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Cable Routing Optimization Under Uncertainty

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Raphaël Guardies, Virgile Foussereau,

Contents

Introduction

Wind power is a crucial component in the shift towards sustainable and renewable energy to combat the effects of climate change. The design of a wind farm, however, involves a multifaceted process about the layout of the farm [1] but also the cable routing [2]. This last problem is about how to electrically connect the turbines of the farm in order to minimize the cost of the cables. Indeed, the expenses for electrical infrastructure account for 15–30% of the overall initial costs of an offshore wind farm [3]. This shows the importance of an effective optimization study about the cable routing of a wind farm.

The work presented in this report starts from the Mixed Integer Linear Programming (MILP) framework presented in the section 2 of [2]. First we extend the MILP to model the uncertainty related to the power production of the turbines. Then we present a two-stage stochastic optimization method that we compare to a more robust optimization based on a pessimistic approach.

1 Uncertainty in cable routing

Previous papers have considered the power production of each turbine as a value known in advance and constant. In [2], they consider two cases: either all turbines have the same power production or the turbines are allowed to have different production depending on their position. In both cases, they are considered to be definitely known before building the cable network. In this study, we consider that this is not the case. Indeed, power production of renewable energies such as wind power are very difficult to predict with precision. The power production distribution may change after construction of the wind farm, due to uncontrolled or unknown effects at the time of construction. Some part of the farm may produce less than expected while another part may produce more. In this case, the total power production will decrease if the cable routing was not designed to accommodate such changes.

For more clarity we take a toy example to illustrate the problem described above. In figure 1 we present a simple wind farm optimized in a deterministic way (i.e we do not consider stochastic power production). Each turbine (blue or red) has a power production of one while each cable has a capacity of five. The total production is therefore 8 units of power production.

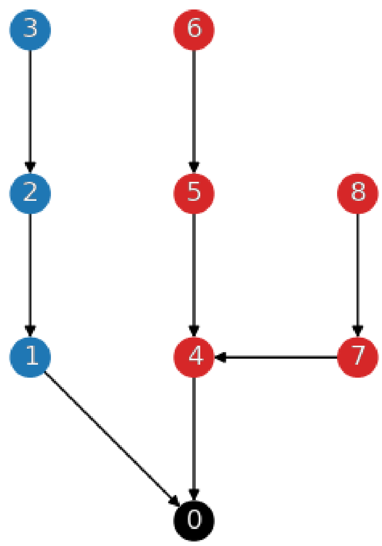


Figure 1: Toy example with deterministic optimization

Now, we consider that after construction, the power production has changed in a stochastic way. Turbines represented by a blue circle now produce only 0.5 while the ones in red now produce 1.5 unit of power production. The left part of the wind farm produces less but the right part cannot compensates because the power retrieved is limited by the capacity of the cable between 4 and 0 The total production drop to 6.5. In the long-term, this lower production could be very costly. To avoid that, the operator of the wind farm can do work to adapt the cable routing to the new situation, for example by linking 4 to 1 to redirect a part of the central flow. However, this is quite costly.

To avoid this situation, we can consider the possible change of the power production when deciding the initial cable routing. In figure 2 we present a different initial routing where stochastic changes of power production have been considered. By linking 7 to 0 instead of 4, this cable routing has a slightly higher initial cost but is more adaptable. If the changes described above happen, no adaptation work would be needed. In this case, the final cost will be lesser than the one of the deterministic cable routing presented in 1.

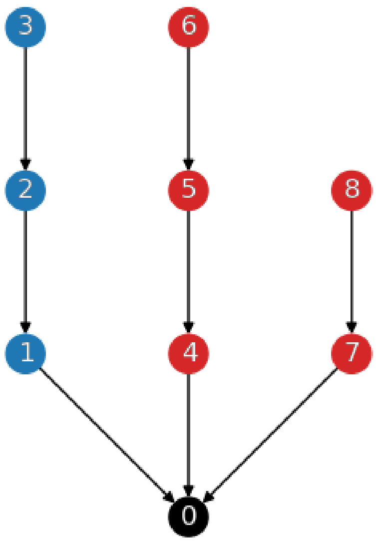


Figure 2: Toy example with stochastic optimization

2 Model description

2.1 Deterministic model

To model the situation described in 1, we start from the deterministic model presented in section 2 of [2] but we remove the no cable crossing constraint for the sake of simplicity. The description of this part of the model is issued from their paper [2].

