Lecture 3 - EE 359: Wireless Communications - Fall 2009

Shadowing, Combined Path Loss/Shadowing, Cell Coverage Area, Model Parameters.

Lecture Outline

- **Log Normal Shadowing**
- **Combined Path Loss and Shadowing**
- **Outage Probability**
- **Cell Coverage Area**
- **Model Parameters from Empirical Data**

1. **Log-normal shadowing:**
   - Statistical model for variations in the received signal amplitude due to blockage.
   - The received signal power with the combined effect of path loss (power falloff model) and shadowing is, in dB, given by

   $$P_r(\mathrm{dB}) = P_u(\mathrm{dB}) + 10\log_{10} K - 10\gamma\log_{10}(d/d_0) + \psi(\mathrm{dB}).$$

   - Empirical measurements support the log-normal distribution for ψ:

   $$p(\psi_{\mathrm{dB}}) = \frac{1}{\sqrt{2\pi}\sigma_{\psi_{dB}}}\exp\left[-\frac{(\psi_{\mathrm{dB}} - \mu_{\psi_{dB}})^2}{2\sigma^2_{\psi_{dB}}}\right].$$

   - This empirical distribution can be justified by a LLN argument.
   - The autocorrelation based on measurements follows an autoregressive model:

   $$A_\psi(\delta) = \sigma^2_{\psi_{dB}} e^{-\delta/X_c} = \sigma^2_{\psi_{dB}} e^{-v\tau/X_c},$$

   where X_c is the decorrelation distance, which depends on the environment.

2. **Combined Path Loss and Shadowing**
   - Linear Model:

   $$\frac{P_r}{P_t} = K\left(\frac{d_0}{d}\right)^\gamma \psi.$$

   - dB Model:

   $$\frac{P_r}{P_t}(dB) = 10\log_{10} K - 10\gamma\log_{10}(d_0/d) + \psi_{dB}.$$

3. **Outage Probability under Path Loss and Shadowing**
   - With path loss and shadowing, the received power at any given distance between transmitter and receiver is random.
   - Outage probability $p_{out}(P_{min}, d)$ is defined as the probability that the received power at a given distance d, $P_r(d)$, is below a target P_{min}: $p_{out}(P_{min}, d) = p(P_r(d) < P_{min})$.

- For the simplified path loss model and log normal shadowing this becomes

$$p(P_r(d) \le P_{min}) = 1 - Q\left(\frac{P_{min} - (P_t + 10\log_{10} K - 10\gamma \log_{10}(d/d_0))}{\sigma_{\psi_{dB}}}\right).$$

4. **Cell Coverage Area:**
   - Cellular systems designed for a given average received power $\overline{P}_r(R)$ at cell boundary.
   - Cell coverage area dictates the percentage of locations within the cell with $P_r \ge \overline{P}_r(R)$.
   - Analysis yields:

   $$C = Q(a) + \exp\left(\frac{2-2ab}{b^2}\right) Q\left(\frac{2-ab}{b}\right), \quad a = \frac{P_{min} - \overline{P}_r(R)}{\sigma_{\psi_{dB}}}, \quad b = \frac{10\gamma \log_{10}(e)}{\sigma_{\psi_{dB}}}.$$

   - When the minimum power P_{min} equals the average power at the cell boundary $\overline{P}_r(R)$, $a = 0$, and

   $$C = \frac{1}{2} + \exp\left(\frac{2}{b^2}\right) Q\left(\frac{2}{b}\right).$$

   - Coverage area increases as σ_ψ decreases.
   - Making $\overline{P}_r$ much greater than required received power increases coverage area but causes more intereference between cells.

5. **Model Parameters from Empirical Data:**
   - Constant K obtained from measurement at distance d_0.
   - Power falloff exponent γ obtained by minimizing the MSE of the predicted model versus the data.
   - The resulting path loss model will include average attenuation, so $\mu_{\psi_{dB}} = 0$.
   - The shadowing variance $\sigma^2_{\psi_{dB}}$ obtained by determining MSE of the data versus the empirical path loss model with the optimizing γ.

Main Points

- Random attenuation due to shadowing modeled as log-normal (empirical parameters).
- Shadowing decorrelates over decorrelation distance.
- Combined path loss and shadowing leads to outage and amoeba-like cell shapes.
- Shadowing affects cell coverage area, defined as the percentage of locations within a cell with acceptable received power.
- Path loss and shadowing parameters are obtained from empirical measurements.