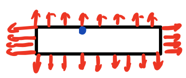
有限元法自学教程第三讲

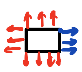
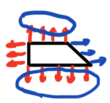
弹性力学方程 $\Longrightarrow$ 能量方程 $\xRightarrow{\text{近似解}}$ 有限元法

刚体力学物理量：$\vec{F}$，$\vec{u}$，t，　$\vec{F}=m\frac{d^2\vec{u}}{dt^2}$

弹性力学物理量：$u, v, w, F_x, F_y, F_z, \sigma_{xx}, \sigma_{yy}, \sigma_{zz}, \sigma_{xz}, \sigma_{yz}, \sigma_{xy}, \varepsilon_{xx}, \varepsilon_{yy}, \cdots$

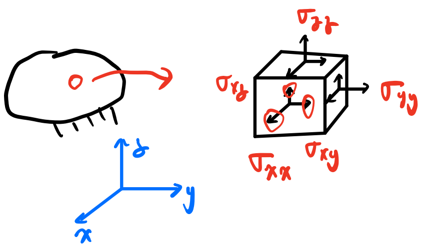
F无法描述复杂变形体的局部受力

例  平衡

正着切  斜着切  x  ✓

同一个点，不同的分割方式会导致力不同

必须引入应力的概念



σ_{ij}　第一个指标代表面的法向

第二个指标代表应力的方向

σ_{xx}　σ_{yy}　σ_{zz}（正应力）　σ_{xy}，σ_{xz}，σ_{yz}（剪应力）

$\sigma_{yx}=\sigma_{xy}$，　$\sigma_{zx}=\sigma_{xz}$，　$\sigma_{yz}=\sigma_{zy}$（剪应力互等定理）

弹簧胡克定律　$F=kx$　　k为材料参数

在弹性体中　$k \Rightarrow (E, \nu)$，　$F \Rightarrow \sigma_{ij}$　，x为宏观量，为了与σ_{ij}匹配，x也要

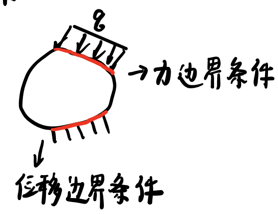
微元化

应变 ε_{xx}, ε_{yy} ε_{zz}, γ_{xy}, γ_{xz}, γ_{yz}

在刚体力学中：

在弹性力学中：

$$\begin{tikzcd}[column sep=huge, row sep=huge]
F \arrow[r, leftrightarrow, "\text{牛顿运动定律}"] \arrow[d, leftrightarrow, "\text{力边界条件}"] & u \arrow[d, leftrightarrow, "\text{几何方程}"] & \text{位移边界条件} \arrow[l] \\
\sigma \arrow[r, leftrightarrow, "\text{胡克定律}"] & \varepsilon &
\end{tikzcd}$$



牛顿运动定律 + 胡克定律 + 几何方程 + 边界条件 = 弹性力学方程

平面问题 $x.\ y$

$F_x, F_y,\quad \sigma_{xx}\quad \sigma_{yy}\quad \sigma_{xy},\ \varepsilon_{xx}, \varepsilon_{yy}, \gamma_{xy},\ u, v$

弹性力学力方程

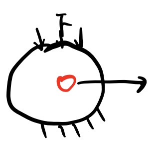
① 胡克定律： $\varepsilon_{xx} = \frac{1}{E}(\sigma_{xx} - \nu\sigma_{yy})$ $\varepsilon_{yy} = \frac{1}{E}(\sigma_{yy} - \nu\sigma_{xx})$ $\gamma_{xy} = \frac{\sigma_{xy}}{G}$ $G = \frac{E}{2(1+\nu)}$

$\sigma \Leftrightarrow \varepsilon$

② 几何方程 $\varepsilon_{xx} = \frac{\partial u}{\partial x}$, $\varepsilon_{yy} = \frac{\partial v}{\partial y}$, $\varepsilon_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}$

$\varepsilon \Leftrightarrow u$

③ 牛顿运动定律(只列平衡状态) $\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} = 0$, $\frac{\partial \sigma_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} = 0$

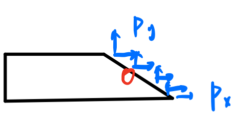


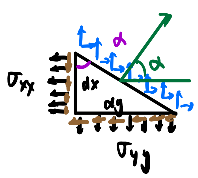
每个微元都要受力平衡

非平衡 $\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} = \rho\frac{\partial^2 u}{\partial t^2}$

④ 力边界条件： $\sigma_{xx}\cos\alpha + \sigma_{xy}\cos\beta = F_x$

$\sigma_{xy}\cos\alpha + \sigma_{yy}\cos\beta = F_y$





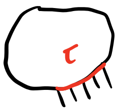
$\Sigma F_x = 0$ $\sigma_{xx}\cdot dx + \sigma_{xy}\cdot dy = P_x\cdot\sqrt{dx^2+dy^2}$

令 $\frac{dx}{\sqrt{dx^2+dy^2}} = \cos\alpha$ $\frac{dy}{\sqrt{dx^2+dy^2}} = \cos\beta$

$\sigma_{xx}\cos\alpha + \sigma_{xy}\cos\beta = P_x$

$\cos\alpha$, $\cos\beta$ 为边界外法线的方向余弦

⑤ 位移边界条件



$u=0,\ v=0$ ，在 Γ 上

弹性力学方程：

① $\dfrac{\partial \sigma_{xx}}{\partial x} + \dfrac{\partial \sigma_{xy}}{\partial y} = 0$

$\dfrac{\partial \sigma_{xy}}{\partial x} + \dfrac{\partial \sigma_{yy}}{\partial y} = 0$

② $\varepsilon_{xx} = \dfrac{1}{E}(\sigma_{xx} - \nu \sigma_{yy})$

$\varepsilon_{yy} = \dfrac{1}{E}(\sigma_{yy} - \nu \sigma_{xx})$

$\varepsilon_{xy} = \dfrac{\sigma_{xy}}{G}$

③ $\varepsilon_{xx} = \dfrac{\partial u}{\partial x}$, $\varepsilon_{yy} = \dfrac{\partial v}{\partial y}$, $\varepsilon_{xy} = \dfrac{\partial u}{\partial y} + \dfrac{\partial v}{\partial x}$

④ $\sigma_{xx}\cos\alpha + \sigma_{xy}\cos\beta = P_x$, $\sigma_{xy}\cos\alpha + \sigma_{yy}\cos\beta = P_y$

⑤ $u = v = 0$,（在 Γ 上）

比 $F=ma$ 要复杂的多