Assuming that the best turbine positions have been identified, the next step for the deterministic method is to to find an optimal cable connection among all turbines and the given substation(s), minimizing the total cable cost. The deterministic model is based on the following requirements:

- the energy flow leaving a turbine must be supported by a single cable;
- different cables, with different capacities and costs, are available;
- the energy flow on each connection cannot exceed the capacity of the installed cable;
- a given maximum number of cables, say C, can be connected to each substation;

Let us consider the turbine positions as the nodes of a complete and loop-free directed graph $G = (V, A)$, and all possible connections between them as directed arcs. Some nodes correspond to the substations that are considered as the roots of the distribution network, and are the only nodes that collect energy. All nodes $h \in V$ have associated coordinates in the plane, that are used to compute distances between nodes as well as to determine whether two given line segments $[i, j]$ and $[h, k]$ cross each other, where $[a, b]$ denotes the line segment in the plane having nodes $a, b \in V$ as endpoints.

The node set V is partitioned into (V_T, V_0), where V_T contains the nodes corresponding to the turbines and V_0 contains the nodes corresponding to the substation(s). Furthermore, let $P_h \geq 0$ denote the power production at node $h \in V$, where $P_h > 0$ for $h \in V_T$ and (P_h being immaterial for $h \in V_0$).

Let T denote the set of different types of cable that can be used. Each cable type $t \in T$ has a given capacity $k_t \geq 0$ and a unit cost $u_t \geq 0$. Arc costs $c^t_{i,j} = u_t \cdot \text{dist}(i, j)$ are defined for each arc$(i, j) \in A$ and for each type $t \in T$, where dist(i, j) is the Euclidean distance between nodes i and j.

In the deterministic model, for each arc$(i, j) \in A$ a continuous variable $f_{i,j} \geq 0$ is used to represent the (directed) energy flow from i to j, and the binary variable $x^t_{i,j}$ with the following meaning:

$$x^t_{i,j} = \begin{cases} 1 \text{ if } \text{arc}(i,j) \text{ is constructed with cable type t} \quad (i,j) \in A, t \in T \\ 0 \text{ otherwise} \end{cases}$$

Finally, binary variables $y_{i,j}$ indicate whether an arc(i,j) is built with any type of cable, i.e. $y_{i,j} = \sum_{t \in T} x^t_{ij}, \quad (i,j) \in A$.

The deterministic part of our MILP model then reads:

$$\min \sum_{(i,j) \in A} \sum_{t \in T} c^t_{i,j} x^t_{i,j} \tag{1}$$

$$\sum_{t \in T} x^t_{i,j} = y_{i,j}, (i,j) \in A \tag{2}$$

$$\sum_{i \in V : i \neq h} (f_{h,i} - f_{i,h}) = P_h, h \in V_T \tag{3}$$

$$\sum_{t \in T} k_t x^t_{i,j} \geq f_{i,j}, (i,j) \in A \tag{4}$$

$$\sum_{j \in V : j \neq h} y_{h,j} = 1, h \in V_T \tag{5}$$

$$\sum_{j \in V : j \neq h} y_{h,j} = 0, h \in V_0 \tag{6}$$

$$\sum_{i \in V : i \neq h} y_{i,h} \leq C, h \in V_0 \tag{7}$$

$$x^t_{i,j} \in \{0,1\}, (i,j) \in A, t \in T \tag{8}$$

$$y_{i,j} \in \{0,1\}, (i,j) \in A \tag{9}$$

$$f_{i,j} \geq 0, (i,j) \in A \tag{10}$$

The objective function (1) minimizes the total cable layout cost. Constraints (2) impose that only one type of cable can be selected for each built arc, and define the $y_{i,j}$ variables. Constraints (3) are flow conservation constraints: the energy (flow) exiting each node h is equal to the energy entering h plus the power production of that node. Note that these constraints are not imposed for $h \in V_0$, i.e., when h corresponds to a substation. Constraints (4) ensure that the flow does not exceed the capacity of the installed cable. Constraints (5) impose that only one cable can exit a turbine and constraints (6) that none can exit the substation (tree structure with root in the substation). Constraint (7) imposes the maximum number of cables (C) that can enter each substation.

Note that the above constraints imply that the y variables define a set of connected components (one for each substation), each component containing a directed tree with out-degree at most one (i.e. an anti-arborescence) rooted at a substation. Circuits are not explicitly forbidden in the model, however because of (3) they cannot arise among our nodes.

2.2 Stochastic model

Now that we have define the initial deterministic part of our model, we can model the stochastic change of production power we want to introduce in this report.

We first a set of scenarios W and a parameter $P_{h,w}^{deviation}$ which is the deviation of power production for each turbine $h \in V$ in each considered scenario $w \in W$. In each scenario, if necessary, the wind farm operator has to adapt its cable routing handle the change in power production.

