Testing of Hypotheses

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1 Introduction

Consider a coin that looks normal, but may or may not be fair – i.e., the probability that it comes up heads when tossed may or may not be $\frac{1}{2}$. How can we ascertain whether it is indeed fair? If we toss it a hundred times, and heads appear exactly fifty times out of these hundred tosses, then we may *feel* sure that it is a fair coin. However, even a fair coin does not always give a perfectly fifty-fifty results every time. And a biased coin may in fact give a fifty-fifty result once in a while. Statistical testing of hypotheses deals with making formal tests of such claims using the theory of probability and statistics.

For instance, in this example, let X be a random variable associated with the result of a single toss of the given coin – let X be 1 if the result is heads and 0 if the result is tails. If θ is the (unknown) probability of heads, then we have $f(1) = \theta$ and $f(0) = 1 - \theta$, where $f(x)$ is the probability mass function of X. Thus, we see that the probability distribution of X has a known type[1], but depends on an unknown parameter θ. That is, if the value of θ were given, it would completely determine the probability distribution of X.

Now, suppose we suspect that the coin is biased, and has a higher probability for heads – i.e., that $\theta > 0.5$. We wish to test this *hypothesis*, which we may denote as $H_1 \colon \theta > 0.5$. We mean to test this *against* the hypothesis that the coin is fair – i.e., that $\theta = 0.5$. This hypothesis can be denoted as $H_0 \colon \theta = 0.5$. We call H_0 the *null hypothesis* and H_1 the *alternative hypothesis*.

In order to test these hypotheses, we can conduct an experiment of tossing the coin a fixed n number of times and observing the result each time – or equivalently, of noting the value of X each time. Thus, we get n different values of X, say, $X_1, X_2, \ldots, X_n$ (each one being either 0 or 1). In other words, we have a sample $(X_1, X_2, \ldots, X_n)$[2] from the population defined by the random variable X.

The question now is, how can we decide from the results of the experiment whether the coin is fair or not? As we noted earlier, an exact fifty-fifty distribution cannot be expected from a fair coin, and at the same time, such a result is possible, if rare, even in the case of a biased coin. So we must allow for a margin of error, luck, or chance in the final result. For instance, we may decide that even if the number of heads obtained is up to 60% of the total number of tosses, the coin may still be fair – but that if the number of heads is above 60%, then it is most likely not fair. Such a choice is certainly arbitrary. But, firstly, there is no theory that can tell us exactly what the cut-off should be. And secondly, what truly matters is how certain we can be of the results of the test, after we choose a particular cut-off (or some other criterion). Measuring this amount of certainty and the probabilities of errors is the real essence of this subject – since all we can control is the amount of certainty we have about the results we obtain.

Since X is 1 when the result is heads and 0 otherwise, the sum $X_1 + X_2 + \cdots + X_n$ is exactly the number of heads. Recall that we have decided to declare the coin to be fair if the percentage of heads obtained is at most 60% – and biased if it is more than 60%. The first case is equivalent to having $\sum_{i=1}^{n} \frac{X_i}{n} \leq 0.6$, that is, $\overline{X} \leq 0.6$, where $\overline{X}$ is the sample mean. And therefore the second case is equivalent to having $\overline{X} > 0.6$. Thus, we will *accept the null hypothesis* H_0 if $\overline{X} \leq 0.6$ and we will *reject the*

[1] Here, $X \sim B(1, \theta)$ – a special case of the binomial distribution (i.e., with $n = 1$), called the Bernoulli distribution.

[2] Written in this form, $(X_1, X_2, \ldots, X_n)$ denotes the as yet unknown value of the sample, which we are considering *before* having conducted the experiment – thus, $(X_1, \ldots, X_n)$ is a (multidimensional) random variable. After conducting the experiment, however, we will obtain *particular* values $x_1, \ldots, x_n$ of these random variables, and thus we will have a *sample point* $(x_1, \ldots, x_n)$. The set of all possible sample points is exactly the *sample space* associated with this experiment. This subtle difference in notation is not new – we usually use X (and other upper case letters) to denote the random variable and x (or corresponding lower case letters) to denote a possible *value* taken by the random variable.

null hypothesis H_0 (and accept the alternative hypothesis H_1) if $\overline{X} > 0.6$. This is an example of what is formally called a *statistical test*.

