# Practical Aspects of Adaptive MQAM MIMO and Space/Time Communications

## Lecture Outline

- Finite Constellations
- Update Rate in Adaptive Modulation
- Effects of Estimation Error on Adaptive MQAM
- Effects of Feedback Delay on Adaptive MQAM
- Introduction to MIMO Systems.
- Parallel Decomposition of MIMO Channels
- MIMO Channel Capacity

## 1. Finite Constellations

- Constellation restricted to finite set  $\{M_0 = 0, M_1, \dots, M_{N-1}\}$
- Divide the fading range of  $\gamma$  into N discrete fading regions  $R_i$ .
- Within each region "conservatively" assign constellation  $M_j: M_j \leq M(\gamma) \leq M_{j+1}$ , where  $M(\gamma) = \gamma/\gamma_K^*$  for some optimized  $\gamma_K^*$ .
- Power control based on channel inversion maintains constant BER within region  $R_i$ .
- Using large enough constellation set results in near-optimal performance.
- Additional power penalty of 1.5-2 dB if each constellation restricted to a single transmit power.

## 2. Update Rate in Adaptive Modulation

- Rate at which constellation size changes (should be much more than a symbol time).
- Approximate as the average dwell time in each of the fading regions  $R_i$ .
- Using a Markov model approximation for Rayleigh fading, this average dwell time is  $\overline{\tau_j} = \pi_j/(N_{j+1} + N_j)$ , where  $\pi_j$  is the probability of being in region  $R_j$  and and  $N_j$   $(N_{j+1})$  is the level crossing rate at the minimum (maximum) fade level in the region.

#### 3. Effects of Estimation Error

- Can have estimation error at receiver, transmitter, or both.
- Receiver estimation generally more accurate, since doesn't need to be causal. Generally neglect this error for slowly changing channels.
- Transmitter estimate must be causal, so is generally less accurate. The effect on BER of imperfect transmitter channel estimates (assuming perfect receiver channel estimates) is BER( $\gamma, \hat{\gamma}$ ) = .2[5BER<sub>0</sub>] $^{\gamma/\hat{\gamma}}$ , where  $\gamma$  is the true channel SNR,  $\hat{\gamma}$  is its estimate at the receiver, and BER<sub>0</sub> is the target BER.
- The average BER is obtained by integrating this expression over the joint distribution of  $\gamma$  and  $\hat{\gamma}$  or the distribution of the ratio  $\gamma/\hat{\gamma}$ . This distribution is not easy to obtain and depends on the estimation process.

## 4. Effects of Estimation Delay

- Assume a fixed estimation delay so that the transmitter and receiver are synchronized to the same constellation.
- BER degredataion due to delay  $i_d$  can be derived as BER $(\gamma[i], \gamma[i-i_d]) = .2[5BER_0]^{\gamma[i]/\gamma[i-i_d]}$ .
- The average BER is obtained by integrating this expression over the joint distribution of  $\gamma[i]$  and  $\gamma[i-i_d]$ .
- A closed-form expression for the ratio  $\gamma[i]/\gamma[i-i_d]$  in Nakagami-m (and Rayleigh) fading is known.

## 5. MIMO Systems: Parallel Channel Decomposition

- MIMO systems have multiple antennas at both the transmitter and receiver.
- With perfect channel estimates at the transmitter and receiver, the MIMO channel decomposes into  $R_{\mathbf{H}}$  independent parallel channels, where  $R_{\mathbf{H}}$  is the rank of the channel matrix (min( $M_t, M_r$ ) for  $M_t$  transmit and  $M_r$  receive antennas under rich scattering).

# 6. MIMO Channel Capacity

- Capacity depends on whether the channel is static or fading, and what is known about the channel at the transmitter and receiver.
- For a static channel known at the transmitter and receiver capacity is given by

$$C = \max_{P_i: \sum_i P_i \leq P} \sum_i B \log_2 \left(1 + \frac{\sigma_i^2 P_i}{\sigma_n^2}\right) = \max_{P_i: \sum_i P_i \leq P} \sum_i B \log_2 \left(1 + \frac{P_i \gamma_i}{P}\right).$$

This leads to a water-filling power allocation in space.

• When channel is unknown at transmitter, uniform power allocation is optimal, but this leads to an outage probability since the transmitter doesn't know what rate to transmit at:

$$P_{out} = p\left(\mathbf{H} : B \log_2 \det \left[\mathbf{I}_{M_r} + \frac{\rho}{M_t} \mathbf{H} \mathbf{H}^H\right] > C\right).$$

- In fading, capacity with both transmitter and receiver knowledge is the average of the capacity for the static channel, with power allocated either by a short-term or long-term power constraint.
- Without transmitter knowledge, outage probability is the right metric for capacity.

## **Main Points**

- Finite constellations has little impact on adaptive MQAM throughput
- Adaptive modulation need not update more frequently then every 10-100 symbols.
- Estimation error and delay in adaptive MQAM lead to irreducible error floors.
- Multiple antennas at both the transmitter and receiver greatly increase channel capacity or provide diversity gain.