figure 2, it is also possible to draw out the first stage of the transmissibility selection process: (i) solid lines, i.e. the first column and last column represent the boundary nodes. Normally, it is not a good choice to use these two columns, especially when the boundary conditions are simply supported or clamped. Note that these two columns are close to the boundary. However, if the initial node and final node held at a distance from the boundary, the algorithm goes directly to step (ii). (ii) Dotted lines, i.e. the first row and last row are normally not chosen solely for constructing a damage sensitive feature. If transmissibility in 'Zone I' or 'Zone II' (Zone I and II mean upper and lower parts of the transmissibility for the whole structure if the diagonal is chosen as separating line) can be taken into consideration, they will play the same role. Note that the diagonal should not be chosen for constructing a feature since they always hold value '1'. Normally one row or one column is chosen for constructing a damage sensitive feature, as one row/column may be sufficient to include all the information of all the measured nodes. Attention should be paid to the real engineering structures, where frequently transmissibility for the whole structure can not be fully extracted, and therefore one

might choose one segment of one row/column in developing damage feature, which will largely depend on the engineer personal experience. (iii) These ideas described above are only from experiences, i.e. there are no strict laws, so that they can only be used as references. Further conclusions will depend on the real structures, loading conditions, boundary conditions and so on.

In order to give a general transmissibility selection approach, the next section will describe the feasibility of using PCA in transmissibility selection.

## 2.2. PCA based data compression

The key idea of PCA is to reduce the dimensionality of the data set by transferring the original data set into a new set of variables by ensuring that much information as possible of the original data set is kept. The main reason for this step is that the original data set might contain interrelated variables. This gives a possibility to extract the un-interrelated variables by transforming the data set into a new coordinate system. And the first few variables will maintain the ones with the higher variation existing in the original data sets.

For the transmissibilities selected above in Stage 1, via PCA, one can write:

$$\mathbf{S} = \mathbf{U}\mathbf{T} \quad (6)$$

where  $\mathbf{U}$  is often called principal components that are eigenvectors of the covariance matrix, and  $\mathbf{S}$  means the projection of the input on these vectors, and so called Projected on Principal Components (PPCs) and  $\mathbf{T}$  means the transmissibility for each state selected in Stage 1.

In order to give a detailed expression, transmissibility for each state (in complete or in part) can be rewritten as:

$$T^P(\omega)_D = (T_1, T_2, \dots, T_N) \quad (7)$$

where

$$T_j = \begin{bmatrix} T_{1,j} \\ T_{2,j} \\ \dots \\ T_{N,j} \end{bmatrix}, \quad j = 1, 2, \dots, N \quad (8)$$

And then for the PCA, from equation (6), the measured transmissibilities to new global variables can be expressed as:

$$\begin{cases} S_1 = u_{11}T_1 + u_{12}T_2 + \dots + u_{1N}T_N \\ S_2 = u_{21}T_1 + u_{22}T_2 + \dots + u_{2N}T_N \\ \dots \\ S_Q = u_{Q1}T_1 + u_{Q2}T_2 + \dots + u_{QN}T_N \end{cases} \quad (9)$$

For simplification, equation (9) can also be written as:

$$S_j = u_{j1}T_1 + u_{j2}T_2 + \dots + u_{jN}T_N, \quad j = 1, 2, \dots, Q \quad (10)$$

where  $Q$  represents the number of PCs. Note that this model should fulfill the following requirements:

- $S_i$  and  $S_j$  are independent and uncorrelated ( $i \neq j$ ,  $i, j = 1, 2, \dots, Q$ ) variables;
- The variances of all  $S$  are holding the rule:

$$\text{if } i < j, \sigma_i > \sigma_j; \quad i, j \in [1, 2, \dots, Q]$$

- $u_{n1}^2 + u_{n2}^2 + \dots + u_{nN}^2 = 1; \quad n \in [1, 2, \dots, Q]$

Herein, it is necessary to give some comments for the functionality of PCA in this study: (i) The PCA firstly reduces the calculation by just choosing the first few PCs, thus reducing the computational costs, especially in long-term SHM and for large structures. (ii) PCA gives the possibility of choosing transmissibility after the measured transmissibilities are cleansed. (iii) PCA gives the possibility to other multivariate analysis tools for further analysis with the extracted PPCs, such as distance measure if PCA itself cannot give recognition for outlier.

### 3. Damage detection scheme

#### 3.1. Distance measure

Distance measure has had a key role in outlier analysis. Several investigations tried to use distance measures for different applications, such as damage detection, quantification and so on (Figueiredo et al., 2009;

Chen et al., 2003; Zhou et al., 2015a; Worden, 1997). On the other hand, a distance measure can also be combined with other algorithms or methodologies, such as artificial neural networks (ANNs) in outlier analysis (Chen et al., 1994). Although distance measures have been commonly used, very few investigations can be found for the comparison of different distance measures in performing outlier analysis. This study intends to do so, and gives an understanding of different distance measures in SHM under the same environment conditions in order to check their ability in discarding uncertainties resulted from operational and systematic errors. Several frequently used distances for pairs of data sets can be illustrated as follows.

For two data sets  $Y_1$  and  $Y_2$  with  $Z$  elements in each set, different distances are defined as:

*City Block distance.*

$$d_{12} = \sum_{z=1}^Z |Y_1(z) - Y_2(z)| \quad (11)$$

*Chebyshev distance.*

$$d_{12} = \lim_{e \rightarrow \infty} \left( \sum_{z=1}^Z |Y_1(z) - Y_2(z)|^e \right)^{1/e} \quad (12)$$

*Minkowski distance.*

$$d_{12} = \sqrt[r]{\sum_{z=1}^Z |Y_1(z) - Y_2(z)|^r} \quad (13)$$

Herein, note that ' $r$ ' is a variable. And if  $r=1$ , this will become City Block distance; if  $r=2$ , this will become Euclidean's distance; if  $r \rightarrow \infty$ , it becomes Chebyshev's distance. In this study, considering the accuracy,  $r$  is set to 4 as empirical result. Certainly one may set  $r$  as higher value, while it does not make too much difference.

