EE359 – Lecture 9 Outline

- Announcements
  - MT tentatively scheduled Wed. Nov. 4 8:45-10:45am.
- Capacity of Freq. Selective Fading Channels
- Linear Modulation Review
- Linear Modulation Performance in AWGN
- Modulation Performance in Fading
  - Outage Probability
  - Average P_s (P_b)

Review of Last Lecture

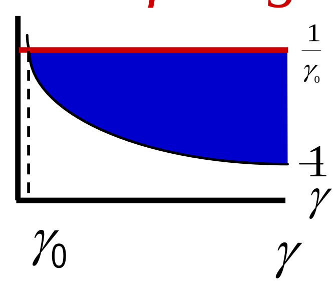
- Capacity of Flat-Fading Channels
  - Fading Statistics Known (few results)
  - Fading Known at RX (average capacity)

$$C=\int_0^\infty B\log_2(1+\gamma)p(\gamma)d\gamma \le B\log_2(1+\bar{\gamma})$$

  - Fading Known at TX and RX
    - Optimal Rate and Power Adaptation

$$C=\max_{S(\gamma):E[S(\gamma)]=\bar{S}}\int_0^\infty B\log_2\left(1+\frac{\gamma S(\gamma)}{\bar{S}}\right)p(\gamma)d\gamma$$

Waterfilling



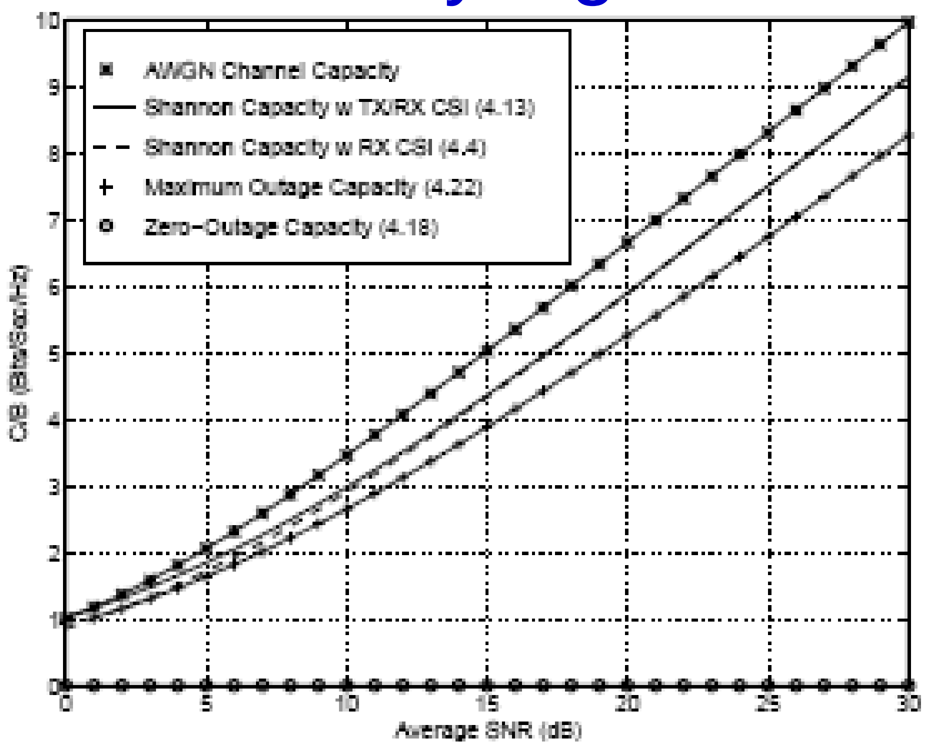
$$\frac{C}{B}=\int_{\gamma_0}^\infty \log_2\left(\frac{\gamma}{\gamma_0}\right)p(\gamma)d\gamma.$$