To model this, we introduce three new variables which are: $x_{i,j,t,w}^{add}$, $y_{i,j,w}^{add}$ and $f_{i,j,w}^{new}$ for $(i,j) \in A, t \in T, w \in W$. $x_{i,j,t,w}^{add}$ is a binary variable indicating if a cable of type t is build between nodes i and j in scenario w, in addition to the initial cable routing to adapt it to the scenario w. As in the first part of the model, binary variables $y_{i,j,w}^{add}$ indicate whether an arc(i,j) is built in scenario w with any type of cable, i.e., $y_{i,j,w}^{add} = \sum_{t \in T} x_{i,j,t,w}^{add}, \quad (i,j) \in A, w \in W$. Finally, $f_{i,j,w}^{new}$ denote the new power flow between nodes i and j, after adaptation work due to scenario w.

These variables are subject to similar constraints than their deterministic counterpart with a slight modification on the one corresponding to equation (3). For the new variables with have instead of equation (3):

$$\sum_{i \in V: i \neq h} (f_{h,i,w} - f_{i,h,w}) \leq P_h + P_{h,w}^{deviation}, h \in V_T, w \in W$$

$$\sum_{i \in V: i \neq h} f_{i,h,w} \geq P_w^{min}, h \in V_0, w \in W$$

The first equation corresponds to (3) but allows the model to not retrieve all power produced by a turbine. In return, the second equation ensures that we retrieve at the substation a sufficient amount of power. The definition of P_w^{min} is a choice:

(A) - If we choose $P_w^{min} = \sum_{h \in V} P_h + P_{h,w}^{deviation}, w \in W$, it means we want to retrieve all power produced by every turbine (as in the deterministic model).
(B) - However we could also choose $P_w^{min} = min(\sum_{h \in V} P_h, \sum_{h \in V} P_h + P_{h,w}^{deviation}), w \in W$. In this case, we only want to retrieve at least as much power as initially planned or, if it is no possible, all the power we can.

For example, in a scenario where all turbines have their power production increased, choice (A) would lead to a lot of adaptation work while choice (B) would not necessitate any adaptation work. This choice depends of the wind farm operator preference between power production or cost. In the rest of our study, we will consider that choice (A) has been made: we want to retrieve all power produced by the turbines, even after perturbation.

Finally, the objective function (1) is changed to include the cost of the possible adaptation works. We consider two cases:

- Stochastic optimization: we add the weighted mean of the adaptation work cost across all scenarios. In this case, all scenarios' adaptation costs are weighted by their probability p_w and

summed. We call this cost the **average stochastic cost**. It is added to the initial construction cost of the cable routing, that we will from now on call **deterministic cost**.
- Pessimistic optimization: we consider only the scenario with the worst adaptation work cost, that we will call **pessimistic cost**, and add it to the deterministic cost.

Our complete MILP model then reads with the new stochastic part in red:

$$\min \sum_{(i,j)\in A} \sum_{t\in T} c_{i,j}^t x_{i,j}^t + \sum_{w\in W} p_w \sum_{(i,j)\in A} \sum_{t\in T} c_{i,j}^t x_{i,j,t,w}^{add} \tag{1}$$

$$\sum_{t\in T} x_{i,j}^t = y_{i,j}, \ (i,j)\in A \tag{2}$$

$$\sum_{i\in V: i\neq h} (f_{h,i} - f_{i,h}) = P_h, \ h\in V_T \tag{3}$$

$$\sum_{t\in T} k_t x_{i,j}^t \geq f_{i,j}, \ (i,j)\in A \tag{4}$$

$$\sum_{j\in V: j\neq h} y_{h,j} = 1, \ h\in V_T \tag{5}$$

$$\sum_{j\in V: j\neq h} y_{h,j} = 0, \ h\in V_0 \tag{6}$$

$$\sum_{i\in V: i\neq h} y_{i,h} \leq C, \ h\in V_0 \tag{7}$$

$$x_{i,j}^t \in \{0,1\}, \ (i,j)\in A, t\in T \tag{8}$$

$$y_{i,j} \in \{0,1\}, \ (i,j)\in A \tag{9}$$

$$f_{i,j} \geq 0, \ (i,j)\in A \tag{10}$$

$$\sum_{t\in T} x_{i,j,t,w}^{add} = y_{i,j,w}^{add}, \ (i,j)\in A, w\in W \tag{11}$$

$$\sum_{i\in V: i\neq h} \left(f_{h,i,w}^{new} - f_{i,h,w}^{new}\right) \leq P_h + P_{h,w}^{deviation}, \quad h\in V_T, w\in W \tag{12}$$