*Mahalanobis distance.*

$$d_{12} = \sqrt{(\mathbf{Y}_1 - \bar{\mathbf{Y}}_2)^T \Sigma^{-1} (\mathbf{Y}_1 - \bar{\mathbf{Y}}_2)} \quad (14)$$

where  $\Sigma^{-1} = \text{Cov}^{-1}$ , i.e. the inverse covariance matrix of  $\bar{\mathbf{Y}}_2$ . Note that this covariance calculation should be carefully considered as sometimes inversion of matrix might be challenging, for which one may refer to (Zhou et al., 2015a). Note that here when  $\Sigma = I$ , the Mahalanobis distance coincides with the Euclidean distance. In this study,  $Y_1$  and  $Y_2$  represent the PPCs derived from the discussion above, and  $Z$  means the elements contained in the PPCs.

### 3.2. Damage sensitive indicator

In order to compare the proposed distance-based measures and to give a better understanding of the structural health, herein, a damage sensitive indicator is constructed:

$$DI(r) = \frac{d_{12}^d(r) - d_{12}^u}{\max(d_{12}^d(r) - d_{12}^u)} \quad (15)$$

where ‘ $d^d$ ’ and ‘ $d^u$ ’ indicate the structural state under damage scenarios or intact scenarios, ‘ $r$ ’ means the  $r$ th measurement of the experiment. Herein, the  $DI$  varies between [0, 1]. If no damage exists,  $DI$  will be equal to ‘0’ and if damage arises,  $DI$  will approach ‘1’.

For Mahalanobis’ distance, as a vector, and for comparison with the damage indicator of other measure distances, inspired from the previous investigation (Worden et al., 2003), in this study, a further form of damage indicator can be expressed as:

$$MDI(r) = 1 - MAC(\mathbf{d}_{12}^d(r), \mathbf{d}_{12}^u) \quad (16)$$

where

$$MAC(\mathbf{d}_{12}^d(r), \mathbf{d}_{12}^u) = \frac{((\mathbf{d}_{12}^d(r))^T (\mathbf{d}_{12}^u))^2}{((\mathbf{d}_{12}^d(r))^T (\mathbf{d}_{12}^d(r))) ((\mathbf{d}_{12}^u)^T (\mathbf{d}_{12}^u))} \quad (17)$$

Herein,  $MDI$  will vary between [0, 1]. If no damage exists,  $MDI$  will be equal to ‘0’, and if damage appears,  $MDI$  depends on the damage severity and will tend to ‘1’ for fully damaged structure.

In order to show the performance of the proposed methodology, the above proposed damage detection indices will be compared with a conventional index - Euclidean distance between two feature vectors of PCA mapping FRFs (Trendafilova et al., 2008), which can be expressed as:

$$D_q = \|PFRF - PFRF_q\| \quad (18)$$

Note that  $PFRF$  means the  $FRF$  compressed by PCA,  $q$  means the  $q$ th  $PFRF$ ,  $D_q$  means the distance between the  $q$ th  $PFRF$  and reference  $PFRF$  (normally the one under intact condition).

Considering the value range, it is better to use the normalized form that can be expressed as:

$$ND_q = \frac{D_q - \min(D_q)}{\max(D_q) - \min(D_q)} \quad (19)$$

which varies from 0 to 1. If  $ND_q$  is close to 0, it means that no damage occurred, while if it approaches 1, it means that damage has certainly took place.

### 3.3. Damage detection procedure

The damage detection scheme will be conducted as follows:

*Step 1: Transmissibility derivation.* As to the intact structural states, transmissibility is derived from equations (4) and (5) by considering different references and estimated from the structural responses;

*Step 2: Transmissibility selection.* In this step, by using the multiple-stage selection procedure proposed in sections 2.1.3 and 2.2, the PCs of transmissibility are selected in a new coordinate system; and then the PPCs will be drawn out for further damage indicator construction;

*Step 3: Distance measure construction.* According to equations (11) to (14), distance-based measures are explored not only for the undamaged state but also for the damaged scenarios;

*Step 4: Damage sensitive indicators estimation.* By using equations (15) to (19), damage sensitive indicators are estimated for all the structural states and for all the different distance-based measures;

*Step 5: Damage detection.* The damage is detected from the damage sensitive indicators estimated in the previous step. Note that in this step, one might need to set a threshold for clarifying the damaged scenarios apart from the intact states. In addition to using engineering experience, one might also use a group of testing for showing the influence of uncertainties in the damage sensitive indicator.

In summary, a flowchart is shown in Figure 3 for better understanding the damage detection procedure.

