Growing the efficient frontier on panel trees

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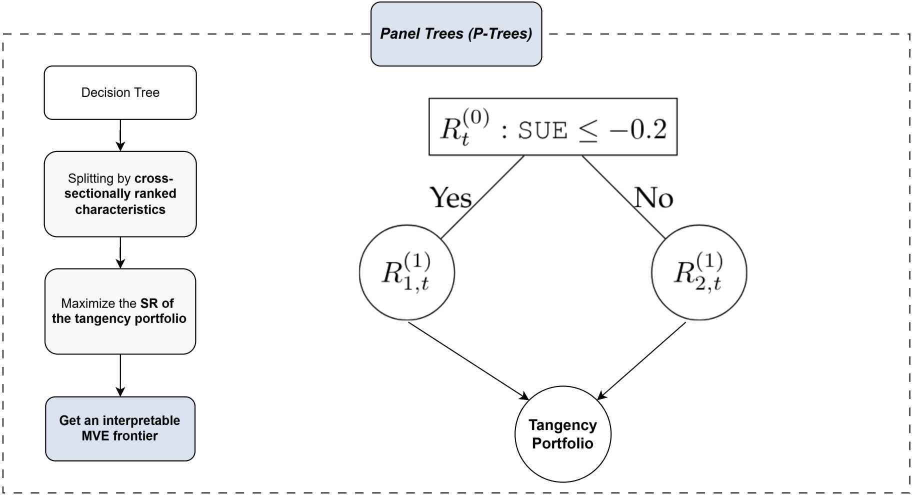
Overview

Summary


Panel Trees (P-Trees)
Decision Tree
Splitting by **cross-sectionally ranked characteristics**
Maximize the **SR of the tangency portfolio**
Get an interpretable MVE frontier
$R_t^{(0)}$: SUE ≤ -0.2
Yes
No
$R_{1,t}^{(1)}$
$R_{2,t}^{(1)}$
Tangency Portfolio

Question

- How to estimate an **interpretable mean–variance efficient (MVE) frontier?**

Background

- Estimating the **mean–variance efficient (MVE) frontier** is crucial for asset pricing
- Estimating the MVE frontier **using individual stocks is impracticable**
  - Sensitive to the noise in individual asset returns (Best and Grauer, 1991, RFS)
  - $N > T \implies$ non-invertible covariance matrix (Fan et al., 2008, JOE)
  - Unbalance panel $\implies$ many missing values in covariance matrix
- Empirical studies typically rely on **characteristic-based portfolios (test assets)** to estimate the MVE frontier

Motivation

- **Challenge**: Existing portfolios face a **trade-off between MVE and interpretability**
  - Sorted portfolios (Fama and French, 1993, JFE) $\Longrightarrow$ overlooking nonlinearity
  - Deep learning (Chen et al., 2024, MS) $\Longrightarrow$ overlooking interpretability
- **Question**: How to **simultaneously attain these dual objectives?**
- **This paper proposes panel trees (P-Trees) to address this issue**

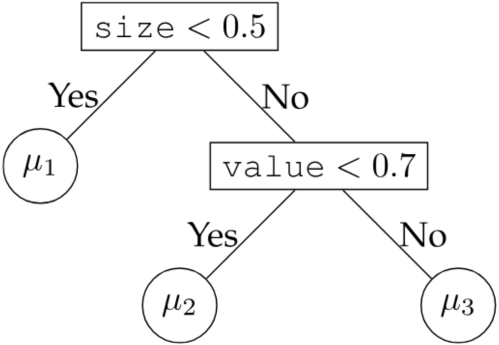
Marginal contribution

- Machine learning in cross-sectional pricing
  - Prior studies
    - **Overlook the panel structure** of the data during the optimization process (Gu et al., 2020, RFS; Bianchi et al., 2021, RFS; Bali et al., 2023, RFS)
  - This paper
    - **Incorporate cross-sectional and time-series information** from panel data during the optimization and achieve **higher Sharpe ratios**

Hypothesis

- H1: P-Trees can construct **more effective and interpretable MVE frontiers**
- H2: P-Trees' **tangency portfolios cannot be spanned by popular factor models**
- H3: P-Trees exhibit **superior out-of-sample performance to existing machine learning methods**
  - The **panel data** structure **smooths out idiosyncratic noise in individual stock returns** and mitigates the risk of overfitting