Review Continued: Channel Inversion

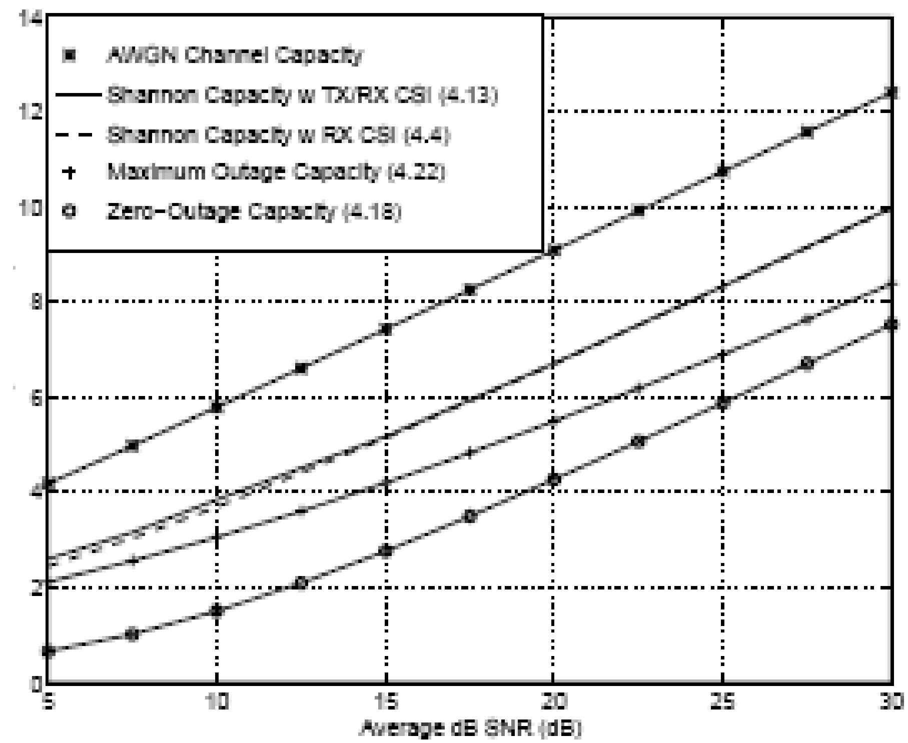
- Fading inverted to maintain constant SNR
- Simplifies design (fixed rate)
- Greatly reduces capacity
  - Capacity is zero in Rayleigh fading
- Truncated inversion
  - Invert channel above cutoff fade depth
  - Constant SNR (fixed rate) above cutoff
  - Cutoff greatly increases capacity
    - Close to optimal

Capacity in Flat-Fading

Rayleigh

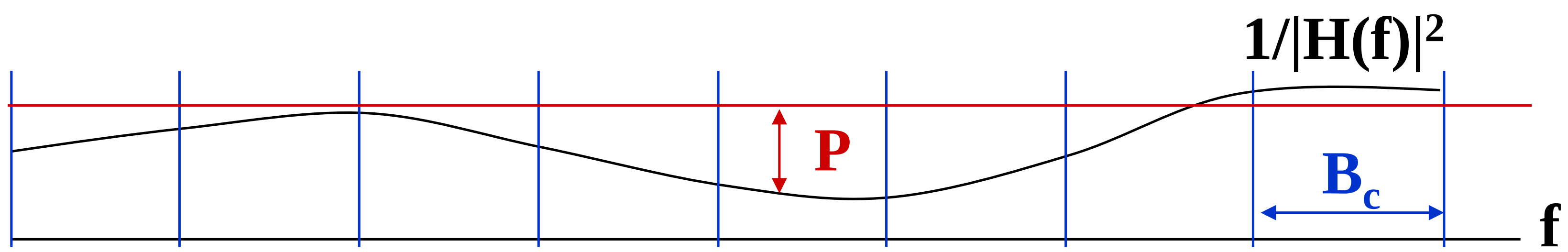


Log-Normal



Frequency Selective Fading Channels

- For TI channels, capacity achieved by water-filling in frequency
- Capacity of time-varying channel unknown
- Approximate by dividing into subbands
  - Each subband has width B_c (like MCM).
  - Independent fading in each subband
  - Capacity is the sum of subband capacities



Passband Modulation Tradeoffs

- Want high rates, high spectral efficiency, high power efficiency, robust to channel, cheap.

