Assignment-1 Number Systems and Integer Arithmetic

Full Score: 20 pts | Due: Sunday 2/06

Q1. Answer the following questions with the detailed procedures

1) What are the decimal and hexadecimal representations for the value 0b01101110?

In decimal: 128 64 32 16 8 4 2 1 = 64 + 32 + 8 + 4 + 2 = 110_{10}

0 1 1 0 1 1 1 0

In hexdec: 4 2 -||- 8 4 2

6 14

6 E

$6E_{16}$

2) What are the binary and hexadecimal representations for the value 339?

339_{10}

339 : 2 = 169.5 = 169 remainder 1

169 : 2 = 84.5 = 84 remainder 1

84 : 2 = 42 remainder 0

42 : 2 = 21 remainder 0

21 : 2 = 10.5 remainder 1

10 : 2 = 5 remainder 0

5 : 2 = 2.5 remainder 1

2 : 2 = 1 remainder 0

1 : 2 = 0.5 remainder 1

In binary: 101010011_2

339_{10}

339 : 16 = 21.1875 remainder 3

21 : 16 = 1.3125 remainder 5

1 : 16 = 0.0625 remainder 1

In hexadecimal 153_{16}

3) As a five-armed creature, Sally the starfish prefers to represent numbers using a base 5 number system. If Sally gives you the base 5 number 1533, what is the equivalent decimal value?

1533 : 5 = 306.6 remainder 3

306 : 5 = 61.2 remainder 1

61 : 5 = 12.2 remainder 1

12 : 5 = 2.4 remainder 2

2 : 5 = 0 reminder 2

In base 5: 22113_5

Q2. Answer the following questions with the detailed procedures

1) Express the following bit patterns in hexadecimal.

a. **0110 0101 0011 0101**
4 2 4 1 2 1 4 1
6 5 3 5
6535_{16}

b. 1010100010101111

c. 1010 1000 1010 1111

d. 82 8 82 8421

e. A. 8. A. F

f. $A8AF_{16}$

2) Express the following hexadecimal in binary.

a. 79ab

7 9 10 11
8421. 8421. 8421. 8421
0111. 1001. 1010. 1011
0111100110101011_2

b. 7025

7 0 2 5
8421. 8421. 8421. 8421
0111. 0000 0010. 0101
0111000000100101_2

3) How many bits are represented by each of the following?

a. $abcdfefe_{16}$

One hexadecimal digit equal to 4 binary digits aka bits. So it is 4 4 4 4 4 4 4 4 = 32 bits

b. 1110_2

It is 4 bits

c. 1111_{16}

It is 16 bits

4) How many hexadecimal digits are required to represent each of the following?

a. six bits ═══ 2 hexadecimal digits
b. eight bits ═══ 2 hexadecimal digits
c. twenty bits ═══ 5 hexadecimal digits
d. thirty-two bits ═══ 8 hexadecimal digits

Q3. Conversion between number systems by hand

1) Convert the following binary numbers to unsigned decimal integer:

a. 1 0 1 0 1 0 1 1
128 64 32 16. 8 4 2 1
128. 32 8. 2. 1
171_{10}

b. 0. 1. 0. 1. 0. 1. 0. 0
128 64 32 16. 8 4 2 1
64. 16. 4.
84_{10}

c. 1 0. 1. 0. 1. 0. 1. 1. 0. 0 1 1 1. 1. 0. 1
32768 16384 8192 4096 2048 1024 512 256 128 64 32 16. 8 4 2 1
32768. 8192. 2048 512. 256. 32. 16. 8. 4 1
43837_{10}

d. 0. 0. 0. 1. 0. 1. 1. 0. 1. 0. 1. 1. 0. 1. 0 0
32768 16384 8192 4096 2048 1024 512 256 128 64 32 16. 8 4 2 1
4096. 1024. 512. 128. 32. 16. 4
5812_{10}

2) Develop an algorithm to convert hexadecimal to decimal, and then convert the following hexadecimal number to unsigned decimal integer:

a. b. 0. 1. 0
16^3 16^2 16^1 16^0
11*4096. + 1*16
45072_{10}

b. a5f5
16^3 16^2 16^1 16^0
$10*16^3$ $5*16^2$ $15*16^1$ $5*16^0$
40960 +1536+240+80
42816_{10}

