

LATTICE THEORY PROBLEMS

1. If x and y are two elements of a lattice, show that $x \wedge y = y$ if and only if $x \vee y = x$.
2. If x, y , and z are elements of a lattice, show that

$$x \vee (y \wedge z) \leq (x \vee y) \wedge (x \vee z)$$

$$(x \wedge y) \vee (x \wedge z) \leq x \wedge (y \vee z).$$

3. Show that in an algebraic system (L, \oplus, \otimes) , where \oplus and \otimes are binary operations satisfying the absorption law, \oplus and \otimes are idempotent.
4. Let a, b, c be elements in a lattice (L, \leq) . Show that $a \leq b$ if and only if

$$a \vee (b \wedge c) \leq b \wedge (a \vee c).$$

5. Show that a lattice L is distributive if and only if for all elements $x, y, z \in L$, $(x \vee y) \wedge z \leq x \vee (y \wedge z)$.
6. Show that every chain is a distributive lattice. Which chains are Boolean lattices?
7. Give an example of a complemented lattice that is not distributive.