

### MAT 2155: PROBLEM SET 4

1. The coefficient of  $x^6$  in the expansion of
  - (i)  $(1 + x)^8$ .
  - (ii)  $(1 - x)^{-8}$ .
  - (iii)  $(1 + x^2)^4$ .
  - (iv)  $(1 - x^2)^{-4}$ .
  - (v)  $(1 + x + x^2)^4$ .
  - (vi)  $(1 + x^2)(1 + x^2 + x^4 + \cdots)$ .
2. Using generating functions, find the number of ways of selecting 6 objects from
  - (i) 8 distinct objects.
  - (ii) 8 types of object, with any number of objects of each type.
  - (iii) 4 types of objects, such that zero or two objects are chosen from each type.
  - (iv) 4 types of objects, such that an even number of objects is chosen from each type.
  - (v) 4 types of objects, such that not more than two objects are chosen from each type.
  - (vi) 2 types of objects, such that zero or two objects of the first type are chosen, and an even number of objects is chosen from the second type.
3. Using generating functions, find the number of integer solutions of the equation
  - (i)  $x_1 + \cdots + x_8 = 6, x_i \in \{0, 1\}, i = 1, \dots, 8$ .
  - (ii)  $x_1 + \cdots + x_8 = 6, x_i \geq 0, i = 1, \dots, 8$ .
  - (iii)  $x_1 + \cdots + x_4 = 6, x_i \in \{0, 2\}, i = 1, \dots, 4$ .
  - (iv)  $x_1 + \cdots + x_4 = 6, x_i \geq 0, x_i$  is even,  $i = 1, \dots, 4$ .
  - (v)  $x_1 + \cdots + x_4 = 6, 0 \leq x_i \leq 2, i = 1, \dots, 4$ .
  - (vi)  $x + y = 6, x \in \{0, 2\}, y \geq 0, y$  is even.
4. Number of ways of distributing 30 identical objects into 3 distinct boxes such that no box is empty.
5. Number of ways of distributing 30 identical marbles into 6 boxes with at most 10 marbles in the first box.
6. Number of ways of selecting 12 flowers for a bouquet from roses, lilacs, tulips, and lilies, with between 2 and 5 of each kind.

7. Number of ways to select 10 marbles from a large pile of red, white, and blue marbles if
  - (i) the selection has at least 2 marbles of each colour.
  - (ii) the selection has at most 2 red marbles.
  - (iii) the selection has an even number of blue marbles.
8. Number of ways to place an order of 12 chocolate sundaes if there are 5 types of sundaes, and at most 4 sundaes of one type are allowed.
9. Number of ways to get a sum of 25 when 10 distinct dice are rolled.
10. Number of ways to select 300 chocolate candies from 7 types of candy if each type comes in boxes of 20, and at least 1 but not more than 5 boxes of each type are chosen.
11. Number of ways of distributing 30 distinct objects into 3 boxes such that no box is empty.
12. Using generating functions, find the number of  $r$ -permutations of objects chosen from unlimited supplies of  $n$  types of objects.
13. Number of  $r$ -digit quaternary sequences (with digits 0, 1, 2, 3) having an even number of 0s and an odd number of 1s.
14. Write the exponential generating function for the number of arrangements of  $k$  objects chosen from  $n$  types with at most 4 objects of each type.
15. Number of  $n$ -digit ternary sequences with
  - (i) an even number of 0s.
  - (ii) an even number of 0s and an even number of 1s.
  - (iii) 0 and 1 occurring a positive even number of times.
  - (iv) at least one 0 and at least one 1.
  - (v) the total number of 0s and 1s being even.
  - (vi) no digit occurs exactly twice.