EE359 – Lecture 6 Outline

- Announcements:
  - Makeup lecture Friday (with pizza): last one
  - Extra OHs for me: Friday after class and by appt.
- Review of Last Lecture
- Signal Envelope Distributions
- Level Crossing Rate
- Average Fade Duration
- Markov Models
- Wideband Multipath Channels
- Scattering Function

Review of Last Lecture

- For a narrowband fading model (T_m<<1/B), received signal has a random, complex amplitude gain
- Under CLT approximation, in-phase and quadrature signal components of received signal are Gaussian processes
  - For ϕ_n~U[0,2π], the processes are zero mean, WSS, with

$$A_{r_I}(\tau) = .5\Omega_p E_{\theta_n}[\cos 2\pi f_{D_n}\tau] = A_{r_Q}(\tau), \quad f_{D_n} = v\cos\theta_n/\lambda$$

$$A_{r_I,r_Q}(\tau) = .5\Omega_p E_{\theta_n}[\sin 2\pi f_{D_n}\tau] = -A_{r_I,r_Q}(\tau) \mathbf{=0}$$

$$A_r(\tau) = A_{r_I}(\tau)\cos(2\pi f_c\tau) - \cancel{A_{r_I,r_Q}(\tau)\sin(2\pi f_c\tau)}$$

- **Auto and cross correlation depends on AOAs of multipath**

Review of Last Lecture (Cont'd)

- Uniform AoAs in Narrowband Model
  - In-phase/quad comps have no cross correlation and

$$A_{r_I}(\tau) = A_{r_Q}(\tau) = PJ_0(2\pi f_D \tau)$$

Decorrelates over roughly half a wavelength

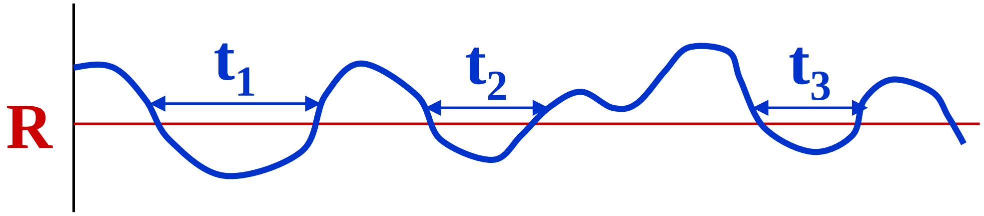
  - PSD is maximum at the maximum Doppler frequency
    - PSD used to generate simulation values

Signal Envelope Distribution

- CLT approx. leads to Rayleigh distribution (power is exponential)
- When LOS component present, Ricean distribution is used
- Measurements support Nakagami distribution in some environments
  - Similar to Ricean, but models “worse than Rayleigh”
  - Lends itself better to closed form BER expressions

Level crossing rate and Average Fade Duration

- LCR: rate at which the signal crosses a fade value
- AFD: How long a signal stays below target R/SNR
  - Derived from LCR

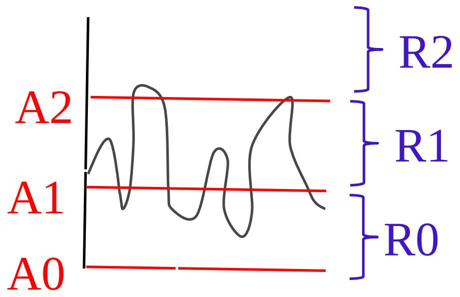


- For Rayleigh fading

$$\bar{t}_R = (e^{\rho^2} - 1)/(\rho f_D \sqrt{2\pi})$$

  - Depends on ratio of target to average level (ρ)
  - Inversely proportional to Doppler frequency

Markov Models for Fading

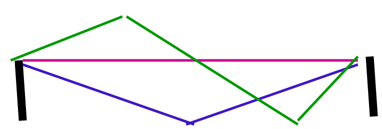


- Model for fading dynamics
  - Simplifies performance analysis
- Divides range of fading power into discrete regions $R_j=\{\gamma: A_j \le \gamma < A_{j+1}\}$
  - A_j s and # of regions are functions of model
- Transition probabilities (L_j is LCR at A_j):

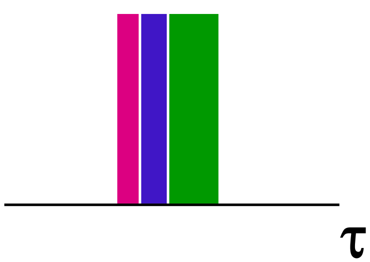
$$p_{j,j+1} = \frac{L_{j+1}T}{\pi_j},\ p_{j,j-1} = \frac{L_j T}{\pi_j},\ p_{j,j} = 1 - p_{j,j+1} - p_{j,j-1}$$

Wideband Channels

- Individual multipath components resolvable
- True when time difference between components exceeds signal bandwidth

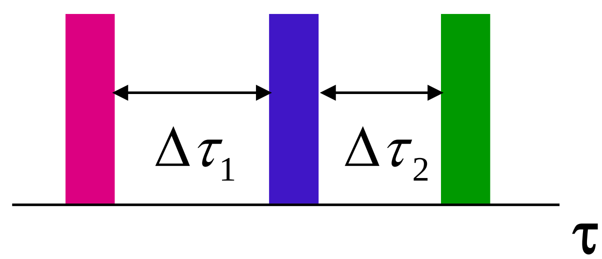


$\Delta\tau << 1/\boldsymbol{B_u}$



Narrowband

$\Delta\tau >> 1/\boldsymbol{B_u}$

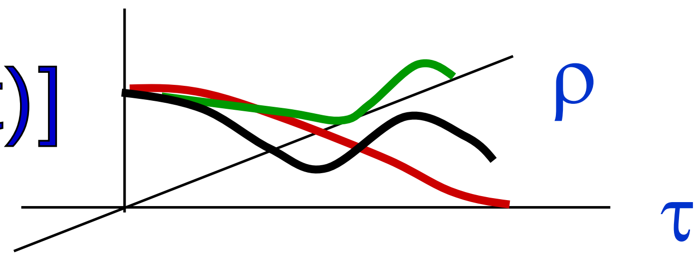


Wideband

Scattering Function

- Fourier transform of $c(\tau,t)$ relative to t
- Typically characterize its statistics, since $c(\tau,t)$ is different in different environments
- Underlying process WSS and Gaussian, so only characterize mean (0) and correlation
- Autocorrelation is $A_c(\tau_1,\tau_2,\Delta t)=A_c(\tau,\Delta t)$
- Statistical scattering function:

$$s(\tau,\rho)=F_{\Delta t}[A_c(\tau,\Delta t)]$$



Main Points

- Narrowband fading distribution depends on environment
  - Rayleigh, Ricean, and Nakagami all common
- Average fade duration determines how long a user is in continuous outage (e.g. for coding design)
- Markov model approximates fading dynamics.
- Scattering function characterizes rms delay and Doppler spread. Key parameters for system design.