Lecture 12 - EE 359: Wireless Communications - Fall 2009

Diversity Combining Techniques and Their Analysis

Lecture Outline

- **Selection Combining (SC) and its Performance**
- **Maximal Ratio Combining (MRC)**
- **Performance of MRC with i.i.d. Rayleigh fading**
- **MGF Analysis of MRC**
- **Equal Gain Combining**
- **Transmit Diversity**

1. **Techniques for Combining Independent Fading Paths**
   - Selection Combining: largest fading path chosen.
   - Maximal Ratio Combining: all paths cophased and summed with optimal weighting to maximize SNR at combiner output.
   - Equal Gain Combining: all paths cophased and summed with equal weighting.

2. **Selection Combining (SC) and its Performance**
   - Combiner SNR γ_Σ is the maximum of the branch SNRs.
   - This gives diminishing returns, in terms of power gain, as the number of antennas increases.
   - CDF of γ_Σ easy to obtain, then pdf found by differentiating.
   - Typically get 10-15 dB of gain for 2-3 antennas.

3. **Maximal Ratio Combining (MRC)**
   - Branch weights optimized to maximize output SNR of combiner.
   - Optimal weights are proportional to branch SNR.
   - Resulting combiner SNR γ_Σ is sum of branch SNRs.
   - Distribution obtained by characteristic function analysis (can be hard).

4. **Performance of MRC with i.i.d. Rayleigh fading**
   - For M branch diversity with i.i.d. Rayleigh fading on each branch, γ_Σ is chi-squared with $2M$ degrees of freedom.
   - Can obtain P_{out} and $\overline{P}_s$ from this distribution.
   - For BPSK, get 15 dB gain at 10^{-3} BER. Larger gains obtained at lower BERs.

5. **MGF Approach to MRC Diversity Analysis.**

   - Distribution of γ_Σ hard to obtain when fading is not Rayleigh or not identically distributed.
   - Can use alternate Q function representation to greatly simplify $\overline{P}_b$ calculation.
   - Using alternate representation and switching order of integration yields

   $$\overline{P}_b = \frac{1}{\pi} \int_0^{.5\pi} \prod_{i=1}^{M} \mathcal{M}_i \left[\frac{-g}{\sin^2 \phi}, \gamma_i \right] d\phi,$$

   for BPSK, where $\mathcal{M}_i$ is the moment generating function for the distribution of the ith branch SNR γ_i. Similar formulas for more general modulations using P_s approximation.

6. **Equal Gain Combining**

   - In EGC, each branch is co-phased and equally weighted.
   - Easier to implement than MRC, but harder to analyze.
   - Distribution of combiner SNR γ_Σ complicated for more than two branches. Hard to obtain P_{out} and $\overline{P}_s$.
   - EGC is about 1 dB worse in performance than MRC.

7. **Transmit Diversity**

   - When channel known at transmitter, similar to receiver diversity. Get same array and diversity gain.
   - When channel unknown at transmitter, can use clever Alamouti scheme over two symbol times to obtain full diversity gain, but no array gain.

Main Points

- SC vs. MRC offer different levels of complexity vs. performance.
- Performance analysis of MRC greatly simplified using MGF approach.
- Performance of EGC about 1 dB worse than MRC, with somewhat less complexity.
- Transmit diversity can obtain diversity even without channel information at the transmitter using clever Alamouti scheme.