PROBABILITY AND STATISTICS – PROBLEM SET 6

1. (X, Y) is a two dimensional random variable having joint pdf $f(x, y) = 3xe^{-(x+3y)}$, $x, y > 0$. If $Z = X$ and $W = 2X + Y$, determine the distribution (Z, W).
2. Let (X, Y) be uniformly distributed in the unit square $0 \leqslant x, y \leqslant 1$. Find the pdf of $Z = X + Y$.
3. If $X \sim \mathcal{E}(2)$ and $Y \sim \mathcal{E}(1)$ are independent, find the pdf of:
   (a) $Z = X + Y$.
   (b) $Z = X/Y$.
4. If (X, Y) has the joint pdf $f(x, y) = 10xy^2$, $0 < x < y < 1$, determine the pdf of $Z = X/Y$.
5. Find the pdf of $Z = X/Y$, if (X, Y) has joint pdf $f(x, y) = 8xy$, $0 < x < y < 1$.
6. (X, Y) is uniformly distributed over the unit disc $x^2 + y^2 \geqslant 1$. Find the pdf of $R = \sqrt{X^2 + Y^2}$.
7. Let X be a random variable with pdf $f(x) = \dfrac{5}{x^2}$, $x > 5$. If X_1 and X_2 are two independent random variables following this distribution, find the pdf of $Y = X_1/X_2$.
8. Let $W \sim N(0, 1)$ and $V \sim \chi^2_n$. Compute the distribution of $T = \dfrac{W}{\sqrt{V/n}}$.
9. If (X_1, X_2) has the joint pdf $f(x_1, x_2) = 2e^{-(x_1+x_2)}$, $x_1 > x_2 > 0$, find the joint pdf of $Y_1 = X_1 - X_2$, $Y_2 = 2X_2$.