PROBABILITY AND STATISTICS – PROBLEM SET 9

1. If $\overline{X}$ is the mean of a random sample of size 5 from a normal distribution with $\mu = 0$ and $\sigma^2 = 125$, determine c such that $\mathrm{P}[P[\overline{X} < c] = 0.9$.
2. If $X \sim N(\mu, 100)$, find n such that $\mathrm{P}[|\overline{X} - \mu| < 5] = 0.954$.
3. Let $\overline{X}$ be the mean of a sample of size 75 from a distribution that has pdf $f(x) = 1$, $0 < x < 1$. Find an approximate probability $\mathrm{P}[0.45 < \overline{X} < 0.55]$.
4. Compute $\mathrm{P}[2.3 < S^2 < 22.2]$, where S^2 is the variance of a sample of size 6 from a normal distribution with variance 12.
5. If $X \sim N(0, 16))$ and $Y \sim N(1, 9)$ respectively, and $\overline{X}$ and $\overline{Y}$ are the means of samples of size 25 each from these two distributions respectively, find $\mathrm{P}[\overline{X} > \overline{Y}]$.
6. If X has a distribution defined by the pdf $f(x) = 3x^2$, $0 < x < 1$, compute an approximate probability that the mean of a sample of size 15 taken from this population lies between $\frac{3}{5}$ and $\frac{4}{5}$.
7. If $Y \sim B(n, 0.55)$, find an approximation for the smallest value of n such that $\mathrm{P}\left[\frac{Y}{n} > \frac{1}{2}\right] \geq 0.95$.
8. Let Y be the sum of the outcomes obtained when a fair, six-sided die is rolled 12 times. Using Central Limit Theorem, compute $\mathrm{P}[36 \leq Y \leq 48]$ approximately.
9. Consider a random sample of size 72 from a distribution having pdf $f(x) = \frac{1}{x^2}$, $x > 1$. What is the probability that more than 50 of the observations in the sample are less than 3?
10. Forty-eight measurements are recorded to several decimal places, and each of these is rounded off to the nearest integer. The sum of the original 48 measurements is approximated by the sum of these integers. Assuming that the round-off errors are independent and each is uniformly distribution in $(-0.5, 0.5)$, compute an approximate probability that the sum of the integers is within two units of the true sum.
11. If $X \sim N(10, 9)$ and $Y \sim N(3, 4)$, find $\mathrm{P}[\overline{X} > 2\overline{Y}]$ where both samples have size 4.
12. Compute $\mathrm{P}[0 < \overline{X} < 6, 55.2 < S^2 < 145.6]$, if $X \sim N(3, 100)$ and $n = 25$.
13. Approximate the probability that the sum of 16 independent random variables, each uniformly distributed in $[0, 1]$, exceeds 10.

14. Compute an approximate probability that $\overline{X}$ is between 7 and 9, if $X \sim \Gamma(2, \frac{1}{4})$ and $n = 128$.
15. A fair, six-sided die is rolled until the total of all outcomes exceeds 400. Approximate the probability that this will require more than 140 rolls.
16. The lifetime, in hours, of a type of electric bulb has expected value 500 and standard deviation 80. Approximate the probability that the mean of the lifetimes of a sample of n such bulbs is greater than 525, when
    (i) $n = 4$
    (ii) $n = 16$
    (iii) $n = 36$
    (iv) $n = 64$.