PROBABILITY AND STATISTICS – PROBLEM SET 10

1. If $X_1, \dots, X_n$ is an independent sample of size n from a population defined by the pmf $f(x) = \theta^x (1-\theta)^{1-x}$, $x = 0, 1$, where $0 < \theta < 1$, find the maximum likelihood estimator for θ.

2. Find the maximum likelihood estimator for θ in each of the following cases:
   (a) $f(x;\theta) = \theta x^{\theta-1}$, $0 < x < 1$, where $\theta > 0$.
   (b) $f(x;\theta) = \theta e^{-\theta x}$, $x > 0$, where $\theta > 0$.
   (c) $f(x;\theta) = e^{-(x-\theta)}$, $x \geqslant \theta$, where θ is any real number.
   (d) $f(x;\theta) = \frac{1}{2}e^{-|x-\theta|}$, where θ is any real number.
   (e) $f(x;\theta) = \dfrac{2x}{\theta^2}$, $0 \leqslant x \leqslant \theta$, where $\theta > 0$.

3. Find the mles of θ_1 and θ_2 in each of the following cases:
   (a) $X \sim N(\theta_1, \theta_2)$
   (b) $f(x;\theta_1,\theta_2) = \dfrac{1}{\theta_2} e^{-\frac{(x-\theta_1)}{\theta_2}}$, $x > \theta_1$
   where θ_1 is any real number and $\theta_2 > 0$.

4. Find the mle of θ in each of the following cases:
   (a) $X \sim B(m, \theta)$.
   (b) $X \sim \mathcal{P}(\theta)$.

5. Let $X \sim U[0, \theta]$, where $\theta > 0$. Compute the mle for θ and show that it is unbiased.

6. Show that $\overline{X}$ is an unbiased and consistent estimator of population mean.

7. Let Y_n be an unbiased estimator of θ, such that $V(Y_n) \to 0$ as $n \to \mathrm{inf}$. Then show that Y_n is consistent.