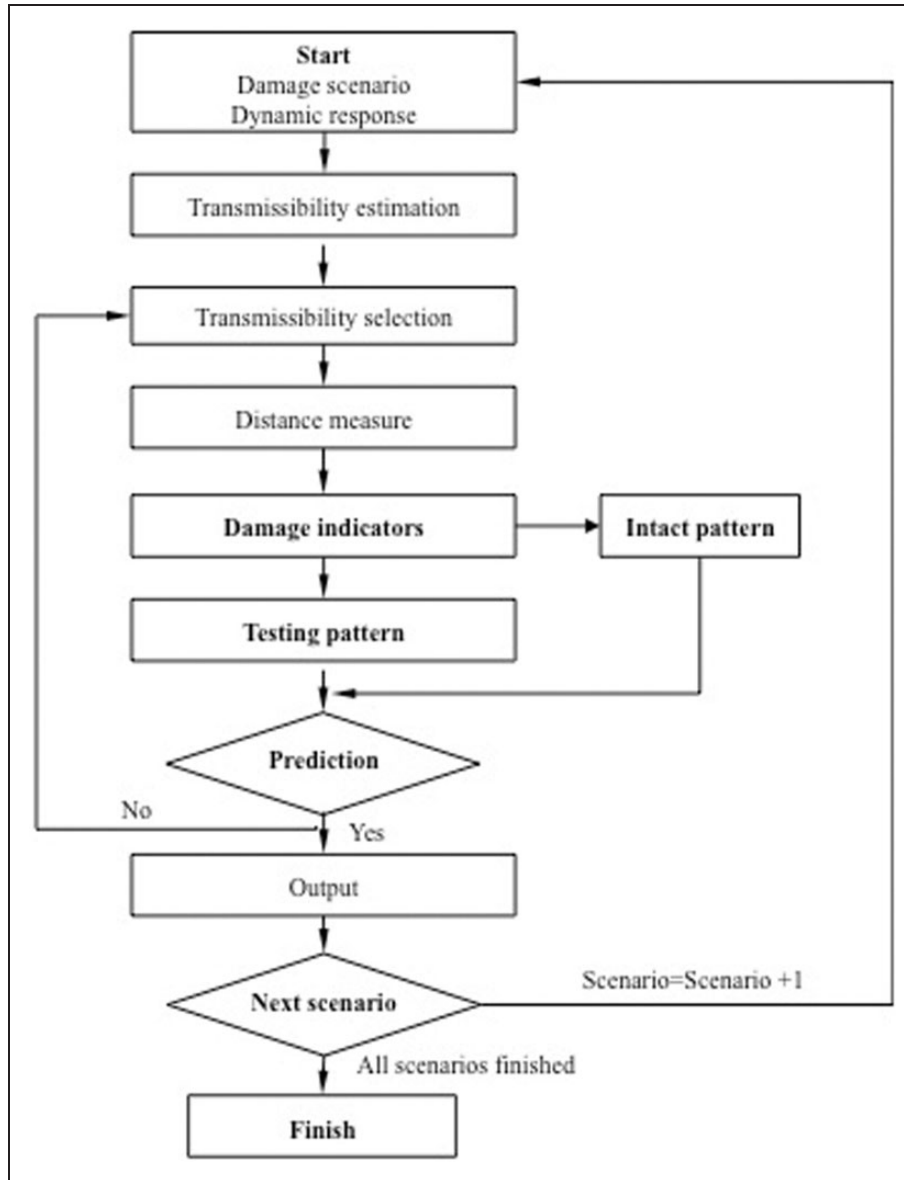
## 4. Numerical study: clamped-clamped beam

### 4.1. Model description

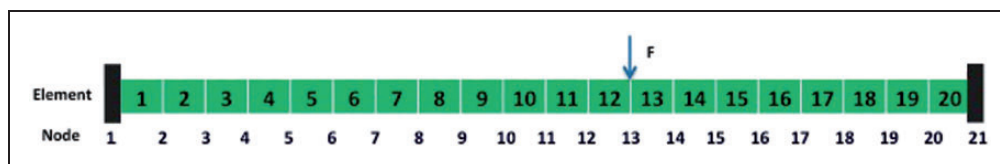
Numerical simulations are carried out for a clamped-clamped beam, which was adopted to examine the performance of the proposed damage indices. A schematic diagram of this beam with its geometric dimensions and material properties is shown in Figure 4. Different simulations were carried out considering a mesh of 20 beam elements. The beam is assumed to have a small constant damping ratio. The loading point is node 13 with a vertical impulse force with amplitude 10 N. Detailed physical information of the beam is shown in Table 1. Matlab is used for all the analysis in this study including FEA and data analysis.

Damage is simulated by a stiffness reduction from the baseline in the third element (between node three and four) and 15<sup>th</sup> element (node 15 and 16). The intact state is denoted as State #1, and damage scenarios are





**Figure 3.** Flowchart of the damage detection procedure.



**Figure 4.** A schematic representation of clamped-clamped beam.

denoted as State #2, State #3, State #4, and State #5 with stiffness reductions 2%, 5%, 10% and 20%, simultaneously. In this study, white noise is not taken into account as this has been verified by a free-free beam experiment with fan-induced airflow (Zhou et al., 2016).

## 4.2. Results

In this section, for highlighting the applicability of the proposed damage detection procedure, the damage detection results are discussed. Note that for the damage detection procedure, in this case, transmissibility

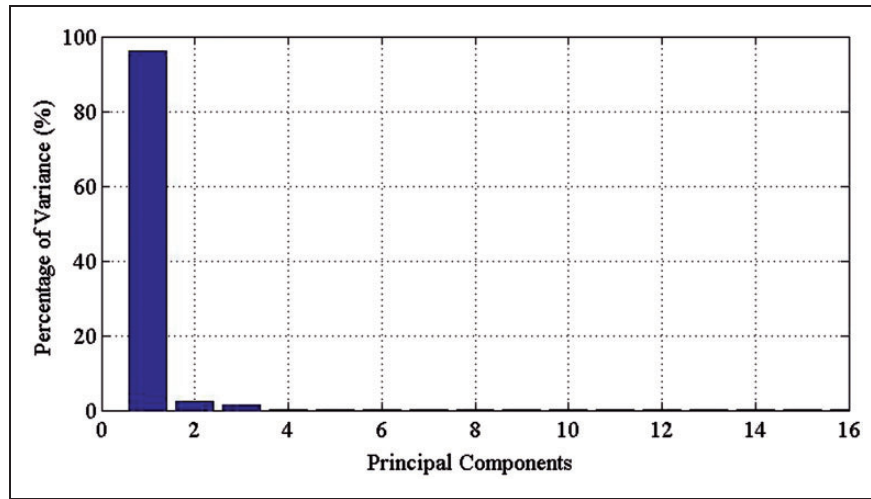
is calculated from structural responses with equation (4), thus node 13 is reference. And then 20 transmissibility elements ( $T_{(1,13)}$ ,  $T_{(2,13)}$ , ...,  $T_{(20,13)}$ ) will be derived, then frequency band [500, 1000] Hz. Furthermore, this input

