

Section - 6

Array Representations

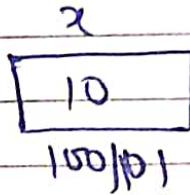
Introduction to Array :-

Arrays :-

`int x=10`



scalar variable



Array is a collection of elements of similar data types.

`int A[5];`

vector
variable

A

0	1	2	3	4
100/1	203/3	15	204/5	206/7

$$A[2] = 15$$

- Declaration of Arrays :-

① `int A[5];`

A	0	1	2	3	4
	?	?	?	?	?

garbage

② `int A[5] = {2, 4, 6, 8, 10};`

A	0	1	2	3	4
	2	4	6	8	10

③ `int A[5] = {2, 4};`

A	0	1	2	3	4
	2	4	0	0	0

④ `int A[5] = {0};`

A	0	1	2	3	4
	0	0	0	0	0

⑤ `int A[] = {2, 4, 6, 8, 10, 12};`

A	0	1	2	3	4	5
	2	4	6	8	10	12

int A[5] = {2, 5, 4, 9, 8};

A	0	1	2	3	4
	200/1	202/3	204/5	206/7	208/9

cout << A[0];

cout << A[i];

for (i=0; i<5; i++)

cout << A[i];

cout << A[2];

or

cout << a[A];

cout << *(A+2)

Static vs Dynamic Array.

Static array means size of array is static.

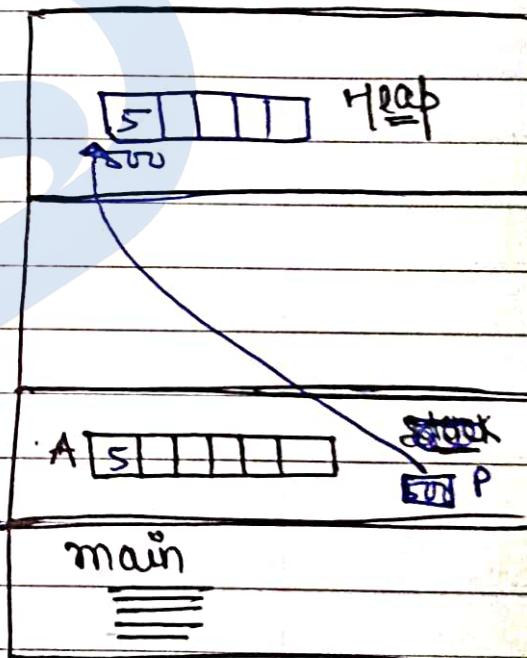
Dynamic array means size of array is dynamic.

Static

Void main ()

{
 int A[5]; → C

size of array is decided at run time.



+ + int n; cin > n;

int B[n]; → C++

↓
size of array is decided at run time

For Accessing Heap, we need pointer

Dynamic Array

void main()

{

int A[5];

int *p;

p = new int[5]; → C++

p = (int *)malloc(5 * sizeof(int)); → C

If unused memory is not deleted then
it cause memory leak.

delete [] p; → C++

free(p); → C

How access the array in heap :-

int A[5];

using int *p; → C/C++

p = new int[5];

$A[0] = 5;$

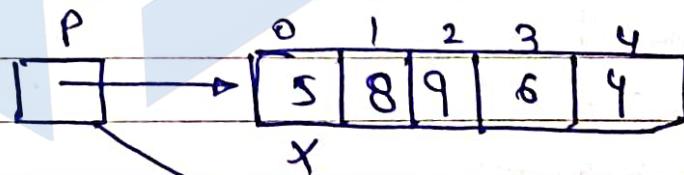
$P[0] = 5;$
↑

It acts as Heap Array name.

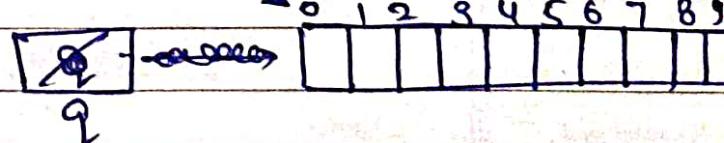
Stack Array size can be resize.

How to Increase Array Size :-

`int *p = new int[5];`



`int *q = new int[10]`



`for(i=0; i<5; i++)`

{

$q[i] = p[i];$

}

delete [] p;

p = q

q = NULL; what will be

array is not grown because memory size is
contiguous.