To summarise what we have so far, a coin is given, which we suspect to be biased. To express this mathematically, we associate a random variable X with the coin, which takes value 1 with probability θ – the probability of heads in a single toss – and 0 with probability $1 - \theta$ (which indicates tails in a single toss). The hypothesis we wish to test[3] is $H_1 : \theta > 0.5$, against the null hypothesis $H_0 : \theta = 0.5$. The test that we have decided to use is: Compute the the sample mean $\overline{x}$, and if $\overline{x} > 0.6$, then we reject the null hypothesis H_0 and accept the alternative hypothesis H_1 (note that here we write $\overline{x}$ since we are talking about the *value* of the sample mean obtained after conducting the experiment).

As mentioned earlier, there is no way to fully justify the decision to use 0.6 as the cut-off instead of some other number. What we can do is measure the probability that our test will give us an erroneous result. What does this mean? Remember that while in reality the coin may be fair, there is still a possibility (however small) that in a particular sequence of tosses, a large number of heads is obtained. If this occurs in our experiment, we will reject H_0 and accept H_1 – thus committing an error, although we have no means of detecting it. Another possibility is that the coin is actually biased, but by a similar accident of chance, the number of heads obtained is nearly 50%. Then we will accept H_0 and reject H_1 – which is another type of error, which again we cannot detect. Thus we have the following.

	H_0 **is True**	H_0 **is False**
Accept H_0	`No error`	**Type II error**
Reject H_0	**Type I error**	`No error`

Note. Carefully note the distinction between the *statement* that a hypothesis "is true" and the *decision* to "accept" the hypothesis. The hypothesis' being true (or not) is a fact of reality which, by definition, we cannot know in practice (otherwise there would be no need for any statistical test). On the other hand, accepting (or rejecting) the hypothesis is a decision we make based on a pre-decided rule (namely the hypothesis test) – which is not a statement about reality, but an *action*.

Thus, we wish to compute the probabilities of the two errors – Type I and Type II – in our example. Since Type I error is *rejecting* H_0 when H_0 *is true*, what we want is $P[\text{Reject } H_0 \mid H_0 \text{ True}]$. But we reject H_0 only if $\overline{X} > 0.6$. And H_0 is true only if $\theta = 0.5$. Thus, the probability of Type I error, which we shall denote by α, is $\alpha = P\left[\overline{X} > 0.6 \mid \theta = 0.5\right]$. It is not hard to see that given the distribution

[3] It is a convention that the hypothesis we form and wish to establish is taken as the *alternative* hypothesis H_1, rather than the *null* hypothesis H_0. The latter is to be thought of as a default belief, that is to be *refuted* by our experiment. And if we fail to refute this – i.e., if we fail to prove the alternative hypothesis – then it does not usually mean that we strongly believe the null hypothesis. It means only that we have *not* succeeded in proving our claim. This is in keeping with the basic scientific principle that the onus of proof is on the one making the claim.

of X as stated earlier, the random variable $\sum_{i=1}^{n} X$ follows the binomial distribution $B(n,\theta)$. If $\theta = 0.5$ (i.e., if H_0 is true), then $\sum_{i=1}^{n} X \sim B(n, 0.5)$. And therefore, $P\left[\overline{X} > 0.6\right] = P[\sum X > 0.6n]$, which can be computed in the usual way, given the value of n, the number of tosses. For example, if $n = 10$, then $\alpha = P(\sum X > 6) = \left(\binom{10}{7} + \cdots + \binom{10}{10}\right)(0.5)^{10} \approx 0.17$. That is, the probability of Type I error in our test when $n = 10$ is $\alpha = 0.17$. This probability is known as the *level of significance* of the test. Any particular value of $K(\theta)$ at a given value of θ is called the *power of the test* at that point.

More generally, the probability of rejecting H_0 is a function of the parameter θ. In order to compute α, the level of significance, we have taken $\theta = 0.5$, which is the case when H_0 is true. If we find the probability of rejecting H_0 for an arbitrary value of θ, we get the *power function of the test*, $K(\theta) = P[\text{Reject } H_0 \mid \theta]$. Thus, in our example, $\alpha = K(0.5)$[4].

