

PROBABILITY AND STATISTICS – PROBLEM SET 5

1. Suppose that the number of items produced in a factory during one week is a random variable with mean 500 and variance 100.
 - (a) What can be said about the probability that this week's production will be at least 1000?
 - (b) What can be said about the probability that this week's production will be between 400 and 600?
2. If the time required to complete a task is a random variable with mean 20 minutes and standard deviation 3 minutes, find the smallest time frame such that the probability that the task will be completed within the time frame is at least 0.75.
3. The number of visitors to a webpage per minute follows a Poisson distribution. If the average number of visitors per minute is 4, what is the probability that the webpage receives
 - (i) exactly four visitors in one minute?
 - (ii) at least two visitors in one minute?
4. If X is a Poisson variate such that $3P[X = 2] = 2P[X = 1]$, then what is $E[X]$?
5. It is observed in a communication channel that 90% of the messages sent are received without any error. Find the probability that among 18 messages sent through the channel
 - (i) at least 16 are received without any error
 - (ii) at most 14 are received without any error.
6. If $X \sim B(6, p)$, and $P[X = 2] = 9P[X = 4]$, then find $P[X \leq 3]$.
7. In a certain factory producing blades, there is a small probability of $\frac{1}{500}$ for any blade to be defective. The blades are supplied in packets of 10. Calculate the approximate number of packets containing
 - (i) no defective
 - (ii) one defective
 - (iii) two defective
 - (iv) at least two defectiveblades in a consignment of 10000 packets.
8. One per thousand of a population is subject to certain kinds of accidents each year. Given that an insurance company has insured 5,000 persons from the population, find the probability that at most 2 of them will incur this accident.

9. An airline company, having observed that 5% of the persons making reservations on a flight do not show up for the flight, sells 100 seats on a plane that has 95 seats. What is the probability that there will be a seat available for every person who shows up for the flight?
10. If X is a binomial variate with mean 3, such that $P[X = 3] = 2P[X = 2]$, find $V[X]$.
11. X is a Poisson variate and the probability that X is even is twice the probability that it is odd. Determine $P[X = 0]$.
12. If X is exponentially distributed with $E(X) = 2$, then find the value of a such that $P(X \leq a) = P(X \geq a)$.
Note: This point a is the *median* of X .
13. Suppose that the duration in minutes of a phone call follows an exponential distribution with mean 5 minutes.
 - (a) Find the probability that the duration of a particular call
 - (i) will exceed 5 minutes
 - (ii) will be between 5 and 6 minutes
 - (iii) will be less than 3 minutes
 - (iv) will be less than 6 minutes, given that it was greater than 3 minutes.
 - (b) Suppose exactly 100 such phone calls are received every day. Find the probability that on a given day
 - (i) every phone call lasted longer than 8 minutes
 - (ii) at least 4 phone calls lasted longer than 8 minutes.
 - (c) Find the probability that in one week, there are at least 3 days on each which there were at least 4 phone calls that lasted longer than 8 minutes.
14. A random variable X follows an exponential distribution with parameter $\alpha = 3$. Compute
 - (a) $P(2X > 1)$.
 - (b) $P(X > s + t \mid X > t)$, where $s, t > 0$.
15. If $K \sim U[0, 5]$, then what is the probability that the roots of the equation $4x^2 + 4Kx + K + 2 = 0$ are real?
16. A book of 200 pages contains 100 misprints distributed randomly throughout its pages. If one page is selected at random and examined, what is the probability that it contains
 - (a) no misprints?
 - (b) at least 2 misprints?
17. Buses start from a certain station at intervals of 45 minutes. What is the probability that a person reaching the station at a random point in time will have to wait for at least 30 minutes?