MAT 2155: PROBLEM SET 3

1. In a class of 76 students, 26 have a brother, 29 have a sister, and 20 have both. How many have no siblings?
2. Number of positive integers between 1 and 100 not divisible by any of 2, 3, and 5.
3. Number of positive integers less than or equal to $p^a q^b$, not divisible by either one of p and q, where p and q are distinct prime numbers and a and b are positive integers.
4. Number of derangements of n objects.
5. Show that the proportion of permutations of $1, 2, \ldots, n$ with no consecutive pair i, $i+1$ for any i is $\frac{n+1}{ne}$.
6. Number of distributions of 30 distinct objects into 3 distinct boxes such that no box is empty.
7. Number of subsets of $\{1, 2, \ldots, 2n\}$ such that the sum of all the elements in the subset is odd.
8. If A and B are finite sets of cardinalities n and m respectively, where $n \geq m$, then show that the number of surjective functions from A to B is $\sum_{k=0}^{m} (-1)^k \binom{m}{k} (m-k)^n$.