

## PROBABILITY AND STATISTICS – PROBLEM SET 5

1. Suppose that the number of items produced in a factory during one week is a random variable with mean 500 and variance 100.
  - (a) What can be said about the probability that this week's production will be at least 1000?
  - (b) What can be said about the probability that this week's production will be between 400 and 600?
2. If the time required to complete a task is a random variable with mean 20 minutes and standard deviation 3 minutes, find the smallest time frame such that the probability that the task will be completed within the time frame is at least 0.75.
3. If  $X \sim B(6, p)$ , and  $P[X = 2] = 9P[X = 4]$ , then find  $P[X \leq 3]$ .
4. An urn contains 4 red, 4 green, and 2 blue marbles. One marble is drawn at random from the urn, observed, and placed back. Find the probability that in 8 such draws
  - (a) exactly 3 red marbles are drawn.
  - (b) not more than 3 red marbles are drawn.
  - (c) at least 3 red marbles are drawn.
  - (d) not all marbles drawn are red.
5. The number of visitors to a webpage per minute follows a Poisson distribution. If the average number of visitors per minute is 4, what is the probability that the webpage receives
  - (i) exactly four visitors in one minute?
  - (ii) at least two visitors in one minute?
6. If  $X$  is a Poisson variate such that  $3P[X = 2] = 2P[X = 1]$ , then what is  $E[X]$ ?
7. It is observed in a communication channel that 90% of the messages sent are received without any error. Find the probability that among 18 messages sent through the channel
  - (i) at least 16 are received without any error
  - (ii) at most 14 are received without any error.
8. In a certain factory producing blades, there is a small probability of  $\frac{1}{500}$  for any blade to be defective. The blades are supplied in packets of 10. Calculate the approximate number of packets containing
  - (i) no defective
  - (ii) one defective
  - (iii) two defective

- (iv) at least two defective blades in a consignment of 10000 packets.
9. One per thousand of a population is subject to certain kinds of accidents each year. Given that an insurance company has insured 5,000 persons from the population, find the probability that at most 2 of them will incur this accident.
  10. An airline company, having observed that 5% of the persons making reservations on a flight do not show up for the flight, sells 100 seats on a plane that has 95 seats. What is the probability that there will be a seat available for every person who shows up for the flight?
  11. If  $X$  is a binomial variate with mean 3, such that  $P[X = 3] = 2P[X = 2]$ , find  $V[X]$ .
  12.  $X$  is a Poisson variate and the probability that  $X$  is even is twice the probability that it is odd. Determine  $P[X = 0]$ .
  13. If  $X$  is exponentially distributed with  $E(X) = 2$ , then find the value of  $a$  such that  $P(X \leq a) = P(X \geq a)$ .  
**Note:** This point  $a$  is the *median* of  $X$ .
  14. Suppose that the duration in minutes of a phone call follows an exponential distribution with mean 5 minutes.
    - (a) Find the probability that the duration of a particular call
      - (i) will exceed 5 minutes
      - (ii) will be between 5 and 6 minutes
      - (iii) will be less than 3 minutes
      - (iv) will be less than 6 minutes, given that it was greater than 3 minutes.
    - (b) Suppose exactly 100 such phone calls are received every day. Find the probability that on a given day
      - (i) every phone call lasted longer than 8 minutes
      - (ii) at least 4 phone calls lasted longer than 8 minutes.
    - (c) Find the probability that in one week, there are at least 3 days on each which there were at least 4 phone calls that lasted longer than 8 minutes.
  15. A random variable  $X$  follows an exponential distribution with parameter  $\alpha = 3$ . Compute
    - (a)  $P(2X > 1)$ .
    - (b)  $P(X > s + t \mid X > t)$ , where  $s, t > 0$ .
  16. If  $K \sim U[0, 5]$ , then what is the probability that the roots of the equation  $4x^2 + 4Kx + K + 2 = 0$  are real?
  17. A book of 200 pages contains 100 misprints distributed randomly throughout its pages. If one page is selected at random and examined, what is the probability that it contains
    - (a) no misprints?
    - (b) at least 2 misprints?

18. Buses start from a certain station at intervals of 45 minutes. What is the probability that a person reaching the station at a random point in time will have to wait for at least 30 minutes?