Lecture 15 - EE 359: Wireless Communications - Fall 2009

MIMO and Space/Time Communications.

Lecture Outline

- **Introduction to MIMO Systems**
- **MIMO Channel Decomposition**
- **MIMO Channel Capacity**
- **Capacity with Outage**
- **Beamforming**
- **Diversity/Multiplexing Tradeoffs**

1. **Introduction to MIMO Systems**
   - MIMO systems have multiple antennas at the transmitter and receiver.
   - The antennas can be used for capacity gain and/or diversity gain.
   - MIMO system design and analysis can be complex since it requires vector signal processing.
   - The performance and complexity of MIMO systems depends on what is known about the channel at both the transmitter and receiver

2. **MIMO Systems: Parallel Channel Decomposition**
   - With perfect channel estimates at the transmitter and receiver, the MIMO channel decomposes into $R_{\mathbf{H}}$ independent parallel channels, where $R_{\mathbf{H}}$ is the rank of the channel matrix ($\min(M_t, M_r)$ for M_t transmit and M_r receive antennas under rich scattering).
   - With this decomposition there is no need for vector signal processing.
   - Decomposition is obtained by transmit precoding and receiver shaping.

3. **MIMO Channel Capacity**
   - Capacity depends on whether the channel is static or fading, and what is known about the channel at the transmitter and receiver.
   - For a static channel known at the transmitter and receiver capacity is given by
   $$C = \max_{P_i:\sum_i P_i \le P} \sum_i B\log_2\left(1+\frac{\sigma_i^2 P_i}{\sigma_n^2}\right) = \max_{P_i:\sum_i P_i \le P} \sum_i B\log_2\left(1+\frac{P_i\gamma_i}{P}\right).$$
   This leads to a water-filling power allocation in space.
   - When channel is unknown at transmitter, uniform power allocation is optimal, but this leads to an outage probability since the transmitter doesn't know what rate to transmit at:
   $$P_{out} = p\left(\mathbf{H} : B\log_2\det\left[\mathbf{I}_{M_r} + \frac{\rho}{M_t}\mathbf{H}\mathbf{H}^H\right] > C\right).$$
   - In fading, capacity with both transmitter and receiver knowledge is the average of the capacity for the static channel, with power allocated either by a short-term or long-term power constraint.

- Without transmitter knowledge, outage probability is the right metric for capacity.

4. **MIMO Systems: Beamforming**

   - Beamforming sends the same symbol over each transmit antenna with a different scale factor.
   - At the receiver, all received signals are coherently combined using a different scale factor.
   - This produces a transmit/receiver diversity system, whose SNR can be maximized by optimizing the scale factors (MRC).
   - Beamforming leads to a much higher SNR than on the individual channels in the parallel channel decomposition.
   - Thus, there is a design tradeoff in MIMO systems between capacity and diversity.

5. **Diversity versus Multiplexing in MIMO Systems**

   - Can exchange data rate for probability of error.
   - Define rate scale factor $r = R/\log(SNR)$. Define diversity gain $d = \log)P_e)/\log(SNR)/$.
   - Can show (Zheng/Tse'02) that in high SNR regime, the optimal tradeoff is $d^*(r) = (M_t - r)(M_r - r)$..
   - The optimal operating point on this traedeoff curve depends on the application.

Main Points

- MIMO systems exploit multiple antennas at both TX and RX for capacity and/or diversity gain.

- With both TX and RX CSI, multiple antennas at both transmitter and receiver lead to independent parallel channels.

- With TX and RX CSI, capacity of MIMO channel uses waterfilling in space or space/time - leads to $\min(M_t, M_r)$ capacity gain.

- Without transmitter CSI, use outage as capacity metric.

- Multiple antennas can also be used for diversity via beamforming, this can be optimal for capacity.

- Fundamental tradeoff of diversity vs. multiplexing in MIMO. The optimal use of antennas depends on the application.