On the performance of volatility-managed portfolios

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Motivation

- **strong performance for volatility-managed of factor pricing**
  - single factor strategy: mkt,mom$\cdots$(Ang, 2014$\cdots$) ——$f_{\sigma,t} = \frac{c^*}{\hat{\sigma}^2_{t-1}} f_t$
  - systematic evidence： 9 equity factors(Moreira and Muir,2017)

$$f_{\sigma,t} = \alpha + \beta f_t + \varepsilon_t$$

  - leave reader impression: volatility-managed strategies reliably boost performance
- **Criticism: Not implementable in practice**
  - optimal weight depends on in-sample return moments, the required strategy is not known prior to the end of the sample.
- **What about the actual out-of-sample performance?**

Question

- Does volatility-managed systematically improve performance in real-world?
  - **No**
  - compares volatility-managed and original strategies:
    - 103 portfolios tested; volatility-managed outperforms in 53, only 8 significant
  - alpha test——real time strategies:
    - poor out-of-sample performance of combination strategies. (72 underperform)

Contribution

- contributes to literature on invest performance of volatility-managed strategy
  - Broader Strategy Evaluation:
    - large sample of 103 strategies,no evidence that volatility managed outperform
  - Real-Time Implementability:
    - Positive alphas from spanning regressions (consistent with MM 2017), implied combination strategies are not feasible in real time, perform poorly out-of-sample.
  - Analysis of Out-of-Sample Model Instability:
    - The gap is attributed to structural instability in the spanning regressions.

Design

volatility-managed portfolios

Raw Portfolio: 103
9 factor(MM2017)
+94 factor(Hou2015)

Input: excess return f_t

$$f_{\sigma,t} = \frac{c^*}{\hat{\sigma}_{t-1}^2} f_t$$

$$\hat{\sigma}_t^2 = \frac{22}{J_t} \sum_{j=1}^{J_t} \left(f_t^j\right)^2$$

$c^* \rightarrow f_t$ & $f_{\sigma,t}$ have same unconditional volatility.

Output: $f_{\sigma,t}$

Portfolio performance comparison: Raw v.s. volatility-managed

Direct comparison: 103 portfolios $f_{\sigma,t}\ v.s.\ f_t$

Spanning reg(MM, 2017):

$$f_{\sigma,t} = \alpha + \beta f_t + \varepsilon_t$$

α>0:
better mean-variance frontier
(Gibbons et al. ,1989)

$$maxU = \max\left[E(R) - \frac{1}{2}\gamma\sigma^2\right]$$

$$= \begin{bmatrix}\omega_\sigma^* \\ \omega^*\end{bmatrix}\begin{bmatrix}\bar{f}_{\sigma,t} \\ \bar{f}_t\end{bmatrix} - \frac{\gamma}{2}\begin{bmatrix}\omega_\sigma^* \\ \omega^*\end{bmatrix}\sigma_f^2\begin{bmatrix}1 & \rho \\ \rho & 1\end{bmatrix}\begin{bmatrix}\omega_\sigma^* & \omega^*\end{bmatrix}$$

$$\omega_\sigma^* = \frac{\alpha}{\gamma\sigma_f^2(1-\rho^2)},\ y_t^* = \omega_\sigma^*\left(\frac{c^*}{\hat{\sigma}_{t-1}^2}\right) + \omega^*$$

Out of sample

120 month	121	
120 month	121	122

$\underbrace{\text{120 month} + 121}_{\text{Estimate } f_{\sigma,t},\ \alpha,\ \omega_\sigma^*}$ $\underbrace{122}_{\text{Test } y_t^*}$

SR diff

CER diff

Design-Data base

- Sample: 1926-2016, CRSP stock
- Factor:
  - MM2017: 9 factor: MKT,SMB,HML,MOM,RMW,CMA,ROE,IA,BAB
  - 9 factor + 94 anomaly portfolios(Hou et al.,2015)
- Raw portfolios: 10 groups, value-weighted portfolios

