Estimation of Parameters

1 Confidence Intervals

Definition 1.1. If $(X_1, \dots, X_n)$ is a random sample of size n, then the *sample mean* is defined as

$$\overline{X} = \frac{1}{n} \sum_{i=1}^{n} X_i$$

and the *sample variance* is defined as

$$S^2 = \frac{1}{n} \sum_{i=1}^{n} \left(X_i - \overline{X}\right)^2.$$

Theorem 1.2. *Suppose that $X \sim N(\mu, \sigma^2)$. Then the following hold:*

1. $\dfrac{\overline{X} - \mu}{\sigma/\sqrt{n}} \sim N(0, 1)$.

2. $\dfrac{nS^2}{\sigma^2} \sim \chi^2_{n-1}$.

3. $\dfrac{\overline{X} - \mu}{S/\sqrt{n-1}} \sim t_{n-1}$.

Confidence intervals for the mean and variance of a normal population, under different conditions, are as described in the table given below.

Parameter	**Condition**	**Interval**
μ	σ^2 known	$\left(\overline{x} - \frac{a\sigma}{\sqrt{n}}, \overline{x} + \frac{a\sigma}{\sqrt{n}}\right)$ where $P[-a < Z < a] = p$, $Z \sim N(0,1)$
	σ^2 unknown	$\left(\overline{x} - \frac{bS}{\sqrt{n-1}}, \overline{x} + \frac{bS}{\sqrt{n-1}}\right)$ where $P[-b < T < b] = p$, $T \sim t_{n-1}$
σ^2	μ unknown	$\left(\frac{ns^2}{b}, \frac{ns^2}{a}\right)$ where $P[Z < a] = P[Z > b] = \frac{p}{2}$, $Z \sim \chi^2_{n-1}$.