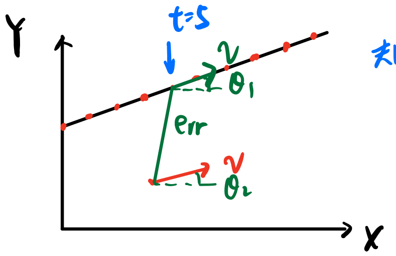
自动驾驶控制算法第四讲

$$\frac{d}{dt}\begin{pmatrix} v_y \\ \dot{\varphi} \end{pmatrix} = \begin{pmatrix} \frac{C_{\alpha f}+C_{\alpha r}}{m v_x} & \frac{aC_{\alpha f}-bC_{\alpha r}}{m v_x} - v_x \\ \frac{aC_{\alpha f}-bC_{\alpha r}}{I v_x} & \frac{a^2C_{\alpha f}+b^2C_{\alpha r}}{I v_x} \end{pmatrix}\begin{pmatrix} v_y \\ \dot{\varphi} \end{pmatrix} + \begin{pmatrix} -\frac{C_{\alpha f}}{m} \\ -\frac{aC_{\alpha f}}{I} \end{pmatrix}\delta$$

$$\dot{x} = Ax + Bu$$



规划 (X_r, Y_r, v_r, θ_r, a_r) 已知

纵向误差，横向误差，航向误差，速度误差，加速度误差

$\vec{X} - \vec{X_r} = \vec{e_{rr}}$ $\vec{X_r}$ 已知，$\vec{X}$ 满足 $\dot{X} = AX + Bu$ $\Rightarrow$ $\dot{e}_{rr} = \bar{A}e_{rr} + \bar{B}u$

物理规律

控制目标：选择合适的u，使得 $\vec{X}$ 与 $\vec{X_r}$ 尽可能接近

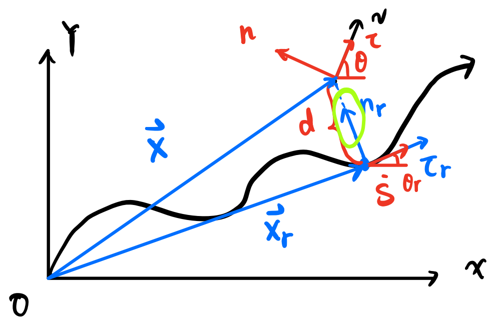
$\Rightarrow$ 让 $|\vec{e_{rr}}|$ 尽可能最小

$J = e_{rr}^2 = \min$ $\Rightarrow$ $J = e_{rr}^2 + u^2$ 最小 $\Rightarrow$ $J = a e_{rr}^2 + b u^2$ 最小

$\Rightarrow$ $J = e_{rr}^T Q e_{rr} + u^T R u$ 最小，Q，R为对角矩阵 e_{rr}，u为列向量

$\Rightarrow$ $J = e_{rr}^T Q e_{rr} + u^T R u$ 在约束 $\underbrace{\dot{e}_{rr} = \bar{A}e_{rr} + \bar{B}u}_{L}$ 的条件下最小 $\Rightarrow$ LQR

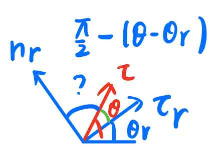
$\dot{x} = Ax + Bu$ $\Rightarrow$ $\dot{e}_{rr} = \bar{A}e_{rr} + \bar{B}u$



横向误差 d

航向误差 $\theta - \theta_r$

投影的速度大小 $\dot{s}$



$\vec{X_r} + d\vec{n_r} = \vec{X}$ $d = (\vec{X} - \vec{X_r}) \cdot \vec{n_r}$

$\dot{d} = (\dot{\vec{x}} - \dot{\vec{x}}_r) \cdot \vec{n_r} + (\vec{x} - \vec{x_r}) \cdot \dot{\vec{n}}_r$

$\vec{X}$为车辆的真实位矢 $\vec{X}=|\vec{v}|\vec{\tau}$

$\vec{X_r}$为投影的位矢 $\dot{\vec{X_r}}=\dot{s}\vec{\tau_r}$

$$\dot{d}=(|\vec{v}|\vec{\tau}-\dot{s}\vec{\tau_r})\cdot\vec{n_r}+(\vec{X}-\vec{X_r})\cdot\frac{d\vec{n_r}}{dt}$$

$$\frac{d\vec{n_r}}{dt}=\frac{d\vec{n_r}}{ds}\cdot\frac{ds}{dt}$$ Frenet 公式 $\frac{d\vec{\tau}}{ds}=k\vec{n}$ $\frac{d\vec{n}}{ds}=-k\vec{\tau}$ k为曲率

$$=\dot{s}(-k\vec{\tau_r})$$

$$\dot{d}=(|\vec{v}|\vec{\tau}\cdot\vec{n_r})+d\vec{n_r}\cdot(-k\dot{s}\vec{\tau_r})$$

$$=|\vec{v}||\vec{\tau}||\vec{n_r}|\cos\langle\vec{\tau},\vec{n_r}\rangle$$

$$=|\vec{v}|\cos\left(\frac{\pi}{2}-(\theta-\theta_r)\right)=|\vec{v}|\sin(\theta-\theta_r)$$

