# EE359 – Lecture 10 Outline

- Announcements:
  - Project proposals due Friday (post; email link)
  - Midterm will be Nov. 4, 8:45-10:45am. Rm TBD.
- Probability of error in fading
- Outage probability
- Average $P_s$ ($P_b$)
- Combined average and outage $P_s$
- $P_s$ due to Doppler and ISI

# Review of Last Lecture

- Capacity of ISI Channels
- Modulation Tradeoffs
- Amplitude / Phase Modulation
  - Constellations, Differential Modulation, bit mappings
- Error Probability for Amp / Phase Modulation

$$P_s \approx \alpha_M Q\left(\sqrt{\beta_M \gamma_s}\right)$$

- Traditional Q function representation

$$Q(z) = p(x > z) = \int_z^\infty \frac{1}{\sqrt{2\pi}} e^{-x^2/2} dx, \quad x \sim N(0,1)$$

  - Infinite integrand with argument in integral limits

# Alternate Q Function Representation

- New representation (Craig'93)

$$Q(z) = \frac{1}{\pi}\int_0^{\pi/2} e^{-z^2/(\sin^2\varphi)}\,d\varphi$$

  - Leads to closed form solution for $P_s$ in PSK
  - Very useful in fading and diversity analysis

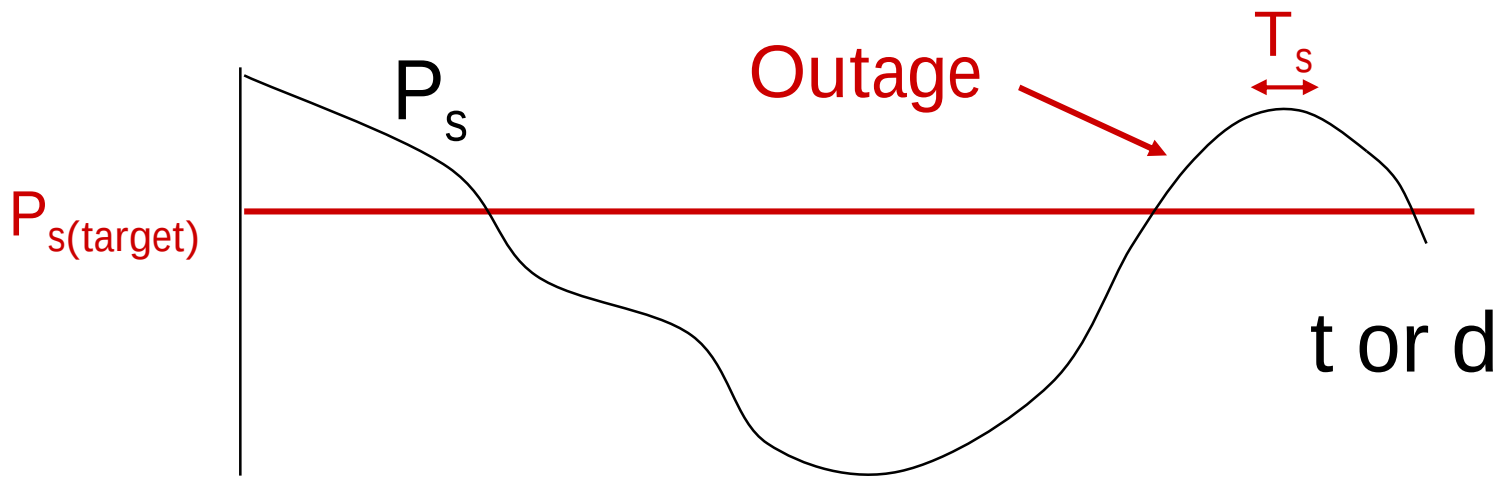
# Linear Modulation in Fading

- In fading $\gamma_s$ and therefore $P_s$ random
- Performance metrics:
  - Outage probability: $p(P_s>P_{target})=p(\gamma<\gamma_{target})$
  - Average $P_s$ , $\overline{P_s}$:

$$\overline{P_s} = \int_0^\infty P_s(\gamma)\, p(\gamma)\, d\gamma$$

  - Combined outage and average $P_s$

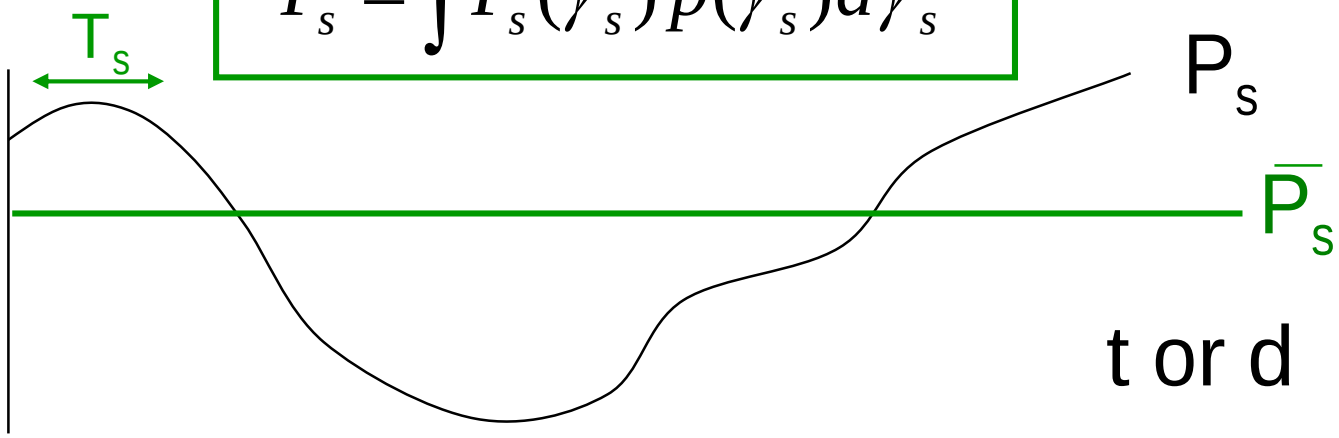
# Outage Probability




- Probability that $P_s$ is above target
- Equivalently, probability $\gamma_s$ below target
- Used when $T_c >> T_s$

