

$$H = \sum_{\substack{i,j \\ i < j}} w_{ij} Z_i Z_j \quad i, j \in \{0, 1, 2, 3\}$$

$$f(x) = X^T H X$$

$$|X\rangle = \sum_i \frac{x_i |i\rangle}{\|X\|}$$

$$\nabla f(x) = 2 H X$$

$$\begin{aligned} |X^{t+1}\rangle &= |X^t\rangle - \gamma_0 \nabla f(x) |X^t\rangle \\ &= \underline{(I - 2\gamma_0 H)} |X^t\rangle \\ &= H^\sharp |X^t\rangle \end{aligned}$$

$$\begin{aligned} H^\sharp &= (I - 2\gamma_0 H) = \\ &= I + \gamma \underbrace{w_{01} Z_0 Z_1}_{\downarrow B_1} + \gamma \underbrace{w_{02} Z_0 Z_2}_{\downarrow B_2} + \gamma \underbrace{w_{12} Z_1 Z_2}_{\downarrow B_3} \\ 2\gamma_0 &= \gamma \underbrace{H_0^\sharp}_{B_0=1} \quad \underbrace{H_1^\sharp}_{B_1} \quad \underbrace{H_2^\sharp}_{B_2} \quad \underbrace{H_3^\sharp}_{B_3} \\ C &= \sqrt{B_0^2 + B_1^2 + B_2^2 + B_3^2 + 0 \dots} \end{aligned}$$

$$Y = \frac{1}{C} \sum_{k=0}^n B_k |k\rangle =$$

$$= \frac{B_0}{C} |00\rangle + \frac{B_1}{C} |01\rangle + \frac{B_2}{C} |10\rangle + \frac{B_3}{C} |11\rangle$$

$$|\phi_0\rangle = |Y\rangle |X^t\rangle$$

$$|\phi_1\rangle = |Y\rangle H^\sharp |X^t\rangle =$$

$$= \frac{B_0}{C} |00\rangle H_0^\sharp |X^t\rangle + \frac{B_1}{C} |01\rangle H_1^\sharp |X^t\rangle +$$

$$\frac{B_2}{C} |10\rangle H_2^\sharp |X^t\rangle + \frac{B_3}{C} |11\rangle H_3^\sharp |X^t\rangle =$$

$$\begin{aligned} |\phi_2\rangle &= \frac{1}{\sqrt{2^2}} \left(\underbrace{\mathcal{H}^2}_{\text{Hadamard}} \frac{B_0}{C} |00\rangle H_0^\sharp |X^t\rangle + \mathcal{H}^2 \frac{B_1}{C} |01\rangle H_1^\sharp |X^t\rangle + \right. \\ &\quad \left. \mathcal{H}^2 \frac{B_2}{C} |10\rangle H_2^\sharp |X^t\rangle + \mathcal{H}^2 \frac{B_3}{C} |11\rangle H_3^\sharp |X^t\rangle \right) \\ &= \frac{1}{2} \frac{1}{C} \left(\underbrace{|00\rangle \mathcal{H}^2 |X^t\rangle}_{H^\sharp |X^t\rangle} + \{ |10\rangle, |11\rangle \} \dots \right) \\ &= H^\sharp |X^t\rangle = |X^{t+1}\rangle \end{aligned}$$