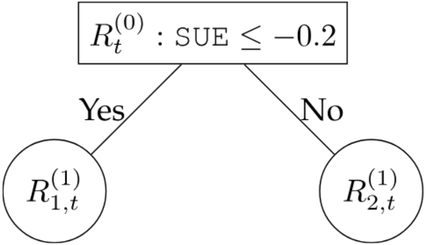
Classification and regression trees (CART)



- **Data distribution:** i.i.d. assumption
- **Split criteria:** local minimization of the sum of squared errors

Panel trees (P-Trees) — split point candidate

- Step 1: **Cross-sectionally** normalize characteristics to $[-1, 1]$ per period
- Step 2: Specify split thresholds as characteristic quintiles: $[-0.6, -0.2, 0.2, 0.6]$
- Step 3: Construct **time-varying value-weighted portfolios** for each leaf node



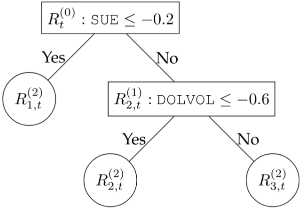
Split criteria of P-Trees — first split

- Step 4: Find the split point that **maximizes the Sharpe ratio of the MVE portfolio**

$$\max \mathrm{SR}(f_t^{(1)}), \quad f_t^{(1)} \propto R_t^{(1)} \underbrace{\widehat{\Sigma}_1^{-1}\widehat{\mu}_1}_{\mathbf{w}} \tag{1}$$

- $\mathbf{R}_t^{(1)} = \left[R_{1,t}^{(1)}, R_{2,t}^{(1)}\right]$ denotes the matrix of portfolio returns $(T \times 2)$
- $\widehat{\mu}_1$ and $\widehat{\Sigma}_1$ are the sample mean and covariance matrix of $\mathbf{R}_t^{(1)}$, respectively
- $f_t^{(1)}$ **is the return series of the tangency portfolio based on** $R_{1,t}^{(1)}$ **and** $R_{2,t}^{(1)}$

Split criteria of P-Trees — second split



- Sequentially search for the split point that **maximizes the Sharpe ratio of $f_t^{(2)}$**
- $f_t^{(2)}$ **is the tangency portfolio based on $R_{1,t}^{(2)}$, $R_{2,t}^{(2)}$ and $R_{3,t}^{(3)}$**
- Maximize the collective SR of the basis portfolios is a **Global criterion**
- Growth termination: leaf count $= 10$ or leaf size < 20

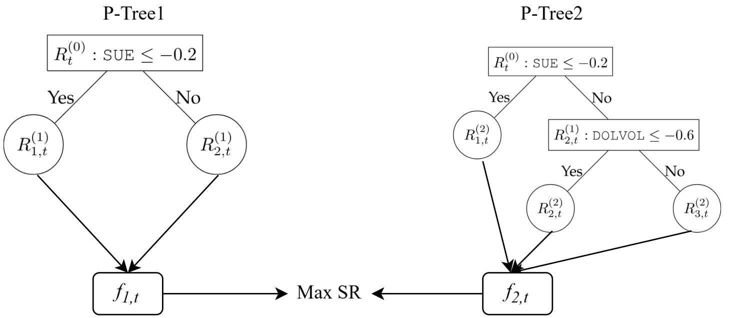
CART VS P-Trees

- P-Trees differ from CART in their **utilization of the panel data structure and split criteria**

Feature	Standard CART	P-Trees
Panel Structure	Ignore	Utilize
Split Criteria	Local	Global

Boosted P-Trees

- Step 1: Generated a P-Tree to get the first pricing factor $f_{1,t}$
- Step 2: Generate the second P-Tree to **maximize the collective SR of** $F = [f_{1,t}, f_{2,t}]$
- Step 3: Repeat Step 2 to sequentially generate P factors $\mathbf{F}_t = [f_{1,t}, \ldots, f_{P,t}]$

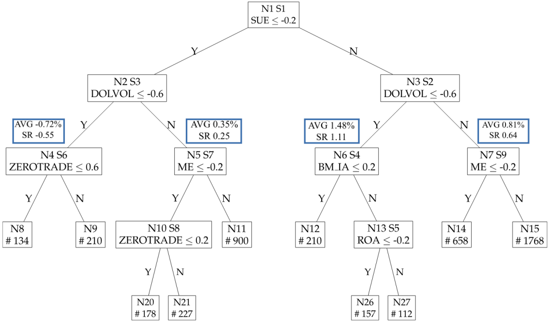


Data

- Focus on the U.S. equity market
- 61 firm characteristics (CRSP and Compustat)
- Characteristics are standardized cross-sectionally to the range [-1, 1]
- The monthly data ranges from 1981 to 2020