**Table 1.** Beam properties.

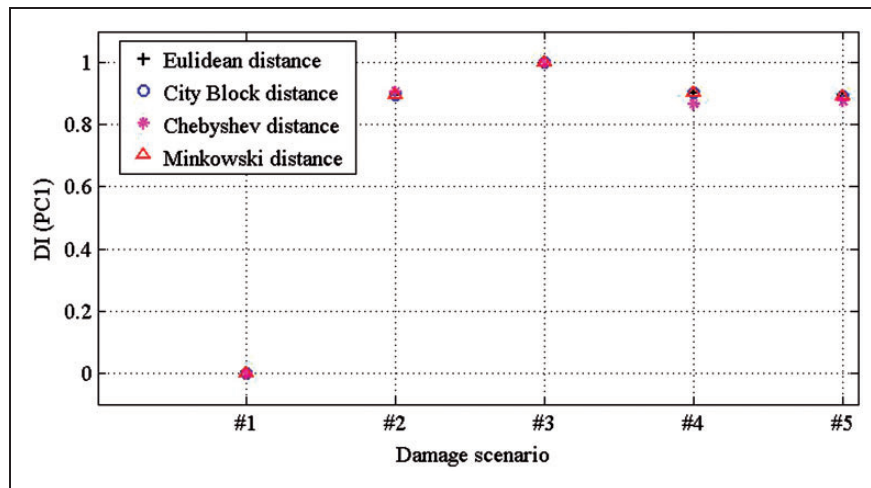
Beam properties	Value
Length of beam (mm)	1000
Width of cross section (mm)	50
Thickness of beam (mm)	6
Density (Kg/m <sup>3</sup> )	7800
Young's modulus (Pa)	$185.2 \times 10^9$
Poisson ratio	0.3
Damping ratio	0.002

will be taken into later procedure to extract PCs and PPCs, and DIs as well. Figure 5 shows the percentage of variance for the first 16 PCs. The total variance of the first three PCs (95.98%, 2.44%, 1.25%) is 99.67%, which suggests that the first few PCs are sufficient in representing the original data. In this clamped-clamped beam study, the first four PCs are selected for calculating the damage indices.

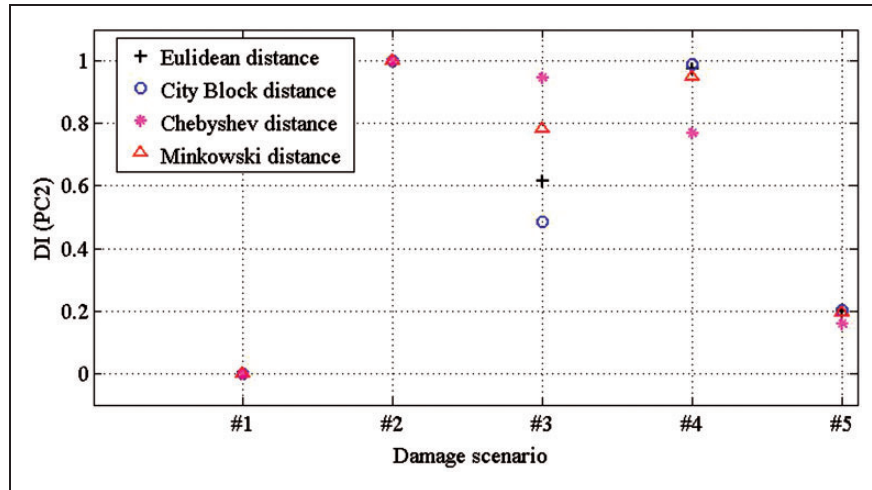
Figures 6 to 9 show the *DI* of the first four PCs calculated from Euclidean's distance, City Block distance, Chebyshev's distance, and Minkowski's distance. From these four figures, one can find that: (i) All the damaged scenarios (States #2 to #5) can be successfully detected, as clear differences can be found between the damaged scenarios and the intact state; (ii) For the second, third and fourth PCs, *DIs* calculated from Euclidean's distance, City Block distance, and



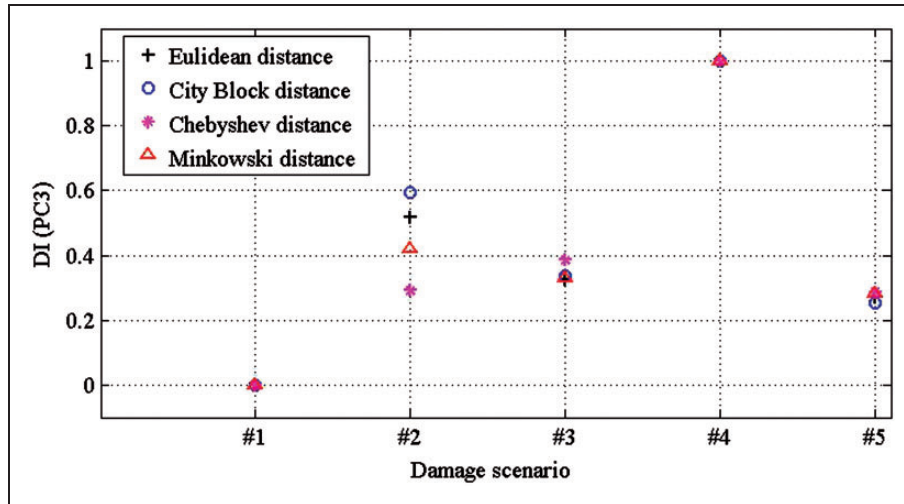
**Figure 5.** Percentage of Variance for PCs in State #1.



