Section 9

Matrices

Section Introduction

Special Matrices

1. Diagonal Matrix
2. Lower Triangular Matrix
3. Upper Triangular Matrix
4. Symmetric Matrix
5. Tridiagonal Matrix
6. Band Matrix
7. Toeptitz Matrix
8. Sparse Matrix

square matrix
↓
n x n
↳ 5 x 5

Diagonal Matrix

A square matrix in which every element except the principal diagonal element is zero is called a Diagonal matrix.

$$M = \begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & 3 & 0 & 0 & 0 & 0 \\ 2 & 0 & 7 & 0 & 0 & 0 \\ 3 & 0 & 0 & 4 & 0 & 0 \\ 4 & 0 & 0 & 0 & 9 & 0 \\ 5 & 0 & 0 & 0 & 0 & 6 \end{array}$$

$M[i][j] = 0$ if $\underline{i \neq j}$

Memory takes

$5 \times 5 = 25$

~~$25 \times 2 = 50$ bytes~~

representation in single Array

A

3	7	4	9	6
0	1	2	3	4

M[i, j] ≠
if(i==j)
A[i-1];

```
int A[5];

void set(int A[], int i, int j, int x)
{
        if(i==j)
        {
            A[i-1]=x;

        }

 }

~~void~~int  get(int A[], int i, int j)
   {

          if(i==j)
          {
              return A[i-1];

          }
          else
          {
              return 0;
           }

    }
```

Lower Triangular Matrix Row-Major Mapping :-

$$\xrightarrow{j}$$

$$i \downarrow \begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & a_{11} & 0 & 0 & 0 & 0 \\ 2 & a_{21} & a_{22} & 0 & 0 & 0 \\ 3 & a_{31} & a_{32} & a_{33} & 0 & 0 \\ 4 & a_{41} & a_{42} & a_{43} & a_{44} & 0 \\ 5 & a_{51} & a_{52} & a_{53} & a_{54} & a_{55} \end{array} \quad 5 \times 5$$

A square matrix in which all the elements above the principal diagonal are zero is called a lower triangular matrix.

$$M[i,j] = 0 \quad \text{if } i < j$$

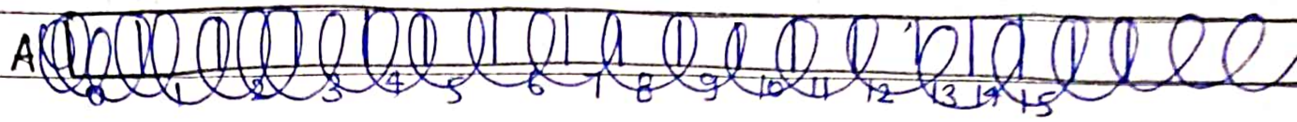
$$M[i,j] = \text{non-zero} \quad \text{if } i >= j$$

No. of Non-zero = $1 + 2 + 3 + 4 + 5$.

for $n \times n$ $= 1 + 2 + 3 + 4 \text{ ---- } n$

$$= \frac{n(n+1)}{2}$$

$$\boxed{\text{Zero element} \Rightarrow n^2 - \frac{n(n+1)}{2} \Rightarrow \frac{n(n-1)}{2}}$$



~~Row May~~

A	a_{11}	a_{21}	a_{22}	a_{31}	a_{32}	a_{33}	a_{41}	a_{42}	a_{43}	a_{44}	a_{51}	a_{52}	a_{53}	a_{54}	a_{55}
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
	row1	row2		row3			row4				row5				

↳ store as rows-wise ↑ (Row major method)

↳ store as column wise

Row Major method

$$\text{Index}(A[4][3]) = [1+2+3]+2 \Rightarrow 8$$

$$\text{Index}(A[5][4]) = [1+2+3+4]+3 \Rightarrow 13$$

$$\text{Index}(A[i][j]) = \left[\frac{i(i-1)}{2}\right]+j-1$$

Lower Triangular matrix column Major Mapping:-

column-major

A	a_{11}	a_{21}	a_{31}	a_{41}	a_{51}	a_{22}	a_{32}	a_{42}	a_{52}	a_{33}	a_{43}	a_{53}	a_{44}	a_{54}	a_{55}
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
			col 1				col 2				col 3		col 4		col 5