Model validation: Interpretability and non-linearity

- SUE (standard unexpected earnings); DOLVOL (dollar trading volume); BM_IA (industry-adjusted book-to-market ratio)
- Reveal **crucial and interpretable asymmetry (nonlinear) pricing effects**



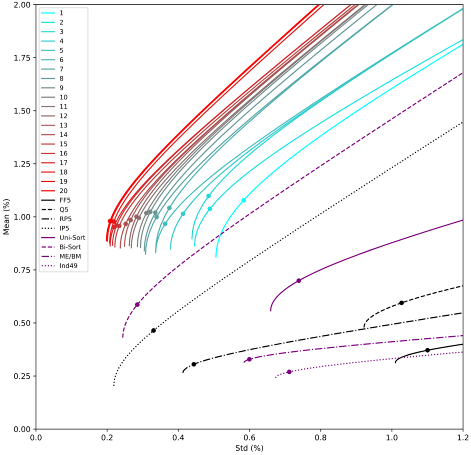
Model validation: Interpretability and non-linearity

- Nonlinear portfolios are challenging to price using prevalent linear factor models

ID	# Median	AVG	STD	α_{CAPM}	β_{CAPM}	R^2_{CAPM}	α_{FF5}	α_{Q5}	α_{RP5}	α_{IP5}
Panel A: 40 Years (1981–2020)										
N8	134	−0.40	5.76	−0.97***	0.84	0.42	−0.91***	−0.66***	−1.51***	−0.58*
N9	210	−0.92***	4.14	−1.35***	0.63	0.47	−1.46***	−1.25***	−1.84***	−0.36*
N20	178	−0.34	8.89	−1.34***	1.45	0.53	−1.10	−0.49*	−1.55***	−1.78***
N21	227	−1.14***	5.90	−1.83***	1.00	0.58	−1.83***	−1.56***	−2.34***	−1.19***
N11	900	0.36*	4.81	−0.35***	1.04	0.93	−0.30***	−0.15**	−0.29***	0.16
N12	210	0.94***	4.63	0.42**	0.76	0.54	0.29**	0.48***	−0.22	1.17***
N26	157	1.77***	6.04	1.16***	0.88	0.43	1.08***	1.37***	0.40**	1.40***
N27	112	2.46***	4.55	1.97***	0.71	0.49	1.80***	2.00***	1.18***	2.18***
N14	658	1.03***	6.88	0.21	1.18	0.59	0.32*	0.64***	−0.33**	0.06
N15	1768	0.81***	4.41	0.14***	0.98	0.98	0.10***	0.03	−0.26***	0.29

Growing the efficient frontier

- H1: P-Trees can construct **more effective and interpretable MVE frontiers**



Are the expanded MVE frontiers explained by popular factor models?

- H2: P-Trees' **tangency portfolios cannot be spanned by popular factor models**
- Each boosted P-Tree adds incremental pricing information to the existing P-Trees

	Sharpe ratio		CAPM test		FF5 test		Expanding factors test			BS test
	Single	Cumu.	α (%)	t-stat	α (%)	t-stat	α (%)	t-stat	R^2	p-value
Panel A: 40 Years (1981–2020)										
1	6.37	6.37	1.39	35.36	1.37	35.81	–	–	–	
2	3.20	7.35	0.52	17.65	0.48	15.27	0.62	9.55	0.01	0.00
3	1.18	7.80	0.34	5.25	0.18	3.28	−0.86	−5.43	0.26	0.00
4	2.06	8.46	0.44	11.22	0.38	9.56	0.69	7.63	0.16	0.00
5	1.99	9.18	0.48	10.25	0.41	9.25	0.84	5.61	0.21	0.00
6	1.01	9.57	0.18	4.24	0.08	2.94	−0.50	−5.47	0.45	0.00
7	1.42	10.11	0.28	7.62	0.22	6.67	0.63	7.03	0.36	0.00
8	1.32	10.40	0.28	7.14	0.20	4.95	−0.50	−5.33	0.41	0.00
9	1.83	10.88	0.53	10.11	0.43	9.38	0.85	6.88	0.34	0.00
10	1.48	11.20	0.44	7.35	0.30	7.30	−0.68	−5.49	0.46	0.00
11	1.78	11.72	0.38	10.33	0.31	8.62	0.72	6.70	0.30	0.00
12	1.02	12.06	0.20	4.68	0.10	2.99	−0.55	−5.76	0.55	0.00
13	1.37	12.57	0.29	8.38	0.22	6.48	0.76	6.01	0.33	0.00
14	1.37	13.01	0.48	5.93	0.32	5.86	−0.91	−5.68	0.60	0.00
15	1.37	13.81	0.31	6.58	0.21	5.66	0.97	7.11	0.48	0.00
16	1.24	14.28	0.28	6.23	0.17	4.11	−0.74	−6.24	0.54	0.00
17	1.54	14.60	0.46	8.16	0.40	7.52	−0.89	−4.80	0.40	0.00
18	1.64	14.92	0.32	8.43	0.27	7.78	−0.65	−5.43	0.34	0.00
19	1.48	15.43	0.43	8.63	0.36	7.47	1.18	5.86	0.34	0.00
20	1.35	15.63	0.33	7.38	0.23	6.19	−0.59	−4.19	0.44	0.00

