

PROBABILITY AND STATISTICS – PROBLEM SET 10

1. If X_1, \dots, X_n is an independent sample of size n from a population defined by the pmf $f(x) = \theta^x(1 - \theta)^{1-x}$, $x = 0, 1$, where $0 < \theta < 1$, find the maximum likelihood estimator for θ .
2. Find the maximum likelihood estimator for θ in each of the following cases:
 - (a) $f(x; \theta) = \theta x^{\theta-1}$, $0 < x < 1$, where $\theta > 0$.
 - (b) $f(x; \theta) = \theta e^{-\theta x}$, $x > 0$, where $\theta > 0$.
 - (c) $f(x; \theta) = e^{-(x-\theta)}$, $x \geq \theta$, where θ is any real number.
 - (d) $f(x; \theta) = \frac{1}{2} e^{-|x-\theta|}$, where θ is any real number.
 - (e) $f(x; \theta) = \frac{2x}{\theta^2}$, $0 \leq x \leq \theta$, where $\theta > 0$.
3. Find the mles of θ_1 and θ_2 in each of the following cases:
 - (a) $X \sim N(\theta_1, \theta_2)$
 - (b) $f(x; \theta_1, \theta_2) = \frac{1}{\theta_2} e^{-\frac{(x-\theta_1)}{\theta_2}}$, $x > \theta_1$
where θ_1 is any real number and $\theta_2 > 0$.
4. Find the mle of θ in each of the following cases:
 - (a) $X \sim B(m, \theta)$.
 - (b) $X \sim \mathcal{P}(\theta)$.
 - (c) $X \sim N(\mu, \theta)$.
5. Let $X \sim U[0, \theta]$, where $\theta > 0$. Compute the mle for θ .
6. Show that the sample mean is an unbiased and consistent estimator of population mean.
7. Show that sample variance is a biased estimator of the population variance.
8. Show that $Y = \frac{1}{n-1} \sum_{i=1}^n (X_i - \bar{X})^2$ is an unbiased estimator of population variance.
9. If $X \sim N(0, \theta)$, show that $Y = \frac{1}{n} \sum_{i=1}^n X_i^2$ is an unbiased estimator of θ and has variance $\frac{2\theta^2}{n}$.

10. Show that \bar{X} is an unbiased estimator of θ if the pdf of X is $f(x; \theta) = \frac{1}{\theta} e^{-\frac{x}{\theta}}, x > 0$, where $\theta > 0$. Also show that \bar{X} has variance $\frac{\theta^2}{n}$ and is therefore a consistent estimator.
11. Let Y_n be an unbiased estimator of θ , such that $V(Y_n) \rightarrow 0$ as $n \rightarrow \infty$. Then show that Y_n is consistent.
12. Let Y_1 and Y_2 be two independent unbiased statistics for θ such that the variance of Y_1 is twice that of Y_2 . Find the constants k_1 and k_2 such that $Z = k_1 Y_1 + k_2 Y_2$ is an unbiased statistic for θ with minimum possible variance for such a linear combination.
13. Find the same size n such that $P[\bar{X} - 1 < \mu < \bar{X} + 1] = 0.9$, given that $X \sim N(\mu, 9)$.
14. If the observed value of the mean of a sample of size 20 from a population having distribution $N(\mu, 80)$ is $\bar{x} = 81.2$, find a 95 percent confidence interval for the population mean.
15. If a random sample of size 17 from a normal distribution $N(\mu, \sigma^2)$ yields $\bar{x} = 4.7$ and $s^2 = 5.76$, determine a 90 percent confidence interval for μ .
16. The observed values of a random sample from a population having distribution $N(8, \sigma^2)$ are 8.6, 7.9, 8.3, 6.4, 8.4, 9.8, 7.2, 7.8, and 7.5. Construct a 90 percent confidence interval for σ^2 .
17. A random sample of size 9 from the distribution $N(\mu, \sigma^2)$ yields $s^2 = 7.63$. Determine a 95 percent confidence interval for σ^2 .
18. Find an approximate 95 percent confidence interval for the mean of a population having variance 100, if the sample size is 25.
19. A random sample of size 15 from a normal population with unknown mean and variance yields $\bar{x} = 3.2$ and $s^2 = 4.24$. Determine a 95 percent confidence interval for σ^2 .
20. If a sample of size 15 from a population with distribution $N(\mu, \sigma^2)$ yields values $\sum_{i=1}^{15} X_i = 8.7$ and $\sum_{i=1}^{15} X_i^2 = 27.3$, obtain a 95 percent confidence interval for σ^2 .
21. Suppose that $X \sim N(8, \sigma^2)$, and the observed values of a sample are of size 9 from this population are 8.6, 7.9, 8.3, 6.4, 8.4, 9.8, 7.2, 7.8, and 7.5. Construct a 90 percent confidence interval for σ^2 .
22. Suppose that $X \sim N(\mu, 4)$. If $\bar{x} = 78.3$ with $n = 25$, obtain a 99 percent confidence interval for μ .