Lecture 5 - EE 359: Wireless Communications - Fall 2009

Narrowband Fading Model

Lecture Outline

- **Narrowband Fading Approximation.**
- **In-Phase and Quad Signal Components under CLT.**
- **Mean, Autocorrelation, and Cross Correlation in Narrowband Fading.**
- **Correlation and PSD under Uniform AOAs**

1. **Narrowband Fading Approximation:**

   - Received signal in general is

   $$r(t) = \Re\left\{\left[\sum_{n=0}^{N(t)} \alpha_n(t)e^{-j\phi_n(t)}u(t-\tau_n(t))\right]e^{j2\pi f_c t}\right\}.$$

   - Define the multipath delay spread as $T_m(t) = \max_n \tau_n(t) - \min_n \tau_n(t)$. For random multipath the delay spread is defined relative to its mean or standard deviation.
   - Assume $T_m(t) << 1/B$ for all t (or the equivalent for random multipath). Then $u(t) \approx u(t-\tau_n(t))$ for all n and t.
   - Received signal simplifies to

   $$r(t) = \Re\left\{u(t)e^{j2\pi f_c t}\left[\sum_{n=0}^{N(t)} \alpha_n(t)e^{-j\phi_n(t)}\right]\right\}$$

   - No signal distortion: multipath only affects complex scale factor in brackets.
   - Characterize scale factor by assuming $s(t) = \Re\left\{u(t)e^{j2\pi f_c t}\right\} = \cos(2\pi f_c t + \phi_0)$ for some phase offset ϕ_0.

2. **In-Phase and Quad Signal Components under CLT**

   - Received signal can be written in terms of in-phase and quadrature components as $r(t) = r_I(t)\cos(2\pi f_c t) + r_Q(t)\sin(2\pi f_c t)$ where $r_I(t) = \sum_{n=0}^{N(t)} \alpha_n(t)\cos(\phi_n(t))$ and $r_Q(t) = \sum_{n=0}^{N(t)} \alpha_n(t)\sin(\phi_n(t))$ for $\phi_n(t) = 2\pi f_c \tau_n(t) - \phi_{D_n} - \phi_0$.
   - If $N(t)$ large then in-phase and quadrature signal components are jointly Gaussian (amplitude of scale factor is Rayleigh).
   - Thus, received signal characterized by mean, autocorrelation, and cross correlation.

3. **Mean, Autocorrelation, and Cross Correlation in Narrowband Fading**

   - Assuming $\phi_n(t)$ uniform, $E[r_I(t)] = E[r_Q(t)] = 0$ and $E[r_I(t)r_Q(t)] = 0$. Thus, $r_I(t)$ and $r_Q(t)$ are uncorrelated, hence independent. Moreover, $E[r(t)] = 0$

- $A_{r_I}(t, t+\tau) = A_{r_I}(\tau) = A_{r_Q}(\tau) = .5\sum_{n=0}^{N-1} E[\alpha_n^2]E_{\theta_n}[\cos(2\pi v\tau/\lambda)\cos\theta_n]$, so the processes are WSS and hence stationary. Note that $.5\sum_n E[\alpha_n^2] = \overline{P}_r$, the total average received power.
- Using a similar analysis, get $A_{r_I,r_Q}(t, t+\tau) = E[r_I(t)r_Q(t+\tau) = \overline{P}_r E_{\theta_n}[\sin(2\pi v\tau/\lambda)\cos\theta_n] = A_{r_I,r_Q}(\tau)$.
- Using these derivations, we get the autocorrelation for the received signal as $A_r(t, t+\tau) = A_{r_I}(\tau)\cos(2\pi f_c\tau) + A_{r_I,r_Q}(\tau)\sin(2\pi f_c\tau) = A_r(\tau)$. Since this only depends on τ, the received signal is WSS and hence stationary.

4. **Correlation and PSD under Uniform AOAs**

- Under uniform scattering (uniform AOAs) the in-phase and quadrature signal components have zero cross correlation and have autocorrelation $A_{r_I}(\tau) = A_{r_Q}(\tau) = \overline{P}_r J_0(2\pi f_D\tau)$.
- Thus the signal decorrelates over roughly one half of a wavelength, but later recorrelates.
- Can obtain the PSD of the received signal by taking the Fourier transform of its autocorrelation.
- This yields $S_r(f) = .25[S_{r_I}(f-f_c) + S_{r_I}(f+f_c)]$ where

$$S_{r_I}(f) = S_{r_Q}(f) = \mathcal{F}[A_{r_I}(\tau)] = \begin{cases} \frac{2\overline{P}_r}{\pi f_D}\frac{1}{\sqrt{1-(f/f_D)^2}} & |f| \le f_D \\ 0 & \text{else} \end{cases}$$

- The PSD is useful in simulating fading channels.

Main Points

- Narrowband model and CLT lead to inphase, quadrature, and received signals that are stationary Gaussian processes with zero mean and an autocorrelation function that depends on the AOAs of the multipath components.
- Uniform scattering leads to in-phase and quadrature signal components that are uncorrelated, hence independent.
- The autocorrelation of these components under uniform scattering follows a Bessel function (decorrelates after half signal wavelength).
- The PSD has a bowel shape centered around the carrier frequency.