Out-of-sample investment performance

- H3: P-Trees exhibit superior out-of-sample performance to existing machine learning methods (Chen et al., 2024, MS; Feng et al., 2024, JFQA)

	SR	α_{CAPM}	α_{FF5}	α_{Q5}	α_{RP5}	α_{IP5}
Panel A: 40 Years (1981–2020)						
P-Tree1	6.37	1.39	1.37	1.36	1.28	1.12
P-Tree1–5	9.19	0.97	0.95	0.93	0.87	0.82
P-Tree1–10	11.21	1.01	1.00	0.98	0.93	0.89
P-Tree1–15	13.83	0.95	0.94	0.93	0.90	0.87
P-Tree1–20	15.64	0.97	0.96	0.95	0.93	0.90
Panel B1: 20 Years In-Sample (1981–2000)						
P-Tree1	7.13	1.86	1.78	1.72	1.62	1.59
P-Tree1–5	12.74	1.54	1.51	1.48	1.37	1.41
P-Tree1–10	19.22	1.51	1.49	1.49	1.43	1.43
P-Tree1–15	28.43	1.42	1.41	1.40	1.37	1.39
P-Tree1–20	38.01	1.36	1.35	1.34	1.32	1.34
Panel B2: 20 Years Out-of-Sample (2001–2020)						
P-Tree1	3.23	1.35	1.31	1.23	1.04	0.93
P-Tree1–5	3.41	1.02	1.00	0.95	0.77	0.62
P-Tree1–10	3.21	0.95	0.94	0.89	0.74	0.56
P-Tree1–15	3.12	0.89	0.89	0.83	0.69	0.48
P-Tree1–20	3.13	0.85	0.84	0.78	0.66	0.49
Panel C1: 20 Years In-Sample (2001–2020)						
P-Tree1	5.83	1.51	1.47	1.50	1.52	1.69
P-Tree1–5	9.32	1.30	1.29	1.28	1.30	1.31
P-Tree1–10	14.35	1.12	1.11	1.11	1.11	1.09
P-Tree1–15	20.64	1.08	1.07	1.08	1.10	1.05
P-Tree1–20	26.57	1.09	1.08	1.08	1.10	1.11
Panel C2: 20 Years Out-of-Sample (1981–2000)						
P-Tree1	4.35	1.50	1.42	1.35	1.60	1.58
P-Tree1–5	3.87	1.18	1.05	0.96	1.23	1.24
P-Tree1–10	4.29	1.02	0.93	0.85	1.14	1.10
P-Tree1–15	4.03	0.96	0.86	0.80	1.07	1.02
P-Tree1–20	3.88	0.96	0.87	0.81	1.08	1.03

Extension

1. Impose limits-to-arbitrage as a regularization constraint
   - Machine learning-based alphas stem primarily from stocks with high limits-to-arbitrage (Avramov et al., 2023, MS)
2. Employ tree-based models for unsupervised asset clustering
   - Minimize intra-cluster characteristic dispersion and maximize inter-cluster heterogeneity
3. Construct high-frequency efficient frontiers

How is the efficient frontier computed?