Result-Direct comparision-9 factor

	Factor								
	MKT (1)	*SMB* (2)	*HML* (3)	*MOM* (4)	*RMW* (5)	*CMA* (6)	*ROE* (7)	*IA* (8)	*BAB* (9)
Panel A: Performance measures for original factors									
Mean	7.80	2.57	4.84	7.94	2.92	3.72	6.52	4.99	8.23
Standard deviation	18.61	11.12	12.14	16.39	7.71	6.97	8.83	6.48	10.71
Sharpe ratio	0.42	0.23	0.40	0.48	0.38	0.53	0.74	0.77	0.77
Panel B: Performance measures for volatility-managed factors									
Mean	9.55	0.86	4.64	16.17	3.94	2.79	9.39	4.69	10.81
Standard deviation	18.61	11.12	12.14	16.39	7.71	6.97	8.83	6.48	10.71
Sharpe ratio	0.51	0.08	0.38	0.99	0.51	0.40	1.06	0.72	1.01
Panel C: Performance comparisons									
Sharpe ratio difference	$\boxed{0.09\ [0.30]}$	−0.15 [0.09]	−0.02 [0.86]	$\boxed{0.50\ [0.00]}$	$\boxed{0.13\ [0.29]}$	−0.13 [0.23]	$\boxed{0.32\ [0.01]}$	−0.05 [0.68]	$\boxed{0.24\ [0.01]}$
Panel D: Properties of volatility-managed factors									
Correlation with original factor	0.63	0.63	0.57	0.48	0.59	0.68	0.68	0.70	0.62
$P_{01}(c^*/\hat{\sigma}^2_{t-1})$	0.04	0.03	0.04	0.04	0.04	0.06	0.06	0.06	0.04
$P_{50}(c^*/\hat{\sigma}^2_{t-1})$	0.96	0.81	1.02	1.01	1.11	0.97	1.08	0.96	1.00
$P_{99}(c^*/\hat{\sigma}^2_{t-1})$	6.47	5.07	5.89	8.64	5.02	4.56	4.73	4.45	5.09

- 5[3] factor better than origin.

Result-Direct comparision-103 factor

Sample (1)	Total (2)	Sharpe ratio difference: $\Delta SR > 0$ [Signif.] (3)	Sharpe ratio difference: $\Delta SR < 0$ [Signif.] (4)
Panel A: Combined sample			
All trading strategies	103	53 [8]	50 [4]
Panel B: By category			
Factors	9	5 [3]	4 [0]
Anomaly portfolios	94	48 [5]	46 [4]
Panel C: By trading strategy type			
Accruals	10	4 [0]	6 [0]
Intangibles	10	3 [0]	7 [0]
Investment	11	3 [0]	8 [1]
Market	1	1 [0]	0 [0]
Momentum	9	9 [5]	0 [0]
Profitability	22	15 [1]	7 [1]
Trading	21	11 [1]	10 [1]
Value	19	7 [1]	12 [1]

