# Statistical Fading Models

## Lecture Outline

- **Model Parameters from Empirical Data.**
- **Random Multipath Model.**
- **Time Varying Channel Impulse Response.**
- **Narrowband Approximation.**
- **In-Phase and Quad Signal Components under CLT.**
- **Mean and Correlation of I and Q Signals.**

1. **Model Parameters from Empirical Data:**
   - Constant $K$ obtained from measurement at distance $d_0$.
   - Power falloff exponent $\gamma$ obtained by minimizing the MSE of the predicted model versus the data.
   - The resulting path loss model will include average attenuation, so $\mu_{\psi_{dB}} = 0$.
   - The shadowing variance $\sigma^2_{\psi_{dB}}$ obtained by determining MSE of the data versus the empirical path loss model with the optimizing $\gamma$.

2. **Statistical Multipath Model:**
   - At each time instant there are a random number $N(t)$ of multipath signal components.
   - Each component has a random amplitude $\alpha_n$, phase $\theta_n$, Doppler shift $f_{D_n} = \frac{v}{\lambda}\cos\theta_n$, and path delay $\tau_n = (d_n - d)/c$.
   - Thus, received signal is

$$r(t) = \Re\left\{\left[\sum_{n=0}^{N(t)} \alpha_n(t) e^{-j\phi_n(t)} u(t-\tau_n(t))\right] e^{j2\pi f_c t}\right\}.$$

3. **Time-Varying Channel Impulse Reponse:**
   - The received signal $r(t)$ is the convolution of the input signal with the equivalent low-pass channel impulse response $c(\tau, t)$:

$$r(t) = \Re\left\{\left(\int_{-\infty}^{\infty} c(\tau, t) u(t-\tau) d\tau\right) e^{j2\pi f_c t}\right\}.$$

   - The channel can thus be modeled as a time-varying linear filter:

$$c(\tau, t) = \sum_{n=1}^{N(t)} \alpha_n(t) e^{-j\phi_n(t)} \delta(\tau - \tau_n(t)),$$

     where $c(\tau, t)$ is the channel response at time $t$ to an impulse at time $t - \tau$.
   - Note there are two time parameters in this expression. The parameter $t$ denotes the time when the impulse response is observed. The parameter $t - \tau$ denotes the time when the impulse was put into the channel relative to the observation time $t$.

- In other words, $\tau$ denotes how long ago the impulse was put into the channel for the current observation.

4. **Properties of Received Signal:**

   - The received signal consists of $N$ multipath components scaled by amplitude $\alpha_n(t)$ and phase $\phi_n(t)$.
   - The amplitude term $\alpha_n(t)$ varies slowly (due to shadowing).
   - The phase $\phi_n(t)$ varies rapidly due to small shifts in the signal component path delays.
   - The phase variation causes rapid variation in the received signal amplitude due to constructive and destructive interference of the multipath components.

5. **Narrowband Approximation:**

   - Define the multipath delay spread as $T_m(t) = \max_n \tau_n(t) - \min_n \tau_n(t)$. For random multipath the delay spread is defined relative to its mean or standard deviation.
   - Assume $T_m(t) << 1/B$ for all $t$ (or the equivalent for random multipath). Then $u(t) \approx u(t - \tau_n(t))$ for all $n$ and $t$.
   - Received signal simplifies to

   $$r(t) = \Re\left\{u(t)e^{j2\pi f_c t}\left[\sum_{n=0}^{N(t)} \alpha_n(t)e^{j\phi_n(t)}\right]\right\}$$

   - No signal distortion: multipath only affects complex scale factor in brackets.
   - Characterize scale factor by assuming $u(t) = 1$

6. **In-Phase and Quad Signal Components under CLT**

   - Received signal can be written in terms of in-phase and quadrature components as $r(t) = r_I(t)\cos(2\pi f_c t) + r_Q(t)\sin(2\pi f_c t)$ where $r_I(t) = \sum_{n=0}^{N(t)} \alpha_n(t)\cos(\phi_n(t))$ and $r_Q(t) = \sum_{n=0}^{N(t)} \alpha_n(t)\sin(\phi_n(t))$.
   - If $N(t)$ large then in-phase and quadrature signal components are jointly Gaussian (amplitude of scale factor is Rayleigh).
   - Thus, received signal characterized by mean, autocorrelation, and cross correlation.
   - Assuming $\phi_n(t)$ uniform, $E[r_I(t)] = E[r_Q(t)] = 0$ and $E[r_I(t)r_Q(t)] = 0$.

## Main Points

- Statistical multipath model leads to a time varying channel impulse response
- The resulting received signal has rapidly varying amplitude due to constructive and destructive multipath combining
- Narrowband model and CLT lead to inphase, quadrature, and received signals that are stationary Gaussian processes with zero mean.