- The **weights of any portfolio on the efficient frontier** can be expressed as a **linear combination of the weights of two other distinct efficient portfolios**

$$W_{new} = \lambda \cdot W_{maxSR} + (1 - \lambda) \cdot W_{minVar} \tag{2}$$

$$w_{maxSR} = \frac{\Sigma^{-1}\mu}{\mathbf{1}^T\Sigma^{-1}\mu}, \quad W_{minVar} = \frac{\Sigma^{-1}\mathbf{1}}{\mathbf{1}^T\Sigma^{-1}\mathbf{1}} \tag{3}$$

Generating test assets via boosted P-Trees

- P-Trees provide more stringent test assets compared to FF5 model

	N	GRS	p-GRS	p-PY	$\overline{\lvert\alpha\rvert}$	$\sqrt{\overline{\alpha^2}}$	$\overline{R^2}$	$\%\alpha_{10\%}$	$\%\alpha_{5\%}$	$\%\alpha_{1\%}$
Panel A: 40 Years (1981–2020)										
P-Tree1	10	141.27	0.00	0.00	0.92	1.11	75	100	90	80
P-Tree1–5	50	60.32	0.00	0.00	0.44	0.61	80	70	62	44
P-Tree6–10	50	4.60	0.00	0.00	0.29	0.37	79	56	50	34
P-Tree11–15	50	4.74	0.00	0.00	0.20	0.26	80	38	36	24
P-Tree16–20	50	4.21	0.00	0.00	0.31	0.42	77	52	44	30
P-Tree1–20	200	41.31	0.00	0.00	0.31	0.43	79	54	48	33
Uni-Sort	150	1.62	0.00	0.00	0.10	0.14	88	25	18	7
Bi-Sort	285	2.50	0.00	0.00	0.12	0.17	89	30	23	15
ME-BM	25	5.01	0.00	0.00	0.12	0.16	92	36	28	20
Ind49	49	1.99	0.00	0.00	0.28	0.35	60	39	31	18

P-Tree with macroeconomic regimes

- P-Trees remain robust under various macro conditions

	Regime	AVG	SR	GRS	p-value	$\overline{\lvert\alpha\rvert}$	$\sqrt{\overline{\alpha^2}}$	$\overline{R^2}$	Top three characteristics		
	Top	1.78	6.14	71.84	0.00	1.27	1.57	52.69	SUE	DOLVOL	CFP
DFY	Btm	1.21	4.56	40.01	0.00	1.08	1.44	60.05	ABR	RVAR_CAPM	STD_DOLVOL
	Com	1.49	5.16								
	Top	1.45	5.84	43.17	0.00	1.03	1.30	73.16	CHPM	BM_IA	ME
DY	Btm	1.30	6.21	101.56	0.00	1.05	1.21	51.45	ABR	ZEROTRADE	DOLVOL
	Com	1.35	6.03								
	Top	0.99	5.51	34.85	0.00	0.94	1.28	78.21	RVAR_CAPM	ME_IA	ME
EP	Btm	1.34	6.04	98.62	0.00	0.94	1.18	56.89	SUE	DOLVOL	BM_IA
	Com	1.24	5.76								
	Top	1.10	5.90	89.15	0.00	0.81	1.04	62.31	RVAR_CAPM	SUE	STD_DOLVOL
ILL	Btm	1.50	7.44	72.13	0.00	1.03	1.30	59.38	SUE	DOLVOL	SP
	Com	1.24	6.21								
	Top	1.16	4.83	30.10	0.00	0.99	1.29	75.20	CFP	ATO	ME
INFL	Btm	1.29	6.61	107.40	0.00	0.93	1.26	57.67	SUE	DOLVOL	CFP
	Com	1.25	5.89								
	Top	1.33	5.10	38.77	0.00	0.89	1.14	65.03	CFP	ALM	STD_DOLVOL
LEV	Btm	1.39	5.36	66.94	0.00	1.23	1.46	52.23	NI	MOM12M	BM_IA
	Com	1.37	5.26								
	Top	1.37	5.05	45.70	0.00	1.67	2.16	53.03	RVAR_CAPM	ME_IA	RD_SALE
NI	Btm	1.51	6.71	93.02	0.00	1.18	1.38	61.79	ROA	DOLVOL	SUE
	Com	1.45	5.81								
	Top	1.24	6.22	72.03	0.00	0.94	1.24	62.66	SUE	DOLVOL	CFP
SVAR	Btm	1.23	4.78	43.92	0.00	1.18	1.38	59.41	ROA	ZEROTRADE	ABR
	Com	1.23	5.36								
	Top	1.09	7.15	59.53	0.00	0.75	0.91	61.98	CHCSHO	SUE	DOLVOL
TBL	Btm	1.30	7.05	134.30	0.00	0.93	1.10	63.44	SUE	DOLVOL	CFP
	Com	1.24	6.97								
	Top	1.17	6.01	67.58	0.00	0.99	1.28	63.45	EP	STD_DOLVOL	SUE
TMS	Btm	1.23	5.82	70.00	0.00	0.86	1.17	52.52	SUE	DOLVOL	BM_IA
	Com	1.20	5.91								