As the result of the test is entirely based on whether the sample point obtained satisfies a certain criterion or not (in our case, whether $\overline{x} > 0.6$ or not), we call the set of all sample points that satisfy the criterion as the *critical region*. In other words, the critical region is that subset C of the sample space such that we reject H_0 and accept H_1 if $(X_1, \ldots, X_n) \in C$. The statistical test is therefore completely determined by specifying the critical region.

1.1 Definitions

Definition 1.1. A *statistical hypothesis* is a claim about the probability distribution of one or more populations (random variables). The hypothesis is *simple* if it completely specifies the distribution, and *composite* otherwise.

Definition 1.2. A *statistical test* of a hypothesis is a rule, stated in terms of a sample, that prescribes whether the hypothesis is to be rejected or accepted, based on the value of the sample point obtained. The *critical region* of the test is the region of the sample space such that if the sample point lies in it, the hypothesis is rejected.

Thus, we may also say that a statistical test is the specification of a subset of the sample space as the critical region, and the prescription that the hypothesis be rejected if the sample point obtained is in this region.

[4] In general, the significance level α is the *maximum* probability of rejecting H_0 when it is true. In our example, H_0 is a *simple hypothesis* where θ takes a particular value (and H_1 is a *composite hypothesis*, where θ lies in a range of values). Therefore in our example, there is only one way in which H_0 could be true and correspondingly there is only one probability of rejecting it when it is true. If H_0 were a composite hypothesis (for example, $\theta \leq 0.5$), then for each possible value of θ that makes H_0 true, we would get a different value of the power $K(\theta)$, and then α would be taken as the maximum (or supremum) of these values.

Definition 1.3. The *power function* K of a test is the probability that the sample point falls in the critical region of the test. If H_0 is the null hypothesis of the test, then K is the probability of rejecting H_0. The value of the power function at a particular point (a particular assignment of values to the unknown parameter(s)) is the *power* of the test at that point.

Definition 1.4. The *level of significance* of a test is the *size* of the critical region of the test – that is, the significance level is the supremum/maximum probability of rejecting the null hypothesis H_0 when H_0 is true.

1.2 Solved Problems

1. X has a pdf of the form $f(x;\theta) = \theta x^{\theta-1}$, $0 < x < 1$, where $\theta > 0$. To test the simple hypothesis $H_0\colon \theta = 1$ against the alternative simple hypothesis $H_1\colon \theta = 2$, it is decided to use a random sample (X_1, X_2) of size $n = 2$ with the critical region as $C = \left\{(x_1, x_2) \mid x_1 x_2 \geq \frac{3}{4}\right\}$. Compute the power function $K(\theta)$ and the significance level α of the test.

   Solution. We know that the power function $K(\theta)$ is the probability of rejecting the null hypothesis, which is the probability that $(X_1, X_2) \in C$. Thus, according to the definition of C given in the question,

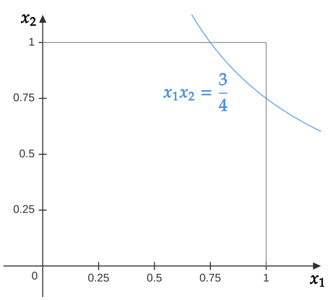
$$K(\theta) = P\left(X_1 X_2 \geq \frac{3}{4}\right).$$

   Since (X_1, X_2) is a sample, X_1 and X_2 are independent, and therefore, the joint pdf of the sample is

$$\begin{aligned} g(x_1, x_2; \theta) &= f(x_1;\theta) f(x_2;\theta) \\ &= \theta^2 (x_1 x_2)^{\theta-1}, \quad 0 < x_1, x_2 < 1. \end{aligned}$$