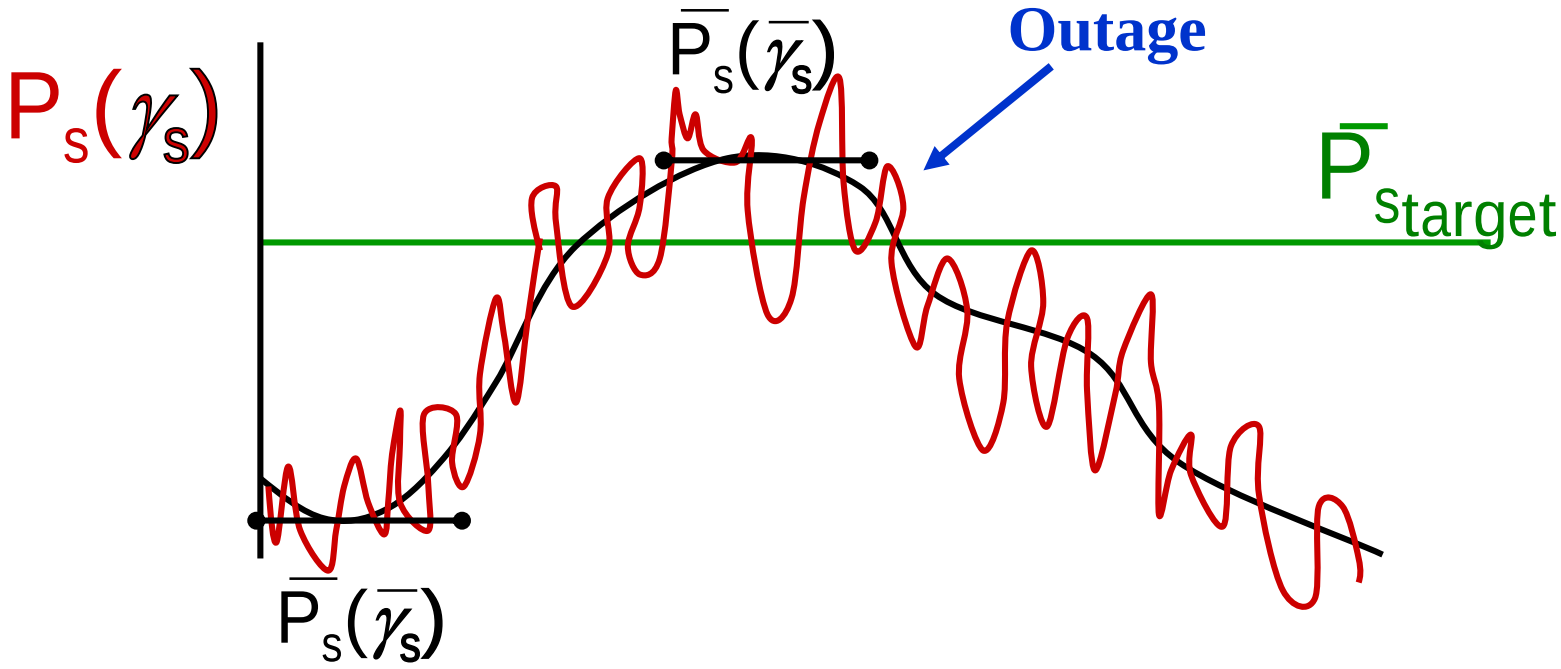
# Average $P_s$

$$\overline{P}_s = \int P_s(\gamma_s)\, p(\gamma_s)\, d\gamma_s$$




- Expected value of random variable $P_s$
- Used when $T_c \sim T_s$
- Error probability much higher than in AWGN alone
- Alternate Q function approach: $$Q(z) = \frac{1}{\pi}\int_0^{\pi/2} e^{-z^2/(\sin^2\varphi)}\, d\varphi$$
  - Simplifies calculations (Get a Laplace Xfm)

# Combined outage and average $P_s$



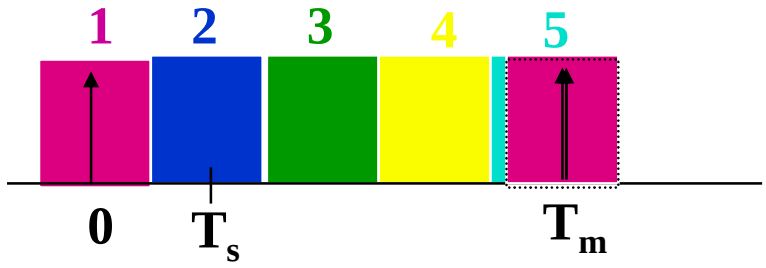

- Used in combined shadowing and flat-fading
- $\overline{P}_s$ varies slowly, locally determined by flat fading
- Declare outage when $\overline{P}_s$ above target value

# Doppler Effects

- High doppler causes channel phase to decorrelate between symbols
- Leads to an irreducible error floor for differential modulation
  - Increasing power does not reduce error
- Error floor depends on $B_d T_s$

# ISI Effects

- Delay spread exceeding a symbol time causes ISI (self interference).




- ISI leads to irreducible error floor
  - Increasing signal power increases ISI power
- ISI requires that $T_s >> T_m$ ($R_s << B_c$)

# Main Points

- In fading $P_s$ is a random variable, characterized by average value, outage, or combined outage/average
  - Outage probability based on target SNR in AWGN.
  - Fading greatly increases average $P_s$.
  - Alternate Q function approach simplifies $P_s$ calculation, especially its average value in fading (Laplace Xfm).
- Doppler spread only impacts differential modulation causing an irreducible error floor at low data rates
- Delay spread causes irreducible error floor or imposes rate limits
- Need to combat flat and frequency-selective fading
  - Focus of the remainder of the course