Index(A[4][4]) = ~~(4+4+3~~
$[5+4+3]+0$

$\Rightarrow$ 12 ✓

Index(A[5][4]) = $(5+4+3)+1$

$\Rightarrow 13$

Index(A[5][3]) = $(5+4$ ~~+02~~$)+2$

$= 11$

$$\boxed{\text{Index}(A[i][j]) = \left[\left(n + n-1 + n-2 \ldots\ldots n-(j-2)\right)\right] + (i-j)}$$

$$= \left[n(j-1) - [1+2+3+ \ldots j-2]\right] + (i-j)$$

$$\boxed{\text{Index}(A[i][j]) = \left[n(j-1) - \frac{(j-2)(j-1)}{2}\right] + [i-j]}$$

C++ Class for Diagonal Matrix

```
class Diagonal
{
      private:
              int n;
              int *A;

        public:
              Diagonal (int n)
              {
                    this->n = n;
                      A = new int[n];

                }
                void  set(int i, int j, int x);
                int  get (int i, int j);
                void  Display ();

                ~Diagonal ( );
                  {      delete[] A;
};                }

void Diagonal :: set (int i, int j, int x)
{
        if (i==j)
        {
              A[i-1] = x;
        }
}
```

```
int Diagonal :: get (int i, int j)
{

        if (i==j)
            return A[i-1];

        else
            return 0;

  }

   void Diagonal :: Display ( )
   {
        for (i=0; i<n; i++)
        {
            for (j=0; j<n; j++)
            {
                if(i==j)
                {
                    cout<< A[i-1];
                }
                else
                {
                    cout<<"0" ;

                }

            }
            cout<<endl;
        }
   }
```

Upper Triangular Matrix Row Major Mapping →

$$\xrightarrow{j}$$

$$M = \begin{array}{cc|ccccc|} & & 1 & 2 & 3 & 4 & 5 \\ i \downarrow & 1 & a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ & 2 & 0 & a_{22} & a_{23} & a_{24} & a_{25} \\ & 3 & 0 & 0 & a_{33} & a_{34} & a_{35} \\ & 4 & 0 & 0 & 0 & a_{44} & a_{45} \\ & 5 & 0 & 0 & 0 & 0 & a_{55} \end{array}$$

5x5

~~M[i][j] =~~

$M[i, j] = 0$ if $i > j$

$M[i, j] =$ non-zero $i <= j$

No. of non-zero $= 5 + 4 + 3 + 2 + 1$

$= n + n-1 + n-2 + \cdots = \frac{n(n+1)}{2}$

zero element $= \frac{n(n-1)}{2}$

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A	a_{11}	a_{12}	a_{13}	a_{14}	a_{15}	a_{22}	a_{23}	a_{24}	a_{25}	a_{33}	a_{34}	a_{35}	a_{44}	a_{45}	a_{55}
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
	row1					row2				row3			row4		row5

$$\text{Index}(A[4][5]) = (5+4+3) + \phi$$

$$= 13$$

~~Index(A[~~

$$\text{Index}(A[i][j]) = (n + n-1 + n-2 + \dots n-(i-2)] + (j-i)$$

$$\boxed{\text{Index}(A[i][j]) = \left[(i-1)*n - \frac{(i-2)(i-1)}{2}\right] + (j-i)}$$

Upper Triangular Matrix column-Major Mapping :-

A	a_{11}	a_{12}	a_{22}	a_{13}	a_{23}	a_{33}	a_{14}	a_{24}	a_{34}	a_{44}	a_{15}	a_{25}	a_{35}	a_{45}	a_{55}
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
	col1	col2		col3			col4				col5				

$$Index(A[4][5]) = (1+2+3+4)+3$$

$$= 13 \checkmark$$

$$Index(A[i][j]) = [1+2+3+ \text{----} \; j-1] + (i-1)$$

$$\boxed{Index(A[i][j]) = \frac{j\times(j-1)}{2} + \underline{\underline{(i-1)}}}$$

