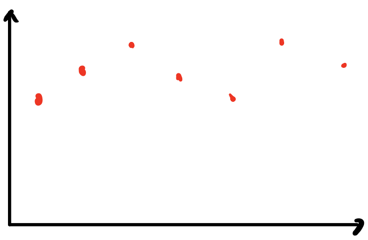
自动驾驶决策规划算法第一章第三节

frenet 坐标与 Cartesian 坐标转换

龙格现象：高次多项式拟合可能会出现震荡，慎用高次多项式



尽可能用分段低次多项式去拟合，而不是高次多项式

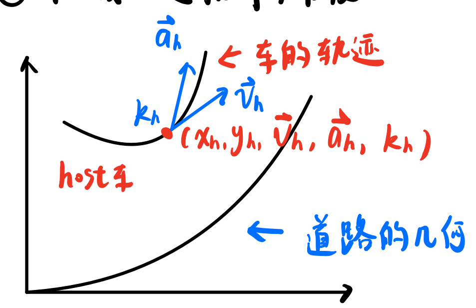
frenet 与 Cartesian 坐标变换

难度巨大，需精通向量微积分

要求：不需要理解，只需要会用即可

① 博客，见评论区，实在看不懂记住结论

② 和我一起推导，难度巨大



已知 车在 Cartesian 坐标的 $x_h, y_h, \vec{v}_h, \vec{a}_h, k_h$，求车在以道路为坐标轴的 frenet 坐标下的

$s, \dot{s}, \ddot{s}, l, l', l''$ (EM planner)

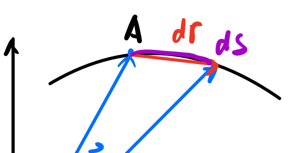
也有 $s, \dot{s}, \ddot{s}, l, \dot{l}, \ddot{l}$ (lattice)

$$\dot{l} = \frac{dl}{dt} = \frac{dl}{ds}\frac{ds}{dt} = \dot{s}l'$$

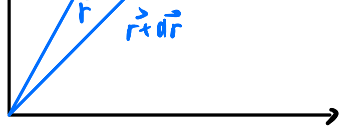
$$\ddot{l} = \frac{d\dot{l}}{dt} = \frac{d(\dot{s}l')}{dt} = \ddot{s}l' + \dot{s}\cdot\frac{dl'}{dt} = \ddot{s}l' + \dot{s}\cdot\frac{dl'}{ds}\cdot\frac{ds}{dt} = \ddot{s}l' + (\dot{s})^2 l''$$

EM Planner，用 $s \ \dot{s} \ \ddot{s} \ \ l \ l' \ l''$

先来点预备知识



证明 $\dot{\vec{r}} = |\vec{v}|\vec{\tau}$ $\quad \vec{\tau}$ 为轨迹在A点的切线方向单位向量



$$\frac{d\vec r}{dt} = \frac{d\vec r}{ds}\cdot\frac{ds}{dt} = |\vec v|\cdot\frac{d\vec r}{ds}$$

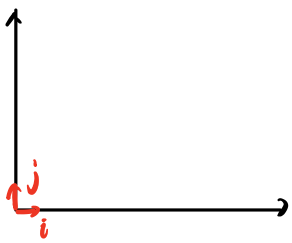
当 $dr\to 0$ 时 $\frac{dr}{ds}\to 1$，且 dr的方向趋于A点的切线方向

∴ $\frac{d\vec r}{ds}=\vec\tau$ ∴ $\frac{d\vec r}{dt}=|\vec v|\vec\tau$

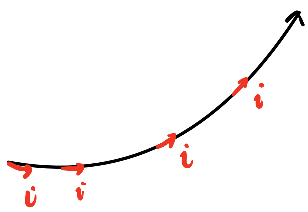
位矢的导数也是一个向量，向量的大小等于质点的速度大小，方向等于质点在轨迹的切线方向