$\vec{X_r}+d\vec{n_r}=\vec{X}$ $\quad\dot{\vec{X_r}}+\dot{d}\vec{n_r}+d\dot{\vec{n_r}}=\dot{\vec{X}}$

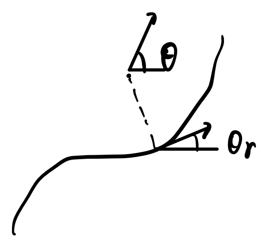
$$\dot{s}\vec{\tau_r}+|\vec{v}|\sin(\theta-\theta_r)\vec{n_r}+d(-k\dot{s}\vec{\tau_r})=|\vec{v}|\vec{\tau}$$

点乘 $\vec{\tau_r}$ $\quad\dot{s}+(-kd\dot{s})=|\vec{v}|\vec{\tau}\cdot\vec{\tau_r}=|\vec{v}|\cos(\theta-\theta_r)$

$$\dot{s}=\frac{|\vec{v}|\cos(\theta-\theta_r)}{1-kd}$$

$$\dot{d}=|\vec{v}|\sin(\theta-\theta_r)$$

$$\dot{s}=\frac{|\vec{v}|\cos(\theta-\theta_r)}{1-kd}$$



与 v_y, φ 联系 $\quad\theta=\varphi+\beta$

$$\dot{d}=|\vec{v}|\sin(\beta+\varphi-\theta_r)=|\vec{v}|\sin\beta\cos(\varphi-\theta_r)+|\vec{v}|\cos\beta\sin(\varphi-\theta_r)$$

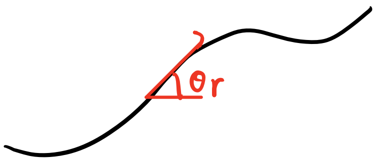
$$=v_y\cos(\varphi-\theta_r)+v_x\sin(\varphi-\theta_r)$$

认为 $\varphi-\theta_r$ 为小量 $\quad\approx v_y+v_x(\varphi-\theta_r)$

d 为~~纵~~横向误差，令 $e_d = d$ $\quad e_\varphi = \varphi - \theta_r$ （e_φ 并不是航向误差）

$$\dot{e}_d = v_x e_\varphi + v_y \qquad v_y = \dot{e}_d - v_x e_\varphi \qquad \dot{v}_y = \ddot{e}_d - v_x \dot{e}_\varphi$$

$$e_\varphi = \varphi - \theta_r \qquad \dot{e}_\varphi = \dot{\varphi} - \dot{\theta}_r \qquad \ddot{e}_\varphi = \ddot{\varphi} - \ddot{\theta}_r \approx \ddot{\varphi}$$



$$\begin{cases} v_y = \dot{e}_d - v_x e_\varphi \\ \dot{v}_y = \ddot{e}_d - v_x \dot{e}_\varphi \\ \dot{\varphi} = \dot{e}_\varphi + \dot{\theta}_r \\ \ddot{\varphi} = \ddot{e}_\varphi \end{cases}$$

$$\dot{v}_y = \frac{C_{\alpha f} + C_{\alpha r}}{m v_x} v_y + \left(\frac{a C_{\alpha f} - b C_{\alpha r}}{m v_x} - v_x \right) \dot{\varphi} - \frac{C_{\alpha f}}{m} \delta$$

$$\ddot{e}_d - v_x \dot{e}_\varphi = \frac{C_{\alpha f} + C_{\alpha r}}{m v_x} (\dot{e}_d - v_x e_\varphi) + \left(\frac{a C_{\alpha f} - b C_{\alpha r}}{m v_x} - v_x \right) (\dot{e}_\varphi + \dot{\theta}_r) - \frac{C_{\alpha f}}{m} \delta$$

$$\ddot{e}_d = \frac{C_{\alpha f} + C_{\alpha r}}{m v_x} \dot{e}_d - \frac{C_{\alpha f} + C_{\alpha r}}{m} e_\varphi + \frac{a C_{\alpha f} - b C_{\alpha r}}{m v_x} \dot{e}_\varphi + \frac{a C_{\alpha f} - b C_{\alpha r}}{m v_x} \dot{\theta}_r - v_x \dot{\theta}_r - \frac{C_{\alpha f}}{m} \delta$$