- 53[8] factor better than origin. ——systematically better? No

Result-spanning reg-9 factor

	Factor								
	MKT (1)	*SMB* (2)	*HML* (3)	*MOM* (4)	*RMW* (5)	*CMA* (6)	*ROE* (7)	*IA* (8)	*BAB* (9)
Panel A: Univariate regressions									
Panel A.1: Regression results									
Alpha, α (%)	4.63	−0.76	1.87	12.39	2.23	0.26	4.97	1.18	5.74
	(3.08)	(−0.87)	(1.88)	(7.31)	(2.57)	(0.39)	(5.10)	(1.83)	(5.97)
Beta, β	0.63	0.63	0.57	0.48	0.59	0.68	0.68	0.70	0.62
	(11.32)	(7.75)	(7.65)	(7.13)	(7.10)	(13.82)	(11.12)	(13.59)	(12.97)
R^2	0.40	0.40	0.33	0.23	0.34	0.46	0.46	0.50	0.38
Appraisal ratio, *AR*	0.32	−0.09	0.19	0.86	0.36	0.05	0.77	0.26	0.68
Panel A.2: Ex post optimization parameters									
Scaling parameter, c^*	10.33	2.63	2.95	4.60	1.48	1.53	2.06	1.64	3.20
Risky allocation, $x_\sigma^* + x^*$	0.61	0.34	0.82	1.22	1.45	1.60	2.44	0.70	2.05
Relative factor weights									
Vol-managed factor, w_σ^*	0.72	−0.60	0.46	0.98	0.79	0.12	0.97	0.41	0.78
Original factor, w^*	0.28	1.60	0.54	0.02	0.21	0.88	0.03	0.59	0.22
Panel A.3: Portfolio performance measures									
Sharpe ratio									
Original factor	0.42	0.23	0.40	0.48	0.38	0.53	0.74	0.77	0.77
Combination strategy	0.53	0.25	0.44	0.99	0.52	0.54	1.06	0.81	1.03
Difference	0.11	0.02	0.04	0.50	0.14	0.00	0.32	0.04	0.26
CER (%)									
Original factor	1.76	0.53	1.59	2.35	1.44	2.85	5.46	5.92	5.90
Combination strategy	2.79	0.61	1.94	9.74	2.71	2.88	11.32	6.57	10.52
Difference	1.03	0.08	0.35	7.39	1.27	0.03	5.86	0.65	4.63

- $\alpha>0$,SR diff >0——same as MM2017

Result-spanning reg-103 factor

Sample (1)	Total (2)	Univariate regressions: $\alpha > 0$ [Signif.] (3)	Univariate regressions: $\alpha < 0$ [Signif.] (4)
		Panel A: Combined sample	
All trading strategies	103	77 [23]	26 [3]
		Panel B: By category	
Factors	9	8 [5]	1 [0]
Anomaly portfolios	94	69 [18]	25 [3]
		Panel C: By trading strategy type	
Accruals	10	8 [3]	2 [0]
Intangibles	10	6 [1]	4 [0]
Investment	11	7 [1]	4 [1]
Market	1	1 [1]	0 [0]
Momentum	9	9 [9]	0 [0]
Profitability	22	19 [2]	3 [0]
Trading	21	14 [4]	7 [1]
Value	19	13 [2]	6 [1]

- 77[23] factor better than origin.

Result-spanning reg & out of sample-9 factor

	Factor								
	MKT (1)	*SMB* (2)	*HML* (3)	*MOM* (4)	*RMW* (5)	*CMA* (6)	*ROE* (7)	*IA* (8)	*BAB* (9)
Panel A: Real-time combination strategies									
Sharpe ratio									
[S1] Combination strategy (real time)	0.42	0.14	0.38	0.92	0.44	0.52	1.13	0.70	1.09
[S2] Original factor (real time)	0.46	0.19	0.43	0.49	0.31	0.56	0.78	0.68	0.79
Difference, [S1]–[S2]	−0.04	−0.06	−0.06	0.44	0.13	−0.03	0.36	0.02	0.30
	[0.64]	[0.37]	[0.41]	[0.00]	[0.53]	[0.20]	[0.00]	[0.74]	[0.00]
[S3] Combination strategy (ex post optimal)	0.53	0.26	0.50	0.99	0.58	0.64	1.21	0.73	1.11
Difference, [S1]–[S3]	−0.11	−0.12	−0.12	−0.07	−0.14	−0.11	−0.07	−0.03	−0.02
	[0.01]	[0.14]	[0.08]	[0.07]	[0.37]	[0.00]	[0.20]	[0.41]	[0.78]
CER (%)									
[S1] Combination strategy (real time)	1.56	0.00	1.41	8.47	1.96	2.74	12.25	4.19	10.88
[S2] Original factor (real time)	1.75	0.38	1.61	2.29	0.91	3.09	5.44	3.68	6.23
Difference, [S1]–[S2]	−0.19	−0.37	−0.20	6.18	1.04	−0.35	6.81	0.51	4.65
	[0.83]	[0.27]	[0.73]	[0.00]	[0.57]	[0.21]	[0.00]	[0.60]	[0.00]
[S3] Combination strategy (ex post optimal)	2.79	0.67	2.47	9.87	3.42	4.04	14.55	5.36	12.34
Difference, [S1]–[S3]	−1.23	−0.66	−1.06	−1.40	−1.46	−1.30	−2.30	−1.17	−1.46
	[0.01]	[0.13]	[0.10]	[0.07]	[0.39]	[0.03]	[0.15]	[0.25]	[0.30]

- 0 factor better than origin.