   Thus,

$$\begin{aligned} K(\theta) &= P\left(X_1 X_2 \geq \frac{3}{4}\right) \\ &= \int_{x_1=3/4}^{1} \int_{x_2=3/4x_1}^{1} \theta^2 (x_1 x_2)^{\theta-1}\, dx_2\, dx_1 \\ &= \theta^2 \int_{x_1=3/4}^{1} x_1^{\theta-1} \left[\frac{x_2^\theta}{\theta}\right]_{3/4x_1}^{1} dx_1 \\ &= \theta \int_{x_1=3/4}^{1} x_1^{\theta-1} - \frac{1}{x_1}\left(\frac{3}{4}\right)^\theta dx_1 \\ &= 1 - \left(\frac{3}{4}\right)^\theta \left[1 - \theta \log \frac{3}{4}\right]. \end{aligned}$$



Now the significance level is obtained by evaluating the power function at the parameter point where the null hypothesis H_0 is true, that is at $\theta = 1$. Therefore, the significance level is

$$\begin{aligned}\alpha &= K(1)\\ &= \frac{1}{4} + \frac{3}{4}\log\frac{3}{4}\\ &\approx 0.034.\end{aligned}$$

2. Let (X_1, X_2) be a random sample of size 2 from the distribution having pdf $f(x;\theta) = \frac{1}{\theta}e^{-x/\theta}$, $x > 0$, where $\theta > 0$. We reject $H_0\colon \theta = 2$ and accept $H_1\colon \theta = 1$ if the observed value (x_1, x_2) of the sample is such that

$$\frac{f(x_1;2)f(x_2;2)}{f(x_1;1)f(x_2;1)} \le \frac{1}{2}.$$

Find the significance level of the test and the power of the test when H_1 is true.

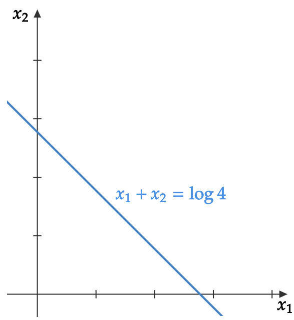
Solution. The critical region is defined by

$$\begin{aligned}\frac{f(x_1;2)f(x_2;2)}{f(x_1;1)f(x_2;1)} \le \frac{1}{2} &\iff\\ \frac{1}{4}\times\frac{e^{-\frac{x_1+x_2}{2}}}{e^{-(x_1+x_2)}} \le \frac{1}{2} &\iff\\ e^{\frac{x_1+x_2}{2}} \le 2 &\iff\\ x_1 + x_2 \le \log 4.\end{aligned}$$

Now, $g(x_1, x_2;\theta) = \frac{1}{\theta^2}e^{-\frac{x_1+x_2}{\theta}}$, $x_1, x_2 > 0$.

The power function is

$$\begin{aligned}K(\theta) &= \frac{1}{\theta^2}\int_0^{\log 4}\int_0^{\log 4 - x_1} e^{-\frac{x_1+x_2}{\theta}}\,dx_2\,dx_1\\ &= \frac{1}{\theta}\int_0^{\log 4} e^{-\frac{x_1}{\theta}}\left[1 - e^{\frac{x_1-\log 4}{\theta}}\right]dx_1\\ &= \frac{1}{\theta}\int_0^{\log 4} e^{-\frac{x_1}{\theta}} - 4^{-\frac{1}{\theta}}\,dx_1\\ &= 1 - 4^{-\frac{1}{\theta}} + 4^{-\frac{1}{\theta}}\log 4^{-\frac{1}{\theta}}.\end{aligned}$$



Then the significance level is

$$\begin{aligned}\alpha &= K(2) \qquad [H_0 : \theta = 2] \\ &= \frac{1}{2} + \frac{1}{2}\log\frac{1}{2} \\ &\approx 0.15.\end{aligned}$$

H_1 is true when $\theta = 1$. Thus, the power of the test when H_1 is true is

$$K(1) = \frac{3}{4} + \frac{1}{4}\log\frac{1}{4} \approx 0.4.$$

3. The life length of a tyre in miles, X, is normally distributed with mean θ and standard deviation 5000. Past experience indicates that $\theta = 30,000$. The manufacturer claims that the tyres made by a new procedure have $\theta > 30,000$ and it is very likely that $\theta = 35,000$. Check the claim by testing $H_0 : \theta \le 30,000$ against $H_1 : \theta > 30,000$. Observe n independent values of X, say $x_1, x_2, \dots, x_n$ and reject H_0 if and only if $\overline{x} > c$. Determine n and c so that the power function $K(\theta)$ has values $K(30,000) = 0.01$ and $K(35,000) = 0.98$.