$$= \left(\frac{C_{\alpha f} + C_{\alpha r}}{m v_x} \right) \dot{e}_d + \left(- \frac{C_{\alpha f} + C_{\alpha r}}{m} \right) e_\varphi + \left(\frac{a C_{\alpha f} - b C_{\alpha r}}{m v_x} \right) \dot{e}_\varphi + \left(\frac{a C_{\alpha f} - b C_{\alpha r}}{m v_x} - v_x \right) \dot{\theta}_r + \left(- \frac{C_{\alpha f}}{m} \right) \delta$$

$$\ddot{\varphi} = \frac{a C_{\alpha f} - b C_{\alpha r}}{I v_x} v_y + \frac{a^2 C_{\alpha f} + b^2 C_{\alpha r}}{I v_x} \dot{\varphi} - \frac{a C_{\alpha f}}{I} \delta$$

$$\ddot{e}_\varphi = \frac{a C_{\alpha f} - b C_{\alpha r}}{I v_x} (\dot{e}_d - v_x e_\varphi) + \frac{a^2 C_{\alpha f} + b^2 C_{\alpha r}}{I v_x} (\dot{e}_\varphi + \dot{\theta}_r) - \frac{a C_{\alpha f}}{I} \delta$$

$$= \left(\frac{a C_{\alpha f} - b C_{\alpha r}}{I v_x} \right) \dot{e}_d + \left(- \frac{a C_{\alpha f} - b C_{\alpha r}}{I} \right) e_\varphi + \left(\frac{a^2 C_{\alpha f} + b^2 C_{\alpha r}}{I v_x} \right) \dot{e}_\varphi + \left(\frac{a^2 C_{\alpha f} + b^2 C_{\alpha r}}{I v_x} \right) \dot{\theta}_r + \left(- \frac{a C_{\alpha f}}{I} \right) \delta$$

$$\ddot{e}_d = 0\cdot e_d + a_1\dot{e}_d + a_2 e_\varphi + a_3\dot{e}_\varphi + b_1\dot{\theta}_r + c_1\delta$$

$$\ddot{e}_\varphi = 0\cdot e_d + a_4\dot{e}_d + a_5 e_\varphi + a_6\dot{e}_\varphi + b_2\dot{\theta}_r + c_2\delta$$ $\Rightarrow$ 线性

$$\dot{e}_d = 0\cdot e_d + \dot{e}_d + 0\cdot e_\varphi + 0\cdot\dot{e}_\varphi + 0\cdot\dot{\theta}_r + 0\cdot\delta$$

$$\dot{e}_\varphi = 0\cdot e_d + 0\cdot\dot{e}_d + 0\cdot e_\varphi + \dot{e}_\varphi + 0\cdot\dot{\theta}_r + 0\delta$$

$$\begin{pmatrix}\dot{e}_d\\ \ddot{e}_d\\ \dot{e}_\varphi\\ \ddot{e}_\varphi\end{pmatrix} = \begin{pmatrix}0 & 1 & 0 & 0\\ 0 & \frac{C_{\alpha f}+C_{\alpha r}}{m v_x} & -\frac{C_{\alpha f}+C_{\alpha r}}{m} & \frac{aC_{\alpha f}-bC_{\alpha r}}{m v_x}\\ 0 & 0 & 0 & 1\\ 0 & \frac{aC_{\alpha f}-bC_{\alpha r}}{I v_x} & -\frac{aC_{\alpha f}-bC_{\alpha r}}{I} & \frac{a^2C_{\alpha f}+b^2C_{\alpha r}}{I v_x}\end{pmatrix}\begin{pmatrix}e_d\\ \dot{e}_d\\ e_\varphi\\ \dot{e}_\varphi\end{pmatrix} + \begin{pmatrix}0\\ -\frac{C_{\alpha f}}{m}\\ 0\\ -\frac{aC_{\alpha f}}{I}\end{pmatrix}\delta$$

$$+\begin{pmatrix}0\\ \frac{aC_{\alpha f}-bC_{\alpha r}}{m v_x} - v_x\\ 0\\ \frac{a^2C_{\alpha f}+b^2C_{\alpha r}}{I v_x}\end{pmatrix}\dot{\theta}_r$$

$\dot{e}_{rr} = Ae_{rr} + Bu + C\dot{\theta}_r$ ✓

$e_\varphi = \varphi - \theta_r$　航向误差 $\theta - \theta_r$ 有无问题?

$\dot{\theta}_r$ 该如何计算?

$\dot{e}_{rr} = Ae_{rr} + Bu \Rightarrow LQR$

$\dot{e}_{rr} = Ae_{rr} + Bu + C\dot{\theta}_r$