Our focus

- Amplitude/Phase Modulation (MPSK,MQAM)
  - Information encoded in amplitude/phase
  - More spectrally efficient than frequency modulation
  - Issues: differential encoding, pulse shaping, bit mapping.
- Frequency Modulation (FSK)
  - Information encoded in frequency
  - Continuous phase (CPFSK) special case of FM
  - Bandwidth determined by Carson's rule (pulse shaping)
  - More robust to channel and amplifier nonlinearities

Amplitude/Phase Modulation

- Signal over ith symbol period:

$$s(t) = s_{i1} g(t)\cos(2\pi f_c t + \phi_0) - s_{i2} g(t)\sin(2\pi f_c t + \phi_0)$$

  - Pulse shape g(t) typically Nyquist
  - Signal constellation defined by (s_{i1}, s_{i2}) pairs
  - Can be differentially encoded
  - M values for $(s_{i1}, s_{i2}) \Rightarrow \log_2 M$ bits per symbol

- P_s depends on
  - Minimum distance d_{min} (depends on γ_s)
  - # of nearest neighbors α_M
  - Approximate expression:

$$\boxed{P_s \approx \alpha_M Q\left(\sqrt{\beta_M \gamma_s}\right)}$$

Alternate Q Function Representation

- Traditional Q function representation

$$Q(z) = p(x > z) = \int_z^\infty \frac{1}{\sqrt{2\pi}} e^{-x^2/2} dx, \quad x \sim N(0,1)$$

  - Infinite integrand
  - Argument in integral limits
- New representation (Craig'93)

$$Q(z) = \frac{1}{\pi} \int_0^{\pi/2} e^{-z^2/(\sin^2\varphi)} d\varphi$$

  - Leads to closed form solution for P_s in PSK
  - Very useful in fading and diversity analysis