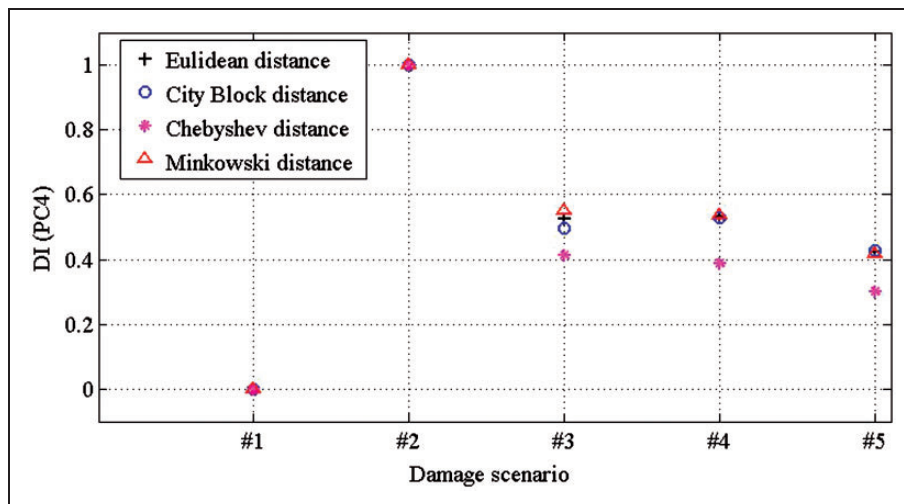
**Figure 6.** DI of PC1 for four kinds of distances for State #1 - #5.



**Figure 7.** DI of PC2 for four kinds of distances for State #1 - #5.



**Figure 8.** DI of PC3 for four kinds of distances for State #1 - #5.



**Figure 9.** DI of PC4 for four kinds of distances for State #1 - #5.

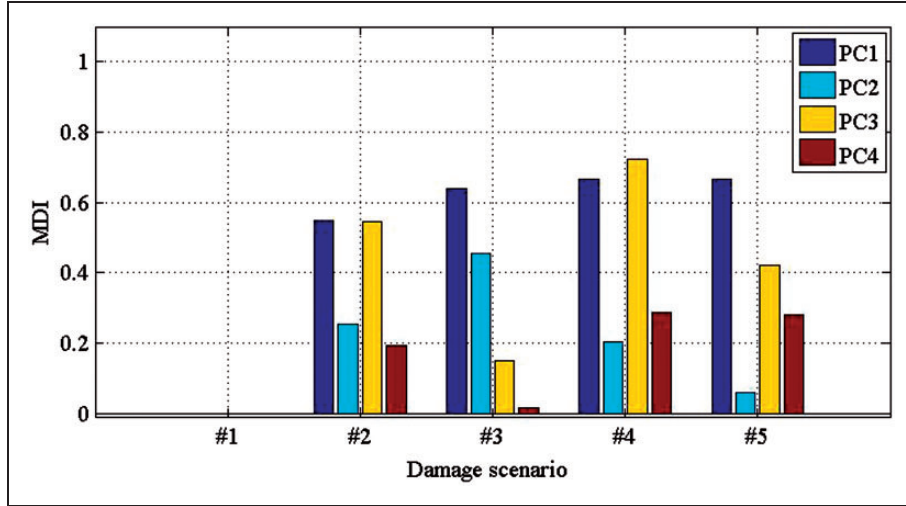


Figure 10. MDI of PC1, PC2, PC3 and PC4 for State #1 - #5.

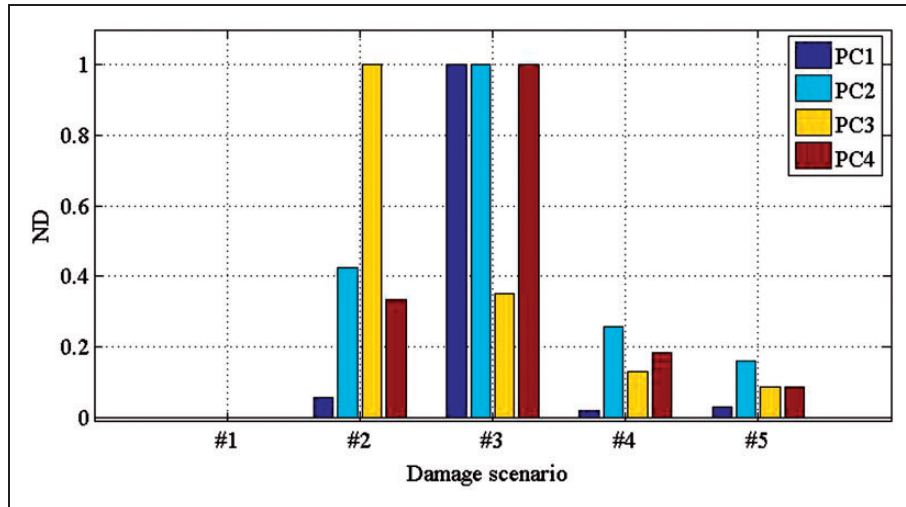


Figure 11. ND of PC1, PC2, PC3 and PC4 for State #1 - #5.

Minkowski's distance are more or less the same, while a big difference for the  $DI$  calculated from Chebyshev's distance can be seen in some states. This is because Chebyshev's distance only takes the maximum value in all the possible pairs of coordinates; (iii) It is challenging to draw out any conclusion for relatively quantifying the damage.

Figure 10 illustrates the MDI of the first four PCs calculated from Mahalanobis's distance, where one can find that: (i) All the damaged scenarios (States #2-#5) can be successfully detected as the differences between damaged scenarios and the intact condition (State #1) are clear; (ii) The MDI of the fourth PC performs worse than the MDI of the other three PCs (PC1, PC2, and PC3), and this implies that the MDI for fourth PC

might not be considered in detecting damage if considering that less variance occupied PC should be excluded as they may vary easily with uncertainties; (iii) Regarding the  $DI$  of the first four PCs as shown in Figures 13 to 16, one can find that both  $DI$  and  $MDI$  can successfully detect damages in multiple damage scenarios.

