CHAPTER

31

The Complex Fourier Transform

Although complex numbers are fundamentally disconnected from our reality, they can be used to solve science and engineering problems in two ways. First, the parameters from a real world problem can be substituted into a complex form, as presented in the last chapter. The second method is much more elegant and powerful, a way of making the complex numbers mathematically equivalent to the physical problem. This approach leads to the *complex Fourier transform*, a more sophisticated version of the *real Fourier transform* discussed in Chapter 8. The complex Fourier transform is important in itself, but also as a stepping stone to more powerful complex techniques, such as the *Laplace* and *z-transforms*. These complex transforms are the foundation of theoretical DSP.

The Real DFT

All four members of the Fourier transform family (DFT, DTFT, Fourier Transform & Fourier Series) can be carried out with either real numbers or complex numbers. Since DSP is mainly concerned with the DFT, we will use it as an example. Before jumping into the complex math, let's review the real DFT with a special emphasis on things that are awkward with the mathematics. In Chapter 8 we defined the *real* version of the Discrete Fourier Transform according to the equations:

EQUATION 31-1
The real DFT. This is the forward transform, calculating the frequency domain from the time domain. In spite of using the names: *real part* and *imaginary part*, these equations only involve ordinary numbers. The frequency index, k, runs from 0 to $N/2$. These are the same equations given in Eq. 8-4, except that the $2/N$ term has been included in the forward transform.

$$Re\,X[k] = \frac{2}{N} \sum_{n=0}^{N-1} x[n] \cos(2\pi k n / N)$$

$$Im\,X[k] = \frac{-2}{N} \sum_{n=0}^{N-1} x[n] \sin(2\pi k n / N)$$

In words, an N sample time domain signal, $x[n]$, is decomposed into a set of $N/2+1$ cosine waves, and $N/2+1$ sine waves, with frequencies given by the