3) Convert the following unsigned decimal integers to 8-bit or 16-bit hexadecimal representation:

a. 103
103:16=6.4375 R 7
6:16 R6
67_{16}

b. 224
224:16= 14 R 0
14:16 R14=E
$E0_{16}$

c. 1060
1060:16=66.25 R4
66:16=4.125 R2

4:16 R4

424_{16}

d. 52765

52765:16 =3297.8125 R 13 (D)

3297:16=206.0625 R 10 (A)

206:16=12.875 R 14 (E)

12:16 R 12 (C)

$CEAD_{16}$

Q4 1) $carryover_0 = 0$

For $i = 0 \ldots, (N-1)$

$sum_i := (bitone_i + bittwo_i + carryover_i) \% 2$

$carryover_{i+1} = (bitone_i + bittwo_i + carryover) / 2$.

2) same thing just change from 2 to 16.

1) $1 = 00000001$;

$+ \ 5 = 00000101$;

$6 = 00000110$

$sum_0 = (1 + 1) \% 2 = 0$

$carryover_0 = (1+1) / 2 = 1$

$sum_1 = (0 + 0 + 1) \% 2 = 1$

$carryover_1 = (0 + 0 + 1) / 2 = 0$

$sum_2 = (0 + 1 + 0) \% 2 = 1$

$carryover_2 = (0 + 1 + 0) \% 2 = 0$

. . . . . - - - - - - - - - 0

Obtain all $sum_i = 00000110$

Q4 2) $(5abc)_{16}$ $(1def)_{16}$

$sum_0 = (c + f) \% 16 = 11 = b$

$carryover_0 = (c + f) / 16 = 1$

$sum_1 = (b + e + 1) \% 16 = 10 = a$

$carryover_1 = (b + e + 1) \% 16 = 1$

$sum_2 = (a + d + 1) \% 16 = 8$

$carryover_2 = (a + d + 1) \% 16 = 1$

$sum_3 = (5 + 1 + 1) \% 16 = 7$

$carryover_3 = (5 + 1 + 1) \% 16 = 0$

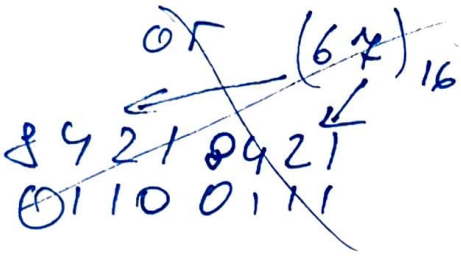
Obtain all sum_i = 7 8 A B.

Q5 1) 8 bit / 16-bit hexadecimal values are stored in 2's complement code. What are the equivalent signed decimal numbers?

a. 67

? ~~Below is a right way of doing it?~~

a) $(67)_{16} = 6 \cdot 16^1 + 7 \cdot 16^0 = 96 + 7 = (103)_{10}$



? why in question exist phrase "2's complement code"? I know 2's complement we need to add 1.

? equivalent signed decimal numbers so we need just to add 1 in front.

b) $(ac)_{16} = 10 \cdot 16^1 + 12 \cdot 16^0 = 160 + 12 = (172)_{10}$

c) $(5567)_{16} = 5 \cdot 16^3 + 5 \cdot 16^2 + 6 \cdot 16^1 + 7 \cdot 16^0 =$

$= 5 \cdot 4096 + 5 \cdot 256 + 6 \cdot 16 + 7 \cdot 1 =$

$= 20480 + 1280 + 96 + 7 =$

$= (21863)_{10}$

d) $(fddc)_{16} = 15 \cdot 16^3 + 13 \cdot 16^2 + 13 \cdot 16^1 + 12 \cdot 16^0 =$

$= 15 \cdot 4096 + 13 \cdot 256 + 13 \cdot 16 + 12 \cdot 1 =$

$= 61440 + 3328 + 208 + 12 =$

$= (64988)_{10}$

Q5. 2) 101 ⟺

101 : 2 = 50 R 1
50 : 2 = 25 R 0
25 : 2 = 12 R 1
12 : 2 = 6 R 0
6 : 2 = 3 R 0
3 : 2 = 1 R 1
1 : 2 = 0 R 1

1 1 0 0 1 0 1
1's: 0 0 1 1 0 1 0
2's: 0 0 1 1 0 1 1

B

For unsigned interger we do not need to do 1's and 2's?