Result-spanning reg & out of sample-103 factor

Panel A: Real-time combination strategies					
		Sharpe ratio difference: Combination strategy (real time) versus		CER difference: Combination strategy (real time) versus	
		Original factor (real time)	Combination strategy (ex post optimal)	Original factor (real time)	Combination strategy (ex post optimal)
Sample (1)	Total (2)	ΔSR +/− (3)	ΔSR +/− (4)	ΔCER +/− (5)	ΔCER +/− (6)
Panel A.1: Combined sample					
All trading strategies	103	45 [8] / 58 [2]	1 [0] / 102 [39]	31 [7] / 72 [7]	0 [0] / 103 [41]
Panel A.2: By category					
Factors	9	5 [3] / 4 [0]	0 [0] / 9 [2]	5 [3] / 4 [0]	0 [0] / 9 [2]
Anomaly portfolios	94	40 [5] / 54 [2]	1 [0] / 93 [37]	26 [4] / 68 [7]	0 [0] / 94 [39]
Panel A.3: By trading strategy type					
Accruals	10	3 [0] / 7 [1]	0 [0] / 10 [5]	3 [0] / 7 [2]	0 [0] / 10 [4]
Intangibles	10	4 [0] / 6 [0]	0 [0] / 10 [0]	1 [0] / 9 [1]	0 [0] / 10 [4]
Investment	11	5 [0] / 6 [0]	0 [0] / 11 [6]	5 [0] / 6 [0]	0 [0] / 11 [5]
Market	1	0 [0] / 1 [0]	0 [0] / 1 [1]	0 [0] / 1 [0]	0 [0] / 1 [1]
Momentum	9	8 [4] / 1 [0]	0 [0] / 9 [5]	8 [5] / 1 [0]	0 [0] / 9 [5]
Profitability	22	10 [1] / 12 [0]	1 [0] / 21 [7]	6 [1] / 16 [1]	0 [0] / 22 [5]
Trading	21	10 [1] / 11 [1]	0 [0] / 21 [6]	6 [1] / 15 [1]	0 [0] / 21 [8]
Value	19	5 [2] / 14 [0]	0 [0] / 19 [9]	2 [0] / 17 [2]	0 [0] / 19 [9]

- 1[0] factor better than origin.

Result-Why is the out-of-sample performance poor?

Description (1)	Total (2)	Frequency distribution for breaks $N_b = 0$ (3)	$N_b = 1$ (4)	$N_b = 2$ (5)	$N_b = 3$ (6)	$N_b \geq 4$ (7)	$\bar{N}_b$ (8)
Panel A: Spanning regressions							
Spanning regressions	103	0	10	52	34	7	2.37
Spanning regressions with FF3 controls	103	1	8	53	35	6	2.37
Panel B: Anomaly regressions							
CAPM regressions	102	15	38	39	9	1	1.44
FF3 regressions	100	10	25	36	21	8	1.92

- structural instability in the spanning regression parameters
  - all 103 reg model break
  - average 2.37 break point > CAPM(1.44)

Idea

- 方法：目前问题：参数结构性变化，样本外表现差
  - 使用机器学习模型来预测 t+1 期的波动率 or 预测 alpha，从而提升波动率管理表现？
  - 构造十分组波动率多空组合；or 十分组先控制收益，再选择波动率
- 进一步：为什么 spanning reg 比 CAPM 更容易突变？
  - 波动率指标本身容易突变？
  - 受到宏观因素、特殊时期的影响？
  - 剔除掉这些时期是否表现会更好？
- 替换 X:
  - 债市的波动率更稳定，可能更方便进行波动率管理

Thanks!