

# EE359 – Lecture 14 Outline

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- Announcements
  - Graded MTs ready for pickup
  - Bonus lecture 11/30 5:15-7:15 in 204 Packard
- Midterm Postmortem and Grade Distribution
- Practical Issues in Adaptive MQAM
  - Update rate
  - Estimation error and delay
- Introduction to MIMO
- MIMO channel capacity

# Midterm Postmortem

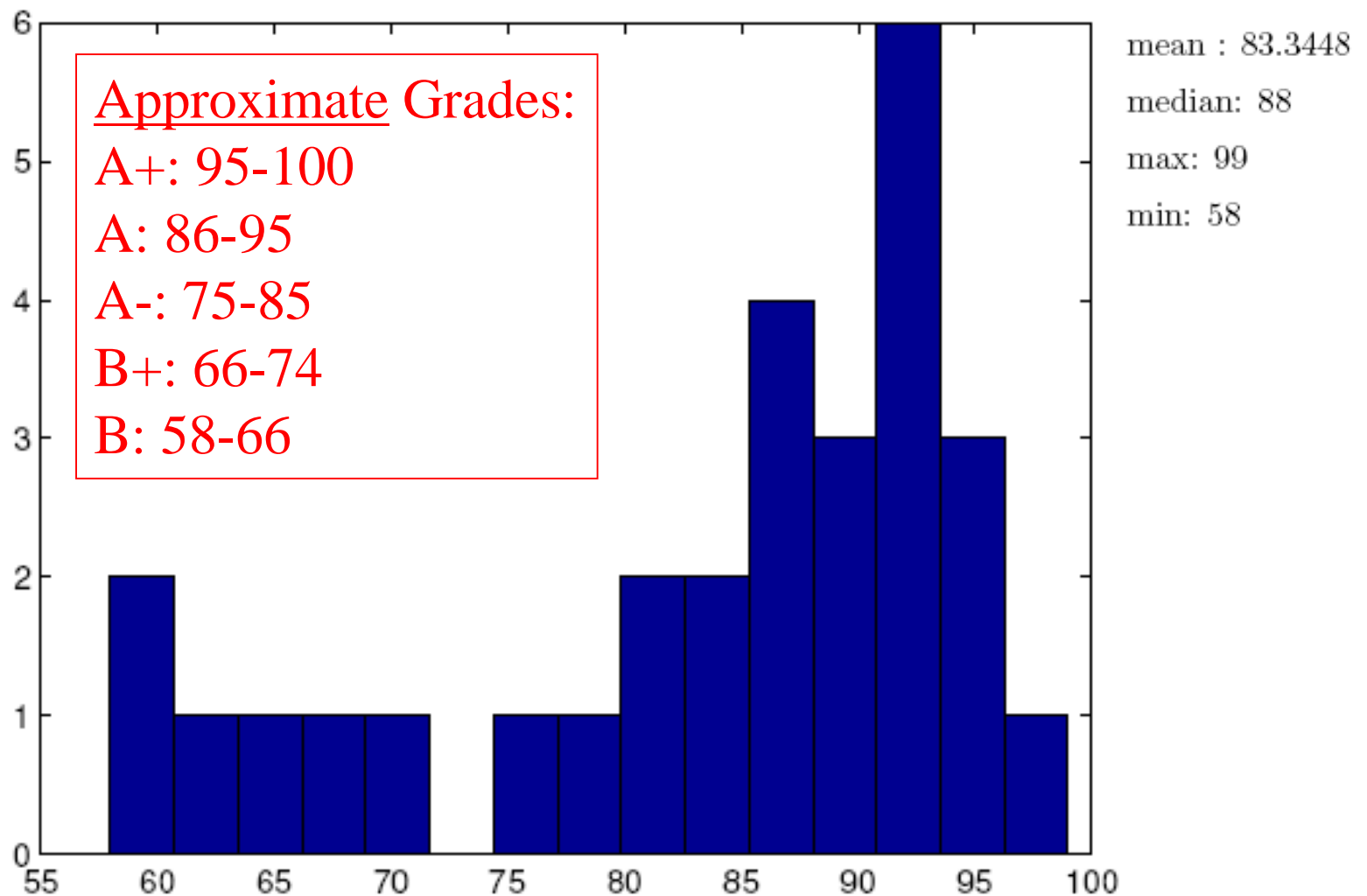
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- Grade distribution typical
- Common Mistakes
  - Prob 1(d): For outage, target SNR must be based on  $P_b$  in AWGN and not average  $P_b$  in fading
  - Prob 2(a)(ii): Instantaneous rate should not be weighted by state probability
  - 2(c): **Transmit** power is fixed, so formula same as capacity w/ RX CSI only (not inversion)

$$C = \int_0^{\infty} B \log_2 \left( 1 + \frac{\gamma P(\gamma)}{\bar{P}} \right) p(\gamma) d\gamma = \int_0^{\infty} B \log_2 (1 + \gamma) p(\gamma) d\gamma$$

- 2(d): Constellation size  $M$  for all channel states is **not** fixed; should adapt  $M$  to SNR.