To convert in hex add 0: 0 1 1 0 0 1 0 1

$(6\ \ 5)_{16}$

b) −16

16 : 2 = 8 R 0
8 : 2 = 4 R 0
4 : 2 = 2 R 0
2 : 2 = 1 R 0
1 : 2 = 0 R 1

decimal 16 = 10000 binary
8 bit binary: 000 10000
1's: 1 1 1 0 1 1 1 1
2's: 1 1 1 1 0 0 0 0
−16 in 2's: 1111 0000

$(F\ \ 0)_{16}$

c) +1033

1033 : 2 = 516 R 1
516 : 2 = 258 R 0
258 : 2 = 129 R 0
129 : 2 = 64 R 1
64 : 2 = 32 R 0
32 : 2 = 16 R 0
16 : 2 = 8 R 0
8 : 2 = 4 R 0
4 : 2 = 2 R 0
2 : 2 = 1 R 0

0 1000 0001 001

$(4\ \ 0\ \ 9)_{16}$

d. -32765

32765 : 2 = 16382 R 1
16382 : 2 = 8191 R 0
8191 : 2 = 4095 R 1
4095 : 2 = 2047 R 1
2047 : 2 = 1023 R 1
1023 : 2 = 511 R 1
511 : 2 = 255 R 1
255 : 2 = 127 R 1
127 : 2 = 63 R 1
63 : 2 = 31 R 1
31 : 2 = 15 R 1
15 : 2 = 7 R 1
7 : 2 = 3 R 1
3 : 2 = 1 R 1
1 : 2 = 0 R 1

1 1 1 1 1 1 1 1 1 1 1 1 1 0 1
1's: 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0
2's: 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1

0000 0000 0000 0011

(0 0 0 3)16

Q6 1) a. 43 + CC

8421
0100

8421
0011

1100

1100

```
  01000011
+ 11001100
-----------
 [1]0000 1111
```

Wrong because of carry over, more than 8-bits

? Do I need to convert it to decimal to check if it is right.. or wrong? (shown below)

Or it will be enough when we see if carry over it is wrong if no carry over it is right?

```
R86432168421
0 1 0 00 0 11 = 67
1 1 0 0 1 1 00 = 204
                 271
1 0000 1111      11
256      8421 = 271
```

Signed values (stored in 2's complement)

? So only signed values (start with 1) convert to 2's complement? (shown below).

```
  1100 1100
1's:0011 0011
2's:0000 0000 +1
   0011 0100
    32168421
   -52
```

First (43) = 0100 0011 = 67

Second (CC) = 0011 0100 = -52

Sum = 0000 1111 = 15

67 - 52 = 15

That's correct since we can verify.

? Do we need to convert back to hex?

```
0000 1111
0    15
     F
```

Q6. 1) b) 44 + f0

0100 0100 1111 0000

```
  0100 0100
+ 1111 0000
-----------
 [1]0011 0100
```

Wrong, because of carry over, more than 8 bits

```
  1111  0000
1's:0000  1111
2's:0001  0000 +1
   (1     0)16
```

Q6. 1) c) 53 + 7a

0101 0011 0111 1010

```
  0101 0011
+ 0111 1010
-----------
  1000 1101
1's:01 11 00 10
2's:01 11 00 11 +1
   (7  3)16
```

Right because there is not carry over.

Q6 2) a) 22cc + ed34

$(22cc)_{16} = (0010\ 0010\ 1100\ 1100)_2$

$(ed34)_{16} = (1110\ 1101\ 0011\ 0100)_2$

① 0001 0000 0000 0000

Wrong because of carry over

$-(ed34)_{16}$ 1's: 0001 0010 1100 1011

2's: 0001 0010 1100 1100 + 1

$(1\ 2\ C\ C)_{16}$

2) b) 7000 + 7000

0111 0000 0000 0000

+ 0111 0000 0000 0000

1000 0000 0000 0000

Right no carry over

1's: 0111 1111 1111 1111

2's: 1000 0000 0000 0000 + 1

$(8\ 0\ 0\ 0)_{16}$