MAC 1103: PROPOSITIONAL CALCULUS

1. Prove that $S \vee R$ follows from the premises $P \vee Q$, $P \to R$, and $Q \to S$.
2. Show that $(P \vee Q) \wedge (Q \to R) \wedge (P \to M) \wedge (\neg M) \implies R \wedge (P \vee Q)$.
3. Show that the premises $P \to (Q \to S)$, $\neg R \vee P$, and Q tautologically imply $R \to S$.
4. Show that the following sets of premises are (each) inconsistent:
   (a) $P \to Q$, $Q \to R$, $R \to \neg P$.
   (b) $A \vee B$, $A \to \neg C$, $C \to \neg B$, C.
   (c) $P \to Q$, $Q \to R$, $Q \to \neg R$, P.
   (d) $A \to (B \to C)$, $D \to (B \wedge \neg C)$, $A \wedge D$.
5. Show that the following premises are inconsistent:
   (a) If Jack misses many classes due to illness, then he fails school.
   (b) If Jack fails school, then he is uneducated.
   (c) If Jack reads a lot of books, then he is not uneducated.
   (d) Jack misses many classes due to illness and reads a lot of books.
6. Let the propositions P, Q, and R be defined as follows:
   P: $\sqrt{2}$ is irrational.
   Q: $\left(\sqrt{2}\right)^{\sqrt{2}}$ is rational.
   R: $\left(\left(\sqrt{2}\right)^{\sqrt{2}}\right)^{\sqrt{2}} = 2$ is rational.
   S: There exist two irrational numbers x and y such that x^y is rational.
   Prove that S follows from the premises P, R, $P \wedge Q \to S$, and $P \wedge (\neg Q) \wedge R \to S$. That is, show that there exist two irrational numbers x and y such that x^y is rational.