Value 1 2 3 4 5 6 7 8 9

p a [p] b [c] d e f g h i

y

so position = p + 10

position = 10

so add 221 to 1107

1109 + 2108

2D Arrays :-

①

$\text{int } *A[3][4]; = \{ \{1, 2, 3, 4\}, \{2, 4, 6, 8\}, \{3, 5, 7, 9\} \}$

		A			
		0	1	2	3
0	0	2/1	1/3	4/5	6/7
	1	8/9	10/11	12/13	14/15
2	16/17	18/19	20/21	22/23	

$$A[1][2] = 15;$$

②

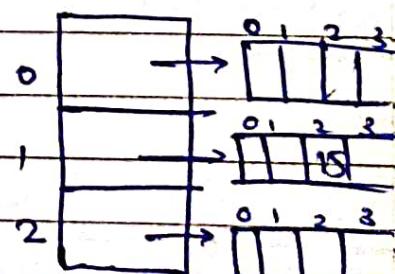
$\text{int } *A[3];$

$A[0] = \text{new int}[4];$

$A[1] = \text{new int}[4];$

$A[2] = \text{new int}[4];$

$$A[1][2] = 15;$$



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(3)

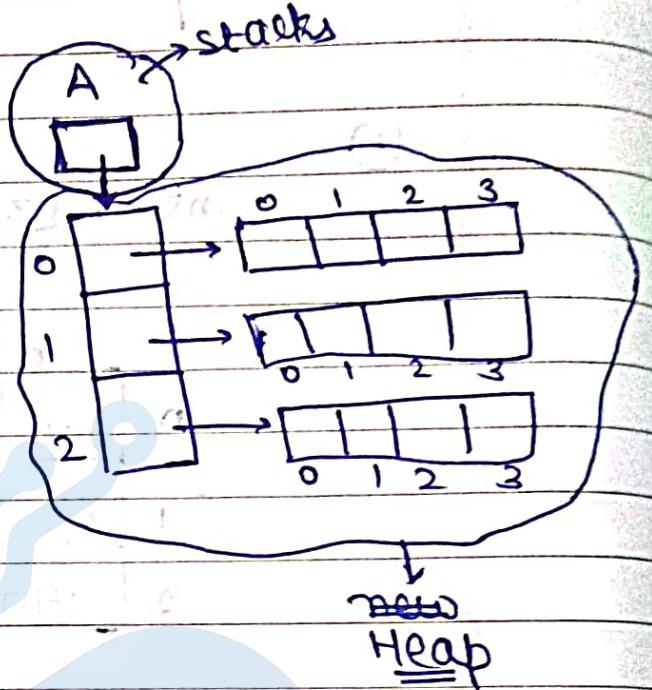
`int **A;`

`5, 3, 8, 10, A = new int*[3];`

`A[0] = new int[4];`

`A[1] = new int[4];`

`A[2] = new int[4];`



`for (i=0; i<3; i++)`
 `{`

`for (j=0; j<4; j++)`
 `{`

`A[i][j] = x; + 70`

`} [EP] for loop = 507A`

`{ [H] for loop = [I] A`

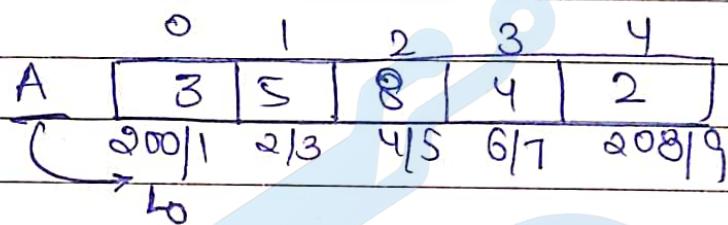
`[EP] for loop = [S] A`

`S = [S][I] A`

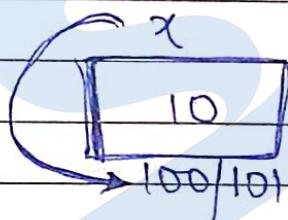
Array Representation by Compiler

Arrays in compilers

int A[5] = {3, 5, 8, 4, 2};



int x = 10;



A[3] = 10;

Compiler needs Address (A[3])

$$\begin{aligned} \text{Address}(A[3]) &= 200 + 3 \times 2 \\ &= 206 \end{aligned}$$

$$\text{Address}(A[3]) = Lo + 3 \times \text{Size of int}$$

$$\text{Address}(A[i]) = \underbrace{Lo}_{\text{base Address}} + i * \underbrace{w}_{\text{index}} \leftarrow \begin{array}{l} \text{size of} \\ \text{data type} \end{array}$$

