PROBABILITY AND STATISTICS – PROBLEM SET 8

1. Compute the moment generating function of the discrete random variable X with pmf $f(x) = \dfrac{1}{2^x}$, $x = 1, 2, \ldots$.
2. If $0 < p < 1$ and $q = 1 - p$, find the mgf of the random variable X with pmf $f(x) = p^{x-1}q$, $x = 1, 2, \ldots$, and hence find $E[X]$ and $V[X]$.
3. Show that if X and Y are independent Poisson variate with means λ and μ respectively, then $X + Y$ is a Poisson variate with mean $\lambda + \mu$.
4. Show that if $X_i \sim N(\mu_i, \sigma_i^2)$, $i = 1, \ldots, n$ are n independent normal variates, and a_i, $i = 1, \ldots, n$ are constants, then $X = \sum_{i=1}^n a_i X_i \sim N(\mu, \sigma^2)$ where $\mu = \sum_{i=1}^n a_i \mu_i$ and $\sigma = \sqrt{\sum_{i=1}^n a_i^2 \sigma_i^2}$.
5. Show that if $Z_i \in N(0, 1)$, $i = 1, \ldots, n$ are n independent standard normal variates, then $\sum_{i=1}^n Z_i^2 \sim \chi_n^2$.