预备知识2. frenet 公式

frenet坐标与 Cartesian坐标最大的不同在于frenet坐标系的基向量不是常向量

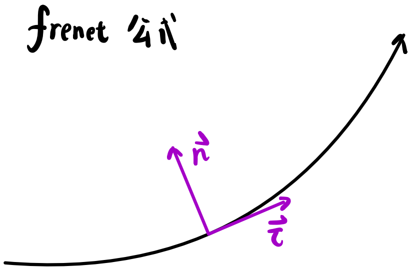


$$\frac{di}{dx}=0$$



求导操作 $\frac{d\vec i}{ds}$ 一般不为零

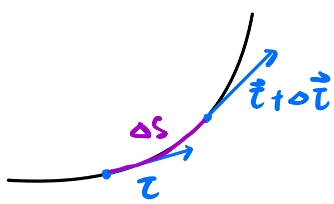
frenet 公式



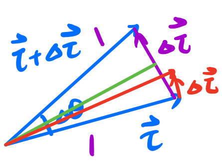
$$\left.\begin{aligned}\frac{d\vec\tau}{ds}&=k\vec n\\ \frac{dn}{ds}&=-k\vec\tau\end{aligned}\right\}\ \text{frenet公式}$$

k为曲率

证:



$$\frac{d\vec\tau}{ds}=\lim_{\Delta s\to 0}\frac{\vec\tau+\Delta\vec\tau-\vec\tau}{\Delta s}=\lim_{\Delta s\to 0}\frac{\Delta\vec\tau}{\Delta s}$$



先看方向，当 $\Delta s\to 0$ 时 $\Delta\vec\tau\to\vec 0$ $\Delta\vec\tau$方向 $\to\vec n$

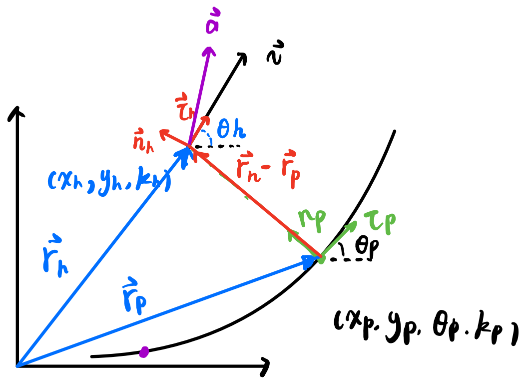
∴ $\lim_{\Delta s\to 0}\frac{\Delta\vec\tau}{\Delta s}=\left(\lim_{\Delta s\to 0}\frac{|\Delta\vec\tau|}{\Delta s}\right)\cdot n$

$\sin\frac{\Delta\theta}{2}=\frac{|\Delta\vec\tau|}{2}$ ⇒ $|\Delta\vec\tau|=2\sin\frac{\Delta\theta}{2}$

$$\therefore \lim_{\Delta s\to 0}\frac{|\Delta\vec\tau|}{\Delta s} = \lim_{\Delta s\to 0}\frac{2\sin\frac{\Delta\theta}{2}}{\Delta s} = \lim_{\Delta s\to 0}\frac{\Delta\theta}{\Delta s} = k$$

$$\therefore \frac{d\vec\tau}{ds} = k\vec n$$

证明 $\frac{d\vec n}{ds} = -k\vec\tau$ 留作过关



关键是找到车在frenet上的投影在Cartesian坐标下的 $(x_p, y_p, \theta_p, k_p)$

假设已找到

车的位矢 $\vec r_h$

投影的位矢 $\vec r_p$

则有 $l = (\vec r_h - \vec r_p)\cdot\vec n_p = (x_h - x_p \quad y_h - y_p)\begin{pmatrix}-\sin\theta_p \\ \cos\theta_p\end{pmatrix}$ 已算出 l

$l = (\vec r_h - \vec r_p)\cdot\vec n_p$ -①

$\vec r_h = \vec r_p + l\vec n_p$ -②

① 对t求导

$\dot l = (\dot{\vec r}_h - \dot{\vec r}_p)\cdot\vec n_p + (\vec r_h - \vec r_p)\cdot\dot{\vec n}_p$

$\dot l = (|\vec v|\vec\tau_h - \dot s\vec\tau_p)\cdot\vec n_p + (l\vec n_p)\cdot\frac{d\vec n_p}{ds}\cdot\frac{ds}{dt}$