$\text{int } A[1.....5]$

$$A[3] = 10;$$

$$\begin{aligned} \text{Address}(A[3]) &= 200 + (3-1) * 2 \\ &= 204 \end{aligned}$$

$$\boxed{\text{Address}(A[i]) = lo + (i-1) * w}$$

Row Major formula in 2D Arrays :-

$\text{int } A[3][4]; \quad A[m][n]$

A	0	1	2	3	4	5	6	7	8	9	10	11
	a ₀₀	a ₀₁	a ₀₂	a ₀₃	a ₁₀	a ₁₁	a ₁₂	a ₁₃	a ₂₀	a ₂₁	a ₂₂	a ₂₃
	8/0	2/3	4/5	6/7	8/9	10/11	12/13	14/15	16/17	18/19	20/21	22/23
	Row 0				Row 1				Row 2			

A 0 1 2 3

0	a ₀₀	a ₀₁	a ₀₂	a ₀₃
1	a ₁₀	a ₁₁	a ₁₂	a ₁₃
2	a ₂₀	a ₂₁	a ₂₂	a ₂₃

Row
Major
Mapping

$$A[1][3] = 10;$$

$$\begin{aligned} \text{Address}(A[1][2]) &= 200 + [4+1+2]*2 \\ &= 200 + 12 \Rightarrow 212 \end{aligned}$$

Two types of mapping :-

- Row-major Mapping
- Column-major Mapping

$$\text{Address}(A[2][3]) = 200 + (8 \times 4 + 3) \times 2$$

$$= 200 + 22$$

$$= \underline{\underline{202}}$$

for Row Major

$$\boxed{\text{Address}(A[i][j]) = L_0 + [i \times n + j] \times w}$$

`int A[1.....3][1.....4]`

$$\boxed{\text{Address}(A[i][j]) = L_0 + [(i-1) \times n + (j-1)] \times w}$$



costlier and take more time as compare
to above formula.

Column Major formula for 2D Arrays :-

	0	1	2	3
0	a ₀₀	a ₀₁	a ₀₂	a ₀₃
1	a ₁₀	a ₁₁	a ₁₂	a ₁₃
2	a ₂₀	a ₂₁	a ₂₂	a ₂₃

0	1	2	3	4	5	6	7	8	9	10	11
a ₀₀	a ₁₀	a ₂₀	a ₀₁	a ₁₁	a ₂₁	a ₀₂	a ₁₂	a ₂₂	a ₀₃	a ₁₃	a ₂₃
200/1	213/4	415/6	617/8	819/10	1011/12	1213/14	1415/16	1617/18	1819/20	2021/22	2223/24

↑ col 0 ↑ col 1 ↑ col 2 ↑ col 3

No. of Col. * size

$$\begin{aligned}
 \text{Address}(A[1][2]) &= 200 + (2 * 3 + 1) * 2 \\
 &= 214, \quad \text{Total element in column}
 \end{aligned}$$

$$\begin{aligned}
 \text{Address}(A[1][3]) &= 200 + (3 * 3 + 1) * 2 \\
 &= 200 + (9 + 1) * 2 \\
 &= 260
 \end{aligned}$$

$$= \underline{\underline{220}}$$

formula for Column Major $\rightarrow A[m][n]$

$$\text{Address}[A[i][j]) = \text{Lo} + [j * m + i] * w$$

\nwarrow right-left

$$\text{Address}[A[i][j]) = \text{Lo} + [i * n + j] * w$$

\nwarrow left-right

Row Major

\rightarrow C/C++ follows row-major

Formulas for nD Arrays :-

Array in compilers

4D

Type $A[d_1][d_2][d_3][d_4]$

$$\text{Addr}(A[i_1][i_2][i_3][i_4]) = L_0 + [i_1 \times d_2 \times d_3 \times d_4 + i_2 \times d_3 \times d_4 + i_3 \times d_4 + i_4]$$

→ Row Major formula for 4D Array

$$= L_0 + \sum_{p=1}^n [i_p * \prod_{q=p+1}^n d_q] * w$$

$$\text{Addr}(A[i_1][i_2][i_3][i_4]) = L_0 + [i_4 \times d_3 \times d_2 \times d_1 + i_3 \times d_2 \times d_1 + i_2 \times d_1 + i_1] * w$$

→ Column Major formula for 4D Array

$$\text{Addr} = L_0 + \sum_{p=n}^1 [i_p * \prod_{q=p+1}^1 d_q] * w$$

General formulae

$$\text{Addr}[A[i] \dots [i_n]] = \text{Lo} + \sum_{p=1}^n [i_p * \sum_{q=p+1}^n d_q] * w$$