Figure 11 shows the ND of the first four PCs for the four damaged scenarios with double damaged elements (State #2-State #5) and intact condition. From Figure 11, one can find that the first two damaged scenarios (State #2 and State #3) are successfully detected, while for State #4, the detection is vague, and for State #5, it is challenging to draw out a confident detection conclusion. Comparing



Figure 11 with Figures 6 to 10, one can clearly find that the distance measure indicators newly proposed in this study perform better than the *ND* in detecting structural damages.



**Figure 12.** Model of the benchmark structure (Dyke et al., 2001).

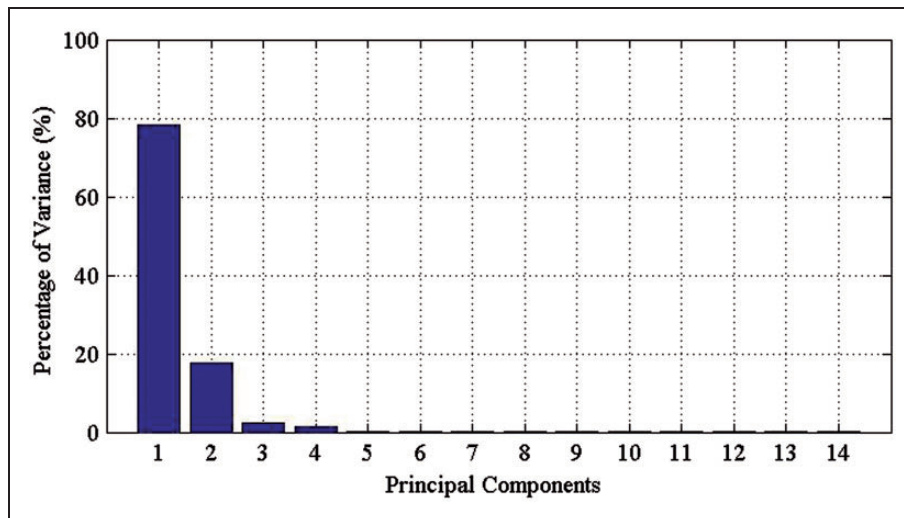
## 5. Experiment validation

### 5.1. Model description

To verify the applicability of the proposed damage detection procedure, the American Society of Civil Engineers (ASCE) benchmark problem (IASC-ASCE SHM Task Group, 2010) shown in Figure 12 is adopted herein to unveil the feasibility and check the performance of the damage detection methodology. This benchmark has been used as a validation tool for several times in the literature, e.g. system identification (Cara et al., 2012), damage detection (Lam et al., 2004) and so on. This structural model is built at the Earthquake Engineering Research Laboratory of the University of British Columbia, Canada, and it is conducted by the International Association for Structural Control (IASC) ASCE Structural Health Monitoring Task Group. Different damage scenarios are modeled either by adding additional mass, changing braces or loosening bolts at the beam-column connections (ASCE Benchmark Group, 2010). Three different types of excitation were considered. In this study, experimental data with an impact hammer of the Phase II are employed, and Table 2 illustrates the nine damage scenarios (Dyke et al., 2001). More information about the experiment can be found in Dyke et al. (2001) and Johnson et al. (2000, 2004).

### 5.2. Results

The results calculated from the proposed damage detection methodology are shown and discussed in this section. The accelerometer labeled 15 is set as reference, and then accelerometers labeled 1–14 are taken to derive transmissibility (14 transmissibility elements



**Figure 13.** Percentage of Variance for PCs in Case #1.

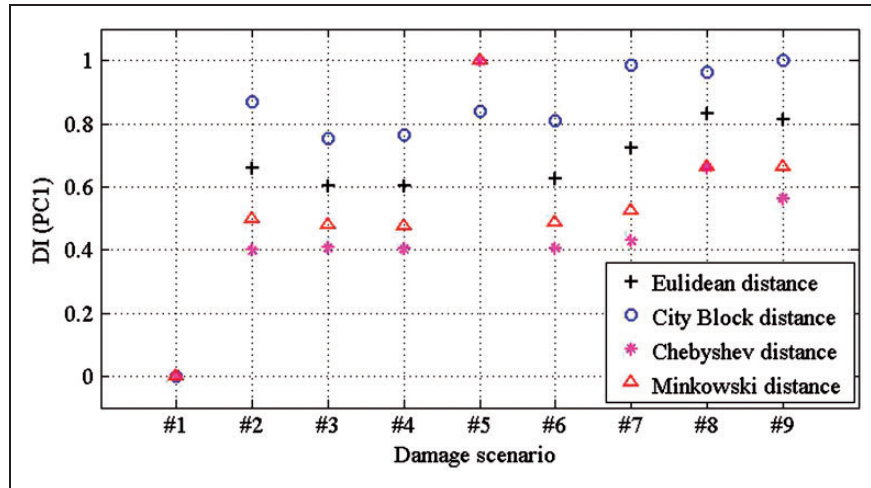


Figure 14. DI of PC1 for four kinds of distances for Case #1 - #9.