   Solution. $K(\theta) = P[\overline{X} > c \mid \theta]$ (where $\overline{X}$ is the sample mean[5]).

   Since $X \sim N(\theta, 5000^2)$, the sample mean $\overline{X} \sim N\left(\theta, \frac{5000^2}{n}\right)$. Therefore, $Z = \dfrac{\overline{X} - \theta}{5000/\sqrt{n}} \sim N(0,1)$.

$$\begin{aligned} K(30,000) = 0.01 &\implies \\ P[\overline{X} > c \mid \theta = 30000] = 0.01 &\implies \\ P\left[Z > \frac{c - 30000}{5000/\sqrt{n}}\right] = 0.01 &\implies \\ P\left[Z \le \frac{c - 30000}{5000/\sqrt{n}}\right] = 0.99 &\implies \\ \frac{c - 30000}{5000/\sqrt{n}} = 2.33 & \qquad (1)\end{aligned}$$

[5]Here we write $\overline{X}$ because it is a random variable, not the particular value $\overline{x}$ obtained after the experiment.

$$
\begin{aligned}
P[\overline{X} > c \mid \theta = 35000] = 0.98 &\implies \\
P\left[Z > \frac{c - 35000}{5000/\sqrt{n}}\right] = 0.98 &\implies \\
P\left[Z \leq \frac{35000 - c}{5000/\sqrt{n}}\right] = 0.98 &\implies \\
\frac{35000 - c}{5000/\sqrt{n}} = 2.06 & \qquad (2)
\end{aligned}
$$

From (1) and (2), $n = 19.2 \approx 19$ and $c = 32653.8$.

1.3 Exercises

1. Let X have a Poisson distribution with mean θ. Test the simple hypothesis $H_0\colon \theta = 0.5$ against the composite hypothesis $\theta < 0.5$ by using a sample $(X_1, \ldots, X_{12})$ of size 12. Reject H_0 if and only if the observed value of $Y = X_1 + \cdots + X_{12} \leq 2$. Find the powers $K(\frac{1}{2})$, $K(\frac{1}{3})$, $K(\frac{1}{4})$, $K(\frac{1}{6})$, and $K(\frac{1}{12})$. What is the significance level of the test?

2. Let Y have a binomial distribution with parameters n and p. We reject $H_0\colon p = \frac{1}{2}$ and accept $H_1\colon p > \frac{1}{2}$ if $Y \geq c$. Find n and c to give a power function $K(p)$ which is such that $K(\frac{1}{2}) = 0.1$ and $K(\frac{2}{3}) = 0.95$, approximately.

3. Let $X \sim U[0, \theta]$. Test $H_0\colon \theta = 1$ against $H_1\colon \theta = 2$ using a sample (X_1, X_2) of size 2, by rejecting H_0 if either $\overline{X} > 0.75$ or at least one of X_1 and X_2 is greater than 1. Compute $K(1)$ and $K(2)$.

2 Chi-Square Tests

In Section 1, we considered the problem of determining if a given coin was fair or biased. Now imagine that the same question is asked about a given six-sided die. Again, we can test the hypothesis that the die is fair, by throwing it n times and observing the distribution of the resulting outcomes. Now, however, we have not two but six different classes into which the outcomes are divided[6] – namely the number of appearances of 1, the number of appearances of 2, ..., the number of appearances of 6. Let these numbers be denoted by $x_1, x_2, \ldots, x_6$. We also have the probabilities $p_1, p_2, \ldots, p_6$ for the occurrence of the respective faces $1, 2, \ldots, 6$ of the

[6] Recall that in the case of a coin, the two classes are number of heads and number of tails – and given the total number of tosses, it is enough to specify just one of these, say, the number of heads obtained.

die in each throw. In the case of a fair die, each of these would be $p_i = \frac{1}{6}$. Since n is the total number of throws, we expect that each observed number x_i should be more or less equal to np_i. For instance, if we throw the die 60 times, then if the die is fair, each face should appear roughly 10 times. Therefore the difference $x_i - np_i$ between the observed and expected number of appearances is a measure of how *false* our hypothesis is likely to be. Therefore we need to design a test that uses these deviations to estimate the probability of the truth or falsehood of the hypothesis.