# Midterm Grade Distribution

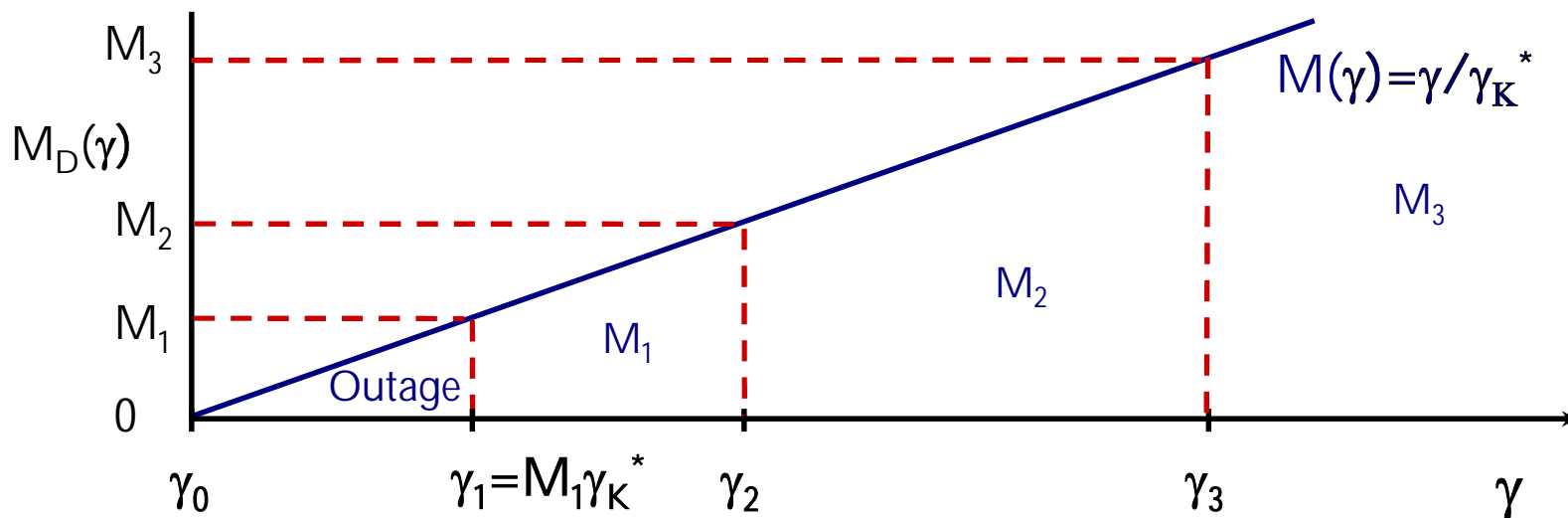


# Review of Last Lecture

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- Introduction to adaptive modulation
- Variable-rate variable-power MQAM
  - Optimal power adaptation is water-filling
  - Optimal rate adaptation is  $R/B = \log(\gamma/\gamma_k)$
- Finite Constellation Sets
  - Use heuristic to assign rates to regions
  - Channel inversion power control in each region

# Constellation Restriction



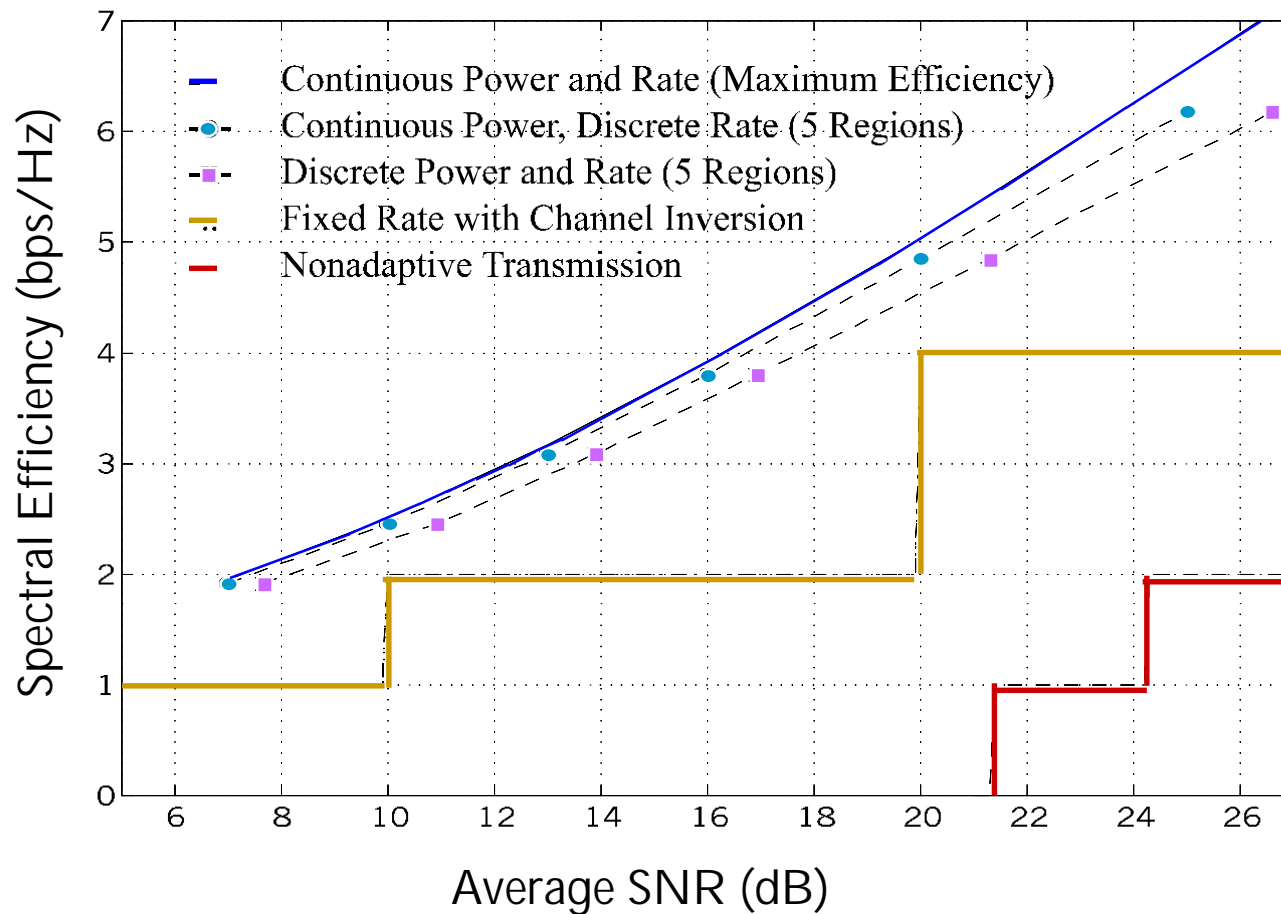
- Power adaptation:

$$\frac{P_j(\gamma)}{P} = \begin{cases} (M_j - 1) / (\gamma K) & \gamma_j \leq \gamma < \gamma_{j+1}, j > 0 \\ 0 & \gamma < \gamma_1 \end{cases}$$

- Average rate:

$$\frac{R}{B} = \sum_{j=1}^N \log_2 M_j p(\gamma_j \leq \gamma < \gamma_{j+1})$$

# Efficiency in Rayleigh Fading



# Practical Constraints

- Constellation updates: fade **region** duration

$$\overline{\tau_j} = \frac{\pi_j}{N_{j+1} + N_j} > T > T_M$$

$\overline{\tau_j}$  = AFRD

$T_M$  = delay spread

$N_j$  = level crossing rate at min fade in region

$N_{j+1}$  = level crossing rate at max fade in region

- Error floor from estimation error
  - Estimation error at RX can cause error in absence of noise (e.g. for MQAM)
  - Estimation error at TX causes mismatch of adaptive power and rate to actual channel
- Error floor from delay: let  $\rho(t, \tau) = \gamma(t - \tau) / \gamma(t)$ .
  - Feedback delay causes mismatch of adaptive power and rate to actual channel

# Detailed Formulas

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- Error floor from estimation error ( $\hat{\gamma} \neq \gamma$ )

$$\bar{P}_b = \int_0^{\infty} \int_{\gamma_K}^{\infty} .2[5BER_{\text{target}}]^{y/\hat{\gamma}} p(\gamma, \hat{\gamma}) d\hat{\gamma} d\gamma$$

- Joint distribution  $p(\gamma, \hat{\gamma})$  depends on estimation: hard to obtain. For PSAM the envelope is bi-variate Rayleigh
- Error floor from delay: let  $\xi = \gamma[i]/\gamma[i-i_d]$ .

$$\bar{P}_b = \int_0^{\infty} \int_0^{\infty} .2[5BER_{\text{target}}]^{\xi} p(\xi | \gamma) p(\gamma) d\xi d\gamma$$

- $p(\xi|\gamma)$  known for Nakagami fading



# Multiple Input Multiple Output (MIMO) Systems

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- MIMO systems have multiple ( $r$ ) transmit and receiver antennas



- With perfect channel estimates at TX and RX, decomposes into  $r$  independent channels
  - $R_H$ -fold capacity increase over SISO system
  - Demodulation complexity reduction
  - Can also use antennas for diversity (beamforming)
  - Leads to capacity versus diversity tradeoff in MIMO

# Capacity of MIMO Systems

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- Depends on what is known at TX and RX and if channel is static or fading
- For static channel with perfect channel knowledge at TX and RX, waterfilling over space is optimal power allocation:
  - Similar idea in fading, based on short-term or long-term power constraint
- Without channel knowledge, capacity metric is based on an outage probability

# Main Points

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- Restricting constellation to a finite set has negligible impact on adaptive MQAM
- Adaptive MQAM need not change more than every 10-100 symbol times.
- Estimation error and delay lead to irreducible error floors in adaptive MQAM
- Multiple antennas at both TX and RX greatly enhance capacity and reduce complexity.
  - Alternatively, can be used for diversity gain