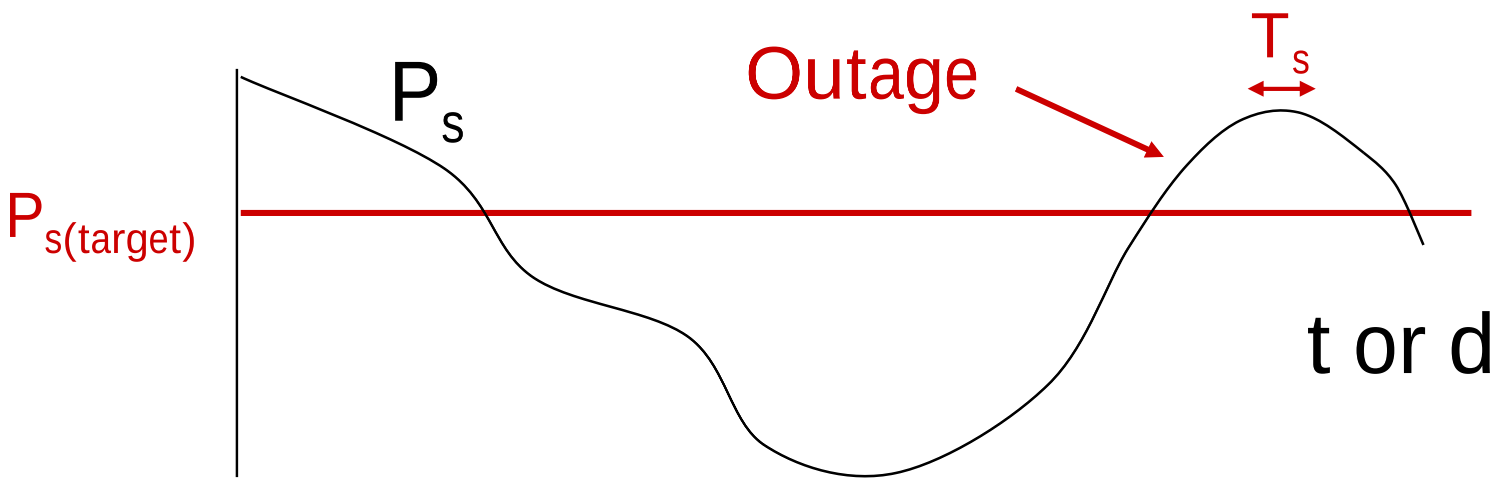
Linear Modulation in Fading

- In fading γ_s and therefore P_s random
- Performance metrics:
  - Outage probability: $p(P_s>P_{target})=p(\gamma<\gamma_{target})$
  - Average P_s , $\overline{P_s}$:

$$\overline{P_s} = \int_0^{\infty} P_s(\gamma)\, p(\gamma)\, d\gamma$$

  - Combined outage and average P_s (next lecture)

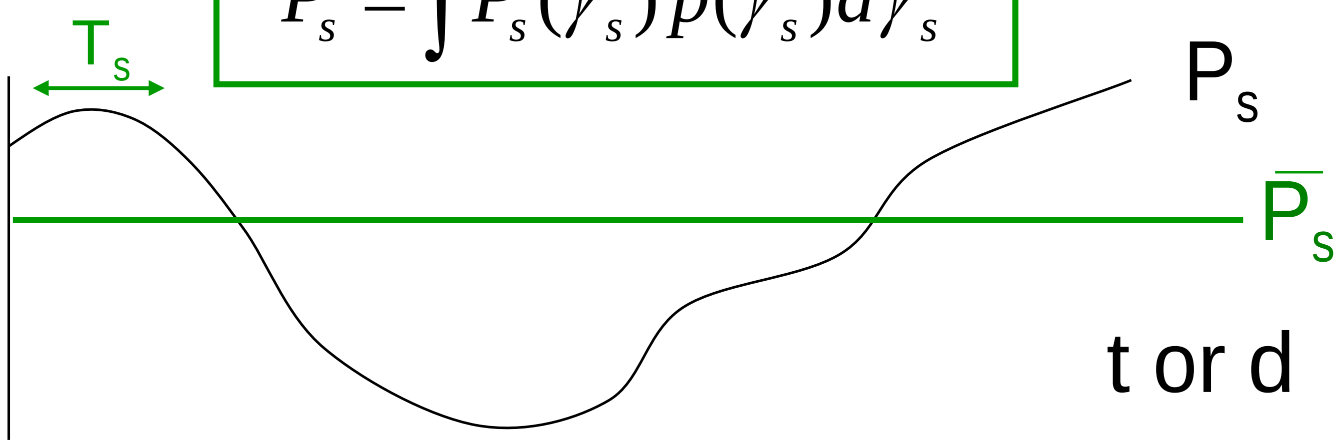
Outage Probability



- Probability that P_s is above target
- Equivalently, probability γ_s below target
- Used when $T_c >> T_s$

Average P_s

$$\boxed{\overline{P}_s = \int P_s(\gamma_s)\, p(\gamma_s)\, d\gamma_s}$$



- Expected value of random variable P_s
- Used when $T_c \sim T_s$
- Error probability much higher than in AWGN alone

Main Points

- Capacity of frequency-selective fading channels obtained by breaking it into subchannels
- Linear modulation more spectrally efficient but less robust than nonlinear modulation
- P_s approximation in AWGN: $\boxed{P_s \approx \alpha_M Q\left(\sqrt{\beta_M \gamma_s}\right)}$
  - Alternate Q function representation simplifies calculations
- In fading P_s is a random variable, characterized by average value, outage, or combined outage / average
- Outage probability based on target SNR in AWGN.
- Fading greatly increases average P_s .