Capacity of Fading Channels

Lecture Outline

- **Capacity of Flat-Fading Channels**
- **Capacity with Fading Known at Transmitter and/or Receiver**
- **Optimal Rate and Power Adaptation (Water Filling)**
- **Channel Inversion with Fixed Rate Transmission**
- **Capacity of Frequency Selective Fading Channels**

1. **Shannon Capacity**
   - The maximum mutual information of a channel. Its significance comes from Shannon's coding theorem and converse, which show that capacity is the maximum error-free data rate a channel can support.
   - Capacity is a channel characteristic - not dependent on transmission or reception techniques or limitation.
   - In AWGN, $C = B\log_2(1+\gamma)$ bps, where B is the signal bandwdith and $\gamma = S/N$ is the received signal-to-noise power ratio.

2. **Capacity of Flat-Fading Channels:**
   - Depends on what is known about the channel.
   - Three cases: 1) Fading statistics known; 2) Fade value known at receiver; 3) Fade value known at transmitter and receiver.
   - When only fading statistics are known, capacity is difficult to compute. Only known results are for Finite State Markov channels, Rayleigh fading channels, and block fading.

3. **Fading Known at the Receiver:**
   - Capacity given by $C = \int_0^\infty B\log_2(1+\gamma)p(\gamma)d\gamma$ bps, where $p(\gamma)$ is the distribution of the fading SNR γ.
   - By Jensen's inequality this capacity always less than that of an AWGN channel.
   - "Average" capacity formula, but transmission rate is fixed.

4. **Capacity with Fading Known at Transmitter and Receiver**
   - For fixed transmit power, same capacity as when only receiver knows fading.
   - By Jensen's inequality, fading reduces capacity w.r.t. AWGN for fixed transmit power.
   - Transmit power as well as rate can be adapted.
   - Under variable rate and power $C = \max_{P(\gamma):\int P(\gamma)p(\gamma)d\gamma = S} \int_0^\infty B\log_2\left(1+\frac{P(\gamma)\gamma}{S}\right)p(\gamma)d\gamma$, where $P(\gamma)$ is power adaptation

5. **Optimal Power and Rate Adaptation**
   - Optimal adaptation found via Lagrangian differentiation.
   - Optimal power adaptation is a "water-filling" in time: power $P(\gamma) = \gamma_0^{-1} - \gamma^{-1}$ increases with channel quality γ above an optimized cutoff value γ_0.
   - Rate adaptation relative to $\gamma \geq \gamma_0$ is $B \log_2(\gamma/\gamma_0)$: also increases with γ above cutoff.
   - Resulting capacity is $C = \int_{\gamma_0}^{\infty} B \log_2(\gamma/\gamma_0) p(\gamma) d\gamma$.
   - Capacity with power and rate adaptation not much larger than when just receiver knows channel, but has lower complexity and yields more insight into practical schemes.
   - Capacity in flat-fading can exceed the capacity in AWGN, typically at low SNRs.

6. **Channel Inversion**
   - Suboptimal transmission strategy where fading is inverted to maintain constant received SNR.
   - Simplifies system design and is used in CDMA systems for power control.
   - Capacity with channel inversion greatly reduced over that with optimal adaptation (capacity equals zero in Rayleigh fading).
   - Truncated inversion: performance greatly improved by inverting above a cutoff γ_0.

7. **Capacity of Frequency Selective Fading Channels**
   - Capacity for time-invariant frequency-selective fading channels is a "water-filling" of power over frequency.
   - For time-varying ISI channels, capacity is unknown in general. Approximate by dividing up the bandwidth subbands of width equal to the coherence bandwidth (same premise as multicarrier modulation) with independent fading in each subband.
   - Capacity in each subband obtained from flat-fading analysis. Power is optimized over both frequency and time.

Main Points

- Capacity of flat-fading channels depends on what is known about the fading at receiver and transmitter.
- Capacity when only the receiver knows the fading is the same as when the transmitter also knows but does not adapt power.
- Capacity-achieving transmission scheme uses variable-rate variable-power transmission with power water-filling in time.
- Power and rate adaptation does not significantly increase capacity, and rate adaptation alone yields no increase. These results may not carry over to practical schemes.
- Channel inversion practical but has poor performance. Performance improved by truncating.
- Capacity of frequency-selective fading channels obtained by breaking up wideband channel into subbands (similar to multicarrier).