PROBABILITY AND STATISTICS – PROBLEM SET 8

1. Compute the moment generating function of the discrete random variable X with pmf $f(x) = \dfrac{1}{2^x}$, $x = 1,2,\ldots$.
2. If $0 < p < 1$ and $q = 1 - p$, find the mgf of the random variable X with pmf $f(x) = p^{x-1}q$, $x = 1,2,\ldots$, and hence find $E[X]$ and $V[X]$.
3. If $E[X^n] = 2^n(n+1)!$, $n = 0,1,\ldots$, then find the mgf of X.
4. Let X be a random variable having pdf $f(x) = ae^{-a(x-b)}$, $x \geq b$, where $a > 0$. Using the mgf of X, determine its mean and variance.
5. If X is a random variable with pdf $f(x) = e^{-2|x|}$, find the mgf of X, and hence compute $E[X]$ and $V[X]$.
6. If $X \sim U[a,b]$, compute $E[X^n]$ using $M_X(t)$. Hence show that if $X \sim U[-a,a]$, then $E[X^{2n}] = \dfrac{a^{2n}}{2n+1}$.
7. If $M_{X_1}(t) = e^{3t+2t^2}$, $M_{X_2}(t) = e^{5t+18t^2}$, $M_{X_3}(t) = e^{4t+8t^2}$, then find the pdf of $Y = 2X_1 + 3X_2 + 4X_3$, given that X_1, X_2, and X_3 are independent.
8. If $X \sim N(0,2)$, then find the moment generating function of $Y = \frac{X^2}{2}$.
9. Find the mean of X, given that its mgf is $M_X(t) = e^{2(e^t-1)}$.
10. Find the variance of X, given that its mgf is $M_X(t) = \left(\frac{3}{4} + \frac{e^t}{4}\right)^{20}$.
11. Show that if X and Y are independent Poisson variate with means λ and μ respectively, then $X+Y$ is a Poisson variate with mean $\lambda + \mu$.
12. Show that if $X_i \sim N(\mu_i, \sigma_i^2)$, $i = 1,\ldots,n$ are n independent normal variates, and a_i, $i = 1,\ldots,n$ are constants, then $X = \sum_{i=1}^n a_i X_i \sim N(\mu, \sigma^2)$ where $\mu = \sum_{i=1}^n a_i \mu_i$ and $\sigma = \sqrt{\sum_{i=1}^n a_i^2 \sigma_i^2}$. Also determine the distribution of $\overline{X} = \frac{X}{n}$.
13. Show that if $Z_i \in N(0,1)$, $i = 1,\ldots,n$ are n independent standard normal variates, then $\sum_{i=1}^n Z_i^2 \sim \chi_n^2$.