$$\sum_{i\in V: i\neq h} f_{i,h,w}^{new} \geq P_w^{min}, \ h\in V_0, w\in W \tag{13}$$

$$\sum_{t\in T} k_t x_{i,j,t,w}^{add} \geq f_{i,j,w}^{add}, \ (i,j)\in A, w\in W \tag{14}$$

$$\sum_{j\in V: j\neq h} y_{h,j,w}^{add} = 0, \ h\in V_0, w\in W \tag{15}$$

$$\sum_{i\in V: i\neq h} y_{i,h,w}^{add} \leq C, \ h\in V_0, w\in W \tag{16}$$

$$x_{i,j,t,w}^{add} \in \{0,1\}, \ (i,j)\in A, t\in T \tag{17}$$

$$y_{i,j,w}^{add} \in \{0,1\}, \ (i,j)\in A, w\in W \tag{18}$$

$$f_{i,j,w}^{new} \geq 0, \ (i,j)\in A, w\in W \tag{19}$$

3 Experiments and results

To test our model, we have developed a pipeline that we used on data from wind farm Kentish Flats, located close to Kent in South East England. The data is issued from [2]. In our pipeline, we randomly samples n scenarios and then try 3 different solutions:

- (1) Deterministic optimization: We solve the deterministic optimization problem presented in 2.1. We fix the solution $(x^t_{i,j}, y_{i,j}, f_{i,j})$ and then solve again using the complete model to find $x^{add}_{i,j,t,w}$, $y^{add}_{i,j,w}$ and $f^{new}_{i,j,w}$.
- (2) Stochastic optimization: We initialize the solver with the previous solution, unfix $(x^t_{i,j}, y_{i,j}, f_{i,j})$ and solve the complete problem presented in 2.2.
- (3) Pessimistic optimization: We initialize the solver with the previous solution but instead of considering the average stochastic cost, we solve considering only the cost of the worst scenario in the objective function.

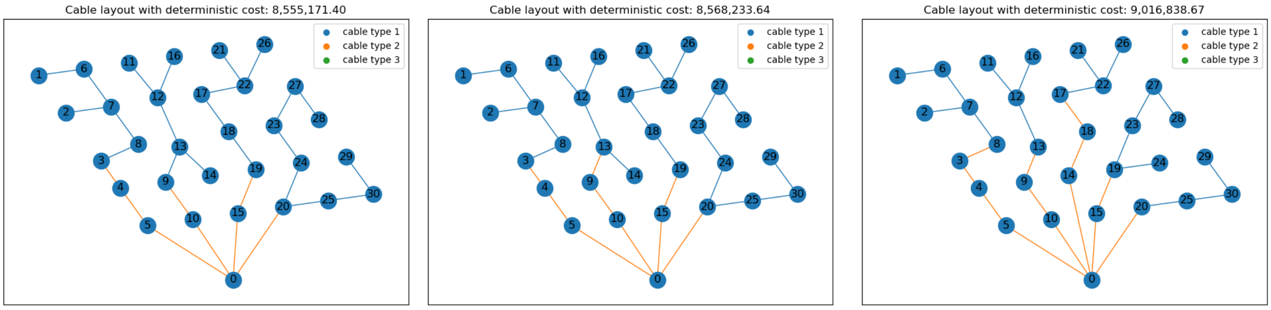
Solution (1) will give a lower bound on the deterministic cost (initial cost of construction) but with a higher stochastic and pessimistic cost. Solution (3) will give a lower bound on the stochastic and pessimistic cost but with a higher deterministic cost. Solution (2) ise a trade-off between the two.

3.1 Experiment 1

In our first experiment, we try our pipeline with $n = 30$ scenarios. We run a cplex solver during 1h for each solution. Using more scenarios would be more accurate, but also more computationally expensive. In the original data, all turbines have a power production of 1. We sample $P^{deviation}_{h,w}$ from a normal distribution with mean 0 and standard deviation 0.5. To avoid outliers which are physically impossible (such as negative power) we cap the deviation between -0.5 and 0.5. The results are presented in table 1 and illustrated in figure 3.

	Deterministic Optimization	Stochastic optimization	Pessimistic optimization
Deterministic cost	8,555,171.40	8,568,233.64	9,016,838.67
Average stochastic cost	748,375.29	679,488.27	179,694.49
Pessimistic cost	1,486,668.04	1,374,026.88	305,508.77

Table 1: Results of our different solutions on experiment 1



(a) Deterministic optimization (b) Stochastic optimization (c) Pessimistic optimization

Figure 3: Cable routing obtained by our three solutions for experiment 1

As we can see in the table 1, the first solution has the lowest deterministic cost (initial cost of construction of the cable network) but could be quite costly to adapt as shown with the high stochastic and pessimistic cost. The solution 2 proposes a cable routing costing only ∼\$13,000 more for the deterministic cost, but more adaptable as it would cost around \$100,000 less in adaptation work in the event that one of the imagined scenarios should occur. Finally, the solution 3 is far more expensive for the initial construction but drastically reduces the cost of a scenario happening, especially if the worst scenario were to happen.