Symmetric Matrix :-

$$M = \begin{array}{c|ccccc} i \downarrow \quad \xrightarrow{} j & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & 2 & 2 & 2 & 2 & 2 \\ 2 & 2 & 3 & 3 & 3 & 3 \\ 3 & 2 & 3 & 4 & 4 & 4 \\ 4 & 2 & 3 & 4 & 5 & 5 \\ 5 & 2 & 3 & 4 & 5 & 6 \end{array}$$

$\underline{5 \times 5}$

if $M[i,j] = M[j,i]$

→ then it is symmetric matrix

Either → lower-triangular

→ Upper-triangular

Tri-Diagonal and Triband Matrix :-

Tri-Diagonal matrix →

$$\xrightarrow{} j$$

$$i \downarrow \quad M = \begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & a_{11} & a_{12} & 0 & 0 & 0 \\ 2 & a_{21} & a_{22} & a_{23} & 0 & 0 \\ 3 & 0 & a_{32} & a_{33} & a_{34} & 0 \\ 4 & 0 & 0 & a_{43} & a_{44} & a_{45} \\ 5 & 0 & 0 & 0 & a_{54} & a_{55} \end{array}$$

5×5

Main Diagonal → $i-j=0$

lower Diagonal → $i-j=1$

Upper Diagonal → $i-j=-1$

$$|i-j| \leq 1$$

$M[i,j]$ = non-zero if $|i-j| \leq 1$

$M[i,j] = 0$ if $|i-j| > 1$

No. of non-zero ⇒ 5+4+4

$= n + n-1 + n-1$

$= 3n-2$

A	a_{21}	a_{32}	a_{43}	a_{54}	a_{11}	a_{22}	a_{33}	a_{44}	a_{55}	a_{12}	a_{23}	a_{34}	a_{45}
	0	1	2	3	4	5	6	7	8	9	10	11	12

lower-diagonal | main-diagonal | upper-diagonal

Index(A[i][j])

case1 if $i-j=1$

index = $i-2$

case 2 if $i-j=0$

index = $n-1+i-1$

case 3 if $i-j=-1$

index = $n-1+n+i-1$

$= 2n-1+i-1$

$= 2n-2+i$

Square Band Matrix

$$\begin{array}{c|cccccccc}
 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\
\hline
1 & & & & & 0 & 0 & 0 & 0 \\
2 & & & & & & 0 & 0 & 0 \\
3 & & & & & & & 0 & 0 \\
4 & & & & & & & & 0 \\
5 & 0 & & & & & & & \\
6 & 0 & 0 & & & & & & \\
7 & 0 & 0 & 0 & & & & & \\
8 & 0 & 0 & 0 & 0 & & & &
\end{array}$$

$\underline{\underline{8 \times 8}}$.

Toeplitz Matrix

$$M = \begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 7 & 2 & 3 & 4 & 5 \\ 3 & 8 & 7 & 2 & 3 & 4 \\ 4 & 9 & 8 & 7 & 2 & 3 \\ 5 & 10 & 9 & 8 & 7 & 2 \end{array}$$

$j-i=2$
$j-i=1$
$i-j=0$

$i-j=3$ $i-j=2$ $i-j=1$

5x5

$$M[i,j] = M[i-1, j-1]$$

Sufficient Element → $n+n-1$

$= 2n-1$

A	2	3	4	5	6	7	8	9	10
	0	1	2	3	4	5	6	7	8

← rows elmt → | ← column elmt →

Index (A[i][j])

case1:

if $i \le j$ (upper triangular)

$A[2][4] = 4-2$
$= 2$

$A[3][4] = 4-3$
$= 1$

$$\boxed{\text{Index} = j - i}$$

case 2:-

if $i > j$ (Lower triangular)

$$\text{Index} = n + i - j - 1$$