Firstly, observe that $x_6 = n - (x_1 + \cdots + x_5)$, so that we only need to consider $x_1, \ldots, x_5$. The sample $(X_1, X_2, \ldots, X_5)$ has what is known as the multinomial distribution[7] with parameters $n, p_1, \ldots, p_5$. Then if we define

$$\begin{aligned} Q_5 = {} & \frac{(X_1 - np_1)^2}{np_1} + \frac{(X_2 - np_2)^2}{np_2} + \frac{(X_3 - np_3)^2}{np_3} + \\ & \frac{(X_4 - np_4)^2}{np_4} + \frac{(X_5 - np_5)^2}{np_5} + \frac{(X_6 - np_6)^2}{np_6}, \end{aligned}$$

$Q_5 \sim \chi^2_5$ approximately for large n (i.e., in the limiting case where $n \to \infty$).

Now the test is as follows. Select a value c, and reject the hypothesis that the die is fair if $Q_5 \geq c$. What should be the ideal value of c? That depends on the significance level of the test. What we can do therefore, is to *set* a desired significance level α and select c such that $P(Q_5 \geq c) = \alpha$. This value of c can be obtained from the chi-square.

The chi-square test in general can therefore be described as follows. Given k classes $C_1, C_2, \ldots, C_k$, to test the hypothesis H_0 that the probability distribution of these classes is $p_1, p_2, \ldots, p_k$ respectively (where $p_1 + \cdots + p_k = 1$), we conduct n trials and observe the frequencies $X_1, X_2, \ldots, X_k$ of the respective classes (so that $X_1 + X_2 + \cdots + X_k = n$). Now define

$$Q_{k-1} = \sum_{i=1}^{k} \frac{(X_i - np_i)^2}{np_i}.$$

Then the chi-square test with significance level α is to reject H_0 if $Q_{k-1} \geq c$, where c is the value such that $P[Q_{k-1} \geq c] = \alpha$. Common values of α are 0.05 and 0.01, written as 5% and 1% respectively.

2.1 Solved Problems

1. A six-sided die is thrown 60 times and the observed frequencies of the faces $1, 2, \ldots, 6$ are $13, 19, 11, 8, 5, 4$ respectively. Test whether the die is fair, at 5%

[7] The multinomial distribution is a generalisation of the binomial distribution – the multinomial distribution with only two parameters n and p_1 is exactly $B(n, p_1)$.

level of significance.

Solution. Here, we have number of classes $k = 6$, and theoretical probabilities $p_1 = p_2 = \cdots = p_6 = \frac{1}{6}$ (since a fair die must have equally likely outcomes), so that $np_i = 10$, $i = 1, 2, \ldots, 10$. Then,

$$\begin{aligned} Q_5 &= \sum_{i=1}^{5} \frac{(x_i - np_i)^2}{np_i} \\ &= \frac{(13-10)^2}{10} + \frac{(19-10)^2}{10} + \frac{(11-10)^2}{10} + \frac{(8-10)^2}{10} + \frac{(5-10)^2}{10} + \frac{(4-10)^2}{10} \\ &= 15.6. \end{aligned}$$

But $\chi_5^2 = 11.1$ at 5% level of significance. Thus, $Q_5 > \chi_5^2$, which means that **we reject the hypothesis that the die is fair**.

2. The Mendelian theory of genetics of crossing two types of peas states that the probabilities of classification of the four resulting types are $\frac{9}{16}$, $\frac{3}{16}$, $\frac{3}{16}$, and $\frac{1}{16}$ respectively. If, from 160 independent observations, the observed frequencies of these classifications are $86, 35, 26, 13$ respectively, test whether the data is consistent with the theory with $\alpha = 0.01$.

