

PROBABILITY AND STATISTICS – PROBLEM SET 10

1. If X_1, \dots, X_n is an independent sample of size n from a population defined by the pmf $f(x) = \theta^x(1 - \theta)^{1-x}$, $x = 0, 1$, where $0 < \theta < 1$, find the maximum likelihood estimator for θ .
2. Find the maximum likelihood estimator for θ in each of the following cases:
 - (a) $f(x; \theta) = \theta x^{\theta-1}$, $0 < x < 1$, where $\theta > 0$.
 - (b) $f(x; \theta) = \theta e^{-\theta x}$, $x > 0$, where $\theta > 0$.
 - (c) $f(x; \theta) = e^{-(x-\theta)}$, $x \geq \theta$, where θ is any real number.
 - (d) $f(x; \theta) = \frac{1}{2}e^{-|x-\theta|}$, where θ is any real number.
 - (e) $f(x; \theta) = \frac{2x}{\theta^2}$, $0 \leq x \leq \theta$, where $\theta > 0$.
3. Find the mles of θ_1 and θ_2 in each of the following cases:
 - (a) $X \sim N(\theta_1, \theta_2)$
 - (b) $f(x; \theta_1, \theta_2) = \frac{1}{\theta_2} e^{-\frac{(x-\theta_1)}{\theta_2}}$, $x > \theta_1$
where θ_1 is any real number and $\theta_2 > 0$.
4. Find the mle of θ in each of the following cases:
 - (a) $X \sim B(m, \theta)$.
 - (b) $X \sim \mathcal{P}(\theta)$.
 - (c) $X \sim B(\mu, \theta)$.
5. Let $X \sim U[0, \theta]$, where $\theta > 0$. Compute the mle for θ and show that it is unbiased.
6. Show that the sample mean is an unbiased and consistent estimator of population mean.
7. Show that sample variance is a biased but consistent estimator of the population variance.
8. Let Y_n be an unbiased estimator of θ , such that $V(Y_n) \rightarrow 0$ as $n \rightarrow \infty$. Then show that Y_n is consistent.
9. Find the same size n such that $P[\bar{X} - 1 < \mu < \bar{X} + 1] = 0.9$, given that $X \sim N(\mu, 9)$.

10. If the observed value of the mean of a sample of size 20 from a population having distribution $N(\mu, 80)$ is $\bar{x} = 81.2$, find a 95 percent confidence interval for the population mean.
11. If a random sample of size 17 from a normal distribution $N(\mu, \sigma^2)$ yields $\bar{x} = 4.7$ and $s^2 = 5.76$, determine a 90 percent confidence interval for μ .
12. Find an approximate 95 percent confidence interval for the mean of a population having variance 100, if the sample size is 25.