

PROBABILITY AND STATISTICS – PROBLEM SET 4

1. (X, Y) is a two dimensional random variable with joint pdf $f(x, y) = x + y, 0 < x < 1, 0 < y < 1$. Compute the following.
 - (a) $P[2X + 2Y < 1]$.
 - (b) $P[2X + 2Y < 3]$.
 - (c) $P[X \geq \frac{1}{2}]$.
 - (d) $P[X \geq \frac{1}{2}, Y \geq \frac{1}{2}]$.
 - (e) $P[Y \geq \frac{1}{2} | X \geq \frac{1}{2}]$.
 - (f) $P[X + Y > 1 | X \geq \frac{1}{2}]$.
2. From a box containing r red, g green, and b blue marbles, n marbles are drawn successively **with replacement**. Let X and Y respectively be the number of blue marbles and number of green marbles obtained in these n draws.
 - (a) Determine the joint pmf of (X, Y) and compute the marginal pmfs.
 - (b) Are X and Y independent?
 - (c) For the particular case $r = 3, g = b = 2$, tabulate the joint pmf values and compute the coefficient of correlation between X and Y .
3. Compute the marginal pdfs of X and Y and show that they are independent, given their joint pdf (X, Y) .
 - (a) $f(x, y) = 3 - 6x - y + 2xy, 0 < x < \frac{1}{2}, 0 < y < 2$.
 - (b) $f(x, y) = e^{-2x - \frac{y}{2}}, x, y > 0$.
4. For each random variable (X, Y) with joint pdf $f(x, y)$ as given below, compute the coefficient of correlation ρ .
 - (a) $f(x, y) = x + y, 0 < x, y < 1$.
 - (b) $f(x, y) = 3x + 3y, 0 < y < x < 1$.
 - (c) $f(x, y) = 2x - xy, 0 < x < 1, 0 < y < 2$.
 - (d) $f(x, y) = \frac{1}{\pi}, x^2 + y^2 \leq 1$ [Uniform distribution in the unit disc centred at the origin].
 - (e) $f(x, y) = \frac{2}{\pi}, y \geq 0, x^2 + y^2 \leq 1$.
 - (f) $f(x, y) = \frac{4}{\pi}, x, y \geq 0, x^2 + y^2 \leq 1$.
 - (g) $f(x, y) = \frac{1}{4}, (x, y) \in R$, where R is the square with vertices $(1, 1), (-1, 1), (-1, -1), (1, -1)$.

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- (h) $f(x, y) = \frac{1}{2}$, $(x, y) \in R$, where R is the square with vertices $(1, 0)$, $(0, 1)$, $(-1, 0)$, $(0, -1)$.

Are X and Y independent in all the cases where $\rho = 0$?