   Solution. Here, $k = 4$, $n = 160$, and $np_1 = 90$, $np_2 = 30$, $np_3 = 30$, $np_4 = 10$. Therefore,

$$\begin{aligned} Q_3 &= \frac{(86-90)^2}{90} + \frac{(35-30)^2}{30} + \frac{(26-30)^2}{30} + \frac{(13-10)^2}{10} \\ &= 2.44 < 11.345 = \chi_3^2 \end{aligned}$$

   at 1% level of significance (since $\alpha = 0.01$).

   Thus, we **accept** the hypothesis – i.e, the data is consistent with Mendelian theory.

2.2 Exercises

1. The table below lists the observed results of $n = 120$ independent throws of a die.

Face	1	2	3	4	5	6
Frequency	a	20	20	20	20	$40 - a$

   For what values of a would the hypothesis that the die is unbiased be rejected at 0.025 level of significance in a chi-square test?

2. A manufacturer of lightbulbs claims that the lightbulbs produced fall into five categories A, B, C, D, and E by quality, from highest to lowest, and that the percentages of lightbulbs in these five categories are 15, 25, 35, 20, and 5 respectively. A contractor who purchases a large number of the lightbulbs tests the claim by taking a random sample of 30 lightbulbs and observes that the numbers of lightbulbs that fall in the categories A, B, C, D, and E are 3, 6, 9, 7, and 5 respectively. Test whether the manufacturer is speaking the truth, using a chi-square test

   (i) at 5% significance level;

   (ii) at 1% significance level.

Appendices

A Chi-Square Distribution

$P[Z \leq z]$ for $Z \sim \chi_n^2$

n	0.010	0.025	0.050	0.100	0.900	0.950	0.975	0.990
1	0.000	0.001	0.004	0.016	2.706	3.841	5.024	6.635
2	0.020	0.051	0.103	0.211	4.605	5.991	7.378	9.210
3	0.115	0.216	0.352	0.584	6.251	7.815	9.348	11.345
4	0.297	0.484	0.711	1.064	7.779	9.488	11.143	13.277
5	0.554	0.831	1.145	1.610	9.236	11.070	12.833	15.086
6	0.872	1.237	1.635	2.204	10.645	12.592	14.449	16.812
7	1.239	1.690	2.167	2.833	12.017	14.067	16.013	18.475
8	1.646	2.180	2.733	3.490	13.362	15.507	17.535	20.090
9	2.088	2.700	3.325	4.168	14.684	16.919	19.023	21.666
10	2.558	3.247	3.940	4.865	15.987	18.307	20.483	23.209
11	3.053	3.816	4.575	5.578	17.275	19.675	21.920	24.725
12	3.571	4.404	5.226	6.304	18.549	21.026	23.337	26.217
13	4.107	5.009	5.892	7.042	19.812	22.362	24.736	27.688
14	4.660	5.629	6.571	7.790	21.064	23.685	26.119	29.141
15	5.229	6.262	7.261	8.547	22.307	24.996	27.488	30.578
16	5.812	6.908	7.962	9.312	23.542	26.296	28.845	32.000
17	6.408	7.564	8.672	10.085	24.769	27.587	30.191	33.409
18	7.015	8.231	9.390	10.865	25.989	28.869	31.526	34.805
19	7.633	8.907	10.117	11.651	27.204	30.144	32.852	36.191
20	8.260	9.591	10.851	12.443	28.412	31.410	34.170	37.566
21	8.897	10.283	11.591	13.240	29.615	32.671	35.479	38.932
22	9.542	10.982	12.338	14.041	30.813	33.924	36.781	40.289
23	10.196	11.689	13.091	14.848	32.007	35.172	38.076	41.638
24	10.856	12.401	13.848	15.659	33.196	36.415	39.364	42.980
25	11.524	13.120	14.611	16.473	34.382	37.652	40.646	44.314
26	12.198	13.844	15.379	17.292	35.563	38.885	41.923	45.642
27	12.879	14.573	16.151	18.114	36.741	40.113	43.195	46.963
28	13.565	15.308	16.928	18.939	37.916	41.337	44.461	48.278
29	14.256	16.047	17.708	19.768	39.087	42.557	45.722	49.588
30	14.953	16.791	18.493	20.599	40.256	43.773	46.979	50.892