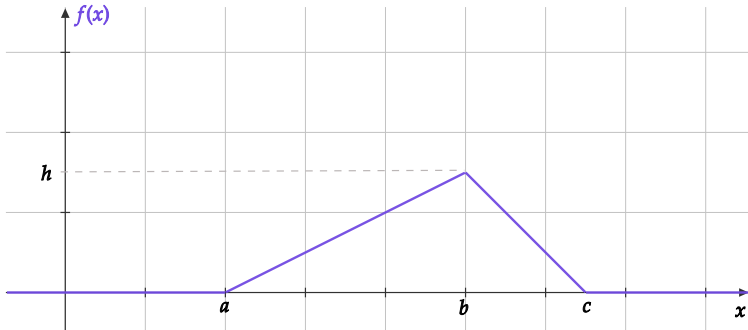


PROBABILITY AND STATISTICS – PROBLEM SET 3

1. $X \sim U[a, b]$, $a < 1 < 2 < b$. If $P[X < 1] = 2 P[X > 2]$, and $5 P[X > 1] = 3 P[X < 2]$, find the values of a and b .
2. X is a discrete random variable taking values $0, 1, 2, \dots$, whose pmf $f(x)$ is such that $\frac{f(x_1)}{f(x_2)} = r^{x_2 - x_1}$ for all x_1, x_2 (where $r \in (0, 1)$ is a constant). Determine $f(x)$ and compute $\mathbb{E}[X]$.
3. Let X be the random variable with pdf $f(x)$ whose graph is shown below (where $a < b < c$ are three real numbers).



Find h , and compute $\mathbb{E}[X]$.

4. Show that the function

$$f(x) = \frac{\lambda^x}{x!} e^{-\lambda}, x = 0, 1, 2, \dots$$

(where $\lambda > 0$ is a constant) is a valid pmf, and compute $\mathbb{E}[X]$ and $\mathbb{V}[X]$.

5. X is a discrete random variable with pmf $f(x) = \frac{6}{\pi^2 x^2}$, $x = 1, 2, \dots$

- (a) Compute $P[X \geq 3]$
- (b) Compute $P[10 \leq X \leq 15]$
- (c) Show that $\mathbb{E}[X]$ does not exist.

6. X is a continuous random variable with pdf $f(x) = \frac{6}{\pi^2} \left(\frac{1}{[x]} \right)^2$, $x > 1$.

- (a) Compute $P[X \geq 3]$

- (b) Compute $P[10 \leq X \leq 15]$
- (c) Show that $\mathbb{E}[X]$ does not exist.
7. X has pdf $f(x) = \frac{x^2}{3}, -1 \leq x \leq 2$. Compute
- $P[X < \frac{3}{2}]$
 - $P[|X| > 1]$
 - $P[X < \frac{3}{2} \mid X > \frac{1}{2}]$
 - $P[X < \frac{3}{2} \mid |X| > 1]$
 - $P[|X| < 1 \mid X < \frac{3}{2}]$.
8. An urn initially has 1 red and 1 blue marble. A marble is drawn at random from the urn, and if it is blue, it is put back and one red marble is added to the urn. This is continued until a red marble is drawn. Let X denote the total number of draws required to obtain a red marble. Determine the probability distribution of X , and find its mean and variance. **Hint:** Compute $\mathbb{E}[X + 1]$ and $\mathbb{E}[X^2 - 1]$.
9. The *median* of a random variable X is the point c such that $P[X \leq c] = P[X \geq c] = \frac{1}{2}$. Compute the median of X with pdf $f(x)$ in each of the following cases.
- $f(x) = \frac{1}{b-a}, a < x < b$.
 - $f(x) = ae^{-ax}, x > 0$ [where $a > 0$ is a constant].
 - $f(x) = \frac{1}{\pi(1+x^2)}$ [Note that this distribution has no mean, but has a median].