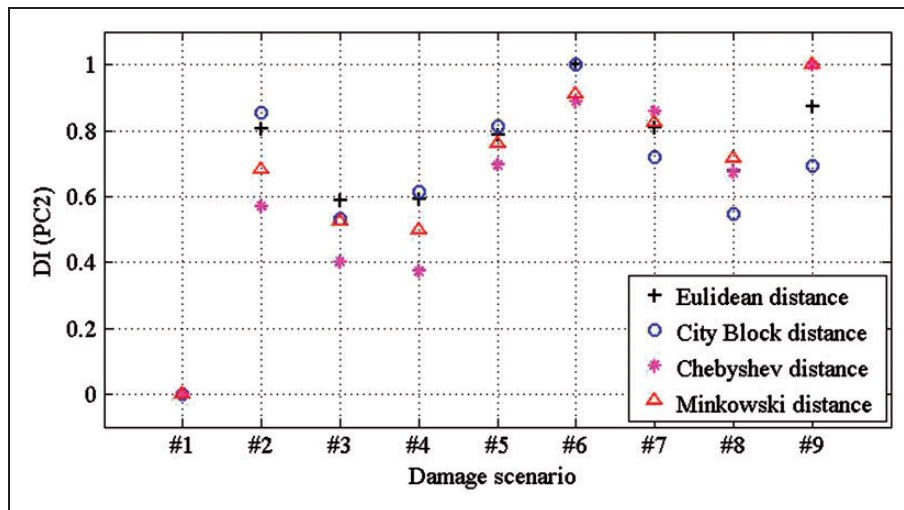


Figure 15. DI of PC2 for four kinds of distances for Case #1 - #9.

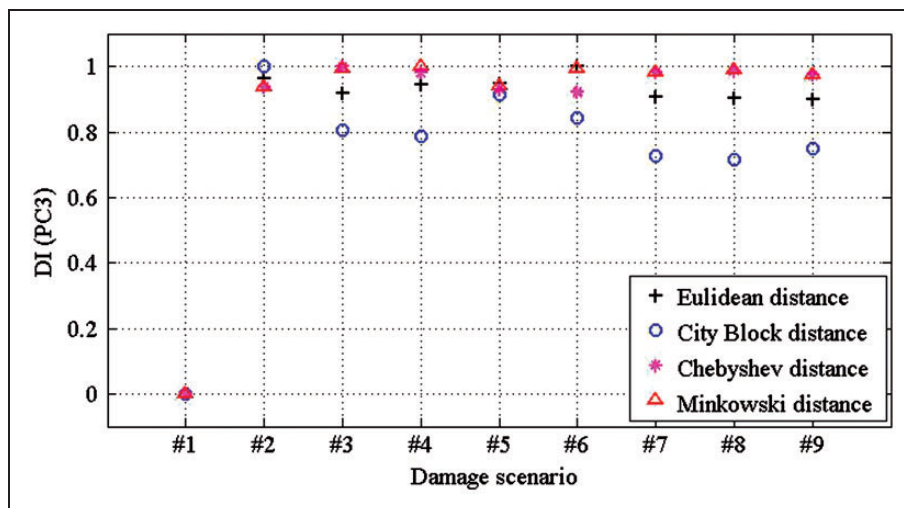


Figure 16. DI of PC3 for four kinds of distances for Case #1 - #9.

**Table 2.** Test scenarios description (Dyke et al., 2001).

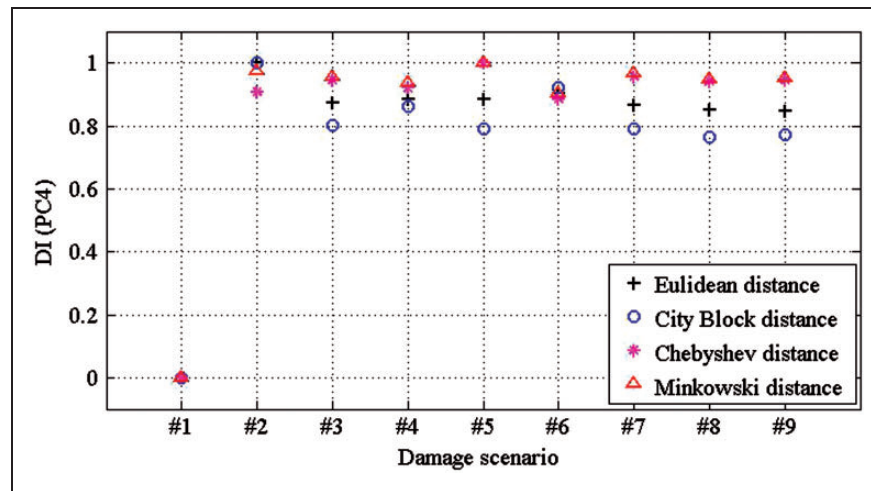
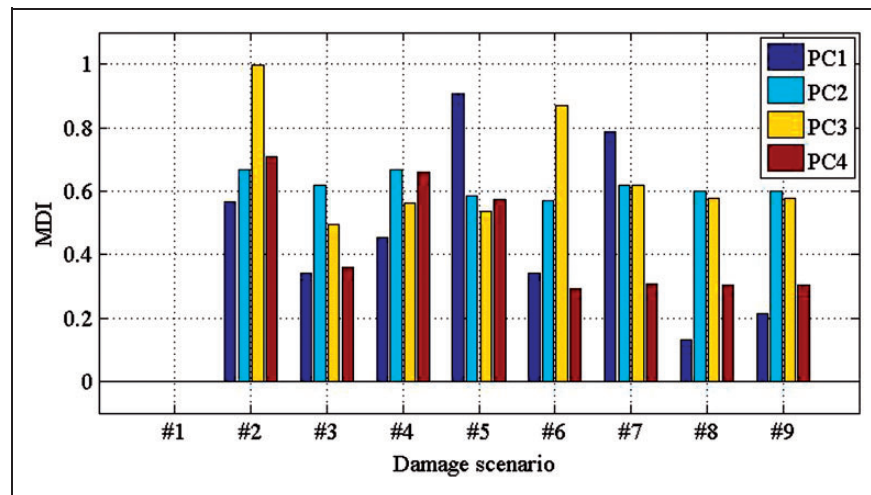
Damage scenarios	Case description
#1	All braces*
#2	Missing all east side braces
#3	Case #2 + remove one brace on floor 1
#4	Case #3 + remove one brace on floor 3
#5	Case #4 + loosen one connection
#6	Remove all braces, tighten loosen connection
#7	Case #6 + loosen one connection
#8	Case #7 + loosen second connection
#9	Reattach beam & repeat Case #6

\*Case A is the only test in which the mass is symmetrically distributed. All other cases have additional masses on the 1st and 2nd floors.

extracted in total). And the frequency band is [0.3, 20] Hz. With this input, by using PCA, damage indicators can be calculated from the PPCs. A common question might be why to choose these parameters? This might depend on the engineer's experience. Note that this might be not the best option, and potential optimization might be considered.