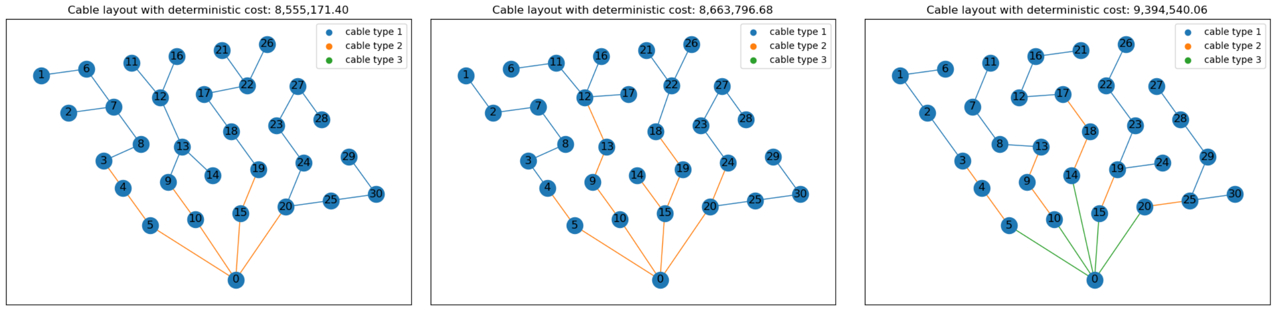
It is interesting to visualize the differences between each cable routing in figure 3. For instance, we can see that both stochastic and pessimistic solution use more cable of type 2 than the solution of the deterministic problem. Indeed, cable of type 2 can more easily handle a change in power production. We also see that the pessimistic optimization creates five smaller clusters connected to the substation instead of five bigger ones, again to increase the adaptability of the cable network. It is also a useful outcome when considering that some cables might break. This is a problem studied by [4], in which they use a close-loop structure to cop with cable failures. While not addressing this issue directly, our model and method improve redundancy against it by preferring smaller clusters, thus making a wind farm less impacted by a cable failure.

3.2 Experiment 2

In our second experiment we wanted to push our model to its limits. Instead of picking random scenarios of change in power production, we manually created an edge-case scenario and tested our pipeline. The results are presented in 2.

	Deterministic Optimization	Stochastic optimization	Pessimistic optimization
Deterministic cost	8,555,171.40	8,663,796.68	9,394,540.06
Average stochastic cost	2,456,297.15	1,417,477.53	0.0
Pessimistic cost	2,456,297.15	1,417,477.53	0.0

Table 2: Results of our different solutions on experiment 2



(a) Deterministic optimization (b) Stochastic optimization (c) Pessimistic optimization

Figure 4: Cable routing obtained by our three solutions for experiment 2

This experiment is very interesting because table 2 shows that for an increased initial cost of only ∼\$8,000, the stochastic solution lower the cost of the potential adaptation work by ∼\$1M. This is an investment that most wind farm operator would probably be willing to make. The pessimistic solution has a far more expensive initial cost but lower the cost of potential adaptation work to 0. This result, illustrated in figure 4 is achieved by using high capacity cables of type 3 on crucial edges between the substation and the clusters, as well as creating smaller clusters as in our first experiment. If the probability of the scenario we created was non-negligible in a real world case, this is the solution we would choose. Otherwise, the trade-off solution is probably a better choice.

Conclusion

In conclusion, the optimization of cable routing is an important consideration in the design of wind farms. This study shows that taking into account the uncertainty of power production can lead to a more efficient cable routing that can accommodate changes in power production without the need for costly adaptations. The proposed model extends the Mixed Integer Linear Programming framework to model the uncertainty related to power production and presents a two-stage stochastic optimization method. This method is compared to a more robust optimization based on a pessimistic approach. The results both our methods taking into account the uncertainty can produce interesting results compared to the deterministic approach. The choice of the scenarios to consider and the approach (stochastic or pessimistic) can be made in real life by the wind farm operator using expert advice for each specific situation. Overall, this study highlights the importance of considering uncertainties in the optimization of cable routing to ensure the long-term sustainability and profitability of wind farms.

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