$\dot l = |\vec v|\vec\tau_h\cdot\vec n_p + l\vec n_p\cdot(-k_p\vec\tau_p)\cdot\dot s$

$= |\vec v|\vec\tau_h\cdot\vec n_p$

$= |\vec v||\vec\tau_h||\vec n_p|\cos\langle\vec\tau_h, \vec n_p\rangle$

$= |\vec v|\cos(\theta_h - (\theta_p + \frac{\pi}{2}))$

$= |\vec v|\cos(\theta_h - \theta_p - \frac{\pi}{2})$

$= |\vec v|\sin(\theta_h - \theta_p)$

$\dot l = \vec v\cdot\vec n_p$

$l = (\vec r_h - \vec r_p)\cdot\vec n_p$

$\dot s = \frac{\vec v\cdot\vec\tau_p}{1 - kl}$

② 对t求导

$$\vec{r}_h = \vec{r}_p + \dot{l}\vec{n}_p + l\dot{\vec{n}}_p$$

$$|\vec{v}|\vec{\tau}_h = \dot{s}\vec{\tau}_p + |\vec{v}|\sin(\theta_h-\theta_p)\cdot\vec{n}_p + (-k_p l)\dot{s}\vec{\tau}_p$$

两边点乘 $\vec{\tau}_p$ $\quad |\vec{v}|\vec{\tau}_h\cdot\vec{\tau}_p = \dot{s} + (-k_p l)\dot{s}$

$$\dot{s} = \frac{|\vec{v}|\vec{\tau}_h\cdot\vec{\tau}_p}{1-k_p l} = \frac{|\vec{v}|\cos(\theta_h-\theta_p)}{1-k_p l}$$

已算出 $\dot{s}$

$$l' = \frac{dl}{ds} = \frac{\frac{dl}{dt}}{\frac{ds}{dt}} = \frac{\dot{l}}{\dot{s}} = \frac{|v|\sin(\theta_h-\theta_p)}{\frac{|v|\cos(\theta_h-\theta_p)}{1-kl}} = (1-k_p l)\tan(\theta_h-\theta_p)$$

$$\dot{s} = \frac{\vec{v}\cdot\vec{\tau}_p}{1-k_p l} \qquad \ddot{s} = \frac{1}{(1-k_p l)^2}\left[(\vec{a}\cdot\vec{\tau}_p + \vec{v}\cdot\dot{\vec{\tau}}_p)(1-k_p l) + (\vec{v}\cdot\vec{\tau}_p\cdot(-\dot{k}_p l - k_p\dot{l}))\right]$$

$$= \frac{\vec{a}\cdot\vec{\tau}_p}{1-k_p l} + \frac{k_p\vec{v}\cdot\vec{n}_p\cdot\dot{s}}{1-k_p l} + \frac{\dot{s}(-\dot{k}_p l - k_p\dot{l})}{1-k_p l}$$

$$= \frac{\vec{a}\cdot\vec{\tau}_p}{1-k_p l} + \frac{k_p|\vec{v}|\cdot\vec{\tau}_h\cdot\vec{n}_p}{1-k_p l} - \frac{\dot{s}^2(k_p' l + k_p l')}{1-k_p l}$$

$$\frac{\vec{a}\cdot\vec{\tau}_p}{1-k_p l} + \frac{k_p\cdot\dot{s}^2 l'}{1-k_p l} - \frac{\dot{s}^2(k_p' l + k_p l')}{1-k_p l}$$

$$\ddot{l} = \frac{d\dot{l}}{dt} = \frac{d(l'\dot{s})}{dt} = l'\ddot{s} + l''\dot{s}^2$$

$$l'' = \frac{\ddot{l} - l'\ddot{s}}{\dot{s}^2} =$$

$$l' = \frac{dl}{ds} = \frac{\dot{l}}{\dot{s}}$$

$$\dot{l} = \vec{v}\cdot\vec{n}_p$$

$$\ddot{l} = \vec{a}\cdot\vec{n}_p + \vec{v}\cdot\dot{s}(k_p\vec{\tau}_p)$$

$$= \vec{a}\cdot\vec{n}_p + k_p\dot{s}(\vec{v}\cdot\vec{\tau}_p)$$