PROBABILITY AND STATISTICS – PROBLEM SET 5

1. Suppose that the number of items produced in a factory during one week is a random variable with mean 500 and variance 100.
   (a) What can be said about the probability that this week's production will be at least 1000?
   (b) What can be said about the probability that this week's production will be between 400 and 600?
2. If the time required to complete a task is a random variable with mean 20 minutes and standard deviation 3 minutes, find the smallest time frame such that the probability that the task will be completed within the time frame is at least 0.75.
3. If $X \sim B(6, p)$, and $P[X = 2] = 9\,P[X = 4]$, then find $P[X \leqslant 3]$.
4. An urn contains 4 red, 4 green, and 2 blue marbles. One marble is drawn at random from the urn, observed, and placed back. Find the probability that in 8 such draws
   (a) exactly 3 red marbles are drawn.
   (b) not more than 3 red marbles are drawn.
   (c) at least 3 red marbles are drawn.
   (d) not all marbles drawn are red.
5. The number of visitors to a webpage per minute follows a Poisson distribution. If the average number of visitors per minute is 4, what is the probability that the webpage receives
   (i) exactly four visitors in one minute?
   (ii) at least two visitors in one minute?
6. If X is a Poisson variate such that $3\,P[X = 2] = 2\,P[X = 1]$, then what is $E[X]$?
7. It is observed in a communication channel that 90% of the messages sent are received without any error. Find the probability that among 18 messages sent through the channel
   (i) at least 16 are received without any error
   (ii) at most 14 are received without any error.
8. In a certain factory producing blades, there is a small probability of $\frac{1}{500}$ for any blade to be defective. The blades are supplied in packets of 10. Calculate the approximate number of packets containing
   (i) no defective
   (ii) one defective
   (iii) two defective

(iv) at least two defective
blades in a consignment of 10000 packets.

9. One per thousand of a population is subject to certain kinds of accidents each year. Given that an insurance company has insured $5,000$ persons from the population, find the probability that at most 2 of them will incur this accident.
10. An airline company, having observed that 5% of the persons making reservations on a flight do not show up for the flight, sells 100 seats on a plane that has 95 seats. What is the probability that there will be a seat available for every person who shows up for the flight?
11. If X is a binomial variate with mean 3, such that $P[X = 3] = 2\,P[X = 2]$, find $V[X]$.
12. X is a Poisson variate and the probability that X is even is twice the probability that it is odd. Determine $P[X = 0]$.
13. If X is exponentially distributed with $E[X] = 2$, then find the value of a such that $P[X \leqslant a] = P[X \geqslant a]$.
    Note: This point a is the *median* of X.
14. Suppose that the duration in minutes of a phone call follows an exponential distribution with mean 5 minutes.
    (a) Find the probability that the duration of a particular call
        (i) will exceed 5 minutes
        (ii) will be between 5 and 6 minutes
        (iii) will be less than 3 minutes
        (iv) will be less than 6 minutes, given that it was greater than 3 minutes.
    (b) Suppose exactly 100 such phone calls are received every day. Find the probability that on a given day
        (i) every phone call lasted longer than 8 minutes
        (ii) at least 4 phone calls lasted longer than 8 minutes.
    (c) Find the probability that in one week, there are at least 3 days on each which there were at least 4 phone calls that lasted longer than 8 minutes.
15. A random variable X follows an exponential distribution with parameter $\alpha = 3$. Compute
    (a) $P[2X > 1]$.
    (b) $P[X > s + t \mid X > t]$, where $s, t > 0$.
16. The daily consumption of milk in a city, in excess of 20,000 gallons, is distributed as a Gamma variate with parameters $\alpha = \frac{1}{10000}$ and $r = 2$. The city has a daily stock of 30000 gallons. What is the probability that the stock is insufficient on a particular day?
17. If $K \sim U[0, 5]$, then what is the probability that the roots of the equation $4x^2 + 4Kx + K + 2 = 0$ are real?

18. A book of 200 pages contains 100 misprints distributed randomly throughout its pages. If one page is selected at random and examined, what is the probability that it contains
    (a) no misprints?
    (b) at least 2 misprints?
19. Suppose that the chances of a traffic accident in a street of a city in a day is 0.001. In how many days in a year can we expect
    (a) no accidents?
    (b) at least one accident, if there are 100 such streets in a city?

    If there are 5 such cities in a state, what is the probability that at least one city will have at least one accident in a day?
20. Buses start from a certain station at intervals of 45 minutes. What is the probability that a person reaching the station at a random point in time will have to wait for at least 30 minutes?
21. In a bombing action, there is a 50% chance that a bomb will hit a target. Two direct hits are needed to destroy the target completely. Estimate the minimum number of bombs to be dropped to give 90% or better chance of completely destroying the target.