

PROBABILITY AND STATISTICS – PROBLEM SET 1

1. VERY EASY

- 1.1. A box contains 10 paper slips, labelled 1, 2, ..., 10. Find the probability that one slip drawn at random contains:
 - (a) the number 9.
 - (b) an even number.
 - (c) an even number or an odd number.
 - (d) an even number or a prime number.
- 1.2. A fair coin is tossed twice. Find the probability that
 - (a) A head is obtained on the first toss.
 - (b) A head is obtained on the first toss and a tail on the second.
 - (c) A head is obtained on at least one of the two tosses.
- 1.3. A fair, six-sided die is rolled. Find the probability that the outcome is
 - (a) 2
 - (b) an odd number.
 - (c) an odd number or an even number.
 - (d) an odd number or a composite number.

2. EASY

- 2.1. A box contains 55 paper slips – one labelled 1, two labelled 2, ..., ten labelled 10 (i.e., k slips labelled k , for each $k = 1, \dots, 10$). Find the probability that one slip drawn at random contains:
 - (a) the number 9.
 - (b) an even number.
 - (c) an even number or an odd number.
 - (d) an even number or a prime number.
- 2.2. A coin with probability $1/3$ for heads and $2/3$ for tails is tossed twice. Find the probability that
 - (a) A head is obtained on the first toss.
 - (b) A head is obtained on the first toss and a tail on the second.
 - (c) A head is obtained on at least one of the two tosses.

- 2.3. A six-sided die is designed in such a way that the probability of occurrence of each face is proportional to the number on that face. Find the probability that the outcome, when the die is rolled once, is
- 2
 - an odd number.
 - an odd number or an even number.
 - an odd number or a composite number.
- 2.4. Let m and n denote the two outcomes when two fair dice are rolled. Find the probability that
- $m = 4$ or $n = 4$.
 - $\max(m, n) = 4$.
 - $\max(m, n) > 4$.
- 2.5. Three marbles are drawn simultaneously at random from a box containing 2 red, 3 green, and 5 blue marbles. Find the probability that
- all three are green.
 - all three are blue.
 - all three are red.
 - at least one is red.
 - each one is green or blue.
 - one is red and two are blue.
- 2.6. A box of 100 lightbulbs manufactured in a factory has 10 defective lightbulbs. An inspector tests 5 lightbulbs selected randomly from the box. What is the probability that a defective one will be found?
- 2.7. A group of $2n$ boys and $2n$ girls is randomly divided into two equal groups. What is the probability that each group has the same number of boys and girls?
- 2.8. A box contains n paper slips, labelled $1, 2, \dots, n$. Find the probability that two slips drawn at random contain consecutive numbers, if they are drawn one after the other
- without replacement.
 - with replacement.

3. NORMAL DIFFICULTY

- 3.1. A box contains 10 paper slips, labelled $1, \dots, 10$. Slips are drawn at random without replacement, until 9 is obtained. Find the probability that 9 is obtained
- in the n^{th} draw (for each $n = 1, \dots, 10$).
 - after the n^{th} draw (for each $n = 1, \dots, 9$). Note: Not necessarily *immediately* after it.
 - after 10 is obtained.
 - immediately after 10 is obtained.
 - immediately before or after 10 is obtained.

- 3.2. A coin with probability $1/3$ for heads and $2/3$ for tails is tossed until a head is obtained. Find the probability that
- exactly n tosses are required ($n = 1, 2, \dots$).
 - the number of tosses required is even.
 - at least n tosses are required.
- 3.3. A fair, six-sided die is rolled until the same face is obtained twice in succession. Find the probability that
- exactly n rolls are required ($n = 2, 3, \dots$).
 - 2 is obtained on the last two rolls (regardless of number of rolls required).
 - 2 is not obtained on any roll.
 - 2 is obtained on the last two rolls, but not before.
- 3.4. Let S be a set of n elements, and $\mathcal{P}(S)$ its power set – the collection of all subsets of S . Let A be a subset of S picked at random from $\mathcal{P}(S)$.
- What is the probability that A has m elements ($0 \leq m \leq n$)?
 - If B is a given subset of S , what is the probability that $A = B$?
- 3.5. Let $S = \{s_1, s_2, \dots, s_n\}$ be a set of n elements. Construct a random subset A of S as follows: For each $i = 1, \dots, n$, toss a fair coin and on heads, include the element s_i in A , and on tails, exclude s_i from A .
- What is the probability that A has m elements ($0 \leq m \leq n$)?
 - If B is a given subset of S , what is the probability that $A = B$?
- 3.6. Let $S = \{s_1, s_2, \dots, s_n\}$ be a set of n elements, and consider a coin weighted such that heads occur twice as often as tails. Construct a random subset A of S as follows: For each $i = 1, \dots, n$, toss the coin and on heads, include the element s_i in A , and on tails, exclude s_i from A .
- What is the probability that A has m elements ($0 \leq m \leq n$)?
 - If B is a given subset of S , what is the probability that $A = B$?

4. SEEMINGLY DIFFICULT

- 4.1. A box contains n paper slips labelled $1, \dots, n$ ($n \geq 9$). Slips are drawn at random with replacement, until 9 is obtained. What is the minimum value of n such that the probability that more than 10 draws are required is at least 0.5? (Take your time to understand this question correctly.)
- 4.2. A coin with probability $1/3$ for heads and $2/3$ for tails is tossed n times. What is the minimum value of n such that the probability that at least 2 heads are obtained is at least 0.5?
- 4.3. A fair, six-sided die is rolled until the total of all outcome 2 is obtained a total of n times (not necessarily in consecutive throws). Find the probability that exactly m throws are required ($m = 1, 2, \dots$).