Figure 13 shows the variance percentage of PCs in Case #1, the total variance of the first four PCs (78.4%, 17.6%, 2.37%, 1.42%) is 99.79%. This indicates the possibility to use the first four PCs in damage detection analysis.

Figures 14 to 17 illustrate the *DI* for the first four PCs. From these four figures, one can find that: (i) In all the four PCs-based *DI*, all damaged scenarios (Cases #2 to #9) are successfully detected, and separated from the intact state, i.e. the baseline state (Case #1). (ii) All the

**Figure 17.** *DI* of PC4 for four kinds of distances for Case #1 - #9.**Figure 18.** MDI of PC1, PC2, PC3, and PC4 for Case #1 - #9.

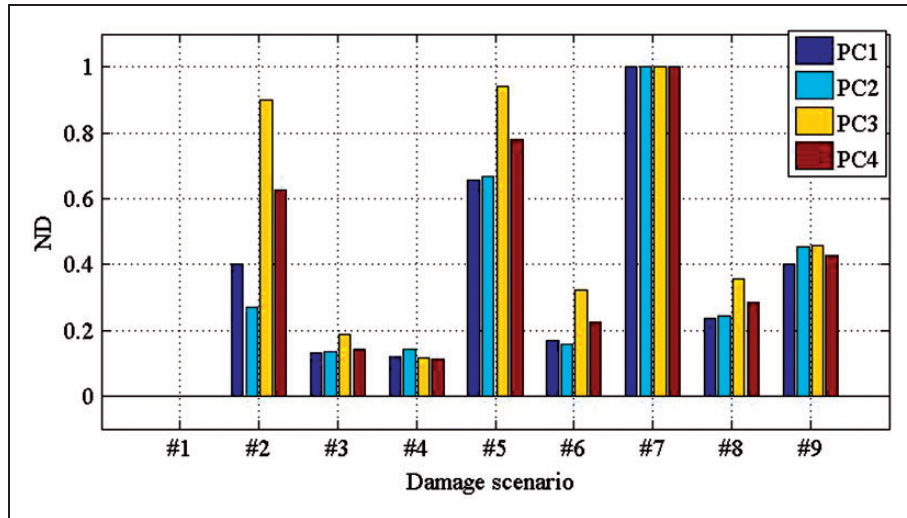


Figure 19. ND of PC1, PC2, PC3, and PC4 for Case #1 - #9.

nine states are clearly identified, while it is still difficult to draw out any conclusion for damage quantification. Further investigation is needed to achieve this goal for better understanding of each state.

Figure 18 shows the *MDI* of the first four PCs for each state. It is evident that damaged scenarios (States #2 to #9) are clearly separated from the baseline state (State #1), and one can also find that: (i) The *MDIs* of PC2 and PC3 perform better than the *MDIs* of PC1 and PC4 as they show higher variation; (ii) The *MDI* of PC1 increases from State #3 to State #5. This agrees with the state condition change, i.e. in Case #4 to remove one brace on floor 3 from Case #3, and in Case #5 one connection is loosened from Case #4. For damage quantification, more investigation should be conducted; (iii) Comparing the *MDI* with the *DI* performance as shown above, one can infer that all damages are successfully detected.

Figure 19 shows the *ND* of PC1, PC2, PC3, and PC4 for the Cases #1–Case #9. From Figure 19, generally speaking, if only comparing the values of *ND* for Cases #2–#9, it is clear that Cases #2–#9 can be successfully identified from the baseline – Case #1. Then, one can find that the *ND* for Cases #3 and #4 are lower than 0.2, which might be challenging in ensuring the damage occurrence, while the *ND* for Cases #2, #5, #7, and #9 are above 0.4, which give a confident damage detection alarm.

Comparing Figure 19 with Figure 14 to 18, it can be found that the *ND* and the distance measure based damage indicators can all detect damages in a general aspect. However, if we consider the amplitude change range, then we can conclude that distance measure-based damage indicators perform better than *ND* since they vary much more than *ND*.

## 6. Conclusion

Transmissibility has the important merit of depending only on the structural operational responses. This gives a possibility in real engineering use for characterizing the structural dynamic properties in order to monitor the structural health condition and give potential suggestions for possible maintenance and rehabilitation.

In this study, transmissibility is systematically introduced from definition, estimation, and selection. PCA was employed in transmissibility selection and performed a feature selection with functionality of data compression. The use of PCA in selecting the main components of transmissibility implemented a new damage detection methodology. In this methodology, via distance measures, damage sensitive indicators were built, including a comparison with a conventional damage norm, and a clamped-clamped beam was numerically analyzed to check the feasibility of the proposed damage detection procedure. Furthermore, a benchmark was employed to unveil the applicability of the proposed procedure in real complex structures. And the well performance of detecting damage in both simulations and experimental data analysis implied a potential usage of the method in real engineering applications comparing with the conventional damage norm.

The key contribution of this study is that it tries to unveil the key issue of transmissibility selection as until now few investigations can be found in addressing how to select transmissibilities. And the usage of PCA in this study gives a possible way for selecting transmissibilities and largely reduces the computation effort. In addition, the proposed damage detection method has proved to be effective for both numerical and experimental data analysis.



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The authors declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

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