→ Row Major formulae for nD Array

~~Analyzing~~

Analyzing the Row Major formulae for nD Array :-

* multiplication :-

$$4D \rightarrow 3+2+1$$

$$5D \rightarrow 4+3+2+1$$

$$nD \rightarrow n-1+n-2+\dots+3+2+1$$

$$= \frac{n(n-1)}{2}$$

$$= O(n^2)$$

Hornel's Rule

$$i_4 + i_3 \times d_4 + i_2 \times d_3 \times d_4 + i_1 \times d_2 \times d_3 \times d_4$$

$$i_4 + d_4 * [i_3 + i_2 * d_3 + i_1 * d_2 * d_3]$$

$$i_4 + d_4 * [i_3 + d_3 (i_2 + i_1 * d_2)]$$

↑ ↑ ↑
 | | |
 i_3 d_3 (i_2 + i_1 * d_2)

for $4D \rightarrow 3$ multiplication

$5D \rightarrow 4$ multiplication

$$nD = n-1$$

$$\underline{\underline{O(n)}}$$

By taking common, we can reduce the time complexity of Row Major formulae.

Column Major:-

$$\text{Addrs}([i_1] - \dots - [i_n]) = L_0 + \sum_{p=n}^1 [i_p * \prod_{q=p-1}^1 d_q] * w$$

↳ Row major formulae for nD array
column

Formulae for 3D Array :-

int A[l][m][n];

Address of $A[i][j][k]$ in Row-major :-

Row-major :-

$$\text{Addr}[A[i][j][k]] = \text{Lo} + [i \times m \times n + j \times n + k] * w$$

left \rightarrow right

Column

Column Major :-

$$\text{Addr}[A[i][j][k]] = \text{Lo} + [k \times m \times l + j \times l + i] * w$$

right - left

int A[m][n]

int A[l][m][n]

Quiz:-

- let A be two-dimensional array declared as follows:

A: array[1.....10][1.....15] of integers

Assuming that each integer takes one memory location. The array is stored in row-major order and the first element of the array is stored at location 100, what is the address of the element $A[i][j]$?



$$L_0 = 100$$

$$L[10][15]$$

$$W = 1$$

$$\text{Address}(A[i][j]) = L_0 + [(i-1)*n + (j-1)] * W$$

$$= 100 + [(i-1)*15 + j-1] * 1$$

$$= 100 + 15i - 15 + j - 1$$

$$= 15i + j + 84$$

what is the output of the following c code?
Assume that the address of x is 2000
(in decimal) and an integer requires four
bytes of memory.

```
int main()
{
```

unsigned int $x[4][3] = \{ \{1, 2, 3\}, \{4, 5, 6\}, \{7, 8, 9\}, \{10, 11, 12\} \};$

$\text{printf}(\%^{4.4}, \%^{4.4}, \%^{4.4}, x+3, *(x+3), *(x+2)+3);$

}



$\text{sizeof}(int) = 4$

$x[4][3] = \{ \{1, 2, 3\}, \{4, 5, 6\}, \{7, 8, 9\}, \{10, 11, 12\} \}$

		0	1	2
0	1	2	3	
1	4	5	6	
2	7	8	9	
3	10	11	12	

$\rightarrow *(*(x+2)+1)$

row ↑ ↓ column

$$x+3 = 2036$$

$$*(x+3) = 2036$$

$$*(x+2)+3 = 2036$$

If A and B are two matrices, for multiplying $A \times B$ which if the following representation will be efficient.

- a) A in Row-major and B in Column-major Representations.
- b) A in Column-major and B in Row-major Representation.
- c) Both A and B in Row Major Representation.
- d) Independent of Representation

→ A and B are matrices

for $A \times B$ which mapping if efficient

Row Major \rightarrow $l_0 + (i * n + j) * w$

Column Major \rightarrow $l_0 + (j * m + i) * w$

(d) Independent of Representation

- if an 3D Array is declared in C language as follows?

$x[?][?][?]$

find data type and dimensions of array
if compiler performs following intermediate operations for finding Address of any location
 $x[i][j][k]$?

$$t_0 = i * 1024$$

$$(t_1 = j * 32) \rightarrow m * w = 32$$

$$(t_2 = k * 4) \rightarrow w = 4$$

$$t_3 = t_0 + t_1$$

$$t_4 = t_3 + t_2$$

$$t_5 = x[t_4]$$

$$\text{Assume } m * n * w = 1024 \\ m = \frac{1024}{32} \Rightarrow 32$$

$$n = 8$$

(Assume int takes 2 bytes, float takes 4 bytes)

→ type $x[l][m][n]$

$$\text{Address}(x[i][j][k]) = L_0 + [i * m * n + j * n * w + k * w]$$

$$= L_0 + i * m * n * w + j * n * w + k * w$$

$$w = 4$$

$$n = 8$$

$$m = 32$$

→ (1)

from \oplus above

Address ($X \in [0, 1]^K$) \Rightarrow

float [$x[?][32][\cancel{8}]$]

Anything